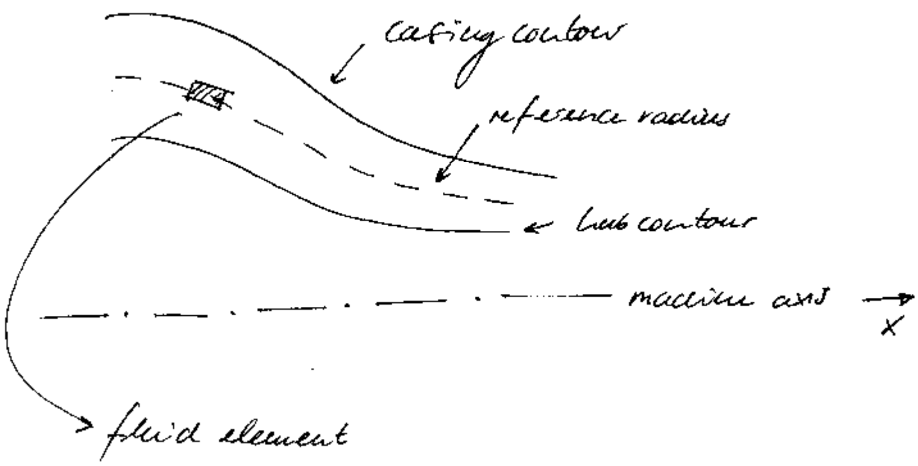
3D Flow

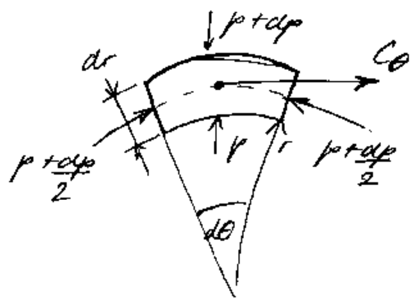
Damian Vogt
Course MJ2430 / MJ2244

Radial Equilibrium

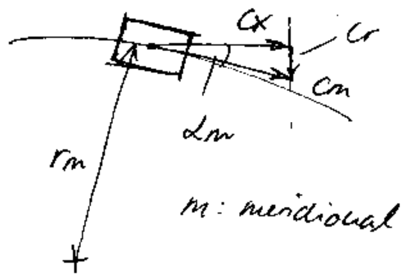
Consider a turbomachine flow passage



axial view



side view



→ conservation of momentum: forces due to fluid inertia must be balanced by pressure forces

↳ centripetal force due to circumf. velocity

$$F_i = dm \frac{c_\theta^2}{r}$$

where $dm = \rho dr\, r d\theta$ (per unit length of fluid elem.)

↳ radial force due to streamline curvature

$$F_{ii} = dm \frac{c_m^2}{r_m} \cdot \cos\alpha_m$$

↳ acceleration along streamline

$$F_{iii} = dm \cdot \frac{dc_m}{dt} \cdot \sin\alpha_m$$

→ total inertial force

$$F_I = \rho dr\, r d\theta \left[\frac{c_\theta^2}{r} + \frac{c_m^2}{r_m} \cdot \cos\alpha_m + \frac{dc_m}{dt} \cdot \sin\alpha_m \right]$$

3

→ balancing pressure forces

$$F_p = (p+dp)(r+dr)d\theta - p r d\theta - \cancel{2}\left(p+\frac{dp}{2}\right)dr \cdot \frac{d\theta}{\cancel{2}}$$

expanding:

$$F_p = \cancel{prd\theta} + \overset{②}{pdrd\theta} + dprd\theta + \overset{0}{dpdrd\theta} - \cancel{prd\theta} - \overset{②}{pdrd\theta} - \overset{0}{\frac{dp}{2}drd\theta}$$

$$\rightarrow F_p = dp\,dr\,d\theta$$

↳ balance: $F_p = F_I$

$$\rightarrow dp\,dr\,d\theta = \rho\,dr\,r\,d\theta\left[\frac{c_\theta^2}{r} + \frac{c_m^2}{r_m}\cdot\cos\alpha_m + \frac{dc_m}{dt}\cdot\sin\alpha_m\right]$$

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{c_\theta^2}{r} + \underbrace{\frac{c_m^2}{r_m}\cdot\cos\alpha_m + \frac{dc_m}{dt}\cdot\sin\alpha_m}$$

in most cases these contributions are $\ll \frac{c_\theta^2}{r}$ → these neglect

$$\rightarrow \boxed{\frac{1}{\rho}\frac{dp}{dr} = \frac{c_\theta^2}{r}}$$ radial equilibrium

→ variation of enthalpy with radius

$$h_0 = h + \frac{c^2}{2} = h + \frac{1}{2}\left(c_x^2 + c_\theta^2\right)$$

$$\frac{dh_0}{dr} = \frac{dh}{dr} + c_x\frac{dc_x}{dr} + c_\theta\frac{dc_\theta}{dr}$$

4

from thermodynamics $T ds = dh - \frac{dp}{\rho}$

$$\hookrightarrow \frac{dh}{dr} = \frac{T ds}{dr} + \underbrace{ds \frac{dT}{dr}}_{\text{neglect}} + \frac{1}{\rho}\frac{dp}{dr} - \underbrace{\frac{1}{\rho^2}\frac{d\rho}{dr} dp}_{\text{neglect}}$$

$$\rightarrow \frac{dh}{dr} = \frac{T ds}{dr} + \frac{1}{\rho}\frac{dp}{dr}$$

$$\rightarrow \text{substitute: } \frac{dh_0}{dr} = \underbrace{\frac{T ds}{dr}}_{\substack{\text{radial variation}\\ \text{of losses} \rightarrow \text{neglect here}}} + \underbrace{\frac{1}{\rho}\frac{dp}{dr}}_{\rightarrow \frac{c_\theta^2}{r} \ (\rightarrow \text{see above})} + c_x \frac{dc_x}{dr} + c_\theta \frac{dc_\theta}{dr}$$

$$\rightarrow \boxed{\frac{dh_0}{dr} = \frac{c_\theta^2}{r} + c_x \frac{dc_x}{dr} + c_\theta \frac{dc_\theta}{dr}}$$ vortex energy equation

→ frequent design condition: constant stag. enthalpy change over blade span

$$\rightarrow \frac{dh_0}{dr} = 0 \quad ; \text{ thus: } \frac{c_\theta^2}{r} + c_x \frac{dc_x}{dr} + c_\theta \frac{dc_\theta}{dr} = 0$$

↳ in case of constant axial velocity $\frac{dc_x}{dr} = 0$

$$\rightarrow \frac{dc_\theta}{dr} = -\frac{c_\theta}{r} \quad \text{or} \quad \frac{dc_\theta}{c_\theta} = -\frac{dr}{r}$$

integrating → $\boxed{c_\theta \cdot r = \text{const}}$ free vortex condition

5

Note: free vortex condition implies change in design parameters over span

$$R = \frac{1}{2} - \frac{c_x}{2u}(\tan\beta_3 + \tan\alpha_2) \rightarrow r\uparrow \rightarrow \begin{matrix}\beta_3\downarrow \\ \alpha_2\downarrow\end{matrix} \rightarrow R\uparrow$$

$$\psi = -1 + \frac{c_x}{u}(\tan\alpha_2 - \tan\beta_3) \qquad \rightarrow \psi\downarrow$$

↳ general: for achieving constant Δh_0 over blade span the following condition must be met

$$u(c_{\theta 2} - c_{\theta 3}) = \text{¢} \qquad \rightarrow \text{Euler}$$

Based on this condition general distributions of $c_{\theta 2}$ and $c_{\theta 3}$ can be formulated that fulfil $\Delta h_0 = \text{¢}$

$$\rightarrow c_{\theta 2} = a \cdot r^n + \frac{b}{r} \;;\; c_{\theta 3} = a \cdot r^n - \frac{b}{r} \qquad a, b \text{ constants}, \; n \text{ exponent}$$

proof: $c_{\theta 2} - c_{\theta 3} = \frac{2b}{r} = \text{const.} \; ✓.$

Δh_0? $\rightarrow u = r \cdot \omega \rightarrow u(c_{\theta 2} - c_{\theta 3}) = 2b \cdot \omega$

the following exponents are usually used

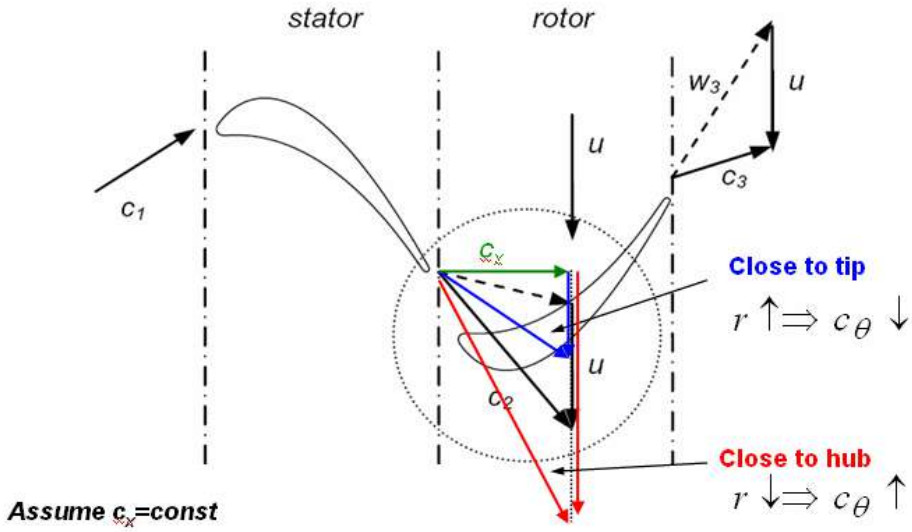
$n = -1$: free vortex
$n = 0$: exponential
$n = +1$: first power

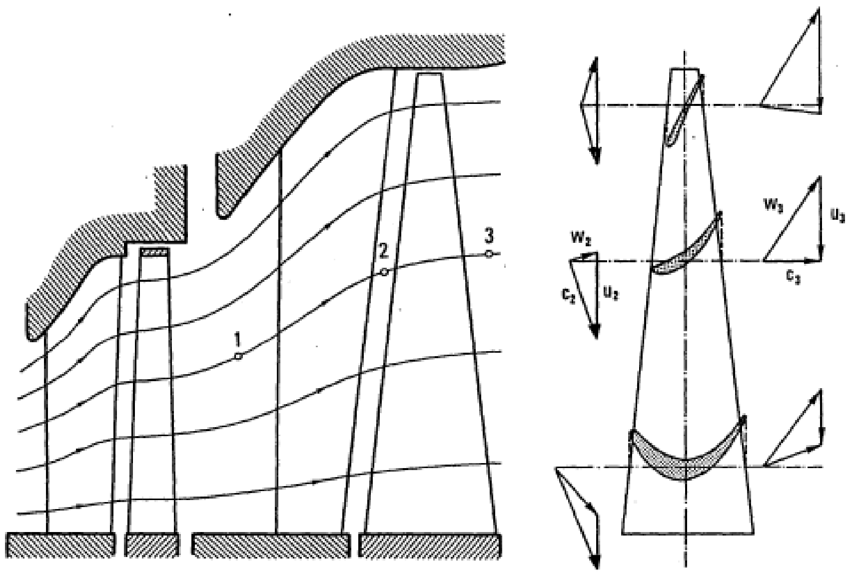
Spanwise Variation of Blade Shape

Spanwise variations in blade shape are due to spanwise changes in velocity triangles such as to fulfill radial equilibrium as well as achieving constant work over blade span.

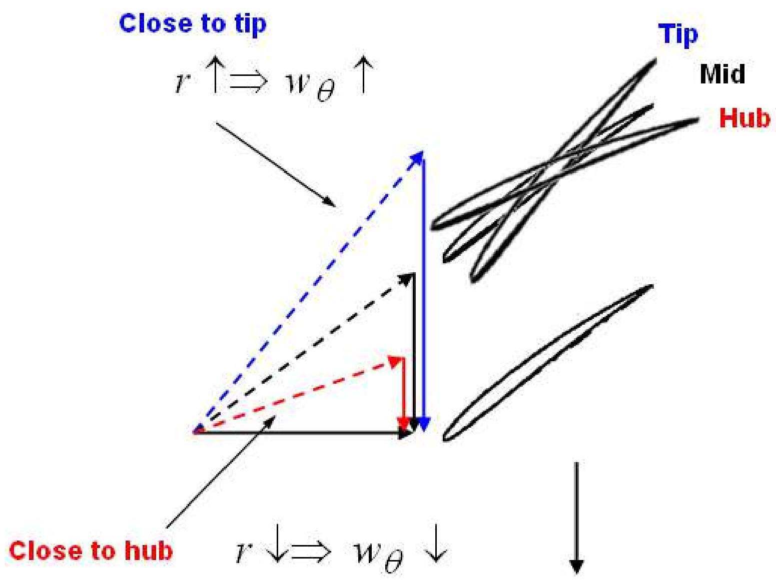
Most common distributions:

- Free Vortex
- Half vortex
- Forced vortex
- General distribution



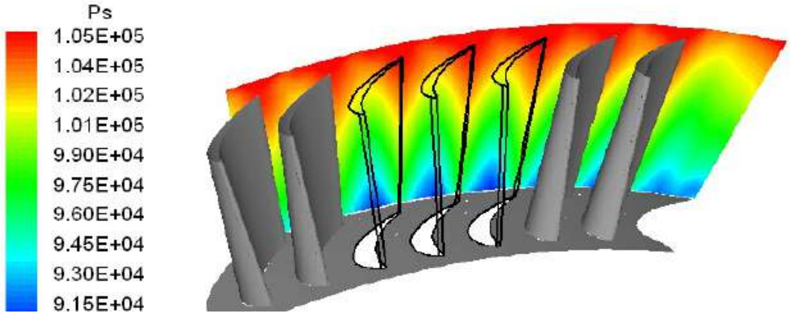


For fan blades the three-dimensional shape is mainly due to minimizing incidence effects as the relative inflow to the rotor blades is changing direction over span.

