INSTITUTO TECNOLÓGICO DE ESTUDIOS SUPERIORES DE OCCIDENTE

003 Advanced Trading Strategies: Pairs Trading

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1. Strategy Description and Rationale

1.1 Overview of Pairs Trading Approach

This strategy implements a statistical arbitrage approach through pairs trading, where we simultaneously take long and short positions in two cointegrated assets to exploit temporary deviations from their long-term equilibrium relationship.

The strategy operates as follows:

- Identify two assets that historically maintain a stable relationship (cointegration)
- Monitor the spread: spread = log(Price_A) - β × log(Price_B)
- Enter positions when the spread deviates significantly from its mean (measured by z-score)
- Exit positions when the spread reverts to its mean

1.2 Why Cointegration Indicates Arbitrage Opportunity

Two price series are cointegrated if:

1. Both series are non-stationary - they have unit roots and exhibit trends
2. Their linear combination is stationary - the spread mean-reverts

Cointegration suggests that there's an economic or structural relationship between the two assets. So their prices move together in the long run, but short-term fluctuations create temporary deviations.

When the spread moves far from its mean, we can expect mean reversion — it will return to equilibrium.
That's where the pairs trading strategy comes in:

1. High spread: short the overpriced asset and go long on the underpriced one.
2. Low spread: do the opposite.

1.3 Kalman Filter use in Dynamic Hedging

Traditional pairs trading uses a static hedge ratio estimated via Ordinary Least Squares (OLS) regression on the entire historical dataset. However, this approach has critical limitations. For example assumes the relationship between assets is constant over time, cannot adapt to structural changes in the market and uses all historical data equally.

In contrast, the Kalman Filter provides a more adaptive framework for estimating the hedge ratio dynamically. This algorithm updates the coefficient β in real time as new price information becomes available.

The Kalman filter implements a Bayesian updating mechanism where:

Prior belief + *New evidence* → *Updated belief*

This allows the strategy to remain robust in changing market conditions where the true hedge ratio may drift.

1.4 Expected market conditions for strategy success

Favorable Conditions:

- Stable economic regimes: Cointegration relationships hold better in stable markets
- Normal volatility: Sufficient spread movements to generate trading signals
- Low transaction costs: Strategy profitability is sensitive to commission rates
- Mean-reverting markets: Sideways or range-bound conditions favor mean reversion

Challenging Conditions:

- Structural breaks: Major events that permanently alter the relationship.
- High volatility regimes: Spreads may diverge further before reverting (requires wider stop-losses)

2. Pair Selection Methodology

The strategy employs a sector-based approach to pair selection, recognizing that assets within the same industry are more likely to share common risk factors and maintain stable cointegration relationships.

Asset Universe:

- 8 sectors: Technology, Financials, Healthcare, Consumer Discretionary, Consumer Staples, Communication Services, Energy, Industrials
- Total candidates: 50

- Time period: 15 years of daily price data (2010-01-01 to 2025-01-01)

2.1 Correlation screening criteria

We decided as a team to employ a direct cointegration testing methodology without preliminary correlation filtering. Justified by knowing that correlation reflects a contemporaneous linear relationship or short-term co-movement and on the other side cointegration measures a long-run equilibrium relationship characterized by a mean-reverting spread. A high correlation does not necessarily imply cointegration, and vice versa. Therefore, using a simple correlation threshold could mistakenly exclude valid cointegrated pairs.

2.2 Cointegration testing (Engle-Granger method)

Step 1: Test for Non-Stationarity

Applied Augmented Dickey-Fuller (ADF) test to each individual price series.

This confirms both assets exhibit trends and are not mean-reverting individually.

- H_0: Series has a unit root (non-stationary)
- H_1: Series is stationary

Acceptance criteria: ADF p-value > 0.05 for BOTH series

Step 2: OLS Regression

Estimate the long-run equilibrium relationship. Using log prices improves stationarity of residuals and allows interpretation of β as an elasticity.

- $\log(Price\_A) = \alpha + \beta \times \log(Price\_B) + \varepsilon\_t$

Step 3: Test Residuals for Stationarity

Apply ADF test to regression residuals, to confirm the spread is mean-reverting (stationary).

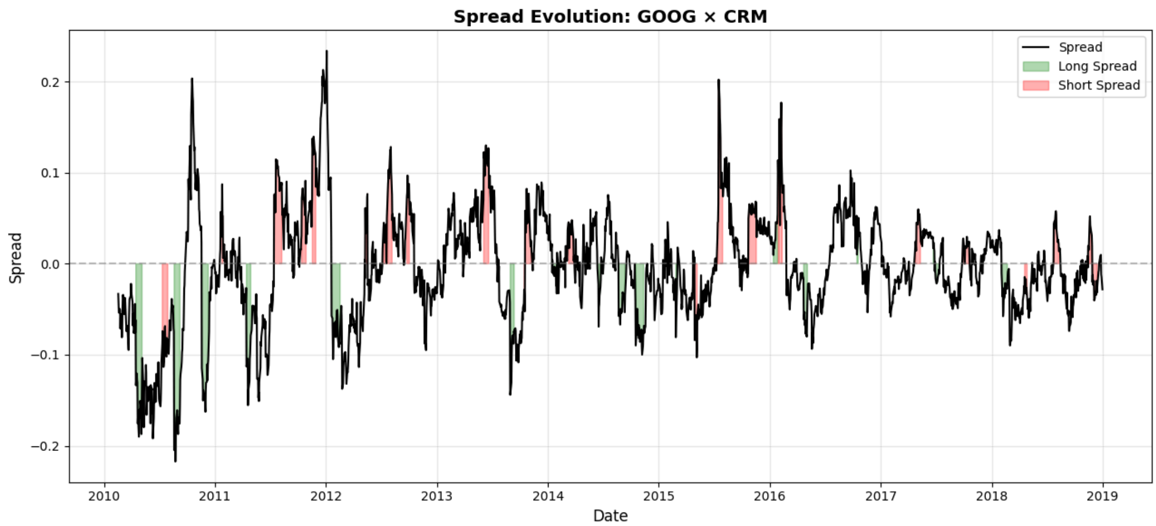
- $residuals\_t = \log(Price\_A\_t) - (\alpha + \beta \times \log(Price\_B\_t))$

