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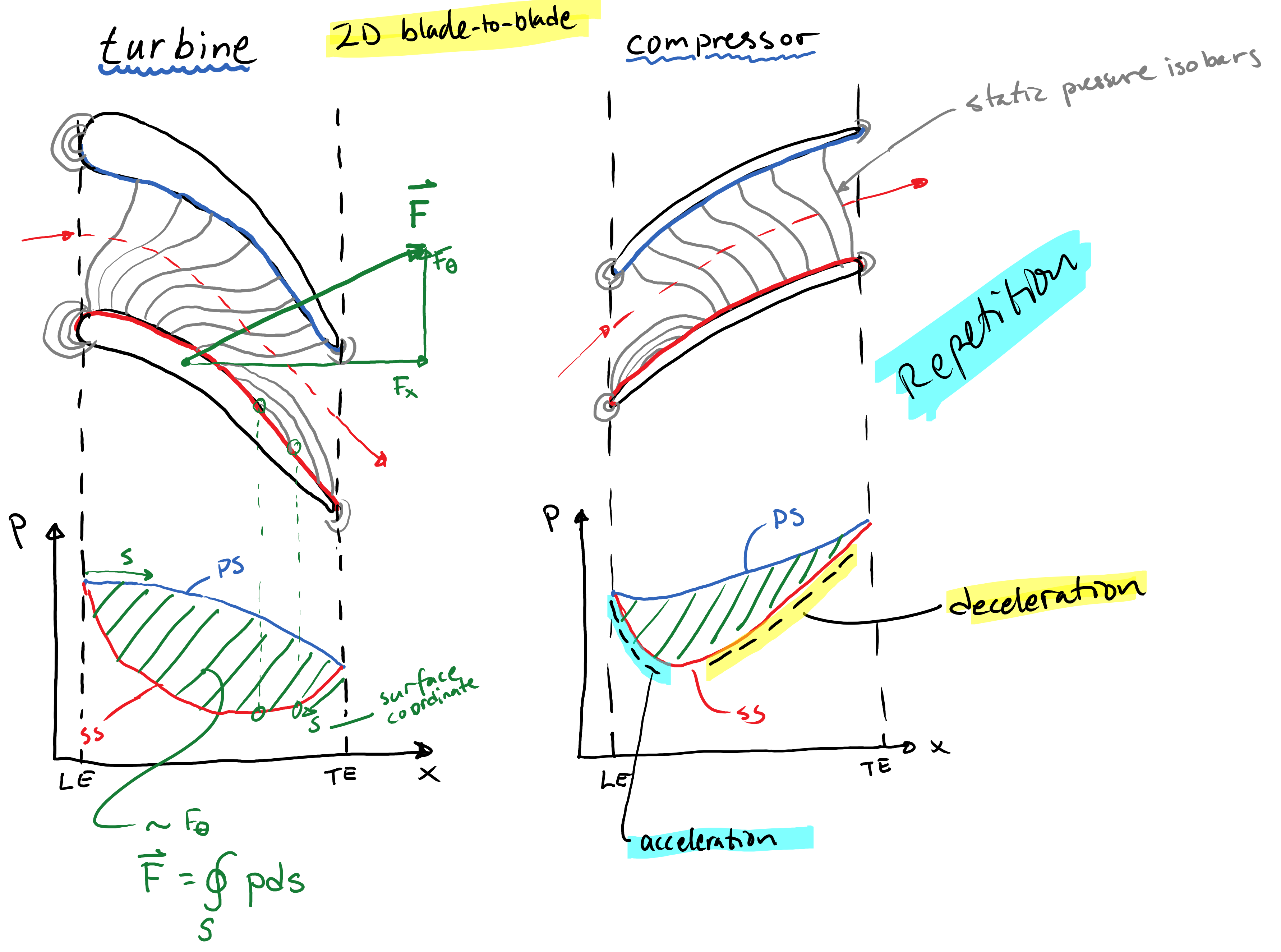
