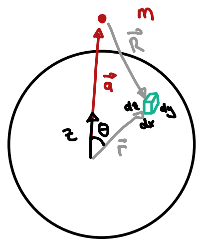
Gravitationsfeld ausgedehnter Körper:

Kugelschale:

$$dE_{pot} = -\int \frac{dM\, m}{R(\theta)} = -\int m \rho \frac{r^2 \sin\theta \, dr\, d\theta\, d\varphi}{R(\theta)}$$

Dicke

$R(\theta) = \sqrt{a^2 + r^2 - 2ar\cos\theta}$ (Kosinussatz)



Radius Kugelschale

$\frac{dR(\theta)}{d\theta} = \frac{ar\sin\theta}{R} \Rightarrow d\theta = dR \cdot R \cdot \frac{1}{ar\sin\theta}$

$\int \frac{\sin\theta}{R(\theta)} d\theta = \int \frac{1}{ar} dR = \frac{1}{ar} R$

$a > r$: $\quad dE_{pot} = -\int m \rho r^2 dr \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \frac{\sin\theta}{R(\theta)}$

$R(\theta=\pi) = \sqrt{a^2+r^2+2ar} = \sqrt{(a+r)^2} = |a+r|$

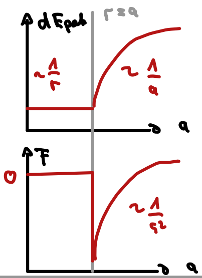
$R(\theta=0) = \sqrt{a^2+r^2-2ar} = \sqrt{(a-r)^2} = |a-r|$

$dM_{KS} = \rho 4\pi r^2 dr$

$= -\int m\, dM_{KS} \frac{1}{2ar} (|a+r| - |a-r|)$

$= \boxed{-\int \frac{m\, dM_{KS}}{a}} \Rightarrow F_z = -\frac{d}{da} dE_{pot} = -\int \frac{m\, dM_{KS}}{a^2}$

$r > a > 0$: $\quad dE_{pot} = \boxed{-\int \frac{m\, dM_{KS}}{r}} \Rightarrow F_z = -\frac{d}{da} dE_{pot} = 0$ (!)



Vollkugel: → Integration über Kugelvolumen

$a > R_0 > r$:

$M = \frac{4\pi}{3} R_0^3 \rho$

$$E_{pot} = -\int m \rho 4\pi \frac{1}{a} \int_0^{R_0} r^2 dr = -\int m \rho \frac{4\pi}{3a} R_0^3$$

$= \boxed{-\int \frac{mM}{a}} \rightarrow F_z = -\frac{d}{da} E_{pot} = -\int \frac{mM}{a^2}$

$dE_{pot} = -\int m\rho\, 4\pi r^2 dr \frac{1}{2ar} (|a+r| - |a-r|) = -\int m\rho\, 4\pi r^2 dr \frac{1}{a}$

$r < a < R_0$ & $a < r < R_0$:

$dE_{pot} = -\int m \rho 4\pi r\, dr$

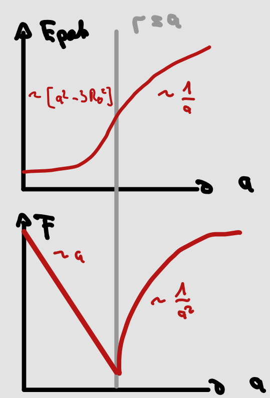
$$E_{pot} = -\int m \rho 4\pi \left[\int_0^a \frac{r^2}{a} dr + \int_a^R r\, dr \right] = -\int m \rho 4\pi \left[\frac{a^2}{3} + \frac{R_0^2}{2} - \frac{a^2}{2} \right]$$

$dE_{pot} = -\int m\rho\, 4\pi r^2 dr \frac{1}{a}$

$= \boxed{\int \frac{mM}{2R_0^3} [a^2 - 3R_0^2]}$

$\rightarrow F_z = -\int \frac{mM}{R_0^3} a$

Vorlesung 18



Mechanik starrer Körper:

Modell: System von MPs mit fixem $|\vec{r}_i - \vec{r}_j|$

↳ Reduktion der Bewegungsgl.

3 Koordinaten des SP

Winkelorientierung (3 Rotationswinkel)

Masse $M = \sum_{i=1}^{N} \Delta m_i$ Dichte $\rho = \frac{\Delta m}{\Delta V} \Rightarrow M = \sum_{i=1}^{N} \rho_i \Delta V_i$

Volumen $V = \sum_{i=1}^{N} \Delta V_i$

Grenzfall $N \to \infty$, $\Delta V_i \to 0$:

$V = \lim_{\substack{N\to\infty \\ \Delta V_i \to 0}} \sum_{i=1}^{N} \Delta V_i =: \int_V dV$

$M = \int_V \rho(x,y,z)\, dV =: \int_V dm$

Volumenintegral: dreifach-Integral

Bsp. Quader $V = \int_{z_1}^{z_2} \int_{y_1}^{y_2} \int_{x_1}^{x_2} dx\, dy\, dz$

Kugel → Polar-Koordinaten

Volumenelement $dV = dx\, dy\, dz$

$= r^2 \sin\theta\, dr\, d\theta\, d\varphi$

Achtung: Wahl Grenzen beschreiben den Körper → Zerlegen in einfache Koordinaten

Wahl der Koordinaten