Diffeomorphic Models for Stochastic Tracking of Deformable Shapes

Valentina Staneva and Laurent Younes Johns Hopkins University Baltimore, USA

Introduction

Goals:

- Construct a stochastic model for the diffeomorphic evolution of 2D shapes in a video sequence.
- Use this model as a prior for tracking the boundary of the shape.

Approach:

- Model the deformation of the boundary as induced by the flow of a finitely generated vector field defined over the domain of the image (similar to the one constructed in [1]).
- Use particle filtering [2] to estimate the unknown parameters of the vector field.

Properties:

- The random deformations are using spatial information about the object which does not allow the boundary to self-intersect.
- By reducing the dimension of the vector field we can parameterize the deformations by only a few variables.

Hidden Markov Model

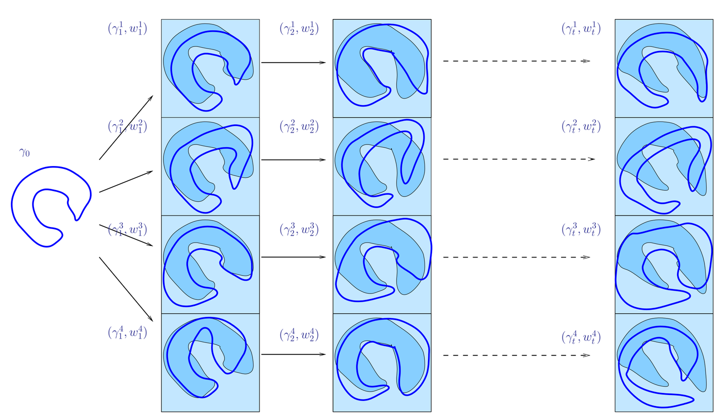
States:	$\gamma_0, ..., \gamma_t \subset \Omega$	*object boundaries*
Observations:	$I_0, ..., I_t : \Omega \to [0, 1]$	*video frames*

Transition density: $p(\gamma_t|\gamma_{t-1})$

Observation likelihood: $p(I_t|\gamma_t)$

Need to estimate the posterior $p(\gamma_1, ..., \gamma_t|I_1, ..., I_t)$.

Particle Filtering



Idea: approximate the posterior by a weighted set of particles $\{\gamma_t^{(i)}, w_t^{(i)}\}_{i=1}^N$.

- sample $\gamma_t^{(i)}$ according to the transition density: $\gamma_t^{(i)} \sim p(\cdot|\gamma_{t-1}^{(i)})$
- associate a weight to $\gamma_t^{(i)}$ using the observation likelihood: $w_t^{(i)} = p(I_t|\gamma_t^{(i)}) w_{t-1}^{(i)}$

Random Diffeomorphic Shapes

Given an initial contour γ_0, we want to generate a new contour γ_1 according to some distribution.

1. Diffeomorphic flows

Define $\gamma_1 = \varphi \circ \gamma_0$, where φ is diffeomorphism generated by the flow of a vector field $v : \Omega \to \mathbb{R}^2$:

$$\frac{\partial \Phi(\cdot,\tau)}{\partial \tau} = v(\Phi(\cdot,\tau)), \qquad \Phi(\cdot,0) = \mathrm{id}(\cdot)$$
$$\Phi(\cdot,1) = \varphi(\cdot).$$

Assume that v belongs to a reproducing kernel Hilbert space with a kernel function

$$K(x_i, x_j) = e^{-\|x_i - x_j\|_2^2/2\sigma_0^2}\mathbb{I}_2.$$

2. Random vector fields

We model v as a zero mean Gaussian random vector field with a covariance function

$$C(x_i, x_j) = e^{-\|x_i - x_j\|_2^2/2\sigma^2}\mathbb{I}_2.$$

Dealing with v is computationally hard, so instead we replace it by its projection onto $span(\{K(\cdot, x_k)\}_{k=1}^n)$, where $\{x_k\}_{k=1}^n$ is a set of control points in Ω:

$$\frac{\partial \Phi(\cdot,\tau)}{\partial \tau} = \bar{v}(\Phi(\cdot,\tau)), \qquad \bar{v}(\cdot) = \sum_{k=1}^n K(\cdot, x_k)\alpha_k.$$

By sampling $\{\alpha_k\}_{k=1}^n \sim \mathcal{N}(0, K^{-1}CK^{-1})$, where $K_{ij} = K(x_i, x_j)$ and $C_{ij} = C(x_i, x_j)$, and numerically integrating the ODE, we obtain samples of random contours.

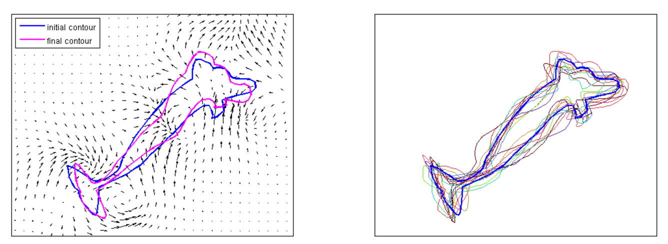


Figure 2. Left: a new contour is generated by moving the boundary of the initial contour along the integral curves of the vector field; right: a sample of countours is generated based on a sample of vector fields.

