A Scrooge McDuck Theory of Wealth Dynamics

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Abstract

Can the rise in the wealth-to-output ratio observed in advanced economies over recent decades be explained by an increase in top income inequality? Recent contributions that advance this interpretation struggle to account for the fact that the increase in wealth was driven by asset prices, rather than capital accumulation. This paper shows that these stylized facts can be reconciled in the presence of *insatiable* preference for wealth. Such preferences induce a cap on optimal consumption—which is more likely to become binding when income inequality is large. When it does, top-income agents accumulate diverging wealth over time while keeping their consumption bounded. This consumption-saving behavior then supports the existence of a uniquely determined rational bubble, which grows at a rate that *exceeds* that of the economy. The bubble crowds out investment, while its diverging price sustains a permanent rise in wealth inequality. In this environment, wealth and capital income taxes are not equivalent, but both can shift the equilibrium from bubbly to non-bubbly—thus being potentially redistributive and growth-*enhancing*. A quantitative exercise suggests that a 1.5 percent wealth tax in the U.S. would raise capital by 4.5 percent over 30 years.

Keywords: Wealth Preferences, Wealth-to-Output Ratio, Inequality, Rational Bubble
JEL Classification: E21, E22

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1 Introduction

Over recent decades, most advanced economies have seen a marked increase in both aggregate wealth and top income inequality. In G7 countries, the average wealth-to-output ratio rose from about three to five between 1980 and today (Figure 1A.). Meanwhile, the pre-tax income share of the top 1% has been rising: in the United States, it almost doubled over the period from 11% to more than 20% (Figure 1B.). Could this change in the income distribution explain the rise in aggregate wealth? Empirical evidence indicates that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019), hence greater income inequality has effectively shifted income from low- to high-saving households. This reallocation may have raised aggregate saving and aggregate wealth.

Yet, models that incorporate this mechanism (e.g., Hubmer et al., 2021; Elina and Huleux, 2023) fail to capture the nature of the observed rise in the wealth-to-output ratio. These models often consider capital, whose price is fixed, as the only available asset. By construction, any increase in wealth mechanically corresponds to an increase in capital. But more fundamentally, in such frameworks, an increase in aggregate savings lowers the interest rate, resulting in further capital investment. This stands in contrast to the data, which shows a stable capital-to-output ratio (Figure 1A.). The observed rise in the wealth-to-output ratio was instead driven by higher asset prices, that is, by capital gains.

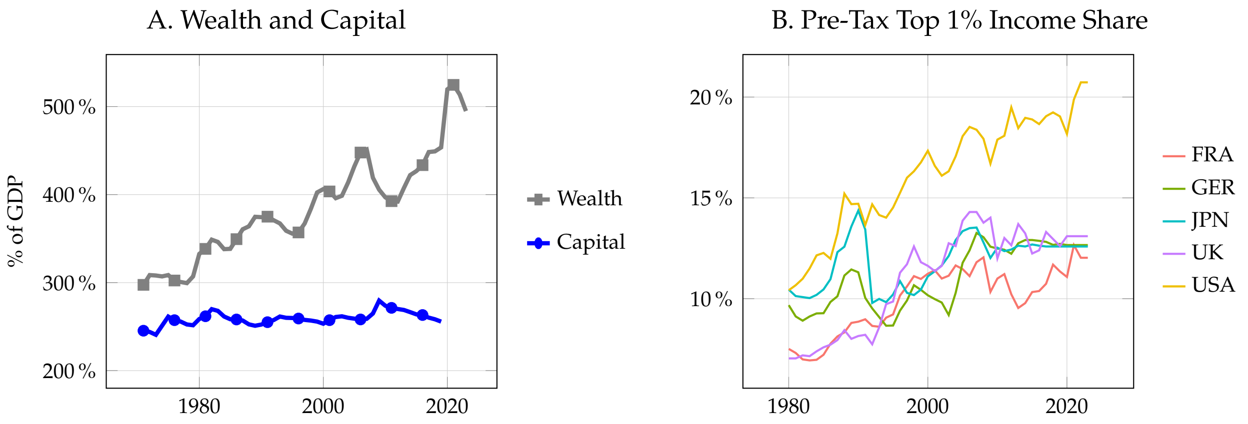
This paper posits insatiable preferences for wealth to generate a capital-gains-driven increase in the wealth-to-output ratio following a rise in income inequality. Preferences for wealth have proven effective in capturing the high saving rates observed at the top of the income distribution (Gaillard et al., 2023). In their standard specification (e.g., Kumhof et al., 2015, Mian et al., 2021), the marginal utility of wealth is assumed to decline more slowly than that of consumption. However, for both wealth and consumption, marginal utility tends to zero as the corresponding argument diverges. We strengthen the asymmetry between utility from wealth and consumption by assuming that the marginal utility of holding wealth remains bounded below by a strictly positive constant, even as wealth diverges.[1] This deviation is parsimonious, to the point that it cannot be tested directly.

We analyze the macroeconomic implications of these preferences within both an endowment and a production economy. The endowment economy helps clarify analytically how asset pricing differs under standard and insatiable preferences for wealth. The production economy, in turn, connects the framework to our motivating questions that require the presence of capital. Both setups include an infinitely lived, rent-generating asset in fixed supply. The price of this asset, which is endogenously determined, lies at the core of our analysis. Furthermore, its presence ensures that the rate of return asymptotically exceeds the economy's growth rate.

In contrast to standard preferences for wealth, under insatiable ones, some agents may accumulate unbounded wealth while keeping consumption bounded relative to output. This occurs because insatiable preferences for wealth induce a lower bound on the marginal utility of saving.

[1] Referring to our preferences for wealth as "insatiable" is perhaps slightly misleading, as it could suggest that standard ones admit a bliss point, which they do not. A more precise, albeit more cumbersome, label would be "asymptotically linear" preferences for wealth.

Figure 1: Wealth dynamics and income inequality for G7 countries



Notes: Panel A reports, as wealth, the aggregate national wealth of G7 countries, measured as the market value of public and private non-financial assets, relative to total G7 GDP, using data from the World Inequality Database. Similarly, it reports, as capital, the aggregate public and private capital stock of G7 countries as a share of their total GDP, obtained from the IMF Investment and Capital Stock Dataset. Panel B presents the pre-tax income share of the top 1 percent in the five main G7 economies, obtained from the World Inequality Database.

As the marginal utility of consumption must not fall below that of saving, this implicitly defines an upper bound on consumption. When income inequality is sufficiently high, some agents at the top of the income distribution asymptotically receive an income that exceeds this bound. As a result, they accumulate unbounded wealth relative to output over time while keeping their consumption bounded. They are referred to as Scrooge McDuck agents. By comparison, under standard preferences for wealth there is no upper bound on optimal consumption and therefore no Scrooge McDuck agents. An agent with diverging wealth would necessarily also have a diverging consumption, albeit typically at a slower rate.

The consumption-saving behavior of Scrooge McDuck agents supports the existence of a diverging rational bubble. The present value of Scrooge McDuck agents' future consumption financed from savings remains below the current value of their wealth. They therefore hold part of their wealth, referred to as surplus wealth, solely for the utility derived from holding it. The associated dividends are fully reinvested. This pushes asset prices above their fundamental value, defined as the present value of future dividends discounted at the rate of return. The difference between the price of the rent-generating asset and its fundamental value constitutes a rational bubble, which coincides with surplus wealth. The return on the bubble arises entirely from the appreciation of its price. Being non-dominated, the bubble grows at the equilibrium rate of return, causing its value to diverge. The economy then does not converge to a steady state, and we develop a novel numerical method to compute the non-balanced growth path in the production economy.

The existence of a rational bubble that expands faster than the economy's growth rate constitutes the central asset-pricing feature generating a wedge between wealth and capital dynamics. The bubble crowds out capital and bubbly equilibria reproduce the key stylized facts motivating this study. They feature a bubble-driven increase in the wealth-to-output ratio without an accompanying rise in the capital-to-output ratio. The recurrent increases in bubble valuation help reconcile the empirical evidence of rising price-to-earnings ratios for various non-reproducible assets with the stability of the return on capital (Gomme et al., 2011; Reis, 2022). Furthermore, the theory provides a unified framework for understanding the rise in the wealth-to-output ratio as a single phenomenon, despite its manifestation on different asset classes across countries. The bubble is formally tied to the rent-generating factor of production in the model, but can be mapped to a broad range of real-world assets such as land, equities, and even public debt.

Whether the equilibrium is bubbly or non-bubbly depends on whether income at the top asymptotically exceeds the upper bound on optimal consumption. If this holds true, agents at the top are necessarily Scrooge McDuck, and the equilibrium is bubbly. In a comparative statics analysis, increasing the insatiable component of preferences for wealth or raising labor income inequality broadens the set of parameters consistent with a bubble. The former operates by lowering the upper bound on optimal consumption, while the latter raises top incomes. In line with our motivating evidence, an increase in top income inequality may therefore shift the equilibrium from non-bubbly to bubbly. For certain parameter values, two equilibria—one of each type—may coexist, in which case equilibrium selection is indeterminate.

This study contributes to the ongoing debate on the welfare implications of capital gains. Existing work has primarily focused on fundamentals-driven increases in the price-to-earnings ratio, resulting from a once-and-for-all decline in the discount rate (Fagereng et al., 2021). These capital gains have often been described as "paper gains" (Cochrane, 2020; Krugman, 2021), as they do not alter dividend flows and cannot sustainably finance higher consumption. In contrast, this analysis focuses on bubble-driven capital gains, which are recurring. This continuous appreciation allows non-Scrooge McDuck agents to realize capital gains in each period to finance consumption. This is particularly relevant as evidence shows that, in the United States, capital gains among the bottom 90 percent have been used largely to finance consumption through increased collateralized borrowing (Mian et al., 2020).

Our framework provides the first microfounded general equilibrium model that jointly delivers diverging wealth inequality and an asymptotic rate of return, r, above the growth rate, g.[2] In a bubbly equilibrium, Scrooge McDuck agents optimally accumulate unbounded wealth, while the wealth of non-Scrooge McDuck agents converges to non-zero but bounded levels. This aligns with Piketty (2014)'s conjecture that wealth inequality diverges when $r > g$, a result that is not immediate in general equilibrium. In particular, under standard preferences for wealth, Michau et al. (2025) find that diverging inequality prevents r from remaining above g.[3] In their model, as more and more wealth accrues to high-saving agents at the top, aggregate savings, and hence capital, increase. Asymptotically, the capital stock converges to the exact level where $r = g$. Under insatiable wealth preferences, diverging inequality breaks the one-to-one link between savings and capital on which this argument rests, as agents invest part of their wealth in the rational bubble.

[2]Diverging wealth inequality is understood here as an increasing absolute gap between agents at the top and bottom of the wealth distribution. A diverging ratio of wealth between these groups arises in many more frameworks, notably when wealth at the bottom converges to zero.

[3]As discussed above, under standard preferences for wealth, agents whose wealth diverges relative to output also have diverging consumption. Their presence does not violate the goods market clearing condition in Michau et al. (2025), only because they account for a vanishingly small share of agents.

The distinction between an economy without a diverging bubble and one with insatiable wealth preferences that sustain a bubble also matters for tax policy. We consider both capital income and wealth taxes. Progressive taxation lowers top incomes, thereby expanding the parameter space in which the upper bound on optimal consumption does not bind. As a result, introducing a progressive tax can shift the equilibrium from bubbly to non-bubbly. This equilibrium-shift effect of taxation has a positive impact on capital accumulation, as it prevents the crowding out associated with the bubble. At the same time, capital income and wealth taxation lower saving incentives, which has a negative effect on the capital stock. The net effect of taxation is ambiguous: the positive equilibrium-shift effect must be weighed against the disincentive to save. The standard efficiency-redistribution trade-off is no longer automatic: certain taxes can simultaneously increase capital accumulation and output while achieving redistribution.

In the present framework, the equivalence between capital income and wealth taxation breaks down. Although all assets yield a homogeneous return, they differ in their taxable income-to-taxable value ratios. Consequently, they are taxed differently under capital income and wealth taxation. The most striking case is that of the rational bubble component of the rent-generating asset. It enters the wealth tax base but not that of the capital income tax, as it pays no dividends. Furthermore, any non-vanishing wealth tax precludes the existence of a bubbly equilibrium. A diverging bubble would yield unbounded tax revenues, the redistribution of which would violate the goods market clearing condition. In a bubbly equilibrium, any wealth tax therefore induces an equilibrium-shift effect. Its disincentive effect, by contrast, increases proportionally with the tax rate. It follows that there exists a range of sufficiently small tax rates for which the equilibrium-shift effect dominates, leading to higher capital accumulation.

Finally, we illustrate the empirical relevance of the mechanism through a simple quantitative exercise for the United States. We calibrate the model parameters to match 1989 data, assuming the economy is initially in a non-bubbly steady state. We then introduce an increase in labor income inequality, calibrated using SCF data, which drives the economy toward a bubbly equilibrium. Despite its simplicity, the model replicates the untargeted evolution of wealth inequality and the capital- and wealth-to-output ratios reasonably well. This is achieved through a bubble amounting to 150% of GDP in 2024. We then consider the introduction of a counterfactual 1.5 percent wealth tax in this economy in 2022. The tax reduces the wealth-to-output ratio by preventing the bubble, while remaining small enough to increase the aggregate capital stock by 4.5 percent over 30 years. The resulting welfare gains in consumption utility, arising from redistribution and a higher capital stock, more than offset the losses associated with the lower utility derived from wealth.

Related Literature. This paper builds on multiple empirical studies documenting, across advanced economies, rising income and wealth inequality (Katz and Murphy, 1992, Piketty and Saez, 2003, Saez and Zucman, 2016; Batty et al., 2019; Chancel et al., 2022; Smith et al., 2023, Blanchet and Martínez-Toledano, 2023), an increase in aggregate wealth driven by capital gains (Piketty and Zucman, 2014, Baselgia and Martínez, 2025), and a declining trend in investment (Gutiérrez and Philippon, 2017). It theoretically contributes to three strands of the literature.

First, motivated by empirical evidence that saving rates increase with income and wealth (Carroll, 2000; Dynan et al., 2004; Straub, 2019; Fagereng et al., 2021) and that aggregate savings are primarily driven by the top of the distribution (Mian et al., 2020; Bauluz et al., 2022), a growing theoretical and quantitative literature highlights the importance of non-homothetic preferences for wealth in explaining wealth inequality and the wealth-to-output ratio (Carroll, 2000; De Nardi, 2004; Kumhof et al., 2015; De Nardi and Yang, 2016; Benhabib et al., 2019; Mian et al., 2021; Elina and Huleux, 2023; Gaillard et al., 2023; Michau et al., 2025). The present framework departs from this literature by positing insatiable preferences for wealth and shows how they alter the predictions for both aggregate and distributional wealth dynamics relative to standard preferences.

Secondly, this work contributes to the rational bubble literature. Previous research has established that rational bubbles can exist in dynamically inefficient economies (Samuelson, 1958; Diamond, 1965; Tirole, 1985; Michau et al., 2023) or under financial frictions (Farhi and Tirole, 2012; Martin and Ventura, 2012; Reis, 2021). In both cases, the rate of return on bubbles is below the economy's growth rate, leading to an asymptotically stationary bubble-to-output ratio. Few studies have explored frameworks with a diverging bubble-to-output ratio, with notable exceptions including Ono (1994) and Kamihigashi (2008). The present analysis departs from these frameworks, which also feature insatiable preferences for liquidity or wealth, by introducing heterogeneous agents that render the equilibrium uniquely determined.

Third, this work contributes to the literature on capital income and wealth taxation (Judd, 1985; Chamley, 1986; Straub and Werning, 2020; Gaillard and Wangner, 2023). It highlights the equilibrium-shift effect that such taxes may have when they move the economy from a bubbly to a non-bubbly equilibrium. This shift raises both capital and consumption, an effect that is absent under standard non-homothetic preferences (Morrison, 2024). Moreover, this paper sheds new light on the equivalence between capital income and wealth taxation (Allais, 1977; Guvenen et al., 2023; Piketty et al., 2023). It shows that this equivalence may break down even under homogeneous returns in the presence of capital gains. Although the argument can be applied more generally, it is developed here for the first time.

The remainder of the paper is organized as follows. Section 2 develops the core intuition for the existence of bubbly equilibria under insatiable preferences, using a tractable endowment economy. Section 3 extends the analysis to a production economy, linking the model to the motivating stylized facts. Section 4 investigates the positive implications of the model for wealth and capital income taxation. Section 5 presents a quantitative exercise to assess the empirical relevance of the Scrooge McDuck mechanism. Section 6 concludes.

2 Endowment Economy

To isolate the mechanism by which insatiable wealth preferences, unlike standard ones, can generate diverging rational bubbles, we embed such preferences in a simple two-agents asset pricing model à la Lucas, 1978. This tractable setting allows us to derive the conditions under which bubbly or non-bubbly equilibria arise, and to characterize the asymptotic behavior of each equilibrium type.

