

Dig-In:

Exponential functions

Exponential functions illuminated.

Exponential functions may seem somewhat esoteric at first, but they model many phenomena in the real-world.

What are exponential functions?

Definition 1. An *exponential function* is a function of the form

$$f(x) = a^x$$

where $a \neq 1$ is a positive real number. The domain of an exponential function is $(-\infty, \infty)$.

Question 1 Is 4^{-x} an exponential function?

Multiple Choice:

- (a) yes ✓
- (b) no

Feedback(attempt): Note that

$$4^{-x} = (4^{-1})^x = \left(\frac{1}{4}\right)^x.$$

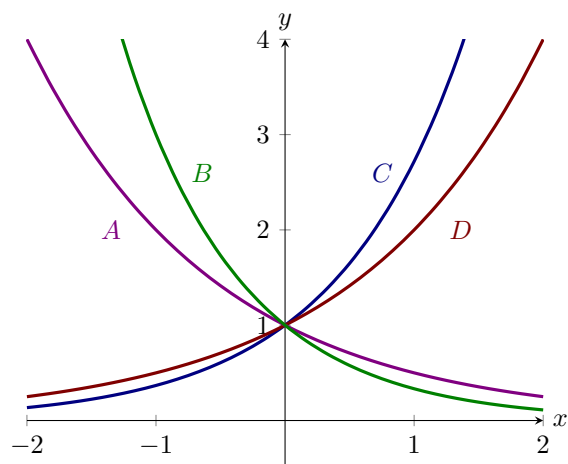
What can the graphs look like?

Graphs of exponential functions

Example 1. Here we see the the graphs of four exponential functions.

Learning outcomes:
Author(s):

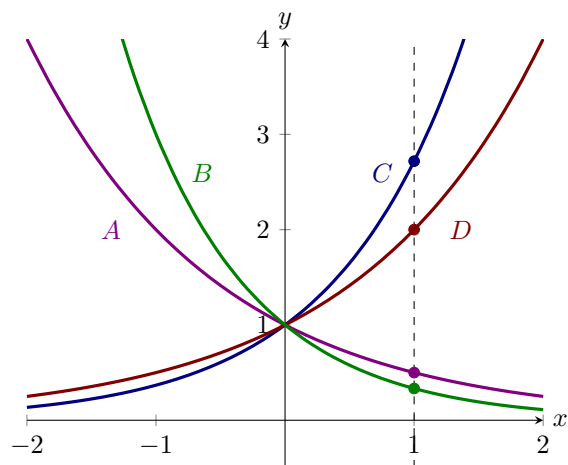
Exponential functions



Match the curves A, B, C, and D with the functions

$$e^x, \quad \left(\frac{1}{2}\right)^x, \quad \left(\frac{1}{3}\right)^x, \quad 2^x.$$

Explanation. One way to solve these problems is to compare these functions along the vertical line $x = 1$,



Note

$$\left(\frac{1}{3}\right)^1 < \left(\frac{1}{2}\right)^1 < 2^1 < e^1.$$

Hence we see:

- $\left(\frac{1}{3}\right)^x$ corresponds to \boxed{B} .
given

- $\left(\frac{1}{2}\right)^x$ corresponds to \boxed{A}_{given} .
- 2^x corresponds to \boxed{D}_{given} .
- e^x corresponds to \boxed{C}_{given} .

Properties of exponential functions

Working with exponential functions is often simplified by applying properties of these functions. We will use these properties throughout the semester.

Properties of exponents

Let a be a positive real number with $a \neq 1$.

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^{-1} = \frac{1}{a}$

Question 2 What exponent makes the following true?

$$2^4 \cdot 2^3 = 2^{\boxed{7}}$$

Hint:

$$(2^4) \cdot (2^3) = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$$

Example 2. Solve the exponential equation.

$$27^x = 9$$

Explanation. Once we have the same base on both sides of the equation, we may equate exponents. In this example, we observe that 27 and 9 are both

powers of 3. That is, $27 = 3^{\boxed{3}}$ and $9 = 3^{\boxed{2}}$, and therefore we can write the exponential equation as

$$3^{\boxed{3x}} = 3^{\boxed{2}}$$

By equating exponents, we arrive at $x = \frac{2}{3}$.