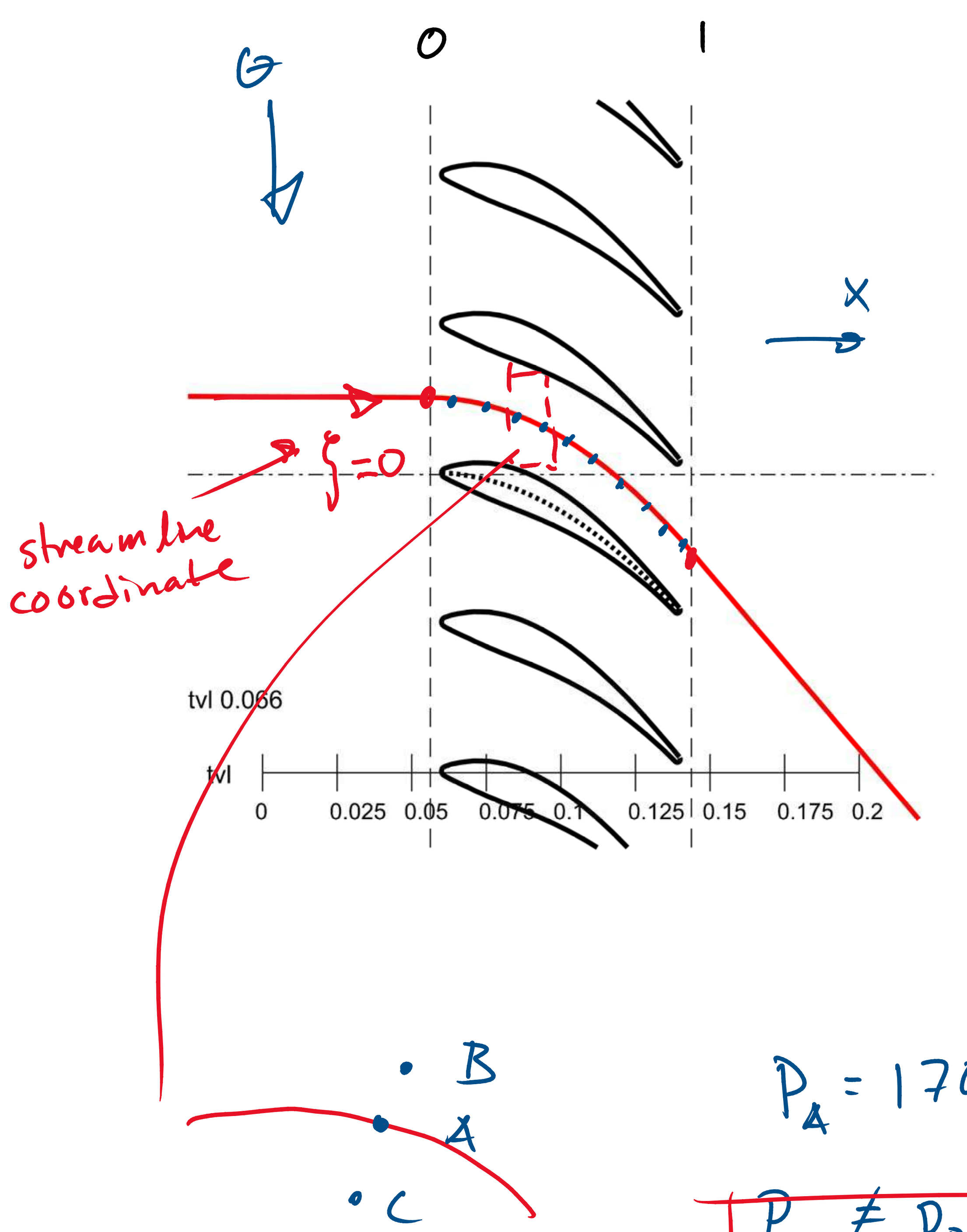
Lect 4 (2D aerodynamics)
den 20 januari 2019 18:46

2D aerodynamics



1D aero

flow quantities change in the direction of the flow

$c_0, c_1 \quad c(\xi)$

$\alpha_0, \alpha_1 \quad \alpha(\xi)$

$p_0, p_1 \quad p(\xi)$

$T_0, T_1 \quad T(\xi)$

$p_A = 170\,\text{kPa}$

$\boxed{p_A \neq p_B \neq p_C}$ ⟹ local variations inside the flow field.

→ There are gradients of flow quantities inside a flow field

$\boxed{2D}$ → gradients in the blade-to-blade direction $(x-\theta)$

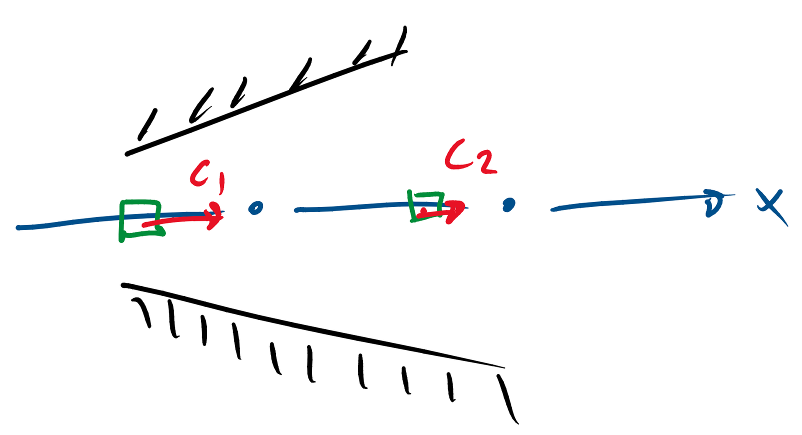
$\boxed{3D}$ → gradients normal to the blade-to-blade surface → radial direction $(r-x)$

