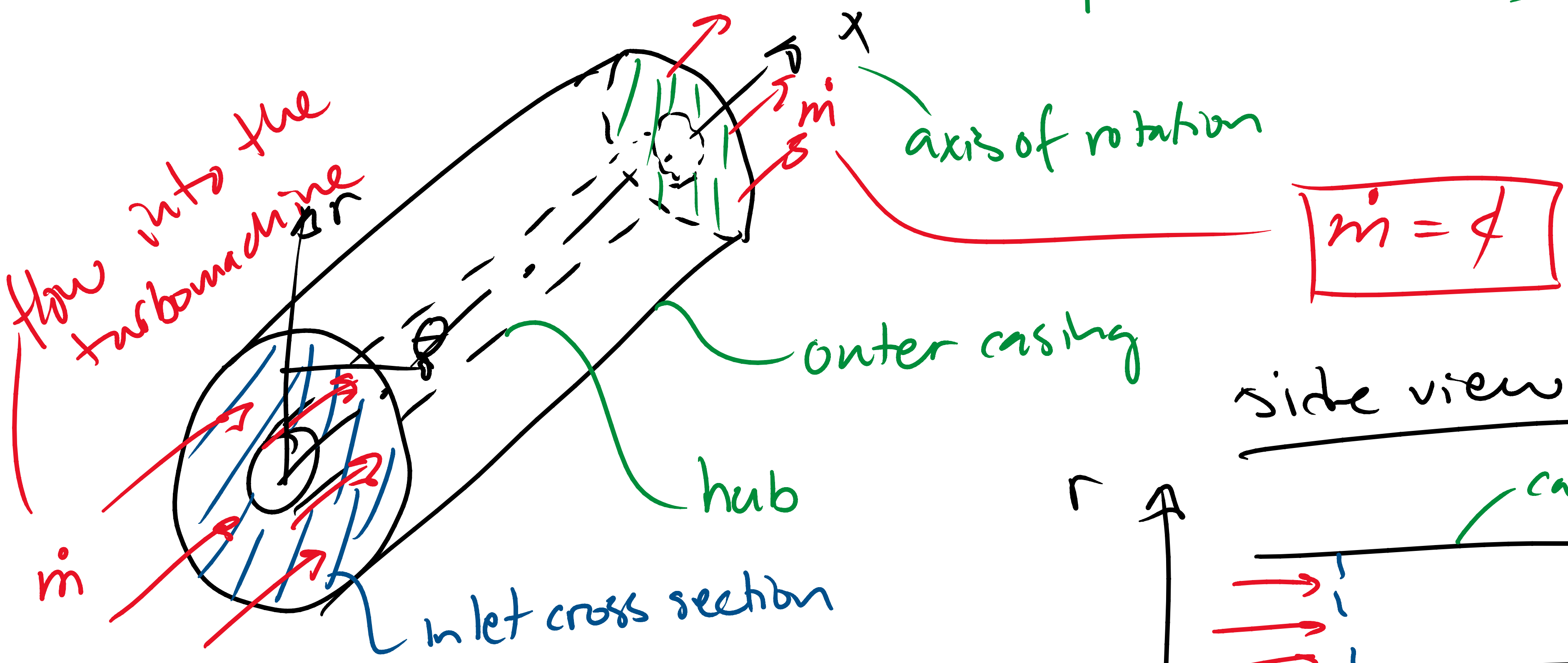
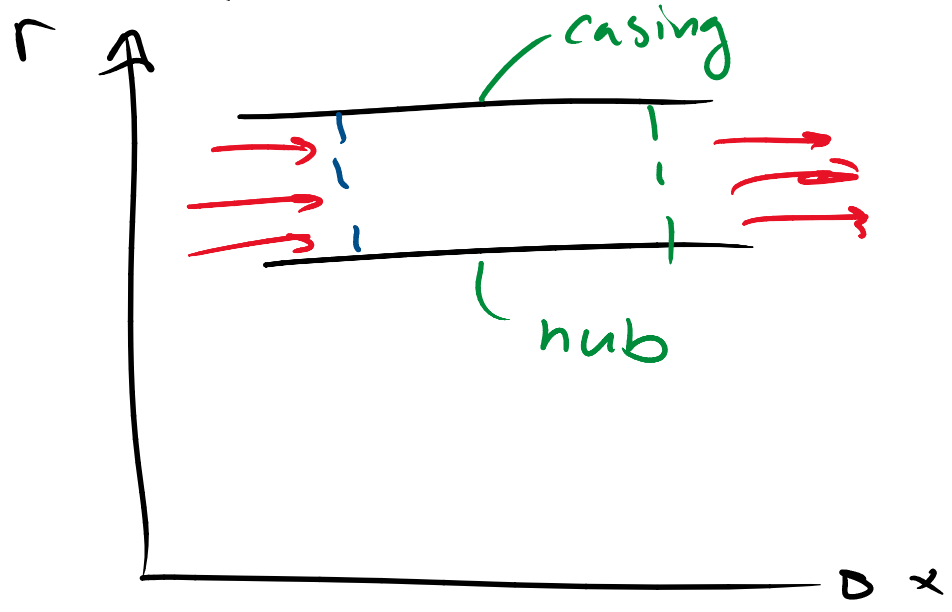
Repetition of basic concepts of turbomachinery

Characteristics of turbomachines

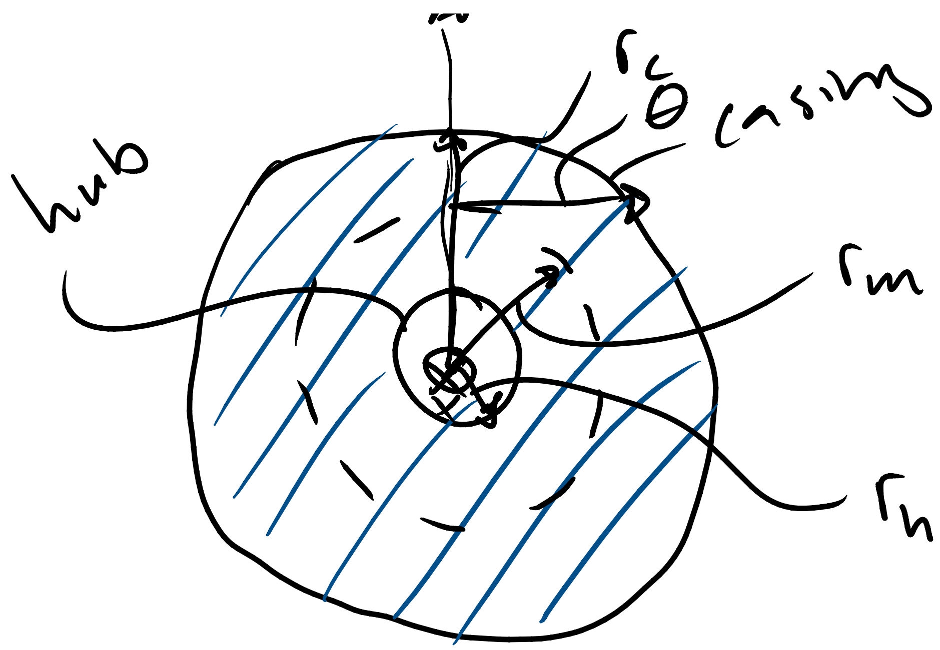
- motion : rotation
- shape : rotational symmetric
- thermodynamics: adiabatic (no heat flows over the boundaries of the machine)



