

Open Source Formal Verification

Linear Temporal Logic - LTL

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July 2025

- 1 Verification engineer mindset
- 2 State machine verification
- 3 Conclusion

LTL: Introduction

- Linear Temporal Logic
 - Allows to express temporal relationships between signals
 - Basic operations:

$\phi ::=$	\top	true
	\perp	false
	$\neg(\phi)$	negation
	p	proposition
	$(\phi \wedge \phi)$	conjunction
	$(\phi \vee \phi)$	disjunction
	$X\phi$	next time ϕ
	$F\phi$	eventually ϕ (strong operator)
	$G\phi$	always ϕ
	$\phi U \psi$	$\phi U \psi$: ϕ until ψ (strong operator)

LTL: Introduction

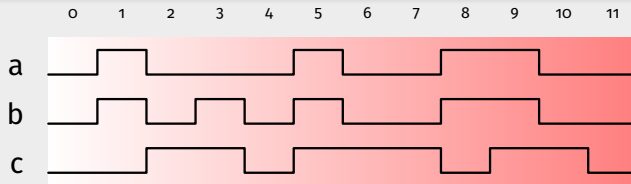
- PSL offers various operators to ease writing temporal properties
 - Presented in the following slides
- *Safety* properties
 - To check the correctness of the system
- *Liveness* properties
 - To check that something eventually happen
 - Not supported by the current tool

next operator and implication (1)

- **next**
 - Advance in time by one clock cycle, so check if its operand is verified on the next clock cycle
- Implication \rightarrow
 - Direct implication, left-hand side in the boolean domain, right-hand side not necessarily
 - Example: $a \rightarrow b$
 - If a then b
 - Example: $a \rightarrow \text{next } b$
 - If a then b on the next clock cycle
- **next moves in time**, and as such can not be on a left-hand side of an implication
 - Not allowed: $(a \text{ and } \text{next } b) \rightarrow c$

next operator and implication (2)

Example



PSL

```
assert always (a -> b);
```

```
assert always (b -> a);
```

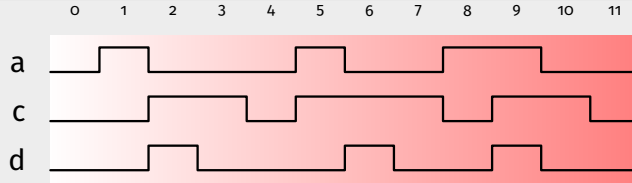
```
assert always (a -> next c);
```

```
assert always (b -> (a or c));
```



next operator and implication (3)

Example



PSL

```
assert always (a -> next c);
```

```
assert always (a -> next d);
```

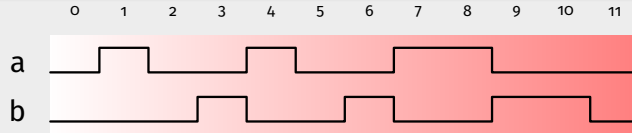


next operator: variants (1)

- `next` [n] (a)
 - Indicates that the property is verified after n clock cycles
 - Example: $a \rightarrow \text{next}[3](b)$
 - If a then b after 3 clock cycles
- `next_a` [n `to` m] (a)
 - Indicates that the property is verified every clock cycles from n to m
 - Warning: The `to` corresponds to VHDL, not PSL.
 - In Verilog it would be [$n:m$]
 - Example: $a \rightarrow \text{next_a}[3 \text{ to } 5](b)$
 - If a then b must be verified after 3, 4 and 5 clock cycles
- `next_e` [n `to` m] (a)
 - Indicates that the property is verified in at least once between n and m clock cycles from the current one
 - Example: $a \rightarrow \text{next_e}[3 \text{ to } 5](b)$
 - If a then b must be verified at least once after 3, 4 or 5 clock cycles

next operator: variants (2)

Example



PSL

```
assert always (a -> next[2] (b));
```

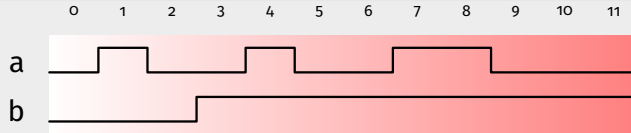
```
assert always (a -> next_a[2 to 4] (b));
```

```
assert always (a -> next_e[2 to 4] (b));
```



next operator: variants (3)

