$\emptyset \vdash \lambda x : Nat . succ(x) : Nat \to Nat$

$\lambda x.\, succ(y)$

$\underbrace{\{y : Nat\}}_{\Delta} \vdash \lambda x : X_1 . succ(y) : X_1 \to Nat$

$\lambda x . x$

$\text{MGU}\{X_2 \to (X_1 \to \text{Bool}) \stackrel{?}{=} X_2 \to X_3\}$

$\xrightarrow{dec} \{X_2 \doteq X_2,\ X_1 \to Bool \doteq X_3\}$

$\xrightarrow{del} \{X_1 \to B \doteq X_3\} \xrightarrow{swap} \{X_3 \doteq X_1 \to B\}$

$\xrightarrow{elim} \{X_3 := X_1 \to B\}\ \{\}$ $\quad S = \{X_3 := X_1 \to Bool\}$

$\text{MGU}\{(X_2 \to X_1) \to \text{Nat} \stackrel{?}{=} X_2 \to X_3\}$

$\xrightarrow{dec} \{X_2 \to X_1 \doteq X_2,\ Nat \doteq X_3\}$

$\xrightarrow{swap} \{X_2 \doteq X_2 \to X_1,\ Nat \doteq X_3\}$ $\times$

Falla x OccursCheck.

$\mathtt{MGU}\{X_1 \to \text{Bool} \stackrel{?}{=} \text{Nat} \to \text{Bool}, X_2 \stackrel{?}{=} X_1 \to X_1\}$

$$\xrightarrow{elim} \{X_2 := X_1 \to X_1\} \quad \{X_1 \to B \doteq N \to B\}$$

$$\xrightarrow{dec} \{X_1 \doteq N, B \doteq B\}$$

$$\xrightarrow{dec} \{X_1 \doteq N\} \xrightarrow{elim} \{X_1 := N\} \quad \{\}$$

$$S = \{X_1 \doteq N\} \circ \{X_2 := X_1 \to X_1\} = \{X_1 := N, X_2 := N \to N\}$$

1. $\lambda x.\, y$
2. f *true*
3. iszero(x)

1) $\{y : X_1\} \vdash \lambda x : X_2 . y : X_2 \to X_1$

2) $\{f : Bool \to X_1\} \vdash f \text{ true} : X_1$

3) $\{x : Nat\} \vdash iszero(x) : Bool$

1 $\mathbb{W}(\lambda f.\, \lambda x.\, f(f\,x))$

$$\emptyset \vdash \lambda f : X_2 \to X_2 .\, \lambda x : X_2 .\, f\,(f\,x) : (X_2 \to X_2) \to X_2 \to X_2$$

$$\{f : X_2 \to X_2\} \vdash \lambda x : X_2 .\, f\,(f\,x) : X_2 \to X_2$$

$$\{f : X_2 \to X_2,\ x : X_2\} \vdash f\,(f\,x) : X_2$$

$$S = MGU\{X_3 \doteq X_4 \to X_5,\ X_3 \doteq X_2 \to X_4\}$$

$$S = \{X_3 := X_2 \to X_2,\ X_4 := X_2,\ X_5 := X_2\}$$

$$\{f : X_3\} \vdash f : X_3 \qquad \{f : X_2 \to X_4,\ x : X_2\} \vdash f \quad x : X_4$$

$$S = MGU\{X_1 \doteq X_2 \to X_4\} = \{X_1 := X_2 \to X_4\}$$

$$\{f : X_1\} \vdash f : X_1 \qquad \{x : X_2\} \vdash x : X_2$$

$$S = MGU\{X_3 \doteq X_4 \to X_5,\ X_3 \doteq X_2 \to X_4\}$$

$$\xrightarrow{elim} \{X_3 := X_4 \to X_5\}\ \{X_4 \to X_5 \doteq X_2 \to X_4\}$$

$$\xrightarrow{dec} \{X_4 \doteq X_2,\ X_5 \doteq X_4\}$$

$$\xrightarrow{elim} \{X_4 := X_2\}\ \{X_5 \doteq X_2\} \xrightarrow{elim} \{X_5 := X_2\}\ \{\}$$

$$S = \{X_3 := X_2 \to X_2,\ X_4 := X_2,\ X_5 := X_2\}$$

2 $\mathbb{W}(x\,(\lambda x.\ \mathrm{succ}(x)))$

$$\{x : (N \to N) \to X_3\} \vdash x\ (\lambda x : Nat.\ \mathrm{succ}(x)) : X_3$$

$$S = MGU\{X_2 \doteq (Nat \to Nat) \to X_3\}$$

$$= \{X_2 := (N \to N) \to X_3\}$$

$$\{x : X_2\} \vdash x : X_2 \qquad \emptyset \vdash \lambda x : Nat.\ \mathrm{succ}(x) : Nat \to Nat$$

