Open Source Formal Verification

Sequential Extended Regular Expressions

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Sequential Extended Regular Expressions

- PSL offers SEREs
- A powerful way to express complex temporal behaviors
- Example:
  - {a;b[4];c}
  - a is active, followed by b active 4 cycles, and then c

Suffix Implication

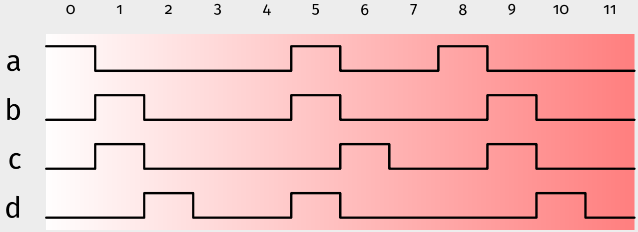
- Two *suffix implication* operators are defined:
    - `|->`
        - When the left-hand side finishes, then the right-hand side starts
    - `|=>`
        - When the left-hand side finishes, the right-hand side starts on the next clock cycle

Example

```
-- Sequence {c;d} is observed when {a;b} finishes
{a;b} |-> {c;d}
-- Sequence {c;d} is observed immediately after {a;b} finishes
{a;b} |=> {c;d}
```

Suffix Implication

Example



PSL

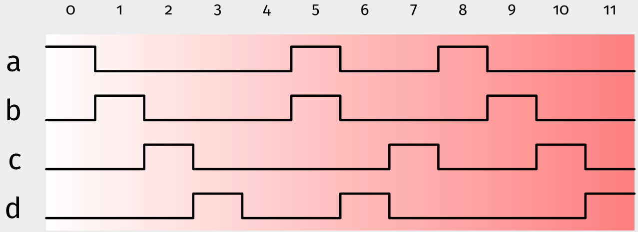
```
assert always({a;b} |-> {c;d});

assert always({a;b} |=> {c;d});

assert always({a and b} |-> {d;c});
```

Suffix Implication

Example



PSL

```
assert always({a;b} |-> {c;d});

assert always({a;b} |=> {c;d});
```

Operator never

- The `never` operator can be used with SEREs to ensure a sequence never happens
- Example:
  - Sequence a, b, c shall never be observed

```
assert never {a;b;c};
```

  - A read shall never precede a write

```
assert never {rd_o;wr_o};
```

  - A signal shall never be asserted on two consecutive cycles

```
assert never {r;r};
```

Repetition operators

- Repeating a sequence like `{a;a;a;a;a}` is not ideal as is

Operator	Description
`[*]`	Repeat a number of times between 0 and ∞
`[+]`	Repeat a number of times between 1 and ∞
`[*n]`	Repeat exactly *n* times
`[*n to m]`	Repeat a number of times between *n* and *m*
`[=n]`	*n* non consecutive repetitions
`[->n]`	*n* non consecutive goto repetitions

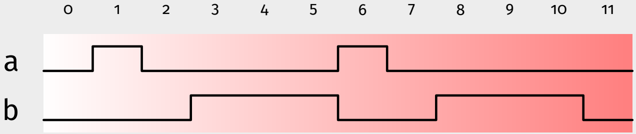
Repetitions: Examples

Sequence	Equivalent to
`{a;a;a;a;a}`	`{a[*5]}`
`{a;b;b;b;c}`	`{a;b[*3];c}`
`{a;b;c;b;c;d}`	`{a;{b;c}[*2];d}`

Sequence	Validated by
`{a;b;b;b;c}`	`{a;b[+];c}`
`{a;c}`	`{a;b[*];c}`
`{a;b;c;b;c;d}`	`{a;b[=2];d}`
`{a;b;c;b;c;d}`	`{a;c[->2];d}`

Repetitions

Exemple



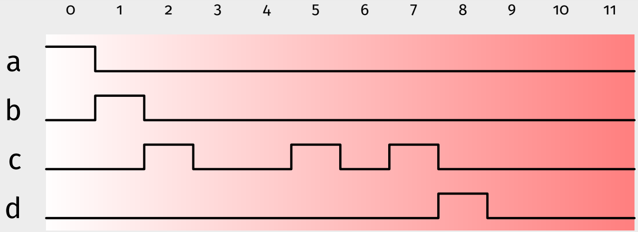
Assertions

```
assert always (a |-> {[*2];b});

assert always (a |-> {[*2];b[*3]});
```

Non consecutive repetitions

Example



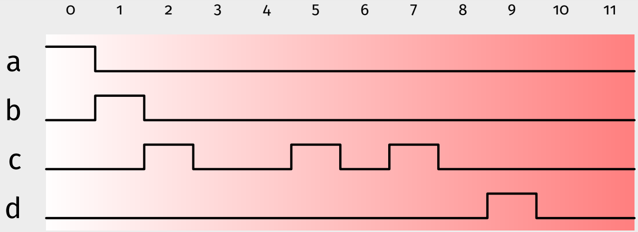
PSL

```
assert always({a} |=> {b;c[=3];d});
```


```
assert always({a} |=> {b;c[->3];d});
```

