Problems from previous Math 340 Exams of mine:

1. (Exam 1 Fall 2014-611: Sauer)

(12 points – 4 points each) With 3-digit rounding after each operation find the value of:

* 1. 0.4 + 0.4 + … + 0.4 + 100 (where the 0.4 is repeated 100 times)
  2. 100+0.4 + 0.4 + … + 0.4 (where the 0.4 is repeated 100 times)
  3. Find the absolute and relative error in parts a and b.

1. (Fall 2013 pre-quiz) If is continuous and differentiable on the interval [0, 10] and attains the values given in the table:

|  |  |
| --- | --- |
| *x* | *f(x)* |
| 0 | 5 |
| 2 | 2 |
| 4 | -3 |
| 6 | 4 |
| 8 | -1 |
| 10 | -4 |

Then on [0, 10]  has at least \_\_\_\_\_\_\_\_ roots. Cite a theorem \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(If a minimum number of roots cannot be given state so and provide reasoning).

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Then on [0, 10]  has at most \_\_\_\_\_\_\_\_ roots. Cite a theorem \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(If a maximum number of roots cannot be given state so and provide reasoning).

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. (Fall 2013 pre-quiz) Find the Taylor polynomial of degree 2 for  around Then find a bound on the error of using this polynomial to approximate  .
2. (Final Spring 2010: A&H)

(10 points) Find the linear and quadratic Taylor polynomials for around *x=*0.

1. (Exam 1 Spring 2010: A&H)

(15pts) Find the Taylor polynomial of degree 2 around  for  . Find the error (remainder) term. Bound it in the interval [0, 2].

1. (Exam 1 Fall 2013: A&H)

(12 points – 6 each part)

* 1. If *a=*1.23 and *b=*34.56 are correct to the number of digits shown, what are the lowest and highest possible values of *a/b.* (Leave your result as a fraction).
  2. Estimate the error and the relative error in approximating  by 

1. (Exam 1 Spring 2014: A&H)

(8 points) Find the error, relative error and percentage error in approximating  by using  when 

1. (Exam 1 Fall 2015: Sauer)

(10 points: 3,7) How many multiplications and how many additions are needed to compute a value of 

* 1. Using the most basic (brute force) method
  2. Using nesting (Show how one would re-write the expression using the nesting process).

1. (Final Spring 2014: A&H)

(15 points) Consider 

* 1. (5) Trying to compute this quotient directly might be problematic near a particular value of *x.* What is that value? (call it  )
  2. (10) Find  (Extra credit, 5 pts, if you do it 2 different valid ways).

1. (Final Spring 2010: A&H)

Given ,

* 1. **(5 points)** Evaluate 
  2. **(10 points)** What should *n* be in the Taylor series to compute  with an accuracy of .

1. (Exam 1 Spring 2010: A&H)

(15pts) Evaluate  within an accuracy of  . Hint: replace by a general Taylor polynomial approximation plus its remainder.

1. (Exam 1 Spring 2010: A&H)

(15pts) How large should n be chosen to have 

1. (Exam 1 Spring 2010: A&H)

(15pts) Use Taylor series to evaluate 

1. (Review questions Spring 2010)

If , why is plugging in values of x not wise if x is near zero? Find a “better” form of this expression that would be more appropriate for computing its value near 

1. (Final Spring 2015: Sauer)

(2,2,5 points) Suppose you were to use the quadratic formula to solve for the roots of the equation .

* 1. What are the solution(s) in terms of *b*?
  2. For large values of *b*,which of the solutions could lead to the numerical issue of large relative error and why?
  3. Suppose you were using a computer that could not tell that two numbers were different if they were no more than 0.01 apart. Approximately how large, *N*, would *b* have to be such that the computer would give zero as one of the roots whenever  ?

1. (Exam 3 Fall 2015: Sauer)

(12 points: 3,3,6) Consider the equation .

* 1. Use the quadratic formula to write the roots of the equation (DO NOT SIMPLIFY).
  2. Which of these roots is subject to loss-of-significance (numerical issues) and why?
  3. Show clearly what you would do to alleviate the loss of significance issue and approximately give its value as a decimal.

1. (Exam 1 Make-up Fall 2015: Sauer)

Write  in base 3 to 4 decimal places. Find the absolute and relative error in your result.

1. (Exam 1 Spring 2015: Sauer)

(10 points) The value in base 10 is represented in binary to 6 places after the decimal point (chopped). Find this binary representation, the error and relative error.

1. (Exam 1 Fall 2015: Sauer)

(16 points: 3,4,3,3,3) (Leave in any roots or pi’s – **DO NOT SIMPLIFY**)  is to be approximated using the first 2 non-zero terms of the Taylor series for  around 

* 1. Write the first 2 non-zero terms of the series in terms of *x*.
  2. What is the approximation of  using the function in part a.
  3. Find an upper bound for the error based on your knowledge of series.
  4. What is the absolute error of the approximation in part b?
  5. What is the relative error of the approximation in part b?

1. (Exam 1 Spring 2015: Sauer)

(12 points) Find the second degree Taylor series, , for around . Then find a bound on the error of using this polynomial to approximate. Do not evaluate or .

1. (Final Fall 2013: A&H)

(15 points) Simplify: 

1. (Exam 1 Spring 2014: A&H)

(16 points) Find the linear Taylor series for around  and find a reasonable bound for the error on the interval  for  small and positive.

1. (Exam 1 Spring 2014: A&H)

(12 points) If  , approximate  by using the first 3 non-zero terms of a series.

1. (Exam 1 Fall 2013: A&H)

(16 points – 8 each part) For the function 

* 1. Find the quadratic Taylor polynomial for  around 
  2. Find the remainder term and use it to find the largest interval around such that the error in using this quadratic to approximate the function value will be no more than 0.001

1. (Review Questions Spring 2010: A&H)
   1. Find the Taylor polynomials of degree 1 and of degree 2 around x=2 for 
   2. Find a bound for the error term in each case on the interval 0 < *x* < 4
2. (Exam 1 Spring 2015: Sauer)

(10 points) Consider the following MATLAB function:

function val = TestFun(x)

xi=[2 3];

yi=[-2 -6];

val = 0;

for i = 1:length(xi)

Li = 1;

for j = 1:length(xi)

if i ~= j

Li = Li\*((x-xi(j))/(xi(i)-xi(j)));

end

end

val = val + Li\*yi(i);

end

end

What is the output of typing: TestFun(1)? (I.e. what is the ending value of val?)

