Reinforcement Learning

Introduction

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Deep Learning with Tensorflow

January 18, 2018

Reinforcement Learning(RL) - Motivation

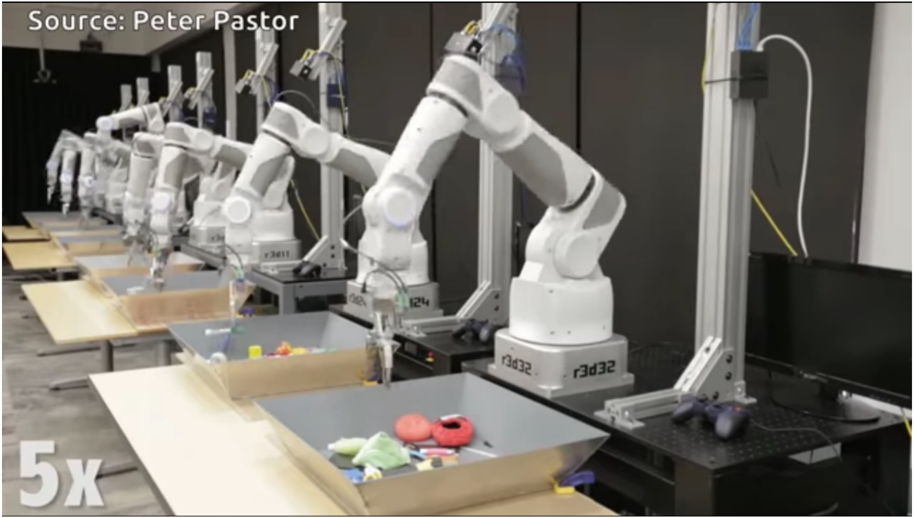

Figure: Application of Reinforcement Learning

What is Reinforcement Learning

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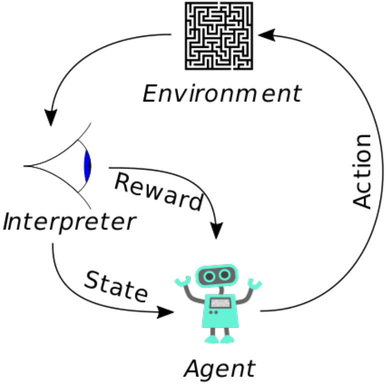
    - **Objective:** choose actions that **maximize future reward**



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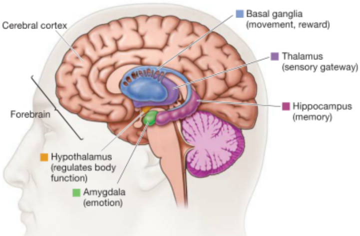
Neural implementation - dopamine dependent learning in the basal ganglia.



Markov decision process (MDP)

Reinforcement learning is defined as a MDP task.
The basic assumption is that:

Definition

A state s_t is Markov iff:

$$P[s_{t+1}|s_t] = P[s_{t+1}|s_1, ..., s_t]$$

This means, that the state captures all relevant information from history.

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Objective : maximizing expected reward, discounted with $\gamma \in [0, 1]$ for numerical stability reasons.

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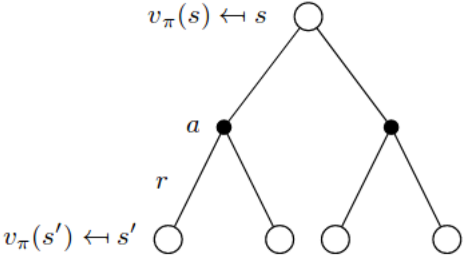

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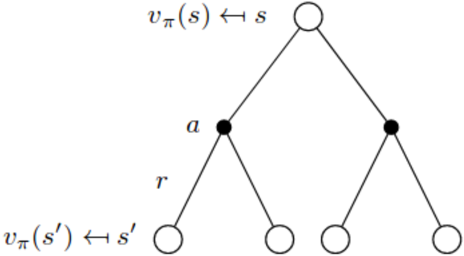

$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s)(\mathcal{R}(s,a) + \gamma \sum_{s' \in S} \mathcal{P}(s,s',a) v_\pi(s'))$$

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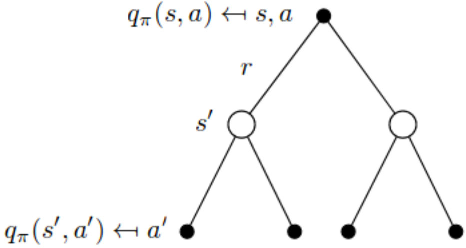

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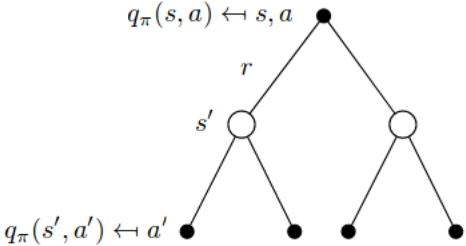

