Theo 3 Angela Bradc 07.05.2024

<u>Aufgabe 1:</u>

1.: z: $\hat{P}$ hermitisch $(\hat{P} = \hat{P}^\dagger)$

$$\hat{P}\hat{P}|x\rangle = \hat{P}|-x\rangle = |x\rangle$$

$$\Rightarrow \hat{P}\hat{P} = \mathbb{1}$$

$$\boxed{\Rightarrow \hat{P} = \hat{P}^{-1}}$$

$$\langle \mathbb{1}x | \mathbb{1}x\rangle$$

$$\langle \hat{P}\hat{P}x | \hat{P}\hat{P}x\rangle \quad \Big| \quad \langle \hat{P}^\dagger x | x\rangle = \langle x | \hat{P}x\rangle$$

$$\langle \hat{P}\hat{P}^\dagger x | \hat{P}\hat{P}^\dagger x\rangle$$

$$\Rightarrow \hat{P}\hat{P}^\dagger = \mathbb{1} = \hat{P}\hat{P}$$

$$\boxed{\Rightarrow \hat{P} = \hat{P}^\dagger}$$

2.: z: $\hat{P}\psi(x) = \lambda\psi(x)$ mit $\lambda = \pm 1$

$(\hat{P} = \hat{P}^{-1} \Rightarrow$ unitär $\Rightarrow |\lambda| = 1) \wedge \lambda \in \mathbb{R}$

$$\boxed{\Rightarrow \lambda = \pm 1}$$

$$\Rightarrow \psi(-x) = \hat{P}\psi(x) = \pm\psi(x)$$

$$\Rightarrow \begin{cases} \text{ungerade}: & \psi(-x) = -\psi(x) \\ \text{gerade}: & \psi(-x) = \psi(x) \end{cases}$$

3.: $[\hat{A}, \hat{P}]\phi_n(x) = 0$

$$\Rightarrow \hat{A}\hat{P}\phi_n(x) = \hat{P}\hat{A}\phi_n(x)$$

$$\hat{A}\phi_n(-x) = \hat{P}E_n\phi_n(x) = \pm E_n\phi_n(x)$$

$\Rightarrow$ $\phi_n(-x) = \pm\phi_n(x)$ sind Eigenfunktion von $\hat{A}$ mit

$$\begin{cases} \text{ungerade}: & \boxed{\phi_n(-x) = \phi_n(x)} \\ \text{gerade}: & \boxed{\phi_n(-x) = -\phi_n(x)} \end{cases}$$