The Multi-State Constraint Kalman Filter

Or, how we learned to stop worrying and love the null space

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AER1513 Course Project

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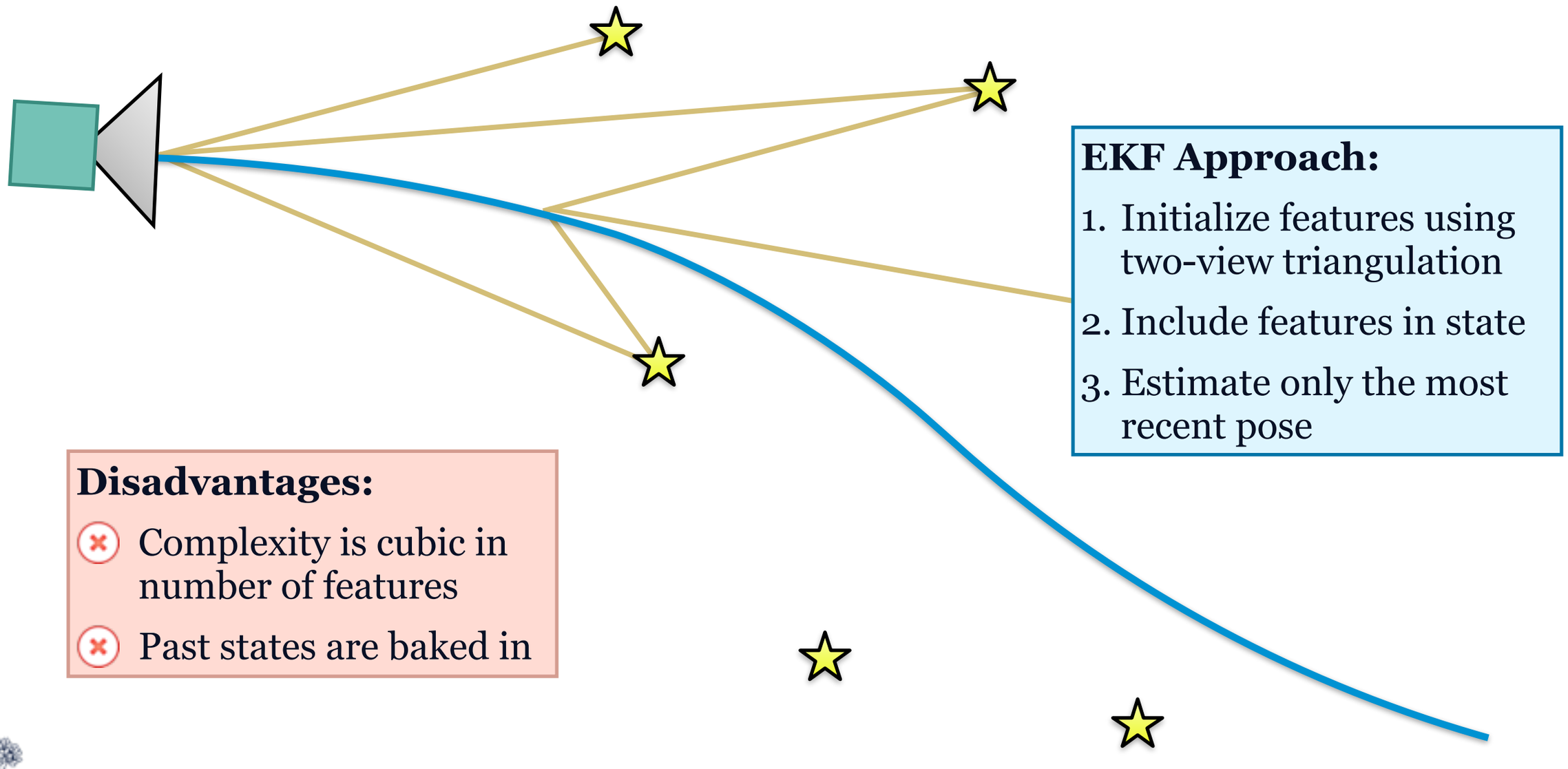
Problem: Mapless Navigation

Goal: Use an IMU with a monocular camera to estimate motion *without a map*.



