Test Course

Valerie Hower

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Dig-In:

1 Exponential functions

Exponential functions illuminated.

Exponential functions may seem somewhat esoteric at first, but they model many phenomena in the real-world.

What are exponential functions?

Definition 1. *An* ***exponential function*** *is a function of the form*

$$f(x) = a^x$$

where $a \neq 1$ is a positive real number. The domain of an exponential function is $(-\infty, \infty)$.

Question 1 *Is 4^{-x} an exponential function?*

Multiple Choice:

(a) *yes* ✓

(b) *no*

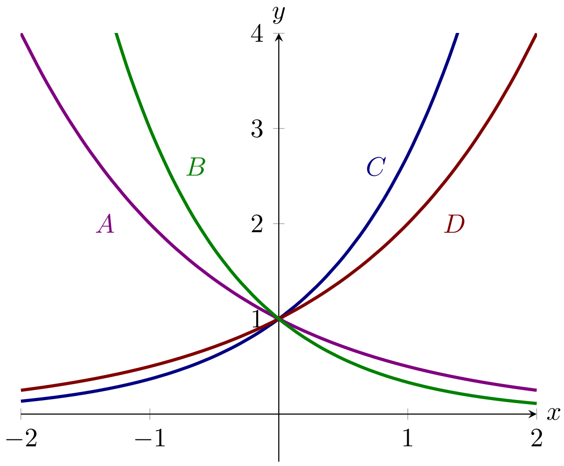
Feedback(*`attempt`*): *Note that*

$$4^{-x} = \left(4^{-1}\right)^x = \left(\frac{1}{4}\right)^x.$$

What can the graphs look like?

Graphs of exponential functions

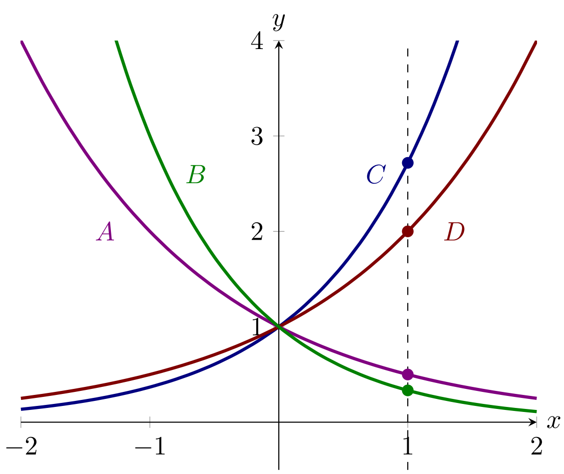
Example 1. *Here we see the the graphs of four exponential functions.*



Match the curves A, B, C, and D with the functions

$$e^x, \qquad \left(\frac{1}{2}\right)^x, \qquad \left(\frac{1}{3}\right)^x, \qquad 2^x.$$

Explanation. *One way to solve these problems is to compare these functions along the vertical line $x = 1$,*



Note

$$\left(\frac{1}{3}\right)^1 < \left(\frac{1}{2}\right)^1 < 2^1 < e^1.$$

Hence we see:

- $\left(\frac{1}{3}\right)^x$ *corresponds to* $\underset{\text{given}}{\boxed{B}}$.

- $\left(\frac{1}{2}\right)^x$ *corresponds to* $\underbrace{\boxed{A}}_{\text{given}}$.
- 2^x *corresponds to* $\underbrace{\boxed{D}}_{\text{given}}$.
- e^x *corresponds to* $\underbrace{\boxed{C}}_{\text{given}}$.

Properties of exponential functions

Working with exponential functions is often simplified by applying properties of these functions. We will use these properties throughout the semester.

Properties of exponents

Let a be a positive real number with $a \neq 1$.

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^{-1} = \frac{1}{a}$

Question 2 *What exponent makes the following true?*

$$2^4 \cdot 2^3 = 2^{\boxed{7}}$$

Hint:

$$(2^4) \cdot (2^3) = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$$

Example 2. *Solve the exponential equation.*

$$27^x = 9$$

Explanation. *in this example, we want to write both sides of the equation as a power of* 3. *Once we have the same base, we may equate exponents. We may write* $27 = 3^{\overset{\boxed{3}}{\text{given}}}$ *and* $9 = 3^{\overset{\boxed{2}}{\text{given}}}$, *therefore we can write the exponential equation as*

$$3^{\overset{\boxed{3x}}{\text{given}}} = 3^{\overset{\boxed{2}}{\text{given}}}.$$

By equating exponents, we arrive at $x = \frac{2}{3}$.

