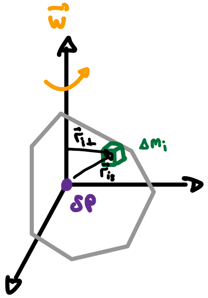
Trägheitsmoment & Rotationsenergie:



$$\Delta E_{kin,i} = \frac{1}{2}\Delta m_i v_{i\perp}^2 = \boxed{\frac{1}{2}\Delta m_i r_{i\perp}^2 \omega^2}$$

Kreisbew.: $\vec{v}_{i\perp} = \vec{\omega} \times \vec{r}_{i\perp}$

$\boxed{(v_{i\perp}^2 = \omega^2 r_{i\perp}^2)}$

Gesamte kin. Energie:

$$E_{rot} = \lim_{\substack{N\to\infty \\ \Delta V_i \to 0}} \left(\frac{1}{2}\sum_i^N \Delta m_i r_{i\perp}^2 \omega^2\right)$$

$$= \underbrace{\boxed{\frac{1}{2}\omega^2 \int_V r_\perp^2 \, dm}} = \frac{1}{2}\omega^2 I$$

I: Trägheitsmoment

$$I = \int_V r_\perp^2 \underbrace{\rho \, dV}_{dm}$$

Translation	Rotation
m	I
v	ω
$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
$E = \frac{p^2}{2m}$ $= \frac{1}{2}mv^2$	$E = \frac{L^2}{2I}$ $= \frac{1}{2}I\omega^2$

Gesamtdrehimpuls:

Massenelement Δm_i: $\vec{L}_i = \vec{r}_{i\perp} \times (\Delta m_i \vec{v}_{i\perp})$

$$= \boxed{r_{i\perp}^2 \Delta m_i \vec{\omega}}$$

$$\Delta m_i (\vec{r}_{i\perp} \times (\vec{\omega} \times \vec{r}_{i\perp}))$$

$$= \Delta m_i \left(r_{i\perp}^2 \vec{\omega} - \vec{r}_{i\perp} (\underbrace{\vec{r}_{i\perp} \cdot \vec{\omega}}_{=0})\right)$$

$$\boxed{\vec{L}_{ges}} = \int_V r_\perp^2 \vec{\omega} \, dm = \vec{\omega} \int_V r_\perp^2 \, dm \boxed{= \vec{\omega} I}$$

$\boxed{\omega^2 = \frac{L^2}{I^2}}$ und $\boxed{E_{rot} = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}}$

Beispiele:

Hohlzylinder:

Höhe h, Radius R
Wanddicke d, z-Richtung
$M = 2\pi R d h \rho$

$$I = \rho \int_V r^2 \, dV = 2\pi\rho h \int_{R-d}^{R} r^2 r \, dr$$

$dV = r\,dr\,d\theta\,dz$

$$\approx 2\pi\rho h R^3 d + \theta(d^2)$$

$\to I = MR^2$

Vollzylinder:

$$I = 2\pi\rho h \int_0^R r^3 dr = \frac{\pi}{2} h\rho R^4$$

$\to I = \frac{M}{2}R^2$

Dünner Stab:

Drehachse bei $\ell/2$ in z-Richt.
ℓ: Länge, A: Endfläche
$dV = A \cdot dx$

$$I = \rho \int_{-\ell/2}^{\ell/2} x^2 A \, dx = 2\rho \int_0^{\ell/2} x^2 A \, dx$$

$$= 2\rho A \frac{1}{3}\left(\frac{\ell}{2}\right)^3 = \frac{1}{12}\overbrace{\rho A \ell}^{M}$$

ℓ^2

$\to I = \frac{1}{12} M \ell^2$

Homogene Kugel: $I = \frac{2}{5} M R^2$ (Radius) **Hohlkugel:** $I = \frac{2}{3} M R^2$

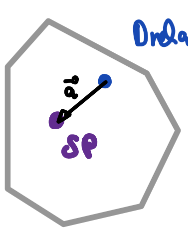
Hantel: $I = 2MR^2$

z-Richtung

Vorlesung 20

Satz von Steiner:

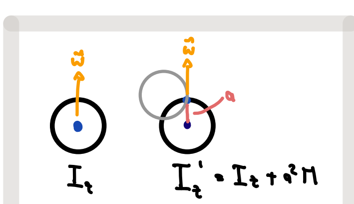
Trägheitsmoment bzgl. Achse, die nicht durch SP geht



$$\boxed{I_z' = I_z + a^2 M}$$

Abstand SP zu Drehachse

Trägheitsmoment bzgl. SP



Bewegungsgl. starrer Rotator:

Rotation um raumfeste Achse, betr. Massenelement Δm_i:

$$\vec{L}_i = (\vec{r}_{i\perp} \times \vec{p}_i) = \Delta m_i r_{i\perp}^2 \vec{\omega}$$

$$\to \dot{\vec{L}}_i = \Delta m_i (\vec{r}_{i\perp} \times \dot{\vec{v}}_i) = \vec{r}_{i\perp} \times \vec{F}_i = \underbrace{\boxed{\vec{D}_i}}_{\text{Drehmoment}}$$

$$\Downarrow$$

$$\boxed{D_i = \Delta m_i r_{i\perp}^2 \frac{d\omega}{dt}}$$

Volumenintegral $\to D = I \frac{d\omega}{dt} = I \frac{d^2\varphi}{dt^2} = I\ddot{\varphi}$

$\Rightarrow$ Wenn nun $D = $ const

$$\Downarrow$$

$$\boxed{\varphi(t) = \frac{D}{2I}t^2 + At + B}$$

Analoge Diff.gleichung wie freier Fall