



**POLITECNICO**  
MILANO 1863

# Displacement and distance sensors

**SENSOR SYSTEMS**

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- Potentiometer
- Capacitive sensors
- Inductive sensors
- Acoustic sensors
- Optical sensors
- Magnetic sensors

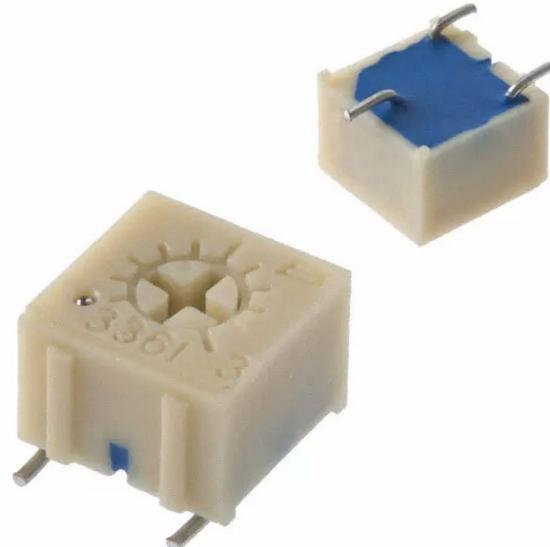
- Linear displacement
  - potentiometer
  - capacitive
  - LVDT (inductive)
  - optical encoder
  - magnetic encoder
- Angular displacement
  - potentiometer
  - optical encoder
  - magnetic encoder
- Proximity (ON/OFF sensor) / distance
  - capacitive
  - inductive
  - ultrasonic
  - optical

# Proximity/distance sensors comparison

| Technology         | Sensing range | Application  |
|--------------------|---------------|--|
| Capacitive sensors | 3mm – 60mm    | Close range detection of non metallic materials                    |
| Inductive sensors  | 4mm – 40mm    | Close range detection of metallic materials                        |
| Acoustic sensors   | 30mm – 3m     | Long range detection of targets with difficult surface properties. |
| Optical sensors    | 1mm-50m       | Long range detection   |

# Potentiometers

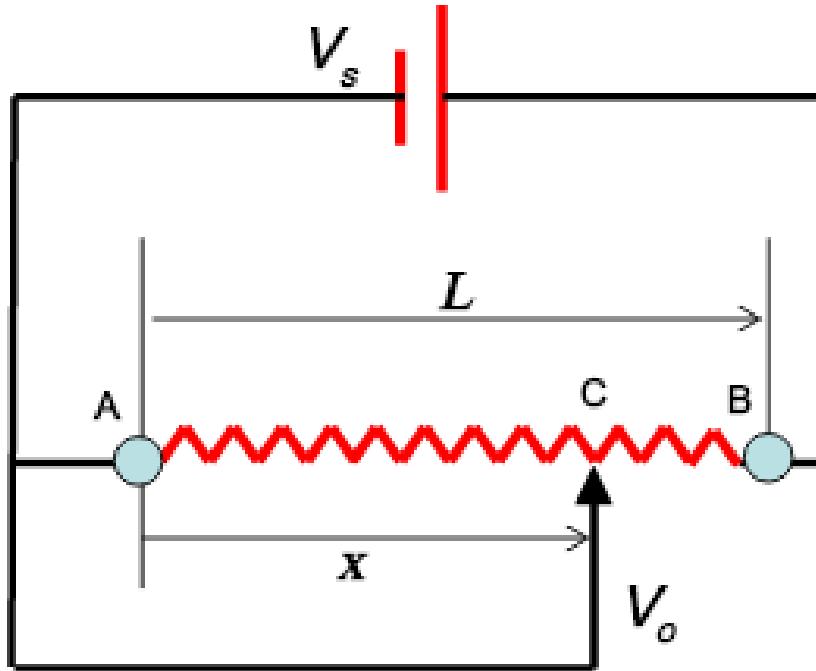
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Linear displacement

Angular displacement

Resistive potentiometer = resistance element with movable contact (slider).



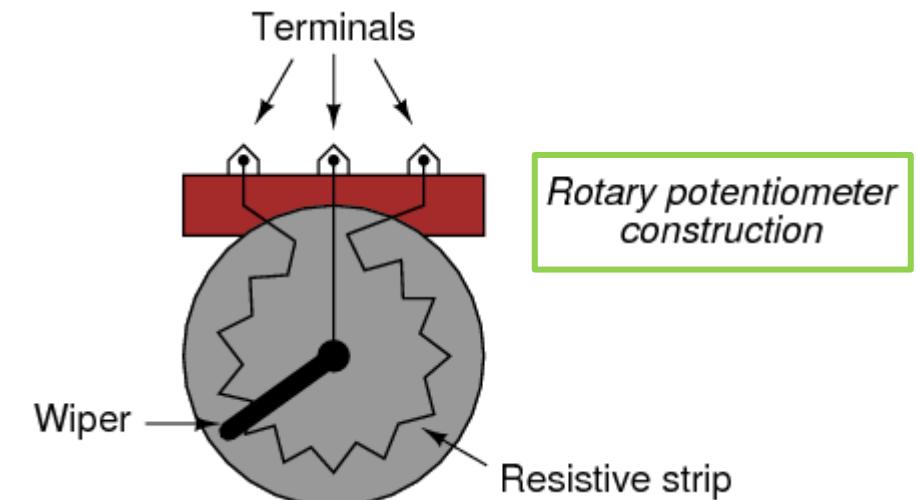
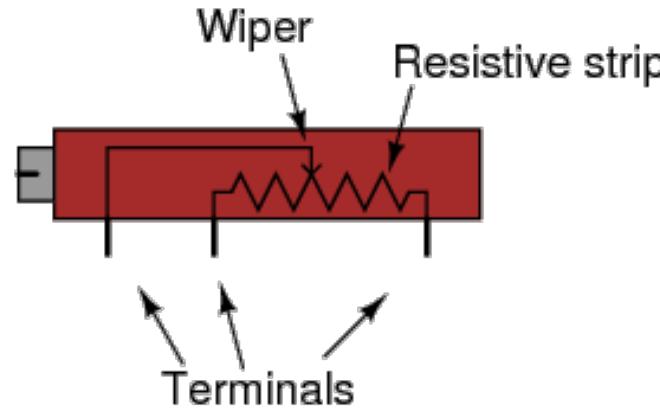
$$V_0 = V_s \cdot \frac{R_{AC}}{R_{AC} + R_{CB}} = V_s \cdot \frac{R_{AC}}{R_{AB}}$$

$V_0$     $\alpha$     $R_{AC}$     $\alpha$    position

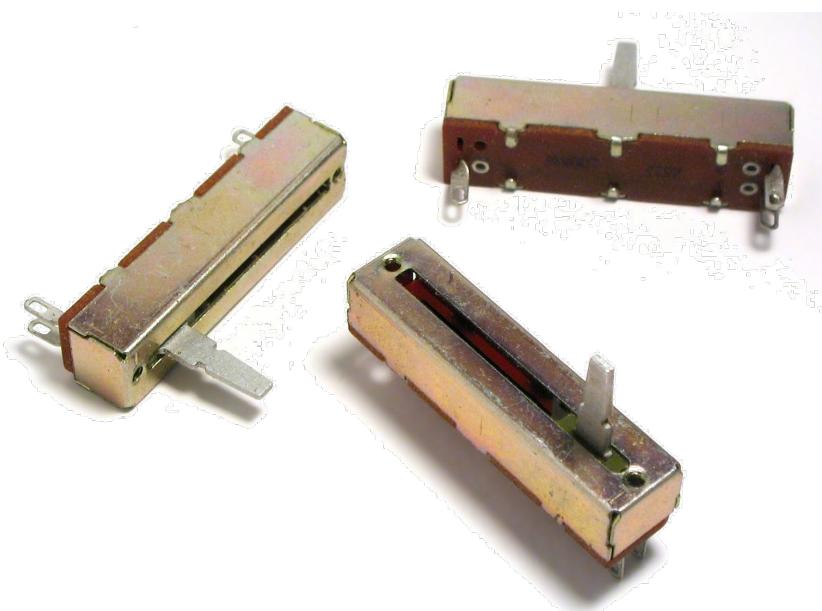
# Potentiometers constructions

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## Linear potentiometer construction

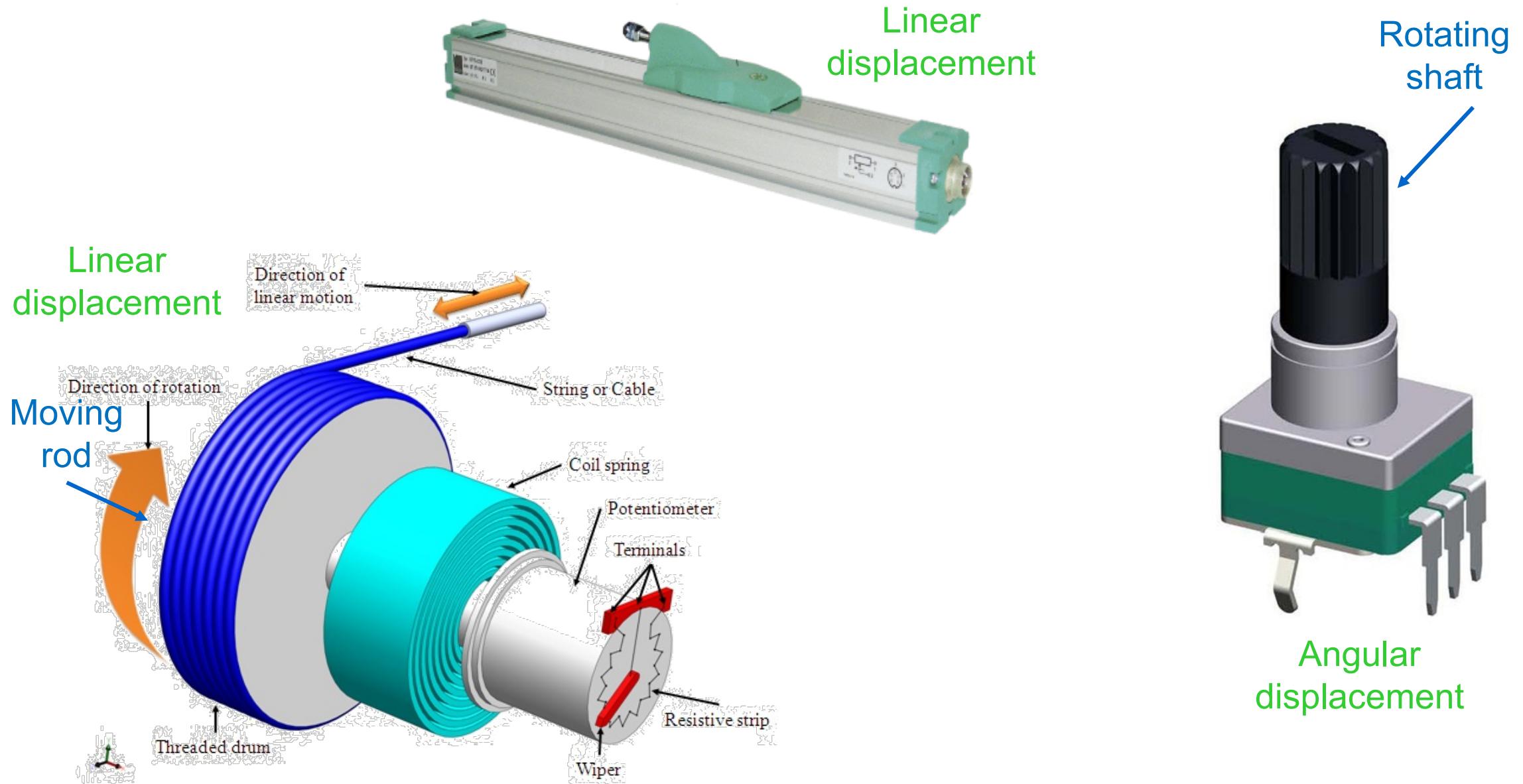


## Rotary potentiometer construction



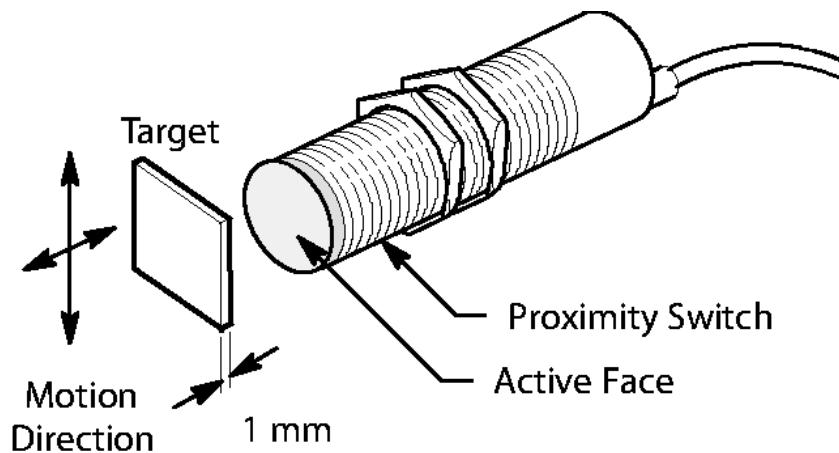
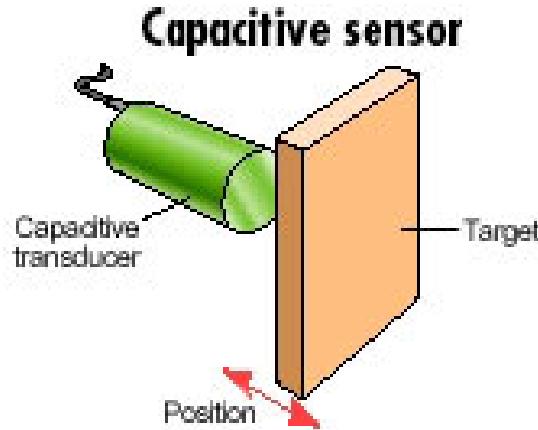
# Mechanical connections

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# Capacitive sensors

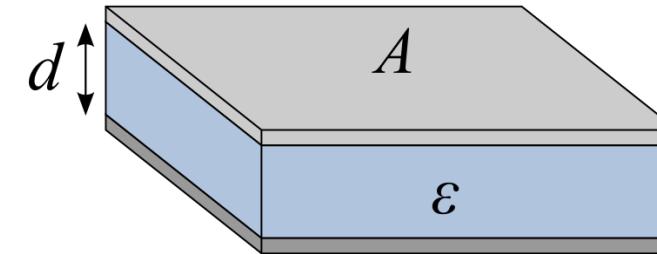
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Linear displacement  
Proximity

Parallel plate capacitor:

$$C = \epsilon_r \epsilon_0 \frac{A}{d}$$



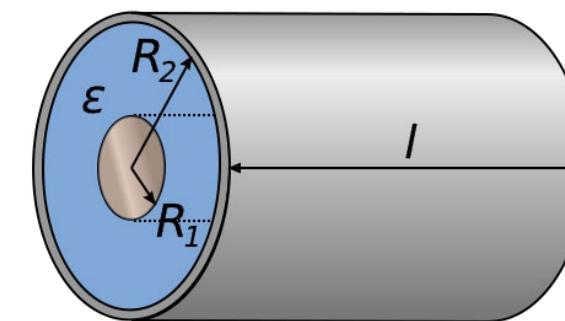
Cylindrical capacitor:

$$C = \frac{2\pi \epsilon_r \epsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)}$$

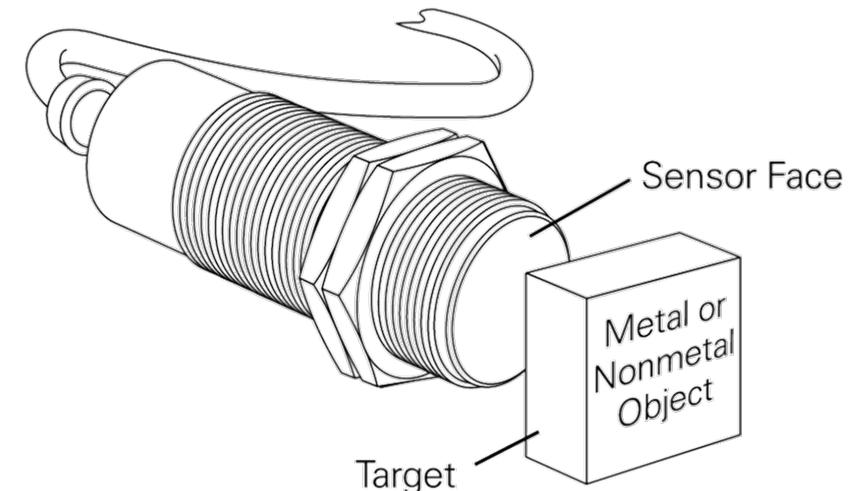
Thin cylindrical capacitor

$$(R_1 \approx R_2 = R, R_1 - R_2 = W \Rightarrow C = \frac{2\pi \epsilon_r \epsilon_0 l}{\ln(1 + W/R)})$$

$$C = \epsilon_r \epsilon_0 \frac{2\pi Rl}{W}$$

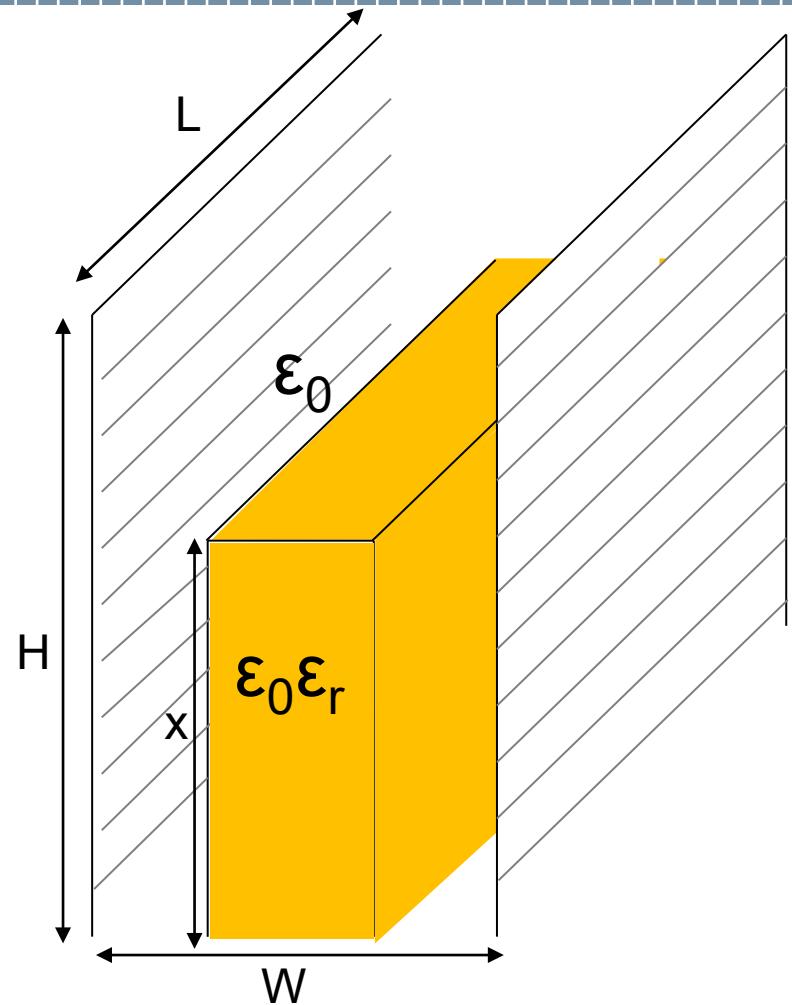


Capacitive sensors produce an electrostatic field, the capacitance is modified by the presence/absence or the displacement of a metallic or non-metallic object at small distance from the sensor.