EKF Approach:

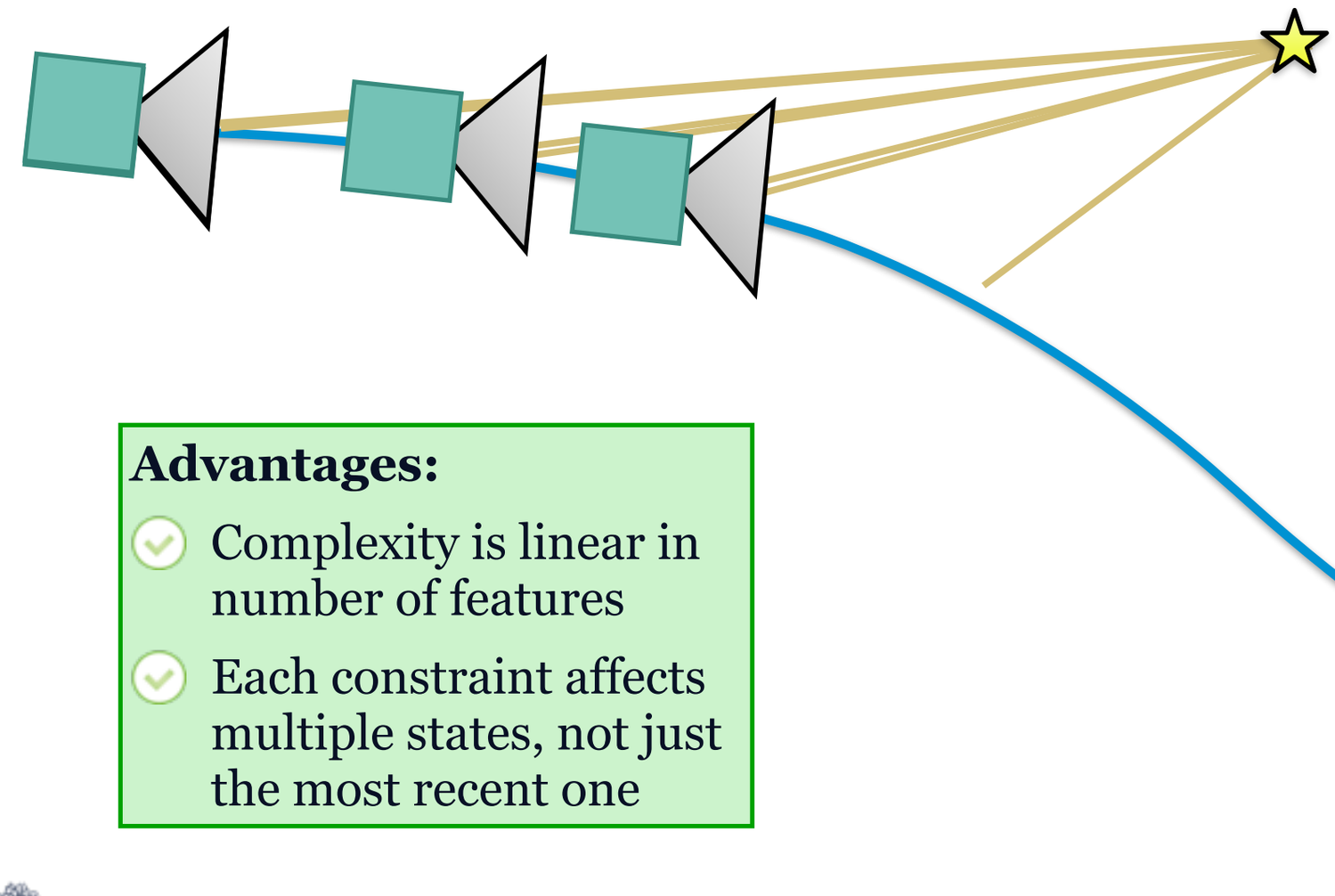
1. Initialize features using two-view triangulation
2. Include features in state
3. Estimate only the most recent pose

Disadvantages:

- Complexity is cubic in number of features
- Past states are baked in

Algorithm: Multi-State Constraint Kalman Filter

MSCKF dead-reckons the vehicle state using interoceptive (IMU) measurements, just like the EKF, but treats exteroceptive (camera) measurements differently.



Hybrid Approach:

1. **Batch component** estimates feature position using Gauss-Newton
2. **Filter component** uses combined observations as one exteroceptive update using null-space trick
3. Repeat for all feature tracks over a **window of poses**

Advantages:

- Complexity is linear in number of features
- Each constraint affects multiple states, not just the most recent one

MSCKF: Null Space Trick

What we want: $\mathbf{e}_{\text{ext}} = \mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \text{noise}$

What we've got: $\mathbf{e}_{\text{ext}} = \mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \mathbf{G}_{\mathbf{p}_f}\delta\mathbf{p}_f + \text{noise}$

