Optimal dynamical stabilization : application to an inverted electromagnetic pendulum

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Kapitza's pendulum

Def. : The notion of dynamic stability involves the time required for a system to regain its static stability after a disturbance.

- Vertically vibrating pivot
- Fast driving frequency
- Constant use of energy

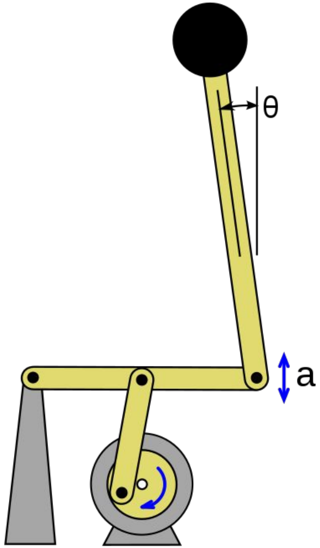

fig. 1 : Kapitza's pendulum [1]

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Plan

I. Characterization

   A. Static stability – homogeneity

   B. Dynamic motion – ON/OFF

II. Dynamic stability – Energy optimization

   A. Floquet theory

   B. Optimal dynamic stabilization

III. Conclusion

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I. Characterization

A. Static stability

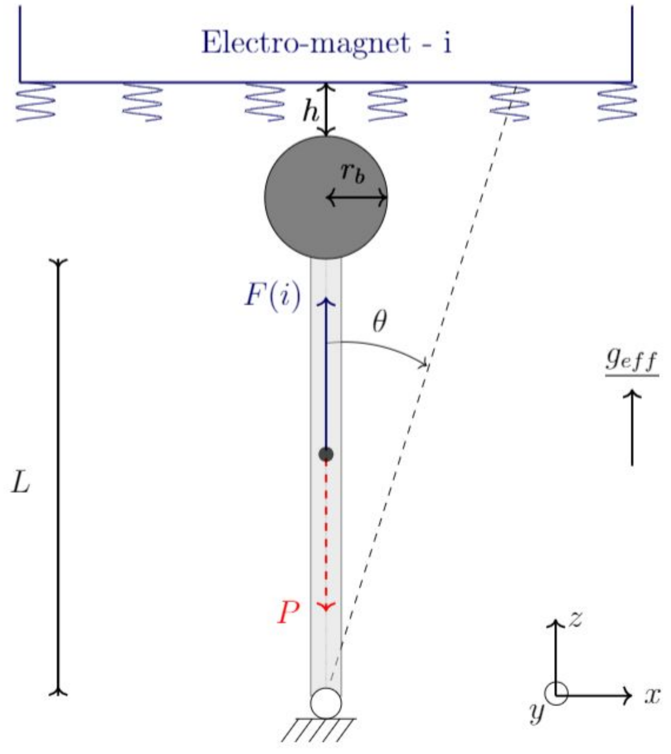


fig. 2 : Model inverted pendulum

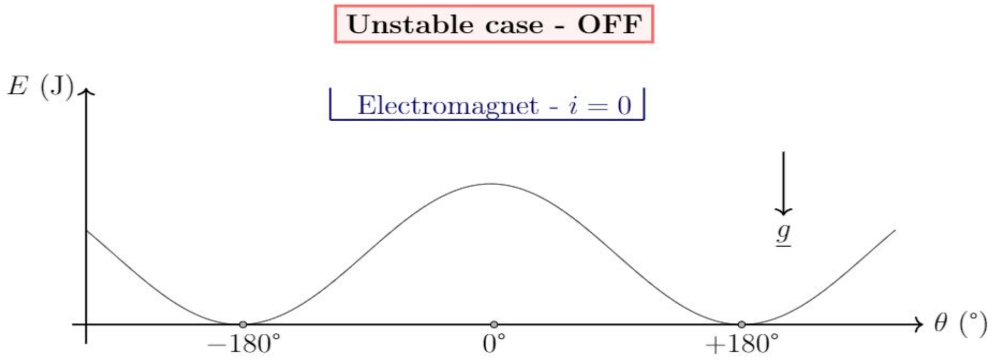


fig. 3 a : Illustration potential energy inverted pendulum

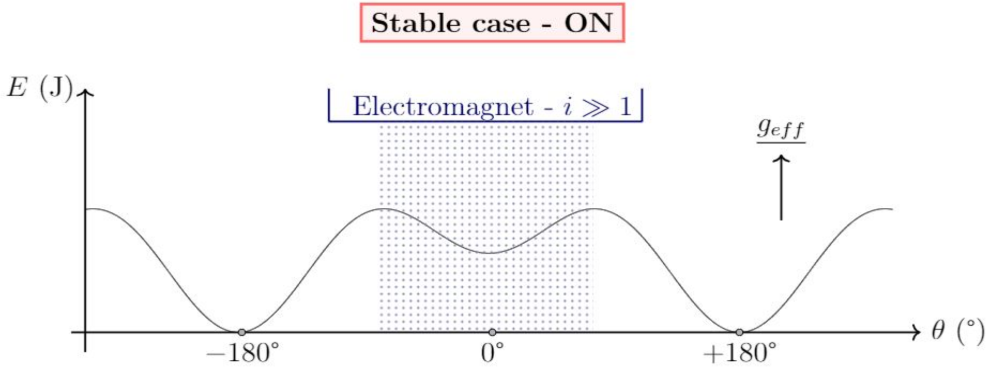
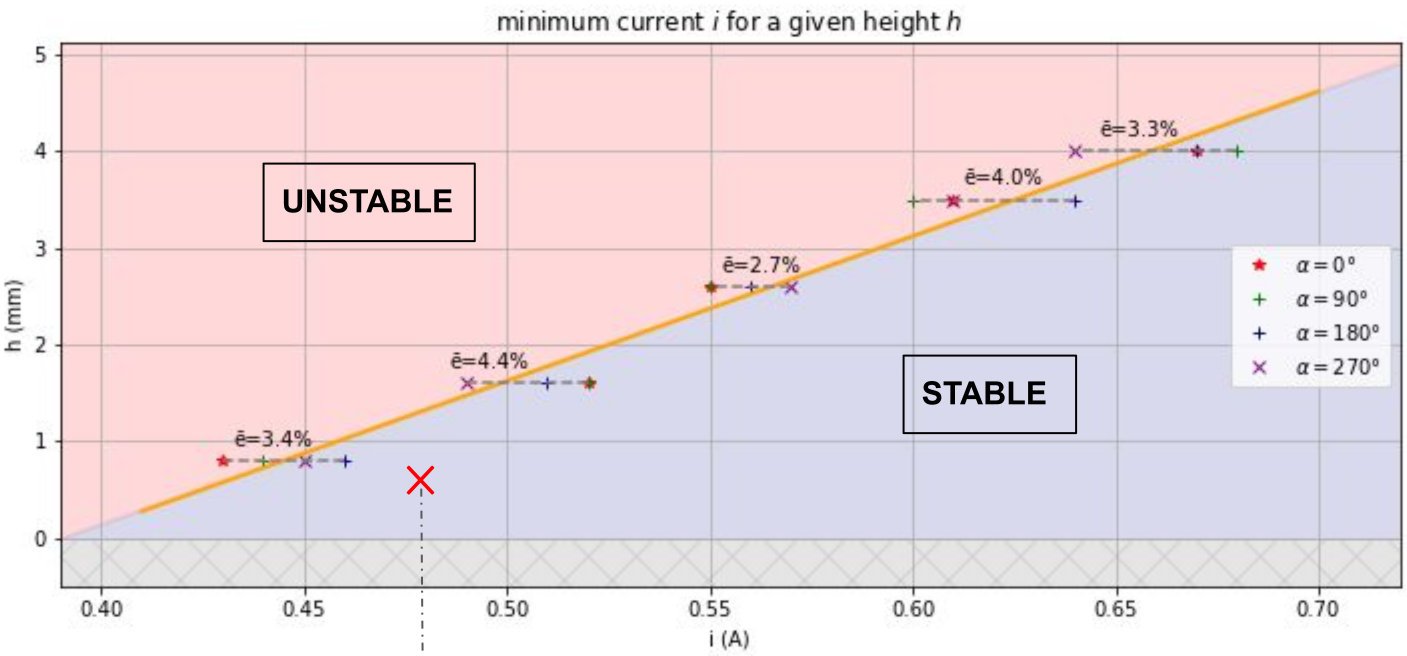


