

Dig-In:

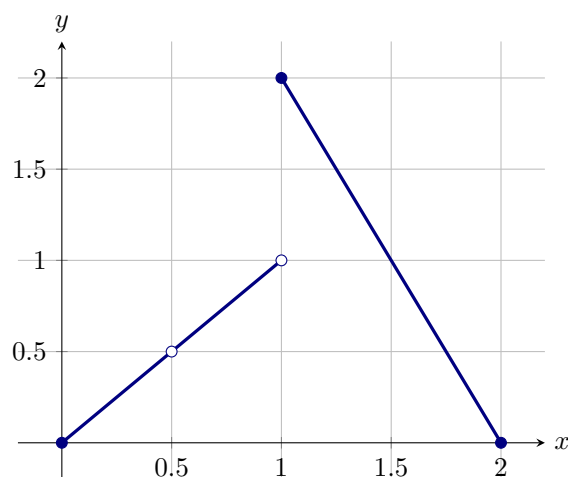
Continuity

The idea of continuity. This section is adapted from source code available at <https://github.com/mooculus/calculus>

Idea 1. A function f is **continuous at** $x = a$ if you can trace through the point $(a, f(a))$ without lifting your pen.

YouTube link: <https://www.youtube.com/watch?v=hXFLvVFQa5k>

Question 1 Consider the graph of the function f



Which of the following are true?

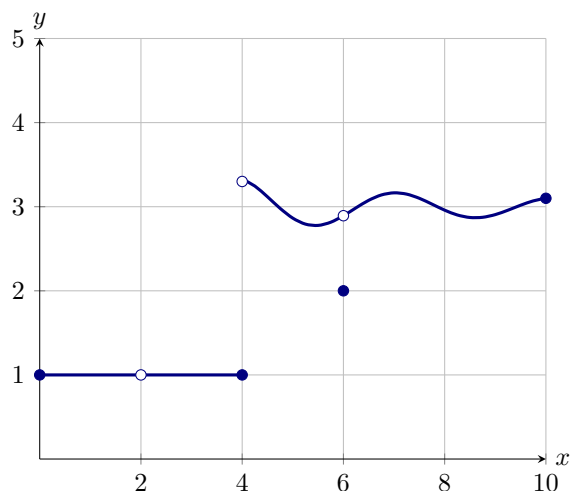
Multiple Choice:

- (a) f is continuous at $x = 0.5$
- (b) f is continuous at $x = 1$
- (c) f is continuous at $x = 1.5$ ✓

Learning outcomes: Identify continuous functions from their graphs. Identify x -values where a function is discontinuous from a graph.

Author(s):

Example 1. Give x -values where the function below is discontinuous (i.e. not continuous).



Explanation. To start, f is not even defined at $x = \boxed{2}_{\text{given}}$, therefore f cannot be continuous at $x = \boxed{2}_{\text{given}}$ as you must lift your pen over the hole in the graph.

Next, from the plot above we see that $\lim_{x \rightarrow 4} f(x)$ does not exist because

$$\lim_{x \rightarrow 4^-} f(x) = \boxed{1}_{\text{given}} \quad \text{and} \quad \lim_{x \rightarrow 4^+} f(x) \approx 3.3$$

This causes a “jump” in the function at $x = 4$ where you must lift your pen. This means f cannot be continuous at $x = 4$.

We also see that $\lim_{x \rightarrow 6} f(x)$ exists and $\lim_{x \rightarrow 6} f(x) \approx 2.9$. However, as $f(6) = \boxed{2}_{\text{given}}$, we must pick up our pen to color in the point $(6, f(6))$ as we trace along the graph. This means f is not continuous at $x = 6$.

Remark 1. The following common functions are continuous at every x -value in their domains. Try visualizing some of their graphs to convince yourself of the continuity.

Constant function $f(x) = k$ for k a real number

Power functions $f(x) = x, f(x) = x^2, f(x) = x^3, \dots$ or more generally $f(x) = x^r$ for $r > 0$

Exponential function $f(x) = a^x$ with base $a > 0$

Logarithmic function $f(x) = \log_a(x)$ with base $a > 0$