Task Sheet

The Complexity Class P

Proseminar Theoretische Informatik WiSe 2020-21
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1. **TIME Classes** *(5 pt.)*

   Given the definition of the TIME class, give equivalent definitions for the class SPACE. How can nondeterminism play a role at analyzing complexity?

2. **Problems in P** *(15 pt.)*

   (a) Research further problems in P. Is the problem PRIMES $\coloneqq \{ \langle x \rangle \mid x \text{ is prime} \}$ in P?

   (b) Prove whether the following problems are in P or SPACE (Check both).

   $$\text{H} \coloneqq \{ \langle M, w \rangle \mid M \text{ is a TM that will halt on the input } w \}$$

   $$\text{TREES} \coloneqq \{ \langle G \rangle \mid G \text{ is an undirected graph and a tree} \}$$

   $$\text{HOARE} \coloneqq \{ \langle w \rangle \mid w \text{ is a correctly functioning iterative program} \}$$

   $$\text{BINOMIALCOEFF} \coloneqq \left\{ \langle x \rangle \mid \text{There are natural numbers } n, k \text{ so that } x = \binom{n}{k} \right\}$$

   $$\text{LINES}_3 \coloneqq \{ \langle L_1, ..., L_k \rangle \mid L_1, ..., L_n \subset \mathbb{R}^3 \text{ are lines that pairwise intersect} \}$$

   (c) Update the landscape of languages according to your new knowledge.

3. **Reducibility in the Context of P** *(10 pt.)*

   Consider the following problem for any fixed $n \in \mathbb{N}$:

   $$\text{PLANES}_n \coloneqq \{ \langle P_1, ..., P_m \rangle \mid P_1, ..., P_m \subset \mathbb{R}^n \text{ are planes that pairwise intersect} \}$$

   Perform a reduction to another problem that seems suitable, so that you can show that:

   $$\text{PLANES}_n \in \text{P}$$