The Relationship between Tactile Graphics and Mathematics for Students with Visual Impairments

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Tactile graphics commonly appear in educational materials adapted for students who have visual impairments. However, several decades of basic research in perceptual psychology has shown that tactile graphics can be difficult to interpret for both blindfolded-sighted participants and those with visual impairments. Educational policy has been enacted to ensure that students with visual impairments are receiving adequate training to effectively use tactile graphics. The current study examines whether these policies have resulted in adequate performance with tactile graphics among 10 braille-reading high-school (grades 8-12) students with visual impairments, and whether there is a relationship between basic tactile-graphics skills and ability in answering math problems with tactile graphics. Results indicated that the participants did poorly with tactile graphics of common objects and those in mathematics problems. There was a statistically significant correlation between the basic tactile graphics recognition skills and performance on a mathematics achievement test that included many tactile graphics, but no significant correlation to performance on a mathematics test that included only a few tactile graphics. These results indicate a need for greater instruction on tactile graphics among students with visual impairments, and greater standardization in using tactile graphics in mathematics education.

Keywords : Haptic, Tactile graphics, Math, Assessment, Academic achievement

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INTRODUCTION

Students with visual impairments tend to underperform academically compared to their sighted peers (Blackorby & Cameto, 2004; Wei, Lenz & Blackorby, 2013), especially in mathematics (Brothers, 1973; Clamp, 1997; Rapp & Rapp, 1992). A potential reason for this is that many educational materials, especially those used in mathematics, are largely inaccessible without sight (Dick & Kubiak, 1997; Karshmer & Bledsoe, 2002). For example, mathematics textbooks often include graphs, tables, diagrams, and charts. For students with visual impairments, these are translated into an accessible format that is collectively referred to as *tactile graphics*. Tactile graphics are typically line drawings with the lines raised to make them accessible to touch. They may also contain textured areas and/or braille text and numbers as labels. Visual elements are commonly overlaid for students who have some useable vision. Figure 1 shows examples of a bar chart and line graph produced on thermoform with raised lines, braille, and visual overlays.

Not only are tactile graphics used within the classroom, they often appear on adapted versions of high-stakes or gatekeeper exams such as the Scholastic Aptitude Test (SAT), an exam that contributes to college admission decisions in the United States and elsewhere. Research on the braille adaptation of the SAT-math showed that among high-achieving, college-bound, high-school students (grades 8-12) with visual impairments mathematics questions were differentially difficult when they contained tactile graphics (Bennett, Rock, & Novatkoski, 1989). In this case, differential item functioning refers to different relative difficulties of items for students who are sighted and visually impaired. This suggests that, when used in academic assessments, tactile graphics are difficult for students with visual impairments to use and interpret.

Unlike the aforementioned SAT study, most research on tactile graphics has been conducted not with educational materials, but instead with depictions of common objects or surfaces (e.g., a raised-line drawing of a cup, hammer, house, or slanted plane). These studies generally agree that tactile graphic depictions are difficult to recognize for both participants who are visually impaired and those who are blindfolded-sighted (Heller, 1989; Lederman, Klatzky, Chataway, & Summers, 1990). This is especially true for participants with visual impairments when the graphics contain vision-specific information, such as self-occlusion (i.e., when one part of an object blocks another part from view), or converging lines to depict depth (Edman, 1992; Morash, Connell Pensky, Urqueta Alfaro, & McKerracher, 2012; Thompson & Chronicle, 2006). In addition, unlike blindfolded-sighted participants, participants who are blind cannot rely on a visual-mediation strategy (i.e., imagining what the graphic would look like if presented visually; Lederman et al., 1990).

Through training or hints (such as revealing the drawing's category; e.g., "fruit"), comprehension of tactile graphics can be improved (Berlá & Butterfield, 1977; Heller, 2002; Heller, Calcaterra, Burson, & Tyler, 1996; Heller et al., 2006; Holmes, Hughes, & Jansson, 1998). This suggests that students with visual impairments can achieve adequate comprehension of educational tactile graphics through training and scaffolding in the classroom. Furthermore, standardization of how visual information is translated into tactile graphics enables students with visual impairments to be trained to use comprehension strategies (Amick & Corcoran, 1997; Gardiner & Perkins, 2003).

