Sorbonne University

MU5MES05 Damage Course - CompMech - 2021/2022

Numerical computation of damage : homogeneous and composite structures

Authors :
Muhammad Faizoud Din ACKBARALLY
muhammad.ackbarally@etu.sorbonne-universite.fr
Valentin Duvivier
valentin.duvivier@etu.sorbonne-universite.fr

Supervisor :
Jeremy Bleyer

February 18, 2022

Contents

I. Introduction

We consider the gradient damage model :

$$\epsilon_{tot}(\underline{u}, d) = \epsilon_{el}(\underline{u}, d) + \epsilon_f(d) = \int_\Omega \psi(\underline{\underline{\epsilon}}(\underline{u}), d)dx + \int_\Omega \frac{G_c}{l_0.c_\omega}.\left(\omega(d) + l_0^2||\underline{\nabla} d||^2\right)dx$$

with

- ϵ_{tot}=total energy in structure; ϵ_{el}=elastic energy ; ϵ_f=fracture energy
- $\psi(\underline{\underline{\epsilon}}(\underline{u})) = (a(d) + \kappa_{res}).\frac{1}{2}\underline{\underline{\epsilon}} : \mathbf{C}_0 : \underline{\underline{\epsilon}}$ the elastic energy density
- $f(d) = Y - Y_c(d) = -\partial_d\psi - Y_c(d) \leq 0$ the damage criterion

The point will be to build a model from which we will be able to study different aspects of damage theory such as dissipated energy, influence l_0 over solution, etc. We will do so for 2 types of problems : homogeneous one and composite one.

Finally, the variables of our model are :

$$a(d) = \frac{(1-d)^2}{1 + K\omega(d)} \tag{I.1}$$

$$\omega(d) = 1 - (1-d)^2 \tag{I.2}$$

with $K \in \mathbb{R}^+$ an additional parameter of the model.

II. Preliminary questions

We use this chapter as an introduction to the model with analytical links between the lectures and this practical case. We here simplify a bit the model by setting $K_{res} = 0$ for the entirety of this chapter.

II.1 Homogeneous solution

We here consider the 1D homogeneous case with $\sigma(\epsilon) = a(d)E\epsilon$ such that

- deformation's homogeneity : $\epsilon \neq \epsilon(\underline{x})$
- damage's homogeneity : $\underline{\nabla} d = \underline{0}$

1. Damage criterion

The damage criterion writes down $f(d) \leq 0$ and we reach it once $f(d) = 0$. Then from directional derivative of total energy this rewrites as:

$$f(d)\{= -\partial_d\psi - Y_c(d)\} = -\frac{a'(d)}{2}E\epsilon_c^2 - \frac{G_c}{c_\omega.l_0}\left(\omega'(d) - \cancel{2.l_0^2.\Delta d}\right) = 0$$

II.2 Critical strain

If we have initially no damage (i.e. $d = 0$), ϵ_c is the critical strain at which damage occurs in the structure. Then, in a few steps :

$$\begin{aligned}
& -\frac{1}{2}E\epsilon_c^2 \times \left[\frac{-2.(1-d).\left(1+K.\omega(d)\right)-(1-d)^2.K\omega'(d)}{\left(1+K.\omega(d)\right)^2}\right] - \frac{G_c}{c_\omega.l_0}(2.(1-d)) = 0 \\
(d=0) \rightarrow \quad & -\frac{1}{2}E\epsilon_c^2 \times \left[\frac{-2.\left(1+K.0\right)-2.(1-0).K}{\left(1+K.0\right)^2}\right] - \frac{2.G_c}{c_\omega.l_0} = 0 \\
\Rightarrow \quad & E\epsilon_c^2 \times (1+K) = \frac{2.G_c}{c_\omega.l_0} \\
\Rightarrow \quad & \epsilon_c = \left(\frac{2.G_c}{E.c_\omega.l_0}.\frac{1}{1+K}\right)^{1/2}
\end{aligned}$$

From this last expression and recalling 1D behavior law $\sigma = E\epsilon$ we deduce the expression of critic stress:

$$\sigma_c\{= E\epsilon_c\} = \left(\frac{2.E.G_c}{c_\omega.l_0}.\frac{1}{1+K}\right)^{1/2} \tag{II.1}$$

Parameter K

From II.1 we can deduce an expression for parameter K:

$$K = \frac{2.l_c}{l_0.c_\omega} - 1 \tag{II.2}$$

with $l_c = \frac{E.G_c}{\sigma_0^2}$, relation giving a length by dimensional analysis.

The relation for K holds as long as $a(d) \geq 0$ and so as long as $1 + K.\omega(d) \geq 0$. However, $\omega(d) \in [0, 1]$ and so :

$$\begin{aligned}
& 0 \leq \omega(d) \leq 1 \\
& 0 \leq K.\omega(d) \leq K \\
& 0 \leq 1 \leq 1 + K.\omega(d) \leq 1 + K
\end{aligned}$$

From II.2 we can write:

$$1 + K = \frac{2.l_c}{l_0.c_\omega} \geq 1 \quad \Rightarrow \quad \frac{l_c}{l_0} \geq \frac{c_\omega}{2}$$

which is a condition ratio l_c/l_0 must respect to be consistent with $a(d) \geq 0$.

II.3 Localized solution

We here consider an infinite 1D bar under traction σ. We look for a damage solution for d(x), respecting following properties:

- $d_{|x=\pm L} = 0$
- $d_{|x=0} = 1$
- $d'_{|x=\pm L} = 0$
- $d(x) \geq 0 \quad \forall x \in \mathbb{R}$

II.3.1 Damage criterion

Damage criterion $f(d)$ is once again deduced from minimization of total energy with respect to damage function $d(x)$. In this case, $\nabla d \neq 0$ and so:

$$\frac{d}{dh}\epsilon_{tot}(u, d + h.\tilde{d})|_{h=0} = \int_\Omega \frac{a'(d)}{2} E\epsilon^2.\tilde{d}\ dx + \frac{G_c}{c_\omega.l_0}\int_\Omega \left(\omega'(d) - 2.l_0^2\Delta d\right)\tilde{d}\ dx$$

and so by application of the fundamental lemma of calculus of variations (LFCV):

$$f(d) = -\frac{a'(d)}{2}E\epsilon^2 - \frac{G_c}{c_\omega}\left(\omega'(d) - 2.l_0^2\Delta d\right) = -\cancel{\frac{a'(d)}{2}\frac{\sigma^2}{E}} - \frac{2.G_c}{c_\omega.l_0}\left((1-d) - l_0^2\Delta d\right) = 0$$

in the case where $\sigma = 0$. Thereby we must solve linear differential equation $(1-d) - l_0^2\Delta d = 0$ or equivalently

$$l_0^2.d''(x) + d - 1 = 0$$

whose solution is

$$d(x) = A.e^{ix/l_0} + B.e^{-ix/l_0} + 1 \equiv \alpha.cos(x/l_0) + \beta.sin(x/l_0) + 1 \quad (\alpha,\beta) \in \mathbb{R}$$

