turboGL: Stochastic Modelling of Weak Lensing

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Weak-lensing basics

$$\underbrace{\Delta m(z)}_{\text{shift in distance modulus}} \simeq 5\log_{10}(1-\underbrace{\kappa(z)}_{\text{effective lens convergence}})$$

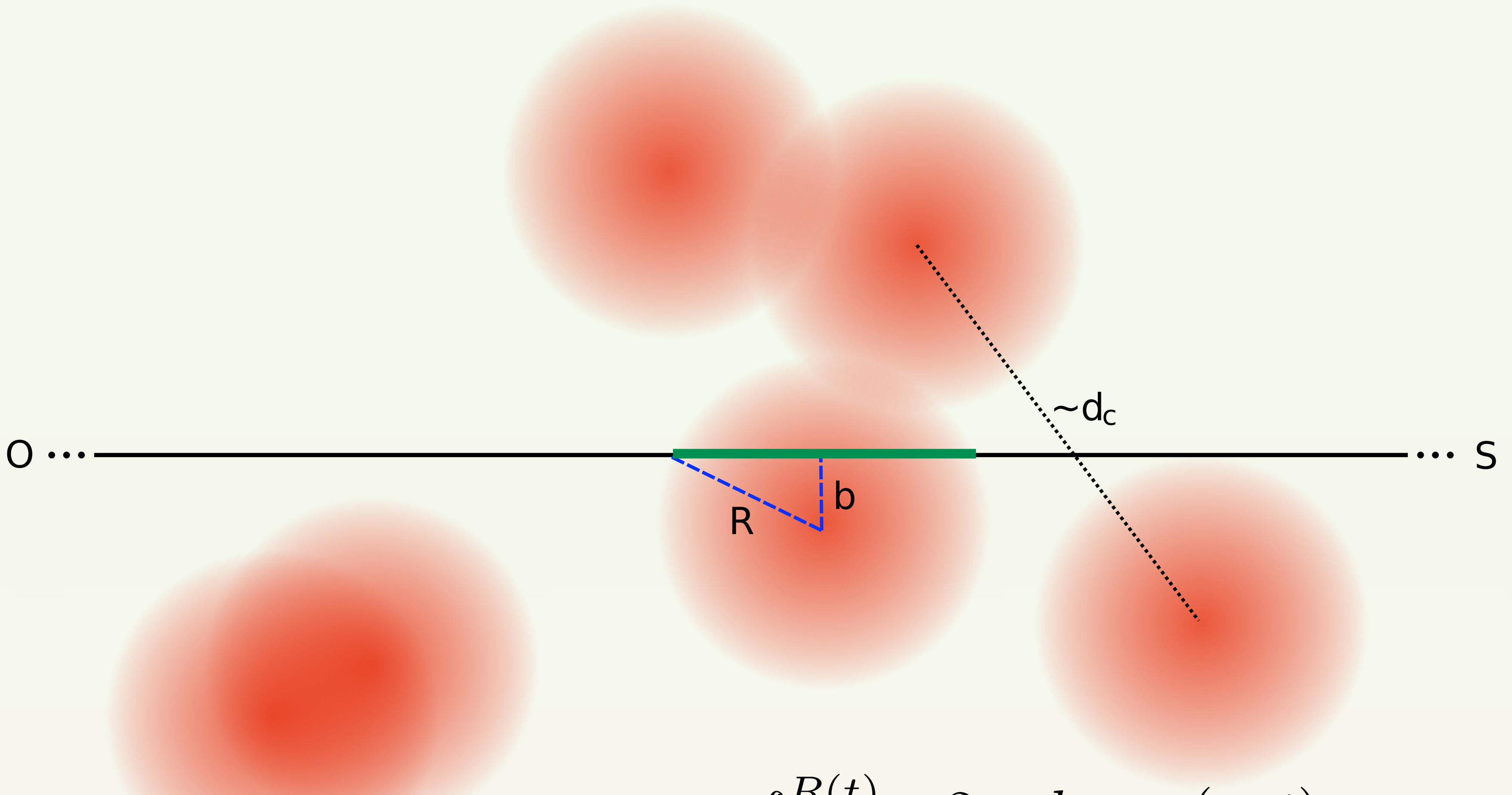
$$\kappa(z) = \int_0^{r_s(z)} dr\, G(r, r_s(z))\, \delta_M(r, t(r))$$

$$G(r, r_s) = \frac{3}{2}\Omega_{M,0}\,\frac{a_0^2 H_0^2}{c^2}\,\frac{r(r_s-r)}{r_s}\,\frac{a_0}{a(t(r))}$$

$\delta_M > 0$ → magnification → $\Delta m < 0$

$\delta_M < 0$ → demagnification → $\Delta m > 0$

Lensing by randomly-placed halos



$$\Gamma(b,t) = \int_b^{R(t)} \frac{2\,x\,dx}{\sqrt{x^2-b^2}}\,\frac{\rho(x,t)}{\bar\rho(t)}$$

