Sorbonne University

M2 Damage Course - CompMech - 2021/2022

Aspects numériques des modèles à gradient d'endommagement

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I. Introduction

We consider the gradient damage model :

$$\epsilon(\underline{u},d) = \epsilon_{el}(\underline{u},d) + \epsilon_f(d) = \int_\Omega \psi(\underline{\underline{\epsilon}}(\underline{u}),d)dx + \int_\Omega \frac{G_c}{l_0.c_\omega}.\left(\omega(d) + l_0^2||\underline{\nabla} d||^2\right)dx$$

with

- $f(d) = Y - Y_c(d) = -\partial_d\psi - Y_c(d) \leq 0$ the damage criterion
- $\psi(\underline{\underline{\epsilon}}(\underline{u})) = ((1-d)^2 + K_{res}).\frac{1}{2}\underline{\underline{\epsilon}} : \mathbf{C} : \underline{\underline{\epsilon}}$ the elastic energy density

II. Preliminary questions

We here consider the 1D homogeneous case with $\sigma(\epsilon) = (1-d)\,E\epsilon$ and

- $K_{res} = 0$
- $\underline{\nabla} d = \underline{0}$

SEE COPY BOOK

III. Homogeneous traction

As we assume theory of discontinuity at break, we subsequently consider no stress on the interface such that $[\underline{\underline{\sigma}}]\underline{n} = \underline{0}$. Therefore, we are most likely to consider traction not to re-derive theory in this case.

<u>Domain</u> : 2D domain $L \times W = [0,1] \times [0,0.1]$
<u>Material</u> : elastico-plastic system that can endure elasticity, hardening and break. We therefore consider a model with possible damage under exterior stimulation.
<u>Forces/Hyp.</u> : we consider plane stress hypothesis as well as a damage from apparition to break of the whole structure.

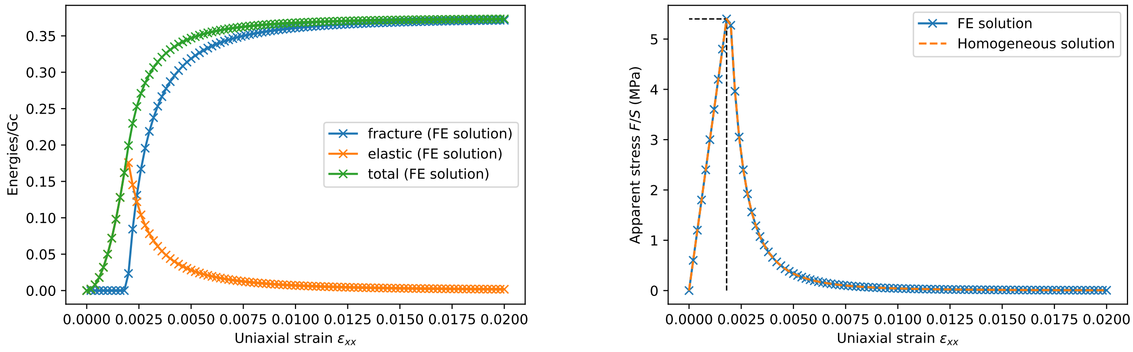
As mentioned in the introduction, we consider traction at the rhs.

We furthermore consider embedded condition to the lhs ($U_x = 0$).

III.1 Q1

We observe behavior of AT1 and AT2 as discussed during the course. We consider $U_{max} = 0.02$m, and get following figures :

