physik 411: Physik IV

Sommer Semester 24

Prof. S. Hofferberth

Dr. N. Stiesdal

Dr. E. Urunuela

Exercise sheet 1

To be handed in on Thursday April 18.

Deadline for this sheet is **Thursday at 10:00 o'clock**. The sheet should be handed in on ecampus in your tutor group. For more information, see ecampus.
The exercises suggested for Lehramt students are marked with a ★.

Do not hesitate to contact your tutors if you have questions!

★ Exercise 1: Mass spectrometry — 10 points

In the following exercise we will consider a simple time-of-flight mass spectrometer. The spectrometer consists of a plate capacitor with inter-plate distance $d = 10$ mm. The applied voltage is $U_0 = 1$ kV. One of the plates has a metal grid section. Isotopes of NO-molecules are prepared in the center of the plate capacitor and ionized with a short laser pulse to a singly charged ion (meaning that only one electron is removed and the ion has charge $q = e$).

The ions are accelerated by the electric field in the capacitor and exit the capacitor through the metal grid. Outside of the capacitor, the ions fly through a field-free region with length $L = 1$ m before they are detected on a micro-channel plate-detector.

The laser pulse that ionizes the molecules has pulse duration $\Delta\tau = 5$ ns (FWHM of the intensity, where FWHM means full width at half maximum, and indicates the pulse width where the pulse reaches half of its full intensity). The interaction region has a width $\Delta x = 0.1$ mm (FWHM).

a) Sketch the mass spectrometer set-up from the above description.

b) Calculate the flight time of the most common NO-isotope $^{14}N^{16}O$. You can assume that all ions are created in one point, meaning that you can neglect the width of the interaction area.

c) Calculate the broadening of the time of flight Δt (FWHM) due to the width of the interaction area.

d) Calculate the maximal mass resolution Δm of the mass spectrometer. Can the mass spectrometer distinguish between different isotopes of NO ($^{14}N^{16}O$, $^{15}N^{16}O$, $^{14}N^{18}O$, ...)?

Exercise 2: Scattering cross-sections **10 points**

In 1947 Estermann, Foner, and Stern (*Phys. Rev. 71, 250 (1947)*) performed a scattering experiment measuring the scattering cross-section between caesium atoms and different noble gasses as well as caesium vapor at room temperature. Concretely, the researchers measured how atoms are lost from a beam of caesium atoms when the beam passes through the different vapors.

The beam intensity of the atomic beam after the vapor cell is given by $I = I_0 e^{-nL\sigma}$ (Beer-Lambert law), where n is the number density of scattering atoms, L is the length of the vapor cell, and σ is the scattering cross-section.

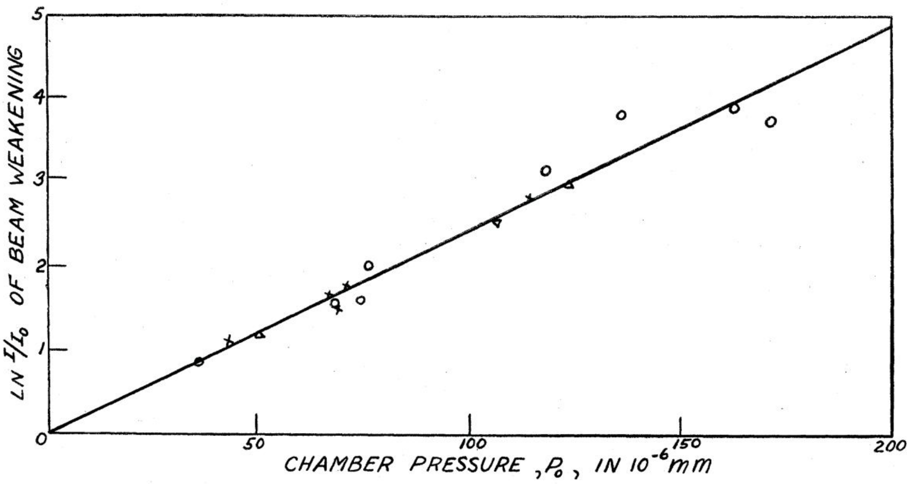

Figure 1: Loss of beam intensity $\ln(I/I_0)$ as function of the pressure (in 10^{-6} mmHg). *Note:* The y-axis actually shows $-\ln(I/I_0) = \ln(I_0/I)$.

a) Use the experimental data for the reduction of beam intensity to determine the scattering cross-section for collisions between caesium atoms. You will need to make some assumptions.

b) Assume that the atoms are hard spheres. How large would the radius of a caesium atom be?

c) The authors found a scattering cross-section of 4.5×10^{-14} cm^2 for collisions between caesium and helium. Use this result and your result from b) to determine the radius of the helium atoms when you again assume the atoms to be hard spheres. Comment on your result.

★ Exercise 3: De Broglie wavelength **10 points**

The de Broglie relation

$$\lambda_{\mathrm{dB}} = h/p \tag{3.1}$$

connects the momentum of a particle to a wavelength. The so-called de Broglie wavelength λ_{dB} is the wavelength of the matter wave of the moving particle.