Applying our BCs, one gets :

$$d(x=0) = \alpha + 1 = 1 \quad \rightarrow \quad \alpha = 0 \tag{II.3}$$

$$d'(x=L) = \frac{1}{l_0}\beta.cos(L/l_0) = 0 \quad \Rightarrow \quad \frac{L}{l_0} = (1/2+n).\pi,\ \ n \in \mathbb{N} \tag{II.4}$$

$$d(x=L) = \beta.sin((1/2+n).\pi) + 1 = 0 \quad \Rightarrow \quad \beta = (-1)^{1-n} \tag{II.5}$$

We observe that to have the aimed damage criterion (i.e. $d(x) > 0$ and a localized damage), we must ensure a local maximum for d in $x = 0$ and so we must have $n = 0$ in above relation. Subsequently, we have for the damage variable:

$$d(x) = 1 - sin\left(\frac{|x|}{l_0}\right) \quad \forall x \in [-L, L] \tag{II.6}$$

with absolute value translating symmetry with respect to $x = 0$ position.

Constant c_ω

With the expression of d(x), we can compute the value of c_ω constant. Indeed, by assuming "dissipated energy" $\epsilon_f(d)$ we can write down $\epsilon_f(d) = G_c.b.l(\alpha)$, and we are left with an equality such that:

$$G_c.S = \frac{G_c}{c_\omega.l_0}\int_\Omega \left(\omega(d) + l_0^2.||\nabla d||^2\right)\ dV \quad \Rightarrow \quad G_c.S = \frac{G_c.S}{c_\omega.l_0}\int_{-L}^{+L} \left(\omega(d) + l_0^2.||\nabla d||^2\right)\ dx$$

$$(symmetry) \Rightarrow \qquad c_\omega = \frac{2}{l_0}\int_0^{+L} \left(1 - (1-d)^2 + l_0^2.d'(x)^2\right)\ dx$$

$$c_\omega = \frac{2}{l_0}\int_0^{+L} \left(1 - \left(1 - 1 + sin\left(\frac{|x|}{l_0}\right)\right)^2 + l_0^2.d'(x)^2\right)\ dx$$

$$c_\omega = \frac{2}{l_0}\int_0^{+L} \left(1 - sin^2\left(\frac{|x|}{l_0}\right) + \frac{l_0^2}{l_0^2}.cos^2\left(\frac{|x|}{l_0}\right)\right)\ dx$$

$$c_\omega = \frac{4}{l_0}\int_0^{+L} cos^2\left(\frac{|x|}{l_0}\right)\ dx = \frac{4}{l_0}.\left[\frac{x + \frac{l_0}{2}.sin\left(\frac{2|x|}{l_0}\right)}{2}\right]_0^L$$

$$\left(\frac{L}{l_0} = \frac{\pi}{2}\right) \quad \Rightarrow \quad c_\omega = 2.\left[\frac{L}{l_0} + 0 - 0\right] = \pi$$

With the model now complete, let's implement it numerically to see what it is worth.

III. Numerical validation

We do the numerical implementation of the model under label 'DM'. To do so, we specify a bunch of variables such as constant c_ω, $\omega(d)$, $d(x)$, etc.

Domain : 2D domain $L \times W = [0,1] \times [0,0.1]$

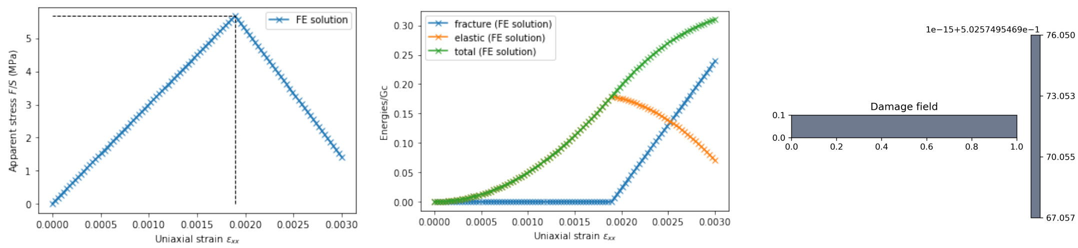
Material : elasto-plastic system that can endure elasticity, hardening and break. We therefore consider a model with possible damage under exterior stimulation.

Forces/Hyp. : we consider plane stress hypothesis as well as a damage from apparition to break of the structure.

As mentioned in the introduction, we consider traction at the rhs. We furthermore consider embedded condition to the lhs ($U_x = 0$).

III.1 Homogeneous solution - no refinement

We starts by doing our simulation for a homogeneous solution by implementing the new damage law for the case **DM**. We run our code for the value of $l_0 = 0.1$m and we obtain the following results:



(a) Stress strain curve (b) Energy variation (c) Damage field

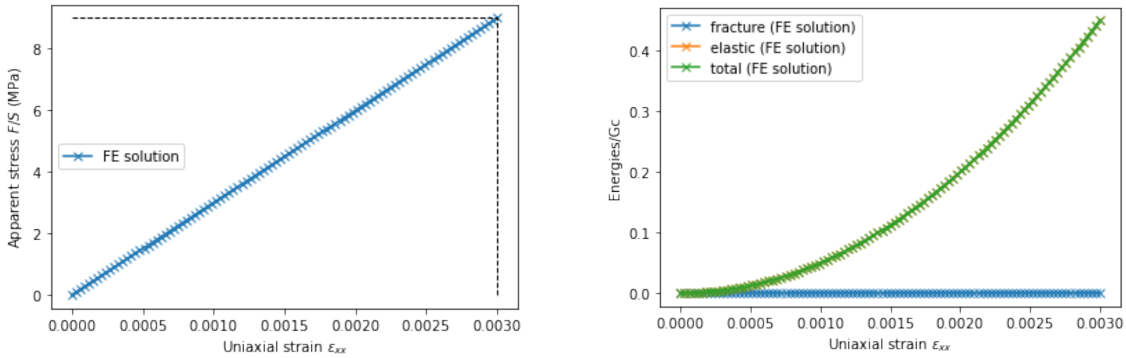
Figure III.1: Numerical simulation results for $l_0 = 0.1$

We observe from figure III.1 that system is linear at first and once we reach characteristic value ϵ_c the system is damaged. From an energetic point of view, we observe an increase in the fracture energy and decrease in the elastic energy from this point. This is because the elastic energy is transferred to fracture energy as the beam is damaged (energy balance principle). For the damage field, a uniform deformation field is observed throughout the beam.

