Theo 3 Angelo Brade 07.05.2024

Aufgabe 1:

1.: z: $\hat{P}$ hermitisch ($\hat{P} = \hat{P}^\dagger$)

$\hat{P}\hat{P}|x\rangle = \hat{P}|-x\rangle$
$= |x\rangle$
$\Rightarrow \hat{P}\hat{P} = \mathbb{1}$
$\Rightarrow \hat{P} = \hat{P}^{-1}$

$\langle \mathbb{1}x | \mathbb{1}x\rangle$
$\langle \hat{P}\hat{P}x | \hat{P}\hat{P}x\rangle \quad | \; \langle \hat{P}^\dagger x | x\rangle = \langle x | \hat{P} x\rangle$
$\langle \hat{P}\hat{P}^\dagger x | \hat{P}\hat{P}^\dagger x\rangle$
$\Rightarrow \hat{P}\hat{P}^\dagger = \mathbb{1}$
$= \hat{P}\hat{P}$
$\Rightarrow \hat{P} = \hat{P}^\dagger$

2.: z: $\hat{P}\psi(x) = \lambda\psi(x)$ mit $\lambda = \pm 1$

$(\hat{P} = \hat{P}^{-1} \Rightarrow$ unitär $\Rightarrow |\lambda| = 1) \wedge \lambda \in \mathbb{R}$

$\Rightarrow \lambda = \pm 1$

$\Rightarrow \psi(-x) = \hat{P}\psi(x)$
$= \pm\psi(x)$

$\Rightarrow \begin{cases} \text{ungerade}: & \psi(-x) = -\psi(x) \\ \text{gerade}: & \psi(-x) = \psi(x) \end{cases}$

3.: $[\hat{H}, \hat{P}]\phi_n(x) = 0$

$\Rightarrow \hat{H}\hat{P}\phi_n(x) = \hat{P}\hat{H}\phi_n(x)$
$\hat{H}\phi_n(-x) = \hat{P}E_n\phi_n(x)$
$= \pm E_n\phi_n(x)$

$\Rightarrow \phi_n(-x) = \pm\phi_n(x)$ sind Eigenfunktion von $\hat{H}$ mit

$\begin{cases} \text{ungerade}: & \phi_n(-x) = \phi_n(x) \\ \text{gerade}: & \phi_n(-x) = -\phi_n(x) \end{cases}$

Aufgabe 2:

1.: Postulat: $\hat{H}$ kann durch ein vollständige ONB dargestellt werden: $\hat{H}\phi_n(x) = E_n\phi_n(x)$.

$[\hat{H}, \hat{P}] = 0$

$\Rightarrow \hat{H}\hat{P}\phi_n(x) = \hat{P}\hat{H}\phi_n(x)$

$\Rightarrow \hat{H}\phi_n(-x) = \hat{P}E_n\phi_n(x)$

$= \pm E_n\phi_n(x)$

$\Rightarrow \hat{H}\psi_n(x) = E_n\psi_n(x)$ mit $\psi_n(x) = \begin{cases} \text{gerade}: & \phi_n(-x) = -\phi_n(x) \\ \text{ungerade}: & \phi_n(-x) = \phi_n(x) \end{cases}$

2.: $\hat{H}\phi_n(|x|<L)\; E_n\phi_n(|x|<L)$, $\hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(x)$

$-\frac{\hbar^2}{2m}\phi_n''(|x|<L) = E_n\phi_n(x<L)$

$-\frac{\hbar^2}{2m}\partial_x^2\left(Ae^{i\frac{p}{\hbar}x} + Be^{-i\frac{p}{\hbar}x}\right) = E_n\left(Ae^{i\frac{p}{\hbar}x} + Be^{-i\frac{p}{\hbar}x}\right)$

$\frac{p^2}{2m}\left(Ae^{i\frac{p}{\hbar}x} + Be^{-i\frac{p}{\hbar}x}\right) = E_n\left(Ae^{i\frac{p}{\hbar}x} + Be^{-i\frac{p}{\hbar}x}\right)$

$\Rightarrow p = \sqrt{2mE_n}$

$\hat{H}\phi_n(|x|>L) = E_n\phi_n(|x|>L)$

$-\frac{\hbar^2}{2m}\phi_n''(|x|>L) = E_n\phi_n(|x|>L)$

$-\frac{\hbar^2}{2m}\left(Ae^{-\frac{u}{\hbar}x} + Be^{\frac{u}{\hbar}x}\right) = [E_n - V_0]\left(Ae^{-\frac{u}{\hbar}x} + Be^{\frac{u}{\hbar}x}\right) \,|\, \lim_{x\to\infty}\psi(x) = 0 \Rightarrow B = 0$ mit $u \in \mathbb{R}^+$

$-\frac{\hbar^2}{2m}Ae^{-\frac{u}{\hbar}x} = Ae^{-\frac{u}{\hbar}x}[E_n - V_0]$

$\Rightarrow -\frac{u^2}{2m} = E_n - V_0$

$\Rightarrow u = \sqrt{2m(V_0 - E_n)}$

$V(x) = \begin{cases} V_0, & |x| > L \\ 0, & |x| < L \end{cases}$

$\Rightarrow \left(Ae^{\pm i\frac{p}{\hbar}L} + Be^{\pm i\frac{p}{\hbar}L}\right) = (Ae$