SORBONNE UNIVERSITÉ

MU5MEF16 - (MEC 655) Numerical Methods for Fluid Mechanics

Comp Mech

Collapse of a column of grain

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Plan

I. Collapse - rheologies

II. μ(I) - rheology

   A. Numerical implementation

      1. Level refinement

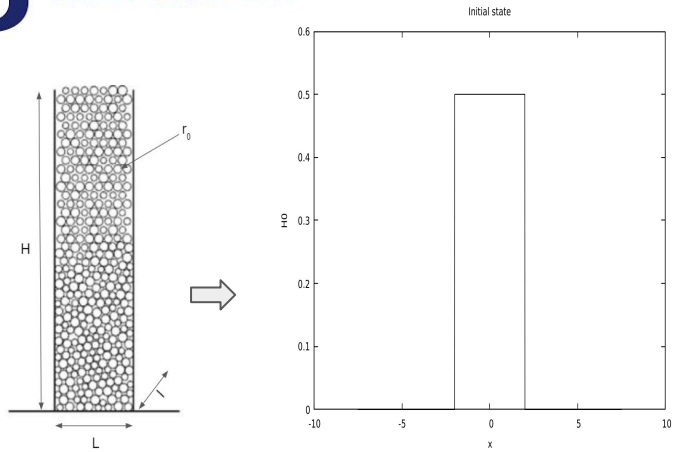
      2. Aspect ratio

      3. Size grain

   B. Consistency model : Discrete solid case

III. Conclusion

I. Recall 1st problem



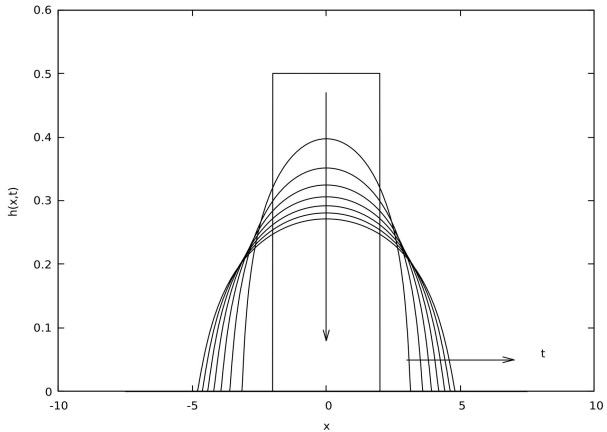


fig 1a : general problem

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{h} + \frac{1}{2}gh^2\right) = -C_f \nu \frac{Q}{h^2}$$