2.1 Model Environment

The economy is deterministic, with discrete time running from $t = 0$ to ∞. The endowment comes from the single unit of Lucas tree, which delivers one unit of the consumption good each period and is priced at q_t at time t after dividend payment. The rate of return of the Lucas tree is defined as:

$$R_{t+1} \equiv \frac{1 + q_{t+1}}{q_t} \tag{1}$$

Households. There is a unit mass of infinitely-lived households, divided into two types $i \in \{1, 2\}$, each representing a strictly positive share λ^i of the total population. Agents of type i hold ℓ_t^i units of the Lucas tree at the beginning of period t and consume c_t^i at time t. Agents differ only in their initial endowment of Lucas tree units, with type 1 agents holding the higher initial endowment: $\ell_0^1 \geq \ell_0^2 > 0$. The wealth of a household of type i at the end of period t is defined as $a_{t+1}^i \equiv q_t \ell_{t+1}^i$, and its budget constraint can be written as:

$$c_t^i + a_{t+1}^i = R_t a_t^i. \tag{2}$$

Both types of agents share identical preferences. They derive utility each period from consumption and from holding wealth. Agents discount the future at rate $\beta \in [0, 1)$ and maximize intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t U(c_t^i, a_{t+1}^i) \qquad \text{with} \qquad U(c, a) = u(c) + v(a). \tag{3}$$

Utility from consumption is represented by the function $u(c)$, which satisfies the Inada conditions. Preference for wealth is captured by $v(a)$, assumed to be twice continuously differentiable, increasing, and concave. The model remains agnostic about the microfoundations of the utility derived from wealth, which may arise from bequest motives, the pursuit of power or social status, or an intrinsic desire to accumulate wealth, as discussed, for instance, by Zou, 1994 and Carroll, 2000.

The solution to the optimization problem for type i households is characterized by the following Euler equation and the transversality condition:[4]

$$u'(c_t^i) = \beta R_{t+1} u'(c_{t+1}^i) + v'(a_{t+1}^i), \tag{4}$$

$$\lim_{t \to \infty} \beta^t [u'(c_t^i) - v'(a_t^i)] a_t^i = 0. \tag{5}$$

Wealth-Dependent Consumption-Saving Behavior. In order to match the empirical evidence that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019), this paper follows the literature on preferences for wealth by assuming that the utility-from-wealth component, $v(a)$, exhibits lower curvature than the utility-from-consumption component, $u(c)$. To formalize this assumption, we first need to define the coefficient of risk aversion of consumption and wealth,

[4] A proof of the necessity of the transversality condition is provided in Appendix A.1

labeled respectively $\theta(c)$ and $\eta(a)$:

$$\theta(c) \equiv -\frac{c\,u''(c)}{u'(c)}, \qquad \eta(a) \equiv -\frac{a\,v''(a)}{v'(a)}. \tag{6}$$

The following assumption induces non-homothetic (i.e., wealth-dependent) consumption-saving behavior, whereby wealthier agents save a larger share of their income.

Assumption 1 *For any $\{c, a\} \in \mathbb{R}^2_+$, the utility from wealth exhibits lower curvature than the utility from consumption. That is,*

$$\eta(a) < \theta(c). \tag{7}$$

Under Assumption 1, wealth is a luxury. Intuitively, as the marginal utility of wealth declines more slowly than that of consumption, high-wealth agents value additional wealth relative to additional consumption more than low-wealth agents do. In equilibrium, this leads type 1 agents to save more than type 2 agents, resulting in type 1 agents purchasing Lucas tree units from type 2 agents in each period.

Standard vs Insatiable Preferences for Wealth. The insatiable preferences for wealth studied in this paper depart from standard preferences by imposing a strictly positive lower bound on the marginal utility of holding wealth. As it is concave and strictly increasing, the utility-from-wealth component $v(a)$ admits a well-defined limit for its marginal utility as wealth tends to infinity. We denote this limit by κ:

$$\lim_{a \to \infty} v'(a) = \kappa. \tag{8}$$

Preferences for wealth are said to be *insatiable* whenever $\kappa > 0$. In contrast, if $\kappa = 0$, we refer to them as *standard* preferences for wealth, as this corresponds to the usual specification in the wealth-in-utility literature. We think of κ as being small, so that insatiable preferences for wealth represent a parsimonious deviation from the standard specification in the literature.[5] In line with this interpretation, and to rule out situations in which all agents would strictly prefer saving over consumption, we impose the technical assumption that $\kappa < \frac{1-\beta}{\beta} u'(1)$.

The insatiability assumption cannot be ruled out empirically. Standard and insatiable preferences for wealth differ only in their asymptotic implications, which are, by definition, not observable. It is therefore fundamentally impossible to distinguish between $\kappa = 0$ and a strictly positive but arbitrarily small value of κ. In principle, one could test whether κ exceeds a given strictly positive constant by exploiting saving rates across wealth levels at the top of the distribution. In practice, however, such a test is ultimately infeasible due to the lack of consumption data for the relevant high-wealth households. Since insatiable and standard preferences for wealth cannot be distinguished using micro-data, this paper proposes to contrast their implications, par-

[5] The crucial assumption of this paper is not that *all* agents have insatiable preferences for wealth, but rather that *some* do. We assume throughout that all agents share these preferences to derive results that are not driven by preference heterogeneity. Importantly, the key mechanisms would still hold under heterogeneous preferences, as long as *some* agents—technically, even a single agent—at the top of the wealth distribution exhibit insatiable preferences for wealth.

ticularly for asset pricing, in order to assess which specification is more consistent with observed macroeconomic stylized facts.

Market Clearing Conditions. In each period, there is one unit of the Lucas tree and one unit of the non-storable consumption good. The market clearing condition for assets is therefore expressed as:

$$\lambda^1 \ell_t^1 + \lambda^2 \ell_t^2 = 1, \tag{9}$$

and the goods market clearing condition is given by:

$$\lambda^1 c_t^1 + \lambda^2 c_t^2 = 1. \tag{10}$$

Equilibrium. Given the initial endowment ℓ_0^1, an equilibrium $\{c_t^1, \ell_t^1, q_t\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems (4, 5) and the asset market clearing condition (9). Goods market clearing holds (10) by Walras's Law.

2.2 Definitions of Key Concepts

We now introduce four key concepts that structure the theoretical analysis: (i) the upper bound on consumption implied by insatiable preferences for wealth; (ii) the behavior of *Scrooge McDuck* agents, who accumulate unbounded wealth while keeping consumption bounded; (iii) the notion of *surplus wealth*, defined as the portion of agents' wealth that does not finance any future consumption; and (iv) the rational bubble that emerges in the price of the Lucas tree as a direct consequence of a positive surplus wealth.

Upper Bound on Consumption. Unlike standard preferences for wealth, insatiable ones implicitly define an upper bound on optimal consumption, $\bar{c}_t$.

Definition 1 *Under insatiable preferences for wealth, the upper bound on optimal consumption, $\bar{c}_t$, is defined as:*

$$\bar{c}_t \equiv u'^{-1}\left(\sum_{s=0}^{\infty} \beta^s \left[\prod_{j=1}^{s} R_{t+j}\right] \kappa\right). \tag{11}$$

Regardless of agent i's wealth, optimal consumption is always such that $c_t^i < \bar{c}_t$. The lower bound on the marginal utility of holding wealth, κ, induces a corresponding lower bound on the marginal utility of saving, as defined by the inner bracket in Equation 11. This lower bound captures the utility derived from saving an additional unit of wealth and reinvesting all associated returns indefinitely, assuming the marginal utility of wealth is fixed to κ. The upper bound $\bar{c}_t$ corresponds to the level of consumption at which the marginal utility of consumption equals the lower bound on the marginal utility of holding wealth. It follows that it is never optimal for any agent i to consume $c_t^i \geq \bar{c}_t$.

As will become clear in the analysis, the fundamental difference between insatiable and standard preferences for wealth stems from the existence of this upper bound on consumption.

From Equation 11, it can be seen that $\bar{c}_t$ diverges in the limit as κ tends to zero. There is therefore no upper bound on consumption under standard preferences for wealth. This paper focuses on insatiable preferences for wealth as the source of the upper bound on consumption, but similar results could arise from alternative preference specifications, as long as they imply an upper bound on optimal consumption. For instance, one could directly impose an upper bound in the utility-from-consumption component $u(c)$, or alternatively, assume preferences over net saving flows.

Scrooge McDuck Agents. Given the upper bound on consumption, some agents may optimally choose to accumulate unbounded wealth over time while keeping consumption bounded; we label them Scrooge McDuck agents.

Definition 2 *An agent i is labeled as a Scrooge McDuck agent, if and only if:*

$$\lim_{t\to\infty} a_t^i = \infty, \quad \textit{and} \quad \exists\, M \in \mathbb{R}_+ \textit{ such that } c_t^i < M \;\forall t. \tag{12}$$

The existence of Scrooge McDuck agents is closely tied to the presence of an upper bound on consumption, and their consumption asymptotically approaches $\bar{c}_t$. As their wealth diverges, the marginal utility of holding wealth converges to κ, and their marginal utility of saving approaches $\sum_{s=0}^{\infty} \beta^s \left[\prod_{j=1}^{s} R_{t+j}\right] \kappa$. By the definition of $\bar{c}_t$ in Equation 11, their asymptotic consumption therefore coincides with the upper bound on optimal consumption. In contrast, under standard preferences for wealth, it is never optimal for an agent to behave as a Scrooge McDuck: diverging wealth necessarily implies diverging consumption, even if consumption may diverge at a slower rate (see, for instance, Michau et al., 2025).

Surplus Wealth. Scrooge McDuck agents hold a portion of their wealth purely for its own sake, referred to as surplus wealth.

Definition 3 *The surplus wealth of an agent of type i at the end of period t, denoted s_{t+1}^i, is defined as the portion of wealth that is not used to finance any future consumption:*

$$s_{t+1}^i \equiv a_{t+1}^i - \sum_{j=1}^{\infty} \frac{c_{t+j}^i}{\prod_{s=1}^{j} R_{t+s}}. \tag{13}$$

In this endowment economy, all consumption is financed out of the Lucas tree. As a result, surplus wealth corresponds to total wealth minus the present value of future consumption. All dividends associated with surplus wealth are reinvested rather than consumed, so that surplus wealth grows at rate R_{t+1} from period t to $t+1$.

Rational Bubble. The presence of Scrooge McDuck agents pushes the price of the Lucas tree above its fundamental value and gives rise to a rational bubble, whose size coincides with that of the surplus wealth.

Definition 4 *The asset price q_t is decomposed into a fundamental value f_t, defined as the present value of future dividends discounted at the rate of return, and a bubble component b_t.*

$$f_t \equiv \sum_{j=1}^{\infty} \frac{1}{\prod_{s=1}^{j} R_{t+s}}, \qquad b_t = \lim_{T\to\infty} \frac{q_T}{\prod_{s=1}^{T} R_{t+s}} \qquad \text{with} \quad q_t = f_t + b_t. \tag{14}$$

The fundamental value derives from both the stream of dividends and the utility of holding it. The latter is reflected in the rate of return R_{t+s}, which is pushed down by the preference for wealth. In contrast, the value of the rational bubble arises solely from the utility of holding it. Since agents derive direct utility from it, one might interpret the bubble as a particular form of fundamental. We nevertheless follow the literature (e.g., Michau et al., 2025) in referring to it as a rational bubble, for two reasons. First, its current value corresponds to its discounted asymptotic value, reflecting the fact that agents value the bubble only because they anticipate future demand from other agents. Second, if multiple infinitely-lived assets were introduced, the model would leave indeterminate which asset the bubble is attached to.

The following proposition establishes that the rational bubble is the asset pricing counterpart of surplus wealth.

Proposition 1 *The rational bubble coincides with the aggregate surplus wealth:*

$$b_t = \sum_i \lambda^i s^i_{t+1}. \tag{15}$$

Proposition 1 follows directly from previous definitions of the surplus wealth (13) and the bubble (14). Intuitively, in the considered frictionless model, a rational bubble that does not finance consumption can exist only if some wealth is held without a consumption motive. Like surplus wealth, the rational bubble grows at rate R_{t+1} from period t to $t+1$, in order to deliver the same rate of return as fundamental wealth.

2.3 Bubbly and Non-bubbly Equilibria

We now describe the asymptotic properties of the possible equilibria. We distinguish between two types of equilibria: those that include a strictly positive bubble and those that do not. Whether the upper bound on optimal consumption asymptotically exceeds the dividend flow of high-wealth type 1 agents determines whether the equilibrium is bubbly or non-bubbly. Since the economy is deterministic, all variables either converge or diverge, and, for notational convenience, we denote asymptotic values without time subscripts.

Asymptotic Lucas Tree Holdings. The non-homotheticity of preferences implies that type 1 agents asymptotically hold the entire Lucas tree.

Proposition 2 *Under Assumption 1, type 1 agents hold the entire Lucas tree asymptotically in every equilibrium:*

$$\lim_{t\to\infty} \ell_t^1 = \frac{1}{\lambda^1}, \qquad \lim_{t\to\infty} \ell_t^2 = 0. \tag{16}$$

Assumption 1 implies that saving rates increase strictly with wealth. Since type 1 agents begin with greater initial wealth, it follows by induction that they exhibit higher saving rates and greater holdings of the Lucas tree than type 2 agents in every period. Their relatively stronger marginal preference for wealth over consumption makes them effectively more patient than type 2 agents. Formally, Proposition 2 can be proved by contradiction. From the combination of the two Euler equations (4), no equilibrium exists in which both type 1 and type 2 agents asymptotically consume exactly their endowment when $\ell^1 > \ell^2 > 0$.

Non-bubbly Equilibrium. A non-bubbly equilibrium is asymptotically characterized by an upper bound on optimal consumption that exceeds the dividend flow of type 1 agents, inducing them to fully consume their endowment.

Proposition 3 *Asymptotically, if an equilibrium features an upper bound on optimal consumption that is above the endowment received by type* 1 *agents,*

$$\lim_{t\to\infty} \bar{c}_t \geq \frac{1}{\lambda^1}, \tag{17}$$

then it is a non-bubbly equilibrium with $b_t = 0\ \forall t$. The asymptotic behavior of the economy is characterized as follows.

(i) *Type 1 agents consume their entire endowment and hold constant wealth:*

$$\lim_{t\to\infty} c_t^1 = \frac{1}{\lambda^1}, \qquad \lim_{t\to\infty} a_t^1 = \frac{1/\lambda^1}{R-1}. \tag{18}$$

(ii) *Type 2 agents' consumption and wealth converge to zero:*

$$\lim_{t\to\infty} c_t^2 = \lim_{t\to\infty} a_t^2 = 0. \tag{19}$$

(iii) *The price of the Lucas tree equals its fundamental value:*

$$\lim_{t\to\infty} q_t = \lim_{t\to\infty} f_t = \frac{1}{R-1} \tag{20}$$

In a non-bubbly equilibrium, all variables converge, and the economy approaches a steady state. Asymptotically, type 1 agents hold the entire stock of the Lucas tree, leaving no additional units to purchase from type 2 agents. As a result, they consume their full endowment, and their wealth converges to the level required to sustain this consumption. They hold no surplus wealth, and no bubble arises in equilibrium, consistent with Proposition 1. Conversely, type 2 agents, whose consumption declines strictly during the transition to the steady state, converge asymptotically to zero consumption and zero wealth.

Bubbly Equilibrium. A bubbly equilibrium is asymptotically characterized by an upper bound on optimal consumption that is below the dividend flow of type 1 agents, leading them to hold a diverging surplus wealth that supports a bubble.

Proposition 4 *Asymptotically, if an equilibrium features an upper bound on optimal consumption that is strictly below the endowment received by type* 1 *agents,*

$$\lim_{t\to\infty} \bar{c}_t < \frac{1}{\lambda^1}, \tag{21}$$

then it is a bubbly equilibrium with $b_t > 0\ \forall t$. The asymptotic behavior of the economy is characterized as follows.

(i) *Type 1 agents consume at the upper bound on optimal consumption and hold diverging wealth:*

$$\lim_{t\to\infty} c_t^1 = \bar{c}, \qquad \lim_{t\to\infty} a_t^1 = \infty. \tag{22}$$

(ii) *Type 2 agents' consumption and wealth converge to positive values:*

$$\lim_{t\to\infty} c_t^2 = c^2 \quad \text{with} \quad c^2 \equiv \frac{1-\lambda^1\bar{c}}{\lambda^2}, \qquad \lim_{t\to\infty} a_t^2 = \frac{c^2}{R-1}. \tag{23}$$

(iii) *The Lucas tree price diverges over time, reflecting the divergence of its bubbly component:*

$$\lim_{t\to\infty} f_t = \frac{1}{R-1}, \qquad \lim_{t\to\infty} b_t = \infty. \tag{24}$$

Asymptotically, type 1 agents receive the entire endowment but choose not to consume all of it, as doing so would exceed the upper bound on consumption. They are Scrooge McDuck agents, and hold a strictly positive surplus wealth. They are willing to exchange the difference between their endowment, $1/\lambda^1$, and their consumption, $\bar{c}$, for Lucas tree units regardless of the price, q_t. This non-vanishing demand for new Lucas tree units by type-1 agents, despite their vanishing supply from type-2 agents, drives the Lucas tree price to diverge. The divergence is fueled by a bubbly component in its valuation which, in line with Proposition 1, coincides with the aggregate surplus wealth held by type 1 agents.

The presence of type 2 agents, who are not Scrooge McDuck, ensures that at most one bubbly equilibrium can exist. Type 2 agents' asymptotic consumption level $\lambda^2 c^2$ corresponds to the endowment not consumed by type 1. They hold a vanishing share of the Lucas tree, whose price diverges. The equilibrium price of the Lucas tree, and thus the size of the bubble, is pinned down by the requirement that type 2 agents retain just enough wealth to sustain their asymptotic consumption c^2. If q_t were above its equilibrium value, type 2 agents would demand more consumption, violating the goods market clearing condition (10); if it were below, some goods would remain unconsumed, which is inconsistent with strictly positive marginal utility of consumption for both agent types. This contrasts with representative-agent models featuring insatiable preferences for wealth, where a marginal propensity to save of one may lead to equilibrium indeterminacy: additional bubble units can be fully saved without affecting consumption (e.g., Kamihigashi, 2008).