$$\{x : Nat\} \vdash \mathrm{succ}(x) : Nat$$

$$S = MGU\{X_1 \doteq Nat\}$$

$$= \{X_1 := Nat\}$$

$$\{x : X_1\} \vdash x : X_1$$

3 $\mathbb{W}(\lambda x.\ x\ y\ x)$

$\lambda x.\ x\ y\ x$

Falla x occurs Check

$x\ y\ x$ $\quad S = MGU\{X_3 \doteq X_4 \to X_5,\ X_2 \to X_3 \doteq X_4\}$

$\{x : X_2 \to X_3,\ y : X_2\} \vdash x\ y : X_3$ $\qquad \{x : X_4\} \vdash x : X_4$

$S = MGU\{X_1 \doteq X_2 \to X_3\}$

$\{x : X_1\} \vdash x : X_1$ $\qquad \{y : X_2\} \vdash y : X_2$

$$S = MGU\{X_3 \doteq X_4 \to X_5,\ X_2 \to X_3 \doteq X_4\}$$

$$\xrightarrow{\text{elim}} \{X_3 := X_4 \to X_5\}\ \{X_2 \to (X_4 \to X_5) \doteq X_4\}$$

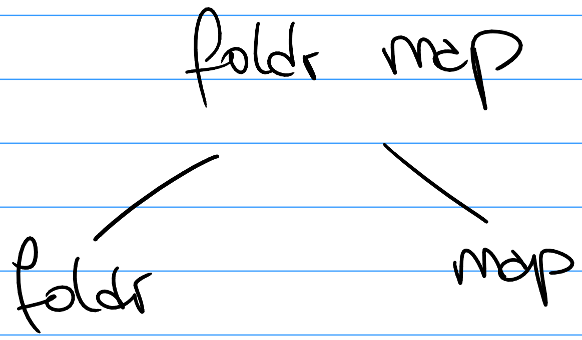
$$\xrightarrow{\text{swap}} \{X_4 \doteq X_2 \to (X_4 \to X_5)\}\ \times$$

$$[X_1] \doteq [X_2] \xrightarrow{\text{dec}} X_1 \doteq X_2$$

$$\mathbb{W}(map) \stackrel{\text{def}}{=} \emptyset \rhd map_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$\mathbb{W}(foldr) \stackrel{\text{def}}{=} \emptyset \rhd foldr_{X_1,X_2} : (X_1 \to X_2 \to X_2) \to X_2 \to [X_1] \to X_2$$

$\mathbb{W}(\text{foldr map})$



$\mathbb{W}(\text{foldr}) = \emptyset \vdash \text{foldr}_{X_1,X_2} : (X_1 \to X_2 \to X_2) \to X_2 \to [X_1] \to X_2$

$\mathbb{W}(\text{map}) = \emptyset \vdash \text{map}_{X_3,X_4} : (X_3 \to X_4) \to [X_3] \to [X_4]$

$S = MGU\{(X_1 \to X_2 \to X_2) \to X_2 \to [X_1] \to X_2 \doteq ((X_3 \to X_4) \to [X_3] \to [X_4]) \to X_5\}$

$\xrightarrow{\text{dec}} \{X_1 \to X_2 \to X_2 \doteq (X_3 \to X_4) \to [X_3] \to [X_4],$
$X_2 \to [X_1] \to X_2 \doteq X_5\}$

$\xrightarrow{\text{swap}} \{X_1 \to X_2 \to X_2 \doteq (X_3 \to X_4) \to [X_3] \to [X_4],$
$X_5 \doteq X_2 \to [X_1] \to X_2\}$

$\xrightarrow{\text{elim}} \{X_5 := X_2 \to [X_1] \to X_2\}$
$\{X_1 \to X_2 \to X_2 \doteq (X_3 \to X_4) \to [X_3] \to [X_4]\},$

$$\xrightarrow{dec} \{X_1 \doteq X_3 \to X_4,\ X_2 \to X_2 \doteq [X_3] \to [X_4]\}$$

$$\xrightarrow{elim}_{\{X_1 := X_3 \to X_4\}} \{X_2 \to X_2 \doteq [X_3] \to [X_4]\}$$

$$\xrightarrow{dec} \{X_2 \doteq [X_3],\ X_2 \doteq [X_4]\}$$

$$\xrightarrow{elim}_{\{X_2 := [X_3]\}} \{[X_3] \doteq [X_4]\}$$

$$\xrightarrow{dec} \{X_3 \doteq X_4\} \xrightarrow{elim}_{\{X_3 := X_4\}} \{\}$$