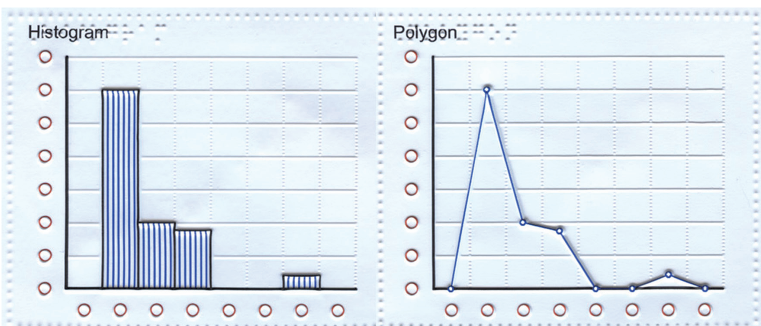

Figure 1. Math Tactile Graphics from the Touch Graphics, Inc., TTT Practical Statistics Handbook

To ensure that students with visual impairments are taught effective tactile-graphic reading skills, tactile graphics have been included in the "compensatory" section of the Expanded Core Curriculum (ECC). The ECC describes skills that sighted students learn incidentally, but students with visual impairments need to be taught explicitly and should be mastered by high-school graduation (Hatlen, 1996). The ECC is supported at a national level for the education of students with visual impairments (Huebner, Merk-Adam, Stryker, & Wolffe, 2004). Furthermore, the state of California has published Braille Mathematics Standards for its students who are visually impaired (California Department of Education, 2006). These standards stipulate that tactile graphics must be made available to students with visual impairments as part of their education, beginning in pre-school (below grade 1), and continuing through high-school education (grade 12). Therefore, students with visual impairments in the United States and California should receive substantial instruction on reading tactile graphics, and be capable of effective tactile-graphic interpretation by high school.

The purpose of the current study is to document whether Californian high-school students with visual impairments are able to effectively interpret tactile graphics of common objects and use tactile graphics in the context of mathematical problems. Over 20 years have passed since difficulties with tactile graphics in mathematics were documented (Bennett, et al., 1989), and in the interim the ECC and Braille Mathematics Standards have been implemented to address this deficit. Therefore, tactile graphics should no longer be problematic for high-achieving students with visual impairments, nor should there be differential item functioning associated with tactile graphics. Continued issues with tactile graphics, in general and in mathematics, would indicate a need to revisit the policy and practice of tactile graphics instruction.

This study also examines the relationship between interpreting tactile graphics of common objects and completing mathematics problems containing tactile graphics. Given that researchers in the field of perceptual psychology have studied the former in great detail, any link between this basic research and the applied use of tactile graphics in mathematics would be useful to future research and practice.

METHODS

Participants

Ten participants who use braille and tactile graphics as their primary literacy media were recruited to take part in the study. The participants were in grades 8-12 in school in California (mean age = 16.2 years), on track to receive their high-school diplomas by age 22, and had no learning or cognitive impairments aside from visual impairment.

ID	Sex	Age (Years)	Conditions	Visual Abilities	Age of Onset (Years)
1	F	16	Retinopathy of Prematurity	Light Direction	4
2	M	20	Glaucoma	Some Form	12
3	F	21	Anophthalmia	None	0
4	M	16	Retinal Detachment	None	<1
5	M	13	Unknown	None	2
6	M	15	Glaucoma and Aniridia	Some Form	0
7	M	18	Retinal Detachment	None	5
8	F	14	Unknown	None	4
9	F	15	Leber's Hereditary Optic Neuropathy	Some Form	0
10	M	14	Retinopathy of Prematurity	Light	0

Table 1. Participant Demographics

Some participants had no visual function, some could see light and/or light direction, and a few could detect the presence of large forms (e.g., a building). Participant information is presented in Table 1. All participants and supervising guardians provided informed consent. The protocol was approved by the University of California, Berkeley Committee for Protection of Human Subjects.

Materials

Tactile Graphics Test

Participants completed a test of their tactile-graphics abilities. Objects and representative tactile graphics were taken from the Setting the Stage for Tactile Understanding Kit (American Printing House for the Blind, 2004). Example items are shown in Figure 2.

The first 6 items in the Tactile Graphics Test asked participants to examine a tactile graphic and then select which of two objects it represented.

