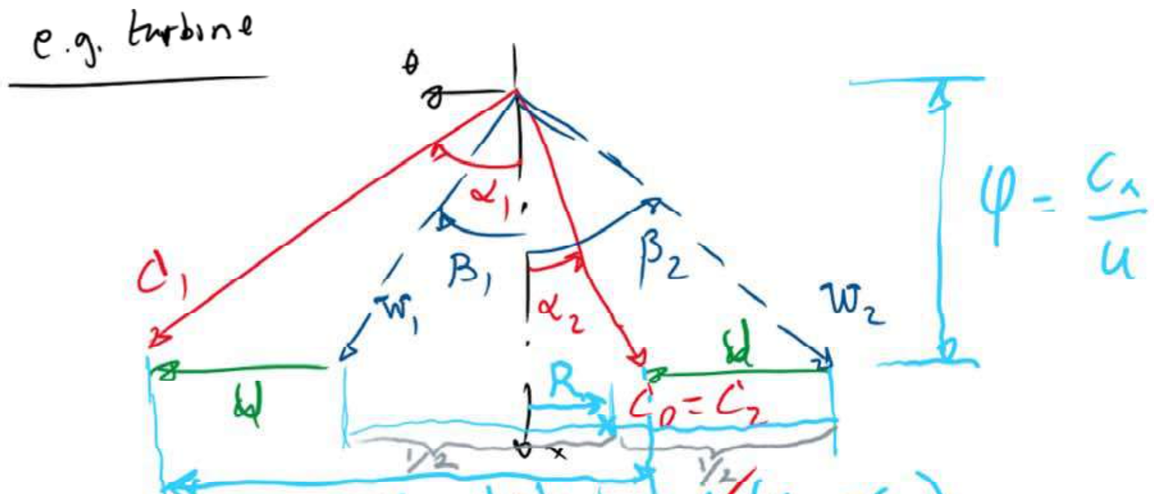
Euler TM eq.

→ general form $\boxed{h_{02}-h_{01} = u_2 \cdot c_{02} - u_1 \cdot c_{01}}$

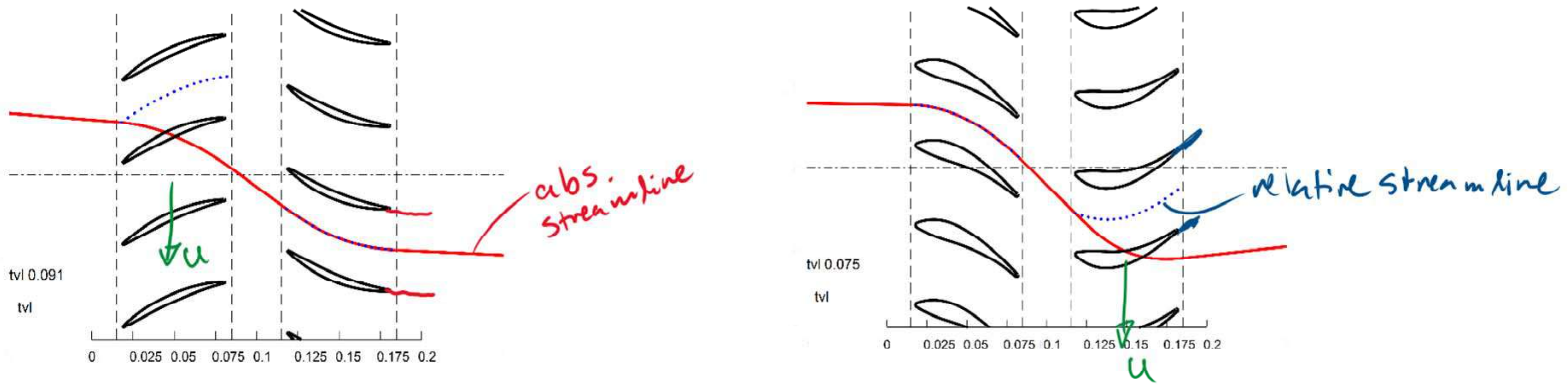
Repetition

e.g. turbine



$\varphi = \frac{c_x}{u}$

$\psi = \frac{|\Delta h_0|}{u^2} = \frac{\cancel{u}(c_{02}-c_{01})}{u^{\cancel{2}}}$