Rate of Return. The following proposition compares the asymptotic equilibrium rate of return between the two types of equilibrium.

Proposition 5 *The asymptotic equilibrium rate of return in any bubbly equilibrium is strictly higher than the asymptotic rate of return in any non-bubbly equilibrium.*

Proposition 5 follows directly from the rewritten Euler equation (4) for type 1 agents:

$$\lim_{t\to\infty} \frac{v'(a_t^1)}{u'(c_t^1)} = 1 - \beta R. \tag{25}$$

In any bubbly equilibrium, type 1 agents consume less than $1/\lambda^1$, which is their consumption level in any non-bubbly equilibrium. Moreover, they hold asymptotically more wealth in the bubbly case, as their wealth diverges. As a result, the left-hand side of Equation 67 is necessarily lower in the presence of a bubble than in its absence, implying that the rate of return must be higher under a bubbly equilibrium than under a non-bubbly one.

The relationship between the existence of rational bubbles and the rate of return is fundamentally different for the bubbles that emerge from the surplus wealth of Scrooge McDuck than for most other types of rational bubbles. Rational bubbles that arise under standard preferences for wealth (Michau et al., 2025), as well as those emerging in overlapping generations models (e.g., Samuelson, 1958, Tirole, 1985) or in the presence of financial frictions (e.g., Martin and Ventura, 2012), require the bubble's rate of return to be *sufficiently low* to lie below the growth rate of the economy. This condition ensures that the bubble-to-output ratio remains bounded, in settings where agents do not optimally choose to accumulate unbounded wealth. In contrast, in the present model, the rate of return exceeds the (zero) growth rate, with $R \in (1, 1/\beta)$ in every equilibrium.[6] Still, in a bubbly equilibrium, the rational bubble exists and diverges over time because holding a diverging wealth does not violate the household optimization problem of Scrooge McDuck agents. Furthermore, the bubble exists only because the rate of return is *sufficiently high* to lower the upper bound on optimal consumption (11) enough to allow for the existence of Scrooge McDuck agents.

2.4 Equilibria Characterization

We now characterize the conditions under which bubbly and non-bubbly equilibria exist. If the insatiable component of preferences for wealth is sufficiently strong, the equilibrium is unique and bubbly. Otherwise, whenever κ is strictly positive, both a non-bubbly and a bubbly equilibrium coexist.

Proposition 6 *The existence and uniqueness of equilibrium depend on the strength of the insatiable component in preferences for wealth, as captured by κ.*

(i) *If $\kappa = 0$, the equilibrium is unique and non-bubbly.*

[6] This follows directly from the definition of R, since the price of the Lucas tree, q_t either converges to a positive constant or diverges.

(ii) *If* $0 < \frac{\beta}{1-\beta}\kappa \leq u'\left(\frac{1}{\lambda^1}\right)$*, two equilibria exist, one bubbly and one non-bubbly, and equilibrium selection is indeterminate.*

(iii) *If* $u'\left(\frac{1}{\lambda^1}\right) < \frac{\beta}{1-\beta}\kappa$*, the equilibrium is unique and bubbly.*

Under standard preferences for wealth (i), there is no upper bound on optimal consumption, and type 1 agents asymptotically choose to consume their entire endowment, resulting in a unique non-bubbly equilibrium. Understanding cases (ii) and (iii) requires considering the lower bound on the marginal utility of savings, $\sum_{s=0}^{\infty} \beta^s \left(\prod_{j=1}^{s} R_{t+j}\right) \kappa$. Since $R \in (1, 1/\beta)$, this expression attains its lowest asymptotic value when R is arbitrarily close to 1, in which case it equals $\frac{\beta}{1-\beta}\kappa$. In case (iii), the marginal utility associated with consuming the full asymptotic endowment of a type 1 agent, $u'(1/\lambda^1)$, lies below this lower bound. The upper bound on optimal consumption must therefore be, asymptotically, strictly below the endowment of type 1 agents. It follows from Proposition 4 that the resulting equilibrium is necessarily bubbly. In case (ii), an additional non-bubbly equilibrium exists, characterized by a lower rate of return and a higher upper bound on consumption than the bubbly equilibrium.

In the case where two equilibria coexist (ii), equilibrium selection is self-fulfilling. If agents anticipate the high rate of return of the bubbly equilibrium, the upper bound on optimal consumption is low, leading type 1 agents to be Scrooge McDuck. Their surplus wealth results in a rational bubble, which in turn raises the rate of return through its continuous appreciation, thereby validating the initial expectation. Conversely, if agents anticipate the low rate of return associated with the non-bubbly equilibrium, the upper bound on optimal consumption is high, type 1 agents do not behave as Scrooge McDuck agents, and no rational bubble arises. A formal proof of this indeterminacy is provided in Appendix A.2.

The existence of multiple equilibria is a strong theoretical result, but its real-world relevance and robustness remain open to discussion. The multiplicity arises from the fact that, under insatiable preferences for wealth, the asymptotic consumption of type 1 agents is particularly responsive to the rate of return. This is evident from the Definition 1 of the upper bound on optimal consumption, which coincides with c^1 in the bubbly case. Empirical evidence is insufficient to assess the plausibility of such a high elasticity of consumption with respect to the rate of return at the top of the income and wealth distribution. The prediction of multiple equilibria also lacks theoretical robustness and may simply reflect that insatiable preferences for wealth constitute an overly stylized preference specification. The high elasticity and the associated multiplicity of equilibria is not found in alternative preference specifications that impose an upper bound on optimal consumption, such as a satiation point in $u(c)$ or preferences defined over new net savings. Consequently, we do not place central importance on the multiplicity of equilibria in our analysis.

To sum up, this simple endowment economy shows that insatiable preferences for wealth differ from standard ones by defining an upper bound on optimal consumption. Whenever this upper bound is asymptotically binding for high-wealth agents, these agents behave as Scrooge McDuck: they accumulate an unbounded surplus wealth, supporting the existence of a rational bubble.

3 Production Economy

We now introduce insatiable preferences for wealth into a growing production economy to show that the existence of bubbly equilibria remains robust in the presence of reproducible capital, and to connect the model to the stylized facts that motivated our analysis. In bubbly equilibria, the wealth-to-output diverges due to a bubble component that crowds out capital, and the capital-to-output ratio converges to a level that does not maximize asymptotic aggregate consumption.

3.1 Model Environment

Production. The production function is a Cobb-Douglas with three inputs: labor, reproducible capital, and a rent-generating factor:

$$Y_t = K_t^{\alpha} L_t^{\gamma} (Z_t N_t)^{1-\alpha-\gamma}, \tag{26}$$

where K_t denotes the capital stock, L_t the rent-generating factor, N_t the labor supply and Z_t the labor productivity. The rent-generating factor is in fixed supply, normalized to $L_t = 1$ for all t, and each of its units is priced at Q_t after production. The capital stock depreciates at a rate δ. It is reproducible, with its price normalized to one by assumption, and therefore cannot be priced above its fundamental value. The rent-generating factor is introduced to study the asset pricing implications of insatiable preferences for wealth. It is interpreted as encompassing all non-reproducible, long-lived assets, for example, land.[7]

The growth rates of productivity and labor supply from period t to $t+1$ are denoted by g_{t+1}^{Z} and g_{t+1}^{N}, respectively:

$$g_{t+1}^{Z} \equiv \frac{Z_{t+1} - Z_t}{Z_t}, \qquad g_{t+1}^{N} \equiv \frac{N_{t+1} - N_t}{N_t}. \tag{27}$$

Asymptotically, the economy grows at a slower rate than the effective labor force. This is due to the presence of the rent-generating factor, which does not scale with output. As the capital-to-output ratio K_t/Y_t converges, the economy tends to grow at the same rate as $(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}$, whose growth rate, denoted by g_t, is given by:

$$g_{t+1} \equiv \left[(1 + g_{t+1}^{Z})(1 + g_{t+1}^{N})\right]^{\frac{1-\alpha-\gamma}{1-\alpha}} - 1. \tag{28}$$

To account for growth, each capital-letter variable X_t has a normalized counterpart x_t, defined as:

$$x_t \equiv \frac{X_t}{(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}}. \tag{29}$$

All figures presented below display variables normalized according to Equation 29.

[7] Its real-world definition could be subject to debate, depending on how one defines reproducibility and longevity—whether, for instance, it includes long-lived intangible assets such as brands or market power.

Factors of production are paid at their marginal productivity. At time t, one unit of labor is paid a wage W_t. By arbitrage, the rates of return on capital and on the rent-generating factor from period t to $t+1$ are equalized and denoted by R_{t+1}:

$$R_{t+1} = 1 + \alpha k_{t+1}^{\alpha-1} - \delta = (1+g_{t+1})\frac{q_{t+1} + \gamma y_t}{q_t}. \tag{30}$$

We further define $\widetilde{R}_t$ as the growth-adjusted rate of return:

$$\widetilde{R}_t \equiv \frac{R_t}{1+g_t}. \tag{31}$$

As in the case of the Lucas tree in the endowment economy, the price of the rent-generating factor can be decomposed into a fundamental component, F_t, which corresponds to the present value of future dividends discounted at the rate of return, and a bubbly component, B_t. Using the normalized versions of F_t and B_t, we obtain:

$$f_t \equiv \sum_{j=1}^{\infty} \frac{\gamma y_{t+j}}{\prod_{s=1}^{j} \widetilde{R}_{t+s}}, \qquad b_t = \lim_{T\to\infty} \frac{q_T}{\prod_{s=1}^{T} \widetilde{R}_{t+s}} \qquad \text{with} \quad q_t = f_t + b_t. \tag{32}$$

We assume the rent-generating factor to be productive in every period with $\gamma > 0$. This assumption prevents the rate of return from falling below the growth rate of the economy, following the argument in Rhee (1991). Since the rent-generating factor's share of output does not converge to zero, its return remains above the growth rate asymptotically. We therefore focus on the empirically relevant case, where rents exist in the economy, at a minimum from land, and the rate of return exceeds the growth rate of the economy (Jordà et al., 2019; Reis, 2021).

Households. The population consists of a unit mass of households, each comprising N_t agents. There are I types of infinitely-lived agents, $i \in \{1, 2, ..., I\}$, differing in their labor productivity, and eventually in their initial endowment. Each type represents a share λ^i of the total population. A household of type i holds K_{t+1}^i units of capital and L_{t+1}^i units of the rent-generating factor at the end of period t.

Agents are ranked in decreasing order of labor productivity and initial endowments. Each agent i supplies ζ_t^i units of labor inelastically in period t, where ζ_t^i reflects productivity differences and satisfies,

$$\zeta_t^i > \zeta_t^j \quad \text{for all } i < j. \tag{33}$$

For convenience and without loss of generality, productivity levels are chosen such that $\sum_i \zeta_t^i \lambda^i = 1$ for all t, ensuring that aggregate labor supply coincides with population size and equals N_t. Initial endowments of capital, K_0^i, and of the rent-generating factor, L_0^i, are weakly decreasing in agent index, satisfying,

$$K_0^i \geq K_0^j, \quad L_0^i \geq L_0^j \quad \text{for all } i < j. \tag{34}$$

The normalized end-of-period wealth of type i households, denoted a^i_{t+1}, is defined as,

$$a^i_{t+1} \equiv k^i_{t+1} + \frac{q_t L^i_{t+1}}{1+g_{t+1}}. \tag{35}$$

Their budget constraint is given by,

$$c^i_t + a^i_{t+1}(1+g_{t+1}) = R_t a^i_t + \zeta^i_t w_t. \tag{36}$$

We assume a functional form for preferences that satisfies Assumption 1 and allows for insatiable preferences for wealth. Households derive utility from consumption and wealth holdings, and maximize the following intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(a_{t+1}) \right], \tag{37}$$

where the period utility functions are given by,

$$u(c) = \begin{cases} \dfrac{c^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1, \\ \log(c) & \text{if } \theta = 1, \end{cases} \quad \text{and} \quad v(a) = \begin{cases} \psi \cdot \dfrac{(a-\underline{a})^{1-\eta}}{1-\eta} + \kappa a & \text{if } \eta \neq 1, \\ \psi \log(a-\underline{a}) + \kappa a & \text{if } \eta = 1. \end{cases} \tag{38}$$

The utility from consumption, $u(c)$, follows a CRRA specification, while the preference for wealth, $v(a)$, consists of two components: a standard concave term and an additional linear component that captures insatiable preferences for wealth. The parameters are chosen such that $\eta < \theta$ and $\underline{a} \geq 0$, ensuring that Assumption 1 holds and that saving rates increase with both income and wealth. The parameter ψ is a scaling parameter that captures the strength of standard preferences for wealth relative to utility from consumption. The linear term in $v(a)$ imposes a lower bound on the marginal utility of holding wealth at κ, that is,

$$\lim_{a \to \infty} \frac{\partial v(a)}{\partial a} = \kappa. \tag{39}$$

We refer to preferences with $\kappa = 0$ as standard preferences for wealth, and to those with $\kappa > 0$ as insatiable preferences for wealth. To avoid the counterfactual implication of ever-increasing aggregate saving rates as the economy expands, the arguments of the utility function are expressed in normalized rather than absolute terms.[8]

The solution to the optimization problem for type i households is characterized by the following Euler equation (40) and the transversality condition (41):

$$c^{i\,-\theta}_t = \beta R_{t+1} \frac{\left(c^i_{t+1}\right)^{-\theta}}{1+g_{t+1}} + \psi \frac{(a^i_{t+1} - \underline{a})^{-\eta}}{1+g_{t+1}} + \frac{\kappa}{1+g_{t+1}}, \tag{40}$$

[8] In the words of Mian et al. (2021), the purpose of preferences for wealth is to break *individual* scale invariance, so that wealthier households exhibit higher saving rates, while preserving *aggregate* scale invariance, thereby preventing the saving rate of the economy as a whole from rising with its size. Aside from the standard preferences-for-wealth case with log utility from consumption, this requires that both consumption and wealth enter the utility function in normalized terms.

$$\lim_{t\to\infty} \beta^t \left[c_t^{i\,-\theta} - \frac{\psi(a_{t+1}^i - \underline{a})^{-\eta} + \kappa}{1 + g_{t+1}} \right] a_{t+1}^i = 0. \tag{41}$$

Definitions. We adapt the definitions of the core elements of the Scrooge McDuck analysis to account for growth and labor income. Similarly to the endowment economy, insatiable preferences for wealth impose a lower bound on the marginal utility of saving, which in turn determines an upper bound on consumption, $\overline{C}_t$. Its normalized counterpart, $\overline{c}_t$, is given by,

$$\overline{c}_t \equiv \left[\frac{1}{1+g_{t+1}} \sum_{s=0}^{\infty} \beta^s \left(\prod_{j=1}^{s} \frac{R_{t+j}}{1+g_{t+j+1}} \right) \kappa \right]^{-\frac{1}{\theta}}, \tag{42}$$

where the term in brackets represents the lower bound on the marginal utility of saving. We refine the definition of Scrooge McDuck agents to use normalized rather than absolute consumption and wealth. Agent i is a Scrooge McDuck if it accumulates unbounded *normalized* wealth, a_t^i, over time while keeping its *normalized* consumption, c_t^i, bounded. Finally, the definition of surplus wealth, still referring to wealth held without consumption motives, is modified to account for the fact that part of future consumption is financed by wages. Its normalized version is given by,

$$s_{t+1}^i \equiv a_{t+1}^i - \sum_{j=1}^{\infty} \frac{c_{t+j}^i - \zeta_t^i w_{t+j}}{\prod_{s=1}^{j} \widetilde{R}_{t+s}}. \tag{43}$$

From the goods and asset market-clearing conditions, it follows that Proposition 1 holds in the production economy, and thus that aggregate surplus wealth coincides with the aggregate bubble.

Market Clearing Conditions. The goods market clearing condition writes as,

$$k_{t+1}(1+g_{t+1}) + \sum_i \lambda^i c_t^i = (1-\delta)k_t + y_t. \tag{44}$$

The asset market clearing conditions for capital and for the rent-generating factor are given by:

$$k_t = \sum_i \lambda^i k_t^i, \tag{45}$$

$$1 = \sum_i \lambda^i L_t^i. \tag{46}$$

Equilibrium. Given the initial endowments $\{K_0^i, L_0^i\}$ for each agent $i \in \{1, 2, \dots, I\}$, an equilibrium $\left\{ \{c_t^i, k_t^i, L_t^i\}_{i\in\{1,2,\dots,I\}}, q_t \right\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems (40, 41) and the market clearing conditions for capital (45) and the rent-generating factor (46). The goods market clearing condition holds by Walras's Law.

Numerical Solution Method. Solving the model numerically raises three main challenges: computing the partial equilibrium with preferences for wealth over long transitions; handling bubbly transitions outside the neighborhood of the steady state; and solving for transitions that do not converge to a steady state but instead follow a divergent path. We propose original methods to tackle each of these issues, detailed in Appendix B.

3.2 Bubbly and Non-bubbly Equilibria

We solve the model numerically and find that bubbly equilibria also exist in the presence of reproducible capital. As the insatiable component of wealth preferences strengthens or the degree of labor income inequality rises, the set of parameters supporting a bubbly equilibrium expands, while that supporting a non-bubbly one shrinks.

A Numerical Illustration. We consider an illustrative exercise in which two transition paths are computed for low and high values of κ, yielding, respectively, a non-bubbly and a bubbly equilibrium. At this stage, we focus on the qualitative implications of the model, while a more complete quantitative analysis is presented in Section 5.