- Item 1: Cup tactile graphic (side view), compared to cup and zipper objects.
- Item 2: Bracelet tactile graphic, compared to bracelet and puzzle-piece objects.
- Item 3: Scissors tactile graphic, compared to key and scissors objects.
- Item 4: Ball tactile graphic, compared to ball and cube objects.
- Item 5: Toothbrush tactile graphic, compared to toothbrush and comb objects.
- Item 6: Crayon tactile graphic, compared to spoon and crayon objects.

Items 7-11 presented the participant with one side of a plastic model house: front, back, left, right, or top, respectively. The participant was presented with a tactile graphic and asked to decide whether the tactile graphic represented the side of the house accurately. If the participant responded that the graphic was inaccurate, they were asked to describe what was wrong with the depiction. The tactile graphic was incorrect for items 8, 10, and 11.

- Item 8: A window was not shown.
- Item 10: A window was shown on the right, rather than left side of the wall.
- Item 11: One roof side was shown with rough texture rather than shingles.

Figure 2. Example Items (1, 3, and 11) from the Tactile Graphics Test, and the House Front (Item 7)

Math Tests

Participants completed two subtests from the KeyMath Revised, which had been adapted with braille and tactile graphics for students with visual impairments (American Printing House for the Blind, 1996). Because the entire KeyMath was normed using a single sample, we were able to compare scores across subtests by calculating the age equivalent of raw scores on the selected subtests. One subtest included only a few simple tactile graphics, and one included frequent and complex graphics. A summary of the graphical aspects of the tests is shown in Figure 3.

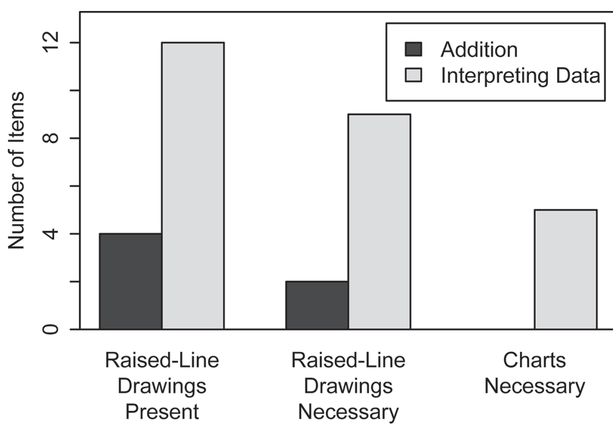

Figure 3. Number of Items (out of 18) That Contained Tactile Graphics, Raised-Line Drawings or Charts, in the KeyMath Subtests

The Addition subtest contained tactile graphics in 4 of its 18 items. 2 of these 4 items included necessary information in the tactile graphic (i.e., the question could not be answered without reading the tactile graphic), while the other 2 items could be answered without reading the tactile graphics. The items on this subtest that required the tactile graphics were simple, similar to: “Two circles are hidden and some circles are shown. How many circles are there total?” The Addition subtest contained no items with charts (tables), which we define as spatial arrangements of braille and numbers in columns and rows.

The Interpreting Data subtest contained raised-line drawings in 12 of its 18 items. 9 of these 12 items included necessary information in the tactile graphic, while the other 3 could be answered without consulting the tactile graphic. In 5 of the 6 items with no raised-line drawings, there was a chart for the participant to consult. Although not all charts contain raised lines, interpreting a chart requires spatial processing similar to that required when reading raised-line drawings. Therefore, charts are included within the definition of a tactile graphic. The 17 tactile graphics contained within the Interpreting Data subtest were more complex than those in the Addition subtest, such as bar charts and line graphs.

Because previous research had shown differential item functioning for braille transcriptions of standardized tests (Bennett, et al., 1989), we suspected that differential item functioning may be present in the braille adaptation of the KeyMath. Specifically, items that are difficult for sighted individuals may be easy for individuals with visual impairments, and vice versa. For this reason, we administered the KeyMath subtests without the stop rule. The stop rule in the KeyMath Examiner’s Manual states that after three consecutive items are answered incorrectly on any given subtest, the tester may discontinue administering items from that subtest. Stop rules rely on the items being ordered from least to most difficult. If items are ordered from easy to difficult and a person is unable to solve three items in a row, it is highly unlikely that they will be able to solve the next, more difficult item. The items on the KeyMath have been ordered from easy to difficult based on norming with sighted children. The braille adaptation of the Keymath was not re-normed with children who have visual impairements. Therefore, to address the possibility of differential item functioning, we administered the KeyMath subtests without the stop rule, and examined how applying the stop rule to the data ex post facto affected the results.