side view :



front view



r_c = casing radius

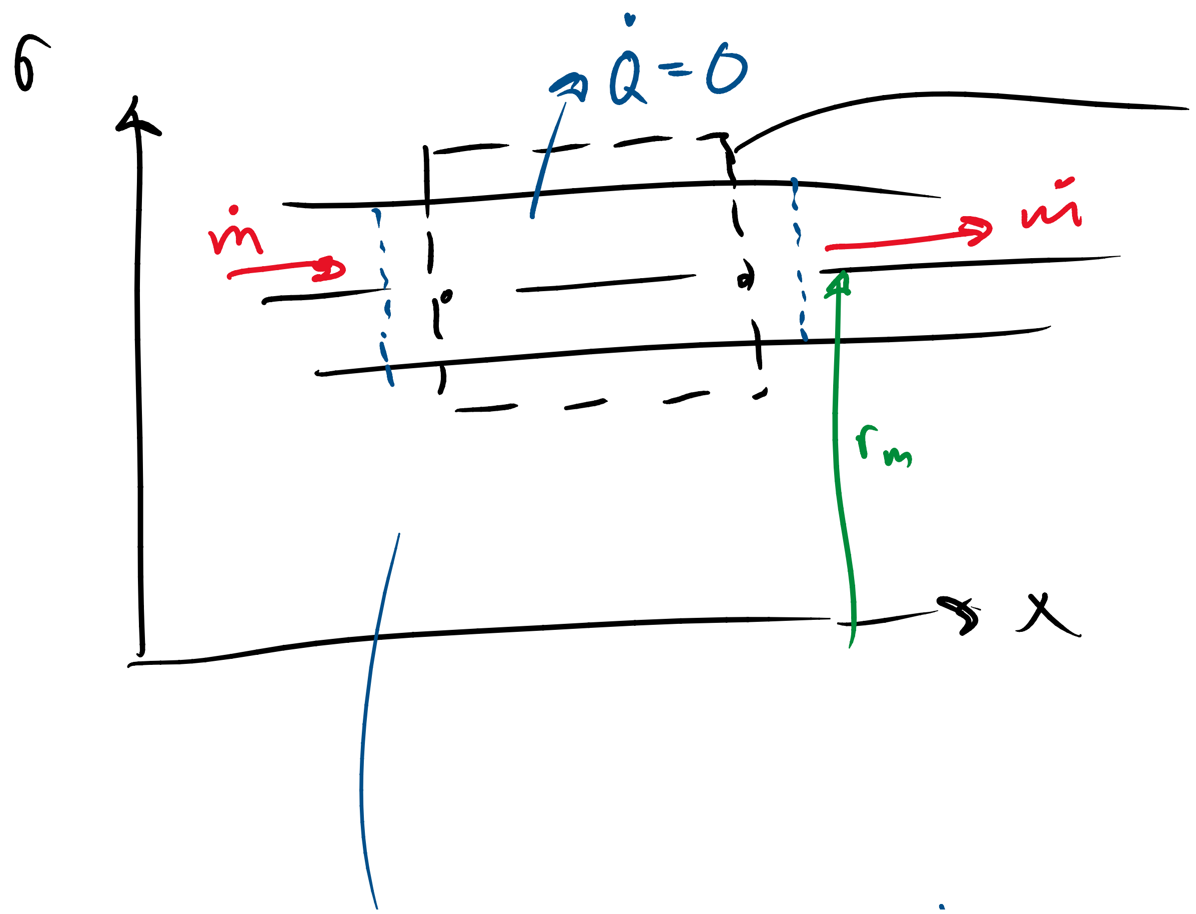
r_h = hub radius

r_m = mean radius

$$r_m = \frac{r_h + r_c}{2}$$

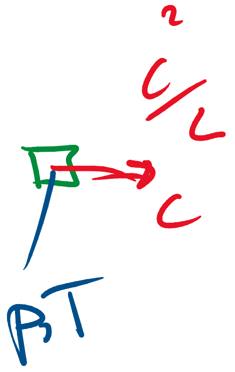
→ task of a turbomachine: change of energy of a fluid

- add energy → compress
- extract energy → expand



inside this box, the energy of the fluid shall be changed

$$\boxed{d\dot{E} = \dot{m} \cdot dh_0}$$



↓ energy passing per unit time $\dot{E} = \dot{m} \cdot h_0$

h_0: total enthalpy

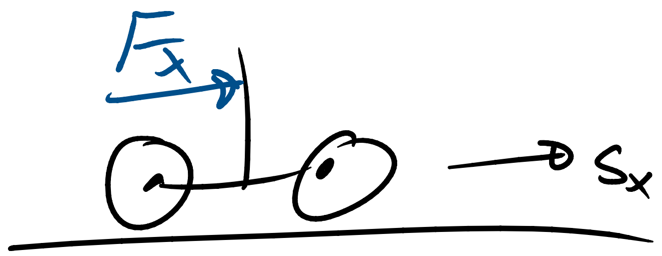
$$h_0 = \underbrace{h}_{\text{static enthalpy}} + \underbrace{\frac{c^2}{2}}_{\text{kinetic energy}}$$

$$h = c_p \cdot T$$

$$d\dot{E} = \cancel{d\dot{Q}}^{=0} - d\dot{W} \Rightarrow \boxed{\underbrace{d\dot{E}}_{\text{internal energy in the system}} = -\underbrace{d\dot{W}}_{\text{mechanical energy added/extracted to/from the fluid}}}$$

How get the mechanical energy into the system?