Let's now see how parameters σ_0 and l_0 influence each other.

Influence of l_0 on σ

We test several values of l_0 to see how it affects the maximum stress and the $stress - strain$ curve:



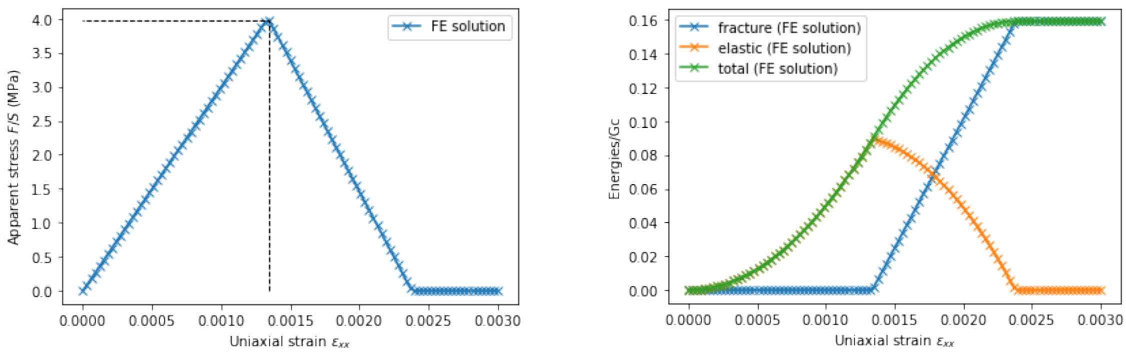
(a) Stress strain curve

(b) Energy variation

Figure III.2: Numerical simulation results for $l_0 = 0.01$m

On figure III.2 we can observe that the beam is never damaged, which translates into a zero fracture energy. Then, for a crack low with respect to domain's length, we observe no apparition nor propagation of damage.

If one increases l_0 from 0.01m to 0.2m, we get:



(a) Stress strain curve

(b) Energy variation

Figure III.3: Numerical simulation results for $l_0 = 0.2$m

This time, figure III.3 tells us that system is linear at first and once we reach characteristic value ϵ_c the system is damaged until fracture occurs. By changing l_0 we then observe damage in the structure.

From an energetic point of view, we conclude on damage of the structure as fracture energy increases, until it remains constant once break of structure occurred.

By testing several values of l_0 it finally comes:

l_0	σ_{max}
0.2	≈ 4.0
0.1	≈ 6.0
0.01	≈ 9.0

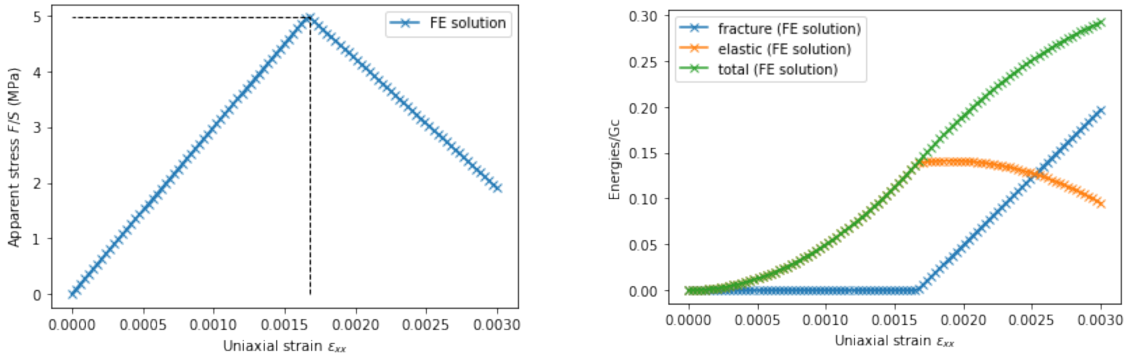
Table III.1: σ_{max} for different values of l_0

Table III.1 tells us that as l_0 decreases the value of σ_{max} increases.

Physically, we in fact observe a singularity whose stress is locally infinite. Therefore, the lower l_0, the bigger the stress. This tells us that homogeneous approach may not be a very physical assumption for the structure.

III.1.1 Influence of σ_0

We now fix $l_0 = 0.1$m and vary σ_0 (damage limit, to differentiate from break limit σ_c) and look for how the deformation changes. From part I we have $\sigma_c = \sqrt{\frac{2.E.G_c}{c_w.l_0}} \approx 7.56$Pa (we take $E = 3e^3$Pa, $\nu = 0.3$, $G_c = 3e^{-3}$Pa.m, $l_0 = 0.1$m, $c_w = \pi$). We consider these values as reference in order to observe what happens when we are below the critical value and above:



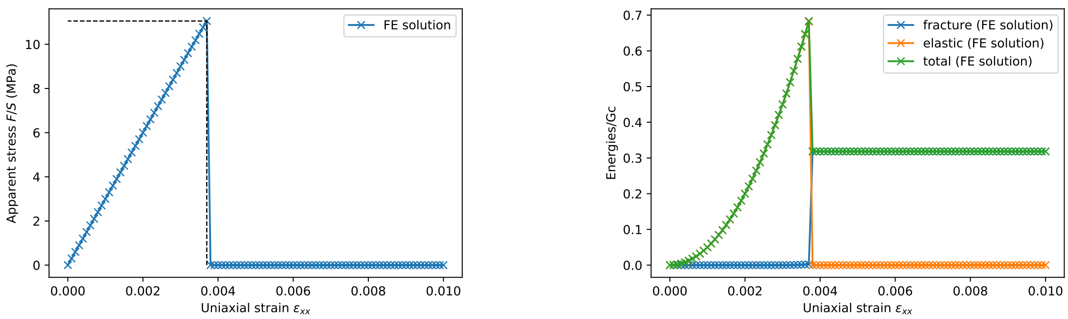
(a) Stress strain curve

(b) Energy variation

Figure III.4: Numerical simulation results for $\sigma_0 = 5\ Pa < \sigma_c$

As we can observe from fig III.4, with a fixed value of l_0 and when we are below the critical value σ_c, the system is linear. Then, we reach characteristic value ϵ_0 at which system is first damaged. We don't observe break here as we don't allow a high enough displacement to the model. From an energetic point of view, we once again observe an increase in the fracture energy and decrease in the elastic energy during damage phase.

