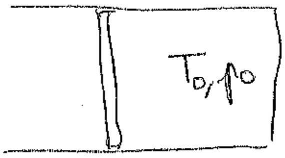
Ex 6

1 mole GP diatomique $\gamma = \frac{7}{5}$.



1) $\underset{A}{(T_0, p_0)} \xrightarrow[\text{compression isotherme}]{T = T_0} \underset{B}{(T_0, p_1)} \xrightarrow[\text{détente adiab. réversible}]{} \underset{C}{(T_1, p_0)}$

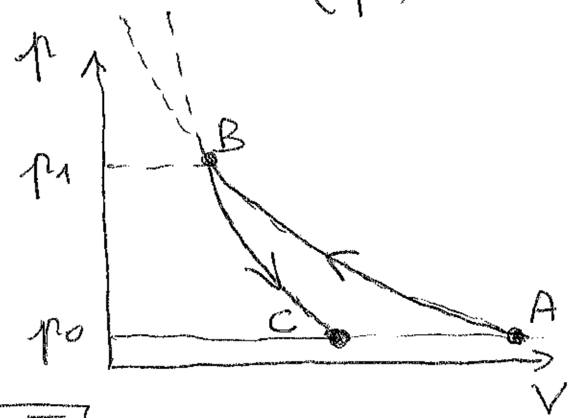
a) $p_B V_B^\gamma = p_C V_C^\gamma \qquad V = \frac{RT}{p}$

$$\Rightarrow p_B \left(\frac{RT_B}{p_B}\right)^\gamma = p_C \left(\frac{RT_C}{p_C}\right)^\gamma \Rightarrow p_B^{1-\gamma} T_B^\gamma = p_C^{1-\gamma} T_C^\gamma$$

$$\Rightarrow T_C = T_B \left(\frac{p_B}{p_C}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_B}{\left(\frac{p_B}{p_C}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\Rightarrow T_1 = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \quad \text{or } p_1 > p_0 \Rightarrow T_1 < T_0.$$

b)



$\boxed{\text{isoT}}$

$$p V = p_B V_B$$

$$\Rightarrow p = \frac{p_B V_B}{V}$$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right) = -\frac{p_B V_B}{V^2}$$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right)_{B\ \text{isoT.}} = -\frac{p_B}{V_B}.$$

$\boxed{\text{adia rev}}$

$$p V^\gamma = p_B V_B^\gamma$$

$$\Rightarrow p = p_B V_B^\gamma \times \frac{1}{V^\gamma}$$

$$\Rightarrow \frac{\partial p}{\partial V} = p_B V_B^\gamma \left(-\gamma V^{-\gamma-1}\right)$$

$$\frac{\partial p}{\partial V} = -\gamma \frac{p_B V_B^\gamma}{V^{\gamma+1}}$$

$$\left(\frac{\partial p}{\partial V}\right)_B = \frac{-\gamma p_B V_B^\gamma}{V_B^{\gamma+1}}$$

$$\left(\frac{\partial p}{\partial V}\right)_{B\ \text{adia rev.}} = -\gamma \frac{p_B}{V_B}.$$

c) $W_i = W_{AB} = -\int_A^B p\,dV = -\int_A^B RT_A \frac{dV}{V} = -RT_A \ln\frac{V_B}{V_A}$

$p_A V_A = p_B V_B \Rightarrow \frac{V_B}{V_A} = \frac{p_A}{p_B}$

$\Rightarrow W_i = -RT_A \ln\frac{p_A}{p_B} = +RT_A \ln\frac{p_B}{p_A} = RT_0 \ln\frac{p_1}{p_0} > 0$

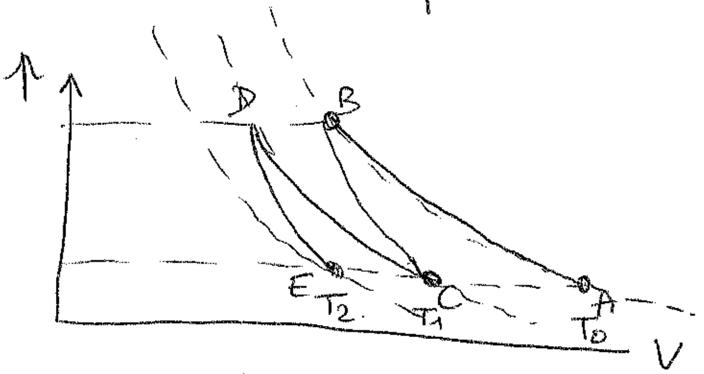
$Q_i = -W_i = -RT_0 \ln\frac{p_1}{p_0} \quad < 0.$

$W_a = W_{BC} = \Delta U_{BC}$ car $Q_a = 0$.

