Laboratory exercise no 1, SG2215 Compressible flow

April 6, 2020

"Compressible pipe flow"

1 Summary

In this laboratory exercise the compressible pipe flow is investigated with regard to its pressure drop, choking condition and the radial distribution of the Mach number at the pipe outlet. The pressure variation along the pipe is measured with eight pressure taps along the pipe and the Mach number distribution with a traversable Pitot tube. The measured values should be compared with analytical results obtained from one-dimensional pipe-flow analysis assuming a constant skin-friction coefficient.

2 Theory

Compressible flow in a pipe with constant cross section can be analyzed through one-dimensional theory if the flow is assumed to be of plug-type, *i.e.* there is no variation of the velocity (or other flow variables) over the pipe cross section. This is a fairly good approximation if the Reynolds number (Re) is high and the flow is turbulent (the higher the Re, the better the approximation). In this case the effect of friction is modeled with a constant friction coefficient c_f defined with local values of the wall friction τ_w, gas density ρ and average velocity u, such that

$$\tau_w = \frac{1}{2}\rho u^2\, c_f\,. \tag{1}$$

For a smooth pipe c_f is typically equal to 0.005. By analyzing the flow over a small length $\mathrm{d}x$ of the pipe, using mass and momentum conservation, it is possible to obtain the following relation

$$\mathrm{d}p + \rho u \mathrm{d}u = -\frac{1}{2}\rho u^2 \frac{4c_f \mathrm{d}x}{D}\,, \tag{2}$$

where p is the pressure and D the diameter of the pipe. It is possible to recast Eq. (2) into an equation only containing the Mach number M giving

$$\frac{4c_f \mathrm{d}x}{D} = \frac{2}{\gamma M^2}(1-M^2)\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}\frac{\mathrm{d}M}{M}\,. \tag{3}$$

Equation (3) can be integrated from x_1 to x_2 and if the inlet Mach number M_1 is known the outlet Mach number M_2 at x_2 can be calculated. Further details of the calculation procedure can be found in Andersson, chapter 3.9.

An interesting aspect of this flow is that the Mach number is increasing along the downstream direction (if $M_1 < 1$). If the pipe is long enough and the pressure difference between the inlet and outlet large enough, the Mach number at the outlet will become equal to one. In that case the flow will become choked and a further increase of the pressure difference (keeping the upstream stagnation pressure constant) will not lead to any increase in the mass flow rate.

2.1 Mass flow determination

In the experiment the mass-flow rate is determined from the pressure change across a nozzle with an area ratio A_1/A_2 of about 4 which is located upstream of the pipe. By assuming that the air at the upstream (large) end of the nozzle is at stagnation conditions we can easily calculate the Mach number at A_2 from the isentropic relationship

$$\frac{p_0}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} . \tag{4}$$

We can also calculate the temperature T_2 by using the adiabatic relationship (if we know the stagnation temperature T_0)

$$\frac{T_0}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 , \tag{5}$$

and thereby obtain both the velocity u_2 from the Mach number as well as the density ρ_2 from the perfect gas law. Finally we obtain the mass flow rate as

$$\dot{m} = \rho_2 u_2 A_2 . \tag{6}$$

Using this value we can obtain a better estimate of the Mach number at A_1 and thereby iterate to obtain an even better estimate of the mass flow rate.

3 Experimental rig and test equipment

The layout of the experiment is sketched in figure 1.