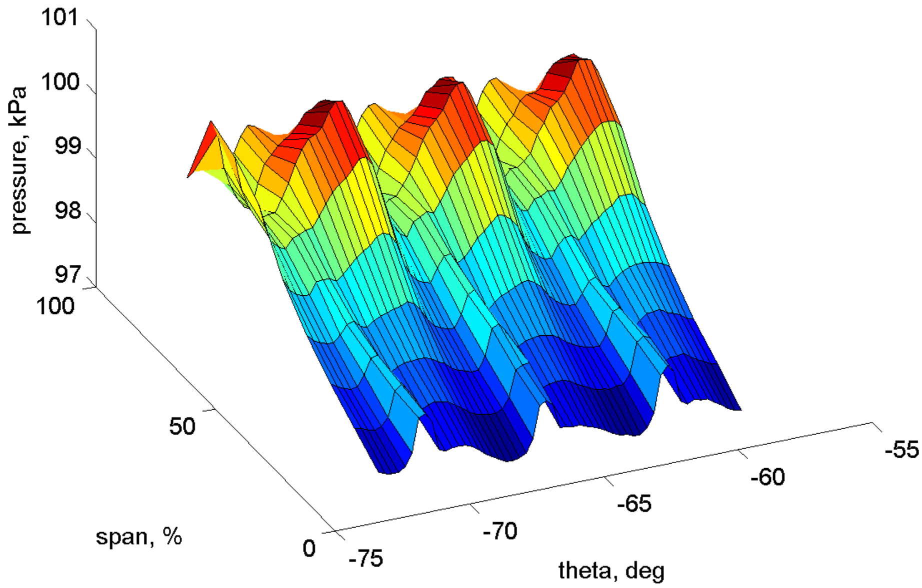

Close to tip
$r \uparrow \Rightarrow w_\theta \uparrow$
Tip
Mid
Hub
Close to hub
$r \downarrow \Rightarrow w_\theta \downarrow$

Radial Pressure Gradient

Due to annular shape of flow channel.



Static pressure distribution downstream (computed)



Static pressure distribution downstream (measured)

Advanced 3D Blade Design

The radial equilibrium provides conditions that allow us to determine the distribution of geometrical parameters along blade span and consequently a 3D blade shape. So far it has however been assumed that the various sections are arranged straight on top of each other such that their respective centers of gravity would lie on one line. This phenomenon is called stacking, which leads to the line connecting the centers of gravity being called the “stacking line”.

The various spanwise profile sections can however be arranged in different ways. This can be done to achieve a certain distribution of flow parameters downstream of the blade row and/or to affect the flow in the passage in a certain way. Another aspect that can profit from a certain stacking condition is the mechanical integrity both with respect to steady loading as well as unsteady loading.

The following stacking conditions are applied:

- Straight (i.e. normal stacking)
- Sweep → inclination of blade in direction of LE or TE
- Lean → inclination of blade to either SS or PS
- Bow → bowing of blade in direction of SS or PS, also referred to as compound lean
- Compound sweep → varying sweep along blade span
- Combinations of the above

The above conditions provide vast possibilities for a turbomachine blade designer. Although the profile sections remain unchanged the final blade can differ considerably in shape as well as operation. Below a brief study is included that analyzes the effects of various advanced 3D blade design conditions.

<u>Base Case</u>

As base case a turbine stator is chosen. A generic blade has been designed using a skeleton line as employed for the 2D flow studies in the present course and overlaying a base profile thickness distribution. Following radial equilibrium conditions a typical twisted shape of the blade is achieved.

The main parameters are the following:

- Hub radius	250mm
- Tip radius	350mm
- # blades	70
- LE angle at hub	-10deg
- LE angle at tip	10deg
- TE angle at hub	55deg
- TE angle at tip	67deg
- Axial chord at hub	45mm
- Axial chord at tip	37mm

