

ON LEARNING PSEUDO-SENSORS TO IMPROVE VISUAL EGOMOTION ESTIMATION

by

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Abstract

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The ability to estimate *egomotion* is at the heart of safe and reliable mobile autonomy. By inferring pose changes from sequential sensor measurements, egomotion estimation forms the basis of mapping and navigation pipelines, and permits mobile robots to self-localize within environments where external localization information may be intermittent or unavailable. Visual egomotion estimation, also known as *visual odometry*, has become ubiquitous in mobile robotics due to the availability of high-quality, compact, and inexpensive cameras that capture rich representations of the world. To remain computationally tractable, ‘classical’ visual odometry pipelines make simplifying assumptions that, while permitting reliable operation in ideal conditions, often lead to systematic error. In this dissertation, we present several data-driven *pseudo-sensors* that serve to augment conventional pipelines by inferring latent information from sensor data. Our approach retains many of the benefits of traditional pipelines, while leveraging high-capacity hyper-parametric models to extract complementary information that can be used to improve uncertainty quantification, correct for systematic bias, and improve robustness to difficult-to-model deleterious effects. We validate our pseudo-sensors on several kilometres of sensor data collected in sundry settings such as urban roads, indoor labs, and planetary analogue sites in the Canadian High Arctic.

Epigraph

A little learning is a dangerous thing;
drink deep, or taste not the Pierian
spring: there shallow draughts
intoxicate the brain, and drinking
largely sobers us again.

ALEXANDER POPE

The universe is no narrow thing and the order within it is not constrained by any latitude in its conception to repeat what exists in one part in any other part. Even in this world more things exist without our knowledge than with it and the order in creation which you see is that which you have put there, like a string in a maze, so that you shall not lose your way. For existence has its own order and that no man's mind can compass, that mind itself being but a fact among others.

CORMAC McCARTHY

Elephants don't play chess.

RODNEY BROOKS

To all those who encouraged (or, at least, *never discouraged*) my intellectual wanderlust.

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Notation

- a : Symbols in this font are real scalars.
- \mathbf{a} : Symbols in this font are real column vectors.
- \mathbf{a} : Symbols in this font are real column vectors in homogeneous coordinates.
- \mathbf{A} : Symbols in this font are real matrices.
- $\mathcal{N}(\boldsymbol{\mu}, \mathbf{R})$: Normally distributed with mean $\boldsymbol{\mu}$ and covariance \mathbf{R} .
- $E[\cdot]$: The expectation operator.
- $\underline{\mathcal{F}}_a$: A reference frame in three dimensions.
- $(\cdot)^\wedge$: An operator associated with the Lie algebra for rotations and poses. It produces a matrix from a column vector.
- $(\cdot)^\vee$: The inverse operation of $(\cdot)^\wedge$.
- $\mathbf{1}$: The identity matrix.
- $\mathbf{0}$: The zero matrix.
- \mathbf{p}_a^{cb} : A vector (resp. homogenous coordinates) from point b to point c (denoted by the superscript) and expressed in $\underline{\mathcal{F}}_a$ (denoted by the subscript).
- \mathbf{C}_{ab} : The 3×3 rotation matrix that transforms vectors from $\underline{\mathcal{F}}_b$ to $\underline{\mathcal{F}}_a$: $\mathbf{p}_a^{cb} = \mathbf{C}_{ab}\mathbf{p}_b^{cb}$.
- \mathbf{T}_{ab} : The 4×4 transformation matrix that transforms homogeneous points from $\underline{\mathcal{F}}_b$ to $\underline{\mathcal{F}}_a$: $\mathbf{p}_a^{ca} = \mathbf{T}_{ab}\mathbf{p}_b^{cb}$.

Chapter 1

Introduction

To be sure, a writer cannot begin with a thesis; he must rather use his writerly sensitivity to intuit what is going on, even if he cannot understand its implications.