Reasons for having gradients

Global features (flight path of a particle)

the flight path is straight, and as a consequence, the fluid quantities change only in the direction of motion

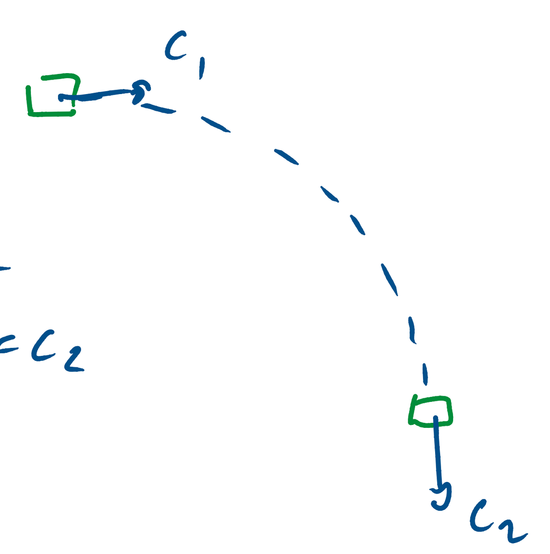
e.g. diffusing flow



increase in pressure

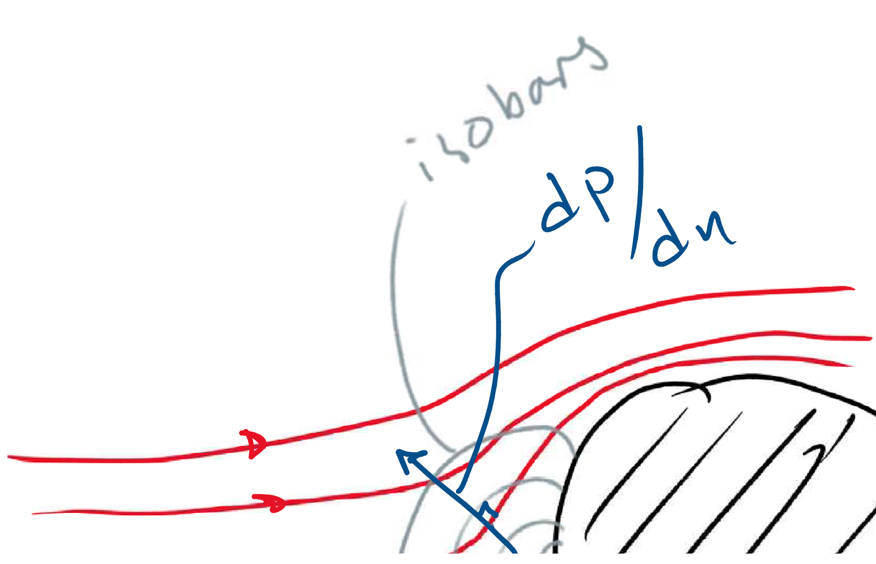
→ $p_2 > p_1$

→ $\frac{dp}{dx} > 0$ pressure gradient

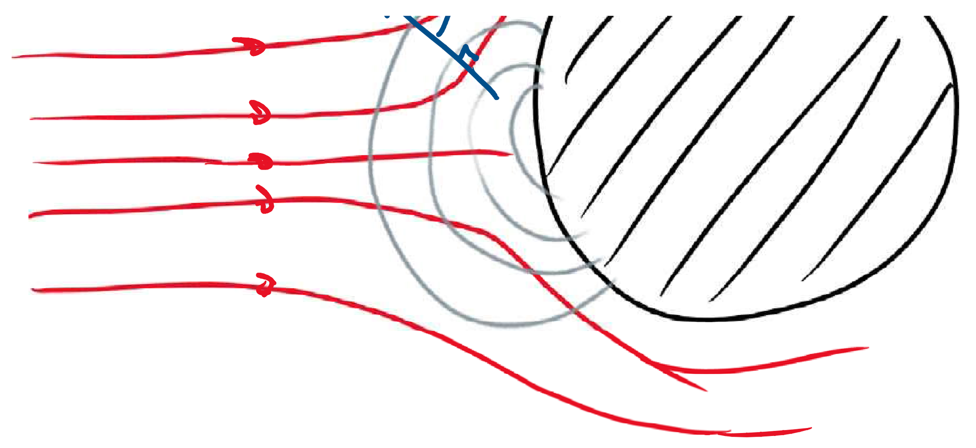


here $c_1 = c_2$

$(p \cdot A = \text{force})$

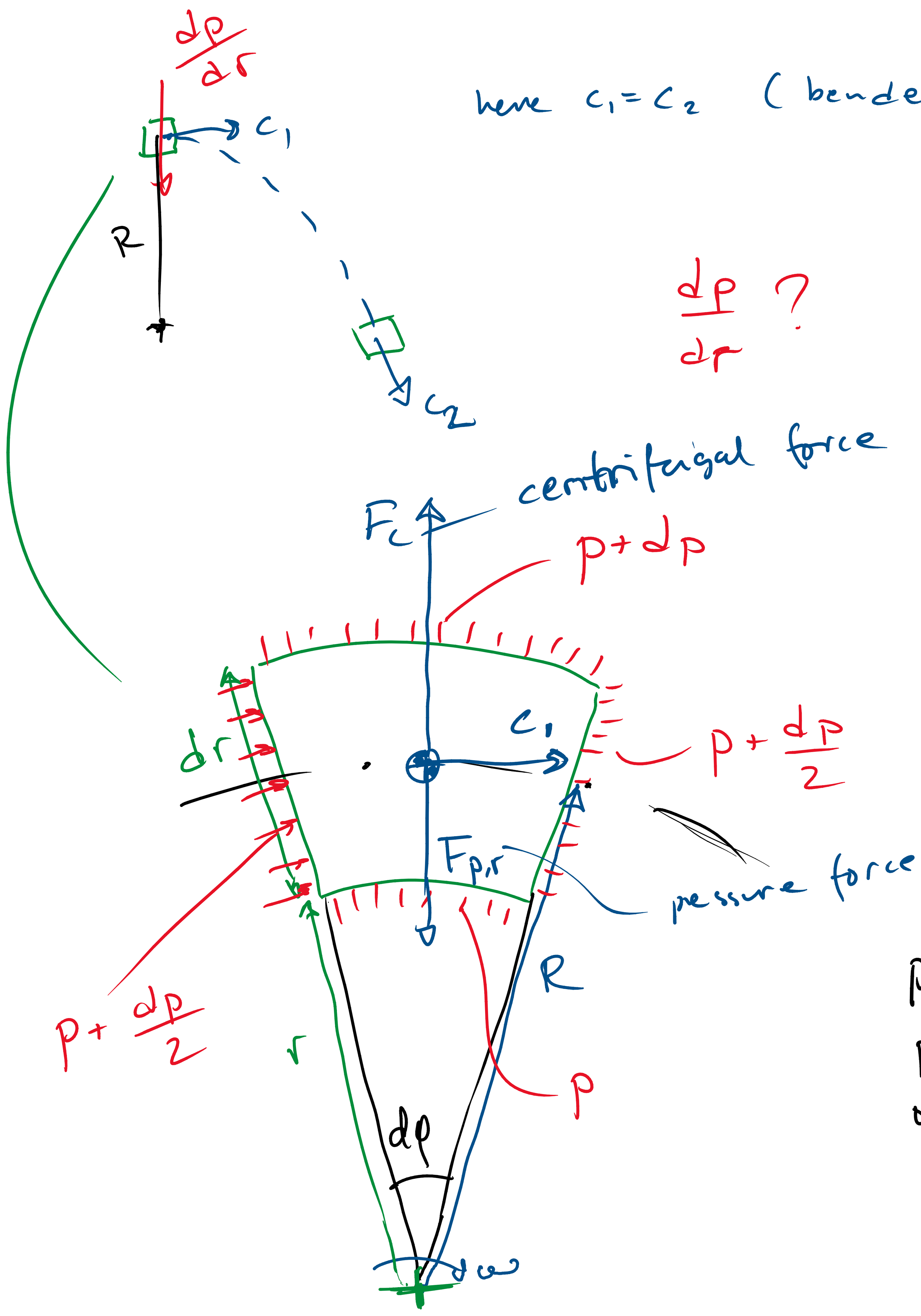


pressure is the only way how information can



information can
be communicated
in a flow field

derive an expression how big the pressure gradient
must be to deviate the flow.



peform the analysis
per unit thickness (dz)
of the fluid particle

Force balance $F_c = F_{p,r}$

i) centrifugal force

$$F_c = \overset{\text{mass}}{dm} \cdot R \cdot \omega^2 \qquad \omega = \frac{c_1}{R}$$

$$F_c = dm \cdot R \cdot \left(\frac{c_1}{R}\right)^2 = dm \cdot \frac{c_1^2}{R}$$

$$dm = \underbrace{dr \cdot r d\varphi \cdot \cancel{dz}}_{\text{volume}} \cdot \rho$$

considering per unit thickness