We now change the value of σ to $\sigma_0 = 9$Pa so above the critical value ; we obtain the following numerical results:



(a) Stress strain curve

(b) Energy variation

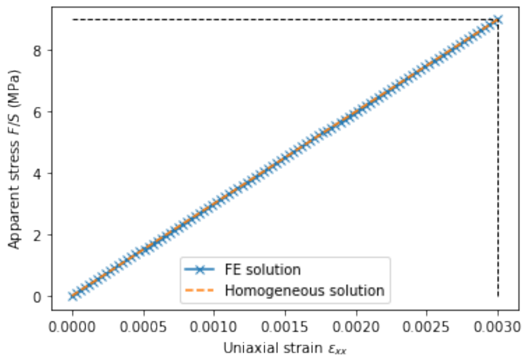
Figure III.5: Numerical simulation results for $\sigma_0 = 9\ Pa > \sigma_c$. We increased U_{max} from $3e^{-3}\ m$ to $1e^{-2}\ m$ in order to observe crack occurring.

The value of σ_c has been calculated analytically in first part. By stating $\sigma_0 = 9$Pa we tell to the system we damage from $\sigma_0 = 9$Pa, but we break will take place at $\sigma_c \approx 7.56$Pa provided we allow displacement high enough. Therefore, by stating $\sigma_0 > \sigma_c$Pa we kind off out-rule the model by changing the value at which crack occurs. One can observe on figure III.5 that by increasing displacement we indeed observe a new break limit (which equals $\sigma_0 = 9$Pa then).

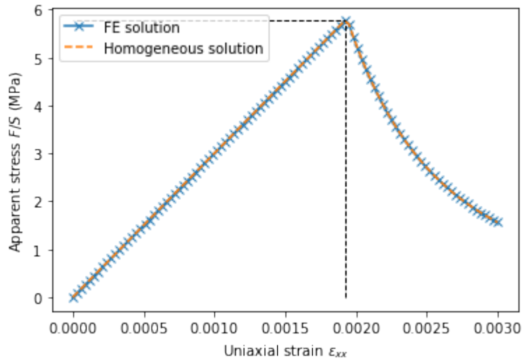
We here see it's important to understand the physics of the problem in order to efficiently compute damage theory. Let's now see how our model compares to AT1 model.

III.2 Comparison with AT1 model

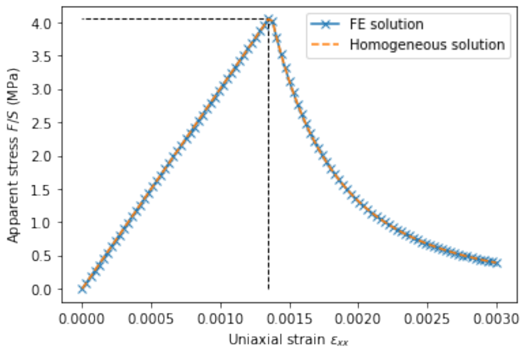
What differentiate both models are $a(d)$ and $w(d)$, and so c_ω. Through these terms, what changes in our model is the formula of f(d) and so d function isn't the same neither. Switching to AT1 model and varying l_0 for same values $l_0 = 0.01, 0.1, 0.2$m, we obtain the following numerical results:



(a) Stress strain curve for $l_0 = 0.01$m



(b) Stress strain curve for $l_0 = 0.1$m



(c) Stress strain curve for $l_0 = 0.2$m

Figure III.6: Numerical simulation results for AT1

Here again the system is linear at first and is damaged once we reach value ϵ_c. Besides, as one increases l_0 we once again have damage occurring at lower strain.

However, as opposed to the model DM, the damage part here is nonlinear. Moreover, comparing case $l_0 = 0.2$ with one of fig III.3, we observe that both models don't share same crack limit.

In addition to this, both models present the issue of singular stress at break, such that one should consider heterogeneous theory, observable using more elements to discretize structure.

III.2.1 Refinement level

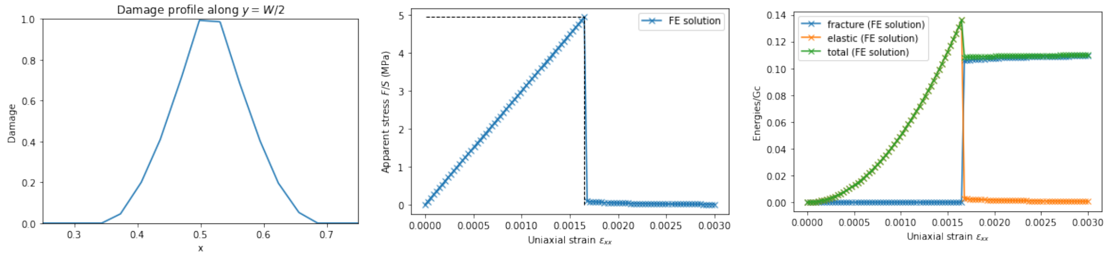
We will now study how the refinement level affects our numerical solution. We once again fix $l_0 = 0.1\ m$ and $\sigma_0 = 5\ Pa$ with $damage - bcs = True$ and we run our simulation for refinement level varying from 4 to 6. The objective of this part is to make sure our numerical computation of the model brings consistent results with respect to analytical solution.

More specifically, we want to recover:

- Dissipated energy: $\epsilon_f = G_c.S$;
- Critical stress $\sigma_0 = 5\ Pa$;
- Crack length $2L = c_\omega.l_0 \approx 0.32\ m$

We moreover verify that we ensure $a(d) > 0$ as analytically $l_0 \approx 0.16 \leq \frac{2l_c}{c_\omega} \approx 0.23$.

From calculation we get: $\epsilon_{cr} = \sqrt{\frac{2G_c}{E(1+K)l_0 c_w}} \approx 1.66e^{-3}$, hence, $\sigma_{cr}\{= \sqrt{\frac{2G_c E}{(1+K)l_0 c_w}}\} = E\epsilon_{cr} \approx 3 \times 1.66 \approx 5.00$:

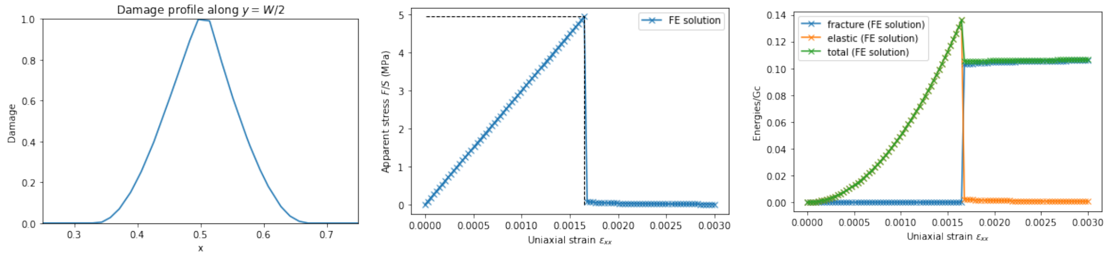


(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure III.7: Numerical simulation results for refinement level 4, $l_0 = 0.1\ m$ and $\sigma_0 = 5Pa$.

