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8.1) Zustandsdichte

a) $I = \int_{-\infty}^{\infty} d^dx \, \exp(-\sum_{i=1}^{d} x_i^2) = \int_{-\infty}^{\infty} dx_1 \dots \int_{-\infty}^{\infty} dx_d \prod_{i=1}^{d} \exp(-x_i^2) = \prod_{i=1}^{d} \int_{-\infty}^{\infty} dx_i \exp(-x_i^2) = \prod_{i=1}^{d} \sqrt{\pi} = \pi^{\frac{d}{2}}$

$\wedge \quad I = \int_0^\infty dr \int_S d\Omega \, \exp(-r^2) \cdot r^{d-1} \qquad \Big| \; x = \sqrt{r} \Rightarrow \frac{dx}{dr} = \frac{1}{2\sqrt{x}}$

$= O(1) \cdot \frac{1}{2} \int_0^\infty dx \, \exp(-x) \cdot x^{\frac{d}{2}-1} \qquad \text{mit } O(1) = \int_S d\Omega = S^d$

$= O(1) \cdot \frac{1}{2} \Gamma(\frac{d}{2}) \qquad \text{mit } \Gamma(z) = \int_0^\infty dt \, e^{-t} t^{z-1}$

$\Rightarrow O(1) = \frac{2I}{\Gamma(\frac{d}{2})} = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} \Rightarrow \underline{\underline{O(R) = O(1) \cdot R^{d-1} = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})} \cdot R^{d-1}}}$

2p

1	2	3	Σ
8	6	10	24
max 8	10	22	40

b) Z: $\int d^dx \int \frac{d^dp}{(2\pi\hbar)^d} f(\varepsilon(p)) = \frac{VS^d}{(2\pi\hbar)^d} \int d\varepsilon \, f(\varepsilon) \begin{cases} m(2m\varepsilon)^{d/2-1} & , \varepsilon = \frac{p^2}{2m} \\ \frac{1}{c^d} \cdot \varepsilon^{d-1} & , \varepsilon = cp \end{cases}$

- $\int d^dx \int \frac{d^dp}{(2\pi\hbar)^d} f(\varepsilon(p)) = \frac{V}{(2\pi\hbar)^d} \int d^dp \, f(\varepsilon(p))$

$= \frac{V}{(2\pi\hbar)^d} \int_0^\infty d|p| \int_S d\Omega \, f(\varepsilon(p)) \, p^{d-1}$

$= \frac{VS^d}{(2\pi\hbar)^d} \int_0^\infty dp \, f(\varepsilon(p)) \, p^{d-1} \qquad \Big| \; p = \begin{cases} \sqrt{2m\varepsilon} \\ \varepsilon/c \end{cases} \Rightarrow \frac{\partial p}{\partial \varepsilon} = \begin{cases} m/\sqrt{2m\varepsilon} \\ 1/c \end{cases}$

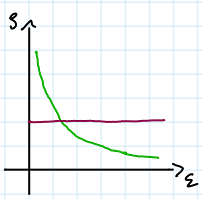
$= \frac{VS^d}{(2\pi\hbar)^d} \int_0^\infty d\varepsilon \, f(\varepsilon) \cdot \begin{cases} \sqrt{2m\varepsilon}^{d-1} \cdot m \sqrt{2m\varepsilon}^{-1} \\ (\varepsilon/c)^{d-1} \cdot 1/c \end{cases}$

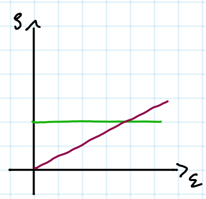
$= \frac{VS^d}{(2\pi\hbar)^d} \int_0^\infty d\varepsilon \, f(\varepsilon) \cdot \begin{cases} m(2m\varepsilon)^{d/2-1} \\ (1/c)^d \cdot \varepsilon^{d-1} \end{cases} \qquad \square$