$$\Rightarrow F_c = \rho \, dr \cdot r d\varphi \cdot \frac{c_1^2}{R}$$

ii) pressure forces acting on the particle, $F_{p,r}$

gen: pressure force = pressure x area

due to inclined surfaces

$$F_{p,r} = \underbrace{(p+dp)(r+dr) \, d\varphi}_{\text{top surface}} - \underbrace{p \cdot r \cdot d\varphi}_{\text{lower surface}} - \underbrace{2\left(p + \frac{dp}{2}\right) dr \cdot \sin\left(\frac{d\varphi}{2}\right)}_{\text{sides}}$$

small $d\varphi$!

$\rightarrow \sin\frac{d\varphi}{2} \approx \frac{d\varphi}{2}$

→ 0 (simplification)

$$F_{p,r} = \underbrace{p\cdot\cancel{r}\cdot d\varphi + dp\cdot r\cdot d\varphi + p\cdot\cancel{dr}\cdot d\varphi + \cancel{dp\cdot dr}\cdot r\cdot d\varphi}_{\text{top}} \xrightarrow{\ } 0 \text{ (simplification)} \quad - \ldots$$

$$\ldots : \; p\cdot\cancel{r}\cdot d\varphi - p\cdot\cancel{dr}\cdot d\varphi - \frac{d\varphi}{2}\cdot\cancel{dr\cdot d\varphi} \to 0 \; = dp\cdot r\cdot d\varphi$$

Force balance $\qquad F_c = F_{p,r}$

$$\rho\cdot dr\cdot r\cdot\cancel{d\varphi}\cdot\frac{c_1^2}{R} = dp\cdot r\cdot\cancel{d\varphi}$$

$$\Rightarrow \quad \boxed{\frac{dp}{dr} = \rho\cdot\frac{c_1^2}{R}}$$

observations:

