SG2215 : COMPRESSIBLE FLOW

Experimental Laboratory 2 - Shocktube

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1 Introduction

In this lab, a shock tube is used as a supersonic wind tunnel to study the flow around a wedge. The experiments are performed for three different flow Mach numbers by varying the pressure ratios of the driver section pressure over the driven section pressure. Moreover, the driver and driven sections are split initially by a Fast Opening Valve to ensure the pressure difference.

The required pressure ratios are calculated before the experiments and the flow Mach numbers are verified from pressure measurements. The flow Mach number is further verified from the shock angle when oblique and normal shocks form around the wedge.

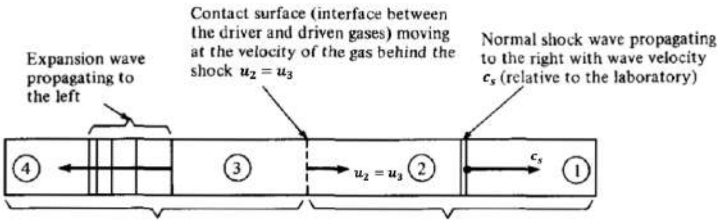

Figure 1: Wave motion inside the shock tube after the Fast Opening Valve (FOV) opening.

In this laboratory exercise, the experiments are performed with three different flow Mach numbers around the wedge, i.e. the Mach number in state 2, M_2. In all the experiments, the temperatures of the different gases in driver and driven sections are assumed to be 300 K (room temperature).

2 Experiment 1 : Subsonic M_2

2.1 Theoretical estimations

In this part, the incident Mach number M_s has to be calculated in order to know what pressure p_1 and p_4 are needed to achieve the targeted M_2 which is 0.46.

To calculate M_s, the following equation has to be used:

$$M_2 = \frac{2(M_s^2 - 1)}{(2\gamma_1 M_s^2 - (\gamma_1 - 1))^{1/2}\,((\gamma_1 - 1)M_s^2 + 2)^{1/2}} \tag{1}$$

where γ_1 is the heat capacity ratio of the air. $\gamma_1 = 1.4$

As it is here, in equation 1, to find the required M_s for the targeted M_2, a numerical resolution of this equation is needed. Therefore, the `MATLAB` function `fsolve` is used and the following result is subsequently found:

$$M_s = 1.3508$$

From there, the ratio $\frac{p_4}{p_1}$ can be calculated, but there is two different equations that are going to be used. The first one is the theoretical $\frac{p_4}{p_1}$:

$$\frac{p_4}{p_1} = \frac{2\gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1}\left(1 - \frac{\gamma_4 - 1}{\gamma_4 + 1}\frac{a_1}{a_4}\left(M_s - \frac{1}{M_s}\right)\right)^{-\frac{2\gamma_4}{\gamma_4 - 1}} \tag{2}$$

where γ_4 is the heat capacity ratio of Helium, a_1 is the speed of sound in the air and a_4 is the speed of sound in Helium. $\gamma_1 = 1.667$ and $\frac{a_1}{a_4} = 0.34$

And the second one is the actual $\frac{p_4}{p_1}$ for the shock tube in KTH:

$$\frac{p_4}{p_1} = \exp\left(0.31M_s^3 - 2.6M_S^2 + 8.1M_s - 5.5\right) \tag{3}$$

By using both equations 2 and 3, the theoretical ratio is $\frac{p_4}{p_1} = 2.6406$ and the actual ratio is $\frac{p_4}{p_1} = 4.3120$ which is quite a big difference between both values. It can be explained by the fact that the experience has not been done in an ideal case. In an ideal case, the valve would have been opened instantly which is not the case actually, so more gas has to be pumped and thereby p_4 should be higher if we want to obtain the target M_2.

2.2 Experimental data analysis

In this part, some data are provided in the file `Oscilloscope_traces.pdf` including figure 2 and some values:

$$p_1 = 49 \text{ [kPa] and } p_2 = 96 \text{ [kPa]}$$

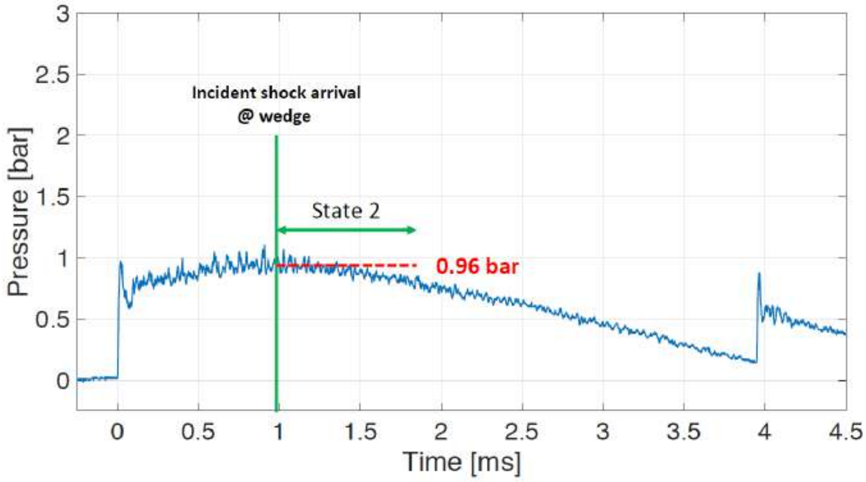

Figure 2: Oscilloscope trace for experiment 1

These values are needed in order to calculate the incident Mach number M_s with the following equation:

$$M_s = \frac{c_s}{a_1} = \left(\frac{\gamma_1 + 1}{2\gamma_1} \left(\frac{p_2}{p_1} - 1 \right) + 1 \right)^{1/2} \tag{4}$$

where c_s is the velocity of the incident shock wave.

By using this latter equation, we obtain:

$$M_{s,\text{exp}} = 1.3499$$

And then can be obtained the experimental M_2 by using this time equation 1:

$$M_{2,\text{exp}} = 0.4590$$

which is very close to the targeted M_2 value of 0.46.

Finally, from the viewing of the video `Exp1.avi`, we can observe that the incident shock wave after its contact with the wedge does not create another shock wave, which confirms a subsonic behavior which means that $M_2 < 1$.

3 Experiment 2 : Supersonic M_2, $\theta > \theta_{\max}$

We now consider a supersonic Mach number $M_2 = 1.04$. However, we consider low enough mach number such that we have theoretically a bow shock in front of the wedge top.

3.1 Theoretical estimations

In this sub-section we are gonna calculate as before the values of pressures and mach number associated with our new M_2.

First, using numerical approach, we can deduce the theoretical M_s out of M_2 value. To do so, we use the equation 1. After numerical resolution, we get that $M_s = 2.1446$.

As before there is now 2 pressure ratios to be deduced : one from theory and one from KTH's formula adjusting the value in order to fit better the physical experience.

From 2, we get that the theoretical ratio is $\frac{p_4}{p_1} = 12.3494$ while from 3 we obtain a corrected pressure ratio equal to $\frac{p_4}{p_1} = 19.5083$.

Once again we see a slight difference between both results as we recall that to avoid the time-position offset of the Mach wave apparition, we have to adjust the pressure ratio, which comes here by an increasing of p_4.

You can note that the corrected pressure ratio is also the one used during the experiment.

3.2 Experimental data analysis

In this part, some data are provided in the file `Oscilloscope_traces.pdf` including figure 3 and pressure values:

$$p_1 = 10 \text{ [kPa] and } p_2 = 52 \text{ [kPa]}$$

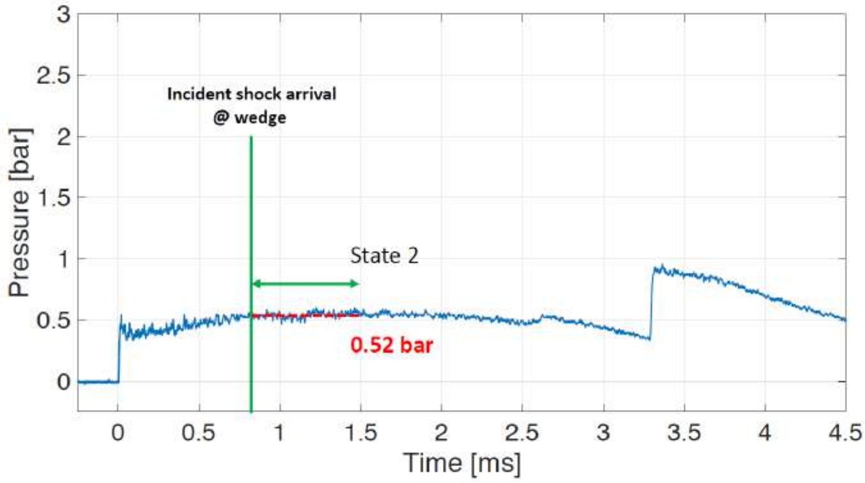

Figure 3: Oscilloscope trace for experiment 2

These values are needed in order to calculate the incident Mach number M_s with equation 4.