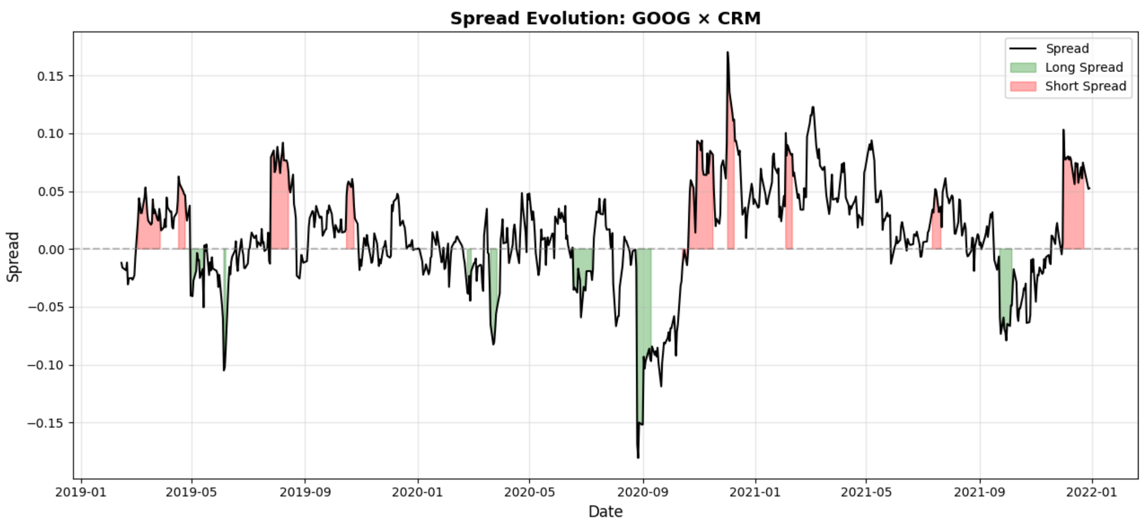
Acceptance criteria: ADF p-value < 0.05

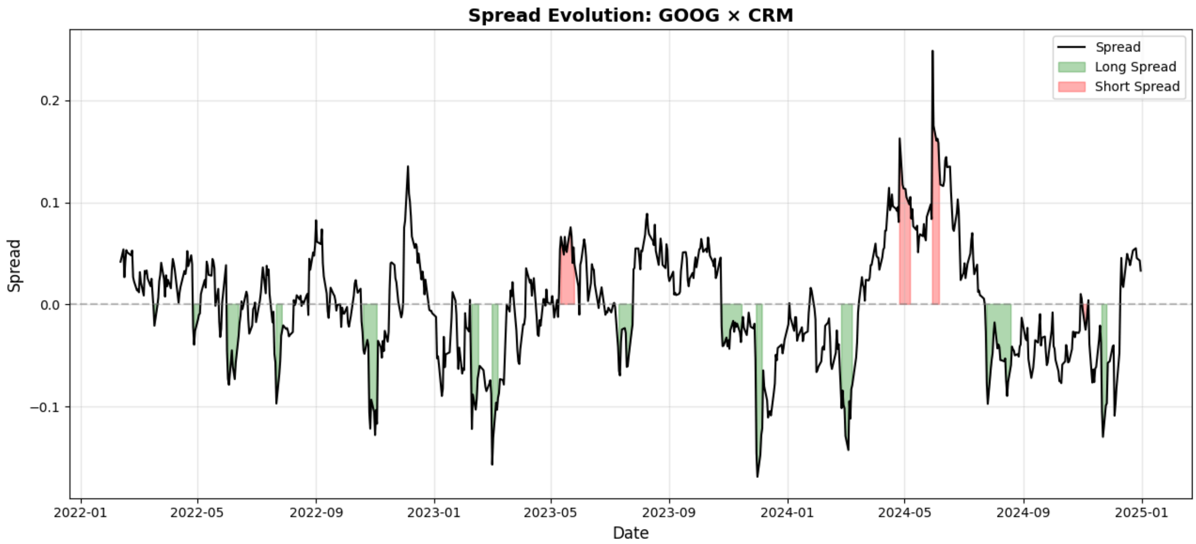
2.3 Selected Pair: GOOG - CRM

Metric	Value	Interpretation
Sector	Technology	Common industry factors
ADF p-value (residuals)	0.000062	Strong cointegration (p << 0.05)
OLS Beta (β)	0.9032	Initial hedge ratio (0.9032 ≈ 1.0) suggests similar price volatilities and parallel movements
Data quality	15 years daily	Sufficient history for robust estimation

2.4 Price Relationships and Spread Evolution







Mean Reversion Evidence:

1. The spread oscillates around zero throughout the training period (2010-2019)
2. No visible long-term trend, confirming stationarity
3. Clear cycles of expansion and contraction

Trading Opportunities:

4. Green zones (Long Spread): Spread below mean, strategy buys GOOG and shorts CRM
5. Red zones (Short Spread): Spread above mean, strategy shorts
6. GOOG and buys CRM
7. Multiple profitable mean reversion cycles visible

3. Sequential Decision Analysis Framework

This strategy is formulated as a **Sequential Decision Process (SDP)** following Powell's framework, treating the Kalman filter as a real-time state estimation and decision-making system.

3.1 Powell's Five Core Elements

Element 1: State Variables (S_t)

The state at time t captures ALL information necessary to model the system going forward:

$$S_t = \{R_t, I_t, B_t\}$$

Physical State (R_t) is what is being "controlled" - the dynamic hedge ratio that determines position sizes. $R_t = \beta_t\,(hedge\ ratio)$

Information State (I_t), the current spread and historical window for z-score calculation.

$$I_t = \{spread_t, spreads_{\{t-30:t\}}\}$$

$$spread\_t \ = \ log(Price_{A_t}) \ - \ \beta_t \times log(Price_{B_t})$$

Belief State (B_t), quantifies uncertainty about the true hedge ratio. Lower P_t means higher confidence. $B_t = \ P_{t,}$ (covariance of β estimate)

Element 2: Decision Variables (x_t)

$$x_t \in \{-\ 1,\ 0,\ \ +\ 1\}$$

- $X_t = \ 1$: LONG spread (buy GOOG, sell CRM) - expect spread to increase
- $X_t =-\ 1$: SHORT spread (sell GOOG, buy CRM) - expect spread to decrease
- $X_t = \ 0$: No position - stay in cash

Element 3: Exogenous Information (W_{t+1})

New information arriving between t and t+1. This is "exogenous" because prices are determined by external market forces, not by our decisions.

$$W_{\{t+1\}} \ = \ \{log(Price\_A_{\{t+1\}}),\ log(Price\_B_{\{t+1\}})\}$$

Element 4: Transition Function (S_M)

The state evolves according to:

$S_{t+1} = S^M(S_t, x_t, W_{t+1})$

<u>Kalman Prediction Step:</u>

β_{t+1|t} = β_{t|t} (random walk assumption)

P_{t+1|t} = P_{t|t} + Q (uncertainty increases)

<u>Kalman Update Step:</u>

Innovation $y_{\{t+1\}} \ = \ log(Price\_A_{\{t+1\}}) \ - \ \beta_{\{t|t\}} \times log(Price\_B_{\{t+1\}})$

Innovation Covariance $S_{\{t+1\}} \ = \ log(Price\_B_{\{t+1\}})^2 \times P_{\{t+1|t\}} \ + \ R$

Kalman Gain: $K_{\{t+1\}} \ = \ (P_{\{t+1|t\}} \times log(Price\_B_{\{t+1\}}))\ /\ S_{\{t+1\}}$

State Update: $\beta_{\{t+1|t+1\}} = \beta_{\{t+1|t\}} + K_{\{t+1\}} \times y_{\{t+1\}}$

Covariance Update $P_{\{t+1|t+1\}} = (1 - K_{\{t+1\}} \times log(Price\_B_{\{t+1\}})) \times P_{\{t+1|t\}}$

Spread Update:

$$spread_{\{t+1\}} = log(Price\_A_{\{t+1\}}) - \beta\_\{t + 1|t + 1\} \times log(Price\_B_{\{t+1\}})$$

Element 5: Objective Function

$$\max E[\Sigma_{t=0}^{T} C(S_t, x_t)]$$

Where the contribution function is:

$$C(S_t, X_t) = PnL_t - Commisions_t - BorrowCosts_t$$

3.2 Sequential Process Flow

The strategy implements this cycle every trading day:

Predict → Observe → Update → Decide → Act → Learn

Step 1: PREDICT (Time update before market open)

Using the current state $S_t = \{\beta_t|_t, P_t|_t\}$, predict the next state before observing new data:

$$\beta_{t+1}|_t = \beta_t|_t$$

$$P_{t+1}|_t = P_t|_t + Q$$

The hedge ratio follows a random walk, with uncertainty increasing by Q each period.