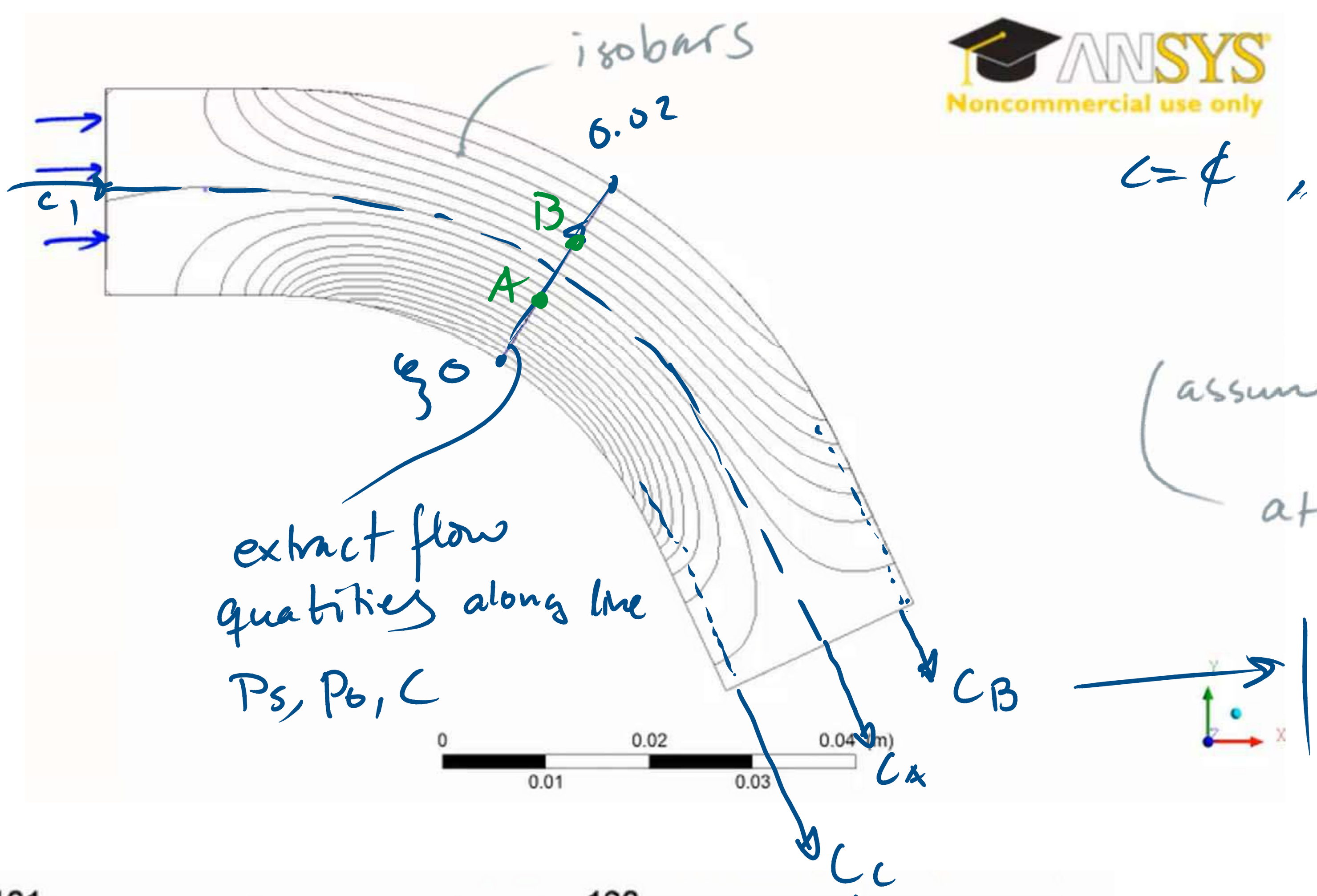
$R \to \infty \to \frac{dp}{dr} \to 0$

$\rho \uparrow \to \frac{dp}{dr} \uparrow$

$c_1^2 \uparrow \to \frac{dp}{dr} \uparrow$

$R \downarrow \to \frac{dp}{dr} \uparrow$

bended channel (const. cross section)