We subsequently obtain:

$$M_{s,\text{exp}} = 2.1448$$

And then, can be obtained the experimental M_2 by using the equation 1 :

$$M_{2,\text{exp}} = 1.0401$$

which is very close to the targeted M_2 value as we have an absolute error of :

$$\frac{|M_2 - M_{2,\text{exp}}|}{M_2} \approx 6 * 10^{-5}$$

Note that M_2 here is the theoretical Mach number.

To finish with experimentation 2, we will now explain the obtained results referring to the absolute error and the θ - β - M curve.

By opening the video, we have the following behavior that is observed :

- The flow is first still ;
- The flow wave makes her appearance and subsequently a bow shock appears before the wedge ;
- The bow shock, once fully developed, stagnates before disappearing with the reflected flow coming back.

The point to highlight here it the fact that we have the creation of a bow shock detached from the wedge. This recall the fact that for a $\theta > \theta_{max}$, we have conditions such that no solution exists for a straight oblique shock wave and therefore the shock will be curved and detached.

Besides, when looking to figure 6, and considering that we have a $\beta \approx 78.2°$ together with $\theta = 8°$ for the wedge's top, and that we get a $\beta \approx 78.4°$ together with $\theta = 4°$ for the wedge's bottom, we see that we have a strong shock, leading in a Mach number inferior to 1 behind the shock.
We also note in the same graph that we do have $\theta > \theta_{\max}$ as mentioned.

To conclude on this part, we have seen numerically and graphically how behave the pressures and the Mach numbers when considering a strong super-critical shock.

4 Experiment 3 : Supersonic M_2, $\theta < \theta_{\max}$

In the third and last experiment, the value of the Mach number that is targeted is $M_2 = 1.34$. We consider big enough mach number such that we have theoretically a normal shock attached to the wedge front.

4.1 Theoretical estimations

First, the incident Mach number M_s is numerically calculated for the given M_2. Thus, using equation 1 :

$$M_s = 2.9353$$

From this value, the theoretical $\frac{p_4}{p_1}$ is calculated with equation 2 :

$$\left(\frac{p_4}{p_1}\right)_{\text{theoretical}} = 40.4616$$

and the actual $\frac{p_4}{p_1}$ can also be obtained with equation 3, leading to :

$$\left(\frac{p_4}{p_1}\right)_{\text{actual}} = 41.0365$$

We note that the difference between the two pressure ratios has now been reduced. In fact, this behavior can be associated with figure 6 from the subject, where we see that bigger the Mach number, fewer the influence of the corrected term. This implies that the correction term is not so necessary for high Mach number.

Anyway, it is still important to use it in order to get a more accurate result and as by working with bigger Mach number than we are now, we would enter in a domain where the perfect gas relations don't stand anymore.

4.2 Experimental data analysis

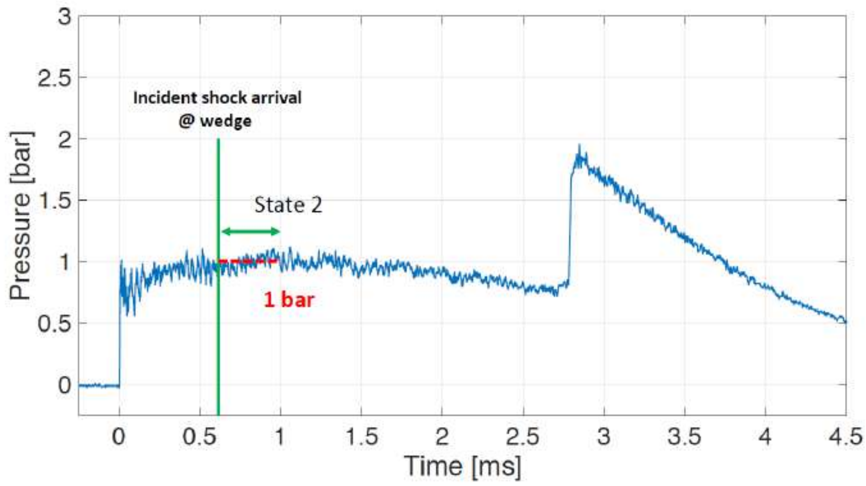

Figure 4: Oscilloscope trace for experiment 3

In this part, we are again going to take a look at the file `Oscilloscope_traces.pdf` that includes figure 4 and the following pressure values :

$$p_1 = 10 \text{ [kPa] and } p_2 = 100 \text{ [kPa]}$$

With these values, the experimental incident Mach number can be calculated with equation 4 :

$$M_{s,\text{exp}} = 2.9520$$

And the associated Mach number M_2 can also be calculated with equation 1 :

$$M_{2,\text{exp}} = 1.3446$$

Contrary to the previous parts, we now introduce a new way of calculating M_2 experimentally. To do so, we introduce the idea of shadow graph, which, from the measurement of angles θ and β, together with figure 6, will give us the value of the associated Mach number M_2.