What is the code doing?

1. (Exam 3 Fall 2015: Sauer)

(15 points: 3,3,4,5) Consider finding the real root of the equation on the interval [2, 3].

* 1. Show that there exists at least one root on the interval?
  2. Show that there is at most one root on the interval?
  3. How many steps of bisection would it take starting with the interval above to guarantee having an approximate value of the root to within 0.0003. (Don’t solve for the actual number – set it up and write your answer with logs in it).
  4. Use two steps of Newton’s Method to approximate the root with initial guess 2.

1. (Final Spring 2010: A&H)

 has exactly one root on the interval [1,2].

(a). **(5 points)** How many steps of the bisection method would be required to guarantee knowing the root to within ?

(b). **(10 points)** Show (prove) that the iteration scheme  converges to a solution of if the initial guess, is between 1 and 2 (without computing any iterates).

1. (Exam 1 Spring 2010: A&H)

(20pts) This problem involves approximating  using the Bisection Method and Newton's Method.

* 1. Find a simple polynomial with integer coefficients for which  is a root.
  2. Find an interval  of length 2 where  are integers such that lies in the interval.
  3. Perform two steps of bisection to find . After performing these two steps, what is your best guess for the value of 
  4. Perform two steps of Newtons Method starting with your original . i.e., find 

1. (Exam 1 Fall 2014-611: Sauer)

(16 points) Set up Newton’s Method to solve for a root of .

* 1. Consider starting guess  and perform 2 steps if possible. If not possible, explain why. If the method does not converge for this starting guess, explain why.
  2. Consider starting guess  and perform 2 steps if possible. If not possible, explain why. If the method does not converge for this starting guess, explain why.
  3. Perform one step of bisection on this function with initial interval [0, 1]. What is the best guess for the root after this step?
  4. How many steps of bisection would be needed to guarantee a solution within 0.001 of the true root on this interval?

1. (Exam 1 Fall 2015: Sauer)

(12points: 3,4,5)

* 1. Find an interval of length 1 where  are integers on which there is a root of 
  2. Use the minimum number of steps of the bisection method using the interval you found in part (a) to find an approximate value of the root with an error guaranteed to be no more than ¼.
  3. Use one step of Newton’s Method with initial guess  to approximate the root.

1. (Final Fall 2013: A&H)

(60 points) Consider 

* 1. Find an interval of length 1, with integer bounds on which  has a root.
  2. Show that  has exactly one root on .
  3. Use one step of bisection to narrow down this interval.
  4. Use one step of secant method using the interval to find an updated iterate.
  5. Use one step of Newton’s Method starting with  to find a new iterate.

1. (Exam 1 Spring 2014: A&H)

(26 points) In this problem we consider methods for approximating (18)1/4 .

* 1. Find a polynomial with integer coefficients which has a root of (18)1/4.
  2. Find an interval  of length 1 where  are integers that brackets the root (18)1/4 of .
  3. Show carefully that there is exactly one root of your polynomial on this interval.
  4. Perform one step of the Secant Method using starting with  to find a new interval .
  5. Use one step of Newton’s Method starting with to find a new iterate .

1. (Exam 1 Fall 2013: A&H)

(48 points: 4, 4, 8, 6, 6, 6, 6, and 8 points respectively per part) In this problem we consider methods for approximating (24)1/3 .

* 1. Find a polynomial with integer coefficients which has a root of (24)1/3.
  2. Find an interval  of length 2 where  are integers that brackets the root (24)1/3 of .
  3. Show that there is exactly one root of your polynomial on this interval.
  4. Perform one step of bisection using  to find a new interval .
  5. How many steps of bisection are needed to be certain that your error < 0.0001?
  6. Perform one step of the Secant Method using starting with  to find a new iterate.
  7. Use one step of Newton’s Method starting with .
  8. Without computing more iterates show that Newton’s Method in this case has quadratic convergence?

1. (Exam 1 Make-up Fall 2015: Sauer)

Consider finding the real root of the equation on the interval [0, 3].

* 1. How do you know that there exists at least one root on the interval?
  2. How do you know that there is at most one root on the interval?
  3. How many steps of bisection would it take starting with the interval above to guarantee having an approximate value of the root to within 0.0001. (Don’t get the number – set it up and solve in terms of logs).
  4. Use one step of Newton’s Method to approximate the root with initial guess 2.
  5. Use two steps of the fixed point method  with initial guess 0 to approximate the root.
  6. Find the exact value of the root. Hint: it is an integer.
  7. Will the method of part e converge to the root if the initial guess is close enough (do not compute iterates – do the analysis)?

1. (Final Spring 2014: A&H)

(60 points) Consider  (DO NOT SIMPLIFY YOUR EXPRESSIONS)

* 1. (10) How many solutions are there to this equation? Show your reasoning clearly.
  2. (5) Find an interval of length 1, with integer bounds on which there is a root.
  3. (5) Use one step of bisection to narrow down this interval.
  4. (10) How many steps of bisection are needed to guarantee an error of no more than 10-3?
  5. (10) Use one step of secant method using the interval to find an updated iterate.
  6. (10) Let  . Use one step of Newton’s Method starting with  to find .
  7. (10) Without finding any iterates, figure out how many steps, *n*, of Newton’s Method would be needed, starting at  to guarantee that  is within 10-3 of the true solution.

1. (Exam 1 Fall 2014: Sauer - 611)

(8 points) Carefully show that has exactly one root on [0, 1] (It is a little trickier than it may seem.)

1. (Final Fall 2014: Sauer - 611)

(10 points) Perform one step of Newton’s Method to approximate a solution of  with initial iterate  .