AT1



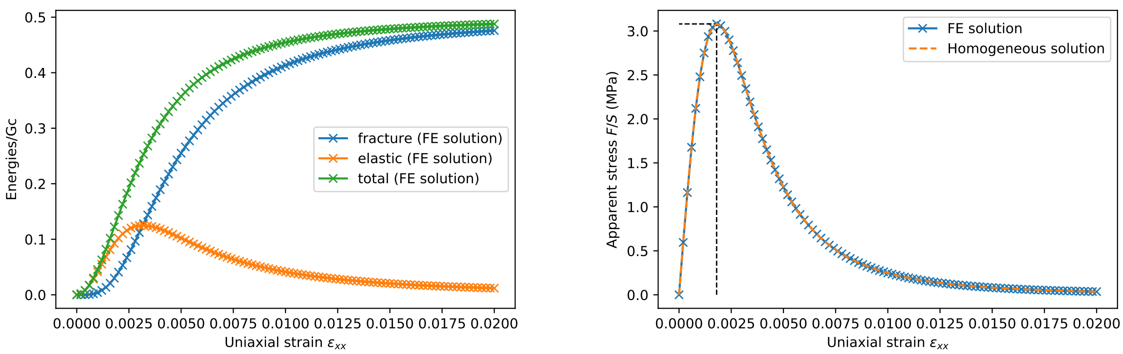
(a) Energy of fracture

(b) Energy of fracture

Figure III.1: Stress/strain curve

We observe that system is linear at first and once we reach characteristic value $\epsilon_c = \epsilon$ the syst is damaged. Then, fracture energy begins to increase and continue as elastic energy is transferred to "break" energy as beam furthermore pulled.

AT2



(a) Energy of fracture

(b) Energy of fracture

Figure III.2: Stress/strain curve

For this case, break appears as soon as one pulls the bar, namely $\epsilon_{c_2} = 0$.

We directly damage or break ? Is it we have Gaussian-like function as it takes time to transfer elastic energy to G term ?

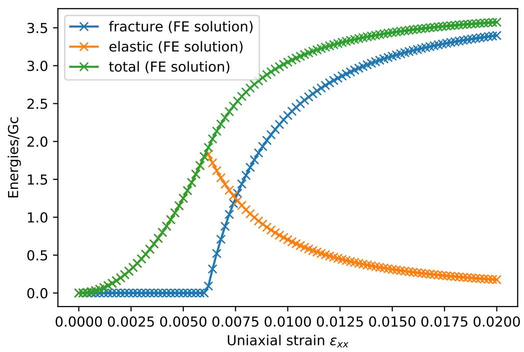
What did I mean by that ?

III.2 Q2

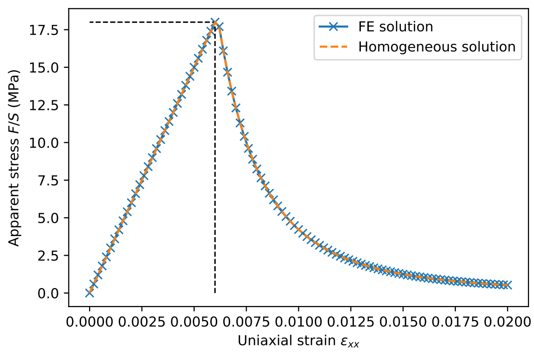
As one makes l_0 tend towards zero, we converge to homogeneous case. We then calculate G_c by ensuring l_0 low enough to have reliable value.

Numerically, we can have homogeneity by considering only a few elements, such that due to BC the strain is constant across system.

J'ai pas fait unloading.



(a) Energy of fracture



(b) Stress/strain curve

Figure III.3: Stress/strain curve

We observe on figIV.2 that we get closer to theoretical case as we reduce l_0. We conclude on the fact theory holds as long as break remains low with respect to structure's size. This point is even more true when we calculate G_c from this method (it's the purpose of the method).

IV. Traction localized

In this chapter we extend previous case to a refined system where we will be able to observe heterogeneous behavior.

IV.1 Q1

As we refine, we allow strain to take different values along mesh (where elements not constrained by BC). It appears a heterogeneous solution (for refinement of 4):

