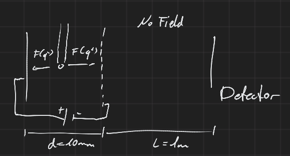
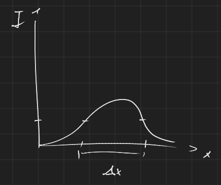
Sheet 1

Exercise 1:

a)





b) $F = qE \qquad E_{kin} = qE\frac{1}{2}d$

$a = \frac{qE}{m} \qquad \Rightarrow v = \sqrt{\frac{qEd}{m}}$

$\Rightarrow \frac{1}{2}d = \frac{1}{2}at_1^2 \qquad \Rightarrow t_2 = \frac{L}{v}$

$\Leftrightarrow t_1 = \sqrt{\frac{d\cdot m}{qE}} \qquad = L\sqrt{\frac{m}{qE\cdot d}}$

$\Rightarrow t = t_1 + t_2 = \sqrt{\frac{dm}{qE}} + L\sqrt{\frac{m}{qEd}} \qquad \mid E = \frac{U_0}{d}$

$t = d\sqrt{\frac{m}{qU_0}} + L\cdot\sqrt{\frac{m}{U_0 q}} \qquad \mid m = m(^{14}N\,^{16}O) = 30u = 30\cdot 1{,}66\cdot 10^{-27}kg$

fehlt nicht noch ein $\frac{1}{2}$ Faktor oder so?

$= \sqrt{\frac{30\cdot 1{,}66\cdot 10^{-27}kg}{1{,}6\cdot 10^{-19}C\cdot 1\cdot 10^{3}V}}\,(10\cdot 10^{-3}m + 1m)$

$= 158{,}78\,ns$

c) $t = \sqrt{\frac{dm}{qE}} + L\sqrt{\frac{m}{qEd}} \qquad \mid d \to d + \Delta x$

$t = \sqrt{\frac{(d+\Delta x)m}{qE}} + L\sqrt{\frac{m}{qE(d+\Delta x)}}$

$= \sqrt{\frac{d(d+\Delta x)m}{qU_0}} + L\sqrt{\frac{dm}{qU_0(d+\Delta x)}}$

$= \sqrt{\frac{dm}{qU_0}}\left(\sqrt{(d+\Delta x)} + L\sqrt{\frac{1}{d+\Delta x}}\right)$

$= 19{,}40\,ns$

$\Rightarrow \Delta t = 139{,}38\,ns$

d) $t = \sqrt{\frac{dm}{qU_0}}\left(\sqrt{(d+\Delta x)} + L\sqrt{\frac{1}{d+\Delta x}}\right)$

$\Leftrightarrow m = \frac{t^2 qU_0}{d\left(\sqrt{(d+\Delta x)} + L\sqrt{\frac{1}{d+\Delta x}}\right)^2} \qquad \mid t = \Delta t$

$= 3{,}3085\cdot 10^{-27}kg$

$= 2{,}068\,u$

$\Rightarrow$ Kann einzelne Isotope auflösen.

Exercise 2:

a) $I = I_0 e^{-nL\sigma}$

$\Leftrightarrow \underbrace{\frac{-\ln(I/I_0)}{p}}_{m_f} = \frac{nL\sigma}{p} \qquad \mid pV = NkT \Leftrightarrow n = \frac{N}{V} = \frac{p}{kT}$

$m_f = \frac{L\sigma}{kT}$

$\Leftrightarrow \sigma = \frac{kT m_f}{L} \qquad \mid$ $L = 3{,}5cm$ (Phys. Rev. 71, 250 (1947))
$T = 273{,}15K + 20K = 293{,}15K$
$k = 1{,}38\cdot 10^{-23}\frac{J}{K}$

$= 2{,}85\cdot 10^{-12}m^2$

$m_f = \frac{4}{(150 + \frac{50}{4})\cdot 10^{-9}m} = 24615384\,\frac{1}{m}$

b) $\sigma = \pi r^2$

$\Rightarrow r = \sqrt{\frac{\sigma}{\pi}}$

$= 3{,}00\cdot 10^{-7}m$

c) $\frac{r_1}{r_2} = \sqrt{\frac{\sigma_1}{\sigma_2}}$

$\Rightarrow r_2 = \sqrt{\frac{\sigma_2}{\sigma_1}}\,r_1$

$= 1{,}197\cdot 10^{-3}\,m$

Das Heliumatom ist um 2 Größenordnungen kleiner.
Dies könnte mit einem Rechenfehler zusammenhängen, oder einfach zeigen,
dass Helium deutlich kleiner als Caesium ist. Dies stimmt zwar konzeptionell,
die Größenordnung ist aber dennoch überraschend.

