A Scrooge McDuck Theory of Wealth Dynamics

Valentin Marchal*

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Abstract

This paper introduces insatiable utility from wealth to jointly explain the rise in the wealth-to-output ratio and investment stagnation in advanced economies as a consequence of increasing top income inequality. These preferences induce an upper bound on optimal consumption, which binds asymptotically for high-income agents. Labelled *Scrooge McDuck*, they hold a portion of their wealth purely for its own sake, with all returns reinvested. This pushes asset prices above fundamental value. A uniquely determined rational bubble exists, which crowds out investment and grows at a rate that exceeds that of the economy. Its price co-moves with income and wealth inequality, all three diverging over time. Any wealth tax prevents these divergent dynamics, which persist under low capital income taxes, thereby breaking the standard equivalence between the two tax types.

Keywords: Wealth Preference, Wealth-to-Output, Inequality, Rational Bubble
JEL Classification: E21, E22

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*Sciences Po. Email: valentin.marchal@sciencespo.fr

1 Introduction

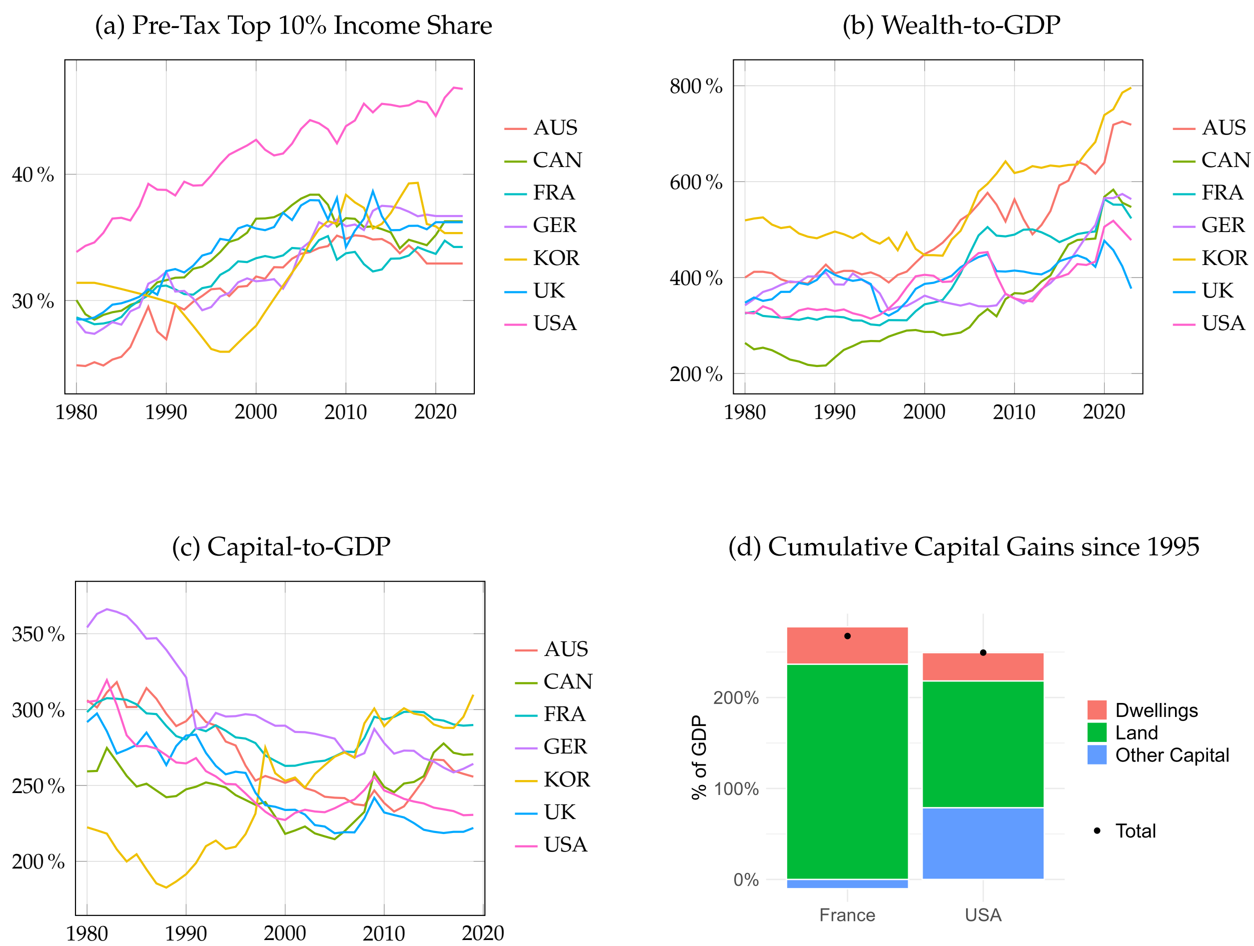
The rise in top income inequality may help explain part of the global upward trend in the wealth-to-output ratio observed in recent decades. Empirical evidence shows that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019). As a result, a more unequal distribution of income shifts resources from low-saving to high-saving agents, thereby raising aggregate savings and, in turn, aggregate wealth, all else equal. Furthermore, the co-occurrence of rising inequality and rising aggregate wealth across many advanced economies (Figures 1a and 1b) reinforces the hypothesis of a structural link between these trends.

Yet, models that aim to explain the increase in the wealth-to-output ratio through rising income inequality (e.g., Hubmer et al., 2021; Elina and Huleux, 2023) fail to capture the nature of the observed rise, attributing it to an increase in capital investment. By construction, these models often consider capital—whose price is fixed—as the only available asset, so that any increase in wealth mechanically corresponds to an increase in capital. More fundamentally, in such frameworks, a rise in the demand for savings lowers the interest rate, resulting in further capital investment. This stands in contrast to the data, which show declining capital-to-output dynamics (Figure 1c), implying that the observed rise in the wealth-to-output ratio was driven instead by rising asset prices, that is, by capital gains.

This paper introduces insatiable preferences for wealth as a parsimonious deviation from standard models, capable of generating a capital gains–driven rise in the wealth-to-output ratio following an increase in income inequality. Preferences for wealth have proven effective to capture the high saving rates observed at the top of the wealth and income distribution (Gaillard et al., 2023). In their standard specification (e.g. Kumhof et al., 2015, Mian et al., 2021), the marginal utility of wealth is assumed to decline more slowly than that of consumption, ensuring that saving rates increase with income. However, in both cases, marginal utility tends to zero as the corresponding argument—wealth or consumption—diverges. We strengthen the asymmetry between utility from wealth and consumption by assuming that the marginal utility of holding wealth remains bounded below by a strictly positive constant, even as wealth diverges. We introduce these preferences in an endowment economy and a production economy featuring both a rent-generating asset that allows to study asset pricing implications.

Under insatiable preferences for wealth, some agents may accumulate unbounded wealth while keeping consumption bounded and are referred to as Scrooge McDuck agents. Insatiable preferences for wealth imply a lower bound on the marginal utility of wealth, which in turn induces a lower bound on the marginal utility of saving. As the marginal utility of consumption must not fall below that of saving, this implicitly defines an upper bound on consumption. When income inequality is sufficiently high, some agents at the top of the income distribution asymptotically receive an income that exceeds the upper bound on consumption. As a result, they accumulate unbounded wealth relative to output over time while keeping their consumption bounded. This consumption–saving behavior would not be optimal under standard preferences for wealth, where diverging wealth necessarily implies diverging consumption—albeit typically at a slower rate. A portion of Scrooge McDuck agents' wealth is purely held for its own sake, and supports

Figure 1: Wealth Dynamics and Income Inequality



Notes: Panel (a) presents the pre-tax income share of the top 10% (source: World Inequality Database). Panel (b) displays the total market value of national non-financial assets (public and private) relative to GDP (source: World Inequality Database). Panel (c) presents the combined public and private capital stock as a share of GDP (source: IMF Investment and Capital Stock Dataset). Panel (d) depicts capital gains by asset type realized in the private non-financial sector (excluding non-profits), calculated using official national accounting data. Details on the construction of these capital gains are provided in Appendix A.1.

the existence of a rational bubble. A corollary of holding unbounded wealth while keeping consumption bounded is that the present value of Scrooge McDuck agents' future consumption financed from savings remains below the current value of their wealth. They therefore hold a share of their wealth, referred to as surplus wealth, solely for the utility derived from holding it. The associated returns are fully reinvested. This pushes asset prices above their fundamental value, defined as the present value of future dividends discounted at the rate of return. As a result, a rational bubble emerges in the pricing of the rent-generating asset. The rational bubble and surplus wealth coincide: in this frictionless model, a bubble that does not finance consumption must be backed by wealth held without a consumption motive.

Both bubbly and non-bubbly equilibria may coexist, with equilibrium selection being self-fulfilling. In the presence of such multiplicity, if agents anticipate a bubbly equilibrium, they also expect high returns driven by capital gains on the bubble. This strengthens saving incentives, increases the lower bound on marginal utility of savings and lowers the upper bound on optimal consumption. Scrooge McDuck consumption-saving behavior is then optimal for agents at the top of the distribution, validating the bubbly equilibrium. Conversely, under expectations of a non-bubbly equilibrium, lower anticipated returns raise the upper bound on consumption, preventing the emergence of Scrooge McDuck behavior and the associated bubble. The existence of bubbly and non-bubbly equilibria depends itself on parameter values, and in particular on the strength of the insatiable component of preferences for wealth.

The existence of a rational bubble is the central asset pricing feature that allows for a wedge between wealth and capital dynamics. Bubbly equilibria replicate the key stylized facts motivating this study by producing a bubble-driven increase in the wealth-to-output ratio without a corresponding rise in the capital-to-output ratio. The bubble crowds out capital, with the capital stock converging to a finite level low enough to ensure strict dynamic efficiency. Accordingly, the asymptotic rate of return on capital remains above the growth rate, in line with empirical evidence (Reis, 2021). Furthermore, the theory offers a unified framework for understanding increasing wealth-to-output ratio as a global phenomenon, despite its occurrence on different asset classes across countries (Figure 1d). The bubble is formally tied to the rent-generating factor of production in the model, but can be mapped to a broad range of real-world assets—including land, equities, and even public debt.

Consistent with our motivating evidence, higher income inequality is associated with a larger bubble and, consequently, a higher wealth-to-output ratio. First, greater labor income inequality broadens the set of parameters consistent with a bubbly equilibrium. Scrooge McDuck behavior arises if, and only if, some agents' income exceeds the asymptotic upper bound on optimal consumption. As labor inequality rises, this threshold becomes easier to satisfy, effectively relaxing the condition for bubbly equilibria. Secondly, conditional on being in a bubbly equilibrium, the diverging wealth of Scrooge McDuck agents results in both higher capital income and greater surplus wealth, which translate into a larger bubble.

The present model yields two uncommon outcomes within a neoclassical framework, the first being diverging wealth inequality. Asymptotically, non–Scrooge McDuck agents, whose income remains below the upper bound on consumption, hold positive but bounded wealth. Otherwise, their intertemporal optimality conditions would be violated, as they would face incentives to increase consumption. In a bubbly equilibrium, wealth inequality diverges over time, driven not by declining wealth at the bottom or middle of the distribution, but by the unbounded accumulation of wealth by Scrooge McDuck agents. By providing a microfounded general equilibrium model in which wealth inequality diverges and the asymptotic rate of return exceeds the growth rate, our framework offers a theoretical foundation for Piketty's (2014) main conjecture that wealth inequality may diverge when $r > g$.

The second distinctive outcome of the model is the existence of a uniquely determined, diverging rational bubble, which generates recurrent capital gains as its price rises over time. A recent debate on the welfare implications of capital gains has centered on fundamental-driven capital gains arising from a one-off decline in the discount rate (Fagereng et al., 2021). These gains, which reflect a revaluation of fundamentals, have often been described as "paper gains" (Cochrane, 2020; Krugman, 2021), as they do not alter dividend flows and cannot sustainably finance higher consumption. In contrast, this analysis focuses on bubble-driven capital gains, which are recurring, as the bubble price continues to rise over time. This continuous appreciation allows non–Scrooge McDuck agents to realize capital gains in each period to finance consumption despite holding only a vanishing share of the bubble.