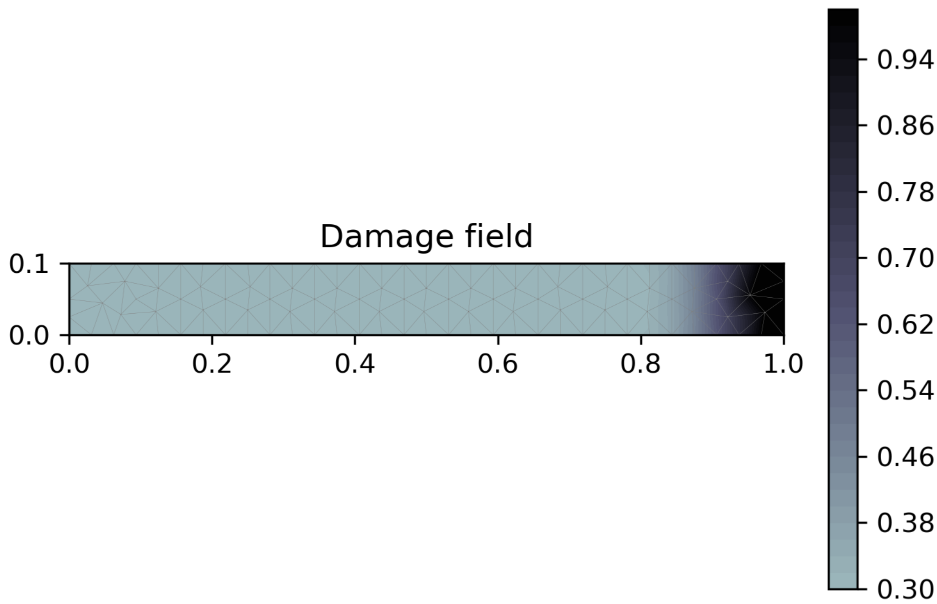
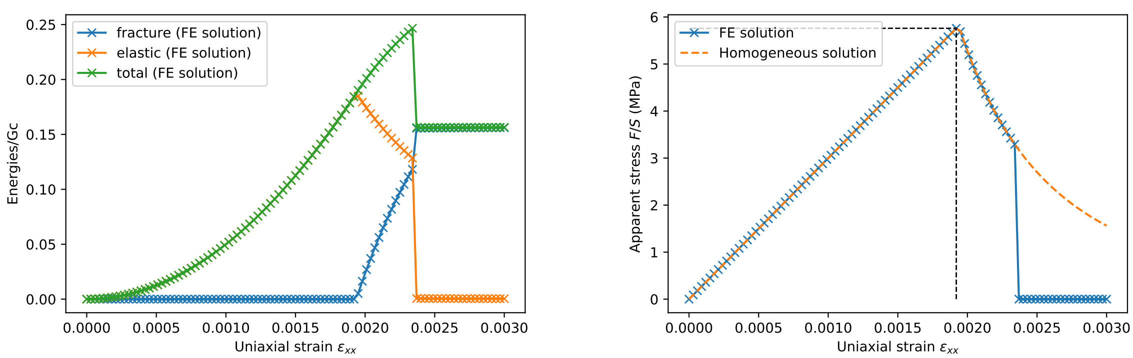


Figure IV.1: damage along beam

On this figure IV.1 we observe a almost homogeneous strain, except at rhs (this point looks to be an issue to calculate G_c as probably to close from edge, where method doesn't hold (as break not on the edge)).

In terms of strain and stress, it does translate an heterogeneous behavior :



(a) Energy of fracture

(b) Stress/strain curve

Figure IV.2: Stress/strain curve

Why is total energy suddenly decreasing ?

We have no direct effect on this in TD2, as opposed to TD1 where alpha allowed to vary how much could the system handle

IV.2 Q2

We add a BC to the rhs and we observe a now sharp fall (or rise) in energies:

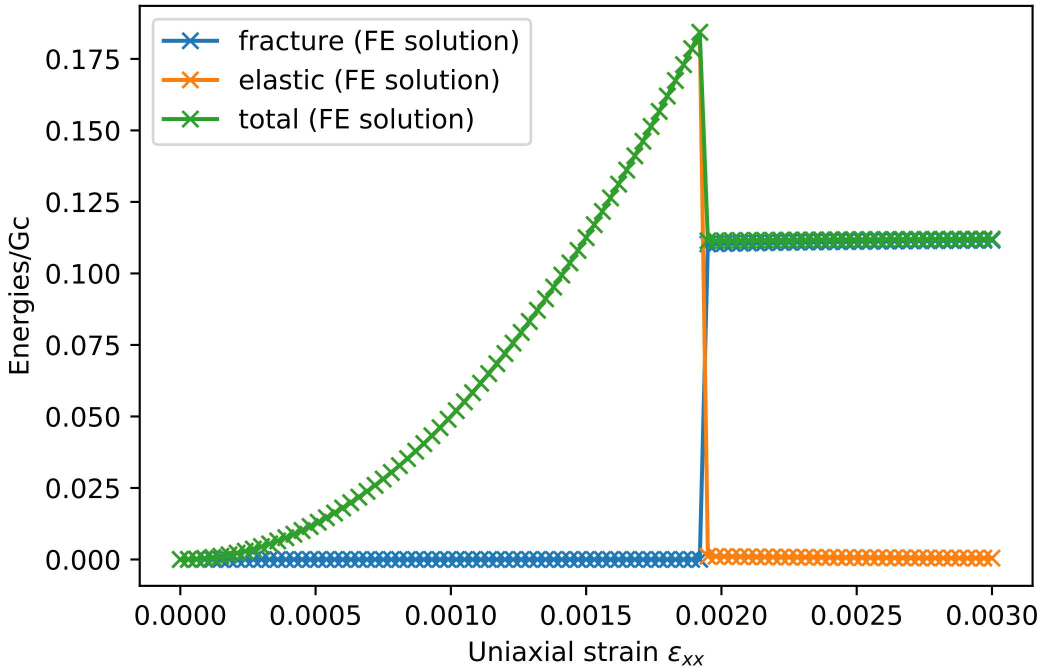


Figure IV.3: energy graph

Physically, we impose $d = 0$ at both end and it result a direct break once damage limit reached. In fact, to add this BC is to ensure theoretical case of Gaussian d. Localized solution was appearing on the edge where $d = 0$, resulting into ill result.

After some testing, it comes that G_c calculation can't be done in this specific case. We fix $U_L = 0$ only to get reliable G_c (physic is affected and we look to a different problem.

IV.3 Q3

We calculate ϵ/G_c, and so we expect in 2D to have $\epsilon = l_0.G_c$ with l_0 the length of crack. As crack goes along entire height, we'll have $l_0 = \omega = 0.1$:

Degree refinement	G(Γ)
4	0.11
5	0.108
6	0.103

Table IV.1: Order of accuracy on Δ_x

We should have obtained $\epsilon_f = 0.1.G_c$. The error is due to the fact the code couldn't reproduce theory identically. Then, we have a Gaussian whose top is at 1 not only in one point but on a few points (SEE NOTES):