1. (Exam 1 Spring 2014: A&H)

(10 points) For a certain iteration scheme it is suspected that convergence is linear. Use Aitken’s method to find an improved iterate. Use your work to find an inequality that could be solved to estimate the number of iterates you would need to get within 0.001 of the true root.

|  |  |  |  |
| --- | --- | --- | --- |
| Iterate | 0 | 1 | 2 |
| Value | 2 | 0 | 1/2 |

1. (Exam 1 Spring 2015: Sauer)

(46 points – each part 4, 8, 6, 6, 6, 8, 8) In this problem we consider methods for approximating (10)1/3 .

* 1. Find the appropriate polynomial with integer coefficients which has a root of (10)1/3.
  2. Consider the initial interval . Perform one step of bisection using  to find a new interval . What is your best guess for the root at this point?
  3. How many steps of bisection are needed to be certain that your error < 0.0001 starting with the interval ?
  4. Perform one step of the Secant Method using starting with  to find a new iterate.
  5. Use one step of Newton’s Method starting with  to find .
  6. Notice that your new iteratein part (e) is further from the root than was. From the shape of in this case it is easy to show that starting with an initial iterate greater than the root will give a new iterate that is closer to the root using Newton’s Method. However, if < (10)1/3 the next iterate will be closer to the root only if is large enough. How large must be such that the new iterate will be closer to the root? (You may assume ).
  7. Consider using the fixed point method to compute (10)1/3. Use this difference scheme with starting value to compute . Then use Aitken’s method to improve the value of .

1. (Exam 1 Fall 2015: Sauer)

(12 points:4 ,8) Consider the iteration scheme 

* 1. Show that this iteration method has fixed points 
  2. **Analyze** whether the iteration scheme converges to these fixed points for an initial guess close enough to the fixed point (but not at the fixed point).

1. (Final Spring 2014: A&H) Consider the fixed point iteration scheme 
   1. (5) Find the quadratic equation that all fixed points of this scheme must satisfy.
   2. (5) Find the solutions of the equation.
   3. (15) Without finding any iterates determine whether the method converges to each of the roots for a starting value close enough to that root.
2. (Final Fall 2014: Sauer - 611)

(15 points) Analyze the following fixed point iteration scheme to determine whether the method converges to the given root if the initial iterate is close enough to the root. Does the method converge to the root? If so, what is the order of convergence? (DO **NOT** COMPUTE ANY ITERATES!!!)

* 1.  convergence to root 
  2.  convergence to root 

1. (Final Fall 2014: Sauer - 611)

(15 points) Find the fixed point of the iteration scheme . For what values of *c* does the scheme converge to the root if the initial iterate is close enough to the root.

1. (Final Fall 2013: A&H)

(25 points) Consider the fixed point iteration scheme 

* 1. Check that the scheme has a fixed point at 
  2. Starting with an initial guess,  , on the interval  , find  such that all iterates from  on will lie on the interval . (Do not compute any iterates. You may leave logarithms in your answer).

1. (Exam 1 Spring 2014: A&H)

(20 points) Analyze the following methods to solve for the root of  [Note that ] by considering the order/rate of convergence to this root for a “good initial” interval or iterate. Do NOT compute iterates. Explain your work carefully.

* 1. Bisection method.
  2. Newton’s Method – writing out Newton’s method for this problem and simplifying before analyzing should be helpful
  3. Re-writing the equation to get the iteration scheme 
  4. Rank these methods from best to worst for finding this root.

1. (Exam 1 Fall 2014: Sauer - 611)

(16 points) Consider the following iteration schemes to solve for the root  of . For each one, determine (**without computing any iterates**) whether the method converges for an initial guess close enough to the root. If it converges, find the order of convergence (linear, quadratic, etc). Rank the methods from best to worst.

* 1. 
  2. 
  3. 

1. (Exam 2 Spring 2010: A&H)

(20 pts) Which of the following iterations will converge to the indicate  . If it coes converge, determine the order of convergence.

1. 
2. 
3. 
4. (Final Spring 2015: Sauer)

(2,7 points) Consider the fixed point method 

* 1. Find all fixed points
  2. Determine whether the method converges to each fixed point and the order of convergence.

1. (Exam 1 Fall 2015: Sauer)

(15 points: 6 ,9) Consider the iteration scheme 

* 1. Find the fixed points of this iteration method.
  2. For each fixed point found, analyze the scheme for convergence (does it converge and, if so, with what order of convergence) for an initial guess close enough to the fixed point.

1. (Exam 1 Spring 2015: Sauer)

(12 points) For the iteration scheme  . Find all the fixed point(s). Determine (without computing iterates) whether the scheme converges to each fixed point for a starting guess close enough. If it converges, find the order of convergence.

1. (Exam 2 Fall 2013: A&H)

**Extra credit:** (5 points each part) For a certain iteration scheme it can be shown that a certain iteration scheme converges to where 

1. How large can the initial error  be and still guarantee the method converges to the root?
2. Create an iteration scheme  that has this form of error.
3. (Exam 2 Fall 2015: Sauer)

(15 points:6 ,9) Consider the iteration scheme 

* 1. Find the fixed points of this iteration method.
  2. For each fixed point found, analyze the scheme for convergence (does it converge and, if so, with what order of convergence) for an initial guess close enough to the fixed point.

1. (Exam 1 Fall 2013: A&H)

(12 points) For the iteration scheme  . Find the fixed point(s). Determine (without computing iterates) whether the scheme converges to each fixed point for a starting guess close enough. If it converges, find the order of convergence.

1. (Exam 1 Spring 2015: Sauer)

(8 points) Newton’s Method is to be used to find the square roots of the numbers from 26 to 99 (except for the perfect squares, as those are easy). The starting guess is to be exactly between 2 whole numbers (for example for the square root of 40, we start with 6.5). How close to the root are we guaranteed to be in two iterations no matter which square root value we are finding?