→ generation of mechanical work $W = F \cdot s$



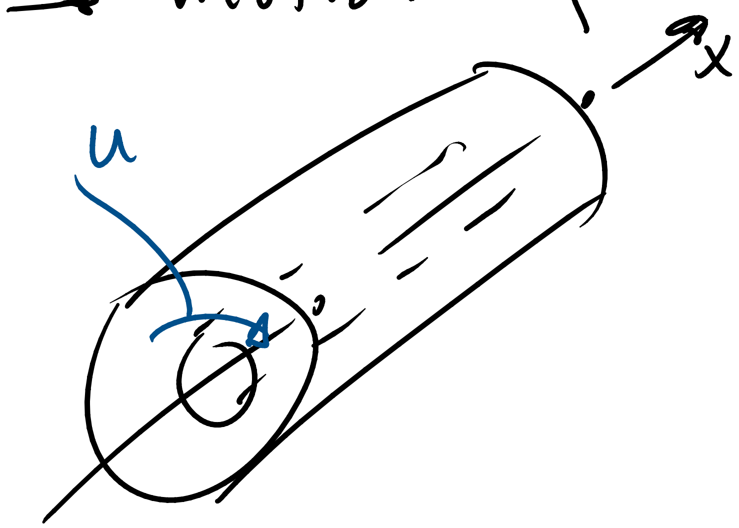
$$dW = F_x \cdot ds_x$$

$$\boxed{d\dot{W} = F_x \cdot V_x}$$

v: velocity

→ only the force in the direction of the motion counts

→ motion of turbomachines → rotating



u: speed of rotation of an object

→ force in direction of rotation

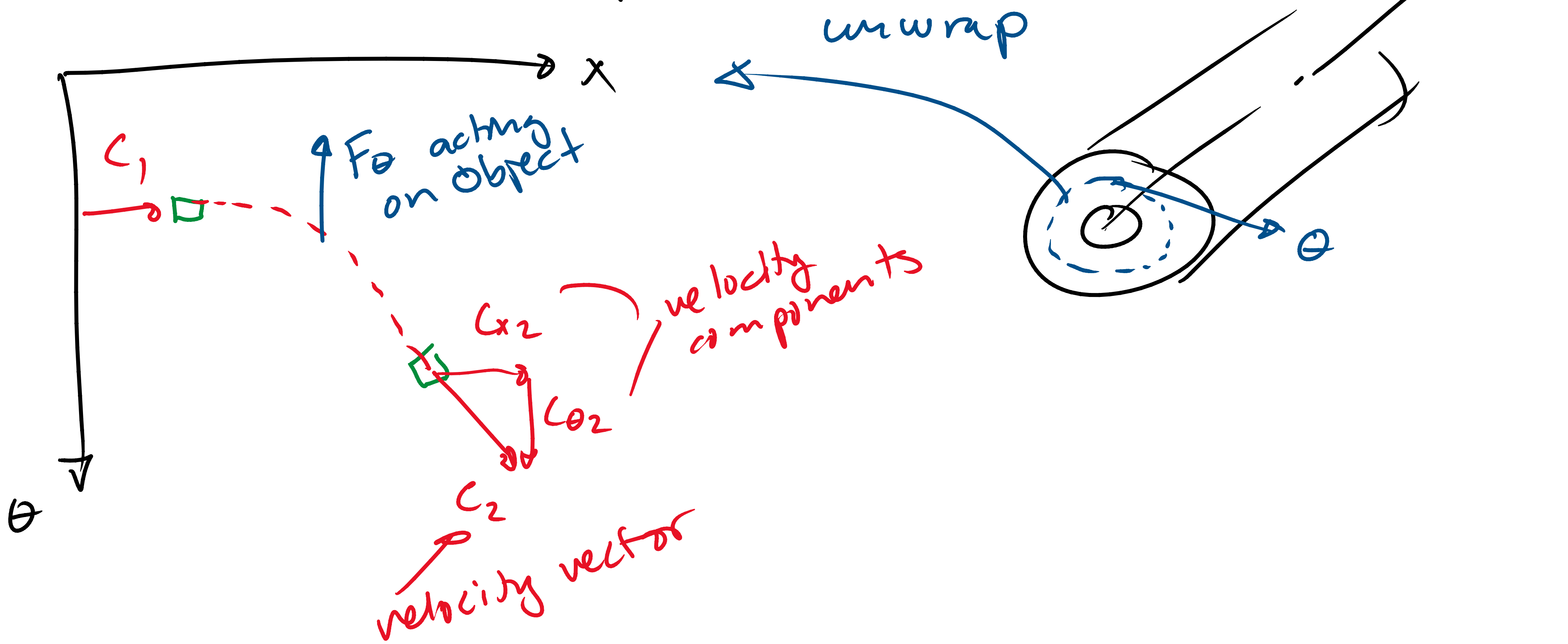
F_θ circumferential direction

$$\boxed{\dot{m}\, dh_0 = -F_\theta \cdot u}$$

F_θ: force acting on the moving part

u: object speed

where does this force F_θ comes from ?



$$\boxed{F_\theta = \dot{m}(c_{\theta 1} - c_{\theta 2})}$$

$$\Rightarrow \quad \cancel{\dot{m}} \cdot dh_0 = -\cancel{\dot{m}}\underbrace{(c_{\theta 1} - c_{\theta 2})}_{F_\theta} \cdot u$$

Euler TM equation

$$\to \quad dh_0 = -(c_{\theta 1} - c_{\theta 2}) \cdot u \quad \to \quad \boxed{h_{02} - h_{01} = \underline{u}\,(\underline{c_{\theta 2} - c_{\theta 1}})}$$

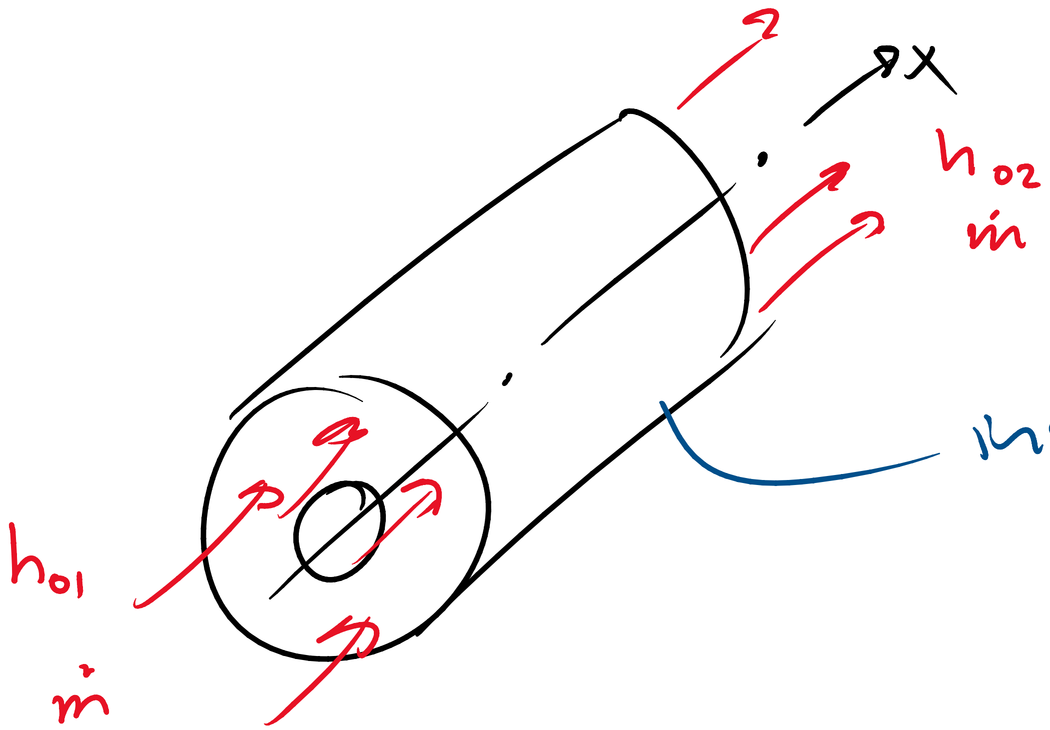
→ in order to change the total enthalpy in a fluid we need to have

i) an object moving (rotating) at a speed of u

ii) a deviation of the flow in circumferential direction

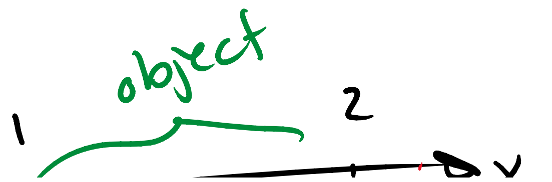
→ general form $\boxed{h_{02}-h_{01} = u_2 \cdot c_{\theta 2} - u_1 \cdot c_{\theta 1}}$

