


# Information sheet

$x$  :   $x$  positions

$dx$  :  scalar cell size

$x_p$  :   $\frac{1}{2}(x[i-1] + x[i])$  scalar nodes position.

$dx_p$  :  velocity cell size

$x_u$  :  Velocity node position.

$dx_u$  :  scalar cell size

$x_p$  : 

↓

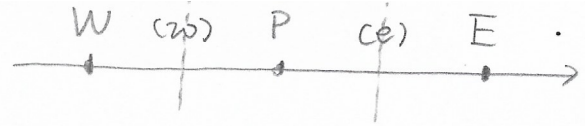
$G_{mx}/\rho$

$x_u$  : 

↓

$G_{mx-ew}/\rho_{ew}$

terme convective:  $\frac{d}{dx}(\rho u \Phi) = 0$



$$\int_w^e \frac{d}{dx}(\rho u \Phi) dx = \rho_e u_e \Phi_e - \rho_w u_w \Phi_w = 0.$$

Schema Upwind:  $\bar{\Phi}_e = \begin{cases} \bar{\Phi}_P & (u_e > 0) \\ \bar{\Phi}_E & (u_e < 0) \end{cases}$   $\bar{\Phi}_w = \begin{cases} \bar{\Phi}_w & (u_w > 0) \\ \bar{\Phi}_P & (u_w < 0) \end{cases}$

\*  $F_i = \rho_i u_i$  \*  $a_P \bar{\Phi}_P = a_E \bar{\Phi}_E + a_w \bar{\Phi}_w + b_P$

$$\Rightarrow \underbrace{[a_E + a_w + (F_e - F_w)]}_{a_P} \bar{\Phi}_P = \underbrace{\max(F_w, 0)}_{a_E} \bar{\Phi}_E + \underbrace{\max(-F_e, 0)}_{a_w} \bar{\Phi}_w.$$

terme diffusive:  $\frac{d}{dx}(\Gamma \frac{d\Phi}{dx}) = 0 \Rightarrow \Gamma_e (\frac{d\Phi}{dx})_e - \Gamma_w (\frac{d\Phi}{dx})_w = 0$

$$\underbrace{\Gamma_e \frac{\bar{\Phi}_E - \bar{\Phi}_P}{\Delta x}}_{D_e} - \underbrace{\Gamma_w \frac{\bar{\Phi}_w - \bar{\Phi}_P}{\Delta x}}_{D_w} = 0 \quad \underbrace{(D_w - D_e)}_{a_P} \bar{\Phi}_P = \underbrace{-D_e}_{a_E} \bar{\Phi}_E + \underbrace{D_w}_{a_w} \bar{\Phi}_w$$

Schema hybride  $R = \frac{\rho u \Delta x}{\Gamma} = \frac{F}{D}$

$\begin{cases} |R| < 2 & \text{Centré} \\ |R| > 2 & \text{upwind + diffu.} \rightarrow 0. \end{cases}$

(\* convective, centrée:  $\frac{F_e - F_w}{2} \bar{\Phi}_P = -\frac{F_e}{2} \bar{\Phi}_E + \frac{F_w}{2} \bar{\Phi}_w$ )

$|R| > 2$  aux limites: (reprenons les formules de la partie diffusion)

WEST:  $\Delta y \left[ \underbrace{\left(-\frac{\Gamma_0}{\Delta x_0} - \frac{2\Gamma_A}{\Delta x_0}\right)}_{a_P} \bar{\Phi}_0 + \underbrace{\frac{\Gamma_0}{\Delta x_0} \bar{\Phi}_1}_{a_E} \right] + \underbrace{\Gamma_A \frac{2\bar{\Phi}_A}{\Delta x_0}}_{-S_A} = 0$

négliger les termes diffusion internes  $\Rightarrow$  supprimer les termes

avec  $\Gamma_0 \Rightarrow a_P = -\frac{2\Gamma_A}{\Delta x_0}$   $a_E = 0$   $S_A = -\Gamma_A \frac{2\bar{\Phi}_A}{\Delta x_0}$

