Report : Strain dependence on exchange interaction February 23, 2018

Valentin BESSE[1], Vladimir VLASOV, Anton GOLOV, Alexandr ALEKHIN, Dmitrii KUZMIN and Vasily TEMNOV

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1 Introduction

In this report we discuss the strain dependence of the exchange interaction. We present the theory that describe the energy and the field of the exchange interaction (see section 2). We introduce the strain dependence on these equations and we predict some results.

We conclude by a proposition of the new tasks regarding this specific interaction.

2 Theory

The energy of the exchange interaction in a ferromagnet can be represented by [1]

$$U_{\text{ex}} = U_{\text{ex},0} + U_{\text{ex,NU}} \tag{1}$$

where $U_{\text{ex},0}$ is the exchange energy when the magnetization is uniform and $U_{\text{ex,NU}}$ is responsible for the change of the exchange energy when the magnetization is non-uniform. The first term of Eq. (1) can be written

$$U_{\text{ex},0} = \frac{1}{2}\mathbf{M}\overleftrightarrow{\Lambda}\mathbf{M} \tag{2}$$

with the magnetization $\mathbf{M}$ and the exchange tensor $\overleftrightarrow{\Lambda}$.

The second term of Eq. (1) can be written

$$U_{\text{ex,NU}} = \frac{1}{2}\sum_{p=1}^{3}\sum_{s=1}^{3} q_{ps}\frac{\partial \mathbf{M}}{\partial x_p}\frac{\partial \mathbf{M}}{\partial x_s} \tag{3}$$

[1] Author of the report

where q_{ps} are the components of a tensor $\overleftrightarrow{q}$.

The effective field is a functional derivative (or variational derivative) of the energy by the magnetization vector

$$H_{\mathbf{eff}} = -\frac{\delta U}{\delta \mathbf{M}} = -\frac{\partial U}{\partial \mathbf{M}} + \sum_{p=1}^{3} \frac{\partial}{\partial x_p} \left[\frac{\partial U}{\partial \left(\partial \mathbf{M} / \partial x_p \right)} \right] \tag{4}$$

with the total free energy U. Using Eqs. (3) and (4) we obtain the contribution of the exchange interaction to the effective field

$$\mathbf{H}_{\text{ex}} = \mathbf{H}_{\text{ex},0} + \mathbf{H}_{\text{ex,NU}} \equiv \overleftrightarrow{\Lambda} \mathbf{M} + \sum_{p=1}^{3} \sum_{s=1}^{3} q_{ps} \frac{\partial^2 \mathbf{M}}{\partial x_p \partial x_s} \tag{5}$$

So now, let's consider the case of an isotropic ferromagnet, it implies that Λ and q are scalars. We write $q = D$. We also consider the equation only for the x-direction. The Eq. (5) becomes

$$\mathbf{H}_{\text{ex}} = \Lambda \mathbf{M} + D \frac{\partial^2 \mathbf{M}}{\partial x^2}. \tag{6}$$

We write the magnetization vector as a Fourier series

$$\mathbf{M} = \mathbf{M}_0 + \sum_{n=0}^{N} \mathbf{M}_n \left(t \right) \cos \left(k_n x \right) \tag{7}$$

where $\mathbf{M}_0$ is the steady states of the magnetization vector, $\mathbf{M}_n$ is the n-th order of the magnetization vector and k_n is the wavevector of the n-th magnetic order. It implies that $\mathbf{M}_n \ll \mathbf{M}_0$. Substitutes Eq. (7) into the non-uniform part of Eq. (6) we obtain

$$\mathbf{H}_{\text{ex,NU}} = D \frac{\partial^2 \mathbf{M}}{\partial x} = -D \sum_{n=0}^{N} k_n^2 \mathbf{M}_n^2 \cos \left(k_n x \right). \tag{8}$$

The lossless Landau-Lifshitz-Gilbert equation is defined by

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} \tag{9}$$

with γ is the gyromagnetic ratio. It is obvious that since $\mathbf{H}_{\text{ex},0}$ depends on $\mathbf{M}$ so it does not play a role

$$\mathbf{M} \times \mathbf{H}_{\text{ex},0} = 0.$$

Adding Eq. (8) into Eq. 9 we obtain

$$\frac{\partial \mathbf{M}}{\partial t} + \gamma D \left[\mathbf{M} \times \frac{\partial^2 \mathbf{M}}{\partial x^2} \right] = 0 \tag{10}$$

Let's consider that the time dependence of the magnetization vector is

$$\mathbf{M}\left(t, \mathbf{r}\right) \sim \mathbf{M}\left(\mathbf{r}\right) \exp\left(i\omega t\right)$$

and that the exchange constant D is modified by the propagation of a strain: the distance between two atoms changes. So the exchange coefficient depends on time and space.

$$i\omega \mathbf{M} + \gamma D\left(t, z\right) \left[\mathbf{M} \times \frac{\partial^2 \mathbf{M}}{\partial x^2} \right] = 0 \tag{11}$$

The Eq. (11) shows that the exchange interaction appears only in parametric terms. It means that this term cannot drive the magnons by itself if $\mathbf{M}_n\left(t = 0, \mathbf{r}\right) = \overrightarrow{0}$.

Now, let's consider that the exchange coefficient is dependant on strain (ϵ) propagation. We write

$$D\left(t, z\right) \sim D_0 \exp\left(-\frac{\Delta r}{R_{\mathrm{D}}}\right) \tag{12}$$

where D_0 is the exchange coefficient equilibrium value at the equilibrium, Δr is the distance between two neighbor electrons which depends on the strain and R_{D} is the typical length of the screening effect. The distance between two neighbor electrons change proportionally to the amplitude of the strain

$$\Delta r \sim r_0 \epsilon$$

where r_0 is the distance between two neighbor electrons at the equilibrium. The Eq. (12) is

$$D\left(t, z\right) \sim D_0 \exp\left(-\frac{r_0 \epsilon}{R_{\mathrm{D}}}\right) \tag{13}$$

We develop in Taylor series the Eq. (13)

$$D\left(t, z\right) \sim ()D_0 \left(1 + \frac{r_0}{R_{\mathrm{D}}} \epsilon\right). \tag{14}$$

The typical length of the screening effect is in the range of the size of the lattice r_0. Consequence Eq. (14) is

$$D\left(t, z\right) \sim D_0\left(1 + \epsilon\right). \tag{15}$$

So the first-order Taylor coefficient is in the order of the strain amplitude, so typically from 0.01% to 0.1%.

3 Conclusion

In conclusion, the exchange interaction cannot drive the magnons by itself. It is necessary to consider it along with the magnetoelastic effect or to use a non-zero initial value for magnetization vector. Also it appears that the Taylor expansion's first-order coefficient of the exchange interaction is in the range of the amplitude of the strain (from 0.01% to 0.1%).

List of abbreviations

Landau-Lifschitz-Gilbert	$\Longrightarrow$	LLG
Ferromagnetic resonance	$\Longrightarrow$	FMR

References

[1] A. G. Gurevich and G. A. Melkov, *Magnetization oscillations and waves*. CRC press, 1996.