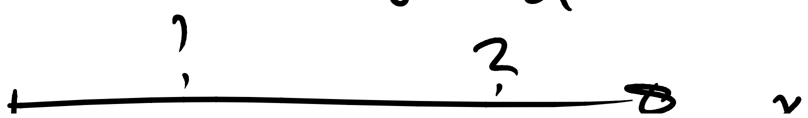
$u = r \cdot \underbrace{\omega}_{\text{rotational frequency}}$

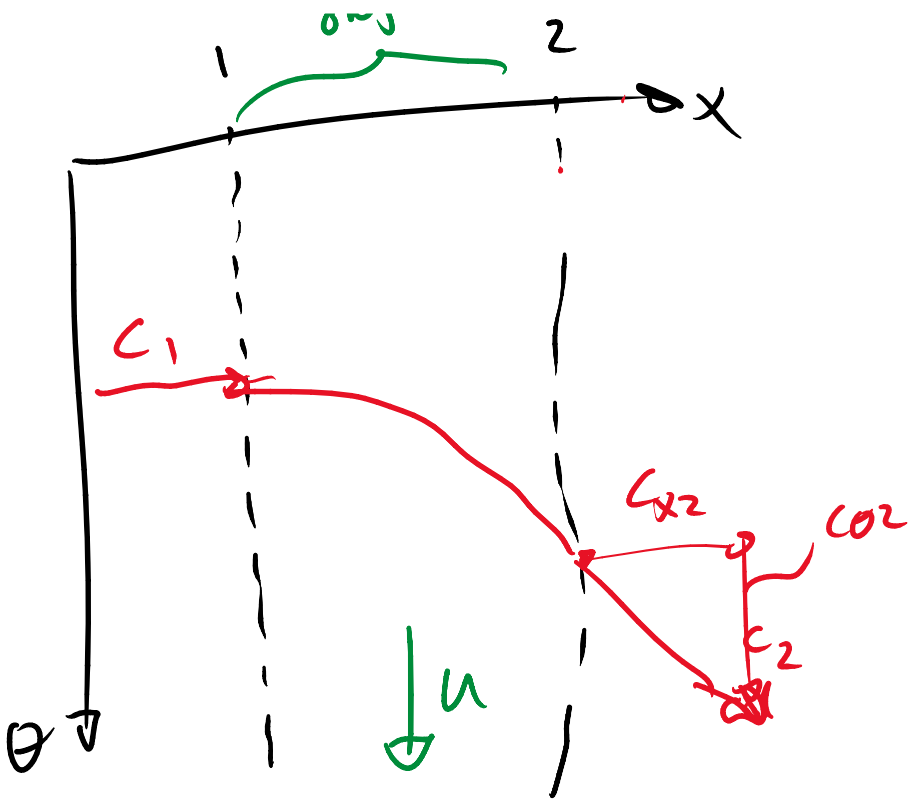


inside: an object moving (rotating) at a speed of u while having a force F_θ acting on it