3D Geometries

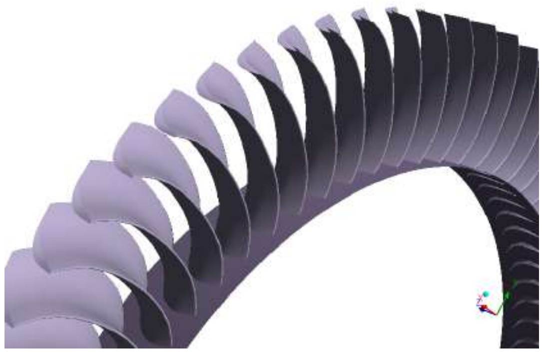
Base

Lean

Sweep

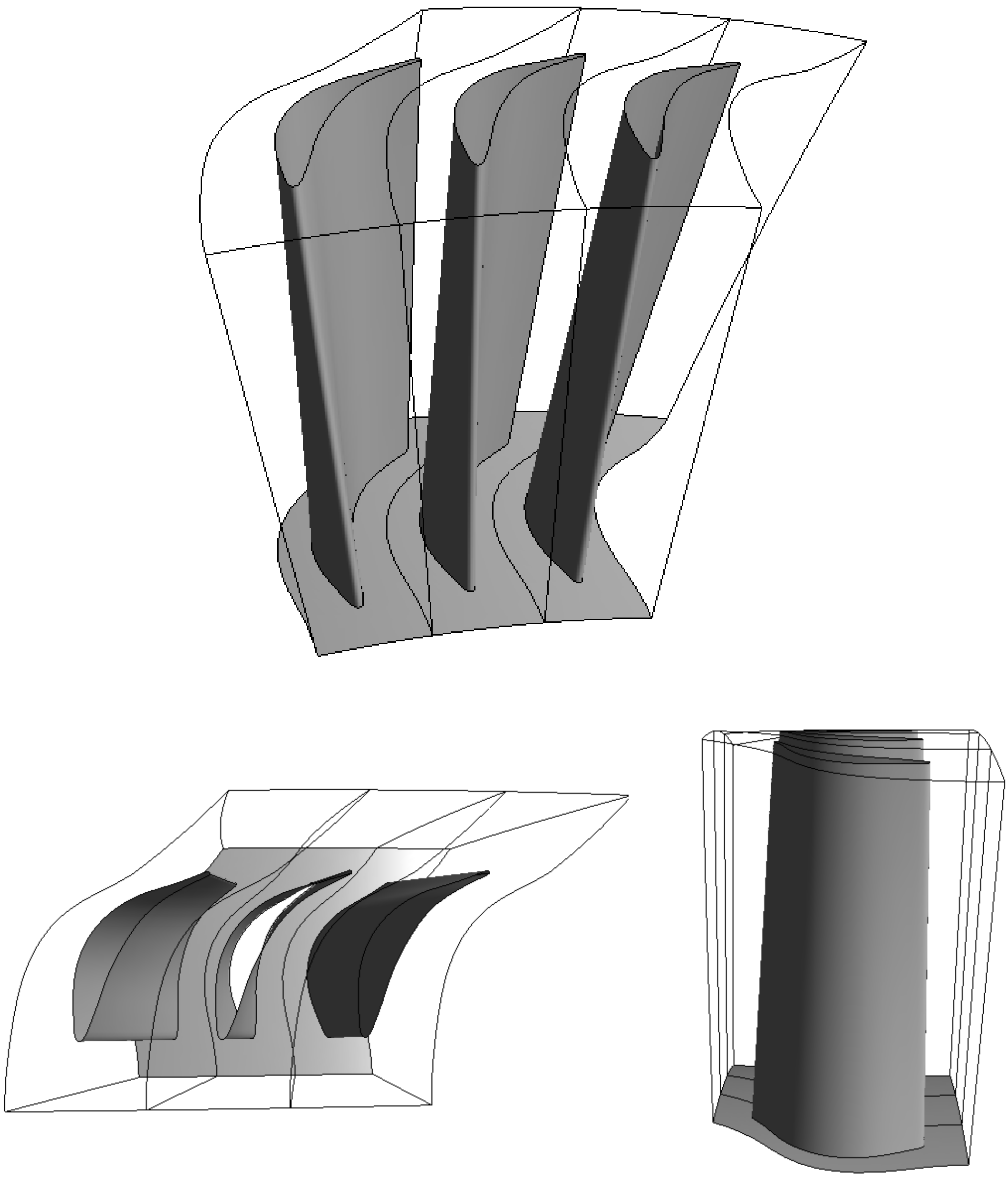
Compound sweep

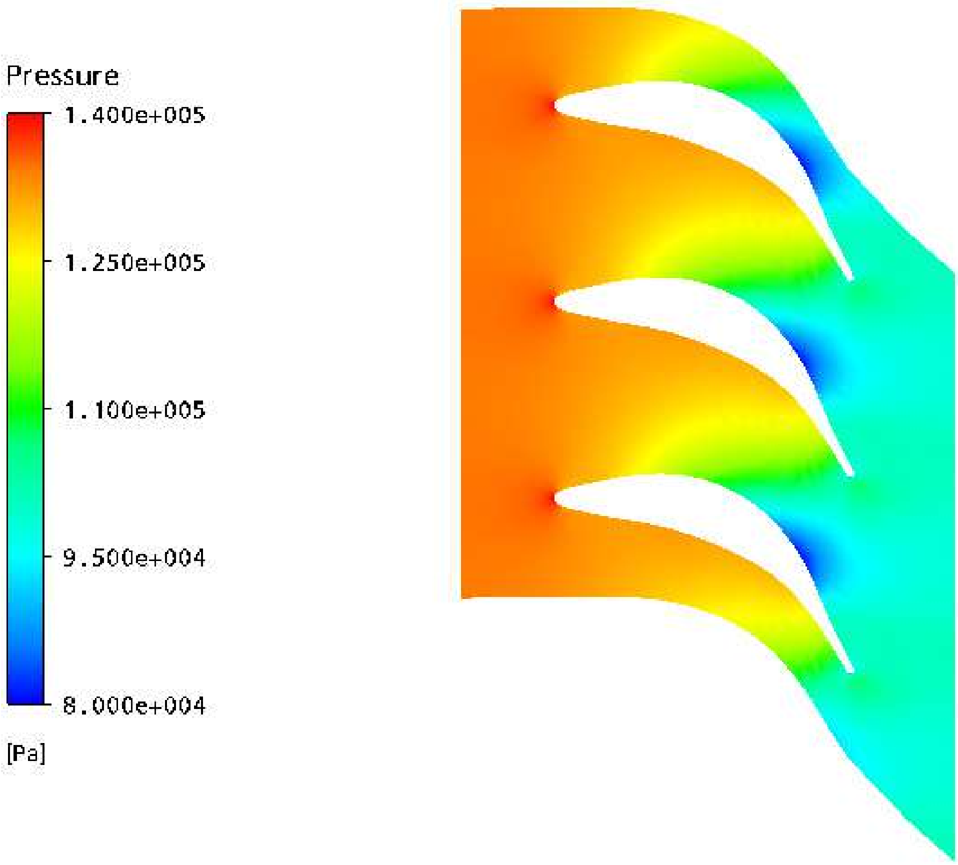
Bow



Bow 2

Case 1: Base case

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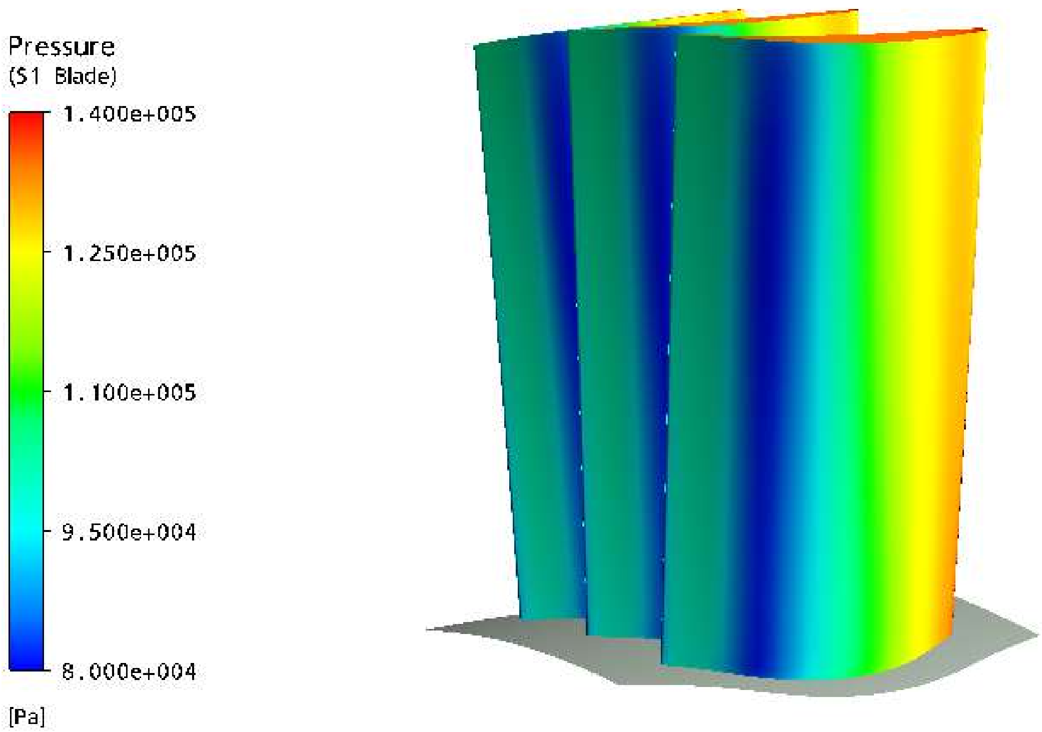
1.250e+005
1.100e+005
9.500e+004
8.000e+004
[Pa]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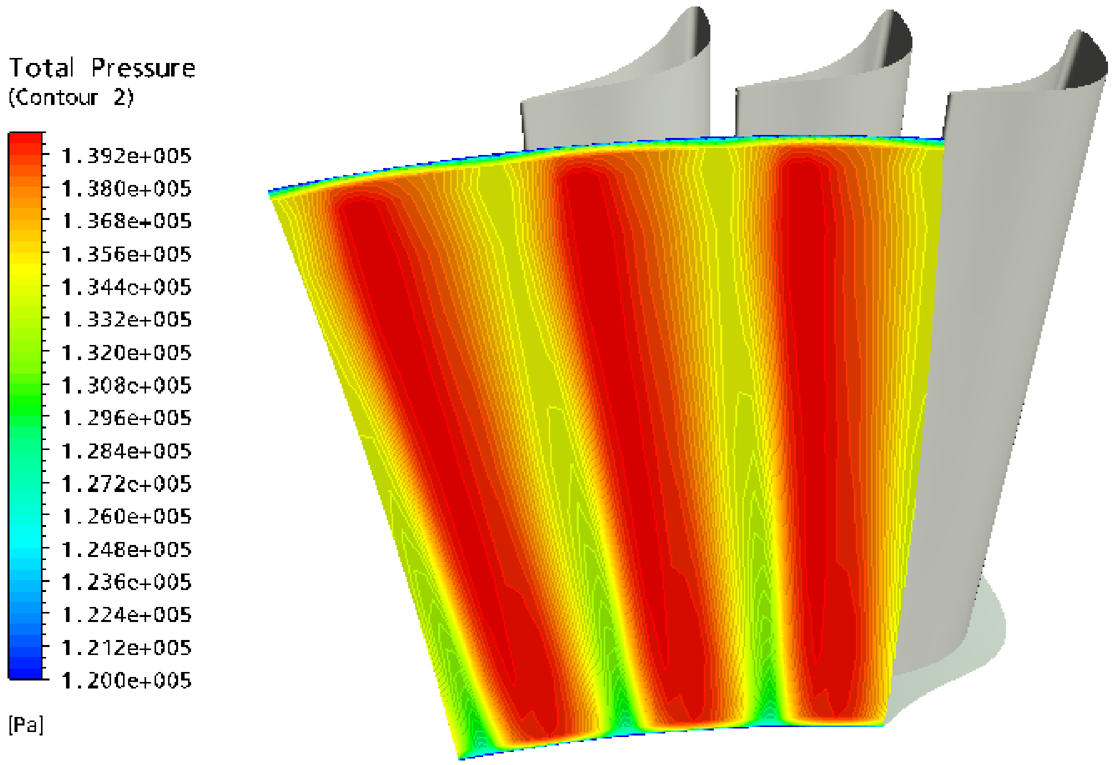
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1.308e+005
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1.284e+005
1.272e+005
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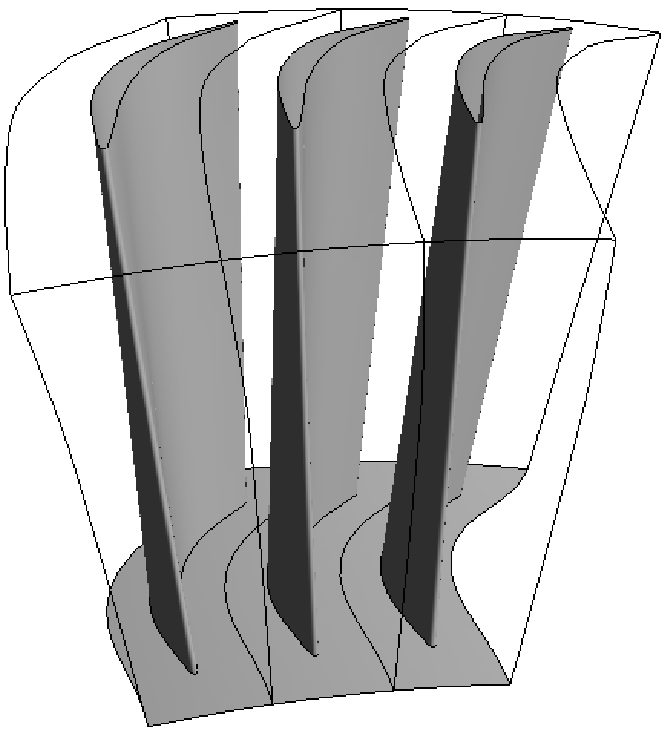
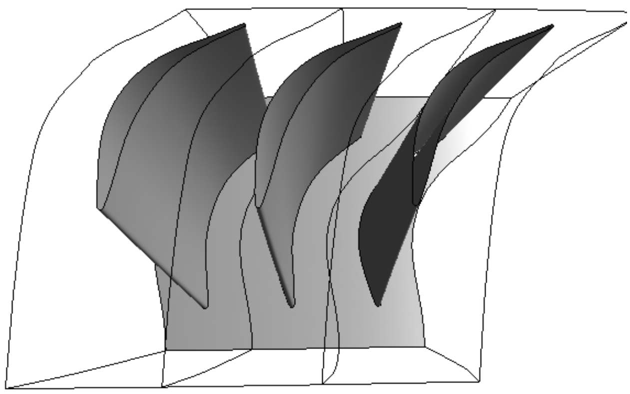
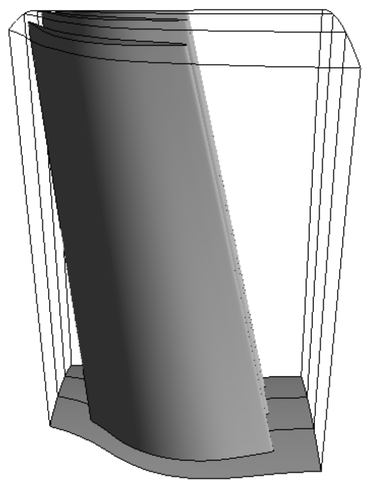
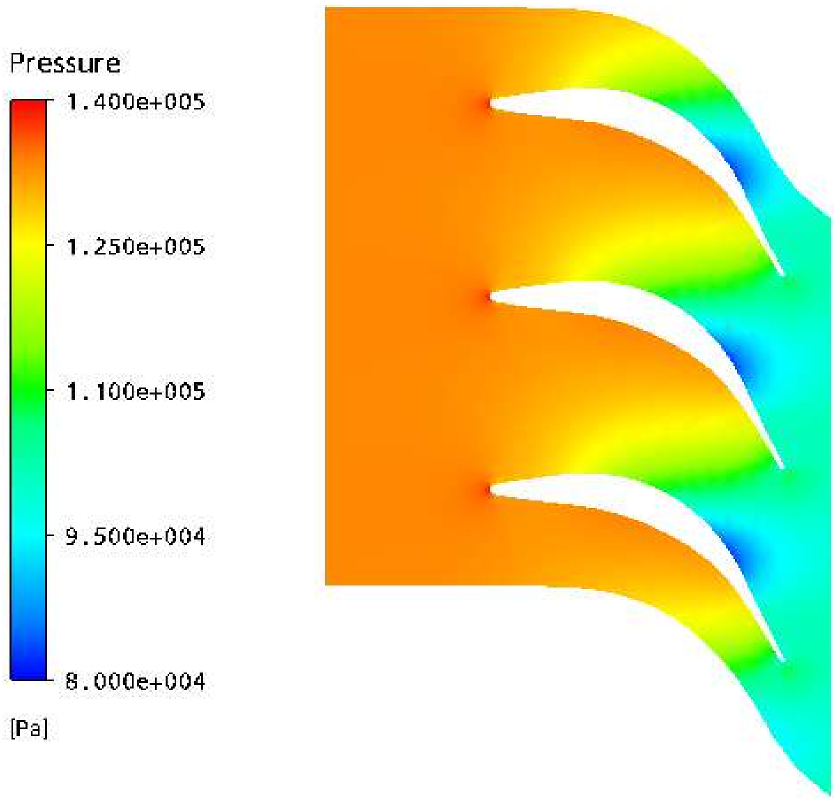
1.248e+005
1.236e+005
1.224e+005
1.212e+005
1.200e+005
[Pa]

Case 2: Sweep





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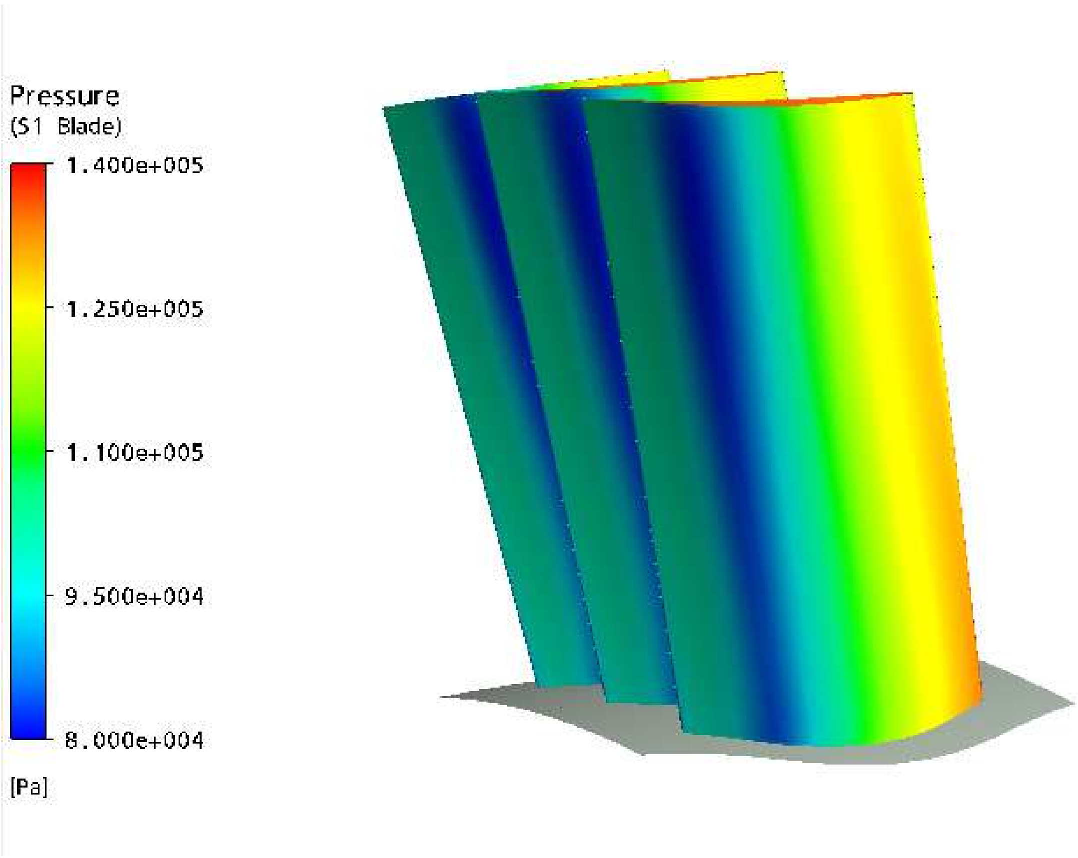
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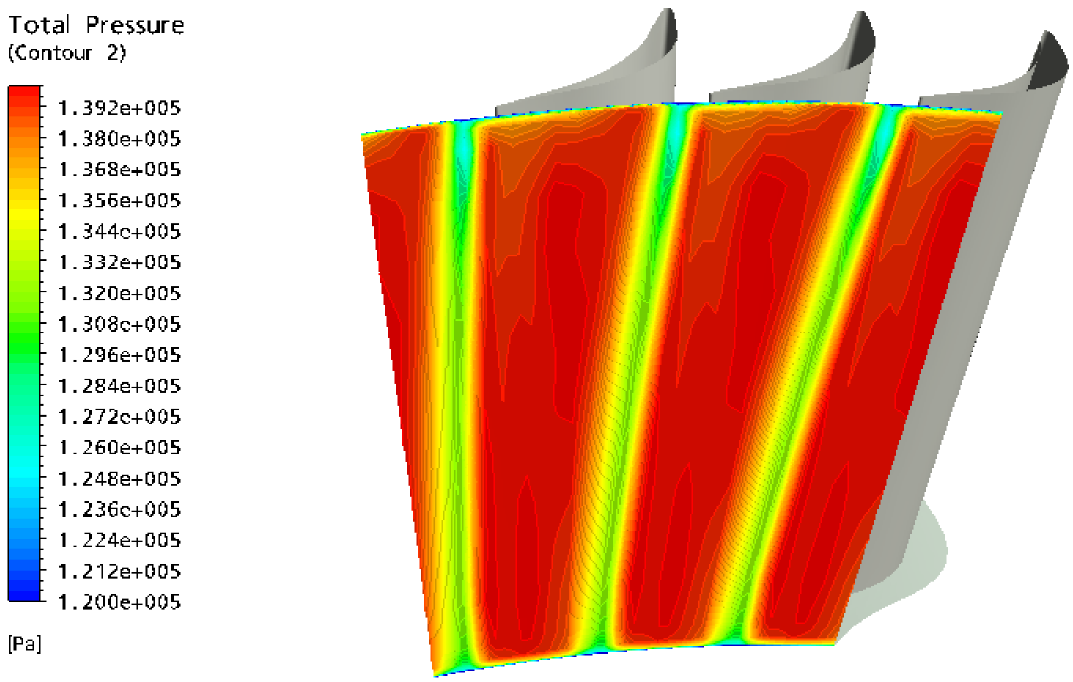
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1.272e+005
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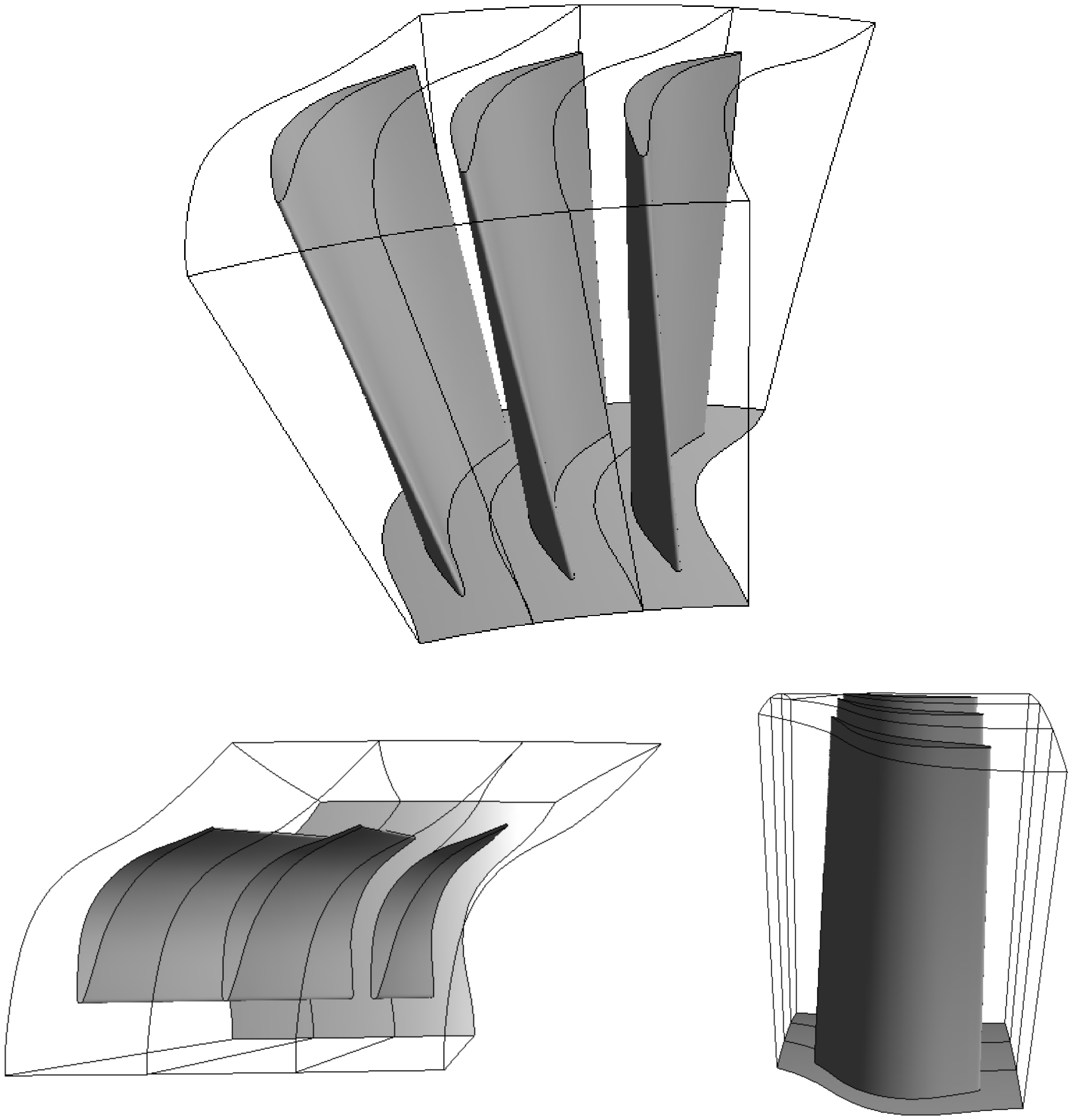
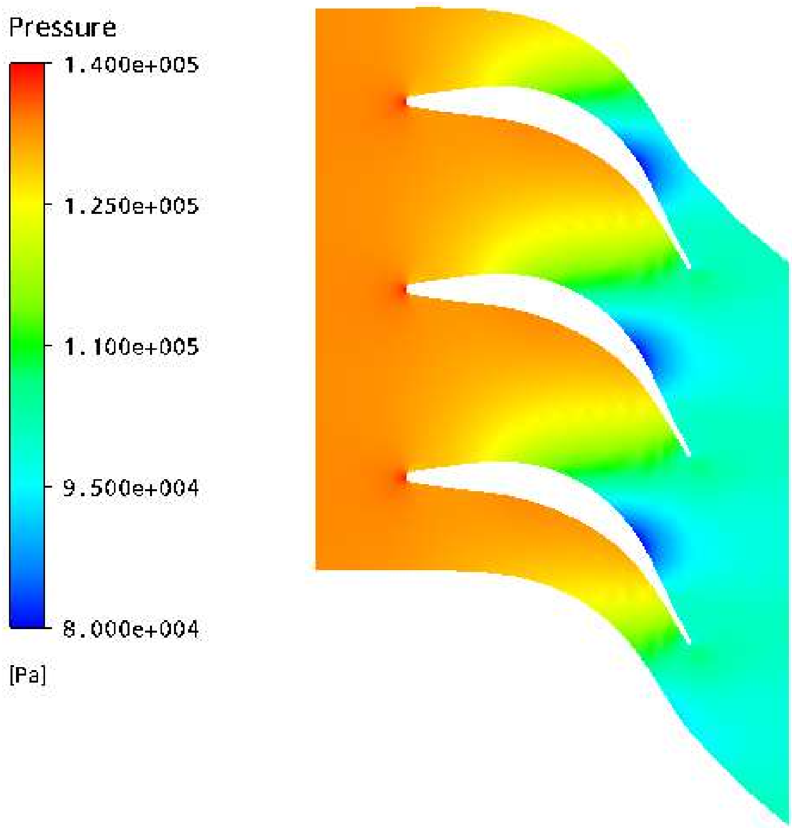
[Pa]

