The Multi-State Constraint Kalman Filter

Or, how we learned to stop worrying and love the null space

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AER1513 Course Project

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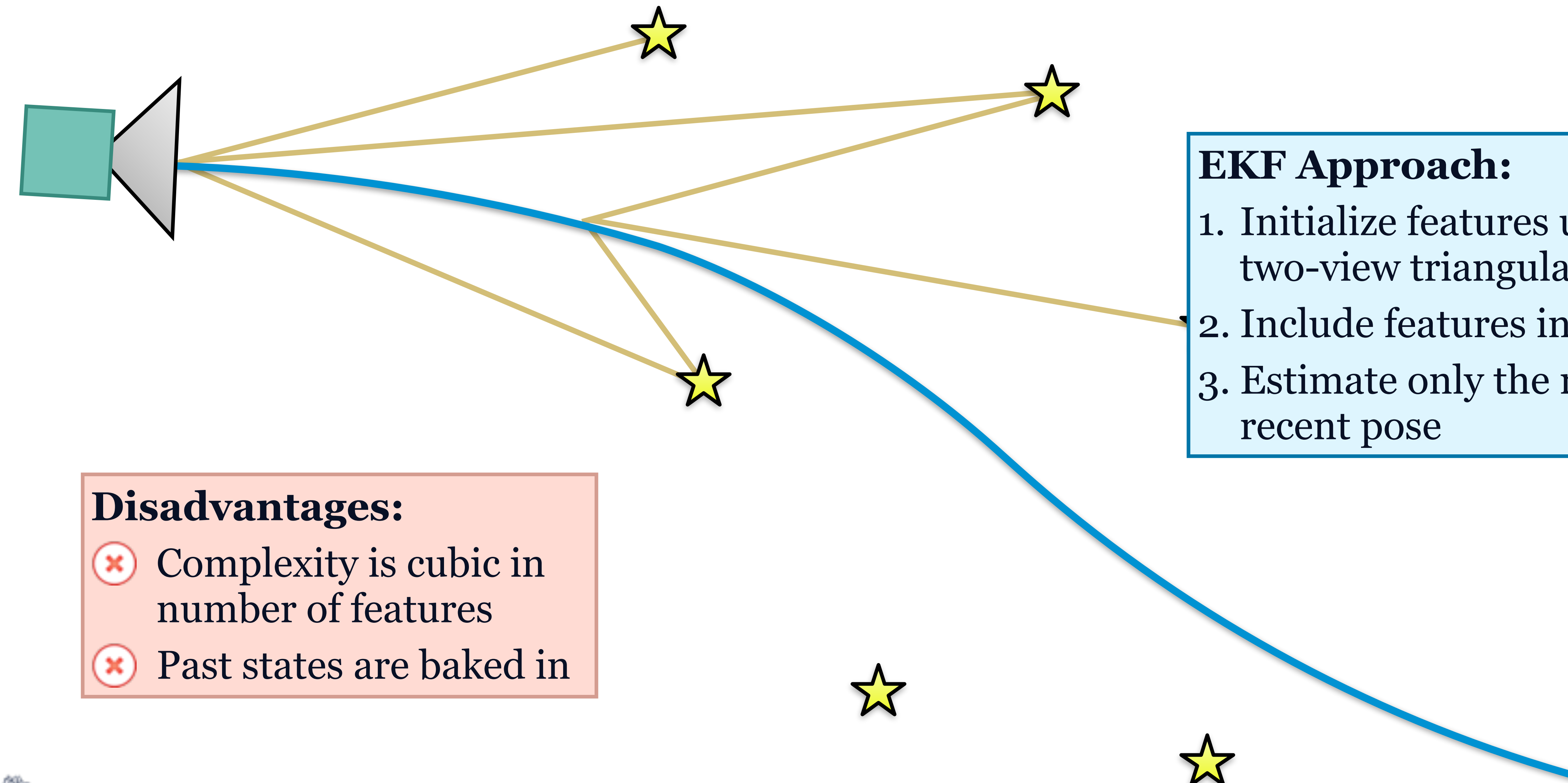
Problem: Mapless Navigation

Goal: Use an IMU with a monocular camera to estimate motion *without a map*.