fig. 3 b : Illustration potential energy stabilized inverted pendulum

A. Magnetic field homogeneity



h	≃	0.6 mm
i	≃	0.48 A

fig. 4 : Static equilibrium zones

B. Dynamic motion

$$\ddot{\theta} + 2\xi(t)\omega(t)\dot{\theta} + \omega^2(t)\theta = 0$$

ON $(i > i_s)$: $\ddot{\theta} + 2\xi(i)\omega(i)\dot{\theta} + \omega^2(i)\theta = 0$

OFF $(i = 0\ A)$: $\ddot{\theta} - \omega_0^2\theta = 0$

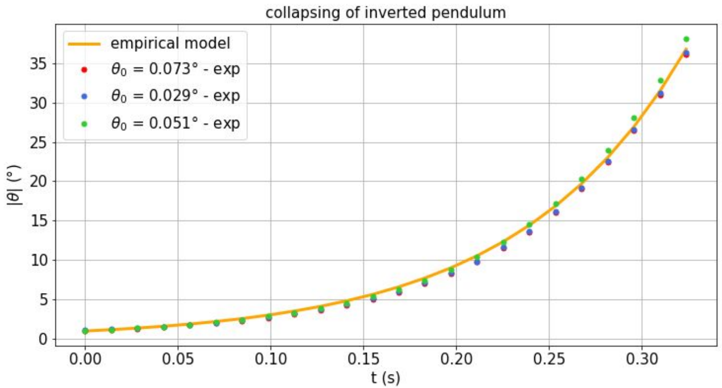