Case 3: Lean

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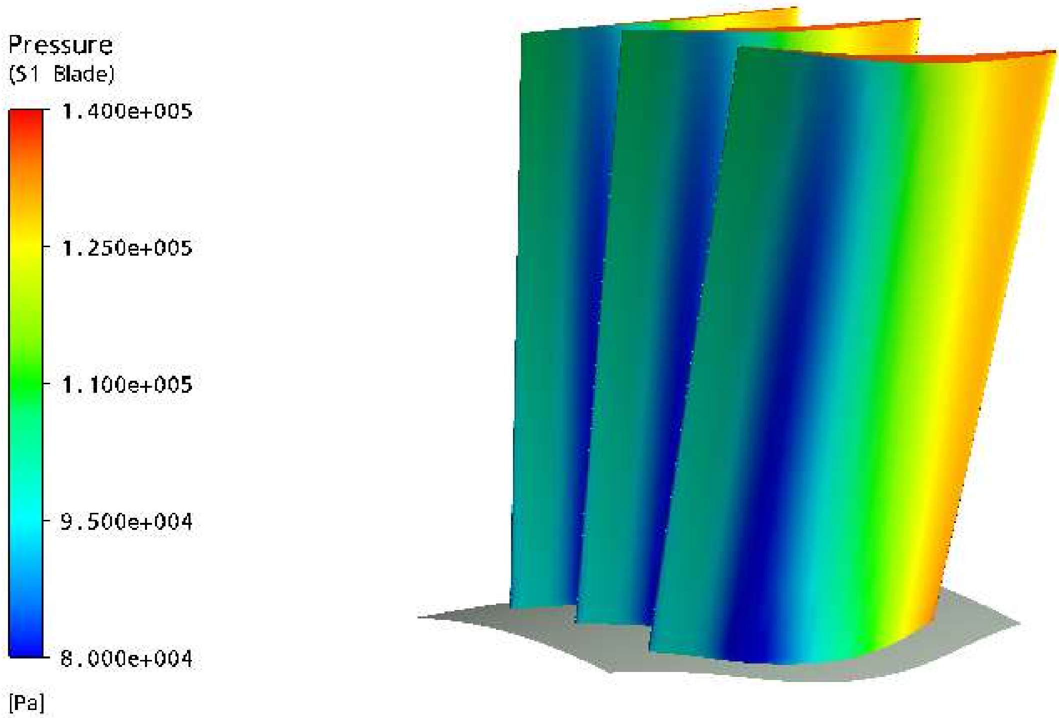
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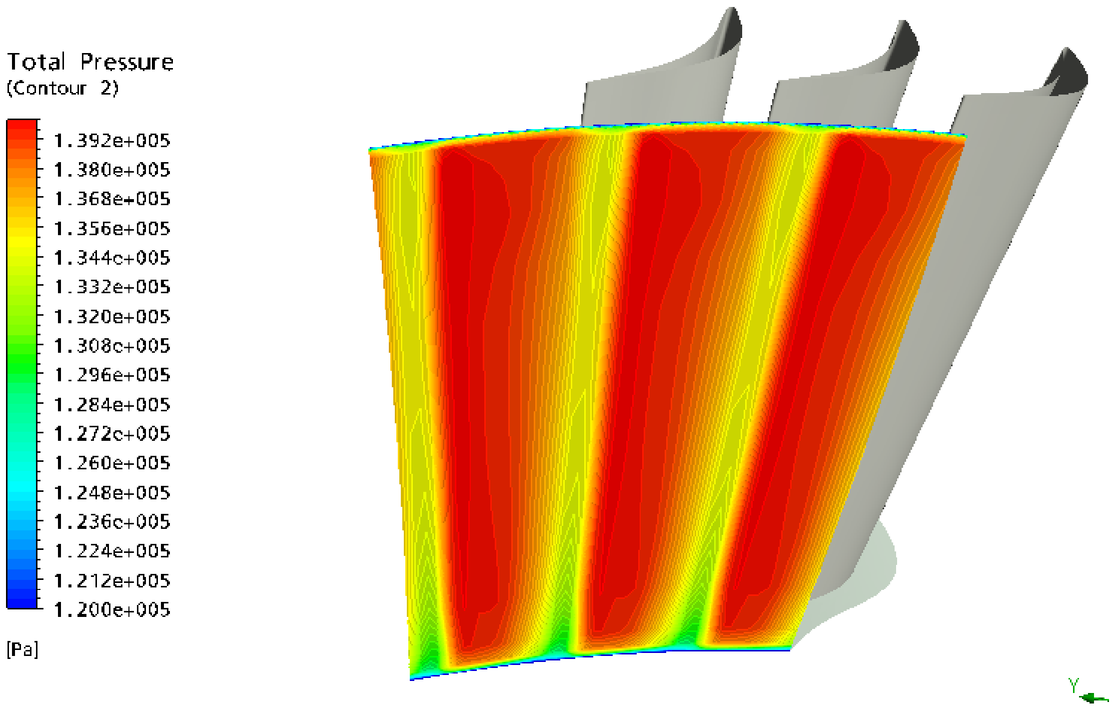
Total Pressure
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1.272e+005
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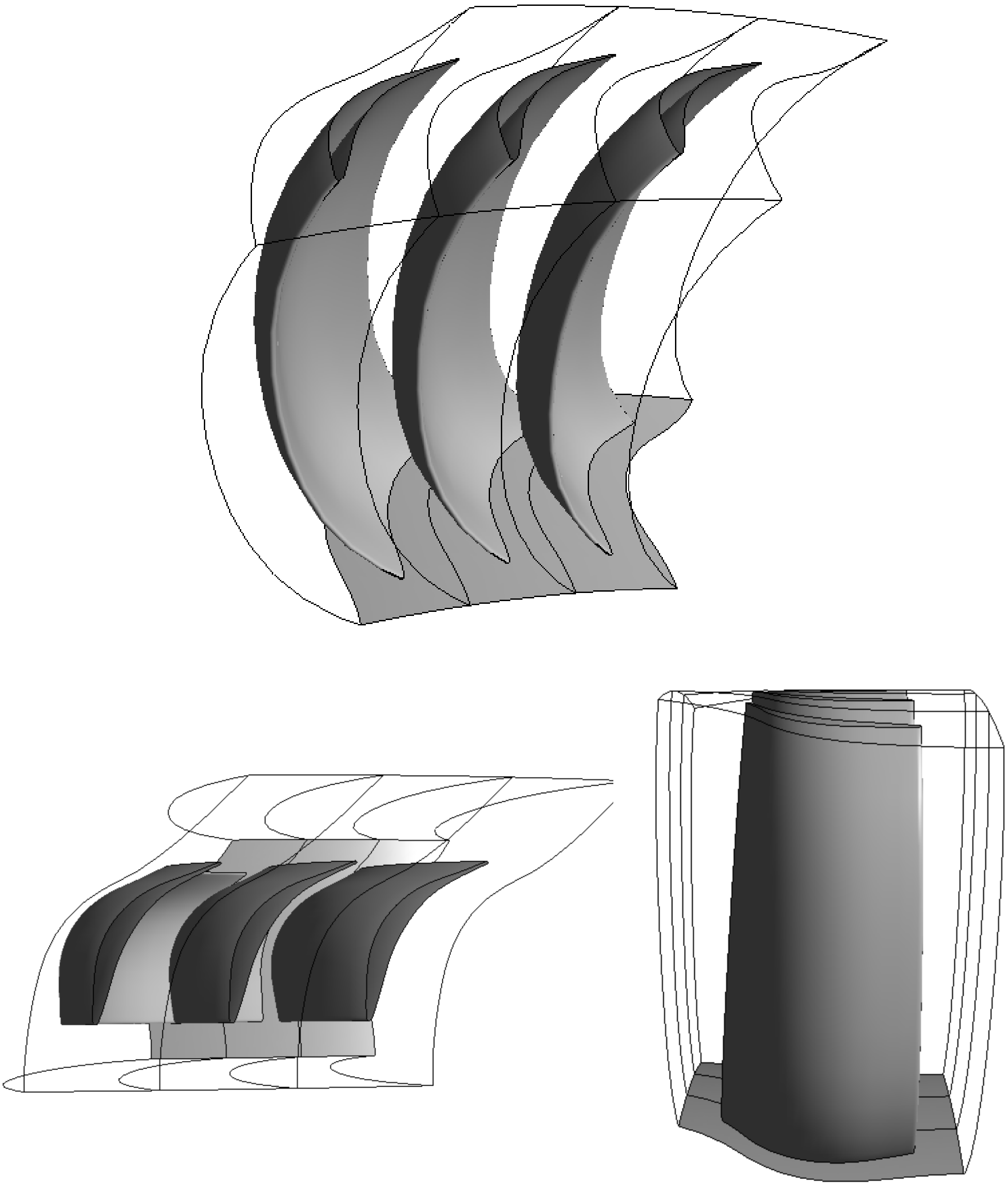
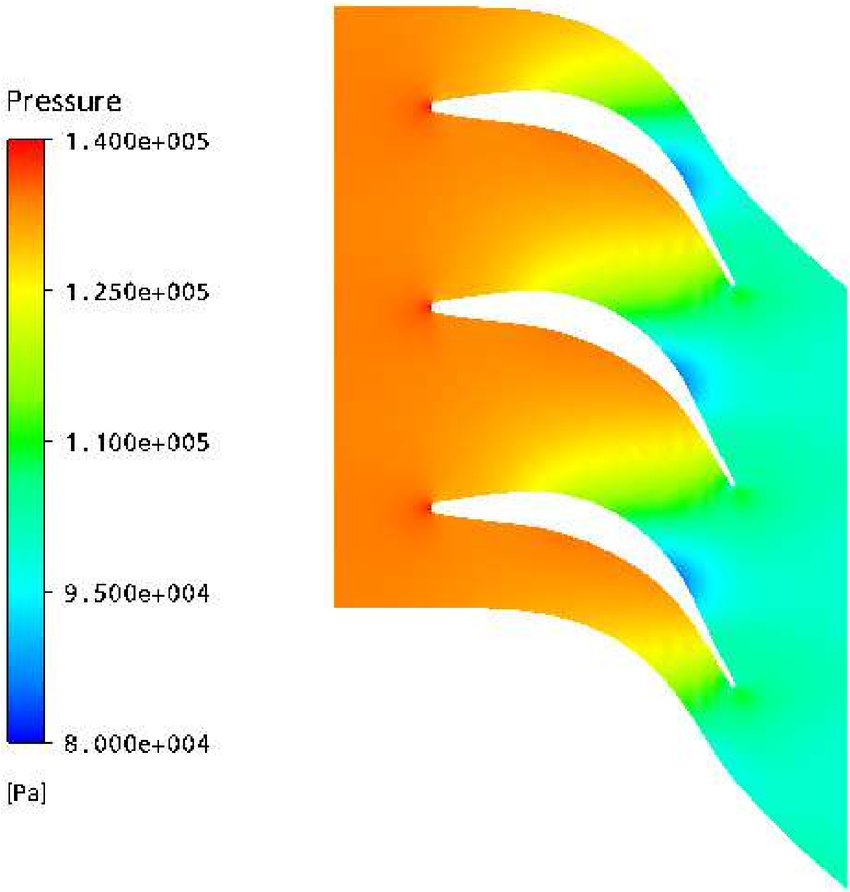
1.236e+005
1.224e+005
1.212e+005
1.200e+005
[Pa]
Y

