Diffeomorphic Shape Tracking

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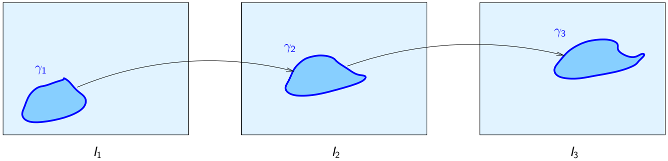
Problem



Goal:

- given a video sequence of a moving and deforming object, obtain an estimate for its boundary at each frame.

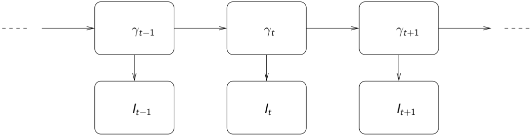
Constraints:

- preserve the topology of the object's boundary.

Desired properties:

- have an algorithm robust to noise and occlusions.

Hidden Markov model



States: $\gamma_0, \gamma_1, ..., \gamma_t$ object boundaries $\gamma_t \subset \Omega \in \mathbb{R}^2$

Observations: $I_0, I_1, ..., I_t$ video frames $I_t : \Omega \to [0, 1]$

Dynamical model: $\gamma_t = f_t \circ \gamma_{t-1}$

Observation model: $I_t = h_t \circ \gamma_t$

Dynamical model

$$\gamma_t = f_t \circ \gamma_{t-1} \quad (1)$$

$$\gamma_t = \varphi_t \circ A_t \circ \gamma_{t-1} \quad (2)$$

A_t – an affine transformation describing the motion of the object
φ_t – a diffeomorphism describing the nonlinear deformation of the object

Affine motion

Two scenarios:

- A is given (it could be either known in advance or estimated from the data by some preliminary process)
- A is unknown (it is modeled through the exponential map: $A = \exp\left(\sum_{i=1}^{6} a_i E_i\right)$ with $\{a_i\}_{i=1}^{6} \sim \mathcal{N}(0, D)$)

Nonlinear deformation

- $\varphi : \Omega \to \Omega$ is a diffeomorphism generated by a smooth vector field $v : \Omega \to \mathbb{R}^2$, i.e. $\varphi(\cdot)$ is the solution of the following autonomous ODE at fixed time τ:

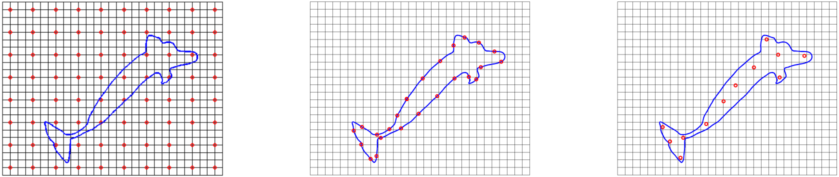
$$\frac{d\phi(x,\tau)}{d\tau} = v(\phi(x,\tau)) \quad x \in \Omega, \quad v : \Omega \to \mathbb{R}^2, \tag{3}$$

subject to the initial condition $\phi(x,0) = x$.

Finitely generated vector fields

- Select a kernel function $K : \Omega \times \Omega \to \mathbb{R}$
- Fix a set of n control points $\{x_i\}_{i=1}^{n}$ in Ω
- Assume $v(\cdot)$ is of the following form:

$$v(\cdot) = \sum_{k=1}^{n} K(\cdot, x_k)\alpha_k \quad \alpha_k \in \mathbb{R}^2 \tag{4}$$

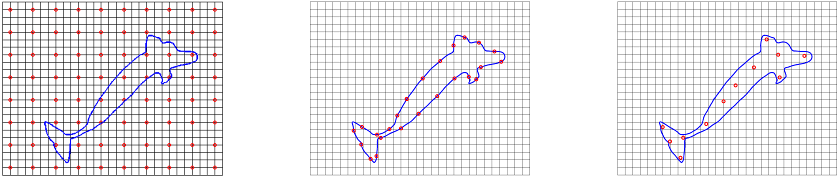


Random vector fields

- Assume $\{\alpha_k\}_{k=1}^{n} \sim \mathcal{N}(0, \Sigma)$, where Σ is such that

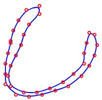
$$Cov(v(x_i), v(x_j)) = \bar{K}(x_i, x_j)\mathbb{I}_2, \quad (5)$$

for some covariance function $\bar{K} : \Omega \times \Omega \to \mathbb{R}^2$.

