TD 0.2 : Finite-element approximations in 1D.

Exercice 1 : bar under traction/compression load

We are going to study here the same structure as in exercice 2 of TD 0.1 and solve the equilibrium problem by using a finite-element approximation instead of Ritz-Galerkin method.
Let us consider a bar of length L occupying the domain $\Omega \equiv (0, L)$. Considering a one-dimensional bar model, let be ES the axial stiffness of the bar and $u(x)$ the axial displacement. The bar is clamped at $x = 0$ and loaded by a axial end-force F at $x = L$ and a distributed axial loading $p(x)$. We suppose at the beginning that ES is independent of x (uniform material and cross-sectional area).

Part I: formulation of the equilibrium problem

- Write the strong formulation of the problem
- Write the weak formulation of the problem
- Write the total potential energy and give the variational formulation of the problem.
- Obtain the the weak and the strong formulation as stationarity conditions of the total potential appearing

Part II: approximation of the equilibrium problem using the FE method

- Let us take $p(x) = p_0 \sin \dfrac{\pi x}{L}$ and $F = F_0$. Determine the analytical solution of the equilibrium problem by integration of the strong formulation.
- Determine an approximate solution of the equilibrium problem using the finite-element approach :
  1. consider a mesh of $N_e = 2$ identical elements with linear approximation: define the elementary vector of degrees of freedom and basis functions;
  2. build the local (elementary) contributions to the approximation of the bilinear and linear forms, $a(u, v)$ and $l(u, v)$, by expressing the elementary stiffness matrix and elementary force vector;
  3. build the approximation of the weak formulation of the equilibrium problem by assembling the elementary contributions over the mesh (express the global stiffness matrix and global force vector);
  4. finally, write the resulting linear system of equations and solve it by applying the displacement boundary conditions.
- Evaluate the error between the approximate and the analytical solution with the following norms: L_∞, L_2, H_1, and energy norm.
- Repeat the finite-element approximate resolution of the problem using a more refined mesh of $N_e = 3$ identical elements with linear approximation: compare the analytical solution and the approximate solutions with $N_e = 2$ and $N_e = 3$ (graphical representation and error estimations).