Sorbonne University

M2 Damage Course - CompMech - 2021/2022

Aspects numériques des modèles d'endommagement locaux

Author:
Valentin Duvivier
valentin.duvivier@etu.sorbonne-universite.fr

January 27, 2022

Contents

List of Figures

List of Tables

I. Introduction

We consider Lemaître-Chaboche-Marigo's damage model :

$$\psi(\epsilon) = \frac{1}{2}(1-d)\,\epsilon : \mathbb{C}_0 : \epsilon \Longleftrightarrow \sigma(\epsilon) = (1-d)\,\mathbb{C}_0 : \epsilon$$

with $f(d) = Y - Y_c(d) = -\partial_d\psi - Y_c(d) \leq 0$ the damage criterion.

II. Preliminary questions

We here consider the 1D case with $\sigma(\epsilon) = (1-d)\,E\epsilon$

SEE COPY BOOK

III. Homogeneous traction

III.1 $U_{max} = 0.003$

When first running the code, the numerical solution is good wrt analytical solution :

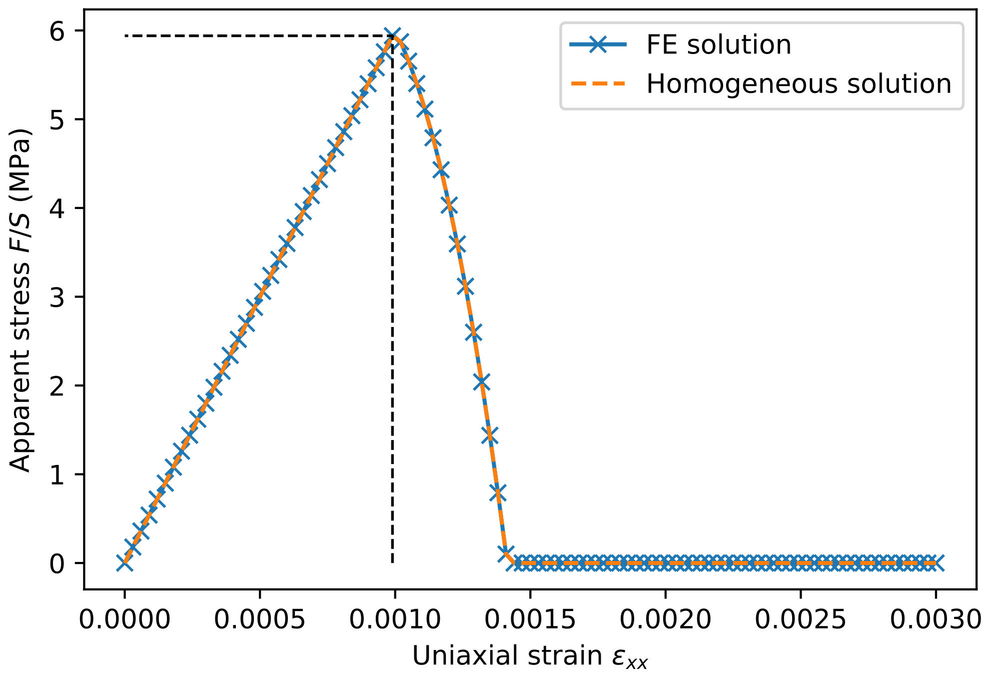


Figure III.1: $\sigma = f(\epsilon) - \alpha = 1$

One can see as we deform the beam (displacement control), the stress evolves linearly until a critical point where the beam suffer strain-softening ?

III.2 $\alpha = 0.1$

To change α is to change how much the system can be damaged. Then, the bigger, the more the damage phase will last.

