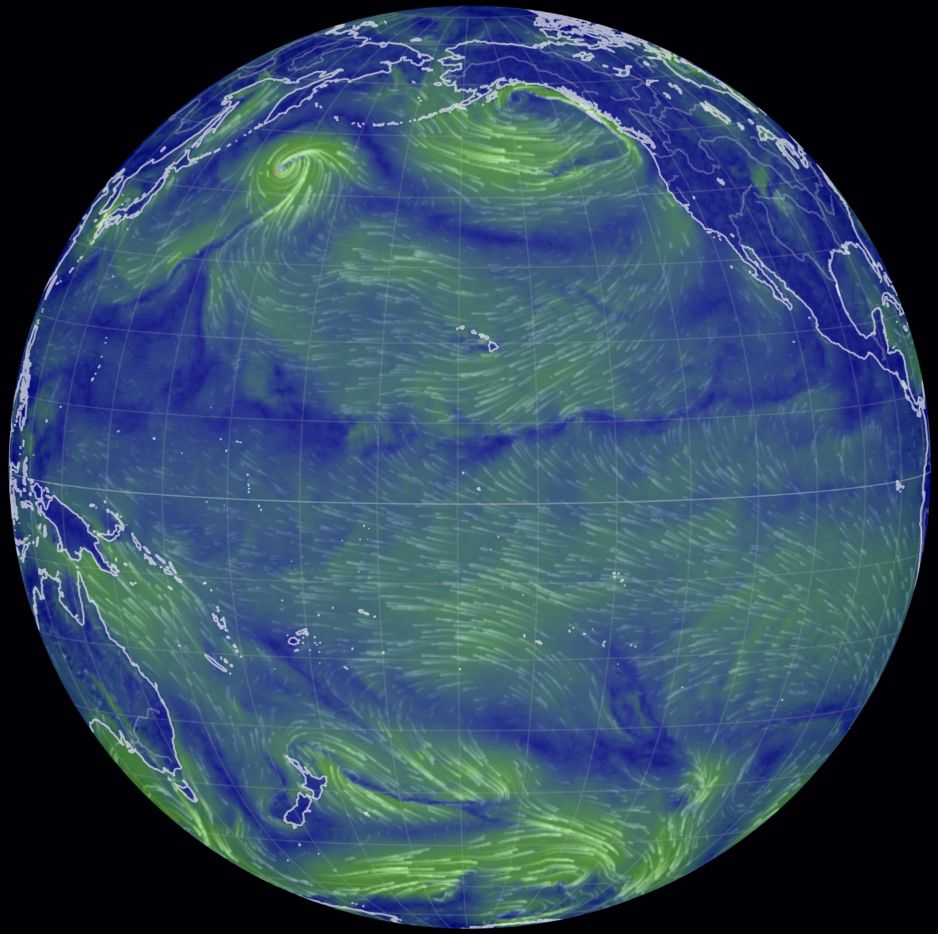
Simulating the Atmosphere

(for our project)

(a very simplified overview)

What can we expect (not) to see?