Dig-In:

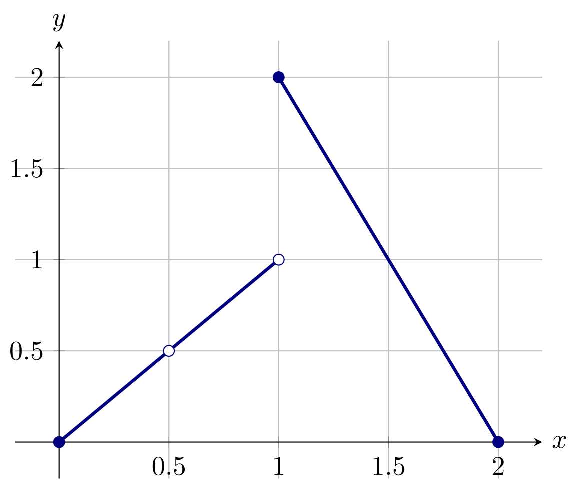
2 Continuity

The idea of continuity.

Idea 1. *A function f is* ***continuous at*** *$x = a$ if you can trace through the point $(a, f(a))$ without lifting your pen.*

YouTube link: `https://www.youtube.com/watch?v=hXFLvVFQa5k`

Question 3 *Consider the graph of the function f*



Which of the following are true?

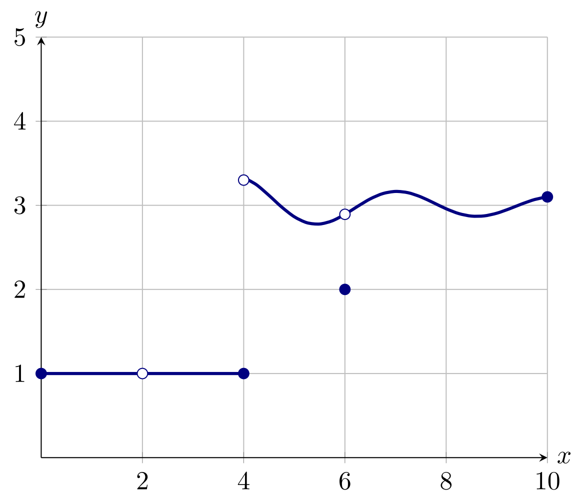
Multiple Choice:

(a) f is continuous at $x = 0.5$

(b) f is continuous at $x = 1$

(c) f is continuous at $x = 1.5$ ✓

Example 3. *Give x-values where the function below is discontinous (i.e. not continuous).*



Explanation. *To start,* f *is not even defined at* $x = \underset{\text{given}}{\boxed{2}}$ *, therefore* f *cannot be continuous at* $x = \underset{\text{given}}{\boxed{2}}$ *as you must lift your pen over the hole.*

Next, from the plot above we see that $\lim\limits_{x \to 4} f(x)$ *does not exist because*

$$\lim_{x \to 4^-} f(x) = \underset{\text{given}}{\boxed{1}} \qquad \textit{and} \qquad \lim_{x \to 4^+} f(x) \approx 3.3$$

This causes a "jump" in the function at $x = 4$ *where you must lift your pen. This means* f *cannot be continuous at* $x = 4$.

We also see that $\lim\limits_{x \to 6} f(x)$ *exists and* $\lim\limits_{x \to 6} f(x) \approx 2.9$ *. However, as* $f(6) = \underset{\text{given}}{\boxed{2}}$ *, we must pick up our pen to color in the point* $(6, f(6))$ *as we trace along the graph. This means* f *is not continuous at* $x = 6$.

Remark 1. *The following common functions are continuous at every* x*-value in their domains. Try visualizing some of their graphs to convince yourself of the continuity.*

Constant function $f(x) = k$ *for* k *a real number*

Power functions $f(x) = x, f(x) = x^2, f(x) = x^3, \cdots$ *or more generally* $f(x) = x^r$ *for* $r > 0$

Exponential function $f(x) = a^x$ *with base* $a > 0$

Logarithmic function $f(x) = \log_a(x)$ *with base* $a > 0$