Considering $\alpha = 0.1$ one gets :

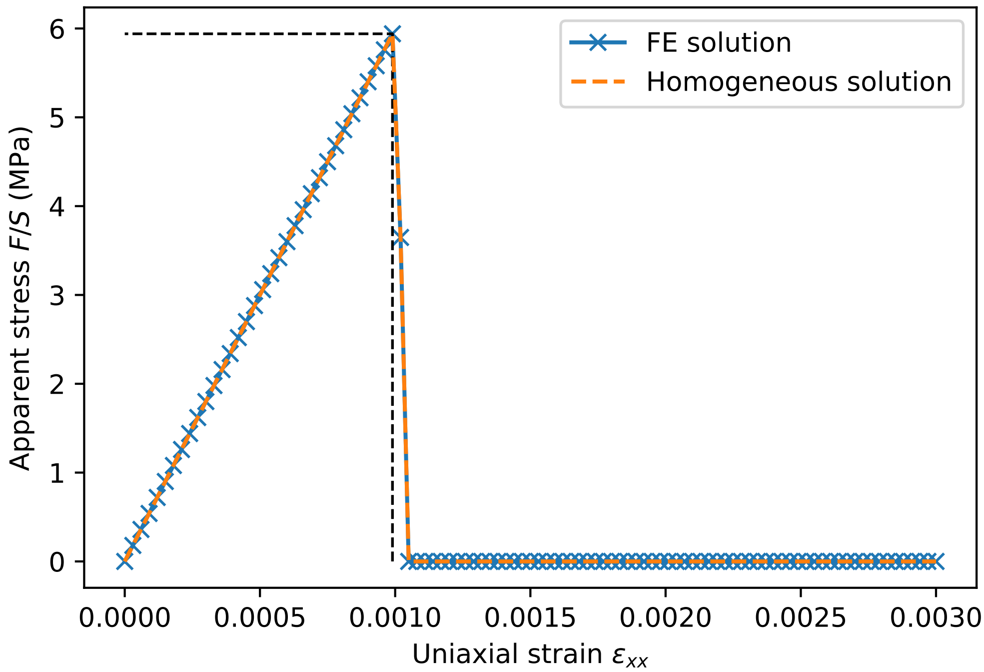


Figure III.2: $\sigma = f(\epsilon) - \alpha = .1$

This case in facts corresponds to the limit case where we break instantly (no time to have strain-hardening or else).

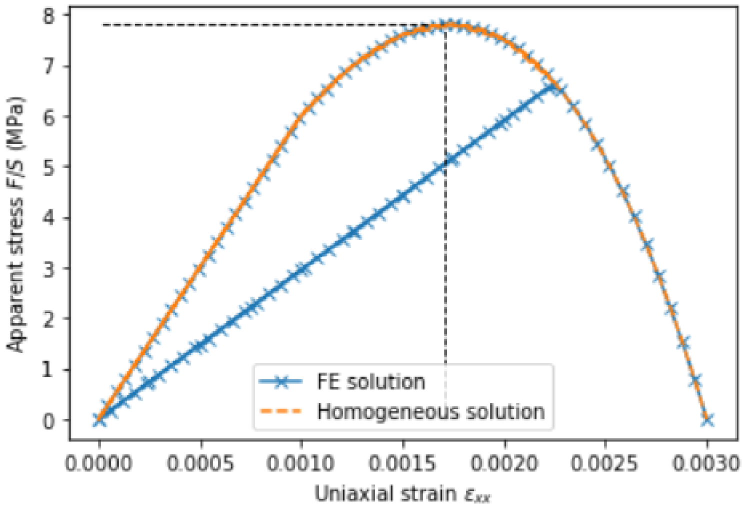
In fact, variable α tells how much the material can handle : the lowest, the more brittle and inversely.

$\Rightarrow$ as one decreasesα, we tend to the case$\epsilon = \epsilon_{max}$

III.3 $\alpha = 8$

We consider a very ductile that can suffers high deformation.

We furthermore want to see how well discharge behavior is modeled. Then, as one deforms the beam and reaches limit value (at which damage first occurs), we get :



