1 State Parametrization

IMU State at Time k (16×1)

$$
\mathbf{X}_{I_k} =
\begin{bmatrix}
\mathbf{q}_{I_k G} \\
\mathbf{b}_{g,k} \\
\mathbf{v}_k^G \\
\mathbf{b}_{a,k} \\
\mathbf{p}_k^G
\end{bmatrix}
\begin{array}{l}
\leftarrow \text{Global to IMU rotation unit quaterion} \\
\leftarrow \text{Gyro bias} \\
\leftarrow \text{IMU velocity in global frame} \\
\leftarrow \text{Accelerometer bias} \\
\leftarrow \text{IMU position in global frame}
\end{array}
\tag{1}
$$

IMU Error State at Time k (15×1)

$$
\widetilde{\mathbf{X}}_{I_k} =
\begin{bmatrix}
\widetilde{\boldsymbol{\theta}}_k^G \\
\widetilde{\mathbf{b}}_{g,k} \\
\widetilde{\mathbf{v}}_k^G \\
\widetilde{\mathbf{b}}_{a,k} \\
\widetilde{\mathbf{p}}_k^G
\end{bmatrix}
=
\begin{bmatrix}
\delta\boldsymbol{\theta}_I \\
\mathbf{b}_{g,k} - \hat{\mathbf{b}}_{g,k} \\
\mathbf{v}_k^G - \hat{\mathbf{v}}_k^G \\
\mathbf{b}_{a,k} - \hat{\mathbf{b}}_{a,k} \\
\mathbf{p}_k^G - \hat{\mathbf{p}}_k^G
\end{bmatrix}
\tag{2}
$$

where

$$
\delta\mathbf{q} = (\hat{\mathbf{q}}_{IG})^{-1} \otimes \mathbf{q}_{IG} \simeq
\begin{bmatrix}
\frac{1}{2}\delta\boldsymbol{\theta} \\
1
\end{bmatrix}
\tag{3}
$$

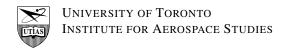**MSCKF State at Time k** $((16 + 7N) \times 1)$

$$
\hat{\mathbf{X}}_k =
\begin{bmatrix}
\hat{\mathbf{X}}_{I_k} \\
\hat{\mathbf{q}}_{C_1 G} \\
\hat{\mathbf{p}}_{C_1}^G \\
\vdots \\
\hat{\mathbf{q}}_{C_N G} \\
\hat{\mathbf{p}}_{C_N}^G
\end{bmatrix}
\begin{array}{l}
\leftarrow \text{IMU State estimate} \\
\leftarrow \text{Camera pose 1 orientation estimate} \\
\leftarrow \text{Camera pose 1 position estimate} \\
\vdots \\
\leftarrow \text{Camera pose N orientation estimate} \\
\leftarrow \text{Camera pose N position estimate}
\end{array}
\tag{4}
$$

MSCKF Error State $((15 + 6N) \times 1)$

$$
\widetilde{\mathbf{X}}_k =
\begin{bmatrix}
\widetilde{\mathbf{X}}_{I_k} \\
\delta\boldsymbol{\theta}_{C_1} \\
\widetilde{\mathbf{p}}_{C_1}^G \\
\vdots \\
\delta\boldsymbol{\theta}_{C_N} \\
\widetilde{\mathbf{p}}_{C_N}^G
\end{bmatrix}
\tag{5}
$$

2 IMU State Estimate Propagation

Integrate these with RK5 from t_{k-1} to $t_{k-1} + T$ where T is the sampling period of the IMU:

$$\dot{\hat{\mathbf{q}}}_{IG} = \frac{1}{2}\boldsymbol{\Omega}\left(\hat{\boldsymbol{\omega}}\right)\hat{\mathbf{q}}_{IG} \tag{6}$$

$$\dot{\hat{\mathbf{b}}}_g = \mathbf{0}_{3\times 1} \tag{7}$$

$$\dot{\hat{\mathbf{v}}}_I^G = \hat{\mathbf{C}}_{IG}^T\left(\mathbf{a}_m - \hat{\mathbf{b}}_a\right) - 2\left(\boldsymbol{\omega}_G^\times\right)\hat{\mathbf{v}}_I^G - \left(\boldsymbol{\omega}_G^\times\right)^2\hat{\mathbf{p}}_I^G + \mathbf{g}^G \tag{8}$$

$$\dot{\hat{\mathbf{b}}}_a = \mathbf{0}_{3\times 1} \tag{9}$$

$$\dot{\hat{\mathbf{p}}}_I^G = \hat{\mathbf{v}}_I^G \tag{10}$$

where

$$\hat{\boldsymbol{\omega}} = \boldsymbol{\omega}_m - \hat{\mathbf{b}}_g - \hat{\mathbf{C}}_{IG}\boldsymbol{\omega}_G, \tag{11}$$

$$\boldsymbol{\Omega}\left(\boldsymbol{\omega}\right) = \begin{bmatrix} -\boldsymbol{\omega}^\times & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix} \text{ is } 4\times 4, \tag{12}$$

