Theo 4; Zettel 07; Marc Hanes, Franka Weronek, Ayda Brade; 28.11.24

<u>Aufgabe 1:</u>

a) zz: $Z(T,N,V) = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$ mit $\lambda = \frac{2\pi\hbar}{\sqrt{2\pi m k_B T}}$

$$Z(T,N,V) = \frac{1}{N!}\underbrace{\int d^3_{x_1}}_{V}\dots\underbrace{d^3_{x_N}}_{V}\int\frac{d^3_{p_1}}{(2\pi\hbar)^3}\dots\frac{d^3_{p_N}}{(2\pi\hbar)^3}e^{-\beta\sum_{i=1}^{N}\frac{p_i^2}{2m}} \quad \text{mit } \beta = \frac{1}{k_B T}$$

$$= \frac{1}{N!}\left[\frac{V}{(2\pi\hbar)^3}\right]^N \cdot \int d^3_{p_1}\dots d^3_{p_N}\prod_{i=1}^{N}e^{-\beta\frac{p_i^2}{2m}}$$

$$= \frac{1}{N!}\left[\frac{V}{(2\pi\hbar)^3}\right]^N \cdot \prod_{i=1}^{N}\underbrace{\int d^3_{p_i}\, e^{-\beta\frac{p_i^2}{2m}}}_{\sqrt{\frac{2m\pi}{\beta}}^3} \quad \Big|\ 3\text{ dim. Gaußintegral}$$

$$= \frac{1}{N!}\left[V\frac{\sqrt{2m\pi k_B T}^3}{(2\pi\hbar)^3}\right]^N$$

$$= \frac{1}{N!}\left[\frac{V}{\lambda^3}\right]^N$$

$\square$

b)

$$w(p) = \frac{1}{N!\cdot Z_c}\int d^3_{x_1}\dots d^3_{x_N}\int\frac{d^3_{p_1}}{(2\pi\hbar)^3}\dots\frac{d^3_{p_N}}{(2\pi\hbar)^3}\,\delta(p_j-p)\,e^{-\beta\sum_{i=1}^{N}\frac{p_i^2}{2m}} \quad \text{mit } \beta = \frac{1}{k_B T}$$

$$= \frac{1}{N!\cdot Z_c}\left(\frac{V}{(2\pi\hbar)^3}\right)^N\prod_{i=1}^{N}\int d^3_{p_i}\,\delta(p_j-p)e^{-\beta\frac{p_i^2}{2m}} \qquad Z_c = \left(\sqrt{n}\,\frac{L}{\lambda_T}\right)^{Nd} \overset{d=3}{=} \left[V\cdot\frac{\sqrt{n}^3}{\lambda_T^3}\right]^N = \left[V\cdot\frac{\sqrt{2\pi m k_B T}^3}{(2\pi\hbar)^3}\right]^N$$

$$= \frac{1}{N!}\frac{1}{\sqrt{2\pi m k_B T}^{3N}}\prod_{i=1}^{N}\int dp_i\, 4\pi p_i^2\,\delta(p_j-p)e^{-\beta\frac{p_i^2}{2m}} \quad \Big|\ N=1$$

warum? Es gibt doch noch andere

$$= \frac{4\pi p^2}{\sqrt{2\pi m k_B T}^3}e^{-\beta\frac{p_i^2}{2m}}$$

$\square$

c)

$$w(v) = \frac{1}{N!\cdot Z_c}\int d^3_{x_1}\dots d^3_{x_N}\int\frac{d^3_{p_1}}{(2\pi\hbar)^3}\dots\frac{d^3_{p_N}}{(2\pi\hbar)^3}\,\delta(v_j-v)\,e^{-\beta\sum_{i=1}^{N}\frac{p_i^2}{2m}} \quad \text{mit } \beta = \frac{1}{k_B T}$$

$$= \frac{1}{N!}\frac{1}{\sqrt{2\pi m k_B T}^{3N}}\prod_{i=1}^{N}\int dp_i\, 4\pi p_i^2\,\delta(v_j-v)e^{-\beta\frac{p_i^2}{2m}} \quad \Big|\ p_i = m v_i,\ dp_i = m\, dv_i$$

$$= \frac{1}{N!}\frac{1}{\sqrt{2\pi m k_B T}^{3N}}\prod_{i=1}^{N}\int dp_i\, 4\pi p_i^2\,\delta(v_j-v)e^{-\beta\frac{p_i^2}{2m}}$$

$$= \frac{1}{N!}\frac{1}{\sqrt{2\pi m k_B T}^{3N}}\prod_{i=1}^{N}\int dv_i\, m^3\, 4\pi v_i^2\,\delta(v_j-v)\,e^{-\beta\frac{m v_i^2}{2}}$$

$$= \frac{1}{N!}\frac{1}{\sqrt{2\pi m k_B T}^{3N}}\prod_{i=1}^{N}m^3\cdot 4\pi v^2 e^{-\beta\frac{m v^2}{2}} \quad \Big|\ N=1$$

$$= \frac{m^3 4\pi v^2 e^{-\beta \frac{mv^2}{2}}}{\sqrt{2\pi m k_B T}^3}$$

ZZ: $w(p)\,dp \to w(v)\,dv$

$$w(p)\,dp = \frac{4\pi p^2}{\sqrt{2\pi m k_B T}^3} e^{-\beta \frac{p^2}{2m}} dp \qquad \Big|\ p = mv \Rightarrow dp = m\,dv$$

$$= \frac{4\pi m^2 v^2}{\sqrt{2\pi m k_B T}^3} e^{-\beta \frac{m \cdot v^2}{2}} dv \qquad \Big|\ w(v) = \frac{m^3 4\pi v^2 e^{-\beta \frac{mv^2}{2}}}{\sqrt{2\pi m k_B T}^3}$$

$$= w(v)\,dv$$

$\square$