Idem pour EST



C.L. upwind

methode 1

$$\begin{array}{c} \Phi_A \quad \Phi_0 \quad \Phi_0^* \quad \Phi_1 \\ | \quad | \quad | \quad | \\ \text{---} \end{array} \quad \left( \rho_e u_e \Phi_e^* - \rho_w u_w \Phi_w^* \right) \Big|_{i=0} = \rho_0 u_0 (\alpha_0 \Phi_0 + \alpha_0^+ \Phi_1) - \rho_A u_A \Phi_A$$

$$= \underbrace{(\rho_0 u_0 \alpha_0) \Phi_0}_{A_p} + \underbrace{(\rho_0 u_0 \alpha_0^+) \Phi_1}_{A_E \text{ (inchange)}} - \underbrace{\rho_A u_A \Phi_A}_{-S_A}$$

Method 2  $\left( \rho_e u_e \Phi_e^* - \rho_w u_w \Phi_w^* \right) \Big|_{i=0} = \rho_0 u_0 (\alpha_0 \Phi_0 + \alpha_0^+ \Phi_1) - \rho_A u_A (\alpha_A \Phi_A + \alpha_A^+ \Phi_0)$

$$= \underbrace{(\rho_0 u_0 \alpha_0 - \rho_A u_A \alpha_A^+) \Phi_0}_{A_p} + \underbrace{\rho_0 u_0 \alpha_0^+ \Phi_1}_{A_E \text{ (inchange)}} - \underbrace{\rho_A u_A \alpha_A \Phi_A}_{-S_A}$$

\* En Générale:  $\left( \rho_e u_e \Phi_e^* - \rho_w u_w \Phi_w^* \right) \Big|_i = \rho_i u_i (\alpha_i \Phi_i + \alpha_i^+ \Phi_{i+1}) - \rho_{i-1} u_{i-1} (\alpha_{i-1} \Phi_{i-1} + \alpha_{i-1}^+ \Phi_i)$

(AA·Φ)<sub>i</sub>

$$= \underbrace{\rho_{i-1} u_{i-1} \alpha_{i-1} \Phi_{i-1}}_{A_w} + \underbrace{(\rho_i u_i \alpha_i - \rho_{i-1} u_{i-1} \alpha_{i-1}^+) \Phi_i}_{A_p} + \underbrace{\rho_i u_i \alpha_i^+ \Phi_{i+1}}_{A_E}$$

$\alpha_i = (u_i > 0) \quad \alpha_i^+ = (u_i < 0)$

$$\begin{array}{c} \Phi_{n-2} \quad \Phi_{n-1}^* \quad \Phi_{n-1} \quad \Phi_B u_B \\ | \quad | \quad | \quad | \\ \text{---} \end{array} \quad \left( \rho_e u_e \Phi_e^* - \rho_w u_w \Phi_w^* \right) \Big|_{i=n-1} = \rho_B u_B \Phi_B - \rho_{n-1} u_{n-1} (\alpha_{n-1} \Phi_{n-2} + \alpha_{n-1}^+ \Phi_{n-1})$$

↑  
methode 1

$$= \underbrace{-\rho_{n-1} u_{n-1} \alpha_{n-1} \Phi_{n-2}}_{A_w} - \underbrace{\rho_{n-1} u_{n-1} \alpha_{n-1}^+ \Phi_{n-1}}_{A_p} + \underbrace{\rho_B u_B \Phi_B}_{-S_B}$$

methode 2:  $\left( \rho_e u_e \Phi_e^* - \rho_w u_w \Phi_w^* \right) \Big|_{i=n-1} = \rho_B u_B (\alpha_B \Phi_{n-1} + \alpha_B^+ \Phi_B) - \rho_{n-1} u_{n-1} (\alpha_{n-1} \Phi_{n-2} + \alpha_{n-1}^+ \Phi_{n-1})$

$$= \underbrace{-\rho_{n-1} u_{n-1} \alpha_{n-1} \Phi_{n-2}}_{A_w} + \underbrace{(\rho_B u_B \alpha_B - \rho_{n-1} u_{n-1} \alpha_{n-1}^+) \Phi_{n-1}}_{A_p} + \underbrace{\rho_B u_B \alpha_B^+ \Phi_B}_{-S_B}$$

