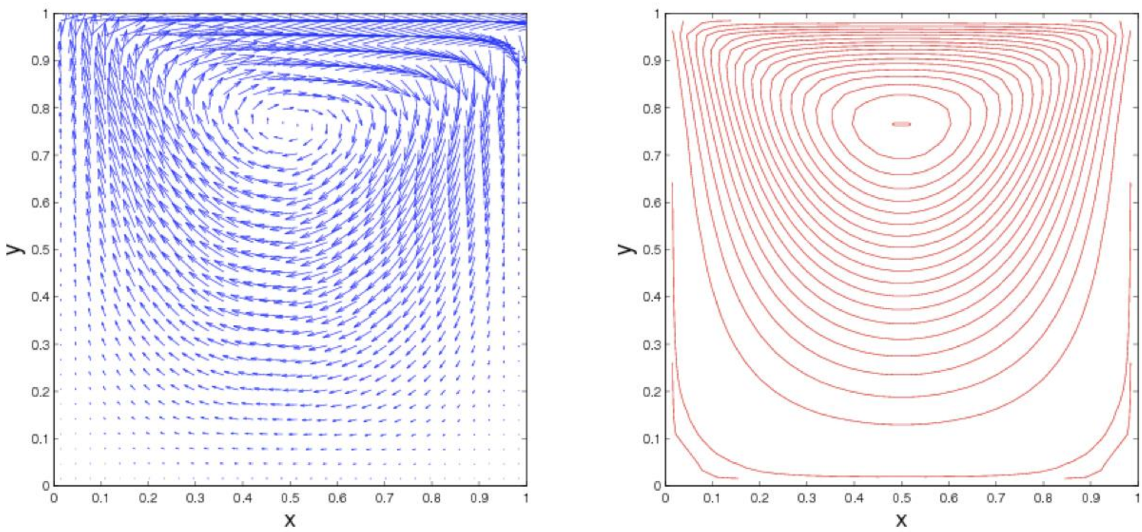
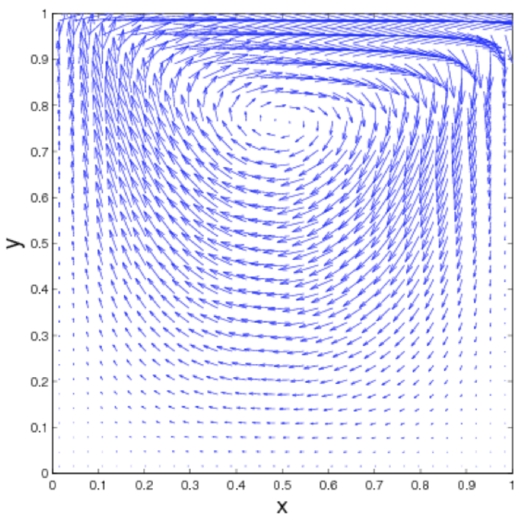
TP#5 : MU4MEM02
Fractional Step/Pressure Projection Method
Stoke's Equation

The objective is to complete a python code for the resolution of the Stoke's equation inside a cavity.

The flow and streamlines are plotted below:

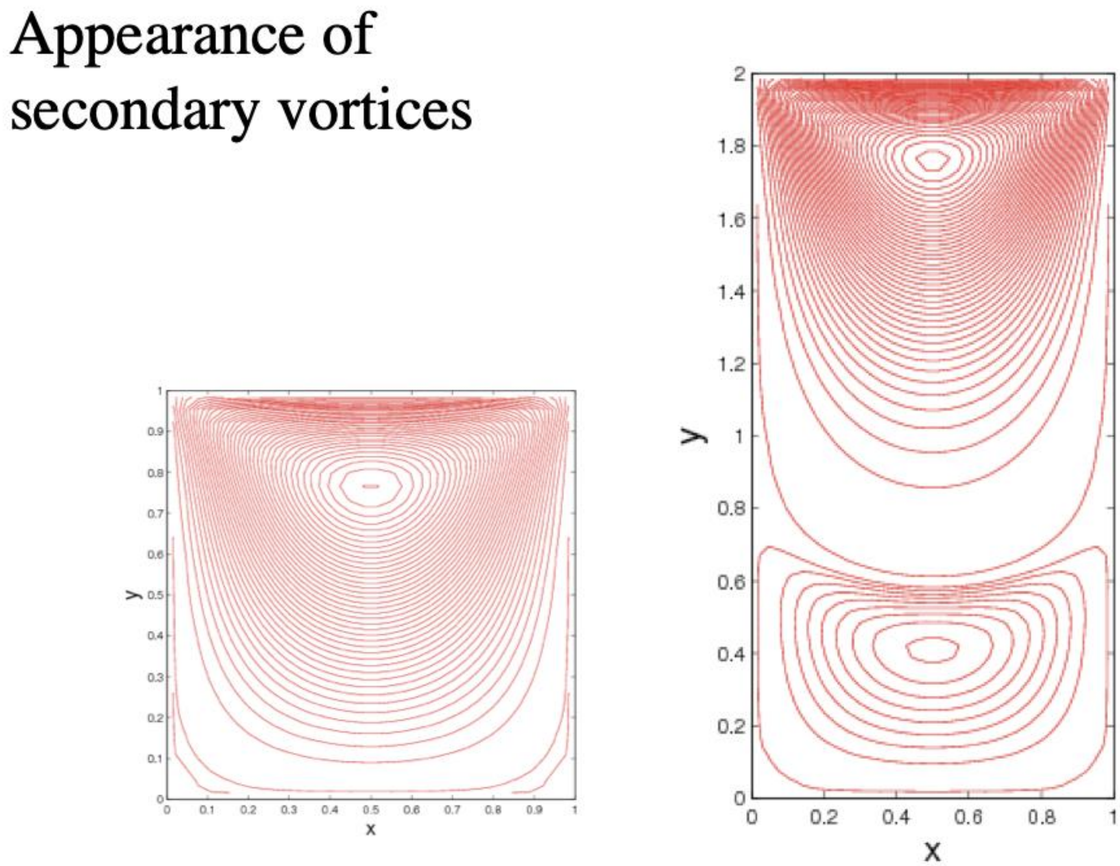


The following characteristics can be identified:



- Symmetric streamlines
- Rapid decay of velocity field
- Strong gradients in upper corners

The effect of the box size:



The equations governing the flow are:

$$\begin{cases} \partial_t u = -\nabla p + \Delta u \\ \nabla \cdot u = 0 \\ u(0 \le x \le 1, 0 \le y \le 1, t = 0) = 0 \\ u = g \quad sur \quad \partial\Omega \quad pour \quad t > 0 \end{cases}$$

Where,

$$g = \begin{cases} 0 \quad pour \quad (0 \le x \le 1, y = 0), (x = 0, 0 \le y < 1), (x = 1, 0 \le y < 1) \\ e_x \quad pour \quad (0 \le x \le 1, y = 1) \end{cases}$$

And (e_x, e_y) are the orthogonal basis of the plane.

For spatial discretization of the diffusion terms, we will use a central difference scheme in both directions. The Laplacian is also discretized following the same discretization as in TP4. For the temporal integration scheme, we will employ a backward Euler scheme. To solve the system of equations a fractional step method, as described in the class, is employed. So, to solve the for u^{n+1}, starting from $u^{n,}$ we follow:

$$
\begin{cases}
\dfrac{u^* - u^n}{\Delta t} = \Delta u^* & (1) \\
u^* = g + \Delta t \nabla p^{n+1} \quad sur \quad \partial\Omega & (2) \\
\Delta p^{n+1} = \dfrac{\nabla \cdot u^*}{\Delta t} & (3) \\
\partial_n p^{n+1} = 0 & (4) \\
\dfrac{u^{n+1} - u^*}{\Delta t} = -\nabla p^{n+1} & (5)
\end{cases}
$$

Q1: Complete the notebook with the necessary steps.

Q2: What is a cell-centered scheme? Comment on how the restriction and prolongation algorithms differ from those of TP#4.

Q3: How do we account for Neuman and Dirichlet boundary conditions?

Q4: Considering bc_diri_U and bc_diri_V in the code, explain the method for implementing the boundary condition on a « cell-centered » grid. Code bc_diri_U2 and bc_diri_V2 such that:

$$
g = \begin{cases}
0 \quad pour \quad (0 \le x \le 1, y = 0), (x = 0, 0 \le y < 1) \\
e_y \quad pour \quad (x = 1, 0 \le y < 1) \\
e_x \quad pour \quad (0 \le x \le 1, y = 1)
\end{cases}
$$