Example



PSL

```
assert always (a -> next[2] (b));
```

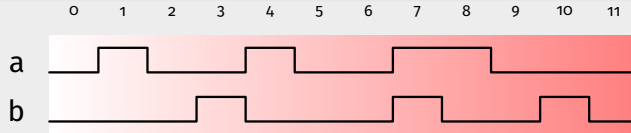
```
assert always (a -> next_a[2 to 4] (b));
```

```
assert always (a -> next_e[2 to 4] (b));
```



next operator: variants (4)

Example



PSL

```
assert always (a -> next[2] (b));
```

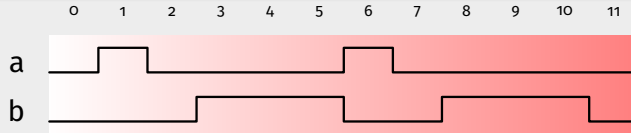
```
assert always (a -> next_a[2 to 4] (b));
```

```
assert always (a -> next_e[2 to 4] (b));
```



next operator: variants (5)

Exemple



PSL

```
assert always (a -> next[2] (b));
```

```
assert always (a -> next_a[2 to 4] (b));
```

```
assert always (a -> next_e[2 to 4] (b));
```

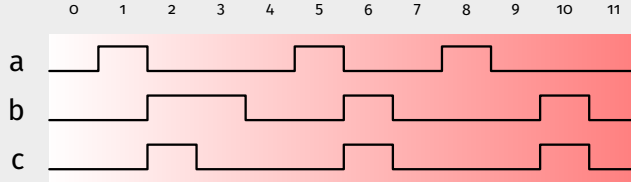


next_event operator (1)

- `next_event (a) (b)`
 - Verifies that b holds the next time a does
 - Example: $c \rightarrow \text{next_event } (a) (b)$
 - If c then next time a is observed, then b must hold
 - Warning: if a holds on the same cycle as c , `next_event` does not trigger
- `next_event (a) [i] (b)`
 - Verifies that b holds on the i^{th} occurrence of a
 - Example: $c \rightarrow \text{next_event } (a) [3] b$
 - If c then b must be true on the third time a is observed

next_event operator (2)

Example



PSL

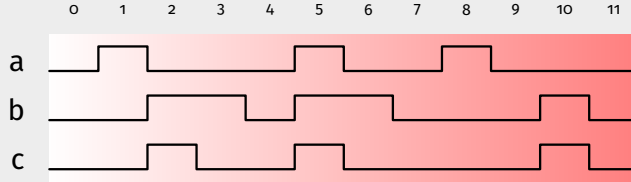
```
assert always (a -> next_event(b) (c));
```

```
assert always (a -> next next_event(b) (c));
```



next_event operator (3)

Example



PSL

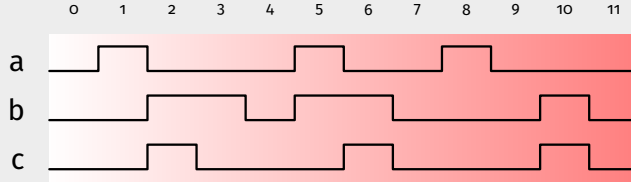
```
assert always (a -> next_event(b) (c));
```

```
assert always (a -> next next_event(b) (c));
```



next_event operator (4)

Example



PSL

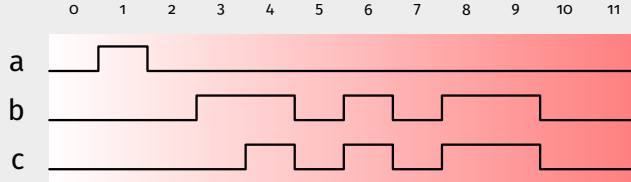
```
assert always (a -> next_event(b) (c));
```

```
assert always (a -> next next_event(b) (c));
```



next_event operator (5)