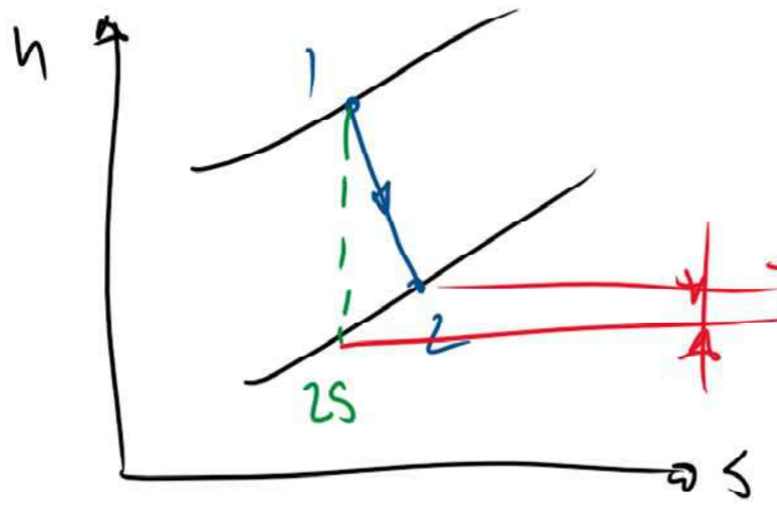


compressor   turbine

φ, ψ, R → flow kinematics c, w, α, β → flow velocities, flow deviation → losses → efficiency η

u → efficiency η

→ simplified loss models



$\Delta h_{loss} = \xi \cdot \frac{c^2}{2}$ — loss coefficient

correlation for loss coefficient

$\xi = 0.04 + 0.06 \cdot \left(\frac{\varepsilon}{100}\right)^2$

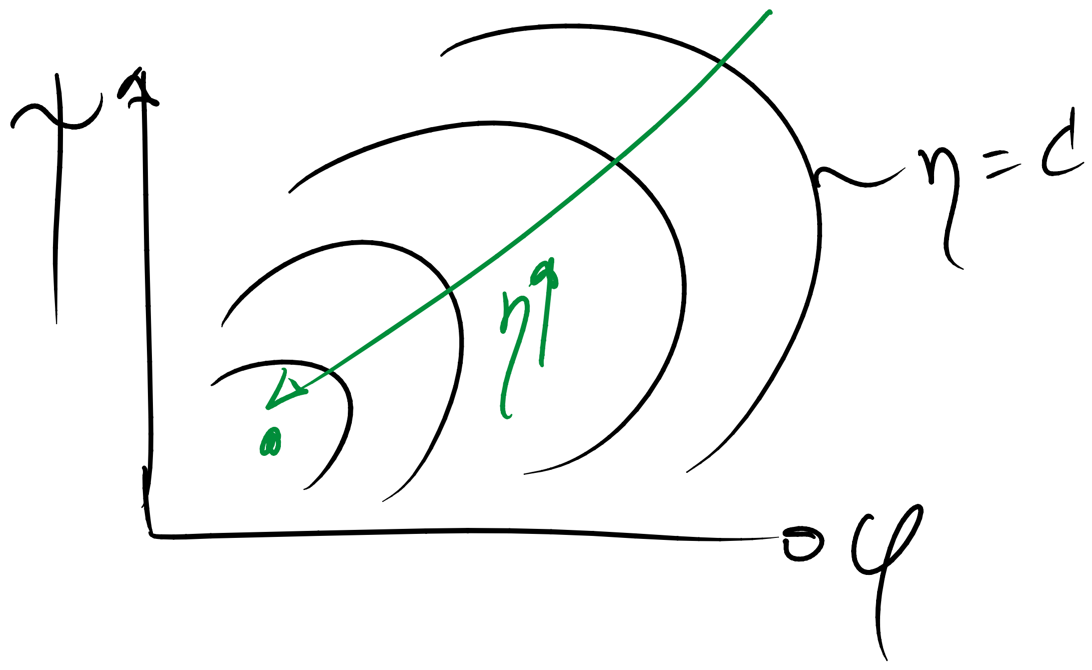
Soderbergh's loss correlation

where

ε: flow deviation in a blade row

$$\begin{cases} \varepsilon_N = \alpha_0 - \alpha_1 \\ \varepsilon_R = \beta_1 - \beta_2 \end{cases}$$

