HA11

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④

a)

$E_{rot} = \frac{1}{2} I \omega^2$

$I = \frac{2}{5} m r^2$

$f_0 = 2{,}1\ s^{-1}$

$\dot{f} = -10^{-15}\ s^{-2}$

$\frac{d E_{rot}(\omega(t))}{dt} = \frac{d \frac{1}{2} I \omega(t)^2}{dt}$

$= \frac{1}{2} I \cdot \omega^2 \cdot \frac{d}{dt} \quad (\omega(t)^2)' = 2\omega \cdot \frac{d\omega}{dt}$

$\omega = f \cdot 2\pi \wedge \dot{\omega} = \dot{f}\, 2\pi = \omega \frac{d}{dt}$

$f = f_0 + \dot{f} \cdot t$

$= I \cdot \omega \cdot \frac{d\omega}{dt}$ | setze für ω und $\dot{\omega}$ ein

$\omega = f_0 \cdot 2\pi + \dot{f} \cdot t \cdot 2\pi$

$= 2\pi (f_0 + \dot{f} \cdot t)$

$= 4 I \pi^2 (f_0 + \dot{f} \cdot t) \cdot \dot{f}$

$= 4 I \pi^2 f_0 \dot{f} + 4 I \pi^2 \dot{f}^2 \cdot t$

$\dot{E}_{rot} = \frac{8}{5} m r^2 \pi^2 f_0 \dot{f} + \frac{8}{5} \pi^2 \dot{f}^2 \cdot t$

$\dot{E}_{rot} = \frac{8}{5} m r^2 \pi^2 \dot{f} (f_0 + \dot{f} t) \Rightarrow \dot{E}_{rot} = -1{,}99 \cdot 10^{25}\ J s^{-1} + 9{,}47 \cdot 10^{9}\ J s^{-2} \cdot t$

b) Wenn $\dot{E}_{rot}$ abhängig von t:

$E_{rot} = \frac{1}{2} I \omega^2$

$\dot{E}_{rot} \overset{!}{=} 0$

$0 = \frac{8}{5} m r^2 \pi^2 \dot{f} (f_0 + \dot{f} t)$

$\Leftrightarrow 0 = f_0 + \dot{f} t$

$\Leftrightarrow t = -\frac{f_0}{\dot{f}}$

$t \approx 2{,}1 \cdot 10^{15} s \approx 66{,}54$ mio. Jahre

Wenn $\dot{E}_{rot}$ konst.:

$\Rightarrow \dot{E}_{rot} = -1{,}99 \cdot 10^{25}\ J s^{-1}$

$E_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{5} m r^2 \omega^2$

$\Rightarrow 0 \overset{!}{=} E_{rot} + \dot{E}_{rot} \cdot t \Leftrightarrow -E_{rot} = \dot{E}_{rot} \cdot t \Leftrightarrow -\frac{E_{rot}}{\dot{E}_{rot}} = t$

$\Rightarrow t = -\frac{E_{rot}}{\dot{E}_{rot}} \approx 1{,}05 \cdot 10^{15} s \approx 33{,}27$ mio. Jahre

②

a) $E_{kin} + E_{rot} = E_{pot}$

$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = g \cdot m \cdot h$

$\frac{1}{2} m v^2 + \frac{1}{4} m r^2 \omega^2 = g m h$

$\frac{1}{2} v^2 + \frac{1}{4} r^2 \omega^2 = g \cdot h$

$$\frac{1}{2}mv^2+\frac{1}{4}mr^2\omega^2 = gmh$$

$$\frac{1}{2}v^2+\frac{1}{4}r^2\omega^2 = g\cdot h$$

$$\frac{1}{2}v^2+\frac{1}{4}v^2 = g\cdot h$$

$$\frac{3}{4}v^2 = g\cdot h$$

$$\frac{3}{4}\dot{s}^2 = g\cdot h$$

$$\frac{3}{4}\dot{s}^2\frac{d}{dt} = g\cdot h\frac{d}{dt}$$

$$\Leftrightarrow \quad \frac{3}{2}\dot{s}\ddot{s} = g\cdot h\frac{d}{dt}$$

$$\ddot{s} = \frac{2}{3}\frac{d(g\cdot h)}{dt}\cdot\dot{s}^{-1}$$

$$\ddot{s} = \frac{2}{3}g\frac{dh}{dt}\cdot\dot{s}^{-1}$$

$$\ddot{s} = \frac{2}{3}g\,\dot{s}\cdot\dot{s}^{-1}$$

$$\ddot{s} = \frac{2}{3}g$$

b)