From our numerical simulation we obtain $\sigma_{max} = 4.9500$, $\epsilon_{max} = 0.0016$ and $G_c = 0.1094$ which are consistent with our analytical values. On fig III.7, from the damage profile we can observe that we get $2L_{num} \approx 0.35$ which is close to the analytical value of $2L_{analytic} \approx 0.32$.

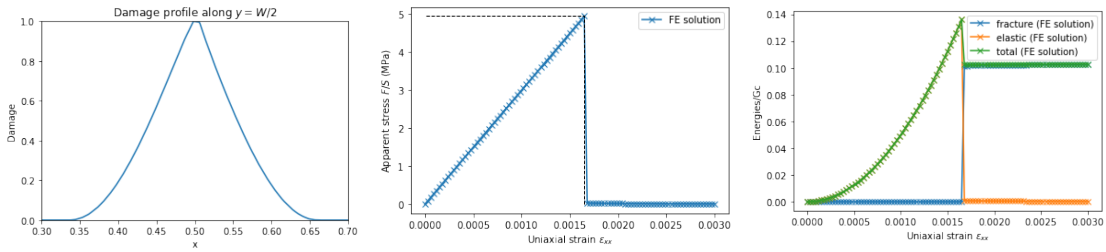
We however see a difference with damage profile. Indeed, the code couldn't reproduce theory identically and thus we have a Gaussian whose top is at 1 not only in one point but on a few points (non-local). Let's see if to refine furthermore may ameliorate this result:



(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure III.8: Numerical simulation results for refinement level 5

When we change the refinement level we can observe on fig III.8 from the damage profile that the value of $2L_{num}$ now equals $\approx 0.32\ m$. The numerical solution approaches the analytical one:



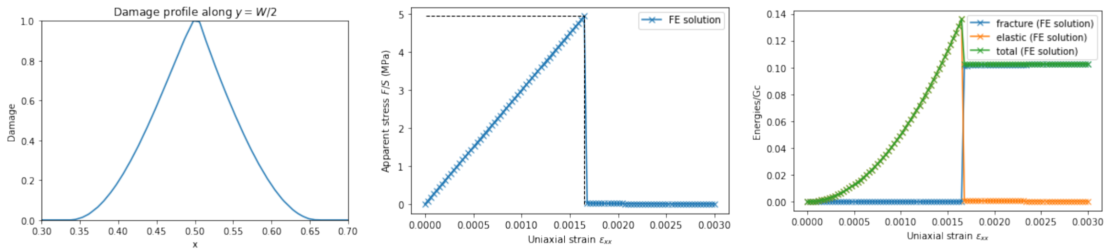
(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure III.9: Numerical simulation results for refinement level 6

In this last figure, the case when the refinement level is now fixed at 6, we have from the damage profile $2L_{num} \approx 0.32$. We also have $\sigma_{max} = 4.9500$, $\epsilon_{max} = 0.0016$ and the value of $G_c = 0.1026$. Hence we can say that the higher the refinement level, the more the numerical results converges to the analytical results.

III.2.2 Influence l_0

We fix the value of σ_0 to 5 Pa and the refinement level at 6. Analytically, we calculate the value of $l_c = \frac{EG_c}{\sigma_0^2} = 0.36$. We then have the condition on l_0 such that $l_0 \leq \frac{2l_c}{c_w} \approx 0.23$. Let's study what happens when l_0 is below or above this criterion:

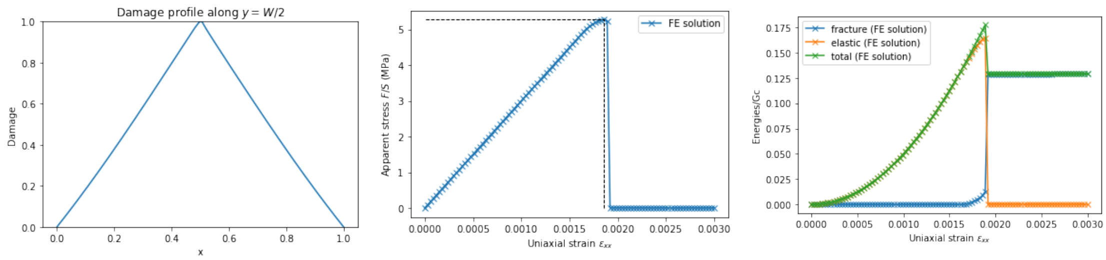


(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure III.10: Numerical simulation results for $l_0 = 0.1 < 0.23$

From our analytical solution we know that $\frac{G_c S}{l_0 c_w}(2L) = G_c S$. Thereby $2L = l_0 c_w$. Considering $c_w = \pi$ and $l_0 = 0.1$ we found $2L \approx 0.3$.

After calculation we deduced $\sigma_{cr} \approx 5.00\ Pa$. As we can observe from fig III.10, we have from the damage profile $2L_{num} \approx 0.3$. We also have $\sigma_{max} = 4.9500$, $\epsilon_{max} = 0.0016$. Our numerical results then are coherent with the analytical ones when we respect the criteria of $l_0 \leq \frac{2l_c}{c_w} \approx 0.23$. If one now considers $l_0 = 0.8\ m > 0.23\ m$:



(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure III.11: Numerical simulation results for $l_0 = 0.8\ m > 0.23\ m$

As we can observe on fig III.11, $2L_{num} = 1.0$. If we calculate the analytical value of 2L for $l_0 = 0.8$, we have $2L_{ana} = l_0 c_w \approx 2.5$. As we can observe the numerical solution is no longer coherent with the analytical one. Once again we highlight that the model is nothing if don't compute it carefully.

III.3 Conclusion

The model DM, unlike the AT1 and AT2 models, has a linear damage phase. For the AT2 model, we have a linear phase, then a non linear strain hardening phase, and a linear damage phase, then fracture. For the AT1 model, we have a linear phase at first but the damaged phase is non linear.

The DM model has its limits because once we reach a length l_0 large enough, we have more damage. Then these model describe different physical behaviors. Together with their high dependence on parameters, it's up to the user to compare when possible experience or analytical results to numerical results in order to make computation as close as possible to physical problem.