GARY MORSON, *How the great truth dawned*

In this dissertation, we present a general approach to improve visual egomotion estimation for mobile autonomous platforms. Such mobile *automata* have been a part of human culture since antiquity. In ancient Greek mythology, Hephaestus—the ‘patron of invention and technology’ (Mayor, 2019)—was said to have forged autonomous handmaidens that assisted him in his workshop. In ancient India, the king Ajatashatru was said to use *bhuta vahana yanta* (‘spirit movement machines’) to protect the relics of Gautama Buddha after his death in the fourth century BCE. According to Burmese legend, the *bhuta vahana yanta* of Ajatashatru were made with stolen secrets from a group of Greco-Roman ‘roboticists’ named the *yantakara*. The methods of the *yantakara* were closely guarded, and mechanical assassins were said to pursue those who attempted to disseminate them¹ (Mayor, 2019).

In the millennia since, mobile automata have been largely documented only in isolated demonstrations (e.g., the programmable cart of Hero of Alexandria or the ‘autonomous’ knight of Leonardo da Vinci) or imagined through cautionary tales in literature (e.g., Mary Shelley’s *Frankenstein*). Although the secrets of the *yantakara* may never be rediscovered, the ancient pursuit of helpful automata has found new life towards the turn of the twenty-first century and yielded machinery and algorithms that show great promise in aiding humanity.

¹Disseminate this document at your own risk.

1.1 Egomotion Estimation

One such algorithm is *egomotion* estimation: the process of computing changes in position and orientation of a moving platform from onboard measurements. In the literature, this process is also sometimes referred to as *self-localization* or *odometry*. The basic principle of egomotion estimation—the method of *dead reckoning*—has been used by marine navigators of ancient times to determine the longitude of a ship at sea. Unlike latitude, which navigators could determine by measuring the angle of the Sun with respect to the horizon at noon, or by measuring the altitude of Polaris (the north star) at night, the problem of how to compute longitude from celestial measurements was not solved until the 18th century with the development of the marine chronometer (Barfoot, 2017). As a result, early seafarers could only *dead reckon* east-west motion by considering the ship’s speed relative to water (measured in knots through a tool called a *chip log*, Figure 1.1) and accounting for estimates of local water currents and the ship’s magnetic heading.

In a similar process, early aviators computed egomotion through magnetic heading, airspeed, and an estimate of prevailing winds. While all dead-reckoning-based egomotion estimates exhibit unbounded error growth, these early methods were particularly inaccurate and required regular corrections through known landmarks.² In the mid twentieth century, the goals of intercontinental flight and space exploration necessitated the development of more accurate egomotion sensors (e.g., gimballed inertial sensors like accelerometers and gyroscopes) and an associated set of estimation techniques that could compute egomotion without human intervention (e.g., the Kalman filter (Grewal and Andrews, 2010)).

By the late twentieth century, unmanned Lunar and Martian exploration motivated the development of a new approach to egomotion estimation. Although ground vehicles like extra-

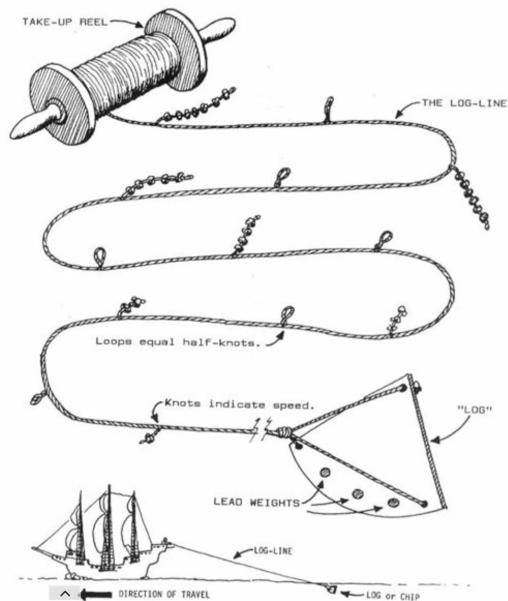


Figure 1.1: A *chip log* was a tool used to measure ship speed. The *log* or *chip* was tossed into the water, and speed was measured by the amount of *knots* that unravelled in a set time interval (credit: [oceano-motion.org](http://ocean-motion.org)).

²Perhaps the most famous example of dead-reckoning error was made by Christopher Columbus in 1492. He believed he had reached the *Indies* (modern Indonesia) but had really arrived in the Bahamas.