## Capacitance variations:

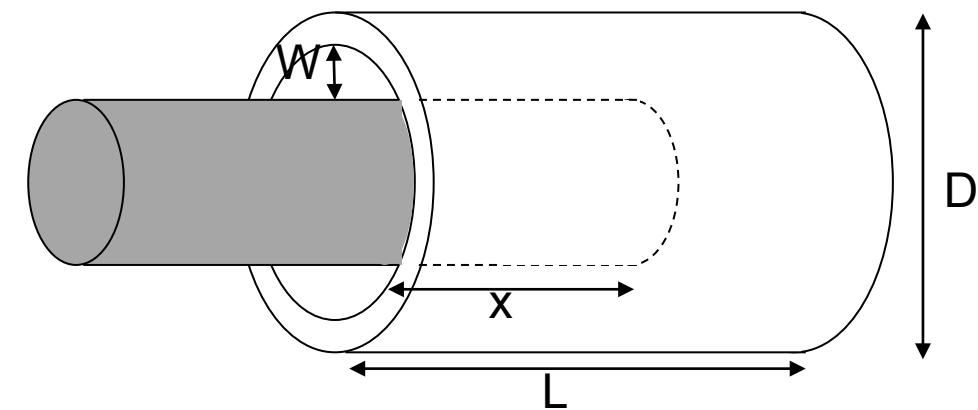
- Changes in the dielectric constant
- Changes in the distance between two plates
- Changes in the plates area



Changes in the dielectric constant

$$C_{TOT} = \frac{\epsilon_0 L}{W} [H + x(\epsilon_r - 1)]$$

1 μm resolution

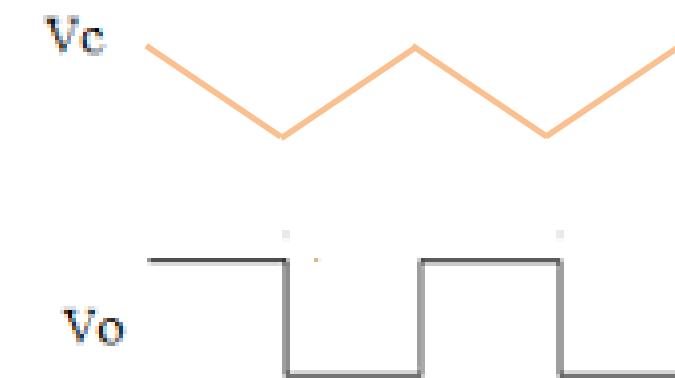
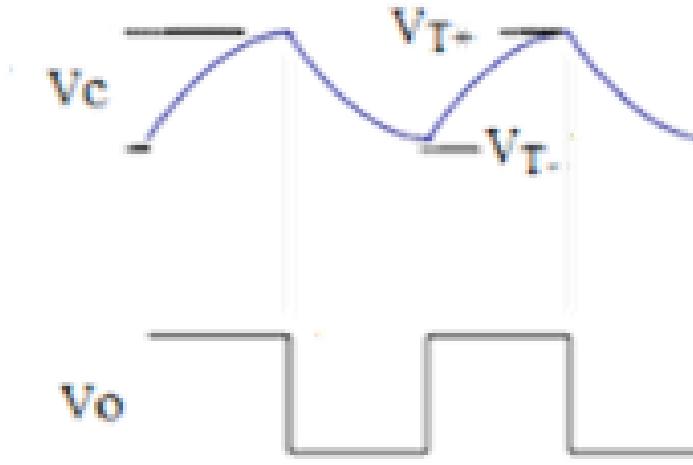
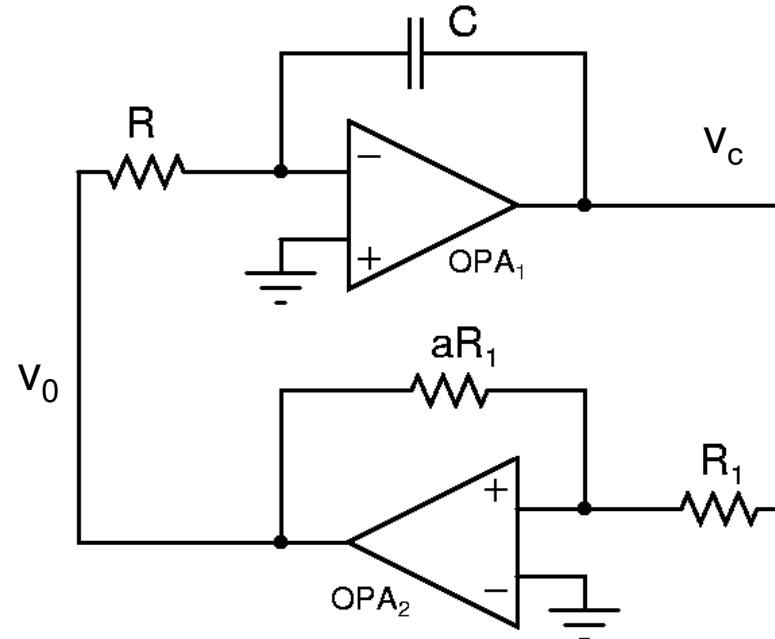
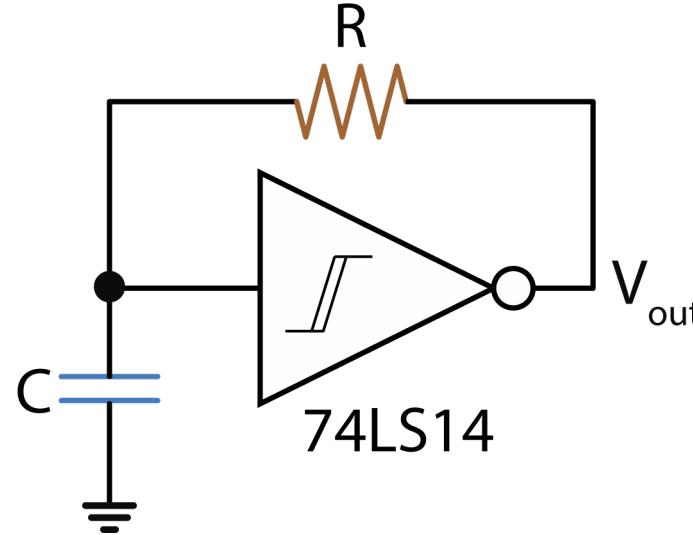


Changes in the plates area

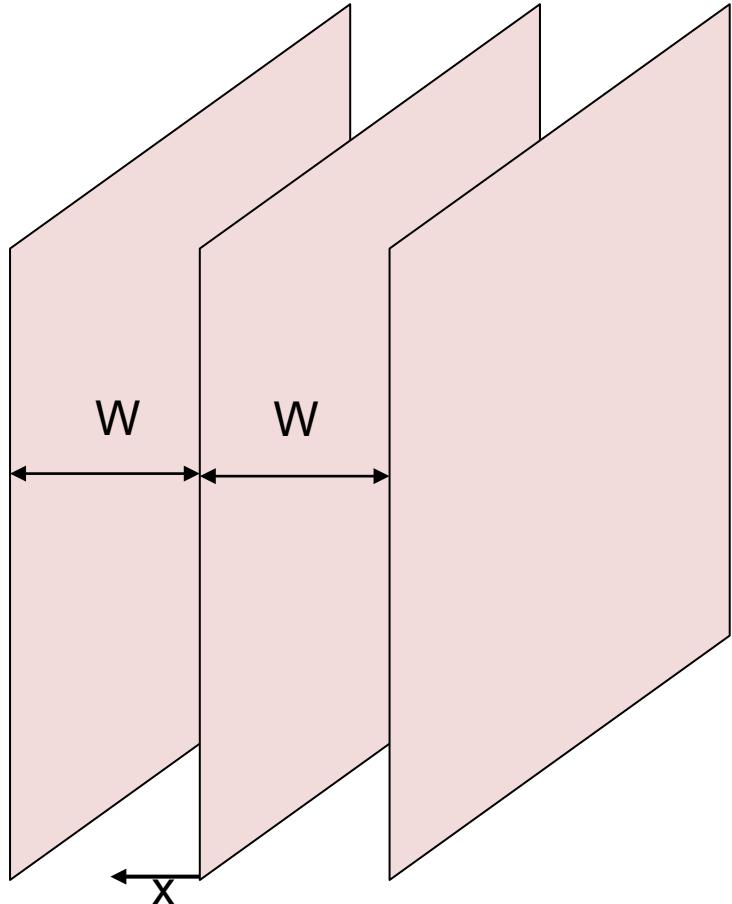
$$\text{Thin cylinder: } C_{TOT} = \epsilon_0 \epsilon_r \frac{\pi D x}{W}$$

# Square wave oscillator

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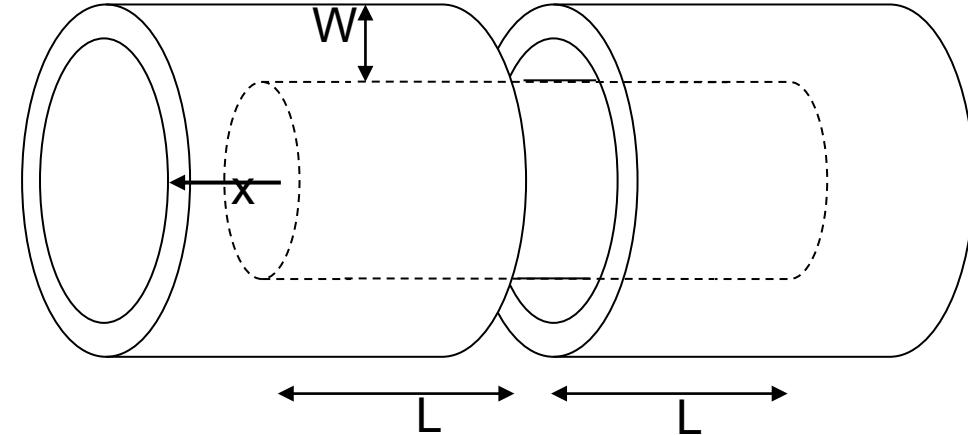
With an object in proximity C increase  
→ oscillation frequency decreases



$$C_1 = \varepsilon_0 \cdot \frac{A}{W-x} \quad C_2 = \varepsilon_0 \cdot \frac{A}{W+x}$$

With Wheatstone bridge:

$$V_{out} = V_A \cdot \frac{x}{2W}$$

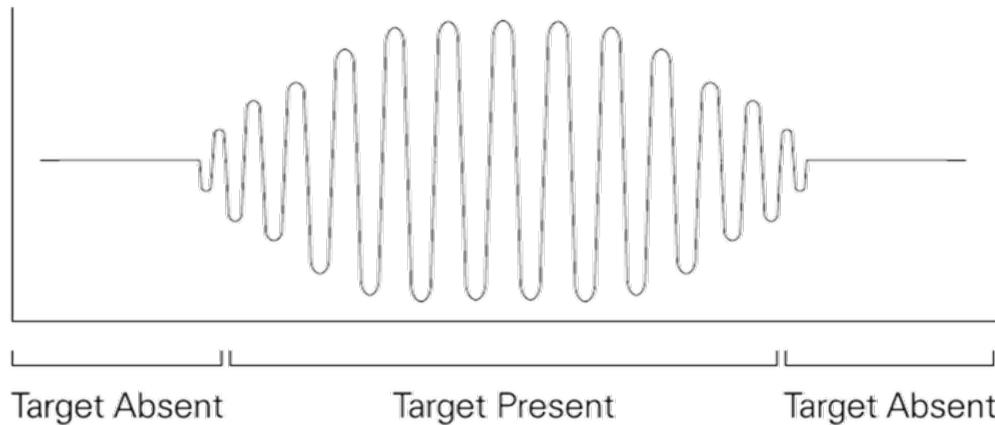


$$C_1 = \frac{\varepsilon_0 \pi D}{W} \cdot (L + x) \quad C_2 = \frac{\varepsilon_0 \pi D}{W} \cdot (L - x)$$

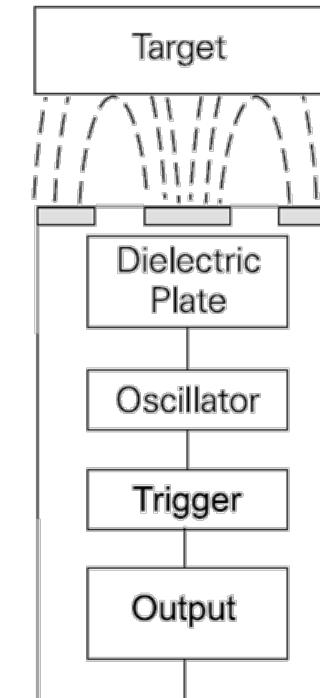
With Wheatstone bridge:

$$V_{out} = V_A \cdot \frac{x}{2L}$$

- 1- The sensing surface is formed by two concentrically shaped metal electrodes of a planar capacitor.
- 2- When an object nears the sensing surface it enters the electrostatic field of the electrodes and changes the capacitance.
- 3- As a result, an oscillator begins oscillating. The trigger circuit reads the oscillator's amplitude and the output state of the sensor changes.
- 4- As the target moves away from the sensor the oscillator's amplitude/ oscillation frequency decreases, switching the sensor output back to its original state.