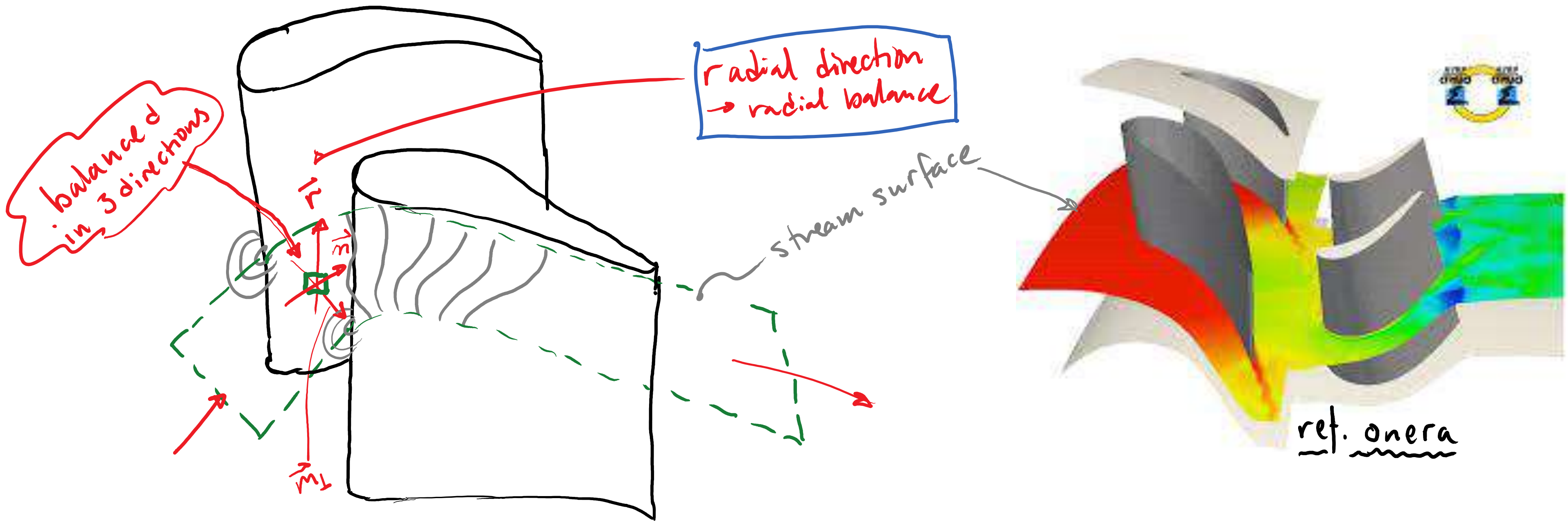
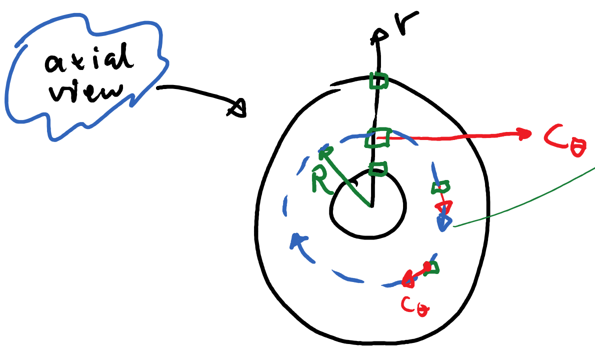