EKF Approach:

1. Initialize features using two-view triangulation
2. Include features in state
3. Estimate only the most recent pose

Disadvantages:

- ⊗ Complexity is cubic in number of features
- ⊗ Past states are baked in

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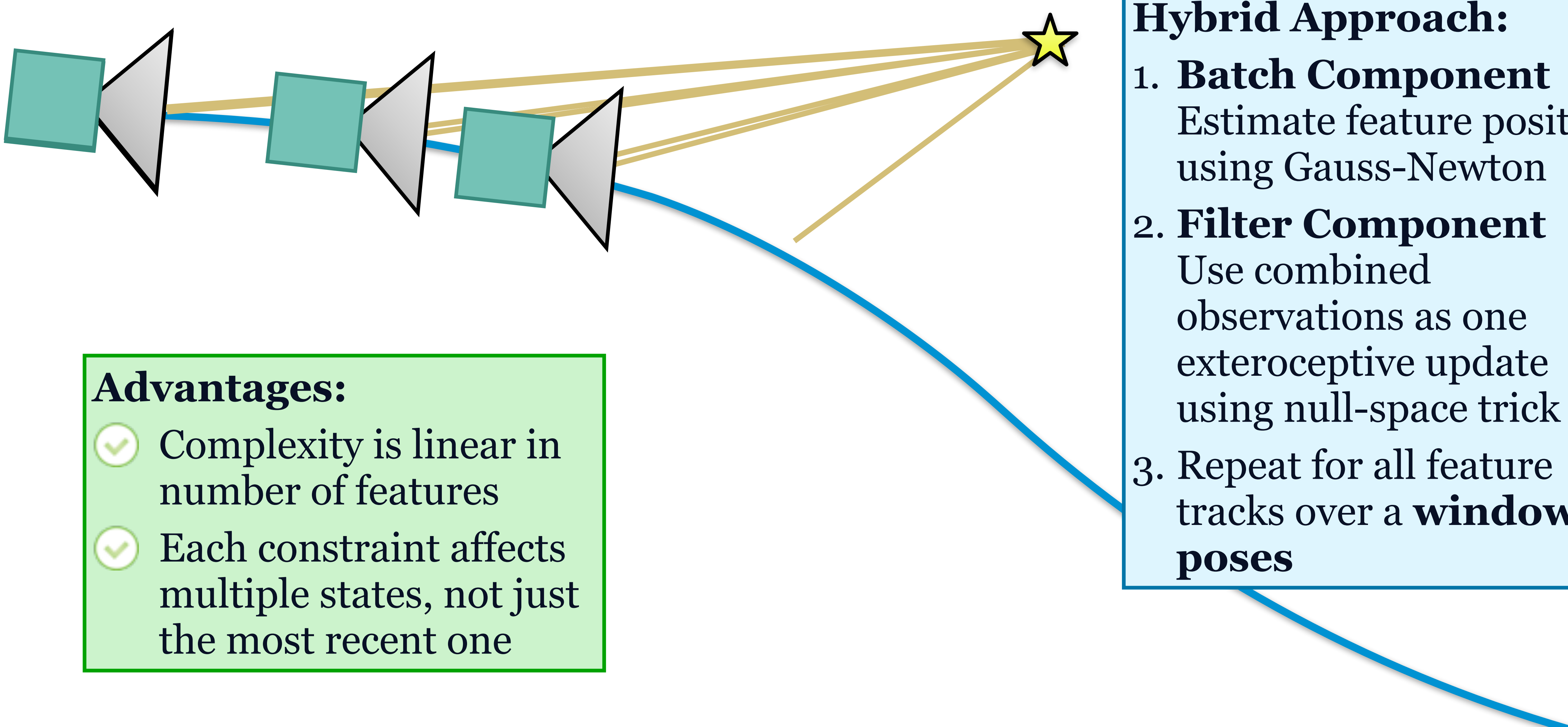
Algorithm: Multi-State Constraint Kalman Filter

MSCKF dead-reckons the vehicle state using interoceptive (IMU) measurements, just like the EKF, but treats exteroceptive (camera) measurements differently.



Hybrid Approach:

1. **Batch Component**
   Estimate feature position using Gauss-Newton
2. **Filter Component**
   Use combined observations as one exteroceptive update using null-space trick
3. Repeat for all feature tracks over a **window of poses**

Advantages:

- Complexity is linear in number of features
- Each constraint affects multiple states, not just the most recent one

MSCKF: Null Space Trick

What we want: $\mathbf{e}_{\text{ext}} = \mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \text{noise}$