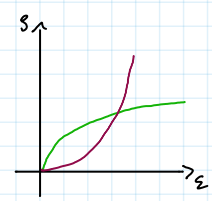
3p

c) $g(\varepsilon) = \int d^dx \int \frac{d^dp}{(2\pi\hbar)^d} \delta(\varepsilon - \varepsilon(p,x)) = \frac{VS^d}{(2\pi\hbar)^d} \begin{cases} m(2m\varepsilon)^{d/2-1} \\ \frac{1}{c^d} \cdot \varepsilon^{d-1} \end{cases}$

- 1 Dim: $g(\varepsilon) = \frac{VS^1}{2\pi\hbar} \cdot \begin{cases} m/\sqrt{2m\varepsilon} \\ 1/c \end{cases}$
- 2 Dim: $g(\varepsilon) = \frac{VS^2}{(2\pi\hbar)^2} \cdot \begin{cases} m \\ \varepsilon/c^2 \end{cases}$
- 3 Dim: $g(\varepsilon) = \frac{VS^3}{(2\pi\hbar)^3} \cdot \begin{cases} m\sqrt{2m\varepsilon} \\ \varepsilon^2/c^3 \end{cases}$







3p

8.2) Das ideale Fermigas

a) Z: $N = \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{\beta(\varepsilon-\mu)}+1}$

- $N = \frac{\partial \Omega}{\partial \mu} = \frac{\partial}{\partial \mu}\left[-\frac{1}{\beta} \ln(Z_G)\right] \qquad \Big| \; Z_G = \prod_k \left(1 + e^{-\beta(\varepsilon_k - \mu)}\right)$

$\Rightarrow N = \frac{\partial}{\partial \mu} \left\{ -\frac{1}{\beta} \ln\left[\prod_k \left(1 + e^{-\beta(\varepsilon_k-\mu)}\right)\right] \right\}$

$= -\frac{1}{\beta} \sum_k \frac{\partial}{\partial \mu} \ln\left[1 + e^{-\beta(\varepsilon_k-\mu)}\right]$

$= \sum_k \frac{e^{-\beta(\varepsilon_k-\mu)}}{1+e^{-\beta(\varepsilon_k-\mu)}}$

$= \sum_k \frac{1}{e^{\beta(\varepsilon_k-\mu)}+1}$

$= \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{\beta(\varepsilon_k-\mu)}+1}$

1p

b) $N(T=0) = \int_0^{\varepsilon_F} d\varepsilon \, g(\varepsilon) \qquad \left(\text{da } \left(e^{(\varepsilon_k-\mu)/k_BT}\right)^{-1} \xrightarrow{T\to 0} 0\right) \qquad \Big| \; g(\varepsilon) = \begin{cases} \frac{V\sqrt{2m}}{2\pi\hbar} \frac{1}{\sqrt{\varepsilon}} & , d=1 \\ \frac{Vm}{2\pi\hbar^2} & , d=2 \\ \frac{V(2m)^{3/2}}{4\pi^2\hbar^3} \sqrt{\varepsilon} & , d=3 \end{cases}$

- 1 Dim: $N(T=0) = \frac{V\sqrt{2m}}{2\pi\hbar} \int_0^{\varepsilon_F} d\varepsilon \, \frac{1}{\sqrt{\varepsilon}} = \frac{V\sqrt{2m}}{\pi\hbar} \sqrt{\varepsilon_F} \Rightarrow \varepsilon_F = \frac{(N\pi\hbar)^2}{2mV^2}$
- 2 Dim: $N(T=0) = \frac{Vm}{2\pi\hbar^2} \int_0^{\varepsilon_F} d\varepsilon = \frac{Vm}{2\pi\hbar^2} \cdot \varepsilon_F \Rightarrow \varepsilon_F = \frac{2\pi\hbar^2 N}{Vm}$
- 3 Dim: $N(T=0) = \frac{V(2m)^{3/2}}{4\pi^2\hbar^3} \int_0^{\varepsilon_F} d\varepsilon \, \sqrt{\varepsilon} = \frac{V(2m)^{3/2}}{6\pi^2\hbar^3} \varepsilon_F^{\frac{3}{2}} \Rightarrow \varepsilon_F = \frac{(6\pi^2\hbar^3 N)^{\frac{2}{3}}}{V^{2/3} \, 2m}$