→ Build 3D aerodynamics by stacking a number of 2D blade-to-blade flow sheets under the respect of radial balance.

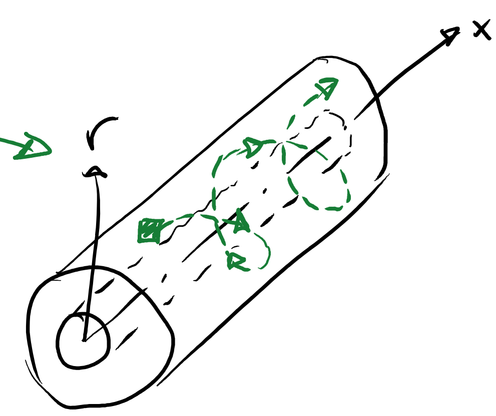


Consider the 3D curvature of the flow path



→ swirling particle

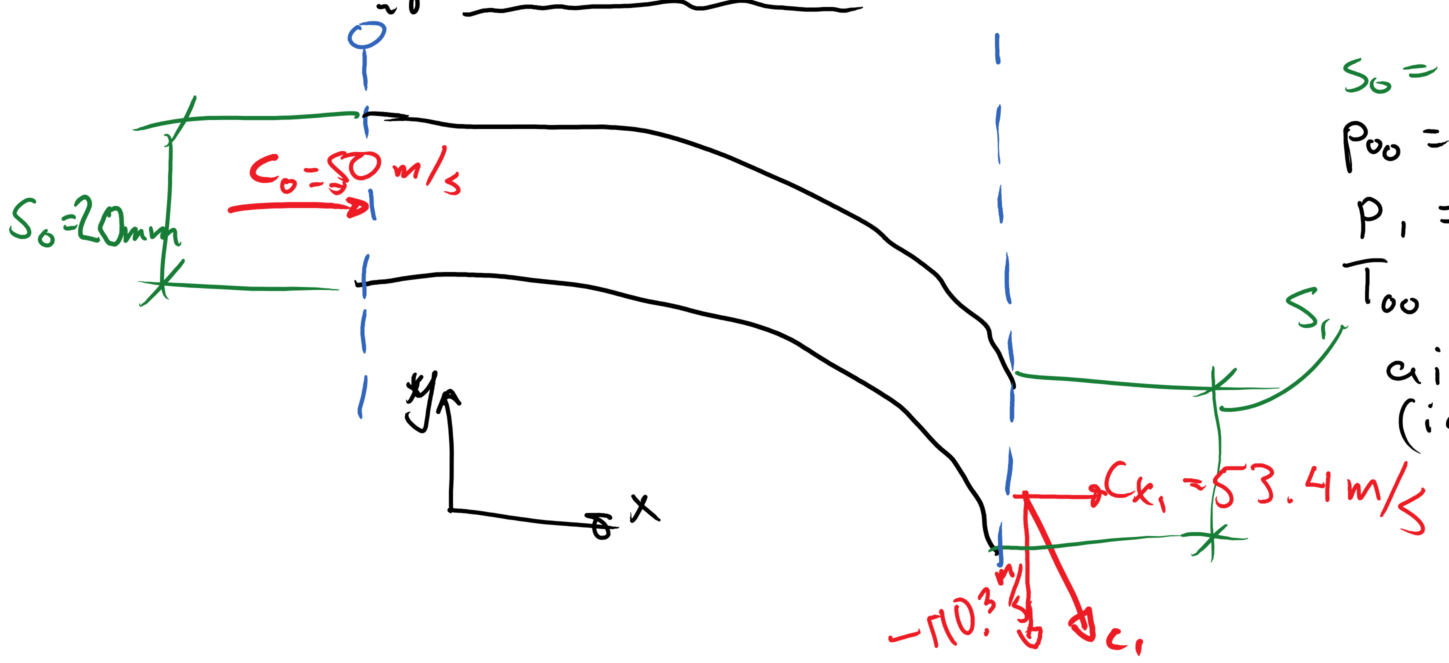
$$\boxed{\frac{dp}{dr} = \rho \cdot \frac{C_\theta^2}{R}}$$



consequence:
the static pressure needs to increase from the hub to the casing

PoW 3 Fluid forces

generic nozzle



$S_0 = S_1$

$p_{00} = 109.4\ kPa$

$p_1 = 100\ kPa$

$T_{00} = 303\ K$

air (ideal gas) → $c_p = 1004.5\ J/kgK$, $\gamma = 1.4$, $R = 287\ J/kgK$

determine quantities per unit height

- calculate fluid forces < conservation of momentum a) / pressure loading b)

a)

$$(1)\quad \underset{\text{momentum}}{F_{x_m}} = \underbrace{\dot{m}\,(c_{x_0} - c_{x_1})}_{\text{due to the acceleration and deviation}} + \underbrace{p_0 \cdot \overset{S_0}{A_0} - p_1 \cdot \overset{S_1}{A_1}}_{\text{external pressure forces}}$$