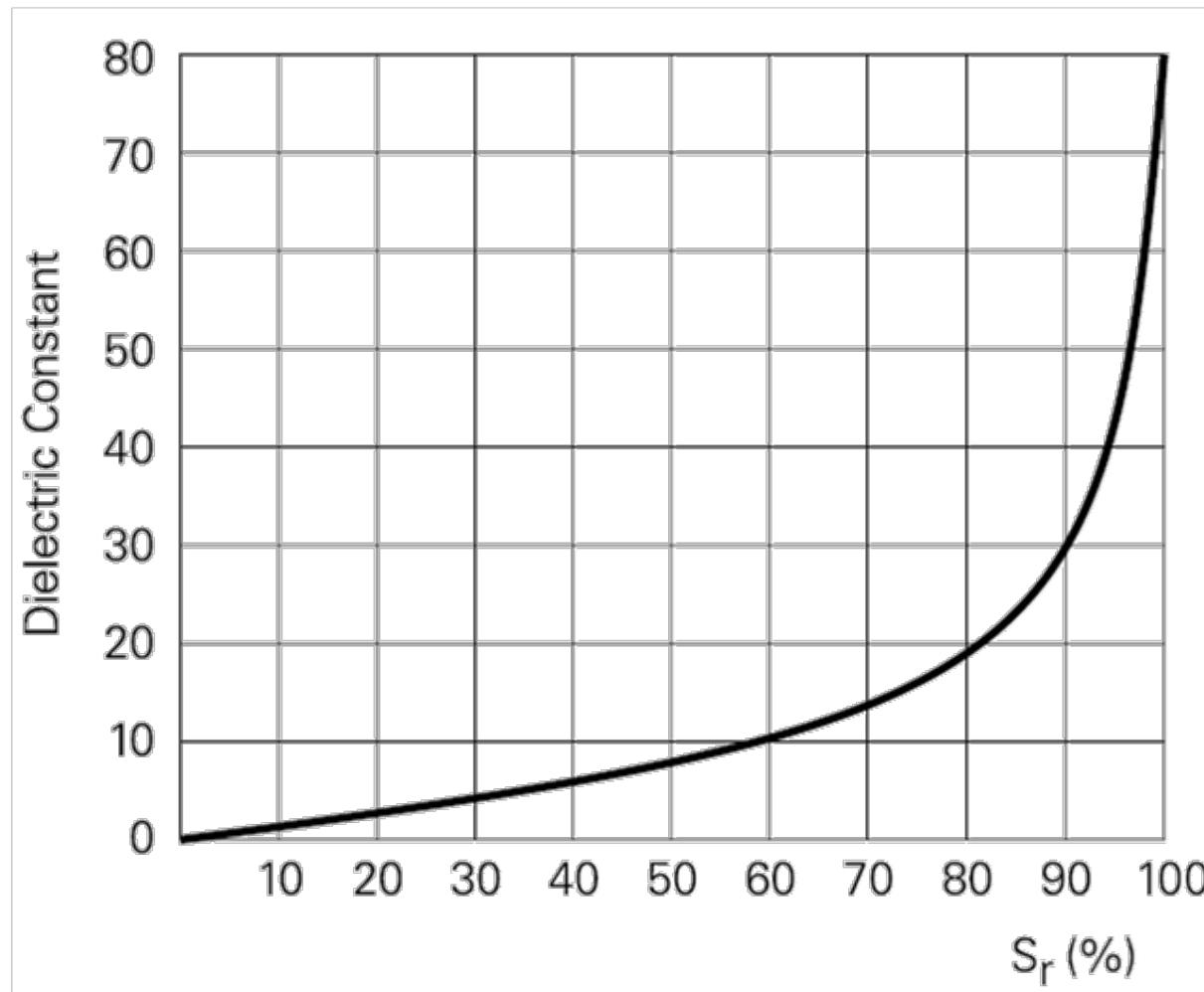


C increases



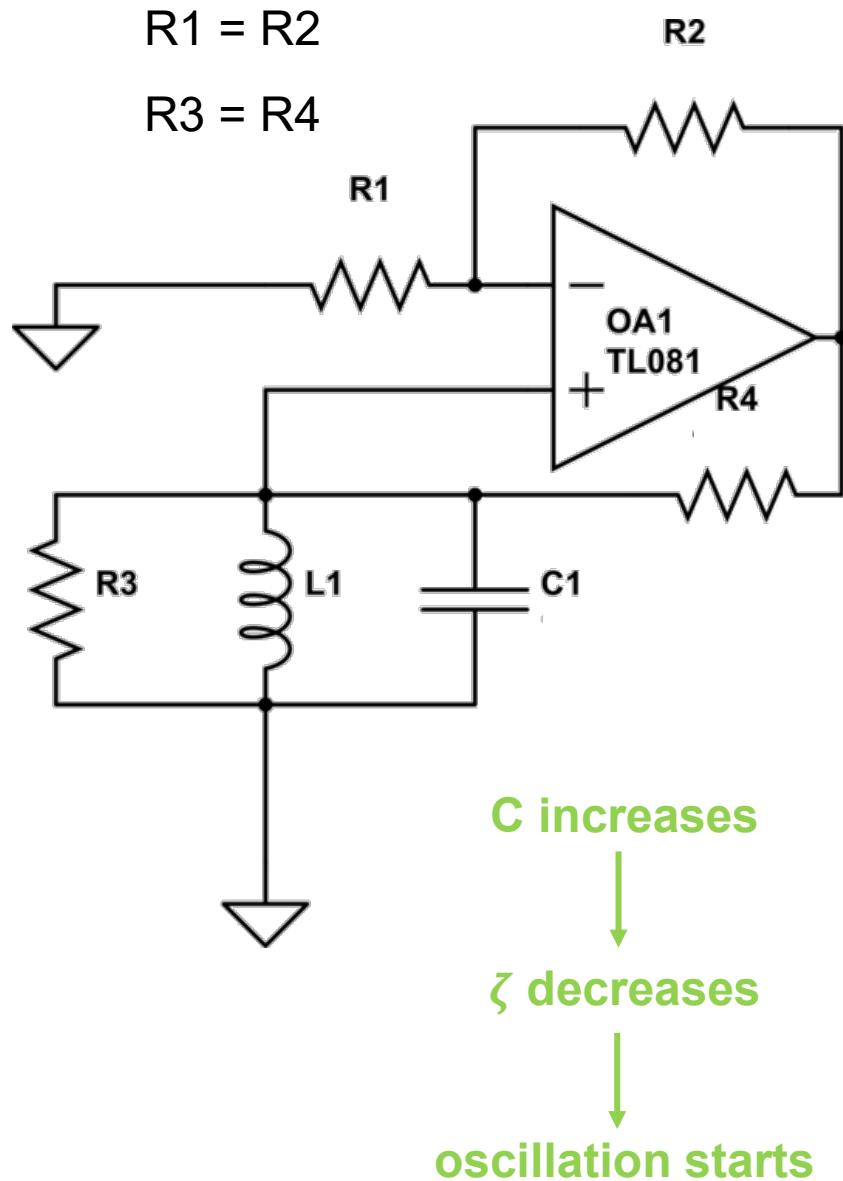
# Dielectric constant

| Material               | Dielectric Constant | Material           | Dielectric Constant |
|------------------------|---------------------|--------------------|---------------------|
| Alcohol                | 25.8                | Polyamide          | 5                   |
| Araldite               | 3.6                 | Polyethylene       | 2.3                 |
| Bakelite               | 3.6                 | Polypropylene      | 2.3                 |
| Glass                  | 5                   | Polystyrene        | 3                   |
| Mica                   | 6                   | Polyvinyl Chloride | 2.9                 |
| Hard Rubber            | 4                   | Porcelain          | 4.4                 |
| Paper-Based Laminate   | 4.5                 | Pressboard         | 4                   |
| Wood                   | 2.7                 | Silica Glass       | 3.7                 |
| Cable Casting Compound | 2.5                 | Silica Sand        | 4.5                 |
| Air, Vacuum            | 1                   | Silicone Rubber    | 2.8                 |
| Marble                 | 8                   | Teflon             | 2                   |
| Oil-Impregnated Paper  | 4                   | Turpentine Oil     | 2.2                 |
| Paper                  | 2.3                 | Transformer Oil    | 2.2                 |
| Paraffin               | 2.2                 | Water              | 80                  |
| Petroleum              | 2.2                 | Soft Rubber        | 2.5                 |
| Plexiglas              | 3.2                 | Celluloid          | 3                   |



In capacitive sensors the sensing distance is rated for water ( $\epsilon_r=80$ ).

E.g., capacitive sensor rated for 10 mm, with alcohol ( $\epsilon_r=25.8$ ) the effective sensing distance is 8.5 mm.



Resonant frequency:  $\omega_0 = \frac{1}{\sqrt{LC}}$

Damping:  $\zeta = \frac{1}{2Q} = \frac{1}{2R} \sqrt{\frac{L}{C}}$   
(Q=quality factor)

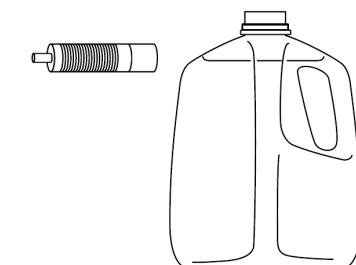
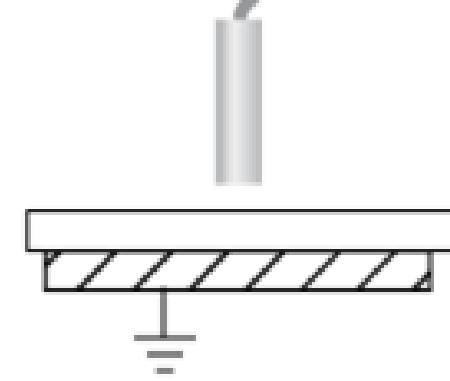
$\zeta < 1$  under-damped

$\zeta = 1$  critically damped

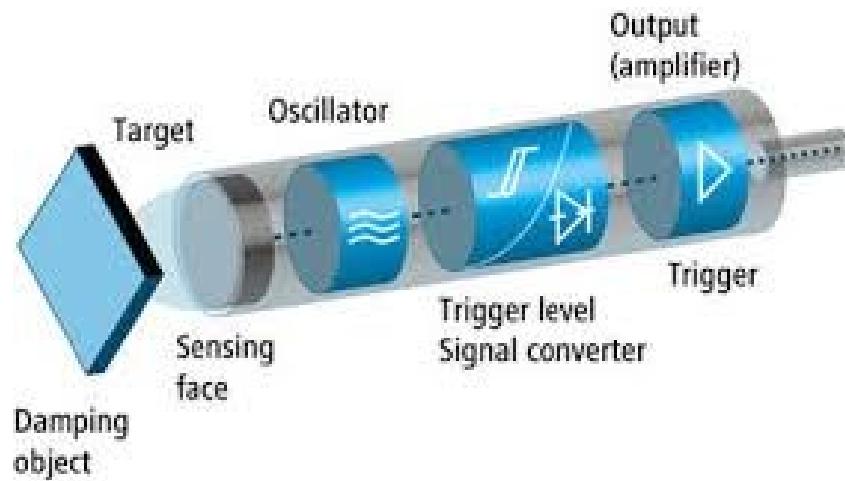
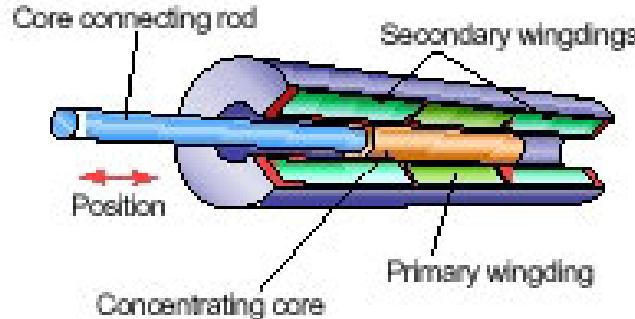
$\zeta > 1$  over-damped

at  $\omega_0$  negative and positive feedback have the same strength

- Position Measurement  
(Automation requiring precise location, Precision stage positioning)
- Dynamic Motion  
(Vibration measurements)
- Nonconductive Thickness  
(Label counting, Sensing water-based fluids applied to materials)
- Assembly testing  
(Capacitive sensors have a much higher sensitivity to conductors than to nonconductors. Therefore, they can be used to detect the presence/absence of metallic subassemblies in completed assemblies.)
- Detection through barriers



## Inductive sensor (LVDT)



Linear displacement  
Proximity

Faraday-Neumann-Lenz law:

$$V_i = -n \cdot \frac{d\Phi(\vec{B})}{dt}$$

$$\Phi(\vec{B}) = \vec{B} \cdot \vec{A} = BA \cdot \cos\alpha$$

With  $\vec{B}$  normal to A:

$$V_i = -n \cdot A \cdot \frac{dB}{dt}$$

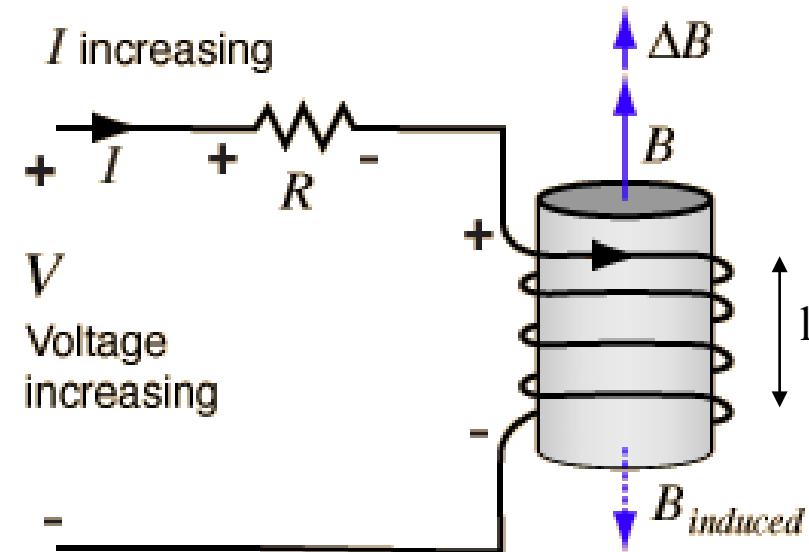
For a solenoid:

$$B = \mu_0 \mu_r \cdot \frac{n}{l} \cdot I$$

thus:

$$V_i = -\frac{\mu_0 \mu_r \cdot n^2 \cdot A}{l} \cdot \frac{dI}{dt} = -L \cdot \frac{dI}{dt}$$

Self-inductance:  $L = \frac{\mu_0 \mu_r \cdot n^2 \cdot A}{l}$



I in (1) induce  $\vec{B}$  in (2):

$$B_2 = k \cdot B_1 = k \cdot \mu_0 \mu_r \cdot \frac{n_1}{l} \cdot I_1$$

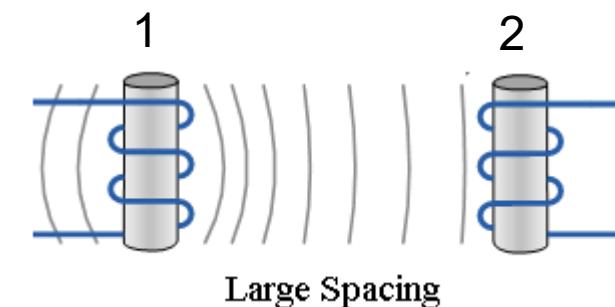
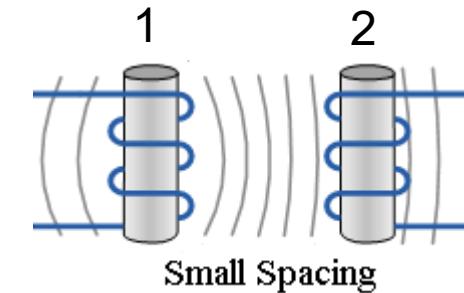
thus:

$$V_{i2} = -n_2 \cdot A \cdot \frac{dB_2}{dt} = -\frac{k \cdot \mu_0 \mu_r \cdot n_1 n_2 \cdot A}{l} \cdot \frac{dI_1}{dt}$$

$$V_{i2} = -M \cdot \frac{dI_1}{dt}$$

Mutual-inductance:  $M = \frac{k \cdot \mu_0 \mu_r \cdot n_1 n_2 \cdot A}{l} = k \sqrt{L_1 L_2}$

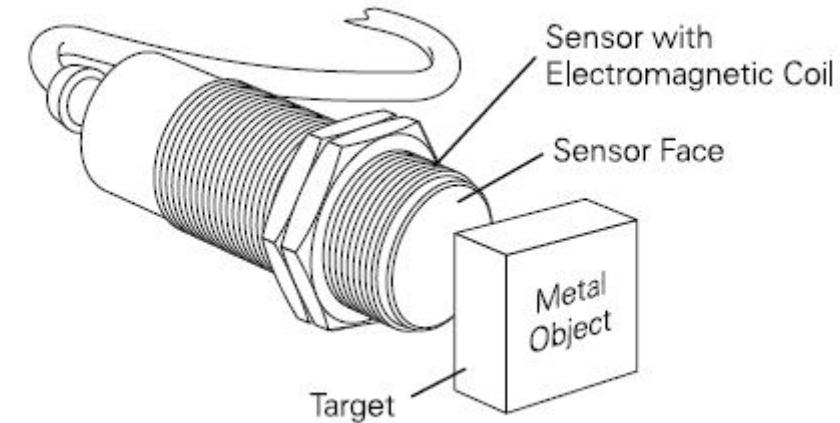
$k$  = coupling coefficient ( $0 < k < 1$ )



$$l_1 = l_2 = l$$

$$A_1 = A_2 = A$$

Inductive sensors produce an electromagnetic field, the inductance is modified by the presence/absence or the displacement of a metallic object at small distance from the sensor.



## Inductance variations:

- Changes in coupling coefficient
- Dumping

Measure linear displacement

1 primary coil excited by an oscillator

→ generates the magnetic field

2 secondary coils with same  $N_2$  and same  $I$

→ induced voltage

Ferromagnetic core (moving element)

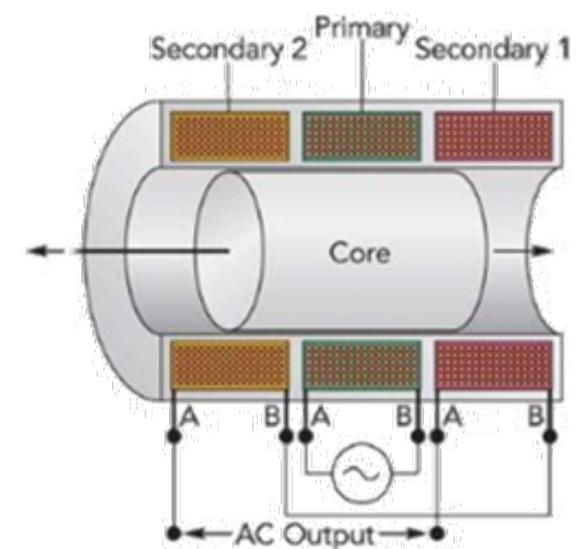
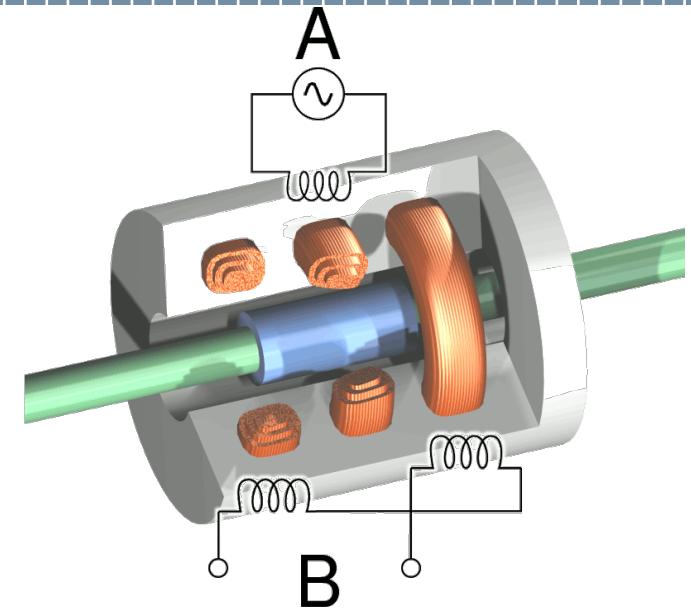
→ determines the amount of coupling