1. (Exam 2 Fall 2015: Sauer)

(10 points) Consider solving using Newton’s Method with starting iterate,  somewhere on the interval [3, 4]. Estimate an upper bound for how close to the root (the iterate after 2 steps) will be. Show your reasoning carefully.

1. (Exam 1 Make-up Fall 2015: Sauer)

Given that for Newton’s Method  . Consider solving using Newton’s Method with iterates between 5 and 6. Estimate an upper bound for how close to the root the iterate will be after 2 steps.

1. (Review questions Spring 2010)

If f is such that and if the error in the initial guess for a root is less than 1/3, what is an upper bound for the error of the next 3 iterates? (Newton error )

1. (Exam 1 Fall 2013: A&H)

(12 points) Error analysis of Newton’s Method shows that:  . If we use Newton’s Method to solve for the root of  lying on the interval  and start with a value for  on this interval, find an appropriate upper bound for the error of the third iterate, i.e. . (Do not compute any iterates).

1. (Exam 2 Fall 2015: Sauer)

(6 points) There is a root of on the interval [3, 4]. Use one step of the Secant Method starting with this interval to find an approximation to the root.

1. (Exam 1 Fall 2013: A&H)

(10 points) Consider applying Newton’s Method to find the root  of . Suppose Newton’s Method will converge for any value of where *r* is as large as possible. Find an equation that one could use to solve in order to find *r.* (Don’t try to solve the equation). (Note: there may be more than one acceptable answer, explain your reasoning clearly).

1. (Exam 1 Fall 2014: Sauer - 611)

(16 points) Solve using LU decomposition.

1. (Exam 1 Fall 2014: Sauer - 611)

(12 points) Use 2 steps of the Gauss Seidel Method with initial guess  in solving . Investigate convergence to the exact solution.

1. (Exam 1 Fall 2014: Sauer - 611)

(12 points) Perform one step of Newton’s Method for systems of equations for and  with the initial guess .

1. (Exam 1 Fall 2014: Sauer - 611)

(8 points) Find analytically the intersection in the first quadrant of the circle and the parabola . Find the forward error of the initial guess and the forward error of the result using one step of Newton’s Method above.

1. (Final Fall 2014: Sauer - 611)

(20 points) Perform one step of the Gauss-Seidel method to approximate a solution of:

 starting with initial guess  . Find the relevant matrix to analyze convergence to the solution. Analyze the matrix to find the factor by which the error is reduced at each iteration?

1. (Exam 2 Fall 2013: A&H)

(16 points) Approximate the value of *f*(0) where *f*(*x*) has the values given in the table (Note: Building a Newton’s Divided Difference table may be helpful.)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | -2 | -1 | 1 | 2 |
| *f*(*x*) | -4 | -2 | 14 | 40 |

* 1. By finding the lowest degree polynomial through the final 3 points (*x=* -1, 1, 2)
  2. By finding the lowest degree polynomial through all 4 points.

1. (Exam 3 Fall 2015: Sauer)

(8 points: 2,6) Consider using 5 equi-spaced points on  to approximate  using a Lagrange Polynomial

* 1. What is the spacing between the points?
  2. Give the theoretical upper bound for the error in using this polynomial to approximate . (DO NOT MULTIPLY IT ALL OUT).

1. (Final Fall 2014: Sauer - 611)

(10 points) If a table is to be constructed of values of  on the interval [1, 2], how small should the spacing between the *x* values be to guarantee that piecewise linear interpolation will give an error of no more than 1.5\*10-6?

1. (Exam 2 Spring 2010: A&H)

(20 pts) Consider the function  .

* 1. Find the linear interpolating polynomial  through the points where  and .
  2. Find the quadratic interpolating polynomial  through the points where .
  3. Bound the error  .
  4. Bound the error  .

1. (Final Spring 2010: A&H)

Consider the function 

(a). **(10 points)** Find the quadratic (Lagrange) interpolating polynomial, , through the points when *x=*0, 1, and 3. Use your quadratic to approximate the value of .

(b). **(5 points)** Bound the error of at *x=*2.

1. (Exam 2 Fall 2015: Sauer)

(19 points: 5,5,3,6) Consider the set of points (-2, 20) (0, 4) and (2,12)

* 1. Find the quadratic Lagrange Polynomial through these points using the standard Lagrange Polynomial technique. (DO NOT SIMPLIFY)
  2. Use Newton’s Divided Differences to the quadratic Lagrange Polynomial through the points. (DO NOT SIMPLIFY)
  3. Check your work in parts b and c by computing your estimate for  in each case.
  4. Given that on the interval [-2, 1] that 

Find an upper bound for the error in part a.

1. (Exam 2 Spring 2014: Sauer)

(28 points) Consider the function 

* 1. Write the first degree Lagrange polynomial through the endpoints. (Simplify).
  2. Find an upper bound (using the formula) for the error at .
  3. Write the second degree Lagrange polynomial through the points where 
  4. Use the quadratic found in part c to approximate  and find the relative error.

1. (Exam 2 Fall 2015: Sauer)

(10 points) Piecewise (Lagrange Polynomial) quadratic interpolation using 5 quadratics was performed on an equispaced set of points on the interval [0, 1]. Suppose the true function being interpolated is  while our interpolation method gives us the piecewise quadratic  .

* 1. What *x-*values were used to find ? What is the step size *h* or  ?
  2. What is the order of the error term?
  3. If it is known that  on [0, 1], find an appropriate bound on the error  at any point on the interval. (DO NOT SIMPLIFY).

1. (Exam 1 Fall 2014: Sauer - 611)

(16 points) (Do not simplify)

* 1. Find the Lagrange Polynomial through the points (1, 6) (3, 6) and (6, 21).
  2. Use divided differences to find a polynomial through (1, 6) (3, 6) and (6, 21).
  3. Use divided differences to find a polynomial through (1, 6) (6, 21) and (3, 6).
  4. Must the three polynomials be the same? Explain why or why not without simplifying your results in parts a, b, and c.

