Stage Thermodynamics and Kinematics

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Course MJ2430 / MJ2244

Nomenclature

Symbol	**Denotation**	**Unit**
I	Rothalpy	J/kg
W	Mechanical energy	J
T	Temperature	K
c	Absolute velocity	m/s
h	Enthalpy	J/kg
p	Pressure	Pa
u	Tangential velocity	m/s
w	Relative velocity	m/s
ρ	Density	kg/m^3

Subscripts

0	Total
1	Position 1 (compressor: rotor inlet, turbine: stator inlet)
2	Position 2 (compressor: rotor outlet/stator inlet, turbine: stator outlet/rotor inlet)
3	Position 3(compressor: stator outlet, turbine: rotor outlet)
c	Total in absolute frame of reference
w	Total in relative frame of reference
θ	Tangential component

Introduction

The purpose of this document is to serve as self-study guide to learn about the following aspects of turbomachinery:

- Stage thermodynamics
- Stage kinematics

Under stage thermodynamics the determination of thermodynamic quantities such as pressures and temperatures is understood. Stage kinematics aims at the determination of flow velocities in a stage, which then finally yield the stage velocity triangle (i.e. 1D aerodynamics).

Instructions

The present document is self-explaining. It guides the student through the aforementioned issues. In order to apply the knowledge a calculation exercise is included at the end.

Basic Thermodynamic Relations

This section focuses on the basics of thermodynamics with respect to a continuous flow process. The following shall be assumed:

- The process is steady in time (i.e. quantities are not varying over time)
- The process is adiabatic (i.e. there is no heat flow over the system boundaries)
- The fluid is ideal gas (i.e. compressible flow indicating that the fluid density is variable)
- Gravity effects are neglected

Let us first focus on a certain point in a steady flow process. As the fluid is moving we can identify the following to states

- Static state
- Stagnation state (also referred to as “total state”)

Each of these states features its respective temperatures, pressures and enthalpies. The relation between the two states is given by the fluid velocity as follows

$$h_0 = h + \frac{c^2}{2} \qquad \text{Eq. 1}$$

, where h0 is the total enthalpy, h the static enthalpy and c the fluid velocity. This equation tells us the following:

There is a total energy present in the fluid containing a static part as well as a contribution from the fluid motion (kinetic energy)
At zero fluid velocity the static enthalpy is equal to the total enthalpy. The fluid is then at rest (also called “stagnating”). Therefore the total enthalpy is also referred to as “stagnation enthalpy”

For an ideal gas with the internal energy (enthalpy) is a function of the temperature as follows

$$h = cp \cdot T \qquad \text{Eq. 2}$$

with cp being the specific heat at constant pressure. Note that this relations is only correct in certain temperature intervals in which cp can be regarded as constant.

From the above two equations we recognize a relation between temperatures and velocities as follows

$$T_0 = T + \frac{c^2}{2cp} \qquad \text{Eq. 3}$$

In order to relate the pressures to the temperatures we employ the isentropic relation as follows

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{Eq. 4}$$

At this stage we have the necessary expressions to determine the following:

- Enthalpy difference total-static due to fluid motion
- Temperature difference total-static due to fluid motion
- Pressure difference total-static due to fluid motion

Example 1: assume having a flow with a fluid velocity *c=100m/s*. The total conditions are T_0*=300K*, p_0*=250kPa*. Determine the following:

a) Static temperature and pressure
b) Total and static enthalpy

Assume an ideal gas with c_p*=1004.5J/kgK*, γ*=1.4*, and *R=287J/kgK*. Note also that the following is valid for such a gas

$$cp = \frac{\gamma \cdot R}{\gamma - 1} \qquad \text{Eq. 5}$$

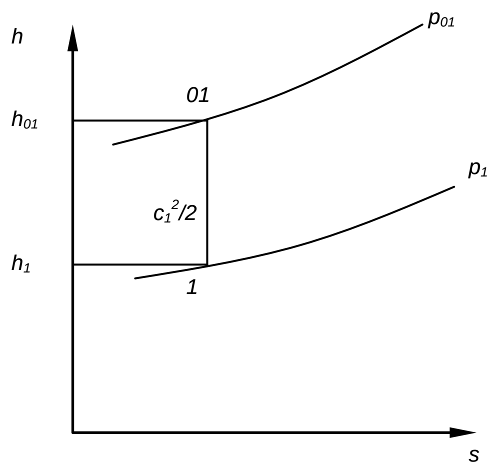
, where R stands for the gas constant.