Case 4: Bow

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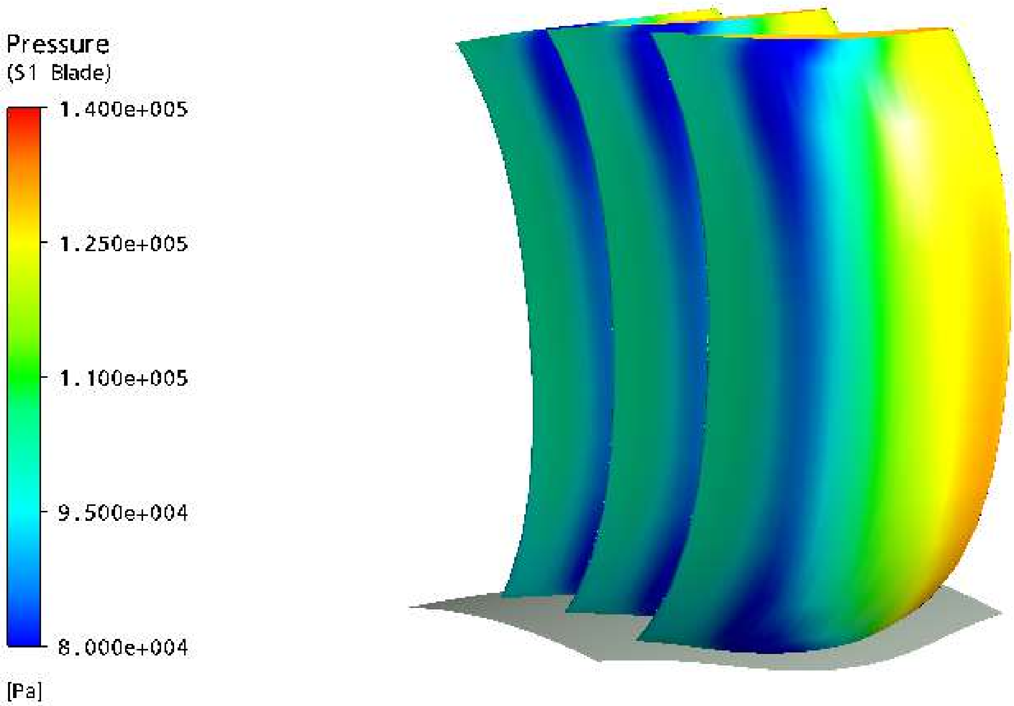
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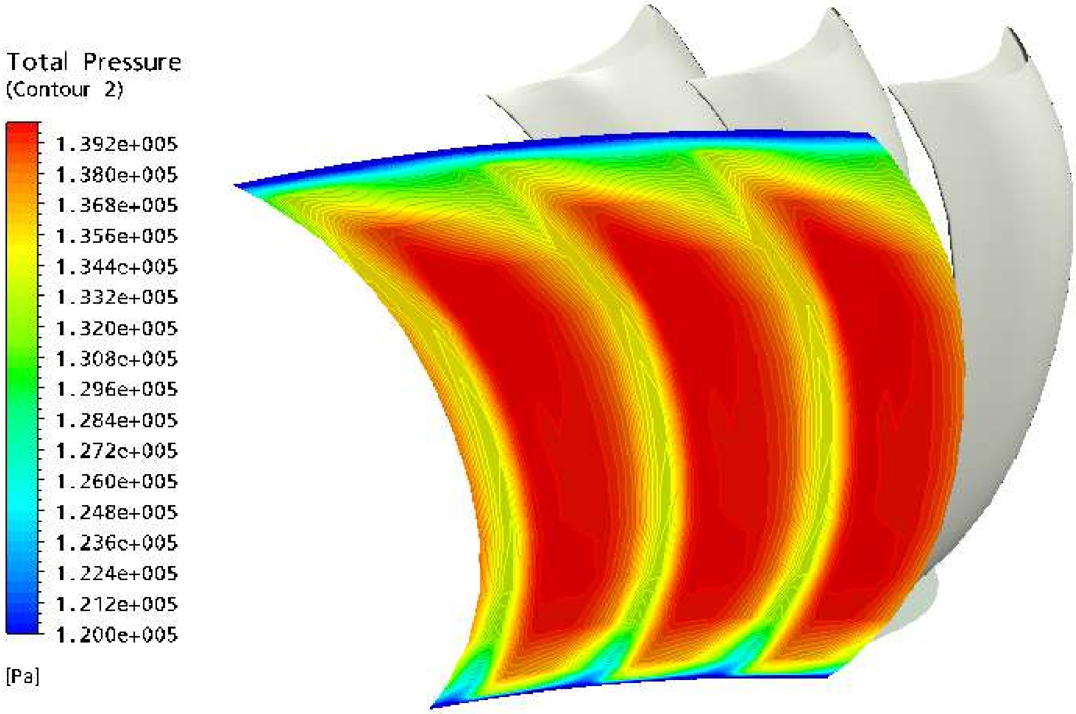
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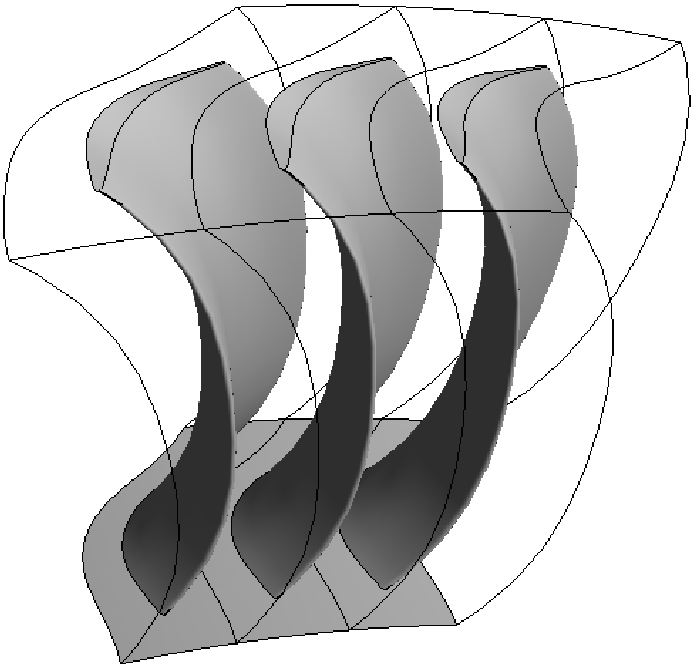
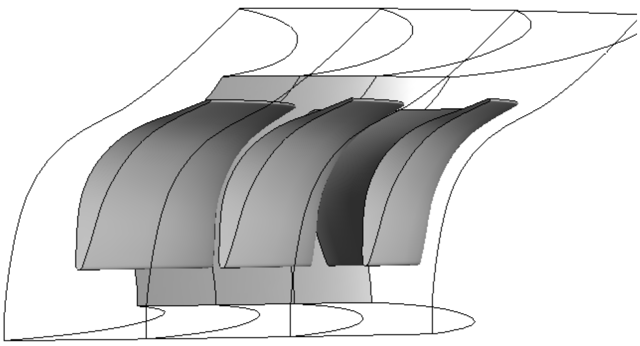
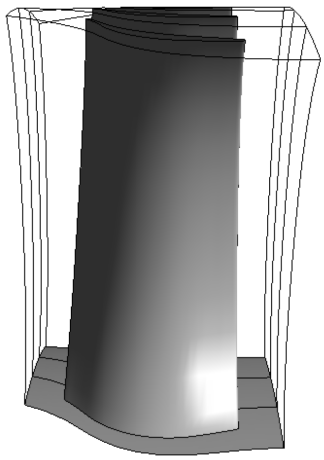
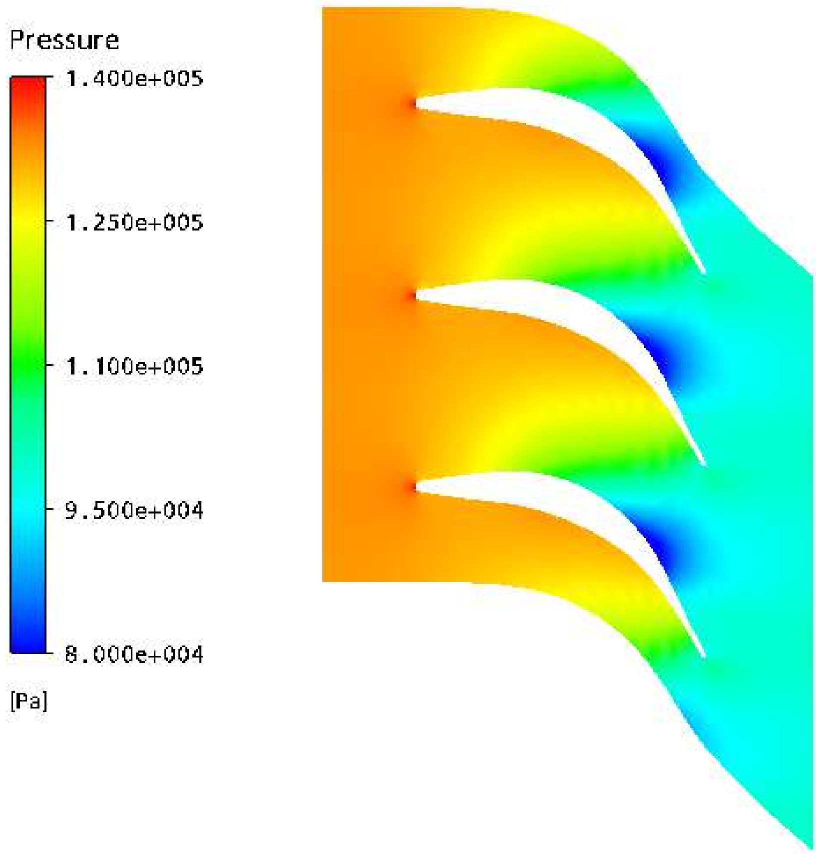
[Pa]