What we've got: $\mathbf{e}_{\text{ext}} = \mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \mathbf{G}_{\mathbf{p}_f}\delta\mathbf{p}_f + \text{noise}$

$$\mathbf{N} := \text{Null}\left(\mathbf{G}_{\mathbf{p}_f}\right)$$

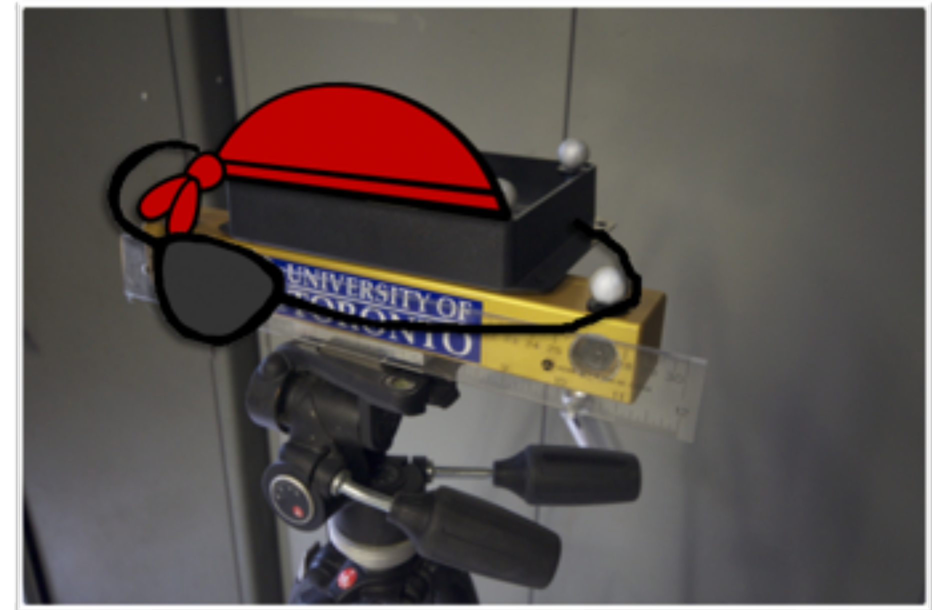
$$\mathbf{0} \quad \because \quad \left(\mathbf{N}^T\mathbf{G}_{\mathbf{p}_f} = \mathbf{0}\right)$$

$$\mathbf{N}^T\mathbf{e}_{\text{ext}} = \mathbf{N}^T\left(\mathbf{G}_{\mathbf{x}}\delta\mathbf{x} + \mathbf{G}_{\mathbf{p}_f}\delta\mathbf{p}_f + \text{noise}\right)$$

$$\mathbf{e}'_{\text{ext}} := \mathbf{G}'_{\mathbf{x}}\delta\mathbf{x} + \text{noise}'$$

Dataset: Starry Night (Assignment 3)

- ☑ Perfect data association
- ☑ Ground truth for landmark positions
- ☑ Pre-integrated IMU measurements



Planned Analysis: MSCKF vs. Sliding Window

We will **investigate** MSCKF parameters:

1. Feature track length
2. Maximum window size

We will **compare**:

MSCKF *vs.* Sliding Window
Batch Estimation *without a map*

Multi-**S**treet **C**omparison **K**ickboxing **F**ight

Proof of Progress

- MSCKF.m
- propagateImuState.m
- propagateMsckfCova...
- augmentState.m
- updateState.m
- calcF.m
- calcG.m
- calcJ.m
- calcHoj.m
- calcTH.m
- calcGNPosEst.m
- calcResidual.m
- getGoodGrade.m

```
%==========================STATE PROPAGATION========================%

%Propagate state and covariance
msckfState = propagateMsckfCovar(msckfState, measurements{state_k}, noiseParams);

%Add camera pose to msckfState
msckfState = augmentState(msckfState, camera);


%==========================FEATURE TRACKING========================%
% Add observations to the feature tracks, or initialize a new one
% If an observation is -1, add the track to featureTracksToResidualize
featureTracksToResidualize = {};
for featureId = 1:20
    meas_k = measurements{state_k}.y(:, featureId);
    if ismember(featureId, trackedFeatureIds)
        if meas_k(1,1) == -1
            %Add to residualize queue and remove from the original
            %struct
            featureTracksToResidualize{end+1} = featureTracks{trackedFeatureIds == featureId};
            featureTracks = featureTracks(trackedFeatureIds ~= featureId);
            trackedFeatureIds(trackedFeatureIds == featureId) = [];
        else
            %Append observation and increase k2
            featureTracks{trackedFeatureIds == featureId}.observations(:, end+1) = meas_k;
            featureTracks{trackedFeatureIds == featureId}.k2 = featureTracks{trackedFeatureIds == featureId}.k2 + 1;
        end
    else
        %Track new feature
        track.featureId = featureId;
        track.observations = meas_k;
        track.k1 = state_k;
        track.k2 = state_k;
        featureTracks{end+1} = track;
        trackedFeatureIds(end+1) = featureId;
    end
end

%==========================FEATURE RESIDUAL CORRECTIONS========================%

featuresToResidualize = []; %1xN matrix of feature ids (this is just the column of y_k_j)
```

Thanks!