The model also carries implications for tax policy: notably, any non-vanishing wealth tax prevents the emergence of a bubbly equilibrium, and its introduction can lead to a higher capital stock. In this framework, the standard equivalence between capital income and wealth taxation under homogeneous return no longer holds. While bubble assets are subject to wealth taxation, they do not generate taxable flows under a capital income tax. Moreover, under any wealth tax with a non-vanishing rate, a diverging bubble would generate unbounded tax revenues, violating the goods market clearing condition. This implies that any positive wealth tax rules out the existence of a bubbly equilibrium. If the economy initially features a bubble, introducing a sufficiently small wealth tax can thus foster capital accumulation by eliminating the crowding-out effect associated with the bubble.

To illustrate the empirical relevance of the proposed mechanism, we conduct a simple quantitative exercise for the U.S. Specifically, we perform an optimal taxation exercise in which the social planner must, beyond the usual trade-offs, weigh the benefits of the rational bubble—such as higher returns and utility from wealth—against its drawbacks, notably the crowding-out of capital. *[In progress]*

Related Literature. This paper builds on multiple empirical studies documenting, across advanced economies, rising income and wealth inequality (Katz and Murphy, 1992, Piketty and Saez, 2003, Saez and Zucman, 2016; Batty et al., 2019; Chancel et al., 2022; Smith et al., 2023), an increase in aggregate wealth driven by capital gains (Piketty and Zucman, 2014), and a declining trend in investment (Gutiérrez and Philippon, 2017). It theoretically contributes to two strands of the literature.

First, on empirical evidence that saving rates increase with income and wealth (Carroll, 2000; Dynan et al., 2004; Straub, 2019; Fagereng et al., 2021) and that aggregate savings are primarily driven by the top of the distribution (Mian et al., 2020; Bauluz et al., 2022), a growing theoretical and quantitative literature highlights the importance of non-homothetic preferences for wealth in explaining wealth inequality and the wealth-to-output ratio (Carroll, 2000; De Nardi, 2004; Kumhof et al., 2015; De Nardi and Yang, 2016; Benhabib et al., 2019; Mian et al., 2021; Elina and Huleux, 2023; Gaillard et al., 2023; Michau et al., 2023a). The present framework departs from this literature by assuming insatiable preferences for wealth, a critical assumption for the existence of rational bubbles that drive investmentless increases in wealth. Building on this mechanism, it is the first to predict ever-growing wealth inequality, driven by a non-zero mass of agents holding diverging levels of wealth.[1]

Secondly, this work contributes to the rational bubble literature. Previous research has established that rational bubbles can exist in dynamically inefficient economies (Samuelson, 1958; Diamond, 1965; Tirole, 1985; Michau et al., 2023b) or under financial frictions (Farhi and Tirole, 2012; Martin and Ventura, 2012; Reis, 2021). In both cases, the rate of return on bubbles is below the economy's growth rate, leading to an asymptotically stationary bubble-to-output ratio. In contrast, few studies have explored frameworks with a diverging bubble-to-output ratio. Notable exceptions include Ono (1994) and Kamihigashi (2008), which examine representative agent models with insatiable preferences for liquidity or wealth, finding an infinite set of diverging bubbly equilibria. By introducing heterogeneous agents, the present analysis departs from these approaches and results in a uniquely determined diverging equilibrium.

The remainder of the paper is organized as follows. Section 2 develops the core intuition for the existence of bubbly equilibria under insatiable preferences, using a tractable endowment economy. Section 3 extends the analysis to a production economy, linking the model to the motivating stylized facts. Section 4 investigates the positive implications of the model for wealth and capital income taxation. Section 5 presents a quantitative exercise to assess the empirical relevance of the Scrooge McDuck mechanism and derives optimal taxation. Section 6 concludes.

2 Endowment Economy

To isolate the mechanism by which insatiable wealth preferences—unlike standard ones—can generate diverging rational bubbles, we embed such preferences in a simple two-agents asset pricing model à la Lucas, 1978. This tractable setting allows us to derive the conditions under which bubbly or non-bubbly equilibria arise, and to characterize the asymptotic behavior of each equilibrium type.

2.1 Model Environment

The economy is deterministic, with discrete time running from $t = 0$ to ∞. The endowment comes from a single unit of Lucas tree, which delivers one unit of the consumption good each

[1] By featuring an asymptotic rate of return above the economy's growth rate, this paper provides a theoretical foundation for Piketty (2014)'s conjecture, which states that "if the rate of return on capital remains significantly above the growth rate for an extended period, then the risk of divergence in the distribution of wealth is very high" (Piketty, 2014, page 34).

period and is priced at q_t at time t after dividend payment. The rate of return of the Lucas tree is defined as $R_{t+1} \equiv \frac{1+q_{t+1}}{q_t}$.

Households. There is a unit mass of infinitely lived households, divided into two types $i \in \{1, 2\}$, each representing a strictly positive share λ^i of the total population. Agents of type i hold ℓ_t^i units of the Lucas tree at the beginning of period t and consume c_t^i at time t. Agents differ only in their initial endowment of Lucas tree units, with type 1 agents holding the higher initial endowment: $\ell_0^1 \geq \ell_0^2 > 0$. The wealth of a household of type i at the end of period t is defined as $a_{t+1}^i \equiv q_t \ell_{t+1}^i$, and its budget constraint can be written as:

$$c_t^i + a_{t+1}^i = R_t a_t^i. \tag{1}$$

To ensure the absence of Ponzi schemes, borrowing is ruled out, as all household resources are derived from their Lucas tree units:

$$\ell_t^i \geq 0. \tag{2}$$

Both types of agents share identical preferences. They derive utility each period from consumption and from holding wealth. Agents discount the future at rate $\beta \in [0, 1)$ and maximize intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t U(c_t^i, a_{t+1}^i) \qquad \text{with} \qquad U(c, a) = u(c) + v(a). \tag{3}$$

Utility from consumption is represented by the function $u(c)$, which satisfies the Inada conditions. Preference for wealth is captured by $v(a)$, assumed to be twice continuously differentiable, increasing, and concave.

The solution to the optimization problem for type i households is characterized by the following Euler equation and the transversality condition[2]:

$$u'(c_t^i) = \beta R_{t+1} u'(c_{t+1}^i) + v'(a_{t+1}^i), \tag{4}$$

$$\lim_{t \to \infty} \beta^t [u'(c_t^i) - v'(a_t^i)] a_t^i = 0. \tag{5}$$

Wealth-Dependent Consumption-Saving Behavior. In order to match the empirical evidence that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019), this paper follows the literature on preferences for wealth by assuming that the utility-from-wealth component, $v(a)$, exhibits lower curvature than the utility-from-consumption component, $u(c)$. To fomalize this assumption, we first need to define the coefficient of risk aversion of consumption and wealth, labelled respectively $\theta(c)$ and $\eta(a)$:

$$\theta(c) \equiv -\frac{c\, u''(c)}{u'(c)}, \qquad \eta(a) \equiv -\frac{a\, v''(a)}{v'(a)}. \tag{6}$$

[2] A proof of the necessity of the transversality condition is provided in Appendix B.1.1

The following assumption induces non-homothetic (i.e., wealth-dependent) consumption-saving behavior, whereby wealthier agents save a larger share of their income.

Assumption 1 *For any $\{c, a\} \in \mathbb{R}^2_+$, the utility from wealth exhibits lower curvature than the utility from consumption. That is,*

$$\eta(a) < \theta(c). \tag{7}$$

Under assumption 1, wealth is a luxury. Intuitively, as the marginal utility of wealth declines more slowly than that of consumption, high-wealth agents value additional wealth relative to additional consumption more than low-wealth agents do. In equilibrium, this leads type 1 agents to save more than type 2 agents, resulting in type 1 agents purchasing Lucas tree units from type 2 agents in each period.

Standard vs Insatiable Preferences for Wealth. The insatiable preferences for wealth studied in this paper depart from standard preferences by imposing a strictly positive lower bound on the marginal utility of holding wealth. As it is concave and strictly increasing, the utility-from-wealth component $v(a)$ admits a well-defined limit for its marginal utility as wealth tends to infinity. We denote this limit by κ:

$$\lim_{a \to \infty} v'(a) = \kappa. \tag{8}$$

Preferences for wealth are said to be *insatiable* whenever $\kappa > 0$. In contrast, if $\kappa = 0$, we refer to them as *standard* preferences for wealth, as this corresponds to the usual specification in the wealth-in-utility literature. We think of κ as being small, so that insatiable preferences for wealth represent a parsimonious deviation from the standard specification in the literature[3]. In line with this interpretation, and to rule out situations in which all agents would strictly prefer saving over consumption, we impose the technical assumption that $\kappa < \frac{1-\beta}{\beta} u'(1)$.

The insatiability assumption cannot be ruled out empirically. Standard and insatiable preferences for wealth differ only in their asymptotic implications, which are, by definition, not observable. It is therefore fundamentally impossible to distinguish between $\kappa = 0$ and a strictly positive but arbitrarily small value of κ. Furthermore, even if one could envision testing whether κ is significantly greater than zero based on saving rates across wealth levels at the top of the distribution, such a test is ultimately infeasible due to the lack of consumption data for the relevant high-wealth households. Since insatiable and standard preferences for wealth cannot be distinguished using micro-data, this paper proposes to contrast their implications, particularly for asset pricing, in order to assess which specification is more consistent with observed macroeconomic stylised facts.

Market Clearing Conditions. In each period, there is one unit of the Lucas tree and one unit of the non-storable consumption good. The market clearing condition for assets is therefore expressed

[3]The crucial assumption of this paper is not that *all* agents have insatiable preferences for wealth, but rather that *some* do. We assume throughout that all agents share these preferences to derive results that are not driven by preference heterogeneity. Importantly, the key mechanisms would still hold under heterogeneous preferences, as long as *some* agents—technically, even a single agent—at the top of the wealth distribution exhibit insatiable preferences for wealth.

as:

$$\lambda^1 \ell_t^1 + \lambda^2 \ell_t^2 = 1, \tag{9}$$

and the goods market clearing condition is given by:

$$\lambda^1 c_t^1 + \lambda^2 c_t^2 = 1. \tag{10}$$

Equilibrium. Given the initial endowment ℓ_0^1, an equilibrium $\{c_t^1, \ell_t^1, q_t\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems and the asset market clearing condition.

2.2 Definitions of Key Concepts

We now introduce four key concepts that structure the theoretical analysis: (i) the upper bound on consumption implied by insatiable preferences for wealth; (ii) the behavior of *Scrooge McDuck* agents, who accumulate unbounded wealth while keeping consumption bounded; (iii) the notion of *surplus wealth*, defined as the portion of agents' wealth that does not finance any future consumption; and (iv) the rational bubble that emerges in the price of the Lucas tree as a direct consequence of a positive surplus wealth.

Upper Bound on Consumption Unlike standard preferences for wealth, insatiable ones implicitly define an upper bound on optimal consumption, $\bar{c}_t$.

Definition 1 *Under insatiable preferences for wealth, the upper bound on optimal consumption, $\bar{c}_t$, is defined as:*

$$\bar{c}_t \equiv u'^{-1}\left(\sum_{s=0}^{\infty} \beta^s \left[\prod_{j=1}^{s} R_{t+j}\right] \kappa\right). \tag{11}$$

Regardless of agent i's wealth, consumption is always such that $c_t^i < \bar{c}_t$. The lower bound on the marginal utility of holding wealth, κ, induces a corresponding lower bound on the marginal utility of saving, as defined by the inner bracket in Equation 11. This lower bound captures the utility derived from saving an additional unit of wealth and reinvesting all associated returns indefinitely, assuming the marginal utility of wealth is fixed to κ. The upper bound $\bar{c}_t$ corresponds to the level of consumption at which the marginal utility of consumption equals the lower bound on the marginal utility of holding wealth. It follows that it is never optimal for any agent i to consume $c_t^i \geq \bar{c}_t$.

As will become clear in the analysis, the fundamental difference between insatiable and standard preferences for wealth stems from the existence of this upper bound on consumption. From Equation 11, it can be seen that diverges as κ tends to zero. From Equation 11, it can be seen that $\bar{c}_t$ diverges in the limit as κ tends to zero. There is therefore no upper bound on consumption under standard preferences for wealth. This paper focuses on insatiable preferences for wealth as the source of the upper bound on consumption, but similar results could arise from alternative preference specifications, as long as they imply an upper bound on optimal consumption. For instance, one could directly impose an upper bound in the utility-from-consumption component $u(c)$, or alternatively, assume preferences over net saving flows.

Scrooge McDuck Agents Given the upper bound on consumption, some agents may optimally choose to accumulate unbounded wealth over time while keeping consumption bounded; we label them Scrooge McDuck agents.

