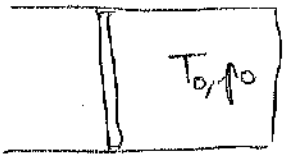
Ex 6 — 1 mole GP diatomique $\gamma = \frac{7}{5}$.



1) $(T_0, p_0)_A \xrightarrow[\text{compression isotherme}]{T = T_0} (T_0, p_1)_B \xrightarrow[\text{détente adiab. réversible}]{} (T_1, p_0)_C$

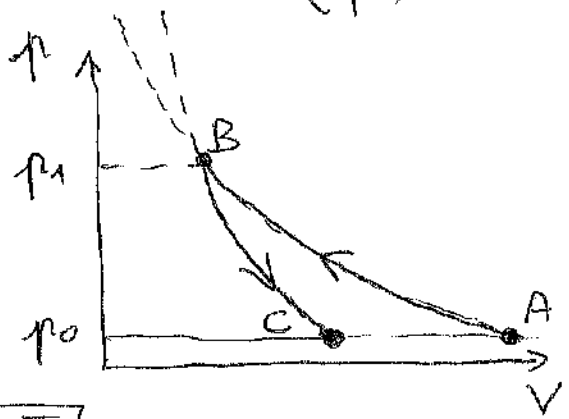
a) $p_B V_B^\gamma = p_C V_C^\gamma \qquad V = \frac{RT}{p}$

$$\Rightarrow p_B \left(\frac{RT_B}{p_B}\right)^\gamma = p_C \left(\frac{RT_C}{p_C}\right)^\gamma \Rightarrow p_B^{1-\gamma} T_B^\gamma = p_C^{1-\gamma} T_C^\gamma$$

$$\Rightarrow T_C = T_B \left(\frac{p_B}{p_C}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_B}{\left(\frac{p_B}{p_C}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\Rightarrow T_1 = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \quad \text{or } p_1 > p_0 \Rightarrow T_1 < T_0.$$

b)



isoT

$p V = p_B V_B$

$$\Rightarrow p = \frac{p_B V_B}{V}$$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right) = -\frac{p_B V_B}{V^2}$$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right)_{B\ \text{isoT.}} = -\frac{p_B}{V_B}.$$

adia rev

$p V^\gamma = \text{cte} = p_B V_B^\gamma$

$$\Rightarrow p = p_B V_B^\gamma \times \frac{1}{V^\gamma}$$

$$\Rightarrow \frac{\partial p}{\partial V} = p_B V_B^\gamma \left(-\gamma V^{-\gamma-1}\right)$$

$$\frac{\partial p}{\partial V} = -\gamma \frac{p_B V_B^\gamma}{V^{\gamma+1}}$$

$$\left(\frac{\partial p}{\partial V}\right)_B = \frac{-\gamma p_B V_B^\gamma}{V_B^{\gamma+1}}$$

$$\left(\frac{\partial p}{\partial V}\right)_{B\ \text{adia rev.}} = -\gamma \frac{p_B}{V_B}$$

c) $W_i = W_{AB} = -\int_A^B p\,dV = -\int_A^B RT_A \frac{dV}{V} = -RT_A \ln\frac{V_B}{V_A}$

$p_A V_A = p_B V_B \Rightarrow \frac{V_B}{V_A} = \frac{p_A}{p_B}$

$\Rightarrow W_i = -RT_A \ln\frac{p_A}{p_B} = +RT_A \ln\frac{p_B}{p_A} = RT_0 \ln\frac{p_1}{p_0} > 0$

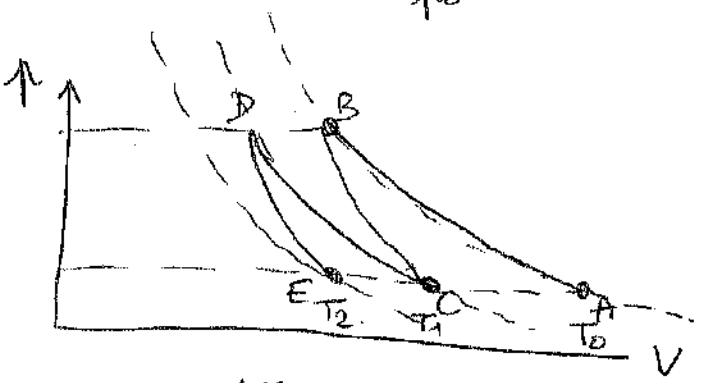
$Q_i = -W_i = -RT_0 \ln\frac{p_1}{p_0} < 0.$

$W_a = W_{BC} = \Delta U_{BC}$ car $Q_a = 0$.

$= \bar{C}_v (T_C - T_B) = \bar{C}_v (T_1 - T_0)$. < 0 $\quad \bar{C}_v = \frac{R}{\gamma - 1}$

$W_a + W_i = RT_0 \ln\frac{p_1}{p_0} + \bar{C}_v (T_1 - T_0) > 0$

2) a)



$T_2 = T_E = T_D \left(\frac{p_D}{p_E}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_C}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{T_1}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{2\frac{(\gamma-1)}{\gamma}}}$

b) $T_n = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{n\left(\frac{\gamma-1}{\gamma}\right)}}$

c) $T_0 = 300$ K $\quad p_0 = 1$ atm $\quad p_1 = 3$ atm $\Rightarrow \frac{p_1}{p_0} = 3$.