$$(2)\quad F_{y_m} = \dot{m}\,(c_{y_0} - c_{y_1})$$

where $A_0 = S_0 = 0{,}02\ m = S_1$

need to calc. $p_0 \rightarrow \dot{m}$

$$c_1 = \sqrt{c_{x_1}^2 + c_{y_1}^2} = \sqrt{53.4^2 + (-110.3)^2} \approx 122.55\ m/s$$

$$T_0 = T_{00} - \frac{c_0^2}{2 \cdot c_p} = 303 - \frac{50^2}{2 \cdot 1004.5} = 301.76\ K$$

isentropic relation

$$p_0 = p_{00} \cdot \left(\frac{T_{00}}{T_0}\right)^{-\frac{\gamma}{\gamma-1}} = \ldots = 107.835\ kPa$$

$$\rightarrow \dot{m} = \rho_0 \cdot c_{x_0} \cdot \overset{S_0}{A_0} \quad \text{where} \quad \rho_0 = \frac{p_0}{R \cdot T_0} = \frac{107.855 \cdot 10^3}{287 \cdot 301.76} = 1.245\ kg/m^3$$

$$\rightarrow \dot{m} = 1.245 \cdot 50 \cdot 0.02 = 1.245\ kg/s$$

$$(1) \rightarrow \underline{F_{x_m}} = 1.245 \cdot (50 - 53.4) + 107835 \cdot 0.02 - 100000 \cdot 0.02 = \underline{152.47\ N/m}$$

$$(2) \rightarrow F_{y_m} = 1.245 \cdot (0 - (-110.3)) = \underline{137.34\ N/m}$$

per unit thickness

(2) $\rightarrow$ $F_{ym} = 1.245 \cdot (0 - (-110.3)) = 137.34\ N/m$ per unit thickness

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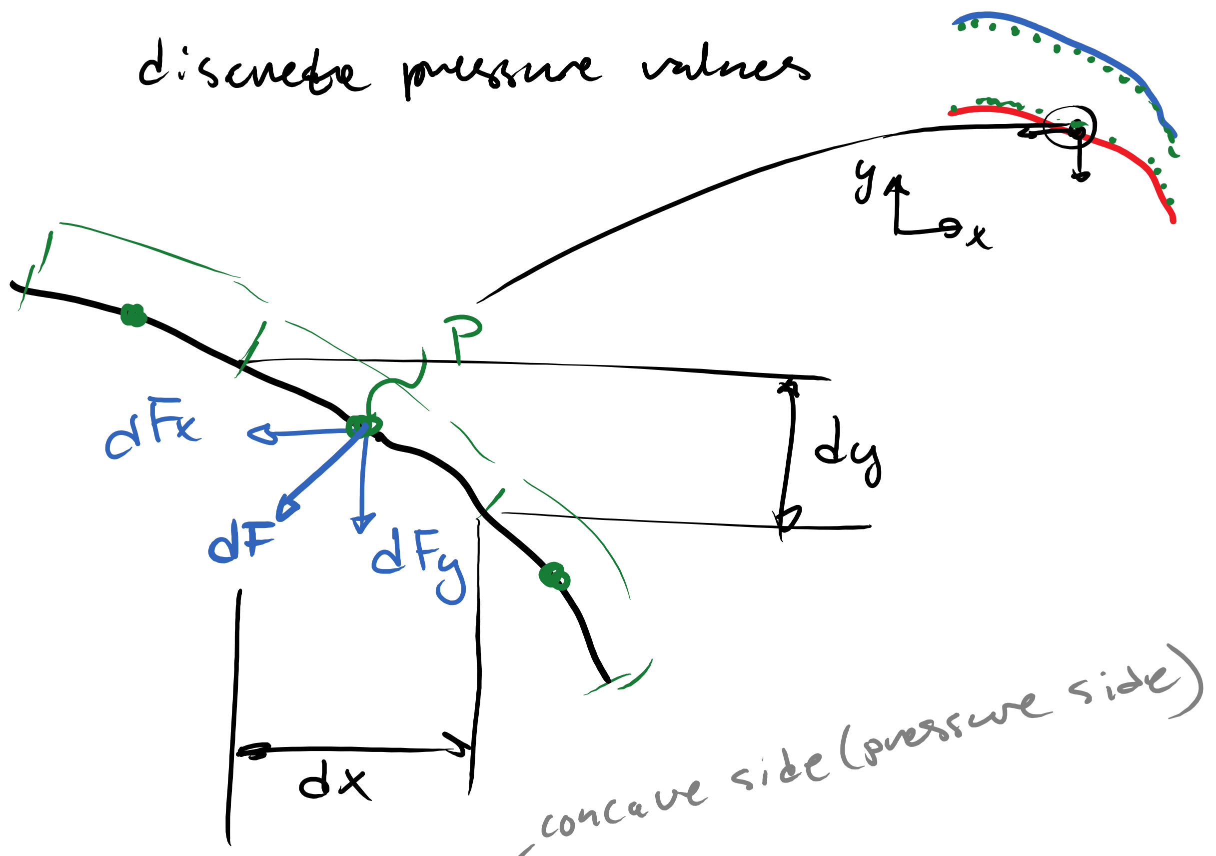
b) from the pressure loading.