Definition 2 *An agent i is labelled as a Scrooge McDuck agent, if and only if:*

$$\lim_{t\to\infty} a_t^i = \infty, \quad \textit{and} \quad \exists\, M \in \mathbb{R}_+ \textit{ such that } c_t < M\ \forall t. \tag{12}$$

The existence of Scrooge McDuck agents is closely tied to the presence of an upper bound on consumption, and their consumption asymptotically approaches $\bar{c}_t$. As their wealth diverges, the marginal utility of holding wealth converges to κ, and their marginal utility of saving approaches $\sum_{s=0}^{\infty} \beta^s \left[\prod_{j=1}^{s} R_{t+j}\right] \kappa$. By the definition of $\bar{c}_t$ in Equation 11, their asymptotic consumption therefore coincides with the upper bound on optimal consumption. In contrast, under standard preferences for wealth, it is never optimal for an agent to behave as a Scrooge McDuck: diverging wealth necessarily implies diverging consumption, even if, as in Michau et al. (2023a), consumption may diverge at a slower rate.

Surplus Wealth Scrooge McDuck agents hold a portion of their wealth purely for its own sake, referred to as surplus wealth.

Definition 3 *The surplus wealth of an agent of type i at the end of period t, denoted s_{t+1}^i, is defined as the portion of wealth that is not used to finance any future consumption:*

$$s_{t+1}^i \equiv a_{t+1}^i - \sum_{j=1}^{\infty} \frac{c_{t+j}^i}{\prod_{s=1}^{j} R_{t+s}}. \tag{13}$$

In this endowment economy, all consumption is financed out of the Lucas tree. As a result, surplus wealth corresponds to total wealth minus the present value of future consumption, as shown in Equation 13. By definition, this surplus wealth is not used to finance future consumption, and all associated dividends are reinvested. As a result, surplus wealth grows at rate R_{t+1} from period t to $t+1$.

Rational Bubble The presence of Scrooge McDuck agents pushes the price of the Lucas tree above its fundamental value and gives rise to a rational bubble, whose size coincides with that of the surplus wealth.

Definition 4 *The asset price q_t is decomposed into a fundamental value f_t, defined as the present value of future dividends discounted at the rate of return, and a bubble component b_t.*

$$f_t \equiv \sum_{j=1}^{\infty} \frac{1}{\prod_{s=1}^{j} R_{t+s}}, \qquad b_t = \lim_{T\to\infty} \frac{q_T}{\prod_{s=1}^{T} R_{t+s}} \qquad \textit{with} \quad q_t = f_t + b_t.$$

The fundamental value derives from both the stream of dividends and the utility of holding it, the latter being reflected in the rate of return R_{t+s}, which is pushed down by the preference for wealth. In contrast, the value of the rational bubble arises solely from the utility of holding it. We refer to it as a rational bubble for two reasons. First, its current value corresponds to its discounted asymptotic value, reflecting the fact that some agents are willing to value the bubble only because they anticipate future demand from other agents. Second, if multiple infinitely-lived assets were introduced, the model would leave indeterminate which asset the bubble is attached to.

The following proposition establishes that the rational bubble is the asset pricing counterpart of surplus wealth.

Proposition 1 *The rational bubble coincides with the aggregate surplus wealth:*

$$b_t = \sum_i \lambda^i s^i_{t+1}. \tag{14}$$

Proposition 1 follows directly from previous definitions of the surplus wealth (13) and the bubble (4). Intuitively, in the considered frictionless model, a rational bubble that does not finance consumption can exist only if some wealth is held without a consumption motive. Like surplus wealth, the rational bubble grows at rate R_{t+1} from period t to $t+1$, in order to deliver the same rate of return as fundamental wealth.

2.3 Bubbly and Non-bubbly Equilibria

We now describe the asymptotic properties of the possible equilibria. We distinguish between two types of equilibria: those that include a strictly positive bubble and those that do not. Whether the upper bound on optimal consumption asymptotically exceeds the dividend flow of high-wealth type 1 agents determines whether the equilibrium is bubbly or non-bubbly. Since the economy is deterministic, all variables either converge or diverge, and, for notational convenience, we denote asymptotic values without time subscripts.

Asymptotic Lucas Tree Holdings In every equilibrium, the non-homotheticity of preferences leads type 1 agents to hold the entire Lucas tree asymptotically. Assumption 1 implies that saving rates increase strictly with wealth. Since type 1 agents begin with greater initial wealth, it follows by induction that they exhibit higher saving rates and greater holdings of the Lucas tree than type 2 agents in every period. Their relatively stronger marginal preference for wealth over consumption makes them effectively more patient than type 2 agents. As in patient–impatient models à la Becker

(1980), the more patient type ultimately acquires the entire asset stock[4]. Accordingly, we obtain:

$$\lim_{t\to\infty} \ell_t^1 = \frac{1}{\lambda^1}, \qquad \lim_{t\to\infty} \ell_t^2 = 0. \tag{15}$$

Non-bubbly Equilibrium A non-bubbly equilibrium is asymptotically characterized by an upper bound on optimal consumption that exceeds the dividend flow of type 1 agents, inducing them to fully consume their endowment.

Proposition 2 *Asymptotically, if an equilibrium features an upper bound on optimal consumption that is above the endowment received by type* 1 *agents,*

$$\lim_{t\to\infty} \bar{c}_t \geq \frac{1}{\lambda^1}, \tag{16}$$

then it is a non-bubbly equilibrium with $b_t = 0\ \forall t$. The asymptotic behavior of the economy is characterized as follows.

(i) *: Type 1 agents consume their entire endowment and hold constant wealth:*

$$\lim_{t\to\infty} c_t^1 = \frac{1}{\lambda^1}, \qquad \lim_{t\to\infty} a_t^1 = \frac{1/\lambda^1}{R-1}. \tag{17}$$

(ii) *Type 2 agents' consumption and wealth converge to zero:*

$$\lim_{t\to\infty} c_t^2 = \lim_{t\to\infty} a_t^2 = 0. \tag{18}$$

(iii) *The price of the Lucas tree equals its fundamental value:*

$$\lim_{t\to\infty} q_t = \lim_{t\to\infty} f_t = \frac{1}{R-1} \tag{19}$$

In a non-bubbly equilibrium, all variables converge, and the economy approaches a steady state. Asymptotically, type 1 agents hold the entire stock of the Lucas tree, leaving no additional units to purchase from type 2 agents. As a result, they consume their full endowment, and their wealth converges to the level required to sustain this consumption. They hold no surplus wealth, and no bubble arises in equilibrium, consistent with Proposition 1. Conversely, type 2 agents, whose consumption declines strictly during the transition to the steady state, converge asymptotically to zero consumption and zero wealth.

Bubbly Equilibrium A bubbly equilibrium is asymptotically characterized by an upper bound on optimal consumption that is below the dividend flow of type 1 agents, leading them to hold a diverging surplus wealth that supports the bubble.

[4]It can be shown by contradiction that no equilibrium exists in which both type 1 and type 2 agents asymptotically consume exactly their endowment, when $\ell^1 > \ell^2 > 0$.

Proposition 3 *Asymptotically, if an equilibrium features an upper bound on optimal consumption that is strictly below the endowment received by type* 1 *agents,*

$$\lim_{t\to\infty} \bar{c}_t < \frac{1}{\lambda^1}, \tag{20}$$

then it is a bubbly equilibrium with $b_t > 0\ \forall t$*. The asymptotic behavior of the economy is characterized as follows.*

(i) *Type 1 agents consume at the upper bound on optimal consumption and hold diverging wealth:*

$$\lim_{t\to\infty} c_t^1 = \bar{c}, \qquad \lim_{t\to\infty} a_t^1 = \infty. \tag{21}$$

(ii) *Type 2 agents' consumption and wealth converge to positive value:*

$$\lim_{t\to\infty} c_t^2 = c^2 \quad \textit{with} \quad c^2 \equiv \frac{1-\lambda^1\bar{c}}{\lambda^2}, \qquad \lim_{t\to\infty} a_t^2 = \frac{c^2}{R-1}. \tag{22}$$

(iii) *The Lucas tree price diverges over time, reflecting the divergence of its bubbly component:*

$$\lim_{t\to\infty} f_t = \frac{1}{R-1}, \qquad \lim_{t\to\infty} b_t = \infty. \tag{23}$$

Asymptotically, type 1 agents receive the entire endowment but choose not to consume all of it, as doing so would exceed the upper bound on consumption. They are Scrooge McDuck agents, and hold a strictly positive surplus wealth. They are willing to exchange the difference between their endowment, $1/\lambda^1$, and their consumption, $\bar{c}$, for an arbitrarily small quantity of Lucas tree units. This persistent demand for a vanishing supply of Lucas tree units drives its price to diverge. The divergence is fueled by a bubbly component in its valuation which, in line with Proposition 1, coincides with the aggregate surplus wealth held by type 1 agents.

The presence of type 2 agents, who are not Scrooge McDuck, ensures that at most one bubbly equilibrium can exist. Type 2 agents' asymptotic consumption level $\lambda^2 c^2$ corresponds to the endowment not consumed by type 1. They hold a vanishing share of the Lucas tree, whose price diverges. The equilibrium price of the Lucas tree, and thus the size of the bubble, is pinned down by the requirement that type 2 agents retain just enough wealth to sustain their asymptotic consumption c^2. If q_t were above its equilibrium value, type 2 agents would demand more consumption, violating the goods market clearing condition (10); if it were below, some goods would remain unconsumed, which is inconsistent with strictly positive marginal utility of consumption for both agent types. This contrasts with representative-agent models featuring insatiable preferences for wealth, where a marginal propensity to save of one may lead to equilibrium indeterminacy: additional bubble units can be saved without affecting consumption (e.g., Kamihigashi, 2008).

Rate of Return The asymptotic equilibrium rate of return in any bubbly equilibrium is strictly higher than the asymptotic rate of return in any non-bubbly equilibrium. This follows directly from the rewritten Euler equation (4) for type 1 agents:

$$\lim_{t\to\infty} \frac{v'(a_t^1)}{u'(c_t^1)} = \frac{1}{\beta} - R. \tag{24}$$

In any bubbly equilibrium, type 1 agents consume less than $1/\lambda^1$, which is their consumption level in any non-bubbly equilibrium. Moreover, they hold asymptotically more wealth in the bubbly case, as their wealth diverges. As a result, the left-hand side of Equation 24 is necessarily lower in the presence of a bubble than in its absence, implying that the rate of return must be higher under a bubbly equilibrium than under a non-bubbly one.

The relationship between the existence of rational bubbles and the rate of return is fundamentally different for the bubbles that emerge from the surplus wealth of Scrooge McDuck than for most other types of rational bubbles. Rational bubbles that arise under standard preferences for wealth (Michau et al., 2023a), as well as those emerging in overlapping generations models (e.g., Samuelson, 1958, Tirole, 1985) or in the presence of financial frictions (e.g., Martin and Ventura, 2012), require the rate of return to be *sufficiently low* to lie below the growth rate of the economy. This condition ensures that the rational bubble can be a non-dominated asset while the bubble-to-output ratio remains bounded, in settings where agents do not optimally choose to accumulate unbounded wealth. In contrast, in the present model, the rate of return exceeds the (zero) growth rate, with $R \in [1 : 1/\beta]$ in every equilibrium[5]. Still, in a bubbly equilibrium, the rational bubble exists and diverges over time because holding a diverging wealth does not violate the household optimization problem of Scrooge McDuck agents. Furthermore, the bubble exists only because the rate of return is *sufficiently high* to lower the upper bound on optimal consumption enough to allow for the existence of Scrooge McDuck agents.

2.4 Equilibria Characterization

We now characterize the conditions under which bubbly and non-bubbly equilibria exist. If the insatiable component of preferences for wealth is sufficiently strong, the equilibrium is unique and bubbly. Otherwise, both a non-bubbly and a bubbly equilibrium coexist.