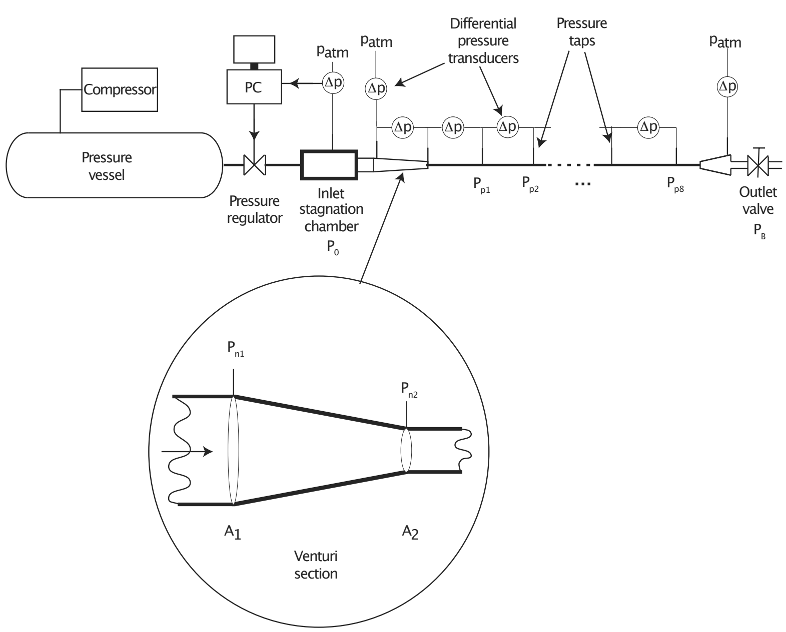

Figure 1: Pressure vessel and pipe with pressure taps

The experimental rig consists of a large pressure vessel connected to a smaller stagnation chamber with a pressure regulator in between. The air flows from the chamber into a Venturi which in turn is connected to a straight pipe. Air is supplied to the pressure vessel by a 7.35 kW compressor at the rate of 25 g/s at maximum. The pressure in the stagnation chamber is held constant by means of a computer controlled pressure regulator. The mass-flow rate is determined by the Venturi section (in this case a conical tube) mounted between the stagnation chamber and the entrance of the pipe. A valve at the outlet of the pipe regulates the back pressure.

The pressure along the pipe is determined through differential pressure transducers connected to pressure taps along the pipe. To increase the accuracy the differential pressure between neighbouring taps along the pipe is measured and this allows pressure transducers with higher sensitivity to be used. The measured pressures are then converted to absolute pressures by referencing the absolute pressure obtained at the inlet of the Venturi section, p_{n1}, using a computer programme. These transducers handle a maximum pressure difference of 100 kPa (Motorola MPX 5100DP). The pressure relative to the atmosphere is measured at the Venturi inlet and in the outlet stagnation chamber by transducers covering pressure differences up to 700 kPa (Motorola MPX 7000DP). These pressures are converted to absolute pressures by adding the value of the atmospheric pressure. The pressure taps in the Venturi are positioned at cross sections where the diameters are 19.5 mm and 10.0 mm, respectively, giving an area ratio A_1/A_2 of 3.80.

The pipe used is 2670 mm long with an inner diameter of 6.4 mm. The location of the pressure taps is given in Table 1.

Tap no.	x (mm)	x/D
1	76	12
2	760	119
3	1520	238
4	2280	356
5	2400	375
6	2530	395
7	2600	406
8	2630	411

Table 1: Pressure tap location in absolute and relative measure. The x distance is measured from the pipe inlet

The pressure taps are connected to the transducers using 2 mm inner diameter plastic tubes. Due to rather long tube lengths and small pressure holes it is necessary to wait for the pressure to settle before making a pressure registration. The transducers are electrically connected to a computer controlled 16 channel, 16 bits AD-card and the measured differential pressures are directly written to a file as well as shown on the computer screen.

At the outlet of the pipe the experimental rig is also equipped with a Pitot tube, *i.e.* a stagnation pressure probe made of a flat nosed hypodermic needle, mounted on a vertical traversing unit. Dismounting the outlet valve from the pipe the velocity profile of the emerging jet can be measured by traversing the Pitot tube through the jet. The same differential pressure transducer, capable of measuring up to 700 kPa, is connected to the stagnation probe with the atmospheric pressure as the reference pressure.

4 Home assignment

The home assignments below should be carried out in advance and brought to the laboratory session. The laboratory assistant in charge needs to approve the home assignment in order for the student to do the laboratory exercise. Fanno flow table can be used to obtain the solutions, however, it is recommended to calculate yourself for better estimations and preparation for the exercises during the laboratory session. Bring your own laptop with Matlab installed for the data analysis part during the laboratory session.

1. Calculate the inlet Mach number that is required to have a Mach number equal to one at the outlet of the pipe.

2. What inlet pressure is required to drive the flow to choked condition (*i.e.* $M_2 = 1$) if the pressure at the outlet is atmospheric (*i.e.* p_{atm}=100 kPa)?

3. Determine the theoretical pressure distribution ($p_i/p0_{inlet}$) and the Mach number distribution at the location (i) for the different pressure taps as given in Table 1, for the case where $M_2 = 1$. Write down the results in a table so that a direct comparison with the measured values is possible.

5 Experiments

Three different experimental tasks should be carried out.

1. **Determination of mass flow rate for choked conditions**
The inlet stagnation pressure, $p0_{inlet}$, is set to $\simeq$ 450 kPa and the stagnation pressure in the outlet chamber, $p0_{outlet}$ (or p_B in the data file), is adjusted by opening the outlet valve in steps, until atmospheric pressure is reached ($p0_{outlet} \simeq$ 100 kPa). During the procedure the pressure at the inlet and outlet should be registered at all steps.

2. **Pressure drop characteristics**
In this task the pressure drop as a function of distance from the pipe inlet is measured. The outlet stagnation chamber is removed and the pipe flow is run so that critical flow is obtained at the outlet. Only one inlet stagnation pressure is used. The pressure distribution along the pipe is recorded and compared to theory.