Example



PSL

```
assert always (a -> next_event(b) (c));
```

```
assert always (a -> next_event(b) [3] (c));
```

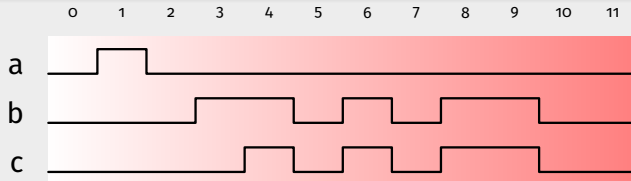


next_event operator: variants (1)

- `next_event_a(b) [n to m] (c)`
 - Indicates that c must be verified at occurrences n to m of b
 - Example: $a \rightarrow \text{next_event_a}(b) [3 \text{ to } 5] (c)$
 - If a then c must be verified at occurrences 3,4 and 5 of b
- `next_event_e(b) [n to m] (c)`
 - Indicates that c must be verified on at least one occurrence of b from n to m
 - Example: $a \rightarrow \text{next_event_e}(b) [3 \text{ to } 5] (c)$
 - If a then c must be verified at least once within occurrences 3, 4 or 5 of b

next_event operator: variants (2)

Example



PSL

```
assert always (a -> next_event_a(b) [2 to 4] (c));
```

```
assert always (a -> next_event_a(b) [2 to 5] (c));
```

```
assert always (a -> next_event_e(b) [1 to 2] (c));
```

until operator (1)

- `until`

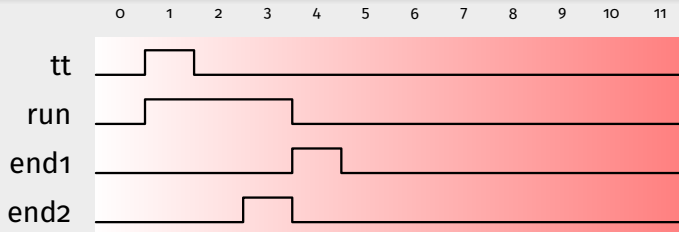
- Indicates that a condition must hold until a second one does (does not include the cycle where the second holds)
- Example: `a until b;`
 - Condition *a* must be verified until *b* is.

- `until_`

- Indicates that a condition must hold until a second one does, including the cycle the second does
- Example: `a until_ b;`
 - Condition *a* must be verified until *b* holds, including the cycle where *b* holds

until operator (2)

Example



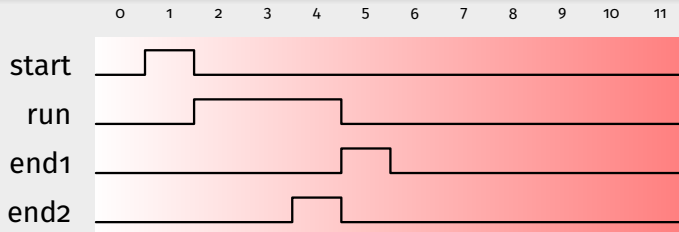
PSL

```
assert always (start -> run until end1);  
  
assert always (start -> run until_ end1);  
  
assert always (start -> run until_ end2);
```



until operator (3)

Example



PSL

```
assert always (start -> next (run until end1));  
  
assert always (start -> next (run until_ end1));  
  
assert always (start -> next (run until_ end2));
```



until operator (4)

Example



PSL

```
assert always (start -> next (run until end1));
```

```
assert always (start -> next (run until_ end1));
```



before operator (1)

- **before**

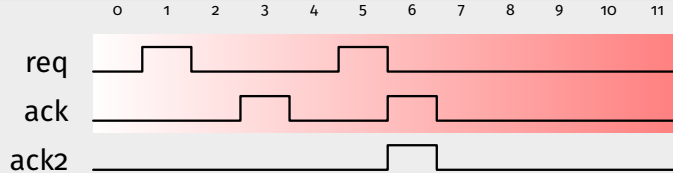
- Indicates that a condition must be verified strictly before a second one is
- Example: $a \text{ before } b$;
 - Condition a must be verified strictly before b