First we do some warm-up calculations of de Broglie wavelengths of different objects.

a) A take-away box with eintopf from the Mensa in Poppelsdorf drops from the Mensa balcony. Assume that the mass of the take-away box with eintopf is 1 kg and that the balcony is seven meters above the pavement. What is the de Broglie wavelength of the eintopf-take-away box system? You can calculate the maximum speed of the box just before it hits the pavement with classical mechanics assuming no air resistance.

b) Electron beam lithography is a commonly used method of writing nano structures for commercial and scientific purposes. PhD students from the Linden-group in the Physical Institute use an electron beam lithography machine to create, for instance, waveguides for surface excitations. The electrons from the beam have an energy of 10 keV. What is the de Broglie wavelength of an electron in the electron beam?

   Do you think the de Broglie wavelength of the electrons from the electron beam is the primary limitation on the resolution of structures that can be written with electron beam lithography?

In the Hofferberth-group at the Institute of Applied Physics, a gas of Ytterbium atoms (^{174}Yb) is laser cooled to $T = 10\,\mu$K and confined in an optical trap. In this exercise we consider a small volume of $20\,\mu\text{m} \times 20\,\mu\text{m} \times 20\,\mu\text{m}$ in the center of the trap. This volume contains 10^5 atoms, and we assume that the density distribution of atoms in this volume is uniform.

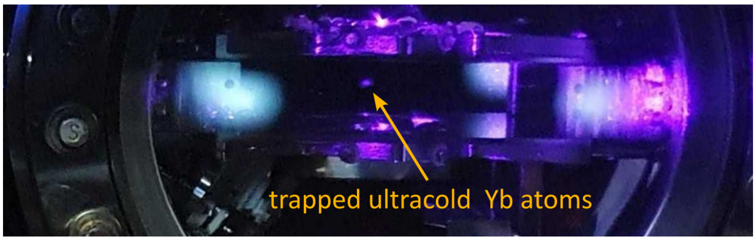

Figure 2: Picture of trapped ultracold Ytterbium atoms in an experiment of the Hoffferberth-group.

The atomic velocity is related to the temperature of an ensemble by a Maxwell-Boltzmann distribution (through the virial theorem), and in three dimensions for the most probable atom velocity v_p it holds that

$$\frac{mv_p^2}{2} = k_B T, \tag{3.2}$$

where m is the mass of the atom, k_B is Boltzmann's constant, and T is the temperature.
Note: You will learn about laser cooling and optical trapping of atoms later in this course. In the following exercise you will do simple calculations of how to reach a de Broglie wavelength on the order of interatomic spacing.

c) Calculate the de Broglie wavelength of the Yb atoms at the most probable velocity.

d) Calculate the average distance d between two Yb atoms in the volume in the center of the trap. At what temperature is the de Broglie wavelength of an atom at the most probable velocity on the same order of magnitude as d? You can assume that the atoms are evenly distributed in the volume. What can you say about the matter wave of the atoms at the critical point $d = \lambda_{\text{dB}}$?

e) The fundamental limit of temperature that can be reached when ^{174}Yb is laser cooled is $4\,\mu$K. This is still higher than the temperature you found in the previous exercise (if not, go back and check). The condition $d = \lambda_{\text{dB}}$ can instead be reached by increasing the atom number density n (number of atoms per volume). Plot the required atom number density (in atoms per cubic centimeter) to fulfill $d = \lambda_{\text{dB}}$ as a function of temperature in the range 50 nK to 1 mK.

f) An important physical quantity is the phase-space density, defined as $\rho = n\lambda_{\mathrm{dB}}^3$. Which is the critical phase-space density for the condition $d = \lambda_{\mathrm{dB}}$? You can calculate it for the different cases that you encountered above in c) to e), or solve the general case. Comment on your result.

Exercise 4: Electron in circular orbit **10 points**

In the following we will imagine an electron in a classical circular orbit around a proton. We assume that classical electrodynamics apply. Further, we assume that the distance between the proton and the electron is given by

$$R = \frac{\epsilon_0 h^2}{\pi m_e e^2} \tag{4.1}$$

where ϵ_0 is the permittivity of vacuum, m_e is the electron mass, and e is the electron charge.

a) Calculate the time it takes for the electron to orbit the proton by equating the centrifugal force with the Coulomb force.

b) How much energy does the electron radiate per time?

c) Assume that the electron will crash into the core once it has radiated all its kinetic energy. You can assume that it maintains its orbit while radiating the kinetic energy away (so no spiral trajectories). What will be the lifetime of the electron?

You have already encountered the de Broglie wavelength. It is defined in equation 3.1.

d) Calculate the de Broglie wavelength of the electron. How does the radius we assumed in eq. 4.1 compare to the de Broglie wavelength of the electron? Interpret your result.

★ Exercise 5: Write on the wiki **≤ 8 bonus points**

It is possible to earn bonus points by contributing to he lecture script on the Wiki-page. To earn bonus points, you need to contribute to the 'Additional material' section on one of the **Physics 4** lecture subjects on the Wiki.

On ecampus you find a description of how you get access to the Wiki. Before you start editing, read rules for the entries on the rule page carefully.

When you hand in the exercise sheet, include a link to the page(s) and your username(s) such that we can verify your contribution through the page history. You can obtain up to 10 points *for this sheet*, but your work on the wiki has to be noted *on this hand-in to count.*

Note: The number of points you get for your contribution depends on the quality and originality of the material.

Note: We do not accept solutions to the course exercises on the Wiki.

Note: You need to provide references for what you write on the Wiki, *no matter what you are writing about.* Please read the rules on plagiarism very carefully.