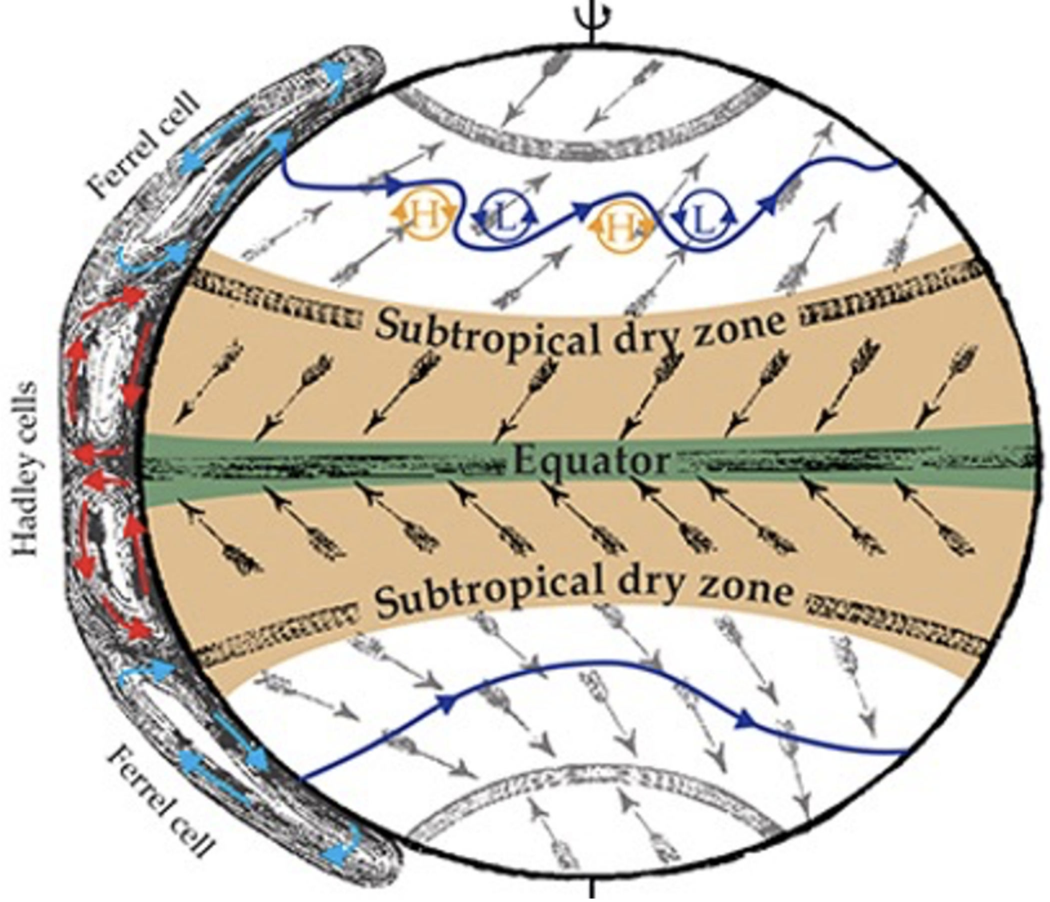


Possible:

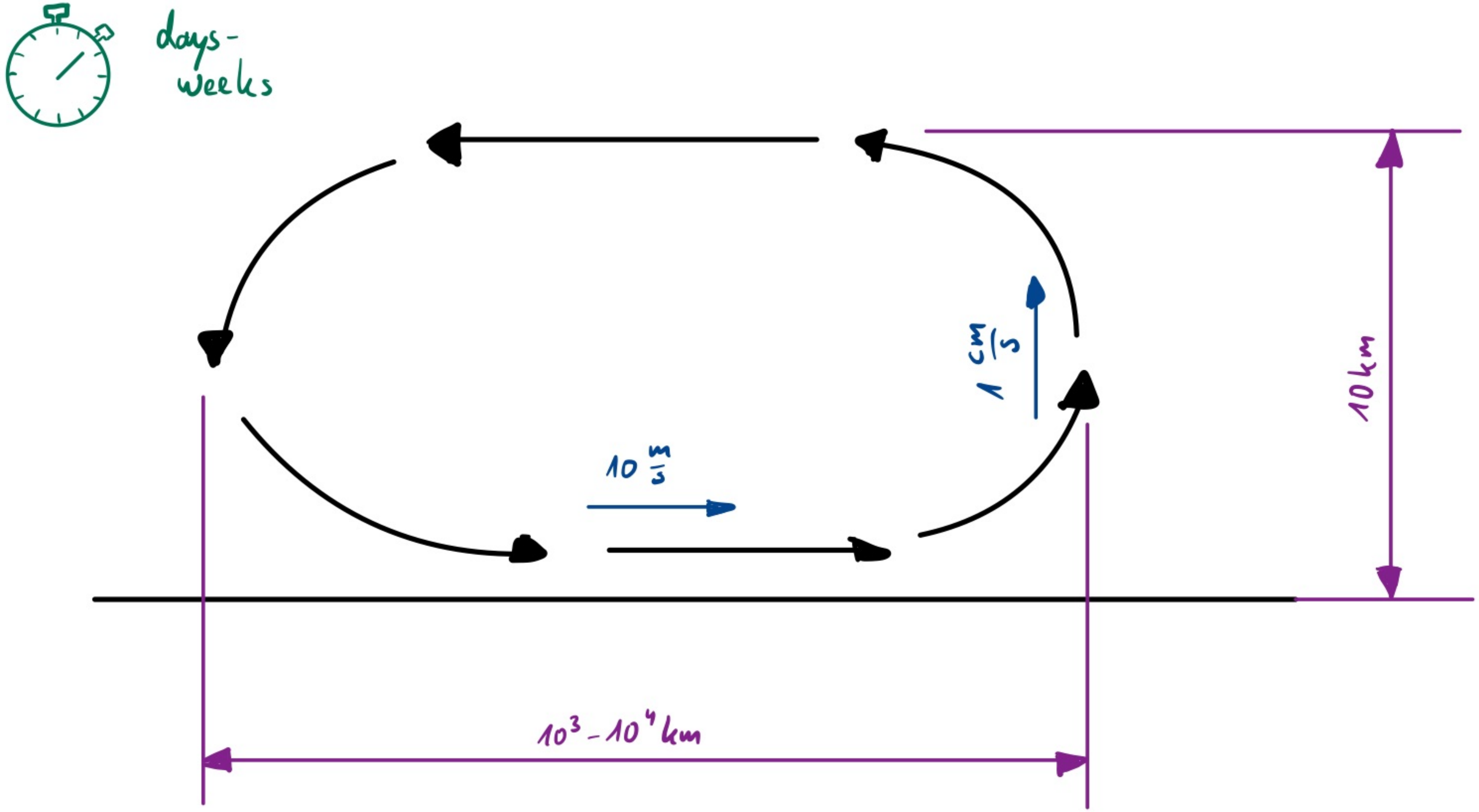
- meridional overturning circulation (Hadley / Ferrel)
- some form of water evaporation / rainfall