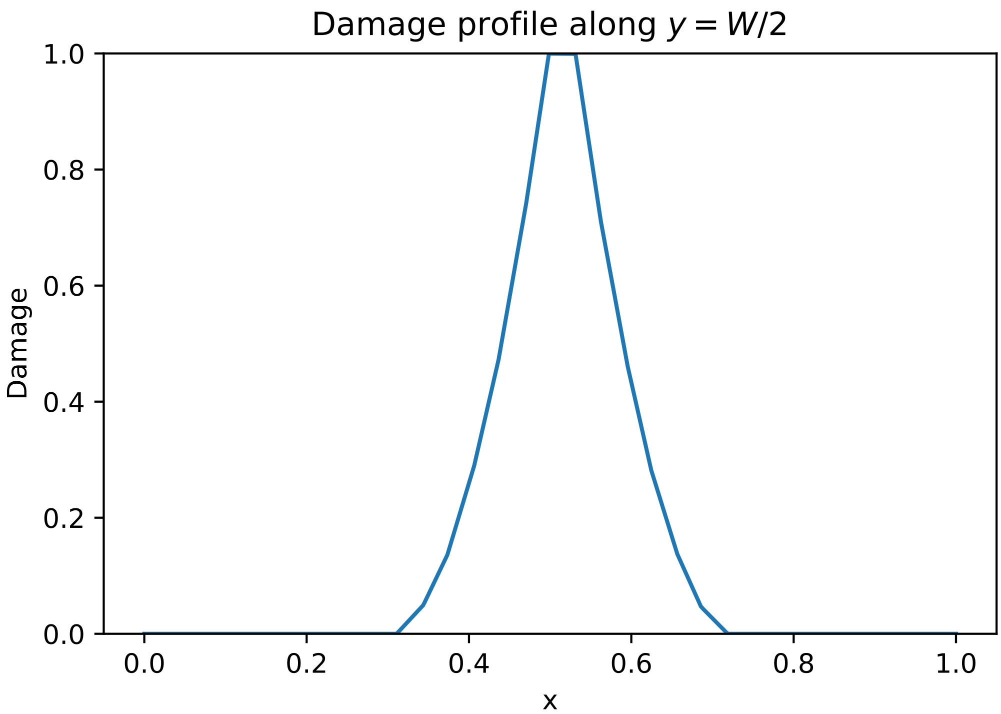


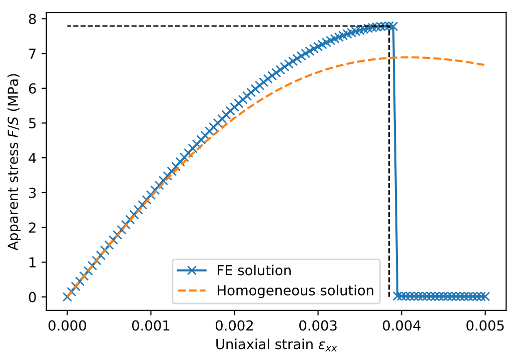
Figure IV.4: damage profile

IV.4 Q4

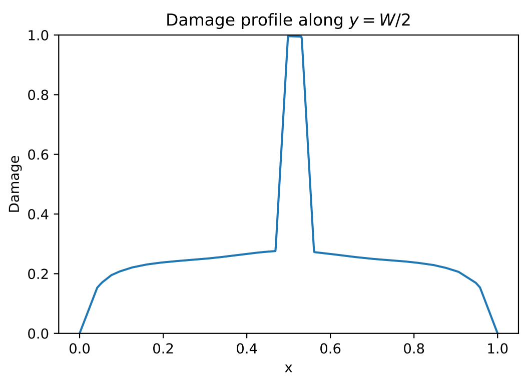
Eventually, as one refines or lowers l_0 value one tends towards theoretical value of G_c.

IV.5 AT2

For AT2, as we fix BC at rhs, even though we normally have a criterion at 1 always as damage directly, we still manage to have the Gaussian like curve:



(a) Energy of fracture



(b) Stress/strain curve

Figure IV.5: Stress/strain curve

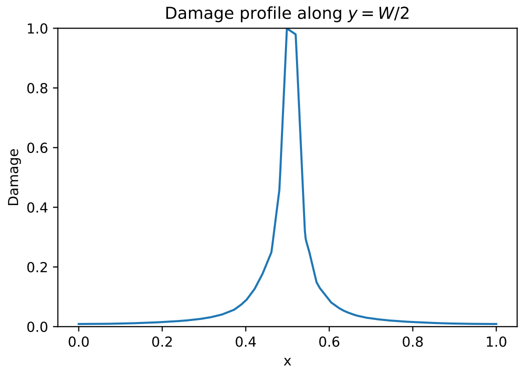
This method (AT2) looks less adapted to the study of this system. We note that we don't follow theoretical curve as we modified BC, and so we don't look at same problem.

V. Damage plate

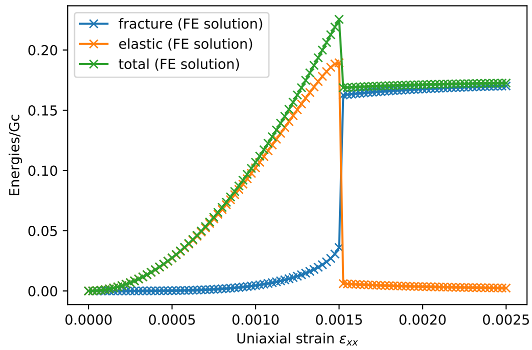
We consider perforated plate to study creation + propagation of break.

