# Test Course

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# Contents

**Dig-In:**

# 1 Exponential functions

*Exponential functions illuminated.*

Exponential functions may seem somewhat esoteric at first, but they model many phenomena in the real-world.

## What are exponential functions?

**Definition 1.** *An* ***exponential function*** *is a function of the form*

$$f(x) = a^x$$

*where $a \neq 1$ is a positive real number. The domain of an exponential function is $(-\infty, \infty)$.*

**Question 1** *Is $4^{-x}$ an exponential function?*

***Multiple Choice:***

(a) *yes* ✓

(b) *no*

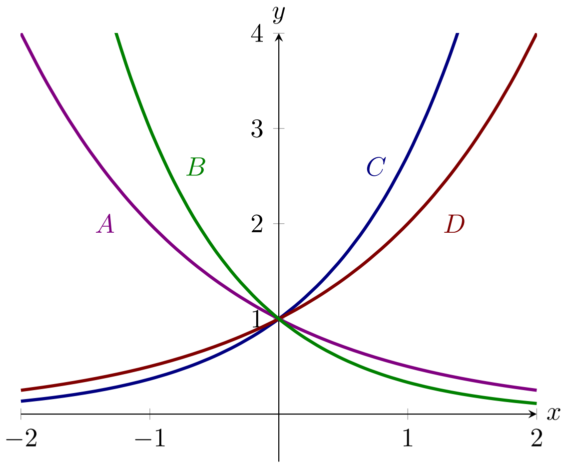
***Feedback(*`attempt`*):*** *Note that*

$$4^{-x} = \left(4^{-1}\right)^x = \left(\frac{1}{4}\right)^x.$$

## What can the graphs look like?

### Graphs of exponential functions

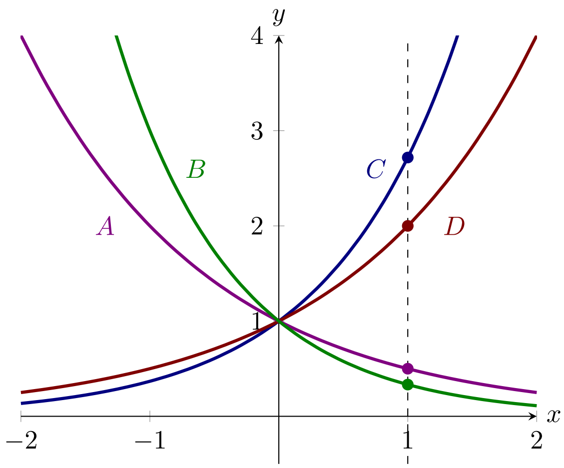
**Example 1.** *Here we see the the graphs of four exponential functions.*




*Match the curves A, B, C, and D with the functions*

$$e^x, \qquad \left(\frac{1}{2}\right)^x, \qquad \left(\frac{1}{3}\right)^x, \qquad 2^x.$$

**Explanation.** *One way to solve these problems is to compare these functions along the vertical line $x = 1$,*




*Note*

$$\left(\frac{1}{3}\right)^1 < \left(\frac{1}{2}\right)^1 < 2^1 < e^1.$$

*Hence we see:*

- $\left(\frac{1}{3}\right)^x$ *corresponds to* $\underset{\text{given}}{\boxed{B}}$.

- $\left(\frac{1}{2}\right)^x$ *corresponds to* $\underset{\text{given}}{\boxed{A}}$.
- $2^x$ *corresponds to* $\underset{\text{given}}{\boxed{D}}$.
- $e^x$ *corresponds to* $\underset{\text{given}}{\boxed{C}}$.

# Properties of exponential functions

Working with exponential functions is often simplified by applying properties of these functions. We will use these properties throughout the semester.

## Properties of exponents

Let $a$ be a positive real number with $a \neq 1$.

- $a^m \cdot a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^{-1} = \frac{1}{a}$

**Question 2** *What exponent makes the following true?*

$$2^4 \cdot 2^3 = 2^{\boxed{7}}$$

***Hint:***

$$(2^4) \cdot (2^3) = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$$

**Example 2.** *Solve the exponential equation.*

$$27^x = 9$$

**Explanation.** *in this example, we want to write both sides of the equation as a power of* 3*. Once we have the same base, we may equate exponents. We may write* $27 = 3^{\overset{\boxed{3}}{\text{given}}}$ *and* $9 = 3^{\overset{\boxed{2}}{\text{given}}}$*, therefore we can write the exponential equation as*

$$3^{\overset{\boxed{3x}}{\text{given}}} = 3^{\overset{\boxed{2}}{\text{given}}}.$$

*By equating exponents, we arrive at* $x = \frac{2}{3}$.