1. (Final Fall 2013: A&H)

(15 points) Find the quadratic polynomial  satisfying:



1. (Exam 1 Spring 2015: Sauer)

(12 points – 6 points each part)

* 1. The number of new undergraduate students at NJIT each year is given in the table below. Use Lagrange Polynomials to fit the data to a quadratic function and then find an approximation for the number of new students in Fall 2012. You might want to simplify to using time = 0 for 2011 and shift the number of students down by 1800 to simplify the calculations – and then shift back up at the end.
  2. Check your result in part (a) by doing the same using Newton’s Divided Differences

|  |  |
| --- | --- |
| Year | Students |
| Fall 2011 | 1799 |
| Fall 2013 | 1814 |
| Fall 2014 | 1826 |

1. (Exam 1 Spring 2014: A&H)

(8 points) Find the lowest possible degree polynomial satisfying the conditions:



1. (Exam 4 Fall 2015: Sauer)

(10 points) If a section of a cubic spline ** on [1, 3] what conditions must the next section of the spline, **,satisfy if it is defined on the interval [3, 5] and terminates at the point  .

1. (Exam 2 Spring 2010: A&H)

(15 points) Find, the values of *a*, *b*, *c*, and *d* such that

is a natural cubic spline.

1. (Final Fall 2013: A&H)

(15 points) A cubic spline on is given by:  Find *a*, *b*, and *c*.

1. (Exam 2 Fall 2013: A&H)

(22 points) Splines:

* 1. A cubic polynomial,  , is a natural cubic spline on  and passes through the points (0, 2) and (4, 6). Find the equations for .
  2. The function  from part (a) along with another cubic polynomial make up a cubic spline on the interval where is defined on  and the spline passes through (5, 10). Find .
  3. Use your result in parts a and/or b to answer: If a clamped cubic spline is to pass through the points (0, 2) (4, 6) and (5, 10) how should  and be defined so that the result would be the cubics on [0, 4] and  on [4, 5].

1. (Final Spring 2010: A&H)

**(10 points)** Find *k,* so that *S(x)* is a cubic spline:

{

1. (Final Fall 2014: Sauer - 611)

(15 points) Find, if possible, the values of *a*, *b*, *c*, and *d* such that the function

is a natural cubic spline.

1. (Exam 2 Spring 2014: Sauer)

(22 points)

* 1. Write out the equations for the second derivatives (the *M*’s) of a natural spline that passes through the points (0, 1) (2, 5) (4, 1) and (6, -35).
  2. Verify that the spline below satisfies the equations found in part a (don’t find  ) 

1. (Exam 2 Spring 2015: Sauer)

(16 points)

(a). The number of new undergraduate students at NJIT each year is given in the table below. Set up the system of equations (for the *c*s) to find the cubic spline polynomials given that the second derivative on the left is 6 and the second derivative is -2 on the right. (DO NOT SOLVE).

|  |  |
| --- | --- |
| Year | Students |
| Fall 2011 | 1799 |
| Fall 2013 | 1814 |
| Fall 2014 | 1826 |

(b). Find *a*, *b*, *c* and *d* such that



is a natural cubic spline.

1. (Final Spring 2015: Sauer)

(7 points) Are there values of *a* and *b*  such that: 

can satisfy the conditions for a “cubic” spline (even though it’s only quadratic)? If so, find those values. If not, clearly show why not.

1. (Final Spring 2014: A&H)

(15 points) A clamped cubic spline,  on passes through (0, 2) and (3, -1) and has  and. Find .

1. (Exam 4 Fall 2015: Sauer)

(10 points) An expression of the form  is to be used to approximate . DO EITHER PART a OR PART b

* 1. Set up the equations needed to find the values of  using Taylor Series and matching up terms. (DO NOT SOLVE THE EQUATIONS)
  2. Use Newton’s Divided Differences to find the approximation method (DO NOT SIMPLIFY)

1. (Final Spring 2015: Sauer)

(10 points) An expression of the form  is to be used to approximate . Find the values of .

1. (Exam 2 Fall 2013: A&H)

(24 points) Consider **

* 1. Approximate  using the centered difference method with *h=*1.
  2. How small would *h* have to be sure that the error in using this centered difference formula to compute is no more than about 0.5 \* 10-4?
  3. *f*(*x*) is to be is to be approximated by a quadratic Lagrange polynomial through the points (5/2,1/5) (3, 1/6) and (7/2, 1/7). Give the appropriate upper bound for the error of the approximation at of the interval. (DO NOT SIMPLIFY!!!)

1. (Final Fall 2014: Sauer - 611)

(10 points) Find the leading order error term in using  to approximate  .

1. (Final Spring 2014: A&H)

(20 points) Use Taylor Series expansions around *x* to analyze what the expression:



approximates. What is the leading error term? Use this error term to answer: if  is a polynomial, what is the highest degree polynomial that this expression is exact for? (Do NOT use a set of polynomials to do this part).

1. (Final Spring 2014: A&H)

(15) Approximate where  using the centered difference scheme with *h=*2 and bound the error.

1. (Exam 3 Spring 2014: A&H)

(30 points: 8, 16, 6) Consider the difference scheme



* 1. Use this method to approximate  for  where *h*=1 and *h=*1/2.
  2. Find the form of the leading term of the error in this approximation scheme (for general functions)
  3. Use Richardson extrapolation and your results in parts (a) and (b) to improve your result for . (If you cannot solve part (b), note it and assume you found the error to be  ).

1. (Final Fall 2013: A&H)

(25 points) The quadrature formula  is to have degree of precision as high as possible. Find .

1. (Exam 3 Spring 2014: A&H)

(15 points) The quadrature formula



has the highest degree of precision possible. Determine  and the degree of precision. (Think about the results: do you notice anything surprising? If so, for extra credit, what?)

1. (Review questions Spring 2010)

What does the difference scheme difference scheme approximate (at x) and find its error order.

1. (Final Fall 2013: A&H)

(20 points) What does  approximate at *x*? What is the leading error term?

