Dig-In:

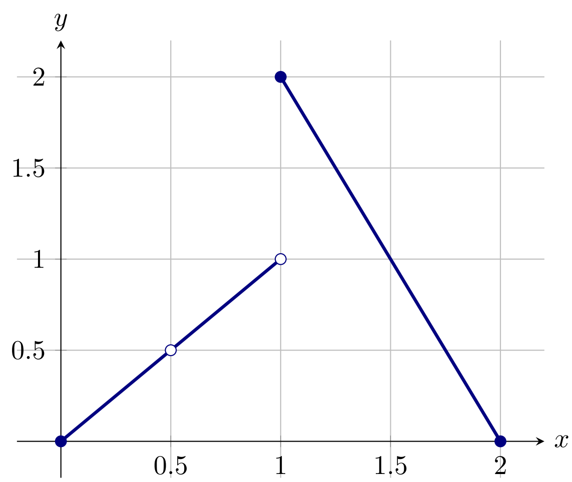
Continuity

The idea of continuity. This section is adapted from the source code calculus/whatIsALimit/digInContinuity.tex *available at* https://github.com/mooculus/calculus.

Idea 1. *A function f is* ***continuous at*** *$x = a$ if you can trace through the point $(a, f(a))$ without lifting your pen.*

YouTube link: https://www.youtube.com/watch?v=hXFLvVFQa5k

Question 1 *Consider the graph of the function f*



Which of the following are true?

Multiple Choice:

(a) *f is continuous at $x = 0.5$*

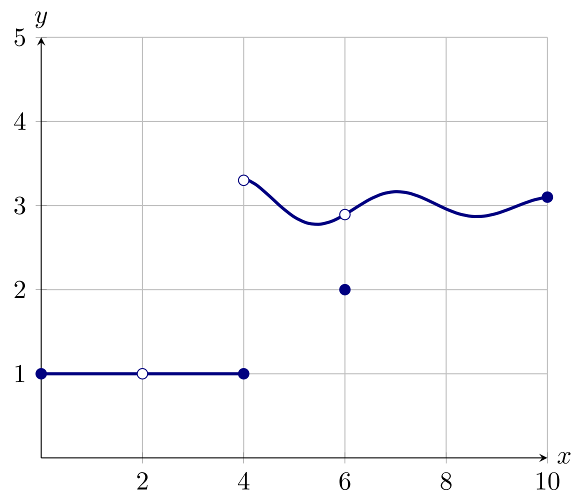
(b) *f is continuous at $x = 1$*

(c) *f is continuous at $x = 1.5$* ✓

Learning outcomes: Identify continuous functions from their graphs. Identify x-values where a function is discontinuous from a graph.

Author(s):

Example 1. *Give x-values where the function below is discontinous (i.e. not continuous).*



Explanation. *To start, f is not even defined at $x = \underset{\text{given}}{\boxed{2}}$, therefore f cannot be continuous at $x = \underset{\text{given}}{\boxed{2}}$ as you must lift your pen over the hole in the graph.*

Next, from the plot above we see that $\lim_{x\to 4} f(x)$ does not exist because

$$\lim_{x\to 4^-} f(x) = \underset{\text{given}}{\boxed{1}} \qquad \text{and} \qquad \lim_{x\to 4^+} f(x) \approx 3.3$$

This causes a "jump" in the function at $x = 4$ where you must lift your pen. This means f cannot be continuous at $x = 4$.

We also see that $\lim_{x\to 6} f(x)$ exists and $\lim_{x\to 6} f(x) \approx 2.9$. However, as $f(6) = \underset{\text{given}}{\boxed{2}}$, we must pick up our pen to color in the point $(6, f(6))$ as we trace along the graph. This means f is not continuous at $x = 6$.

Remark 1. *The following common functions are continuous at every x-value in their domains. Try visualizing some of their graphs to convince yourself of the continuity.*

Constant function $f(x) = k$ *for k a real number*

Power functions $f(x) = x, f(x) = x^2, f(x) = x^3, \cdots$ *or more generally* $f(x) = x^r$ *for $r > 0$*

Exponential function $f(x) = a^x$ *with base $a > 0$*

Logarithmic function $f(x) = \log_a(x)$ *with base $a > 0$*