Inferencia de Tipos

Paradigmas (de Lenguajes) de Programación

Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires

18 de octubre de 2024

Inferencia de tipos

Motivación

Dada una expresión: ¿Tiene tipo? ¿Cuál es el tipo? ¿Es el más general? ¿Qué necesitamos saber del contexto?

Introducción

**¿Tiene tipo? ¿Cuál es el tipo? ¿Es el más general?
¿Qué necesitamos saber del contexto?**

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- $\lambda x.\ \mathit{succ}(x)$

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- $(\lambda x.\, \mathit{isZero}(x))\ \mathit{true}$
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- $\emptyset \triangleright \lambda x : \mathit{Nat}.\, x : \mathit{Nat} \to \mathit{Nat}$

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- $(\lambda x.\, isZero(x))\ true$
- $\lambda x.\, succ(x)$
- $\lambda x.\, succ(y)$
- $\emptyset \ \triangleright\ \lambda x : Nat.\, x : Nat \to Nat$
- $\emptyset \ \triangleright\ \lambda x : X_1.\, x : X_1 \to X_1$

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- $\emptyset \rhd \lambda x : Nat.\, x : Nat \to Nat$
- $\emptyset \rhd \lambda x : Bool.\, x : Bool \to Bool$

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- $\emptyset \rhd \lambda x : Nat.\, x : Nat \to Nat$
- $\emptyset \rhd \lambda x : Bool.\, x : Bool \to Bool$
- $\{y : Bool\} \rhd \lambda x : X_2 \to Nat.\, x : (X_2 \to Nat) \to X_2 \to Nat$

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- $\emptyset \vartriangleright \lambda x : \mathit{Nat}.\, x : \mathit{Nat} \to \mathit{Nat}$
- $\emptyset \vartriangleright \lambda x : \mathit{Bool}.\, x : \mathit{Bool} \to \mathit{Bool}$
- $\{y : \mathit{Bool}\} \vartriangleright \lambda x : X_2 \to \mathit{Nat}.\, x : (X_2 \to \mathit{Nat}) \to X_2 \to \mathit{Nat}$
- ...

Ejemplos a ojo

Inferir el juicio de tipado de las siguientes expresiones:

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1. $\lambda x.\, y$
2. $f\ true$
3. $\mathsf{iszero}(x)$

Algoritmo de Martelli Montanari

Determinar el resultado de aplicar el algoritmo MGU sobre las siguientes ecuaciones:

1. $\texttt{MGU}\{X_2 \to X_1 \to \mathsf{Bool} \stackrel{?}{=} X_2 \to X_3\}$

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4. $\texttt{MGU}\{X_1 \to X_2 \overset{?}{=} X_3 \to X_4, X_3 \overset{?}{=} X_2 \to X_1\}$

Algoritmo de inferencia

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1. $\lambda f.\, \lambda x.\, f(f\ x)$
2. $x\ (\lambda x.\ \mathsf{succ}(x))$
3. $\lambda x.\, x\ y\ x$

Algoritmo de inferencia

¿Qué tipo tienen las siguientes expresiones?

1. $\mathbb{W}(\lambda f.\, \lambda x.\, f(f\ x))$
2. $\mathbb{W}(x\,(\lambda x.\ \mathsf{succ}(x)))$
3. $\mathbb{W}(\lambda x.\, x\ y\ x)$

Algoritmo de inferencia

Ejercicio

Dada la siguiente extensión al conjunto de términos para el cálculo λ con listas:

$$M ::= \ldots | map_{\sigma,\tau} | foldr_{\sigma,\tau}$$

La modificación al sistema de tipos es la introducción de dos axiomas de tipado para $map_{\sigma,\tau}$ y $foldr_{\sigma,\tau}$:

$$\mathbb{W}(map) \stackrel{\text{def}}{=} \emptyset \triangleright map_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$\mathbb{W}(foldr) \stackrel{\text{def}}{=} \emptyset \triangleright foldr_{X_1,X_2} : (X_1 \to X_2 \to X_2) \to X_2 \to [X_1] \to X_2$$

siendo X_1 y X_2 variables de tipo frescas. Se asumen dadas las extensiones correspondientes para `Erase` y `mgu`. Usar el algoritmo $\mathbb{W}()$ con esta nueva extensión para tipar la siguiente expresión:

$$foldr\ map$$

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$S = MGU\{???\}$

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$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = ??$$

$$\mathbb{W}(\mathit{foldr}) = \emptyset \triangleright \mathit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$$
$$\mathbb{W}(\mathit{map}) = \emptyset \triangleright \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1]\ /X_4\} \circ \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1]\ /X_4\} \circ \{X_1 \to X_2\ /X_3\}$$