C.L. diffusive

$$\begin{array}{c} \Phi_A \quad \Phi_0 \quad \Phi_0^* \quad \Phi_1 \\ | \quad | \quad | \quad | \\ \text{---} \end{array} \quad \Delta y \left( \Gamma_e \frac{d\Phi}{dx} \Big|_e - \Gamma_w \frac{d\Phi}{dx} \Big|_w \right) = 0$$

$$= \Delta y \left( \Gamma_0 \frac{\Phi_1 - \Phi_0}{\Delta x_{p_0}} - \Gamma_A \frac{\Phi_0 - \Phi_A}{\Delta x_{0/2}} \right)$$

$$= \Delta y \left[ \underbrace{\left( -\frac{\Gamma_0}{\Delta x_{p_0}} - \frac{2\Gamma_A}{\Delta x_0} \right) \Phi_0}_{A_p} + \underbrace{\frac{\Gamma_0}{\Delta x_{p_0}} \Phi_1}_{A_E} \right] + \underbrace{\Gamma_A \frac{2\Phi_A}{\Delta x_0}}_{-S_A}$$

$$\begin{array}{c} \Phi_{n-2} \quad \Phi_{n-1}^* \quad \Phi_{n-1} \quad \Phi_B \\ | \quad | \quad | \quad | \\ \text{---} \end{array} \quad \Delta y \left( \Gamma_e \frac{d\Phi}{dx} \Big|_e - \Gamma_w \frac{d\Phi}{dx} \Big|_w \right) = \Delta y \left( \Gamma_B \frac{\Phi_{n-1} - \Phi_B}{\Delta x_{n-1/2}} - \Gamma_{n-1} \frac{\Phi_{n-2} - \Phi_{n-1}}{\Delta x_{p_{n-1}}} \right)$$

$$= \Delta y \left[ \underbrace{\left( -\frac{\Gamma_{n-1}}{\Delta x_{p_{n-1}}} \right) \Phi_{n-2}}_{A_w} + \underbrace{\left( \frac{2\Gamma_B}{\Delta x_{n-1}} + \frac{\Gamma_{n-1}}{\Delta x_{p_{n-1}}} \right) \Phi_{n-1}}_{A_p} - \underbrace{\frac{2\Gamma_B}{\Delta x_{n-1}} \Phi_B}_{-S_B} \right]$$

# Condition aux limites Quick:

$$\underline{\text{Int}} = (\alpha_i) \begin{bmatrix} W & P & E & EE \\ \alpha_i & \alpha_i^+ & \alpha_i^{++} \\ \alpha_i^- & & \alpha_i^+ \\ & & & \alpha_i^{++} \end{bmatrix}$$

$$\alpha_i^- = (-\frac{1}{8}) \times (U_e > 0)$$

$$\alpha_i = \frac{3}{8} + (\frac{3}{8}) \times (U_e > 0)$$

$$\alpha_i^+ = \frac{3}{8} + (\frac{3}{8}) \times (U_e < 0)$$

$$\alpha_i^{++} = (-\frac{1}{8}) \times (U_e < 0)$$

	W	P	E	EE
indice	i-1	i	i+1	i+2
$U_e > 0$	$-\frac{1}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	
$U_e < 0$		$\frac{3}{8}$	$\frac{6}{8}$	$-\frac{1}{8}$

\* (General)

$$\begin{aligned} \underline{AA} \bar{\Phi}_i &= P_{i,i} U_{i,i} \bar{\Phi}_i^* - P_{i-1,i} U_{i-1,i} \bar{\Phi}_{i-1}^* \\ &= P_{i,i} U_{i,i} (\alpha_i^- \bar{\Phi}_{i-1} + \alpha_i \bar{\Phi}_i + \alpha_i^+ \bar{\Phi}_{i+1} + \alpha_i^{++} \bar{\Phi}_{i+2}) \\ &\quad - P_{i-1,i} U_{i-1,i} (\alpha_{i-1}^- \bar{\Phi}_{i-2} + \alpha_{i-1} \bar{\Phi}_{i-1} + \alpha_{i-1}^+ \bar{\Phi}_i + \alpha_{i-1}^{++} \bar{\Phi}_{i+1}) \\ &= \underbrace{(P_{i-1,i} U_{i-1,i} \alpha_{i-1}^-)}_{a_{WW}} \bar{\Phi}_{i-2} + \underbrace{(P_{i,i} U_{i,i} \alpha_i^- - P_{i-1,i} U_{i-1,i} \alpha_{i-1}^-)}_{a_W} \bar{\Phi}_{i-1} \\ &\quad + \underbrace{(P_{i,i} U_{i,i} \alpha_i - P_{i-1,i} U_{i-1,i} \alpha_{i-1}^+)}_{a_P} \bar{\Phi}_i + \underbrace{(P_{i,i} U_{i,i} \alpha_i^+ - P_{i-1,i} U_{i-1,i} \alpha_{i-1}^{++})}_{a_E} \bar{\Phi}_{i+1} + \underbrace{(P_{i,i} U_{i,i} \alpha_i^{++})}_{a_{EE}} \bar{\Phi}_{i+2} \end{aligned}$$

