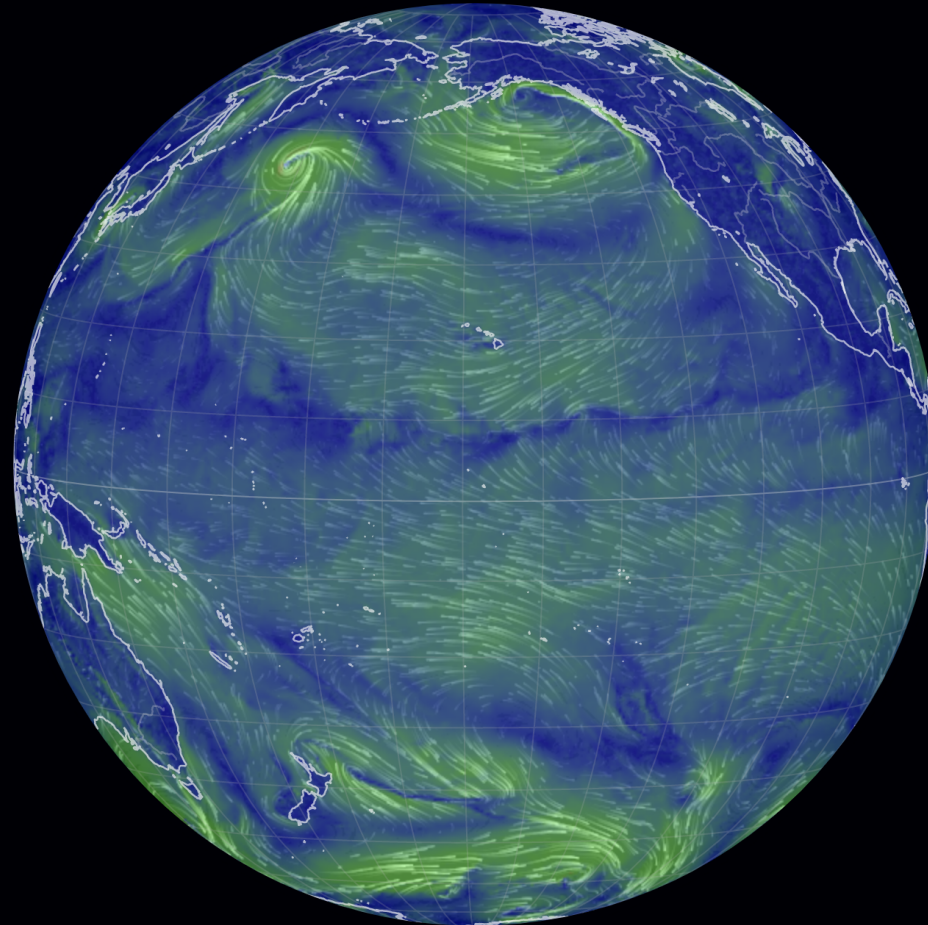


# **Simulating the Atmosphere**

**(for our project)**

**(a very simplified overview)**

# What can we expect (not) to see?