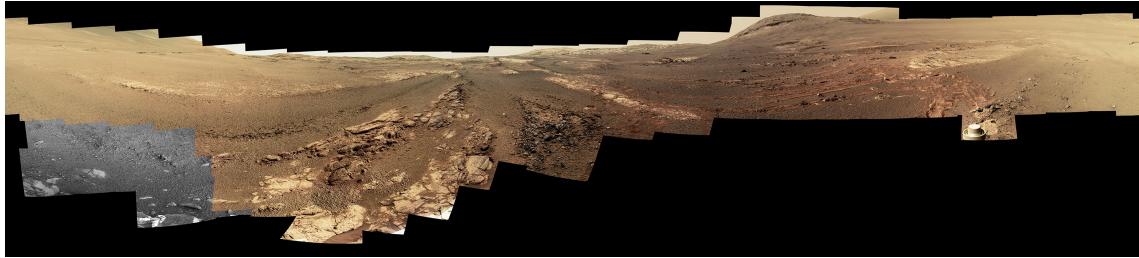


Figure 1.3: The last 360 degree panorama of the rocky Martian surface taken by the PanCam apparatus of the Mars Exploration Rover, *Opportunity*, at its final resting place, the western rim of the Endeavour Crater (*credit: NASA/JPL-Caltech/Cornell/ASU*).

planetary rovers could infer egomotion through *wheel odometry* (integrating wheel rates and using wheel orientation to drive a kinematic model), this approach was highly inaccurate on surfaces that induce wheel slip (e.g., the rock and sand covered surface of Mars, Figure 1.3). To address this, a number of researchers in the 1980s developed the technique of *visual odometry* (Scaramuzza and Fraundorfer, 2011) (or VO), as a way to infer egomotion from sequentially-collected images. The mathematical basis of VO was closely tied to the techniques of photogrammetric *bundle adjustment* (Triggs et al., 2000) and *structure from motion* (SfM) that were initially developed to automate the reconstruction of cold-war-era reconnaissance imagery.

In the nearly four decades since the first development of VO, the proliferation of compact, relatively-inexpensive, high resolution cameras has made vision-based sensing techniques ubiquitous in mobile autonomous applications. In addition to computing egomotion, camera data can be used to build detailed maps of an environment (which can be computed in tandem with egomotion through a technique called visual Simultaneous Localization and Mapping, or visual SLAM (Cadena et al., 2016)), as well as detect, track and avoid other objects (Figure 1.2).

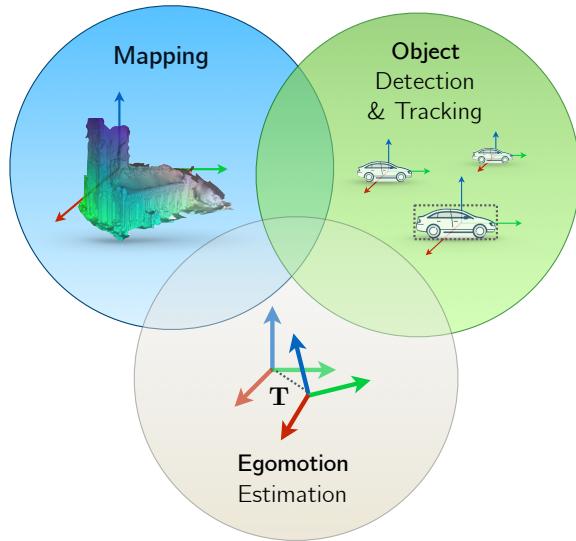


Figure 1.2: Visual egomotion estimation can be useful to create maps and to detect and track other objects.

1.2 A Visual Pipeline

Central to *classical* visual odometry algorithms (which, in this context, refers to the bulk of VO research published during what Cadena et al. (2016) call the *classical* and *algorithmic*-