C.L (Est)  $\xrightarrow{U_0}$   $\begin{array}{c} W \quad \bar{\Phi}_A \quad \bar{\Phi}_0 \quad \bar{\Phi}_1 \\ \times \quad \circ \quad \circ \quad \circ \\ \hline U_A \quad x_p[0] \quad x_p[1] \end{array}$

$$\underline{AA} \bar{\Phi}|_{i=0} = P_0 U_0 \bar{\Phi}_0^* - P_A U_A \bar{\Phi}_A ; \quad \bar{\Phi}_0^* = \begin{cases} U_e < 0 & \frac{3}{8} \bar{\Phi}_0 + \frac{6}{8} \bar{\Phi}_1 - \frac{1}{8} \bar{\Phi}_2 \quad (\checkmark) \\ U_e > 0 & -\frac{1}{8} \bar{\Phi}_{-1} + \frac{6}{8} \bar{\Phi}_0 + \frac{3}{8} \bar{\Phi}_1 \quad (\times) \end{cases}$$

②  $U_e > 0 \Rightarrow$  Interpolation:

$$\begin{array}{c} \bar{\Phi}_A \quad \bar{\Phi}_0 \quad \bar{\Phi}_0^* \quad \bar{\Phi}_1 \\ \circ \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \\ \hline x = \frac{x}{\Delta x} \end{array}$$

$$\bar{\Phi}(x) = \frac{(x-\frac{1}{2})(x-\frac{3}{2})}{(0-\frac{1}{2})(0-\frac{3}{2})} \bar{\Phi}_A + \frac{(x-0)(x-\frac{3}{2})}{(\frac{1}{2}-0)(\frac{1}{2}-\frac{3}{2})} \bar{\Phi}_0 + \frac{(x-0)(x-\frac{1}{2})}{(\frac{3}{2}-0)(\frac{3}{2}-\frac{1}{2})} \bar{\Phi}_1$$

$$\Rightarrow \bar{\Phi}_0^* = \bar{\Phi}(x=1) = -\frac{1}{3} \bar{\Phi}_A + \bar{\Phi}_0 + \frac{1}{3} \bar{\Phi}_1$$

$$\begin{aligned} \Rightarrow \underline{AA} \bar{\Phi}|_{i=0} &= P_0 U_0 (-\frac{1}{3} \bar{\Phi}_A + \bar{\Phi}_0 + \frac{1}{3} \bar{\Phi}_1) - P_A U_A \bar{\Phi}_A \\ &= \underbrace{P_0 U_0 \bar{\Phi}_0}_{a_P^*} + \underbrace{\frac{1}{3} P_0 U_0 \bar{\Phi}_1}_{a_E^*} - \underbrace{(\frac{1}{3} P_0 U_0 + P_A U_A) \bar{\Phi}_A}_{-S_A^*} \end{aligned}$$

\* observ.

$a_{EE}^*$  inchangé.

$$\begin{aligned} \text{if } U_e < 0 : \underline{AA} \bar{\Phi}|_{i=0} &= P_0 U_0 (\frac{3}{8} \bar{\Phi}_0 + \frac{6}{8} \bar{\Phi}_1 - \frac{1}{8} \bar{\Phi}_2) - P_A U_A \bar{\Phi}_A \\ &= \underbrace{\frac{3}{8} P_0 U_0 \bar{\Phi}_0}_{a_P^*} + \underbrace{\frac{6}{8} P_0 U_0 \bar{\Phi}_1}_{a_E^*} - \underbrace{\frac{1}{8} P_0 U_0 \bar{\Phi}_2}_{a_{EE}^*} - \underbrace{P_A U_A \bar{\Phi}_A}_{-S_A^*} \end{aligned}$$



$$\underline{AA} \underline{\Phi} |_{i=0} = \rho_0 U_0 \bar{\Phi}_0^* - \rho_A U_A \bar{\Phi}_A \quad \text{if } \bar{\Phi}_0^* = -\frac{11}{8} \bar{\Phi}_{-1} + \frac{6}{8} \bar{\Phi}_0 + \frac{3}{8} \bar{\Phi}_1 \quad (U_0 > 0)$$

