PW2 - Perforated plate.

1 Introduction

The structure of a plane must be mechanically reliable while being as light as possible. It is commonly made of plates and shells in which holes are drilled in order to lower the global weight of the structure. In this configuration plates are non-homogeneous, and as a consequence their local stress fields are non longer uniform, even when submitted to uniform loads.

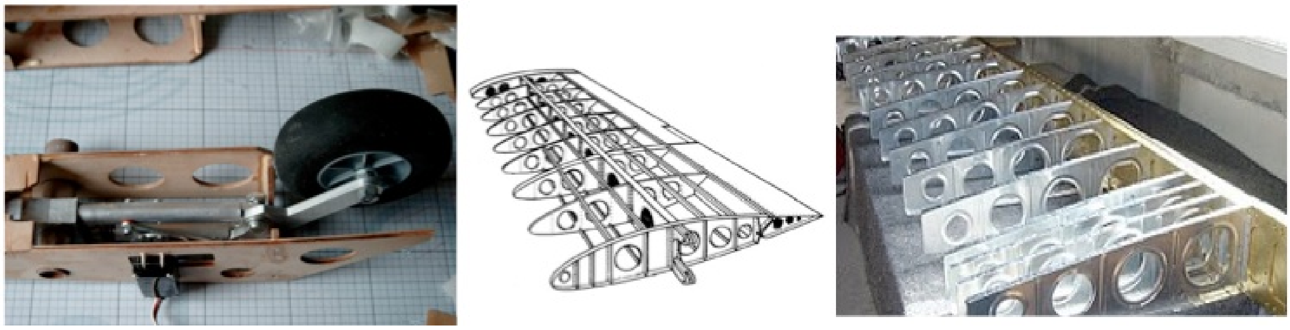

Figure 1: Examples of perforated plates used on sporting aeroplanes or scale models.

The purpose of this work is to study the impact of holes on the local stress field in a plate submitted to a symmetric traction load. High stress concentration induced by the presence of a hole may indeed lead to cracking and to the subsequent ruin of the plate.

It is first supposed that there are not too many holes in the plate and that they are rather scattered. As a consequence the stress field around a hole is not influenced by its neighbors. The study is therefore limited to a square plate in which a single hole is drilled, and the radius of the hole is small compared to the length of the external edges of the plate.
Note : Interactions between holes are in fact important, they are usually accounted for by studying more complex representative volumes elements (RVE) or by applying periodic boundary conditions to a simple RVE.

2 Problem definition

The structure studied is a square plate in which a hole is drilled (Figure 2) and whose dimensions are the following:

- $L = 40$ cm
- $a = 2.5$ cm

The plate is made of an isotropic homogeneous linear elastic material whose mechanical properties are listed below:

- $E = 135$ GPa
- $\nu = 0.35$

The plate is submitted to two surface force densities (i.e. distributed forces) of the same modulus $f_s = 1$MPa but with opposite directions.

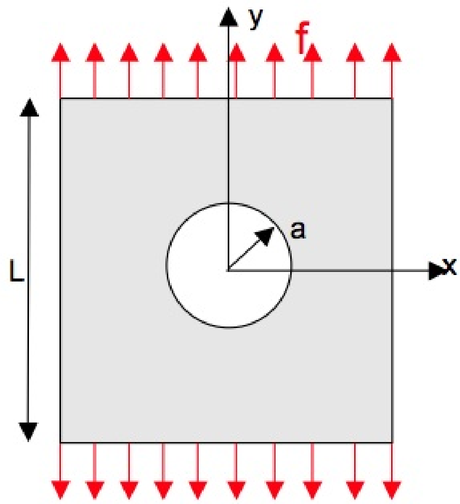

Figure 2: Schematic representation of the drilled plate and its load.

3 Preliminary simulation

1. Is the problem completely defined at this point i.e. is there an unique solution to this problem? In case of a negative answer which characteristic of the problem can be used to make the solution unique?
2. Create the file *Plate.geo*, and write a main script *Plate.py* allowing to solve the problem using the finite element library *wombat*. Justify the choices you made (structure of the mesh, type of elements[1], plane elasticity hypothesis, etc).
3. Analyze the obtained stress field. Knowing that cracks nucleate and propagate preferentially in the planes orthogonal to principal normal stresses (mode I failure), in which direction cracks may appear and grow in the plate?