$\hat{\mathbf{C}}_{IG} := \mathbf{C}(\hat{\mathbf{q}}_{IG})$ is the rotation matrix form of the quaternion $\hat{\mathbf{q}}_{IG}$,

$$\mathbf{F} = \begin{bmatrix} -\hat{\boldsymbol{\omega}}^\times & -\mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ -\hat{\mathbf{C}}_{IG}^T\hat{\mathbf{a}}^\times & \mathbf{0}_{3\times 3} & -2\boldsymbol{\omega}_G^\times & -\hat{\mathbf{C}}_{IG}^T & -\left(\boldsymbol{\omega}_G^\times\right)^2 \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \text{ is } 15\times 15, \tag{13}$$

$$\mathbf{G} = \begin{bmatrix} -\mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & -\hat{\mathbf{C}}_{IG}^T & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{1}_3 \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \text{ is } 15\times 12, \tag{14}$$

and

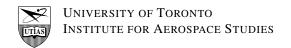$$\mathbf{n}_I = \begin{bmatrix} \mathbf{n}_g \\ \mathbf{n}_{wg} \\ \mathbf{n}_a \\ \mathbf{n}_{wa} \end{bmatrix} \begin{matrix} \leftarrow \text{ Gyro noise} \\ \leftarrow \text{ Gyro-bias rate-of-change noise} \\ \leftarrow \text{ Accelerometer noise} \\ \leftarrow \text{ Accelerometer-bias rate-of-change noise} \end{matrix} \tag{15}$$

is the system noise whose covariance matrix $\mathbf{Q}_I$ is computed offline during calibration. Note, $\boldsymbol{\omega}_G$ is the angular velocity of the spinning Earth in our global frame. The magnitude (i.e. rotational speed) is equal to $7.292 \times 10^5 \frac{\text{rad}}{s}$.

For the covariance,

$$\hat{\mathbf{P}}_k^- = \begin{bmatrix} \hat{\mathbf{P}}_{II,k}^- & \mathbf{\Phi}\left(t_{k-1} + T, t_{k-1}\right)\hat{\mathbf{P}}_{IC,k-1} \\ \hat{\mathbf{P}}_{IC,k-1}^T \mathbf{\Phi}\left(t_{k-1} + T, t_{k-1}\right)^T & \hat{\mathbf{P}}_{CC,k-1} \end{bmatrix} \text{ is } (15 + 6N) \times (15 + 6N) \tag{16}$$

where:

- $\hat{\mathbf{P}}_{CC,k-1}$ is the $6N \times 6N$ covariance matrix of the camera pose estimates (see section on State Augmentation),

- $\hat{\mathbf{P}}_{IC,k-1}$ is the correlation between the errors in the IMU state and the camera pose estimates,

- the state transition matrix $\mathbf{\Phi}\left(t_{k-1} + T, t_{k-1}\right)$ is computed by integrating (with RK5)

$$\dot{\mathbf{\Phi}}\left(t_{k-1} + \tau, t_{k-1}\right) = \mathbf{F}\mathbf{\Phi}\left(t_{k-1} + \tau, t_{k-1}\right), \qquad\qquad \tau \in [0, T] \tag{17}$$

with initial condition $\mathbf{\Phi}\left(t_{k-1}, t_{k-1}\right) = \mathbf{1}_{15}$, and

- $\hat{\mathbf{P}}_{II,k}^-$ is obtained by integrating (with RK5)

$$\dot{\hat{\mathbf{P}}}_{II}\left(t_{k-1}, t_{k-1} + \tau\right) = \mathbf{F}\hat{\mathbf{P}}_{II}\left(t_{k-1}, t_{k-1} + \tau\right) + \hat{\mathbf{P}}_{II}\left(t_{k-1}, t_{k-1} + \tau\right)\mathbf{F}^T + \mathbf{G}\mathbf{Q}_I\mathbf{G}^T, \qquad \tau \in [0, T] \tag{18}$$

with initial condition $\mathbf{P}_{II}\left(t_{k-1}, t_{k-1}\right) = \hat{\mathbf{P}}_{II,k-1}$.