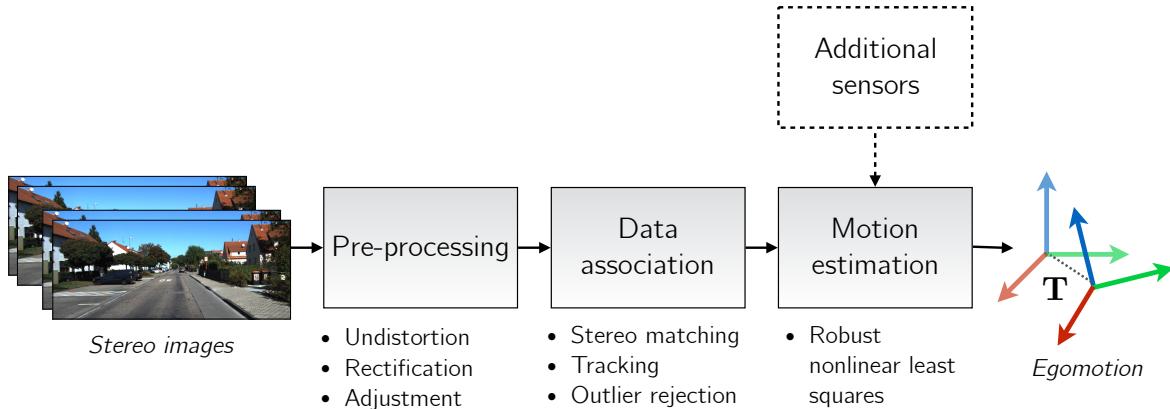


Figure 1.4: A ‘classical’ visual odometry pipeline consists of several distinct components with interpretable inputs and outputs. We list common examples of each component.

analysis ages of VO and SLAM research between 1986 and 2015) is the idea of a processing pipeline. A pipeline consists of several connected computational ‘blocks’ or ‘stages’ that have interpretable inputs and outputs. By carefully processing information contained within raw sensor data, pipelines facilitate the construction of complex state estimation architectures that can fuse visual observations with other sensors of varied modality to create maps and models of the external world and infer the egomotion of a mobile platform within it. In this dissertation, we will largely deal with the improvement of a canonical visual odometry pipeline— we illustrate its major components in Figure 1.4.

Broadly, VO solutions based on the idea of hand-crafted pipelines ([Leutenegger et al., 2015](#); [Cvišić and Petrović, 2015](#); [Tsotsos et al., 2015](#); [Alcantarilla and Woodford, 2016](#); [Forster et al., 2014](#)) have achieved impressive localization accuracy within a variety of settings. They have been used to compute the egomotion of self-driving cars through urban roads, and track autonomous drones through aggressive flying maneuvers.

Building on momentum from the computer vision community, a significant part of the visual state estimation literature has also considered replacing classical pipelines with parametric modelling through deep convolutional neural networks (CNNs) and data-driven learning. Although initially developed for image classification ([LeCun et al., 2015](#)), CNN-based measurement models have been applied to numerous problems in visual state estimation including homography estimation ([DeTone et al., 2016](#)), single image depth reconstruction ([Garg et al., 2016](#)), camera re-localization ([Kendall and Cipolla, 2016](#)), and place recognition ([Sünderhauf et al., 2015](#)). A number of recent CNN-based approaches have also tackled the problem of egomotion estimation, often purporting to obviate the need for classical visual localization pipelines by learning pose changes *end-to-end*, without requiring intermediate

outputs (Melekhov et al., 2017; Handa et al., 2016; Oliveira et al., 2017).

In light of these new approaches, debate has emerged within the robotics and computer vision communities regarding the extent to which data-driven parametric models should replace pipelines. Although classical approaches can achieve high accuracy under nominal conditions, deep data-driven networks have the potential to improve upon them in two respects.

First, owing to their representational power, deep parametric networks have the potential to learn bespoke data associations that are robust to (1) large viewpoint changes, (2) moving objects, and (3) self-similar visual textures (e.g., indoor walls, grass, sky, etc.). Such robust, invariant data associations have been the holy-grail of hand-crafted pipelines, and traditional data association methods must be carefully tuned to work well in a given setting.

Second, deep parametric regression of egomotion has the potential to more efficiently use dense high-dimensional visual data. To remain computationally tractable, classical VO pipelines typically take one of two approaches. First, some pipelines (Leutenegger et al., 2015; Cvišić and Petrović, 2015) choose to indirectly summarize visual data by extracting and matching a set of sparsely-distributed salient *features*. These features may be robust to some of the effects mentioned above, but, by construction, they discard large portions of visual data that may inform more accurate egomotion estimates. Alternatively, other methods (Forster et al., 2014) may instead choose to match individual pixels through an assumption of photometric consistency. This relatively simple schema permits dense data association but can be a poor assumption in the presence of large viewpoint changes or non-Lambertian surfaces. Further, this approach yields a high number of data associations and produces a highly non-convex objective that requires care to optimize.