Example 2: In order to get a feeling for thermodynamic quantities determine the total-static enthalpy differences for the following fluid velocities (assuming same fluid as above):

a) 10m/s
b) 50m/s
c) 200m/s
d) 300m/s

Let us stay for a while with the static and total conditions such as to understand what they express and when we use them. The total conditions provide us with the necessary thermodynamic information of a gas a rest, e.g. having compressed gas in a plenum. Once the gas is put into motion we have seen that static temperature and static pressure start to decrease. Assuming that we have an ideal process without losses we could accelerate a fluid from stagnation conditions such as to locally achieve a lower static pressure and then decelerate it again to stagnation conditions. Note that the static conditions are always equal to or lower than the total conditions but never greater than.

Having the above information it is straightforward to indicate total and static conditions in an h-s diagram. This is done as follows:



Note that the corresponding total and static conditions always are located at the same entropy. As this is the case we can use the isentropic relation as outlined above.

Next we draw the attention to the change of quantities along a flow path. Assume that we have a component (stator, rotor) with a steady flow. We would like to know how the thermodynamic quantities change along the flow path. Here we assume that the quantities are only changing in direction of the flow and not normal to it. This is called a 1-D analysis.

From an overall perspective we can differ the following two cases:

- The flow is in a stationary component (stator)
- The flow is in a rotating component (rotor)

In order to relate two or more points in a fluid we need to apply the conservation laws (mass, energy, momentum). We have seen that the Euler turbine equation yields from a combination of energy and momentum balance as follows

$$h_{01} - h_{02} = u_1 c_{\theta 1} - u_2 c_{\theta 2} \qquad \text{Eq. 6}$$

From the aforementioned differentiation we get the following two relations

- Stator: $u = 0 \rightarrow h_{01} = h_{02}$
- Rotor: $u \neq 0 \rightarrow h_{01} - u_1 c_{\theta 1} = h_{02} - u_2 c_{\theta 2} = I$

The quantity I is called “rothalpy” and is a fictitious enthalpy.

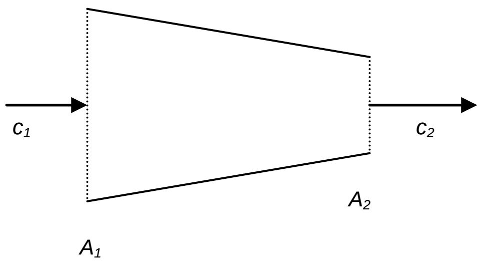
From these relations we can now easily relate various points on the flow path in either a stator or a rotor as long as we assume ideal flow (no losses).

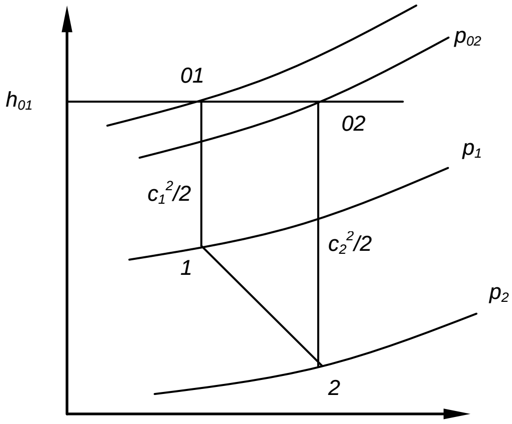
Example 3: assume having a flow in a stator with total conditions are T0=350K, p0=180kPa. From one point to another the flow velocity changes from 167m/s to 388m/s. Determine the respective static conditions (*p, T* and ρ). Sketch it in an h-s diagram. Assume an ideal gas with c_p*=1004.5J/kgK*, γ*=1.4*, and *R=287J/kgK*. The density is obtained as follows

$$\rho = \frac{p}{RT} \qquad \text{Eq. 7}$$

Example 4: we focus on a rotor of an ideal machine with the total inlet conditions T0=291K, p0=101.3kPa (still same fluid as above). Assume that we have a blade speed of 300m/s. What change in tangential flow speed is needed in order to achieve a compression with a total pressure ration of 1.7?