1. (Review questions Spring 2010)

Given a smooth continuous function, f(x), for which f(0)=8, f(1)=5, f(2)=3, f(3)=2 and f(4)=3

* 1. Compute the approximate value of f’’(2) using a step size of h=2.
  2. Compute the approximate value of f’’(2) using a step size of h=1.
  3. Use Richardson extrapolation to improve the results of the previous steps.
  4. Note: 

1. (Review questions Spring 2010)

The difference scheme  approximates f’(x) (at x) with error order . If you know the information in the following table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 2 | 3 | 4 | 5 | 6 |
| f(x) | 16 | 11 | 4 | 7 | 32 |

Use the difference scheme with x=4 and h=2 to approximate f’(4).

Use the difference scheme with x=4 and h=1 to approximate f’(4).

Use Richardson extrapolation to improve your approximation of f’(4).

1. (Final Spring 2014: A&H)

(70 points). Consider the function: .

* 1. (10) Use a divided difference table or Lagrange Polynomials to find the lowest degree polynomial passing through the curve at *x*=2, 4, and 6. (Do not simplify)
  2. (10) Evaluate your polynomial at *x* =3 and bound the error there.
  3. (10) Find the actual error and relative error in your polynomial approximation for .
  4. (10) Approximate  using the trapezoidal rule with *h=*2
  5. (10) Bound the error of using the trapezoidal rule with *h=*2 to approximate .
  6. (10) Find the **asymptotic** error of using the trapezoidal rule with *h=*2 for .
  7. (10) Use Simpson’s rule with *h=*2 to approximate .

1. (Exam 2 Spring 2010: A&H)

(20pts) Use 4 subintervals (*n* = 4) with Trapezoidal rule to approximate 

and bound the error of the approximation. Compare with the asymptotic error.

1. (Exam 2 Spring 2010: A&H)

(20pts) Use 4 subintervals (*n* = 4) with Simpson's rule to approximate 

and bound the error of the approximation. Compare with the asymptotic error. Note that you

may use the identity .

1. (Exam 4 Fall 2015: Sauer)

(18 points: 3, 4, 4, 4, 3 points) Use the Trapezoidal Rule to approximate 

* 1. with step size 
  2. with step size 
  3. Find an upper bound for the error when 
  4. Find the asymptotic error when .
  5. Find the absolute error when (DO NOT SIMPLIFY)

**Extra Credit:** In the this problem, based on your understanding of the behavior of the error, without calculating it, estimate the value of the integral that the Trapezoidal Rule will produce if the step size is .

1. (Final Fall 2013: A&H)

(50 points). Consider the table

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 0 | 2 | 4 |
| *f*(*x*) | 2 | 2 | -6 |

* 1. Find the Lagrange Polynomial passing through all the points
  2. Give a bound for the error in using the polynomial found in part (a) to approximate  given that 
  3. Approximate  using the trapezoidal rule with *h=*2
  4. Approximate  using the trapezoidal rule with *h=*2
  5. Approximate  using the centered difference scheme

1. (Exam 2 Spring 2014: Sauer)

(42 points) Consider  Approximate :

* 1. Using the trapezoidal rule with *h=*4
  2. Using the trapezoidal rule with *h=*2
  3. Find the asymptotic error for part b.
  4. Use the result of parts a and b along with Richardson extrapolation to get an improved estimate for the value of the integral.
  5. How small do the subintervals need to be using the Trapezoidal Rule to be guaranteed that the error in computing this integral is smaller than 0.03?
  6. Use Simpson’s rule to approximate the value of the integral using *h=*2 and then with *h=*1.
  7. (Extra credit) Comment on the results of parts (d) and (f).

1. (Exam 2 Spring 2015: Sauer)

(38 points) Consider 

* 1. Approximate with step size 1 using the forward difference method
  2. Approximate  with step size 1 using the centered difference method
  3. Find the error bound in each case and comment on the results in (a) and (b).
  4. Use the trapezoidal rule to approximate  with the largest step size possible. Find the asymptotic error and use it to give a better estimate of the true integral.
  5. Use Simpson’s rule to approximate  with the largest step size possible. Find the asymptotic error and use it to give a better estimate of the true integral.
  6. Make an intelligent comment on the results of parts (d) and (e).

1. (Exam 2 Spring 2014: Sauer)

(8 points) The trapezoidal rule (with a single subinterval) applied to  gives the value 4 while Simpson’s Rule (with as large a step size as possible) gives 1. Find  .

1. (Exam 2 Fall 2013: A&H)

(26 points) Use the value of *f*(0) you found in 1b along with the values in the table in problem 1 to approximate :

* 1. Using the trapezoidal rule with *h=*2
  2. Using the trapezoidal rule with *h=*1
  3. Use Richardson extrapolation on your results in parts a. and b. to improve your result.
  4. Use Simpson’s rule to approximate the value of the integral using *h=*1

1. (Exam 2 Spring 2015: Sauer)

(12 points) Consider the following MATLAB code:

function Bukiet = Method(f,a,b,n)

h = (b-a)/n;

S = 0;

x = a + 0.5 \* h;

for i = 1:n

S = S + f(x);

x = x+h;

end

Bukiet = h\*S;

end

If we set f=@(x) sin(\*x) and then call Method(f,1,4,3)

1. What is the output? (show each step of what occurs).
2. What is being computed and what is the method called?
3. (Exam 2 Spring 2015: Sauer)

(10 points) The following is output of a MATLAB code for integration.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Step (i) | Grid spacing (*h*) | Approximate Value of the Integral (Ii) | Change from previous value (Ii- Ii-1) | Ratio of previous change divided by new change  (Ii-1- Ii-2) / (Ii- Ii-1) |
| 1 | 2.5 | 0.712116 |  |  |
| 2 | 1.25 | 0.928919 | 0.216803 |  |
| 3 | 0.625 | 0.95720412424 | 0.02828541 | 7.664808 |
| 4 | 0.3125 | 0.95941569451 | 0.00221157 | 12.789743 |
| 5 | 0.15625 | 0.95956238722 | 0.00014669 | 15.076211 |
| 6 | 0.078125 | 0.95957169508 | 0.00000930 | 15.760087 |
| 7 | 0.03906250 | 0.95957227903 | 0.00000058 | 15.939436 |

1. What is the order of the leading error term? Explain.
2. Which of the method(s) you’ve learned is likely to have been used to compute these results?