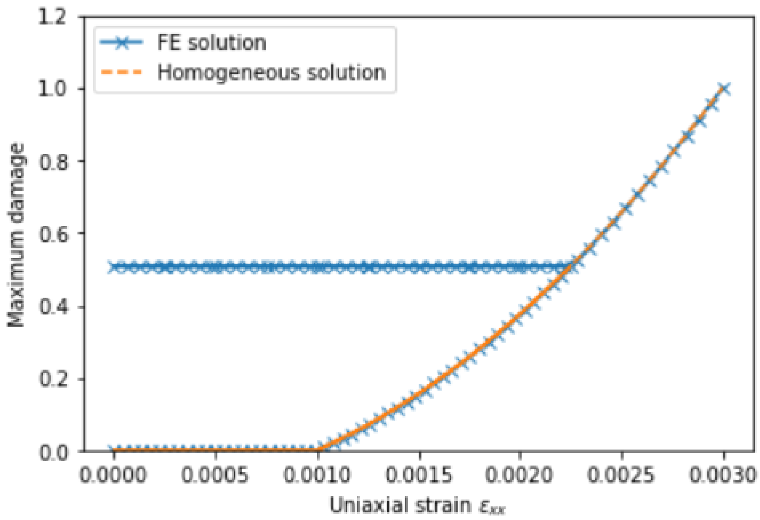


Figure III.3: $\sigma = f(\epsilon) with discharge - \alpha = 8$

First and foremost, we observe that once we reach ϵ_c, the beam suffers strain-hardening and then strain-softening.

Then, as one discharge, as opposed to plastic case where the discharge is linear but follows an affine line, there the discharge is purely linear, even after damage occurs.

$\Rightarrow$ by opposition, as one increasesα, we allow the structure to suffer higher deformation before breaking
$\Rightarrow$ if one discharge, it works as long asαbig enough for one to reach$\epsilon = \epsilon_c$($\alpha$is our loading parameter here, ϵ_c

III.4 Refinement=1

We here refine the mesh in order to test precision result in function of refinement.

Once we apply let's say a refinement of 1 (i.e. we double nb of elements, so 4 now), we get :

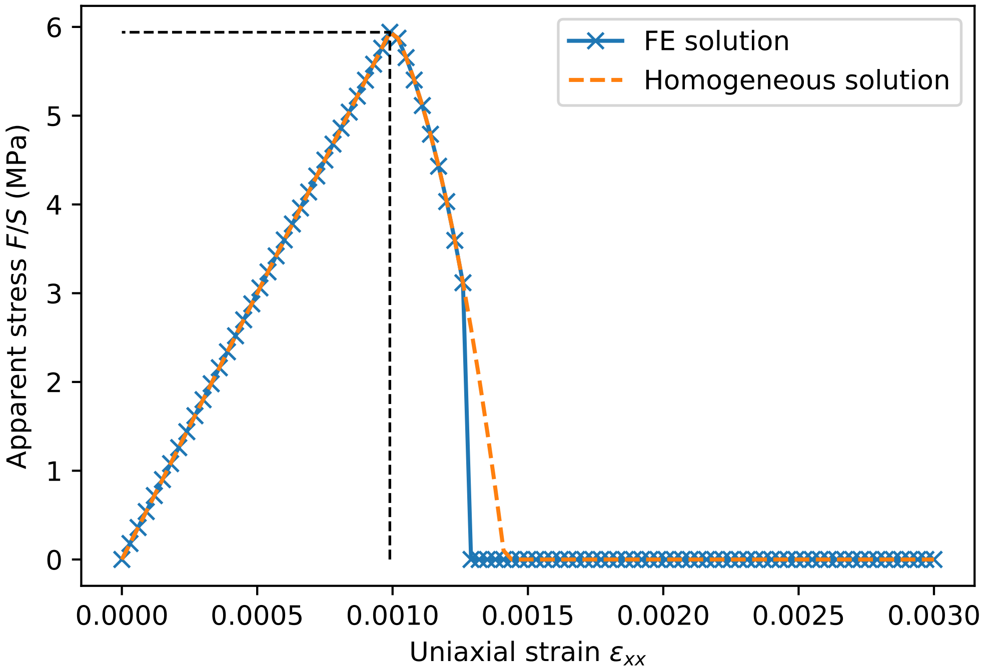


Figure III.4: $\sigma = f(\epsilon) with refinement - \alpha = 1$

We observe a jump in the graph once we reach a certain value.

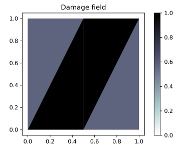
By refining the mesh, we add element but that brings an "error". In fact, we then highlight the beam is heterogeneous such that we have a damaged phase and an healthy one at the same time in the beam. The case with more elements is closer to reality, and the jump we observe corresponds to the point where we go from damage beam to healthy one.