Not Possible:

- zonal circulation (e.g., jet streams)
- eddies (Rossby waves)



A Matter of Scales



Modeling Assumptions

using [Held & Hou, 1980]

- "pizza model" $\leftrightarrow$ axi-symmetric model $\Rightarrow$ no day/night effects, no cross-equatorial flow
- spherical coordinates
- neglect land/water surface height
- neglect effects of water content on dynamics and radiation $\Rightarrow$ dry model equations
- solar radiation $\leftrightarrow$ relaxation towards equilibrium temperature Θ_e

Navier-Stokes Equations - Basic Idea

$$\frac{D(\cdot)}{Dt} = \sum_i Q_i$$

where:

$$\frac{D(\cdot)}{Dt} = \left(\partial_t + \nabla \cdot \begin{pmatrix} v \\ w \end{pmatrix}\right)(\cdot)$$

Notation

- z, φ: coordinates
- u, v, w: wind speeds in zonal, meridional, vertical directions
- $\Theta, \Theta_E, \Theta_0, \tau$: potential temperature, equilibrium temperature, mean of Θ_E, radiative damping time
- $a, f = 2\Omega sin(\varphi), \Phi = gz$: planet radius, Coriolis parameter, geopotential
- $\nabla = \begin{pmatrix} (a\cos(\varphi))^{-1}\partial_\varphi \\ \partial_z \end{pmatrix}$

Model Equations (cf. Held, Hou 1980)

continuity equation / conservation of mass:

$$0 = -\nabla \cdot \begin{pmatrix} v \\ w \end{pmatrix}$$

(implicit assumption: $\partial_t \varrho = 0$)

Model Equations (cf. Held, Hou 1980)

equations of motion:

$$\partial_t u = -\nabla \cdot \begin{pmatrix} vu \\ wu \end{pmatrix} + fv + \frac{uv \tan\varphi}{a} + \partial_z \left(\nu \partial_z u\right)$$

$$\partial_t v = -\nabla \cdot \begin{pmatrix} vv \\ wv \end{pmatrix} - fu - \frac{u^2 \tan\varphi}{a} - \frac{1}{a}\partial_\varphi \Phi + \partial_z \left(\nu \partial_z v\right)$$

$$\partial_t w = -\nabla \cdot \begin{pmatrix} vu \\ wu \end{pmatrix} - \partial_z \Phi + \frac{g\Theta}{\Theta_0}$$

Model Equations (cf. Held, You 1980)

first law of thermodynamics:

$$\partial_t \Theta = -\nabla \cdot \begin{pmatrix} v\Theta \\ w\Theta \end{pmatrix} - (\Theta - \Theta_E)\tau^{-1} + \partial_z \left(\nu \partial_z \Theta\right)$$

Discretization Suggestions

- Finite Differences in space
- staggered grid for stability reasons
- implicit time integration method

See equations.pdf

Implementation Considerations I

- input data
  - physical parameters
- interface data:
  - water evaporating at the ground
  - rainfall
- output data:
  - u, v, w, Θ fields, water content in the grid

Implementation Considerations II

If everything fails, what is our MVP?

- a system which conserves mass, momentum, and energy
- fluid rising at the equator, travelling polewards, descending and going back to the equator
- water entering and leaving as we want to
- $\Rightarrow$ just implement a steady-state movement roughly resembling this!

Literature / References

see repository

Image Credits

1: "Atmosphere | Atmosphäre" by Astro_Alex is licensed with CC BY-SA 2.0. License Copy.

2: Screenshot from https://earth.nullschool.net/. (2021-09-17, 10:57)

3: Adapted from Fig. 2 in: Birner, Davis, Seidel: Physics Today 67, 38-44 (2014). DOI: 10.1063/PT.3.2620.