discrete pressure values



$$F_x = \sum_{i=1}^{n_{ps}} dy \cdot dp_{ps} - \sum_{i=1}^{n_{ss}} dy \cdot dp_{ss} = \underline{151.4} \ N/m$$

$$F_y = \sum_{i=1}^{n} dx \cdot dp_{ss} - \sum_{i=1}^{n} dx \cdot dp_{ps} = \underline{139.55} \ N/m$$

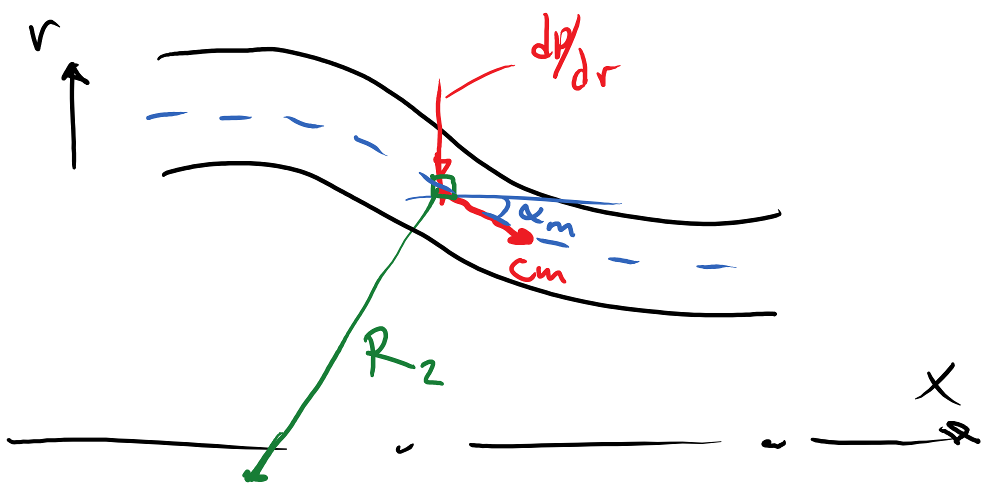
the values do not match

possible reasons → discrete points (interpolation in between)

→ defining correct start and end points at LE, TE

3D flow (global aspects)

flow: streamline curvature



$$\boxed{\frac{dp}{dr} = \rho \cdot \frac{c_m^2}{R_2} \cdot \cos\alpha_m}$$

contribution due to swirling of the flow is usally >> streamline curvature effects

Radial equilibrium (simple) $\boxed{\frac{dp}{dr} = \rho \cdot \frac{c_\theta^2}{R}}$

$$\boxed{\frac{dp}{dr} = \rho \cdot \frac{c_\theta^2}{R} + \rho \cdot \frac{c_m^2}{R_2} \cdot \cos\alpha_m + \ldots}$$

swirling + curvature effects

acceleration along streamline (lect. notes)

