Test 26/10/2012

1 Statique

1. $$\begin{cases} \dfrac{\partial p}{\partial r} = \omega^2 r \\ \dfrac{1}{r}\dfrac{\partial p}{\partial \theta} = 0 \quad \Rightarrow \quad p \neq f(\theta) \\ \dfrac{\partial p}{\partial z} = -g \end{cases}$$

2. $p = f(r, z)$

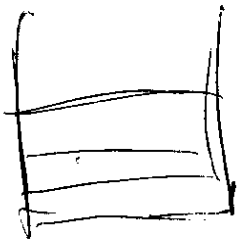
$$p = \omega^2 \frac{r^2}{2} - gz + Cte.$$

isobare $P = cte \Rightarrow \dfrac{\omega^2 r^2}{2} - gz = C.$

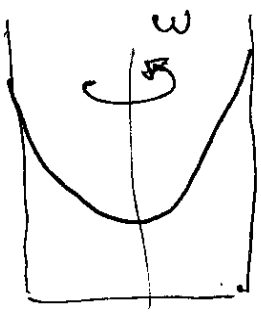
$$\boxed{z = \frac{\omega^2}{2g} r^2 + C}$$

3 ω petit $\quad p \simeq -gz$

$\Rightarrow$ isobares $z = cte$



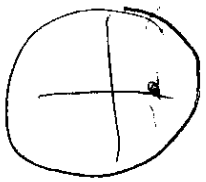
ω grand



$x_0 = A\cos(\varphi)$ $\quad x = f(x_0, y_0, t)$

$y_0 = A\sin(\varphi)$ $\quad y = f(x_0, y_0, t)$

4 $\quad x^2 + y^2 = A^2(\cos^2 + \sin^2) = A^2$



Tournoi – ext.

1. même qu'avant

$$u_e = -\frac{B y}{x^2+y^2}\,\bar{e}_x + \frac{B x}{x^2+y^2}\,\bar{e}_y$$

2 $\quad \mathrm{div}(u_e) = -\frac{w y^2 x}{(x^2+y^2)^{3/2}} + \frac{w x^2 y}{(x^2+y^2)^{3/2}} = 0$

$$\mathrm{rot}(u_e) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$= \frac{B}{x^2+y^2} - \frac{B x^2 2}{(x^2+y^2)^2} + \frac{B}{(x^2+y^2)} - \frac{B y^2}{} = 0$$

3. $\quad \frac{\partial \psi}{\partial x} = -v \qquad \frac{\partial \psi}{\partial y} = u$ $\quad\Big|\quad \partial\psi = -\frac{B y}{x^2+y^2}\,dy$

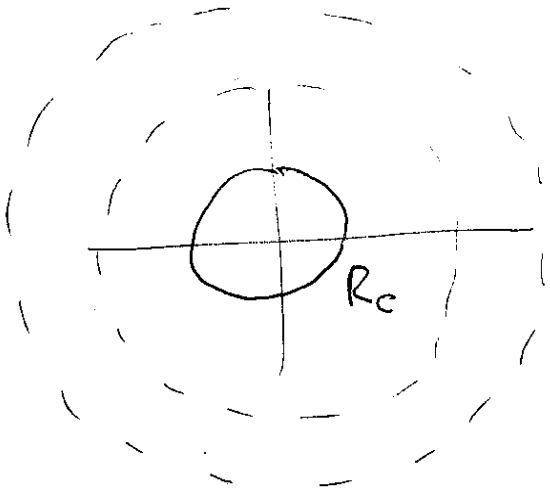
$u = y^2$
$du = 2y\,dy$

$\partial\psi = -\frac{1}{2}\frac{B\,du}{x^2+u} \Rightarrow \psi = -\frac{1}{2} B \ln(x^2+y^2)$

$\Big|\ \psi = -B \ln(x^2+y^2)^{1/2}$

en polaires

$$\frac{\partial \psi}{\partial r} = -v_\theta = -\frac{B}{r} \qquad \boxed{\psi = -B \ln(r)}$$



$\psi = cte$

$\Rightarrow r = cte$

avec $r \geq R_c$

4

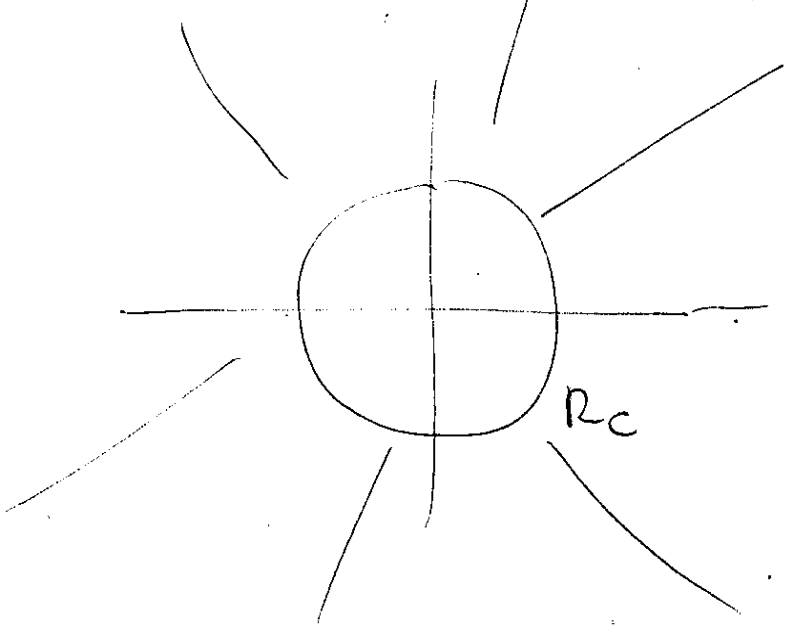
$$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = v_\theta = -\frac{B}{r}$$

$$\Rightarrow \frac{\partial \phi}{\partial \theta} = -B \qquad \phi = -B\theta$$

$\phi = cte \Rightarrow \phi = cte$

$r \geq R_c$

des droites



$$\Gamma = \int_L \vec{\pi} \cdot \vec{n} \, dL \qquad \vec{\pi} = \frac{B}{\Gamma} \vec{e_\theta}$$

$$\vec{n} = \vec{e_\theta}$$

$$= \int_0^{2\pi} \frac{B}{\Gamma} \, \Gamma \, d\theta$$

$$\boxed{\Gamma = 2\pi B}$$

Conclusion : indépendant du rayon choisi.