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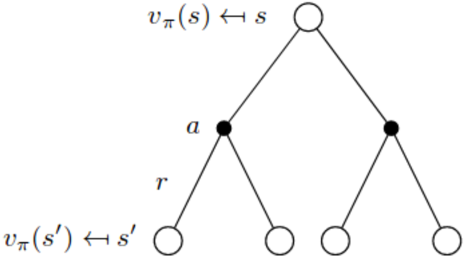

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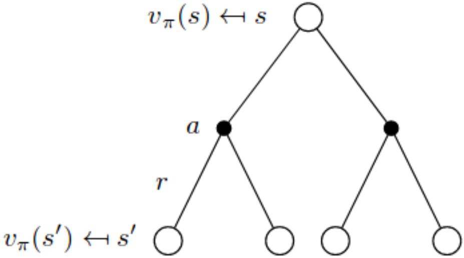

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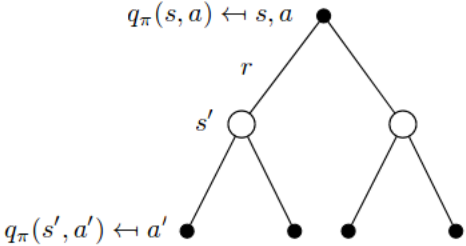

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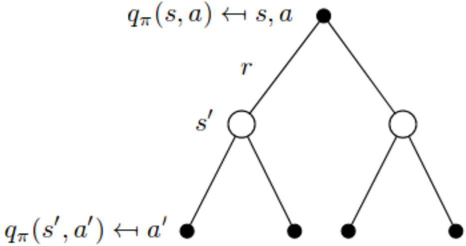

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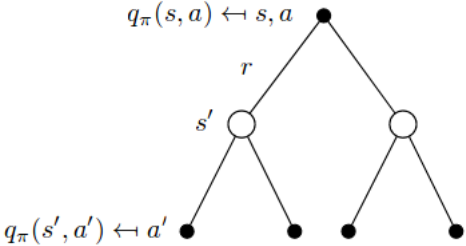

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Both can be solved by iterative algorithm:

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- If no value changes by more than some $\epsilon > 0$, we stop.

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And the Q-Learning is the approach to implement the *value iteration update* in this case, having the *learning rate* α:

$$q(s_t, a_t) = (1 - \alpha)q(s_t, a_t) + \alpha(r_t + \gamma max_{a \in \mathcal{A}} q(s_{t+1}, a))$$

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which together with the Banach fixed-point theorem gives us the desired statement.

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Model-free RL can solve these issues!

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- $$R = \begin{array}{c|cccccc} & \multicolumn{6}{c}{\textbf{Action}} \\ \textbf{State} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 0 & -1 & 100 \\ 2 & -1 & -1 & -1 & 0 & -1 & -1 \\ 3 & -1 & 0 & 0 & -1 & 0 & -1 \\ 4 & 0 & -1 & -1 & 0 & -1 & 100 \\ 5 & -1 & 0 & -1 & -1 & 0 & 100 \end{array}$$

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  (Action: columns 0–5)

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- $$Q = \begin{array}{c|c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{bmatrix} \end{array}$$

Q-learning example by hand

- $$R = \begin{array}{c|c} & \text{Action} \\ \text{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4\\5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 64 & 0 & 100 \\ 0 & 0 & 0 & 64 & 0 & 0 \\ 0 & 80 & 51 & 0 & 80 & 0 \\ 64 & 0 & 0 & 64 & 0 & 100 \\ 0 & 80 & 0 & 0 & 80 & 100 \end{bmatrix} \end{array}$$

Q-learning example by hand

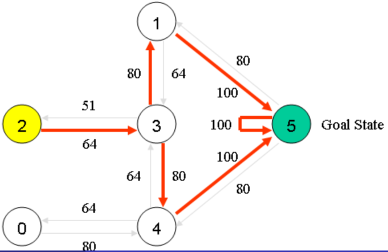
- $$R = \begin{array}{cc} & \text{Action} \\ \text{State} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{bmatrix} \end{array}$$

- $$Q = \begin{array}{cc} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 64 & 0 & 100 \\ 0 & 0 & 0 & 64 & 0 & 0 \\ 0 & 80 & 51 & 0 & 80 & 0 \\ 64 & 0 & 0 & 64 & 0 & 100 \\ 0 & 80 & 0 & 0 & 80 & 100 \end{bmatrix} \end{array}$$

- 

Q-learning - code example

```
import numpy.random as rnd
learning_rate0 = 0.05
learning_rate_decay = 0.1
n_iterations = 20000
s = 0 # start in state 0
Q = np.full((3, 3), -np.inf)
# -inf for impossible actions

for state, actions in enumerate(possible_actions):

    Q[state, actions] = 0.0
```

Q-learning - code example

```
for iteration in range(n_iterations):

    a = rnd.choice(possible_actions[s])
    # choose an action (randomly)
    sp = rnd.choice(range(3), p=T[s, a])
    # pick next state using T[s, a]
    reward = R[s, a, sp]
    learning_rate = learning_rate0 /\
    (1 + iteration * learning_rate_decay)
    print learning_rate
    Q[s, a] = learning_rate * Q[s, a] \
    + (1 - learning_rate) * ( reward + \
    discount_rate * np.max(Q[sp]) )
    s = sp # move to next state
```