V.1 1-2

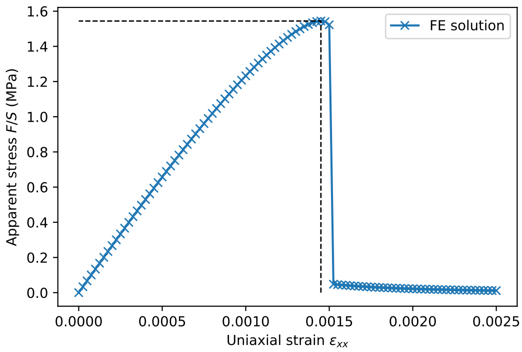
Break where lowest distance:



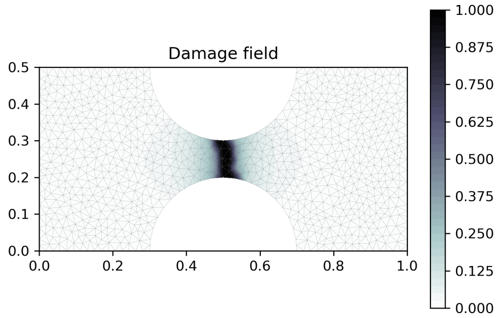
(a) Damage profile



(b) Energy of fracture



(c) Strain/stress curve



(d) Damage value

Figure V.1: Q2 - $l_0 = 0.02$m

NOTE : we don't need to impose BC at rhs anymore, as this case looks more physical.

G_c is much bigger here. If we consider a lower l_0 to tend towards convergence, one should as well consider a thinner mesh in order to ensure l_0 is the scale along which break develops, and not to account for additional energies.

Equivalently, the distance between the two holes could be reduced (i.e. ratio hole/system) in order to have $\epsilon_{f_{numerical}} = theory$.

As a conclusion, we can either

- have l_0 low enough to measure G_c. Then, it might be l_0 too low for the mesh so pay attention to size mesh wrt l_0 value

- have a lower break wrt system's size, namely reduce size of holes.

V.2 3

Notch crack propagation case is modeled by setting a high aspect ratio and low radius.

We are in a singularity case (Griffith theory). This case is equivalent to a pre-damaged system where one studies propagation only (namely not creation).

We therefore don't deal with l_0 here (can't converge to G_c by diminishing l_0 or refining ??). We observe that after break occurs (sudden fall in stress), the propagation still asks for some energy. Then, once creation of crack, the crack furthermore develops by use of some energy ($\equiv$ some rhs force)

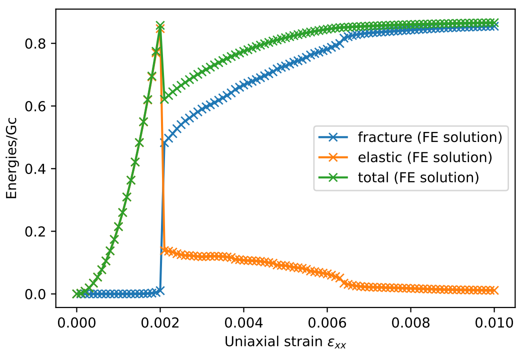
Paraview Post-process

We post-process the break to see how it propagates in AT2 case. SEE VIDEO.
It appears that l_0 doesn't influence much. probably it comes with some necessary refinement as well.

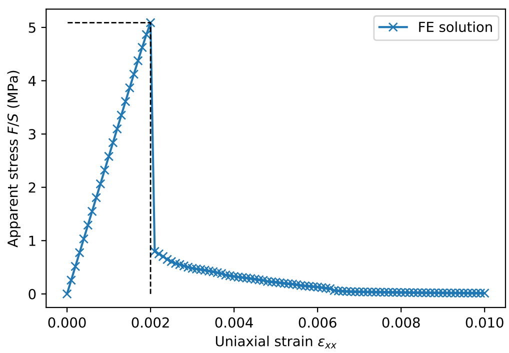
At this stage I don't really know what I'm doing anymore

