Exam 26/10/2012

1 Statique

1.
$$\begin{cases} \dfrac{\partial p}{\partial r} = \omega^2 r \\ \dfrac{1}{r}\dfrac{\partial p}{\partial \theta} = 0 \quad \Rightarrow \quad P \neq f(\theta) \\ \dfrac{\partial p}{\partial z} = -g \end{cases}$$

2. $p = f(r, z)$

$$p = \omega^2 \frac{r^2}{2} - gz + \text{cte}$$

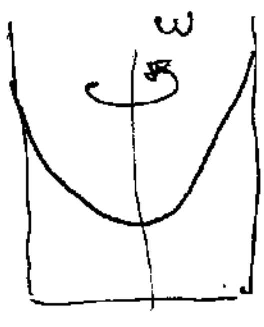
isobare $P = \text{cte} \Rightarrow \dfrac{\omega^2 r^2}{2} - gz = C$

$$\boxed{z = \frac{\omega^2}{2g} r^2 + C}$$

3. ω petit $\quad p = -gz$

$\Rightarrow$ isobares $z = \text{cte}$

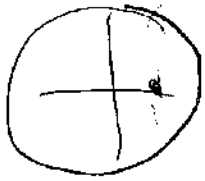
ω grand



$$x_0 = A \cos(\varphi)$$
$$y_0 = A \sin(\varphi_0)$$

$x = f(x_0, y_0, t)$; $y = f(x_0, y_0, t)$

4 $$x^2 + y^2 = A^2 (\cos^2 + \sin^2) = A^2$$



Tornade - ext.

1. même qu'avant

$$u_e = -\frac{B y}{x^2+y^2} \bar{e_x} + \frac{B x}{x^2+y^2} \bar{e_y}$$

2 $$\mathrm{div}(u_e) = -\frac{\omega y 2x}{(x^2+y^2)^2} + \frac{\omega x 2y}{(x^2+y^2)^2} = 0$$

$$\mathrm{rot}(u_e) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$= \frac{B}{x^2+y^2} - \frac{B x^2 2}{(x^2+y^2)^2} + \frac{B}{(x^2+y^2)} - \frac{B y^2}{} = 0$$

3 $$\frac{\partial \psi}{\partial x} = v \qquad \frac{\partial \psi}{\partial y} = u$$

$$\partial \psi = -\frac{B y}{x^2+y^2} dy$$

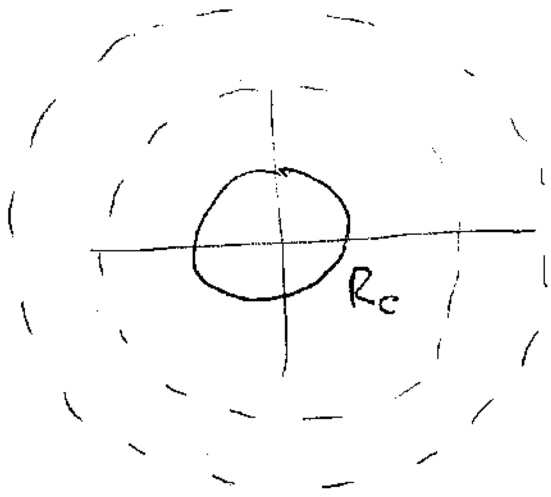
$u = y^2$
$du = 2y\,dy$

$$\partial \psi = -\frac{1}{2} \frac{B\, du}{x^2+u} \Rightarrow \psi = -\frac{1}{2} B \ln(x^2+y^2)$$

$$\psi = -B \ln(x^2+y^2)^{1/2}$$

en polaires

$$\frac{\partial \psi}{\partial r} = - v_\theta = - \frac{B}{r} \qquad \boxed{\psi = - B \ln(r)}$$



$\psi = cte$

$\Rightarrow r = cte$

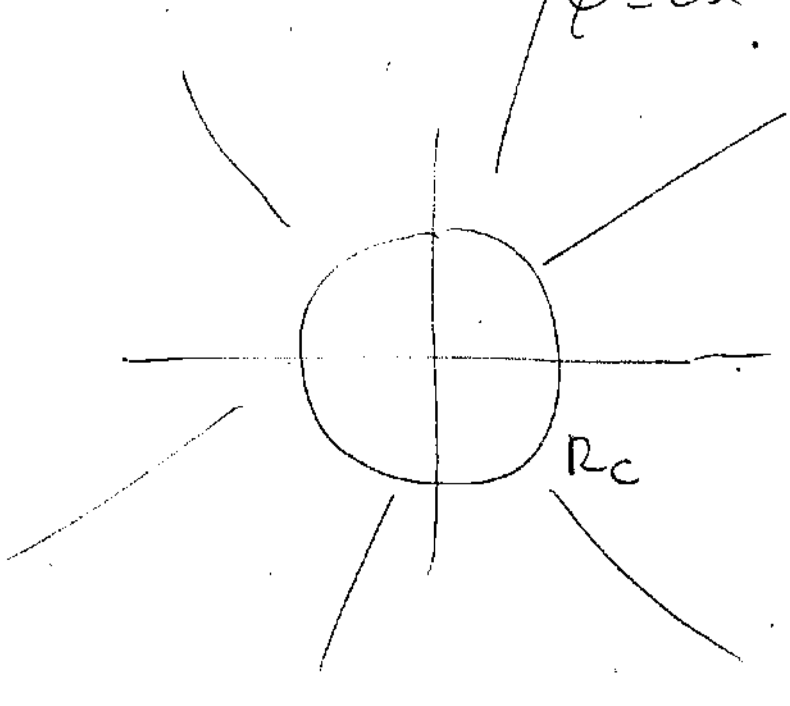
avec $r \geq R_c$

4

$$\frac{1}{r} \frac{\partial \phi}{\partial \theta} = v_\theta = - \frac{B}{r}$$

$$\Rightarrow \frac{\partial \phi}{\partial \theta} = - B \qquad \phi = - B \theta$$

$\phi = cte \Rightarrow \theta = cte$



$r \geq R_c$

des droites

$\underline{\underline{5}}$ $\Gamma = \int_L \bar{v} \, \bar{n} \, dL$ $\bar{v} = \frac{B}{r} \bar{e}_\theta$

$\bar{n} = \bar{e}_\theta$

$= \int_0^{2\pi} \frac{B}{r} \, r \, d\theta$

$$\boxed{\Gamma = 2\pi B}$$

Conclusion : indépendant du rayon choisi.