The Dynamics of Shapes: Modeling, Estimation, Learning

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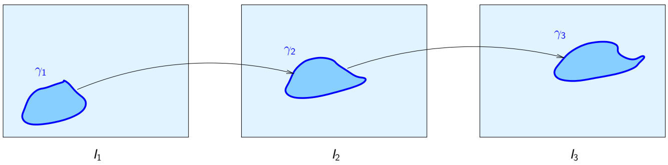
The dynamics of shapes



Modeling = building probabilistic models for the evolution of shapes
Estimation = extracting the boundary of the shape from a sequence of observations
Learning = determining the parameters of the probabilistic models from training data

In search of the posterior

Hidden Markov Model

Hidden variables:	$\gamma_0, \gamma_1, \ldots, \gamma_t$	*object boundaries*
Observed variables:	$I_0, I_1, \ldots, I_t$	*image frames*

Transition probability: $\gamma_{t+1|\gamma_t} \sim Q(\cdot, \gamma_t)$

Observation likelihood: $I_{t|\gamma_t} \sim p_I(\cdot|\gamma_t)$

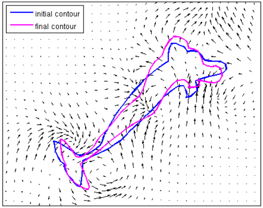
Filtering problem:

- we are looking for an "optimal" estimate for $\gamma_1, \ldots, \gamma_t$, given $I_1, \ldots, I_t$
- our goal is to estimate the posterior probability $P(\gamma_1, \ldots, \gamma_t \in B | I_1, \ldots, I_t)$
- as a final estimate of γ_t we can take the mean or mode of the posterior

Other problems:

- interpolation, prediction, smoothing ...

Curve flows



We assume curves deform under the action of diffeomorphisms:

$$\gamma_t = \varphi^{v_t} \circ \gamma_{t-1}, \tag{1}$$

$\varphi^{v_t} : \Omega \to \Omega$ is a diffeomorphism generated by the flow Φ of a vector field $v_t : \Omega \to \mathbb{R}^2$:

$$\frac{d\Phi(x,\tau)}{d\tau} = v_t(\Phi(x,\tau)) \qquad \Phi(x,0) = \mathrm{id}(x), \tag{2}$$

$$\Phi(x,T) = \varphi^{v_t}(x). \tag{3}$$

Finitely-generated flows

- let χ_t be a set of n control points $\{x_i\}_{i=1}^n$in Ω,
- let $K : \Omega \times \Omega \to \mathbb{R}$ be a positive definite function
- define $V(\chi_t)$ as

$$V(\chi_t) = \left\{ v(\cdot) : v(\cdot) = \sum_{k=1}^{n} K(\cdot, x_k)\alpha_k \quad \alpha_k \in \mathbb{R}^2 \right\}, \tag{4}$$

- set $v_t \in V(\chi_t)$:

$$\frac{d\Phi(x,\tau)}{d\tau} = \sum_{i=1}^{n} K(\Phi(x,\tau), x_i)\alpha_i \tag{5}$$

Choices of K:

$K(x, y) = e^{-\frac{\|x-y\|^2}{2\sigma^2}}$ (nonlinear deformations)

$K_{aff}(x, y) = K(x, y) + \frac{yx^T}{2\lambda_1} + \frac{1}{2\lambda_2}\begin{bmatrix} x_2y_2 & x_1y_2 \\ x_2y_1 & x_1y_1 \end{bmatrix} + \frac{x^Ty - xy^T}{2\lambda_3} + \frac{1}{2\lambda_4}\begin{bmatrix} x_1y_1 & -x_2y_1 \\ -x_1y_2 & x_2y_2 \end{bmatrix}$.

(affine at infinity)

Random curves

Let v be such that for any two control points x_i and x_j

$$Cov(v(x_i), v(y_j)) = C(x_i, y_j)\mathbb{I}_2, \tag{6}$$

where $C(x, y) = e^{-\frac{\|x-y\|^2}{2\sigma^2}}$.
Then $\{\alpha_k\}_{k=1}^n \sim \mathcal{N}(0, \Sigma(\chi))$, where $\Sigma(\chi) = K(\chi)^{-1}C(\chi)K(\chi)^{-1}$.