3. **Mach number distribution across the emerging jet at the pipe exit**
Using the inlet setting in task 2, the stagnation pressure distribution in the jet at the pipe outlet is determined. The Mach number distribution is sought from stagnation pressure data and compared with theory. Note that the relation between static and total pressure is different depending on whether the flow is sub- or supersonic.

6 Evaluation of data

The data obtained from 3 different experiments is analysed during the laboratory session. The data is in ASCII format (.txt), named with the corresponding experiment number (Exp 1 to 3). Each column represents different sampling. The pressures are acquired in kilopascals and the rows are organised in the order of:

p_0:	Stagnation pressure at the inlet stagnation chamber
p_{n1}, p_{n2}:	Inlet and outlet pressures at the Venturi section
$p_{p1}, \ldots, p_{p8}$:	Pressures along the pipe
p_B:	Stagnation pressure at the outlet stagnation chamber
p_{pitot}:	Pressure from the Pitot tube

1. **Determination of mass flow rate for choked conditions**
   Open the file from the first experiment, Exp 1.

   a. Plot the mass flow rate calculated from the pressures measured at the Venturi section against the back pressure, p_B. The calculation may be improved by iteration as mentioned in section 2.1.

   b. Plot pressure profile along the pipe (p_{p1} to p_{p8}) for different valve positions.

2. **Pressure drop characteristics**
   Open the file from the second experiment, Exp 2.

   a. Plot the pressures measured along the pipe, p_{p1} to p_{p8}, against their corresponding distance from the pipe inlet. Use adequate non-dimensional values.

   b. Calculate the Mach number at each pressure tap and plot them against the distance from the inlet.

   c. In the same figure created in **2a** and **2b**, plot your theoretical estimation from the home assignments. Compare and discuss your observations.

3. **Mach number distribution across the emerging jet at the pipe exit**
   Open the file from the third experiment, Exp 3. Note that the Pitot tube was traversed with equidistant steps across the pipe outlet.

   a. Plot the Mach number distribution using the isentropic ratio, p_{01}/p_1.

   b. Plot the Mach number distribution using the *Rayleigh Pitot tube formula* in the same figure. Discuss your observations. Figure 2 may aid your discussion.

$$\frac{p_{02}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)} \right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1}$$

4. **Uncertainty assessment on the experimental data**
   Overall, reflect on how well the experiment matched with the theory. Discuss possible sources of discrepancy in the experiment and the theoretical estimation.

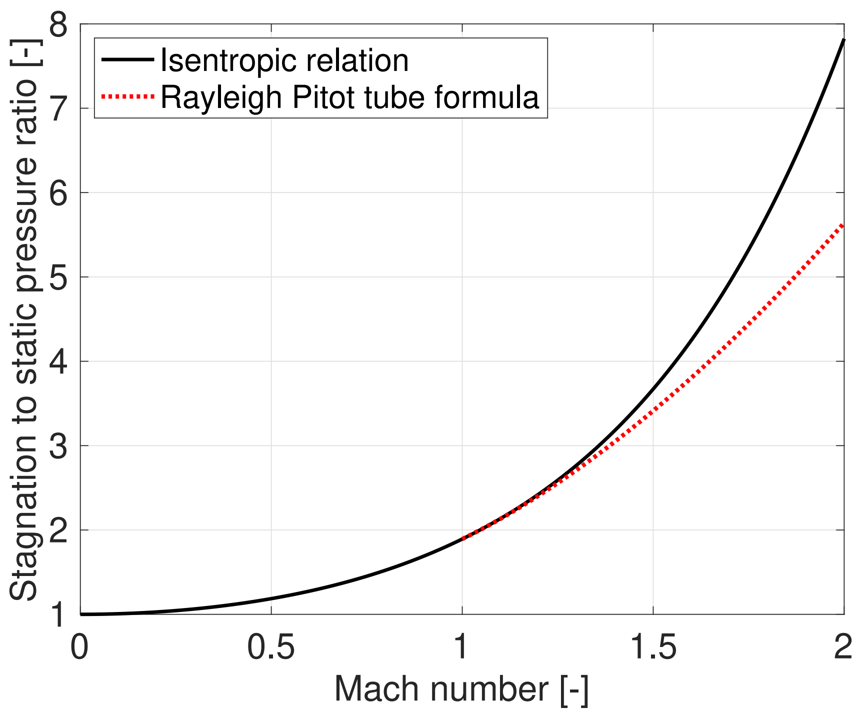

Figure 2: Isentropic relation and Rayleigh Pitot tube formula