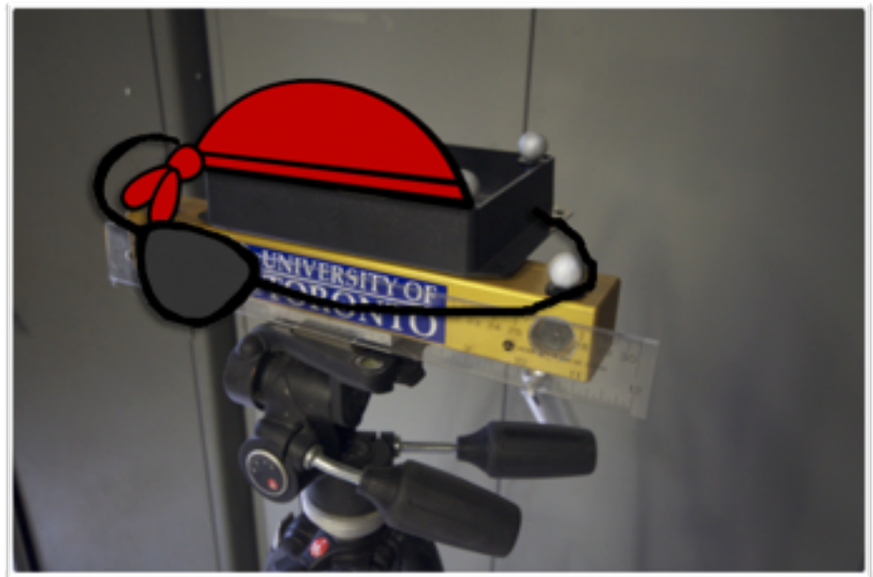
$$\mathbf{N} := \text{Null}\left(\mathbf{G}_{\mathbf{p}_f}\right)$$

$$\mathbf{N}^T\mathbf{e}_{\text{ext}} = \mathbf{N}^T\left(\mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \mathbf{G}_{\mathbf{p}_f}\delta\mathbf{p}_f + \text{noise}\right)$$

$$\mathbf{0} \quad \because \quad \left(\mathbf{N}^T\mathbf{G}_{\mathbf{p}_f} = \mathbf{0}\right)$$

$$\mathbf{e}'_{\text{ext}} := \mathbf{G}'_{\mathbf{x}}\delta\mathbf{x} + \text{noise}'$$

Dataset: Starry Night (Assignment 3)



- ☑ Perfect data association
- ☑ Ground truth for landmark positions
- ☑ Pre-integrated IMU measurements

Planned Analysis: MSCKF vs. Sliding Window

We will **investigate** MSCKF parameters:

1. Feature track length
2. Maximum window size

We will **compare**:

MSCKF *vs.* Sliding Window Batch Estimation *without a map*

Multi-**S**treet **C**omparison **K**ickboxing **F**ight

Proof of Progress

MSCKF.m
propagateImuState.m
propagateMsckfCova...
augmentState.m
updateState.m
calcF.m
calcG.m
calcJ.m
calcHoj.m
calcTH.m
calcGNPosEst.m
calcResidual.m
getGoodGrade.m
+

```
74
75      %==========================STATE PROPAGATION=======================%
76
77      %Propagate state and covariance
78 -    msckfState = propagateMsckfCovar(msckfState, measurements{state_k}, noiseParams);
79
80      %Add camera pose to msckfState
81 -    msckfState = augmentState(msckfState, camera);
82
83
84      %==========================FEATURE TRACKING========================%
85      % Add observations to the feature tracks, or initialize a new one
86      % If an observation is -1, add the track to featureTracksToResidualize
87 -    featureTracksToResidualize = {};
88 -    for featureId = 1:20
89 -        meas_k = measurements{state_k}.y(:, featureId);
90 -        if ismember(featureId, trackedFeatureIds)
91 -            if meas_k(1,1) == -1
92                  %Add to residualize queue and remove from the original
93                  %struct
94 -                featureTracksToResidualize{end+1} = featureTracks{trackedFeatureIds == featureId};
95 -                featureTracks = featureTracks(trackedFeatureIds ~= featureId);
96 -                trackedFeatureIds(trackedFeatureIds == featureId) = [];
97 -            else
98                  %Append observation and increase k2
99 -                featureTracks{trackedFeatureIds == featureId}.observations(:, end+1) = meas_k;
100 -                featureTracks{trackedFeatureIds == featureId}.k2 = featureTracks{trackedFeatureIds == featureId}.k2 + 1;
101 -            end
102 -        else
103              %Track new feature
104 -            track.featureId = featureId;
105 -            track.observations = meas_k;
106 -            track.k1 = state_k;
107 -            track.k2 = state_k;
108 -            featureTracks{end+1} = track;
109 -            trackedFeatureIds(end+1) = featureId;
110 -        end
111 -    end
112
113     %==========================FEATURE RESIDUAL CORRECTIONS=======================%
114
115 -    featuresToResidualize = []; %1xN matrix of feature ids (this is just the column of y_k_j)
```

Thanks!

