physik 411: Physik IV

Sommer Semester 24

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Nonlinear Quantum Optics

Exercise sheet 3

To be handed in on Thursday May 2.

Deadline for this sheet is **Thursday at 10:00 o'clock**. The sheet should be handed in on ecampus in your tutor group.
The exercises suggested for Lehramt students are marked with a ★.

Do not hesitate to contact your tutors if you have questions!

★ Exercise 1: Quantum numbers of hydrogen atoms — 13 points

a) List for all states of hydrogen with $n \leq 4$ the possible values of l and m_l. Which electron states are possible for $n = 4$? You can neglect the electron spins.

b) Make two plots showing the radial wave function $R_{nl}(r)$ as a function of the distance r of the electron from the core. The first plot should show $R_{nl}(r)$ for $1s$, $2s$, $3s$, and $4s$. The second plot should show $R_{nl}(r)$ for $4s$, $4p$, $4d$, and $4f$.

c) Plot for the same states the probability density distribution $r^2 R_{nl}^2(r)$.

d) Discuss based on the plots the influence of the quantum numbers n and l on the radial wave function and in particular on the distance between the electron and the core.

Exercise 2: Expectation values for hydrogen atoms — 10 points

As discussed in the lecture one uses the norm-square of the wave function $\psi^*(\vec{r})\psi(\vec{r})$ to determine the probability density $P(\vec{r})$ for finding a particle at a specific point $\vec{r}$. This is called Born's rule. This way we can determine the expectation value of some physical quantity a, such as the particle position, by solving the integral

$$\langle a \rangle = \int_{\mathbb{R}^3} a P(\vec{r})\, d^3r = \int_{\mathbb{R}^3} a\psi^*(\vec{r})\psi(\vec{r})\, d^3r. \tag{2.1}$$

In the following exercises you should use equation 2.1 to determine the expectation value for different quantities for the states $1s$ and $2p$ of hydrogen.

Since the probability density for a particular electron state in the hydrogen atom is constant over time, you will not need to consider any time-dependencies in this exercise.

a) Determine for both states the expectation values of $\langle r \rangle$, $\langle r^2 \rangle$ and $\langle 1/r \rangle$.
Hints: In general $1/\langle r \rangle \neq \langle 1/r \rangle$. Keep in mind that the radial and the angular part of the wavefunction can be separated, and that only the radial part of the wavefunction depends on r.

b) Find for both states the most probable distance between the electron and the core.

Exercise 3: Rydberg atoms **10 points**

Rydberg atoms are atoms that have been excited to states with very large principal quantum numbers n. This excitation is normally done through collision processes or laser pulses.

a) Calculate the expectation value of the electron orbital radius and the energy of the Rydberg states of hydrogen with $n = 70$ for $l = 0$ and for $l = 69$.

b) Calculate the wavelength of the radiation that would be radiated for a transition from $n = 70$ to $n = 69$. In what spectral region is this radiation?

c) Use classical mechanics to calculate the frequency and wavelength that an electron would radiate if it would be in a circular orbit around the core with the radius you found in exercise a)?

d) Look up the formula for the radius as described by the Bohr model. Calculate the radius for $n = 70$. How does this compare to the radius obtained in the previous exercises. Compare for $l = 0$ and $l = 69$.

★ Exercise 4: Spherical harmonics **7 points**

The quantum mechanical operators of angular momentum $\hat{L}_z$ and $\hat{L}^2$ can be written in sperical coordinates as

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi} \tag{4.1}$$

and

$$\hat{L}^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\varphi^2} \right]. \tag{4.2}$$

They can be expressed in terms of spherical harmonics that we have encountered in the lecture.

a) Show explicitly that the sperical harmonics $Y_{1,0}$, $Y_{1,-1}$ and $Y_{3,2}$ are orthogonal to each other, that is, that their scalar product

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} Y^*_{l',m'_l}(\phi,\theta) Y_{l,m_l}(\phi,\theta) \sin\theta \, d\phi d\theta \tag{4.3}$$

vanishes.

b) Determine the mean squared error of the measurement value $\hat{L}^2$, given by $(\Delta\hat{L}^2)^2 = \langle(\hat{L}^2)^2\rangle - \langle(\hat{L}^2)\rangle^2$ for the three functions.

Hint: You find the explicit form of the spherical harmonics on for instance Wikipedia.

★ Exercise 5: Write on the wiki **≤ 8 bonus points**

It is possible to earn bonus points by contributing to he lecture script on the Wiki-page. To earn bonus points, you need to contribute to the 'Additional material' section on one of the **Physics 4** lecture subjects on the Wiki.

On ecampus you find a description of how you get access to the Wiki. Before you start editing, read rules for the entries on the rule page carefully.

When you hand in the exercise sheet, include a link to the page(s) and your username(s) such that we can verify your contribution through the page history. You can obtain up to 10 points *for this sheet*, but your work on the wiki has to be noted *on this hand-in to count.*

Note: The number of points you get for your contribution depends on the quality and originality of the material.

Note: We do not accept solutions to the course exercises on the Wiki.

Note: You need to provide references for what you write on the Wiki, *no matter what you are writing about.* Please read the rules on plagiarism very carefully.