fig. 5 a : Motion collapsing

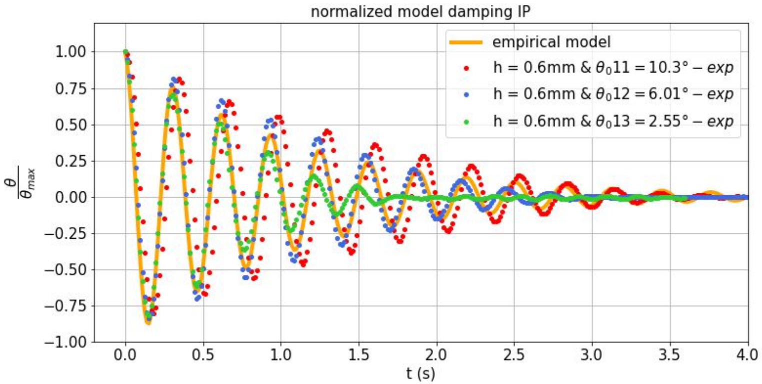


fig. 5 b : Motion damped vibration

$$\omega_0 \approx 11.1\ rad.s^{-1} \approx \sqrt{\frac{g}{L}}$$

Framework

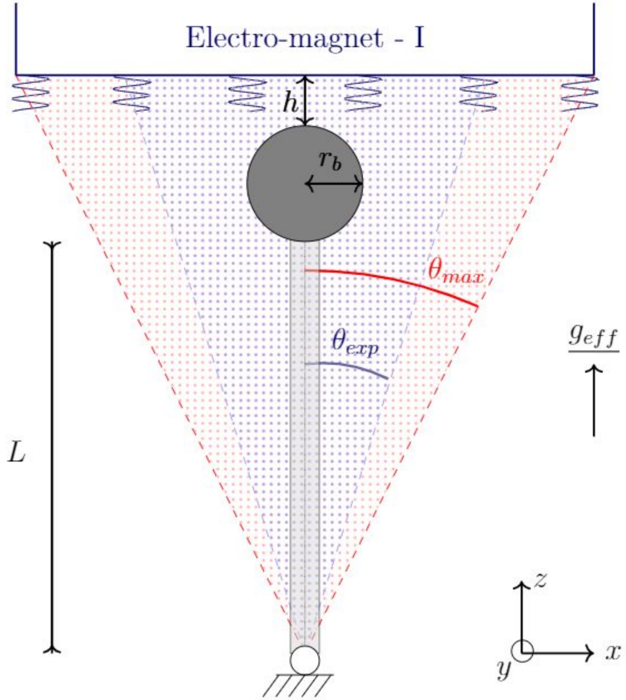


fig. 6 : Framework homogeneity

- θ_{max} = 10°
- θ_{exp} = 6°

- Framework reduction
- Set-up validated

II. Dynamic stability – Energy optimization

ON + OFF motions

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Testing parameters