Proposition 4 *The existence and uniqueness of equilibrium depend on the strength of the insatiable component in preferences for wealth, as captured by κ.*

(i) *If $\frac{\beta}{1-\beta}\kappa > u'\left(\frac{1}{\lambda^1}\right)$, the equilibrium is unique and bubbly.*

(ii) *If $0 < \frac{\beta}{1-\beta}\kappa \leq u'\left(\frac{1}{\lambda^1}\right)$, two equilibria exist, one bubbly and one non-bubbly, and equilibrium selection is indeterminate.*

(iii) *If $\kappa = 0$, the equilibrium is unique and non-bubbly.*

As $R \in [1, 1/\beta]$, the lower bound on the marginal utility of savings, $\sum_{s=0}^{\infty} \beta^s (\prod_{j=1}^{s} R_{t+j})\kappa$, takes its lowest asymptotic value when $R = 1$, in which case it equals $\frac{\beta}{1-\beta}\kappa$. In case (i), when the marginal

[5] This follows directly from the definition of R, since the price of the Lucas tree, q_t either converges to a positive constant or diverges.

utility associated with consuming the full asymptotic endowment of a type 1 agent, $u'(1/\lambda)$, lies below this lower bound, the upper bound on optimal consumption must, asymptotically, be strictly below the endowment of type 1 agents. It follows from Proposition 3 that the resulting equilibrium is necessarily bubbly. In case (ii), an additional non-bubbly equilibrium exists. Under standard preferences for wealth (iii), there is no upper bound on optimal consumption, and type 1 agents asymptotically choose to consume their entire endowment, resulting in a unique non-bubbly equilibrium. A formal proof of Proposition 4 is provided in Appendix B.1.2.

In the case where two equilibria coexist (ii), equilibrium selection is self-fulfilling. If agents anticipate the high rate of return of the bubbly equilibrium, the upper bound on optimal consumption is low, leading type 1 agents to be Scrooge McDuck. Their surplus wealth results in a rational bubble, which in turn raises the rate of return through its continuous appreciation, thereby validating the initial expectation. Conversely, if agents anticipate the low rate of return associated with the non-bubbly equilibrium, the upper bound on optimal consumption is high, type 1 agents do not behave as Scrooge McDuck agents, and no rational bubble arises.

The existence of multiple equilibria is a strong theoretical result, but its real-world relevance and robustness remain open to discussion. The multiplicity arises from the fact that, under insatiable preferences for wealth, the asymptotic consumption of type 1 agents is particularly responsive to the rate of return. This is evident from the Definition 1 of the upper bound on optimal consumption, which coincides with c^1 in the bubbly case. However, we lack empirical evidence on the elasticity of consumption with respect to the rate of return at the top of the income and wealth distribution. It remains unclear whether the relatively strong elasticity predicted by insatiable preferences for wealth is realistic, or merely a consequence of an overly simplified preference representation. For instance, alternative preference specifications that impose an upper bound on optimal consumption, such as a satiation point in $u(c)$ or preferences defined over new net savings, typically do not generate such multiplicity of equilibria. Consequently, we do not place central importance on the multiplicity of equilibria in our analysis.

To sum up, this simple endowment economy shows that insatiable preferences for wealth differ from standard ones by defining an upper bound on optimal consumption. Whenever this upper bound is asymptotically binding for high-wealth agents, these agents behave as Scrooge McDuck: they accumulate an unbounded surplus wealth, supporting the existence of a rational bubble.

3 Production Economy

We now introduce insatiable preferences for wealth into a growing production economy to show that the existence of bubbly equilibria remains robust in the presence of reproducible capital, and to connect the model to the stylized facts that motivated our analysis. In bubbly equilibria, the wealth-to-output diverges due to a bubble component that crowds out capital, and the capital-to-output ratio ratio converges to a level that that does not maximize asymptotic aggregate consumption.

3.1 Model Environment

Production The production function is a Cobb-Douglas with three inputs: labor, reproducible capital, and a rent-generating factor:

$$Y_t = K_t^{\alpha} L_t^{\gamma} (Z_t N_t)^{1-\alpha-\gamma}, \tag{25}$$

where K_t denotes the capital stock, L_t the rent-generating factor, N_t the labor supply and Z_t the labor productivity. The rent-generating factor is in fixed supply, normalized to $L_t = 1$ for all t, and each of its units is priced at Q_t after production. The capital stock depreciates at a rate δ.

The rent-generating factor is introduced to study the asset pricing implications of insatiable preferences for wealth. It is an infinitely lived, non-reproducible asset, which allows for the existence of a bubbly component in its price. In contrast, the only other asset, capital, has its price fixed at 1 by assumption and cannot be priced above its fundamental value, as it is reproducible. The rent-generating factor is interpreted as encompassing all non-reproducible, long-lived assets, for example, land. Its real-world definition could be subject to debate, depending on how one defines reproducibility and longevity—whether, for instance, it includes long-lived intangible assets such as brands.

The growth rates of productivity and labor supply from period t to $t+1$ are denoted by g^Z_{t+1} and g^N_{t+1}, respectively:

$$g^Z_{t+1} \equiv \frac{Z_{t+1} - Z_t}{Z_t}, \qquad g^N_{t+1} \equiv \frac{N_{t+1} - N_t}{N_t}. \tag{26}$$

Asymptotically, the economy grows at a slower rate than the effective labor force. This is due to the presence of the rent-generating factor, which does not scale with output. As the capital-to-output ratio K_t/Y_t converges, the economy tends to grow at the same rate as $(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}$, whose growth rate, denoted by g_t, is given by:

$$g_{t+1} \equiv \left[(1 + g^Z_{t+1})(1 + g^N_{t+1})\right]^{\frac{1-\alpha-\gamma}{1-\alpha}} - 1. \tag{27}$$

To account for growth, each capital-letter variable X_t has a normalized counterpart x_t, defined as:

$$x_t \equiv \frac{X_t}{(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}}. \tag{28}$$

All figures presented below display variables normalized according to Equation 28. Moreover, we denote $\widetilde{R}_t$, the rate of return adjusted for growth:

$$\widetilde{R}_t \equiv \frac{R_t}{1 + g_t}. \tag{29}$$

Factors of production are paid at their their marginal productivity. At time t, one unit of labor is paid a wage W_t. By arbitrage, the rates of return on capital and on the rent-generating factor between period t to $t+1$ are equalized and denoted by R_{t+1}:

$$R_{t+1} = 1 + \alpha k_{t+1}^{\alpha-1} - \delta = (1+g_{t+1})\frac{q_{t+1} + \gamma y_t}{q_t}. \tag{30}$$

As in the case of the Lucas tree in the endowment economy, the price of the rent-generating factor can be decomposed into a fundamental component, F_t, which corresponds to the present value of future dividends discounted at the rate of return, and a bubbly component, B_t. Using the normalized versions of F_t and B_t, we obtain:

$$f_t \equiv \sum_{j=1}^{\infty} \frac{\gamma y_{t+j}}{\prod_{s=1}^{j} \widetilde{R}_{t+s}}, \qquad b_t = \lim_{T\to\infty} \frac{q_T}{\prod_{s=1}^{T} \widetilde{R}_{t+s}} \qquad \text{with} \quad q_t = f_t + b_t. \tag{31}$$

We assume the rent-generating factor to be productive in every period with $\gamma > 0$. This assumption prevents the rate of return from falling below the growth rate of the economy, following the argument in Rhee (1991). Since the rent-generating factor's share of output does not converge to zero, its return remains above the growth rate asymptotically, and by a no-arbitrage condition, so does the return on capital. We therefore focus on the empirically relevant case, where rents exist in the economy, at a minimum from land, and the rate of return on capital exceeds the growth rate of the economy (Reis, 2021).

Households The population consists of a unit mass of households, each comprising N_t agents. There are N types of infinitely-lived agents, $i \in \{1, 2, ..., N\}$, differing in their labor productivity, and eventually in their initial endowment. Each type represents a share λ^i of the total population. A household of type i holds K_{t+1}^i units of capital and L_{t+1}^i units of the rent-generating factor at the end of period t.

Agents are ranked in decreasing order of labor productivity and initial endowments. Each agent i supplies ζ_t^i units of labor inelastically in period t, where ζ_t^i reflects productivity differences and satisfies,

$$\zeta_t^i > \zeta_t^j \quad \text{for all } i < j. \tag{32}$$

For convenience and without loss of generality, productivity levels are chosen such that $\sum_i \zeta_t^i \lambda^i = 1$ for all t, ensuring that aggregate labor supply coincides with population size and equals N_t. Initial endowments of capital, K_0^i, and of the rent-generating factor, L_0^i, are weakly decreasing in agent index, satisfying,

$$K_0^i \geq K_0^j, \quad L_0^i \geq L_0^j \quad \text{for all } i < j. \tag{33}$$

The normalized end-of-period wealth of type i households, denoted a_{t+1}^i, is defined as,

$$a_{t+1}^i \equiv k_{t+1}^i + \frac{q_t L_{t+1}^i}{1+g_{t+1}}. \tag{34}$$

Their budget constraint is given by,

$$c_t^i + a_{t+1}^i(1+g_{t+1}) = R_t a_t^i + \zeta_t^i w_t, \tag{35}$$

subject to a no-borrowing constraint,

$$a^i_{t+1} \geq 0. \tag{36}$$

We assume a functional form for preferences that satisfies Assumption 1 and allows for insatiable preferences for wealth. Households derive utility from consumption and wealth holdings, and maximize the following intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(a_{t+1}) \right], \tag{37}$$

where the period utility functions are given by,

$$u(c) = \begin{cases} \dfrac{c^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1, \\ \log(c) & \text{if } \theta = 1, \end{cases} \quad \text{and} \quad v(a) = \begin{cases} \psi \cdot \dfrac{(a-\underline{a})^{1-\eta}}{1-\eta} + \kappa a & \text{if } \eta \neq 1, \\ \psi \log(a - \underline{a}) + \kappa a & \text{if } \eta = 1. \end{cases} \tag{38}$$

The utility from consumption, $u(c)$, follows a CRRA specification, while the preference for wealth, $v(a)$, consists of two components: a standard concave term and an additional linear component that captures insatiable preferences for wealth. The parameters are chosen such that $\eta < \theta$ and $\underline{a} \geq 0$, ensuring that Assumption 1 holds and that saving rates increase with both income and wealth. The parameter ψ is a scaling parameter that captures the strength of standard preferences for wealth relative to utility from consumption. The linear term in $v(a)$ imposes a lower bound on the marginal utility of holding wealth at κ, that is,

$$\lim_{a \to \infty} \frac{\partial v(a)}{\partial a} = \kappa. \tag{39}$$

We refer to preferences with $\kappa = 0$ as standard preferences for wealth, and to those with $\kappa > 0$ as insatiable preferences for wealth.

To ensure that aggregate saving rates do not continuously increase as the economy grows, the variables entering the utility function are expressed in normalized terms rather than in absolute levels. As discussed in Mian et al. (2021), the purpose of preferences for wealth is to break individual scale invariance, so that wealthier households exhibit higher saving rates, while preserving aggregate scale invariance, thereby preventing the saving rate of the economy as a whole from rising with its size. Indeed, predicting an increase in saving rates as the economy grows would be counterfactual. In frameworks with log utility of consumption (e.g., Mian et al., 2021; Michau et al., 2023a), specifying utility over the absolute level of consumption and the normalized value of wealth is sufficient to achieve aggregate scale invariance. However, this paper also considers cases where $\theta \neq 1$, in which case consumption must likewise be normalized to preserve aggregate scale invariance.

Whenever the no-borrowing constraint (36) is not binding, the solution to the optimization problem for type i households is characterized by the following Euler equation (40) and the transversality condition (41):

$$c_t^{i\,-\theta} = \beta R_{t+1} \frac{(c_{t+1}^i)^{-\theta}}{1+g_{t+1}} + \psi \frac{(a_{t+1}^i - \underline{a})^{-\eta}}{1+g_{t+1}} + \frac{\kappa}{1+g_{t+1}}, \tag{40}$$

$$\lim_{t\to\infty} \beta^t \Big[c_t^{i\,-\theta} - \frac{\psi(a_{t+1}^i - \underline{a})^{-\eta} + \kappa}{1+g_{t+1}} \Big] a_{t+1}^i = 0. \tag{41}$$

Definitions We adapt the definitions of the core elements of the Scrooge McDuck analysis to account for growth and labor income. Similarly to the endowment economy, insatiable preferences for wealth impose a lower bound on the marginal utility of saving, which in turn determines an upper bound on consumption, $\overline{C}_t$. Its normalized counterpart, $\overline{c}_t$, is given by,

$$\overline{c}_t \equiv \left[\frac{1}{1+g_{t+1}} \sum_{s=0}^{\infty} \beta^s \left(\prod_{j=1}^{s} \frac{R_{t+j}}{1+g_{t+j+1}} \right) \kappa \right]^{-\frac{1}{\theta}}, \tag{42}$$

where the term in brackets represents the lower bound on the marginal utility of saving. Because the absolute upper bound on consumption, as well as average absolute consumption and wealth, diverge in a growing economy, we refine the definition of Scrooge McDuck agents to use normalized rather than absolute consumption and wealth. Agent i is a Scrooge McDuck if it accumulates unbounded *normalized* wealth, a_t^i, over time while keeping its *normalized* consumption, c_t^i, bounded. Finally, the definition of surplus wealth, still referring to wealth held without consumption motives, is modified to account for the fact that part of future consumption is financed by wages. Its normalized version is given by,

$$s_{t+1}^i \equiv a_{t+1}^i - \sum_{j=1}^{\infty} \frac{c_{t+j}^i - \zeta^i w_{t+j}}{\prod_{s=1}^{j} \widetilde{R}_{t+s}}. \tag{43}$$

From the goods and asset market-clearing conditions, it follows that Proposition 1 holds in the production economy, and thus that aggregate surplus wealth coincides with the aggregate bubble.