V.3 Q4

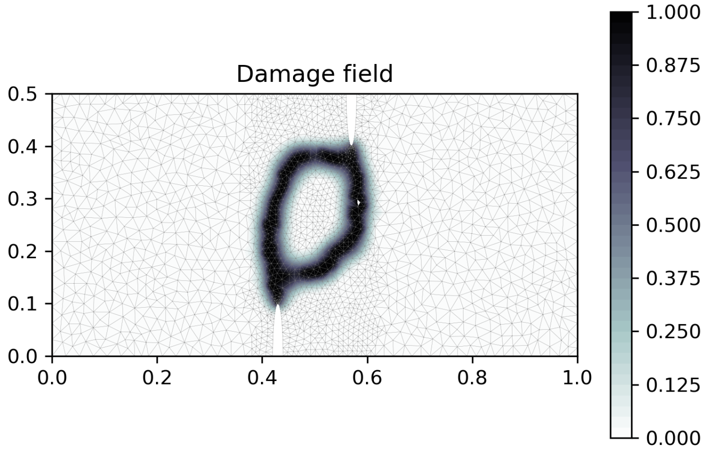
In this case, we offset notch. Then, if breaks are close enough, breaks should join, while otherwise they are more independent. Here are some graphs describing behavior:



(a) Energy of fracture



(b) Strain/stress curve



(c) Damage value

Figure V.2: Q4 - $l_0 = 0.02$m - refinement=1

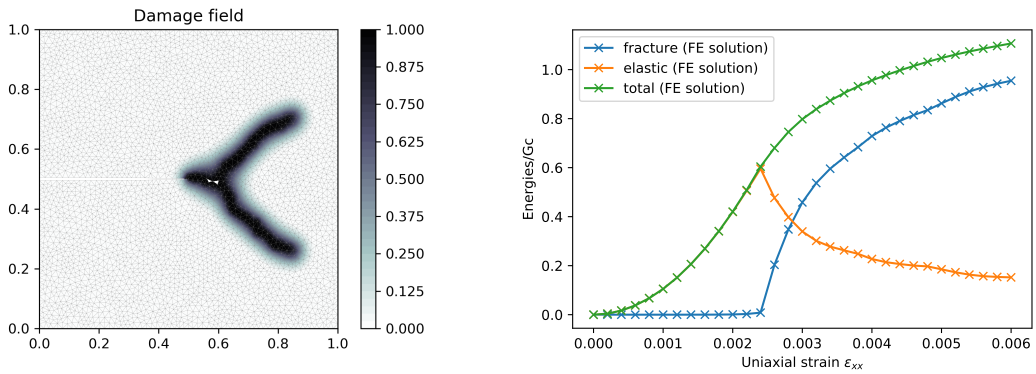
We see breaks don't meet at first, they then develop before joining from behind. It's a usual observation, even though the system remains very BC and geometry-dependant.

VI. Damage in mode II

VI.1 Compression

The model we work with here don't really differentiate traction from compression. In compression, break shouldn't occur while it does for traction.

This aspect must be included as it brings some subtleties. Here is how mode II damage looks like:



(a) Damage

(b) Energy of fracture

Figure VI.1: $l_0 = 0.04$m

We observe as mentioned two breaks, while only bottom one was expected given upper side is in compression while lower one is in traction (only case where break possible).

From energy figure we observe that once break occurs, we propagate break by use of further energy. Then, as we propagate, break propagates, but doesn't cross entire system here as we don't fall at $\sigma_{xx} = 0$.

VII. Conclusion

Where is energy going once we break ?

Anyway we have seen different models and their numerical application in various geometries and different damage behavior (with hardening/softening, without, etc).

Such a study is made possible by describing thoroughly the behavior of the model : ensure "manually" the irreversibility, must have some perspective on max displacement possible, length l_0 to apply when theory works, etc.

Overall, this has been an opportunity to have a first insight in computation of damage, from apparition to propagation. We moreover observed the heterogeneous behavior resulting in damage of the structure.