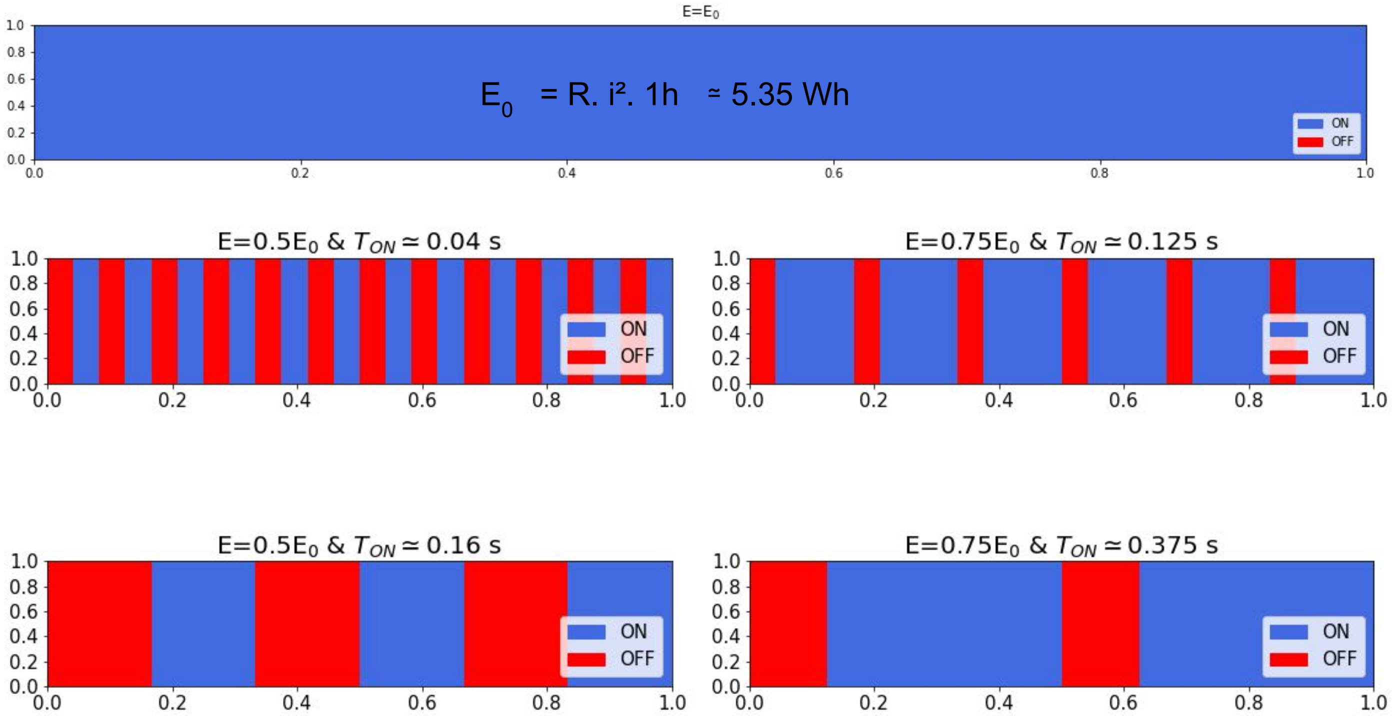

fig. 7 : Stability modulation parameters

The more stable

The more Energy

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Stability motions

Stable

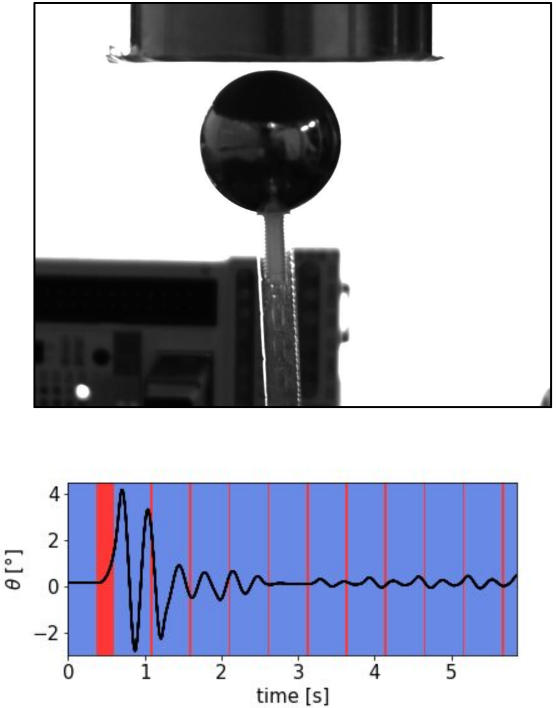

fig. 8 a : Stable motion

Oscillation

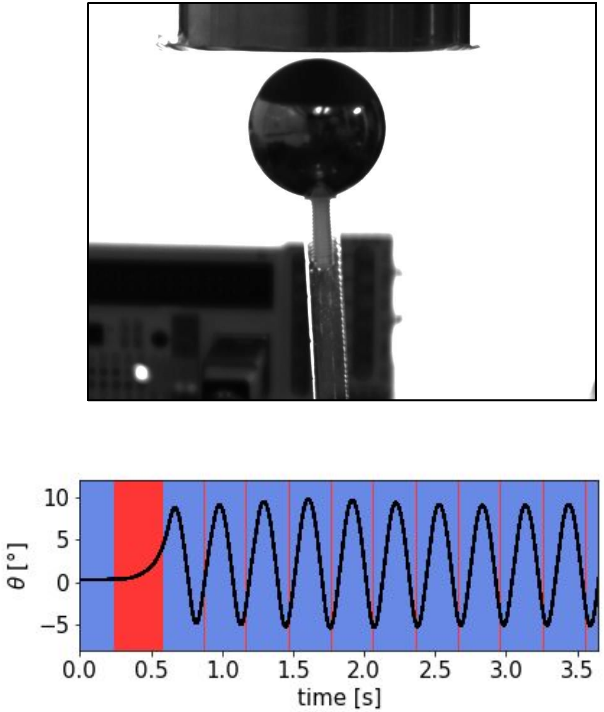

fig. 8 b : Oscillation motion

Unstable

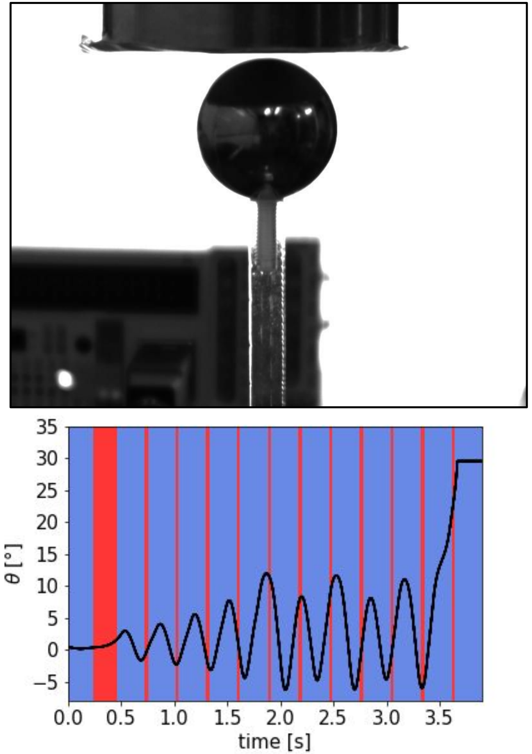