First order model:

$$\begin{aligned} \chi_{t+1} &= \varphi(\alpha_{t+1}, \chi_t) \cdot \chi_t & (7) \\ \gamma_{t+1} &= \varphi(\alpha_{t+1}, \chi_t) \cdot \gamma_t & (8) \end{aligned}$$

More models

- use EPDiff to generate diffeomorphisms - α_{t-1} is automatically transported to $V(\chi_t)$

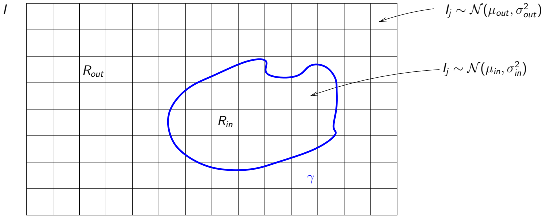
Second order model:

$$\chi_{t+1} = \varphi(\alpha_{t+1}, \chi_t) \cdot \chi_t \quad (9)$$

$$\gamma_{t+1} = \varphi(\alpha_{t+1}, \chi_t) \cdot \gamma_t \quad (10)$$

$$\alpha_{t+1} = \rho_1 \alpha_t + \rho_2 \varepsilon_{t+1} \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma(\chi_t)) \quad (11)$$

Observation model - region based

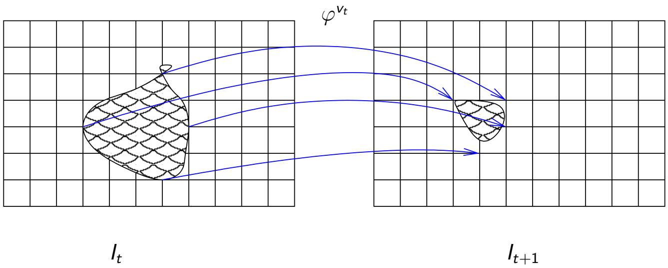


I – the observed image

- Model $p_I(I|\gamma)$ as the joint density of all pixels in I:

$$p_I(I|\gamma) = \text{const} \prod_{I_j \in R_{in}} e^{-\frac{(I_j - \mu_{in})^2}{2\sigma^2_{in}}} \prod_{I_j \in R_{out}} e^{-\frac{(I_j - \mu_{out})^2}{2\sigma^2_{out}}}. \tag{12}$$

Observation model - flow based



For every $x \in$ nbhd(γ_t),

$$I_{t+1}(\varphi^{v}(x)) = I_t(x) + \eta \qquad \eta \sim \mathcal{N}(0, \sigma^2) \tag{13}$$

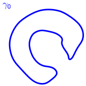
Particle filtering outline

Idea: approximate the posterior by a weighted set of particles $\{\gamma_t^{(i)}, w_t^{(i)}\}_{i=1}^N$

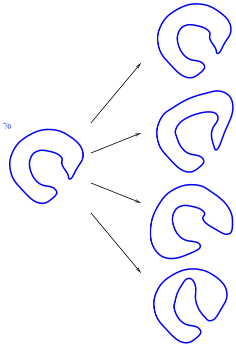
1. sample from the prior $\gamma_{t+1}^{(i)} \sim p_\gamma(\cdot|\gamma_t^{(i)})$
2. update the weights $w_{t+1}^{(i)} = p_I(I_{t+1}|\gamma_{t+1}^{(i)})w_t^{(i)}$
3. resample $\gamma_t^{(i)}$ according to the weights

As $N \to \infty$ the empirical measure converges to the posterior [2].