Case 5: Bow 2





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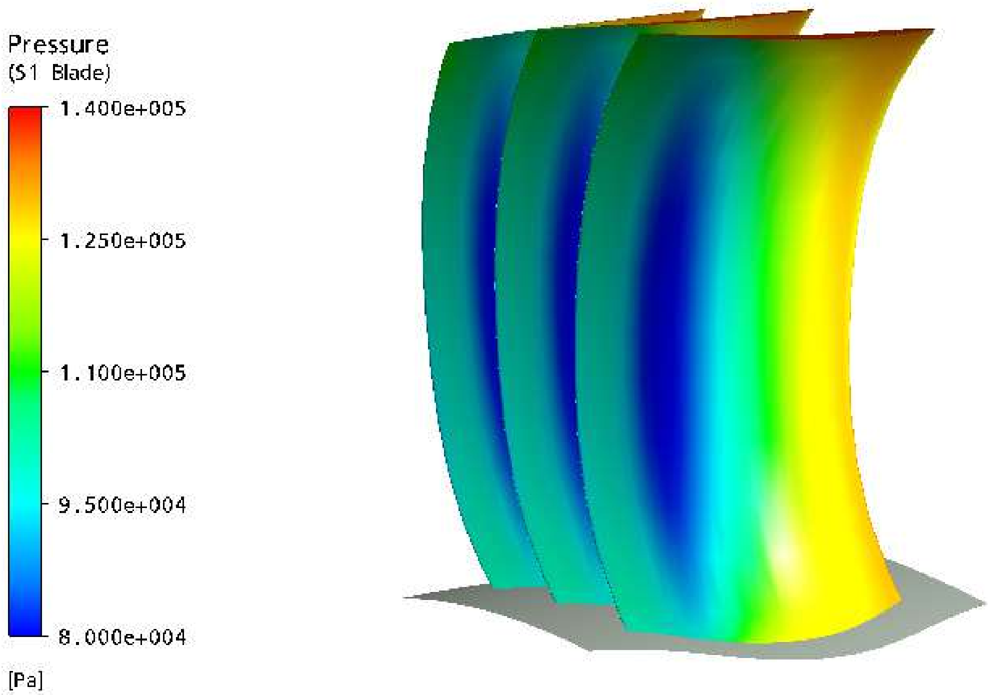
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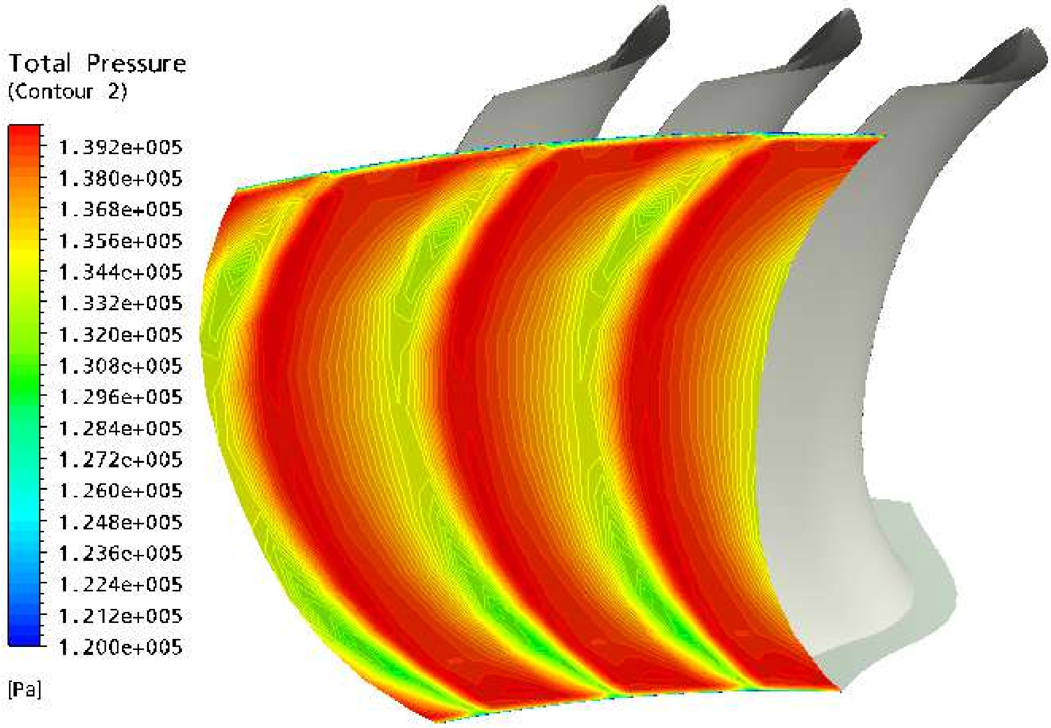
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1.284e+005
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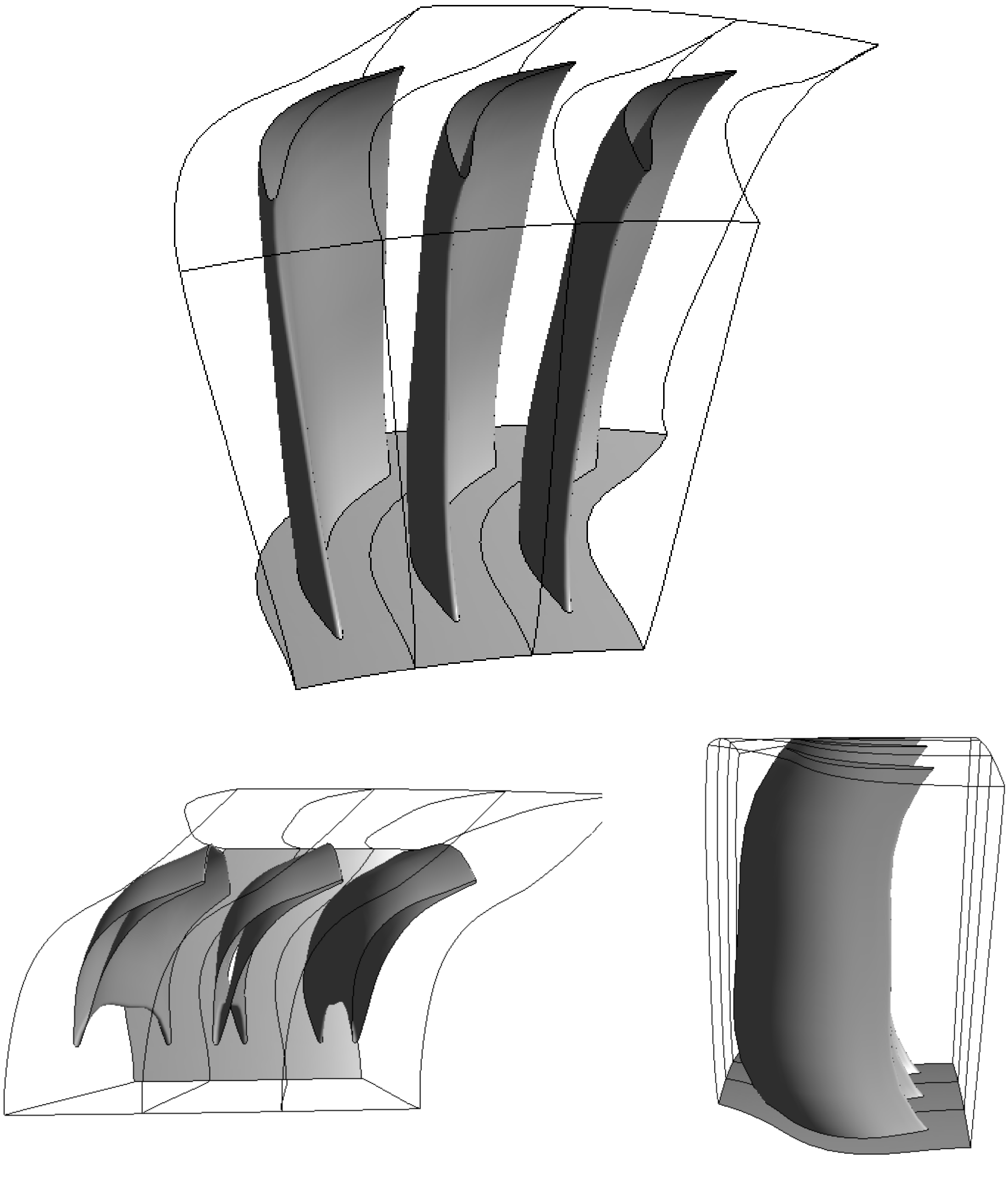
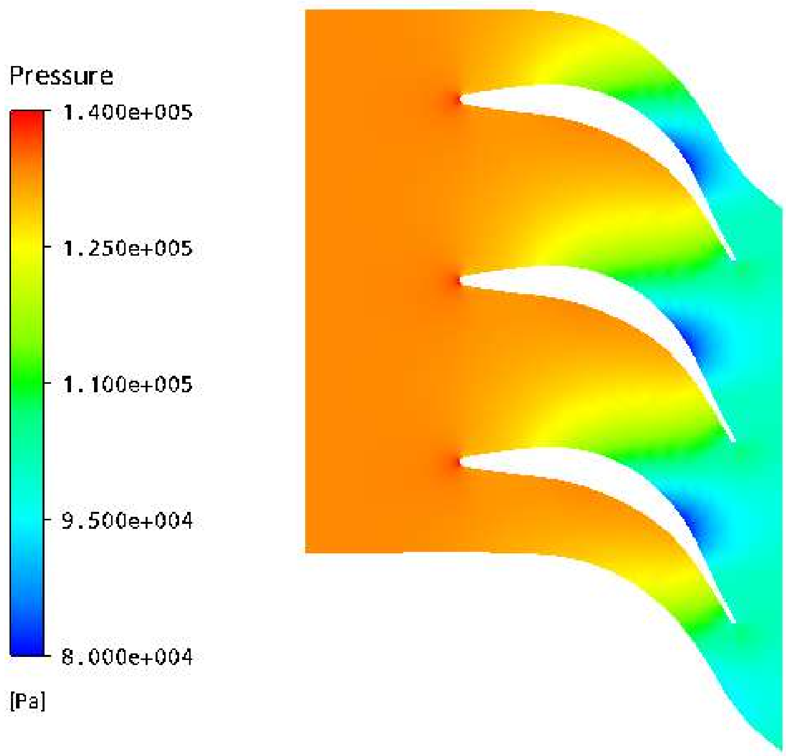
1.200e+005
[Pa]

Case 6: Compound Sweep

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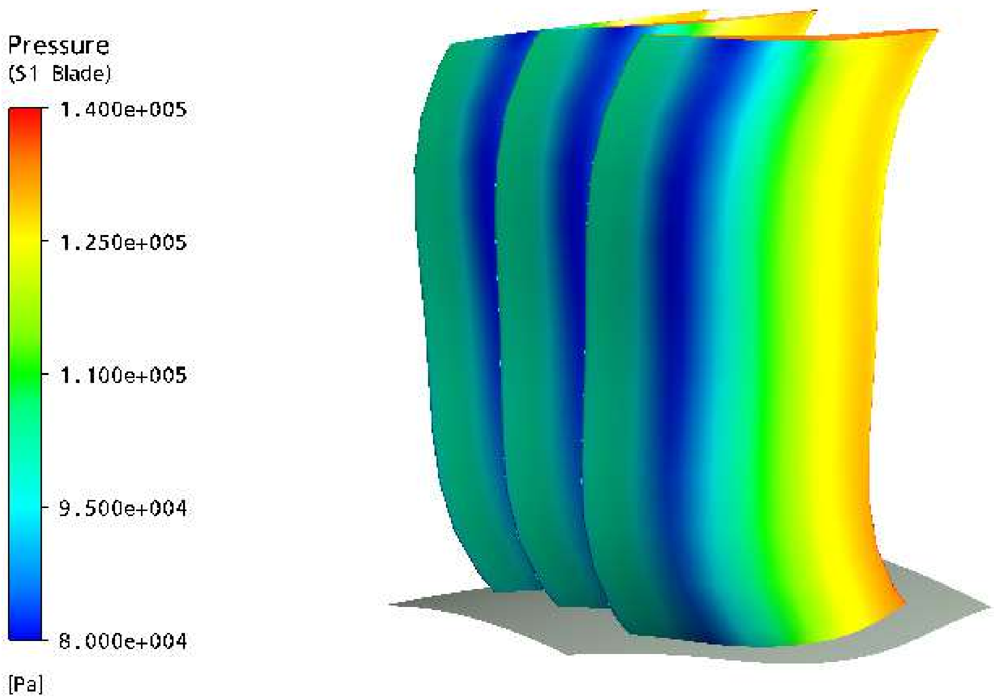
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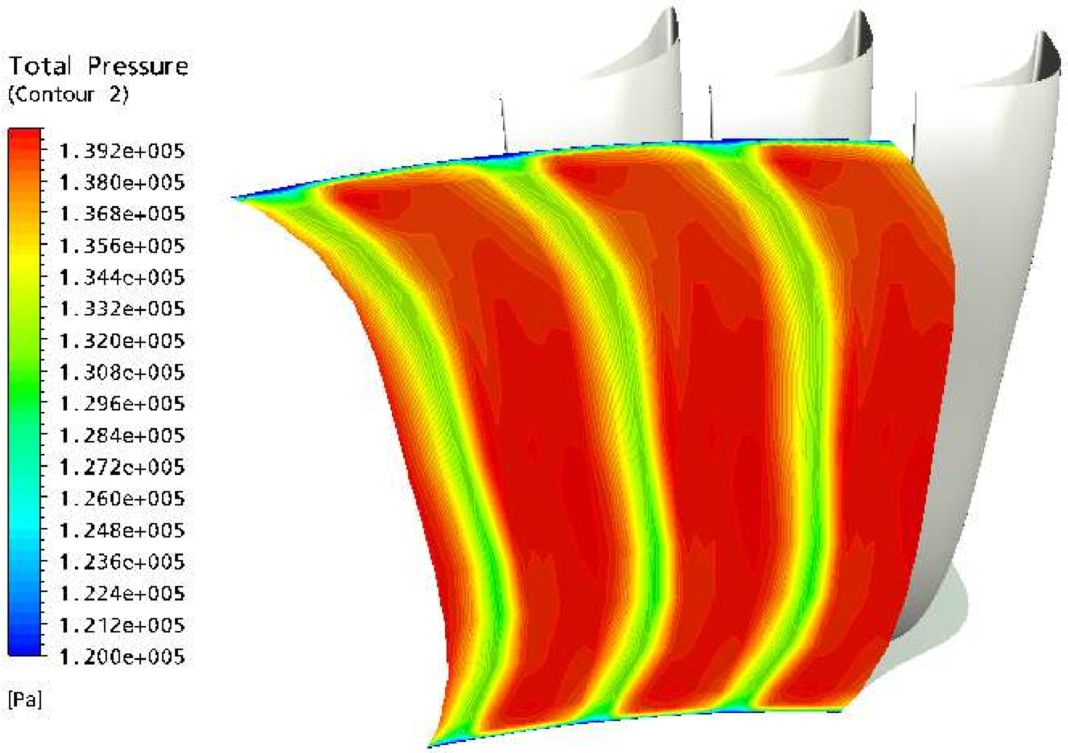
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1.236e+005
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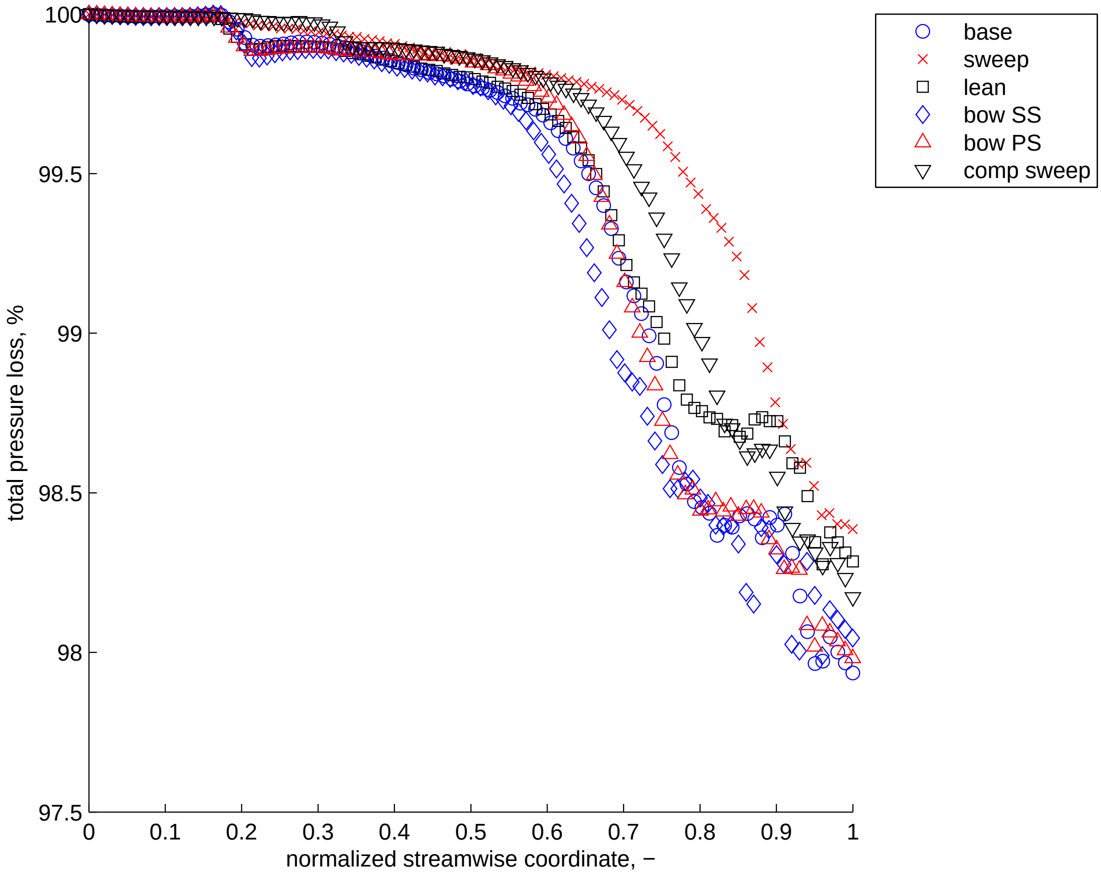
1.200e+005
[Pa]