- **before_**

- Indicates that a condition must be verified before a second one is. Both can hold at the same time.
- Example: $a \text{ before_ } b$;
 - Condition a must be verified strictly before b or at the same time

before operator (2)

Example



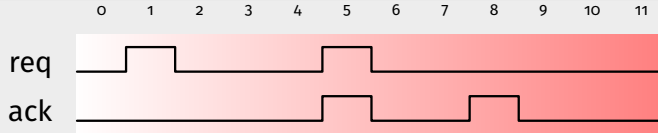
PSL

```
assert always (req -> next (ack before req));  
  
assert always (req -> next (ack2 before req));
```



before operator (3)

Example



PSL

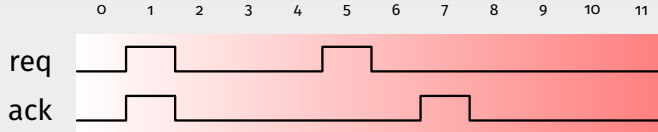
```
assert always (req -> next (ack before req));
```

```
assert always (req -> next (ack before_ req));
```



before operator (4)

Example



PSL

```
assert always (req -> next (ack before req));
```

```
assert always (req -> next (ack before_ req));
```

```
assert always (req -> (ack or next (ack before req)));
```

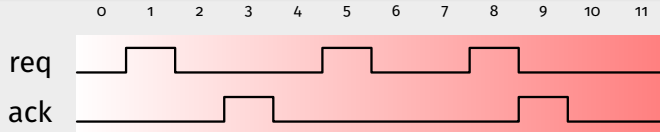


Operator eventually! (1)

- `eventually!`
 - Indicates that a condition must be observed at least once in the future
 - Example: `a -> eventually! b;`
 - Condition `b` must be verified at least once after `a`
- `eventually!` is a strong operator: if `a` occurs, then `b` *must* be observed once

Operator eventually! (2)

Example



PSL

```
assert always (req -> eventually! ack);
```

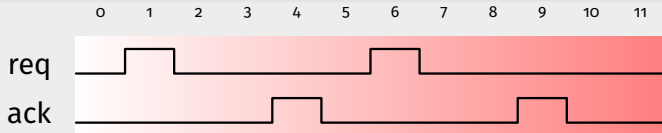


- Actually we'll have to wait a bit for this operator, as GHDL currently has issues with it.

How to think about specs?

- In the specs: `req` active is followed by `ack` active 3 clock cycles later

Example 1



PSL

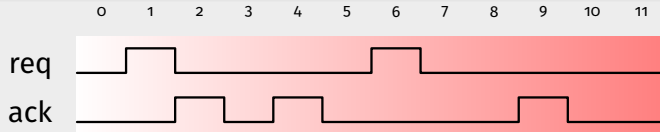
```
assert always (req -> next[3] ack);
```

- Sounds good, no?

How to think about specs?

- In the specs: `req` active is followed by `ack` active 3 clock cycles later

Example 2



PSL

```
assert always (req -> next[3] ack);
```

- Mmh...

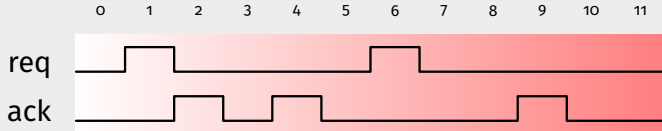
How to think about specs?

- In the specs: `req` active is followed by `ack` active 3 clock cycles later
- What does it mean exactly?
- It only says what it says: `req` \rightarrow `next`[3] `ack`
- But `ack` could be '1' every clock cycle
- Maybe the specs meant also: If `req` was not active 3 clock cycles ago, the `ack` shall not be active
- Was it a mistake in the specs? Most probably
- So... the information about a certain signal being active at a certain time can hide the fact that it shall not be active any other time
- ⚠ That's a big danger for formal verification
- Mathematically speaking:
- $A \Rightarrow B$ only says that A implies B , but it does not say $\bar{A} \Rightarrow \bar{B}$

How to think about specs?