Core in its center position

→ secondary coils are equally coupled

Core displaced from its center position

→ one of the secondary coils is more strongly coupled

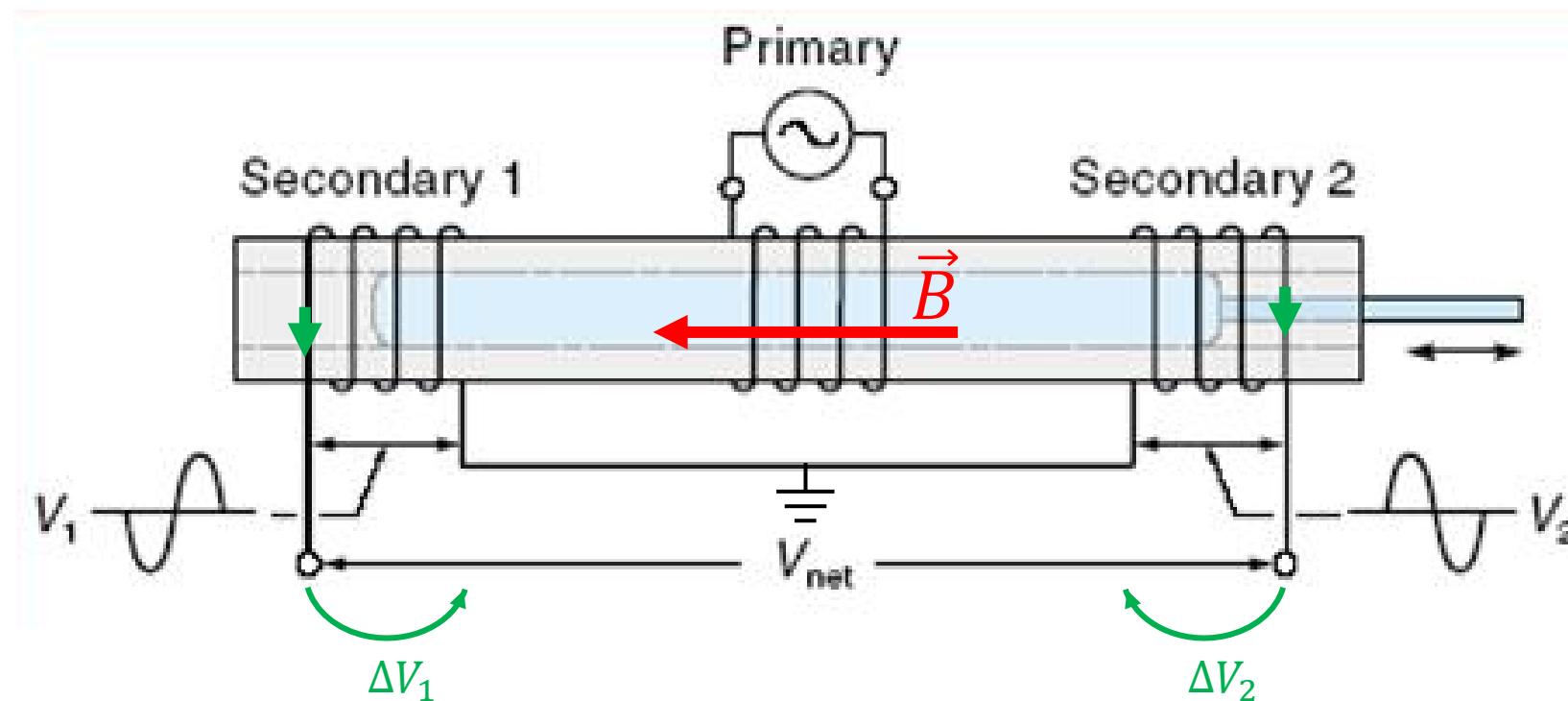


Core in its center position

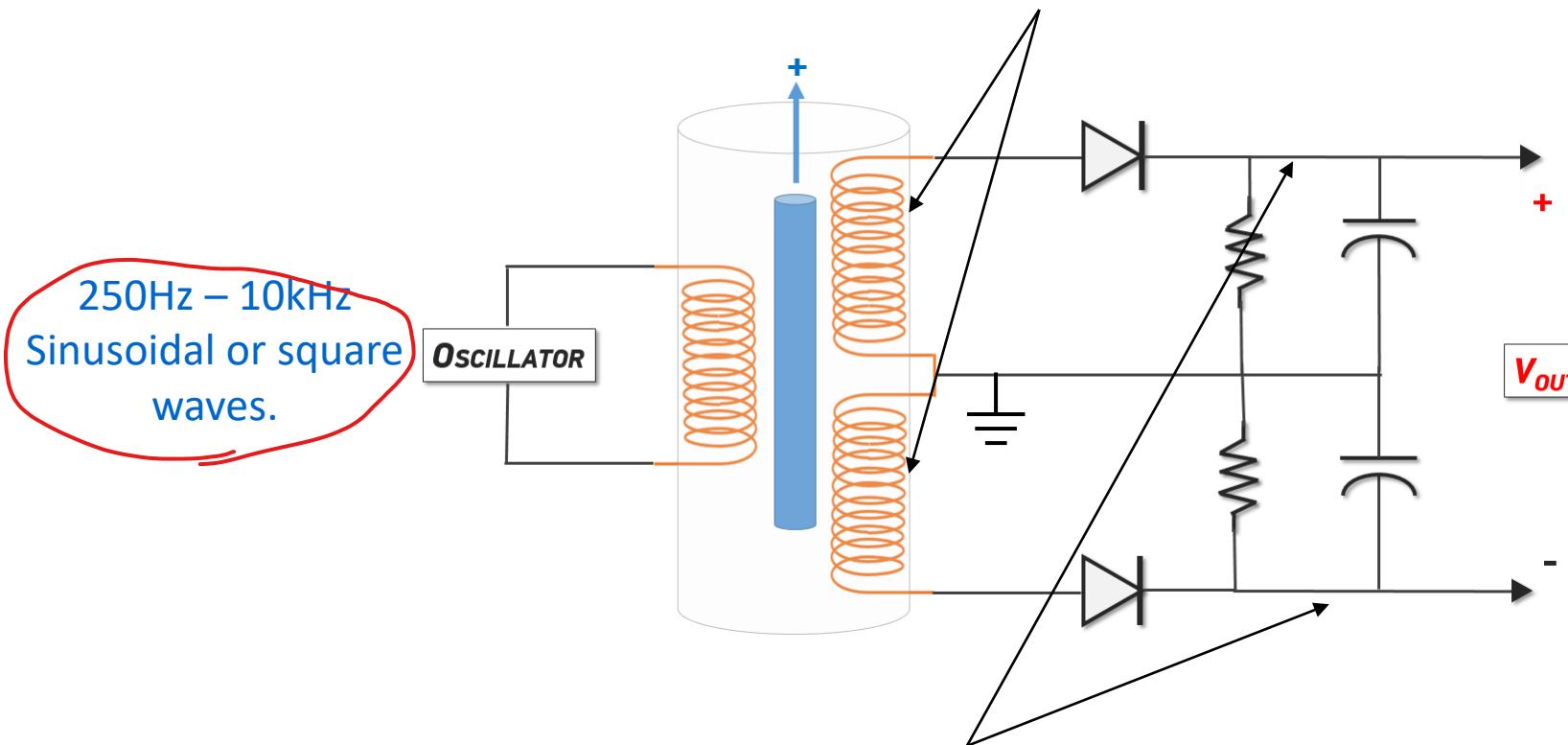
→ secondary coils are equally coupled

Core displaced from its center position

→ one of the secondary coils is more strongly coupled



Secondary coils mounted in opposite direction  
→ output the excitation signal in opposite phase.



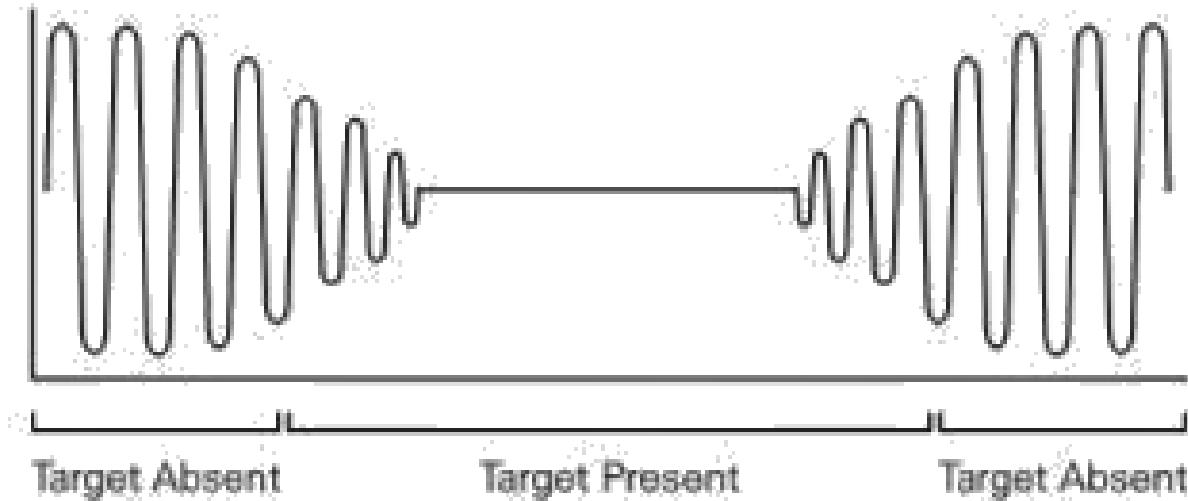
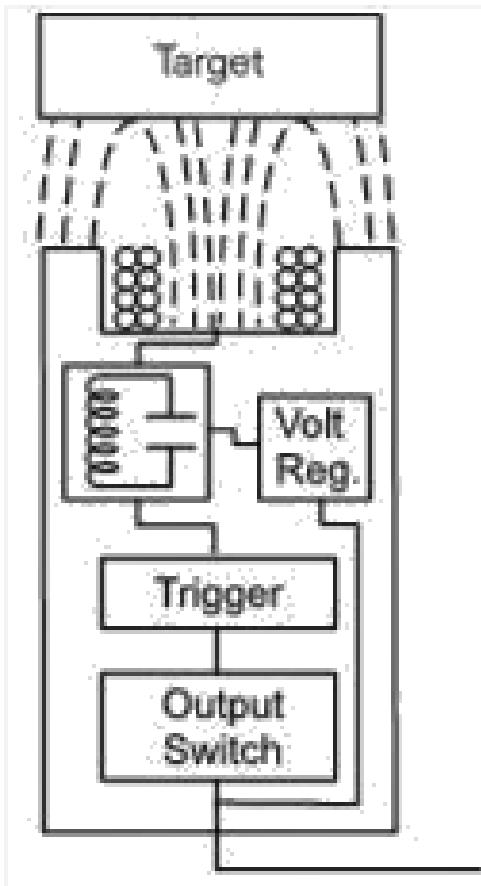
250Hz – 10kHz  
Sinusoidal or square waves.

A differential voltage indicates the direction of the displacement

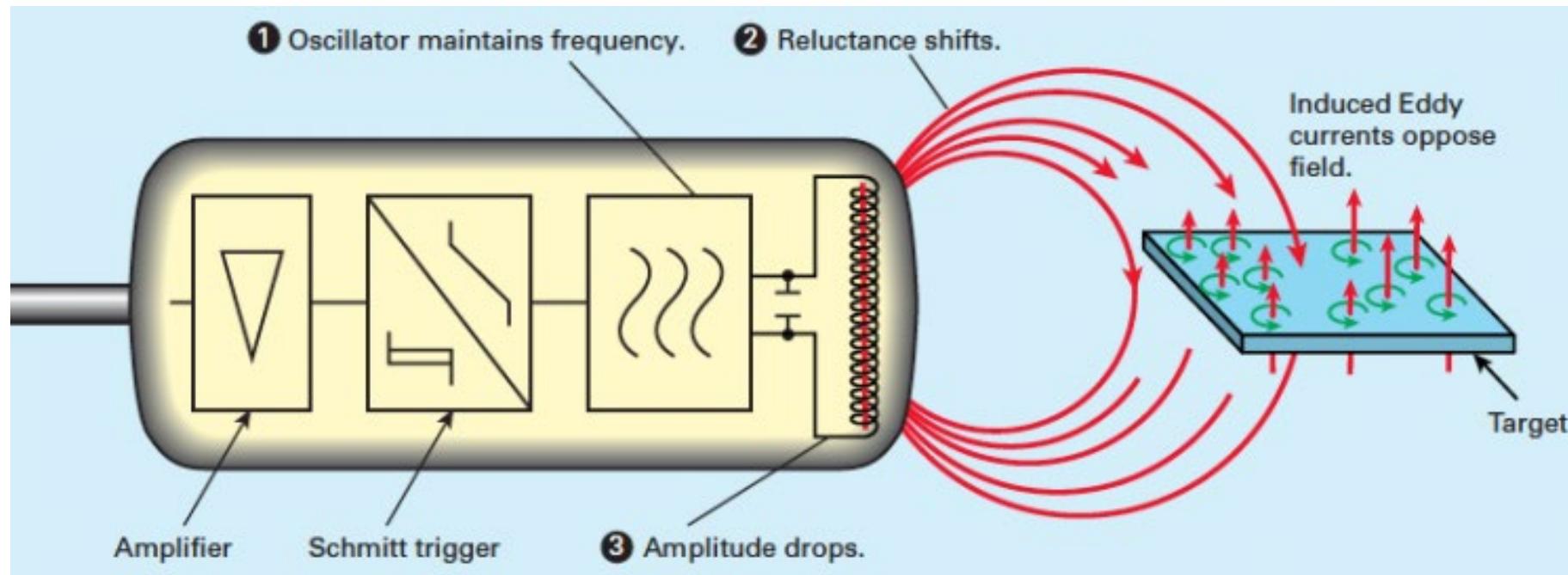
Outputs of the secondary coils rectified and compared  
→ voltage which varies linearly between the minimum/maximum displacement with the center position being at zero volts

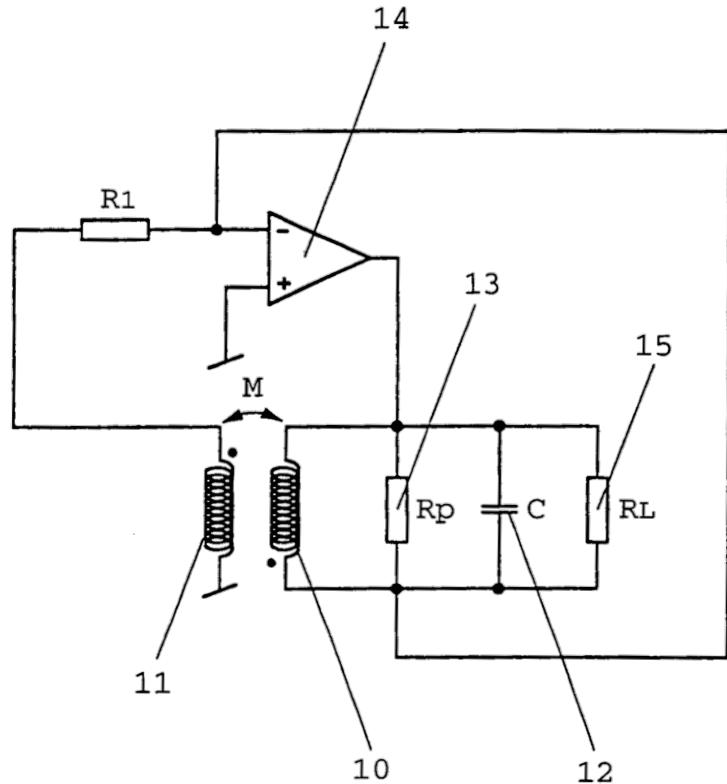
## Sensor components:

- LC oscillating circuit
- signal evaluator
- switching amplifier



- 1- Coil → generates a high-frequency electromagnetic alternating field
- 2- If metallic objects nears the sensing face → Eddy currents are generated
- 3- These losses draw energy from the oscillating circuit and reduce oscillation
- 4- The signal evaluator detects this reduction and converts it into a switching signal.