Considering the linear damage phase of DM model, it's most likely it's a theoretical model that would encounter less physical application than AT1 and AT2 models.

IV. Composite structure

In this chapter we consider a composite structure made of 3 layers from 2 different materials:

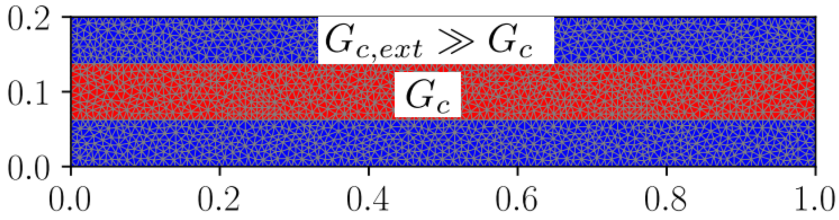


Figure IV.1: Composite structure.

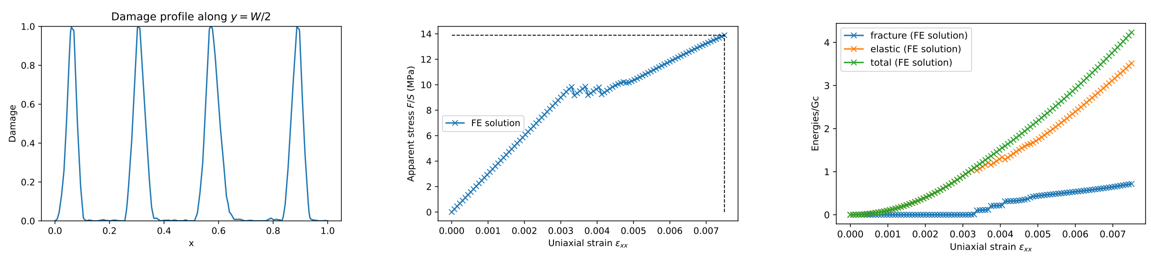
In our model, we would like to consider that only center layer may present break (ensuring good mathematical framework with break inside). To ensure this physically, we consider a high toughness for outer material, namely $G_{c,ext} \gg G_c$ with G_c the toughness of inner material.

In the code, we change some of the variables to adapt to this new problem (be able to observe crack mostly):

- $U_{max} = 7.5e^{-3}\ m$;
- $\sigma_0 = 10\ Pa$

IV.1 Damage field

In this first section we look at how damage field evolves in the no-refinement case with center thickness at $0.1\ m$ and $l_0 = 0.02\ m$:



(a) Damage profile (b) Stress strain curve (c) Energy variation

Figure IV.2: Damage field for center thickness at $0.1\ m$ and $l_0 = 0.02\ m$

In this first figure it appears the damage behavior of composite structure. We observe that damage is occurring in different locations, but still at characteristic stress $\sigma_0 = 10Pa$. This implies the limit we set is below the one at which structure breaks (i.e. $\sigma_0 = 10\ Pa < \sigma_c$).

We see that structure breaks once we reach critical stress ϵ_{cr}, yet we still have strain-hardening after that. Then once a part of the bar is damaged, the rest of the bar is still under rhs imposed displacement and so structure is furthermore damaged.

When these breaks take place, stress/strain curve presents a fall confirming the fact we then damage. The subsequent increasing stress as a function of uniaxial stress as well confirms the fact we continue damaging until entire structure is damaged (i.e. break then).

Looking to the damage field this translates in localised damage criterion reached at 4 locations.

IV.2 Dissipated energy

By post processing *results-data.csv* we extract data to display the energy of fracture as a function of imposed displacement at rhs:

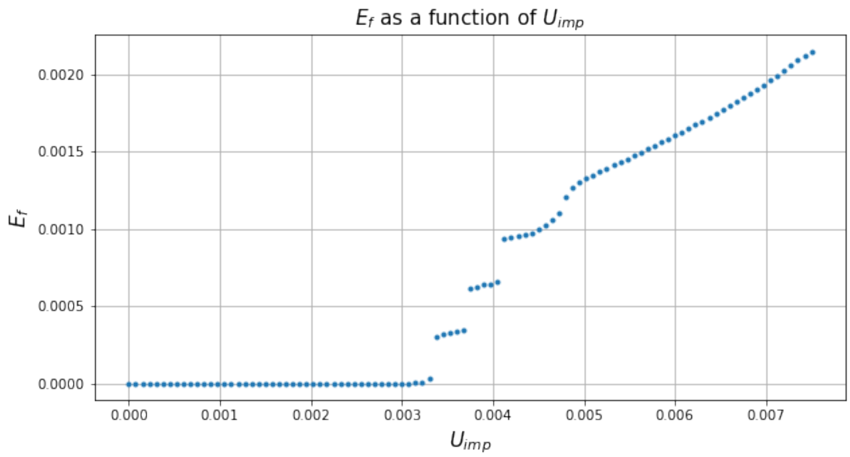


Figure IV.3: $\epsilon_f = f(U_{imp})$ for $l_0 = 0.02\ m$, $center - thickness = 0.1\ m$ and no refinement (2 elements). The post-process from python uses same data than figure IV.2.

We confirm that we have a step by step behavior, as one can see in figure IV.2 c). Figure IV.2 tells us that every time the inner structure is damaged (here a vertical crack), we have a jump in dissipated energy. It then breaks in an instant (i.e. for an infinitesimal value of displacement) and results in sudden raise of dissipated energy.

However, for the last crack we observe almost continuity in figure IV.2 which **translates** one **didn't need same energy than for previous cracks**. In the next section we will check if it's the low refinement that caused the issue ; and so we now **refine mesh**.