fig. 8 c : Unstable motion

Experimental stability

The more stable

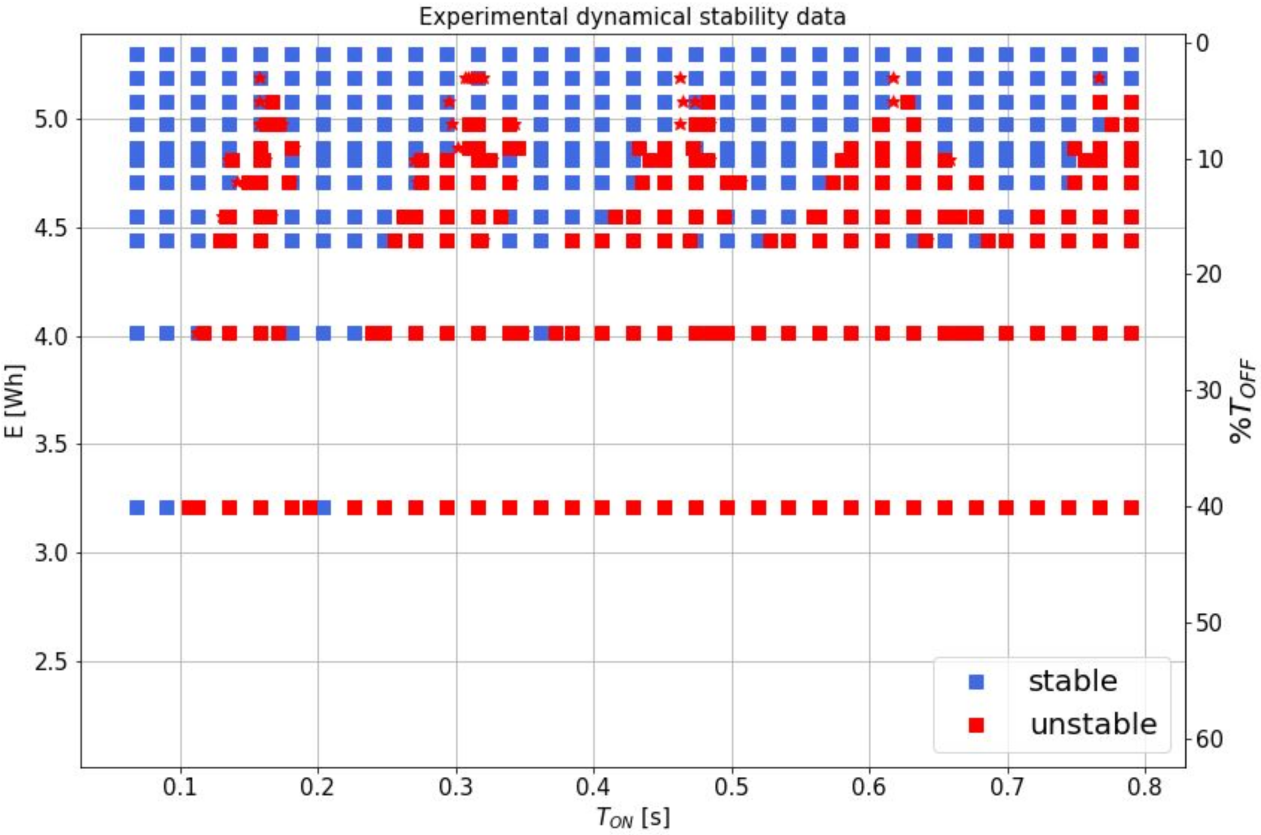

fig. 9 : Experimental dynamic stability

The more Energy

Analytical stability - Floquet theory

$$\begin{cases} \ddot{\theta} + 2\xi(i)\omega(i)\dot{\theta} + \omega^2(i)\theta = 0 \\ \ddot{\theta} \qquad\qquad - \omega_0^2\theta = 0 \end{cases} \iff \{\dot{X}(t)\} = \begin{bmatrix} 0 & 1 \\ -\omega^2(t) & 0 \end{bmatrix} \cdot \{X(t)\}$$