fig 1b : fluid system

Convective terms

Poiseuille viscous term

I. Navier-Stokes

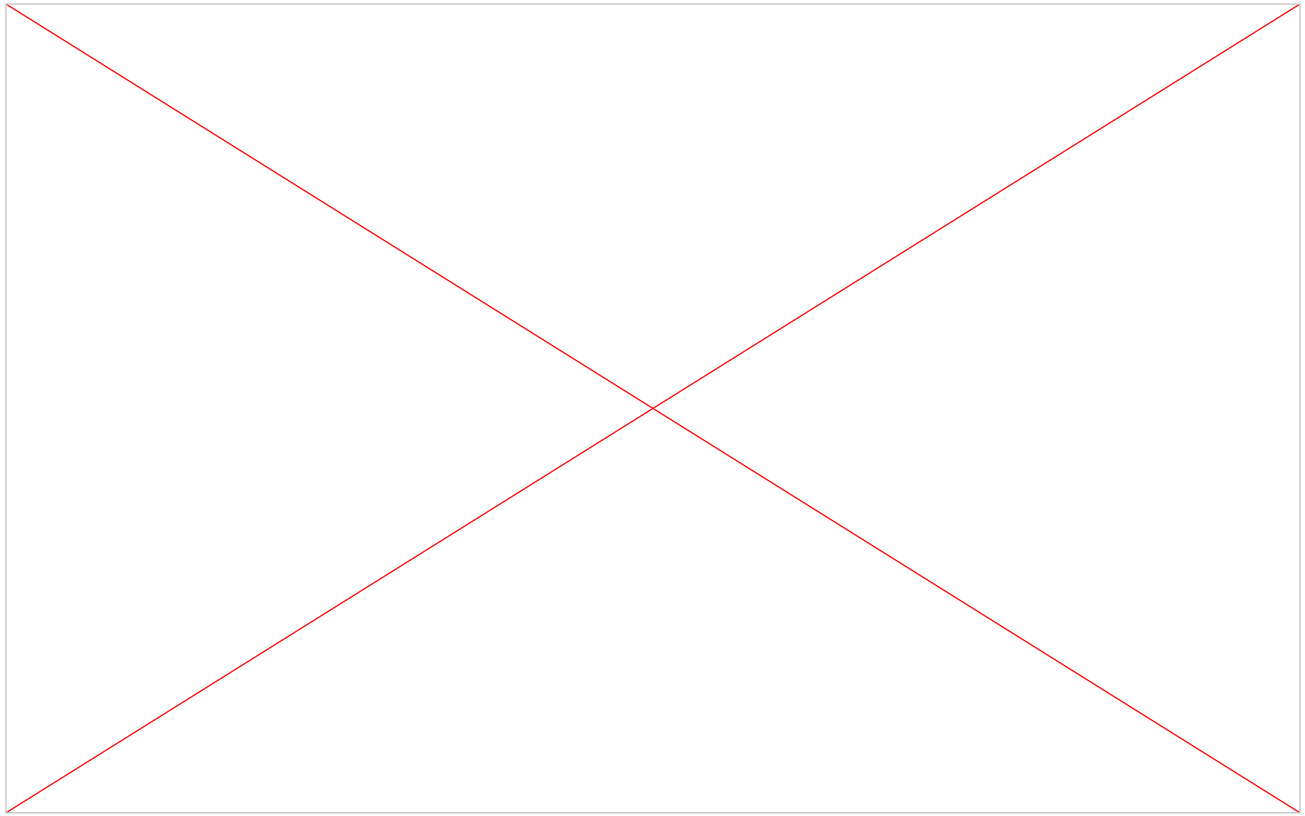


fig 2 : Navier-Stokes

- constant viscosity
- stability can’t ensure good physics
- not well adapted for our case

A. Numerical implementation

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho \mathbf{g}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0$$

- incompressible
- movement quantity conservation
- color function

$$\eta = \max\left(\frac{\mu(I)}{\sqrt{2}D_2}p\ ,\ 0\right),\ \text{with } I = d\sqrt{2}D_2/\sqrt{(|p|/\rho)}$$

$$\mu(I) = \mu_s + \frac{\Delta\mu}{I_0/I + 1} \qquad I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{p/\rho}}$$