Market Clearing Conditions The goods market clearing condition writes as,

$$k_{t+1}(1+g_{t+1}) + \sum_i \lambda^i c_t^i = (1-\delta)k_t + y_t. \tag{44}$$

The asset market clearing conditions for capital and for the rent-generating factor are given by:

$$k_t = \sum_i \lambda^i k_t^i, \tag{45}$$

$$1 = \sum_i \lambda^i L_t^i. \tag{46}$$

Equilibrium Given the initial endowments $\{K_0^i, L_0^i\}$ for each agent $i \in \{1, 2, \ldots, N\}$, an equilibrium $\left\{\{c_t^i, k_t^i, L_t^i\}_{i\in\{1,2,\ldots,N\}}, q_t\right\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems and the market clearing conditions for capital and the rent-generating factor. The goods market clearing condition holds by Walras's Law.

3.2 Bubbly and Non-bubbly Equilibria

We solve the model numerically and find that bubbly equilibria also exist in the presence of reproducible capital. As the insatiable component of wealth preferences strengthens or the degree of labor income inequality rises, the set of parameters supporting a bubbly equilibrium expands, while that supporting a non-bubbly one shrinks.

A Numerical Illustration We consider an illustrative exercise in which two transition paths are computed for low and high values of κ, yielding, respectively, a bubbly and a non-bubbly equilibrium. At this stage, our focus is purely theoretical, with a more complete quantitative exercise presented in Section 5.

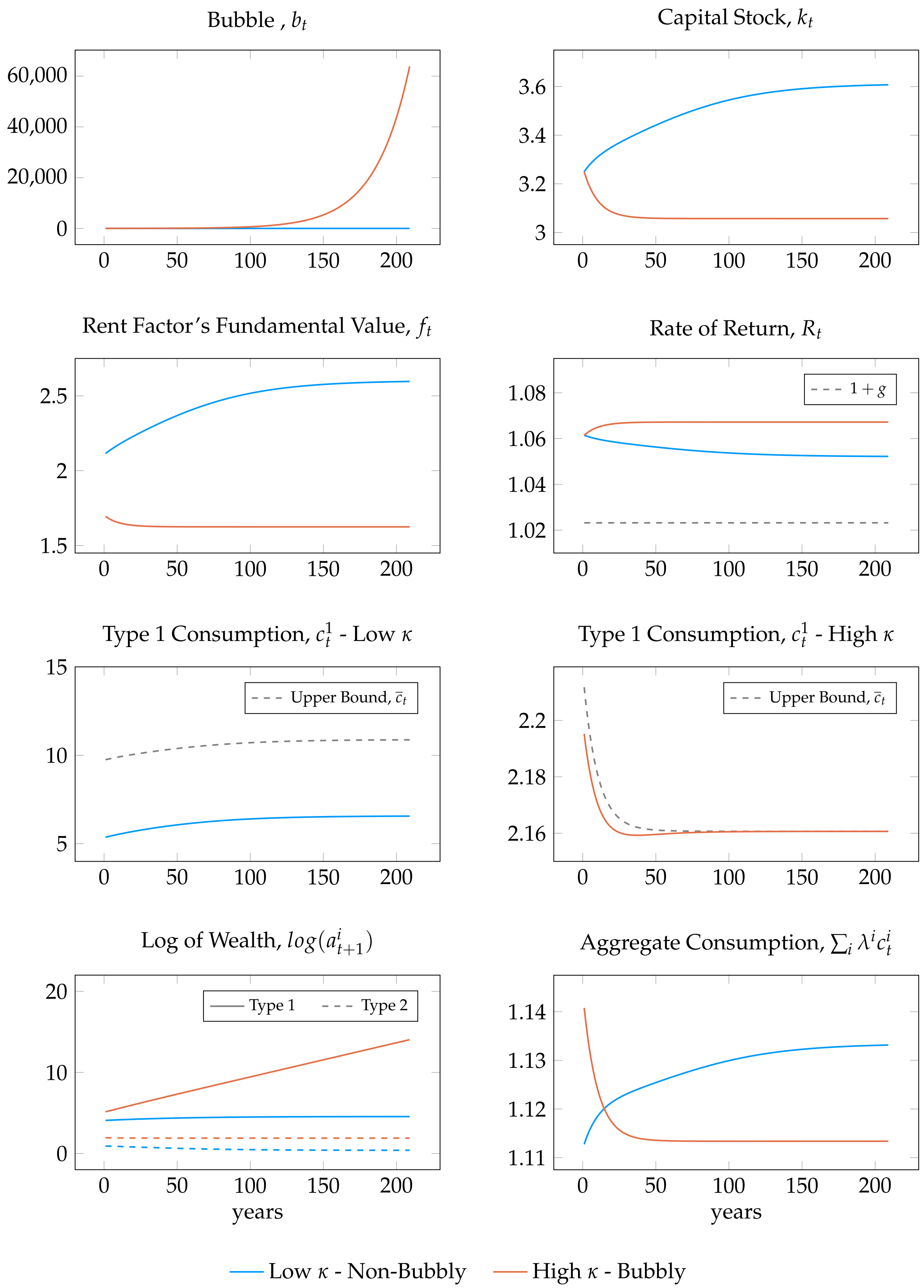
The population is divided into two types. Type 1 represents 5% of the population, $\lambda^1 = 0.05$, earns 25% of labor income in every period, $\zeta_t^1 = 5\ \forall t$, and holds 55% of the initial stock of capital and rent-generating factor endowments. Production parameters are set to $\gamma = 0.05$, $\alpha = 0.3$, $\delta = 0.07$, $g_t^Z = 1.5\%$, and $g_t^N = 1\%$ for all t. The discount factor is $\beta = 0.95$, and the consumption utility and standard preference-for-wealth parameters follow Kumhof et al. (2015): $\theta = 2$, $\eta = 1.09$, and $\psi = 0.05$. We consider two values for the insatiable preference-for-wealth parameter: a low value, $\kappa = 0.0002$, and a high value, $\kappa = 0.002$. Starting from an initial capital stock of $k_0 = 3.25$, the transition dynamics are shown in Figure 2.

When κ is high, the upper bound on optimal consumption binds for type 1 agents, which are Scrooge McDuck. From Equation 42, the upper bound on optimal consumption, $\bar{c}_t$, decreases with the rate of return and with the strength of the insatiable preference for wealth. As a result, in our example it binds asymptotically for type 1 agents exclusively in the high-κ case, where it is lower than under low κ[6]. Under high κ, type 1 agents' consumption remains bounded, and, in normalized terms, their income persistently exceeds their consumption. It follows that their their wealth diverges over time. They behave as Scrooge McDucks and hold positive surplus wealth, $s_t > 0$, for all t.

When strong enough to generate a bubbly equilibrium, insatiable preferences for wealth produce a diverging wealth-to-output ratio, while the capital-to-output ratio converges. Under high κ, the surplus wealth held by type 1 agents coincides in the aggregate with a rational bubble, $b_t = \lambda^1 s_{t+1}^1$. Since the bubble grows at the rate of return, which exceeds the economy's growth rate, its normalized size diverges. Asymptotically, this divergence in the bubble-to-output ratio raises the wealth-to-output ratio without affecting the capital-to-output ratio. The rational bubble is the key asset-pricing mechanism that disconnects wealth dynamics from those of capital. Bubbly

[6] For simplicity, we do not consider cases in which the upper bound on optimal consumption bind for both types.

Figure 2: Transition Dynamics in the Production Economy: Bubbly vs. Non-Bubbly Equilibrium



Transitions starts from the same initial capital stock, $k_0 = 3.25$, and asset distribution, with type 1 agents holding 55% of initial wealth, under either low ($\kappa = 0.0002$, blue line) or high ($\kappa = 0.002$, red line) insatiable preferences for wealth. Purely illustrative graphs.

equilibria thus align with the wealth and capital trends observed in many advanced economies in recent decades, motivating this paper.

In contrast, under low insatiable preferences for wealth—of which standard preferences are a special case— the equilibrium is non-bubbly. In the low-κ case, the upper bound on optimal consumption is not binding for type 1 agents, who do not accumulate unbounded wealth. There are no Scrooge McDuck agents, no associated surplus wealth, and thus no rational bubble. Given our initial k_0, the wealth-to-output ratio increases over the transition but does not diverge. This rise is intrinsically linked to the increase in the capital-to-output ratio. The latter raises the wealth-to-output ratio both directly, and indirectly by increasing the fundamental value of the rent-generating factor through a lower rate of return, R_t, and higher associated rents.

The presence of a rational bubble crowds out capital. As the capital stock comparison shows, despite a stronger preference for wealth, the high-κ case features a lower capital stock. This occurs because, under high κ, part of agents' savings goes into the bubble, but not under low κ. In turn, lower capital stock levels imply lower output and higher rates of return. Type 2 agents' wages are lower in the high-κ case, while their wealth income benefits from the capital gains associated with the rational bubble. The parametrization determines the relative strength of these channels for asymptotic consumption and, in particular, whether type 2 agents consume more in the low-κ non-bubbly case or in the high-κ bubbly one. By reducing rents and increasing the discount factor, the lower capital stock under high κ also fully explains why the fundamental value of the rent-generating factor is higher under low κ.

The bubble's crowding-out of capital reduces asymptotic aggregate consumption. A well-established result implies that, whenever $R > 1 + g$, an increase in the capital stock raises asymptotic consumption, whereas it lowers it when $R < 1 + g$[7]. By the assumption of a productive rent-generating factor, we are necessarily in the case where the rate of return exceeds the growth rate, and the bubble's crowding-out reduces asymptotic consumption. This can be seen in Figure 2, where aggregate consumption is asymptotically higher in the non-bubbly, low-κ scenario than in the bubbly, high-κ scenario. However, during the transition to this asymptotic state, aggregate consumption is initially higher under high κ, as fewer resources are allocated to capital investment. Notice that neither of the two equilibria considered reaches the capital level that maximizes asymptotic consumption and would imply $R = 1 + g$.

Comparative Statics When the insatiable component of wealth preferences κ is sufficiently low, the economy admits a non-bubbly equilibrium; when it is sufficiently high, a bubbly equilibrium exists. The left panel of Figure 3 shows the different equilibria and their associated capital levels as a function of κ. At $\kappa = 0$, which corresponds to standard preferences for wealth, the equilibrium is necessarily unique and non-bubbly. More generally, there exist two threshold values of κ: below the lower threshold the equilibrium is unique and non-bubbly, above the upper threshold it is unique and bubbly. In the intermediate range, one bubbly and one non-bubbly equilibrium coexist.

[7] This result traces back to Diamond (1965). Asymptotically, a one-unit increase in the capital stock raises output by $\alpha k^{\alpha-1}$ and investment by $\delta + g$ in normalized terms. If $\alpha k^{\alpha-1} > \delta + g$, it raises aggregate consumption—a condition that can be rewritten as $R > 1 + g$.

As in the endowment economy, equilibrium selection is self-fulfilling: if agents expect high returns, the upper bound on consumption is low and binding, leading to a bubble that sustains high returns; if they expect low returns, the opposite holds.

