MATH 340 - Lab Instructor: Valeria Barra

LAB 12 Assignment

DUE Tuesday 04-19-2016

ODEs:

Runge-Kutta (R-K2) also called Explicit Trapezoidal Rule

Recalling the notation from your textbook, we discretize the time interval $[t_0, t_f]$ with a grid of $n+1$ points $t_0 < t_1 < \ldots < t_n$ with equal step size h (or in other words $t_i = t_0 + ih, \; i = 0, 1, \ldots, n$). We seek the discrete approximated solution to an initial value problem, say $w(t_n)$ (it can be also denoted by the equivalent notations $w_h(t_n)$ or w_n). Runge-Kutta of order two, is given by the following set of equations:

$$w_{i+1} = w_i + \frac{h}{2}(v_1 + v_2) \tag{1}$$

where

$$\begin{aligned} v_1 &= f(t_i, w_i) \\ v_2 &= f(t_i + h, w_i + hv_1) \end{aligned}$$

with initial condition given by $w_0 = y_0$.

Runge-Kutta (R-K4)

Runge-Kutta method of order four is given by the following set of equations:

$$w_{i+1} = w_i + \frac{h}{6}(v_1 + 2v_2 + 2v_3 + v_4) \tag{2}$$

where

$$
\begin{aligned}
v_1 &= f(t_i, w_i) \\
v_2 &= f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}v_1\right) \\
v_3 &= f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}v_2\right) \\
v_4 &= f(t_i + h, w_i + hv_3)
\end{aligned}
$$

with initial condition given by $w_0 = y_0$.

<u>**Problem 1**</u>:

Implement your own R-K2 and R-K4 methods in `MATLAB`. Use them to solve the same initial value problem seen last week

$$
\begin{cases} y' &= ty + t^3 \\ y(0) &= 1 \end{cases}
$$

Print your results, for each method, in a table like the one made last week, with columns: t_i (where the values in each row are $t_i = 0, 0.2, 0.4, 0.6, 0.8, 1.0$), the approximated solutions $w_{h=0.2}(t_i)$, $w_{h=0.1}(t_i)$, $w_{h=0.05}(t_i)$, the actual solution y_i and the error $e_i = |y(t_i) - w_{h=0.05}(t_i)|$. To compute the error at each grid point use the value for the actual solution given $y(t) = 3e^{t^2/2} - t^2 - 2$.
For each method, plot in the same figure the approximations $w_{h=0.2}, w_{h=0.1}, w_{h=0.05}$ you have found together with the actual solution $y(t)$ for $t \in [0, 1]$.
Find, for each method, the order of convergence p at the point $t_i = 1$ in the same fashion you did last week, given by:

$$
p = \log_2\left(\frac{w_{h=0.1}(t_i) - w_{h=0.2}(t_i)}{w_{h=0.05}(t_i) - w_{h=0.1}(t_i)}\right) \tag{3}
$$

Comment on your results. How do the methods RK-2 and RK-4 perform comparing to Euler's explicit method?