Aufgabe 1 – Neutronenstrahlung

Tutoriumsmitschrift

$$\frac{1}{2} m_n v_n^2 = \frac{1}{2} m_n v_n'^2 + \frac{1}{2} m_k v_k'^2 \iff m_n(v_n^2 - v_n'^2) = m_k v_k'^2$$

$$m_n v_n = m_n v_n' + m_k v_k' \iff m_n(v_n - v_n') = m_k v_k'$$

$$\Rightarrow \frac{v_n^2 - v_n'^2}{v_n - v_n'} = v_k' \Rightarrow v_n + v_n' = v_k' \Rightarrow m_n(v_n - v_k') = m_k v_k' \iff \frac{2 m_n v_n}{m_n + m_k}$$

$$\frac{v_H}{v_N} = \frac{m_n + m_N}{m_n + m_H} \Rightarrow m_n = \frac{m_H\left(14 - \frac{v_H}{v_N}\right)}{\left(\frac{v_H}{v_N} - 1\right)} = \frac{7}{6} m_H = 1066 \frac{MeV}{c^2}$$ Masse des Neutrons

$m_H = 937{,}9 \frac{MeV}{c^2} \approx 1u$

$m_N = 13047{,}9 \frac{MeV}{c^2} \approx 14u = 14 m_H$

Energie des Neutrons: $\Rightarrow v_n = \frac{m_n + m_H}{2 m_n} v_k'$

$$E_{kin,H} = \frac{1}{2} m_H v_H^2 \iff v_H = \sqrt{\frac{2 E_{kin,H}}{m_H}}$$

$$\Rightarrow v_H = 0{,}1$$

$$\Rightarrow E_{kin,H} = \frac{1}{2} m_H v_H^2 = 5{,}65 \frac{MeV}{c^2}$$

Aufgabe 2 – Relativistische Kinematik

Vierervektoren

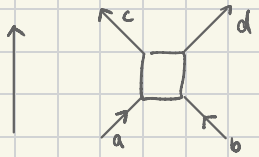
$a + b \to R \to c + d$ || $P_A + P_B = P_R = P_C + P_D$ → Lösung 1)

$$\begin{pmatrix} E_A \\ \vec{P}_A \end{pmatrix} + \begin{pmatrix} E_B \\ \vec{P}_B \end{pmatrix} = \begin{pmatrix} E_R \\ \vec{P}_R \end{pmatrix} = \begin{pmatrix} E_C \\ \vec{P}_C \end{pmatrix} + \begin{pmatrix} E_D \\ \vec{P}_D \end{pmatrix}$$ ← Bezugssystem abhängige Größen

Lorentz-Skalar, invariant

$$P^2 = P \cdot P = E^2 - \vec{P} \cdot \vec{P}$$
$$= E^2 - |\vec{P}|^2 = m^2 = \left[(E - E_{kin})^2\right]$$

2)
$$s = (P_a + P_b)^2$$
$$t = (P_a - P_c)^2$$
$$u = (P_a - P_d)^2$$



s-Kanal:

t-Kanal:

u-Kanal:

a-c → Austauschteilchen

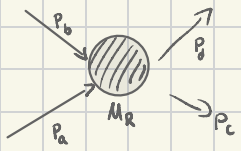
$$s + t + u = P_a^2 + P_b^2 + 2P_aP_b + P_a^2 + P_c^2 - 2P_aP_c + P_a^2 + P_d^2 - 2P_aP_d$$
$$= 3P_a^2 + P_b^2 + P_c^2 + P_d^2 - 2P_a \underbrace{(P_c + P_d + P_b)}_{= P_a \text{ (Impulserhaltung)}} = P_a^2 + P_b^2 + P_c^2 + P_d^2 = m_a^2 + m_b^2 + m_c^2 + m_d^2$$

3) $P_a + P_b = P_R$, definiere Schwerpunktsystem so, dass $\sum \vec{P}_i = \vec{0}$ $\Rightarrow P_R = \begin{pmatrix} E_R \\ \vec{0} \end{pmatrix} = \begin{pmatrix} \sqrt{m_R^2 + |\vec{P}_R|^2} \\ |\vec{P}_R|^2 \end{pmatrix} = \begin{pmatrix} m \\ \vec{0} \end{pmatrix}$

R = Schwerpunkt

$$s = (P_a + P_b)^2 = P_R^2 = m^2$$

$\Rightarrow$ Gesamtenergie Schwerpunktsystem $= \sqrt{s}$ $\Rightarrow$ kann man zur "Produktion" neuer Teilchen verwenden



$$P_R = P_c + P_d$$

$$= \begin{pmatrix} E_c \\ \vec{p}_c \end{pmatrix} + \begin{pmatrix} E_d \\ \vec{p}_d \end{pmatrix} \qquad \vec{p}_c = -\vec{p}_d \qquad |\vec{p}_c| = |\vec{p}_d| = p$$

$$= \begin{pmatrix} \sqrt{m_c^2 + p^2} \\ \vec{p}_c \end{pmatrix} + \begin{pmatrix} \sqrt{m_d^2 + p^2} \\ \vec{p}_d \end{pmatrix}$$

$$\Rightarrow s = m_R^2 = P_R^2 = \Big(\underbrace{\sqrt{m_c^2 + p^2}}_{E_c} + \underbrace{\sqrt{m_d^2 + p^2}}_{E_d} \Big)^2$$

Für größtmögliche Teilchenmasse wird der Impuls null.