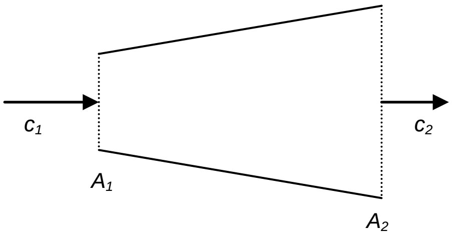
The question is now how losses come into the picture. Losses are due to internal friction in the fluid (note that the fluids that we work with are viscous!) and are observed as reduction in total pressure. In any real application there are losses meaning that we reduce total pressure. At the same time the entropy is increasing. It is very essential that this relation is understood in detail. Let us focus on an expansion process as sketched below.

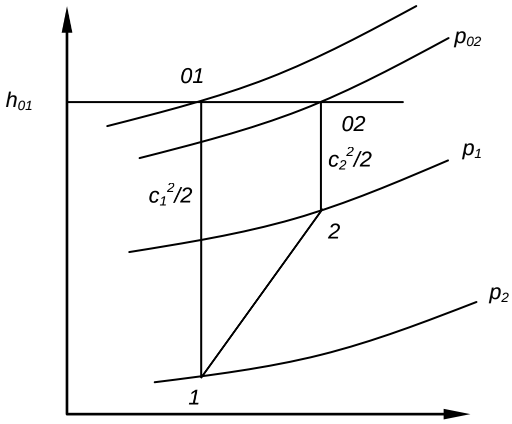




It gets obvious that a reduction in total pressure also signifies an increase in entropy. As we have an adiabatic machine the entropy can only increase. In the best case (ideal process) it will not increase and hence the total pressure will not change.

Let us try to understand what the implication is of a loss in total pressure. Assume that we have the above expansion process in which we accelerate the fluid from point 1 to point 2. The static pressure p2 as well as the total conditions p01, T01 are fixed. The implication of a loss in total pressure is a reduction in flow velocity c2. This makes sense. With increasing losses point 02 travels horizontally to the right (increasing entropy, constant total enthalpy) while point 2 climbs up on isobar p2. The vertical distance between 02 and 2 (i.e. kinetic energy at position 2) decreases. We have losses; the process is no longer ideal. In similar manner it is seen for a deceleration, see below.



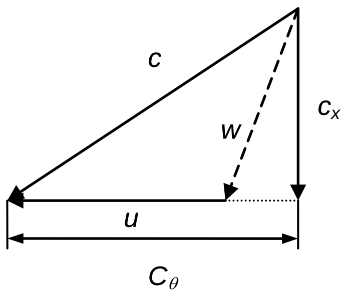


So far we have focused on 1D flow in a stationary component. The application to a rotating component is similar. Instead of the toal enthalpy being constant it is the rothalpy, which is constant.

The general notation of rothalpy is

$$I = h + \frac{1}{2}c^2 - u \cdot c_\theta \qquad \text{Eq. 8}$$

This expression can be reformulated by expressing the velocities in the relative frame of reference as follows



$$c_\theta - u = w_\theta \rightarrow c_\theta = w_\theta + u \qquad \text{Eq. 9}$$

$$c^2 = c_x^{\ 2} + c_\theta^{\ 2} = c_x^{\ 2} + w_\theta^{\ 2} + 2w_\theta u + u^2 \qquad \text{Eq. 10}$$

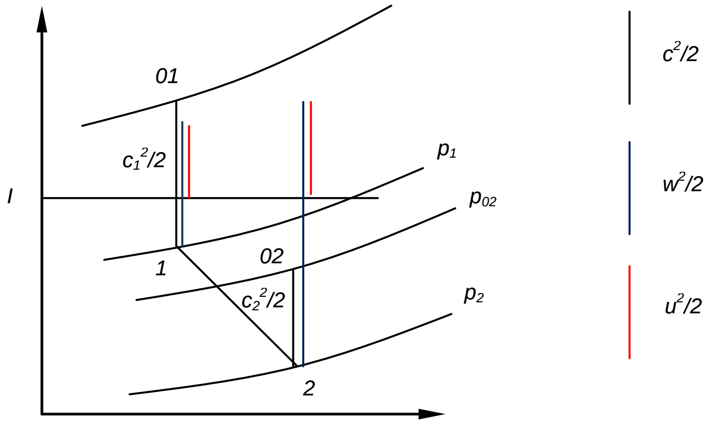
Substituting c^2 and c_θ in the expression of rothalpy

$$I = h + \frac{1}{2}\left[c_x^{\ 2} + w_\theta^{\ 2} + 2w_\theta u + u^2\right] - w_\theta u - u^2 = h + \frac{1}{2}\left(c_x^{\ 2} + w_\theta^{\ 2}\right) - \frac{u^2}{2} \qquad \text{Eq. 11}$$