Divide the distance r_s to the source and the radius R of the halo into N_S and N_R bins of widths Δr_i and Δb_m so that the convergence due to a halo placed within the bin $(i,\, m)$ is:

$$\kappa_{1im} = G(r_i, r_s)\,\Gamma(b_m, t_i)$$

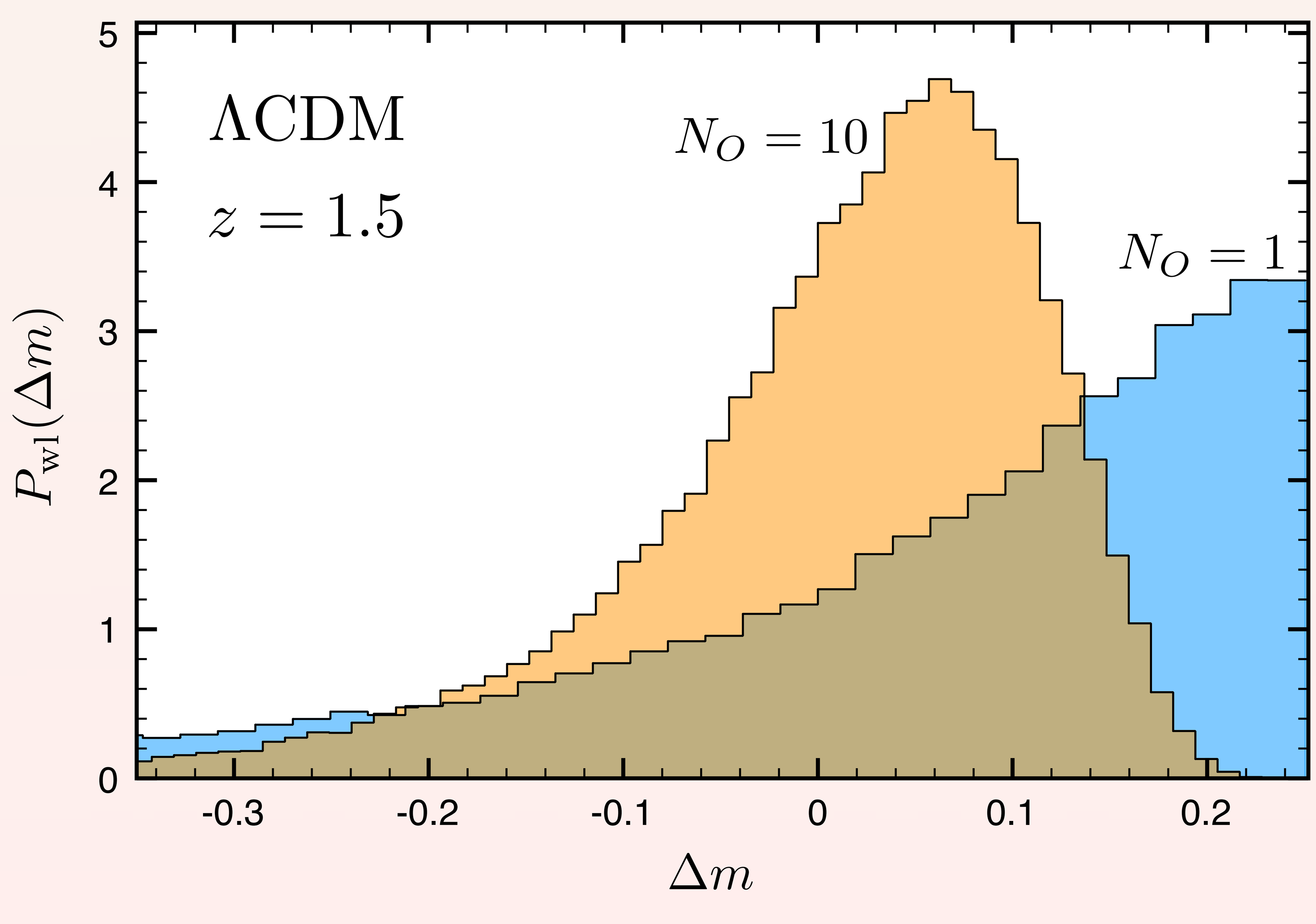
halo surface density

Stochastic simulation

$$\kappa(\{k_{im}\}) = \sum_{i=1}^{N_S}\sum_{m=1}^{N_R} \kappa_{1im}\left(\frac{k[N_O\Delta N]_{im}}{N_O} - \Delta N_{im}\right)$$

- $\kappa(\{k_{im}\})$: the convergence PDF is generated by the configurations $\{k_{im}\}$ of Poisson random numbers.
- κ_{1im}: convergence due to one halo
- $k[N_O\Delta N]_{im}$: Poisson random variable of parameter $N_O\Delta N_{im}$
- N_O: # observations at redshift z
- ΔN_{im}: expected number of halos in ΔV_{im}: $\Delta N_{im} = n_c \Delta V_{im}$, $\Delta V_{im} = 2\pi b_m \Delta b_m \Delta r_i$

- Expected convergence is zero: photon conservation
- PDF approaches δ-function at $\kappa = 0$ for $N_O \to \infty$
- **VERY FAST!! ~1s**
- Only weak-lensing approximation: $\lesssim 5\%$ of error



EXAMPLE:
SIS halo profile
$d_c = 10h^{-1}\mathrm{Mpc}$
$R = 700h^{-1}\mathrm{kpc}$
$M_H = 4.6\cdot 10^{14} h^{-1} M_\odot$

Full details at: arXiv:0909.0822

Code available at: www.turboGL.org