Exercise 3:

a) $E_{kin} = E_{pot}$

$\Leftrightarrow v = \sqrt{2gh}$

$\Rightarrow \lambda_{dB} = \frac{h}{m\sqrt{2gh}}$

$= 5{,}654\cdot 10^{-35}m$

b) Assuming $E_{kin} \approx 10keV$.

$p = m\sqrt{2\frac{E}{m}}$

$= \sqrt{2Em}$

$\Rightarrow \lambda_{dB} = \frac{h}{\sqrt{2Em}}$

$= 1{,}23\cdot 10^{-11}m$

c)

$\frac{m v_p^2}{2} = k_B T$

$v_p = \sqrt{2\frac{k_B\cdot T}{m}}$

$\lambda_{dB} = \frac{h}{m\cdot\sqrt{2\cdot\frac{k_B\cdot T}{m}}}$

$= \frac{h}{\sqrt{2\cdot k_B\cdot T\cdot m}}$

$= 7{,}421\cdot 10^{-8}m$

d) $V = (20\mu m)^3, \; N = 10^5$

$d = \sqrt[3]{\frac{V}{N}} \qquad \hat{=} \quad n = 1{,}25\cdot 10^{15}\,\frac{1}{m^3}$

$= 4{,}309\cdot 10^{-7}m$

$\hat{=}$ 6 Größenordnungen $= 10^{-d}$

$\lambda_{dB} = \frac{h}{\sqrt{2mk_BT}}$

$T = \frac{h^2}{2mk_B\lambda_{dB}^2} \qquad \mid \lambda_{dB} = 10^{-6}m$

$= 5{,}507\cdot 10^{-8}K$

At $d = \lambda_{dB}$ the Atoms become indistinguishable, since their Wave overlap.

e)

$\sqrt[3]{\frac{V}{\underbrace{N}_{n^{-1}}}} = \frac{h}{\sqrt{2mk_BT}}$

$\Leftrightarrow n = \left(\frac{h}{\sqrt{2mk_BT}}\right)^{-3}$

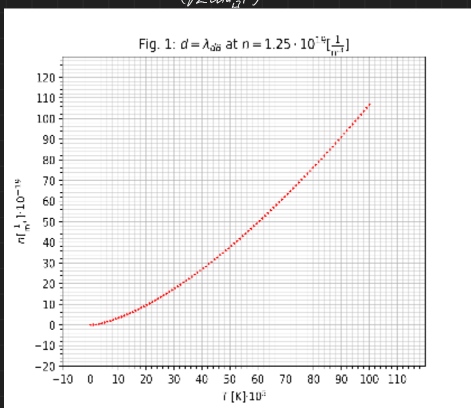


Fig. 1: $d = \lambda_{dB}$ at $n = 1.25\cdot 10^{-15}[\frac{1}{m^3}]$

f) $\lambda_{dB,1} = 7{,}421\cdot 10^{-8}m \Rightarrow \rho = n\cdot\lambda_{dB} = 927{,}54\cdot 10^{10}\,\frac{1}{m^2}$

$\lambda_{dB,2}\; 4{,}309\cdot 10^{-7}m \Rightarrow \rho = 538{,}68\cdot 10^{10}\,\frac{1}{m^2}$

Exercise 4:

a) $F_z = F_c$

$$m_e r \omega^2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$m_e \frac{4\pi^2}{T^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}$$

$$T = \sqrt{\frac{16\pi^3 \varepsilon_0 r^3}{q_1 q_2} m_e} \quad \Big| \; r = \frac{\varepsilon_0 h^2}{\pi m_e e^2}$$

$$T = \sqrt{\frac{16\varepsilon_0^4 h^6}{m_e^2 e^8}}$$

$$= 1{,}49 \cdot 10^{-16}\,s$$

b)

$$P = \frac{2q^2 a^2 \gamma^4}{3c^3} \quad \Big| \; a = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2 m_e} = \frac{m_e q^6}{4\varepsilon_0^3 h^4}$$

$$= \frac{2q^{14} m_e^2}{3c^3 16\varepsilon_0^6 h^8} \frac{1}{(1-(\frac{v}{c})^2)^2} \quad \Big| \; v = \frac{2\pi r}{T} = \frac{\varepsilon_0 h^2}{\pi m_e e^2 \cdot T} =$$

$$=$$