→ $h_{01} \neq h_{02}$







$c_{\theta 1} = 0$

$c_{\theta 2} > 0$

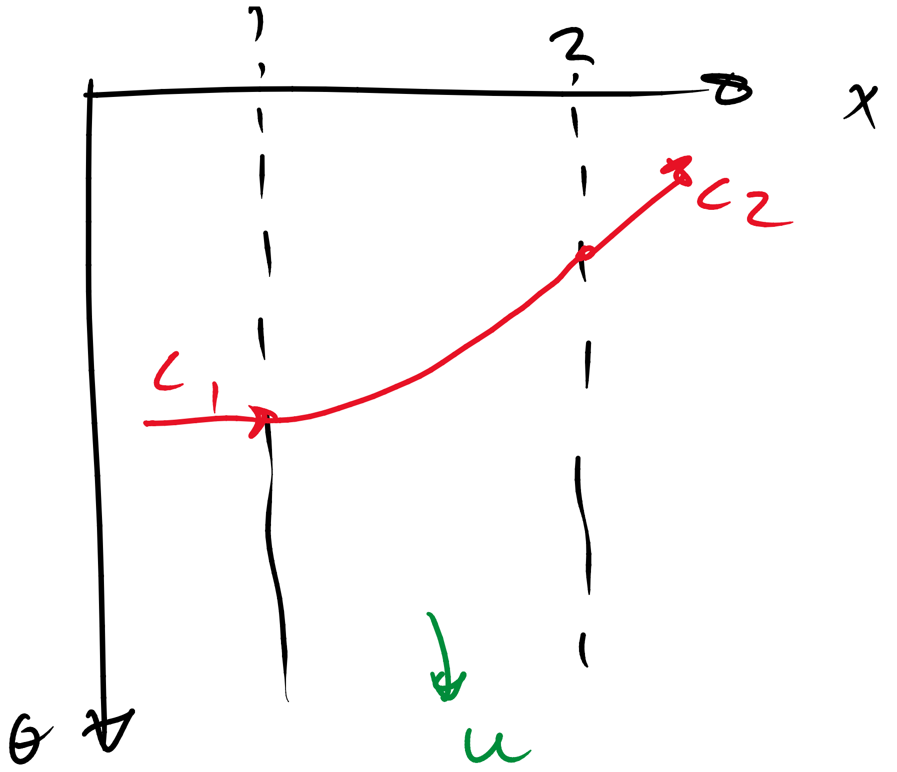
$u > 0$

$dh_0 = u \cdot (c_{\theta 2} - \cancel{c_{\theta 1}})$ 0

→ $\boxed{dh_0 = u \cdot c_{\theta 2} > 0}$

→ adding energy

→ compressor



$c_{\theta 1} = 0$

$c_{\theta 2} < 0$

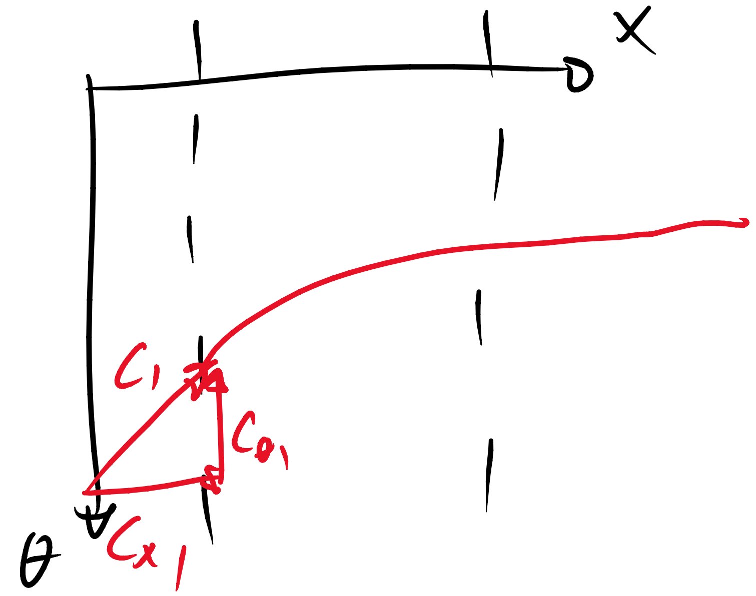
$u > 0$

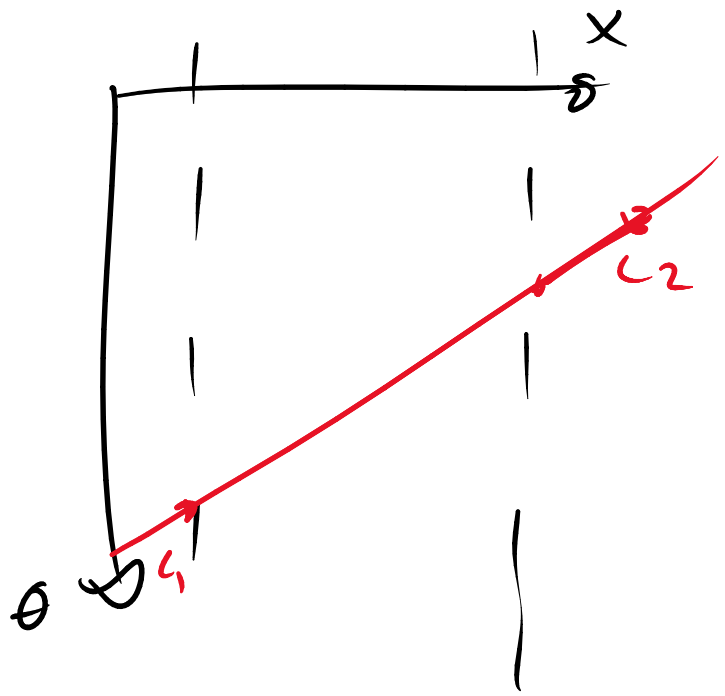
$dh_0 = u(c_{\theta 2} - \cancel{c_{\theta 1}})$ 0

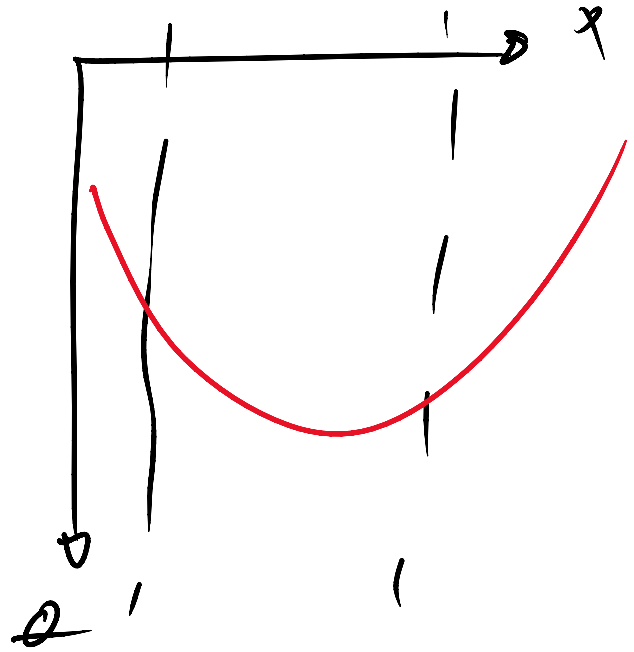
→ $\boxed{dh_0 = u \cdot c_{\theta 2} < 0}$

