Ex 1.

1° $\frac{dy}{dx} = -10y \Rightarrow y = e^{-10x} + C$ / $y(0) = e^{0} + C = 1 \Rightarrow C = 0$

$\Rightarrow y = e^{-10x}$

EDO

2° $y_{n+1} = y_n + 0{,}05 \times (-10 y_n) = y_n - 0{,}5 y_n = \frac{1}{2} y_n$

$\Rightarrow y_1 = \frac{1}{2} y_0 = \frac{1}{2}$ $\quad x_1 = 1 \times h = 0{,}05$

$y_2 = \frac{1}{2} y_1 = (\frac{1}{2})^2$ $\quad x_2 = 2 \times h = 0{,}1$

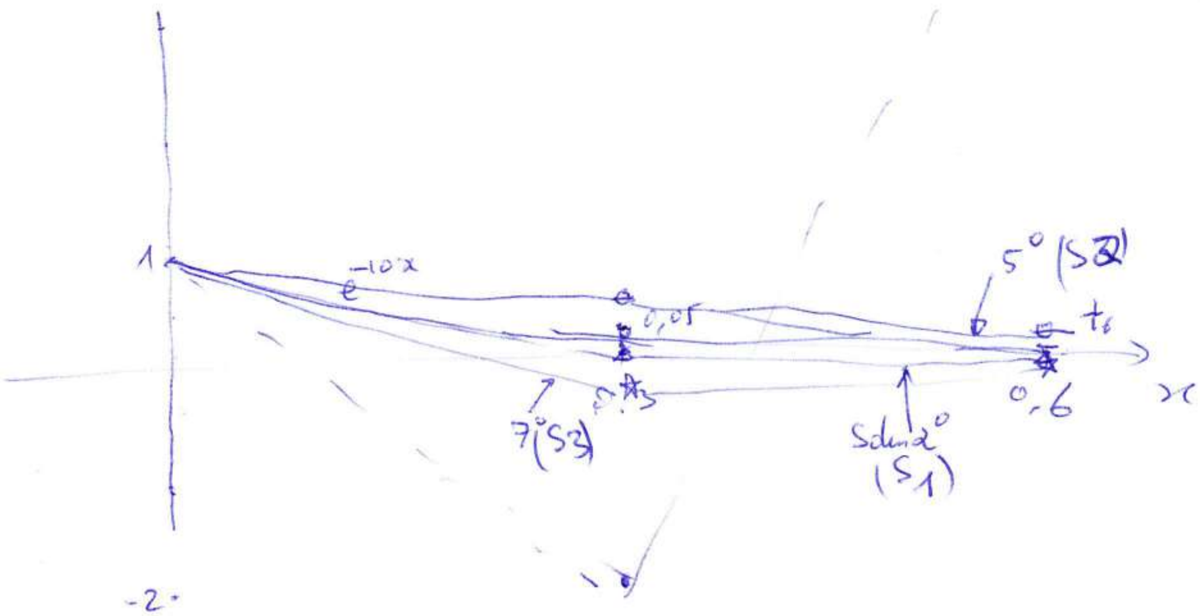
quand $x = 0{,}3 \rightarrow k_1 = \frac{0{,}3}{h} = \frac{0{,}3}{0{,}05} = 6$

$y_6 = (\frac{1}{2})^6 \Rightarrow y(x = 0{,}3) = (\frac{1}{2})^6$

$x = 0.6 \Rightarrow k_2 = 12 \quad y(x = 0{,}6) = (\frac{1}{2})^{12}$

3° si $h = 0{,}3$ $\quad y_{n+1} = y_n + 0{,}3 \times (-10 y_n) = -2 y_n$

$y_1 = -2 \; , \; y_2 = 4$ ← Solution 3° S2 $h = 0{,}3$



4° la Solution 2° suit solution exact et 3° oscille parce que 2° et 3° sont Explicite, leur stabilité depend du pas de temp h

5°
$$y_{n+1} = y_n + 0{,}3 \cancel{(y_{n+1})} \times (-10\, y_{n+1})$$

$$\Rightarrow 4\, y_{n+1} = y_n \Rightarrow y_{n+1} = \frac{y_n}{4}$$

$$y_1 = \frac{1}{4}$$

$$y_2 = \frac{1}{16}$$

6° le schéma 5° est un schéma implicite, leur stabilité ne dépend pas du pas de temps.

7° S_3 $\cancel{y_1 = \frac{y_1 f^0 + y_1 f^0}{2}}$ $y_{n+1} = y_n + \frac{0{,}3}{2}\left(-10\, y_n - 10\, y_{n+1}\right)$

$$\Rightarrow \cancel{(1+3)\, y_{n+1} = y_n} \; \left(\frac{3}{2}+1\right) y_{n+1} = y_n \left(1 - \frac{3}{2}\right)$$

$$\Rightarrow y_{n+1} = -y_n \frac{1}{5}$$

$$y_1 = -\frac{1}{5}$$

$$y_2 = \frac{1}{25}$$

8° le schéma est aussi implicite donc stable sans condit[ion]
il est de l'ordre 2, normal plus précis que 5°