Algoritmo de inferencia

$$\mathbb{W}(\textit{foldr map}) = ??$$

$$\mathbb{W}(\textit{foldr}) = \emptyset \triangleright \textit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$$
$$\mathbb{W}(\textit{map}) \ = \emptyset \triangleright \textit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = ??$$

$$\mathbb{W}(\mathit{foldr}) = \emptyset \triangleright \mathit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$$
$$\mathbb{W}(\mathit{map}) = \emptyset \triangleright \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1]\ /X_4\} \circ \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1]\ /X_4\} \circ \{X_1 \to X_2\ /X_3\}$$

$$\longmapsto^4 \{[X_2] \to [X_2 \to X_2] \to [X_2] \stackrel{?}{=} X_5\} \mid \{X_2\ /X_1\} \circ \{[X_1]\ /X_4\} \circ \{X_1 \to X_2\ /X_3\}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr\ map}) = ??$$

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$$\mathbb{W}(\mathit{map}) \ = \emptyset \rhd \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_2] \to [X_2 \to X_2] \to [X_2] \stackrel{?}{=} X_5\} \mid \{X_2 \,/ X_1\} \circ \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^3 \{X_5 \stackrel{?}{=} [X_2] \to [X_2 \to X_2] \to [X_2]\} \mid \{X_2 \,/ X_1\} \circ \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

Algoritmo de inferencia

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$$\mathbb{W}(\mathit{map}) \ = \emptyset \rhd \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \stackrel{?}{=} ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$

$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^4 \{[X_2] \to [X_2 \to X_2] \to [X_2] \stackrel{?}{=} X_5\} \mid \{X_2 \,/ X_1\} \circ \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

$$\longmapsto^3 \{X_5 \stackrel{?}{=} [X_2] \to [X_2 \to X_2] \to [X_2]\} \mid \{X_2 \,/ X_1\} \circ \{[X_1] \,/ X_4\} \circ \{X_1 \to X_2 \,/ X_3\}$$

Algoritmo de inferencia

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$$\longmapsto^1 \{X_3 \to X_4 \to X_4 \stackrel{?}{=} (X_1 \to X_2) \to [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^1 \{X_3 \stackrel{?}{=} X_1 \to X_2,\ X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_3] \to X_4 \stackrel{?}{=} X_5\}$$

$$\longmapsto^4 \{X_4 \to X_4 \stackrel{?}{=} [X_1] \to [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^1 \{X_4 \stackrel{?}{=} [X_1],\ X_4 \stackrel{?}{=} [X_2],\ X_4 \to [X_1 \to X_2] \to X_4 \stackrel{?}{=} X_5\} \mid \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^4 \{[X_1] \stackrel{?}{=} [X_2],\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/X_4\} \circ \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^1 \{X_1 \stackrel{?}{=} X_2,\ [X_1] \to [X_1 \to X_2] \to [X_1] \stackrel{?}{=} X_5\} \mid \{[X_1] \,/X_4\} \circ \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^4 \{[X_2] \to [X_2 \to X_2] \to [X_2] \stackrel{?}{=} X_5\} \mid \{X_2 \,/X_1\} \circ \{[X_1] \,/X_4\} \circ \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^3 \{X_5 \stackrel{?}{=} [X_2] \to [X_2 \to X_2] \to [X_2]\} \mid \{X_2 \,/X_1\} \circ \{[X_1] \,/X_4\} \circ \{X_1 \to X_2 \,/X_3\}$$

$$\longmapsto^4 \{\} \mid \{[X_2] \to [X_2 \to X_2] \to [X_2] \,/X_5\} \circ \{X_2 \,/X_1\} \circ \{[X_1] \,/X_4\} \circ \{X_1 \to X_2 \,/X_3\}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = ??$$