$= \bar{C}_V (T_C - T_B) = \bar{C}_V (T_1 - T_0) < 0 \qquad \bar{C}_V = \frac{R}{\gamma - 1}$

$W_a + W_i = RT_0 \ln\frac{p_1}{p_0} + \bar{C}_V (T_1 - T_0) > 0$

2) a)



$T_2 = T_E = T_D \left(\frac{p_D}{p_E}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_C}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{T_1}{\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{2\left(\frac{\gamma-1}{\gamma}\right)}}$

b) $T_n = \frac{T_0}{\left(\frac{p_1}{p_0}\right)^{n\left(\frac{\gamma-1}{\gamma}\right)}}$

c) $T_0 = 300$ K $\quad p_0 = 1$ atm $\quad p_1 = 3$ atm $\Rightarrow \frac{p_1}{p_0} = 3.$

$T_n \leq \frac{T_0}{3}$; $\Rightarrow \frac{T_0}{3^{n\left(\frac{\gamma-1}{\gamma}\right)}} \leq \frac{T_0}{3} \Rightarrow 3^{n\left(\frac{\gamma-1}{\gamma}\right)} \geq 3$

$\frac{\gamma-1}{\gamma} = \frac{2}{7}$ $\quad \boxed{n=4} \quad \boxed{n\left(\frac{\gamma-1}{\gamma}\right) \geq 1}$

$\Rightarrow e^{n\left(\frac{\gamma-1}{\gamma}\right)\ln 3} \geq e^{\ln 3}.$

$\Rightarrow n\left(\frac{\gamma-1}{\gamma}\right)\ln 3 \geq \ln 3 \Rightarrow n \geq \frac{\gamma}{\gamma-1} = \frac{7/5}{\frac{7}{5}-\frac{5}{5}}$

$n \geq \frac{7}{2} = 3{,}5 \quad \Rightarrow \boxed{n=4.}$

3)

$$\underset{A'}{\boxed{T_0 \quad p_0}} \xrightarrow[\substack{\text{irréversible} \\ p_{ext} = p_1 = Cte}]{\text{compression}} \underset{B'}{\boxed{T_0 \quad p_1}} \xrightarrow[\substack{\text{irréversible} \\ p_{ext} = p_0 = Cte}]{\text{détente adia.}} \underset{C'}{\boxed{T'_1 \quad p_0}}$$

a) $W'_i = -p_1 (V_{B'} - V_{A'})$

on a $p_0 V_0 = R T_0$ $V_{A'} = V_0$

$p_1 V_{B'} = R T_0$.

$\Rightarrow V_{B'} = \dfrac{R T_0}{p_1}$

$\Delta U_{A'B'} = 0$

$Q'_i = -W'_i < 0$.

$$W'_i = -p_1 \left(\frac{RT_0}{p_1} - \frac{RT_0}{p_0} \right) = -RT_0 + \frac{p_1}{p_0} RT_0$$

$$= RT_0 \left(\frac{p_1}{p_0} - 1 \right) > 0 .$$

b) $Q'_a = 0$.

$W'_a = -p_0 (V_{C'} - V_{B'})$

$p_0 V_{C'} = R T'_1$

$V_{B'} = R \dfrac{T_0}{p_1}$

$$= -p_0 \left(\frac{RT'_1}{p_0} - \frac{RT_0}{p_1} \right)$$

$$= -RT'_1 + RT_0 \left(\frac{p_0}{p_1} \right)$$

posons $x = \dfrac{p_1}{p_0}$.

$$= -RT'_1 + \frac{RT_0}{x}$$

$$\Delta U_{B'C'} = \frac{R}{\gamma - 1} (T'_1 - T_0) .$$

$$\Rightarrow \frac{R}{\gamma-1} (T'_1 - T_0) = -RT'_1 + \frac{RT_0}{x}$$

$$\Rightarrow \left(\frac{R}{\gamma-1} + R \right) T'_1 = \left(\frac{R}{\gamma-1} + \frac{R}{x} \right) T_0 .$$

$$\Rightarrow \left(\frac{\cancel{1} + \gamma - \cancel{1}}{\cancel{\gamma-1}} \right) T'_1 = \frac{x + \gamma - 1}{x \cancel{(\gamma-1)}} T_0$$

$$\Rightarrow \boxed{T'_1 = \frac{x + \gamma - 1}{x (\gamma)} T_0}$$

AN:

AN $$T_1' = \frac{3 + \frac{7}{5} - 1}{3 \times \frac{7}{5}} \times 300$$

$$= \frac{15 + 7 - 5}{21} \times 300$$

$$= \underbrace{\frac{17}{21}}_{\approx 0{,}8} \times 300 = 242{,}85 \text{ K}$$

on avait $$T_1 = \frac{300}{3^{\frac{\frac{7}{5}-1}{7/5}}} = \frac{300}{3^{\frac{2}{7}}} = 219 \text{ K}$$

$$\Longrightarrow \boxed{T_1' > T_1}$$

|| On récupère moins en irréversible.

$$W_a' = \bar{C}_v (T_1' - T_0) \Rightarrow \boxed{|W_a'| < |W_a|}$$

$$W_i' = RT_0 \left(\frac{p_1}{p_0} - 1 \right) \qquad W_i = RT_0 \ln\left(\frac{p_1}{p_0}\right)$$

$$W_i = RT_0 \ln\left(\frac{p_1}{p_0} + 1 - 1 \right) \approx RT_0 \left(\frac{p_1}{p_0} - 1 \right) \underbrace{- \frac{RT_0}{2} \left(\frac{p_1}{p_0} - 1 \right)^2}_{<0}$$

$$\Longrightarrow \boxed{W_i < W_i'}$$

|| On doit fournir + de travail en irréversible.

$$\ln(x+1) = 1 - \frac{x^2}{2} + \dots$$