3 State Augmentation

3.1 Camera Poses

For the $(N + 1)^{\text{th}}$ image, the camera pose is estimated as

$$\hat{\mathbf{q}}_{CG} = \mathbf{q}_{CI} \otimes \hat{\mathbf{q}}_{IG} \tag{19}$$

$$\hat{\mathbf{p}}_C^G = \hat{\mathbf{p}}_I^G + \hat{\mathbf{C}}_{IG}^T \, \mathbf{p}_C^I \tag{20}$$

and the EKF covariance matrix is augmented according to

$$\hat{\mathbf{P}}_{k-1} \leftarrow \begin{bmatrix} \mathbf{1}_{15+6N} \\ \mathbf{J} \end{bmatrix} \hat{\mathbf{P}}_{k-1} \begin{bmatrix} \mathbf{1}_{15+6N} \\ \mathbf{J} \end{bmatrix}^T \tag{21}$$

where the Jacobian

$$\mathbf{J} = \begin{bmatrix} \hat{\mathbf{C}}_{CI} & \mathbf{0}_{3\times9} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times6N} \\ \left(\hat{\mathbf{C}}_{IG}^T \, \mathbf{p}_C^I\right)^{\times} & \mathbf{0}_{3\times9} & \mathbf{1}_3 & \mathbf{0}_{3\times6N} \end{bmatrix} \text{ is } 6 \times (15 + 6N) \tag{22}$$

4 Correction Equations

Residuals (pretend everything has a k subscript...)

$$\mathbf{r}_n = \mathbf{T}_H \widetilde{\mathbf{X}} + \mathbf{n}_n \tag{23}$$

where $\mathbf{T}_H$ (an upper-triangular matrix) is obtained from the QR decomposition of $\mathbf{H}_o$ ($\mathbf{H_x}$ in the original tech report, but I think that's an error...)

$$\mathbf{H}_o = \begin{bmatrix} \mathbf{H}_o^{(1)} \\ \vdots \\ \mathbf{H}_o^{(L)} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \end{bmatrix} \begin{bmatrix} \mathbf{T}_H \\ \mathbf{0} \end{bmatrix}. \tag{24}$$

Each $\mathbf{H}_o^{(j)}$ is the projection of $\mathbf{H_x}^{(j)}$ onto the left nullspace of $\mathbf{H}_f^{(j)}$

T_H and $\mathbf{r}_n$ can be computed in $O(r^2 d)$ time using Givens rotations without having to form $\mathbf{Q}_1$ explicitly. [FIGURE OUT HOW TO DO THIS]

Each $\mathbf{H_x}^{(j)}$ is a stack of Jacobians $\mathbf{H}_{\mathbf{x}_i}^{(j)}$ of the ith measurement of feature j with respect to the state (only the entries corresponding to pose i are non-zero):

$$\mathbf{H}_{\mathbf{x}_i}^{(j)} = \begin{bmatrix} \mathbf{0}_{2\times 15} & \mathbf{0}_{2\times 6} & \dots & \mathbf{J}_i^{(j)} \left(\hat{\mathbf{X}}_{f_j}^{C_i}\right)^{\times} & -\mathbf{J}_i^{(j)}\hat{\mathbf{C}}_{C_i G} & \dots & \mathbf{0}_{2\times 6} \end{bmatrix} \text{ is } 2 \times (15 + 6N) \tag{25}$$

Each $\mathbf{H}_f^{(j)}$ is a stack of Jacobians $\mathbf{H}_{f_i}^{(j)}$ of the ith measurement of feature j with respect to the feature position:

$$\mathbf{H}_{f_i}^{(j)} = \mathbf{J}_i^{(j)}\hat{\mathbf{C}}_{C_i G} \text{ is } 2 \times 3 \tag{26}$$

In both equations,

$$\mathbf{J}_i^{(j)} = \frac{1}{\hat{Z}_j^{C_i}} \begin{bmatrix} 1 & 0 & -\frac{\hat{X}_j^{C_i}}{\hat{Z}_j^{C_i}} \\ 0 & 1 & -\frac{\hat{Y}_j^{C_i}}{\hat{Z}_j^{C_i}} \end{bmatrix} \tag{27}$$

where

$$\begin{bmatrix} \hat{X}_j^{C_i} \\ \hat{Y}_j^{C_i} \\ \hat{Z}_j^{C_i} \end{bmatrix} = \hat{\mathbf{C}}_{C_i G} \left(\hat{\mathbf{p}}_{f_j}^G - \hat{\mathbf{p}}_{C_i}^G \right) \tag{28}$$

are the 3D coordinates of feature j in the frame of image i.