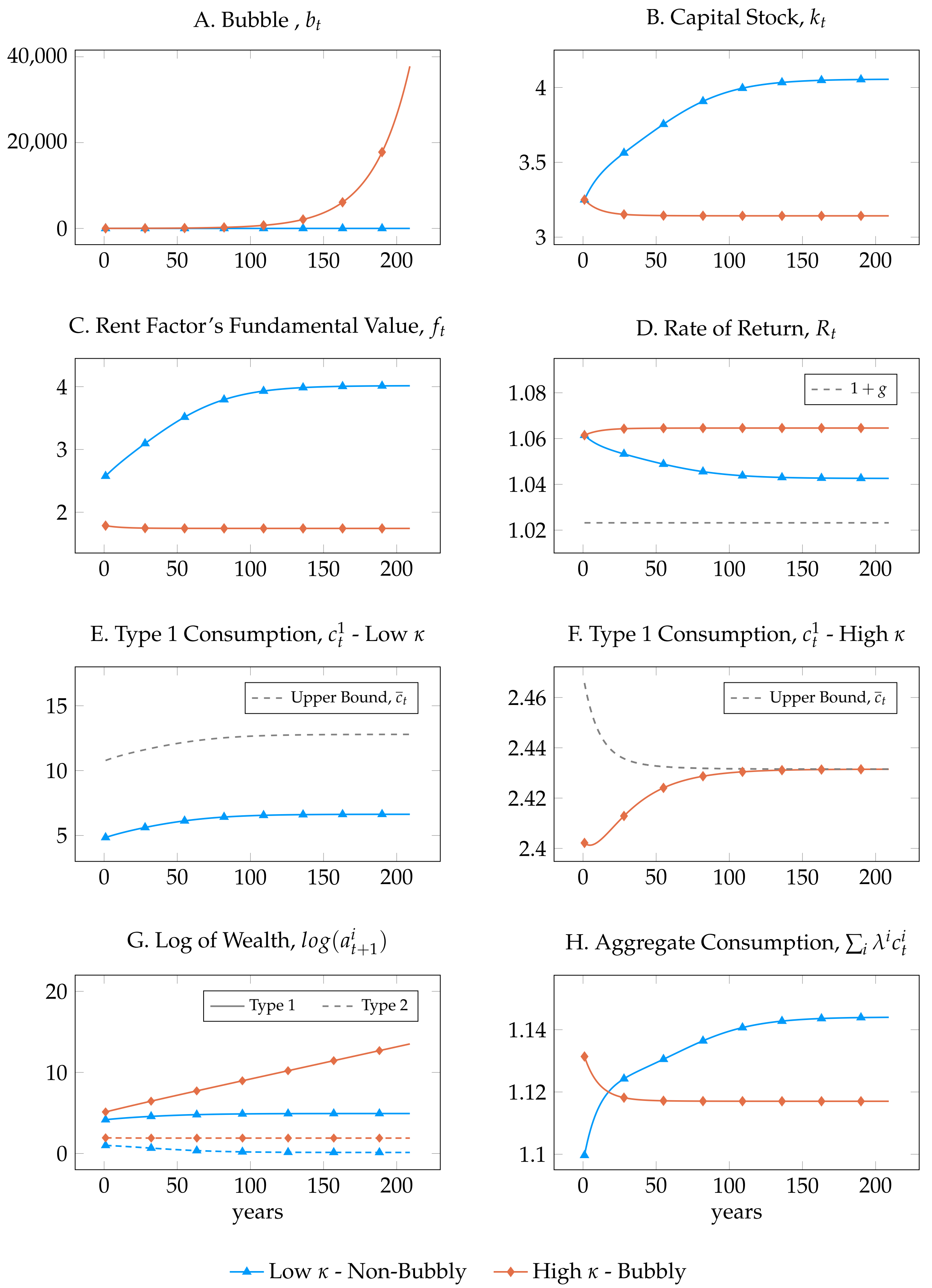
The population is divided into two types. Type 1 represents 5% of the population, $\lambda^1 = 0.05$, earns 25% of labor income in every period, $\zeta_t^1 = 5\ \forall t$, and holds 55% of the initial stock of capital and rent-generating factor endowments. Production parameters are set to $\gamma = 0.05$, $\alpha = 0.3$, $\delta = 0.07$, $g_t^Z = 1.5\%$, and $g_t^N = 1\%$ for all t. The discount factor is $\beta = 0.95$, and the consumption utility and standard preference-for-wealth parameters follow Kumhof et al. (2015): $\theta = 2$, $\eta = 0.92$, and $\psi = 0.05$. We consider two values for the insatiable preference-for-wealth parameter: a low value, $\kappa = 0.0002$, and a high value, $\kappa = 0.002$. Starting from an initial capital stock of $k_0 = 3.25$, the transition dynamics are shown in Figure 2.

When κ is high, the upper bound on optimal consumption binds for type 1 agents, which are Scrooge McDuck. From Equation 42, the upper bound on optimal consumption, $\bar{c}_t$, decreases with the rate of return and with the strength of the insatiable preference for wealth. As a result, in our example it binds asymptotically for type 1 agents exclusively in the high-κ case, where it is lower than under low κ.[9] Under high κ, type 1 agents' normalized consumption remains bounded, and their income persistently exceeds their consumption. It follows that their wealth diverges over time. They are Scrooge McDuck agents and hold positive surplus wealth, $s_t > 0$, for all t.

When strong enough to generate a bubbly equilibrium, insatiable preferences for wealth produce a diverging wealth-to-output ratio, while the capital-to-output ratio converges. Under high κ, the surplus wealth held by type 1 agents coincides in the aggregate with a rational bubble, $b_t = \lambda^1 s_{t+1}^1$. Since the bubble grows at the rate of return, which exceeds the economy's growth rate, its normalized size diverges. Asymptotically, this divergence in the bubble-to-output ratio raises the wealth-to-output ratio without affecting the capital-to-output ratio. The rational bubble is the key asset-pricing mechanism that disconnects wealth dynamics from those of capital. Bubbly equilibria thus align with the wealth and capital trends observed in many advanced economies in recent decades, motivating this paper.

[9] For simplicity, we do not consider cases in which the upper bound on optimal consumption bind for both types.

Figure 2: Transition Dynamics in the Production Economy: Bubbly vs. Non-Bubbly Equilibrium



Notes: The transition starts from the same initial capital stock, $k_0 = 3.25$, and asset distribution, with type 1 agents holding 55% of initial wealth, under either low ($\kappa = 0.0002$, blue line) or high ($\kappa = 0.002$, red line) insatiable preferences for wealth.

In contrast, under low insatiable preferences for wealth—of which standard preferences are a special case—the equilibrium is non-bubbly. In the low-κ case, the upper bound on optimal consumption is not binding for type 1 agents, who do not accumulate unbounded wealth. There are no Scrooge McDuck agents, no associated surplus wealth, and thus no rational bubble. Given our initial k_0, the wealth-to-output ratio increases over the transition but does not diverge. This rise is intrinsically linked to the increase in the capital-to-output ratio. The latter raises the wealth-to-output ratio both directly, and indirectly by increasing the fundamental value of the rent-generating factor through a lower rate of return, R_t, and higher associated rents.

The presence of a rational bubble crowds out capital. As the capital stock comparison shows, despite a stronger preference for wealth, the high-κ case features a lower capital stock. This occurs because, under high κ, part of agents' savings goes into the bubble, but not under low κ. In turn, lower capital stock levels imply lower output and higher rates of return. It follows that type 2 agents may receive higher asymptotic income in either the low-κ non-bubbly case or the high-κ bubbly case, depending on the relative strength of two channels. On the one hand, they earn lower wages in the high-κ case; on the other hand, their wealth income benefits from the capital gains associated with the rational bubble. The parametrization determines which effect dominates and, consequently, whether type 2 agents consume asymptotically more in the low- or high-κ case.

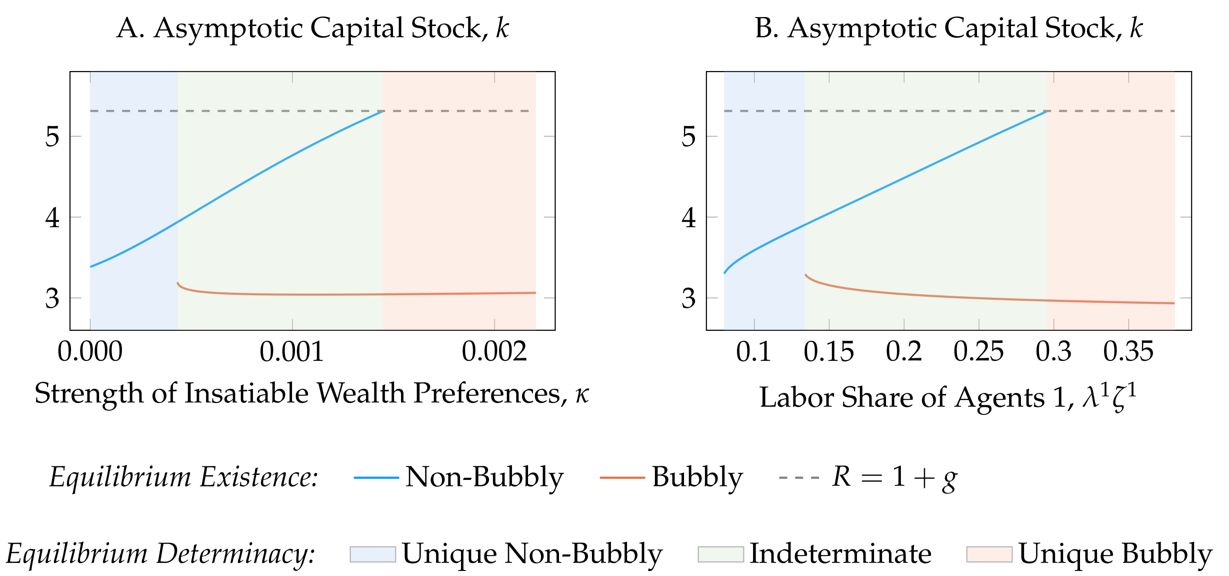
The bubble's crowding-out of capital reduces asymptotic aggregate consumption. A well-established result implies that, whenever $R > 1 + g$, an increase in the capital stock raises asymptotic consumption, whereas it lowers it when $R < 1 + g$.[10] By the assumption of a productive rent-generating factor, we are necessarily in the case where the rate of return exceeds the growth rate, and the bubble's crowding-out reduces asymptotic consumption. This can be seen in Figure 2, where aggregate consumption is asymptotically higher in the non-bubbly, low-κ scenario than in the bubbly, high-κ scenario. However, during the transition to this asymptotic state, aggregate consumption is initially higher under high κ, as fewer resources are allocated to capital investment. Notice that neither of the two equilibria considered reaches the capital level that maximizes asymptotic consumption and would imply $R = 1 + g$.

Comparative Statics. We now turn to comparative statics to determine the levels of insatiable wealth preference strength and labor income inequality under which non-bubbly and bubbly equilibria exist, along with the corresponding asymptotic capital stock.

When the insatiable component of wealth preferences κ is sufficiently low, the economy admits a non-bubbly equilibrium; when it is sufficiently high, a bubbly equilibrium exists. The left panel of Figure 3 shows the different equilibria and their associated capital levels as a function of κ. At $\kappa = 0$, which corresponds to standard preferences for wealth, the equilibrium is necessarily unique and non-bubbly. More generally, there exist two threshold values of κ: below the lower threshold, the equilibrium is unique and non-bubbly (blue zone), while above the upper threshold, it is unique and bubbly (red zone). In the intermediate range, one bubbly and one non-bubbly

[10] This result traces back to Diamond (1965). Asymptotically, a one-unit increase in the capital stock raises output by $\alpha k^{\alpha-1}$ and investment by $\delta + g$ in normalized terms. If $\alpha k^{\alpha-1} > \delta + g$, it raises aggregate consumption, a condition that can be rewritten as $R > 1 + g$.

Figure 3: Capital Accumulation by Strength of Insatiable Wealth Preferences and Labor Inequality



Notes: This figure shows the existence and type of equilibria, along with the corresponding asymptotic capital stock k, under alternative parameter values for (Panel A) the strength of insatiable wealth preferences, κ, and (Panel B) the labor share of agents 1, $\lambda^1\zeta^1$. Non-bubbly and bubbly equilibria are represented by the blue and red lines, respectively. The dashed line indicates the capital level at which the rate of return equals the growth rate. Shaded areas indicate equilibrium regimes: regions with a unique non-bubbly equilibrium (blue), a unique bubbly equilibrium (red), or coexistence of both equilibria with indeterminate selection (slate green).

equilibrium coexist (green zone). As in the endowment economy, equilibrium selection is in that case self-fulfilling: if agents expect high returns, the upper bound on consumption is low and binding, leading to a bubble that sustains high returns; if they expect low returns, the opposite holds.

The asymptotic capital stock rises with stronger insatiable preferences for wealth, as long as the equilibrium does not turn bubbly. A stronger preference for wealth strengthens saving incentives and, consequently, aggregate savings. In the absence of a bubble, this inevitably translates into a larger capital stock.[11] The non-bubbly equilibrium disappears once κ exceeds the threshold at which further capital accumulation would drive the asymptotic return below the growth rate. This is indeed precluded by the presence of a rent-generating factor of production, as discussed in Subsection 3.1. Furthermore, any marginal increase in κ that shifts the economy from a non-bubbly to a bubbly equilibrium necessarily lowers the asymptotic capital stock, reflecting the crowding-out effect of the bubble.

The comparative statics with respect to labor income inequality yield results analogous to those obtained for κ. In this simplified two-agent framework, labor income inequality is captured entirely by the labor share of type-1 agents. The right panel of Figure 3 shows how the equilibria and their associated capital levels depend on this measure. When labor inequality is low, type-1 agents earn insufficient income to hit the asymptotic upper bound on consumption, which therefore never

[11] A similar result is obtained when the taste for wealth is increased through the standard wealth-preference component of utility (for instance, a higher value of ψ) rather than through the insatiable one.

binds. In this case, the equilibrium is unique and non-bubbly (blue zone). Conversely, when labor inequality exceeds a given threshold, the upper bound always binds and the equilibrium is unique and bubbly (red zone). In the intermediate range, a bubbly and a non-bubbly equilibrium coexist with self-fulfilling equilibrium selection (green zone). Finally, increasing labor inequality raises aggregate savings by redistributing income from low-saving to high-saving agents. This translates into a higher capital stock, conditional on the equilibrium being non-bubbly.

This comparative static exercise suggests that the widely observed capital-gains-driven increase in the wealth-to-output ratio could stem from the concomitant rise in top income inequality. This is illustrated in Figure 3, where rising labor income inequality may translate into the existence, or even the necessity, of a bubbly equilibrium. It is important to note that the mechanism at play is not one in which a one-time increase in income inequality leads to a one-time rise in wealth through valuation effects. Rather, higher inequality shifts the economy from a non-bubbly regime to a bubbly one, generating persistent capital gains. In our model, when unrealized capital gains are excluded from the definition of income, income inequality converges in bubbly equilibria while the wealth-to-output ratio diverges. This could rationalize why the wealth-to-output ratio has continued to rise in many economies where income inequality appears to have converged, under a definition of income that excludes unrealized capital gains (Figures 1A. and 1B.).

3.3 Discussion

The model generates two unusual outcomes within a neoclassical framework—a non-zero mass of agents holding diverging wealth and a diverging rational bubble—and, in doing so, provides insights into several strands of the literature.

Return on Wealth. Although the safe rate of return has declined steadily since the 1980s, the evolution of the average return on wealth remains considerably less clear. Several studies examining risk premia or returns on capital argue that the ex-ante expected return on wealth has remained broadly stable, or has declined only slightly (Duarte and Rosa, 2015; Caballero et al., 2017; Reis, 2022). However, under the assumption that assets are priced at their fundamental value, this pattern is difficult to reconcile with evidence of increasing price-to-earning ratios (Kuvshinov and Zimmermann, 2021; Eggertsson et al., 2021; Greenwald et al., 2025). These ratios suggest that the average rate of return has moved in line with the safe rate and declined over time.

The rational bubble developed in this model provides a coherent explanation for the seemingly conflicting evidence regarding the average return on wealth. The bubble is a non-dominated asset because its absolute value rises by R_t between periods t and $t+1$. Hence, it generates new capital gains in every period. On the one hand, these capital gains raise the average return on wealth above the ratio of earnings from wealth to total wealth. This is consistent with evidence that the average return on wealth has remained relatively stable. On the other hand, because the bubble grows at a rate exceeding that of the economy, it increases the ratio of wealth to earnings from wealth over time. This aligns with the observed rise in price-to-earnings ratios.

A Unified Framework for Rising Wealth-to-Output Ratios. Another appealing feature of the framework is that it offers a unified explanation for the widely observed rise in the wealth-to-output ratio. Large capital gains represent a robust stylized fact across advanced economies, yet the types of assets benefiting from them vary substantially across regions. The latter aspect has led some lines of research to develop country-specific explanations for a global phenomenon. For instance, in the U.S., the rise in stock values is often attributed to increasing market power (Farhi and Gourio, 2018; Eggertsson et al., 2021), while the low elasticity of housing supply is frequently cited to explain the rise in real estate prices in Europe (Hilber and Vermeulen, 2016; Muellbauer, 2018). While these mechanisms account for part of the observed dynamics, the Scrooge McDuck theory links the widespread capital gains to an equally widespread and simultaneous rise in top income inequality. Furthermore, it does so through a rational bubble that can arise in a wide range of real-world assets.

