MATH 340 - Lab Instructor: Valeria Barra

LAB 3 Assignment

DUE Tuesday 02-09-2015

Fixed Point Iteration

A general fixed point iteration is a recursive scheme such as:

$$x_{n+1} = g(x_n) \qquad , \; n \geq 0\,.$$

This is not guaranteed to converge to a *fixed point* r, s. t. $r = g(r)$, unless some assumptions are satisfied.

You can visualize this through a "**cobweb**" graph of all iterations, drawing the lines joining the couples of points (x_n, x_n) [a point on the bisector line $y = x$] and $(x_n, g(x_n))$. See the figure (1) for an example of just a few iterations and corresponding lines.

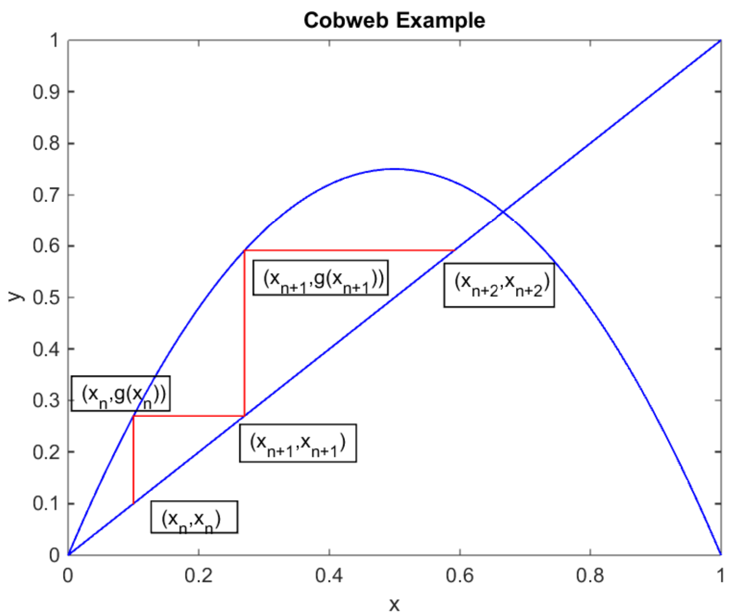

Figure 1: Example of a cobweb graph for the logistic function.

Assuming we don't know the actual fixed point r, we can approximate the error by $E_n = |x_{n+1} - x_n|$.

Problem 1:

Write your own program that computes the fixed point of a given function. Check whether your program converges to a fixed point or if it diverges. The convergence criterion for your program is given by the condition $|x_{n+1} - x_n| < tol$, where tol is a given tolerance. In case it diverges, it will run up to a maximum number N of iterations. Print out a message in either case to say how many times your program has run, and what solution has found. Use your program to solve the following:

(1.1) Do problem 17 of your homework by using your code. Use an initial guess $x_0 = 1$, a tolerance of 10^{-6}, and a maximum number of steps $N = 50$.

(1.2) Solve the fixed point iteration $g(x_n) = (2+(x_n - 2)^2)$, up to a tolerance 10^{-6} or maximum number of iterations $N = 50$, with **(a)** $x_0 = 2.8$, **(b)** $x_0 = 3.1$. Which execution is better? And why?

(1.3) Solve the fixed point iteration $g(x_n) = (2+(x_n - 2)^3)$, up to a tolerance 10^{-6} or maximum number of iterations $N = 50$, with **(a)** $x_0 = 2.8$, **(b)** $x_0 = 3.1$. Which execution is better? And why?

Extra Credit (+4):

For better visualization of the iterations, plot a cobweb graph in your program.

Root Finding: Newton's Method

This method uses a recursive formula to find the approximated solutions, called iterates, of an equation of the type $f(x) = 0$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad , n = 0, 1, 2, \ldots \qquad \text{for } f'(x_n) \neq 0 \tag{1}$$

In this case too, we consider an estimate for the error to be the difference between to consecutive iterates $Err_n = |x_{n+1} - x_n|$. This method is not guaranteed to converge if the initial guess x_0 is not close enough to the root r we want to find.

Problem 2

Implement your own Newton's method to solve Problems 19 and 21 in your Homework, with $N = 50$. Compare your results and the performance of your code with the one obtained through the bisection method in the Lab 2 Problems 1 and 3, respectively.