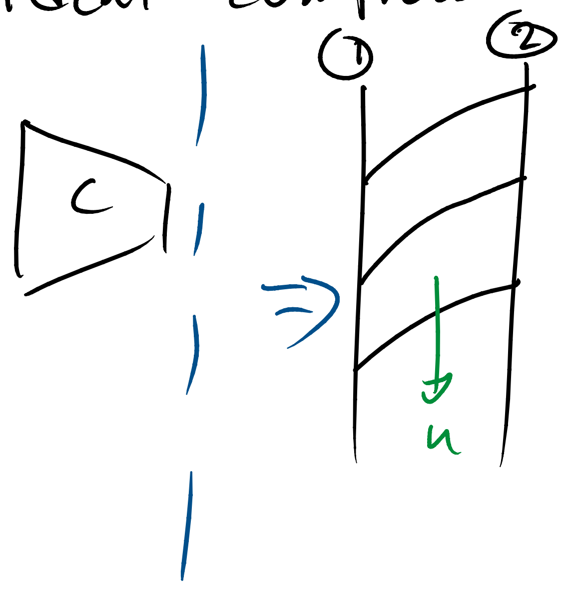
→ Smith charts



for a reaction degree of 0.5

Example 4) from "stage thermodynamics" hand-out

ideal compressor rotor $\Pi = \frac{p_{02}}{p_{01}} = 1.7$



$T_{01} = 291\ K$

$p_{01} = 101.3\ kPa$

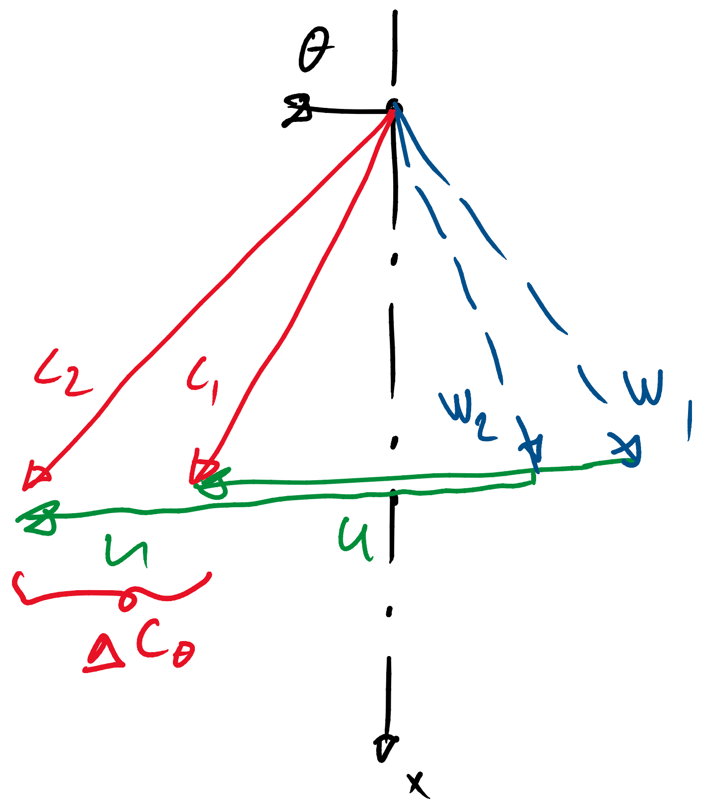
ideal gas (air) $\gamma = 1.4$

$c_p = 1004.5\ J/kgK$

$u = 300\ m/s$

what change in tang. velocity component is required for $\Pi = 1.7$?

ΔC_θ



Euler : $\Delta h_0 = u \cdot \Delta C_\theta$ (2)

ideal gas : $\Delta h_0 = c_p \Delta T_0$ (1)

Since it's an ideal compressor (reversible - no losses)
→ isentropic relation between total conditions

(1) => $\Delta h_0 = c_p T_0 = 1004.5 (T_{02} - T_{01})$

isentropic relationship => $\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ => $T_{02} = 291 \cdot (1.7)^{\frac{1.4-1}{1.4}} = 338.6\ K$

(1) => $\Delta h_0 = c_p T_0 = 1004.5 (338.6 - 291) = 47.8\ kJ/kg$

$$(2) \Rightarrow \quad \Delta c_\theta = \frac{\Delta h_0}{u} = \frac{47800}{300} = 159.5 \ m/s$$