1p

c) Zu best: $S, U, C_V, p, (\Delta N)^2$ $\quad (T \to 0, d = 1,2,3)$

$$S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu} \qquad \Omega = \frac{1}{\beta}\sum_k \ln\left(1 + e^{-\beta(\varepsilon_k - \mu)}\right)$$

$\mu \to \varepsilon_F$

$$= -\left(-k_B \sum_k \ln\left(1 + e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}\right)\right) - \left(\cancel{k_BT} \sum_k \frac{-\frac{\varepsilon_k-\varepsilon_F}{k_BT^2}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}{1+e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}\right)$$

$$= k_B \sum_k \ln\Big(1 + \underbrace{e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}}_{\to\infty ?}\Big) + \sum_k \frac{(\varepsilon_k-\varepsilon_F)}{T}\cdot \frac{1}{1+\underbrace{e^{\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}_{0}}$$

$T \to 0$, $\varepsilon_k < \varepsilon_F$

$$\overset{T\to 0}{=} \infty - \infty = 0 \;✓$$

etwas sketchy, aber mit L'Hospital kommt man nicht weiter

$T \to 0 \Rightarrow \frac{1}{T} \to \infty \Rightarrow e^{\frac{1}{T}} \to \infty$

$e^{-\frac{1}{T}} \to 0$

vereinfacht für L'Hospital

$$\underbrace{\ln\left(1+e^{\frac{1}{x}}\right)}_{f(x)} - \underbrace{\frac{1}{x}}_{g(x)}$$

$$\lim_{x\to\infty} \frac{\frac{1}{\frac{1}{x}} - \frac{1}{\ln\left(1+e^{\frac{1}{x}}\right)}}{\frac{1}{\ln\left(1+e^{\frac{1}{x}}\right) - \frac{1}{x}}} = \lim_{x\to\infty} \frac{1 - \frac{\sqrt[x]{e}}{\left(\sqrt[x]{e}+1\right)x^2\left(\ln\left(\sqrt[x]{e}+1\right)\right)^2}}{\left(\sqrt[x]{e}+1\right)\left(x\ln\left(\sqrt[x]{e}+1\right)-1\right)^2}$$

$\to$ das scheint keinen Sinn zu ergeben :(da $\lim 0^+ = \infty$ $\quad \lim 0^- = -\infty$

$$U = \Omega + T\underbrace{S}_{=0} + \mu N$$

$$= \Omega + \mu N$$

$$= \frac{1}{\beta}\sum_k \ln\left(1+e^{-\beta(\varepsilon_k-\mu)}\right) + \mu N$$

$$= \underbrace{k_BT}_{\to 0}\sum_k \ln\Big(1+\underbrace{e^{-\frac{(\varepsilon_k-\mu)}{k_BT}}}_{\to\infty}\Big) + \underbrace{\mu}_{\to\varepsilon_F} N$$

$$\lim_{T\to 0} k_BT \cdot \ln\left(1+e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}\right) = \lim_{T\to 0} \frac{\ln\left(1+e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}\right)}{\frac{1}{k_BT}}$$

$$\lim_{T\to 0} \frac{\frac{1}{1+e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}}\cdot\frac{(\varepsilon_k-\varepsilon_F)}{\cancel{k_B}T^2}}{-\frac{1}{\cancel{k_B}T^2}} = -\underbrace{(\varepsilon_k-\varepsilon_F)}_{<0}\cdot\frac{1}{1+\underbrace{e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}_{\to\infty \text{ für } T\to 0,\ \varepsilon_k<\varepsilon_F}} = 0$$