Comparison of 3D Geometries

In the figure below the mass averaged total pressure in streamwise direction is compared.



Observations

- The base case features the highest losses
- Applying sweep geometry leads to the greatest reductions in total pressure loss. Compared to the base case losses the reduction is 22%
- Lean and compound sweep also lead to a considerable reduction in total pressure losses
- The streamwise distribution of total pressure losses is noticeably different for the two bowed cases within the blade passage. This indicates that the development of losses is quite different

Example: Advanced 3D Blade Shaping for a Fan

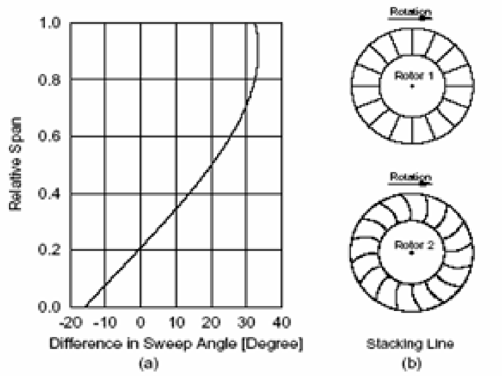
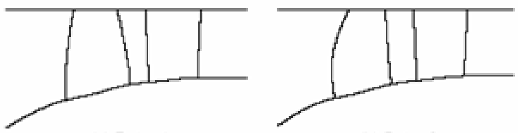
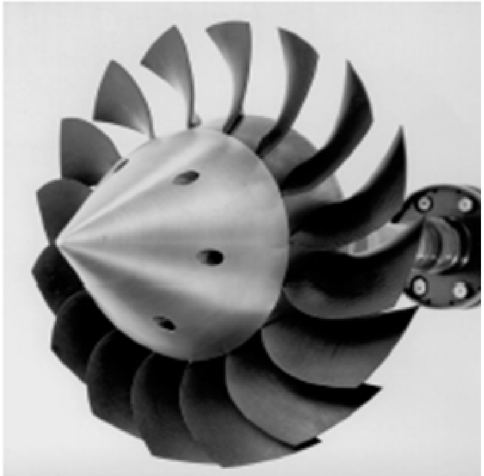


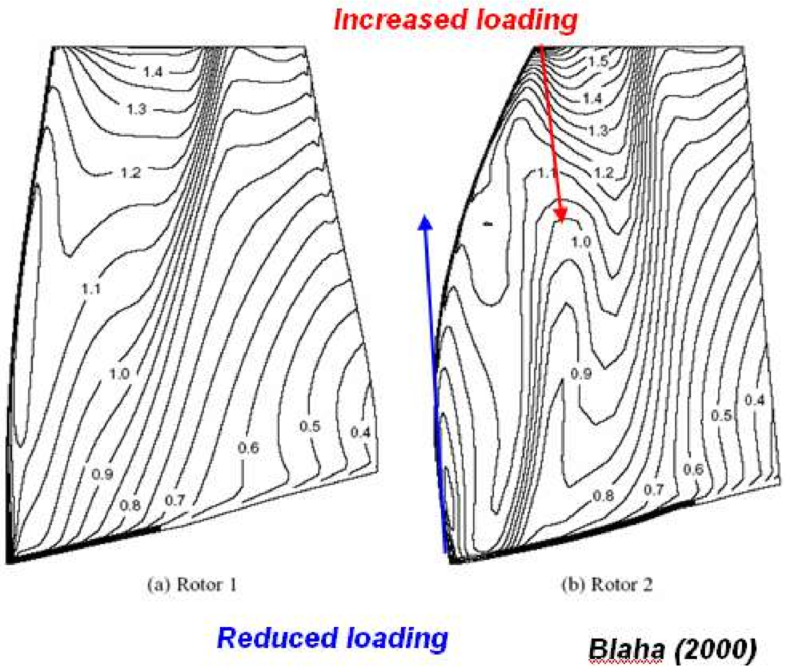
Figure 1. Difference in aerodynamic sweep angle ($\Delta\lambda$) and stacking lines for both rotors.



(a) Rotor 1 (b) Rotor 2

Figure 2. Meridional views of investigated rotors.



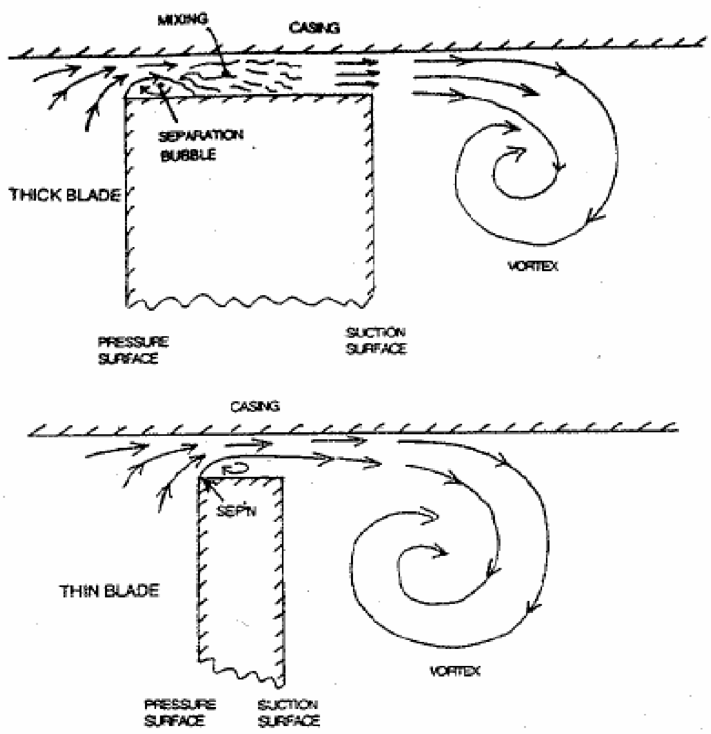


Blaha (2000)

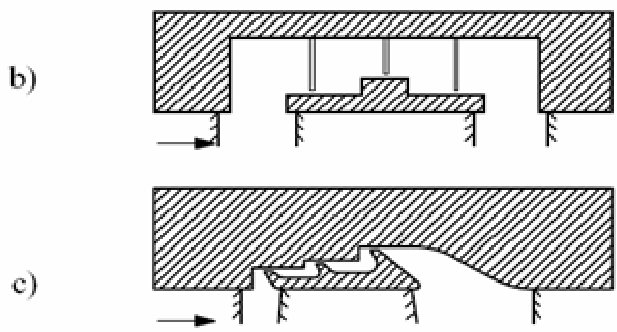
3D Flow Phenomena Due to Geometrical Features

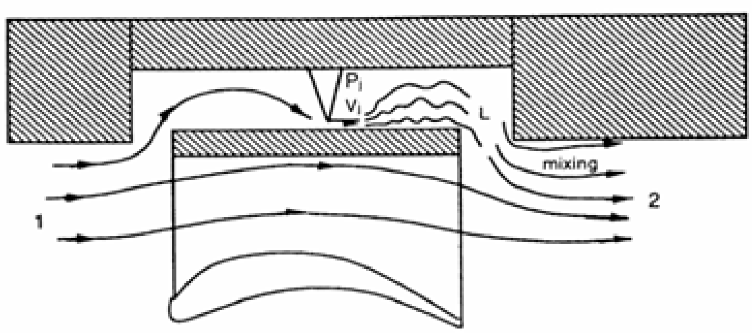
Possible features that can lead to 3D flow phenomena are gaps, slots etc.

