Ex 1.

1) $3x^2 - 1 = 0 \iff \xi_{0,1} = \pm \frac{1}{\sqrt{3}}$ $\quad \xi_0 = -\frac{1}{\sqrt{3}}$, $\xi_1 = \frac{1}{\sqrt{3}}$

$$\tilde{f}(x) = \frac{x - \xi_1}{\xi_0 - \xi_1} f(\xi_0) + \frac{x - \xi_0}{\xi_1 - \xi_0} f(\xi_1) = \frac{x - 1/\sqrt{3}}{-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}} f(-\tfrac{1}{\sqrt{3}}) + \frac{x + 1/\sqrt{3}}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} f(\tfrac{1}{\sqrt{3}})$$

$$\tilde{f}(x) = \frac{-\sqrt{3}\,x + 1}{2} f(-\tfrac{1}{\sqrt{3}}) + \frac{\sqrt{3}x + 1}{2} f(\tfrac{1}{\sqrt{3}})$$

2) $$\int_{-1}^{1} \tilde{f}(x)\,dx = \frac{1}{2}\left[\int_{-1}^{1}(-\sqrt{3}\,x + 1)\,dx \cdot f(-\tfrac{1}{\sqrt{3}}) + \int_{-1}^{1}(\sqrt{3}x + 1)\,dx\, f(\tfrac{1}{\sqrt{3}})\right]$$

or $\int_{-1}^{1} \pm\sqrt{3}\,x\,dx = 0$ et $\int_{-1}^{1} dx = 2 \implies \int_{-1}^{1} f(x)\,dx \cong f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$

3) $$\int_0^1 f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f(\tfrac{1}{2}u + \tfrac{1}{2})\,du = \frac{1}{2}\left[f(\tfrac{1}{2} - \tfrac{1}{2\sqrt{3}}) + f(\tfrac{1}{2} + \tfrac{1}{2\sqrt{3}})\right]$$

$x = \frac{1}{2}u + \frac{1}{2} \implies$ $u = -1 \implies x = 0$
$u = 1 \implies x = 1$

4) Cette formule de quadrature est exacte pour les pol. de degré ≤ 3 $(= 2\times 1 + 1)$ – En effet, il y a 4 paramètres : ξ_0, ξ_1 et $w_0^1, w_1^1 \implies$ 4 éq^{ns} $\implies$ degré ≤ 3 –
$\implies$ degré de précision $= 3$.

5) $$\int_0^1 x^3\,dx = \frac{1}{4} = \frac{1}{2}\left[\left(\frac{1}{2} - \frac{1}{2\sqrt{3}}\right)^3 + \left(\frac{1}{2} + \frac{1}{2\sqrt{3}}\right)^3\right]$$

$$= \frac{1}{2}\left[\frac{(1 - 1/\sqrt{3})^3}{8} + \frac{(1 + 1/\sqrt{3})^3}{8}\right]$$

$$= \frac{1}{16}\left[\left(1 - \frac{1}{\sqrt{3}}\right)^2\left(1 - \frac{1}{\sqrt{3}}\right) + \left(1 + \frac{1}{\sqrt{3}}\right)^2\left(1 + \frac{1}{\sqrt{3}}\right)\right]$$

$$= \frac{1}{16}\left[\left(1 - \frac{2}{\sqrt{3}} + \frac{1}{3}\right)\left(1 - \frac{1}{\sqrt{3}}\right) + \left(1 + \frac{2}{\sqrt{3}} + \frac{1}{3}\right)\left(1 + \frac{1}{\sqrt{3}}\right)\right]$$

$$= \frac{1}{16}\left[1 - \cancel{\frac{2}{\sqrt{3}}} + \frac{1}{3} - \cancel{\frac{1}{\sqrt{3}}} + \frac{2}{3} - \cancel{\frac{1}{3\sqrt{3}}} + 1 + \cancel{\frac{2}{\sqrt{3}}} + \frac{1}{3} + \cancel{\frac{1}{\sqrt{3}}} + \frac{2}{3} + \cancel{\frac{1}{3\sqrt{3}}}\right]$$

$$= \frac{1}{16}\left[2 + \frac{6}{3}\right] = \frac{1}{16}[4] = \frac{1}{4}$$ <u>oui</u>

Ex 2.

2.1 $a = 0$, $T = 3$, $n = 3$, $h = 1$, $f(t,y) = 2t + y$

s_1

$$y_{n+1} = y_n + \frac{1}{2}\left[2t_n + y_n + 2(t_{n+1}) + y_{n+1}\right]$$

$$\Rightarrow \frac{1}{2} y_{n+1} = \frac{3}{2} y_n + t_n + t_{n+1}$$

$$\Rightarrow \boxed{y_{n+1} = 3y_n + 2(t_n + t_{n+1})}$$

$n=0$ $\quad y_0 = a = 0 \qquad t_0 = 0, \; t_1 = h = 1, \; t_2 = 2, \; t_3 = 3$

$y_1 = 3a + 2(0 + h) = 3a + 2h = 2h = 2$

$n=1$ $\quad y_2 = 3 \times 2 + 2(\times 3) = 12$

$n=2$ $\quad y_3 = 3 \times 12 + 2(2+3) = 36 + 10 = 46.$

s_2

$$y_{n+1} = y_n + \frac{1}{2}\left[2t_n + y_n + 2t_{n+1} + (y_n + 2t_n + y_n)\right]$$

$$= y_n + t_n + \frac{y_n}{2} + t_{n+1} + \frac{y_n}{2} + t_n + \frac{y_n}{2}$$

$$y_{n+1} = \frac{5}{2} y_n + 2t_n + t_{n+1}$$

$n=0$ $\quad y_0 = a = 0$

$y_1 = \frac{5}{2} \times a + 2 \times 0 + 1 = 1$

$n=1$ $\quad y_2 = \frac{5}{2} \times 1 + 2 \times 1 + 2 = \frac{5}{2} + 4 = \frac{13}{2} = 6{,}5$