$$\mathbb{W}(\mathit{foldr}) = \emptyset \triangleright \mathit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4$$
$$\mathbb{W}(\mathit{map}) = \emptyset \triangleright \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]$$

$$S = MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\}$$
$$= \{X_1 \to X_2 \,/X_3,\ [X_1] \,/X_4,\ X_2 \,/X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2] \,/X_5\}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = S\emptyset \cup S\emptyset \rhd S\ (\mathit{foldr}_{X_3,X_4}\ \mathit{map}_{X_1,X_2}) : Se$$

$$\begin{aligned}
\mathbb{W}(\mathit{foldr}) &= \emptyset \rhd \mathit{foldr}_{X_3,X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \\
\mathbb{W}(\mathit{map}) &= \emptyset \rhd \mathit{map}_{X_1,X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]
\end{aligned}$$

$$\begin{aligned}
S &= MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\} \\
&= \{X_2 \to X_2\ /X_3,\ [X_2]\ /X_4,\ X_2\ /X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2]\ /X_5\}
\end{aligned}$$

Algoritmo de inferencia

$$\mathbb{W}(\mathit{foldr}\ \mathit{map}) = \emptyset \triangleright \mathit{foldr}_{X_2 \to X_2, [X_2]}\ \mathit{map}_{X_2, X_2} : [X_2] \to [X_2 \to X_2] \to [X_2]$$

$$\begin{aligned}
\mathbb{W}(\mathit{foldr}) &= \emptyset \triangleright \mathit{foldr}_{X_3, X_4} : (X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \\
\mathbb{W}(\mathit{map})\ &= \emptyset \triangleright \mathit{map}_{X_1, X_2} : (X_1 \to X_2) \to [X_1] \to [X_2]
\end{aligned}$$

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S &= MGU\{(X_3 \to X_4 \to X_4) \to X_4 \to [X_3] \to X_4 \doteq ((X_1 \to X_2) \to [X_1] \to [X_2]) \to X_5\} \\
&= \{X_2 \to X_2\, / X_3,\ [X_2]\, / X_4,\ X_2\, / X_1,\ [X_2] \to [X_2 \to X_2] \to [X_2]\, / X_5\}
\end{aligned}$$

Extensión del algoritmo de inferencia

Listas

$$\sigma ::= \ldots \mid [\sigma]$$

$$M, N, O ::= \ldots \mid [\,]_\sigma \mid M :: N \mid \mathit{Case}\ M\ \mathit{of}\ [\,] \rightsquigarrow N\ ; h :: t \rightsquigarrow O$$

$$\frac{}{\Gamma \triangleright [\,]_\sigma \ : [\sigma]} \qquad \frac{\Gamma \triangleright M : \sigma \quad \Gamma \triangleright N : [\sigma]}{\Gamma \triangleright M :: N : [\sigma]}$$

$$\frac{\Gamma \triangleright M : [\sigma] \quad \Gamma \triangleright N : \tau \quad \Gamma \cup \{h : \sigma, t : [\sigma]\} \triangleright O : \tau}{\Gamma \triangleright \mathit{Case}\ M\ \mathit{of}\ [\,] \rightsquigarrow N\ ; h :: t \rightsquigarrow O : \tau}$$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \triangleright [\,]_X : [X]$ con X variable fresca

Extensión del algoritmo de inferencia

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$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \triangleright S(M :: N) : S[\sigma]$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma$

$\mathbb{W}(V) = \Gamma_2 \triangleright N : [\sigma]$

$S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

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$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \triangleright S(M :: N) :$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma$

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$S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \rhd [\,]_X : [X]$ con X variable fresca

$\mathbb{W}(U :: V) \stackrel{\text{def}}{=} S\Gamma_1 \cup S\Gamma_2 \rhd S(M :: N) : SX_2$

$\mathbb{W}(U) = \Gamma_1 \rhd M : X_1$

$\mathbb{W}(V) = \Gamma_2 \rhd N : X_2$

$S = MGU\{X_2 \stackrel{?}{=} [X_1]\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \triangleright [\,]_X : [X]$ con X variable fresca

