Turbomachinery – Summary of Equations

Course MJ2430/MJ2244

Gas Equations

Polytropic relation $pv^n = const.$

Polytropic coefficient n

Expansion process $\frac{n-1}{n} = \eta_p \frac{\gamma - 1}{\gamma}$

Compression process $\frac{n-1}{n} = \frac{1}{\eta_p}\frac{\gamma - 1}{\gamma}$

Note: for $\eta_p = 1$ → $n = \gamma$ → isentropic (i.e. ideal)

Equivalent identities $p^{\frac{1-n}{n}} T = const.$ → isentropic $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$

$p\rho^{-n} = const.$

$c_p T = \frac{\gamma}{\gamma - 1} pv$ $\quad \frac{T_{01}}{T_{02s}} = \left(\frac{p_{01}}{p_{02s}}\right)^{\frac{\gamma-1}{\gamma}}$

Enthalpy $h = c_p \cdot T$

Enthalpy difference (polytropic, expansion) $\Delta h_{01\to 02} = c_p(T_{01} - T_{02}) = c_p T_{01}\left[1 - \left(\frac{p_{02}}{p_{01}}\right)^{\eta_p \frac{\gamma-1}{\gamma}}\right]$

Enthalpy difference (isentropic) $\Delta h_{01\to 02} = \eta_{tt} c_p T_{01}\left[1 - \left(\frac{p_{02s}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]$

Compressible Analysis

Mach number $M = \left[\frac{2}{\gamma - 1} \left[\left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right]^{\frac{1}{2}} = \left[\frac{2}{\gamma - 1} \left[\left(\frac{T_0}{T} \right) - 1 \right] \right]^{\frac{1}{2}}$

Speed of sound $a = \sqrt{\gamma R T}$

Velocity $c = M \cdot a = \left[\frac{2\gamma R T}{\gamma - 1} \left[\left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right]^{\frac{1}{2}}$

Energy, Mass and Momentum Balance

Power $\dot{W} = \dot{m} \Delta h_0 \qquad \dot{W} \to \left[\frac{J}{s} \right],\ \dot{m} \to \left[\frac{kg}{s} \right],\ \Delta h_0 \to \left[\frac{J}{kg} \right]$

Expressed by torque $\dot{W} = M\omega \qquad M \to [Nm],\ \omega \to \left[\frac{rad}{s} \right]$

Rotational frequency $\omega = \frac{\pi n}{30} \qquad n \to [rpm]$

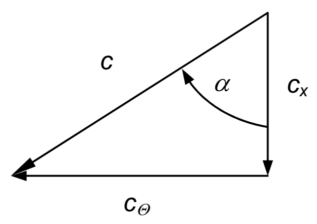
Mass balance $\dot{m} = \Omega \cdot \rho \cdot cn = \Omega \cdot \rho \cdot \phi \cdot u$

Euler equation $h_{01} - h_{02} = u_1 c_{\theta 1} - u_2 c_{\theta 2}$

Stagnation enthalpy (→ constant in stator) $h_0 = h + \frac{1}{2} c^2$

Rothalpy (→ constant in rotor) $I = h + \frac{1}{2} c^2 - u \cdot c_\theta = h + \frac{1}{2} w^2 - \frac{1}{2} u^2$

