Exercice 4 :

Pour de la turbulence homogène et isotrope, toutes les dérivées spatiales des quantités moyennes sont nulles.

On a alors qu'une quantité moyenne générique

$\phi = \phi(t)$ seulement. Le terme de production est nul (pas de gradient de vitesse moyen)

1) Modèle K-Kl :

$$\frac{\partial K}{\partial t} + \bar{u}_J \underbrace{\cancel{\frac{\partial K}{\partial x_J}}}_{0} = 0 - C_D \frac{K^{3/2}}{l} + \frac{\partial}{\partial x_J}\left[\underbrace{\cancel{\left(\mu + \frac{\mu_t}{\sigma_K}\right)}}_{0} \frac{\partial K}{\partial x_J}\right]$$

$$\Rightarrow \begin{cases} \dfrac{dK}{dt} = -C_D \dfrac{K^{3/2}}{l} \\ \dfrac{d(Kl)}{dt} = -C_{L2} K^{3/2} \end{cases}$$

Modèle K-τ :

$$\begin{cases} \dfrac{dK}{dt} = -\dfrac{K}{\tau} \\ \dfrac{d\tau}{dt} = (C_{\epsilon 2} - 1) \end{cases}$$

Modèle K-ϵ RNG :

$$\begin{cases} \dfrac{dK}{dt} = -\epsilon \\ \dfrac{d\epsilon}{dt} = -C_{\epsilon 2} \dfrac{\epsilon^2}{K} \end{cases}$$

2) k-kl :

$$\frac{dk}{dt} = -C_D \frac{k^{3/2}}{l} = +\frac{C_D}{C_{L2}} \frac{1}{l} \frac{d(kl)}{dt}$$

$$\Rightarrow \frac{dk}{dt} = \frac{C_D}{C_{L2}} \frac{k}{(kl)} \frac{d(kl)}{dt}$$

$$\Rightarrow \frac{dk}{k} = \frac{C_D}{C_{L2}} \frac{d(kl)}{(kl)} \Rightarrow k = k_0 (kl)^{C_D/C_{L2}}$$

$$\Rightarrow \frac{dk}{dt} = -C_D \frac{k^{5/2}}{(kl)} = -C_D \frac{k^{5/2}}{(k/k_0)^{C_{L2}/C_D}}$$

$$= -\text{const}\ k^{5/2 - \frac{C_{L2}}{C_D}} \Rightarrow \frac{dk}{k^{5/2 - C_{L2}/C_D}} \sim dt$$

$$\Rightarrow k \sim t^{-\frac{5}{2} + \frac{C_{L2}}{C_D} + 1} \Rightarrow n = \frac{3}{2} - \frac{C_{L2}}{C_D} = 0{,}84$$

k-τ :

$$\begin{cases} \dfrac{dk}{dt} = -\dfrac{k}{\tau} \\ \dfrac{d\tau}{dt} = C_{\epsilon 2} - 1 \Rightarrow \tau = (C_{\epsilon 2} - 1)\, t \end{cases}$$

$$\frac{dk}{dt} = -\frac{k}{(C_{\epsilon 2} - 1)t} \Rightarrow \frac{dk}{k} = -\frac{1}{C_{\epsilon 2} - 1} \frac{dt}{t}$$

$$\Rightarrow k \sim t^{-\frac{1}{C_{\epsilon 2} - 1}} \Rightarrow n = \frac{1}{C_{\epsilon 2} - 1} = 0{,}83$$

k-ε :

$$\begin{cases} \dfrac{dk}{dt} = -\epsilon \\ \dfrac{d\epsilon}{dt} = -C_{\epsilon 2} \dfrac{\epsilon^2}{k} \end{cases}$$

$$\Rightarrow \frac{dk}{d\epsilon} = \frac{1}{C_{\epsilon 2}} \frac{k}{\epsilon} \Rightarrow \frac{dk}{k} = \frac{1}{C_{\epsilon 2}} \frac{d\epsilon}{\epsilon}$$

$$\Rightarrow k \sim \epsilon^{1/C_{\epsilon 2}} \Rightarrow \epsilon \sim k^{C_{\epsilon 2}}$$

$$\frac{dk}{dt} \sim -k^{C_{\epsilon 2}} \Rightarrow \frac{dk}{k^{C_{\epsilon 2}}} \sim -dt \Rightarrow k^{1 - C_{\epsilon 2}} \sim t$$

$$\Rightarrow k \sim t^{1-C_{\epsilon 2}} \quad \Rightarrow n = \frac{1}{C_{\epsilon 2}-1} = 1{,}47$$