3. Deformation models

We propose three different deformation models based on the number and position of the control points.

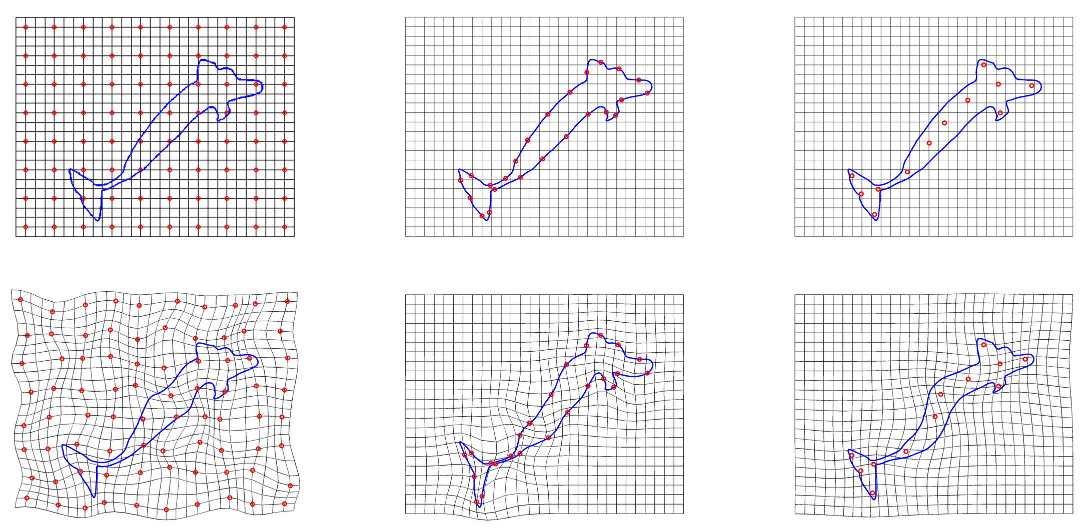
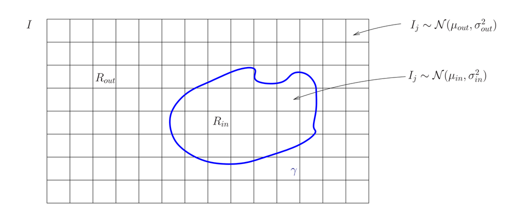


Figure 3. Left: a grid based model - unbiased, but numerically impractical; middle: a boundary based model - useful for small perturbations of the contour; right: skeleton based model - useful for bigger deformations but with some prior knowledge on the locations of the control points.

Observations

The image pixels are modeled as independent normally distributed random variables with parameters (μ_{in}, σ_{in}) or $(\mu_{out}, \sigma_{out})$ according to whether they are inside or outside of the given contour.



$$p(I|\gamma) \propto \prod_{I_j \in R_{in}} e^{-\frac{(I_j - \mu_{in})^2}{2\sigma_{in}^2}} \prod_{I_j \in R_{out}} e^{-\frac{(I_j - \mu_{out})^2}{2\sigma_{out}^2}}$$

Algorithm

1. Construct $\Sigma^{(i)} = K^{-1}CK^{-1}$ based on the control points $x_k^{(i)}(t-1)$.
2. Sample the projection coefficients $\{\alpha_k^{(i)}(t)\}_{k=0}^n \sim \mathcal{N}(0, \Sigma)$.
3. Evolve the boundary points through
$$\frac{\partial}{\partial \tau}\Phi(\gamma_{t-1}^{(i)}, \tau) = \bar{v}_t(\Phi(\gamma_{t-1}^{(i)}, \tau)),$$
and set $\gamma_t^{(i)} = \Phi(\gamma_{t-1}^{(i)}, 1)$.
4. Evolve the control points through
$$\frac{\partial}{\partial \tau}\Phi(x_k^{(i)}(t-1), \tau) = \bar{v}_t(\Phi(x_k^{(i)}(t-1), \tau)),$$
and set $x_k^{(i)}(t) = \Phi(x_k^{(i)}(t-1), 1)$.
5. Compute the observation likelihood $p^{(i)} = p(I_t|\gamma_t^{(i)})$.
6. Update the weights by $\tilde{w}_t^{(i)} = p^{(i)} w_{t-1}^{(i)}$, and normalize by $w_t^{(i)} = \tilde{w}_t^{(i)} / \sum_{i=1}^N \tilde{w}_t^{(i)}$.
7. Resample $\{\gamma_t^{(i)}\}_{i=1}^N$ according to $\{w_t^{(i)}\}_{i=1}^N$ and set $w_t^{(i)} = 1/N$.