Procedure

Participants completed the Tactile Graphics Test and the two KeyMath subtests, Addition and Interpreting Data, in random order. Each participant was allowed to use an abacus and/or Perkins Brailler to complete the KeyMath items, according to the KeyMath Examiner’s Manual. During testing, the participant and experimenter sat across the table from each other, and instructions/questions were delivered with a combination of verbal instruction and graphical/braille materials.

RESULTS

Tactile Graphics Test

Results on the Tactile Graphics Test are shown in Table 2. Some items were frequently answered incorrectly – items 1 and 11 were answered incorrectly by 6 of the 10 participants. We used a Pearson's Chi-Square test, with Monte Carlo simulation to compute the p-value, to examine whether the tendency to be wrong on certain items could be explained by chance. We found that the distribution of inaccuracies across the items could not be explained as chance occurrence ($p = 0.002$).

ID	Missed Items	Accuracy (%)
1	8,11	81.82
2		100.00
3	1, 5, 8, 10, 11	54.55
4	1, 8, 11	72.73
5	1, 10	81.82
6	1, 3, 8, 11	63.64
7	1, 9	81.82
8	11	90.91
9		100.00
10	1, 10, 11	72.73

Table 2. Tactile Graphics Test Performance

Math Tests

Using the KeyMath Revised Examiner's Manual, we calculated the age-equivalents for the raw scores on the Addition and Interpreting Data subtests. We did this twice, once ignoring the stop rule and once following the stop rule. The stop rule did not change the raw scores or age-equivalent scores that participants received on the Addition subtest, but did significantly alter the scores on the Interpreting Data subtest, based paired T-tests (raw scores $p = 0.04$, age-equivalency scores $p = 0.03$). Participant performance on the KeyMath subtests is shown in Table 3.

To compare performance on the Addition and Interpreting Data tests, we converted scores into age equivalents, both ignoring and following the stop rule, using the KeyMath Examiner's Manual. Several students had differences in age equivalent scores that were ranges rather than precise values (see Table 3). Ignoring these participants, performance was significantly better on the Addition subtest than the Interpreting Data subtest, both when ignoring and following the stop rule (paired T-tests, $p = 0.04$ and $p = 0.02$, respectively). Results were unchanged using data from all participants, using the least extreme values within the ranges (paired T-tests, $p = 0.04$ and $p = 0.02$, respectively).

Accuracy (%) Interpreting							Age Equivalence (Years, Months) Interpreting				
ID	Addition	Data (SR)		Difference (SR)		Addition	Data (SR)		Difference (SR)	
1	100.0	77.8	(77.8)	22.2	(22.2)	≥ 15,11	≥ 15,11	(≥15,11)	≈ 0,0	(≈ 0,0)
2	66.7	66.7	(66.7)	0.0	(0.0)	11,7	12,8	(12,08)	≤ -1,1	(≤ -1,1)
3	72.2	16.7	(5.6)	55.6	(68.2)	12,2	6,11	(5,03)	5,3	(6,11)
4	77.8	38.9	(5.6)	38.9	(73.5)	12,11	9,4	(5,03)	3,7	(7,8)
5	83.3	83.3	(83.3)	0.0	(0.0)	14,2	≥ 15,11	(≥15,11)	≤ -1,9	(≤ -1,9)
6	55.6	44.4	(33.3)	11.1	(52.5)	10,8	9,11	(8,08)	0,9	(2,0)
7	83.3	44.4	(5.6)	38.9	(78.7)	14,2	9,11	(5,03)	4,3	(8,11)
8	94.4	88.9	(88.9)	5.6	(5.6)	≥ 15,11	≥ 15,11	(≥15,11)	≈ 0,0	(≈ 0,0)
9	94.4	50.0	(45.3)	44.4	(89.2)	≥ 15,11	10,07	(9,11)	≥ 5,4	(≥ 6,0)
10	94.4	33.3	(22.2)	61.1	(89.2)	≥ 15,11	8,8	(7,06)	≥ 7,3	(≥ 8,5)

Table 3. Accuracy and Age-Equivalent Scores on Math Subtests, SR Indicates Following the Stop Rule for Scoring

Relationship Between Tactile Graphics and Math Tests

We computed correlations between the raw scores on the Tactile Graphics Test and the KeyMath subtests. The relationship between the Tactile Graphics Test and Addition subtest was not significant ($r = 0.39$, $p = 0.26$). The relationship between the Tactile Graphics Test and the Interpreting Data subtest was significant ($r = 0.64$, $p = 0.047$). These relationships are shown in Figure 4.

