MATH 340 - Lab Instructor: Valeria Barra

LAB 11 Assignment

DUE Tuesday 04-12-2016

ODEs:

Euler's Method

We want to solve the initial value problem:

$$\begin{cases} \frac{dy}{dt} &= f(x, y(t)), \qquad t_0 \leq t \leq t_f \\ y(t_0) &= y_0 \end{cases} \tag{1}$$

To solve this numerically, we discretize the time interval $[t_0, t_f]$ with a grid of $n+1$ points $t_0 < t_1 < \ldots < t_n$ with equal step size h (in other words $t_i = t_0 + ih,\ i = 0, 1, \ldots, n$, or equivalently $h = \Delta t$). We seek the discrete approximated solution to this problem, say $w(t_n)$ (it can be also denoted by the equivalent notations $w_h(t_n)$ or w_n). Euler's forward (or explicit) method is defined with a forward finite difference by the iterative scheme:

$$\begin{aligned} \frac{w_{i+1} - w_i}{h} =& f(t_i, w_i), \qquad 0 \leq i \leq n \\ w_{i+1} =& w_i + hf(t_i, w_i), \qquad 0 \leq i \leq n \end{aligned} \tag{2}$$

with initial condition given by $w_0 = y_0$.

Problem 1:

Read carefully the first table in Example 6.2 in your textbook. Implement your own Euler's method. Print out you results in a table with columns: t_i (where the values in each row are $t_i = 0, 0.2, 0.4, 0.6, 0.8, 1.0$), the approximated solutions $w_{h=0.2}(t_i)$, $w_{h=0.1}(t_i)$, $w_{h=0.05}(t_i)$, the actual solution y_i and the error $e_i = |y(t_i) - w_{h=0.05}(t_i)|$. To compute the error at each grid point use the value for the actual solution given $y(t) = 3e^{t^2/2} - t^2 - 2$.

Plot in the same figure the approximations $w_{h=0.2}, w_{h=0.1}, w_{h=0.05}$ you have found together with the actual solution $y(t)$ for $t \in [0, 1]$. Find the order of convergence at the point $t_i = 1$ in the same fashion you did for integration rules applied on coarser or finer meshes, given by:

$$p = \log_2 \left(\frac{w_{h=0.1}(t_i) - w_{h=0.2}(t_i)}{w_{h=0.05}(t_i) - w_{h=0.1}(t_i)} \right) \tag{3}$$

Comment on your results.