$$= \varepsilon_F N \cdot \frac{d}{d+2} \leftarrow \text{fehlt irgendwie}$$

$$= \varepsilon_F \cdot \begin{cases} \frac{V\sqrt{2m\varepsilon_F}}{2\pi\hbar}, & d = 1 \\ \frac{Vm}{4\pi\hbar}\varepsilon_F, & d = 2 \\ \frac{V(2m)^{\frac{3}{2}}}{6\pi^2\hbar^3}\varepsilon_F^{\frac{3}{2}}, & d = 3 \end{cases} \qquad (✓)$$

$$C_V = T\cdot\left(\frac{\partial S}{\partial T}\right)_{V,\mu} = 0 \;✓$$

$$p = -\left(\frac{\partial \Omega}{\partial V}\right)_{T,\mu}$$

$$\Omega_{d1} = k_BT\sum_k \ln\left(1+e^{-\frac{(\varepsilon_k-\varepsilon_F)}{k_BT}}\right)$$

$$= k_BT\sum_k \ln\left(1+\exp\left(-\frac{1}{k_BT}\cdot\left(\varepsilon_k - \left(\frac{N\cdot 2\pi\hbar}{V\sqrt{2m}}\right)^2\right)\right)\right)$$

$$\overset{d=1}{=} \cancel{k_BT}\sum_k \frac{\exp\left(-\frac{1}{k_BT}\left(\varepsilon_k-\left(\frac{N\cdot 2\pi\hbar}{V\sqrt{2m}}\right)^2\right)\right)\cdot 2\left(+\frac{1}{\cancel{k_BT}}\left(\frac{N\cdot 2\pi\hbar}{V\sqrt{2m}}\right)\left(\frac{N\cdot 2\pi\hbar}{V^2\sqrt{2m}}\right)\right)}{1+\exp\left(-\frac{1}{k_BT}\cdot\left(\varepsilon_k-\left(\frac{N\cdot 2\pi\hbar}{V\sqrt{2m}}\right)^2\right)\right)}$$

$$\boxed{\text{für } d=1 \quad \varepsilon_F = \left(\frac{N\cdot 2\pi\hbar}{V\sqrt{2m}}\right)^2}$$

$$= \sum_k \frac{-2\frac{N^2 4\pi^2\hbar^2}{2m}\cdot\frac{1}{V^2}\cdot \overbrace{\exp\left(-\frac{1}{k_BT}(\varepsilon_k-\varepsilon_F)\right)}^{\to\infty}}{1+\underbrace{\exp\left(-\frac{1}{k_BT}(\varepsilon_k-\varepsilon_F)\right)}_{\to\infty}}$$

$$\lim_{T\to\infty}\frac{\exp\left(\frac{1}{k_BT}\right)}{1+\exp\left(\frac{1}{k_BT}\right)} \overset{\text{L'Hospital}}{=} \frac{-\frac{1}{k_BT^2}\cdot\exp\left(\frac{1}{k_BT}\right)}{-\frac{1}{k_BT^2}\exp\left(\frac{1}{k_BT}\right)} = 1$$

$$= \sum_k -2\frac{N^2 4\pi^2\hbar^2}{2m\cdot V^2}$$

$$= -\frac{4\pi^2 N^3\hbar^2}{2mV^2}$$

sollte >0 sein und von Dimension d abhängen...

3P

d)
$$\Omega = \frac{1}{\beta}\sum_k \ln\left(1+e^{-\beta(\varepsilon_k-\mu)}\right)$$

$$\lim_{T\to 0}\frac{1}{k_BT} = \infty$$

$$= \underbrace{\frac{1}{k_BT}}_{\to 0}\sum_k \ln\Big(1+\underbrace{e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}_{\to\infty}\Big) \quad \to \text{L'Hospital}$$