4) a) $\gamma \rightsquigarrow p$ $\qquad \vec{p}_p = 0$

$P_\gamma = E_\gamma \cdot \vec{e}_z$

Eingangsimpuls $\Rightarrow$ = Ausgangsimpuls $\Rightarrow$

Impuls des SP-Systems

Schwerpunktsystem:

$$\sqrt{s} = \sqrt{(P_\gamma + P_p)^2} = \sqrt{P_\gamma^2 + P_p^2 + 2P_\gamma P_p} = \sqrt{0^2 + m_p^2 + 2P_\gamma P_p}$$

$$= \sqrt{m_p^2 + 2E_\gamma \underbrace{E_p}_{=m_p} - \vec{P}_\gamma \cdot \vec{P}_p}_{=0}$$

$$= \sqrt{m_p^2 + 2E_\gamma m_p} = 2{,}15 \text{ GeV}$$

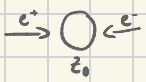
(viel kleiner als Gesamtenergie, da "Impulsenergie" nicht zur Verfügung für Masse steht)

b) $$s = (P_p + P_{\bar{p}})^2 = \left[\begin{pmatrix} E_p \\ \vec{p}_p \end{pmatrix} + \begin{pmatrix} E_{\bar{p}} \\ -\vec{p}_p \end{pmatrix} \right]^2 = \begin{pmatrix} E_p + E_{\bar{p}} \\ \vec{0} \end{pmatrix}^2 = \left[\begin{pmatrix} E_{kin,p} + m_p \\ \vec{0} \end{pmatrix} + \begin{pmatrix} E_{kin,\bar{p}} + m_p \\ \vec{0} \end{pmatrix} \right]^2$$

$\vec{p}_p = -\vec{p}_{\bar{p}}$

$$= 2^2 (E_{kin,p} + m_p)^2 \Rightarrow \sqrt{s} = 2\,(E_{kin,p} + m_p) = 3{,}88 \text{ GeV}$$

5) Z^0 Produktion



$$P_R = \begin{pmatrix} m_{Z_0} \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E_{e^-} \\ \vec{p}_{e^-} \end{pmatrix} + \begin{pmatrix} E_{e^+} \\ \vec{p}_{e^+} \end{pmatrix} = \begin{pmatrix} 2E \\ \vec{0} \end{pmatrix}$$

$$\Rightarrow m_{Z_0} = 2E$$

$$\Rightarrow E = \frac{m_{Z_0}}{2} = \sqrt{s} = 45 \text{ GeV}$$

Aufgabe 3 - invariante Masse

1) $$M_{\gamma_1\gamma_2}^2 = (P_{\gamma_1} + P_{\gamma_2})^2$$

$$P_{\gamma_1} = (E_{\gamma_1}, E_{\gamma_1} \sin\theta \cos\phi, E_{\gamma_1} \sin\theta \sin\phi, E_{\gamma_1} \cos\theta) \qquad P_{\gamma_1} = \begin{pmatrix} E_\gamma \\ E_1 \cos\varphi_1 \sin\theta_1 \\ E_1 \sin\varphi_1 \sin\theta_1 \\ \cos\theta_1 \end{pmatrix}$$

$P_{\gamma_2} = \ldots$ (analog zu P_{γ_1})

$$P_{\gamma_1} = (0{,}661 \text{ GeV}, 0{,}646 \text{ GeV}; 0{,}095 \text{ GeV}; 0{,}100 \text{ GeV})$$

$$P_{\gamma_2} = (0{,}061 \text{ GeV}, 0{,}039 \text{ GeV}; 0{,}043 \text{ GeV}; 0{,}018 \text{ GeV})$$

$$\sqrt{M_{\gamma_1\gamma_2}^2} = \sqrt{(E_{\gamma_1}+E_{\gamma_2})^2 - (P_{x\gamma_1}+P_{x\gamma_2})^2 - (P_{y\gamma_1}+P_{y\gamma_2})^2 - (P_{z\gamma_1}+P_{z\gamma_2})^2} = 0{,}138 \text{ GeV}$$

$$P_{in} = P_\gamma + P_p$$

$$P_{out} = P_x + P_{\gamma 1} + P_{\gamma 2}$$

$$P_{in} = P_{out} \Rightarrow P_x = P_\gamma + P_p - (P_{\gamma 1} + P_{\gamma 2})$$

$$M_x^2 = P_x^2 = (P_\gamma + P_p - P_{\gamma 1} - P_{\gamma 2})^2$$

$$\Rightarrow M_x = 938 \text{ MeV}$$ $\rightarrow$ Proton