Exchange Magnons in Ferromagnetic Films Excited by Picosecond Acoustic Pulses

Supplementary Information

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The supplementary information is split in ... sections. The first section is dedicated to the calculation of the model we numerically resolve in the main text.

MAGNETIZATION DYNAMICS EXCITED BY ACOUSTIC PULSES

We consider the Landau-Lifschitz-Gilbert equation in the Cartesian coordinate system (x, y, z)

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\mu_0 \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}, \tag{1}$$

which describe the magnetization dynamics in a ferromagnetic film, where $\mathbf{m}$ is the unit magnetization vector, γ is the gyromagnetic ratio, μ_0 is the vacuum permeability and $\mathbf{H}_{eff}$ is the effective magnetic field. It is the functional derivative of the free energy density

$$\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_0}\frac{\partial U}{\partial \mathbf{m}} + \frac{1}{M_0}\sum_{p=1}^{3}\frac{\partial}{\partial x_p}\frac{\partial U}{\partial\left(\frac{\partial \mathbf{m}}{\partial x_p}\right)}, \tag{2}$$

where M_0 is the magnetic saturation. We define the free density energy to be

$$U = U_{de} + U_z + U_{me} + U_{ex}, \tag{3}$$

which include the demagnetizing field's energy

$$U_{de} = \frac{1}{2}\mu_0 M_0^2 \mathbf{m}\cdot\mathbf{N}\cdot\mathbf{m}, \tag{4}$$

the external magnetic field's energy

$$U_z = -\mu_0 M_0 \mathbf{m}\cdot\mathbf{H}, \tag{5}$$

the magnetoelastic field energy

$$U_{me} = b_1 \sum_{p=1}^{3} m_p^2 \epsilon_{pp} \tag{6}$$

and the exchange magnetic field energy

$$U_{ex} = \frac{1}{2}M_0 \sum_{p=1}^{3} D\left(\frac{\partial \mathbf{m}}{\partial x_p}\right)^2. \tag{7}$$

The magnetoelastic constant is b_1, while the exchange constant is D. In this section, we will consider that the exchange magnetic field energy does not depend on the strain.

We consider the situation shown in the Fig. 1 of the main text. It implies that the unit magnetic vector $\mathbf{m}$ and the external magnetic vector $\mathbf{H}$ are in the plane (xOz). The consequence is that the demagnetization tensor is

$$\mathbf{N} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

and the acoustic strain is

$$\sum_{p=1}^{3} \epsilon_{pp} = \epsilon_{zz}. \tag{9}$$

By using Eqs. (2), (3), (4), (5), (6) and (7) and in accordance with the Fig. 1, we obtain

$$\begin{cases} H_{\mathrm{eff,x}} = H\cos(\xi) + M_0 D \dfrac{\partial^2 m_{\mathrm{x}}}{\partial z^2} & (10) \\ H_{\mathrm{eff,y}} = M_0 D \dfrac{\partial^2 m_{\mathrm{y}}}{\partial z^2} & (11) \\ H_{\mathrm{eff,z}} = H\sin(\xi) - \dfrac{2b_1}{\mu_0 M_0} m_{\mathrm{z}} \epsilon_{\mathrm{zz}} + M_0 D \dfrac{\partial^2 m_z}{\partial z^2} - M_0 \mathrm{m_z} & (12) \end{cases}$$

First, we consider that $\alpha = 0$, so we can rewrite 1:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{eff}. \tag{13}$$

We represent the unit magnetization vector as a Fourier series

$$\mathbf{m} = \mathbf{m}^0 + \sum_{\mathrm{n}=0}^{\mathrm{N}} \mathbf{m}_{\mathrm{n}}(t) \cos\left(\frac{\pi n}{L} z\right), \tag{14}$$

where $\mathbf{m}^0$ is the stable state equilibrium without any perturbation, n is the magnon mode number, L is the thickness of ferromagnetic film. The constant component of the magnetization vector tilted by a θ angle with respect to the x axis (see Fig. 1 in the main manuscript). We set the boundary condition to

$$\left.\frac{\partial m_i}{\partial z}\right|_{z=0,L} = 0. \tag{15}$$

MAGNETOELASTIC FIELD

STRAIN DEPENDENCE OF EXCHANGE FIELD

In this section, we will consider that the exchange field energy is a strain dependent process

The energy of the exchange interaction in a ferromagnet can be represented by [1]

$$U_{\text{ex}} = U_{\text{ex},0} + U_{\text{ex,NU}} \tag{16}$$

where $U_{\text{ex},0}$ is the exchange energy when the magnetization is uniform and $U_{\text{ex,NU}}$ is responsible for the change of the exchange energy when the magnetization is non-uniform. The first term of Eq. (16) can be written

$$U_{\text{ex},0} = \frac{1}{2}\mathbf{M}\overleftrightarrow{\Lambda}\mathbf{M} \tag{17}$$

with the magnetization $\mathbf{M}$ and the exchange tensor $\overleftrightarrow{\Lambda}$.

The second term of Eq. (16) can be written

$$U_{\text{ex,NU}} = \frac{1}{2}\sum_{p=1}^{3}\sum_{s=1}^{3} q_{ps}\frac{\partial \mathbf{M}}{\partial x_p}\frac{\partial \mathbf{M}}{\partial x_s} \tag{18}$$

where q_{ps} are the components of a tensor $\overleftrightarrow{q}$.