Without target:  $|g \cdot k| \geq 1$   
 $(R_p$  infinite)

With target:  $|g \cdot k| < 1$   
 $(R_p$  decreases)  
 $\rightarrow$  oscillation stops

- 10: primary winding
- 13: represent the losses by eddy currents  
in the case where a target is near.
- 11: secondary winding  
coupled with the primary winding

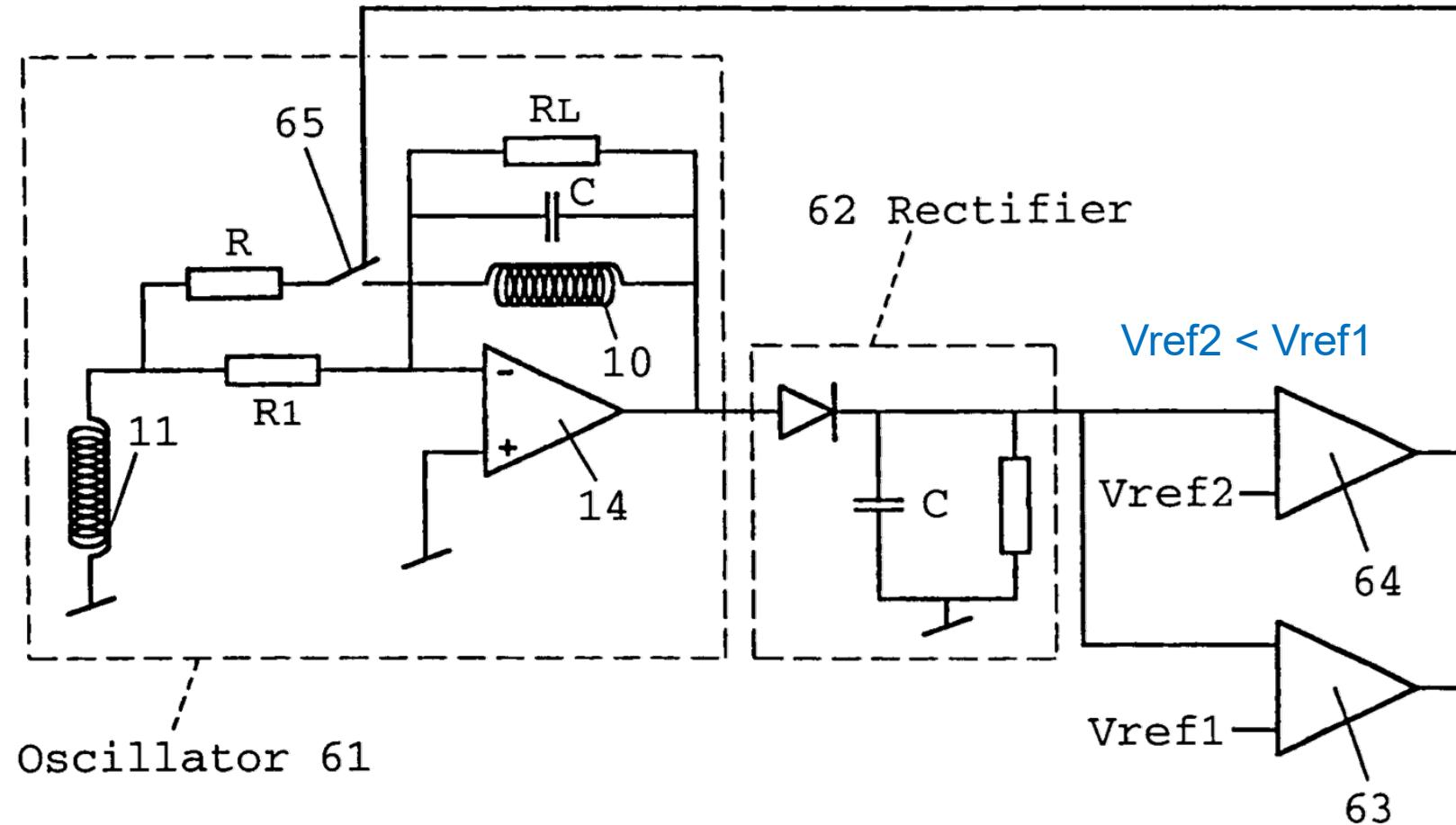
Without the inductive reaction:

$$g = -\frac{R_L//R_p}{R_1}$$

Ratio of the primary and secondary voltage:

$$k = \frac{V_2}{V_1}$$

The condition of oscillation is given by:  
 $|g \cdot k| \geq 1$



Comparator **64** closes switch **65**, increasing the gain of amplifier **14**, thus restarting the oscillation.

LVDT:

resolution down to 1 mm

→ Industrial, military, and aerospace applications

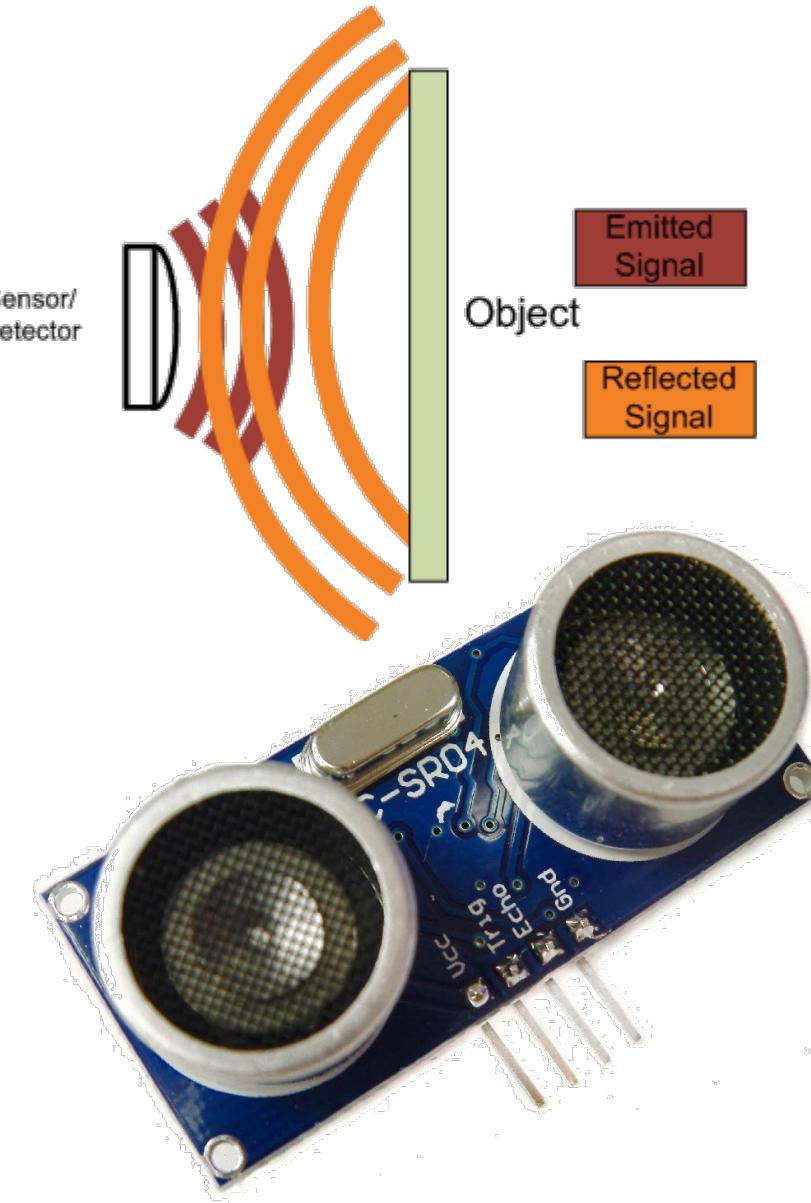
- aircraft wing flaps
- off-axis rotational movement of wheels

Proximity:

Safety and warning systems

- parking sensors
- ground proximity warning system





WeBeep: 15 – Ultrasonic distance sensors



Proximity  
Distance

Ultrasonic waves are sounds which cannot be heard by humans

→ frequencies of above 20 kHz

Sound velocity in air:  $c = 331.5 + 0.607 \cdot T$  (m/s) (T temperature in °C)

(@ 20°C:  $c = 344$  m/s)

→ need for temperature compensation

Wavelength:  $\lambda = \frac{c}{f}$  → shorter  $\lambda$  better resolution

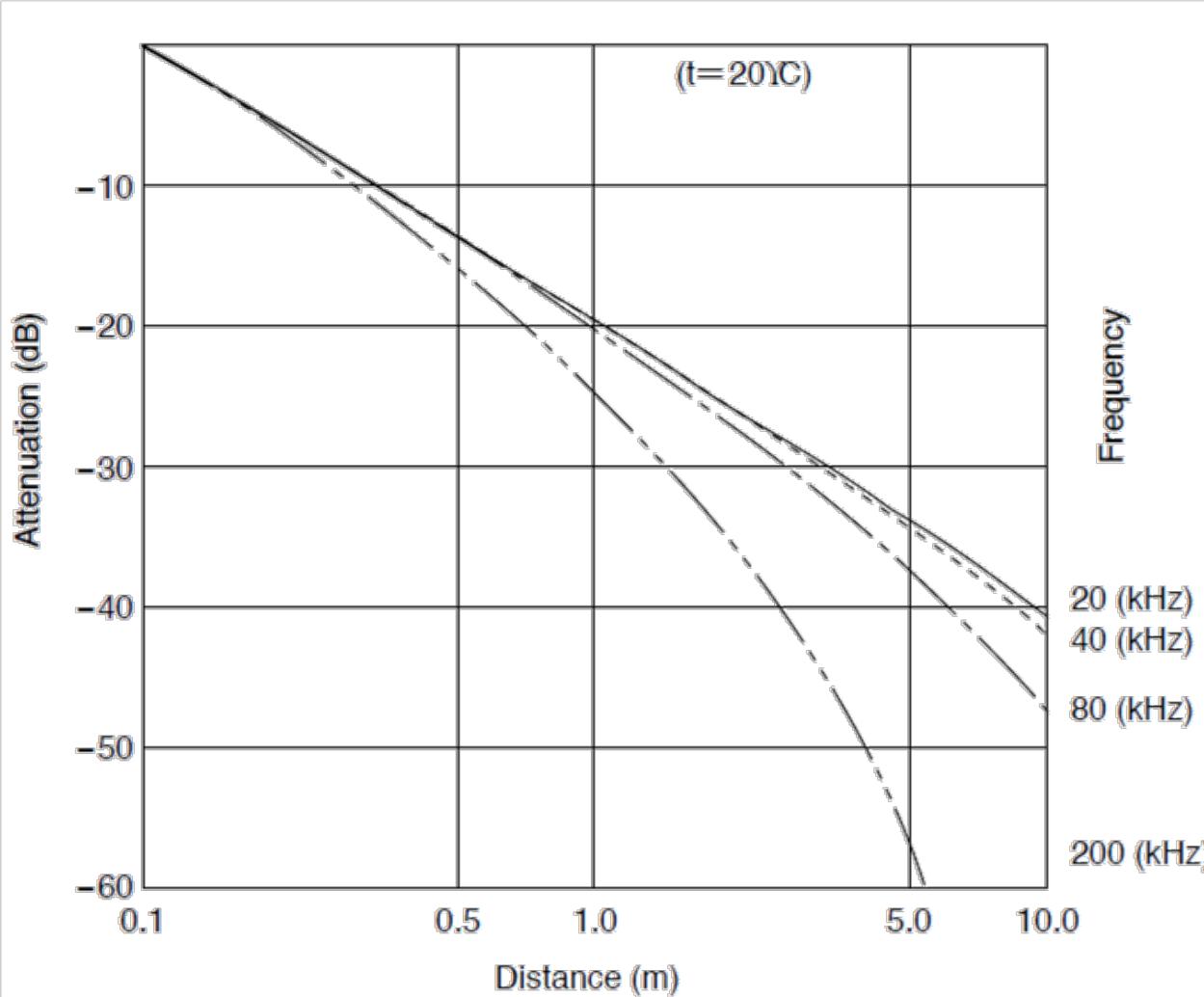
@ 20°C  $f = 20$  kHz →  $\lambda = 17.2$  mm

$f = 80$  kHz →  $\lambda = 4.3$  mm

$f = 200$  kHz →  $\lambda = 1.7$  mm

Reflection:

- ~100%: metal, wood, concrete, glass, rubber and paper
- ~ few %: cloth, cotton, wool



- diffusion loss on a spherical surface due to diffraction phenomenon
- absorption loss, i.e. energy is absorbed by medium

higher frequency  
↓  
higher absorption  
↓  
shorter distance

Sound pressure level (SPL) = volume of sound, expressed by:

$$SPL = 20 \cdot \log \frac{P}{P_0} \text{ (dB)}$$

P = sensor sound pressure

P<sub>0</sub> = reference sound pressure (20μPa)

Sensitivity in linear scale (V/Pa) is expressed by:

$$S_{lin} = \frac{V_{out}}{P_{in}} \text{ (V/Pa)}$$

V<sub>out</sub> = output voltage

P<sub>in</sub> = input sound pressure

Typically sensitivity is expressed in dB:

$$S_{dB} = 20 \cdot \log \frac{S_{lin}}{S_{ref}} \text{ (dB)}$$

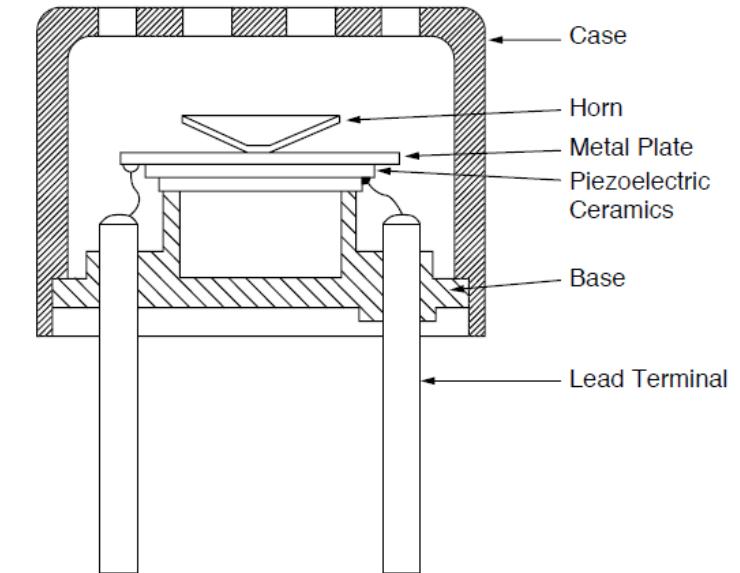
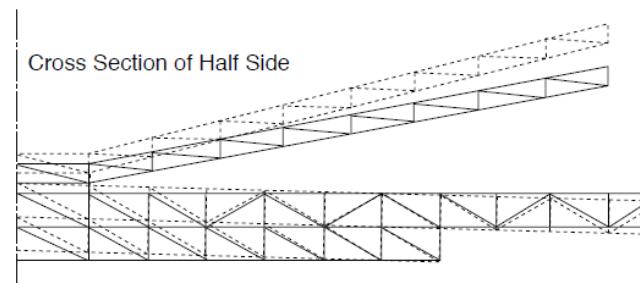
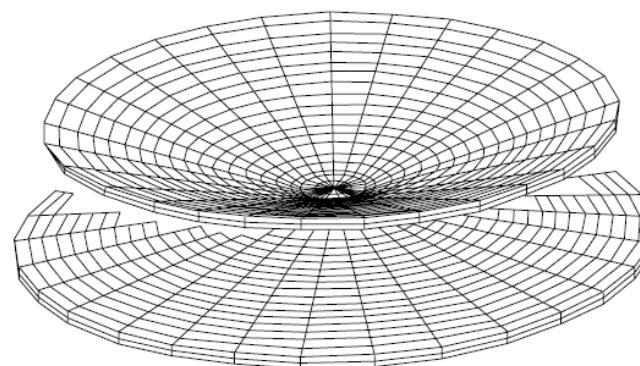
with S<sub>ref</sub> = 1V/Pa

A **multiple vibrator** is fixed elastically to the base.

Multiple vibrator = resonator + vibrator:

- resonator → composed of a metal sheet (conical in order to efficiently radiate and concentrate the ultrasonic waves)

- vibrator → composed of a piezoelectric ceramics sheet  
(generate and sense the ultrasonic waves)

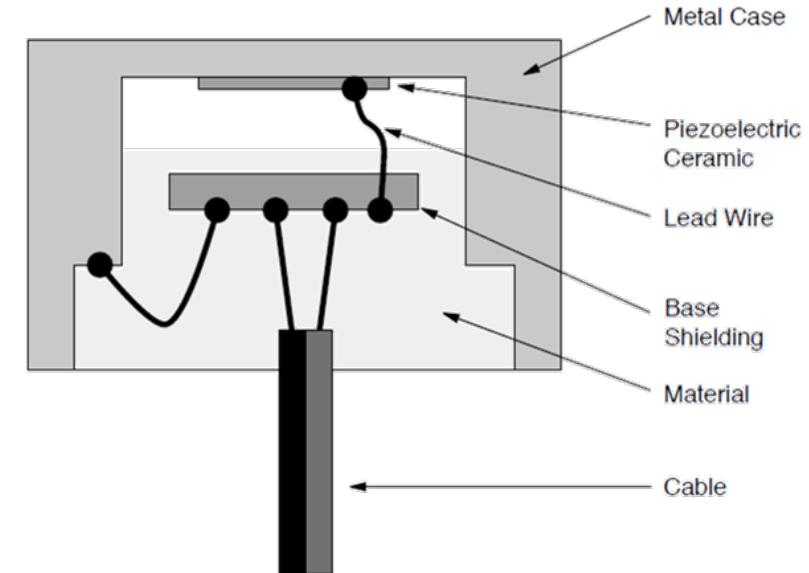


## Enclosed structure

for outdoors use

→ sealed to protect from rain and dust.

Piezoelectric ceramics are attached to the top inside of the metal case.



