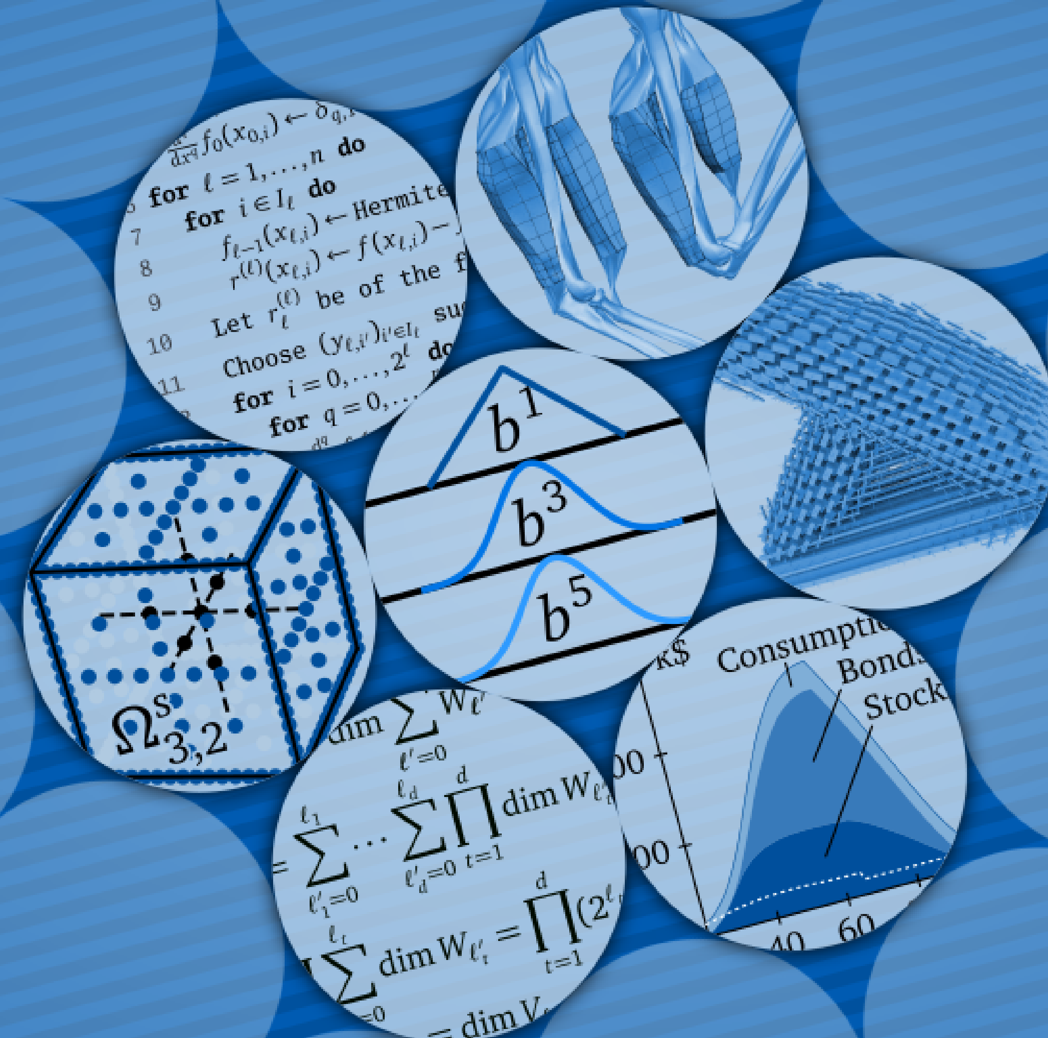

Julian Valentin

B-SPLINES FOR SPARSE GRIDS

Algorithms and Application to Higher-Dimensional Optimization

In simulation technology, computationally expensive objective functions are often replaced by cheap surrogates, which can be obtained by interpolation. Full grid interpolation methods suffer from the so-called *curse of dimensionality,* rendering them infeasible if the parameter domain of the function is higher-dimensional (four or more parameters). *Sparse grids* constitute a discretization method that drastically eases the curse, while the approximation quality deteriorates only insignificantly. However, conventional basis functions such as piecewise linear functions are not smooth (continuously differentiable). Hence, these basis functions are unsuitable for applications in which gradients are required. One example for such an application is gradient-based optimization, in which the availability of gradients greatly improves the speed of convergence and the accuracy of the results.

This thesis demonstrates that *hierarchical B-splines on sparse grids* are well-suited for obtaining smooth interpolants for higher dimensionalities. The thesis is organized in two main parts: In the first part, we derive new B-spline bases on sparse grids and study their implications on theory and algorithms. In the second part, we consider three real-world applications in optimization: topology optimization, biomechanical continuum-mechanics, and dynamic portfolio choice models in finance. The results reveal that the optimization problems of these applications can be solved accurately and efficiently with hierarchical B-splines on sparse grids.

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