The effective field is a functional derivative (or variational derivative) of the energy by the magnetization vector

$$H_{\mathbf{eff}} = -\frac{\delta U}{\delta \mathbf{M}} = -\frac{\partial U}{\partial \mathbf{M}} + \sum_{p=1}^{3}\frac{\partial}{\partial x_p}\left[\frac{\partial U}{\partial\left(\partial \mathbf{M}/\partial x_p\right)}\right] \tag{19}$$

with the total free energy U. Using Eqs. (18) and (19) we obtain the contribution of the exchange interaction to the effective field

$$\mathbf{H}_{\text{ex}} = \mathbf{H}_{\text{ex},0} + \mathbf{H}_{\text{ex,NU}} \equiv \overleftrightarrow{\Lambda}\mathbf{M} + \sum_{p=1}^{3}\sum_{s=1}^{3} q_{ps}\frac{\partial^2 \mathbf{M}}{\partial x_p \partial x_s} \tag{20}$$

So now, let's consider the case of an isotropic ferromagnet, it implies that Λ and q are scalars. We write $q = D$. We also consider the equation only for the x-direction. The Eq. (20) becomes

$$\mathbf{H}_{\text{ex}} = \Lambda\mathbf{M} + D\frac{\partial^2 \mathbf{M}}{\partial x^2}. \tag{21}$$

We write the magnetization vector as a Fourier series

$$\mathbf{M} = \mathbf{M}_0 + \sum_{n=0}^{N}\mathbf{M}_n\left(t\right)\cos\left(k_n x\right) \tag{22}$$

where $\mathbf{M}_0$ is the steady states of the magnetization vector, $\mathbf{M}_n$ is the n-th order of the magnetization vector and k_n is the wavevector of the n-th magnetic order. It implies that $\mathbf{M}_n \ll \mathbf{M}_0$. Substitutes Eq. (22) into the non-uniform part of Eq. (21) we obtain

$$\mathbf{H}_{\mathrm{ex,NU}} = D\frac{\partial^2 \mathbf{M}}{\partial x} = -D\sum_{n=0}^{N} k_n^2 \mathbf{M}_n^2 \cos\left(k_n x\right). \tag{23}$$

The lossless Landau-Lifshitz-Gilbert equation is defined by

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\mathrm{eff}} \tag{24}$$

with γ is the gyromagnetic ratio. It is obvious that since $\mathbf{H}_{\mathrm{ex},0}$ depends on $\mathbf{M}$ so it does not play a role

$$\mathbf{M} \times \mathbf{H}_{\mathrm{ex},0} = 0.$$

Adding Eq. (23) into Eq. 24 we obtain

$$\frac{\partial \mathbf{M}}{\partial t} + \gamma D\left[\mathbf{M} \times \frac{\partial^2 \mathbf{M}}{\partial x^2}\right] = 0 \tag{25}$$

Let's consider that the time dependence of the magnetization vector is

$$\mathbf{M}\left(t, \mathbf{r}\right) \sim \mathbf{M}\left(\mathbf{r}\right) \exp\left(i\omega t\right)$$

and that the exchange constant D is modified by the propagation of a strain: the distance between two atoms changes. So the exchange coefficient depends on time and space.

$$i\omega \mathbf{M} + \gamma D\left(t, z\right)\left[\mathbf{M} \times \frac{\partial^2 \mathbf{M}}{\partial x^2}\right] = 0 \tag{26}$$

The Eq. (26) shows that the exchange interaction appears only in parametric terms. It means that this term cannot drive the magnons by itself if $\mathbf{M}_n\left(t = 0, \mathbf{r}\right) = \overrightarrow{0}$.

Now, let's consider that the exchange coefficient is dependant on strain (ϵ) propagation. We write

$$D\left(t, z\right) \sim D_0 \exp\left(-\frac{\Delta r}{R_{\mathrm{D}}}\right) \tag{27}$$

where D_0 is the exchange coefficient equilibrium value at the equilibrium, Δr is the distance between two neighbor electrons which depends on the strain and R_{D} is the typical length of the screening effect. The distance between two neighbor electrons change proportionally to the amplitude of the strain

$$\Delta r \sim r_0 \epsilon$$

where r_0 is the distance between two neighbor electrons at the equilibrium. The Eq. (27) is

$$D\left(t,z\right) \sim D_0 \exp\left(-\frac{r_0\epsilon}{R_{\mathrm{D}}}\right) \tag{28}$$

We develop in Taylor series the Eq. (28)

$$D\left(t,z\right) \sim ()D_0\left(1+\frac{r_0}{R_{\mathrm{D}}}\epsilon\right). \tag{29}$$

The typical length of the screening effect is in the range of the size of the lattice r_0. Consequence Eq. (29) is

$$D\left(t,z\right) \sim D_0\left(1+\epsilon\right). \tag{30}$$

So the first-order Taylor coefficient is in the order of the strain amplitude, so typically from 0.01% to 0.1%.

PARAMETERS USED IN THE SIMULATION

The relevant parameters are given in Table I.

Group	Parameter	Value
Nickel	Thickness	30 nm
	H-field	484.1×10^3 A/m
	Sound velocity c_{s}	5.6×10^3 m/s
	Exchange constant D	430 meV.A^2
	Magnetoelastic constant b_1	1×10^7
	Electron gyromagnetic ratio γ	1.76×10^{11} rad/s.T
Adjustable	Magnetic field angle ξ	45°
	Acoustic pulse duration	1 ps
	Maximal propagation time	400 ps

TABLE I. Parameters value used for the numerical simulation

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