**Dig-In:**

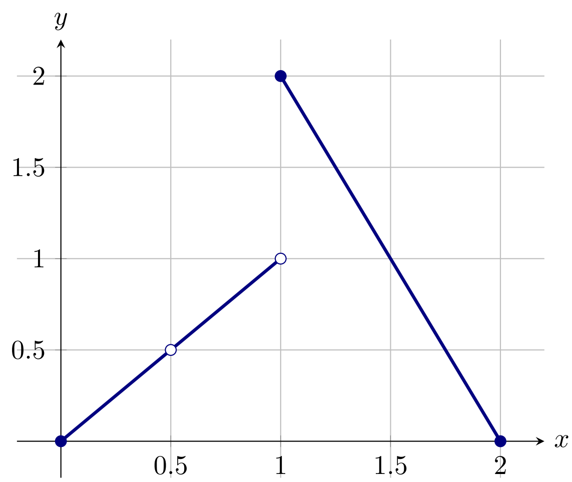
# 2 Continuity

*The idea of continuity.*

**Idea 1.** *A function $f$ is* ***continuous at*** *$x = a$ if you can trace through the point $(a, f(a))$ without lifting your pen.*

*YouTube link:* https://www.youtube.com/watch?v=hXFLvVFQa5k

**Question 3** *Consider the graph of the function $f$*




*Which of the following are true?*

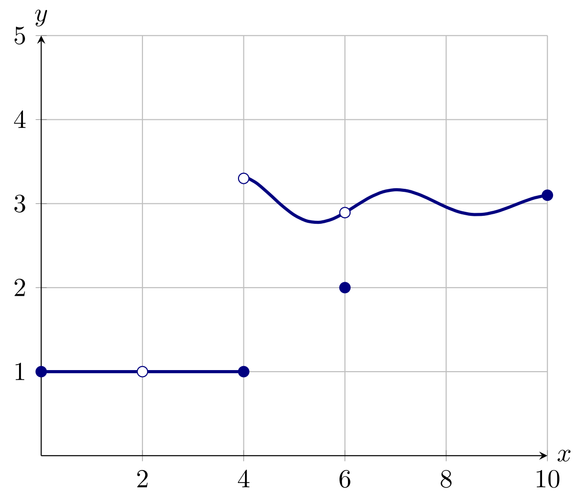
***Multiple Choice:***

(a) $f$ is continuous at $x = 0.5$

(b) $f$ is continuous at $x = 1$

(c) $f$ is continuous at $x = 1.5$ ✓

**Example 3.** *Give $x$-values where the function below is discontinous (i.e. not continuous).*




**Explanation.** *To start, $f$ is not even defined at $x = \underbrace{\boxed{2}}_{\text{given}}$, therefore $f$ cannot be continuous at $x = \underbrace{\boxed{2}}_{\text{given}}$ as you must lift your pen over the hole.*

*Next, from the plot above we see that $\lim_{x\to 4} f(x)$ does not exist because*

$$\lim_{x\to 4^-} f(x) = \underbrace{\boxed{1}}_{\text{given}} \qquad \textit{and} \qquad \lim_{x\to 4^+} f(x) \approx 3.3$$

*This causes a "jump" in the function at $x = 4$ where you must lift your pen. This means $f$ cannot be continuous at $x = 4$.*

*We also see that $\lim_{x\to 6} f(x)$ exists and $\lim_{x\to 6} f(x) \approx 2.9$ . However, as $f(6) = \underbrace{\boxed{2}}_{\text{given}}$, we must pick up our pen to color in the point $(6, f(6))$ as we trace along the graph. This means $f$ is not continuous at $x = 6$.*

**Remark 1.** *The following common functions are continuous at every $x$-value in their domains. Try visualizing some of their graphs to convince yourself of the continuity.*

**Constant function** $f(x) = k$ *for $k$ a real number*

**Power functions** $f(x) = x, f(x) = x^2, f(x) = x^3, \cdots$ *or more generally* $f(x) = x^r$ *for* $r > 0$

**Exponential function** $f(x) = a^x$ *with base* $a > 0$

**Logarithmic function** $f(x) = \log_a(x)$ *with base* $a > 0$