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Aufgabe 1:

b)

i)

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, e^{-a(x^2+y^2)} = \int_0^{2\pi} d\varphi \int_0^{\infty} dr \cdot r \cdot e^{-ar^2} = 2\pi \int_0^{\infty} du\, \tfrac{1}{2} e^{-au}$$

$r^2 = u \Rightarrow \frac{du}{dr} = 2r$

$$= \pi \left[\frac{1}{-a} e^{-au} \right]_0^{\infty}$$

$\longrightarrow a$

$$= \frac{\pi}{a}$$

$$\left(\int_{-\infty}^{\infty} dx\, e^{-ax^2} \right)^2 = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, e^{-a(x^2+y^2)} = \frac{\pi}{a}$$

$$\Rightarrow \int_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$$

ii)

$u = \frac{x}{\sqrt{a}} - \frac{J}{\sqrt{2a}} \Rightarrow du = \frac{1}{\sqrt{a}} dx$

$$Z(J) = \int_{-\infty}^{\infty} e^{-ax^2+Jx} dx = \int_{-\infty}^{\infty} dx\, e^{-(ax^2 - Jx + \frac{J^2}{2a} - \frac{J^2}{2a})} = \int_{-\infty}^{\infty} dx\, e^{-(\frac{x}{\sqrt{a}} - \frac{J}{\sqrt{2a}})^2 + \frac{J^2}{2a}} = e^{\frac{J^2}{2a}} \cdot \int_{-\infty}^{\infty} a^{-u^2} \cdot \sqrt{a}\, du$$

$$= e^{\frac{J^2}{2a}} \sqrt{a\pi}$$

iii)

$$\left.\frac{\partial^n Z(J)}{\partial J^n}\right|_{J=0} = \left.\frac{\partial^n}{\partial J^n} e^{\frac{J^2}{2a}} \sqrt{a\pi}\right|_{J=0} \qquad = \sqrt{a\pi}$$

$$\frac{\partial Z(J)}{\partial J} = \sqrt{a\pi} \left.\left(\frac{J}{a} e^{\frac{J^2}{2a}} \right)\right|_{J=0} \qquad = 0$$

$$\frac{\partial^2 Z(J)}{\partial J^2} = \sqrt{a\pi}\, \exp(J^2/2a) \left.\left(\frac{J^2}{a} + \frac{1}{a} \right)\right|_{J=0} \qquad = \sqrt{\frac{\pi}{a}}$$

$$\frac{\partial^3 Z(J)}{\partial J^3} = \sqrt{a\pi}\, \exp(J^2/2a) \left.\left(\frac{J^3}{a^2} + \frac{J}{a^2} + 2\frac{J}{a} \right)\right|_{J=0} \qquad = 0$$

$$\frac{\partial Z(J)}{\partial J^4} = \sqrt{a\pi}\, \exp(J^2/2a) \left.\left(\frac{J^4}{a^3} + \frac{J^2}{a^3} + 2\frac{J}{a} + 3\frac{J^2}{a^2} + \frac{1}{a^2} + \frac{2}{a} \right)\right|_{J=0} \qquad = \sqrt{a\pi} \left(\frac{1}{a^2} + \frac{2}{a} \right)$$

symm. Interval & asym. Funktion

$$\langle X \rangle = \int_{\mathbb{R}} e^{-ax^2} \cdot x\, dx = 0$$

$$(\Delta x)^2 = \int_{\mathbb{R}} e^{-ax^2} x^2 dx - 0 = \int_{\mathbb{R}} \frac{\partial}{\partial a} (-e^{-ax^2}) dx = -\frac{\partial}{\partial a} \sqrt{\frac{\pi}{a}} = \frac{1}{2} \left(\frac{\pi}{a} \right)^{-\frac{3}{2}}$$

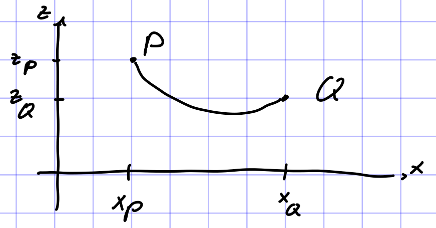
$$\langle X^4 \rangle = \int_{\mathbb{R}} e^{-ax^2} x^4 dx = \int_{\mathbb{R}} \frac{\partial^2}{\partial a^2} e^{-ax^2} dx = \frac{\partial^2}{\partial a^2} \sqrt{\frac{\pi}{a}} = \frac{\partial^2}{\partial a^2} \frac{1}{2} \left(\frac{\pi}{a}\right)^{-\frac{3}{2}} = -\frac{3}{4}\left(\frac{\pi}{a}\right)^{-\frac{5}{2}}$$

c)

i) $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$

$$\Gamma(n+1) = \int_{\mathbb{R}^+} x^{(n+1)-1} e^{-x} dx = \int_{\mathbb{R}^+} x \cdot x^{n-1} e^{-x} dx =$$

Aufgabe 2:



$$U_0 = \sum_{ds} m(ds) \cdot g \cdot h(ds)$$

$$= \int_{s_0}^{s_1} \rho \cdot ds \cdot g \cdot z(ds)$$

$$= \rho \cdot g \int_{s_0}^{s_1} z(s)\, ds \quad \Big| \quad s = s(x, z)$$

$s = s(x,z)$, infinites. Seile, ds, $s' = s'(x', z')$ $\Rightarrow$ $ds = s' - s \approx \sqrt{(x'-x)^2 + (z'-z)^2}$

$$ds \approx \sqrt{dx^2 + dz^2}$$

$$= \rho \cdot g \int_\gamma z \sqrt{dx^2 + dz^2}$$

$$= \rho \cdot g \int_\gamma z \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\, dx$$

$$= \rho \cdot g \int_{x_P}^{x_Q} z \sqrt{1 + z'^2}\, dx$$

warum unabhängig von x?

woher?

$$U = U_0 + \lambda \cdot f(x, z) \quad \Big| \quad f(x, z) = -g \cdot \rho \cdot l = -g \cdot \rho \int_{x_P}^{x_Q} dx \sqrt{1 + z'^2}$$

$$= \rho \cdot g \int_{x_P}^{x_Q} (z - \lambda) \sqrt{1 + z'^2}\, dx$$

$$= \rho \cdot g \int_{x_P}^{x_Q} dx\, \mathcal{F}(z, z', x) \quad \text{mit } \mathcal{F}(z, z', x) = (z - \lambda) \sqrt{1 + z'^2} \text{ und } z' = \frac{dz}{dx}.$$

b)

Tailor z in x

$$\mathcal{F}(z, z', x) = (z - \lambda) \sqrt{1 + z'^2} = (x - \lambda) \sqrt{1 + z'^2} + \sqrt{1 + z'^2}(z - x)$$

$$= (z - \lambda) \sqrt{1 + z'^2}$$

$\Rightarrow$