1. Refinement

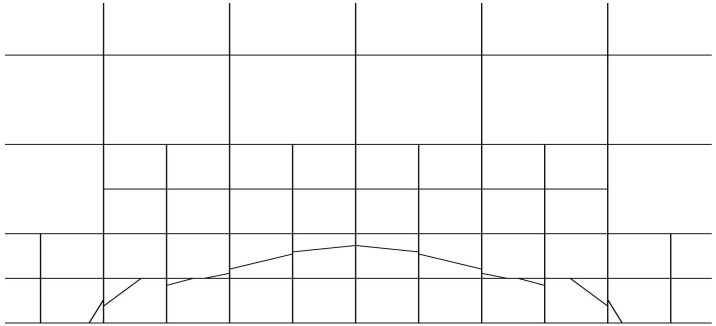


fig 3a : LEVEL=5

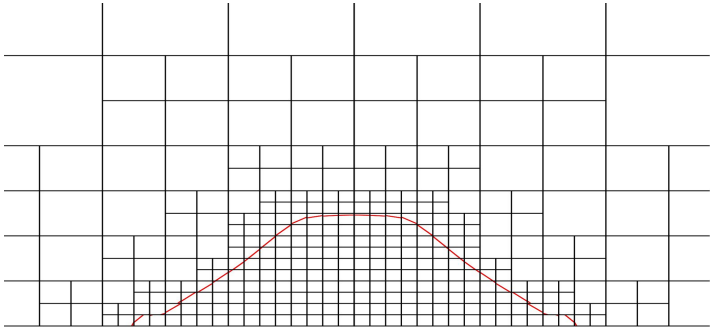


fig 3b : LEVEL=7

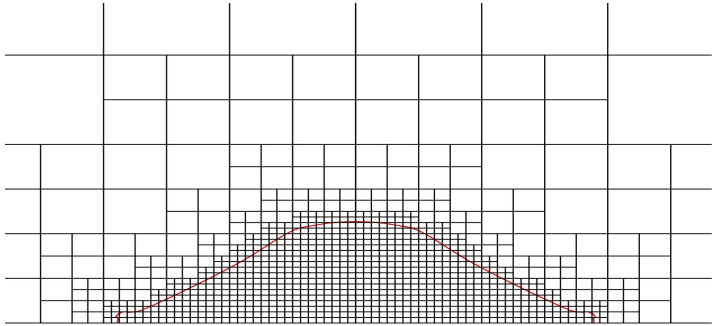


fig 3c : LEVEL=8

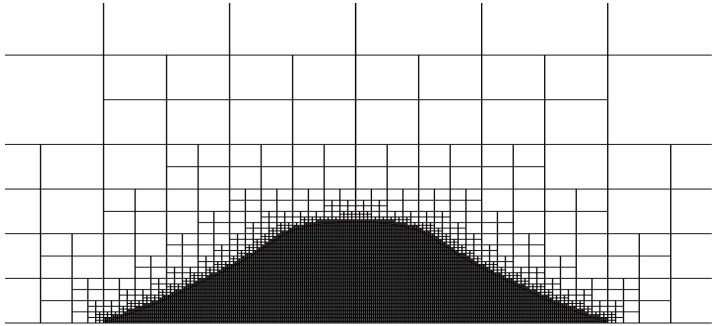


fig 3d : LEVEL=10

2. Aspect ratio

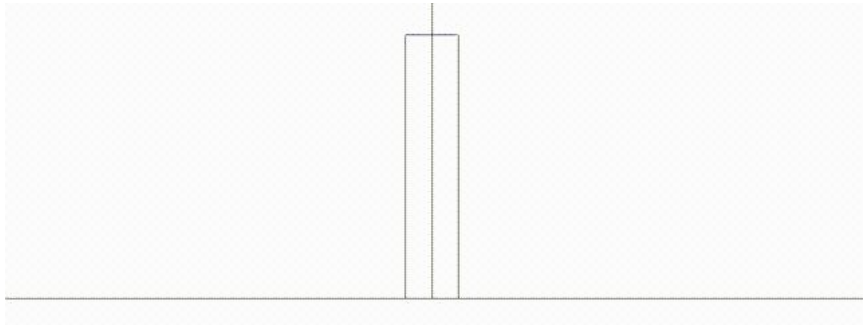


fig 4a : ratio 10/1

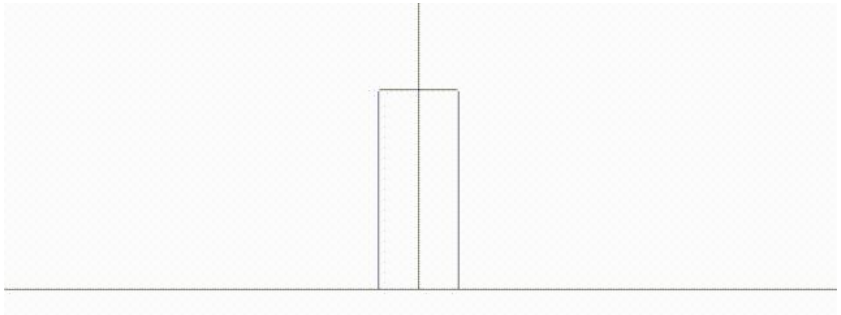


fig 4b : ratio 5/1

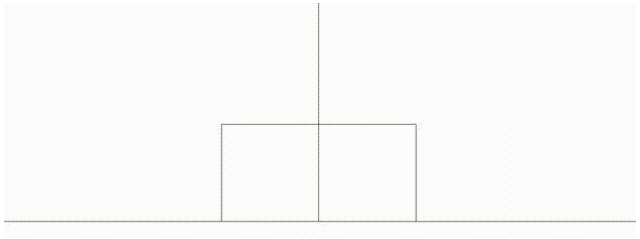


fig 4c : ratio 1/1

- physical behavior for several ratio
  - LEVEL=10
  - d_{grains} = 0.04 m

1st Balance sheet

- Refinement adapted from LEVEL = 8
- method good for several aspect ratio
- What details on solid influence ?