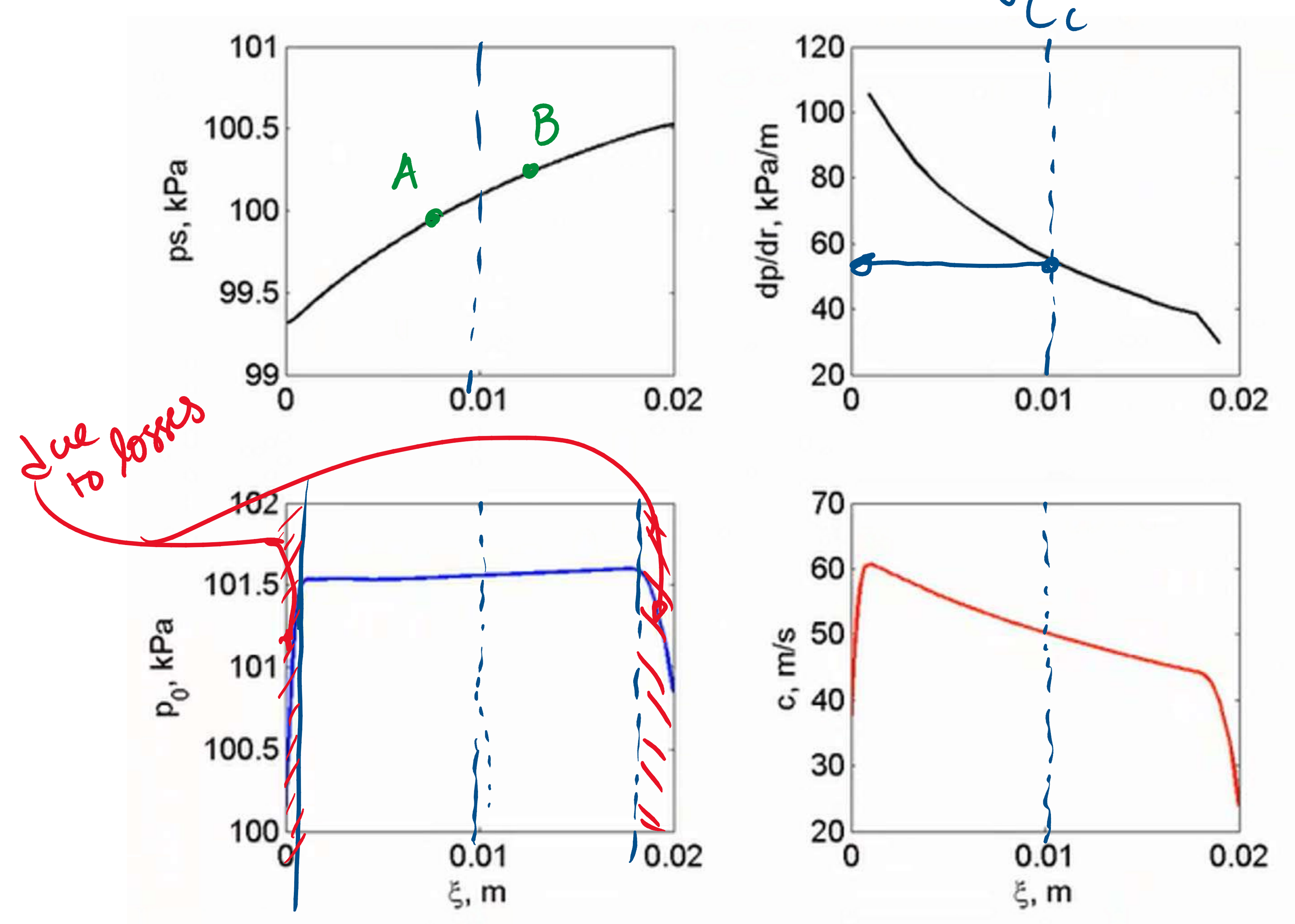


$c = \text{const}, \; c_1 = c_2 = c_A$

$P_1 = P_2 = P_A$

(assume no losses $P_0 = \text{const}$ at center line)

