MATH 340 - Lab Instructor: Valeria Barra

LAB 9 Assignment

DUE Tuesday 03-29-2016

Numerical Integration:

Trapezoid Rule

To approximate $I(f) = \int_a^b f(x)dx$, let the number of subintervals to be n and let

$$h = \frac{b-a}{n}$$

to be the length of each subinterval. The endpoints (equispaced) of the subintervals are given by

$$x_j = a + hj, \qquad j = 0\,, \ldots\,, n$$

Then the approimation of $I(f)$ by the Trapezoid rule is given by the formula

$$T_n(f) = h\left[\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \ldots + f(x_{n-2}) + f(x_{n-1}) + \frac{1}{2}f(x_n)\right] \tag{1}$$

Simpson's Rule

For the same number of subintervals and grid points the Simpson's rule follows:

$$S_n(x) = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right] \tag{2}$$

Problem 1:

Use both methods above to find the approximations $I_n(f)$ of integrals $I(f) = \int_a^b f(x)dx$, with $n = 2^1, 2^2, 2^3, \ldots, 2^9$, for the functions:

$$(a) \int_0^{\pi} e^x \cos(4x)dx = \frac{e^{\pi} - 1}{17}$$
$$(b) \int_0^1 x^{5/2}dx = \frac{2}{7}$$
$$(c) \int_0^5 \frac{1}{1 + (x - \pi)^2}dx = \arctan(5 - \pi) + \arctan(\pi)$$
$$(d) \int_{-\pi}^{\pi} e^{\cos x}dx = 7.95492652101284$$
$$(e) \int_0^{\pi/4} (e^{\cos x})dx = 1.93973485062365$$
$$(f) \int_0^1 \sqrt{x}dx = \frac{2}{3}$$

For each of the approximated integrals $I_n(f)$ calculate the error as the difference from the exact value of the integral $I(f)$, given by

$$Err_n = |I(f) - I_n(f)|$$

For each exercise (a)-(f) make one table. In each table, each row should have the number n of points used, the approximation of the integral by Trapezoidal Rule, its error Err_n, and its ratio $Ratio_n = \frac{Err_{n-1}}{Err_n}$, respectively, then the approximation by Simpson's Rule, its error and its ratio, respectively.

This format can be printed this way:

```
fprintf('n  Tr Rule  Err_n Trap  Ratio_n Trap Simpon  Err_n Simps  Ratio_n Simps\n
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Comment on your results. For each execution $(a)-(f)$, which method is better? And why?

Boole's Rule

The degree four Newton-Cotes rule, often called Boole's rule, approximates $I(f) = \int_{x_0}^{x_4} f(x)dx$ as follows:

$$B_n(x) = \frac{2h}{45}\left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)\right] \tag{3}$$

Problem 2:

Use your code to find the degree of precision of Trapezoidal Rule, Simpson's Rule and Boole's Rule. To recall how you find the degree of precision of an integration formula, please refer to definition 5.2 on page 258 and example 5.7 on page 259 of your textbook (you should test your rule with monomials $1, x, x^2, x^3, \ldots$).