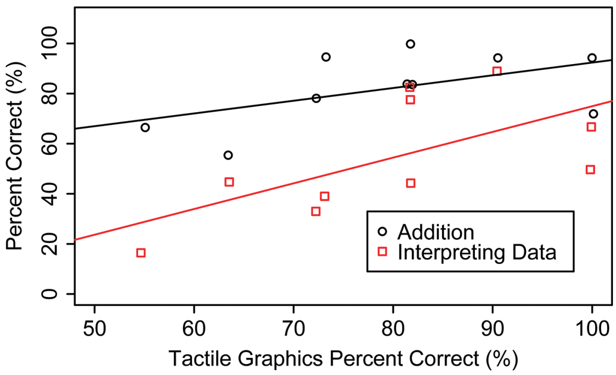

Figure 4. Raw Scores on the Tactile Graphics Test vs. Math Subtests

DISCUSSION

The participants in our study had greater difficulty with the basic Tactile Graphics Test than we expected. The items on the Tactile Graphics Test were taken from a kit designed for young children (ages 5 and up) that contained objects and graphics that were simple and/or familiar. As students with visual impairments on a high-school diploma track, who should have received substantial tactile graphics training as indicated by the Expanded Core Curriculum (ECC) and Californian Braille Mathematics Standards, we expected the participants would have been near-perfect at translating three-dimensional objects into two-dimensional representations. However, mean performance on the test was 80 % accuracy (chance at 50 %).

We believe that our participants' poor results in translating between common objects and tactile graphics are comparable to what would be found with high-school aged students with visual impairments located outside of California. Our participants were not held to unusually low tactile-graphics standards, as they were subject to expectations set forth by both the national ECC and California Braille Mathematics Standards. Furthermore, recent results indicate that a large proportion of teachers of students with visual impairments in North America feel that tactile graphics are not appropriately adapted from their print counterparts in assessments and classroom materials (Zebehazy & Wilton, 2014). This explains why students with visual impairments would perform more poorly than expected based on educational policy.

Examining the performance on each tactile-graphics item revealed that certain items were more difficult than others, and this pattern of differential difficulty could not be explained by chance. Items 1 and 11 were most frequently wrong (accuracy 40 % on both). Item 1 provided the participants with a raised-line drawing of a cup, viewed from the side, and an actual cup and zipper. The participants routinely (6 out of 10) indicated that the drawing was of the zipper and not the cup. Many of the participants explained that the hard lines drawn on the side of the cup in the drawing could not belong to a cup, which was smooth all the way around. This was a failure in recognizing self-occlusion, as has been regularly reported in the research literature (e.g., Heller et al., 2006).

Item 11 asked the participants to indicate whether a top-down view of a plastic house's roof was correct. In the three-dimensional house, the two sides of the roof were not contained within the same plane, they met at roughly a 45° angle, but the drawing depicted the roof in a single plane with no foreshortening (the type of projection from three-dimensions to two-dimensions was orthogonal projection). Errors on this item may have reflected failures in understanding projection, a common issue among individuals with visual impairments who are inexperienced with tactile graphics (e.g., Heller et al., 2006). Additionally, several students provided incorrect reasons why the top-down view of the plastic house's roof did not match the tactile graphic. The students indicated that the house should be drawn in a fold-out fashion when viewed from above, with the sides of the house shown next to the roof sides, like a folded-out box. This erroneous three-dimensional to two-dimensional transformation has been noted among children with visual impairments before (Kennedy, 1993 ; Heller et al., 2006).