Unshrouded rotor

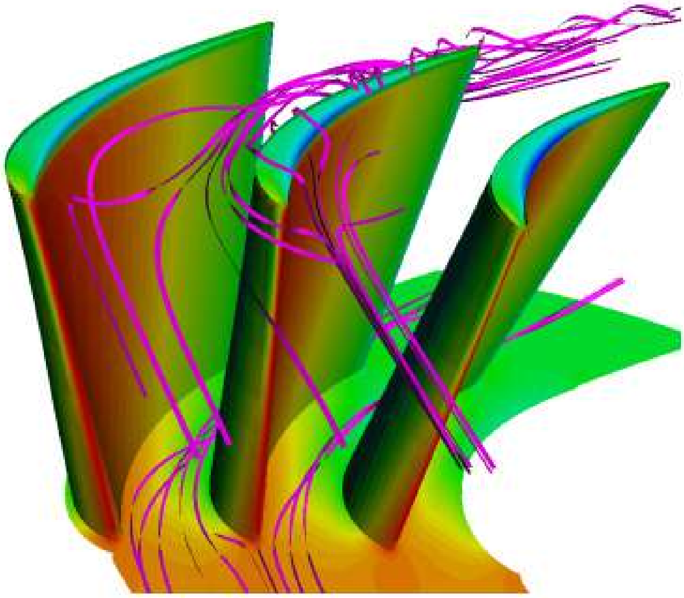


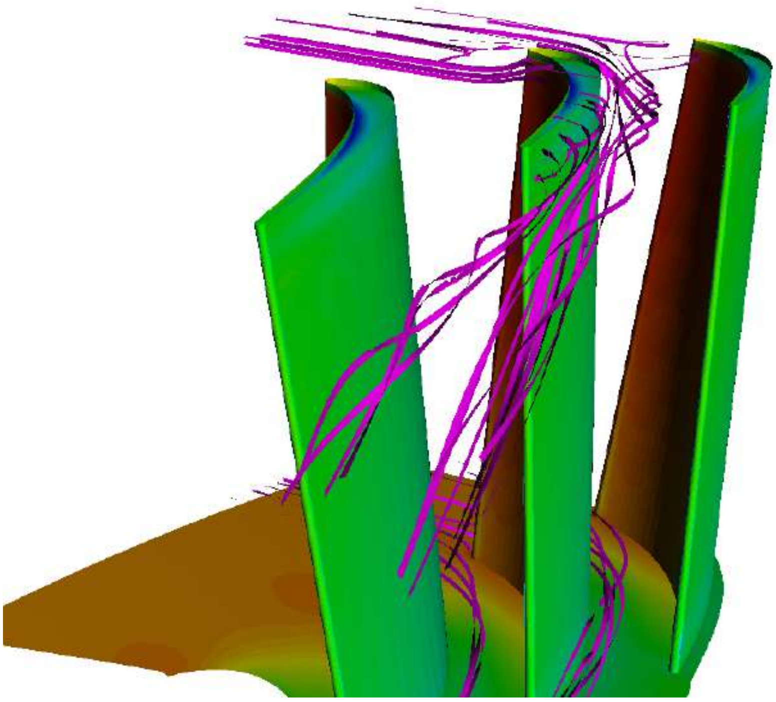
Shrouded rotor



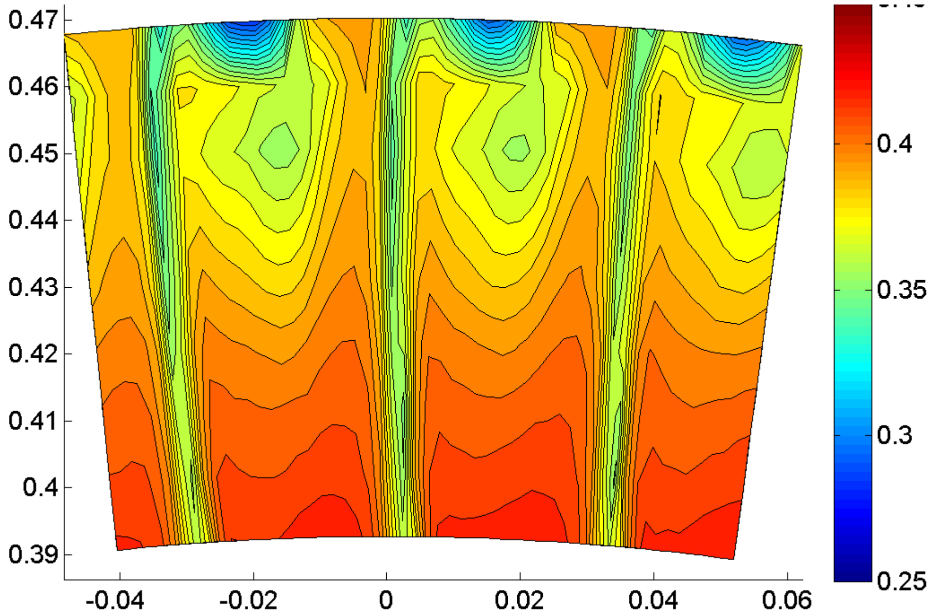


Predicted tip leakage flow

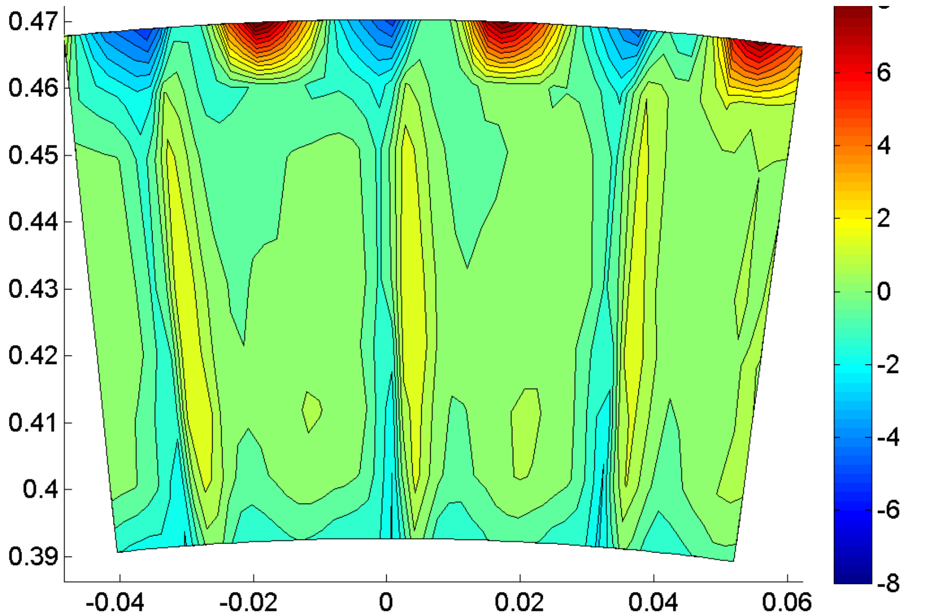




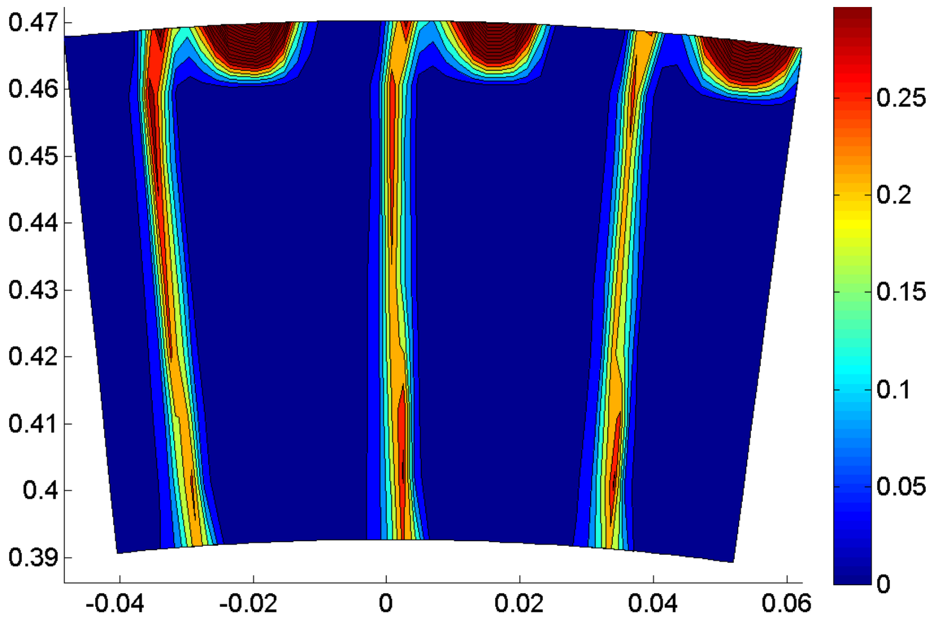
Effect on cascade outlet flow field



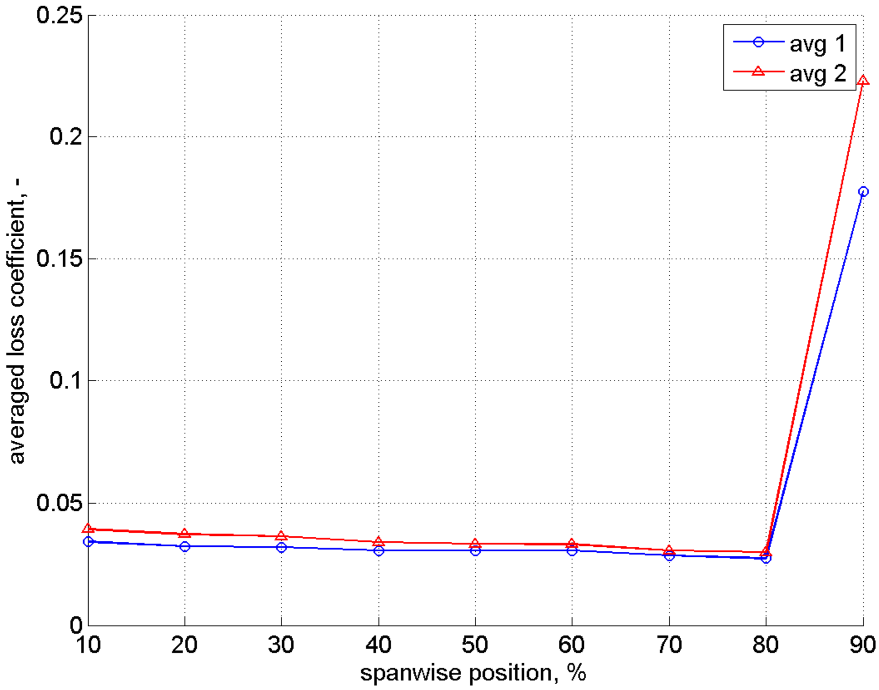
Mach number distribution downstream



Secondary flow angle distribution downstream



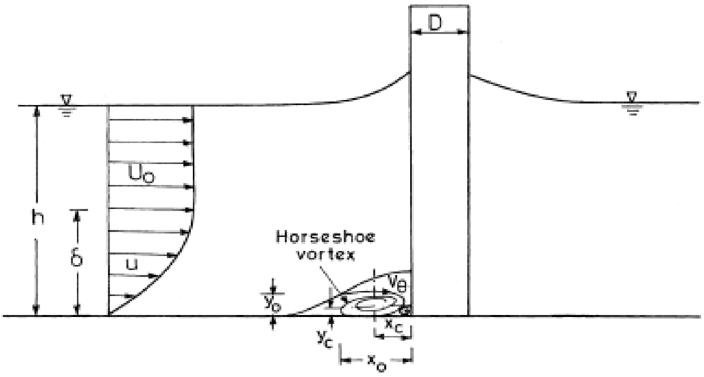
Distribution of loss coefficient downstream of cascade

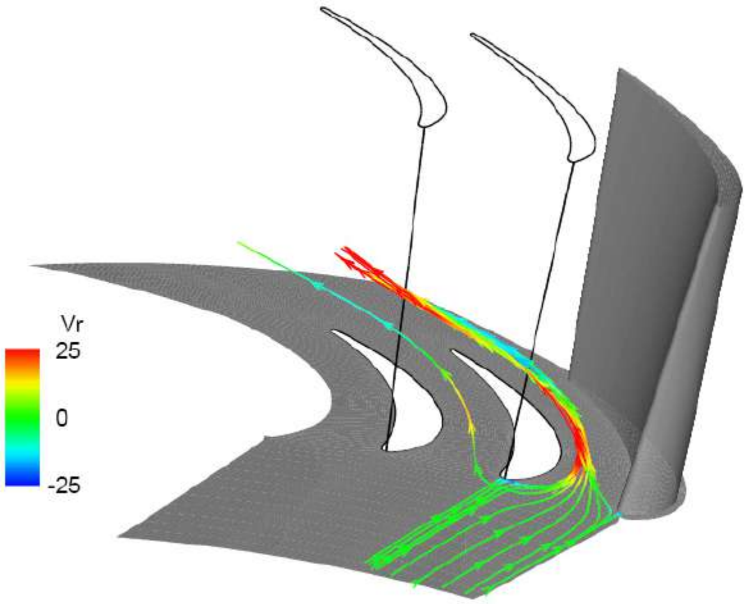


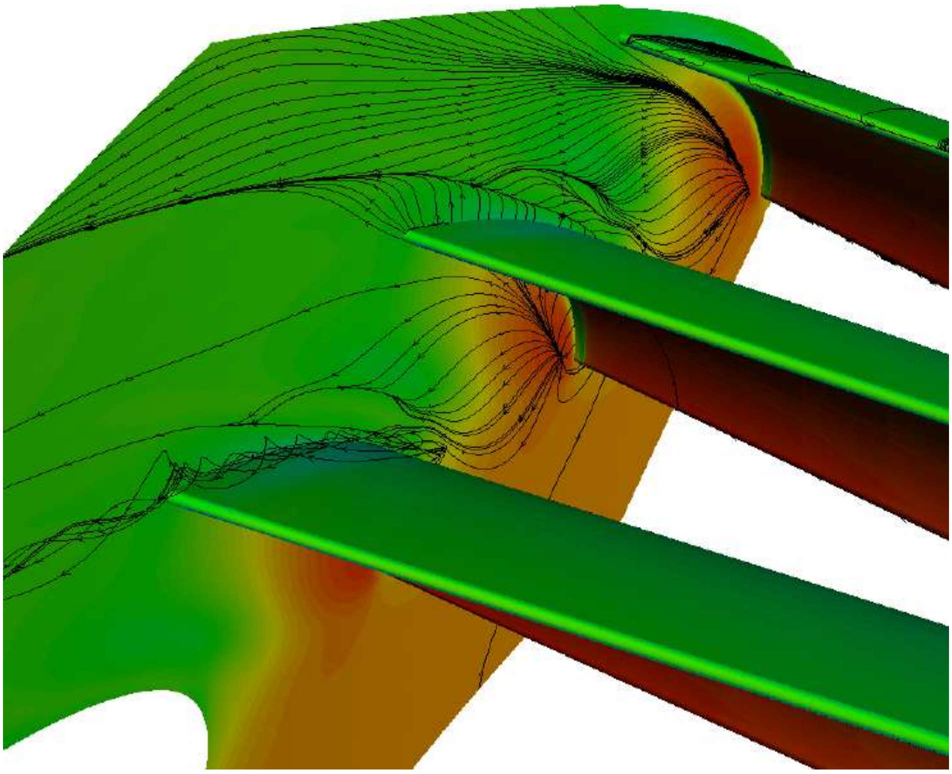
Spanwise distribution of averaged loss coefficient

3D Flow Phenomena Due to Vortices and Cross Flows

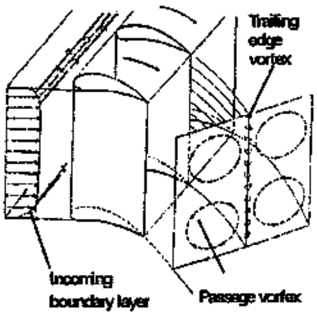
Horseshoe vortex



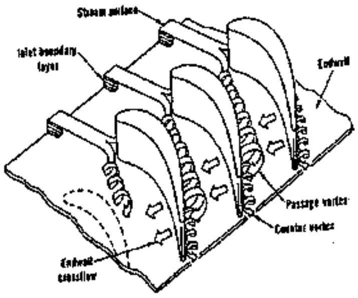




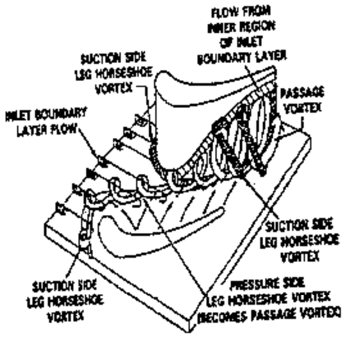
Secondary flow models



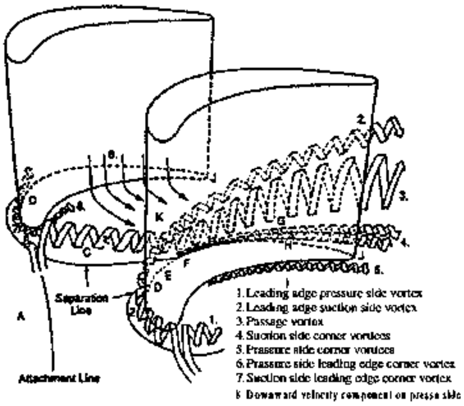
(a) *Model of Hawthorne(1955)*



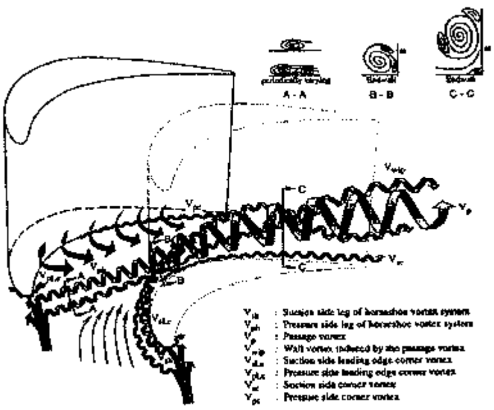
(b) *Model of Langston(1980)*



(c) *Model of Sharma(1987)*



(d) *Model of Goldstein(1988)*



(e) *Model of Wang(1995)*