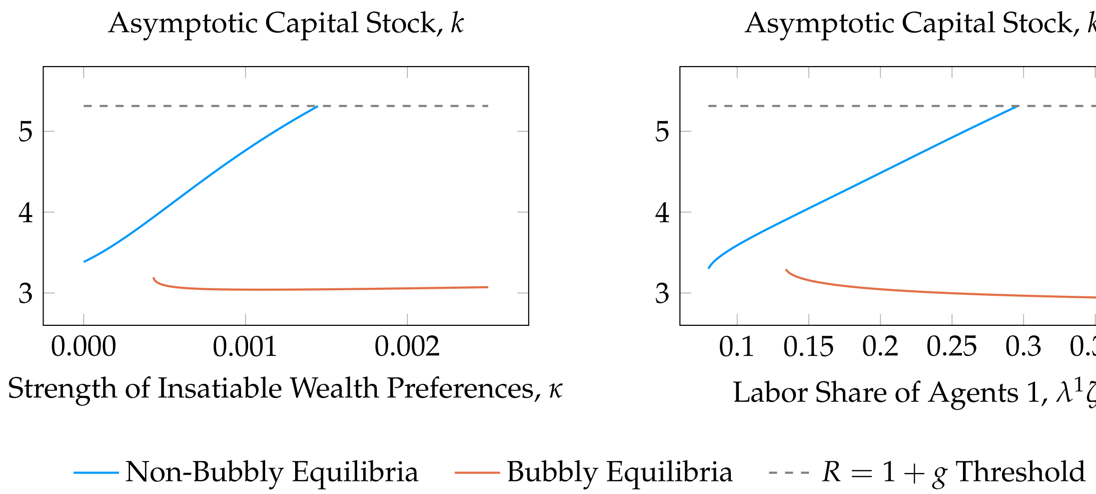
The asymptotic capital stock rises with stronger insatiable preferences for wealth, as long as the equilibrium does not turn bubbly. A stronger preference for wealth strengthens saving incentives and, consequently, aggregate savings. In the absence of bubbles, this inevitably translates into a larger capital stock[8]. The non-bubbly equilibrium disappears once κ exceeds the threshold at which further capital accumulation would drive the asymptotic return below the growth rate. This case is precluded by the presence of a rent-generating factor of production, as discussed in Subsection 3.1. Furthermore, any marginal increase in κ that shifts the economy from a non-bubbly to a bubbly equilibrium necessarily lowers the asymptotic capital stock, reflecting the crowding-out effect of the bubble.

The comparative statics with respect to labor income inequality yield results analogous to those obtained for κ. In this simplified two-agent framework, labor income inequality is captured entirely by the labor share of type-1 agents. The right panel of Figure 3 shows how the equilibria and their associated capital levels depend on this measure. When labor inequality is low, type-1 agents earn insufficient income to hit the asymptotic upper bound on consumption, which therefore never binds. In this case, the equilibrium is unique and non-bubbly. Conversely, when labor inequality exceeds a given threshold, the upper bound always binds and the equilibrium is unique and non-bubbly. In the intermediate range, a bubbly and a non-bubbly equilbrium coexist with self-fulfilling equilibrium selection. Finally, increasing labor inequality raises aggregate savings by redistributing income from low-saving to high-saving agents. This translates into a higher capital stock, as long as the equilibrium remains non-bubbly.

This comparative static exercise suggests that the widely observed capital-gains–driven increase in the wealth-to-output ratio could stem from the concomitant rise in top income inequality. This is illustrated in Figure 3, where rising labor income inequality may translate into the existence, or even the necessity, of a bubbly equilibrium. It is important to note that the mechanism at play is not one in which a one-time increase in income inequality leads to a one-time rise in wealth through valuation effects. Rather, higher inequality shifts the economy from a non-bubbly regime to a bubbly one, generating persistent capital gains. In our model, when unrealized capital gains are excluded from the definition of income, income inequality converges in bubbly equilibria while the wealth-to-output ratio diverges. This could rationalize why the wealth-to-output ratio has continued to rise in many economies where income inequality appears to have converged, under a definition of income that excludes unrealized capital gains (Figures 1a and 1b).

[8] A similar result is obtained when the taste for wealth is increased through the standard wealth-preference component of utility (for instance, a higher value of ψ) rather than through the insatiable one.

Figure 3: Capital Accumulation by Strength of Insatiable Wealth Preferences and Labor Inequality



3.3 Discussion

The model generates two unusual outcomes within a neoclassical framework—a non-zero mass of agents holding diverging wealth and a diverging rational bubble—and, in doing so, provides insights to several strands of the literature.

Capital Gains The substantial capital gains observed across many advanced economies since the 1980s have given rise to a growing debate on their welfare implications. This debate has mostly centered on asset price increases driven by a decline in the discount rate, which raises the fundamental value of assets. Such capital gains mechanically increase absolute wealth inequality without affecting the distribution of capital income. As long as households do not buy or sell assets, such capital gains do not affect income and thus cannot be used to sustainably finance consumption. They have been interpreted in various ways: as a pure increase in welfare inequality (Saez et al., 2021), as mere "paper gains" with no welfare effect (Cochrane, 2020; Krugman, 2021), or as a mix of both, depending on net asset sales (Fagereng et al., 2024).

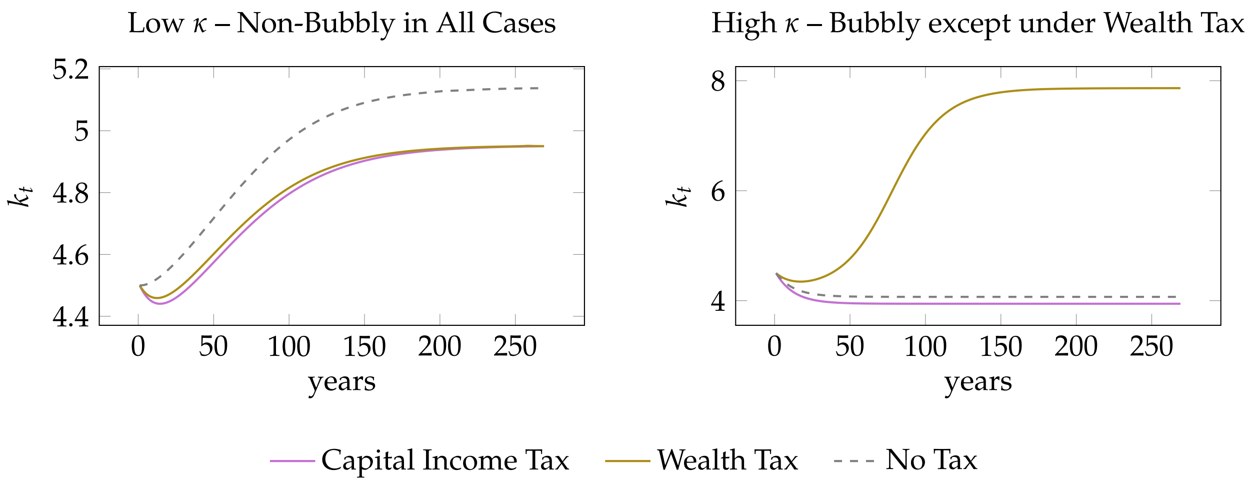
This paper proposes a theory in which part of the capital gains arise from a rational bubble and, in contrast to discount-rate–driven fundamental capital gains, can be used to permanently finance some consumption. The rational bubble is a non-dominated asset because its absolute value rises by R_t between periods t and $t+1$. Hence, it generates capital gains in every period. This allows non–Scrooge McDuck agents to steadily finance part of their consumption by selling some bubble units each period. This result is especially relevant given empirical evidence from the middle 40% of the wealth distribution, which has experienced substantial capital gains—particularly in housing—across many advanced economies since the 1980s (Bauluz et al., 2022; Blanchet and Martínez-Toledano, 2023). Furthermore, Mian et al. (2020) shows that, in the US, capital gains realized by the bottom 90% have been used to finance consumption through increased collateralized debt. This can be interpreted as a real-world illustration of agents using capital gains to finance consumption.

Return on Wealth Trend Interpreting a significant share of the capital gains of recent decades as bubble-driven attenuates the interpretation that the risky discount rate has been on a decreasing trend. If assets are assumed to be priced at their fundamental value, the ex-ante expected rate of return appears to have declined in recent decades, as indicated by declining rent- or dividend-to-price ratios (Kuvshinov and Zimmermann, 2021) and quantitative analyses (Eggertsson et al., 2021). This decline in the discount rate would have contributed to the observed capital gains, temporarily increasing the ex-post rate of return on wealth. In contrast, if these capital gains had been observed on a rational bubble, agents would anticipate future capital gains, implying that the ex-ante expected rate of return would be higher than under the assumption of no bubble. This insight is particularly relevant in light of studies arguing that the ex-ante expected rate of return may have remained stable or only slightly declined (Duarte and Rosa, 2015; Caballero et al., 2017; Reis, 2022).

A Unified Framework for Rising Wealth-to-Output Ratios This paper proposes a unified explanation for the widely observed rise in the wealth-to-output ratio—attributing it to a global and simultaneous phenomenon: the increase in permanent income inequality. Large capital gains are a prevalent stylized fact, but as noted in the introduction, the types of assets benefiting from them vary substantially across regions (Figure 1d). The latter aspect has led some lines of research to develop country-specific explanations for a global phenomenon. For instance, in the US, the rise in stock values is often attributed to increasing market power (Farhi and Gourio, 2018; Eggertsson et al., 2021), while the low elasticity of housing supply is frequently cited to explain the rise in real estate prices in Europe (Hilber and Vermeulen, 2016; Muellbauer, 2018). In contrast, this paper offers a framework in which widespread capital gains stem from an equally widespread cause—rising income inequality—through an asset-pricing feature, the rational bubble, which can emerge in a broad range of real-world assets.

Relation to Piketty's Capital in the Twenty-First Century By offering a microfounded general equilibrium model in which the rate of return exceeds the growth rate and wealth inequality diverges, this framework contributes to the discussion initiated by Piketty (2014) on the role of the $r - g$ gap in driving wealth inequality. A key conjecture of Piketty (2014) (p. 34) posits that "if the rate of return on capital remains significantly above the growth rate for an extended period, the risk of divergence in the distribution of wealth is very high". Addressing this conjecture in a model with standard preferences for wealth, Michau et al. (2023a) refutes the possibility of r remaining asymptotically above g when wealth inequality is diverging. As wealth at the top of the distribution diverges, aggregate savings should rise, resulting in greater capital accumulation and a corresponding decline in the rate of return. In contrast, by introducing a diverging rational bubble, this paper breaks the link between higher wealth inequality and higher capital stock. In doing so, it provides the first micro-founded general equilibrium model in which the rate of return remains asymptotically above the growth rate and wealth inequality diverges, thereby aligning with Piketty's conjecture.

Figure 4: Capital Dynamics under Taxation of Capital Income vs Wealth



Notes

4 Tax Policy Implications

Whether the economy is in a bubbly or non-bubbly equilibrium has important implications for taxation. Any non-vanishing wealth tax—and certain capital income taxes—rule out the existence of rational bubbles, and their introduction can foster capital accumulation.

4.1 Capital Tax Instruments: Income vs. Wealth Taxation

We introduce capital income and wealth taxation into the previous model. The standard equivalence between the two breaks down in the presence of a bubble, which enters the tax base of the wealth tax but not that of the capital income tax.

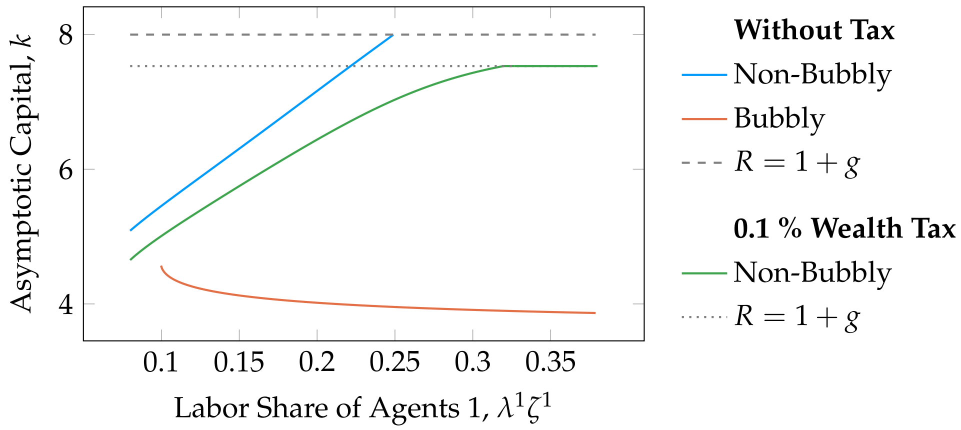
4.2 Capital Accumulation and Tax Policy

A bubbly equilibrium is ruled out in the presence of a non-vanishing wealth tax, as it would imply unbounded tax revenues and violate the goods market clearing condition. By deterring the formation of a bubble, the introduction of some sufficiently small taxes can prevent the bubbly crowding-out and supports higher capital accumulation.