$$S = \{X_3 := X_4\} \circ \{X_2 := [X_3]\} \circ \{X_1 := X_3 \to X_4\} \circ \{X_5 := X_2 \to [X_1] \to X_2\}$$

$$= \{X_3 := X_4,\ X_2 := [X_4],\ X_1 := X_4 \to X_4,\ X_5 := [X_4] \to [X_4 \to X_4] \to [X_4]\}$$

$$\mathbb{W}(\text{foldr map}) = \emptyset \vdash \text{foldr}_{X_4 \to X_4, [X_4]}$$

$$\text{map}_{X_4, X_4} : [X_4] \to [X_4 \to X_4] \to [X_4]$$

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \rhd S(M :: N) : S[\sigma]$

$\mathbb{W}(U) = \Gamma_1 \rhd M : \sigma$

$\mathbb{W}(V) = \Gamma_2 \rhd N : [\sigma]$

$S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

$M \leftrightarrow x \qquad \sigma \leftrightarrow X_1$

$\underline{0} :: x$

$X_1 \doteq [N]$

$\emptyset \vdash \underline{0} : N \qquad \{x : X_1\} \vdash x : X_1$

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \rhd S(M :: N) : SX_2$

$\mathbb{W}(U) = \Gamma_1 \rhd \cancel{M} : \cancel{X_1}$ Bool

$\mathbb{W}(V) = \Gamma_2 \rhd N : \cancel{X_2}$ [Bool]

$S = MGU\{X_2 \stackrel{?}{=} [X_1]\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1$

$\tau_x = Nat$

$\tau_y = X_5$

$\{x : [Nat]\} \vdash$ Case $\text{succ}(0) :: x$ of $[\,] \rightsquigarrow x\,;\, x :: y \rightsquigarrow \text{succ}(x) :: [\,]_{Nat} : [Nat]$

$S = MGU\{[N] \doteq [N],\ X_2 \doteq [N],\ X_5 \doteq [N]\}$

$\{x : [N]\} \vdash \text{succ}(\underline{0}) :: x : [Nat] \qquad \{x : X_2\} \vdash x : X_2 \qquad \{x : N\} \vdash \text{succ}(x) :: [\,]_{Nat} : [N]$

$S = MGU\{X_1 \doteq [Nat]\}$

$\{X_1 := [Nat]\}$

$MGU\{[X_4] \doteq [N]\}$

$S = \{X_4 := N\}$

$\emptyset \vdash \text{succ}(\underline{0}) : Nat \qquad \{x : X_1\} \vdash x : X_1 \qquad \{x : N\} \vdash \text{succ}(x) : Nat \qquad \emptyset \vdash [\,]_{X_4} : [X_4]$

$S = \{X_3 := Nat\}$

$\emptyset \vdash \underline{0} : Nat \qquad \{x : X_3\} \vdash x : X_3$

$[\text{succ}(x) \mid x \leftarrow \underline{0} :: \underline{1} :: [\,]_{Nat},\ \text{isZero}(x)]$

$\xrightarrow{*} \underline{1} :: [\,]_N.$

$\tau_{x_1} \doteq B \quad \tau_{x_2} \doteq X_4$

$\{x : N\} \vdash [\text{ if } x \text{ then } \underline{0} \text{ else } \underline{1} \mid x \leftarrow \mathit{false} :: \text{iszero}(x) :: [\,]_B, \mathit{true} \,] : [Nat]$

$S = MGU\{B \doteq X_4, [B] \doteq [B], B \doteq B\} = \{X_4 := Bool\}$

$\{x : B\} \vdash \text{if } x \text{ then } \underline{0} \text{ else } \underline{1} : Nat$

$\{x : N\} \vdash \mathit{false} :: \text{iszero}(x) :: [\,]_B : [B]$

$\emptyset \vdash \mathit{true} : Bool$

$S = MGU\{X_3 \doteq B, N \doteq N\} = \{X_3 := B\}$

$S = MGU\{[B] \doteq [B]\} = \{\}$

$\{x : X_3\} \vdash x : X_3$

$\emptyset \vdash \underline{0} : Nat$

$\emptyset \vdash \text{succ}(\underline{0}) : Nat$

$\emptyset \vdash \mathit{false} : Bool$

$\{x : N\} \vdash \text{iszero}(x) :: [\,]_B : [B]$

$S = MGU\{N \doteq N\} = \{\}$

$\emptyset \vdash \underline{0} : Nat$

$S = MGU\{[X_2] \doteq [B]\} = \{X_2 := Bool\}$

$\{x : N\} \vdash \text{iszero}(x) : Bool$

$\emptyset \vdash [\,]_{X_2} : [X_2]$

$S = \{X_1 := N\}$

$\{x : X_1\} \vdash x : X_1$