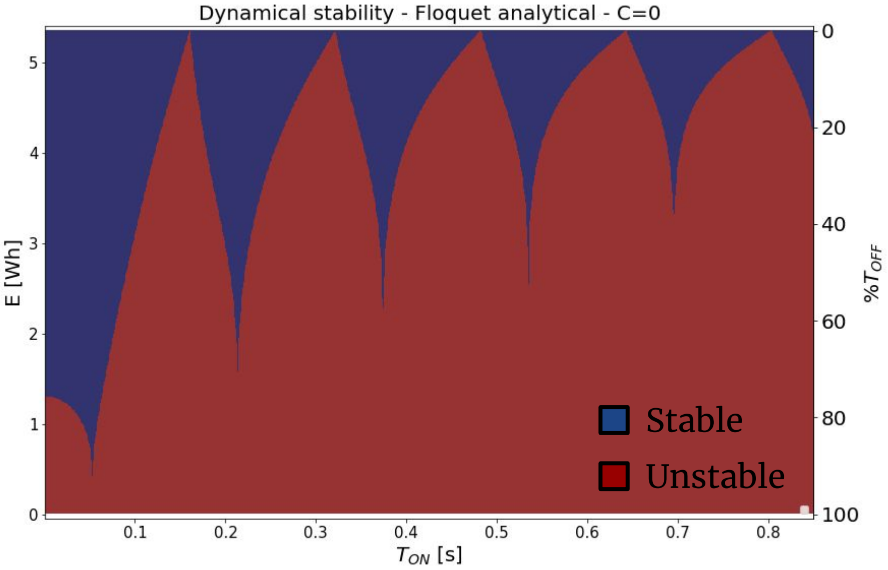

fig. 10 a : Analytical dynamic stability

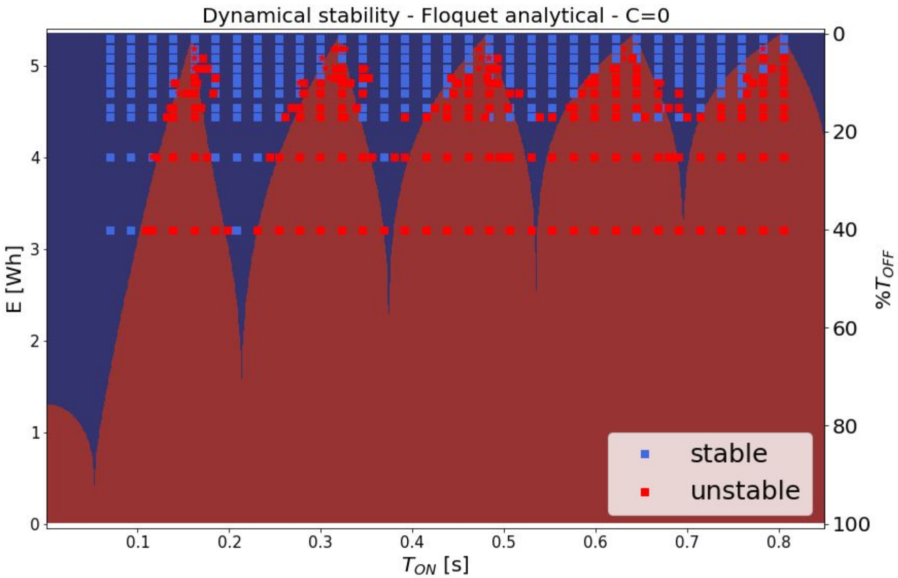

fig. 10 b : Superposition exp / theory

Equation stability periods

$T_{res} = 0.5 \times T_{\omega}$

$T_{opt} = 0.5 \times T_{\omega} + b$