$$\lim_{T\to 0}\frac{\ln\left(1+e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}\right)}{k_BT} = \frac{\overbrace{\frac{\varepsilon_k-\varepsilon_F}{k_BT^2}}^{\to\infty}\cdot \overbrace{e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}^{\to\infty}}{k_B\left(1+\underbrace{e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}_{\to\infty}\right)} \overset{\text{L'Hospital}}{=} \frac{-\frac{\varepsilon_k-\varepsilon_F}{k_BT^3}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}} + \frac{\varepsilon_k-\varepsilon_F}{k_BT^2}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}{\frac{\varepsilon_k-\varepsilon_F}{k_BT^2}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}} \overset{\text{L'Hospital}}{=} \frac{-\frac{\varepsilon_k-\varepsilon_F}{k_BT^3}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}}{\frac{\varepsilon_k-\varepsilon_F}{k_BT^2}\cdot e^{-\frac{\varepsilon_k-\varepsilon_F}{k_BT}}} + 1 =$$

$$= -\frac{1}{T} + 1 = -\infty + 1 = -\infty \quad f$$

1P

Theo 4; Zettel 8; Marc Hauer, Franka Weronek, Angelo Brade; 03.12.24

Aufgabe 3:

a)

Weitere Freiheitsgrade in Quantenzahlen

$$Z = \underbrace{\int d^3x_1 \dots \int d^3x_N}_{V^N} \int \frac{d^3p_1}{(2\pi\hbar)^3} \dots \int \frac{d^3p_N}{(2\pi\hbar)^3}\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}} \cdot Z_\alpha \quad \Big| \; Z_\alpha = \sum_{\alpha \in (\alpha_n)_n} e^{-\beta \sum^{N} \varepsilon_{i,n}(\alpha_n)}$$

$(2\pi\hbar)^{3N}$

$\hat{H}_{i,n}|i\rangle = \varepsilon_{i,n}|i\rangle$

aus $H = \sum_{n=1}^{N} \left(\frac{\vec{p}_n^2}{2m} + H_{i,n} \right)$

$$= \left(\frac{V}{(2\pi\hbar)^3}\right)^N \int d^3p_1 \dots \int d^3p_N\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}} \cdot \sum_{\alpha \in (\alpha_n)_n} e^{-\beta \sum^{N} \varepsilon_{i,n}(\alpha_n)} \quad \Big| \; \beta = \frac{1}{k_B T}$$

$$= \left(\frac{V}{(2\pi\hbar)^3}\right)^N \int d^3p_1 \dots \int d^3p_N\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}} \cdot \sum_{\alpha \in (\alpha_n)_n} \prod_n^N e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}} \quad \Big| \text{ keine Interaktion}$$

$$= \left(\frac{V}{(2\pi\hbar)^3}\right)^N \int d^3p_1 \dots \int d^3p_N\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}} \cdot \prod_n^N \sum_{\alpha_n} e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}}$$

✓ 1P

b)

$$Z = \left(\frac{V}{(2\pi\hbar)^3}\right)^N \underbrace{\int d^3p_1 \dots \int d^3p_N\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}}}_{\overset{NR}{=} \sqrt{2m k_B T \pi}^{3N}} \cdot \prod_n^N \sum_{\alpha_n} e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}}$$

Teilchenindeze

Da jedes Teilchen gleich ist, ist $\sum_{\alpha_n} e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}}$ für jedes n auch gleich: $\prod_n^N \sum_{\alpha_n} e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}} = z_i^N$ mit $z_i = \sum_{\alpha_n} e^{-\frac{\varepsilon_{i,n}(\alpha_n)}{k_B T}}$, wobei $n \in \{1, \dots, N\}$

$$= \left(\frac{V}{\lambda^3} z_i\right)^N \text{ mit } \lambda = \frac{2\sqrt{\pi}\,\hbar}{\sqrt{2m k_B T}}$$