The idea here will then be to compare two ways of calculating the mach number : one using a piezoelectric sensor, and an other one using shadow graph. Besides, we will then have the distinction between mach numbers at the top and at the bottom of the wedge in the case of the shadow graph, providing useful information on the flow field.

In the end we will know which of the method is the most relevant one and discuss the assumptions.

The shadow graph that is gonna be used here is the following one :

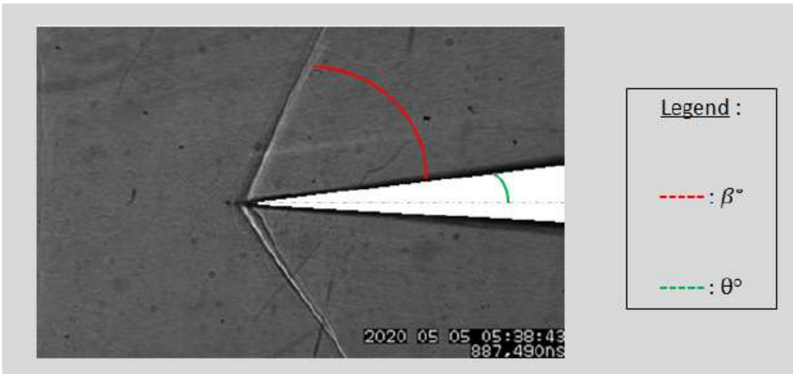

Figure 5: Snapshot of experiment 3

NOTE : the angles β and θ mentioned on the graph serve as illustration and we have the same way of measuring the angle for the wedge's bottom.

From the file `Exp3_snapshot.jpg`, we measure the following angles :

	β	θ
TOP	65.8°	8°
BOTTOM	55°	4°

Table 1: Angles from shadow graph, top and bottom

Finally, from the graph 6, we get that :

$$M_{2,\text{TOP}} \approx 1.35$$

and

$$M_{2,\text{BOTTOM}} \approx 1.32$$

We can already note that these value look to be close to the target one.

Nevertheless, before doing the final calculation of the error, we will exploit the video called `Exp3.avi` in order to visually understand what is behind the screen picture 5.

By opening the video, we have the following behavior that is observed :

- The flow is first still ;

- The flow wave makes her appearance and subsequently a normal shock appears at the wedge front.

- The normal shock, once fully developed, stagnates before disappearing with the reflected flow coming back

The point to highlight here it the fact that we have the creation of a normal shock attached to the wedge. In fact, from the values obtained from `Exp3_snapshot.jpg` we have a weak shock, which is in fact likely to occur in flow geometries with an angle "opened" as it is here.
We thereby have a shock closer to the wedge's surface.

For what is up to the precision of the different methods, we will now compare the method of pressure measurement and shadow graph with respect to the target value.

First, by looking to the experimental value, we have the following absolute error :

$$\frac{M_{2,\text{exp}} - M_2}{M_2} = \frac{|1.3446 - 1.34|}{1.34} \approx 3.4 * 10^-3$$

For the Mach numbers obtained through graph 6, we have the cases for top and bottom of the wedge :

TOP

$$\frac{M_{2,\text{TOP}} - M_2}{M_2} = \frac{|1.35 - 1.34|}{1.34} \approx 7.5 * 10^-3$$

BOTTOM

$$\frac{M_{2,\text{BOTTOM}} - M_2}{M_2} = \frac{|1.32 - 1.34|}{1.34} \approx 1.5 * 10^-2$$

We can note that for the two cases the order of the absolute error is the same, arguing that the method are close in term of precision. This is furthermore what we had seen when looking to the obtained Mach numbers.

We can thereby argue that these two methods match similarly with the target value.

What will make the difference from the two of them in term of relevance is the assumptions that have been taken to come to these results.

Indeed, while the experimental Mach number is based a correcting formula, the shadow graph method provides a Mach number through a graphic interpretation. Thereby, the results coming with the shadow graph come with uncertainties linked to the camera resolution and moreover to the human eye precision.