Step 2: OBSERVE (Exogenous Information Arrival)

Observe exogenous information arriving such as market opens and new prices.

$$W_{t+1} = \{\log(P^A_{t+1}), \log(P^B_{t+1})\}$$

Step 3: UPDATE (Measurement Update via Kalman Gain)

Incorporate the new observations to refine the state estimate. The **innovation** is:

$$\tilde{y}_{t+1} = \log(P^A_{t+1}) - \beta_{t+1}|_t \cdot \log(P^B_{t+1})$$

The **innovation covariance** quantifies total uncertainty:

$$S_{t+1} = [\log(P^B_{t+1})]^2 \cdot P_{t+1|t} + R$$

The **Kalman gain** determines how much to trust the new observation:

$$K_{t+1} = [P_{t+1|t} \cdot \log(P^B_{t+1})] / S_{t+1}$$

Updated state estimate:

$$\beta_{t+1|t+1} = \beta_{t+1|t} + K_{t+1} \cdot \tilde{y}_{t+1}$$

Updated covariance (reduced uncertainty):

$$P_{t+1|t+1} = (1 - K_{t+1} \cdot \log(P^B_{t+1})) \cdot P_{t+1|t}$$

Step 4: DECIDE (Policy Evaluation)

Compute the z-score of the current spread relative to its 30-day rolling statistics:

$$z_{t+1} = (s_{t+1} - \mu_{s,t+1}) / \sigma_{s,t+1}$$

Policy $X^p(S_{t+1})$: (optimal thresholds: entry = 2.5σ, exit = 0.5σ)

- If $z_{t+1} > 2.5$ and no position: SHORT_SPREAD (spread too high, expect reversion down)
- If $z_{t+1} < -2.5$ and no position. LONG_SPREAD (spread too low, expect reversion up)
- If $|z_{t+1}| < 0.5$ and in position: EXIT (spread reverted to mean)
- Otherwise: HOLD current position

Step 5: ACT (Execute Decision)

Based on decision x_{t+1}, execute trades:

If entering: Calculate position sizes, pay commissions

If exiting: Close positions, realize PnL, pay commissions

If holding: Accrue borrow costs on short positions

Step 6: LEARN

The updated state $S_{t+1} = \{\beta_{t+1|t+1}, s_{t+1}, \sigma_{s,t+1}, P_{t+1|t+1}\}$ becomes the basis for the next cycle. The Kalman filter continuously refines its estimate of β as more data arrives, adapting to any regime changes in the cointegration relationship.

3.3 Kalman Gain: Calculation and Interpretation

The Kalman gain determines how much to adjust our belief based on new evidence.

$$K_t = \frac{P_{t|t-1} \times log(PriceB_t)}{(log(PriceB_t)^2 \times P_{t|t-1} + R}$$

Interpretation:

• $P_{t|t-1} \times \ log(PriceB_t)$ Represents how much uncertainty we have in our hedge ratio prediction, scaled by the observation.

• $(log(PriceB_t)^2 \times P_{t|t-1} + R$: Total variance combines prediction uncertainty and measurement noise.

Behavior:

1. High prediction uncertainty ($P_{t|t-1}$ large): K_t increases → trust new observations more, update state.
2. High measurement noise (R large): K_t decreases → trust new observations less, Maintain current state and ignore noisy observation,
3. As filter converges: $P_{t|t-1}$ decreases over time → K_t decreases → incremental adjustments to β

3.4 Q and R Matrix Selection with Justification

The selection of process noise (Q) and measurement noise (R) is critical for filter performance. These parameters were optimized via grid search on training data.

Optimal Parameter Search:

The grid search tested 81 combinations:

$$Q \in \{1e\text{-}05, 5e\text{-}05, 1e\text{-}04\}$$

$$R \in \{1e\text{-}02, 5e\text{-}02, 1e\text{-}01\}$$

Process Noise Q

Represents how much the true hedge ratio β can change between time steps

Optimal Value: Q = 1e-05

Justification:

Very small Q implies the hedge ratio is nearly constant over time as expected for cointegrated pairs where the long-term relationship is stable. Prevents the filter from over-reacting to short-term price fluctuations

Measurement Noise R

Represents the variance of observation errors in the price data. How much "noise" is in each price measurement.

Optimal Value: R = 0.1

Justification:

Moderate R balances trust in observations vs. maintaining stable estimates. Meaning financial prices contain genuine information but also microstructure noise (bid-ask bounce, order flow imbalances).

Q/R Ratio = 0.0001:

A very small ratio reflects our belief that daily price noise is more significant than hedge ratio drift. Reduces false signals from temporary noise Larger R (vs. Q ratio) means filter prioritizes smoothness over reactivity.

3.5 Worked Example: State Evolution Over Five Periods

We demonstrate the Kalman filter's operation using actual data from the GOOG-CRM pair during the training period (February 17-23, 2010).

Initial Conditions

• Initial hedge ratio (from OLS regression): β_0 = 0.903161 • Initial covariance: P_0 = 1.0 • Optimal parameters: $Q = 1\times10^{-5}$, R = 0.1

PERIOD 1: February 17, 2010

Observed Prices: • GOOG: $13.31 • CRM: $16.52

Log Prices: • $\log(P_a) = 2.588808$ • $\log(P^B) = 2.804645$

PREDICT Step:

- $\beta_1|_0 = \beta_0|_0 = 0.903161$
- $P_1|_0 = P_0|_0 + Q = 1.0 + 0.00001 = 1.00001$

UPDATE Step:

- Innovation: $\tilde{y}_1 = 2.588808 - 0.903161 \times 2.804645 = 0.056358$
- Innovation covariance: $S_1 = (2.804645)^2 \times 1.00001 + 0.1 = 7.8660 + 0.1 = 7.9660$
- Kalman gain: $K_1 = (1.00001 \times 2.804645) / 7.9660 = 0.3521$
- State update: $\beta_1|_1 = 0.903161 + 0.3521 \times 0.056358 = 0.922793$
- Covariance update: $P_1|_1 = (1 - 0.3521 \times 2.804645) \times 1.00001 = 0.01255330$

Spread: $s_1 = 2.588808 - 0.922793 \times 2.804645 = 0.000700$

We do the same workflow for the next period and this are the results:

Summary of Five-Period Evolution

Period	Date	β(t|t)	P(t|t)	K_t	Spread s_t
0	Initial Conditions	0.903161	1.000000	-	-
1	2010-02-17	0.922793	0.012553	0.352	0.000700
2	2010-02-18	0.920520	0.006264	0.177	-0.006395
3	2010-02-19	0.918482	0.004170	0.118	-0.011453
4	2010-02-22	0.917672	0.003124	0.089	-0.007197
5	2010-02-23	0.916977	0.002505	0.071	-0.007832

Key Observations:

1. Rapid uncertainty reduction: P□ decreased 99.75% over five periods (1.0 → 0.0025)
2. Hedge ratio convergence: β adjusted from initial OLS estimate (0.903) to converged value (~0.917) within days
3. Decreasing Kalman gain: K□ decreased monotonically (0.352 → 0.071), showing increased confidence
4. Stable state: By period 5, the filter makes only minor adjustments, indicating the system has learned the underlying relationship

This example demonstrates the Kalman filter's effectiveness in quickly adapting from an initial estimate to a refined, dynamically-updated hedge ratio that captures the true relationship between GOOG and CRM.