extraction of energy

→ turbine







$c_{\theta 1} < 0$

$c_{\theta 2} = 0$

$dh_0 = u(\overset{0}{c_{\theta 2}} - \overset{<0}{c_{\theta 1}})$

$dh_0 > 0$

$c_{\theta 1} < 0$

$c_{\theta 2} < 0$

$\to c_{\theta 1} = c_{\theta 2}$

$dh_0 = u(\underbrace{c_{\theta 2} - c_{\theta 1}}_{0})$

$dh_0 = 0$

$c_{\theta 1} > 0$

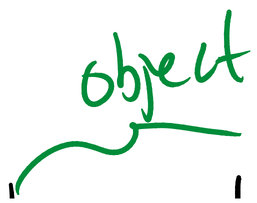
$c_{\theta 2} < 0$

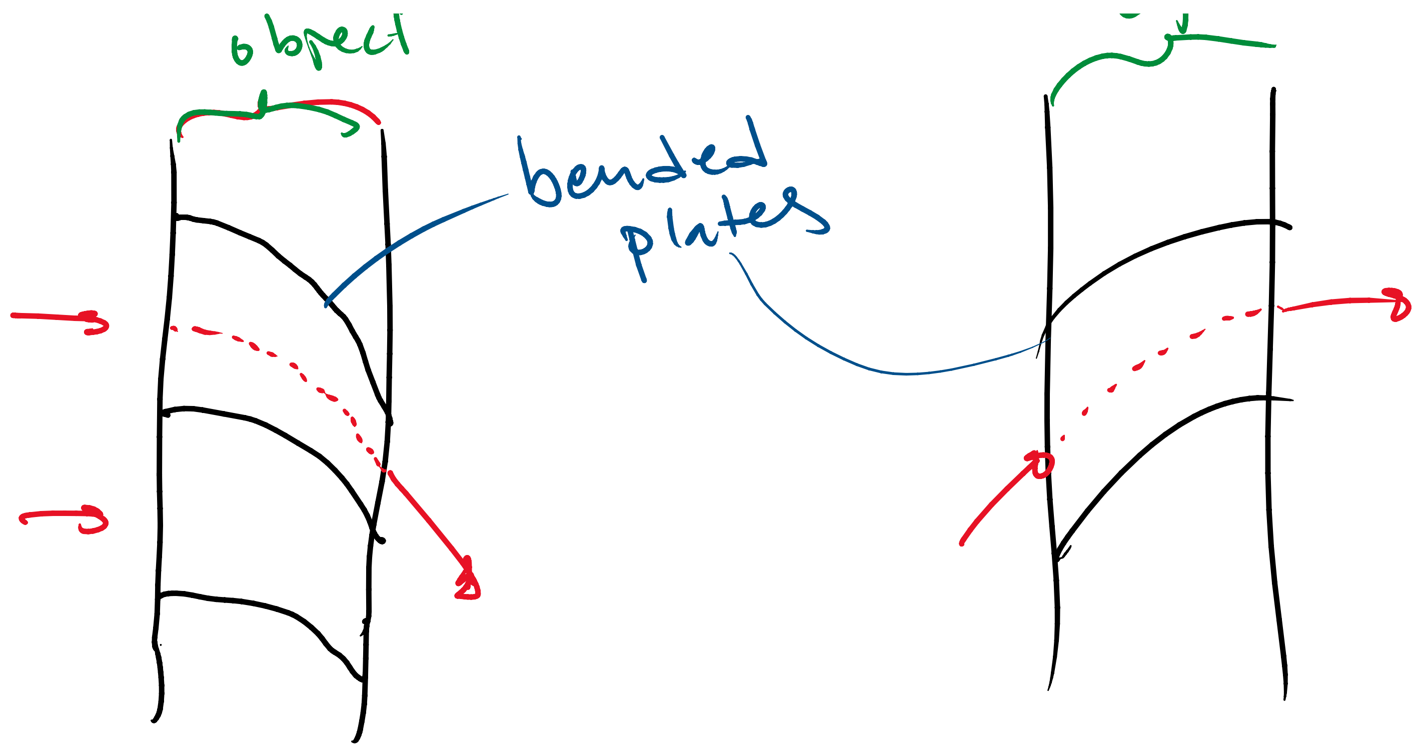
$dh_0 = u(c_{\theta 2} - c_{\theta 1})$

$\boxed{dh_0 < 0}$

what is the aforemention object?

- deviates the flow
- rotates



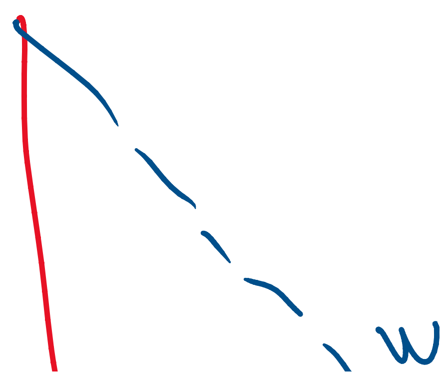


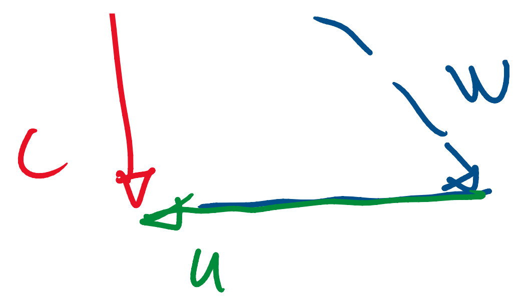
principle of absolute and relative velocities

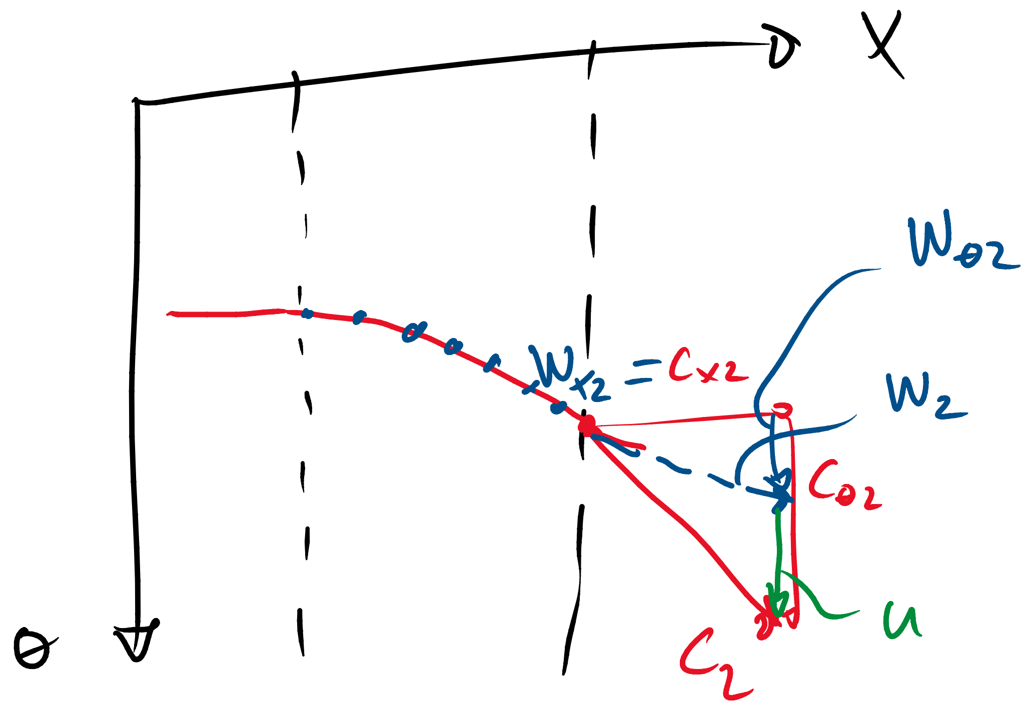
c absolute velocity: velocity in the absolute frame of reference

w relative velocity: velocity experienced if I as an observer on the moving object