$\Rightarrow$ Homogeneous phase and then heterogeneous one ?? or rather homogeneous damage and then other phase where no damage.

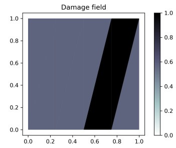
Note : it's possible to get position of transition in simple analytical cases by considering $\frac{\partial u}{\partial x} = t$ as our loading parameter.

III.5 Refinement - $\alpha = 8$

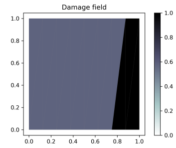
(a) 2 ele



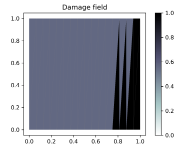
(b) 4 ele



(c) 8 ele



(d) 16 ele



(e) 16 ele

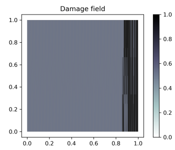
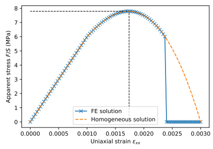
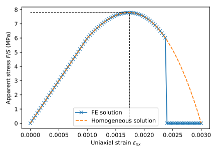


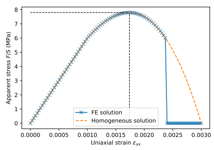
Figure III.5: Position phase transition as a function of refinement

(a) ref 1

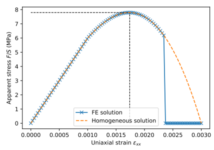




(b) YP5



(c) YP10



(d) YP30

Figure III.6: stress/strain graph for different refinement

By increasing refinement (i.e. increasing nb of elements), we verify we converge towards a unique transition point.

We should moreover have deformed transition at rhs of the beam even though it's not really the case here (numerical error ? maybe more elements in both directions ?).

What did I mean by that ?

On the other hand stress remains globally piecewise continuous.

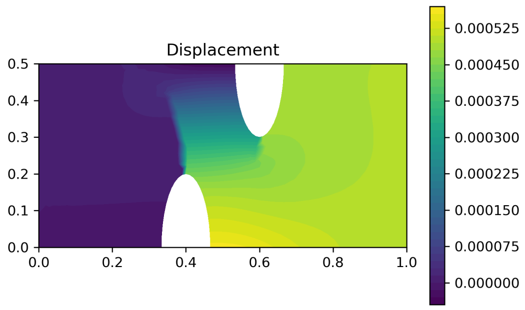
IV. Notched plate

We here consider another geometry where we'd like to observe the fracture in damage material ; and as well deduce some interrogation on the fractures' thickness.

IV.1 R=0.2

When running the code, we observe it takes more iteration to solve some of the time increment.

(a) displacement



(b) break path (1 moves, 0 static)

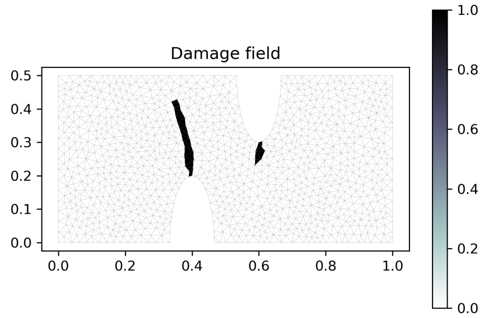


Figure IV.1: displacement + break path

The displacement is imposed to the rhs ? and so crack first occurs to lower hole ? We note that as crack appears to the rhs of the structure, the lhs remains untouched and the break line represents a limit above which we note no displacement in the structure. Probably energy applied to rhs is "absorbed" by, transferred to, the crack.

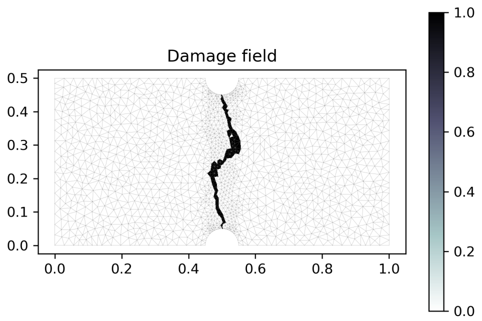
Let's see how this behavior evolves with refinement.

