Hausaufgabenblatt 1

Aufgabe 1

(i) $(1+3i)^2 = 1 - 9 + 6i = -8 + 6i$

(ii) $\dfrac{1}{(5+i)} = \dfrac{1}{(5+i)} \dfrac{(5-i)}{(5-i)} = \dfrac{5-i}{26} = \dfrac{5}{26} - \dfrac{1}{26}i$

(iii) $\dfrac{(1+2i)}{(3+4i)} = \dfrac{(3-4i)}{(3-4i)} \dfrac{(1+2i)}{(3+4i)} = \dfrac{3 + 6i - 4i + 8}{9 + 16}$

$\qquad = \dfrac{11}{25} + \dfrac{2}{25}i$

(iv) $\dfrac{1}{(2+i)(2+2i)} = \dfrac{1}{4 + 4i + 2i - 2}$

$\qquad = \dfrac{1}{2 + 6i} = \dfrac{1}{2}\dfrac{(1-3i)}{(1+3i)(1-3i)}$

$\qquad = \dfrac{1}{20} - \dfrac{3}{20}i$

Aufgabe 2

(i) $100 = 100\, e^{i0}$

(ii) $5i = 5 e^{i\frac{\pi}{2}}$

(iii) $3+2i = \sqrt{9+4}\left(\dfrac{3}{\sqrt{9+4}} + \dfrac{2}{\sqrt{9+4}}i \right) = \sqrt{13}\, e^{i\arccos\left(\frac{3}{\sqrt{13}}\right)}$

(iv) $\dfrac{1}{6+i} = \dfrac{1}{\sqrt{37}\left(\dfrac{6}{\sqrt{37}} + \dfrac{1}{\sqrt{37}}i \right)} = \dfrac{1}{\sqrt{37}}\, e^{-i\arccos\left(\frac{6}{\sqrt{37}}\right)}$

(wobei $\arccos\left(\dfrac{3}{\sqrt{13}}\right) \approx 0{,}59$ und $\arccos\left(\dfrac{6}{\sqrt{37}}\right) \approx 0{,}17$)

Aufgabe 3

(i) $z^4 = 1$, die Lösung sind gegeben durch

$$z_k = e^{i\frac{(0 + 2\pi k)}{4}} = e^{i\frac{\pi k}{2}} \qquad k = 0, 1, 2, 3$$

(ii) $z^4 = -1$

$$z_k = e^{i\frac{(\pi + 2\pi k)}{4}} \qquad k = 0, 1, 2, 3.$$

(iii) $$z_1 = \frac{1 + i\sqrt{3}}{2}$$

$$z_2 = \frac{1 - i\sqrt{3}}{2}$$

(iv) Sei $u = z^3$, wir lösen erstmal:

$$u^2 + u + 1 = 0$$

$$u_1 = -\frac{1 + i\sqrt{3}}{2} = e^{i\frac{2\pi}{3}}$$

$$u_2 = -\frac{1 - i\sqrt{3}}{2} = e^{i\frac{4\pi}{3}}$$

$$\Rightarrow \begin{cases} z_k = e^{i\left(\frac{4\pi}{3} + \frac{2\pi k}{3}\right)} & k \in \{0, 1, 2\} \\ \\ z'_k = e^{i\left(\frac{2\pi}{3} + 2\pi k\right)\frac{1}{3}} & k \in \{0, 1, 2\} \end{cases}$$

sind die Lösungen

(v) $z^2 = 10i \Rightarrow z_k = \sqrt{10}\ e^{\frac{i\left(\frac{\pi}{2} + 2\pi k\right)}{2}}$ $k \in \{0,1\}$.