$T_n \leqslant \frac{T_0}{3} \Rightarrow \frac{T_0}{3^{n\left(\frac{\gamma-1}{\gamma}\right)}} \leqslant \frac{T_0}{3} \Rightarrow 3^{n\left(\frac{\gamma-1}{\gamma}\right)} \geqslant 3$

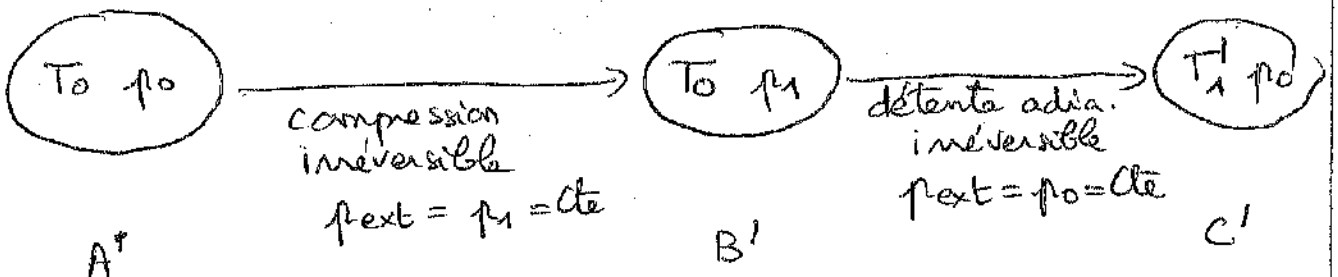
$\frac{\gamma-1}{\gamma} = \frac{2}{7}$ $\quad n\left(\frac{\gamma-1}{\gamma}\right) \geqslant 1$ $\quad n = 4$

$\Rightarrow e^{n\left(\frac{\gamma-1}{\gamma}\right)\ln 3} \geqslant e^{\ln 3}$.

$\Rightarrow n\left(\frac{\gamma-1}{\gamma}\right)\ln 3 \geqslant \ln 3 \Rightarrow n \geqslant \frac{\gamma}{\gamma-1} = \frac{7/5}{\frac{7}{5}-\frac{5}{5}}$

$n \geqslant \frac{7}{2} = 3{,}5 \Rightarrow n = 4$.

3)



a) $W_i' = -p_1 (V_{B'} - V_{A'})$ on a $p_0 V_0 = RT_0$ $V_{A'} = V_0$

$p_1 V_{B'} = RT_0$.

$\Delta U_{A'B'} = 0$ $\Rightarrow V_{B'} = \frac{RT_0}{p_1}$

$Q_i' = -W_i' < 0$.

$$W_i' = -p_1 \left(\frac{RT_0}{p_1} - \frac{RT_0}{p_0} \right) = -RT_0 + \frac{p_1}{p_0} RT_0$$

$$= RT_0 \left(\frac{p_1}{p_0} - 1 \right) > 0.$$

b) $Q_a' = 0$.

$W_a' = -p_0 (V_{C'} - V_{B'})$ $p_0 V_{C'} = R T_1'$

$V_{B'} = R \frac{T_0}{p_1}$

$$= -p_0 \left(\frac{RT_1'}{p_0} - \frac{RT_0}{p_1} \right)$$

$$= -RT_1' + RT_0 \left(\frac{p_0}{p_1} \right)$$ posons $\chi = \frac{p_1}{p_0}$.

$$= -RT_1' + \frac{RT_0}{\chi}$$

$$\Delta U_{B'C'} = \frac{R}{\gamma - 1} (T_1' - T_0).$$

$$\Rightarrow \frac{R}{\gamma-1} (T_1' - T_0) = -RT_1' + \frac{RT_0}{\chi}$$

$$\Rightarrow \left(\frac{R}{\gamma-1} + R \right) T_1' = \left(\frac{R}{\gamma-1} + \frac{R}{\chi} \right) T_0.$$

$$\Rightarrow \left(\frac{\chi + \gamma - \chi}{\gamma - 1} \right) T_1' = \frac{\chi + \gamma - 1}{\chi(\gamma-1)} T_0$$

$$\Rightarrow \boxed{T_1' = \frac{\chi + \gamma - 1}{\chi(\gamma)} T_0}$$ — AN:

AN $\quad T_1' = \dfrac{3 + \frac{7}{5} - 1}{3 \times \frac{7}{5}} \times 300.$

$$= \frac{15+7-5}{21} \times 300$$

$$= \frac{17}{21} \times 300 = 242{,}85 \text{ K}$$

$\frac{17}{21} \approx 0{,}8.$

on avait $\quad T_1 = \dfrac{300}{3^{\frac{\frac{1}{5}-1}{\frac{1}{5}}}} = \dfrac{300}{3^{\frac{2}{7}}} = 219 \text{ K}$

$$\Rightarrow \boxed{T_1' > T_1}$$

// On récupère moins en irréversible.

$$W_a' = \bar{C}_v (T_1' - T_0) \quad \Rightarrow \boxed{|W_a'| < |W_a|}$$

$$W_i' = RT_0 \left(\frac{p_1}{p_0} - 1\right) \qquad W_i = RT_0 \ln\left(\frac{p_1}{p_0}\right)$$

$$W_i = RT_0 \ln\left(\frac{p_1}{p_0} + 1 - 1\right) \approx RT_0 \left(\frac{p_1}{p_0} - 1\right) - \underbrace{\frac{RT_0}{2}\left(\frac{p_1}{p_0} - 1\right)^2}_{<0.}$$

$$\Rightarrow \boxed{W_i < W_i'}$$

// On doit fournir + de travail en irréversible.

$\ln(x+1) = 1 - \frac{x^2}{2} + \dots$