$$* \quad \bar{\Phi}_{-1} = \frac{\bar{\Phi}_0 + \bar{\Phi}_1}{2} \Rightarrow \boxed{\bar{\Phi}_{-1} = 2\bar{\Phi}_A - \bar{\Phi}_0} \quad \left(-\frac{1}{8} \bar{\Phi}_A - \bar{\Phi}_0 + \bar{\Phi}_1\right)$$

$$\begin{aligned} \textcircled{1} U_0 > 0 \quad \underline{AA} \underline{\Phi} |_{i=0} &= \rho_0 U_0 \left[ -\frac{1}{8} (2\bar{\Phi}_A - \bar{\Phi}_0) + \frac{6}{8} \bar{\Phi}_0 + \frac{3}{8} \bar{\Phi}_1 \right] - \rho_A U_A \bar{\Phi}_A \\ &= \underbrace{\frac{7}{8} \rho_0 U_0 \bar{\Phi}_0}_{A_P} + \underbrace{\frac{3}{8} \rho_0 U_0 \bar{\Phi}_1}_{A_E} - \underbrace{\left( \frac{\rho_0 U_0}{4} + \rho_A U_A \right) \bar{\Phi}_A}_{-S_A} \end{aligned}$$

$$\begin{aligned} \textcircled{2} U_0 < 0 \quad \underline{AA} \underline{\Phi} |_{i=0} &= \rho_0 U_0 \left( \frac{3}{8} \bar{\Phi}_0 + \frac{6}{8} \bar{\Phi}_1 - \frac{1}{8} \bar{\Phi}_2 \right) - \rho_A U_A \bar{\Phi}_A \\ &= \underbrace{\frac{3}{8} \rho_0 U_0 \bar{\Phi}_0}_{A_P^*} + \underbrace{\frac{6}{8} \rho_0 U_0 \bar{\Phi}_1}_{A_E^*} - \underbrace{\frac{1}{8} \rho_0 U_0 \bar{\Phi}_2}_{A_{EE}^*} - \underbrace{\rho_A U_A \bar{\Phi}_A}_{-S_A} \end{aligned}$$

$$\begin{aligned} \underline{AA} \underline{\Phi} |_{i=1} &= \rho_1 U_1 \bar{\Phi}_1^* - \rho_0 U_0 \bar{\Phi}_0^* = \left( * \quad U_0 > 0 \quad \bar{\Phi}_0^* = \left( -\frac{1}{4} \bar{\Phi}_A + \frac{7}{8} \bar{\Phi}_0 + \frac{3}{8} \bar{\Phi}_1 \right) \right) \\ &= \rho_1 U_1 (\alpha_1^- \bar{\Phi}_0 + \alpha_1 \bar{\Phi}_1 + \alpha_1^+ \bar{\Phi}_2 + \alpha_1^{++} \bar{\Phi}_3) - \rho_0 U_0 \left( -\frac{1}{4} \bar{\Phi}_A + \frac{7}{8} \bar{\Phi}_0 + \frac{3}{8} \bar{\Phi}_1 \right) \\ &= \underbrace{(\rho_1 U_1 \alpha_1^- - \frac{7}{8} \rho_0 U_0) \bar{\Phi}_0}_{A_W} + \underbrace{(\rho_1 U_1 \alpha_1 - \frac{3}{8} \rho_0 U_0) \bar{\Phi}_1}_{\text{inchange}} + (xx) \bar{\Phi}_2 + (xx) \bar{\Phi}_3 + \underbrace{\frac{\rho_0 U_0}{4} \bar{\Phi}_A}_{-S_1} \end{aligned}$$

$$* \quad U_0 < 0 : \text{inchange} \Rightarrow A_W = \rho_1 U_1 \alpha_1^- - \rho_0 U_0 \alpha_0 = \rho_1 U_1 \alpha_1^- - \rho_0 U_0 \frac{8}{3}$$

$$\underline{AA} \underline{\Phi} |_{i=-1} = \rho_B U_B \bar{\Phi}_B - \rho_{n-1} U_{n-1} \bar{\Phi}_{n-2}^*$$

$$\bar{\Phi}_{n-2}^* = (\alpha_{n-1}^- \bar{\Phi}_{n-3} + \alpha_{n-1} \bar{\Phi}_{n-2} + \alpha_{n-1}^+ \bar{\Phi}_{n-1} + \alpha_{n-1}^{++} \bar{\Phi}_n)$$

$$U_{n-1} > 0 \Rightarrow \text{OK} \quad U_{n-1} < 0 \quad \bar{\Phi}_n? \Rightarrow \bar{\Phi}_n = \bar{\Phi}_E = 2\bar{\Phi}_B - \bar{\Phi}_{n-1}$$