$$\boxed{P_B > P_A > P_C}$$

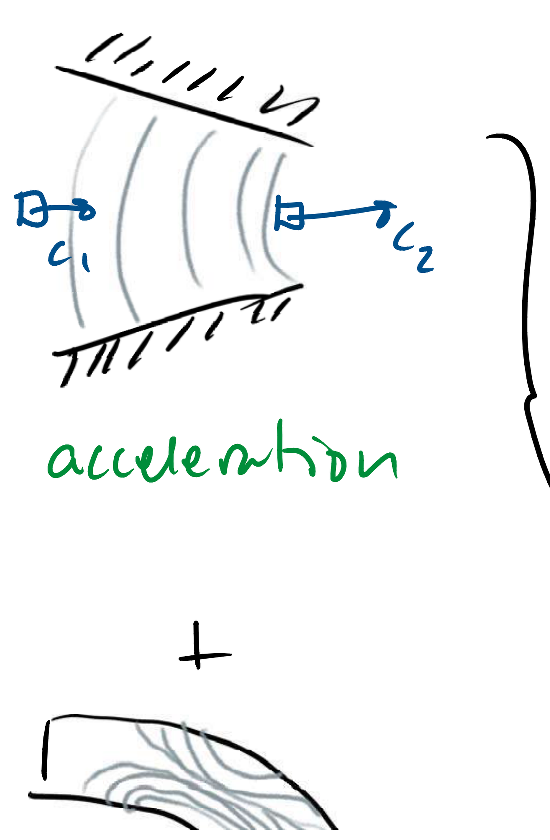


$$\frac{dp}{dr} = \rho\cdot\frac{c^2}{R}$$

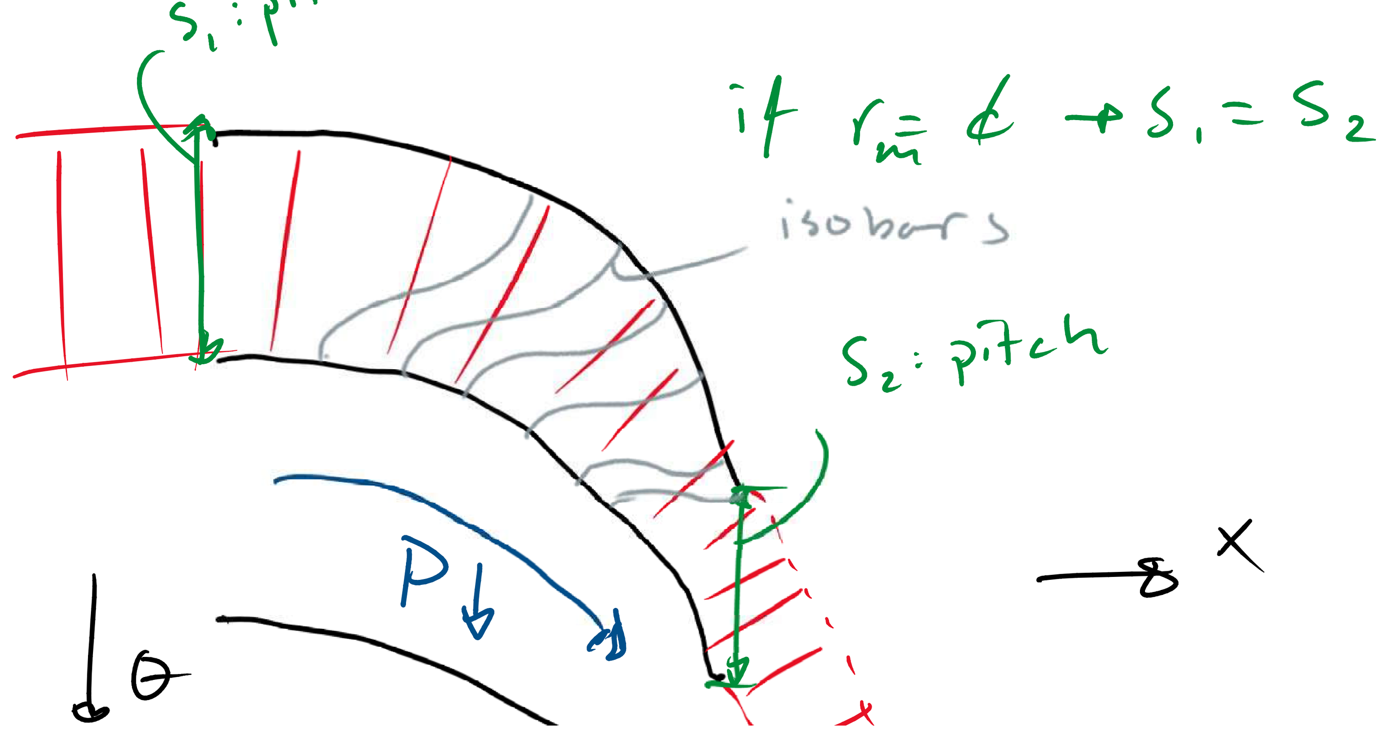
$$\begin{cases} \rho = 1.15 \text{ kg/m}^3 \\ R = 50 \text{ mm} \\ c = 50 \text{ m/s} \end{cases}$$

