Dig-In:

Exponential functions

Exponential functions illuminated.

Exponential functions may seem somewhat esoteric at first, but they model many phenomena in the real-world.

What are exponential functions?

Definition 1. *An* ***exponential function*** *is a function of the form*

$$f(x) = a^x$$

where $a \neq 1$ is a positive real number. The domain of an exponential function is $(-\infty, \infty)$.

Question 1 *Is 4^{-x} an exponential function?*

Multiple Choice:

(a) *yes* ✓

(b) *no*

Feedback(*`attempt`*): *Note that*

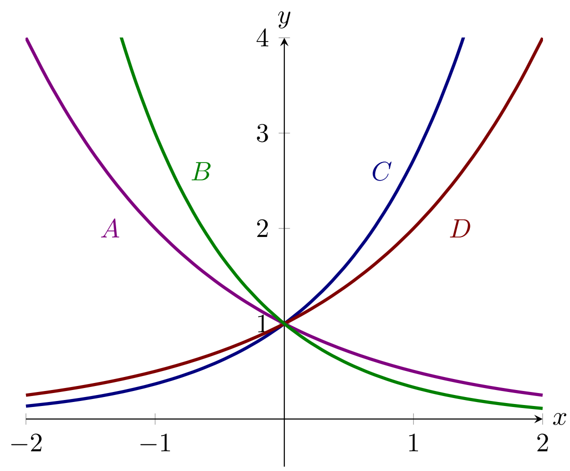
$$4^{-x} = \left(4^{-1}\right)^x = \left(\frac{1}{4}\right)^x.$$

What can the graphs look like?

Graphs of exponential functions

Example 1. *Here we see the the graphs of four exponential functions.*

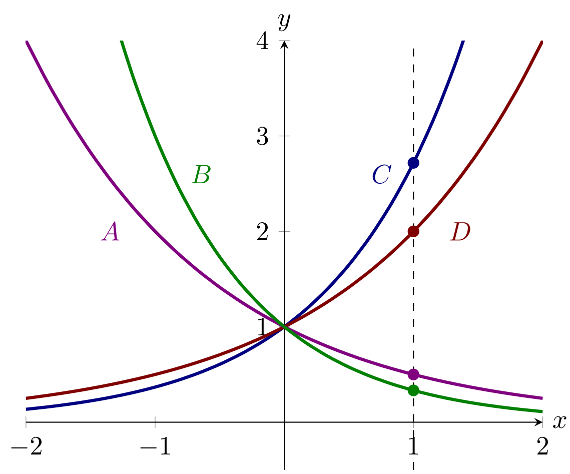
Learning outcomes:
Author(s):



Match the curves A, B, C, and D with the functions

$$e^x, \qquad \left(\frac{1}{2}\right)^x, \qquad \left(\frac{1}{3}\right)^x, \qquad 2^x.$$

Explanation. *One way to solve these problems is to compare these functions along the vertical line $x = 1$,*



Note

$$\left(\frac{1}{3}\right)^1 < \left(\frac{1}{2}\right)^1 < 2^1 < e^1.$$

Hence we see:

- $\left(\frac{1}{3}\right)^x$ *corresponds to* $\underbrace{\boxed{B}}_{\text{given}}$.

- $\left(\frac{1}{2}\right)^x$ *corresponds to* $\underset{\text{given}}{\boxed{A}}$.
- 2^x *corresponds to* $\underset{\text{given}}{\boxed{D}}$.
- e^x *corresponds to* $\underset{\text{given}}{\boxed{C}}$.

Properties of exponential functions

Working with exponential functions is often simplified by applying properties of these functions. We will use these properties throughout the semester.

Properties of exponents

Let a be a positive real number with $a \neq 1$.

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^{-1} = \frac{1}{a}$

Question 2 *What exponent makes the following true?*

$$2^4 \cdot 2^3 = 2^{\boxed{7}}$$

Hint:

$$(2^4) \cdot (2^3) = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$$

Example 2. *Solve the exponential equation.*

$$27^x = 9$$

Explanation. *In this example, we observe that 27 and 9 are both powers of 3. Once we have the same base on both sides of the equation, we may equate exponents. We use that* $27 = 3^{\overset{\boxed{3}}{\text{given}}}$ *and* $9 = 3^{\overset{\boxed{2}}{\text{given}}}$, *therefore we can write the exponential equation as*

$$3^{\overset{\boxed{3x}}{\text{given}}} = 3^{\overset{\boxed{2}}{\text{given}}}.$$

By equating exponents, we arrive at $x = \frac{2}{3}$.