## High frequency (up to several hundred kHz)

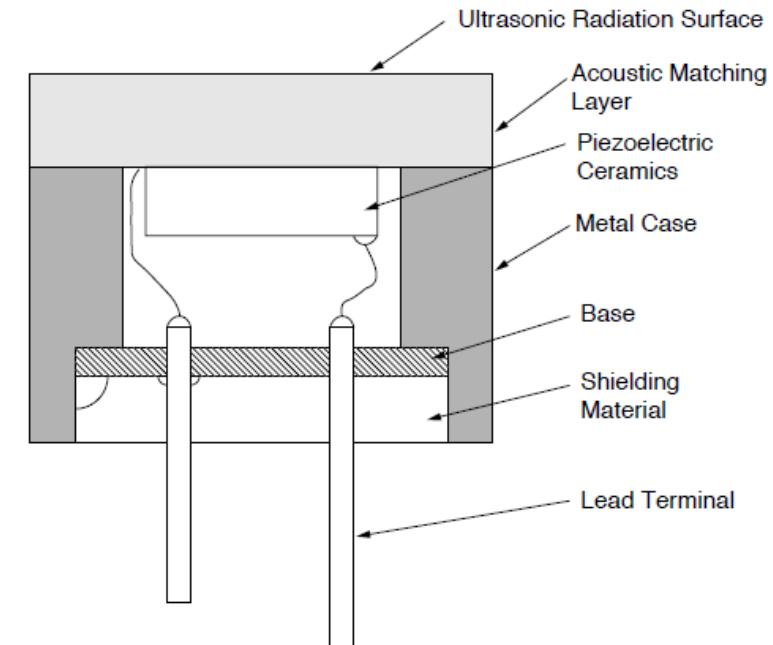
for use in industrial robots (accuracy as precise as 1mm)

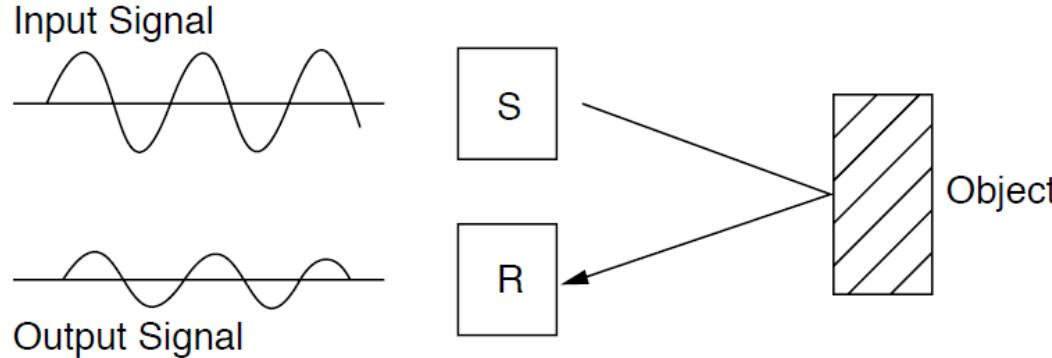
Conventional vibrator < 70kHz

→ vertical thickness vibration  
for high frequency

$z_{\text{piezoelectric ceramics}} = 2.6 \times 10^7 \text{ Pa}\cdot\text{s}/\text{m}$   $z_{\text{air}} = 4.3 \times 10^2 \text{ Pa}\cdot\text{s}/\text{m}$

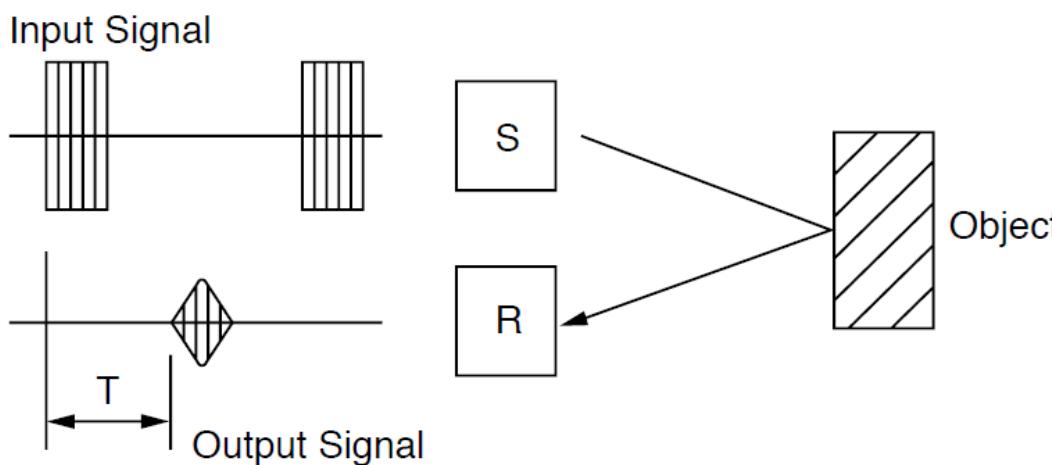
→ large loss → acoustic matching layer





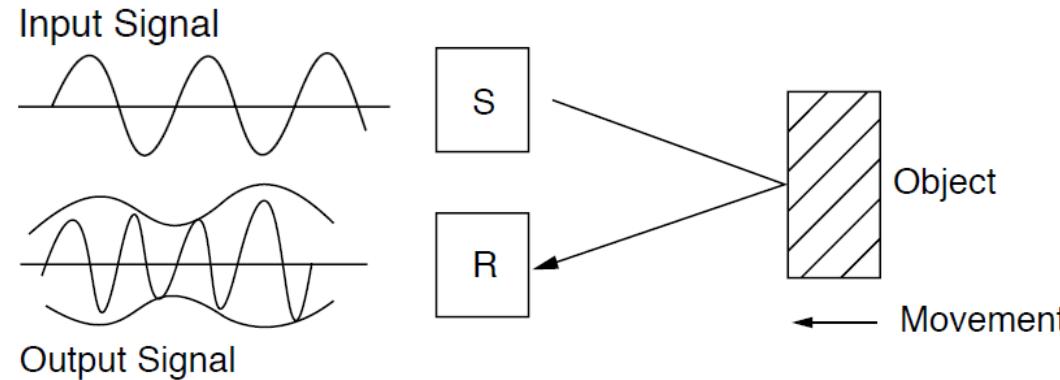
## Proximity sensor

Detect the amplitude of the return signal



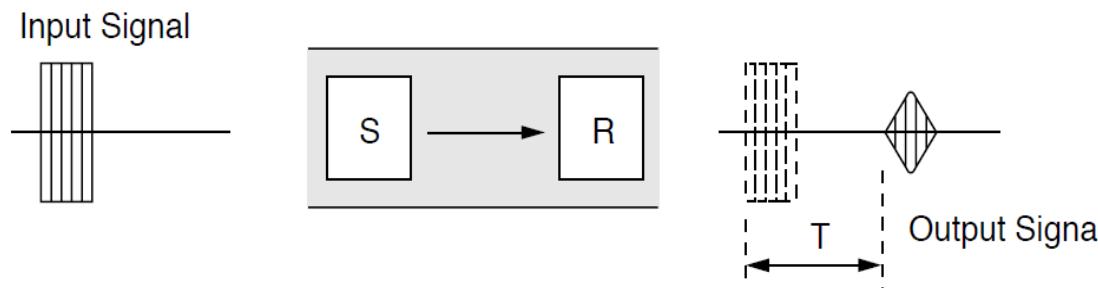
## Distance meter

Measure the time of flight of the return signal



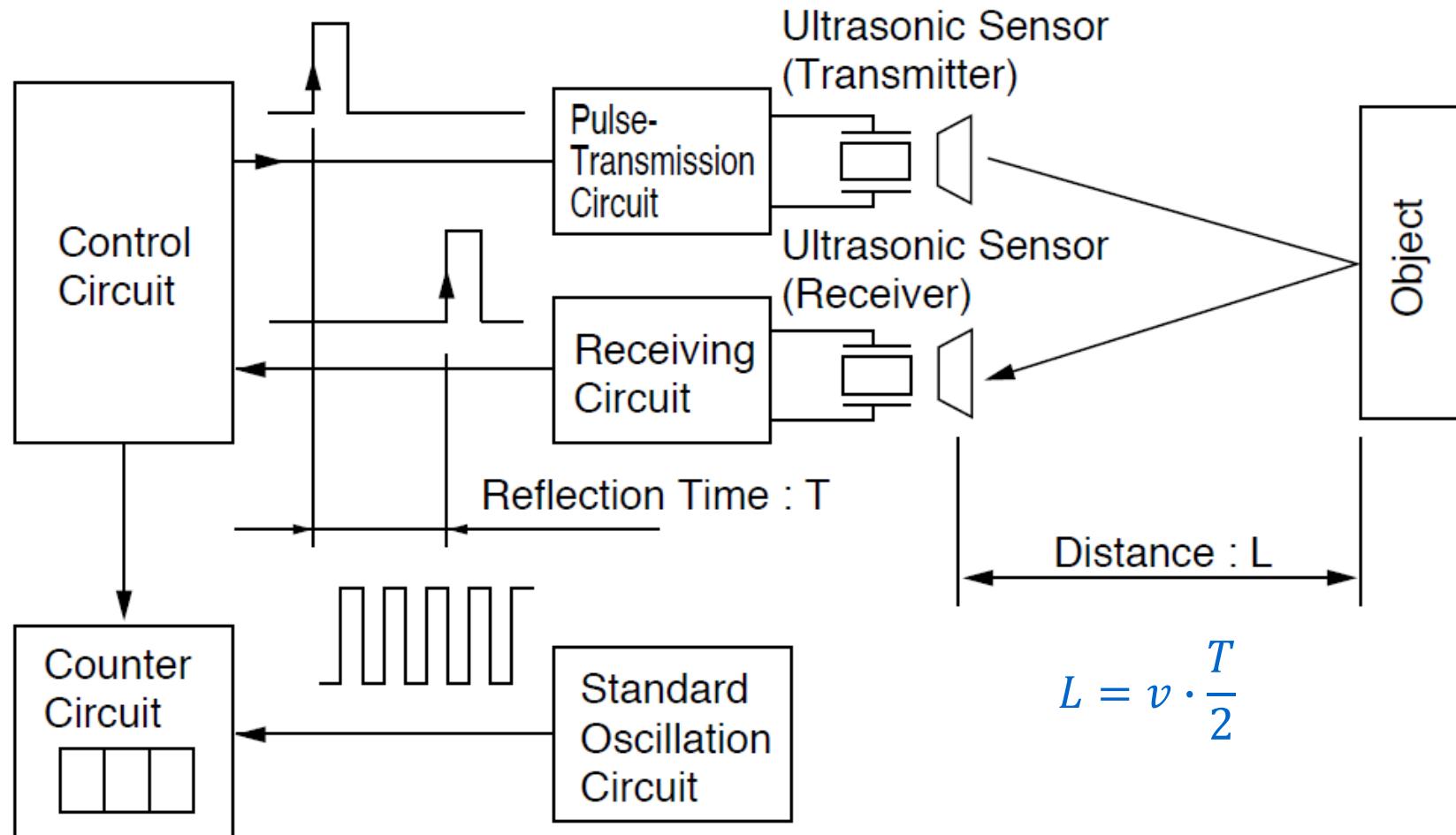
### Displacement sensor

Exploits Doppler effect to measure the reciprocal movements between sensor and object

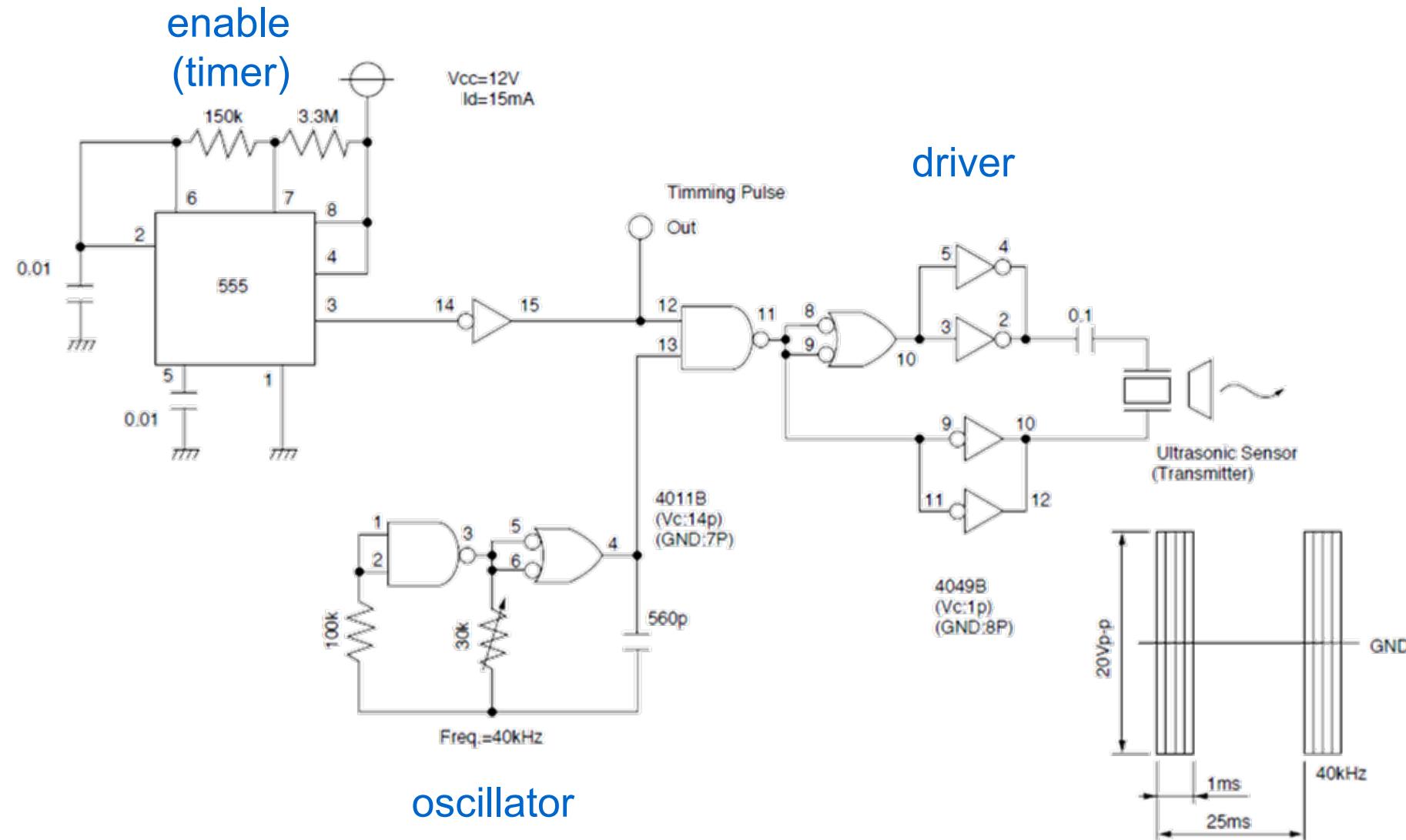


### Direct propagation time

Utilizes the change of sound velocity to measure density of a medium

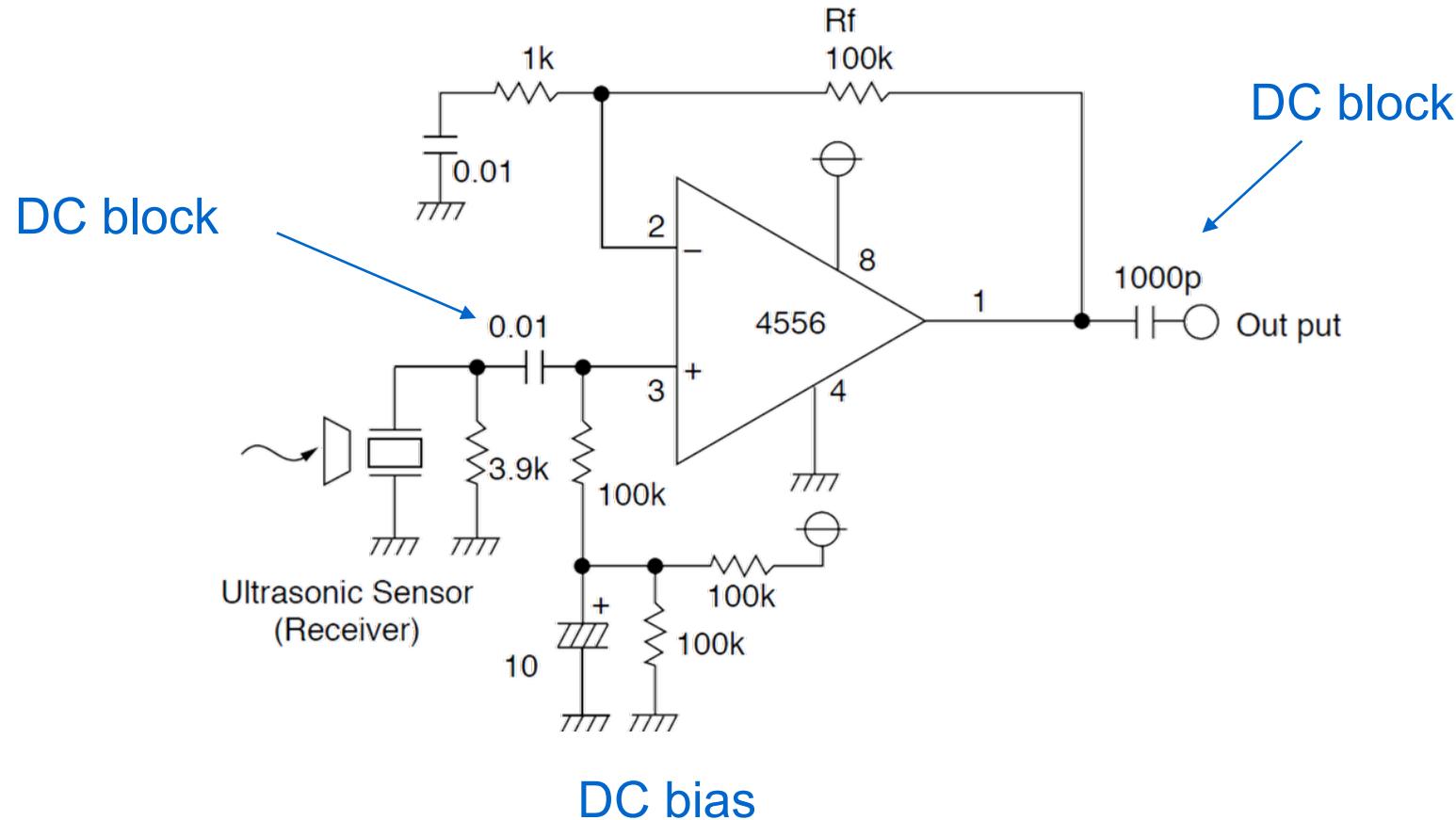


TDC = Time-to-Digital Converter ( $10\text{ ns} \rightarrow \sim 2\mu\text{m}$ )