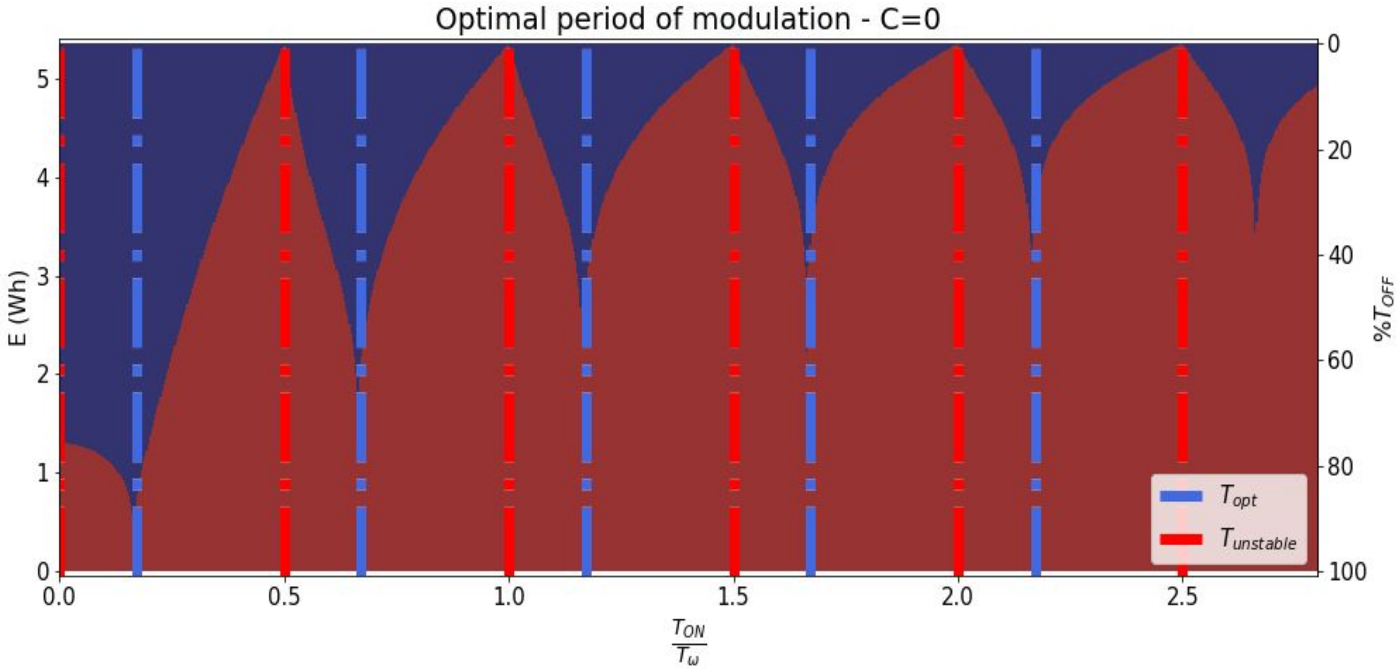

fig. 11 : Optimal modulation period

Conclusion

- Static stability
- Framework reduction
- Dynamic stability
- Energy optimization

References

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