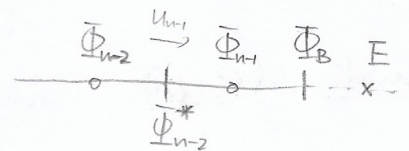
$$\begin{aligned} \textcircled{1} U_{n-1} < 0 : \rho_B U_B \bar{\Phi}_B - \rho_{n-1} U_{n-1} \left[ \frac{3}{8} \bar{\Phi}_{n-2} + \frac{6}{8} \bar{\Phi}_{n-1} - \frac{1}{8} (2\bar{\Phi}_B - \bar{\Phi}_{n-1}) \right] &= 0 \\ \underbrace{-\rho_{n-1} U_{n-1} \times \frac{3}{8} \bar{\Phi}_{n-2}}_{A_W} - \underbrace{\rho_{n-1} U_{n-1} \frac{7}{8} \bar{\Phi}_{n-1}}_{A_P} + \underbrace{(\rho_B U_B + \frac{1}{4} \rho_{n-1} U_{n-1}) \bar{\Phi}_B}_{-S_B} &= 0 \end{aligned}$$

$$\begin{aligned} \textcircled{2} U_{n-1} > 0 \quad \rho_B U_B \bar{\Phi}_B - \rho_{n-1} U_{n-1} \left( -\frac{1}{8} \bar{\Phi}_{n-3} + \frac{6}{8} \bar{\Phi}_{n-2} + \frac{3}{8} \bar{\Phi}_{n-1} \right) &= 0 \\ \underbrace{+\frac{1}{8} \rho_{n-1} U_{n-1} \bar{\Phi}_{n-3}}_{A_{WW}} - \underbrace{\frac{6}{8} \rho_{n-1} U_{n-1} \bar{\Phi}_{n-2}}_{A_W} - \underbrace{\frac{3}{8} \rho_{n-1} U_{n-1} \bar{\Phi}_{n-1}}_{A_P} + \underbrace{\rho_B U_B \bar{\Phi}_B}_{-S_B} &= 0 \end{aligned}$$

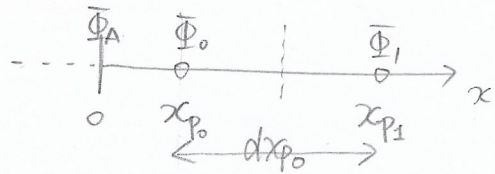
$$\underline{AA} \underline{\Phi} |_{i=-2} \text{ (inchange quand } U_{n-2} > 0 \text{)}$$

$$* \quad U_{n-2} < 0 : A_E \text{ va changer : } A_E = \rho_{n-1} U_{n-1} \frac{7}{8} - \rho_{n-2} U_{n-2} \alpha_{n-2}^{++} \quad S_{-2} = \frac{\rho_{n-1} U_{n-1}}{4} \bar{\Phi}_B$$

$$* \quad U_{n-2} > 0 \text{ (inchange) } A_E = \rho_{n-1} U_{n-1} \frac{3}{8} - \rho_{n-2} U_{n-2} \alpha_{n-2}^{++}$$



# diffusion pour Quick (interpolation 3 pts)



$$\bar{\Phi}(x) = \bar{\Phi}_A \frac{(x-x_{p_1})(x-x_{p_2})}{x_{p_0}x_{p_1}} + \bar{\Phi}_0 \frac{x(x-x_{p_2})}{-x_{p_0}(x_{p_0}-x_{p_1})} + \bar{\Phi}_1 \frac{x(x-x_{p_0})}{x_{p_1}(x_{p_1}-x_{p_0})} \quad (dx_{p_0} = x_{p_1} - x_{p_0})$$