With $c_x = w_x$ and $w^2 = {w_x}^2 + {w_\theta}^2$ the reformulated expression of the rothalpy yields

$$I = h + \frac{1}{2}w^2 - \frac{1}{2}u^2 \qquad \text{Eq. 12}$$

Practically this is seen as follows in an h-s diagram (here an expansion is shown).



Important observations:

- The rothalpy is constant
- The relative velocities are not equal to the absolute velocities
- The stagnation enthalpies at position 1 and 2 are not equal (here we have a reduction, hence we have a positive work output → turbine)

Example 5: consider an axial turbine stage with constant reference diameter (i.e. constant blade speed u).

The following parameters are given:

- Axial velocity $c_x=100m/s$, constant throughout stage
- Inflow velocity to stage equal to outflow velocity $c_1=c_3$ (this is called “repetition stage”)
- Absolute velocity at rotor inlet $c_2=300m/s$
- Relative velocity at rotor outlet $w_3=200m/s$
- Circumferential speed at mean radius $u=150m/s$
- Static temperature at stage inlet $T=800K$
- Gas specific heat $c_p=1148J/kgK$

Determine the following:

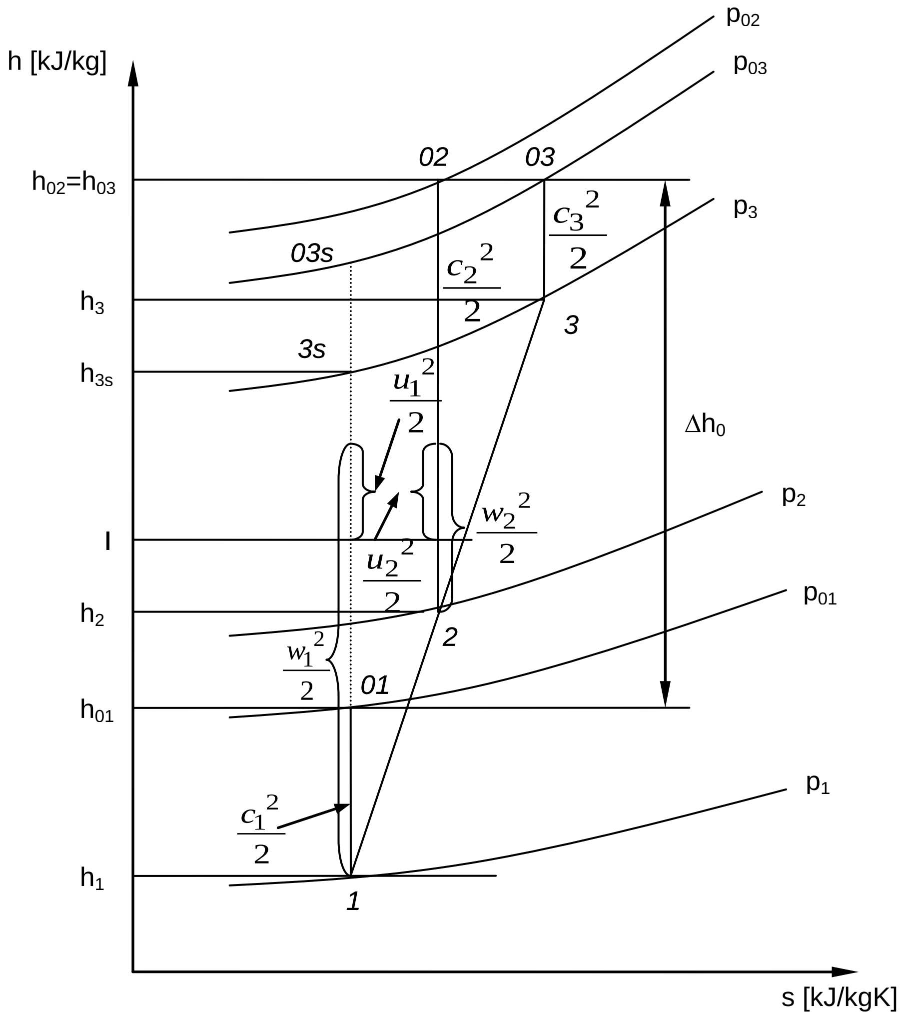
a) Static enthalpies throughout the stage, i.e. at positions 1, 2 and 3
b) Total enthalpies (in the absolute frame of reference) at stage inlet and outlet
c) Stage specific power output calculated from enthalpy differences
d) Stage specific power output calculated from Euler turbine equation

