physik 411: Physik IV

Sommer Semester 24

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Nonlinear Quantum Optics

Exercise sheet 2

To be handed in on Thursday April 25.

Deadline for this sheet is **Thursday at 10:00 o'clock**. The sheet should be handed in on ecampus in your tutor group.
The exercises suggested for Lehramt students are marked with a ★.

Do not hesitate to contact your tutors if you have questions!

★ Exercise 1: Millikan's oil-drop experiment — 10 points

In 1910 Robert Millikan and his student Harvey Fletcher were able to perform the first measurements of the elementary charge with the so-called oil-drop experiment (in German it is called the Millikan experiment). The experimental setup is sketched in figure 1. In the experiment tiny, charged oil droplets were sprayed into the gap between two capacitor plates. Millikan and Fletcher determined the elementary charge e by observing how quickly the droplets rose and fell between the plates.

Robert Millikan was awarded the Nobel prize in 1923 in part for his work on the elementary charge.

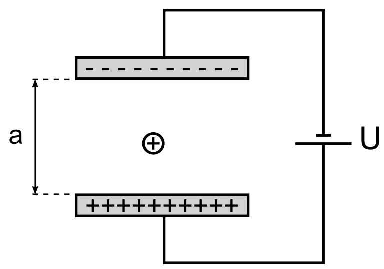

Figure 1: Sketch of the experimental set-up for the Millikan experiment.

a) Write down the equilibrium of forces such that a positively charged droplet is at rest in the capacitor when a voltage U is applied to the capacitor. Don't forget the buoyancy of air. What quantity would you need to measure to determine the charge of the droplet?

b) Assume that you turn off the voltage supply. The droplet will start to drop, and then falls with constant velocity. What are now the equilibrium forces? Don't forget the air resistance.

c) The drop is spherical, and it falls with constant velocity after you turned off the voltage supply. After 10 s it has dropped by 1.38 mm. Calculate the diameter of the droplet. The relevant densities are $\rho_{oil} = 0.8129\,\mathrm{g/cm^3}$, $\rho_{air} = 0.0013\,\mathrm{g/cm^3}$, and the viscosity of air is $\eta_{air} = 18.2\,\mu\mathrm{Pas}$.

d) To stop the droplet from falling, you turn back on the power supply and apply $U = 500\,\mathrm{V}$ to the capacitor. The plate distance is $1\,\mathrm{cm}$. Calculate the charge of the droplet in units of e.

Exercise 2: The Kibble-balance **12 points**

Figure 2 shows a sketch of a so-called Kibble-balance. The Kibble-balance can be used to measure the mass of an object, not by comparing one mass to another, well-known mass, but by compensating the weight of the object with an electromagnetic force.

The sketch shows how a coil (here with a single loop and length L) is placed in a magnetic field. In this case we will assume that the magnetic field B is constantly increasing in the radial direction. The test mass m is connected to the coil with a wire over a pulley.

Two experimental steps are necessary to weighing the test-mass and to calibrate the scale.

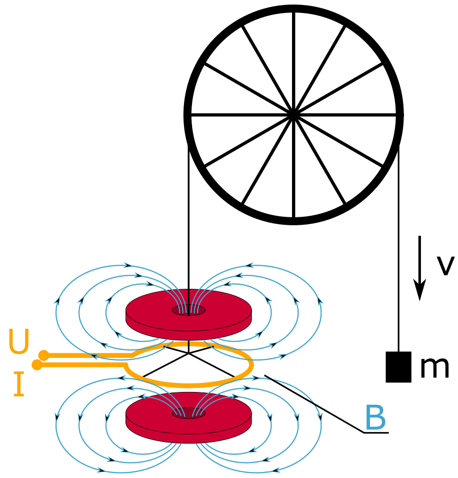

Figure 2: Sketch of a Kibble balance or Watt scale.

a) When the Kibble balance is used as a scale, a current runs through the coil to compensate the weight of the test-mass. Derive the relationship between current I, test-mass m, radial magnetic field strength B, and wire length of the coil loop L.