Trigonometry

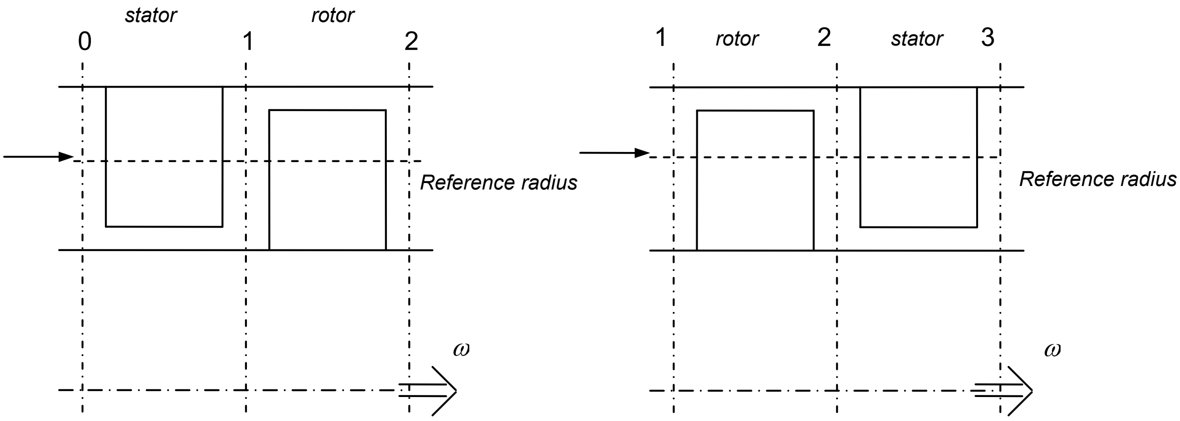


$$\tan \alpha = \frac{c_\theta}{c_x}$$

$$\cos \alpha = \frac{c_x}{c}$$

$$\sin \alpha = \frac{c_\theta}{c}$$

Turbomachinery Stages



Turbine Compressor

Design Parameters

Assumption: normal repetition stage: turbine: $\vec{c}_0 = \vec{c}_2$, $c_{x,0} = c_{x,1} = c_{x,2} = const$, $u_0 = u_1 = u_2$

compressor $\vec{c}_1 = \vec{c}_3$, $c_{x,1} = c_{x,2} = c_{x,3} = const$, $u_1 = u_2 = u_3$

Parameter	Turbine	Compressor
Degree of reaction $R = \frac{\Delta h_{rotor}}{\Delta h_{stage}}$	$R = -\frac{1}{2u}\left(w_{\theta,2} + w_{\theta,1}\right)$	$R = -\frac{1}{2u}\left(w_{\theta,1} + w_{\theta,2}\right)$
	$R = \frac{1}{2} - \frac{c_x}{2u}\left(\tan\beta_2 + \tan\alpha_1\right)$	$R = \frac{1}{2} - \frac{c_x}{2u}\left(\tan\alpha_1 + \tan\beta_2\right)$
Loading coefficient $\psi = \frac{\Delta h_0}{u^2}$	$\psi = \frac{c_{\theta,1} - c_{\theta,2}}{u}$	$\psi = \frac{c_{\theta,2} - c_{\theta,1}}{u}$
	$\psi = -1 + \frac{c_x}{u}\left(\tan\alpha_1 - \tan\beta_2\right)$	$\psi = 1 + \frac{c_x}{u}\left(\tan\beta_2 - \tan\alpha_1\right)$
Flow coefficient $\phi = \frac{c_x}{u}$	$\phi = \frac{c_x}{u}$	$\phi = \frac{c_x}{u}$

Multistage machine with z similar stages $\Delta h_{0,\alpha\to\omega} = \sum_{i=1}^{z} \psi_i u_i^2 = z \cdot \psi \cdot u^2$

Engine Geometry

Radius ratio	$Y = \frac{r_s}{r_h}$	Hub radius	$r_h = \frac{2r_m}{Y+1}$
Blade length	$s = r_s - r_h$	Shroud radius	$r_s = \frac{2Yr_m}{Y+1}$
Mean radius	$r_m = \frac{r_h + r_s}{2}$	Cross section	$\Omega = \pi\left(r_s^2 - r_h^2\right) = 4\pi r_m^2 \frac{Y-1}{Y+1} = 2\pi \cdot r_m s$

Radial compressors

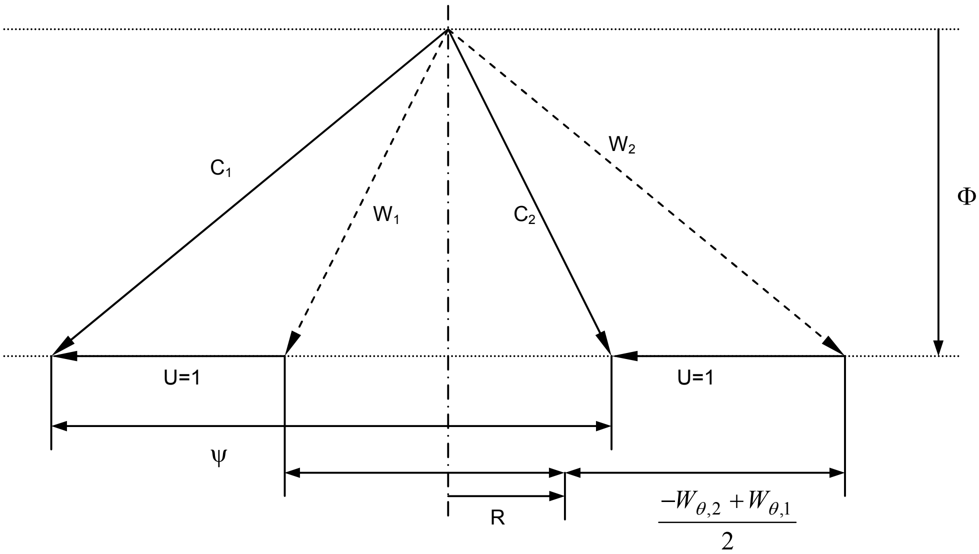
Slip factor definition $\sigma = \frac{c_{\theta 2}}{c_{\theta 2}'}$

