MATH 340 - Lab Instructor: Valeria Barra

LAB 7 Assignment

DUE Tuesday 03-08-2016

Interpolation using Cubic Clamped Splines

We want to best approximate a given function $f(x)$ on an interval $[a, b]$. In class you have seen how cubic splines are derived. We will find a cubic spline interpolating polynomial $P_S(x)$, clamped at the boundary of the domain. This means that we enforce a particular value for the slope (first derivative) at the endpoints of the domain, namely v_l and v_r, respectively given by:

$$f'(a) = v_l \qquad f'(b) = v_r \tag{1}$$

Problem 1)

Interpolate the function $f(x) = \frac{1}{1+25x^2}$ on $[-1, 1]$ with 11 equi-spaced points, using the built-in Matlab `spline` function. To use this command, you want to first define the set of discrete data points `Xi`, find the corresponding y-values `Yi=f(Xi)`, and find the values v_l, v_r by differentiating the function $f(x)$. Use the `diff` command that we have learnt in Matlab, do not calculate this by hand. Find the coefficients for the splines via the command `cs = spline(Xi,[`v_l `Yi` v_r`])`. Then, define a domain of points, e. g. using `domain = linspace(a,b,101);`, and finally find the interpolating polynomial with the command `P=ppval(cs,domain);`

After you have found your interpolating polynomial via the cubic splines, $P_S(x)$, plot it against the domain, and in the same figure plot also the actual function $f(x)$ and the discrete points (x_i, y_i). Also, plot in the same figure the 10^{th}-degree polynomial $P_L(x)$ found with equi-spaced Lagrange interpolation, and $P_C(x)$ found with the Chebyshev nodes.

To compare the performance of all the three methods, calculate for each of them the error by

$$Err_S = \max_{x\in[a,b]} |f(x) - P_S(x)| \tag{2}$$

$$Err_L = \max_{x\in[a,b]} |f(x) - P_L(x)| \tag{3}$$

$$Err_C = \max_{x\in[a,b]} |f(x) - P_C(x)| \tag{4}$$

Rank the method performances from best to worst and comment on your results.