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$\mathbb{W}(V) = \Gamma_2 \triangleright N : X_2$

$S = MGU\{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

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$\mathbb{W}(U) = \Gamma_1 \rhd M : \sigma$

$\mathbb{W}(V) = \Gamma_2 \rhd N : \tau$

$S = MGU\{\tau \stackrel{?}{=} [\sigma]\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\text{def}}{=} \emptyset \rhd [\,]_X : [X]$ con X variable fresca

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$\mathbb{W}(\mathit{Case}\ U\ \mathit{of}\ [\,] \rightsquigarrow V\ ; h :: t \rightsquigarrow W) \stackrel{\text{def}}{=}$

$S\Gamma_1 \cup S\Gamma_2 \cup S\Gamma_{3'} \rhd S\,(\mathit{Case}\ M\ \mathit{of}\ [\,] \rightsquigarrow N\ ; h :: t \rightsquigarrow O) : S\tau$

$\mathbb{W}(U) = \Gamma_1 \rhd M : \sigma \qquad \mathbb{W}(V) = \Gamma_2 \rhd N : \tau \qquad \mathbb{W}(W) = \Gamma_3 \rhd O : \rho$

$\tau_h = \begin{cases} \alpha \text{ si } h : \alpha \in \Gamma_3, \\ \text{var fresca si no} \end{cases} \qquad \tau_t = \begin{cases} \beta \text{ si } t : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$

$\Gamma_{3'} = \Gamma_3 \ominus \{h, t\}$

$S = MGU(\{\sigma \stackrel{?}{=} [\tau_h],\ \rho \stackrel{?}{=} \tau,\ \tau_t \stackrel{?}{=} \sigma\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_i,\ x : \sigma_2 \in \Gamma_j,\ i, j \in \{1, 2, 3'\}\})$

Extensión del algoritmo de inferencia

$\mathbb{W}([\,]) \stackrel{\mathrm{def}}{=} \emptyset \triangleright [\,]_X : [X]$ con X variable fresca

$\mathbb{W}(U :: V) \stackrel{\mathrm{def}}{=} S\Gamma_1 \cup S\Gamma_2 \triangleright S(M :: N) : S\tau$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma$

$\mathbb{W}(V) = \Gamma_2 \triangleright N : \tau$

$S = MGU\{\tau \stackrel{?}{=} [\sigma]\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

$\mathbb{W}(\mathit{Case}\ U\ \mathit{of}\ [\,] \leadsto V\ ; h :: t \leadsto W) \stackrel{\mathrm{def}}{=}$

$S\Gamma_1 \cup S\Gamma_2 \cup S\Gamma_{3'} \triangleright S\,(\mathit{Case}\ M\ \mathit{of}\ [\,] \leadsto N\ ; h :: t \leadsto O) : S\tau$

$\mathbb{W}(U) = \Gamma_1 \triangleright M : \sigma \qquad \mathbb{W}(V) = \Gamma_2 \triangleright N : \tau \qquad \mathbb{W}(W) = \Gamma_3 \triangleright O : \rho$

$\tau_h = \begin{cases} \alpha \text{ si } h : \alpha \in \Gamma_3, \\ \text{var fresca si no} \end{cases} \qquad \tau_t = \begin{cases} \beta \text{ si } t : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$

$\Gamma_{3'} = \Gamma_3 \ominus \{h, t\}$

$S = MGU(\{\sigma \stackrel{?}{=} [\tau_h],\ \rho \stackrel{?}{=} \tau,\ \tau_t \stackrel{?}{=} \sigma\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_i,\ x : \sigma_2 \in \Gamma_j,\ i, j \in \{1, 2, 3'\}\})$

Dar el tipo de: $\mathit{Case}\ \mathrm{succ}(0) :: x\ \mathit{of}\ [\,] \leadsto x\ ; x :: y \leadsto \mathrm{succ}(x) :: [\,]$

Extensión del algoritmo de inferencia

Listas por comprensión

$$M ::= \ldots \mid \; [\; M \mid x \leftarrow M, M \;]$$

Consideremos el Cálculo Lambda extendido con las listas por comprensión vistas en la práctica 4.