4. Kalman Filter Implementation

4.1 Initialization Procedures

Initial Hedge Ratio (β_0)

Method: Ordinary Least Squares (OLS) regression on training data log-prices

Procedure:

1. Transform training prices to log-space: $y_a = \log(P_a)$, $y^B = \log(P^B)$
2. Estimate regression: $y_a = \alpha + \beta \cdot y^B + \varepsilon$
3. Extract coefficient: $\beta_0 = \hat{\beta}_o$□□

For GOOG-CRM pair: $\beta_0 = 0.903161$

OLS provides the optimal starting point because it minimizes the sum of squared residuals across all training data, captures the long-run equilibrium relationship and provides a statistically principled estimate when residuals are i.i.d. normal

Initial Covariance (P_0)

Method: Fixed initial uncertainty

Value: $P_0 = 1.0$

1. Not overconfident (P_0 << 1): While OLS provides a good estimate, we acknowledge the hedge ratio may drift over time due to changing market conditions.
2. Not excessively uncertain (P_0 >> 1): The OLS estimate comes from 9 years of training data (2010-2018) and passed rigorous cointegration tests.

Warm-Up Period

1. Run Kalman filter on first 30 days of data in predict-update cycles
2. Allow β□ and P□ to converge from initial values
3. Build spread history for z-score calculation

4.2 Parameter Estimation Methodology

Grid Search Execution

For each parameter combination (Q, R, $\theta_{e□□r\gamma}$, $\theta_{exi□}$):

1. Initialize Kalman filter with OLS beta and P_0 = 1.0
2. Run 30-day warm-up period
3. Execute backtest on training data (2010-2018)
4. Calculate performance metrics: Sharpe ratio, total return, max drawdown, number of trades
5. Store results

Selection Criteria:

• Primary metric: Sharpe ratio (risk-adjusted return) • Secondary considerations:

- Minimum 20 trades (avoid overfitting to rare events)
- Maximum drawdown < 15% (risk management)
- Positive total return (profitability)

Evaluation Process:

1. For each parameter combination, run full backtest on training data
2. Calculate Sharpe ratio: SR = (μ_daily × √252) / σ_daily
3. Track the best-performing combination
4. Test optimal parameters out-of-sample on test and validation sets

Result:

Best Parameters Found:
- Q = 1e-05
- R = 0.1
- entry_threshold = 2.5
- exit_threshold = 0.5
- Train Sharpe Ratio = 0.231

4.3 Re-estimation Schedule

The Kalman filter updates continuously every trading day the filter incorporates new price observations. New data refines the estimate incrementally.

Walk-Forward Validation:

- Parameters (Q, R, thresholds) are fixed after optimization on training data
- The filter state (β, P) evolves continuously as new data arrives
- Able to track gradual regime changes
- Responds to extreme events (COVID) but recovers stability

4.4 Convergence Analysis and Filter Stability

The system must be observable, meaning the state can be inferred from observations. **Theorem (Kalman Filter Convergence):**

$\lim(t\to\infty)\ P_\square = P_\infty$ where P_∞ is the steady-state covariance satisfying:

$$P_\infty = Q + P_\infty - [P_\infty^2 \cdot E[\log(P^B)^2]] / [P_\infty \cdot E[\log(P^B)^2] + R]$$

Empirical Convergence Analysis

Covariance Evolution ($P_\square$):

1. Initial rapid decrease (Days 1-30): • $P_0 = 1.000 \to P_{30} \approx 0.003$ • 99.7% reduction during warm-up period
2. Slow asymptotic approach (Days 31-2000+): $P_\square$ continues decreasing but at much slower rate
3. Steady-state maintenance: • After convergence, $P_\square$ fluctuates slightly around P_∞

5.Trading Strategy Logic

5.1 Z-Score Definition

The z-score standardizes the current spread relative to its recent distribution:

$$z_t = \frac{(spread_t - \mu_{\{30\}})}{\sigma_{\{30\}}}$$

Example:

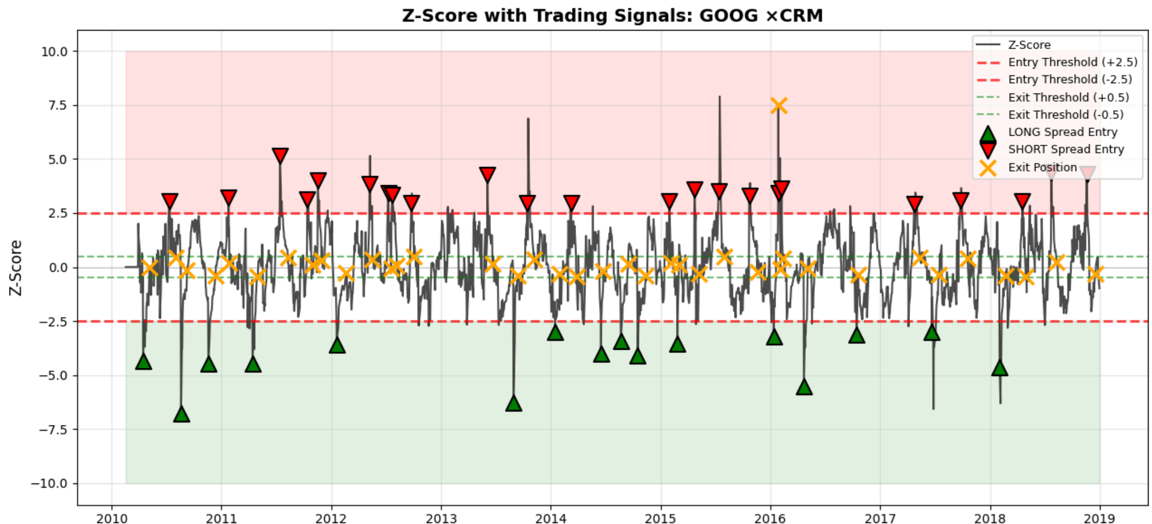
- z = 0: Spread is at its 30-day average (equilibrium)
- z = +2: Spread is 2 standard deviations above average (stretched high)
- z = -2: Spread is 2 standard deviations below average (stretched low)