[1]T6 elements are here suggested. Download the corresponding library *SolidT6.py* and place it in the *element* file if needed

4 Stress field accuracy

In order to validate the accuracy of the finite element approximation for this problem a comparison must be made with an analytical solution. For an isolated hole in an infinite plate (that is a very good approximation of a small hole far from the edges of the plate, especially at the vicinity of the hole), the analytical expressions of the stress field components in the cylindrical basis defined on Figure (3) are reported below:

$$\sigma_{rr} = \frac{f}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{f}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos\left(2\left[\theta - \frac{\pi}{2}\right]\right)$$
$$\sigma_{\theta\theta} = \frac{f}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{f}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos\left(2\left[\theta - \frac{\pi}{2}\right]\right)$$
$$\sigma_{r\theta} = -\frac{f}{2}\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin\left(2\left[\theta - \frac{\pi}{2}\right]\right)$$

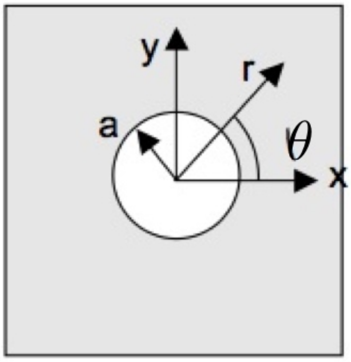

Figure 3: Definition of the local cylindrical basis used to express analytical solutions.

In order to simplify the comparison process, it will be held on only two specific axis defined by $\theta = 0$ and $\theta = \pi/2$. The point is now to estimate and plot the evolution of the components of the stress tensor $\underline{\underline{\sigma}}$.

4. Compute and plot the analytical solutions (σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$) along both recommended axis.

Tips:

- π can be called using `np.pi`.
- To create a uniform distribution of 10 values between two values a and b one can use the operator `np.linspace(a,b,10)`.
- Syntax for the plot (if `matplotlib.pyplot` is imported as `plt`):

```
plt.plot(list_absc, list_ord,'bo')
plt.xlabel('x-axis legend')
plt.ylabel('y-axis legend')
plt.legend(('name of curve'))
plt.show()
```

Stress components values must now be collected from the numerical results obtained after solving and compared to analytical solutions. The difficulty here is that stresses are calculated at Gauss points and not nodes. A first step is therefore to collect the values of stress components extrapolated at nodes.

5. Write a method called `extract_Sig(self, mesh, V, theta, ...)` for the class `specific_res_treat` in *res_ treat.py*. The input arguments suggested for this method are :

   - `mesh` : the mesh as defined in the main script,
   - `V` : the scalar field (i.e. stress component) dealt with.
   - `theta` : the angle corresponding to the axis on which you want to extract the values of the scalar field.

   Note that you can add other input argument at your convenience.
   The output data of the method must be :

   - the list of radial coordinates for the nodes on the axis considered
   - the list of the values of the scalar field `V` estimated at those nodes.

   Tips:

   - Get strong inspiration from the method `plot_field` in the file *post_ process.py* in the case of *non uniform field.*
   - Values to collect are from nodes whose coordinates are specific think about it...
   - Nodes coordinates of an element can be found thanks to its attribute `e.node_coor()`.
   - Note that a node usually belongs to several elements. The contribution of each element connected to a node must be arithmetically summed to evaluate a scalar field at one node.
   - Pay attention to the number of Gauss points used depending on the type of element chosen (T3 or T6). When several Gauss points are involved, shape functions N_k^σ (that are *not* the same than the displacement shape functions N_k) can be used to interpolate in a way that:

     $$\underbrace{\underline{\underline{\sigma}}(\underline{a}_g)}_{\text{at Gauss points}} = \sum_k N_k^\sigma(\underline{a}_g) \underbrace{\underline{\underline{\sigma}}^{(k)}}_{\text{nodal values}}$$

     Nodal values can be recovered from values at Gauss points by inverting this linear system: see the example provided in the method `plot_field`.
   - Values must be stored into lists or numpy arrays before plotting.
     Reminder: Lists can be declared and appended the following way:
     `NameOfList = []; NameOfList.append(NewValue)`
     while arrays can be appended using the function `np.append`.

6. When the nodal values of the stress components fields are obtained, write a second method called `plot_extract_Sig` for the class `specific_res_treat` in order to plot the extracted values from the FEM analysis and compare them to analytical solutions by superimposing curves. Analyze the results thus obtained.

5 Stress concentration factor for elliposidal holes

7. K_G is the stress concentration factor. It is defined as the ratio between the maximum stress component around the hole σ_M and the nominal stress σ_G far from the hole:

$$K_G = \frac{\sigma_M}{\sigma_G}$$

Evaluate the stress concentration factor for a circular hole.

8. Compare it to the results obtained for an elliptic hole with a ratio $r = \frac{a}{b} = 2$, with b and a the semi-minor and semi-major axis of the ellipse respectively.
Tip from GMSH documentation:
`Ellipse(expression) = {expression, expression, expression, expression <, ...> };`
Creates an ellipse arc. The four expressions on the right-hand-side define the start point, the center point, a major axis point and the end point of the ellipse.

9. Study the limit case where the ellipse has a higher ratio and flattens towards a crack shape (perpendicular to the load direction).

6 One step further: interacting holes

10. Study the case of two interacting holes located on the x-axis of the plate. How is the stress field impacted ? What consequences on the nucleation of cracks ?

Tip: Think of the simplest way to create two holes in the whole plate...