5 Quantitative Assessment

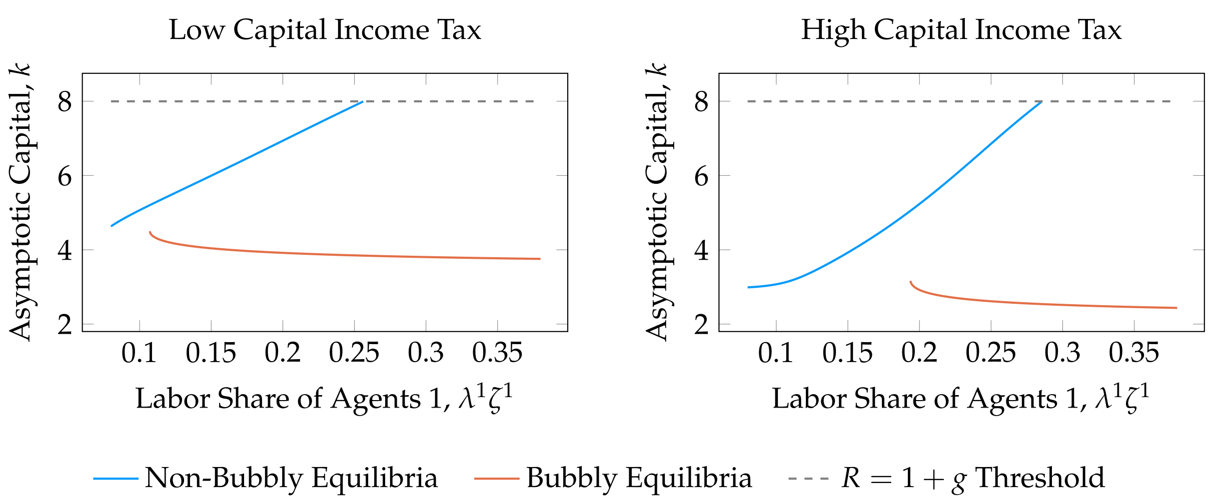
We conduct a simple calibration exercise to assess the quantitative relevance of the Scrooge McDuck mechanism. We then derive optimal tax levels for a range of social planners, depending on their preferences over wealth and the social weight assigned to the top 1 percent of the wealth distribution.

Figure 5: Equilibria with and without a Wealth Tax



Notes

Figure 6: Equilibria under Low vs. High Capital Income Taxation



Notes: Shift vers la droite ... et vers le bas !

[In progress]

5.1 Main Exercise

A simple calibration exercise is now conducted to assess the quantitative relevance of the Scrooge McDuck mechanism. The model is calibrated to the U.S. economy. It features four agent types corresponding to the top 1% $[99, 100]$, the top next 9% $[90, 99)$, the middle 40% $[50, 90)$, and the bottom 50% $[0, 50)$ percentiles of both the labor income and wealth distributions. Due to the absence of idiosyncratic shocks, the income and wealth rankings are fully aligned across agent types. The analysis abstracts from wealth or bequest taxation, which are shut down in the baseline. Moreover, to ensure that the results are not driven by land, it is assumed to be unproductive.

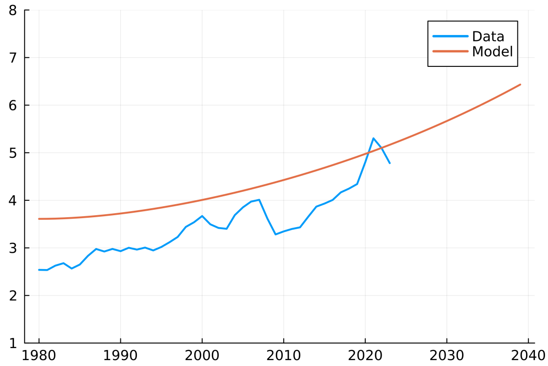
The initial steady state is set to replicate the economic conditions of 1980, with parameters chosen to match the wealth-to-output ratio, capital-to-output ratio, and the distribution of wealth at that time. Labor income, aggregate wealth levels and wealth shares are taken from Blanchet et al. (2022). The value of the capital stock corresponds to private fixed assets as measured by the Bureau of Economic Analysis (BEA), while GDP data are taken from the Federal Reserve Economic Data (FRED) database. A full characterization of the calibration is provided in Table 1. A permanent increase in labor income inequality is then introduced by assuming the pre-tax labor income distribution corresponds to that observed in 2000 for every subsequent period. Following this shock, the economy transitions to a diverging equilibrium, as shown in Fig 7.

Due to the perfect alignment between income and wealth distributions, the present stylised model cannot accurately match empirical wealth share levels. In the initial steady state, the wealth share of the top 9% is underestimated (26% instead of 42%), while the shares of the top 1% and the middle 40% are overestimated (33% and 42% instead of 23% and 34%, respectively). Given these discrepancies, the analysis focuses on the evolution of wealth shares rather than their exact levels. While the model overestimates the increase in the wealth share of the top 1% and the decline in the share of the middle 40%, it still captures the qualitative dynamics reasonably well.

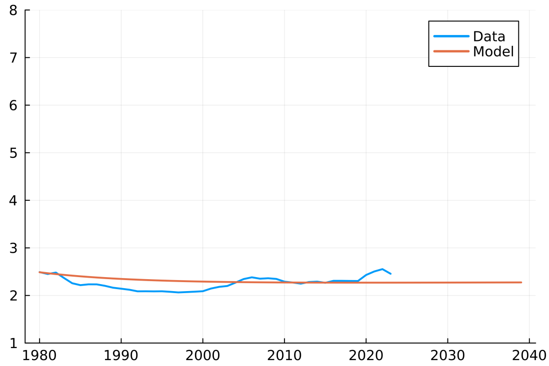
Regarding the dynamics that motivate this analysis, the simulated wealth-to-output ratio tends to lie above its value in the data. This is mostly driven by an artificial jump in 1980, when the increase in labor income inequality triggers a transition from a non-bubbly to a bubbly equilibrium, accompanied by the emergence of the bubble. By predicting a converging capital-to-output ratio, the model performs well in fitting the data and captures the disconnection between the wealth-to-output and capital-to-output ratios.

This initial quantitative exercise therefore yields encouraging results, and further work will aim to refine the calibration and extend the analysis. Natural next steps include incorporating progressive labor income taxation, introducing a wealth tax, and allowing land to be a productive asset.

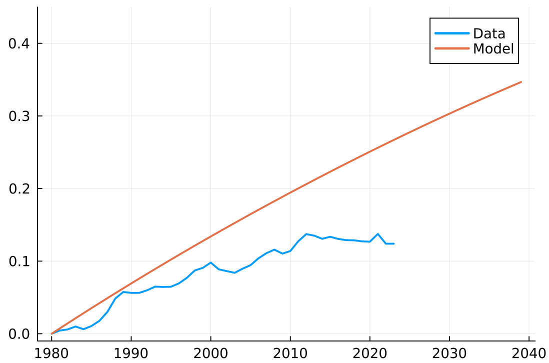
Figure 7: Quantitative Assessment for the U.S., 1980–2040: Baseline Scenario (in progress)



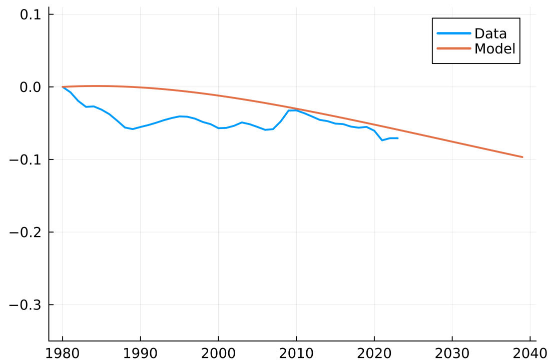
(a) Wealth-to-Output Ratio



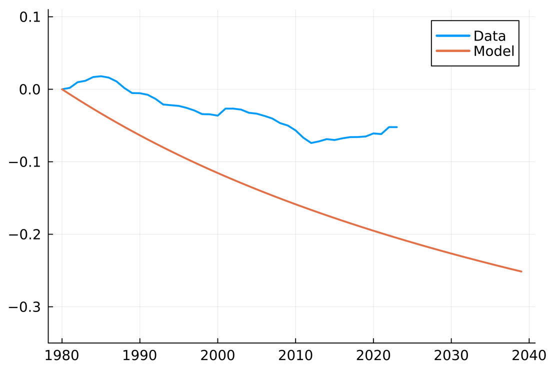
(b) Capital-to-Output Ratio



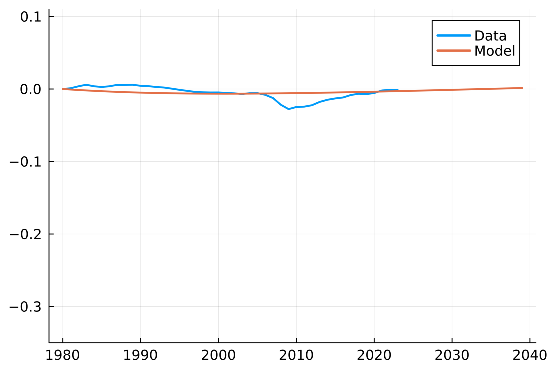
(c) Top 1% Wealth Share Change (p.p.)



(d) Next 9% Wealth Share Change (p.p.)



(e) Middle 40% Wealth Share Change (p.p.)



(f) Bottom 50% Wealth Share Change (p.p.)

Table 1: Calibration of the Baseline Model

Parameters	Description	Value	Source
Production			
α	Capital factor share	0.33	
γ	Land factor share	0.0	
δ	Capital depreciation rate	0.07	
ζ^i	Pre-tax labor income share in 1980 and 2000 and linear tax of 0.28		Blanchet et al. (2022)
g_t^N	Labor force growth rate - long-term average (1980-2023)	1.1%	U.S. Bureau of Labor Statistics
g_t^Z	Labor productivity growth rate - long-term average (1980-2023)	1.8%	Calculated as a residual
Preferences			
β	Discount rate	0.925	
ψ	Weight on taste for wealth		Internally calibrated
θ	CRRA coefficient for consumption	2	
η	CRRA coefficient for wealth		Internally calibrated
a_{min}	Stone-Geary parameter for wealth		Internally calibrated
κ	Linear coefficient in the utility from wealth		Internally calibrated
Taxation			
τ^{KI}	Capital income tax rate	0.35	Brun and González (2017)
τ^W	Wealth tax rate	0.0	

5.2 Welfare and Optimal Taxation

Beyond the standard efficiency–redistribution trade-off, taxation can also prevent rational bubbles. A marginal increase in taxation that removes or deters a bubble lowers aggregate wealth but raises capital accumulation. Its effect on welfare is ambiguous, as it increases wages and reduces returns, and it varies across agents, since the wealth-to-income ratio rises along the wealth distribution.

[In progress]

6 Conclusion

This paper proposes a parsimonious explanation for the rise in the wealth-to-output ratio, the stagnation (or decline) of the capital-to-output ratio, and the increase in wealth inequality observed across advanced economies in recent decades. The core assumption of this framework is that agents derive *insatiable* utility from wealth. As a result, agents at the top of the income distribution accumulate a *surplus wealth* solely for the sake of holding it. They are defined as *Scrooge McDuck* agents, as they ultimately hold unbounded wealth while maintaining bounded consumption relative to output. Their surplus wealth drives asset prices above their fundamental value, leading to a rational bubble. The latter grows at a rate exceeding the rate of return, enabling a disconnect between the wealth-to-output ratio and the capital-to-output ratio.

The Scrooge McDuck theory carries several implications. It suggests that wealth inequality may follow a diverging trajectory; that increases in income or wealth inequality do not necessarily translate into higher investment; and that the rate of return may remain permanently above the dividend-to-price ratio due to capital gains on a diverging rational bubble. By preventing unbounded wealth accumulation, a wealth tax—or sufficiently high capital income taxation—eliminates the Scrooge McDuck mechanism and the associated rational bubble, potentially fostering capital accumulation. The next steps in this research involve refining the quantitative exercise, conducting a welfare analysis of a wealth tax, and examining the model's theoretical implications for public debt.

References

Batty, Michael et al. (2019). Introducing the distributional financial accounts of the United States. In.

Bauluz, Luis, Filip Novokmet and Moritz Schularick (2022). The Anatomy of the Global Saving Glut. In.

Becker, Robert A (1980). On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. In: The Quarterly Journal of Economics 95.2, 375–382.

Benhabib, Jess, Alberto Bisin and Mi Luo (2019). Wealth distribution and social mobility in the US: A quantitative approach. In: American Economic Review 109.5, 1623–1647.

Blanchet, Thomas and Clara Martínez-Toledano (2023). Wealth inequality dynamics in Europe and the United States: Understanding the determinants. In: Journal of Monetary Economics 133, 25–43.