- In our case, we can add a new assertion: `ack -> prev(req, 3)`

Example 2



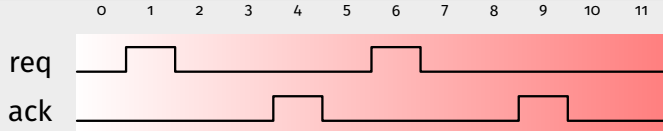
PSL

```
assert always (req -> next[3] ack);
assert always (ack -> prev(req, 3));
```

How to think about specs?

- In our case, we can add a new assertion: `ack -> prev(req, 3)`

Example 1



PSL

```
assert always (req -> next[3] ack);
assert always (ack -> prev(req, 3));
```

State machine verification

- Let's consider a finite state machine with potentially some internal counters
- How to verify its behavior?
 - Are there assumptions on the inputs?
 - How to express complex output behaviors?
 - Should we test the internal architecture?

State machine - assumptions

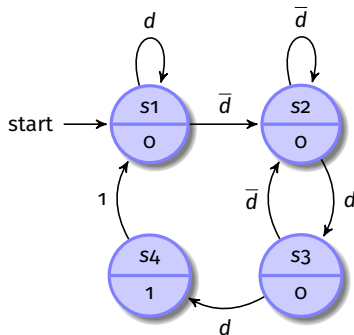
- Usually a state machine interacts with other parts of the system
- Are there really assumptions on the inputs?
 - The fewer assumptions the more reliable the FSM
 - Really depends on the system
 - Be careful not to restrict too much the inputs
- Helper code can be used to model part of the outside world

State machine - output behaviors

- Not always easy to test the outputs
- Using sequences can be an option, in comparison with LTL
- **Start to write what you do expect and then implement the corresponding assertions** (and maybe assumptions)
- Do not forget things like:
 - `ack` shall be '1' 3 cycles after `req`
 - Does it mean it shall be '0' any other time?
- FSM often have such kind of inputs

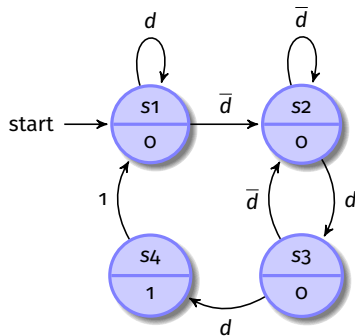
State machine - transitions

- Example: A sequence detector (011)
- The output shall go high when the sequence is detected (Moore machine)



State machine - transitions

- If we have access to the internal architecture
- Detect invalid transitions



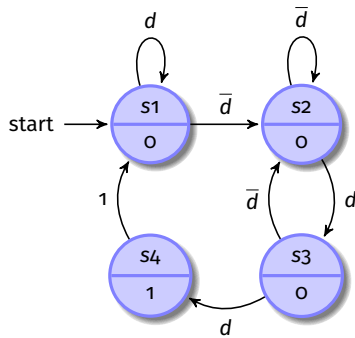
Check allowed transitions

```

assert always (
  ((state = S1) -> next((state = S1) or (state = S2)))
  abort rst_i);
assert always (
  ((state = S2) -> next((state = S2) or (state = S3)))
  abort rst_i);
assert always
  (((state = S3) -> next((state = S2) or (state = S4)))
  abort rst_i);
assert always (
  ((state = S4) -> next(state = S1))
  abort rst_i);
  
```

State machine - transitions

- If we have access to the internal architecture
- Detect transitions with the help of the inputs (often too complex)



Check allowed transitions

```

assert always (
  (((state = S1) and d) -> next(state = S1))
  abort rst_i);
assert always (
  (((state = S1) and not d) -> next(state = S2))
  abort rst_i);
assert always (
  (((state = S2) and d) -> next(state = S3))
  abort rst_i);
assert always (
  (((state = S2) and not d) -> next(state = S2))
  abort rst_i);
...
  
```


Key takeaway

Check the expected behavior

- In our example: `ack` must be high 3 cycles after `req`

Do not forget to check the unwritten behavior

- In our example: `ack` should be low every other clock cycle