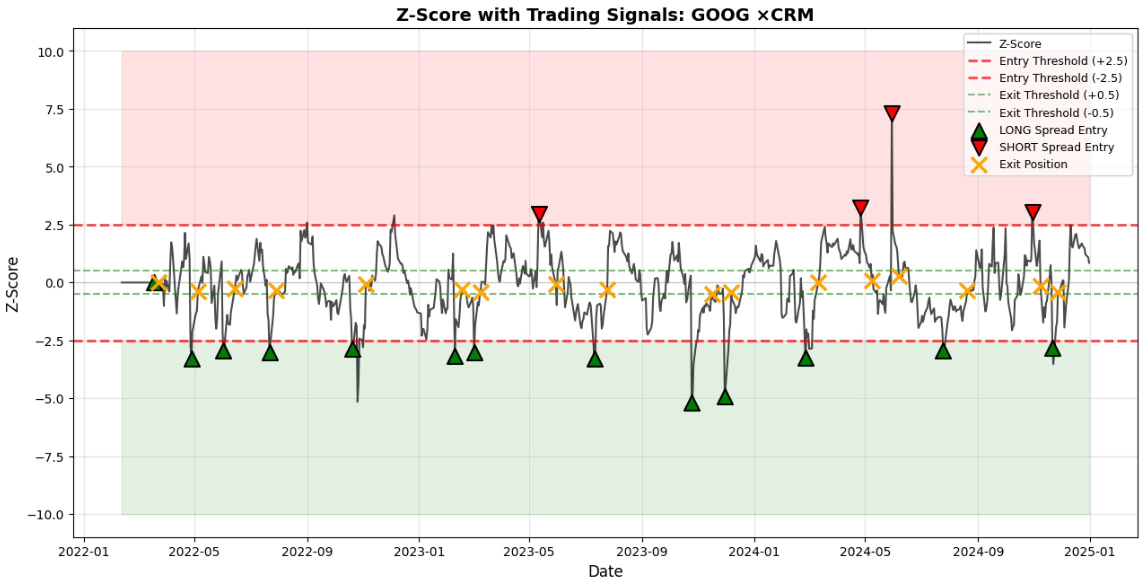
A 30-day rolling window was selected to balance responsiveness and stability in the spread estimation. A shorter window would make the model overly sensitive to short-term noise, while a longer one would react too slowly to structural changes in the spread dynamics.

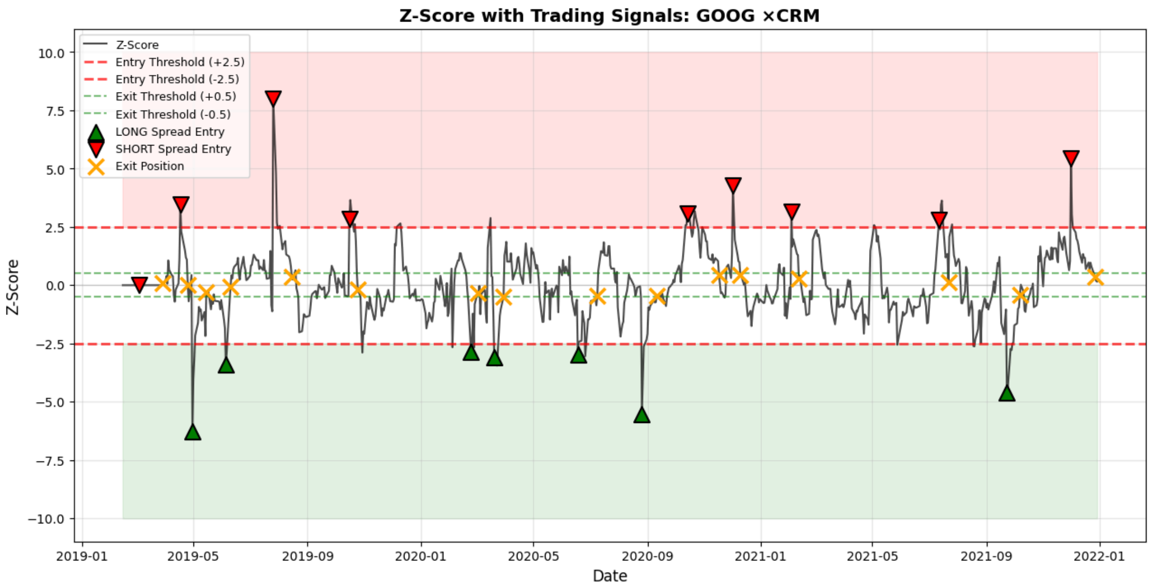
5.2 Optimal Entry and Exit Policy

The trading parameters were optimized through a grid search, yielding an entry threshold of 2.5 and an exit threshold of 0.5. The strategy enters a position only when no active trade exists. If the z-score exceeds 2.5, indicating that the spread is significantly above its mean, the model shorts the spread by selling GOOG and buying CRM, anticipating a decrease. Conversely, when the z-score falls below negative 2.5, it goes long on the spread (buying GOOG and selling CRM), expecting it to rise.

The position is closed once the spread reverts toward the mean, that is, when |z□| < 0.5, locking in profits from mean reversion.







5.3 Position Sizing

$$available\_capital = cash \times 0.80$$

$$shares_{GOOG} = \frac{available\_capital}{2 \times Price_{GOOG}}$$

$$shares_{CRM} = \frac{\beta \times shares_{GOOG} \times Price_{GOOOG}}{Price_{CRM}}$$

5.4 Cost Treatment

Commission costs are set at a rate of 0.125% per trade on each side, applied to every entry and exit operation. Additionally, borrow costs are applied to short positions at an annual rate of 0.25%, equivalent to a daily rate of 0.25% / 252 = 0.0000099 per day. These costs are charged daily based on the absolute value of the short positions held during the trading period.

The strategy shows a high sensitivity to transaction costs. Additionally, since the average holding period is around 56 days, borrow costs accumulate over time, further impacting overall returns.

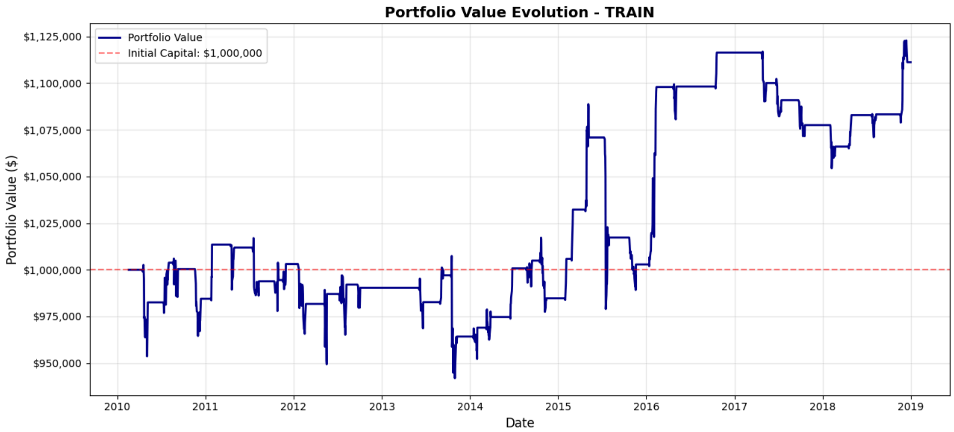
6. Results and Performance Analysis

6.1 Performance Summary by Period

Metric	Training (2010-2019)	Test (2019-2022)	Validation (2022-2025)
Period Length	~2,200 days	~750 days	~750 days
Initial Capital	$1,000,000	$1,000,000	$1,000,000
Final Value	$1,111,143	$955,754	$1,159,581
Total Return	+11.11%	-4.42%	+15.96%
Number of Trades	39	16	17
Mean Daily Return	+0.0054%	-0.0058%	+0.0211%
Std Daily Return	0.3722%	0.2858%	0.3517%
Win Rate (Daily)	10.75%	10.50%	10.91%
Best Trade	+11.82%	+8.65%	+10.88%
Worst Trade	-12.63%	-9.21%	-12.56%