$$\Rightarrow \frac{dp}{dr} = 1.15\,\frac{50^2}{0.05} \approx 58 \text{ kPa/m}$$

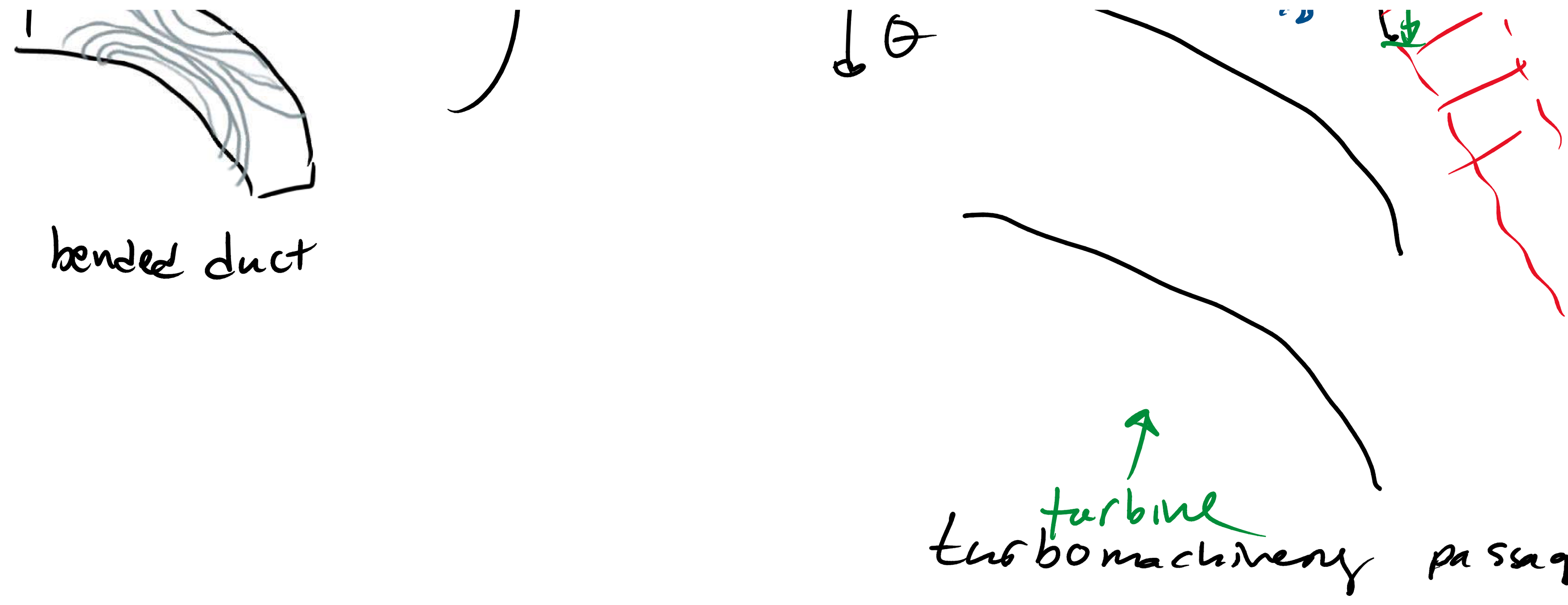
Put the global 2D together



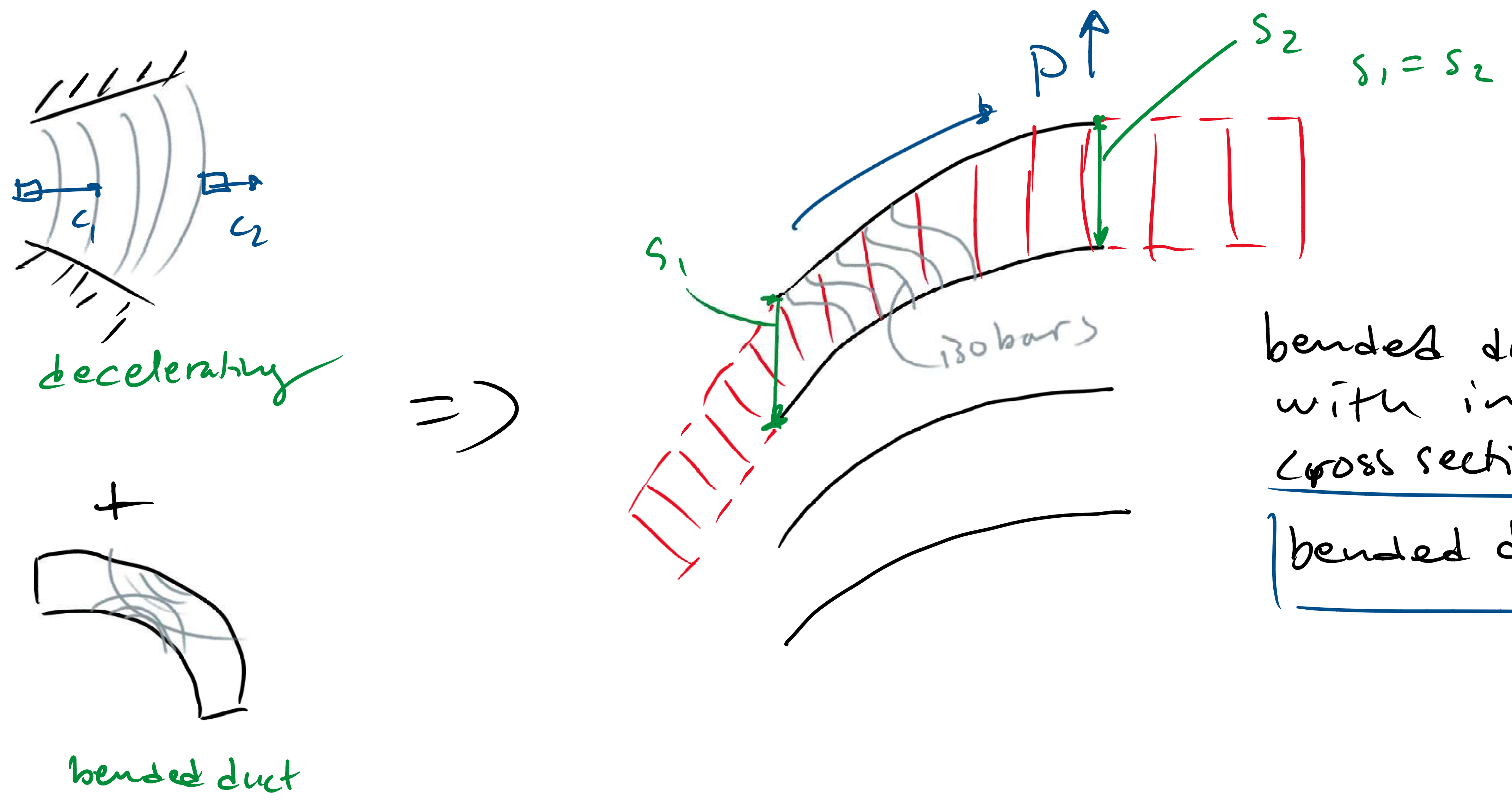
$\Rightarrow$



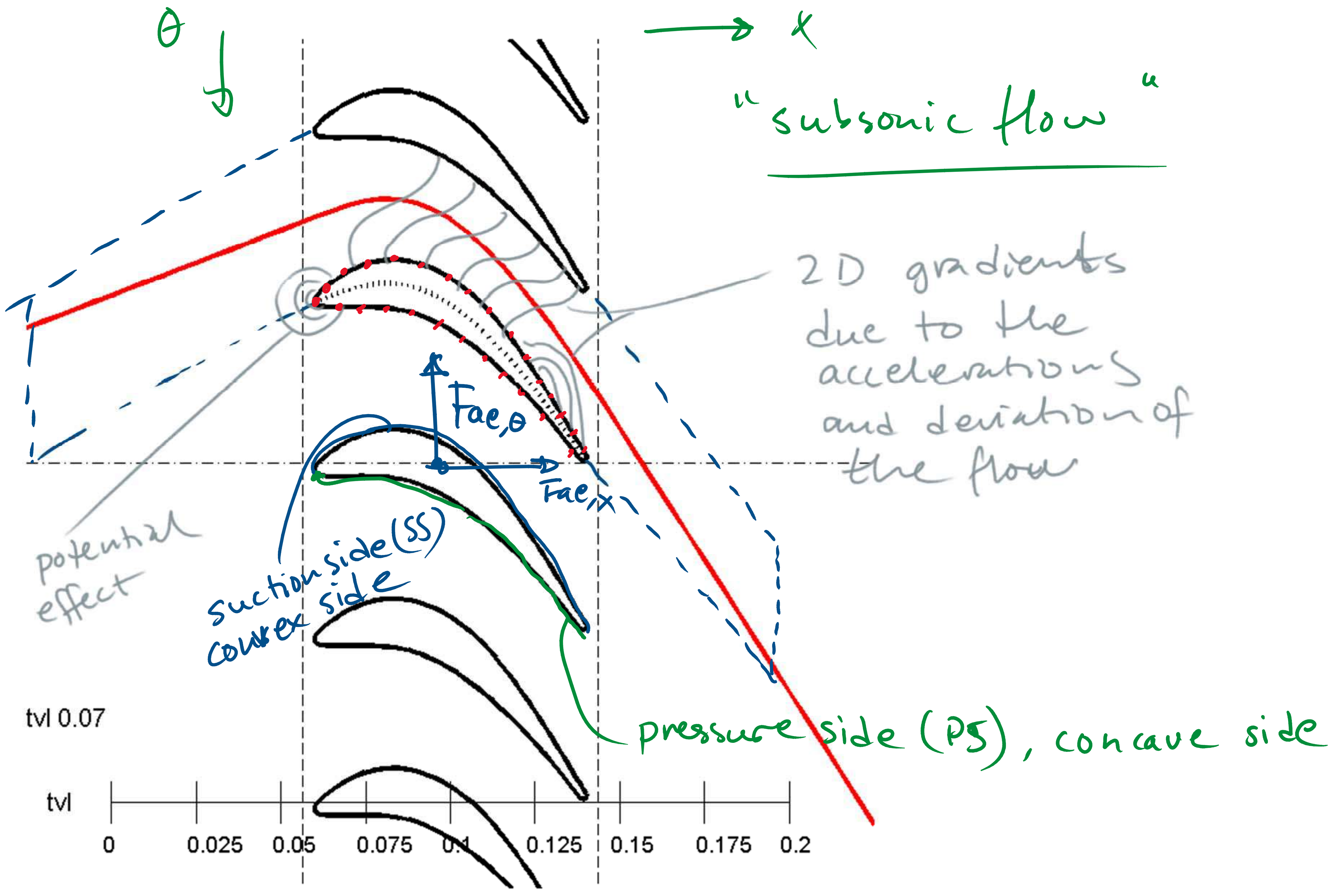
if $r_m = \text{const} \to S_1 = S_2$