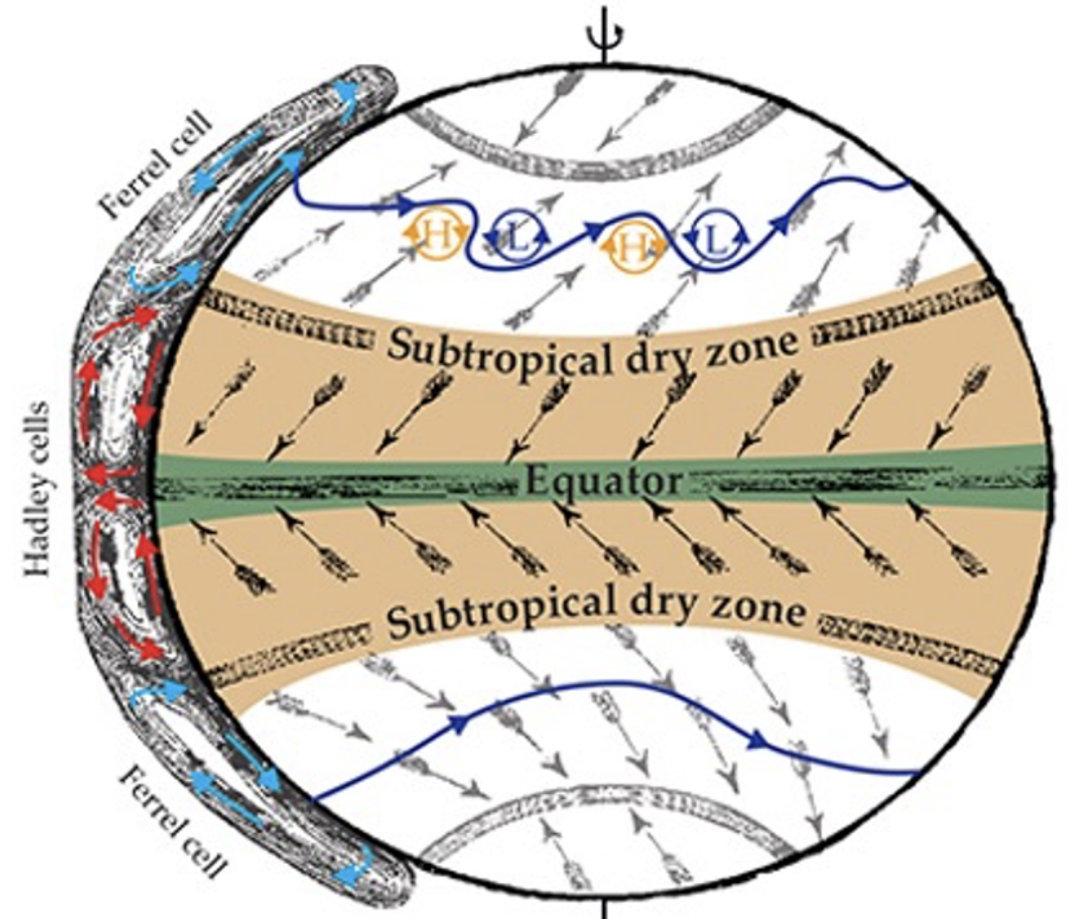


## Possible:

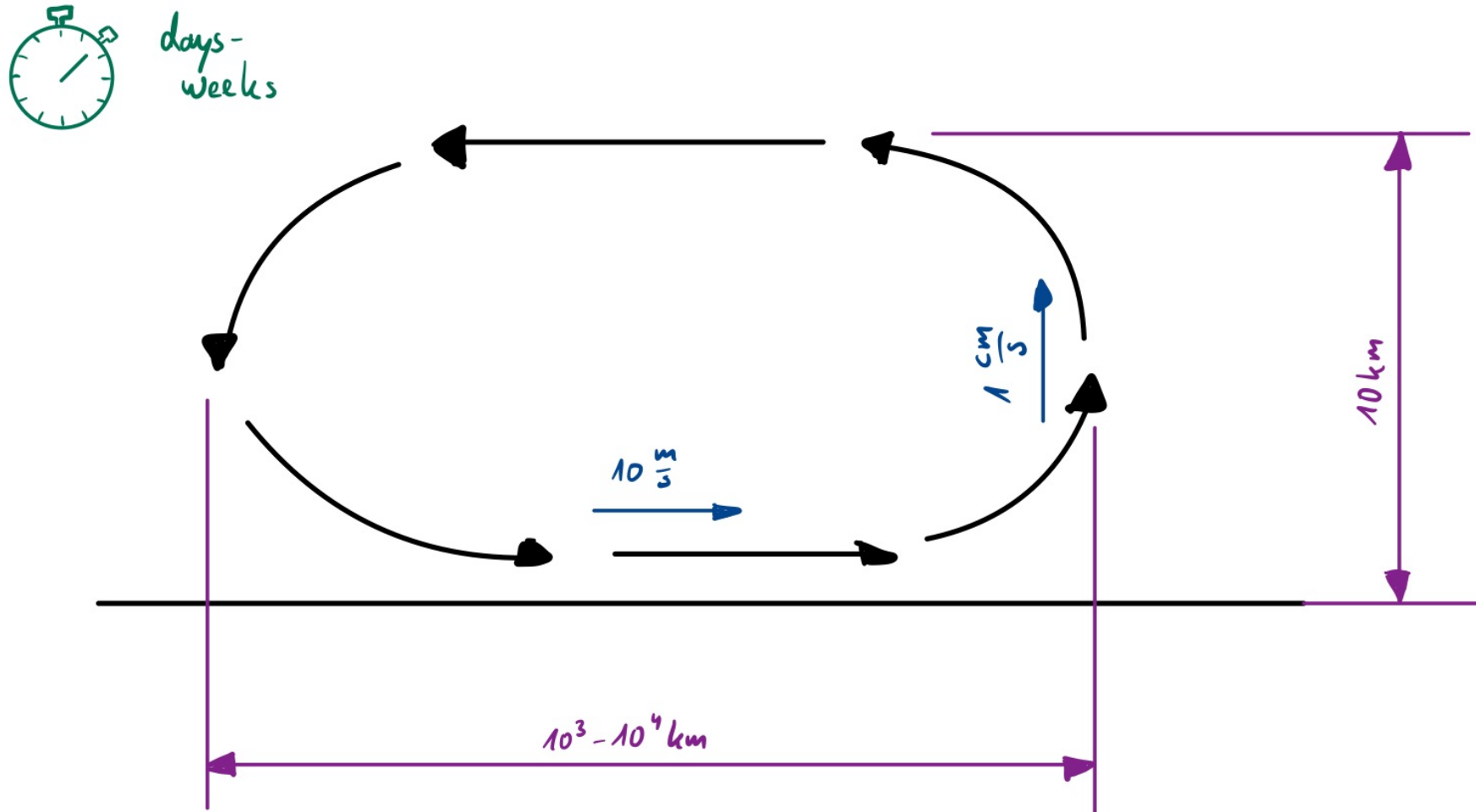
- meridional overturning circulation (Hadley / Ferrel)
- some form of water evaporation / rainfall