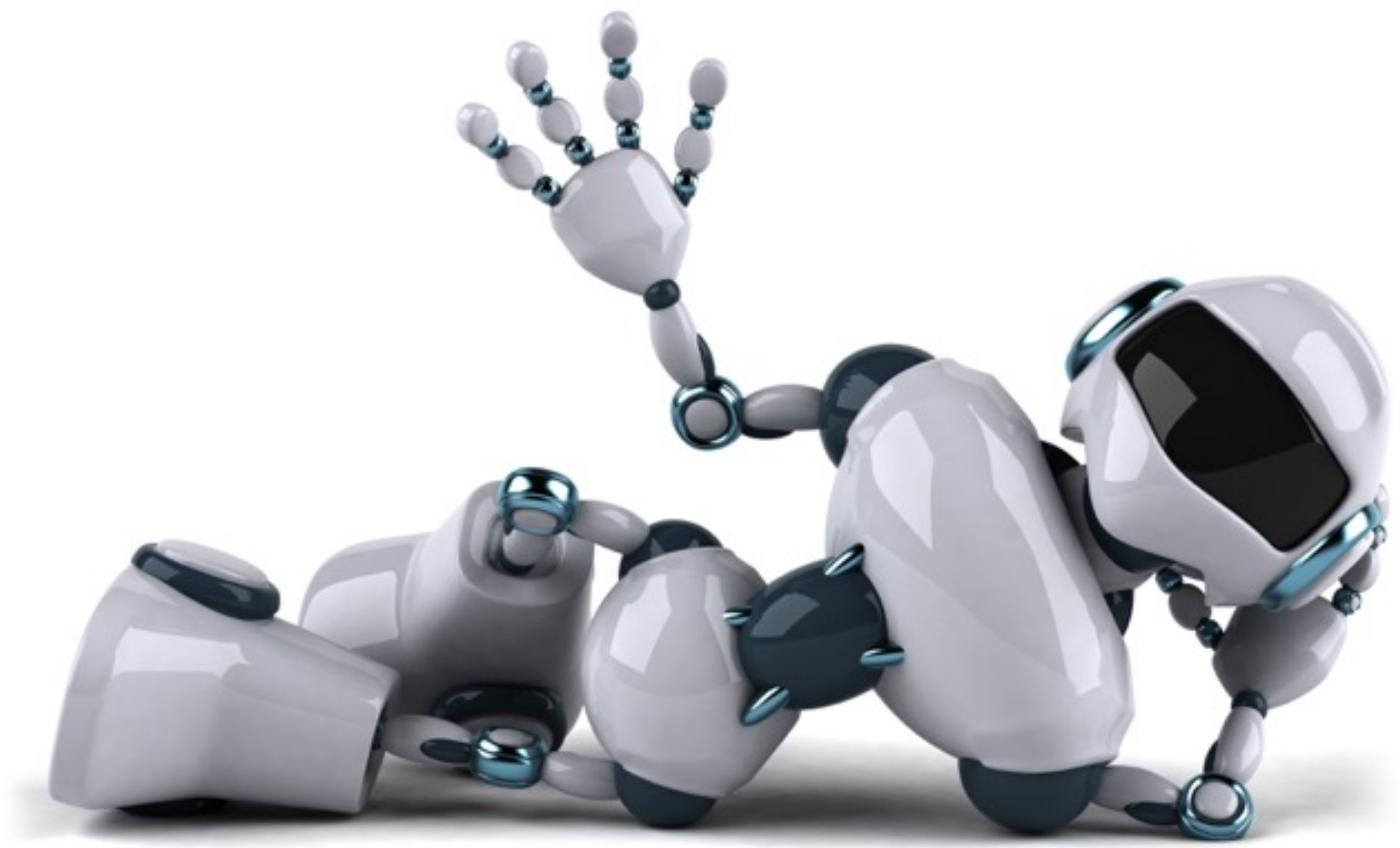


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Image credit: galleryhip.com

Extra Slides

Algorithm: Multi-State Constraint Kalman Filter

Idea: Use a hybrid batch/recursive filter to incorporate all observations of a feature *without storing it in the state vector*.

Batch component: Track each feature until it goes out of view, then compute its position from *all available measurements* using Gauss-Newton optimization.

Recursive component: Use landmark position and all of its measurements (with null space trick) *to constrain motion*.

Advantages over plain EKF:

- **Sliding window of poses** allows each constraint to affect multiples states
- **Computational complexity is linear** in number of landmarks instead of cubic for plain EKF SLAM.