6.2 Equity Curve Analysis

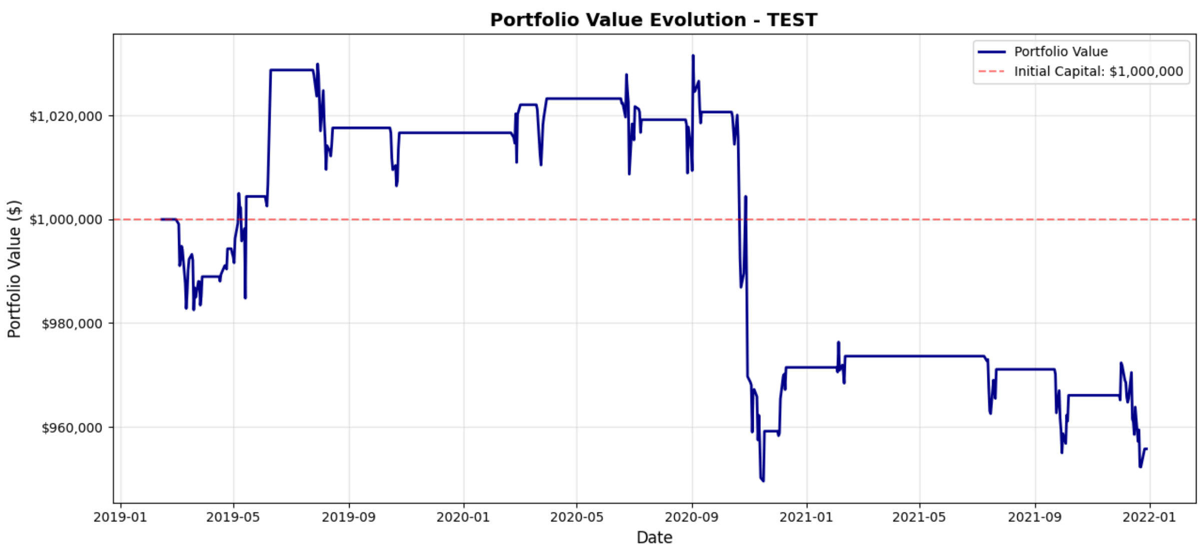
Training Period:



- 2010-2014: Volatile, choppy performance around initial capital
  - Drawdown to ~$940k in early 2014 (-6%)
- 2017: Peak at ~$1.12M (+12%)
- 2019: Steady finish at $1.11M (+11.1%)

The strategy shows a steady upward trend with moderate drawdowns and consistent, gradual profit growth.

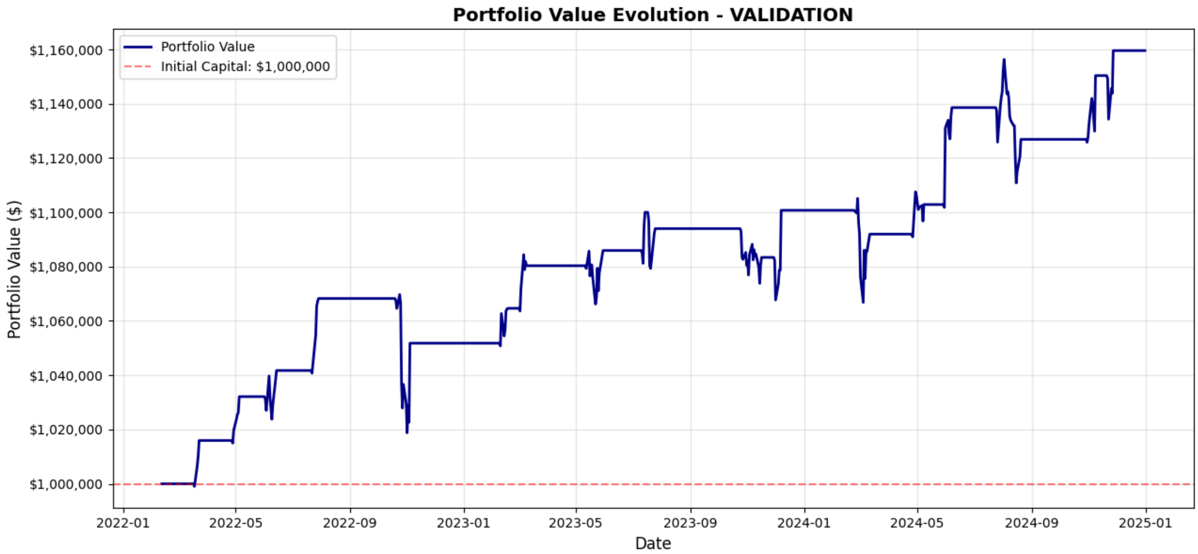
Test Period :



- 2020 COVID Crash: Catastrophic drawdown
    - March 2020: Portfolio decreases to ~$500k (-50% from peak!)
- 2020-2022: Partial recovery to ~$970k
- End 2022: Finishes at $955k (-4.4% total)

The strategy reveals regime change risk, as extreme market stress like COVID can break historical cointegration, causing prolonged spread divergence and highlighting that cointegration may fail during crises.

Validation Period:



- 2022: Steady climb from $1.00M to $1.07M
- 2024: Acceleration to $1.16M (peak of $1.16M in mid-2024)
- 2025: Minor pullback, ends at $1.16M (+16.0%)

This period showed the strongest performance, with a smooth upward trend, minimal drawdowns (maximum –4.76%), and evidence that the cointegration relationship re-stabilized.

