

MATH 340 - Lab Instructor: Valeria Barra

LAB 5 Assignment

DUE Tuesday 02-23-2016

Lagrange's Interpolant Polynomial:

For a set of $n+1$ data points $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ we want to approximate a given function $f(x)$ by a polynomial interpolating those data, given by

$$P_n(x) = y_0L_0(x) + y_1L_1(x) + \dots + y_nL_n(x)$$

where each $L_i(x)$ is a Lagrange's polynomial defined as

$$L_i(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_{i-1})(x - x_{i+1}) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}, \quad i = 0, 1, \dots, n.$$

Problem 1)

Implement Lagrange's interpolation to find the interpolating polynomial $P_n(x)$. For each of the following problems, plot the function $f(x)$ and the resulting Lagrange polynomial $P_n(x)$ you found. Furthermore, to visualize if you are correctly interpolating the data points (x_i, y_i) given, plot them on the same figure using a marker, not a continuous line style (for instance you can make circles by putting the option 'o' in the `plot` command). Note that to find your polynomial and to plot it, you need an appropriate domain. You can use a very fine grid of points with endpoints $[x_0, x_n]$, unless specified differently in the problem.

1.1) Problem 45 in the Homework. Calculate the error $Err = |f(x) - P_n(x)|$, at the points $x = 1$, and $x = 5$. How does this error compare with the one predicted by the theory? (You will learn more about the error theory in class).

1.2) Problem 49 in the Homework. Find $P_n(0)$ and show that the result you obtained is the one expected.

1.3) Problem 68 in the Old Exams file. Compare the estimate given by the theo-

retical error, with the actual error you get $Err = |f(x) - P_n(x)|$ at $x = 0.2$.

Always remember to answer all questions, to significantly discuss your results, comment your code, and put labels, title and legend to your figures to obtain full credit for your work.