In contrast, deep networks built on basic image operations with modern computationally-efficient implementations have the potential to avoid the pitfalls of both of these approaches while remaining tractable (assuming appropriate hardware). Despite this potential, current end-to-end learning techniques for egomotion estimation have a number of disadvantages (see Table 1.1). They often generalize poorly to new environments, come with few analytical guarantees, often provide only point estimates of latent parameters, and do not allow for intermediate representations that have been shown to improve generalization performance on visual tasks (Zhou et al., 2019). Indeed, according to one benchmark, the most accurate visual egomotion approach at the time of writing³ remains a classical pipeline based carefully selected sparse features.

³Based on the KITTI Odometry benchmark leaderboard at http://www.cvlibs.net/datasets/kitti/eval_odometry.php.

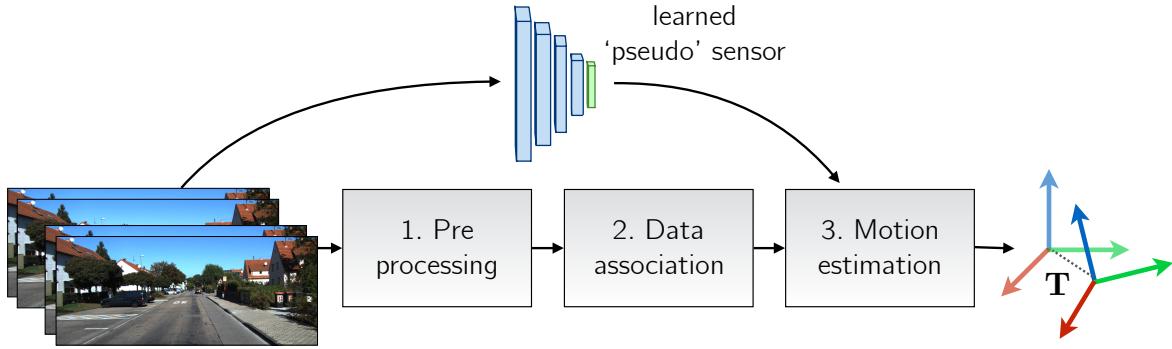


Figure 1.5: A learned *pseudo-sensor* extracts latent information from the same data stream.

Table 1.1: A comparison of pipelines and end-to-end deep models for visual egomotion estimation.

	Classical Pipelines	Deep Models
<i>Maturity</i>	Decades of literature & domain knowledge	Nascent with few uses in mobile autonomy
<i>Interpretability</i>	Good, each component has interpretable input and output	Poor, often with no interpretable intermediate outputs
<i>Uncertainty</i>	Foundational to <i>probabilistic robotics</i>	Few nascent methods (Monte-carlo Dropout (Gal and Ghahramani, 2016b), Bootstrap (Osband et al., 2016))
<i>Robustness</i>	Empirically generalizable (Zhou et al., 2019)	Highly dependant on training data
<i>Flexibility</i>	Limited by ingenuity of designer	Limited by training data

1.3 The Learned Pseudo-Sensor

As mobile autonomy enters the robust-perception age ([Cadena et al., 2016](#)), classical pipelines that work in limited contexts will need to be adapted and augmented to ensure they can operate over longer time-periods, and through challenging environments. However, replacing these performant pipelines completely with learned approaches, is in our view, unnecessary. Instead, we propose the paradigm of the learned *pseudo-sensor* (Figure 1.5). Learned pseudo-sensors leverage the representational power of modern data-driven learning techniques to extract useful quantities that make existing classical pipelines more consistent and accurate

in a given environment. In this dissertation, we present four such pseudo-sensors. In each case, the output of the pseudo-sensor is fused with a classical visual odometry pipeline to produce better motion estimates. To accomplish this fusion, we rely on two approaches. The first, which is used by Predictive Robust Estimation (PROBE, Chapter 4), uses the paradigm of a pseudo-sensor to build a heteroscedastic noise model from extant sensor data. By predicting uncertainty information, PROBE effectively re-scales a robust loss function to better account for deleterious visual effects. The second approach (used by Sun-BCNN, DPC-Net, and HydraNet, Chapters 5 to 7 respectively) produces geometric quantities (probabilistic estimates of an illumination source, SE(3) corrections to existing egomotion estimates, and independent probabilistic rotation estimates, respectively), that can be fused with the original egomotion estimate through pose graph optimization.