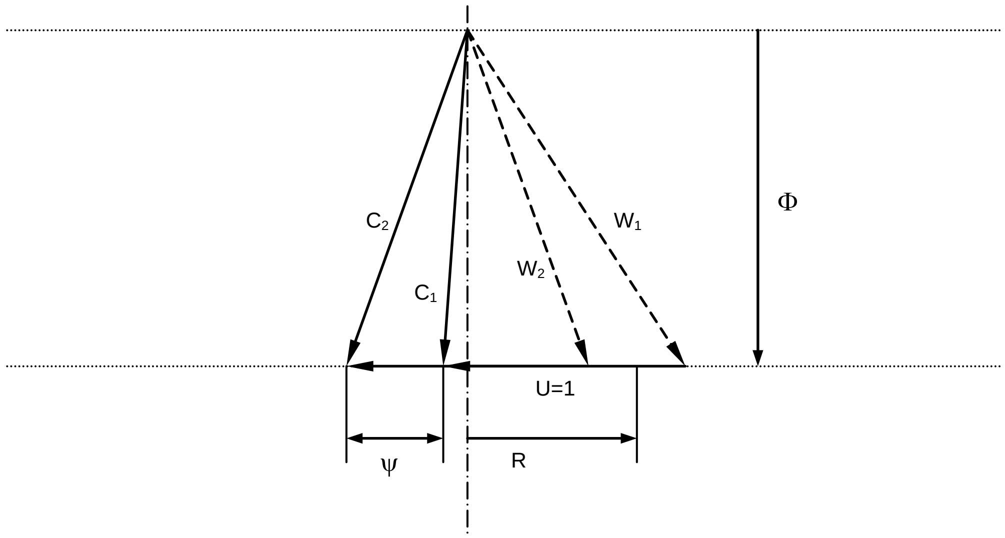
Review your results under the following aspects:

- What quantities are constant in the stator?
- What quantities are constant as you move from stator to rotor?
- How do the results of points c) and d) compare
- What would the power output be for a mass flow rate of 20kg/s?

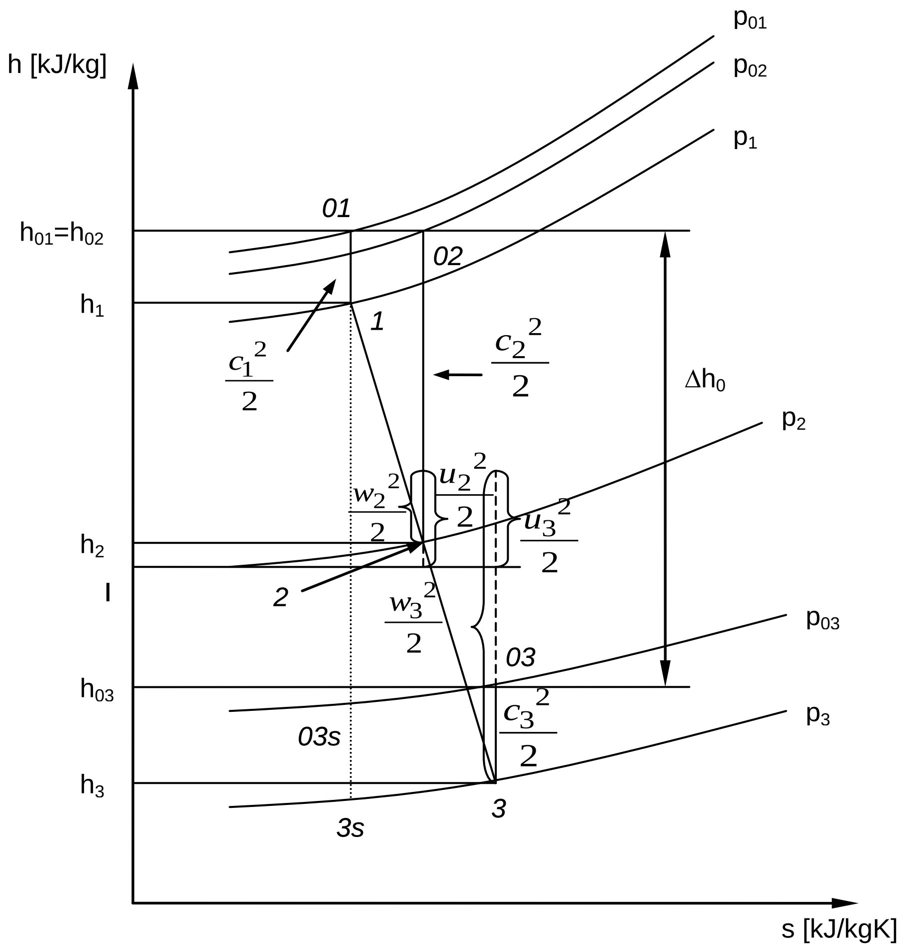
As a final step draw realistic expansion lines including static and total points qualitatively in an h-s diagram.