We also examined the students' performance with tactile graphics in the context of mathematics problems. The students completed two subtests from the braille version of the KeyMath math achievement test: Addition, which contained almost no tactile graphics, and Interpreting Data, which contained a substantial number of tactile graphics. Because raw scores on these tests are not informative (i.e., a raw score of five on the Addition subtest is not necessarily equivalent to a raw score of five on the Interpreting Data subtest), we translated each score to its age equivalent, using the conversion table provided in the KeyMath Examiner's Manual. Differences in age equivalent scores are meaningful in that they reflect differences in student understanding versus differences in test difficulty. The equivalent ages were significantly lower on the Interpreting Data subtest than on the Addition subtest (true when both ignoring and following the stop rule). This indicates that the Interpreting Data subtest was more difficult for the participants in our sample than the Addition subtest. This is indication that students with visual impairments may have greater difficulty with mathematics when the content contains tactile graphics.

Scores on the Interpreting Data subtest, but not the Addition subtest, changed when following the KeyMath's stop rule. The stop rule stipulates that no further items from a subtest be administered after three consecutive wrong answers. This assumes that the items are ordered from least to most difficult, and, therefore, a participant is extremely unlikely to answer an item correctly following three consecutive wrong answers. This assumption was supported for the Addition subtest, where no student managed to answer an item correctly following three consecutive wrong answers. However, students were able to provide correct answers following three consecutive wrong answers on the Interpreting Data subtest. Thereby, following the stop rule significantly lowered the students' raw scores and age equivalent scores on the Interpreting Data subtest.

The items on the KeyMath were ordered from least to most difficult based on normative data from sighted participants. Therefore, changes in scores due to the stop rule suggest differential item functioning among our participants with visual impairments. Notably, we found no evidence to suggest differential item functioning on the Addition subtest, which contained very few tactile graphics (4 of 18 items, Figure 3). In contrast, the results from the Interpreting Data subtest, which contained a substantial number of tactile graphics (17 of 18 items, Figure 3), were suggestive of differential item functioning. This finding provides further evidence that mathematics problems containing tactile graphics remain differentially difficult for students with visual impairments. This is the same result as what was found over 20 years ago on the SAT-math (Bennett, et al., 1989).

Lastly, we examined the relationship between interpreting tactile graphics of common objects in the Tactile Graphics Test and answering mathematics problems containing and not containing tactile graphics. A priori, it was unclear whether poor performance on the Tactile Graphics Test, which indicates difficulty in transforming three-dimensional objects into two-dimensional depictions, would correspond with poor interpretation of mathematical tactile graphics. The most frequent errors on the Tactile Graphics Test were errors in interpreting visual information related to viewpoint: self-occlusion (item 1) and projection of a three-dimensional configuration into two dimensions (item 11). Educational tactile graphics, such as bar charts and line graphs, do not represent three-dimensional objects and do not contain these visual cues. Therefore, one would not necessarily predict a strong relationship between performance on the Tactile Graphics Test and mathematics problems containing tactile graphics.

Performance on the Tactile Graphics Test was significantly correlated with raw scores on the Interpreting Data subtest but not the Addition subtest. To our knowledge, this is the first evidence of a relationship between basic tactile graphic abilities and the ability to use complex tactile graphics in a mathematical context. It's possible that both of these skills are related to general exposure to tactile graphics. Alternatively, basic tactile-graphic skills may provide a necessary foundation for comprehending the complex tactile graphics used in mathematics. If future research continues to support this relationship, it would be prudent for specialist teachers, curriculum developers, policy makers, and other individuals involved in the education of students with visual impairments to become better versed in perceptual psychology research on basic tactile graphics abilities, especially common mistakes and effective strategies.

In conclusion, our results suggest the need for additional research to verify, in a broader sample, that students who have visual impairments exhibit deficits in basic tactile graphics skills and poorer relative performance on mathematics problems that contain tactile graphics. Our results suggest a need to better standardize the production of tactile graphics used in educational materials to facilitate understanding and clarity for students with visual impairments. Further, lessons that include strategies for interpreting tactile graphics should be developed and made available to all students with visual impairments using tactile graphics in their coursework.

Future research may also further explore whether there is differential item functioning for students with visual impairments on achievement and gatekeeper exams, such as the KeyMath and SAT. Evidence of differential item functioning demonstrates a need for future braille/tactile-graphics adaptations of standardized academic assessments to be re-normed with a sample of students who have visual impairments, rather than using normative data from sighted students. Practitioners should be skeptical of the applicability of tests that have been normed with sighted students to their students who have visual impairments.

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