Slip factor by Stodola $\sigma = 1 - \frac{\pi}{n} \cdot \frac{\cos \beta_2'}{1 - \phi_2 \tan \beta_2'}$

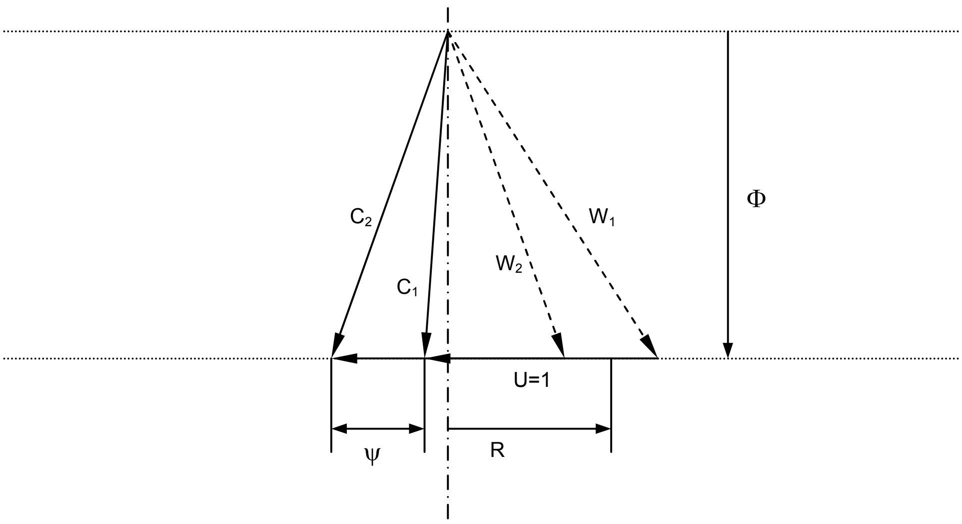
Slip factor by Stanitz $\sigma = 1 - \frac{\pi}{n} \cdot \frac{0.63}{1 - \phi_2 \tan \beta_2'}$

Velocity Triangles

Normalized velocity triangle turbine stage



Normalized velocity triangle compressor stage



Jet Engine Cycles

Total pressure at intake outlet

$$p_{01} = p_a \left[1 + \eta_i \frac{c_a^2}{2 c_p T_a} \right]^{\gamma / (\gamma - 1)}$$

Critical nozzle pressure ratio

$$\frac{p_{04}}{p_c} = \frac{1}{\left[1 - \frac{1}{\eta_j} \left(\frac{\gamma - 1}{\gamma + 1} \right) \right]^{\gamma / (\gamma - 1)}}$$

Specific thrust (turbojet, choked nozzle)

$$F_s = \left(c_j - c_a \right) + \frac{A_j}{\dot{m}} \left(p_c - p_a \right)$$

Specific thrust (turbojet, unchoked nozzle)

$$F_s = \left(c_j - c_a \right)$$

Specific thrust (turbofan, unchoked nozzle)

$$F_s = \left(c_{jc} - c_a \right) + BPR \cdot \left(c_{jb} - c_a \right)$$

Energy balance fan – LPT

$$\left(1 + BPR \right) \left| \Delta h_{0,fan} \right| = \left| \Delta h_{0,LPT} \right| \cdot \eta_{mech}$$

Bypass ratio

$$BPR = \frac{\dot{m}_{bypass}}{\dot{m}_{core}}$$

Fan mass flow

$$\dot{m}_{fan} = \dot{m}_{core} \cdot \left(1 + BPR \right)$$

p_a	ambient static pressure	η_i	isentropic intake efficiency
T_a	ambient static temperature	η_j	isentropic nozzle efficiency
p_{01}	total pressure at intake outlet	η_{mech}	mechanical efficiency
p_{04}	total pressure at nozzle inlet	F_s	specific thrust
p_c	critical static pressure (at nozzle outlet)	A_j	nozzle outlet area
c_a	flight speed	BPR	bypass ratio
c_j	jet velocity	$\dot{m}$	mass flow rate
c_{jc}	core flow jet velocity	γ	ratio of specific heats
c_{jb}	bypass flow jet velocity	c_p	specific heat at constant pressure