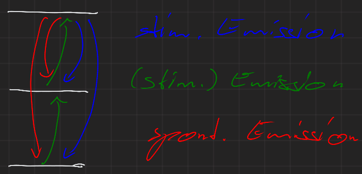
Ex4 Blatt 8 Angelo Brade, Noa Goetke, Jonas Wortmann 19.06.2024 23:54:00

Aufgabe 2:

a)



$N = N_e + N_l + N_g$

spont. Em. (stim.) Abs. stim. Em.

$$\dot N_g = A_{eg}N_e + A_{lg}N_l - P_{ge}N_g - P_{gl}N_g + P_{lg}N_l + P_{eg}N_e$$
$$\dot N_l = A_{el}N_e - A_{lg}N_l - P_{le}N_l + P_{gl}N_g - P_{lg}N_l + P_{el}N_e$$
$$\dot N_e = -A_{el}N_e - A_{eg}N_e + P_{le}N_l + P_{ge}N_g - P_{eg}N_e - P_{el}N_e$$

b) $P_{eg} = P_{el} = P_{le} = P_{ge} = 0$

$$\Rightarrow \begin{cases} \dot N_g = A_{eg}N_e + A_{lg}N_l - P_{ge}N_g - P_{gl}N_g + P_{lg}N_l \\ \dot N_l = A_{el}N_e - A_{lg}N_l - P_{lg}N_l \\ \dot N_e = -A_{el}N_e - A_{eg}N_e + P_{ge}N_g \end{cases}$$

c) $\dot N_g = \dot N_l = \dot N_e = 0$

$$0 \overset{!}{=} \dot N_g = A_{eg}N_e + A_{lg}N_l - P_{ge}N_g - P_{gl}N_g + P_{lg}N_l$$
$$\Rightarrow N_e = \frac{1}{A_{eg}}\left(P_{ge}N_g + P_{gl}N_g - P_{lg}N_l - A_{lg}N_l\right)$$
$$0 \overset{!}{=} \dot N_l = A_{el}N_e - A_{lg}N_l - P_{lg}N_l$$
$$\Rightarrow N_l = \frac{A_{el}}{P_{lg}+A_{lg}}\cdot\frac{1}{A_{eg}}\left(P_{ge}N_g + P_{gl}N_g - P_{lg}N_l - A_{lg}N_l\right)$$
$$= \frac{A_{el}}{P_{lg}+A_{lg}}\cdot\frac{N_g}{A_{eg}}(P_{ge}+P_{gl}) - \frac{A_{el}}{P_{lg}+A_{lg}}\cdot\frac{N_l}{A_{eg}}(P_{lg}+A_{lg})$$
$$= \frac{\frac{A_{el}}{P_{lg}+A_{lg}}\cdot\frac{N_g}{A_{eg}}(P_{ge}+P_{gl})}{1 + \frac{A_{el}}{P_{lg}+A_{lg}}\cdot\frac{1}{A_{eg}}(P_{lg}+A_{lg})}$$
$$= N_g \frac{(P_{ge}+P_{gl})}{\frac{A_{eg}(P_{lg}+A_{lg})}{A_{el}} + (P_{lg}+A_{lg})}$$
$$\Leftrightarrow \frac{N_l}{N_g} = \frac{(P_{ge}+P_{gl})}{\frac{A_{eg}(P_{lg}+A_{lg})}{A_{el}} + (P_{lg}+A_{lg})}$$

d)

$$\Rightarrow \frac{A_{eg}(P_{lg}+A_{lg})}{A_{el}} = (P_{ge}+P_{gl})\frac{N_g}{N_l} - P_{lg} - A_{lg}$$
$$\Rightarrow A_{eg} = \frac{A_{el}}{P_{lg}+A_{lg}}\left[(P_{ge}+P_{gl})\frac{N_g}{N_l} - P_{lg} - A_{lg}\right]$$
$$P_{eg} - A_{el} \gtrsim \frac{A_{el}}{P_{lg}+A_{lg}}\left[(P_{ge}+P_{gl})\frac{N_g}{N_l} - P_{lg} - A_{lg}\right]$$
$$\Rightarrow P_{eg} \gtrsim A_{el}\left[\frac{1}{P_{lg}+A_{lg}}\left[(P_{ge}+P_{gl})\frac{N_g}{N_l} - P_{lg} - A_{lg}\right] + 1\right]$$
$$\Rightarrow \left(1 - \frac{P_{eg}}{A_{el}}\right)(P_{lg}+A_{lg}) + P_{lg} - (P_{ge}+P_{gl})\frac{N_g}{N_l} \gtrsim A_{lg}$$
$$\Rightarrow \frac{N_g}{N_l} \gtrsim -\frac{A_{lg} + \left(\frac{P_{eg}}{A_{el}} - 1\right)(P_{lg}+A_{lg}) - P_{lg}}{(P_{ge}+P_{gl})}$$
$$\Rightarrow \frac{N_g}{N_l} \gtrsim \frac{P_{lg} - A_{lg} - \left(\frac{P_{eg}}{A_{el}} - 1\right)(P_{lg}+A_{lg})}{(P_{ge}+P_{gl})}$$

Die dargelegte Beziehung muss für Populationsinversion gelten. Das heißt, |l⟩ muss möglichst gut erreichbar von |e⟩ und |g⟩ ein.