Capital Gains. The substantial capital gains observed since the 1980s have given rise to a growing debate on their welfare implications. This debate has mostly centered on asset price increases driven by a once-and-for-all decline in the discount rate, which raises the fundamental value of assets. Such capital gains mechanically increase absolute wealth inequality without affecting the distribution of income from wealth. These valuation effects therefore cannot be used to sustainably finance consumption. They have been interpreted in various ways: as a pure increase in welfare inequality (Saez et al., 2021), as mere "paper gains" with no welfare effect (Cochrane, 2020; Krugman, 2021), or as a mix of both, depending on net asset sales (Fagereng et al., 2024).

In contrast, this paper examines the consumption implications of the recurrent capital gains generated by a rational bubble. As new capital gains emerge each period, non-Scrooge McDuck agents can draw on them to finance a fraction of their consumption. This result is especially relevant given empirical evidence showing that a large fraction of the population finances consumption through capital gains. In the United States, Mian et al. (2020) show that capital gains realized by the bottom 90 percent of the wealth distribution have been used to a large extent to finance consumption through increased collateralized debt. This study suggests that in a frictionless bubbly world, collateralized debt could continue to rise without bound. In practice, this continued rise could ultimately be halted by a binding financial constraint, the analysis of which lies beyond the scope of this paper.

Relation to Piketty's *Capital in the Twenty-First Century*. The present analysis also contributes to the discussion initiated by Piketty (2014) on how the gap between the rate of return, r, and the growth rate, g, influences the dynamics of wealth inequality. A key conjecture of Piketty (2014) posits that "if the rate of return on capital remains significantly above the growth rate for an extended period, the risk of divergence in the distribution of wealth is very high" (p. 34). Addressing this conjecture in a model with standard preferences for wealth, Michau et al. (2025) refutes the possibility of r remaining asymptotically above g when wealth inequality is diverging. As wealth at the top of the distribution diverges, aggregate savings should rise, resulting in greater capital accumulation. The corresponding decline in the rate of return drives it asymptotically toward the growth rate.

In contrast, this paper develops the first microfounded general equilibrium model in which the rate of return remains asymptotically above the growth rate and wealth inequality diverges. The divergence of wealth for a non-zero fraction of agents implies that aggregate wealth diverges. Under fundamental valuation, this would require the discount rate to approach the growth rate. The presence of a rational bubble growing at a rate higher than that of the economy breaks this result and allows the model to align with Piketty (2014). This suggests that the longer an economy experiences rising wealth inequality alongside a rate of return significantly above the growth rate, the more likely it is to be on a bubbly path.[12] This distinction is crucial, since, as shown above, the dynamics of capital accumulation vary sharply between bubbly and non-bubbly equilibria. It also carries significant implications for taxation, as discussed in the next section.

4 Tax Policy Implications

Whether the economy is in a bubbly or non-bubbly equilibrium has important implications for taxation. The introduction of any wealth tax—and, in some cases, capital income taxes—in a bubbly equilibrium shifts the economy to a non-bubbly one. This can foster capital accumulation by preventing the crowding-out associated with a bubble.

4.1 Capital Tax Instruments: Income vs. Wealth Taxation

We introduce capital income and wealth taxation into the previous model. The standard equivalence between the two breaks down in the presence of a bubble, which enters the tax base of the wealth tax but not that of the capital income tax.

Capital Income Tax. Let us first define the capital income tax. In many sectors, the tax authority typically does not (and cannot) distinguish between rent and capital income, taxing them identically under the so-called capital income tax. Following this terminology, we define the capital income tax as one that applies to both capital and rent income. The net-of-depreciation income paid to capital and to the rent-generating factor is taxed at a fixed rate τ^r. The post-capital income tax rate of return from period t to $t+1$ on capital is denoted by $R_{t+1}^{\text{post-CIT}}$:

$$R_{t+1}^{\text{post-CIT}} = 1 + (1-\tau^r)(\alpha k_{t+1}^{\alpha-1} - \delta). \tag{47}$$

By the no-arbitrage condition, the post-capital income tax rate of return on the rent-generating factor is also equal to $R_{t+1}^{\text{post-CIT}}$:

$$R_{t+1}^{\text{post-CIT}} = (1+g_{t+1})\frac{q_{t+1} + (1-\tau^r)\gamma y_t}{q_t}. \tag{48}$$

[12]This interpretation is only suggestive: the divergence of aggregate wealth or inequality is inherently untestable, given that the asymptote is never observed.

The tax revenues from the capital income tax are given by T_t^r:

$$T_t^r = \tau^r \left[(\alpha k_t^\alpha - \delta k_t) + \gamma k_t^\alpha \right]. \tag{49}$$

Wealth Tax. We introduce a wealth tax in addition to the capital income tax. Before the production process, a fraction τ^a of each agent's wealth is collected. We define $R_{t+1}^{\text{after-tax}}$ as the rate of return between periods t and $t+1$ net of both the wealth and capital income taxes:

$$R_{t+1}^{\text{after-tax}} = (1 - \tau^a) R_{t+1}^{\text{post-CIT}}. \tag{50}$$

We denote wealth tax revenues by T_t^a:

$$T_t^a = \tau^a R_t^{\text{post-CIT}} \sum_i \lambda^i a_t^i. \tag{51}$$

Revenues from both the capital income tax and the wealth tax are assumed to be redistributed equally across households through lump-sum transfers. The budget constraint of households of type i then becomes:

$$c_t^i + a_{t+1}^i (1 + g_{t+1}) = R_t^{\text{after-tax}} a_t^i + \zeta_t^i w_t + T_t^r + T_t^a. \tag{52}$$

(Non-)Equivalence Between Wealth and Capital Income Tax. When rates of return are homogeneous, capital income and wealth taxation are generally equivalent. A wealth tax at rate τ^a collects τ^a units of the consumption good for each unit of wealth. By contrast, a capital income tax at rate τ^r on the net-of-depreciation return yields revenues of $\tau^r(R-1)$ per unit of wealth. For any wealth tax, there thus exists an equivalent capital income tax such that $\tau^r = \tau^a/(R-1)$. This equivalence and its breakdown under heterogeneous returns are discussed extensively in Guvenen et al. (2023). With heterogeneous returns, a wealth tax penalizes low-return agents more than a capital income tax, while the reverse holds for high-return agents.

In fact, the equivalence requires a condition more restrictive than mere return homogeneity: at each point in time, the ratio of income paid to price must be uniform across assets. Two assets priced at their fundamental value can have the same value and rate of return, yet differ in the timing of their income payments. In that case, they are taxed the same under a wealth tax but differently under a capital income tax. This is what happens here when comparing capital and rent-generating factor taxation in a non-bubbly equilibrium. The no-arbitrage condition (30) ensures homogeneous returns, but does not necessarily imply that the following equality holds:

$$\frac{\gamma y_t}{f_{t-1}/(1+g_{t-1})} = \frac{\alpha y_t - \delta k_t}{k_t}. \tag{53}$$

Equation 53 illustrates the case in which the ratio of taxable income to taxable wealth is equalized across both assets. In the presence of capital gains on the rent-generating factor ($f_t > f_{t-1}$), this equality no longer holds, and the income-to-value ratio at time t is lower for the rent-generating factor.[13]

This mechanism can generate persistent differences in the effective taxation of capital and the rent-generating factor. As the economy grows asymptotically at rate g_t, so do rents and the fundamental value of the rent-generating factor. These capital gains on f_t are taxed under a wealth tax but not under a capital income tax. Taxing wealth amounts to taxing the rent-generating factor's fundamental value slightly more heavily than capital, compared with taxing capital income. This asymmetry is asymptotically proportional to the growth rate of the economy: the higher g, the greater the asymmetry. This consideration is, however, of secondary importance for the purposes of this paper.

More importantly for the present analysis, the rational bubble enters the tax base of the wealth tax but not that of the capital income tax. The rational bubble does not enter the production process and pays no dividends. Its taxable income-to-value ratio equals zero, reflecting the fact that it is held solely for its expected appreciation. It is therefore not subject to the capital income tax and is included in the wealth tax base only. This distinction is central to why wealth and capital income taxes have different implications for capital accumulation and the equilibrium type in this model, which will be examined in the next subsection. Before turning to this analysis, it is worth noting that the requirement of homogeneous taxable income-to-value ratios for the equivalence between capital income and wealth taxation extends beyond the specific context of insatiable preferences for wealth. Its generalization and detailed analysis are outside the scope of this paper and are left for future research.

4.2 Capital Accumulation and Tax Policy

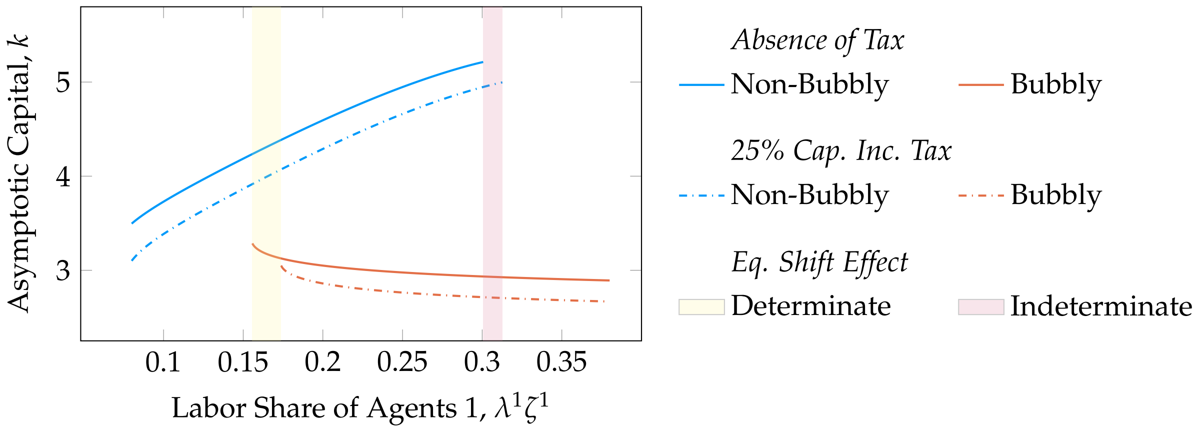
The effect of introducing a tax on capital accumulation is ambiguous. Two mechanisms are at play. While the standard disincentive effect of taxation reduces the capital stock, its *equilibrium shift effect*—from bubbly to non-bubbly—operates in the opposite direction when present.

Asymptotic Capital under a Capital Income Tax. We first examine the effect of introducing a capital income tax on capital accumulation. We focus on the asymptotic level of capital as a summary measure of the economy's long-run intensity of capital accumulation, abstracting from transition dynamics. We replicate the comparative exercise from Figure 3, extending it to include a capital income tax. Figure 4 displays the asymptotic capital stock as a function of labor income inequality, contrasting the no-tax scenario (solid lines) with a 25% capital income tax (dash-dotted lines, , $\tau^r = 0.25$). In both cases, the wealth tax rate remains set to zero.

Introducing a capital income tax shifts the curves representing the existence of non-bubbly and bubbly equilibria both downward and to the right. These two movements reflect the channels through which taxation influences the capital stock, with their magnitude increasing as the tax

[13] One way to restore the equivalence between the two taxes would be to include capital gains, both realized and unrealized ones, as part of capital income. However, since capital gains tend to be taxed only upon realization and often at different rates than capital income, this non-equivalence is likely to be important in practice. Accordingly, our definition of the capital income tax excludes capital gains from its tax base.

Figure 4: Equilibria with and without a 25% Capital Income Tax



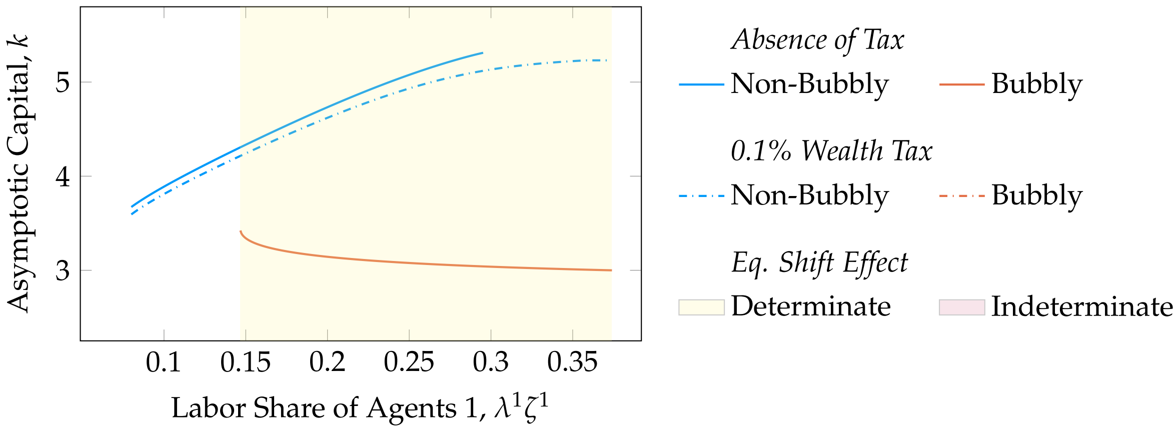
Notes: This figure shows the equilibrium type and corresponding asymptotic capital stock k as the labor share of agents 1, $\lambda^1\zeta^1$, varies, comparing a baseline without taxation to a scenario with a 25% capital income tax. Blue and red lines represent the non-bubbly and bubbly equilibria, respectively; dash-dotted lines correspond to the taxed case. Shaded areas indicate levels of labor income inequality at which introducing a 25% capital income tax in the bubbly no-tax equilibrium would necessarily (yellow) or possibly, depending on equilibrium selection (purple), shift the economy to a non-bubbly equilibrium.

rate rises. The first channel is the standard disincentive effect. It is reflected in the downward shift of the dash-dotted lines relative to the solid ones. For a given level of inequality and a given equilibrium type, the capital stock is lower under a capital income tax. Taxation makes saving less attractive by lowering the post-tax rate of return. As a result, agents save less, which translates into a lower asymptotic capital stock.

The second channel is the equilibrium shift effect: a capital income tax can shift the equilibrium from bubbly to non-bubbly. The tax being progressive, it reduces income at the top of the distribution, and relaxes the upper-bound constraint on optimal consumption. It follows that the introduction of a tax increases the range of inequality levels for which a non-bubbly equilibrium exists and reduces the range for which a bubbly equilibrium does not exist. Graphically, this effect is reflected in the rightward shift of the curves. For a given level of taxation and labor inequality, the capital stock is always higher in the absence of a bubble, and the equilibrium shift effect has a positive impact on capital accumulation. In its presence, the impact of taxation on the capital stock is ambiguous and depends on the relative magnitude of the disincentive and equilibrium shift effects.

The equilibrium shift effect of a capital income tax operates only within a limited range of labor inequality levels. The levels of labor inequality, where the introduction of a 25% capital income tax can shift the equilibrium from bubbly to non-bubbly corresponds to the shaded area of Figure 4. We distinguish between two subcases of these zones. The yellow-shaded area corresponds to levels of inequality for which two equilibria exist in the absence of taxation, but only one non-bubbly equilibrium remains when a 25% capital income tax is introduced. In this zone, conditional on the economy initially being in a bubbly equilibrium, the equilibrium shift effect is determinate: the economy cannot remain in a bubbly state, as no bubbly equilibrium exists anymore. In contrast, the purple-shaded region identifies inequality levels with a single bubbly equilibrium in the absence

Figure 5: Equilibria with and without a 0.1% Wealth Tax



Notes: This figure shows the equilibrium type and corresponding asymptotic capital stock k as the labor share of agents 1, $\lambda^1\zeta^1$, varies, comparing a baseline without taxation to a scenario with a 0.1% wealth tax. Blue and red lines represent the non-bubbly and bubbly equilibria, respectively, while dashdotted lines correspond to the taxed case. Yellow-shaded areas indicate levels of labor income inequality at which introducing a 0.1% wealth tax in the bubbly no-tax equilibrium would necessarily shift the economy to a non-bubbly equilibrium.

of taxation, but two equilibria under a 25% capital income tax. In this zone, conditional on the economy initially being in a bubbly equilibrium, the equilibrium shift effect is indeterminate: the economy may either remain in the bubbly state or transition to the non-bubbly one.

Asymptotic Capital under a Wealth Tax. We now turn to the analysis of the effects of wealth taxation on asymptotic capital. To do so, we once again perform a comparative statics exercise with and without taxation. Figure 5 displays the asymptotic capital stock as a function of labor income inequality, contrasting the no-tax case with a 0.1% wealth tax. This relatively small rate is chosen for reasons that will become clear in the analysis. We keep the same legend styles as in Figure 4 to indicate bubbly and non-bubbly equilibria, as well as the inequality levels for which the equilibrium shift effect of the tax is determinate or indeterminate.

Figure 5 reveals a striking pattern: the wealth tax rules out any bubbly equilibrium. This is evident from the yellow-shaded area, which represents the range where the tax necessarily shifts a bubbly equilibrium to a non-bubbly one. It fully overlaps with the set of values for which a bubbly equilibrium exists in the absence of taxation. This result holds for any positive wealth tax rate. The value of τ^a is set to 0.1% to demonstrate that the result remains valid even for very small taxes. In this framework, a bubbly equilibrium generates a bubble whose size diverges relative to output. Taxing a fixed fraction of it, τ^a, would asymptotically yield diverging tax revenues, redistributed equally across households. Since not all agents are Scrooge McDuck—this is ruled out by assumption in the parametrization—this redistribution would raise asymptotic aggregate consumption above feasible levels, thereby violating the goods market clearing condition.