→ variation of enthalpy with radius (energy analysis)

$$\underbrace{h_0(r) = h(r)}_{\text{static enthalpy}} + \underbrace{\frac{c^2}{2}(r)}_{\text{kinetic energy}} \quad \text{here} \quad \underbrace{\frac{c^2}{2} = \frac{1}{2}\left(c_x^2 + c_\theta^2\right)}_{\text{here } c_r \text{ is neglected}}$$

derivate →

$$\frac{dh_0}{dr} = \frac{dh}{dr} + \underbrace{c_x \cdot \frac{dc_x}{dr} + c_\theta \cdot \frac{dc_\theta}{dr}}_{\text{inner derivative}} \qquad (1)$$

$dh = Tds + vdp$ apply thermo 2:nd law

$$= \underbrace{T \cdot \frac{ds}{dr}}_{\text{radial variation of losses}} + \frac{1}{\rho} \cdot \frac{dp}{dr}$$

$\cancel{+ \, s \frac{dT}{dr}}$ $\cancel{- \frac{1}{\rho^2} \frac{d\rho}{dr}} \cdot dp$ neglected.

radial pressure gradient $\frac{dp}{dr} = \rho \cdot \frac{c_\theta^2}{r}$

– – →

assume constant losses vs blade span

$$\frac{ds}{dr} = 0 \Rightarrow \frac{dh}{dr} = \frac{1}{\rho}\cdot\frac{dp}{dr}$$

where $\frac{dp}{dr} = \rho\frac{c_\theta^2}{r} \Rightarrow \boxed{\frac{dh}{dr} = \frac{c_\theta^2}{r}}$

Thus (1) $\Rightarrow$ $\boxed{\frac{dh_0}{dr} = \frac{c_\theta^2}{r} + c_x\frac{dc_x}{dr} + c_\theta\frac{dc_\theta}{dr}}$

