Dig-In:

The Chain Rule

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The Chain Rule will let us find the derivative of a composition.

Example 1. *Find the derivative of* $y = (4x^3 + 15x)^2$

This is not a simple polynomial, so we cant use the power rule yet. This function is a product, so we could write it as $(4x^3 + 15x)(4x^3 + 15x)$ *and use the product rule. Or we could multiply/F.O.I.L it out and simply differentiate the resulting polynomial. Let's do the second way:*

$$y = (4x^3 + 15x)(4x^3 + 15x) = 16x^6 + 120x^4 + 225x^2$$

$$y' = 64x^5 + 480x^3 + 450x$$

Now suppose we want to find the derivative of $y = (4x^3 + 15x)^{20}$. We **could** write it as a product with 20 factors and use the product rule, or we **could** multiply it out. But I don't want to do that, do you?

We need an easier way, a rule that will handle a composition like this. The Chain Rule is a little complicated, but it saves us the much more complicated algebra of multiplying something like this out. It will also handle compositions where it wouldn't be possible to "multiply it out."

The Chain Rule is the most common place for students to make mistakes. Part of the reason is that the notation takes a little getting used to. And part of the reason is that students often forget to use it when they should. When should you use the Chain Rule? Almost every time you take a derivative.

Formula 1 (Chain Rule)**.** *Here* $f(x)$ *and* $g(x)$ *are differentiable functions and* $y = f(g(x))$.

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

in words: The derivative of a composition is the derivative of the outside, with the inside staying the same, TIMES the derivative of the inside function.

Example 2. *Find the derivative of* $y = (4x^3 + 15x)^2$.

This is the same one we did before by multiplying out. This time, lets use the Chain Rule: The inside function is what appears inside the parentheses:

Learning outcomes:
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$g(x) = \underbrace{\boxed{4x^3 + 15x}}_{\text{given}}$. *In our mind, we can replace the inside function with an "x" and read off the outside function* $f(x) = \underbrace{\boxed{x^2}}_{\text{given}}$.