In the presence of a wealth tax, higher labor inequality always translate into a higher asymptotic capital stock. Conditional on remaining in a non-bubbly equilibrium, increasing labor inequality raises aggregate savings and the capital stock by redistributing income from low-saving to high-saving agents. In the absence of taxation, there are levels of inequality for which the capital stock would need to satisfy $R < 1 + g$ in order to absorb all savings in a non-bubbly equilibrium. This is precluded by the presence of the rent-generating factor. However, when R approaches $1 + g$ in the presence of a wealth tax, tax revenues become arbitrarily large, since so does f_t. This redistributes income from the high-saving type 1 agents t o the type 2 agents, thereby lowering aggregate savings. This mechanism explains why, under a wealth tax, a non-bubbly equilibrium exists for any level of labor income inequality.[14]

In a bubbly equilibrium, one can always find a sufficiently low wealth tax rate that, if implemented, raises the capital stock. We have seen that, regardless of the tax rate, a wealth tax always generates an equilibrium shift effect that increases the capital stock. Moreover, much like the capital income tax, the wealth tax entails a disincentive effect. Graphically, for any given level of inequality, the non-bubbly equilibrium features a higher capital stock in the absence of taxation than under a wealth tax. This disincentive effect is proportional to the tax rate. A small τ^a, such as the one considered in Figure 5, generates only a minor disincentive effect, which explains why the curves with and without taxation are close to each other. It follows that, in a bubbly equilibrium, there always exists a range of sufficiently low wealth tax rates for which the disincentive effect is dominated by the rate-invariant equilibrium shift effect.

5 Quantitative Assessment

Thus far, we have focused on the qualitative features of a framework with insatiable preferences for wealth. We now turn to assessing the quantitative relevance of the Scrooge McDuck mechanism. The production economy is calibrated to the U.S. economy from 1989 onward. Despite being highly stylized, it reproduces the untargeted dynamics of aggregate wealth and wealth inequality. It serves as a benchmark for the counterfactual analysis of a 1.5 percent wealth tax.

5.1 Overview of the Exercise and Calibration

The economy is assumed to start in a non-bubbly steady state calibrated to match the economic conditions of 1989. This steady state serves as the basis for identifying the values of the internally calibrated parameters. The latter are chosen to match the wealth-to-output ratio, capital-to-output ratio, and the distribution of wealth. The economy is then subjected to an exogenous increase in labor income inequality, calibrated to match its empirical counterpart. After the shock, the equilibrium can either shift to a bubbly state or remain non-bubbly. The model remains deliberately stylized and features four agent types corresponding to the top 1% $[99, 100]$, the top next 9% $[90, 99)$, the middle 40% $[50, 90)$, and the bottom 50% $[0, 50)$ percentiles of the wealth distribution.

[14] Note that a bequest tax and a wealth tax are equivalent in the context of a model with infinitely-lived agents. This could shed light on a country that appears to have followed a non-diverging path since the early 1990s: Japan. Japan exhibits stable wealth inequality, a growing capital-to-output ratio, and a declining wealth-to-output ratio since the burst of the Japanese asset price bubble. At the same time, it has a significant and highly progressive bequest tax, which accounts for more than 1% of total tax revenues (OECD, 2021) while taxing less than 9% of descendants (National Tax Agency, 2022).

Table 1: Baseline Calibration

Parameters	Description	Value	Source
Production			
γ	Rent factor share	0.035	Match Initial W/Y ratio
α	Capital factor share	0.325	Labor Share of 0.64
δ	Capital depreciation rate	0.075	Standard
ζ_t^i	Pre-tax labor income share		SCF
Growth Rate	Averages for 1989-2004 and 2004-2023		
g_t^N	Labor Input growth rate	1.3% - 1.2%	Bureau of Labor Statistics (BLS)
g_t^Z	Labor productivity growth rate	1.8% - 1.0%	Calculated from the TFP estimate of the BLS
Taxation			
τ^r	Capital income tax rate	0.25	McDaniel (2007)
τ^w	Labor tax rate	0.225	McDaniel (2007)
τ^a	Wealth tax rate	0.0	
Preferences			
β	Discount rate	0.9	Mian et al. (2021)
θ	CRRA coefficient for consumption	2	Standard
ψ	Weight on taste for wealth	0.76	Joint Calibration
η	CRRA coefficient for wealth	1.43	Joint Calibration
$\underline{a}$	Stone-Geary parameter for wealth	-2.46	Joint Calibration
κ	Linear coefficient in the utility from wealth	0.0008	Joint Calibration

Notes: The internally calibrated preference parameters are jointly selected to match the wealth-to-output ratio, the capital-to-output ratio, and the wealth shares observed in 1989, assuming the economy was in a non-bubbly steady state at that time. The rent-generating factor share γ is chosen such that, in a steady state where the capital-to-output ratio equals 2.16 (its 1989 value), total wealth equals 3.16 (its 1989 value). To focus on long-term dynamics rather than business-cycle fluctuations, TFP and labor input growth are smoothed over time. In the initial steady state and from 1989 to 2004, it is set to its 1989-2004 average; from 2004 onward, it is set to its 2004-2023 average. This 2004 break accounts for the growth slowdown observed in the United States over the period considered.

By construction, income and wealth rankings are perfectly aligned across agent types. The levels of effective labor supply are selected to reflect this alignment. The parameter ζ_t^i measures the effective labor of type i relative to the population average at time t. It is calibrated by dividing the population into groups based on their position in the *wealth distribution* and measuring the corresponding labor income of each group. This is done using the Survey of Consumer Finances, which reports both individual income and wealth. As the survey begins in its current form in 1989, this year is chosen as the starting point for the analysis. Because the model abstracts from life-cycle considerations, we calibrate it using data for households aged 35 to 54. Since only pre-tax income is observed, the post-tax effective labor income is proxied using the transformation

$$\zeta_t^i = (1 - \tau^w)\zeta_t^{i,pre} + \tau^w, \tag{54}$$

where $\zeta_t^{i,pre}$ denotes non-tax-adjusted relative effective labor supply. This corresponds to a linear labor income tax at rate τ^w, rebated lump-sum.

Of the externally calibrated parameters, several warrant particular attention. The first is the discount factor, set to a relatively low value of $\beta = 0.9$. This calibration is consistent with the preference-for-wealth literature, where agents exhibit an additional saving motive beyond intertemporal consumption smoothing. In equilibrium, our parameterization will imply a rate of return of about 5 percent, which is the typical targeted moment for choosing a discount factor around 0.95 in absence of preferences for wealth. Second, following Guvenen et al. (2023), we calibrate the capital and labor income tax rates using the estimates reported by McDaniel (2007). The analysis abstracts from wealth or bequest taxation, which are shut down in the baseline. Finally, to account for the growth slowdown observed over the sample period, we set the growth rates of labor-augmenting productivity, g_t^N, and TFP, g_t^Z, equal to their 1989-2004 averages from 1989 to 2004, and to their 2004-2023 averages thereafter. Although this adjustment represents an additional shock to the initial steady state, it is neither necessary nor sufficient to drive the economy from a non-bubbly to a bubbly equilibrium.

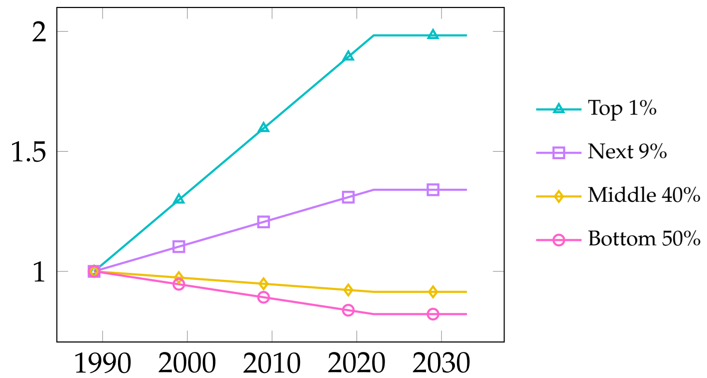
The internal calibration proceeds in two steps. First, the factor shares of rents and capital, γ and α, are chosen so that a non-bubbly steady state matching the 1989 wealth-to-output ratio also reproduces the 1989 capital-to-output ratio given a standard labor share of 64%. The rent share parameter is set to $\gamma = 3.5\%$, a relatively low value given that land rents alone are generally estimated to account for 5-10 percent of output. This modest value likely reflects capital mismeasurement, similar to the mechanisms that have historically driven the macro Tobin's q below one (see Eggertsson et al., 2021). The final calibration step jointly determines the preference parameters ψ, η, $\underline{a}$, and κ so as to match the 1989 wealth-to-output ratio and the corresponding wealth distribution. All the calibration details are summarized in Table 1

The fit of all targeted moments is summarized in Table 2. The model performs well overall, except for the wealth shares of the next 9 percent and the middle 40 percent. Owing to the perfect alignment between income and wealth distributions, this stylized framework cannot simultaneously match both groups' empirical shares: it underestimates the wealth share of the next 9 percent by about 9 percentage points and overestimates that of the middle 40 percent by a similar magnitude.

Table 2: Calibration Targets: Initial Steady-State vs. Data (1989)

Moments	Data	Model
Wealth-to-output	3.16	3.12
Capital-to-output	2.16	2.15
Top 1% Wealth Share	27%	29%
Next 9% Wealth Share	35%	26%
Middle 40% Wealth Share	33 %	42%
Bottom 50% Wealth Share	5 %	3%

Figure 6: Labor Inequality Shock: Dynamics of ζ_t^i (1989 = 1)



Notes: This figure depicts the exogenous shock to ζ_t^i. Effective labor supply is computed from pre-tax labor income data across the wealth distribution, using the Survey of Consumer Finances and adjusted for a 22.5% linear labor income tax as in equation 54.

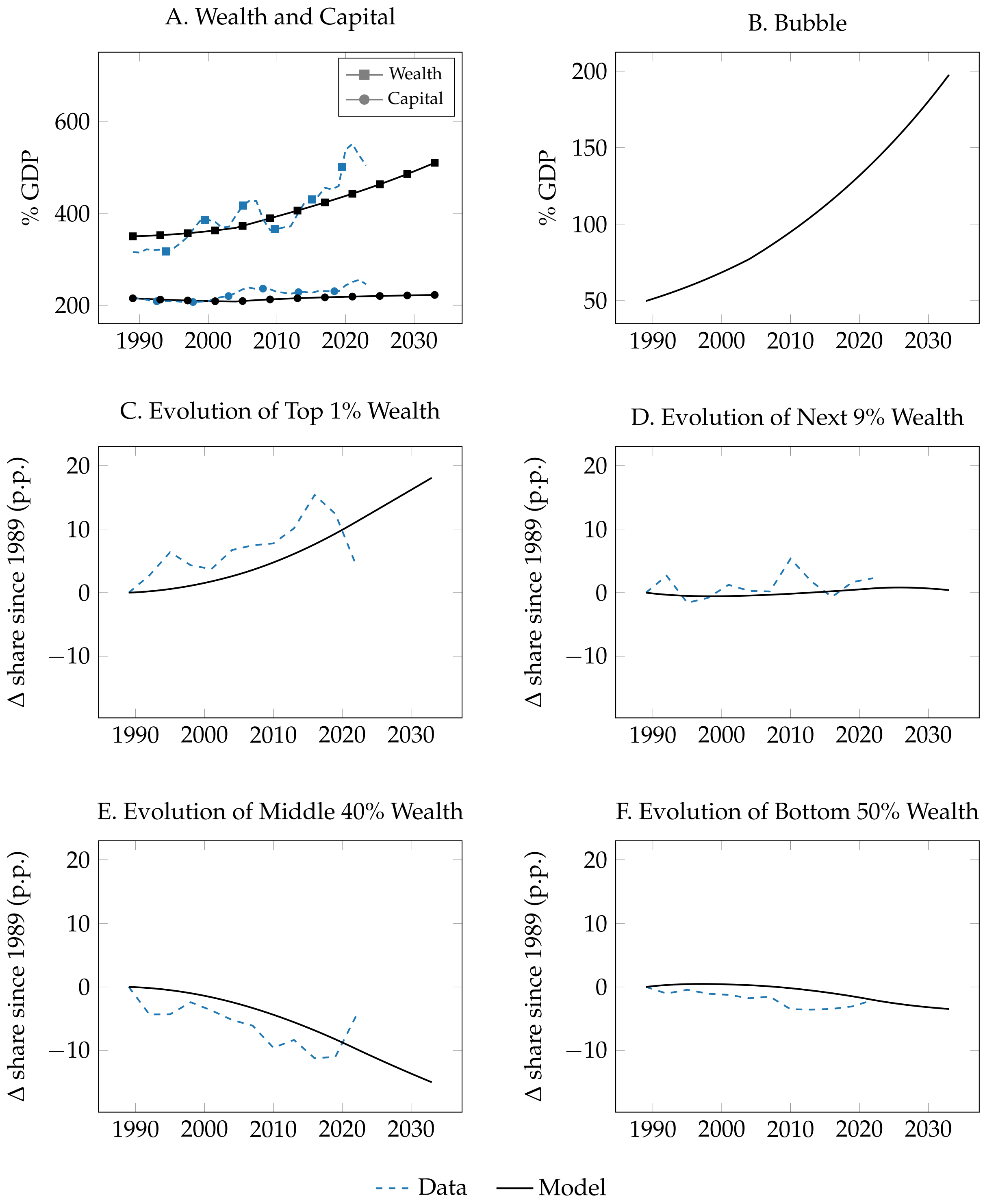
This 1989 steady state is subsequently shocked by the rise in effective labor supply inequality illustrated in Figure 6. We assume a smooth transition from 1989 to 2022 between the ζ_t^i values of 1989, computed using Equation 54, and their 2022 counterparts. The change in ζ_t^i, computed from the data, amplifies inequality: the higher an agent's position in the wealth distribution, the larger the increase (or the smaller the decrease) in its effective labor supply.

5.2 Results

Figure 7 displays the model's predicted adjustment following the labor income shock. Despite being calibrated to match the non-bubbly steady state, the model endogenously switches to a bubbly equilibrium after the shock. The transition path is uniquely determined, and no equilibrium-selection assumption underlies these results.

The model predicts an increase in wealth-to-output ratio driven by a rational bubble. Since agents have perfect foresight and there is no uncertainty, a bubbly equilibrium requires the bubble to be present from the first period onward. The bubble therefore jumps, by assumption, from its steady-state value of zero to a positive value in 1989. This accounts for the model's predicted wealth-to-output ratio starting above its 1989 value. Designed to capture long-run dynamics, the model consistently underestimates wealth during booms and overestimates it during busts. Overall, the model replicates reasonably well both the speed and the magnitude of the increase

Figure 7: Transition Dynamics in the Baseline Scenario



Notes: Model's transition starts from the 1989 non-bubbly steady state. It departs from this steady state and evolves along a bubbly path as the economy experiences an increase in effective labor supply inequality, depicted in Figure 6. In addition, the growth rates of labor productivity and labor input exhibit a slowdown after 2004, as reported in Table 1. The transition path is uniquely determined. Data on wealth are obtained from the World Inequality Database, capital stock data from the Bureau of Economic Analysis, and wealth share data from the Survey of Consumer Finances (conducted every three years).

in the wealth-to-output ratio observed in the data. In the simulated economy, this rise is largely driven by the rational bubble, which reaches about 150 percent of GDP by 2024.

The growth slowdown accelerates slightly the normalized bubble and capital growth. The growth rate of $(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}$, denoted g_t, closely tracks that of the overall economy and equals 3 percent before 2004 and 2.1 percent thereafter. This break in trend makes it possible to visualize the effects of slower growth on the model's dynamics. The normalized bubble then evolves at rate $R_t - (1 + g_t)$, with the return on capital, $R_t - 1$, oscillating around 5.5 percent both before and after 2004. Overall, $R_t - (1 + g_t)$ rises slightly after 2004, accounting for the modest acceleration in the growth of the bubble-to-output ratio. Moreover, the slowdown in growth mechanically induces a modest rebound in the capital-to-output ratio. This rebound may help explain the rise in the capital-to-output ratio documented in the data.

Not only the aggregate wealth dynamics is well matched by the model but also its distribution. In both the data and the model, the main change in relative wealth shares occurs between the top 1 percent and the middle 40 percent. By 2019, the wealth share of the top 1 percent had increased by 12.4 percentage points, while that of the middle 40 percent had declined by 11 percentage points in the data. In the model, the corresponding changes are an increase of 9.29 percentage points and a decrease of 8.25 percentage points. The wealth share of the top 1 percent rises as they are Scrooge McDuck, while the rest of the population is not. As a result, their share follows a path that converges asymptotically to one. At the other end of the distribution, some agents must lose in terms of wealth share, and these losses are concentrated among the middle 40 percent. Note that this trend partially reverses in the last SCF wave (2022), likely reflecting temporary factors related to the COVID crisis and its aftermath that are not captured by the model.