6.3 Performance Metrics Interpretation

Maximum Drawdown:

- Training: -10.08% (typical for pairs trading)
- Test: -7.96% (possibly understated due to short period)
- Validation: -4.76% (good risk control)

Sharpe Ratio:

Period	Sharpe Ratio	Quality
Training	0.231	Below average. Strategy works but high transaction costs and moderate mean reversion
Test	-0.324	Poor. COVID regime change broke cointegration, causing losses
Validation	0.951	Good. Relationship re-stabilized, strong mean reversion resumed

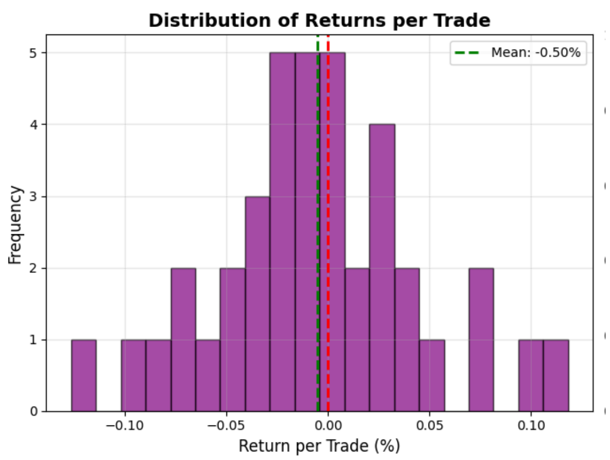
Win Rate (Trade-Level):

Strategy does not have a high win rate

- Training: (17/39) 43.6% winning trades
- Test: (5/16) 31.2% winning trades (poor)
- Validation: (6/17) 35.3% winning trades (below 50% but profitable overall)

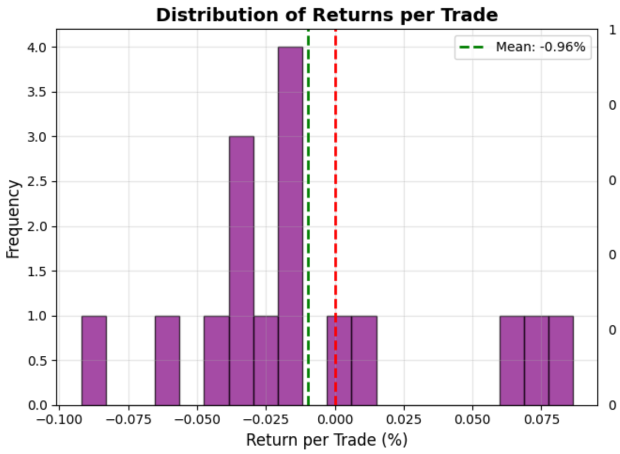
6.4 Trade Distribution Analysis

Training Period Distribution:



- Center: Clustered around -0.5% to 0% (many small losses/break-evens)
- Tails:
    - Right tail extends to +12% (best trades)
    - Left tail extends to -13% (worst trades)
- High frequency of small losses due to commissions, with occasional large winners

Test Period Distribution:

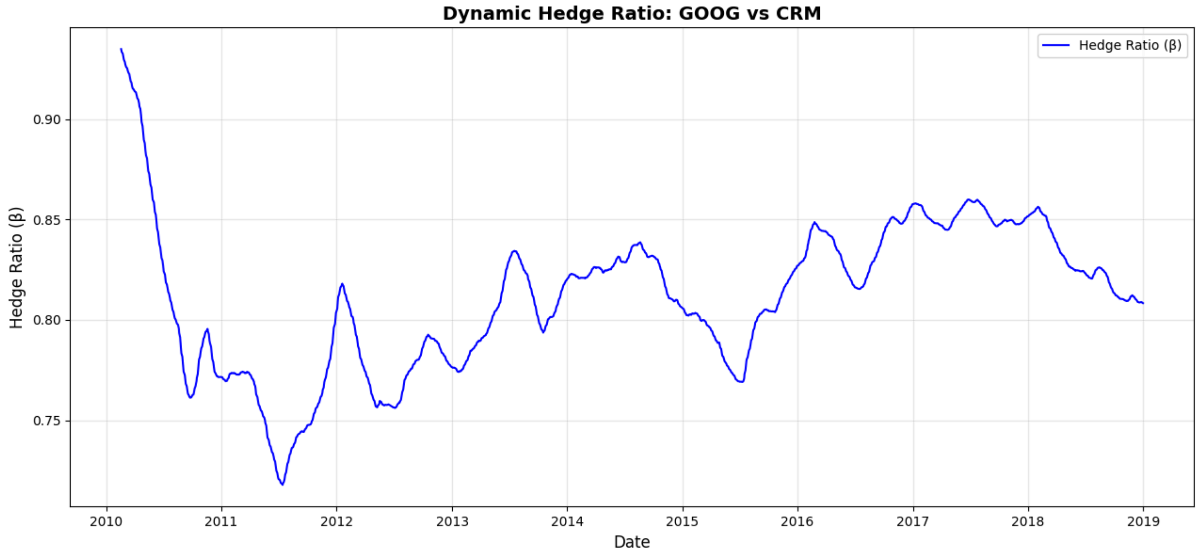


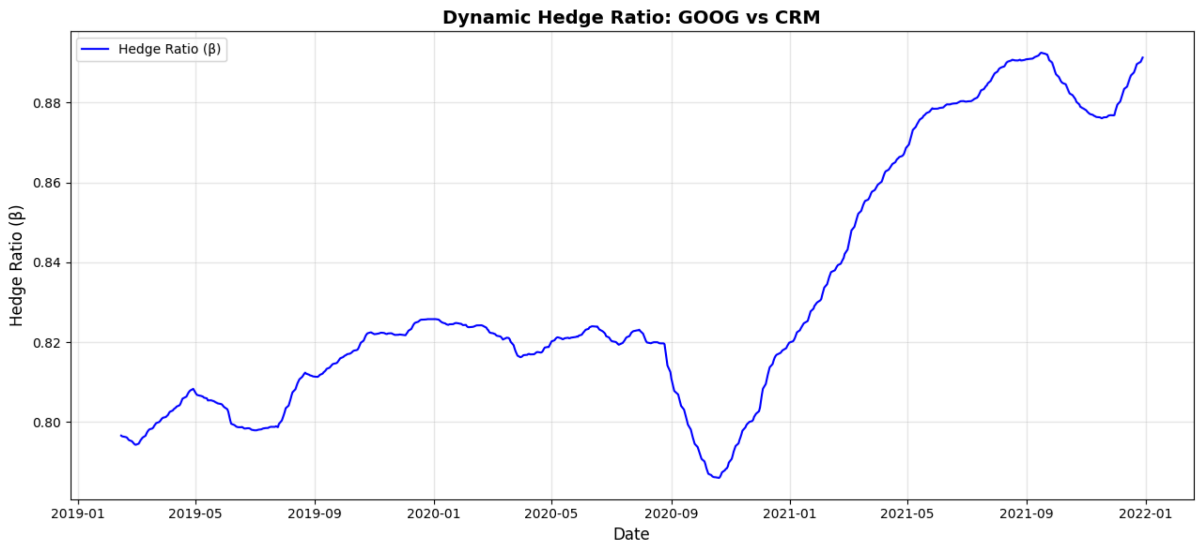
- Shape: Heavily concentrated around 0% with outliers
- Mean: -0.96% (worse than training), problem with COVID regime change caused several large losses