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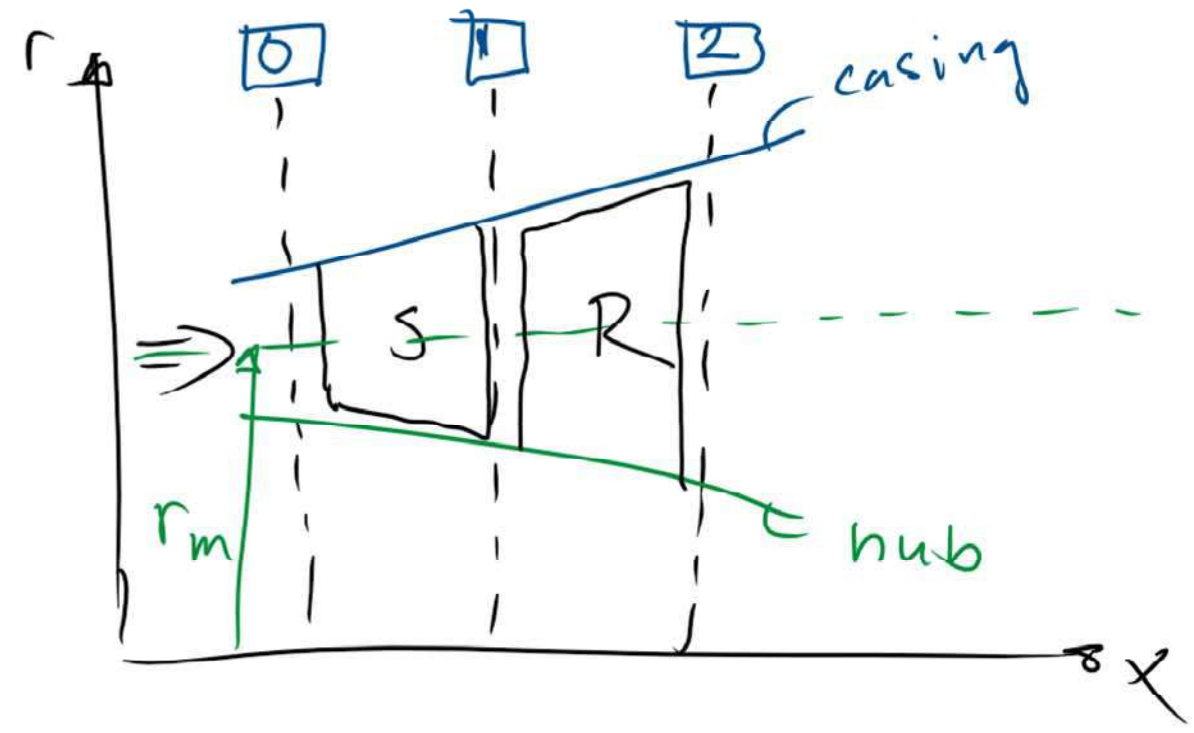
problem 5) ... in the same hand out

Axial turbine stage

$r_m = c \rightarrow u = c$ because $u = r_m \cdot \omega$, $\omega = \frac{2\pi \cdot N}{60}$ ← rotational speed