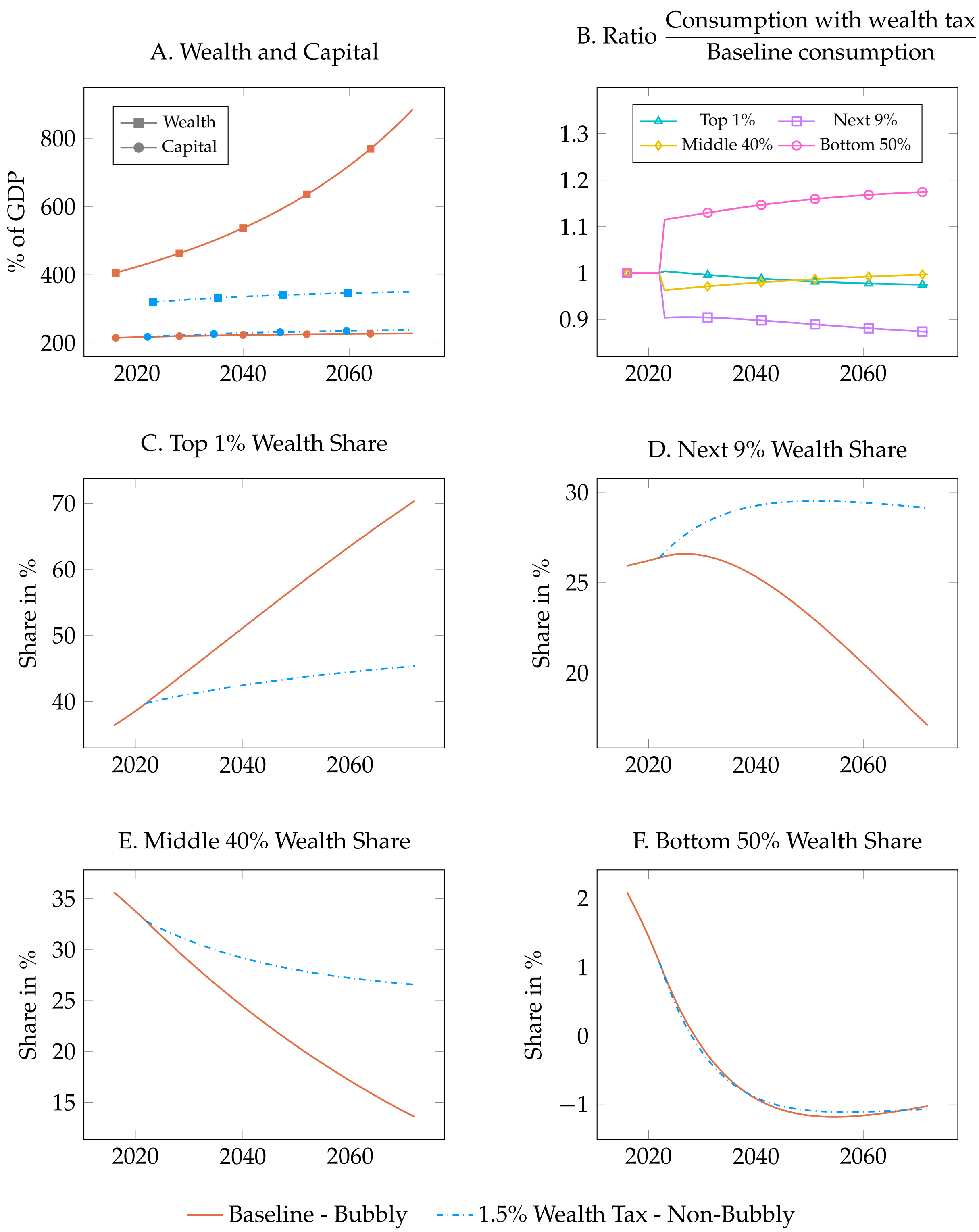
In both the data and the model, the wealth shares of the next 9 percent and the bottom 50 percent remain relatively stable, though for distinct reasons. The wealth share of the bottom 50 percent declines but cannot fall as much as that of the middle 40 percent, given its already low initial level. For the next 9 percent, two opposing forces offset each other over the period considered in the model. On the one hand, as with the middle 40 percent, their wealth share should decline since they are not Scrooge McDuck. On the other hand, they benefit from an increase in relative labor income after 1989 (Figure 6), which raises their savings. Yet this second effect is temporary, and their wealth share is projected to decline after 2030.

5.3 Introduction of a 1.5 Percent Wealth Tax

Having established that the model quantitatively replicates the observed wealth dynamics, it can now be used to conduct counterfactual analysis. We consider the introduction of a 1.5 percent wealth tax from 2022 onward. The tax is assumed to be unanticipated prior to its implementation. Figure 8 illustrates how the economy's trajectory under this policy departs from the baseline between 2016 and 2072.

The primary effect of the wealth tax is to eliminate the bubble. As shown in Section 4, any positive wealth tax rules out bubbly equilibria. Accordingly, the 1.5 percent tax therefore induces an immediate drop in the wealth-to-output ratio and halts its divergence, which had previously

Figure 8: Prospective Transition Dynamics: Baseline Scenario vs. 1.5% Wealth Tax



Notes: The figure contrasts the baseline scenario (in red, continuing the model series from Figure 7) with a counterfactual scenario featuring a 1.5 percent wealth tax implemented in 2022 (in blue). Panel B shows, for each period and each type, the ratio of consumption c_t^i under the wealth tax to its baseline counterpart.

been driven by the bubble. The burst of the bubble further affects capital stock, as the crowding-out effect of the bubble disappears. The wealth tax rate is small enough that its disincentive effect on capital accumulation is dominated by this equilibrium shift effect. As a result, the capital stock path lies above the baseline in the presence of the tax. The capital-to-output ratio is 1.8 percent higher in 2032 and 4.5 percent higher in 2052 under the tax than in its absence.

The wealth tax also modifies the evolution of wealth inequality. The presence of a wealth tax prevents any agent from accumulating unbounded wealth. Because the tax is proportional to wealth, doing so would entail an asymptotically infinite tax burden exceeding aggregate output. Consequently, in the presence of a wealth tax, type 1 agents no longer exhibit Scrooge McDuck consumption-saving behavior. Their wealth share no longer converges to one but stabilizes below 50 percent. The wealth shares lost by the top 1 percent accrue to the next 9 percent and the middle 40 percent. As the wealth of the bottom 50 percent oscillates around zero, their wealth share does as well and remains largely unaffected by the wealth tax.

This counterfactual experiment shows that the standard efficiency-redistribution trade-off vanishes in a bubbly equilibrium for sufficiently low wealth tax rates. In standard model, introducing a tax has a positive effect through redistribution but comes at the cost of lower output due to the disincentive effects of taxation. This mechanism can be further amplified in the presence of non-homothetic saving rates, as discussed in Morrison (2024). In contrast, within the present framework, the capital stock increases following the introduction of the 1.5 percent wealth tax. The capital-to-output ratio is, for instance, 4.4 percent higher with the tax than without it in 2052. There is therefore no longer an output cost associated with taxation, while its redistributive effect remains. This can be seen in Panel B of Figure 8: the higher an agent's position in the wealth distribution, the lower their consumption with the tax relative to without it. The only exception to this pattern is the top 1 percent. Their consumption declines to a lesser extent than that of the next 9 percent. As they cease to be Scrooge McDuck and to hold excess wealth, the resulting reduction in saving boosts their consumption.

Yet a new trade-off may appear for the social planner between utility over consumption versus over wealth. Since small taxes do not entail an efficiency-redistribution trade-off, a social planner maximizing utility from consumption only would optimally implement a wealth tax. The 1.5 percent wealth tax considered here, for instance, would result in a consumption-equivalent gain of 7.26 percent. However, a social planner assigning the same weight to preferences for wealth as agents do would face a welfare cost due to the lower wealth-to-output ratio under taxation. In the present case, this cost is outweighed by the utility gains from consumption, with the consumption-equivalent welfare gain from the tax amounting to 4.7 percent. The weight that a social planner should assign to preferences for wealth likely depends on the micro-foundations of these preferences. Some motives, such as status or power seeking, suggest that preferences for wealth are essentially a zero-sum game and should not be valued by the planner. Others, such as the desire to accumulate or to leave bequests, are positive-sum motives, which would lead the planner to place a positive weight on preferences for wealth.

6 Conclusion

This paper proposes a parsimonious explanation for the rise in the wealth-to-output ratio, the stagnation of the capital-to-output ratio, and the increase in wealth inequality observed across advanced economies in recent decades. The core assumption of this framework is that agents derive *insatiable* utility from wealth. As a result, agents at the top of the income distribution accumulate a *surplus wealth* solely for the sake of holding it. They are defined as *Scrooge McDuck* agents, as they ultimately hold unbounded wealth while maintaining bounded consumption relative to output. Their surplus wealth drives asset prices above their fundamental value, leading to a rational bubble. The latter grows at a rate exceeding the economy's growth rate, enabling a disconnect between the wealth-to-output ratio and the capital-to-output ratio.

The Scrooge McDuck theory carries several implications. It suggests that wealth inequality may follow a diverging trajectory; that increases in income or wealth inequality do not necessarily translate into higher investment; and that the rate of return may remain permanently above the dividend-to-price ratio due to capital gains on a diverging rational bubble. By preventing unbounded wealth accumulation, a wealth tax, or some capital income taxation, shift the equilibrium from bubbly to non-bubbly, potentially fostering capital accumulation. In that case the standard efficiency-redistribution trade-off is replaced a trade-off between preferences for consumption and preferences for wealth .

This paper opens a research agenda for studying the wealth-to-output ratio as a diverging rather than a slowly converging process. Natural extensions include modeling boom-bust cycles of bubbles to better understand financially driven business cycles. Another promising avenue is the study of public debt sustainability when the debt-to-output ratio diverges while the debt-to-wealth ratio remains stable, as has been the case in the U.S. since 2009.

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A Proofs for Section 2

A.1 Proof of the necessity of the transversality condition

We first prove the necessity of the transversality condition (Equation 5) in the endowment economy. The maximization problem of household i is rewritten as:

$$\begin{cases} \max\limits_{\{a_t^i\}_{t=1}^{\infty}} & \sum\limits_{t=0}^{\infty} g_t^i(a_t^i, a_{t+1}^i) \\ \text{s.t.} & \forall t \in \mathbb{Z}_+, (a_t^i, a_{t+1}^i) \in X_t, \qquad \text{with } a_0^i \text{ given,} \end{cases} \tag{55}$$

with $g_t(a_t^i, a_{t+1}^i) \equiv \beta^t \left[u\left(a_t^i R_t - a_{t+1}^i\right) + v(a_{t+1}^i) \right]$, and X_t corresponds to the set of pairs (a_t^i, a_{t+1}^i) satisfying the budget constraint, that is such that: $a_{t+1}^i \leq R_{t+1} a_t$.

The proof follows Kamihigashi (2002), which identifies five conditions under which the transversality condition is a necessary condition. Solely interior solutions of this problem are considered, as it can be easily shown that solutions with $a_t^i = 0$ for some t are dominated by interior solutions. The five conditions identified by Kamihigashi (2002) to prove the necessity of the transversality condition are then satisfied.

Condition 1. $\exists n$, such that $a_0^i \in \mathbb{R}_+^n$ and $\forall t \in \mathbb{Z}_+$, $X_t \subset \mathbb{R}_+^n \times \mathbb{R}_+^n$

This condition is fulfilled for $n = 1$.

Condition 2. $\forall t \in \mathbb{Z}$, X_t is convex and $(0, 0) \in X_t$

It can be shown that if (y, z), $(y', z') \in X_t$, then, for all $\gamma \in [0; 1]$, $(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)z') \in X_t$

Condition 3. $\forall t \in \mathbb{Z}$, $g_t : X_t \to \mathbb{R}$ is C^1 on $\overset{\circ}{X}_t$ and concave.

As $u(c)$ and the function $a_{t+1}^i :\to v(a_{t+1}^i)$ are C^1, g_t is C^1. Moreover, the consumption level implied by $(a_t^i, a_{t+1}^i) = (y, z)$ is defined as $c(y, z) = yR_t - z$. As $u(c)$ is concave, for all $(y, z) \in \overset{\circ}{X}_t$, it can be shown that $\forall t \in \mathbb{Z}$ and $\forall \gamma \in [0; 1]$

$$\gamma u\Big(c(y,z)\Big) + (1-\gamma) u\Big(c(y',z')\Big) \leq u\Big(\gamma c(y,z) + (1-\gamma) c(y',z')\Big) \tag{56}$$

$$= u\bigg(c\Big(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)\Big)\bigg) \tag{57}$$

Given the concavity of both preference terms for consumption and wealth, it follows that:

$$\gamma g_t(y,z) + (1-\gamma) g_t(y',z') \leq g_t\Big(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)z'\Big), \tag{58}$$

and hence that g_t is concave.

Condition 4. $\forall t \in \mathbb{Z}, \forall (y,z) \in \overset{\circ}{X}_t, g_{t,1}(y,z) \geq 0.$

This condition follows from $u'(c) \geq 0\ \forall c$.

Condition 5. For any feasible path a^i_t,

$$\sum_{t=0}^{\infty} g_t(a^i_t, a^i_{t+1}) \equiv \lim_{T\to\infty} \sum_{t=0}^{T} g_t(a^i_t, a^i_{t+1}), \tag{59}$$

exists in $(-\infty, \infty)$.

The wealth of an agent cannot grow at a rate above R_t. Given that $R_t < 1/\beta$ and does not converge to $1/\beta$, $\lim_{T\to\infty} \sum_{t=0}^{T} g_t(a^i_t, a^i_{t+1})$ is not diverging and the condition is satisfied.

The five conditions of Kamihigashi (2002) being satisfied, the transversality condition is a necessary condition of the household maximization problem in the endowment economy ■

A.2 Proof of Propositions 1-6

Proof of Proposition 1 Proposition 1 follows directly from the definitions of the surplus wealth (13) and the bubble (14). ■

Proof of Proposition 2 It follows from Blackwell's contraction mapping theorem that the sequence ℓ^1_t converges. We can then rule out any trajectory in which ℓ^1_t does not converge to $1/\lambda^1$. First, by the monotonicity of preferences, we have $\ell^1_t > \ell^2_t$ for all t, implying that the economy cannot converge to a steady state with $\ell^1 < \ell^2$. Second, one can show by contradiction that the egalitarian steady state characterized by $\ell^1 = \ell^2 = 1$ is unstable under Assumption 1. If the economy were to converge toward the egalitarian steady state, the growth rate of consumption of type-1 agents would have to be increasing according to the combined Euler equations of both agent types (4), but decreasing according to the law of motion of their Lucas tree holdings,

$$q_t \times (\ell^1_{t+1} - \ell^1_t) = \ell^1 - c^1_t, \tag{60}$$

obtained from their budget constraint (2). Finally, the economy cannot converge to a steady state such that $\ell^1 > \ell^2 > 0$, since this would require, from the combined Euler equations (4), that

$$\frac{v'(\ell^1 q)}{u'(\ell^1)} = \frac{v'(\ell^2 q)}{u'(\ell^2)}, \tag{61}$$

which is violated under Assumption 1. Indeed, the left-hand side is necessarily larger than the right-hand side under this assumption. Proposition 2 is therefore necessarily satisfied. ■

Proof of Proposition 3 If $\bar{c} > 1/\lambda^1$, then the asymptotic value of q_t must be bounded above. Otherwise, type 1 agents would accumulate unbounded wealth and asymptotically demand consumption exceeding their endowment, $1/\lambda^1$. Given that q_t is bounded, the wealth of type 2 agents converges to zero and so does their consumption given $\ell^2 = 0$:

$$\ell^2 = a^2 = c^2 = 0 \tag{62}$$

By the goods market clearing condition (10),

$$c^1 = \frac{1}{\lambda^1}. \tag{63}$$

Substituting into the budget constraint of type 1 agents (2) yields

$$a^1 = \frac{1/\lambda^1}{R-1}. \quad \blacksquare \tag{64}$$

Proof of Proposition 4 Given $\bar{c} < 1/\lambda^1$, it cannot be that type 1 agents consume the whole endowment and it follows that $c^2 > 0$. Since $\ell^2 = 0$, this positive asymptotic level of consumption of agents 2 is only possible if:

$$\lim_{t\to\infty} q_t = \infty, \tag{65}$$

from the budget constraint (2) of type 2 agents. It follows under $R > 1$ that

$$\lim_{t\to\infty} a^1 = s^1 = \infty, \tag{66}$$

and consequently,

$$c^1 = \bar{c}.$$

From the goods market clearing condition (10), we obtain

$$c^2 = \frac{1-\lambda^1\bar{c}}{\lambda^2},$$

and from the budget constraint (2) of type 2 agents

$$a^2 = \frac{c^2}{R-1}. \quad \blacksquare$$

Proof of Proposition 5 As discussed in the main text, Proposition 5 follows directly from the rewritten Euler equation (4) for type 1 agents:

$$\lim_{t\to\infty} \frac{v'(a_t^1)}{u'(c_t^1)} = 1 - \beta R. \quad \blacksquare \tag{67}$$

Proof of Proposition 6

Existence of a Non-Bubbly Equilibrium. Let $\overline{R}$ denote the maximal asymptotic rate of return consistent with a non-bubbly equilibrium, defined implicitly by:

$$u'\left(\frac{1}{\lambda^1}\right) = \frac{\beta\overline{R}}{1-\beta\overline{R}}\kappa. \tag{68}$$

For any $R > \overline{R}$, the marginal utility of consumption at the non-bubbly level $c^1 = 1/\lambda^1$ is lower than the marginal utility from saving. It follows that no non-bubbly equilibrium exists when $R > \overline{R}$. By Equation 68, no non-bubbly equilibrium exists if $u'\left(\frac{1}{\lambda^1}\right) < \frac{\beta}{1-\beta}\kappa$ as $\overline{R}$ is lower than 1 and the return R necessarily exceeds 1 due to positive payoff from the Lucas tree each period.