The experimental precision that can be obtained on the length of the coil L and the local magnetic field B limits the precision to which the weight of the test-mass can be measured. By calibrating the Kibble-balance, the weight measurements can be made independent of L and B.

b) To calibrate the kibble balance the coil is moved through the magnetic field with a constant, known velocity v. This motion induces a voltage U that can be measured. How are U, B, L and v related? Rewrite the expression you found in exercise a) to express m independently of B and L.

The precision of the measurement now relies on the determination of U, I, and v. v can be measured with laser-interferometry, and U and I can be expressed in terms of an exact measurable frequency and the Planck constant by measuring them with a volt- and ampere-meter based on Josephson junctions and the quantum Hall effect (you will learn more about this later in this course). This way, the mass m can be expressed in terms of the Planck constant.

Now we consider a real Kibble balance as it has been realized at the National Institute of Standards and Technology (NIST) in the US. The balance uses a $4\,\mathrm{kg}$ coil with a diameter of $43\,\mathrm{cm}$

on which roughly 1.4 km wire is wound. The coil is placed in a 0.55 T static magnetic field from a 1000 kg permanent magnet system. You can read more about the NIST kibble balance on the NIST webpage.

c) Assume that no mass is connected to the coil. What current is required to keep the coil levitated?

d) The current is measured on the Josephson junction and Quantum Hall-based ampere-meter, following the relation $I = U_{Josephson}/R_{Hall} = n_J \cdot n_H \cdot f \cdot e/2$. Here n_J and n_H are natural numbers that measure the discrete steps of the Josephson junction and the Hall effect. f is the frequency of the Josephson junction, and e is the elementary charge. If $f \approx 5\,\mathrm{GHz}$ and $n_J \sim n_H \sim 1$, in what range are the currents that are measured? What does that mean for the mass in our example?

A Kibble balance can even be build out of LEGO, see this YouTube video.

★ Exercise 3: Wave-particle duality with buckyballs — 10 points

On the previous exercise sheet we did quite some calculations of the de Broglie wavelength $\lambda_{\mathrm{dB}} = h/p$. As we have learned in the lecture, the de Broglie wavelength is not only a theoretical concept, but it describes the matter-wave related to a particle moving with a certain velocity. The matter-waves have all the physical properties of waves such as interference. Here we consider the interference pattern created when 'large' objects pass through a transmission grating.

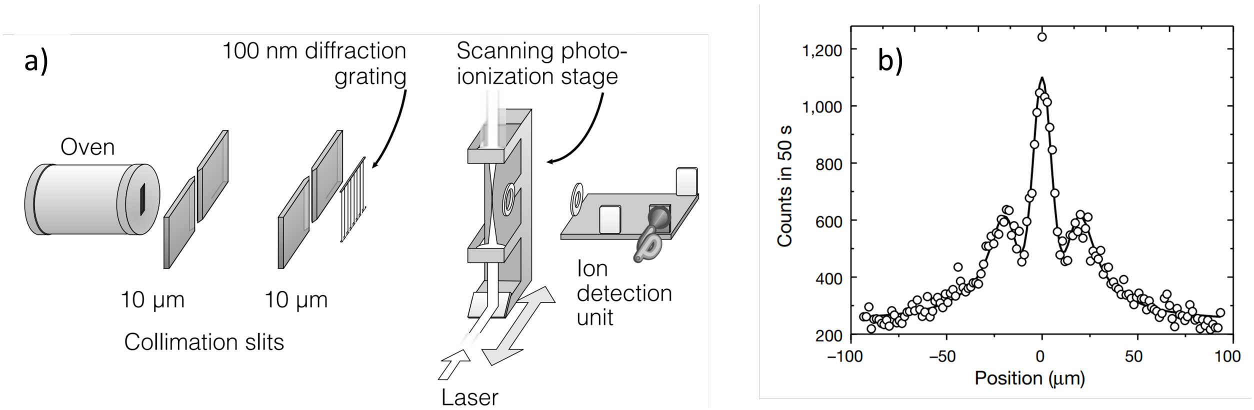

Figure 3: (a) Diagram of the experimental setup by Markus Arndt et al. (1999), and (b) resulting pattern from the interference of C_{60} molecules.

