

SORBONNE UNIVERSITÉ

STAGE M1 - COMPMech - INSTITUT JEAN LE ROND

d'Alembert

Fundamental study of the dynamical stabilisation of an inverted pendulum, an experimental and theoretical approach

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1. Abstract

My name is DUVIVIER Valentin and as part of my master Computational Mechanics, I am part of the internship "*Fundamental study of the dynamical stabilisation of an inverted pendulum (IP), an experimental and theoretical approach*". It takes place amongst d'Alembert laboratory, directed by LAGREE Pierre Yves.

My direct supervisors are Professor LAZARUS Arnaud and Researcher PROTIERE Suzie, as well as the PhD student A. GRANDI Alvaro.

2. Introduction

The characterisation of fundamental mechanical models is an entry point in the world of sciences. In this context, simple pendulums allow one to study oscillating behavior as well as systems' stability.

In this paper we take a slightly more complex approach which consists into stabilizing the naturally unstable upside-down state of a simple pendulum. To do so, we are to use an electro-magnet which will create a magnetic field opposed to the gravity one, and keep a punctual mass in the vertical equilibrium.

Doing so, we force the pendulum to hold by use of a certain amount of energy. One may then want to know if other stable states exist like dynamical ones. The point would then be to create stability on a naturally unstable system and to then try to minimize the energy one needs to do so.

Trough the use of the electro-magnet we make possible to periodically turn ON & OFF the magnetic field and we then offer to the system the possibility to have dynamical stable equilibrium. The process as this relies on the principle of resonance and anti-resonance.

A result of this and the overall objective of this paper is to derive a model that explains how to dynamically stabilize the upside-down pendulum and to observe it experimentally.

We firstly aim to make sure the magnetic field reproduces well the physical expectation of field's homogeneity.

Next, we will separately study the behavior when the electro-magnet is ON & OFF. A model will be deduced from it and will serve as a basis for our next studies.

Then, we will tackle the oscillation aspect of the IP (namely when we periodically turn ON & OFF the electro-magnet) through experimental measures and analytical calculations regarding IP's behavior in function of (T_{ON}, T_{OFF}) .

Afterwards, we will extend our first model to the general case and try to derive a general formula on dynamical stability.

We will conclude by summarizing the results made on dynamical stability and therefore energy minimisation.