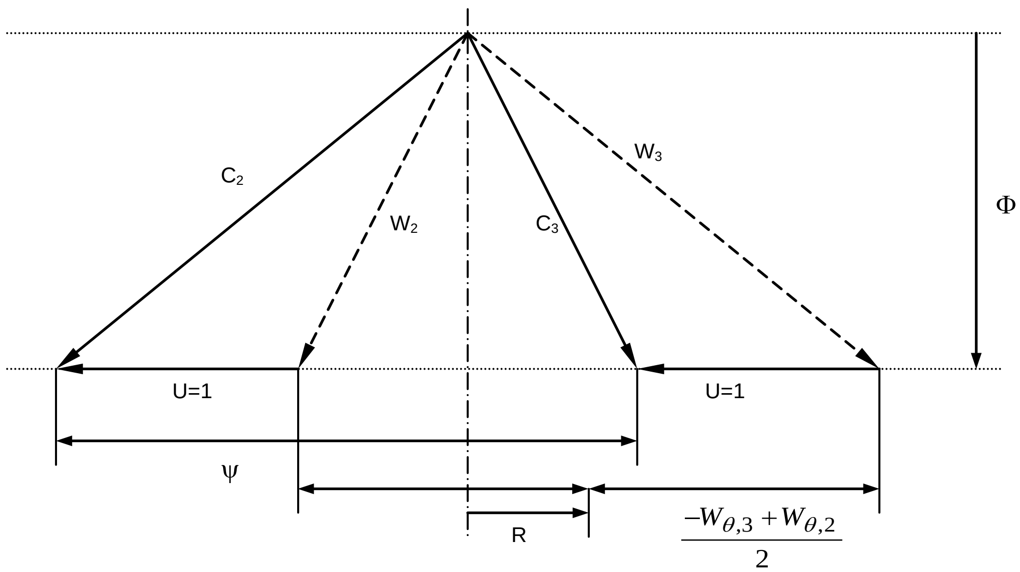
Compressor Stage Thermodynamics and Velocity Triangles





Turbine Stage Thermodynamics and Velocity Triangles





Solutions

1. a) $T=295K$, $p=235.8kPa$, b) $h_0=301.4kJ/kg$, $h=296.4kJ/kg$
2. a) $50J/kg$, b) $1250J/kg$, c) $20kJ/kg$, d) $45kJ/kg$
3. $T_1=336.2K$, $p_1=156.2kPa$, $\rho_1=1.62kg/m^3$, $T_2=275K$, $p_2=77.5kPa$, $\rho_2=0.98kg/m^3$
4. $\Delta c_\theta=-159.5m/s$
5. a) $h_1=918.4kJ/kg$; $h_2=878.7kJ/kg$; $h3=872.5kJ/kg$, b) $h_{01}=923.7kJ/kg$; $h_{03}=877.8kJ/kg$, c) $\Delta h = 45.9kJ/kg$, d) $\Delta h = u(c_{\theta 2} - c_{\theta 3}) = 45.9kJ/kg$