La regla de tipado es la siguiente:

$$\frac{\Gamma \cup \{x : \sigma\} \;\triangleright M : \tau \quad \Gamma \triangleright N : [\sigma] \quad \Gamma \cup \{x : \sigma\} \;\triangleright\; O : \mathsf{Bool}}{\Gamma \triangleright [\; M \mid x \leftarrow N, O \;] : \; [\tau]}$$

Extensión del algoritmo de inferencia

Listas por Comprensión

$$\mathbb{W}([\ U \mid x \leftarrow V, W\ ]) \stackrel{\mathrm{def}}{=} S\Gamma_{1'} \cup S\Gamma_2 \cup S\Gamma_{3'} \ \rhd S\,([\ M \mid X \leftarrow N, O\ ]) : S[\sigma_1]$$

$$\begin{aligned} \mathbb{W}(U) &= \Gamma_1 \rhd M : \sigma_1 \\ \mathbb{W}(V) &= \Gamma_2 \rhd N : \sigma_2 \\ \mathbb{W}(W) &= \Gamma_3 \rhd O : \sigma_3 \end{aligned}$$

$$\tau_{x1} = \begin{cases} \alpha \text{ si } x : \alpha \in \Gamma_1, \\ \text{var fresca si no} \end{cases} \qquad \tau_{x2} = \begin{cases} \beta \text{ si } x : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$$

$$\Gamma_{1'} = \Gamma_1 \ominus \{x\} \qquad \Gamma_{3'} = \Gamma_3 \ominus \{x\}$$

$$\begin{aligned} S = MGU(&\{\tau_{x1} \stackrel{?}{=} \tau_{x2},\ \sigma_2 \stackrel{?}{=} [\tau_{x1}],\ \sigma_3 \stackrel{?}{=} \text{Bool}\} \\ &\cup \{\rho_1 \stackrel{?}{=} \rho_2 \mid y : \rho_1 \in \Gamma_i,\ y : \rho_2 \in \Gamma_j,\ i, j \in \{1', 2, 3'\}\}) \end{aligned}$$

Extensión del algoritmo de inferencia

Listas por Comprensión

$$\mathbb{W}([\ U \mid x \leftarrow V, W\ ]) \stackrel{\text{def}}{=} S\Gamma_{1'} \cup S\Gamma_2 \cup S\Gamma_{3'}\ \triangleright S\,([\ M \mid X \leftarrow N, O\ ]) : S[\sigma_1]$$

$$\begin{aligned} \mathbb{W}(U)\ &= \Gamma_1 \triangleright M : \sigma_1 \\ \mathbb{W}(V)\ &= \Gamma_2 \triangleright N : \sigma_2 \\ \mathbb{W}(W) &= \Gamma_3 \triangleright O : \sigma_3 \end{aligned}$$

$$\tau_{x1} = \begin{cases} \alpha \text{ si } x : \alpha \in \Gamma_1, \\ \text{var fresca si no} \end{cases} \qquad \tau_{x2} = \begin{cases} \beta \text{ si } x : \beta \in \Gamma_3, \\ \text{var fresca si no} \end{cases}$$

$$\Gamma_{1'} = \Gamma_1 \ominus \{x\} \qquad \Gamma_{3'} = \Gamma_3 \ominus \{x\}$$

$$\begin{aligned} S = MGU(&\{\tau_{x1} \stackrel{?}{=} \tau_{x2},\ \sigma_2 \stackrel{?}{=} [\tau_{x1}],\ \sigma_3 \stackrel{?}{=} \text{Bool}\} \\ &\cup \{\rho_1 \stackrel{?}{=} \rho_2 \mid y : \rho_1 \in \Gamma_i,\ y : \rho_2 \in \Gamma_j,\ i,j \in \{1', 2, 3'\}\}) \end{aligned}$$

Dar el tipo de: $[\ \text{if } x \text{ then } \underline{0} \text{ else } \underline{1} \mid x \leftarrow \mathit{false} :: \text{iszero}(x) :: [\ ], \mathit{true}\ ]$

Fin

¿ ¿ ¿ ¿ ¿ ¿Preguntas? ? ? ? ? ?