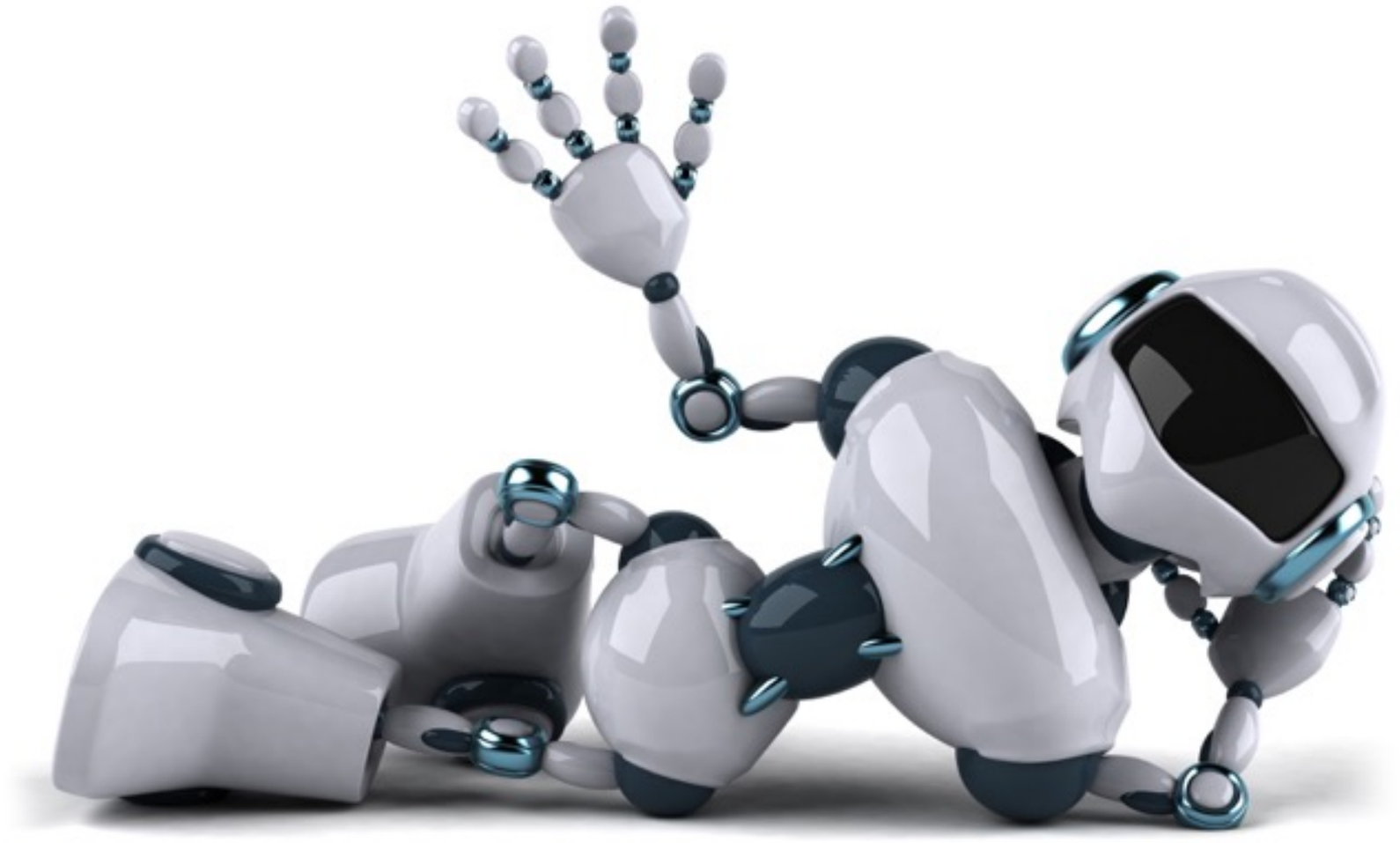


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Image credit: galleryhip.com

Extra Slides

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Algorithm: Multi-State Constraint Kalman Filter

Idea: Use a hybrid batch/recursive filter to incorporate all observations of a feature *without storing it in the state vector*.

Batch component: Track each feature until it goes out of view, then compute its position from *all available measurements* using Gauss-Newton optimization.

Recursive component: Use landmark position and all of its measurements (with null space trick) *to constrain motion*.

Advantages over plain EKF:

- **Sliding window of poses** allows each constraint to affect multiples states
- **Computational complexity is linear** in number of landmarks instead of cubic for plain EKF SLAM.