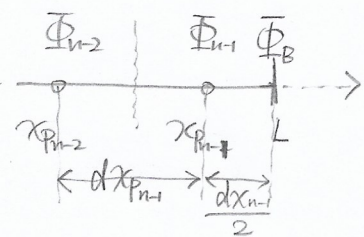
$$\Rightarrow \bar{\Phi}'(x) = \frac{\bar{\Phi}_A}{x_{p_0}x_{p_1}} (2x - (x_{p_0} + x_{p_1})) - \frac{\bar{\Phi}_0}{x_{p_0}dx_{p_0}} (2x - x_{p_1}) + \frac{\bar{\Phi}_1 (2x - x_{p_0})}{x_{p_1}dx_{p_0}}$$

$$\bar{\Phi}'(x=0) = \frac{d\bar{\Phi}}{dx} \Big|_A = -\bar{\Phi}_A \frac{(x_{p_0} + x_{p_1})}{x_{p_0}x_{p_1}} + \frac{x_{p_1}}{x_{p_0}} \frac{\bar{\Phi}_0}{dx_{p_0}} - \frac{x_{p_0}}{x_{p_1}} \frac{\bar{\Phi}_1}{dx_{p_0}}$$

diffusion:  $\Gamma_e \bar{\Phi}'_e - \Gamma_w \bar{\Phi}'_w \Big|_{i=0} = \Gamma_0 \frac{\bar{\Phi}_1 - \bar{\Phi}_0}{dx_{p_0}} - \Gamma_A \left[ -\bar{\Phi}_A \frac{x_{p_0} + x_{p_1}}{x_{p_0}x_{p_1}} + \frac{x_{p_1}}{x_{p_0}} \frac{\bar{\Phi}_0}{dx_{p_0}} - \frac{x_{p_0}}{x_{p_1}} \frac{\bar{\Phi}_1}{dx_{p_0}} \right]$

$$= \underbrace{\left( -\frac{\Gamma_A}{dx_{p_0}} \frac{x_{p_1}}{x_{p_0}} - \frac{\Gamma_0}{dx_{p_0}} \right)}_{a_p} \bar{\Phi}_0 + \underbrace{\left( \frac{x_{p_0}}{x_{p_1}} \frac{\Gamma_A}{dx_{p_0}} + \frac{\Gamma_0}{dx_{p_0}} \right)}_{a_E} \bar{\Phi}_1 + \underbrace{\Gamma_A \frac{x_{p_0} + x_{p_1}}{x_{p_1}x_{p_0}}}_{-S_A} \bar{\Phi}_A$$

Idem (Est)



$$\bar{\Phi}'_B = \bar{\Phi}_{n-2} \frac{dx_{p_{n-1}}/2}{dx_{p_{n-1}}(dx_{p_{n-1}} + dx_{p_{n-1}}/2)} - \frac{dx_{p_{n-1}} + \frac{dx_{p_{n-1}}}{2}}{dx_{p_{n-1}} \times \frac{dx_{p_{n-1}}}{2}} \bar{\Phi}_{n-1} + \frac{dx_{p_{n-1}} + dx_{p_{n-1}}}{\frac{dx_{p_{n-1}}}{2}(dx_{p_{n-1}} + \frac{dx_{p_{n-1}}}{2})} \bar{\Phi}_B$$

diffusion:  $\Gamma_B \bar{\Phi}'_B - \Gamma_{n-1} \frac{\bar{\Phi}_{n-1} - \bar{\Phi}_{n-2}}{dx_{p_{n-1}}}$

$$= \left( \Gamma_B \frac{dx_{p_{n-1}}/2}{dx_{p_{n-1}}(dx_{p_{n-1}} + dx_{p_{n-1}}/2)} + \frac{\Gamma_{n-1}}{dx_{p_{n-1}}} \right) \bar{\Phi}_{n-2} - \left( \Gamma_B \frac{dx_{p_{n-1}} + \frac{dx_{p_{n-1}}}{2}}{dx_{p_{n-1}} \times \frac{dx_{p_{n-1}}}{2}} + \frac{\Gamma_{n-1}}{dx_{p_{n-1}}} \right) \bar{\Phi}_{n-1} + \frac{\Gamma_B (dx_{p_{n-1}} + dx_{p_{n-1}})}{\frac{dx_{p_{n-1}}}{2}(dx_{p_{n-1}} + \frac{dx_{p_{n-1}}}{2})} \bar{\Phi}_B$$