STA380 Project Proposal: Monte Carlo Simulation of a Hypothesis Test of Binomially Distributed Samples

Siling Cheng, Junyi Hou, Valerie Pan, Tianhong Shen, Feiyang Xue

2026-02-11

Project Topic

Monte Carlo Simulation of a Hypothesis Test of Binomially Distributed Samples [One-sample z-test for a binomial proportion]

Simulation vs. Dataset

We will using pure simulation

Project Detail

Hypothesis tests for binomial data are widely used in applications involving binary outcomes, such as quality control and clinical trials. Monte Carlo simulation allows practitioners to assess whether the normal approximation performs adequately for their specific sample size and parameter settings.

$$X \sim \text{Binomial}(n, p),$$

We'll consider the following Hypothesis:

$$H_0 : p = p_0 \quad \text{vs.} \quad H_a : p \neq p_0$$

To estimate Type I error, Type II error, and power, we will be following Monte Carlo procedure[?]:

- Repeatedly generate binomial samples under a specified parameter value p'.

- For each simulated sample, perform a normal approximation test and record the resulting p-value.

- Use the empirical distribution of p-values to approximate probabilities related to the test's performance.

The following outputs will be provided:

- A histogram of simulated p-values, with a vertical line indicating the chosen significance level α.

- A ggplot of the power curve

- A table showing Type I error rate, Type II error rate and power of the test

- For more visualization, we would provide a ggplot of the power curve, a plot of type I and II error trade-off.

User Input(Shiny Components)

Core Simulation Controls

- **Simulation Sample Size**:
  - Users can set the number of Monte Carlo simulations (range: 1,000 to 10,000) via a dual slider or numeric input.
  - Demonstrates how increasing the sample size stabilizes Type I error and power estimates, reducing simulation-based variance.

- **Random Seed**:
  - An integer input field to initialize the pseudo-random number generator.
  - Ensures reproducibility in scientific research, allowing users to generate identical results for verification and ecological modeling.

- **Number of Trials** (n):
  - A slider (range: 10–500) representing the sample size of individual units.
  - Affects the denominator of the z-score; illustrates how larger n reduces standard error and improves test sensitivity.

- **Success Probability** (p):
  - A slider (range: 0 to 1) for the true population proportion.
  - Allows users to explore how different effect sizes impact the statistical power of the test.

Advanced Features

- **Significance Level** (α):
  - An adjustable threshold input for statistical significance.
  - Dynamically updates the rejection region on the plot and recalculates the estimated Type I error rate.

- **Hypothesis Test Directionality**:
  - A toggle switch between **one-sided** and **two-sided** tests.
  - Shows how directional hypotheses change results and p-values without altering the sample size or effect size.

- **Visualization Toggles**:
  - Buttons to switch between different plot types (e.g., Binomial PMF vs. simulated sampling distribution).

- Provides multiple perspectives on the data, from theoretical distributions to empirical simulation results.

- **Color Themes**:

    - A selection menu with various color palettes (including palettes from Nichols).
    - Enhances accessibility for color-blind users and allows for clear visual distinction between plot elements. ## Reference

[Chang et al., 2025] [R Core Team, 2026] [Wickham et al., 2026] [Nichols]

Generative AI Usage Statement

References

David Nichols. Coloring for colorblindness. URL https://davidmathlogic.com/colorblind/#%23D81B60-%231E88E5-%23FFC107-%23004D40.

Winston Chang, Joe Cheng, JJ Allaire, Carson Sievert, Barret Schloerke, Garrick Aden-Buie, Yihui Xie, Jeff Allen, Jonathan McPherson, Alan Dipert, and Barbara Borges. *shiny: Web Application Framework for R*, 2025. URL https://CRAN.R-project.org/package=shiny. R package version 1.12.1.

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2026. URL https://www.R-project.org/.

Hadley Wickham, Winston Chang, Lionel Henry, Thomas Lin Pedersen, Kohske Takahashi, Claus Wilke, Kara Woo, Hiroaki Yutani, Dewey Dunnington, and Teun van den Brand. *ggplot2: Create Elegant Data Visualisations Using the Grammar of Graphics*, 2026. URL https://CRAN.R-project.org/package=ggplot2. R package.