✓ 1P

NR:
$$\int d^3p_1 \dots \int d^3p_N\, e^{-\beta \sum_{n=1}^{N} \frac{\vec{p}_n^2}{2m}} = \prod_{n=1}^{N} \int d^3p_n\, e^{-\beta \frac{\vec{p}_n^2}{2m}} \quad \Big| \; \vec{p}_n^2 = p_{n,x}^2 + p_{n,y}^2 + p_{n,z}^2$$

$$= \prod_{n=1}^{N} \int d^3p_n\, e^{-\beta \frac{(p_{n,x}^2 + p_{n,y}^2 + p_{n,z}^2)}{2m}}$$

$$= \prod_{n=1}^{N} \prod_{i=1}^{3} \int dp_{n,i} \prod_{i=1}^{3} e^{-\beta \frac{p_{n,i}^2}{2m}} \quad \Big| \; p_i \text{ ist unabhängig von } p_j$$

$$= \prod_{n=1}^{N} \prod_{i=1}^{3} \underbrace{\int dp_{n,i}\, e^{-\beta \frac{p_{n,i}^2}{2m}}}_{\sqrt{2 k_B T m \pi}} \quad \Big| \; \beta = \frac{1}{k_B T}$$

$$= \prod_{n=1}^{N} \sqrt{2 k_B T m \pi}^{3}$$

$$= \sqrt{2 k_B T m \pi}^{3N}$$

c) $Z(V,T,N)$ abhängig von V, T und N => kanonisches Ensemble

$$\Rightarrow F = -k_B T \ln(Z(V,T,N)) \quad | \; Z(V,T,N) = \left(\frac{V}{\lambda^3}\cdot z_i\right)^N$$

$$= -k_B T N \ln\left(\frac{V}{\lambda^3}\cdot z_i\right) \quad | \; \ln(x\cdot y) = \ln x + \ln y$$

$$= -k_B T N \left[\ln\frac{V}{\lambda^3} + \ln z_i\right] \quad \square$$

2p

d)

$$p = -\left(\frac{\partial \langle E\rangle}{\partial V}\right)_{N,T}$$

$$= -\left(\frac{\partial (F+TS)}{\partial V}\right)_{N,T}$$

$$= \frac{k_B T N \lambda^3}{V}\cdot\frac{1}{\lambda^3}$$

$$\Rightarrow pV = N k_B T$$

✓ 1p

e)

$$S = -\frac{\partial F}{\partial T}$$

$$= k_B N\left[\ln\frac{V}{\lambda^3} + \ln z_i + \underbrace{T\frac{\partial}{\partial T}\ln\frac{V}{\lambda^3}}_{=\frac{3}{2}} + T\frac{\partial \ln z_i}{\partial T}\right]$$

offensichtlich (habe ich für nachgerechnet [trust me bro]) (I)

$$= k_B N\left[\ln\frac{V}{\lambda^3} + \ln z_i + \frac{3}{2} + T\frac{\partial \ln z_i}{\partial T}\right]$$

$$E = F + TS$$

$$= -k_B T N\left[\cancel{\ln\frac{V}{\lambda^3}} + \cancel{\ln z_i}\right] + T k_B N\left[\cancel{\ln\frac{V}{\lambda^3}} + \cancel{\ln z_i} + \frac{3}{2} + T\frac{\partial \ln z_i}{\partial T}\right]$$

$$= T k_B N\left[\frac{3}{2} + T\frac{\partial \ln z_i}{\partial T}\right]$$

✓ 1p

I: $$\frac{\partial \ln\frac{V}{\lambda^3}}{\partial T} = \underbrace{\frac{\lambda^3}{V}\left(-3\frac{V}{\lambda^4}\right)\left(\frac{2\sqrt{\pi}\hbar}{\sqrt{2\pi m k_B}}\cdot\frac{-1}{2}\cdot\frac{1}{\sqrt{T}^3}\right)}_{= \frac{3}{\lambda}\cdot\frac{\sqrt{\pi}\hbar}{2\pi m k_B}\cdot\frac{1}{\sqrt{T}}\cdot\frac{1}{T} = \frac{3}{2}\frac{1}{T}}$$