1.4 Original Contributions

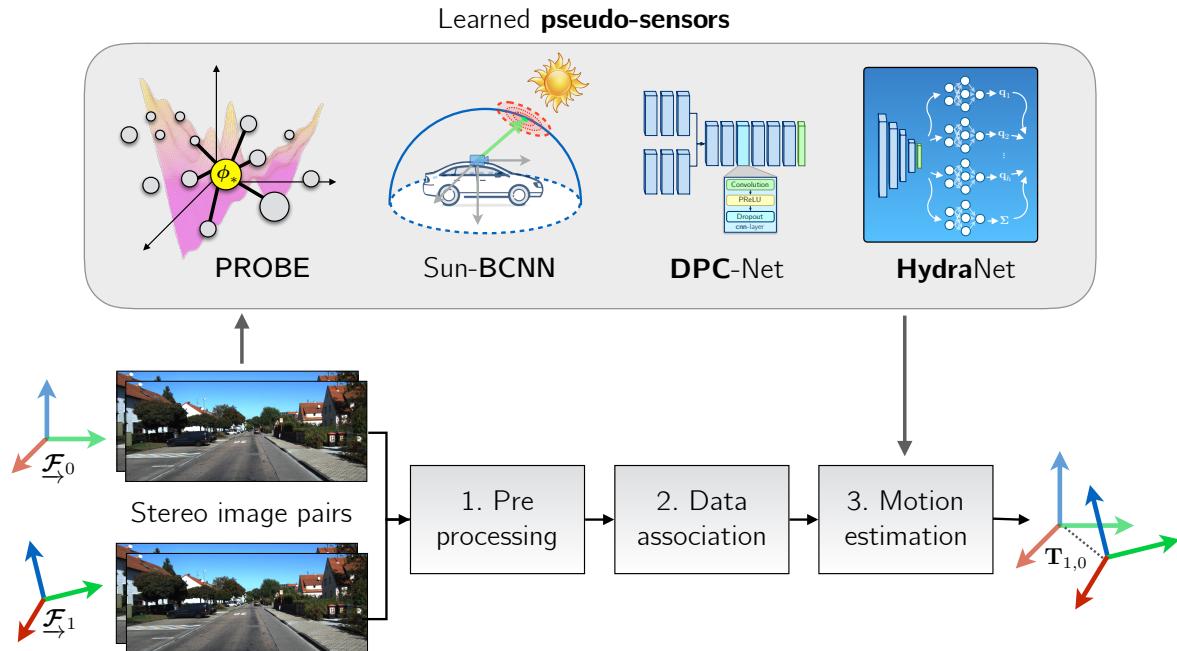


Figure 1.6: This dissertation details four examples of *pseudo-sensors* that improve 'classical' egomotion estimation through data-driven learning.

This dissertation consists of several published contributions under the umbrella of *learned pseudo-sensors* that improve a canonical visual egomotion pipeline. Before detailing each pseudo-sensor, we present some mathematical foundations (Chapter 2) and a common baseline for a feature-based stereo visual odometry pipeline (Chapter 3) which all four methods

build upon. In total, there are two journal papers and five conference papers associated with our work. Below, we briefly summarize each of the pseudo-sensors and list the publications that are associated with each.

1. Predictive Robust Estimation (PROBE),

Predictive Robust Estimation (Chapter 4, Appendix A) builds a predictive model for observation uncertainty from training data. To do this, we collaborate with William Vega-Brown at MIT to adapt the technique of Generalized Kernel (GK) estimation ([Vega-Brown et al., 2014](#)) to visual odometry. Generalized Kernel estimation allows us to build an efficient Bayesian model for stereo tracking uncertainty. By setting a prior on covariance, we derive a ‘robust’ objective that can be *predictively* scaled to improve the accuracy and consistency of a feature-based stereo visual odometry pipeline. PROBE is associated with three publications listed below. The first two publications (and Appendix A) explore useful predictors for uncertainty and build a non-Bayesian isotropic covariance model. The latter publication presents the Bayesian GK approach.