$n=2$ $\quad y_3 = \frac{5}{2} \times \frac{13}{2} + 2 \times 2 + 3 = \frac{65}{4} + 7 = 16{,}25 + 7 = 23{,}25$

2.2 s_1 : schéma implicite — 1 éq^{n} à résoudre à chaque pas

s_2 : schéma explicite —

2.3 <u>ordre 2.</u> (mais ne se verra que lorsque $h \to 0$ or ici $h = 1$).

Ex 3.

$$A = \begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

3.1 $\det[A - \lambda I] = \det \begin{vmatrix} a-\lambda & 0 & 0 \\ d & b-\lambda & 0 \\ 0 & 0 & c-\lambda \end{vmatrix} = (a-\lambda)(b-\lambda)(c-\lambda)$

$\Rightarrow \det(A - \lambda I) = 0 \iff \lambda = a, b$ ou c

3.2 $A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix} = c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\quad \vec{x}_c = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

3.3 Puiss. itérée $\Longrightarrow c, \vec{x}_c$ (valeur propre de + grand module + vecteur propre associé).

3.4 $\Longrightarrow b, \vec{x}_b$ (car $\vec{x}_0$ n'a pas de composante suivant $\vec{x}_c$ donc on tombe sur la val. propre suivante et le vecteur propre associé.)

3.5 $B = A - aI = \begin{bmatrix} 0 & 0 & 0 \\ d & b-a & 0 \\ 0 & 0 & c-a \end{bmatrix}$

Puiss. itérée $\Longrightarrow (c-a)$, et $\vec{x}_c$ $\quad$ { car $c-a$ est la val. propre de + gd module

3.6 $\det B = 0 \Longrightarrow$ non inversible $\Longrightarrow$ non.

3.7 $C = A - dI = \begin{bmatrix} a-d & 0 & 0 \\ d & b-d & 0 \\ 0 & 0 & c-d \end{bmatrix}$

Puiss. itérée $\Longrightarrow (a-d), \vec{x}_{a-d}$ car $|a-d| > |b-d| > |c-d|$

3.8 $\det C \neq 0 \Longrightarrow$ Puiss. itérée OK $\Longrightarrow \dfrac{1}{(c-d)}, \vec{x}_{c-d}$

Ex 4.

1) $\frac{\partial F}{\partial \alpha} = 0 = 2 \sum_{i=0}^{n} (x_i^2 + \alpha y_i^2 + \beta y_i + \gamma) y_i^2$

$\frac{\partial F}{\partial \beta} = 0 = 2 \sum_{i=0}^{n} (x_i^2 + \alpha y_i^2 + \beta y_i + \gamma) y_i$

$\frac{\partial F}{\partial \gamma} = 0 = 2 \sum_{i=0}^{n} (x_i^2 + \alpha y_i^2 + \beta y_i + \gamma)$

$$\Rightarrow \begin{bmatrix} \Sigma y_i^4 & \Sigma y_i^3 & \Sigma y_i^2 \\ \Sigma y_i^3 & \Sigma y_i^2 & \Sigma y_i \\ \Sigma y_i^2 & \Sigma y_i & (n+1) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} -\Sigma x_i^2 y_i^2 \\ -\Sigma x_i^2 y_i \\ -\Sigma x_i^2 \end{bmatrix}$$

2) $\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix}$ $\quad x_0 y_0 = x_1 y_1 = x_2 y_2 = x_3 y_3 = 0$

3) $\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ & u_{22} & u_{23} \\ & & u_{33} \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & & \\ l_{21} & 1 & \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{pmatrix}$$

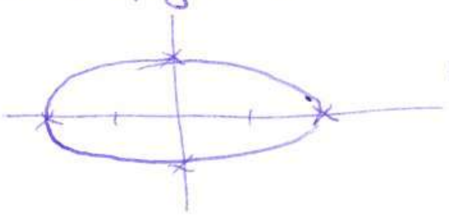
$$\Rightarrow \begin{cases} u_{11} = 2 & u_{12} = 0 & u_{13} = 2 \\ l_{21} = 0 & u_{22} = 2 - 0 = 2 & u_{23} = 0 - 0 = 0 \\ l_{31} = 2/2 = 1 & l_{32} = 0 - 1 \times 0 = 0 & u_{33} = 4 - 1 \times 2 - 0 = 2 \end{cases}$$

$$L = \begin{pmatrix} 1 & & \\ 0 & 1 & \\ 1 & 0 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

4) $Ly = b \quad \begin{pmatrix} 1 & & \\ 0 & 1 & \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix} \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = 0 \\ y_3 = -8 \end{cases}$

$Ux = y \quad \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix} \Rightarrow \begin{cases} \gamma = -4 \\ \beta = 0 \\ \alpha = 0 - \frac{2 \times -4}{2} = 4. \end{cases}$

5) $x^2 + 4y^2 - 4 = 0 \Rightarrow x^2 + (2y)^2 = 2^2 \Rightarrow \left(\frac{x}{2}\right)^2 + y^2 = 1.$



ellipse passe par les points

$F = 0$.