Non consecutive repetitions

Example



PSL

```
assert always({a} |=> {b;c[=3];d});

assert always({a} |=> {b;c[->3];d});
```

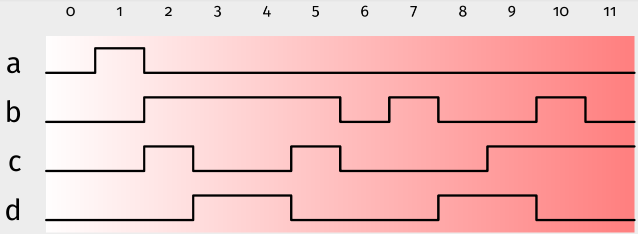
Sequences Combinations

Function	Description
`s1 ; s2`	concatenation of two sequences
`s1 : s2`	fusion of two sequences[1]
`s1 & s2`	non-length-matching *and* operator
`s1 && s2`	length-matching *and* operator
`s1 | s2`	*Or* operator
`s1 within s2`	s1 shall be observed during the execution of s2

[1]Concatenation, but s2 starts on the last cycle of s1. Not really used

Non-length-matching *and* operator

Exemple



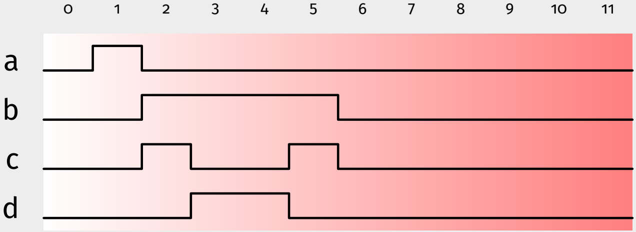
Assertions

```
assert always {a}|=>{{b[*4]} & {c; d}};

assert always {d[*2]}|=>{{c} & {b; not b}};
```

Length-matching *and* operator

Exemple



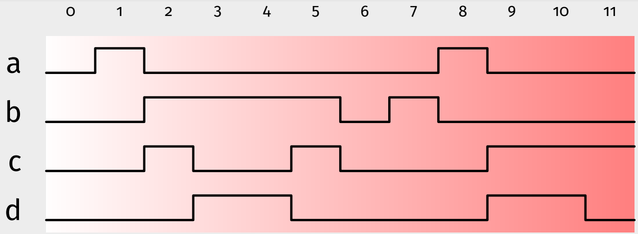
Assertions

```
assert always {a}|=>{{b[*4]} && {c; d[*2]; c}};
```


```
assert always {a}|=>{{b[*4]} && {c[*1 to 2]; d[*1 to 3]}};
```

Or operator

Exemple



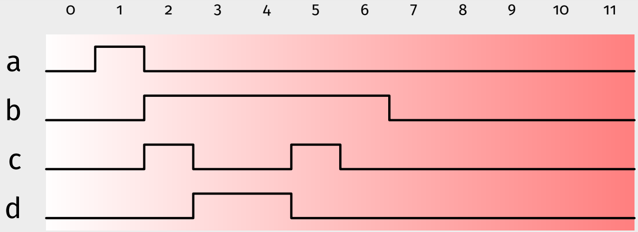
Assertions

```
assert always {a}|=>{{b[*4]} | {d; c}};

assert always {a}|=>{{[*1];d; b} | {d[*2]}};
```

s1 within s2

Example



PSL

```
assert always({a} |=> {{c;d[*2];c} within {b[*5]}});

assert always({a} |=> {{d[*2]} within {c;d[*2];c}});
```

Named sequences

- Like the properties, sequences can be named and then reused
  - Named sequences with parameters are not currently supported
- Particularly useful if a sequence is used in different contexts
  - For instance in an assert and a cover

PSL

```
sequence seq is {a; not a; a};

assert always({b} |=> {seq;d[*2];c});

cover seq;
```

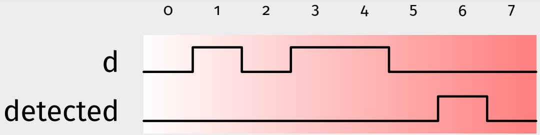
Detecting the end of a sequence

- Dealing with a signal that shall be asserted at the end of a sequence is tricky
- Examples:
  - A timer triggers after a certain number of clock cycles
  - A sequence detector triggers at the end of the sequence
- Some constructs exist, but are not supported by SBY (`ended()`)
- So, how to solve it?