In 1999 Markus Arndt and colleagues performed an experiment (Nature 401, 680–682 (1999)) with molecular balls of 60 carbon atoms (C_{60}). These carbon molecules are called fullerenes and nick-named buckyballs.

The experimental setup is shown in figure 3 a). In the experiment the buckyballs were produced in an oven. The hot balls escape the oven as a molecular beam through an opening, pass through two additional slits and then through a transmission grating. Afterwards, the molecules are ionized and the ions are detected.

a) The width of the two slits in the experimental setup can be adjusted. What is the use for the two slits in the setup? Explain in one or two sentences.

b) Assume that a C_{60} fullerene molecule has velocity of 220 m/s. You can assume the mass to be 720 AMU. What is the de Broglie wavelength of this molecule? Compare the λ_{dB} to the size of the molecule. The matter-wave of the molecule is now incident on the transmission grating. Assume that the grating period is $a \approx 100\,\mathrm{nm}$. How large is the angle of the first order of diffraction? *Hint:* Use the grating formula from optics.

Figure 3 b) shows the actual interference pattern from the buckyballs. The interference was measured by position dependent ionization of the buckyball and subsequent detection of the ions. The ion count directly reflects the interference pattern. The ionization took place 1.25 m behind the grating.

c) Similar experiments have been performed with even larger molecules. What happens to the diffraction angle for heavier objects? What spacing between the zeroth and first order of diffraction would you expect to measure for the same experiment performed with C_{70}?

Exercise 4: Phase and group velocity of matter waves **8 points**

The one-dimensional Schrödinger equation for a free particle ($U = 0$) with mass m and fixed momentum p is given by

$$E\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \frac{\hat{p}^2}{2m}\Psi(x,t) \quad \text{with} \quad \hat{p} = -i\hbar\frac{\partial}{\partial x}.$$

a) Using the Ansatz $\Psi(x,t) = \exp i(kx - \omega t)$ solve the above equation. How do k and ω depend on E and p? What is the de Broglie wavelength λ_{dB} as a function of the particle momentum p? How large is the phase velocity v_{Ph} of a wave in general as a function of k and ω, and how large is it for the "matter wave" as a function of the classical particle velocity v?

b) Now superimpose two matter waves with minimally different wave numbers k and $k + \mathrm{d}k$ and correspondingly minimally different frequencies ω and $\omega + \mathrm{d}\omega$ (the linear superposition of two solutions of the Schrödinger equation is again a solution). Calculate and sketch the absolute value squared of this superposition. Show that the "wave packets" move with the speed $v_g = \mathrm{d}\omega/\mathrm{d}k$ (group velocity). How large is v_g for free particles as a function of the classical particle velocity v?

★ Exercise 5: Write on the wiki **≤ 8 bonus points**

It is possible to earn bonus points by contributing to he lecture script on the Wiki-page. To earn bonus points, you need to contribute to the 'Additional material' section on one of the **Physics 4** lecture subjects on the Wiki.

On ecampus you find a description of how you get access to the Wiki. Before you start editing, read rules for the entries on the rule page carefully.

When you hand in the exercise sheet, include a link to the page(s) and your username(s) such that we can verify your contribution through the page history. You can obtain up to 10 points *for this sheet*, but your work on the wiki has to be noted *on this hand-in to count.*

Note: The number of points you get for your contribution depends on the quality and originality of the material.

Note: We do not accept solutions to the course exercises on the Wiki.

Note: You need to provide references for what you write on the Wiki, *no matter what you are writing about.* Please read the rules on plagiarism very carefully.