IV.3 Dissipated energy - refined

We refine up to $refinement - level = 1$ and get following graph:

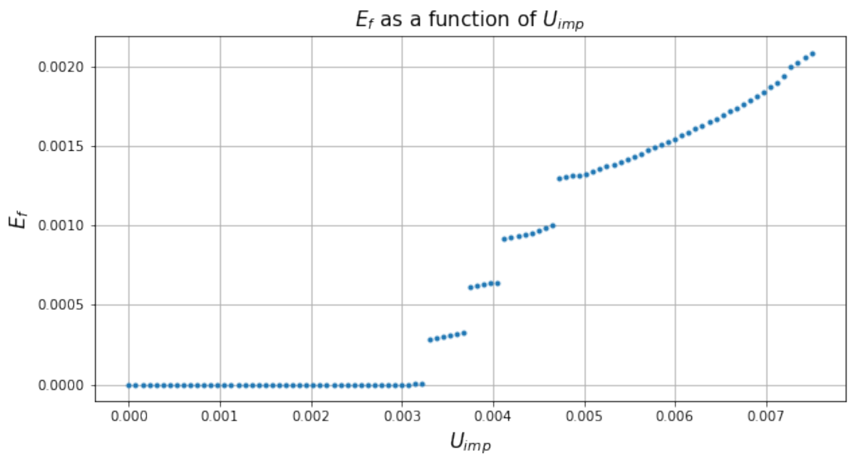


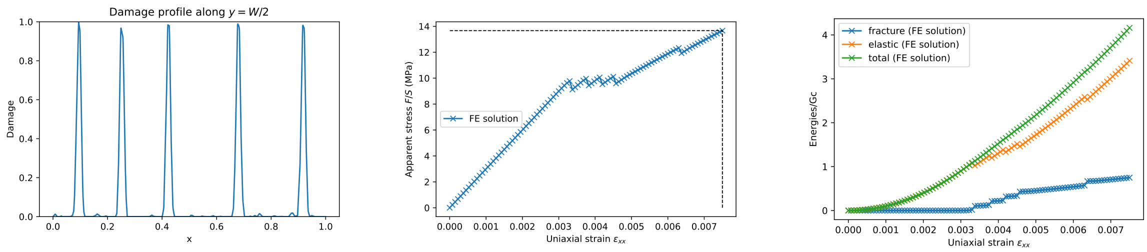
Figure IV.4: $\epsilon_f = f(U_{imp})$ for $l_0 = 0.02\ m$, $center - thickness = 0.1\ m$ and $refinement = 1$ (4 elements).

Now that we refined mesh, and for same variables, we have indeed a jump as well for last crack. This confirms the structure isn't deteriorated, namely $d = 0$ or $d = 1$ only. Eventually we conclude on the fact we model a system that presents instantaneous break, a result one can observe on several of the displayed graphs so far.

Now that we know refinement brings more consistency with respect to aimed modelisation. What we would like to know is the influence of other parameters such as l_0.

IV.4 Lower l_0

We now want to see what effects it has to lower the thickness of crack l_0. To see its effect, we remain at a refinement of 1 and look at a same graphs than figure IV.2:



(a) Damage profile (b) Stress strain curve (c) Energy variation

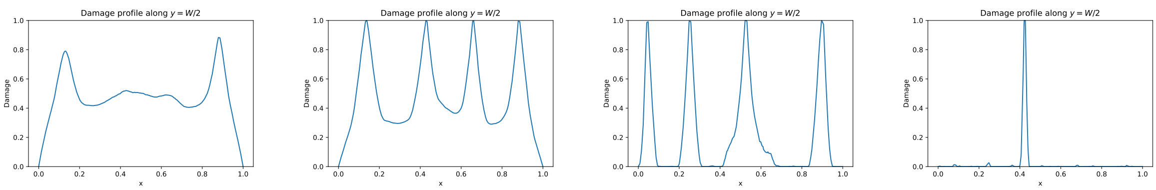
Figure IV.5: Damage field for $center - thickness = 0.1\ m$, $l_0 = 0.01\ m$ and a refinement of 1 (4 elements)

We observe from figure IV.5 a) that damage is more localized, which was expected by reducing crack's thickness (theory:$l_0 \to 0$). On the other hand, by reducing thickness of the cracks, a fifth crack appeared. More importantly it affects the cracks positions.

Therefore, for one to reduce l_0 implies major changes in term of damage state. This case is supposedly closer to theoretical case where $l_0 \to 0$. Let's see how well does AT1 model do with composite structure.

IV.5 AT1 model - l_0 influence

In another case, we look at how AT1 model compares with DM model in the study of composite structure's damage. To do so we consider both the damage field and the graph strain/Uimp to see how l_0 values influence the results :



(a) $l_0 = 0.1\ m$ (b) $l_0 = 0.05\ m$ (c) $l_0 = 0.02\ m$ (d) $l_0 = 0.01\ m$

Figure IV.6: Damage field for $l_0 = 0.1; 0.05; 0.02; 0.01\ m$ and a refinement of 1 (4 elements), and $U_{max} = 7.5.e^{-3}\ m$

On figure IV.6 we observe an evolution of AT1 results as a function of l_0 value. It then comes that once again as one reduced thickness l_0, the model presents more breaks. We conclude at this stage on the fact that the lower the thickness of a crack, the better we model damage of the system (then number of cracks should be finite for $l_0 \to 0$).

Eventually, what is observable on figure IV.6 d) is that ultimately a unique crack is in the structure. In fact, as we considered a very low thickness, the crack here takes place in longitudinal direction:

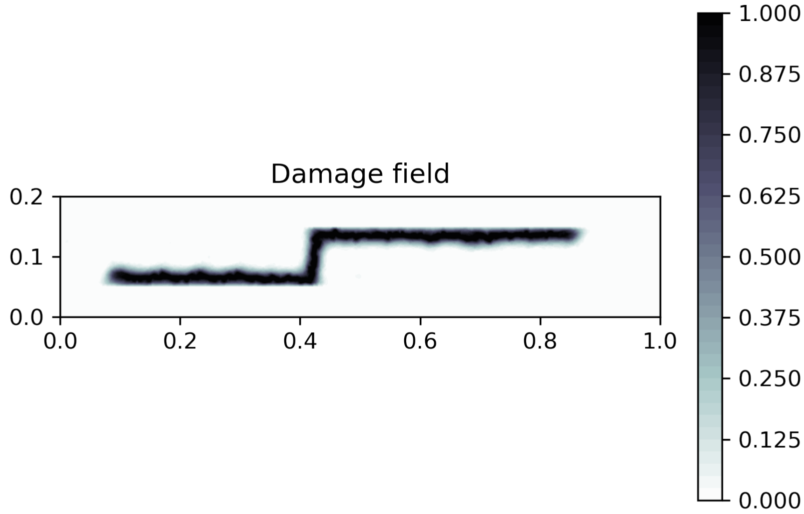


Figure IV.7: Damage profile for $l_0 = 0.01\ m$, $center - thickness = 0.1\ m$ and $refinement = 1$ (4 elements). Displacement is imposed at right hand side.

In fact, the crack first occurs at center of the structure, as initialized, but then propagates at interface between two materials where directional energy looks to be minimized.