$\mathbf{n}_n$ is a noise vector with covariance matrix

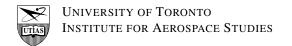$$\mathbf{R}_n = \sigma_{\text{im}}^2 \mathbf{1}_r \tag{29}$$

where r is the number of columns in $\mathbf{Q}_1$.

Kalman Gain

$$\mathbf{K}_k = \hat{\mathbf{P}}_k^- \mathbf{T}_{H,k}^T \left(\mathbf{T}_{H,k}\hat{\mathbf{P}}_k^- \mathbf{T}_{H,k}^T + \mathbf{R}_{n,k} \right)^{-1} \tag{30}$$

State Vector Correction

$$\Delta \mathbf{X}_k = \mathbf{K}_k \mathbf{r}_{n,k} \tag{31}$$

State Covariance Correction

$$\hat{\mathbf{P}}_k = \left(\mathbf{1}_{15+6N} - \mathbf{K}_k\mathbf{T}_{H,k}\right)\hat{\mathbf{P}}_k^-\left(\mathbf{1}_{15+6N} - \mathbf{K}_k\mathbf{T}_{H,k}\right)^T + \mathbf{K}_k\mathbf{R}_{n,k}\mathbf{K}_k^T \tag{32}$$

5 Feature Triangulation

We triangulate features using an inverse depth least squares Gauss-Newton optimization. The procedure takes as input N camera poses and N sets of ideal pixel measurements. We define *ideal* measurements as pixel measurements that have been corrected for the focal length and principal points of the camera:

$$u' = (u - c_u)/f_u \tag{33}$$
$$v' = (v - c_v)/f_v \tag{34}$$
$$\tag{35}$$

We calculate the 3D position of the feature in the first camera frame ($\mathbf{p}_f^{C_1}$) using a linear least squares method with measurements from the first two camera poses:

$$\mathbf{p}_f^{C_1} = \gamma\hat{\mathbf{p}}_f^{C_1} \tag{36}$$

where

$$\gamma = \left[(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{p}_{C_2}^{C_1}\right]_1, \quad \hat{\mathbf{p}}_f^C := \frac{1}{\sqrt{u'^2 + v'^2 + 1}}\begin{bmatrix}u'\\v'\\1\end{bmatrix}, \quad \mathbf{A} := \begin{bmatrix}\hat{\mathbf{p}}_f^{C_1} & -\hat{\mathbf{p}}_f^{C_2}\end{bmatrix} \tag{37}$$

Next, we define a vector of three parameters:

$$\mathbf{x} = \begin{bmatrix}\alpha\\\beta\\\rho\end{bmatrix}, \quad \alpha = \frac{x_f^{C_1}}{z_f^{C_1}}, \quad \beta = \frac{y_f^{C_1}}{z_f^{C_1}}, \quad \rho = \frac{1}{z_f^{C_1}} \tag{38}$$

The feature position in the ith camera frame can be expressed as:

$$\mathbf{p}_f^{C_i} = \mathbf{C}_{in}\mathbf{p}_f^{C_1} + \mathbf{p}_{C_1}^{C_i} \tag{39}$$

With our state parameters, we can use this equation to define three functions:

$$\begin{bmatrix}h_1(\alpha,\beta,\rho)\\h_2(\alpha,\beta,\rho)\\h_3(\alpha,\beta,\rho)\end{bmatrix} = \mathbf{C}_{in}\begin{bmatrix}\alpha\\\beta\\1\end{bmatrix} + \rho\mathbf{p}_{C_1}^{C_i} \tag{40}$$
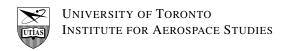
The camera measurement error can then be written as:

$$\mathbf{e} = \hat{\mathbf{z}} - \frac{1}{h_3}\begin{bmatrix}h_1\\h_2\end{bmatrix} \tag{41}$$

The least squares linear system then becomes:

$$(\mathbf{E}\mathbf{W}^{-1}\mathbf{E})\delta\mathbf{x}^* = -\mathbf{E}^T\mathbf{W}^{-1}\mathbf{e}(\bar{\mathbf{x}}) \tag{42}$$

where

$$\mathbf{E} := \frac{\partial\mathbf{e}}{\partial\mathbf{x}} \in \mathbb{R}^{2N\times3}, \quad \mathbf{W} := \begin{bmatrix}\mathbf{R}^{-1} & & \\ & \ddots & \\ & & \mathbf{R}^{-1}\end{bmatrix} \in \mathbb{R}^{2N\times2N} \tag{43}$$