Blanchet, Thomas, Emmanuel Saez and Gabriel Zucman (2022). *Real-time inequality*. Tech. rep. National Bureau of Economic Research.

Brun, Lídia and Ignacio González (2017). Tobin's Q and Inequality. In: Available at SSRN 3069980.

Caballero, Ricardo J, Emmanuel Farhi and Pierre-Olivier Gourinchas (2017). Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. In: American Economic Review 107.5, 614–620.

Carroll, Christopher D (2000). 'Why Do the Rich Save So Much?' In: *Does atlas shrug?: The economic consequences of taxing the rich*. ed. by J. Slemrod, Harvard University Press.

Chancel, Lucas et al. (2022). *World inequality report 2022*. Harvard University Press.

Cochrane, John (2020). Wealth and Taxes, part II. In: Blog post.

De Nardi, Mariacristina (2004). Wealth inequality and intergenerational links. In: The Review of Economic Studies 71.3, 743–768.

De Nardi, Mariacristina and Fang Yang (2016). Wealth inequality, family background, and estate taxation. In: Journal of Monetary Economics 77, 130–145.

Diamond, Peter A (1965). National debt in a neoclassical growth model. In: The American Economic Review 55.5, 1126–1150.

Duarte, Fernando and Carlo Rosa (2015). The equity risk premium: a review of models. In: Economic Policy Review 2, 39–57.

Dynan, Karen E, Jonathan Skinner and Stephen P Zeldes (2004). Do the rich save more? In: Journal of political economy 112.2, 397–444.

Eggertsson, Gauti B, Jacob A Robbins and Ella Getz Wold (2021). Kaldor and Piketty's facts: The rise of monopoly power in the United States. In: Journal of Monetary Economics 124, S19–S38.

Elina, Eustache and Raphaël Huleux (2023). *From Labor Income to Wealth Inequality in the U.S.* Tech. rep.

Fagereng, Andreas et al. (2021). Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains. In.

Fagereng, Andreas et al. (2024). *Asset-price redistribution*. Tech. rep.

Farhi, Emmanuel and François Gourio (2018). Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia. In: Brookings Papers on Economic Activity.

Farhi, Emmanuel and Jean Tirole (2012). Bubbly liquidity. In: The Review of economic studies 79.2, 678–706.

Gaillard, Alexandre et al. (2023). *Consumption, Wealth, and Income Inequality: A Tale of Tails*. Tech. rep.

Gutiérrez, Germán and Thomas Philippon (2017). Investmentless Growth: An Empirical Investigation. In: Brookings Papers on Economic Activity 2017.2, 89–190.

Hilber, Christian AL and Wouter Vermeulen (2016). The impact of supply constraints on house prices in England. In: The Economic Journal 126.591, 358–405.

Hubmer, Joachim, Per Krusell and Anthony A Smith Jr (2021). Sources of US wealth inequality: Past, present, and future. In: NBER Macroeconomics Annual 35.1, 391–455.

Kamihigashi, Takashi (2002). A simple proof of the necessity of the transversality condition. In: Economic theory 20, 427–433.

— (2008). The spirit of capitalism, stock market bubbles and output fluctuations. In: International Journal of Economic Theory 4.1, 3–28.

Katz, Lawrence F and Kevin M Murphy (1992). Changes in relative wages, 1963–1987: supply and demand factors. In: The quarterly journal of economics 107.1, 35–78.

Krugman, Paul (2021). Pride and Prejudice and Asset Prices. In: International New York Times, NA–NA.

Kumhof, Michael, Romain Rancière and Pablo Winant (2015). Inequality, leverage, and crises. In: American economic review 105.3, 1217–1245.

Kuvshinov, Dmitry and Kaspar Zimmermann (2021). The expected return on risky assets: International long-run evidence. In: Available at SSRN 3546005.

Lucas, Robert E (1978). Asset prices in an exchange economy. In: Econometrica: journal of the Econometric Society, 1429–1445.

Martin, Alberto and Jaume Ventura (2012). Economic growth with bubbles. In: American Economic Review 102.6, 3033–3058.

Mian, Atif, Ludwig Straub and Amir Sufi (2020). *The saving glut of the rich*. Tech. rep. National Bureau of Economic Research.

— (2021). Indebted demand. In: The Quarterly Journal of Economics 136.4, 2243–2307.

Michau, Jean-Baptiste, Yoshiyasu Ono and Matthias Schlegl (2023a). *The Preference for Wealth and Inequality: Towards a Piketty Theory of Wealth Inequality*. Tech. rep.

— (2023b). Wealth preference and rational bubbles. In: European Economic Review, 104496.

Muellbauer, John (2018). Housing, debt and the economy: A tale of two countries. In: National Institute Economic Review 245, R20–R33.

Ono, Yoshiyasu (1994). 'Money, interest, and stagnation: Dynamic theory and Keynes's economics'. In: Oxford University Press, USA. Chap. 11.

Piketty, Thomas (2014). *Capital in the twenty-first century*. Harvard University Press.

Piketty, Thomas and Emmanuel Saez (2003). Income inequality in the United States, 1913–1998. In: The Quarterly journal of economics 118.1, 1–41.

Piketty, Thomas and Gabriel Zucman (2014). Capital is back: Wealth-income ratios in rich countries 1700–2010. In: The Quarterly journal of economics 129.3, 1255–1310.

Reis, Ricardo (2021). The constraint on public debt when r¡ g but g¡ m. In: CEPR Discussion Papers 15950.

— (2022). Which r-star, Public Bonds or Private Investment? Measurement and Policy implications. In: LSE manuscript.

Rhee, Changyong (1991). Dynamic inefficiency in an economy with land. In: The Review of Economic Studies 58.4, 791–797.

Saez, Emmanuel, Danny Yagan and Gabriel Zucman (2021). Capital gains withholding. In: University of California Berkeley.

Saez, Emmanuel and Gabriel Zucman (2016). Wealth inequality in the United States since 1913: Evidence from capitalized income tax data. In: The Quarterly Journal of Economics 131.2, 519–578.

Samuelson, Paul A (1958). An exact consumption-loan model of interest with or without the social contrivance of money. In: Journal of political economy 66.6, 467–482.

Smith, Matthew, Owen Zidar and Eric Zwick (2023). Top wealth in america: New estimates under heterogeneous returns. In: The Quarterly Journal of Economics 138.1, 515–573.

Straub, Ludwig (2019). Consumption, savings, and the distribution of permanent income. In: Unpublished manuscript, Harvard University.

Tirole, Jean (1985). Asset bubbles and overlapping generations. In: Econometrica: Journal of the Econometric Society, 1499–1528.

A Data Appendix

A.1 Cumulative Capital Gains

Cumulative capital gains displayed in Figure 1d are calculated from national accounting data for non-financial firms and households (excluding NPISH). This extends the national wealth accumulation decomposition of Piketty and Zucman (2014) to different asset types, expressed as:

$$W_{t+1}^k = W_t^k + S_t^k + KG_t^k,$$

where W_{t+1}^k is the market value of wealth in asset type k at time $t+1$, S_t^k is the net-of-depreciation saving flow in asset type k between time t and $t+1$ (volume effect), and KG_t^k is the capital gain or loss between time t and $t+1$, calculated as a residual. All calculations are performed for total wealth, land, and dwellings. Other domestic capital results are then computed as a residual.

The market value of household's nonfinancial wealth, as well as the market value of land and dwellings of nonfinancial business, are obtained from the nonfinancial assets balance sheets. To account for the fact that Tobin's Q may differ significantly from one (Gutiérrez and Philippon, 2017), the total value of nonfinancial businesses is calculated as total liabilities (including equity) minus total financial assets. Depending on the availability of country-level data, the wealth of the unincorporated sector is either calculated using this methodology or derived directly from the nonfinancial balance sheet.

The net-of-depreciation saving flow of asset k is first calculated separately for nonfinancial businesses and households as gross formation of fixed capital - consumption of fixed capital + other volume changes + acquisitions minus disposals[9]. S_t^k then corresponds to the sum of the net-of-depreciation saving flows from households and nonfinancial businesses.

B Proofs

B.1 Endowment Economy

B.1.1 Proof of the necessity of the transversality condition

The proof follows Kamihigashi (2002), which identifies five conditions under which the transversality condition is a necessary condition.

The maximization problem of household i is rewritten as:

$$\begin{cases} \max\limits_{\{a_t^i\}_{t=0}^{\infty}} & \sum\limits_{t=0}^{\infty} g_t^i(a_t^i, a_{t+1}^i) \\ \text{s.t.} & \forall t \in \mathbb{Z}_+, (a_t^i, a_{t+1}^i) \in X_t, \qquad \text{with } a_0^i \text{ given,} \end{cases} \tag{47}$$

[9] I am currently in the process of obtaining data on acquisitions minus disposals for the US, which are assumed to be zero for now. However, their inclusion should have a moderate overall effect, as acquisitions minus disposals within the considered institutional sectors (nonfinancial firms and households) do not impact the capital gains calculation.

with $g_t(\ell_t^i, \ell_{t+1}^i) \equiv \beta^t \Big[u\Big(a_t^i R_t - a_{t+1}^i\Big) + v(a_{t+1}^i) \Big]$. X_t corresponds to the set of combinaison (a_t^i, a_{t+1}^i) satisying the budget constraint, that is such that: $a_{t+1}^i < R_t a_t$.

Solely interior solution of this problem are considered, as it can be easily shown that solution with $a_t^i = 0$ for some t are dominated by interior solutions. The five conditions identified by Kamihigashi (2002) to prove the necessity of the transversality condition are then satisfied.

Condition 1. $\exists n$, such that $a_0^i \in \mathbb{R}_+^n$ and $\forall t \in \mathbb{Z}_+$, $X_t \subset \mathbb{R}_+^n \times \mathbb{R}_+^n$

This assumption is fulfilled for $n = 1$.

Condition 2. $\forall t \in \mathbb{Z}$, X_t is convex and $(0, 0) \subset X_t$

It can be shown that if (y, z), $(y', z') \in X_t$, then, for all $\gamma \in [0; 1]$, $(\gamma y + (1 - \gamma)y', \gamma z + (1 - \gamma)z') \in X_t$

Condition 3. $\forall t \in \mathbb{Z}$, $g_t : X_t \to \mathbb{R}$ is C^1 on $\overset{\circ}{X}_t$ and concave.

As $u(c)$ and the function $a_t^i :\to v(a_{t+1}^i)$ are C^1, g_t is C^1. Moreover, the consumption level implied by $(a_t^i, a_{t+1}^i) = (y, z)$ is defined as $c(y, z) = yR_t - z$. As $u(c)$ is concave, for all $(y, z) \in \overset{\circ}{X}_t$, it can be shown that $\forall t \in \mathbb{Z}$ and $\forall \gamma \in [0; 1]$

$$\gamma u\Big(c(y, z)\Big) + (1 - \gamma)u\Big(c(y', z')\Big) \leq u\Big(\gamma c(y, z) + (1 - \gamma)c(y', z')\Big) \tag{48}$$

$$= u\Bigg(c\Big(\gamma y + (1 - \gamma)y', \gamma z + (1 - \gamma)\Big)\Bigg) \tag{49}$$

Given the concavity of both preference terms for consumption and wealth, it follows that:

$$\gamma g_t(y, z) + (1 - \gamma)g_t(y', z') \leq g_t\Big(\gamma y + (1 - \gamma)y', \gamma z + (1 - \gamma)z'\Big), \tag{50}$$

and hence that g_t is concave.

Condition 4. $\forall t \in \mathbb{Z}$, $\forall (y, z) \in \overset{\circ}{X}_t$, $g_{t,1}(y, z) \geq 0$.

It follows from $u'(c) \geq 0 \; \forall c$.

Condition 5. For any feasible path a_t^i,

$$\sum_{t=0}^{\infty} g_t(a_t^i, a_{t+1}^i) \equiv \lim_{T \to \infty} \sum_{t=0}^{T} g_t(a_t^i, a_{t+1}^i), \tag{51}$$

exists in $(-\infty, \infty)$.

The wealth of an agent cannot grow at a rate above R_t. Given that $R_t < 1/\beta$ and does not converge to $1/\beta$, $\lim\limits_{T\to\infty} \sum_{t=0}^{T} g_t(a_t^i, a_{t+1}^i)$ is not diverging and the assumption is fulfilled.

The five conditions of Kamihigashi (2002) being fulfilled, the transversality condition is a necessary condition of the household maximization problem in the endowment economy ■

B.1.2 Proof of Proposition 4

[To be written.]