(A)Midpoint method (B) Trapezoidal Rule (C) Simpson’s Rule

1. Boole’s Method (E) More than one of these methods (specify which ones)
2. (Exam 4 Fall 2015: Sauer)

(12 points) The formula is to be used to integrate polynomials. Find the degree of precision of the method (i.e. the number such that the method is exact for  ).

1. (Final Fall 2014: Sauer - 611)

(20 points) Find the values of  such that the integration method  has as high a degree of precision as possible.

(DO NOT FIND THE DEGREE OF PRECISION).

1. (Final Spring 2010: A&H)

Consider the formula .

(a). **(10 points)** Determine the weight and the node  so that the formula is exact for all polynomials to as large a degree as possible.

(b). **(5 points)** What is the degree of precision of the formula?

1. (Final Spring 2015: Sauer)

(6,3 points) Find the values of *c* and such that the integration formula



has the highest degree of precision possible. Find the degree of precision.

1. (Exam 2 Fall 2013: A&H)

(12 points) Find  and  in the quadrature method of the form



such that the method has the highest possible degree of precision.

1. (Final Spring 2014: A&H)

(15 points) Recall the Gaussian quadrature formula . Use this to approximate .

1. (Exam 2 Spring 2015: Sauer)

(16 points)

(a). Write the equations one must solve such that the degree of precision of the integration method  is as large as possible. (DO NOT SOLVE the equations)

1. (Exam 5 Fall 2015: Sauer)

(6 points) What step size (using equi-spaced points) should be used to approximate using Simpson’s rule to guarantee an error no more than 10-6 ? (Do not actually find the step size, just an inequality for finding it).

1. (Final Fall 2014: Sauer - 611)

(10 points) Use Simpson’s Rule with as large an interval size possible to approximate ?

1. (Exam 5 Fall 2015: Sauer)

(8 points)  is approximated using only function values when (in each case using as many of these function values as possible). The Trapezoidal Rule gives the result . Simpson’s Rule gives the result  . What can you say about the result given by the Midpoint Method?

1. (Exam 5 Fall 2015: Sauer)

(8 points) A numerical method has error of the form  and gives the following results:

|  |  |
| --- | --- |
| Step size | Approximated value of solution |
|  | 14 |
|  | 6 |
|  | 4 |

Use Richardson approximation as many times as possible to improve the result.

1. (Final Spring 2015: Sauer)

(3, 5, 4, 2 points) Use the Trapezoidal Rule with step size  and with step size to approximate  and apply Richardson Extrapolation to improve the result. What order should the error be ***after*** applying Richardson Extrapolation?

1. (Exam 2 Spring 2015: Sauer)

(8 points) Suppose that a numerical approximation method has leading error order *h*3. If when the step size *h*=1, the approximation gives the value of 4 and when *h*=1/2 the approximation gives the value of 3, use Richardson extrapolation to get an improved estimate of the desired value.

1. (Final Fall 2014: Sauer - 611)

(15 points) A method to compute integrals *I*(*h*) (where the interval size is *h*) has the property that  Use as many steps of Richardson extrapolation as possible to improve the results given:  What is the order of the error in your final result?

1. (Final Fall 2014: Sauer - 611)

(10 points) Use one step of the 2nd order Taylor Series Method to approximate if

.

1. (Exam 5 Fall 2015: Sauer)

(6 points) The Taylor Series Method of order *q* for solving ordinary differential equations is used to solve . Without actually solving the ODE, find the smallest value of *q*  such that the method will solve the ODE exactly.

1. (Exam 5 Fall 2015: Sauer)

(8 points) Approximate  if  with the largest step size possible

* 1. Using Euler’s Method
  2. Using the 2nd order Runge-Kutta Method called the Trapezoidal Method

**Extra Credit:** Estimate the error in part a without solving the differential equation analytically.

1. (Final Spring 2014: A&H)

(20 points) For the ordinary differential equation , use fourth order Runge-Kutta with step size *h =* 2 to approximate .

1. (Final Spring 2010: A&H)

**(10 points)** Consider the ordinary differential equation:  with Use the second order Runge-Kutta method given by:



with step size *h*=2 to approximate 

1. (Final Spring 2014: A&H)

(25 points) Approximatewhere  and  using Euler’s method with step size *h*=3. Do the same with step size *h*=1. Then use Richardson extrapolation to improve the results of your calculation.

1. (Exam 3 Fall 2013: A&H)

(54 points) Consider the ordinary differential equation .

* 1. Use Euler’s method with step size *h=*2 to approximate y(5).
  2. Use the second order Taylor Series method on the ordinary differential equation with step size *h=*2 to approximate y(5).
  3. Use the fourth order Runge-Kutta method on the ordinary differential equation with step size *h=*2 to approximate y(3).

1. (Exam 3 Fall 2013: A&H)

**Extra Credit**: Write out the 3rd order Taylor series to solve for *y*(1+*h*) for the differential equation:  . (Just 1 step – leave your answer in terms of *h*). Then let . Explain.

1. (Exam 3 Spring 2014: A&H)

(40 points: 6, 10, 12, 12) Consider the ordinary differential equation .

* 1. Use Euler’s method with step size *h=*1 to approximate *y*(1).
  2. Use Euler’s method with step size *h=*1/2 to approximate *y*(1).
  3. Use the second order Taylor series method with *h=*1/2 to approximate *y*(1).
  4. Use the second order Runge Kutta method



with step size *h=*1/2 to approximate y(1).

1. (Exam 3 Fall 2013: A&H)

(30 points) The values in the table represent the (hypothetical) result of using several numerical methods with several step sizes to compute the solutions of some differential equations of the formto approximate.

