TD1 : Thermoelastic behavior of a cylindrical structure with a rectangular section.
Part 1 : Equations of the problem

The structure at stake in this problem is a cylindrical part called Ω with a rectangular section whose dimension ℓ in direction $\underline{e}_3$ is way larger than both dimensions H and $2L$ of its rectangular section in plane $(\underline{e}_1, \underline{e}_2)$.

The part lays on a rigid horizontal stand. The part is supposed to remain in contact with the stand all along its lower plane surface located at $(x_2 = 0)$.

The part is made of an isotropic homogeneous linear elastic material. Its elastic rigidity tensor is referred to as $\mathbb{A}$, while $\mathbb{S}$ stands for its elastic compliance tensor and finally its thermal expansion coefficient is noted α_T.

At last symmetries of the problem in terms of geometry, material properties and load allow to perform the study on only half of the part, namely of subpart located in the half space $x_1 > 0$.

1 Pressure load

The part is first submitted to gravity as well as a pressure load on its upper boundary $(x_2 = H)$, as represented on Figure 1.

Contact between the part and the stand is supposedly characterised by an elastic restoring force, i.e. a surface density force is applied in the plane of the contact surface in a direction opposed to mouvement : $\underline{F} = -k_e \underline{u}_T$, where $\underline{F}$ is the restoring force, $\underline{u}_T = u_1 \underline{e}_1$ is the tangential component of the displacement vector $\underline{u}$, and k is a positive constant $(k > 0)$.

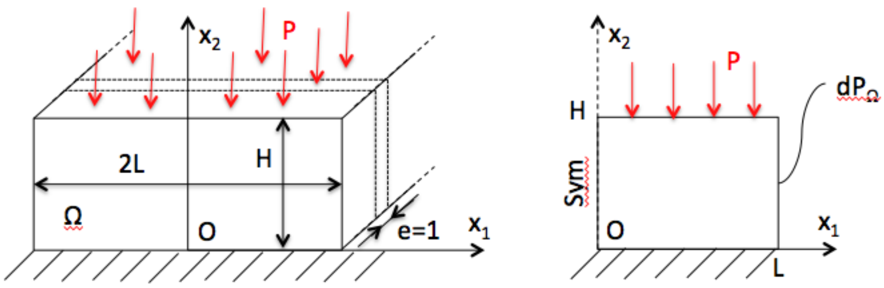

Figure 1: Schematic representation of the structure studied and its pressure load

1. Write the local equations (i.e. strong formulation) of this linear elastic problem.
2. To which physical mechanism do the cases $k_e = 0$ and $k_e \to \infty$ correspond?
3. Define the spaces of admissible fields $\mathcal{U}$, kinematically admissible fields $\mathcal{U}_{ad}$, and kinematically admissible to zero fields $\mathcal{U}^0_{ad}$.
4. Choose a test field $v(\underline{x})$ and write successively the weak and variationnal formulations of the problem.
5. Is it a well defined problem (i.e. does the problem have a unique solution)?

2 Thermomechanical coupling

The part is now supposed to have been submitted to a thermal load priori to its pressure load. As a consequence the temperature in the part is non homogeneous and $T(\underline{x})$ stands for the related thermal field.

1. Indicate how the strong and weak formulations of problem of the part submitted to pressure only are affected

3 One step further : Thermal load

In this section the part is only submitted to its preliminary thermal load (i.e. the pressure load is not applied yet). The thermal load applied is schematized on Figure 2 :

- The upper surface of the part ($x_2 = H$) is maintained at temperature $T = T_1 \neq 0$.
- The stand is maintained at temperatureT_0 which is chosen as the reference temperature.
- The right wall of the part ($x_1 = L$) is in contact with open air at inital temperature $T = T_0$.
- At last the part does not contain any heat source.

It is supposed that a stationnary state has been reached meaning that none of the solution fields depends on time.

The isotropic thermal conductivity tensor of the material is refered to as $k\underline{\underline{I_d}}$ and α_e stands for the exchange coefficient of the walls of the part.

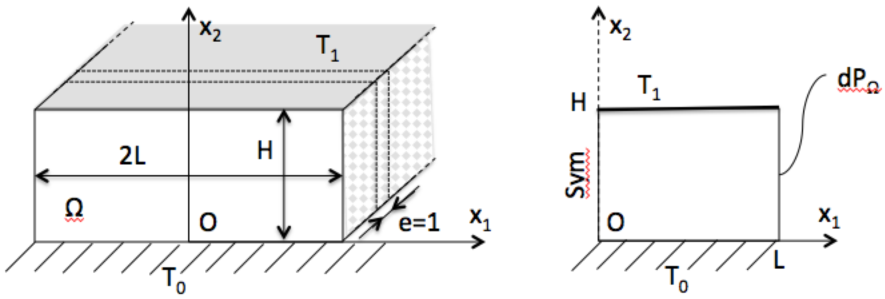

Figure 2: Schematic representation of the structure and its thermal load.

1. Write the local equations (i.e. strong formulation) of this linear thermal problem.
2. Define the space of thermically admissible fields $\mathcal{T}_{ad}$ and thermically admissible to zero fields $\mathcal{T}^0_{ad}$.
3. Choose a test field $\theta(\underline{x})$, and write successively the weak and variationnal formulations of the problem.
4. The right exterior wall of the part ($x_1 = L$) is now supposed to be adiabatic. Modify consequently the strong and weak formulations of the problem.