Interpolation
9°
$$l_0(x) = \frac{(x-0{,}3)(x-0{,}6)}{-0{,}3 \times (-0{,}6)}$$

$$l_1(x) = \frac{(x-0)(x-0{,}6)}{0{,}3 \times (-0{,}3)}$$

$$l_2 = \frac{(x-0)(x-0{,}3)}{0{,}3 \times 0{,}6}$$

10° $y(0) = 1$

$$y(0{,}3) = e^{-10 \times 0{,}3} = e^{-3} = 0{,}05 \qquad \Rightarrow Y(x) = \frac{(x-0{,}3)(x-0{,}6)}{0{,}18} + 0{,}05\, \frac{(x-0{,}6)\cdot x}{0{,}09}$$

$$y(0{,}6) = e^{-10 \times 0{,}3} = e^{-6} = 0$$

$$y(x=0,1) = \frac{(-0,2)\times(-0,5)}{0,18} - 0,05 \cdot \frac{(-0,5)\cdot 0,1}{0,09} = \frac{0,1}{0,18} + \frac{0,1\,(0,05)}{0,18}$$

$$= \frac{10(1+0,05)}{18} \simeq 0,52$$

Intégration

12° $S(f) = \sum_{i=0}^{2} \omega_i f(x_i)$

$$= \omega_0 f(0) + \omega_1 f(0,3) + \cancel{\omega_2 f(0,6)}$$

$$= \int_0^{0,6} l_0(x)\,dx \cdot 1 + \int_0^{0,6} l_1(x)\,dx \cdot 0,05$$

13° c'est Newton-Cotes 3 points (Sympson) $n=2$ pair
précision est $\underline{\underline{3}}$

14° $S(f) = \underline{(b-a)}\left(\frac{1}{6} f(0) + \frac{4}{6} f(0,3)\right) = 0,6\times\left(\frac{1}{6}\cdot 1 + \frac{4}{6}\, 0,05\right)$

$$= 0,12$$

15° $I(f) = \int_0^{0.6} e^{-10x}\,dx = -\frac{1}{10} e^{-10x}\Big|_0^{0.6} = \frac{1}{10}\left(1 - e^{-6}\right) = 0,1$

$R(f)$ $S(f) - I(f) = 0,02$

Ex 2.

1° $\det(B-\lambda I) = (a-\lambda)(b-\lambda)(c-\lambda) = 0$ donc a, b, c sont VP

2° $(B-aI)\vec{x} = 0 \Rightarrow \begin{pmatrix} 0 & -1 & 0 \\ 0 & b-a & -1 \\ 0 & 0 & c-a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$ $\quad \begin{aligned} -x_2 &= 0 \\ (b-a)x_2 - x_3 &= 0 \\ (c-a)x_3 &= 0 \end{aligned}$

$x_2 =$

$\Rightarrow x_1 =$ arbitraire $x_2 = x_3 = 0$

$$x = \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix}$$

3° $x^{(0)}$ donné $\quad x^{(k+1)} = B x^{(k)} \quad \lambda^{(k)} = \dfrac{\vec{x}^{(k)} \cdot B\vec{x}^{(k)}}{\vec{x}^{(k)} \cdot \vec{x}^{(k)}}$

si $|\lambda^{(k)} - \lambda^{(k-1)}| < \varepsilon \longrightarrow \lambda^{(k)} = \lambda_1 = a$

4° $\det(B^t - \lambda I) = (a-\lambda)(b-\lambda)(c-\lambda) = 0$ donc a, b, c sont VP de

$$\begin{pmatrix} a-a & 0 & 0 \\ -1 & b-a & 0 \\ 0 & -1 & c-a \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = 0 \Rightarrow \begin{aligned} -y_1 + (b-a)y_2 &= 0 \\ -y_2 + (c-a)y_3 &= 0 \end{aligned}$$

$$\Rightarrow y_2 = y_1/(b-a)$$

$$y_3 = \frac{y_2}{(c-a)} = \frac{y_1}{(b-a)(c-a)}$$

$$\vec{y} = \begin{pmatrix} y_1 \\ \dfrac{y_1}{(b-a)} \\ \dfrac{y_1}{(b-a)(c-a)} \end{pmatrix}$$

5°

$$C = B - a\frac{1}{x_1 y_1} \begin{pmatrix} x_1 y_1 & \frac{x_1 y_1}{(b-a)} & \frac{x_1 y_1}{(b-a)(c-a)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\left(1 + \frac{a}{b-a}\right) & -\frac{1}{(b-a)(c-a)} \\ 0 & b & -1 \\ 0 & 0 & c \end{pmatrix}$$

6° $\det(C - \lambda I) = (b-\lambda)(c-\lambda)\lambda = 0$ $\quad \lambda_1 = b$, $\lambda_2 = c$, $\lambda_3 = 0$

7° puissance ~~itérée~~ donne $x^{(k+1)} = C x^{(k)}$ donne $\lambda^{(p)} \xrightarrow[p \to \infty]{} = \lambda_1 = b$

8° $\mu_1 = \frac{1}{\lambda_3} = \frac{1}{c}$

9° $\mu_1 = \frac{1}{\lambda_3} = \frac{1}{0} \to \infty$ ne marche pas

Ex 3. condition $\rho(\Omega) < 1$

1° Comme $\Omega \cdot \vec{x}_k = \lambda_k \vec{x}_k$

$|\lambda_k| \vec{x} \le \|\Omega\| \cdot \|\vec{x}\| \Rightarrow \rho(\Omega) < \|\Omega\|$

si $\|\Omega\| < 1 \Rightarrow \rho(\Omega) < 1$ condition suffisante

2° $\|\Omega\|_\infty = \sup\left(\frac{8}{10}, \frac{6}{10}, \frac{9}{10}\right) = \frac{9}{10} < 1$

Convergent.