IV.2 Refinement

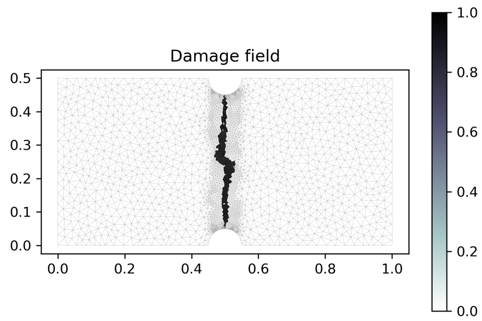
We here below observe crack expansion in function of the number of elements. Note that :

- we changed geometry as code otherwise takes well too long ;
- the refinement does more than double nb of elements, or at least mesh is already refined close to holes.

(a) 2 ele



(b) 4 ele



(c) 8 ele

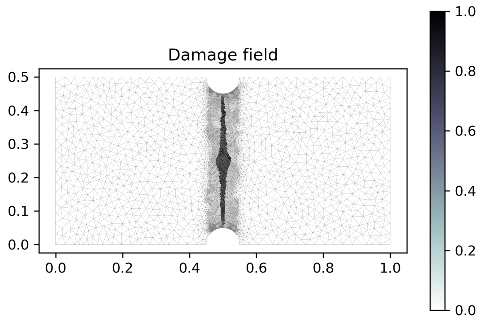


Figure IV.2: break path as a function of Refinement

As one refine, the break looks to better see the constraints of opposite hole and most likely minimizes energy towards it.

For what is up to max stress, it appears that as one refines, its value is increasing. In fact, we look at a singularity such that for $ref \to \infty$, stress goes to infinity.

IV.3 $R = 0.05$ - aspect ratio=10

Notch crack propagation case is modeled by setting a high aspect ratio and low radius.

We note values of dissipated energy for different refinement ($G_c \times \Gamma$):

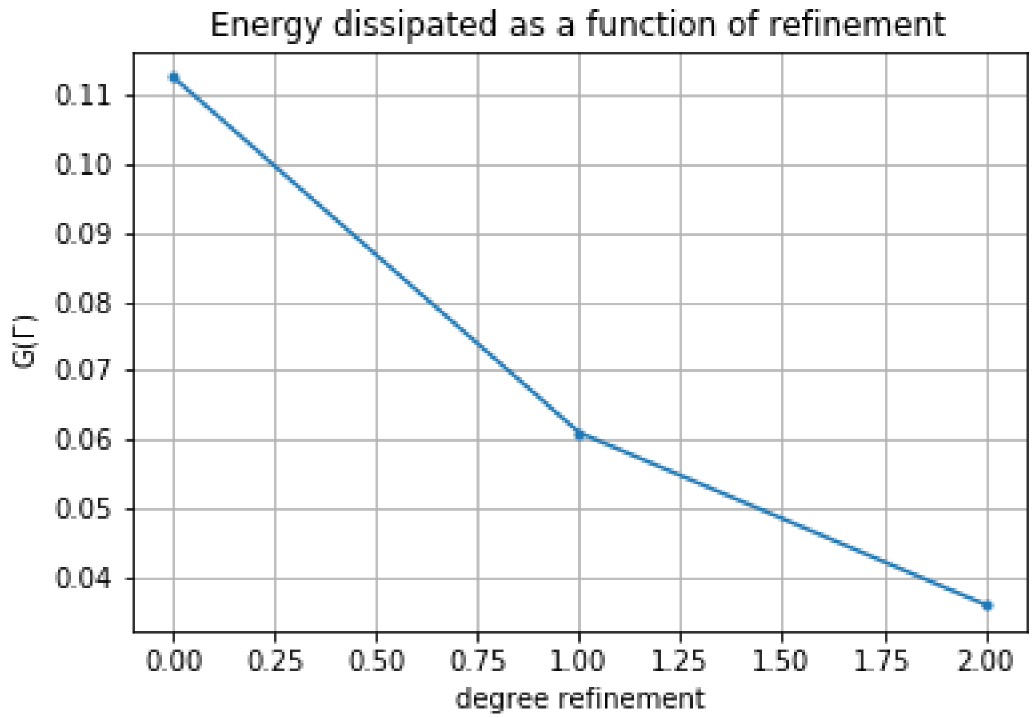
Degree refinement	G(Γ)
0	0.1126
1	0.061
2	0.036

Table IV.1: Order of accuracy on Δ_x

We can see from IV.2 that the path is different depending on the mesh. It then comes that a longer path may come with a larger energy dissipation.

