3.1

$$\langle a\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{m=1}^{N} a = \lim_{N\to\infty} \frac{Na}{N} = a$$

$$\langle aQ\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{m=1} aQ_m = \lim_{N\to\infty} \frac{a}{N}\sum_{m=1}^{\infty} Q_m =$$

$$= a \lim_{N\to\infty} \frac{1}{N}\sum_m Q_m = a\langle Q\rangle$$

$$\langle Q+R\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_m (Q_m+R_m) = \lim_{N\to\infty}\left(\frac{\sum Q_m}{N} + \frac{\sum R_m}{N}\right) =$$

$$= \langle Q\rangle + \langle R\rangle$$

$$\langle\langle Q\rangle\langle R\rangle\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_m \left(\langle Q\rangle + \langle R\rangle\right) =$$

$$= \lim_{N\to\infty} \frac{N}{N}\left(\langle Q\rangle + \langle R\rangle\right) = \langle Q\rangle + \langle R\rangle$$

$\langle\langle Q\rangle R\rangle = \langle aR\rangle$ avec $a = \langle Q\rangle$

et $\langle aR\rangle = a\langle R\rangle = \langle Q\rangle\langle R\rangle$

$$\langle q\rangle = \langle Q - \langle Q\rangle\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_m \left(Q_m - \langle Q\rangle\right) =$$

$$= \lim_{N\to\infty}\left(\frac{\sum Q_m}{N} - \frac{N\langle Q\rangle}{N}\right) = \langle Q\rangle - \langle Q\rangle = 0$$

$$\langle q\langle R\rangle\rangle = \lim_{N\to\infty} \frac{1}{N}\sum_m q_m\langle R\rangle = \lim_{N\to\infty} \frac{\langle R\rangle}{N}\sum \left(Q_m - \langle Q\rangle\right) =$$

$$= \frac{\langle R\rangle}{N} \lim_{N\to\infty} \frac{1}{N}\sum\left(Q_m - \langle Q\rangle\right) = 0$$

3.2 $\quad Q = a + bU$

$$\langle Q\rangle = \langle a + bU\rangle = \lim_{N\to\infty} \frac{1}{N} \sum_{n} (a + bU_n) =$$

$$= \lim_{N\to\infty} \frac{1}{N} \left(Na + b\sum_{n} U_N\right) = a + b\langle U\rangle$$

$$\mathrm{var}(Q) = \langle (Q - \langle Q\rangle)^2\rangle = \langle (\cancel{a} + bU - \cancel{a} - b\langle U\rangle)^2\rangle =$$

$$= \langle b^2 (U - \langle U\rangle)^2\rangle = b^2 \langle (U - \langle U\rangle)^2\rangle = b^2 \mathrm{var}(U)$$

$$\mathrm{sdev}(Q) = \sqrt{\mathrm{var}(Q)} = \sqrt{b^2 \mathrm{var}(U)} = b\sqrt{\mathrm{var}(U)}$$

$$\mathrm{var}(U) = \langle (U - \langle U\rangle)^2\rangle = \langle U^2 + \langle U\rangle^2 - 2U\langle U\rangle\rangle =$$

$$= \langle U^2\rangle + \langle\langle U\rangle^2\rangle - 2\langle U\rangle\langle U\rangle =$$

$$= \langle U^2\rangle - \langle U\rangle^2$$