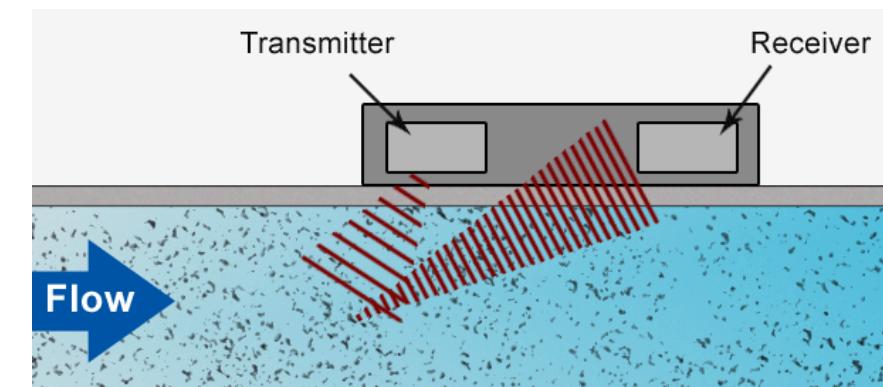
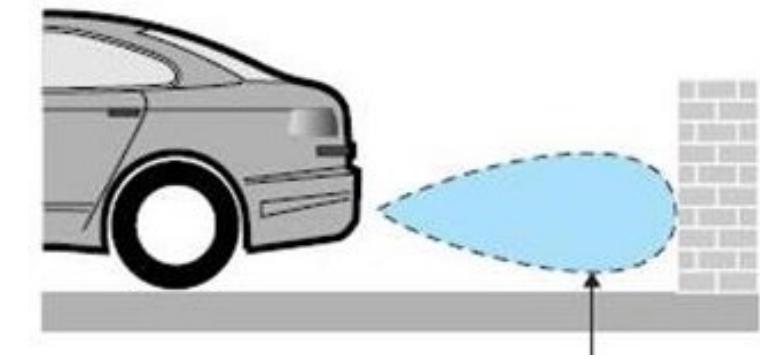


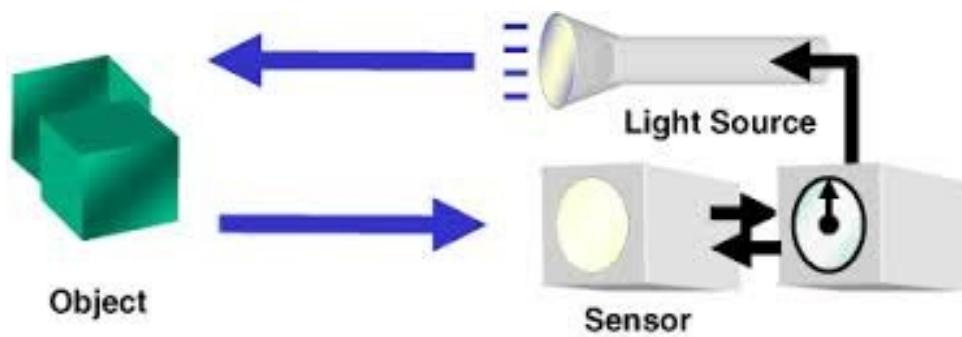
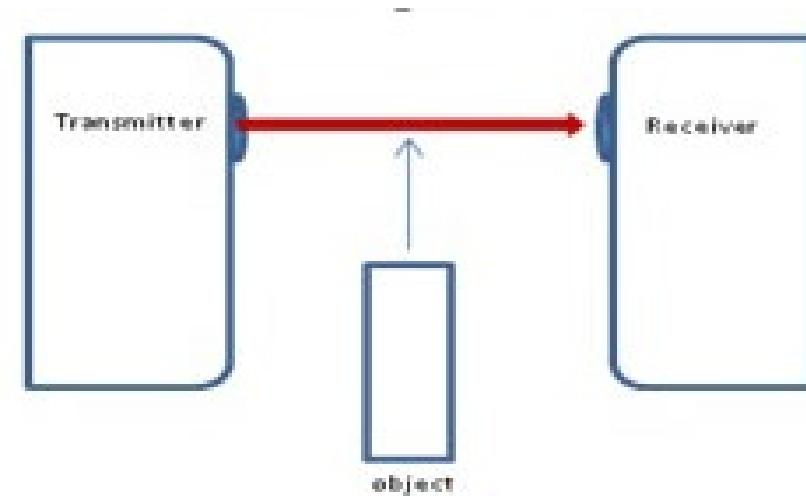
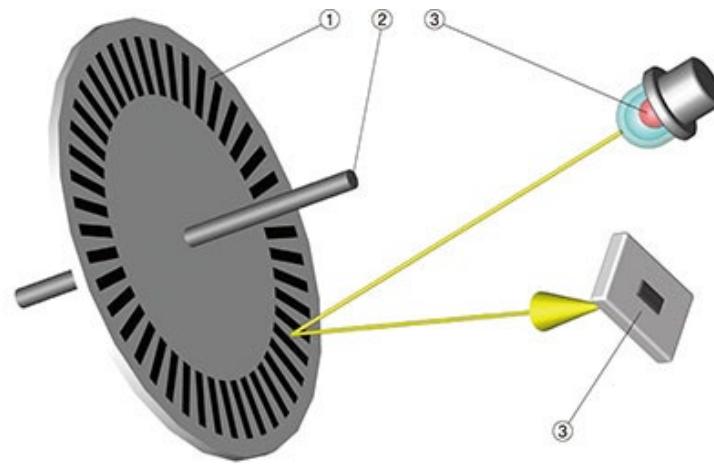
DC gain = 1

AC gain = 101



- Proximity sensor:
  - Counting instruments
  - Access switches
  - Parking meters
- Distance meter
  - Automatic doors
  - Level gauges
  - Back sonars of automobiles
- Displacement sensor
  - Intruder alarm systems
  - Flowmeters
- Direct propagation time
  - Densitometers
  - Flowmeters



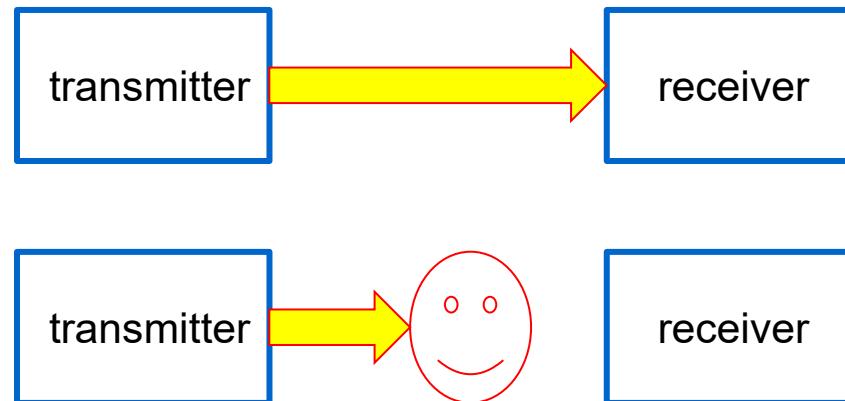


Proximity  
Distance  
Displacement encoder

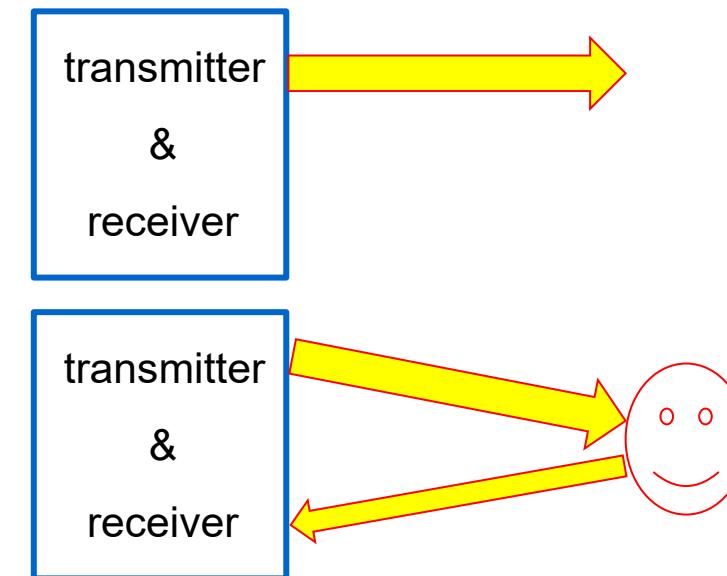
Infrared **transmitter** (LED or laser) + **receiver** (photodiode):

- immunity to electro-magnetic field
- immunity to ambient light
- long range

## Transmission

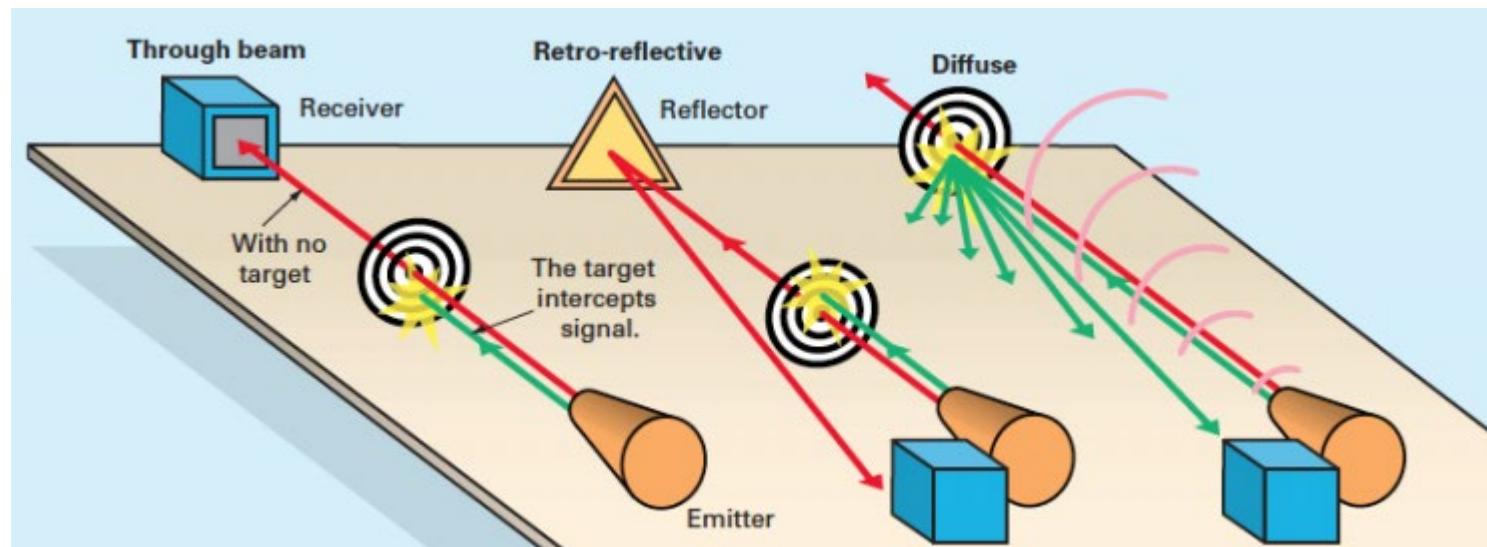


## Reflection



3 possible set-ups:

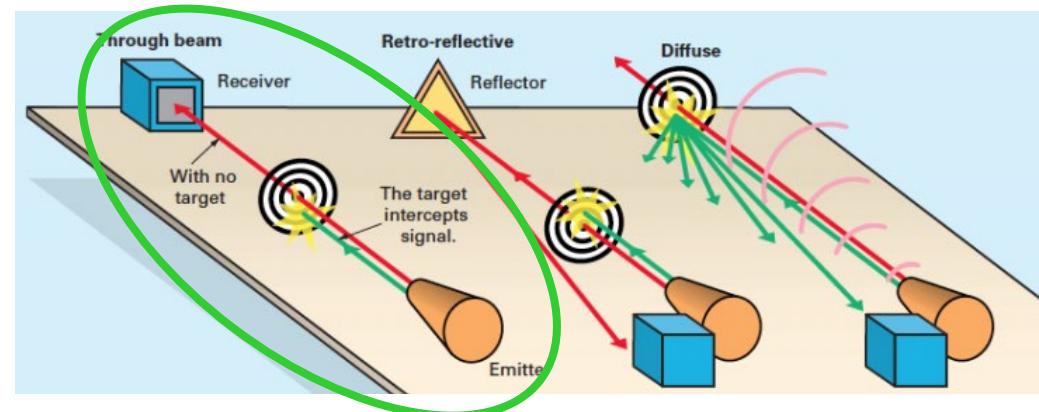
- Through beam
- Retro-reflective
- Diffuse



Separate emitter and receiver

→ detection occurs when an object between them breaks the beam

- most reliable, but least popular  
→ the purchase, installation, and alignment are costly and laborious
- typically offer the longest sensing distance (25 m and over)  
new laser diode emitter can transmit a well-collimated beam at 60 m:
  - at 60 m → detecting an object of few mm;
  - at close range → detecting an object of 0.01 mm.



The emitter produces light beam towards a specially designed reflector, which then deflects the beam back to the receiver.

→ detection occurs when the light path is broken by an object

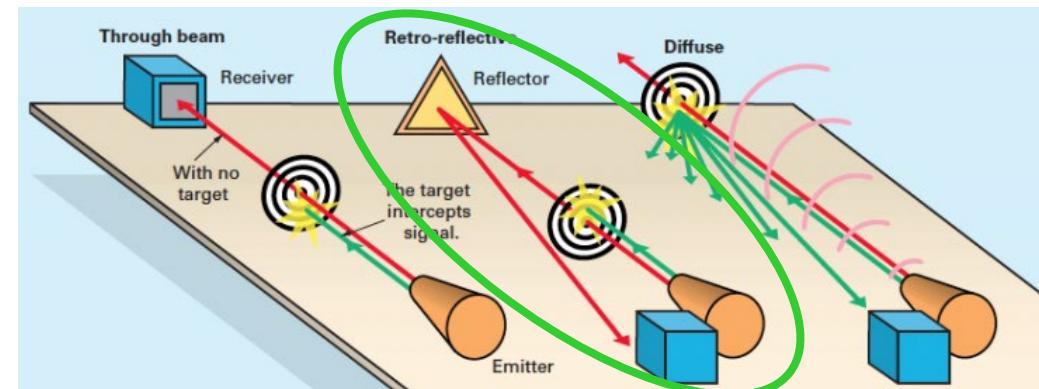
sensing distance up to 10 m

Emitter and receiver are both located in the same housing

→ **one wiring location**, the opposing side only requires reflector mounting.

Very shiny or reflective objects (like mirrors) sometimes reflect enough light to trick the receiver into thinking the beam was not interrupted, causing erroneous outputs.

→ polarization filtering (allows detection only from specially designed reflectors)



The **emitter** sends out a beam of light and the **target** diffuses it in all directions, and part of the backscattered beam reaches the **receiver**.

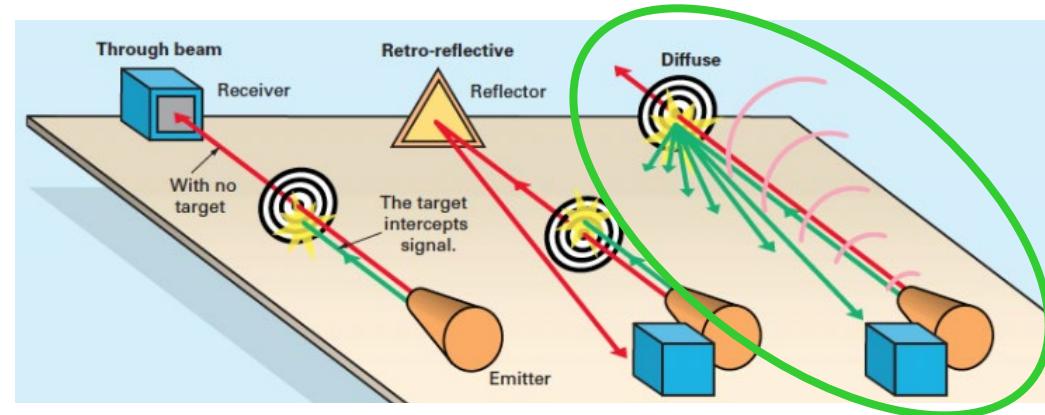
→ detection when light reflected from object.

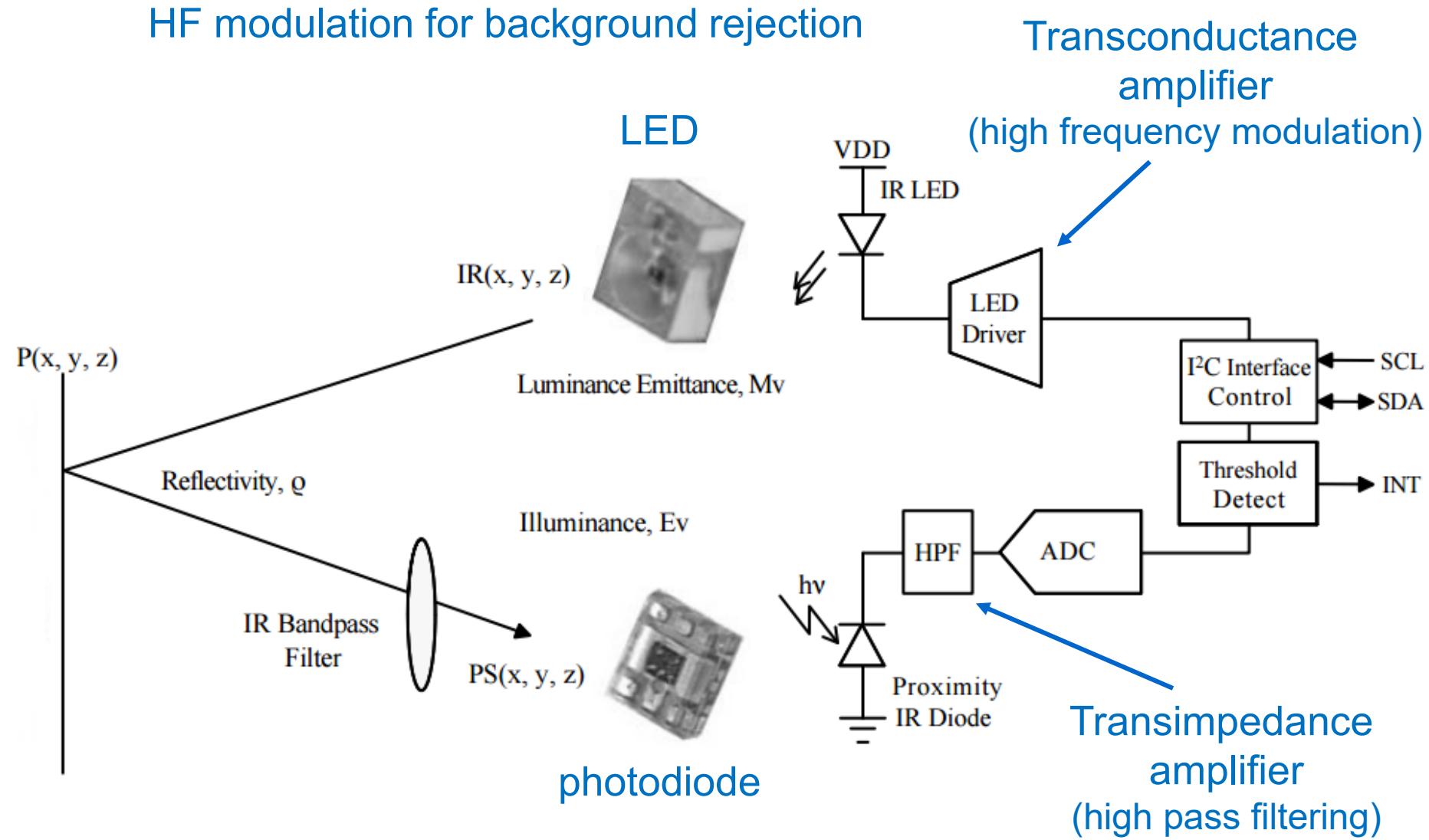
A non-reflective target will have a significantly decreased sensing range as compared to a bright white target → color dependent.