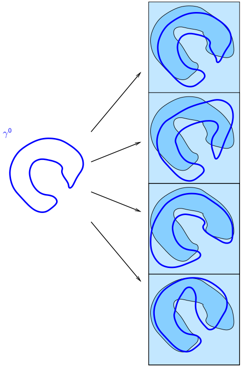
Particle filtering – steps



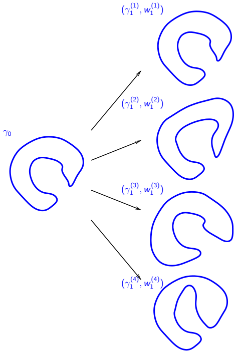
Particle filtering – steps



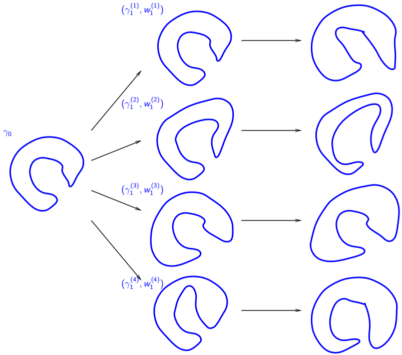
Particle filtering – steps



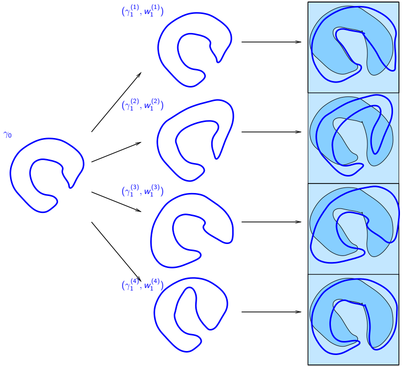
Particle filtering – steps



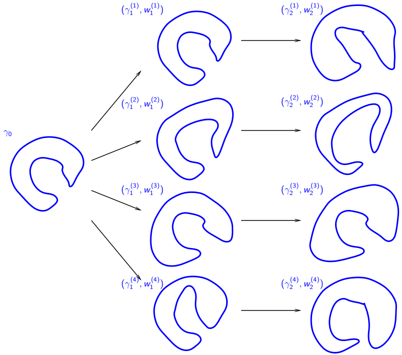
Particle filtering – steps



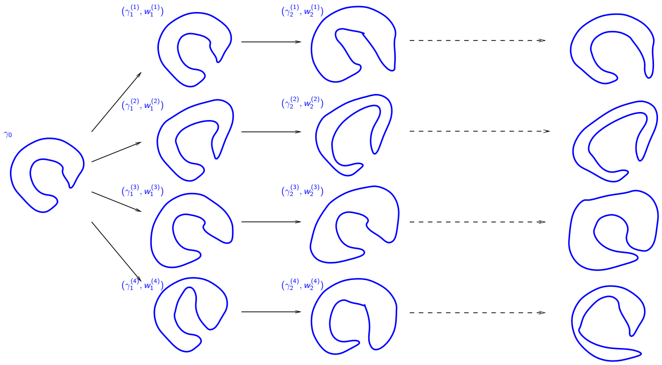
Particle filtering – steps



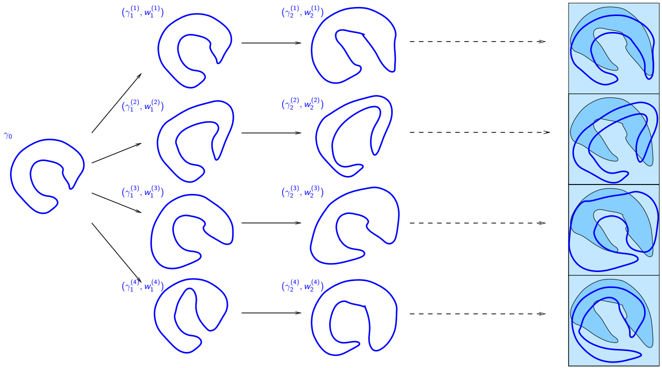
Particle filtering – steps



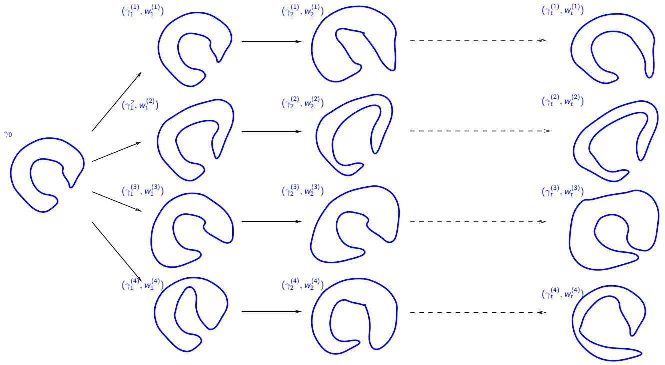
Particle filtering – steps



Particle filtering – steps

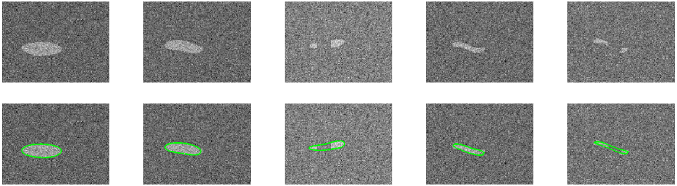


Particle filtering – steps

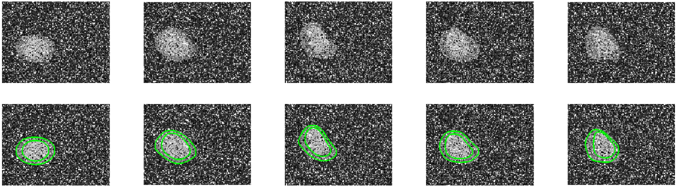


Simulations

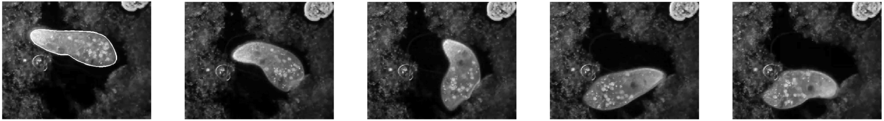
- tracking under occlusions

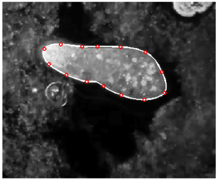


- tracking under salt and pepper noise

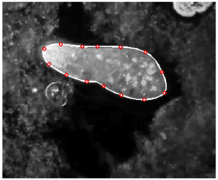


Paramecium sequence





affine tranformation



non-affine deformation

Particle filtering: the good and the bad

Pros:

- updates the posterior sequentially
- generates independent particles $\Rightarrow$ parallelizable

Cons:

- suffers from sample impoverishment
- relies on the proposal being close to the posterior

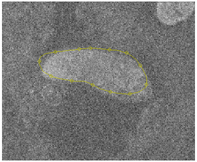
We never have $N \to \infty$...

Incorporating MCMC moves [1]

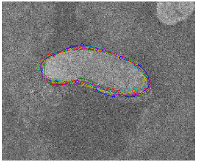
Apply a Markov transition kernel with an invariant distribution $p(\gamma_{1:t}|I_{1:t})$;

- assume that we have a correct estimate of $p(\gamma_{1:t-1}|I_{1:t-1})$
- use Metropolis-Hastings with the current particle filtering estimate as a proposal to sample from $p(\gamma_{1:t}|I_{1:t})$.

