Gedämpfte harmon. Schwingung:

$$m\ddot{z} + \beta\dot{z} + kz = 0$$

neue Reibungskomponente

$$\boxed{\ddot{z} + 2\delta\dot{z} + \omega_0^2 z = 0 \qquad \delta = \frac{\beta}{2m} \quad \omega_0^2 = \frac{k}{m}}$$

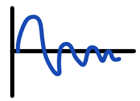
Lösungsansatz:

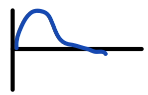
$$z(t) = z_0 e^{i\omega t}$$

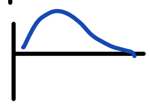
$$\Rightarrow z(t) = e^{-\delta t}\left\{ z_{01}\cdot e^{i\hat{\omega} t} + z_{02}\cdot e^{-i\hat{\omega} t}\right\}$$

mit $\hat{\omega} = \sqrt{\omega_0^2 - \delta^2}$

3 Fälle:
- $\hat{\omega} > 0$ $(\delta < \omega_0)$ → schw. Dämpfung
- $\hat{\omega} = 0$ $(\delta = \omega_0)$ → krit. –"–
- $\hat{\omega} \in \mathbb{C}$ $(\delta > \omega_0)$ → imaginäre –"–


Schwingfall


Schnellstmögl. Dämpfung


Langsamere Dämpfung

ballistische Lsg.

$\dot{z}(0) = v_0$
$z(0) = 0$

$$x(t) = \frac{v_0}{\hat{\omega}} e^{-\delta t} \sin \hat{\omega} t$$

$$x(t) = v_0 t\, e^{-\delta t}$$

$$x(t) = \frac{v_0}{\hat{\omega}_\ell} e^{-\delta t} \sinh(\hat{\omega}_\ell t)$$

mit $\hat{\omega}_\ell = \sqrt{\delta^2 - \omega_0^2}$

Vorlesung 25

Erzwungene harm. Schwingung:

$$m\ddot{z} + \beta\dot{z} + kz = F_0 e^{i\omega_E t}$$

per. Anregung mit Frequenz ω_E, Kraft F_0

$$\boxed{\ddot{z} + 2\delta\dot{z} + \omega_0^2 z = f_0 e^{i\omega_E t}}$$

inhom. Dgl.

$\Rightarrow$ Lsg = hom Lsg. ⊕ part. Lsg

z.B. bal. $x(t) = \frac{v_0}{\hat{\omega}} e^{-\delta t} \sin \hat{\omega} t$

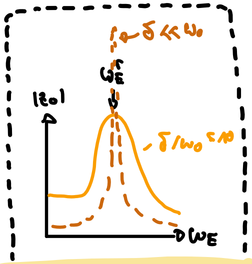
Ansatz: $z_p(t) = z_0 e^{i\omega t}$

$z_0 \in \mathbb{C}$: $z_0 = |z_0| \cdot e^{i\varphi}$

$$= \underbrace{f_0 \frac{\omega_0^2 - \omega_E^2}{(\omega_0^2-\omega_E^2)^2 - (2\delta\omega_E)^2}}_{\text{Realteil}} - \underbrace{f_0 \frac{2i\delta\omega_E}{(\omega_0^2-\omega_E^2)^2 - (2\delta\omega_E)^2}}_{\text{Imaginärteil}}$$

Phase φ: $\tan\varphi = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[z_0]} = -\frac{2\delta\omega_E}{\omega_0^2 - \omega_E^2}$

Ampl. $|z_0|$: $f_0 \frac{1}{\sqrt{(\omega_0^2-\omega_E^2)^2 + (2\delta\omega_E)^2}}$ → Max. bei $\omega_E^r = \sqrt{\omega_0^2 - 2\delta^2}$



$\delta = 0 \Rightarrow |z_0| \to \infty$ wenn $\omega_E = \omega_0$

bal. Anfangsbed: Lsg.

$$x(t) = \frac{v_0}{\hat{\omega}} e^{-\delta t} \sin(\hat{\omega} t) + \frac{f_0 \cos(\omega_E t + \varphi)}{\sqrt{(\omega_0^2-\omega_E^2)^2 + (2\delta\omega_E)^2}}$$

Einschwingvorgang klingt mit $e^{-\delta t}$ ab

Erregerschw. dominant für große $t \gg \frac{1}{\delta}$