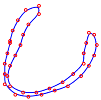


Random spline contours vs. random diffeomorphic contours

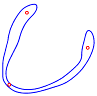
- diffeomorphic deformation guarantees the countours do not to cross
- diffeomorphic deformation can have a low dimensional representation



spline contours



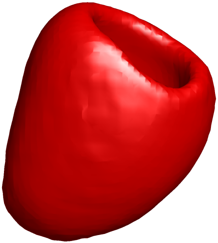
diffeomorphic contours



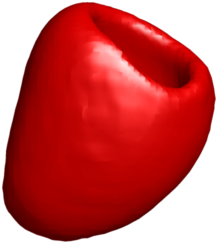
diffeomorphic contours

Extensions to 3D ...

A sample of perturbed shapes:

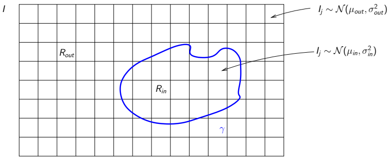


A sample deformation path:



Observation model (region-based)

I – the observed image

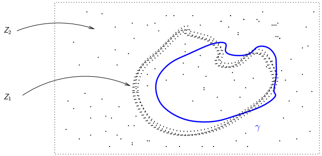


- Model $p(I|\gamma)$ as the joint density of all pixels in I:

$$p(I|\gamma) = \text{const} \prod_{I_j \in R_{in}} e^{-\frac{(I_j - \mu_{in})^2}{2\sigma_{in}^2}} \prod_{I_j \in R_{out}} e^{-\frac{(I_j - \mu_{out})^2}{2\sigma_{out}^2}}.$$

Observation model (edge-based)

Z – the edges in the observed image (a set of m points $x_1, \ldots, x_m$ in Ω)



- Model Z as a superposition of two spatial Poisson point processes:

$$Z = Z_1 \cup Z_2,$$

Z_1 – the boundary process with an intensity $\lambda_1(\cdot) = C_1 e^{-dist(\cdot,\gamma)^2/2\sigma^2}$
Z_2 – the clutter process with a constant intensity $\lambda_2(\cdot) = C_2$
Then $p(Z|\gamma) = p(\{x_1, \ldots, x_m\}|\gamma) = \frac{e^{-\int_\Omega \lambda(x)dx}}{m!} \prod_{i=1}^m \left(C_1 e^{-dist(x_i,\gamma)^2/2\sigma^2} + C_2 \right).$

Summary of the problem

Our dynamical system can be written as:

$$\gamma_{t|\gamma_{t-1}} \sim p(\cdot|\gamma_{t-1}) \qquad \text{transition density}$$
$$I_{t|\gamma_t} \sim p(\cdot|\gamma_t) \qquad \text{observation likelihood}$$

- we can sample from the transition density $\gamma_t \sim p(\cdot|\gamma_{t-1})$;
- we can pointwise evaluate the observation likelihood $p(I_t|\gamma_t)$.

Our goal is to estimate the posterior density $p(\gamma_1, ..., \gamma_t | I_1, ..., I_t)$.

Particle filtering

Idea:

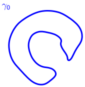
- approximate the posterior through a weighted set of particles $\{\gamma_t^{(i)}, w_t^{(i)}\}_{i=1}^N$.

Steps:

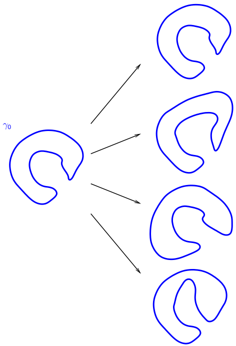
- Set the initial weights to $1/N$ and let $\gamma_0^{(i)} = \gamma_0$.
- For each consequent time t sample N particles $\gamma_t^{(i)} \sim p(\cdot|\gamma_{t-1}^{(i)})$.
- Update their weights by $w_t^{(i)} = p(I_t^{(i)}|\gamma_t^{(i)})w_{t-1}^{(i)}$.

As $N \to \infty$ this approximation converges to the posterior at each step.