f)
$$C_v = T\left(\frac{\partial S}{\partial T}\right)_{V,N}$$
$$= T\frac{\partial}{\partial T} k_B N \left[\ln\frac{V}{\lambda^3} + \ln Z_i + \frac{3}{2} + T\frac{\partial \ln Z_i}{\partial T}\right]$$
$$= k_B N\left[\frac{3}{2} + T\frac{\partial \ln Z_i}{\partial T} + \frac{\partial \ln Z_i}{\partial T} + T\frac{\partial^2 \ln Z_i}{\partial T^2}\right]$$
$$= k_B N\left[\frac{3}{2} + \frac{\partial}{\partial T}(T\ln Z_i) - \ln Z_i + \cancel{\frac{\partial}{\partial T}} Z_i + \frac{\partial}{\partial T}\left(T\frac{\partial \ln Z_i}{\partial T}\right) - \cancel{\frac{\partial}{\partial T}} Z_i\right]$$
$$= k_B N\left[\frac{3}{2} + \frac{\partial}{\partial T}\left(T\left(\ln Z_i + \frac{\partial \ln Z_i}{\partial T}\right)\right)\right]$$
$$= k_B N\left[\frac{3}{2} + \frac{\partial}{\partial T}\left(T\frac{\partial}{\partial T}(T\ln Z_i)\right)\right] \checkmark$$
$$\frac{\partial \ln Z_i}{\partial T} = \frac{\partial \ln Z_i}{\partial \beta}\cdot\frac{\partial \beta}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \ln Z_i}{\partial \beta}$$

Von hier aus weiß ich nicht weiter :C
$$\frac{\partial}{\partial T}(T\ln Z_i) = \ln Z_i + \frac{\partial \beta}{\partial T}\frac{\partial \ln Z_i}{\partial \beta}$$

↳ Tutorium

1P

g)
$$Z_{vib} = \exp\frac{-\sum_n \omega\hbar\left(n+\frac{1}{2}\right)}{k_B T} = \exp\left[-\Theta_{vib}\beta k_B \sum_n\left(n+\frac{1}{2}\right)\right] \text{ mit } \Theta_{vib} = \frac{\hbar\omega}{k_B} \text{ und } \beta = \frac{1}{k_B T}$$

$$Z_{vib} = \sum_n e^{-\beta\hbar\omega\left(n+\frac{1}{2}\right)}$$

0P

h)
$$\langle E\rangle = \sum_n \varepsilon_{n,vib} W_{vib}(n)$$
$$= \sum_n \Theta_{vib} k_B\left(n+\frac{1}{2}\right)\frac{e^{-\frac{\Theta_{vib}}{T}\left(n+\frac{1}{2}\right)}}{Z_{vib}}$$
$$= -k_B T\frac{\partial}{\partial\beta}\ln(Z_{vib}) \checkmark$$

=> ausrechnen mit Ergebnis aus (g)

1P

i)
$$C_V^{vib} = -N k_B\frac{\partial}{\partial T}\left(\frac{1}{k_B}\frac{\partial \ln Z_{vib}}{\partial\beta}\right)$$
$$= -N k_B\frac{\partial}{\partial T}\left(\frac{1}{k_B}\left(-\Theta_{vib}k_B\sum_n\left(n+\frac{1}{2}\right)\exp\left[-\Theta_{vib}\beta k_B\sum_n\left(n+\frac{1}{2}\right)\right]\right)\right)$$
$$= N k_B\Theta_{vib}\sum_n\left(n+\frac{1}{2}\right)\frac{\partial\beta}{\partial T}\left(\frac{\partial}{\partial\beta}\exp\left[-\Theta_{vib}\beta k_B\sum_n\left(n+\frac{1}{2}\right)\right]\right)$$
$$= \frac{N}{T^2}\Theta_{vib}^2\left(\sum_n\left(n+\frac{1}{2}\right)\right)^2 k_B Z_{vib}$$
$$= N k_B\left(\frac{\Theta_{vib}\sum\left(n+\frac{1}{2}\right)}{T}\right)^2 Z_{vib} \quad (\checkmark)$$

gleiches Problem, wegen Fehler in (g)

2P