Conclusion

We introduce a method for tracking deforming objects in a video sequence, which guarantees to preserve their original topology, and thus is expected to handle motions obscured by noise and occlusions. Our framework provides a finite dimensional representation of random nonlinear deformations of shapes which we believe can be useful beyond the scope of the tracking problem. Studying the robustness of the algorithm under different settings and exploring its possible extensions to other computer vision problems are objects of ongoing work.

Results

1. Tracking under noise

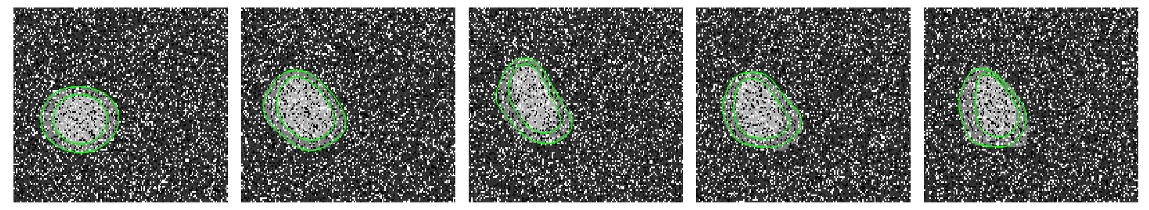


Figure 4 (a): We have simulated a sequence of random deformations of two closed contours. 25% of salt & pepper noise has been added to the images. The boundaries are successfully tracked despite the noise. The diffeomorphic model ensures the two contours never intersect.

2. Tracking under occlusions

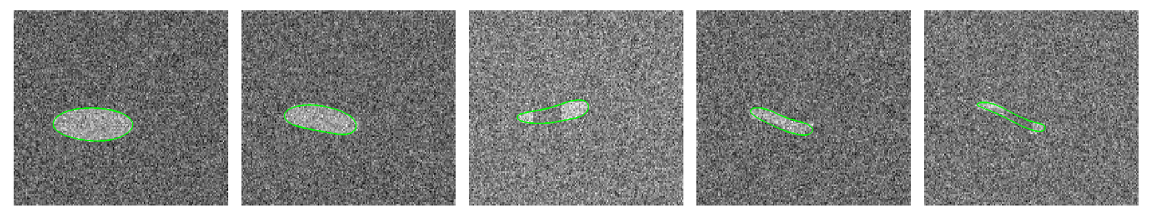


Figure 4 (b): In this sequence a single contour is deformed and in some frames it is occluded by an object having the same color as the background which makes it hard for the algorithm to disinguish from it. The constraints on the deformation in the model allow to preserve the object's shape.

3. Boundary based tracking

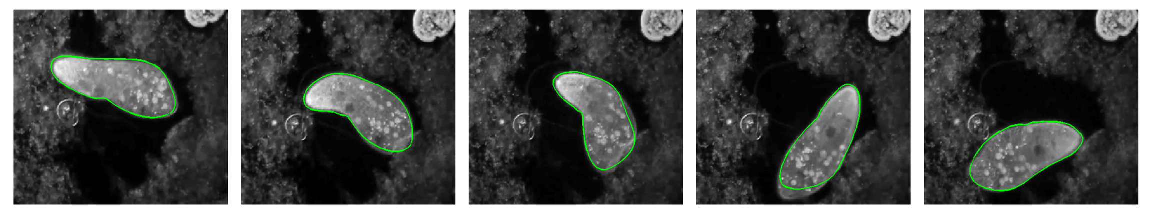


Figure 4 (c): The above sequence displays the motion of a paramecium. Parameciua often change their direction of motion based on the objects they touch, therefore the control points have been allocated equidistantly along the boundary.

4. Skeleton based tracking

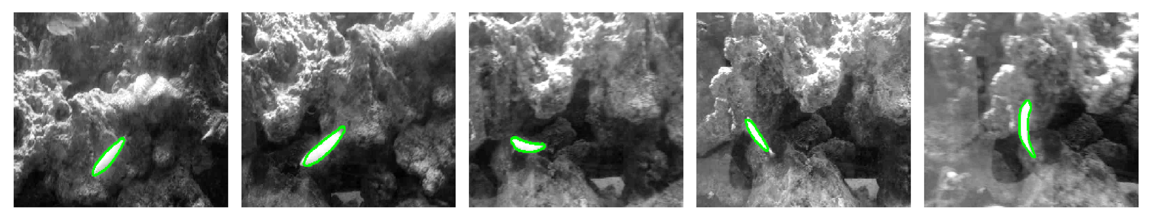


Figure 4 (d): To track the movements of the fish, we first estimate its affine motion, and then we incorporate it in our nonlinear model. When tracking the deformations, we select the control points to be along the medial axis of the shape, since this naturally describes the possible motions of the fish.

Acknowledgements

This work has been partially supported by the Office of Naval Research.

References

[1] F. Arrate, J. T. Ratnanather, and L. Younes,"Diffeomorphic Active Contours," *SIAM J. Imaging Sc.*, vol. 3, pp. 176–198, 2010.

[2] N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/nongaussian bayesian state estimation," *IEE Proceedings-F (Radar and Signal Processing)*, vol. 140(2), pp. 107–113, 1993.