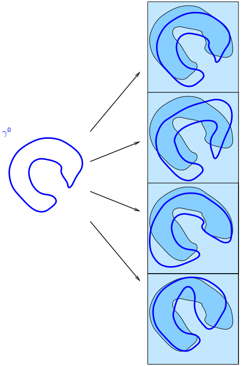
Particle filtering



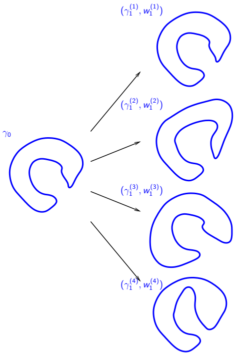
Particle filtering



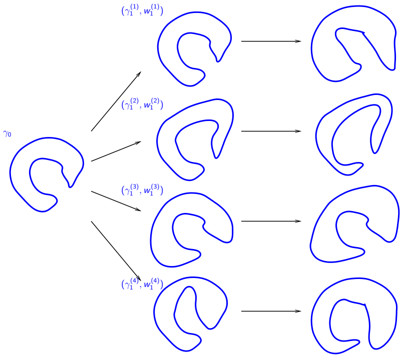
Particle filtering



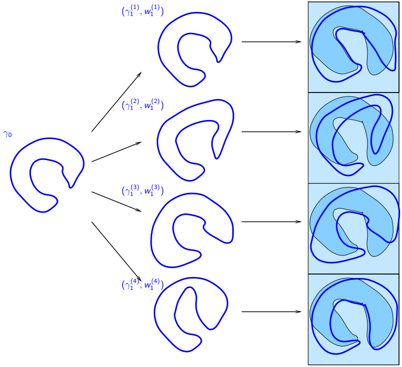
Particle filtering



Particle filtering



Particle filtering



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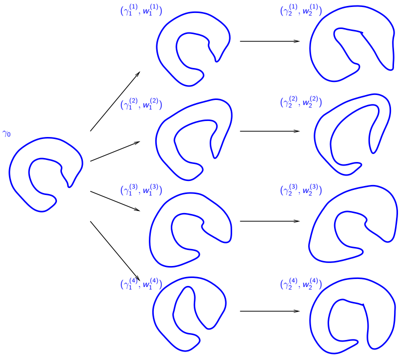

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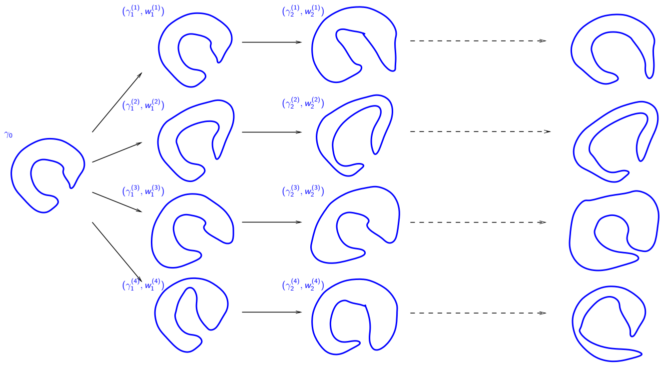
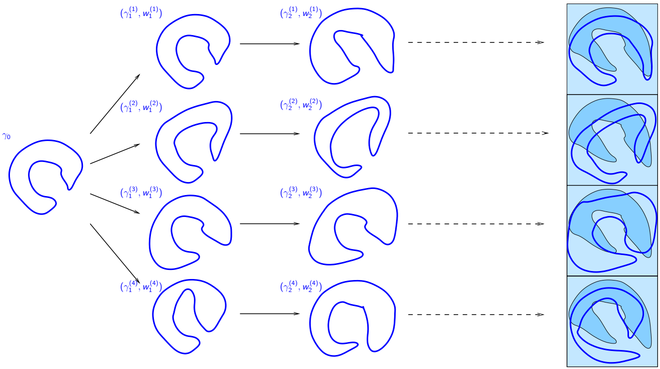
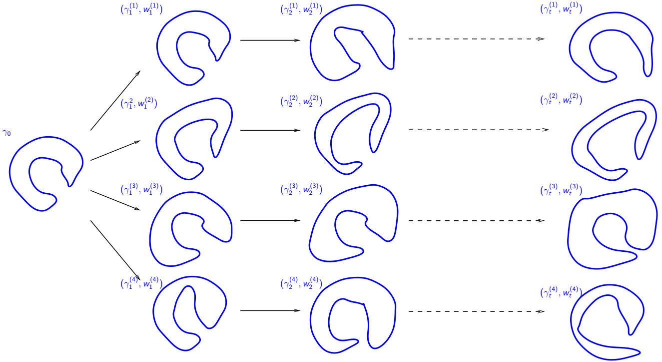


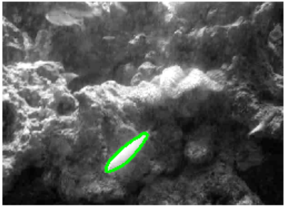
Particle filtering



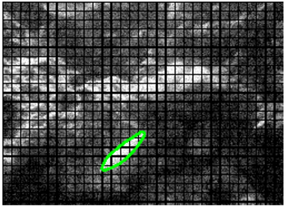
Particle filtering



Region-based tracking



Region-based tracking with noise and occlusions



Future research directions

- find the ‘right’ class of diffeomorphisms for a given problem
- make the proposal density a function of the observations
- improve the prior by learning the dynamics
- incorporate higher order dynamics
- combine particle filtering and parameter estimation
- study possible applications: cell tracking, heart tracking, etc.