$c_x = c = 100 \ m/s$

normal repetition stage, $\vec{c}_0 = \vec{c}_2$

$c_1 = 300 \ m/s$

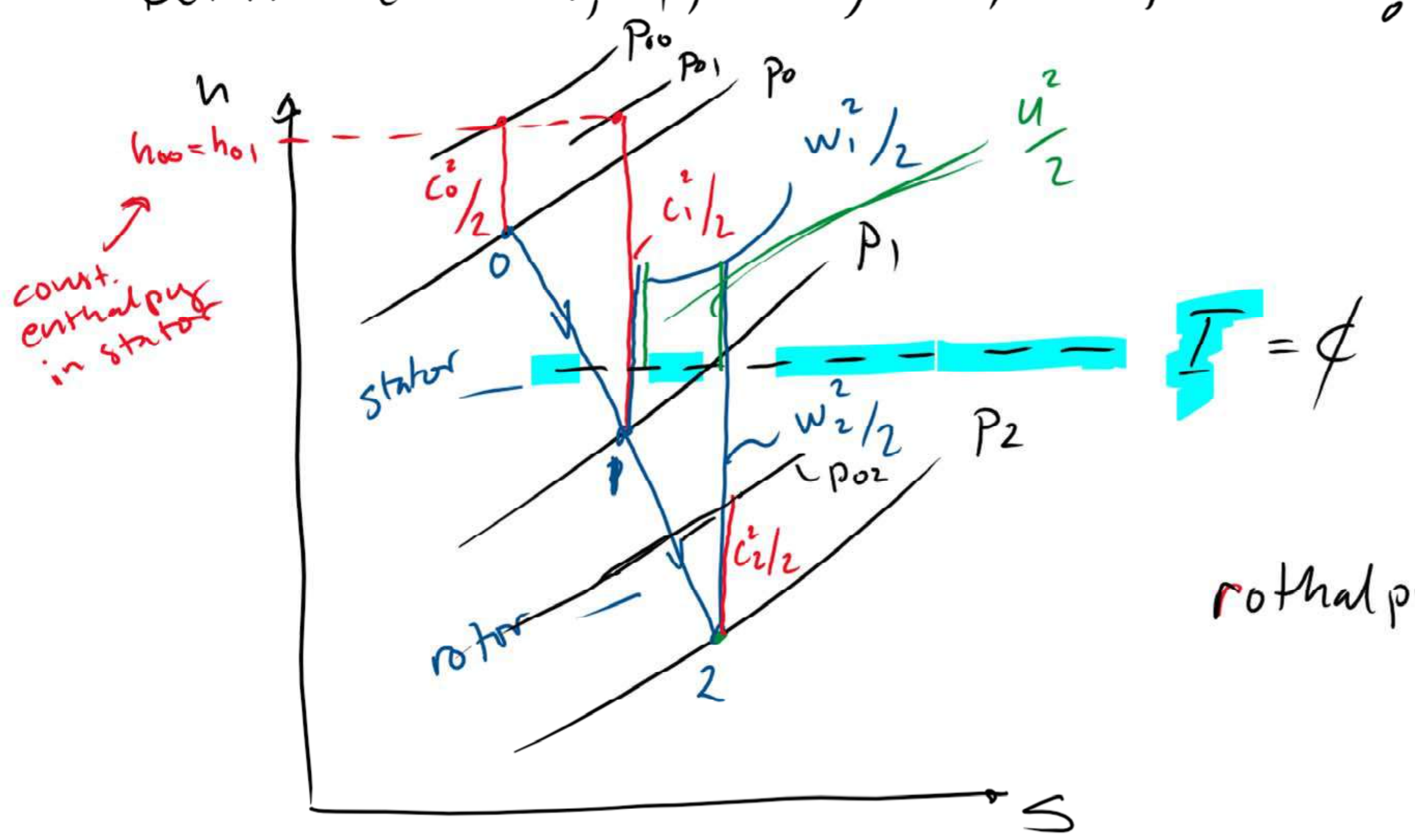
$w_2 = 200 \ m/s$

$u = 150 \ m/s$

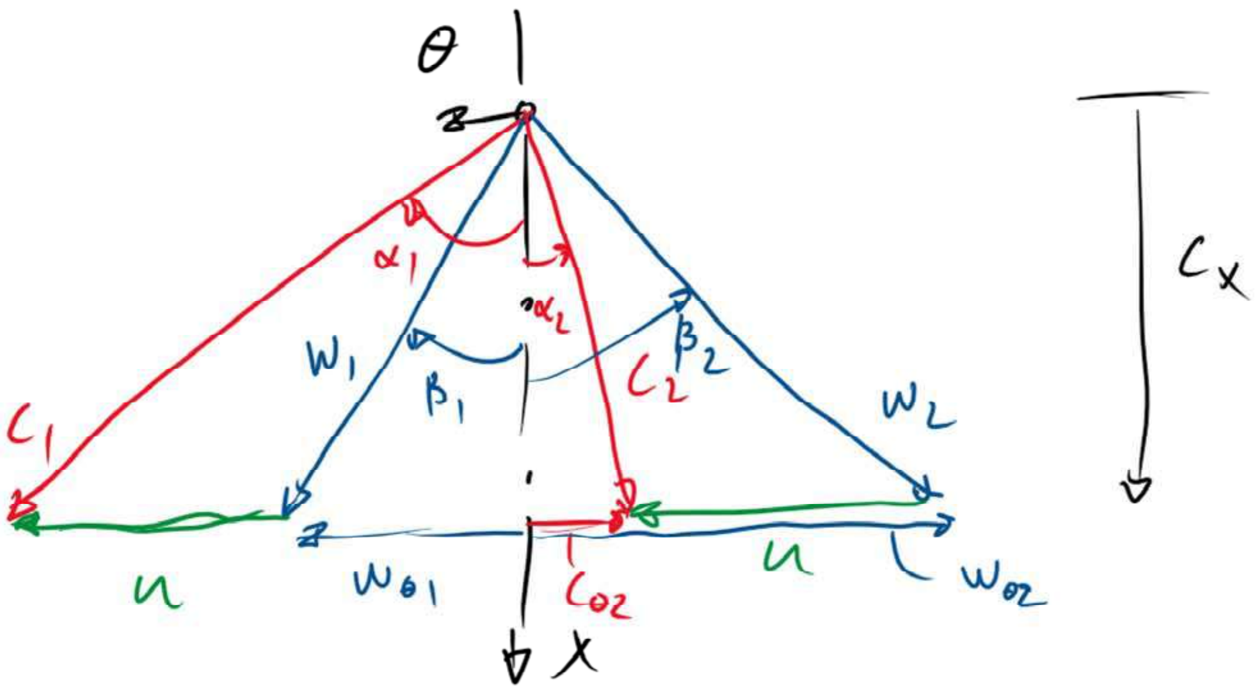
$T_0 = 800 \ K$ static temp. at inlet

$c_p = 1148 \ J/kgK$

Determine: $h_0, h_1, h_2, h_{00}, h_{01}, h_{02}$!



rothalpy, $I = h + \frac{w^2}{2} - \frac{u^2}{2}$



enthalpies throughout the stage

$$h_0 = c_p \cdot T_0 = 1148 \frac{J}{kgK} \cdot 800 K \approx 918.4 \frac{kJ}{kg}$$

$$h_{01} = h_{00} = h_0 + \frac{c_0^2}{2} = 918.4 \frac{kJ}{kg} + \frac{102.7^2}{2} = 923.7 \ kJ/kg$$

$= c_2^2$

$= c_2$

c_0: we have that $c_x = w_x \rightarrow w_{\theta 2} = -\sqrt{w_2^2 + w_x^2} = -173\ m/s$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 200 \quad\quad 100 = c_x$

$$c_{\theta 2} = w_{\theta 2} + u = -173 + 150 = -23.2\ m/s$$

$$\Rightarrow c_2 = \sqrt{c_x^2 + c_{\theta 2}^2} = \sqrt{100^2 + (-23.2)^2} = \underbrace{102.7}_{c_2 = c_0}\ m/s$$

$$h_1 = h_{01} - \frac{c_1^2}{2} = 923.7 - \frac{300^2}{2} = 878.7\ kJ/kg$$

h_2: go throug rothalpy $\left(I = h + \frac{w^2}{2} - \frac{u^2}{2}\right)$