Finally, by looking to these sources of uncertainties it looks that the pressure sensor might more relevant, even if the difference is low enough to consider the two methods to be as consistent.

Nevertheless, it looks like

5 Conclusion

To conclude, we have been calculating the theoretical pressure and the actual pressure that are needed to be applied in the driver section for different targets M_2. We can notice that the larger M_2 is, the less we will need to adjust the actual pressure in comparison to the theoretical one.

After that, we treated the experimental data to verify if all the experimental M_2 were close enough to the aimed one, and in every single case that we studied they were indeed very close.

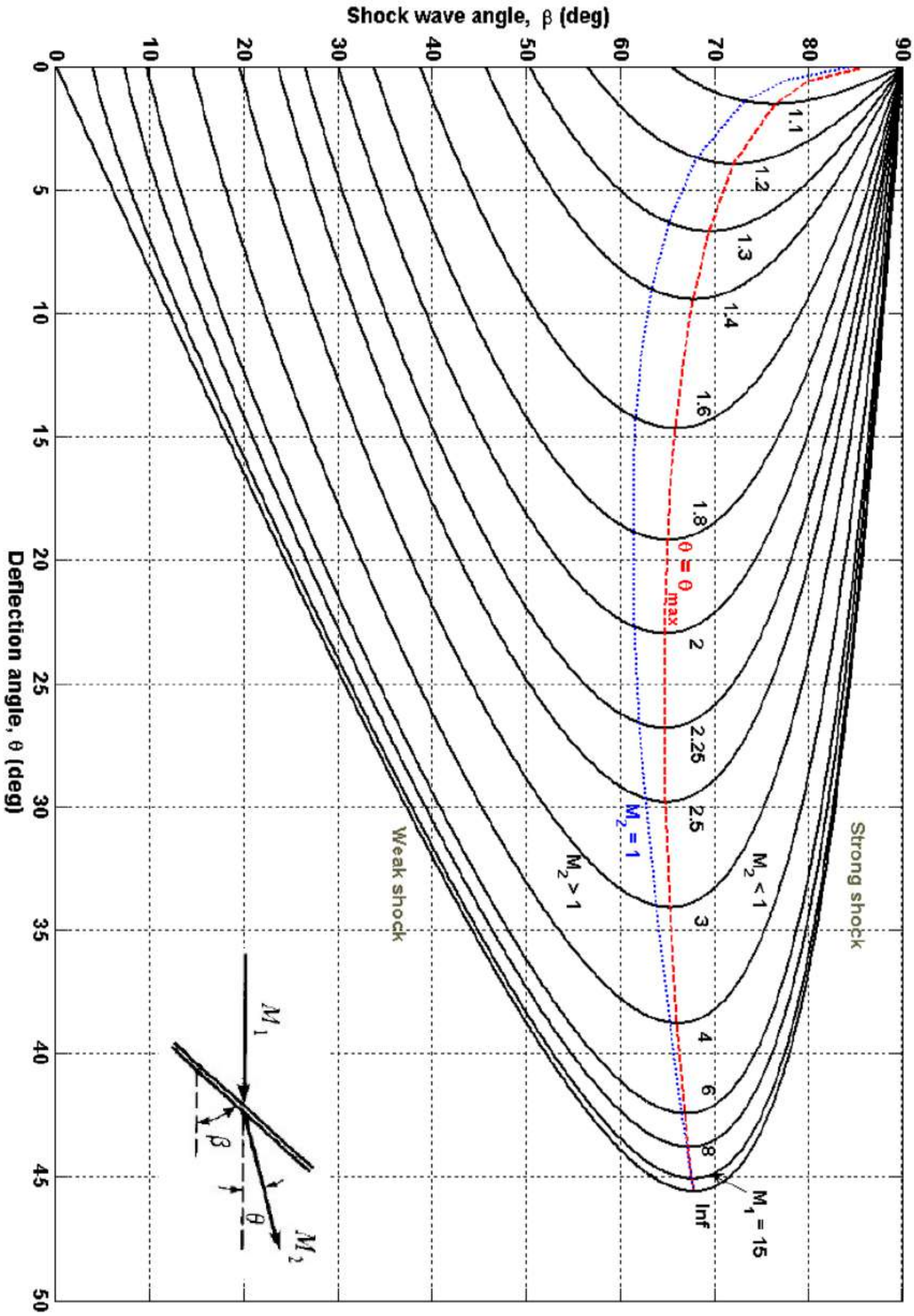

Figure 6: θ-β-M curves from the subject