- Peretroukhin, V., Clement, L., Giamou, M., and Kelly, J. (2015a). PROBE: Predictive robust estimation for visual-inertial navigation. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS’15)*, pages 3668–3675, Hamburg, Germany,
- Peretroukhin, V., Clement, L., and Kelly, J. (2015b). Get to the point: Active covariance scaling for feature tracking through motion blur. In *Proceedings of the IEEE International Conference on Robotics and Automation Workshop on Scaling Up Active Perception*, Seattle, Washington, USA,
- Peretroukhin, V., Vega-Brown, W., Roy, N., and Kelly, J. (2016). PROBE-GK: Predictive robust estimation using generalized kernels. In *Proc. IEEE Int. Conf. Robot. Automat. (ICRA)*, pages 817–824.

2. Sun-BCNN: Learned sun sensor

Sun-BCNN (Chapter 5) is a virtual sun sensor based on a Bayesian Convolutional Neural Network (BCNN) that was developed in collaboration with Lee Clement. Much like a dedicated hardware sun sensor, Sun-BCNN infers a probabilistic estimate of the direction of the sun that can be used to inject orientation information into an egomotion pipeline. However, unlike dedicated sensors, Sun-BCNN requires no additional hardware and can predict both a mean and uncertainty from a single RGB image. It is associated with three publications listed below. The first publication consists of initial exploratory work on virtual sun sensors, while the second presents the BCNN formulation. The final publication is an extended journal article that summarizes the results

of the prior two conference publications and adds experiments from data collected in the Canadian High Arctic and around Oxford, UK, as well as investigating the effect of cloud cover and transfer learning.

- Clement, L., Peretroukhin, V., and Kelly, J. (2017). Improving the accuracy of stereo visual odometry using visual illumination estimation. In Kulic, D., Nakamura, Y., Khatib, O., and Venture, G., editors, *2016 International Symposium on Experimental Robotics*, volume 1 of *Springer Proceedings in Advanced Robotics*, pages 409–419. Springer International Publishing, Berlin Heidelberg. Invited to Journal Special Issue,
- Peretroukhin, V., Clement, L., and Kelly, J. (2017). Reducing drift in visual odometry by inferring sun direction using a bayesian convolutional neural network. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA’17)*, pages 2035–2042, Singapore,
- Peretroukhin, V., Clement, L., and Kelly, J. (2018). Inferring sun direction to improve visual odometry: A deep learning approach. *International Journal of Robotics Research*, 37(9):996–1016.

3. DPC-Net: Learned pose corrections

Deep Pose Correction (Chapter 6) is an approach to improving egomotion estimates through SE(3) pose corrections learned with deep networks. As part of this work, we derive a novel loss function based on Lie theory that permits the learning six degree-of-freedom pose residuals in a supervised learning framework. After training, our Deep Pose Correction Network (DPC-Net) predicts low-rate, ‘small’ *corrections* that can be fused with egomotion estimates from a canonical pipeline. DPC-Net does not require any modification to an existing pipeline, and can learn to correct multi-faceted errors from estimator bias, sensor mis-calibration or environmental effects. It is associated with one journal publication listed below.

- Peretroukhin, V. and Kelly, J. (2018). DPC-Net: Deep pose correction for visual localization. *IEEE Robotics and Automation Letters*, 3(3):2424–2431.

4. HydraNet: Learned probabilistic rotation estimation

Finally, HydraNet (Chapter 7) is a multi-headed network structure that can regress probabilistic estimates of rotation (elements of the matrix Lie group, SO(3)) that account for both aleatoric and epistemic uncertainty. HydraNet builds upon results from both Sun-BCNN and DPC that show that correcting rotation is critical to accurate ego-motion estimation. Towards this end, HydraNet is designed to produce well-calibrated

notions of uncertainty over $\text{SO}(3)$ that facilitate fusion with classical egomotion pipelines through a probabilistic factor graph formulation. It is associated with one publication:

- Peretroukhin, V., Wagstaff, B., and Kelly, J. (2019). Deep probabilistic regression of elements of $\text{SO}(3)$ using quaternion averaging and uncertainty injection. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR'19) Workshop on Uncertainty and Robustness in Deep Visual Learning*, pages 83–86, Long Beach, California, USA.

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