w_1: we have $c_x = w_x = 100\ m/s$

$$\Rightarrow c_{\theta 1} = \sqrt{c_1^2 - c_x^2} = 282.8\ m/s$$

$$w_{\theta 1} = c_{\theta 1} - u = 282.8 - 150 = 132.8\ m/s$$

$$w_1 = \sqrt{w_{\theta 1}^2 + w_x^2} = \sqrt{132.8^2 + 100^2} = 166.3\ m/s$$

$$\Rightarrow h_1 + \frac{w_1^2}{2} - \cancel{\frac{u_1^2}{2}} = h_2 + \frac{w_2^2}{2} - \cancel{\frac{u_2^2}{2}} \quad \text{since } u_1 = u_2$$

$$\Rightarrow h_2 = h_1 + \frac{w_1^2}{2} - \frac{w_2^2}{2} = 878.7 + \frac{166.3^2}{2} \cdot 1000 - \frac{200^2}{2} = 872.5\ kJ/kg$$

$$h_{02} = h_2 + \frac{c_2^2}{2} = 872.5 + \frac{102.7^2}{2} \cdot 1000 = 877.8\ kJ/kg$$

Stage specific power $\Delta h_0 = h_{02} - h_{01} = 877.8 - 923.7 = \boxed{-45.9\ kJ/kg}$

From Euler TM eq.

$$\begin{cases} c_{\theta 1} = 282.8\ m/s \\ c_{\theta 2} = -23.2\ m/s \end{cases}$$

$$u = 150\ m/s$$

$$\Delta h_0 = u \cdot (c_{\theta 2} - c_{\theta 1}) = 150(-23.2 - 282.8) = \boxed{-45.9\ kJ/kg}$$

Power output ($\dot{W}$) or P if $\dot{m} = 20\ kg/s$

$$\Rightarrow \dot{W} = P = \dot{m} \cdot |\Delta h_0| = 20 \cdot |-45.9 \cdot 1000| = 918\ kW$$