In a non-bubbly equilibrium, the asymptotic rate of return R^{nb} is defined by the asymptotic Euler equation (4) of type 1 agents:

$$R^{nb} = \frac{1}{\beta} - \frac{v'(a^1)}{\beta u'(1/\lambda^1)}. \tag{69}$$

R^{nb} is uniquely defined from Equation 69.

We now prove the existence of a non-bubbly equilibrium under $u'\left(\frac{1}{\lambda^1}\right) \geq \frac{\beta}{1-\beta}\kappa$ by showing, by contradiction, that $R^{nb} < \overline{R}$ in this case. Assume, for the sake of contradiction, that $R^{nb} > \overline{R}$. By rearranging Equations 68 and 69, we obtain:

$$\frac{\beta\overline{R}}{1-\beta\overline{R}}\kappa > \frac{v'(a^1)}{1-\beta R}, \tag{70}$$

which can be equivalently rewritten as

$$\frac{\beta\overline{R}}{v'(a^1)}\kappa > \frac{1-\beta\overline{R}}{1-\beta R}. \tag{71}$$

The left-hand side of Equation 71 is strictly below one, while the right-hand side is strictly above one. It follows that $R^{nb} \leq \overline{R}$ and therefore a unique non-bubbly equilibrium exists whenever $u'\left(\frac{1}{\lambda^1}\right) \geq \frac{\beta}{1-\beta}\kappa$.

Existence of a Bubbly Equilibrium. Whenever $\kappa = 0$, $\bar{c}$ is not defined and the condition for the existence of a bubbly equilibrium cannot be fulfilled (21). Hence, we restrict attention to the case $\kappa > 0$. Asymptotically, in a bubbly equilibrium, the consumption of type-1 agents, $c^{1,b}$, is constrained by the upper bound implied by their optimality condition:

$$c^{1,b} = u'^{-1}\left[\frac{\beta R}{1-\beta R}\kappa\right], \tag{72}$$

whereas the consumption of type-2 agents, $c^{2,b}$, is determined by their asymptotic Euler equation (4):

$$1 - \beta R - \frac{v'\left(\frac{c^{2,b}}{R-1}\right)}{u'(c^{2,b})} = 0. \tag{73}$$

We denote $g(R)$, the aggregate consumption in a bubbly equilibrium for a given R:

$$g(R) = \lambda^1 c^{1,b}(R) + \lambda^2 c^{2,b}(R) \tag{74}$$

Combining Equations 72 and 73 with the technical assumption $\kappa < \frac{1-\beta}{\beta} u'(1)$, we obtain

$$\lim_{R \to 1/\beta} g(R) = 0, \quad \text{and} \quad \lim_{R \to 1} g(R) > 1. \tag{75}$$

A bubbly equilibrium exists for each value of $R \in (1, 1/\beta)$ such that

$$g(R) = 1. \tag{76}$$

Given Equation 75, a unique bubbly equilibrium exists if and only if $g(R)$ is strictly decreasing over the interval $(1, 1/\beta)$. Since the consumption of type-1 agents is trivially decreasing in R, this condition holds provided that $c^{2,b}(R)$ is also strictly decreasing over the interval $(1, 1/\beta)$, which is equivalent to:

$$\frac{\partial h(c^{2,b})}{\partial c^{2,b}} < 0, \quad \text{with} \quad h(c^{2,b}) \equiv \frac{v'\left(\frac{c^{2,b}}{R-1}\right)}{u'(c^{2,b})}. \tag{77}$$

This holds true under Assumption 1 and there exists a unique bubbly equilibrium whenever $\kappa > 0$.

Equilibrium Selection Indeterminacy. The proof of equilibrium selection indeterminacy remains to be written in details. It proceeds by contradiction and relies on the following properties:

- Every equilibrium, whether bubbly or non-bubbly, converges to an asymptotic state in which $\ell^1 = 1$.
- At each point in time, the state of the economy is fully characterized by ℓ^1_t.
- Every equilibrium is locally stable with respect to ℓ^1_t.

B Numerical Algorithm for the Production Economy

This appendix describes the algorithm used to compute the diverging transition path in the production economy. The paper introduces three key innovations to solve the model numerically:

1. It provides a new approach to solving the household partial equilibrium over long horizon with preferences for wealth. Indeed, under standard solution methods, two nearly identical initial guesses for consumption can lead to equilibria in which wealth diverges—one positively and the other negatively.
2. It provides a way to solve for bubbly dynamics beyond the neighborhood of a steady state.
3. It develops a method to compute divergent transitions arising from insatiable preferences for wealth.

The main difficulty in solving for the diverging transition dynamics lies in the presence of a diverging rational bubble. Its initial and asymptotic values are both endogenous and unknown, requiring them to be jointly determined with the equilibrium path of the economy. To address this challenge, we iteratively guess the initial value q_0 of the rent-generating factor and update this guess until convergence to its equilibrium value.

In this subsection, we first outline the general structure of the numerical algorithm, describing how the model is solved for a given guess of q_0 and how this guess is subsequently updated. We then detail the solution of the household optimization problem in partial equilibrium.

B.1 General Structure of the Algorithm

B.1.1 Solving the Transition for a Guess of q_0.

We assume that all variables converging asymptotically have reached their steady-state values by period $t = T$. Starting from an initial guess for the paths of consumption $\{c_t^{i,guess}\}_{t=0}^{T-1}$ for all agents i, we iterate on the following steps:

Step 1: Updating the Capital Guess. We determine a guess of the transition path of capital from the consumption paths $\{c_t^{i,guess}\}_{t=0}^{T-1}$ of all agents i. We proceed *backward*. We start from $k_T^{guess} = k$, with k the asymptotic capital stock, whose value is known. Using the goods market-clearing condition, one can then recover the value of k_t^{guess} based on k_{t+1}^{guess} and $\{c_t^{i,guess}\}_i$. This value corresponds to the unique positive k_t^{guess} that satisfies the goods market-clearing condition:

$$(1-\delta)k_t^{guess} + (k_t^{guess})^\alpha = k_{t+1}^{guess}(1+g_{t+1}) + \sum_i \lambda^i c_t^{i,guess}. \quad (78)$$

Through iteration, we obtain a complete guess for the path of capital $\{k_t^{guess}\}_{t=0}^{T}$. By construction, it is consistent with the consumption paths $\{c_t^{i,guess}\}_{t=0}^{T-1}$ and with the equilibrium value of k_t at $t = T$. However, it may be inconsistent with the initial capital stock k_0.

Step 2: Deriving the Truncated Capital Path. This method of step 1 to derive the capital guess can give extreme values of capital. When used directly to compute the solutions to the household optimization problem in partial equilibrium, this approach is likely to trigger a snowball effect. If the level of capital exceeds the value to which it should converge given our guess of q_0, agents receive higher wages and initial wealth. This may translate into too high consumption and thus too high capital, reinforcing the initial deviation.

A modified version of the capital guess must therefore be used to compute the wages and rates of return that enter the household optimization problem in partial equilibrium. We arbitrarily define two levels of capital corresponding to the minimum and maximum values that capital is allowed to take in the partial equilibrium: k_{min} and k_{max}. All values of capital in the modified path lie within the interval $[k_{min}, k_{max}]$, and we refer to this adjusted series as the *truncated* capital path guess. We define t' as the latest period for which k_t^{guess} lies outside the interval $[k_{min}, k_{max}]$. The truncated capital path, $\{k_t^{trunc}\}_{t=0}^{T}$, is defined as follows:

- if $k_{t'}^{guess} < k_{min}$, then $k_t^{trunc} = k_{min}$ for all $t \leq t'$,
- conversely, if $k_{t'}^{guess} > k_{max}$, then $k_t^{trunc} = k_{max}$ for all $t \leq t'$,
- in both cases, $k_t^{trunc} = k_t^{guess}$ for all $t > t'$.

Notice that it is important to fix k_t^{trunc} from period 0 to t', otherwise the model could generate jumps from k_{min} to k_{max} that prevent convergence.

Step 3: Household Optimization Problem in Partial Equilibrium. We then solve for the partial equilibrium of all agents following the method described in Subsubsection B.1.3. Specifically, we use our guess of q_0 and the exogenous values of k_0 and of the wealth shares to compute the initial wealth a_0^i of each agent, while the truncated capital path, $\{k_t^{trunc}\}_{t=0}^{T}$, is used to compute the rates of return, $\{R_t^{trunc}\}_{t=0}^{T}$ and wages $\{w_t^{trunc}\}_{t=0}^{T}$. The resulting consumption paths are denoted by $\{c_t^{i,new}\}_{t=0}^{T-1}$ for all agent types i.

Step 4: Updating the Consumption Guess. We update the consumption guess as a period-by-period weighted average between the previous guess and the newly computed consumption path:

$$c_t^{i,guess'} = \omega c_t^{i,guess} + (1-\omega) c_t^{i,new}, \tag{79}$$

with $\omega \in [0, 1)$.

We then compute the Manhattan distance between $\{c_t^{i,guess}\}_{t=0}^{T-1}$ and $\{c_t^{i,new}\}_{t=0}^{T-1}$. If this distance falls below the tolerance threshold, the algorithm converges, and we take $c_t^{i,guess}{}_{t=0}^{T-1}$ and $\{k_t^{guess}\}_{t=0}^{T}$ as the equilibrium vectors associated with the initial guess q_0. If the distance exceeds the threshold and the maximal number of iterations has not yet been reached, the iteration continues. Otherwise, once the maximal number of iterations is reached, we restart the algorithm from the initial guess with a higher ω. We allow for a limited number of adjustments to the relaxation parameter ω. If the algorithm still fails to converge at the highest ω, we take the final values, obtained at the highest ω and at the maximal number of iterations, of $\{c_t^{i,guess}\}_{t=0}^{T-1}$ and $\{k_t^{guess}\}_{t=0}^{T}$ as those associated with the initial guess q_0.

B.1.2 Finding the Value of q_0.

We initialize the algorithm with two arbitrarily chosen values of q_0, denoted q_0^{min} and q_0^{max}, which are expected to lie below and above the equilibrium value, respectively. For simplicity, we typically set $q_0^{min} = 0$. We then iterate the following steps:

Step 1: Defining a Test Value for q_0. We define an initial value of the bubble to test, denoted q_0^{test}, as the midpoint between q_0^{min} and q_0^{max}:

$$q_0^{test} = \frac{q_0^{min} + q_0^{max}}{2}. \tag{80}$$

Step 2: Computing the Associated Capital Path. Following the method described above (B.1.1), we compute the capital path $\{k_t^{guess}\}_{t=0}^{T}$ associated with q_0^{test}.

Step 3: Updating q_0^{min} or q_0^{max} Depending on the value of k_0^{guess}, either q_0^{min} or q_0^{max} is updated to q_0^{test}:

- If $k_0^{guess} > k_0$, consumption is higher than in equilibrium, implying that the economy must start from a level of capital above the exogenously given k_0. We therefore infer that the bubble guess exceeds its equilibrium value and set $q_0^{max} = q_0^{test}$.
- Conversely, if $k_0^{guess} < k_0$, we set $q_0^{min} = q_0^{test}$.

If, after this update, the difference $q_0^{max} - q_0^{min}$ falls below the tolerance level for the bubble, the algorithm converges and $\frac{q_0^{min}+q_0^{max}}{2}$ and k_0^{guess} are taken as the equilibrium values of the bubble and of capital. Otherwise, the iteration continues.

B.1.3 Solving for Household Optimization Problem in Partial Equilibrium

Now that the general structure of the algorithm has been defined, we turn to a detailed description of how the households' partial equilibrium optimization problems are solved. We distinguish between the solution method for non-Scrooge McDuck agents and that for Scrooge McDuck agents. Whether an agent is a Scrooge McDuck or not is taken as given by the algorithm. It follows directly from whether the equilibrium is bubbly, given that we restrict our analysis to cases in which at most type-1 agents can be Scrooge McDuck.

For Non-Scrooge McDuck Agents Consider the problem of a non-Scrooge McDuck agent i. The presence of preferences for wealth does not change fundamentally the way we want to solve its optimization problem in partial equilibrium. We are looking for the path $\{c_t^{i,\star}\}_{t=0}^{T-1}$ such that:

- the budget constraint (36) and the Euler equation (40) of agent i are satisfied for all $t \in [0 : T-1]$ given the paths of rate of return $\{R_t^{trunc}\}_{t=0}^{T}$, and wages, $\{w_t^{trunc}\}_{t=0}^{T}$,
- $a_T^i = a^i$, with a^i the asymptotic value of the wealth of type i,
- a_0^i is equal to its value determined by the guess of q_0 and the exogenous values of the initial capital stock, k_0, and the wealth shares of each agent type.

To do so, we initialize an algorithm with two arbitrarily chosen values of c_0^i, denoted $c_0^{i,min}$ and $c_0^{i,max}$, which are expected to lie respectively below and above the value $c_0^{i,\star}$. For simplicity, we typically set $c_0^{i,min} = 0$. We then iterate the following steps:

1. We define $c_t^{i,test} = \frac{c_0^{i,min}+c_0^{i,max}}{2}$
2. We compute forward the paths of consumption, $\{c_t^{i,test}\}_{t=0}^{T-1}$, and wealth, $\{a_t^{i,test}\}_{t=0}^{T}$, associated with $c_t^{i,test}$ given the budget constraint (36) and the Euler equation (40) of agent i
3. If $a_T^{i,test} \geq a_T^i$, the agent has underconsumed compared to the solution of the problem and the new value of $c_0^{i,min}$ is set to $c_t^{i,test}$. Conversely, if $a_T^{i,test} \leq a_T^i$, $c_0^{i,max}$ is set to $c_t^{i,test}$.
4. If the difference $c_0^{i,max} - c_0^{i,min}$ falls below a pre-specified tolerance level, the algorithm is stopped.

Without preferences for wealth, this algorithm converges to a solution coherent with the asymptotic level of wealth, a^i, for reasonable precision levels. However, when agents derive utility directly from wealth, a snowball effect can occur: two very close initial consumption choices, c_0^i, may lead to sharply different terminal wealth outcomes, a_T^i. In the path with *slightly lower initial consumption*, the agent starts with higher wealth a_1^i due to the budget constraint, which raises the marginal utility of wealth $v'(a_1^i)$. This, in turn, strengthens the incentive to save and further reduces consumption. As the simulation progresses, wealth a_t^i remains systematically *above* that of the higher-consumption path, while consumption c_t^i remains *below* it. The gap between the two trajectories therefore widens over time, preventing convergence of the simple bisection algorithm. All else equal, the higher the horizon T, the larger the discrepancy between the two terminal wealth values a_T^i.

This is particularly an issue when the two close values of initial consumption, $c_0^{i,min}$ and $c_0^{i,max}$, at which the partial-equilibrium algorithm stops, both generate terminal wealth levels far from the target, i.e. $a_T^i(c_0^{i,min}) \gg a_T^i$ and $a_T^i(c_0^{i,max}) \ll a_T^i$. In this case, the algorithm incorrectly interprets the bracket as having converged, even though both candidate paths are off target. The outline of the new method developed to overcome this problem is as follows. We initialize $t^{conv} = 0$ and iterate the following steps:

1. We run the previous partial equilibrium algorithm from period t^{conv} to T with starting wealth $a^i_{t^{conv}}$. We denote by t^{div} the divergence period, defined as the earliest period for which the difference between the two asset paths, $a^i_{t^{div}}(c^{i,min}_{t^{conv}}) - a^i_{t^{div}}(c^{i,max}_{t^{conv}})$, exceeds a tolerance threshold.
2. We collect in two vectors of length T, the value of consumption and wealth as the average between the one predicted by the two paths from t^{conv} to t^{div} for consumption and from $t^{conv}+1$ to $t^{div}+1$ for wealth
3. We choose a value $a^{intermediate} \in \left[a^i_{t^{div}}(c^{i,max}_{t^{conv}}), a^i_{t^{div}}(c^{i,min}_{t^{conv}})\right]$ such that starting the path from $a^{intermediate}$ at time t^{div} and continuing the simulation with consumption levels $c^{i,min}_{t^{conv}}$ and $c^{i,max}_{t^{conv}}$, the corresponding terminal wealth values are respectively above and below the target.

4. We then set the new convergence period $t^{conv} = t^{div}$ and initialize wealth at that date with $a^i_{t^{conv}} = a^{intermediate}$.

Iterating this loop until $t^{conv} = T$ allows us to compute long transition paths in the presence of preferences for wealth. The solution of the problem is then obtained from the two vectors constructed in the second step.

For Scrooge McDuck Agents

For Scrooge McDuck agents, consumption at time T is known and assumed to be equal to its asymptotic value, $c^1_T = c^1$, while wealth at time T remains unknown. Given a guess for a^1_T, $a^{1,guess}_T$, one can derive the entire paths of consumption, $\{c^{1,predicted}_t\}^{T-1}_{t=0}$, and wealth, $\{a^{1,predicted}_t\}^{T-1}_{t=0}$, consistent with $(a^{1,guess}_T, c^1_T)$ using their budget constraint (36) and their Euler equation (40) backward.

The predicted wealth at $t = 0$, $a^{1,predicted}_0$ is strictly increasing in $a^{1,guess}_T$. The terminal wealth of type-1 agents is chosen such that $a^{1,predicted}_0$ matches the initial wealth a^1_0 implied by q_0, k_0 and the wealth share of type 1 agents at $t = 0$. The corresponding paths of consumption, $\{c^{1,predicted}_t\}^{T-1}_{t=0}$, and wealth, $\{a^{1,predicted}_t\}^{T-1}_{t=0}$ are then the solutions to the household optimization problem used in other stages of the algorithm.