* 1. Use Richardson extrapolation to improve the results of the Euler Method calculation
  2. Use Richardson extrapolation to improve the results of the second order Taylor Series Method calculation
  3. Use Richardson extrapolation to improve the results of the fourth order Runge-Kutta Method calculation

|  |  |  |  |
| --- | --- | --- | --- |
| Step size | Euler approximation for *y*(5) | Taylor Series approximation for *y*(5) | Runge Kutta approximation for *y*(5) |
| 1 | 4 | 1 | -10 |
| 1/2 | 3 | 4 | 5 |

1. (Final Fall 2013: A&H)

(25 points) The values in the table give the (hypothetical) results for  using the second order Runge Kutta Method with 2 step sizes where . Use Richardson extrapolation to improve the results of calculation

|  |  |
| --- | --- |
| Step size | Runge Kutta approximation for *y*(5) |
| 3 | -11 |
| 1 | 5 |

1. (Final Spring 2015: Sauer)

(18 points) Estimate  for the ODE- Initial Value Problem  and .

* 1. Using the explicit method .
  2. Using the (similar) implicit method .
  3. Find the inequality that must be satisfied by the step size *h* to guarantee stability for the explicit method in part (a) for the ODE . DO NOT SOLVE.
  4. Find the inequality that must be satisfied by the step size *h* to guarantee stability for the implicit method in part (b) for the ODE . DO NOT SOLVE.

1. (Final Fall 2014: Sauer - 611)

(10 points) What inequality must the step size *h* satisfy such that the midpoint-based method  is stable for the ODE ? Do NOT solve.

1. (Final Spring 2010: A&H)

**(10 points)** Find the inequality that must be analyzed to determine the stability (condition on step size) of the numerical method for  given by

.

I.e., find the inequality for stability considering the differential equation . Do not analyze or solve the inequality.

Extra credit (5 points): For what values of *h* is the method stable?

1. (Final Spring 2014: A&H)

(20 points) Write the stability conditions for the differential equations schemes for the ordinary differential equation . (DO **NOT** SOLVE THE INEQUALITIES).

* 1. 
  2. 

1. (Exam 3 Fall 2013: A&H)

(16 points) Consider the ordinary differential equation . Find the range of possible step sizes *h* for the method to be **stable** using the second order Taylor Series method. (Hint: consider the second order Taylor Series method for this particular differential equation. Find the condition for stability and solve for *h*.)

1. (Exam 3 Spring 2014: A&H)

(15 points) Consider the ordinary differential equation . Analyze the stability of the method



in order to find the range of possible step sizes *h* for the method to be **stable.**  (Hint: Find the condition for stability and solve for *h*.)

1. (Final Fall 2013: A&H)

(25 points) For the ordinary differential equation , find the inequality that needs to be satisfied by the step size *h* for the following method to be **stable. (Do not solve)**



1. (Final Fall 2014: Sauer - 611)

(15 points) Consider the Backward Euler method  to approximate the solution to the ODE  . Find an equation for the approximation for  using *h*=1. Do NOT solve for but in one sentence write how you might go about evaluating and perform one step of your method.

1. (Final Spring 2015: Sauer)

(6 points) Analyze the stability for the MULTI-STEP method .

1. (Final Fall 2014: Sauer - 611)

(10 points) Analyze whether the multi-step method  is strongly stable, weakly stable or unstable.

1. (Final Spring 2015: Sauer)

EXTRA CREDIT (5 points) Find the values of *a* and *b* such the that method  is consistent and has as high an order error as possible.

1. (Final Spring 2015: Sauer)

(5 points) Write the ODE  as a system of first order ODEs.

1. (Final Fall 2014: Sauer - 611)

(10 points) Perform one step of Euler’s Method for the system of ODEs

 to approximate  .

1. (Final Fall 2013: A&H)

(25 points) Use Euler’s method with step size *h=*2 to approximate *y*(3) and z(3) for



1. (Final Spring 2015: Sauer)

(3, 4, 6 points) Suppose you are solving (numerically) the ODE Boundary Value Problem  with . Since we have only learned methods for solving first order systems of ODE Initial Value Problems, you solve the system with:

with the result that 

with the result that 

with the result that 

1. Given the above information, use Bisection appropriately to get an improved value for .
2. Given the above information, use the Secant Method to find an improved value for  .
3. Given the above information, use the highest degree Lagrange Polynomial possible to find an improved value for  .
4. (Exam 5 Fall 2015: Sauer)

(14 points) Without actually computing approximations for the following integrals give the behavior you should expect to get (choose from column A) and provide the reason for expecting that behavior (from the choices in column B) for each case below:

|  |  |
| --- | --- |
| **Column A** | **Column B** |
| 1. Should give the exact value | 1. The asymptotic error is zero for this function on this particular interval but probably wouldn’t be if the interval was different. |
| 1. Should be better than order convergence | I. Function has enough continuous derivatives on the interval to attain the normal convergence order for the given method. |
| 1. Should be order  convergence | 1. Function is periodic on the interval |
| I. Should be worse than order convergence | 1. The theoretical error bound is zero when applied to this particular function for this method |
| 1. Should be worse than order convergence | T. Function has fewer continuous derivatives than needed on the interval for the method to attain the normal convergence order for the method. |
| 1. Should be order  convergence | 1. Function has many more continuous derivatives than necessary on the interval |
| 1. Should be better than order convergence |  |

Using Simpson’s Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Using Trapezoidal Rule for **Column A**. \_\_\_\_ **Column B**. \_\_\_\_\_

Using Simpson’s Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Using Trapezoidal Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Using Simpson’s Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Using Trapezoidal Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Using Simpson’s Rule to compute  **Column A**. \_\_\_\_\_\_\_ **Column B**. \_\_\_\_\_\_\_

Possibles for final

* 1. Evaluate 
  2. Explain how one can go about evaluating  near  to reduce rounding error and use your result to evaluate .

1. Aitken’s Method problem
2. For a certain iteration scheme it can be shown that: 
   1. How large can the initial error  be such that the method converges to the root?
   2. Create an iteration scheme  that has this form of error.