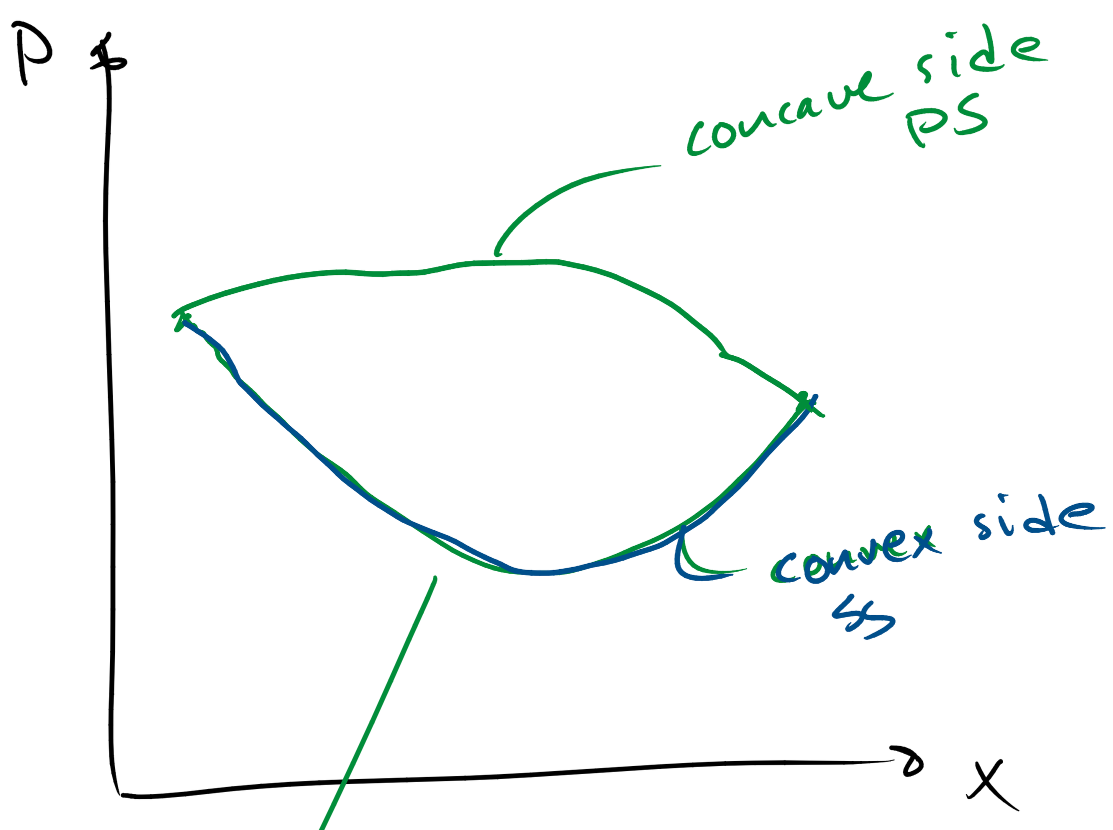
bended duct with decreasing cross section → $\boxed{\text{bended nozzle}}$



bended duct with increasing cross section
$\boxed{\text{bended diffuser}}$



plot the surface pressure



$$\oint_S p \cdot ds = \vec{F}_{ae}$$

$F_{ae,\theta} = \dot{m} \cdot \Delta c_\theta$

$F_{ae,x}$