Note: MCMC based approach without the need to sample from the full posterior.



particle filtering



particle filtering + MCMC moves

Optimal proposal density

The prior density does not use the newly available observation!
Optimal importance density is $p(\gamma_t|\gamma_{t-1}, I_{1:t})$.

Problem: We do not know how to sample from it.
Solution: We can use Laplace approximation.

1. compute the mode of $p(\gamma_t|\gamma_{t-1}, I_{1:t})$
2. sample from $\alpha_t \sim \mathcal{N}(m, H)$, where

$$H = -\nabla^2 \log(p(\gamma_t|\gamma_{t-1}, I_{1:t})) \tag{14}$$

(I use the covariance of the prior instead of H)

Learning dynamical models

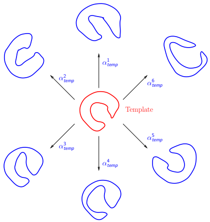
Given a set of training shapes, we would like to estimate the mean μ_t and covariance Σ_t of $\{\alpha_k\}_{k=1}^n$ at time t.

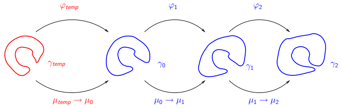
Problems:

- need to select a common reference frame
- need to transport the vector fields

Solution:

- select a template shape γ_{temp}
- compute μ_{temp} and Σ_{temp}
- to transport $\mu_{temp} \rightarrow \mu_0$ and $\Sigma_{temp} \rightarrow \Sigma_0$





- understand shape-dependent and time-dependent deformations
- consider general processes on the space of shapes
- link to the statistical problem

Walter R. Gilks.
Following a moving target: Monte Carlo inference for dynamic Bayesian models.
Journal of Royal Statistical Society, 63:127–146, 2001.

P. Del Moral.
Measure valued processes and interacting particle systems. application to non linear filtering problems.
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