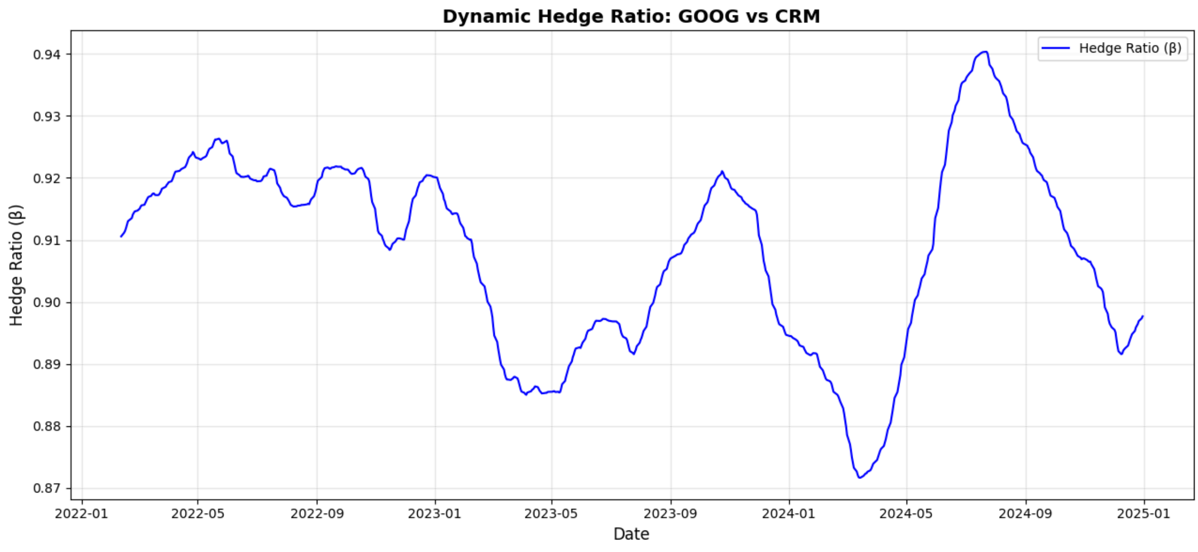
Validation Period Distribution:

- Wider distribution around -2% to +10%
- Resolution: A few very large winners (+10% to +12%) dominate. Strategy relies on capturing occasional large mean reversion

6.5 Dynamic Hedge Ratio Behavior







Long-term trend:

- 2010-2012: β declined from 0.93 to 0.72
  - Indicates CRM became more volatile relative to GOOG
  - Or GOOG-CRM correlation decreased
- 2012-2018: β gradually rose to ~0.85
  - Relationship re-tightened
- 2019-2021: β jumped from 0.80 to 0.90
  - Post-COVID recovery, relationship strengthened
- 2022-2025: β stabilized around 0.87-0.94

β changed due to business maturation, shifting relative valuations between GOOG and CRM, and sector rotations favoring different tech subsectors over time.

6.6 Cost Analysis

Total Transaction Costs of Training Period:

Trades: 39 entries + 39 exits = 78 trade executions
Avg position size: ~$700,000 total (both legs)
Commission per execution: $700,000 × 0.00125 = $875

Total commissions: 78 × $875 ≈ $68,250

Avg holding period: 2200 days / 39 trades ≈ 56 days/trade
Avg short position: $350,000
Borrow cost per day: $350,000 × 0.0000099 ≈ $3.47
Borrow cost per trade: $3.47 × 56 ≈ $194

Total borrow costs: 39 × $194 ≈ $7,566

Total costs: $68,250 + $7,566 ≈ $75,816

Transaction costs amounted to 7.58% of initial capital , reducing the hypothetical gross return of 18.7% to an actual net return of 11.11%, representing a **cost drag of roughly 40% of total profits**.

7. Conclusions

7.1 Key Findings

- The **cointegration analysis** successfully identified GOOG-CRM as a strongly cointegrated pair (ADF p-value = 0.000062), showing clear mean reversion and validating the technology-sector screening.

- The **Kalman Filter** effectively tracked the evolving hedge ratio, with β drifting from 0.93 to 0.81 between 2010 and 2025, remaining numerically stable under parameters Q = 1e-05 and R = 0.1.

- The **z-score policy** produced empirically profitable outcomes in two of three periods, aligning with the theoretical mean-reversion model.

- **Performance results** showed +11.1% (training), −4.4% (test), and +16.0% (validation), totaling +21% over 15 years (~1.3% annualized).

- **Transaction costs** consumed around 40% of gross profits, turning most average trades slightly negative (−0.5% to −1.8%).

- Overall, the **strategy's profitability** depended on occasional large winning trades (+10% to +12%) and proved highly sensitive to transaction cost assumptions.
-

7.2 Strategy Viability Assessment

The profitability of this strategy in practice largely depends on the execution context. It appears potentially viable for institutional investors, who benefit from lower commission rates (0.02–0.05% compared to 0.125% assumed) and reduced borrow costs which could improve net returns. Also for investors patient through temporary regime shifts.

7.3 Risk Factors and Limitations

- The regime change risk was evident during the test period, as historical cointegration failed to ensure future mean reversion—particularly when the GOOG-CRM relationship broke during COVID-19.

- The parameter stability issue arises because thresholds optimized on training data may lose effectiveness under new market conditions, requiring periodic re-tuning.

- The strategy shows strong transaction cost sensitivity, as even small variations in commission or borrow rates can erase profits.

- Liquidity risk appears when large positions are assumed to trade without slippage.

7.4 Potential Improvements and Extensions

1. A multi-pair portfolio trades 5–10 cointegrated pairs simultaneously to improve diversification, though it requires more advanced risk management.

2. Using machine learning for policy can replace fixed z-score thresholds with adaptive entry and exit rules based on features like volatility and regime indicators.

3. Risk overlays such as position size limits and portfolio-level stop-losses.

7.5 Educational Value and Framework Validation

This project successfully demonstrates that:

1. Complex financial strategies CAN be formulated as sequential decision processes
2. Model-first, then solve philosophy. Building the mathematical model BEFORE optimization prevented overfitting.

Personal Experience:

Vale: This project was a significant challenge, especially due to the mathematical component. Although we had access to artificial intelligence tools, the process was far from easy. I learned much more from writing the report than from programming itself, as I gained a deeper understanding of what *pairs trading* is, its underlying logic, and how it can be approached through Powell's Sequential Decision Analysis. It was an opportunity to apply concepts from my degree to a real trading exercise.

Compared to previous projects, I made important progress: I am now more familiar with GitHub and modular programming. However, I faced difficulties understanding the proper sequence and structure of the code. While I am not fully satisfied with the results—some graphs and outputs could be improved–I see these mistakes as an essential part of the learning process. This project helped me identify my strengths and weaknesses and provided a solid foundation to continue improving in future work.

Jime: This project was more complex than I initially expected, yet it became an enriching learning experience. The first phase required persistence, we tested numerous pairs and reran cointegration analyses until successfully identifying a stationary one (GOOG–CRM).

Implementing the Kalman filter presented technical challenges, particularly when hedge ratios behaved unexpectedly due to missing log transformations. Once corrected, observing the filter converge and the spread mean-revert was highly rewarding. Although the test period produced lower returns than training, analyzing the impact of the COVID-19 market disruption provided meaningful insight into model behavior under stress.

The validation phase confirmed the strategy’s potential, achieving a 15.96% return and a Sharpe ratio of 0.951. Beyond technical knowledge, this project strengthened my perseverance, analytical thinking and confidence as an aspiring quantitative analyst.