2. Size grains

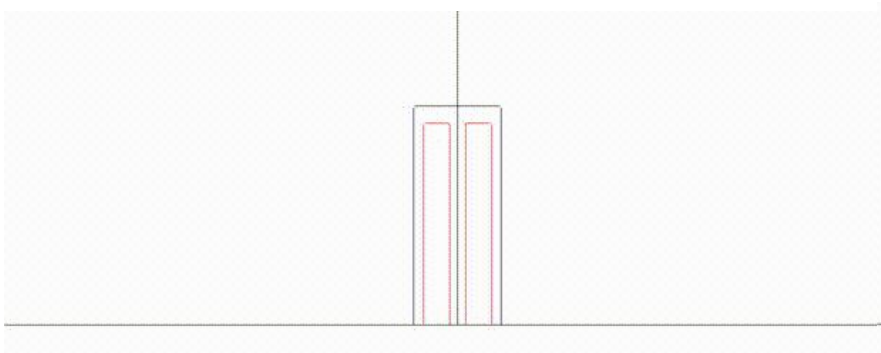


fig 5a : d_{grains} = 0.08 m

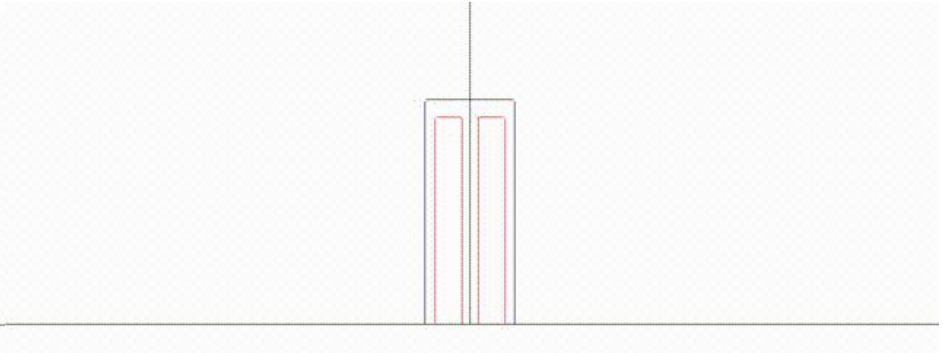


fig 5b : d_{grains} = 0.16 m

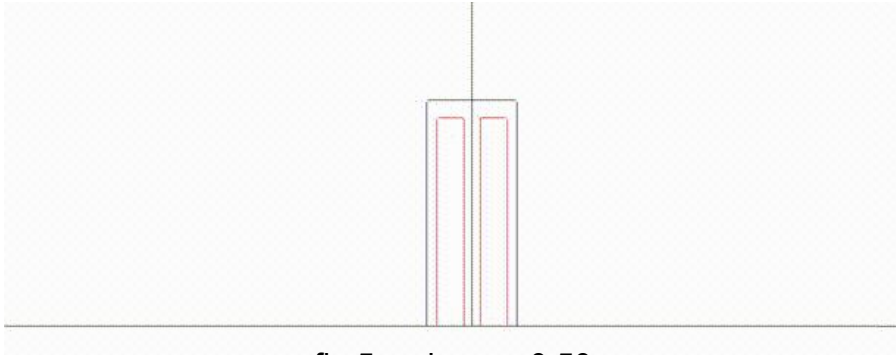


fig 5c : d_{grains} = 0.50 m

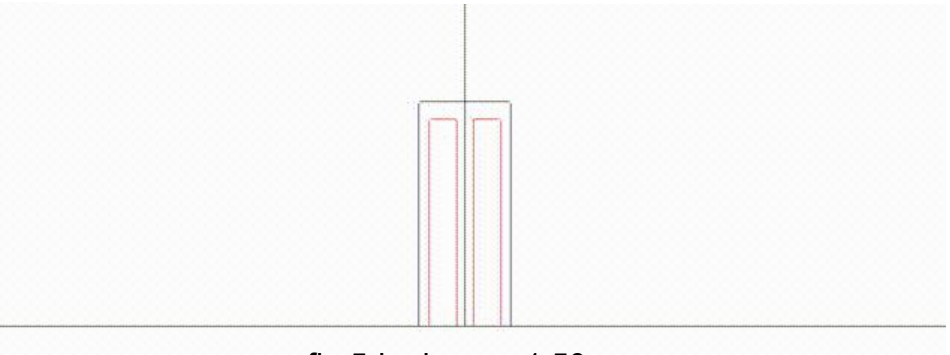


fig 5d : d_{grains} = 1.50 m

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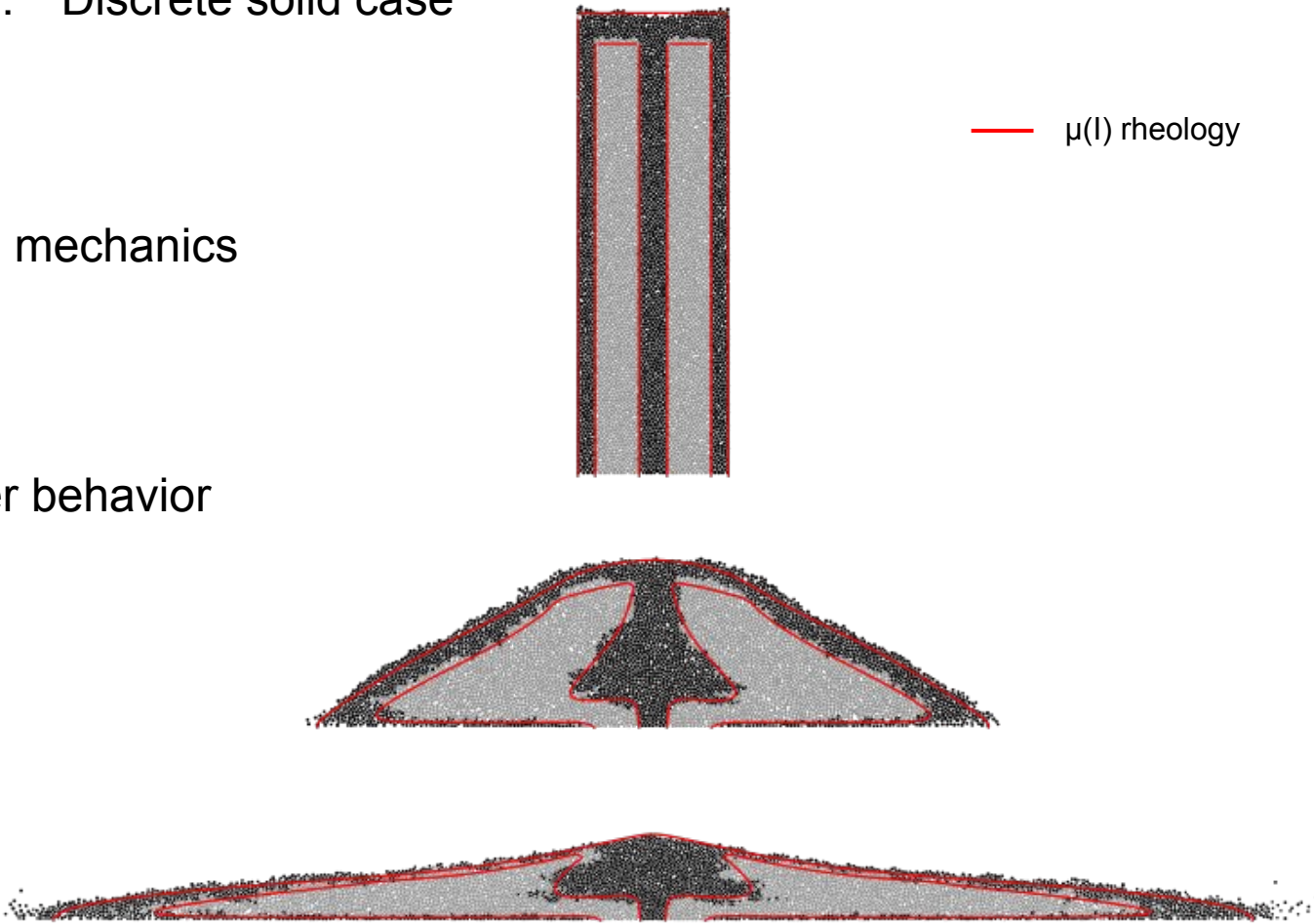
B. Discrete solid case

- μ(I) vs discrete mechanics
- inner and outer behavior



III. Conclusion

- Recovery of most the shape
- Model developed last decade → more to come from slender approach
- Combinatory behaviors and approaches

Bibliography

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- http://www.lmm.jussieu.fr/~lagree/SOURCES/GERRIS/BAGNOLDP/BAGNOLDP.pdf
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