Looking at energy dissipation with refinement this information is confirmed :
Side note : the number of iteration looks to diminish once either crack propagates or as been propagating a lot (maybe direction then is known ?).

Figure IV.3: Energy dissipation



On the other hand, it's possible the lower the size of an element, the less energy accounted for : lower element, we may go along a line and count energy for the line only as opposed to coarse mesh where we may go straight but count more then necessary path.

It's most likely the energy will tend towards infinity as one increases precision of the mesh.

The method then isn't reliable to get value of energy needed to propagate break as if we consider $refinement \to \infty$, then it asks for a zero force to do so.

NOTE : WE CONSIDERED $U_{max} = 3e^{-3}$ instead of asked max displacement. However, overall observations should be affected by this.

IV.4 Ductile material - $\alpha = 8$ & $U_{max} = 1e^{-3}$

We know consider a more ductile material (i.e. $\alpha = 8$) to see if previous observations are affected by this characteristic or if the method we use can be applied to some specific materials (the more ductile then).

First, when running the code we observe non convergence of some of the iteration for too much elements. It then becomes much more difficult to gather enough data to make a consistent comparison. Looking at lower degrees of refinement we get :

Degree refinement	G(Γ)
0	0.27
1	0.15
2	0.081

Table IV.2: Order of accuracy on Δ_x

We fix a low enough U_{max} to prevent strain-hardening, softening. We thus compare two similar elastic and damage behaviors.

Finally, we see that here again energy isn't converging but only decreasing, which can be explained through :

$$G(\Gamma) = G_c.\Gamma \equiv cst.l$$

and so as one decreases size of element, we in fact decrease l. A method to correct would be to get G_c for a material and deduce l from it and see the adapted mesh size. There is probably other method that allow one to change mesh's size while converging.

IV.5 Offset holes

We here consider a horizontal offset between the two holes. To this aim we set "hole-spacing" at 0.2 and we get the result from IV. In term of stress-strain, here is a comparison between usual case and offset one :

(a) hole-spacing=0

(b) hole-spacing=0.2

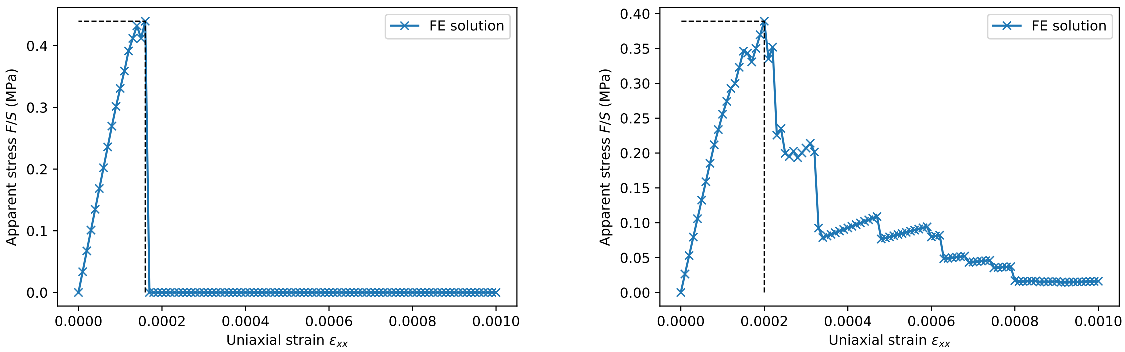


Figure IV.4: stress/strain graphs comparison

The offset one presents strain-hardening : in fact there is strain-hardening in the beam as long as material can endure enough displacement. Here, even though we have a fix $\alpha = 1$, in the offset case the break appears long after the usual case, and even then the break doesn't propagate immediately but rather in a step motion, allowing material to plastify.

The main difference then is that in the usual case, the two holes "communicate" (i.e. break of one is influenced by the other), such that it's then easier to break ; while the offset case necessitates more deformation for the break to occur and develop.