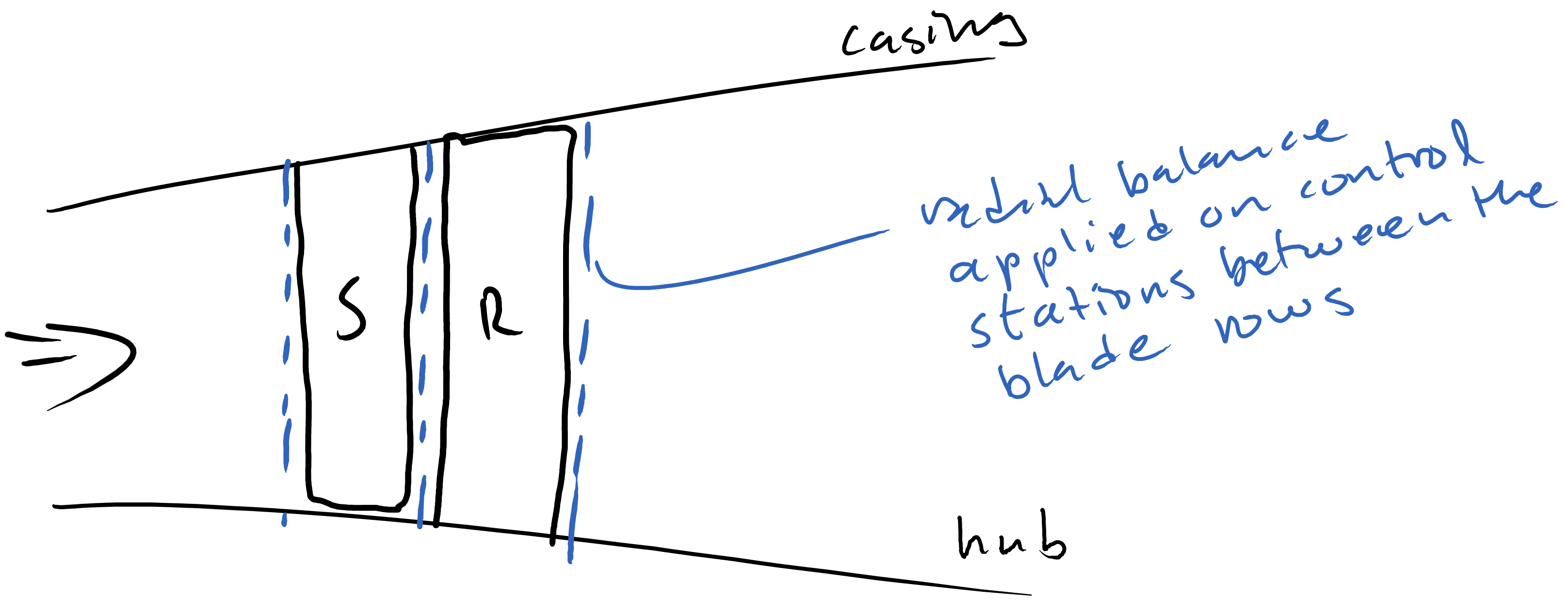
vortex energy equation

next step: design choices

a) $\boxed{\frac{dh_0}{dr} = 0}$ $\rightarrow$ constant total enthalpy vs radius

b) constant axial velocity vs radius

$\boxed{\frac{dc_x}{dr} = 0}$



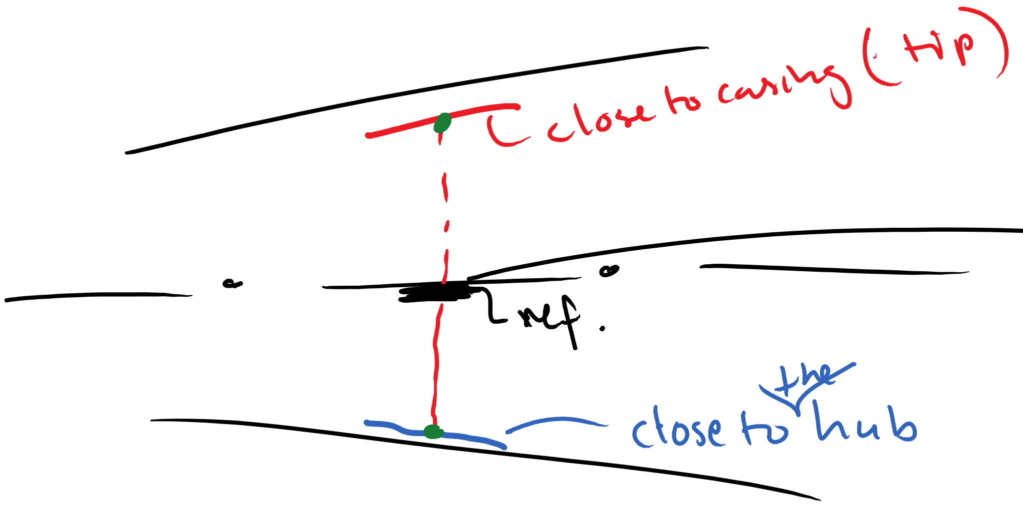
vortex energy eq. 0 a) assumpt. 0 b) assumption

$$\Rightarrow \cancel{\frac{dh_0}{dr}} = \frac{c_\theta^2}{r} + c_x\cancel{\frac{dc_x}{dr}} + c_\theta\cdot\frac{dc_\theta}{dr}$$

$$0 = \frac{c_\theta^{\cancel{2}}}{r} + \cancel{c_\theta}\cdot\frac{dc_\theta}{dr} = \frac{c_\theta}{r} + \frac{dc_\theta}{dr} \Rightarrow$$

$$\Rightarrow \quad \frac{dC_\theta}{dr} = -\frac{C_\theta}{r} \quad \text{or} \quad \frac{dc_\theta}{c_\theta} = -\frac{dr}{r}$$

integrate $\Rightarrow$ $\boxed{C_\theta \cdot r = \text{¢}}$ free vortex condition



values from 1D design at midspan $C_x, C_\theta, p, T \ldots$

1D $\begin{bmatrix} C_x \\ C_\theta \end{bmatrix}$ $\longrightarrow$ 3D $\boxed{\begin{array}{l} C_x = \text{¢} \\ C_\theta(r) = C_{\theta,ref} \cdot \dfrac{r_{ref}}{r} \end{array}}$

e.g. at tip C_θ is less due to that r increases