## Not Possible:

- zonal circulation (e.g., jet streams)
- eddies (Rossby waves)



# A Matter of Scales



# Modeling Assumptions

using [Held & Hou, 1980]

- "pizza model"  $\leftrightarrow$  axi-symmetric model  $\Rightarrow$  no day/night effects, no cross-equatorial flow
- spherical coordinates
- neglect land/water surface height
- neglect effects of water content on dynamics and radiation  $\Rightarrow$  dry model equations
- solar radiation  $\leftrightarrow$  relaxation towards equilibrium temperature  $\Theta_e$

# Navier-Stokes Equations - Basic Idea

$$\frac{D(\cdot)}{Dt} = \sum_i Q_i$$

where:

$$\frac{D(\cdot)}{Dt} = \left( \partial_t + \nabla \cdot \begin{pmatrix} v \\ w \end{pmatrix} \right) (\cdot)$$



# Notation

- $z, \varphi$ : coordinates
- $u, v, w$ : wind speeds in zonal, meridional, vertical directions
- $\Theta, \Theta_E, \Theta_0, \tau$ : potential temperature, equilibrium temperature, mean of  $\Theta_E$ , radiative damping time
- $a, f = 2\Omega \sin(\varphi), \Phi = gz$ : planet radius, Coriolis parameter, geopotential
- $\nabla = \begin{pmatrix} (a \cos(\varphi))^{-1} \partial_\varphi \\ \partial_z \end{pmatrix}$

# Model Equations (cf. Held, Hou 1980)

continuity equation / conservation of mass:

$$0 = -\nabla \cdot \begin{pmatrix} v \\ w \end{pmatrix}$$

(implicit assumption:  $\partial_t \varrho = 0$ )



# Model Equations (cf. Held, Hou 1980)

equations of motion:

$$\partial_t u = -\nabla \cdot \begin{pmatrix} vu \\ wu \end{pmatrix} + fv + \frac{uv \tan \varphi}{a} + \partial_z (\nu \partial_z u)$$

$$\partial_t v = -\nabla \cdot \begin{pmatrix} vv \\ wv \end{pmatrix} - fu - \frac{u^2 \tan \varphi}{a} - \frac{1}{a} \partial_\varphi \Phi + \partial_z (\nu \partial_z v)$$

$$\partial_t w = -\nabla \cdot \begin{pmatrix} vu \\ wu \end{pmatrix} - \partial_z \Phi + \frac{g\Theta}{\Theta_0}$$

# Model Equations (cf. Held, You 1980)

first law of thermodynamics:

$$\partial_t \Theta = -\nabla \cdot \begin{pmatrix} v\Theta \\ w\Theta \end{pmatrix} - (\Theta - \Theta_E)\tau^{-1} + \partial_z (\nu \partial_z \Theta)$$

# Discretization Suggestions

- Finite Differences in space
- staggered grid for stability reasons
- implicit time integration method

See equations.pdf

# Implementation Considerations I

- input data
  - physical parameters
- interface data:
  - water evaporating at the ground
  - rainfall
- output data:
  - $u, v, w, \Theta$  fields, water content in the grid

# Implementation Considerations II

If everything fails, what is our MVP?

- a system which conserves mass, momentum, and energy
- fluid rising at the equator, travelling polewards, descending and going back to the equator
- water entering and leaving as we want to
- $\Rightarrow$  just implement a steady-state movement roughly resembling this!

# Literature / References

see [repository](#)

## Image Credits

1: "Atmosphere | Atmosphäre" by Astro\_Alex is licensed with CC BY-SA 2.0. [License Copy](#).

2: Screenshot from <https://earth.nullschool.net/>. (2021-09-17, 10:57)

3: Adapted from Fig. 2 in: Birner, Davis, Seidel: Physics Today 67, 38-44 (2014). DOI: [10.1063/PT.3.2620](https://doi.org/10.1063/PT.3.2620).