MATH 340 - Lab Instructor: Valeria Barra

LAB 13 Assignment

DUE Tuesday 04-26-2016

ODEs:

Implicit Euler

We want to solve the initial value problem:

$$\begin{cases} Y'(t) &= f(x, Y(t)), \qquad t_0 \leq t \leq t_f \\ Y(t_0) &= Y_0 \end{cases} \tag{1}$$

To solve this numerically, we discretize the time interval $[t_0, t_f]$ with a grid of $n+1$ points $t_0 < t_1 < \ldots < t_n$ with equal step size h (or in other words $t_i = t_0 + ih,\ i = 0, 1, \ldots, n$). We seek the discrete approximated solution to this problem, say w_{i+1}. Euler's backward (or implicit) method is defined by the iterative scheme:

$$w_{i+1} = w_i + hf(t_{i+1}, w_{i+1}), \qquad 0 \leq i \leq n \tag{2}$$

with initial condition given by $w_0 = Y_0$.

Problem 1:

Recast the following problem by hand to isolate the w_{i+1} term on the LHS to implement your own Implicit Euler Method for

$$\begin{cases} y'(t) &= -5y, \qquad 0 \leq t \leq 2 \\ y(0) &= 1 \end{cases} \tag{3}$$

whose actual solution is $Y(t) = e^{-5t}$.

Implement your own Implicit Euler's scheme to find the approximated solutions at the point $t = 2$, for $h = 2^0, 2^{-1}, \ldots, 2^{-4}$. Write your results in a table with columns in order: h_i, the approximated solution $w_{h_i}(t = 2)$, the error $e_{h_i}(t = 2) = |Y(t = 2) - w_{h_i}(t = 2)|$, and the ratio $R_{h_i}(t = 2) = \frac{e_{h_{i-1}}(t=2)}{e_{h_i}(t=2)}$. What is the order of convergence? How can you infer it from the table? Plot in the same figure the approximations w_h you have found for all five values of h together with the actual solution $Y(t)$ for $t \in [0, 2]$. Repeat the whole exercise with the code for Explicit Euler's scheme. How do the two method compare?