$$t = \sqrt{\frac{h}{a}}$$

$$\Rightarrow t = \sqrt{\frac{3h}{2g}} \Rightarrow t = 0{,}55\,s$$

c)

$$v = a\cdot t$$

$$\omega = \frac{v}{r}$$

$$\Rightarrow \omega = \frac{a\cdot t}{r}$$

$$\Rightarrow \omega = \frac{2gt}{3r}$$

$$\Rightarrow \omega = 35{,}37\,s^{-1}$$

③ $r = 2m \quad L = 10m \quad \rho = 5 \frac{g}{cm^3} \quad \omega_0 = 50 \cdot \frac{1}{s}$

a) $I = \frac{1}{2} m r^2$

$E_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{2} m r^2 \omega_0^2 = \frac{1}{4} V \rho r^2 \omega_0^2 = \frac{1}{4} \rho L \cdot \pi \, r^4 \omega_0^2$

$= \frac{1}{4} \cdot 5 \frac{10^{-3} kg}{(\frac{1}{100} \cancel{m})^3} \cdot 10 \cancel{m} \cdot \pi \cdot 16 m^{\cancel{4}\,2} \cdot 50^2 \cdot \frac{1}{s^2}$

$\approx 1,57 \, GJ$

b) $P = \frac{1,57 \, GJ}{2s} = 0,785 \, GW = 785 MW$

c) $M_z = 4 \cdot 10^6 kg \qquad V_{zug} = 100 \frac{km}{h} = \frac{100}{3,6} \frac{m}{s}$

$E_{kin,z} = \frac{1}{2} M_z V_z^2 = \frac{1}{2} \cdot 4 \cdot 10^6 kg \left(\frac{100}{3,6} \frac{m}{s} \right)^2$

$W_z = \frac{E_{kin,z}}{2s} = \frac{1}{\cancel{2}s} \cdot \frac{1}{\cancel{2}} \cdot \cancel{4} \cdot 10^6 kg \left(\frac{100}{3,6} \frac{m}{s} \right)^2$

$W_z = 0,772 \, GW = 772 MW$

(1)

$$I = \int_m r^2 \, dm$$

$$= \rho \int_V r^2 \, dV$$

$$= \rho \int_0^h \int_0^{2\pi} \int_0^{r-\frac{r}{h}\cdot z} r'^2 \underbrace{r' \, dr' \, d\varphi \, dz}_{}$$

dV bei Zylinderkoordinaten

$$= \rho \int_0^h \int_0^{2\pi} \left[\frac{1}{4} r'^4 \right]_0^{r-\frac{r}{h}\cdot z} d\varphi \, dz$$

$$= \rho \int_0^h \int_0^{2\pi} \left(\frac{1}{4} \left(r - \frac{r}{h}\cdot z\right)^4 - 0 \right) d\varphi \, dz$$

$$= \frac{\rho}{4} \int_0^h \left(r - \frac{r}{h}\cdot z\right)^4 dz \int_0^{2\pi} d\varphi$$

$$= \frac{2\pi\rho}{4} \left[-\frac{h}{5r} \left(r - \frac{r}{h}\cdot z\right)^5 \right]_0^h$$

$$= \frac{\pi\rho}{2} \cdot \left(0 - \left(-\frac{h}{5r} \cdot r^5 \right) \right)$$

$$= \frac{\pi \rho h r^4}{10}$$

$$\boxed{\begin{array}{l} \text{NR:} \quad \frac{d}{dz} \frac{1}{5} \left(r - \frac{r}{h}\cdot z\right)^5 = \left(r - \frac{r}{h}\cdot z\right)^4 \cdot \left(-\frac{r}{h}\right) \\ \quad\quad\; \frac{d}{dz} \left(-\frac{h}{5r} \left(r - \frac{r}{h}\cdot z\right)^5 \right) = -\frac{h}{r} \left(r - \frac{r}{h}\cdot z\right)^4 \cdot \left(-\frac{r}{h}\right) = \left(r - \frac{r}{h}\cdot z\right)^4 \end{array}}$$