In terms of $E_f/Uimp$ curve:

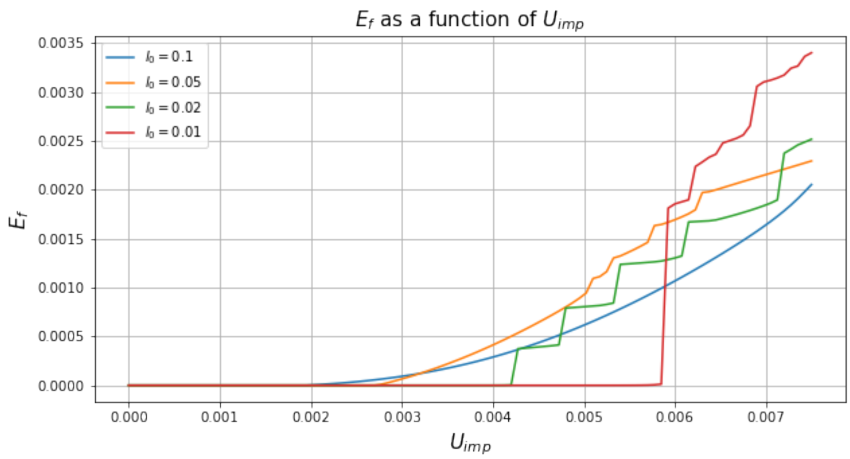


Figure IV.8: Damage profile for $l_0 = 0.01\ m$, $center - thickness = 0.1\ m$ and $refinement = 1$ (4 elements). Displacement is imposed at right hand side and fix at lhs.

This figure IV.8 has for purpose to better understand the dynamic of the crack : how much loading steps ? instantaneous crack ? etc.

We here observe that depending on the value of l_0 we have different crack behaviors and conditions. Then, when l_0 low enough we observe longitudinal crack occurring while it's transverse for higher values.

In details:

- $l_0 = 0.10\ m$: we have 2 damages but whose value doesn't go to 1 (i.e. we damage but didn't break yet). No matter how much crack in the structure, stress keeps increasing as long as there is material to damage (always the case here given composition of outer material) ;
- $l_0 = 0.05\ m$: this time we reach break criterion, here associated to transverse crack ;
- $l_0 = 0.02\ m$: it looks like the lower l_0 the closer to DM model we get. In this third case we have quasi-localized damage ;
- $l_0 = 0.01\ m$: this last case highlights the fact our model comes with different physical applications and observations, namely different crack behaviors.

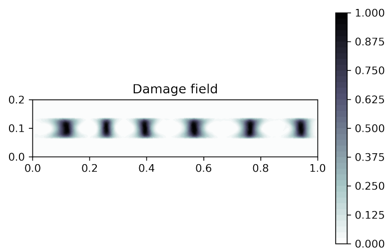
To conclude on l_0 influence, to lower it brings results closer to the aimed local damage behavior, for both tested models. Eventually, the case of "longitudinal break is a real physical behavior observed under certain conditions.

DM and AT1 models can then both be used for the study of crack apparition, respecting initial values (e.g. damage at σ_0) and physical behavior (e.g. one must provide energy for each crack).

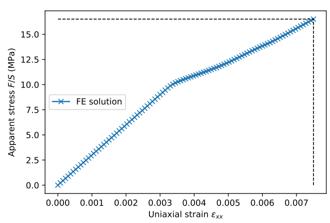
Let's now finish our study by testing a different composition of the composite structure, namely change the proportion of inner/outer material.

IV.6 Center-thickness

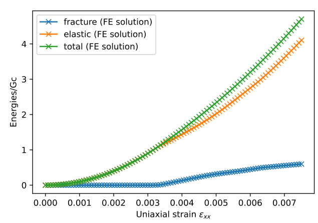
In this last section we test the influence of composite structure's proportion on the apparition and propagation of crack. We consider $l_0 = 0.02\ m$ with a refinement of 1 in order to remain consistent with above computations and observations:



(a) Damage profile

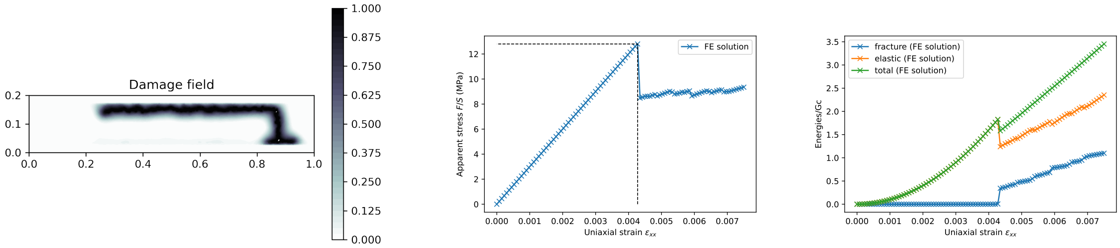


(b) Stress strain curve



(c) Energy variation

Figure IV.9: Graphs for center-thickness of $0.075\ m$ for inner material at $l_0 = 0.02\ m$ and a refinement of 1 (4 elements), and $U_{max} = 7.5.e^{-3}\ m$. a) damage profile associated to damage model DM. b) stress/strain behavior as one applies displacement at rhs. c) energy variations during damage process.



(b) Stress strain curve

(c) Energy variation

(a) Damage profile

Figure IV.10: Graphs for center-thickness of 0.15 m for inner material at $l_0 = 0.02$ m and a refinement of 1 (4 elements), and $U_{max} = 7.5.e^{-3}$ m. a) damage profile associated to damage model DM. b) stress/strain behavior as one applies displacement at rhs. c) energy variations during damage process.

These two figures aim at understanding how does the system evolve as a function of proportion ductile/brittle materials. In fact, we'd like to see if a certain path can be associated to either of the two cases.

On figure IV.9 a) the damage field tells us that break develops along vertical direction. Therefore, when the proportion of brittle material is weak the break occurs in several places of the structure (depending on how much we displace) and so on as one charges until break.

On the contrary, if one considers a thicker inner material, namely a bigger proportion of brittle material, then the direction of crack isn't quiet the same. Indeed, we observe on figure IV.10 that crack first occurs to the rhs and expands to the left. We moreover note that crack propagates and develops again at the interface between the two materials.

Conclusion

During this report we went from an analytical approach to numerical applications and modelisation of a physical subject : composite structures. We have then been able to judge the model DM looking at analytical reference, at the origin of the model, and by testing the dependency of damage response to refinement, l_0 value, etc.

It resulted that a model must be understood physically before one derives conclusion on it. We furthermore observed that both DM and AT1 models had their specificities and could model different physical behaviors.

Eventually, the model we implemented responded well in the tested frames, and it would be a good expansion to this numerical study than to have some experimental data to compare.

In a more personal conclusions, the last application to composite structures has been the opportunity to have some insight on a real application case as well as the post process one must undergo to analyse results.