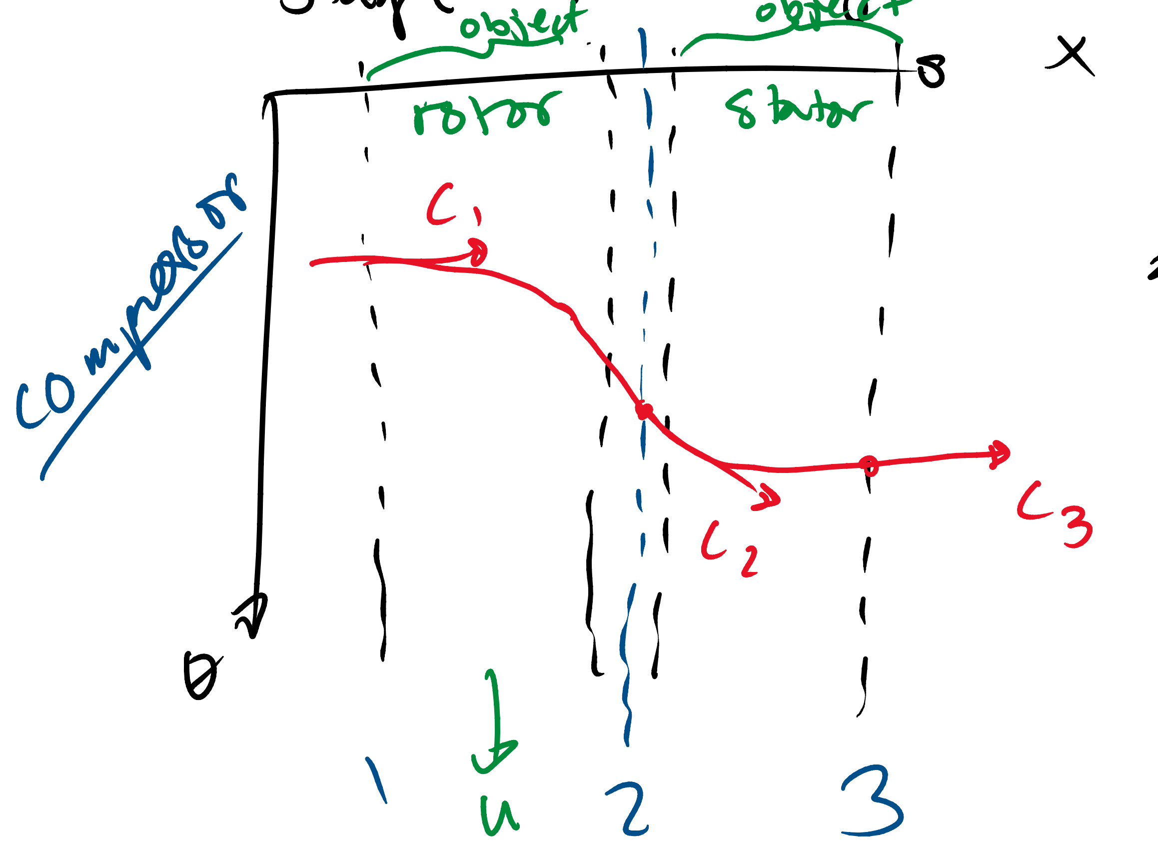
u







Stage thermodynamics



- repetition stage

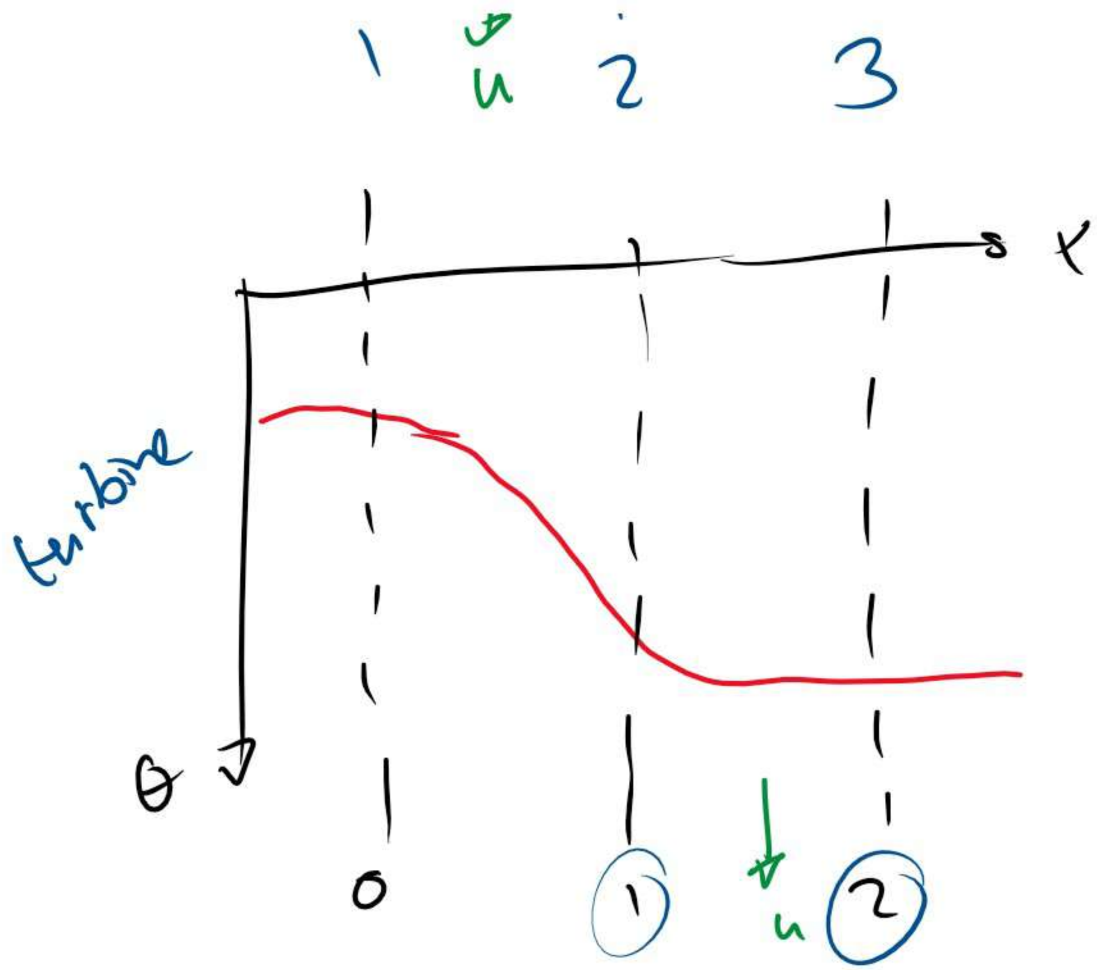
$\vec{c}_1 = \vec{c}_3$

same direction and magnitude

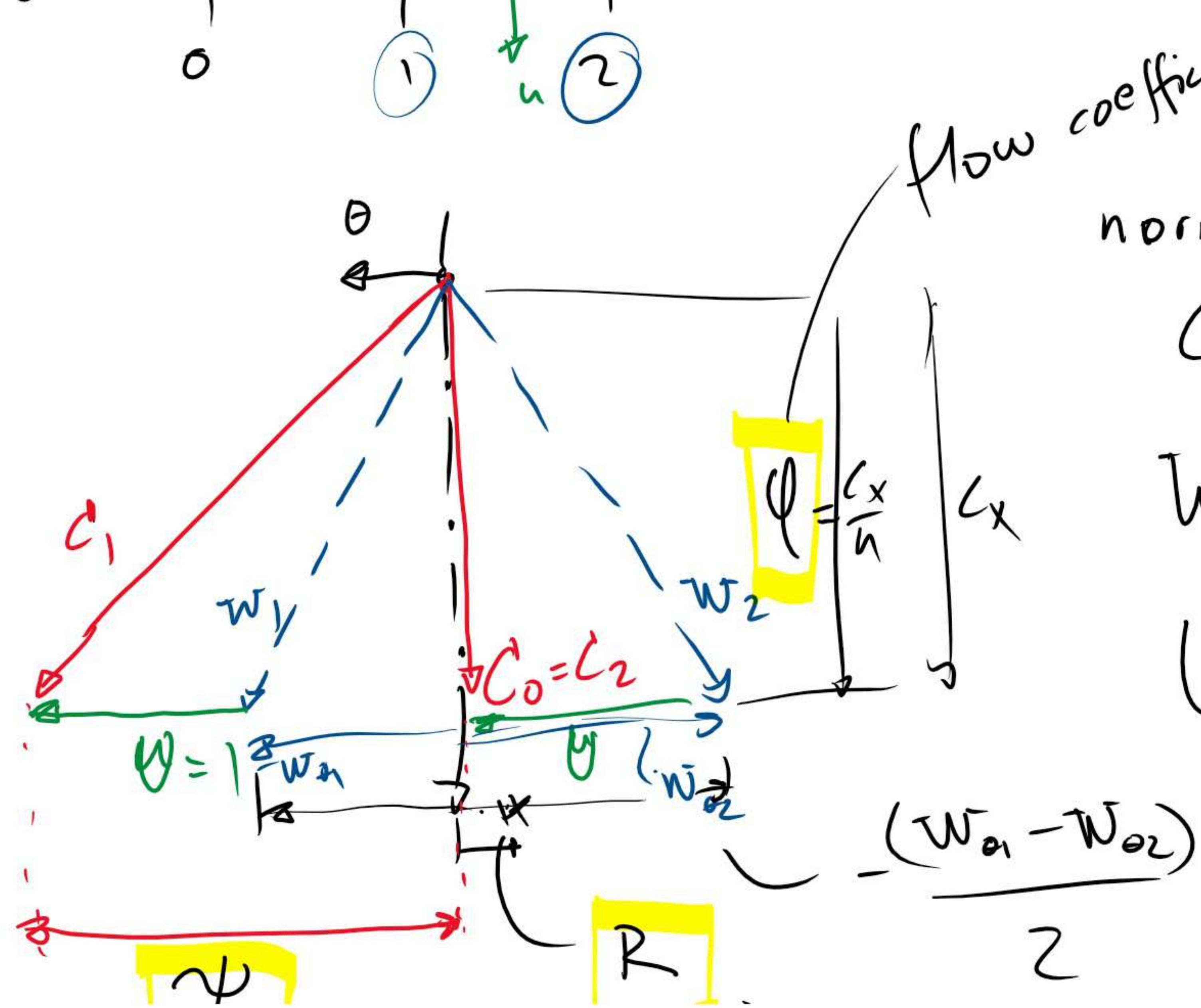
- normal repetition stage

$c_x = \phi$

$c_x = 4$



$$\boxed{h_{02} - h_{01} = u_2 \cdot c_{\theta 2} - u_1 c_{\theta 1}}$$



normalize velocity with u

$$C = \frac{c}{u} \quad [-]$$

$$W = \frac{w}{u}$$

$$U = \frac{u}{u} = 1$$

$\boxed{\Psi}$ — loading coefficient

$\boxed{R}$ — reaction degree

2

$(C_{\theta 2} - C_{\theta 1})$

→ 3 design parameters

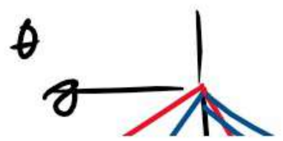
φ: flow coefficient $\quad \varphi = \frac{c_x}{u}$

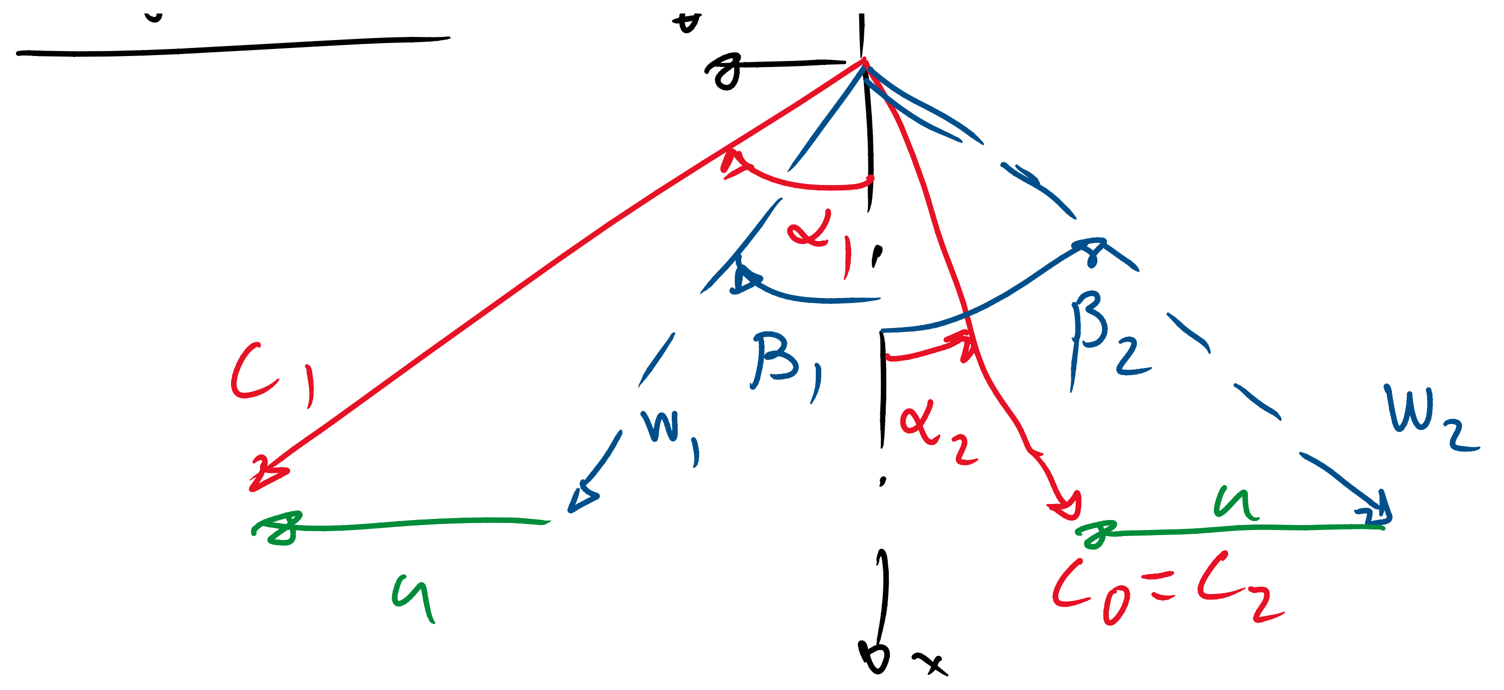
Ψ: loading coefficient $\quad \Psi = \frac{|\Delta h_0|}{u^2}$ — turb $\Delta h_0 < 0$, compr $\Delta h_0 > 0$

R : reaction degree $\quad R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} \quad [0 \dots 1]$

→ when having a normal repetition stage, the velocity triangle of a turbomachine is fully defined by the three design parameters

e.g. turbine





in this case:

$$\begin{cases} \varphi = 1 \\ \psi = 2.1 \\ R = 0.3 \end{cases}$$

example: assume $d_m = 400\,mm \rightarrow r_m = 200\,mm$

$N = 10\,000\,rpm \rightarrow \omega = \frac{\pi \cdot N}{30} \approx 1000\,rad/s$

$\rightarrow u = r_m \cdot \omega = 0.2 \cdot 1000 = 200\,m/s$

since $\varphi = 1$ $u = C_x = 200\,m/s$

$$\Delta h_0 = \frac{2.1}{\psi} \cdot \left(200\,m/s\right)^2 = 84\ kJ/kg$$

$$\Delta h_0 \propto \Pi \quad \text{pressure ratio}$$

$$\propto \frac{T_{00}}{T_{02}}$$