6 AER 1513 Mods

6.1 State Parametrization

IMU State at Time k (13×1)

$$\mathbf{X}_{I_k} = \begin{bmatrix} \mathbf{q}_{I_k G} \\ \mathbf{b}_{g,k} \\ \mathbf{b}_{v,k} \\ \mathbf{p}_k^G \end{bmatrix} \begin{matrix} \leftarrow \text{Global to IMU rotation unit quaterion} \\ \leftarrow \text{Gyro bias} \\ \leftarrow \text{Velocity measurement bias} \\ \leftarrow \text{IMU position in global frame} \end{matrix} \tag{44}$$

IMU Error State at Time k (12×1)

$$\widetilde{\mathbf{X}}_{I_k} = \begin{bmatrix} \widetilde{\boldsymbol{\theta}}_k^G \\ \widetilde{\mathbf{b}}_{g,k} \\ \widetilde{\mathbf{b}}_{v,k} \\ \widetilde{\mathbf{p}}_k^G \end{bmatrix} = \begin{bmatrix} \delta\boldsymbol{\theta}_I \\ \mathbf{b}_{g,k} - \hat{\mathbf{b}}_{g,k} \\ \mathbf{b}_{v,k} - \hat{\mathbf{b}}_{v,k} \\ \mathbf{p}_k^G - \hat{\mathbf{p}}_k^G \end{bmatrix} \tag{45}$$

6.2 IMU State Propagation

$$\dot{\hat{\mathbf{q}}}_{IG} = \frac{1}{2}\boldsymbol{\Omega}\left(\hat{\boldsymbol{\omega}}\right)\hat{\mathbf{q}}_{IG} \tag{46}$$

$$\dot{\hat{\mathbf{b}}}_g = \mathbf{0}_{3\times 1} \tag{47}$$

$$\dot{\hat{\mathbf{b}}}_v = \mathbf{0}_{3\times 1} \tag{48}$$

$$\dot{\hat{\mathbf{p}}}_I^G = \hat{\mathbf{C}}_{IG}^T \left(\hat{\mathbf{v}}_I^G - \hat{\mathbf{b}}_v\right) \tag{49}$$

where

$$\hat{\boldsymbol{\omega}} = \boldsymbol{\omega}_m - \hat{\mathbf{b}}_g, \tag{50}$$

$$\mathbf{F} = \begin{bmatrix} -\hat{\boldsymbol{\omega}}^\times & -\mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & -\hat{\mathbf{C}}_{IG}^T \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \text{ is } 12 \times 12, \tag{51}$$

$$\mathbf{G} = \begin{bmatrix} -\mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_3 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{1}_3 \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \text{ is } 12 \times 12, \tag{52}$$

and

$$
\mathbf{n}_I = \begin{bmatrix} \mathbf{n}_g \\ \mathbf{n}_{wg} \\ \mathbf{n}_v \\ \mathbf{n}_{wv} \end{bmatrix} \quad
\begin{array}{l} \leftarrow \text{Gyro noise} \\ \leftarrow \text{Gyro-bias rate-of-change noise} \\ \leftarrow \text{Velocity measurement noise} \\ \leftarrow \text{Velocity bias rate-of-change noise} \end{array}
\tag{53}
$$

$\hat{\mathbf{P}}_k^-$ is $(12 + 6N) \times (12 + 6N)$.

6.3 State Augmentation

$$
\hat{\mathbf{P}}_{k-1} \leftarrow \begin{bmatrix} \mathbf{1}_{12+6N} \\ \mathbf{J} \end{bmatrix} \hat{\mathbf{P}}_{k-1} \begin{bmatrix} \mathbf{1}_{12+6N} \\ \mathbf{J} \end{bmatrix}^T
\tag{54}
$$

where the Jacobian (not quite sure if I took out the right columns)...

$$
\mathbf{J} = \begin{bmatrix} \hat{\mathbf{C}}_{CI} & \mathbf{0}_{3\times6} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times6N} \\ \left(\hat{\mathbf{C}}_{IG}^T \, \mathbf{p}_C^I\right)^\times & \mathbf{0}_{3\times6} & \mathbf{1}_3 & \mathbf{0}_{3\times6N} \end{bmatrix} \text{ is } 6 \times (12 + 6N)
\tag{55}
$$

6.4 Correction Equations

$$
\mathbf{H}_{\mathbf{x}_i}^{(j)} = \begin{bmatrix} \mathbf{0}_{2\times12} & \mathbf{0}_{2\times6} & \dots & \mathbf{J}_i^{(j)} \left(\hat{\mathbf{X}}_{f_j}^{C_i}\right)^\times & -\mathbf{J}_i^{(j)} \hat{\mathbf{C}}_{C_iG} & \dots & \mathbf{0}_{2\times6} \end{bmatrix} \text{ is } 2 \times (12 + 6N)
\tag{56}
$$