

duda ¿porqué transformad lineal corrige heteroscedasticidad?

with large samples isn't important efficiency

Heteroskedasticity

- No relation with unbiased and consistency \rightarrow tiene que ver con variancia de residuos
 - No relation with fit (R^2)
 - Related to sample size asymptotical analysis
- $E(\hat{\beta})$
 $Var(\hat{\beta})$
MLR1-MLR4
standar errors (se) to estimate t & F distribution

② MELI (MLR1-MLR5)
(best linear unbiased estimator)
OLS is no longer asymptotically efficient but unbiased.

DEF: constant variance MLR5
 $Var(u|x_1, x_2, \dots, x_k) = \sigma^2 I_n$
Important to INFERENCE

① HYPOTHESIS under Gauss Markow assumptions

① TEST

■ OPTION 1 - BREUSH PAGAN \rightarrow heteroscedasticidad lineal

$$H_0: Var(u|x_1, x_2, \dots, x_k) = \sigma^2 \text{ (test MLR5)} \rightarrow H_0: \delta_1 = \delta_2 = \dots = \delta_k = 0$$

Probaremos si u^2 está relacionado con alguna de las variables independientes

$$Var(u|x) \xrightarrow{\text{by MLR1-4}} E(u^2|x) = \sigma^2 = E(u^2)$$

① Regresión lineal de residuos y regresores

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \dots + \delta_k x_k + \text{errores}$$

② Hipótesis conjunta

$$F = \frac{R_{\hat{u}^2}^2}{(1 - R_{\hat{u}^2}^2)} \cdot \frac{(n-k-1)}{k}$$

■ OPTION 2: WHITE TEST \rightarrow heteroscedasticidad no lineal

Probaremos si u^2 está relacionado con todas las variables

① Regresión lineal de residuos y regresores

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \dots + \delta_k x_k + \text{errores}$$

$$\delta_3 x_1^2 + \delta_4 x_1 x_2 + \dots + \delta_{k+2} x_k^2$$

② Hipótesis conjunta

$$F = \frac{R_{\hat{u}^2}^2}{(1 - R_{\hat{u}^2}^2)} \cdot \frac{(n-k-1)}{k}$$

Con k grande y n pequeño

Paso 1: Regreso \hat{u}^2
 $\hat{u}^2 = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \dots + \hat{\beta}_k x_k$

Alternativa \rightarrow pues

Paso 2: Tomo \hat{u}^2 y es predictor de \hat{u}^2
 $\hat{u}^2 = \delta_0 + \delta_1 \hat{u}^2 + \delta_2 \hat{u}^2 + \text{error}$

K restricciones son más entre más variables sean
Ejemplo
Si tengo 3 variables en MLR \rightarrow son 9 restricciones (intercept de x_1, x_2 y x_3)
Si tengo 6 variables \rightarrow 27 restricciones
6 variables \rightarrow 24 regresores
 \rightarrow Necesitare más grados de libertad para estimar \hat{u}^2

② SOLUTIONS

■ ROBUST INFERENCE

- Estimation in presence of heteroskedasticity \rightarrow corregir SE
- Asymptotical Approach ($n \rightarrow \infty$)
- in presence of multicollineality we test RI

RLS
 $y_i = \beta_0 + \beta_1 x_i + u_i$

Heteroskedasticity
 $Var(u_i|x_i) = \sigma_i^2$
 σ_i^2 cambia cuando cambia x_i

■ ROBUST STANDARD ERRORS (WHITE SE)

Tomando (1) & (2)
 $Var(\hat{\beta}_1|x_i) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 \hat{u}_i^2}{[\sum_{i=1}^n (x_i - \bar{x})^2]^2}$
R.L.S

se = $\sqrt{Var(\hat{\beta}_1|x)}$

Empirical regularities

1. If "normal" estimations are significant, probably in robust estimation to.
 2. Robust SE > Normal SE \rightarrow In homoscedasticity, estimators are minimum
 3. Robust SE \approx Normal SE in homoscedasticity
- Los β no cambian, solo SE

■ LM TEST (H_0 : homoskedasticity)

STEP 1. ESTIMATE MLR AND SAVE RESIDUALS
COMPUTE \hat{u}^2 (SQUARED RESIDUALS)

errors are inobservables

STEP 2. ESTIMATE AUXILIAR REGRESSION WITH PREDICTORS (OF RESIDUALS)
 $\hat{u}^2 = \delta_0 + \delta_1 x_1 + \dots + \delta_k x_k + \text{error}$ (repeat with restricted model)

STEP 3. COMPUTE LM (or F) TEST

ML = $\frac{R_{\hat{u}^2}^2 - R_0^2}{(1 - R_{\hat{u}^2}^2)} \cdot \frac{(n-k-1)}{q}$ $\xrightarrow{i.i.d.}$ $F(q, n-k-1)$

Global
 $n \cdot R_0^2 \xrightarrow{\alpha} \chi_k^2$ \rightarrow ML $\approx \chi_k^2$ are similar when $n \rightarrow \infty$
ChiSq-TEST

Con homoscedasticidad $\sigma_i^2 = \sigma^2$
 $Var(\hat{\beta}_1|x_i) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 \sigma^2}{[\sum_{i=1}^n (x_i - \bar{x})^2]^2} = \frac{\sigma^2 \sum_{i=1}^n (x_i - \bar{x})^2}{[\sum_{i=1}^n (x_i - \bar{x})^2]^2} = \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2}$
Con heteroskedasticity
 $Var(\hat{\beta}_1|x_i) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 \sigma_i^2}{[\sum_{i=1}^n (x_i - \bar{x})^2]^2}$
 \downarrow
SSR_x²

SE are more valid
Why compute normal SE.

\rightarrow If homoscedasticity is valid, $se \sim N$, and t & F have exactly distributions independent of the size sample
 \rightarrow Robust SE in high sample

Si no es una aproximación.