Sequence detector

- Let's have a sequence detector that shall assert the signal `detected` when the sequence $d; d; \overline{d}; \overline{d}$ is observed (asserted at the same cycle as the last `not d`)

Example: valid chronogram

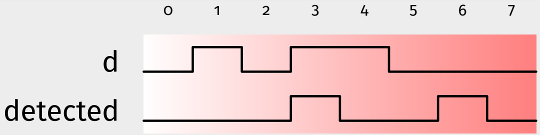


A nice assertion

```
   assert always({d;d;not d;not d} |-> detected);
-- or
   assert always({d[*2];(not d)[*2]} |-> detected);
```

Sequence detector

Invalid chronogram



- What happens then?

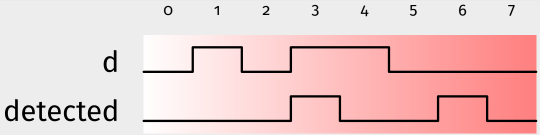
A nice assertion

```
assert always({d[*2];(not d)[*2]} |-> detected);
```

- Is it what we want?
- Well, our assertion does not say when `detected` shall stay low

Sequence detector

Invalid chronogram



- Idea : The signal should not be asserted twice before the sequence ends
  - Use `within` and the non consecutive goto repetition
  - Use the length-matching operator and the non consecutive goto repetition

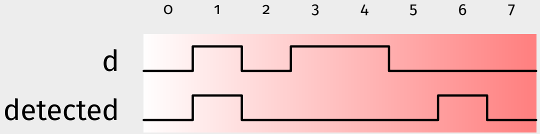
Assert that the signal does not go high twice within the sequence

```
assert never {{detected[->2]} within {d;d;not d;not d}};
assert never {{detected[->2]} && {d;d;not d;not d}};
```

- What is the issue here?

Sequence detector

Invalid chronogram



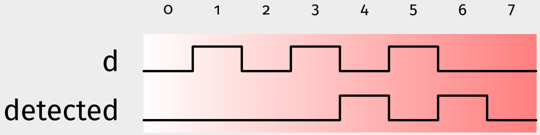
Assert that the signal does not go high twice within the sequence

```
assert never {{detected[->2]} within {d;d;not d;not d}};
assert never {{detected[->2]} && {d;d;not d;not d}};
```

- Well, maybe not the best idea
- And some issues with sequence overlapping

Sequence detector

Sequence now d;not d;d;not d : valid chronogram



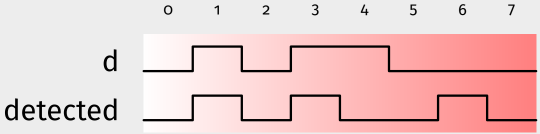
Assert that the signal does not go high twice within the sequence

```
assert never {{detected[->2]} within {d;not d;d;not d}};
assert never {{detected[->2]} && {d;not d;d;not d}};
```

- Well, once again, maybe not the best idea

Sequence detector

Example



- Idea : reverse the implication
  - Instead of `seq |-> det`, write `det |-> seq has occured`

Assert that the signal does not go high before the sequence ends

```
assert always (detected |->
            (d = '0' and prev(d) = '0' and prev(d,2) = '1' and prev(d,3) = '1'));
```

- Not easily generalizable. Imagine if the sequence is `{d[*3 to 12];(not d)[*5 to 15]}`

Sequence detector: Best option

- Use the following construct, specific to SBY:

```
attribute anyseq : boolean;
signal detected_test: std_logic;
attribute anyseq of detected_test : signal is true;   the signal is then a free variable
gen_test: assume always({d[*2];(not d)[*2]} |-> detected_test);
assert_low: assert always (detected -> detected_test);
assert_high: assert always({d[*2];(not d)[*2]} |-> detected);
```

- `gen_test` ensures that `detected_test` is high at the end of the sequence
  - Nothing more
  - So, it could be `'0'` or `'1'` at any other time
- `assert_low` only says that if `detected` is high, then `detected_test` **has to be high**
  - Nothing more, so
  - If `detected` was to be high at another time, as `detected_test` could have any value, `assert_low` would fail
- QED

Sequence detector: Conclusion

- The approach based on the assertion of the sequence detection and the free variable can be applied to any kind of sequence
  - A pure sequence detector
  - Triggering of a timer
  - ...

LTL - SERE equivalences

- LTL formula can be expressed in SERE-style

LTL	SERE
`a until b`	`{a[*];b}`
`a and next b`	`{a;b}`
`next[i] (a)`	`{[*i];a}`
`[next_a[i to j] (b)`	`{[*i];b[*j-i+1]}`
`[next_e[i to j] (b)`	`{[*i to j];b}`

Conclusion

- SEREs are powerful
- But careful with their definition you should be
- Do not be afraid of using coverage to know if a sequence has been observed or not
- Quite often LTL vs SERE is a matter of preference