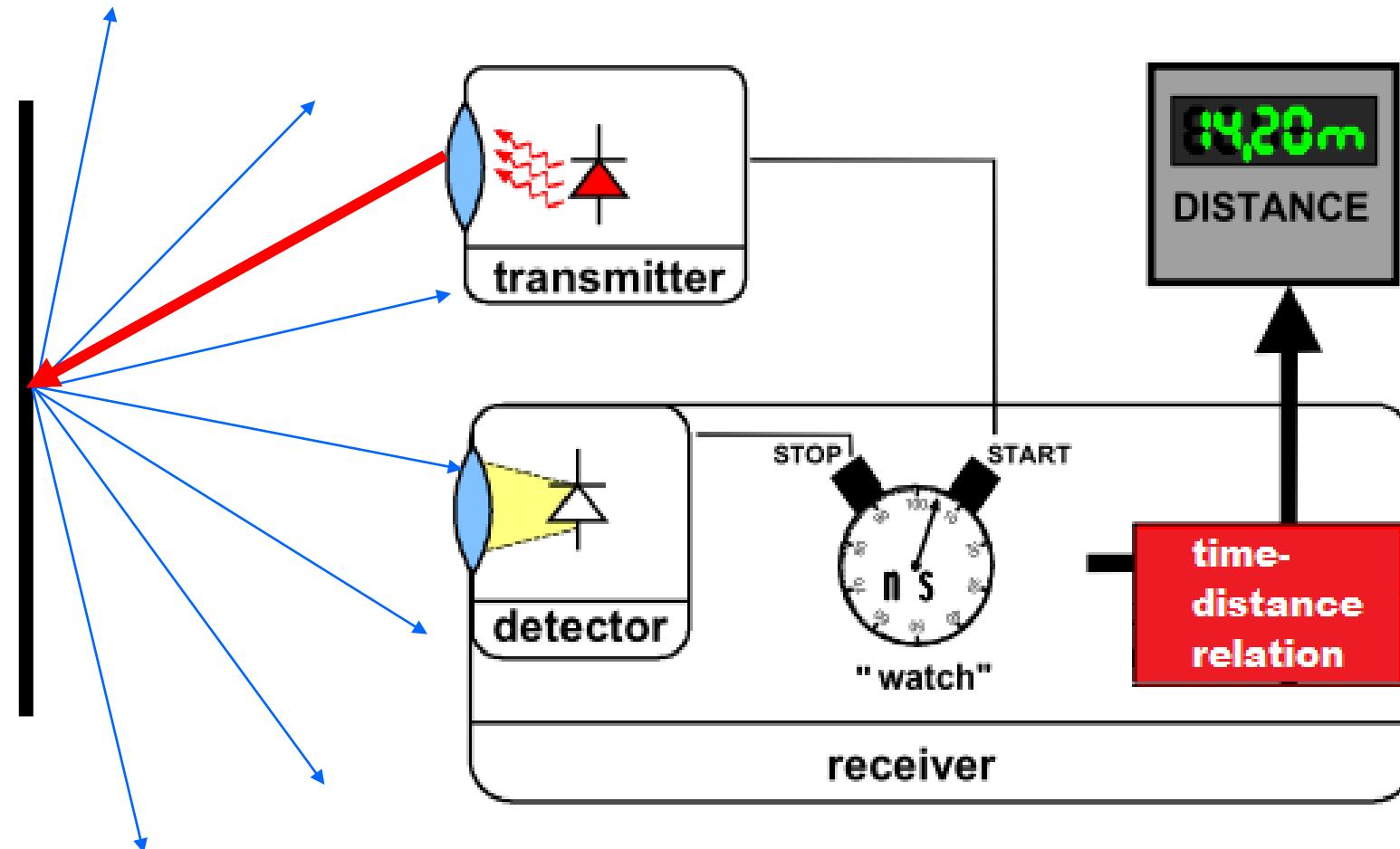
Certain versions are suitable for distinguishing dark and light targets in applications that require sorting or quality control by contrast.

False triggers caused by reflective background → diffuse sensors with focus:

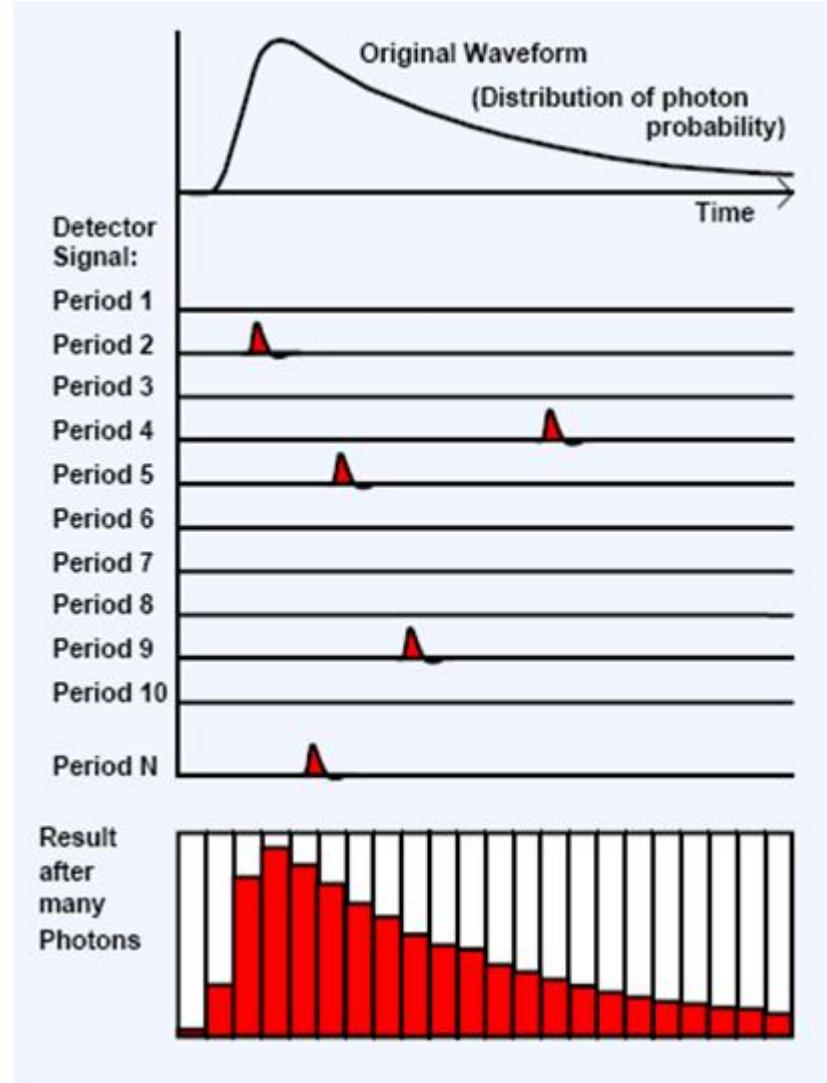
- 2 receivers focused on background and target (compares the intensities)
- Triangulation







$$\text{Distance} = \Delta T \cdot c / 2$$



Linear sensor (photodiode):

- Single shot measurement
- High optical energy per pulse

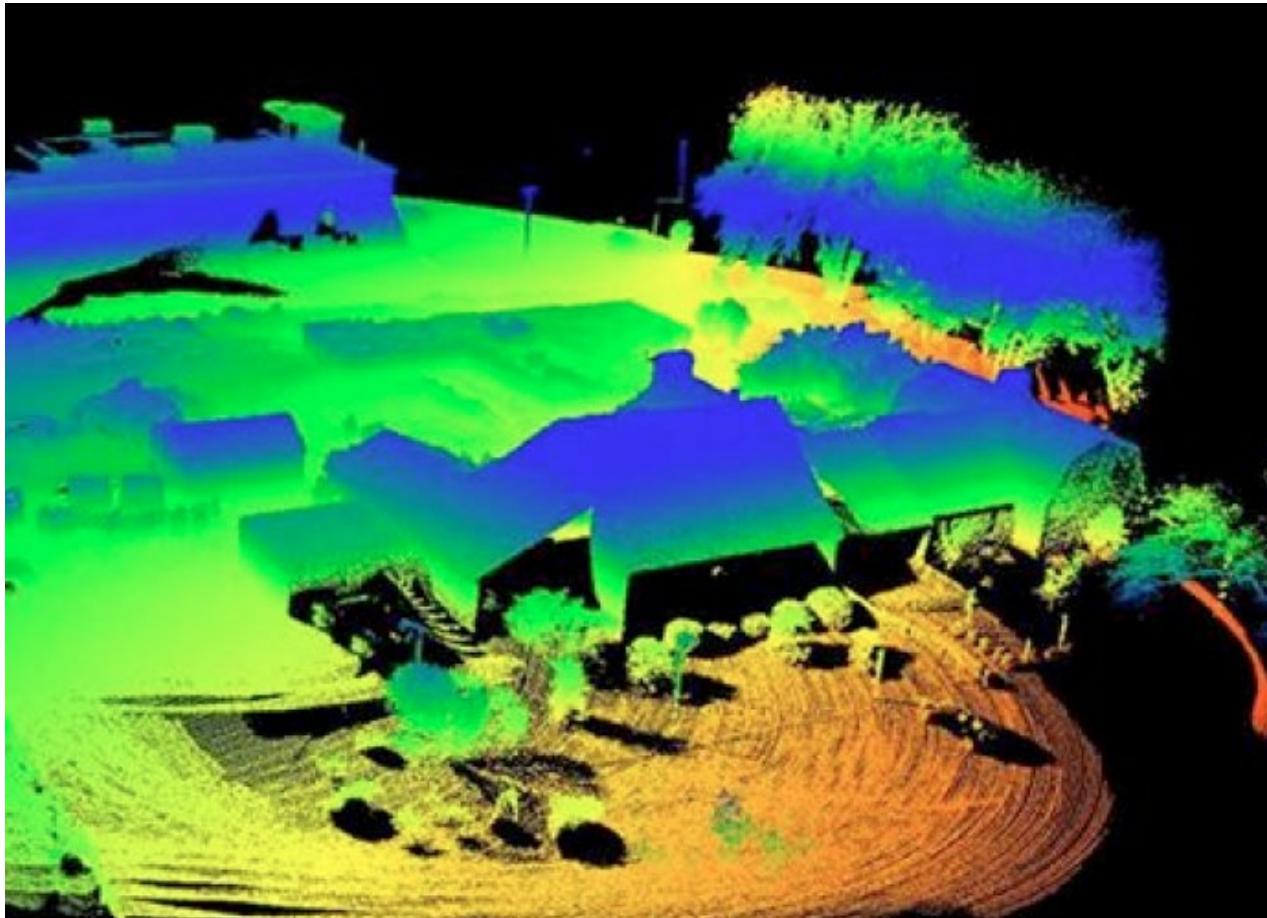
Single-photon sensor (SPAD)

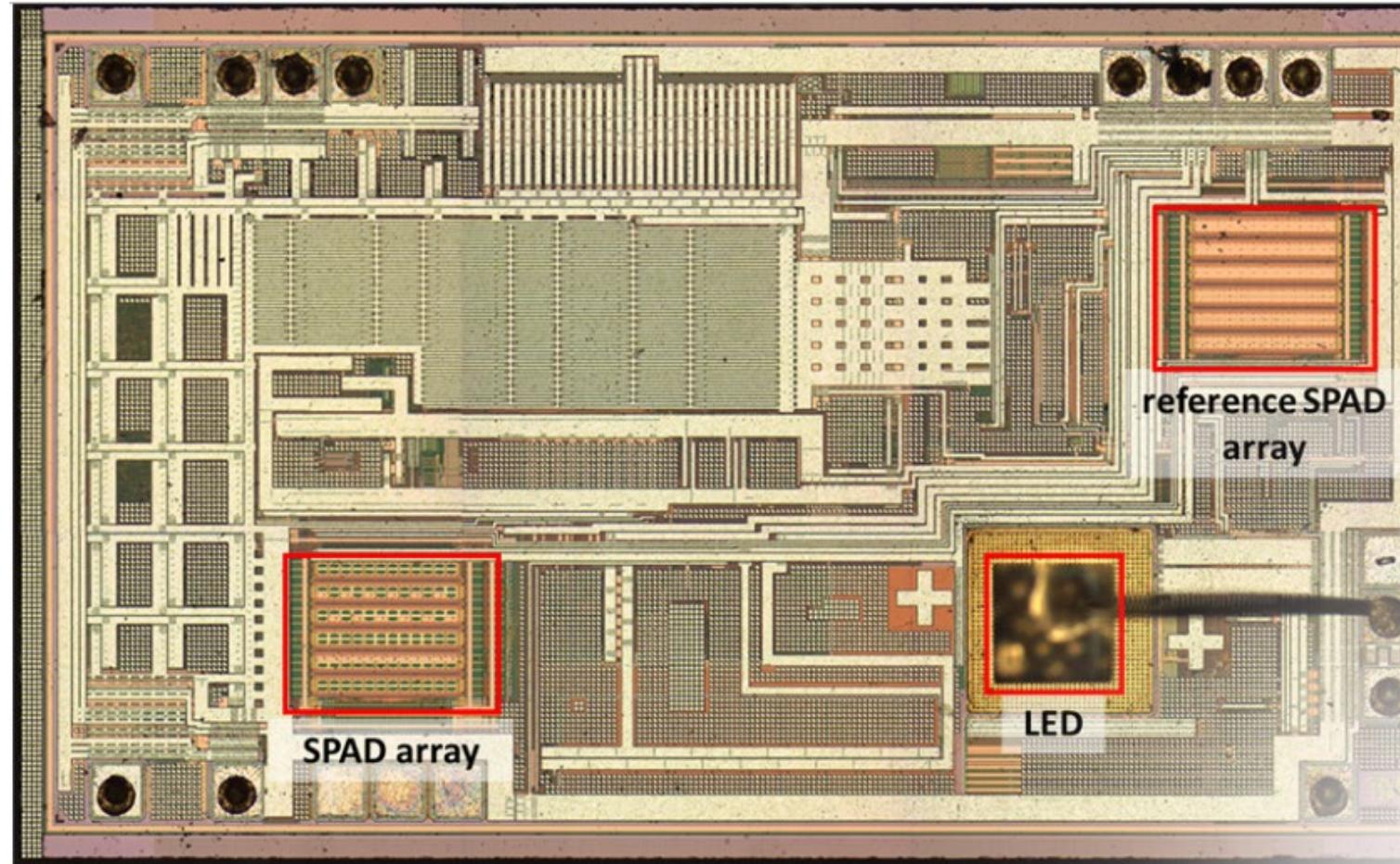
- Many repeated measurements
- Low optical energy per pulse



Time Correlated  
Single Photon Counting  
(TCSPC)

Light Detection and Ranging  
or  
Laser Imaging Detection and Ranging





- Proximity sensor
- Front camera autofocusing

= sensors that generate digital signals in response to movements

Displacement encoders can be:

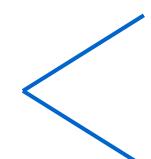
- **rotary encoders** (respond to rotation)
- **linear encoders** (respond to motion in a line)

Both rotary and linear encoders can be:

- **incremental encoders**: generate a series of pulses as they move  
(measure speed or keep track of position)
- **absolute encoders**: generate multi-bit digital words that indicate  
directly actual position

:

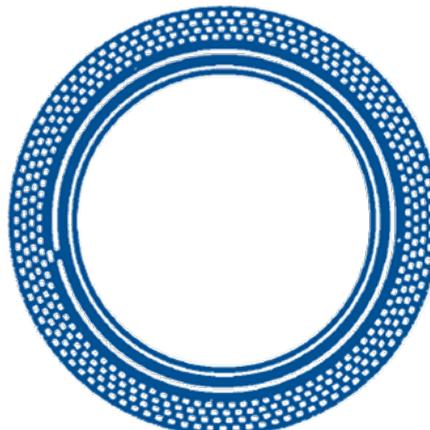
Optical encoders → resolution



Magnetic encoders → reliability in harsh environments

**Glass disk (rotary encoders) or strip (linear encoder)**  
with a pattern of lines deposited on it.

- light from an LED shines through the disk or strip onto one or more photodetectors, which produce the encoder's output
- incremental encoder has one (or many equally spaced) track
- absolute encoder has as many tracks as output bits

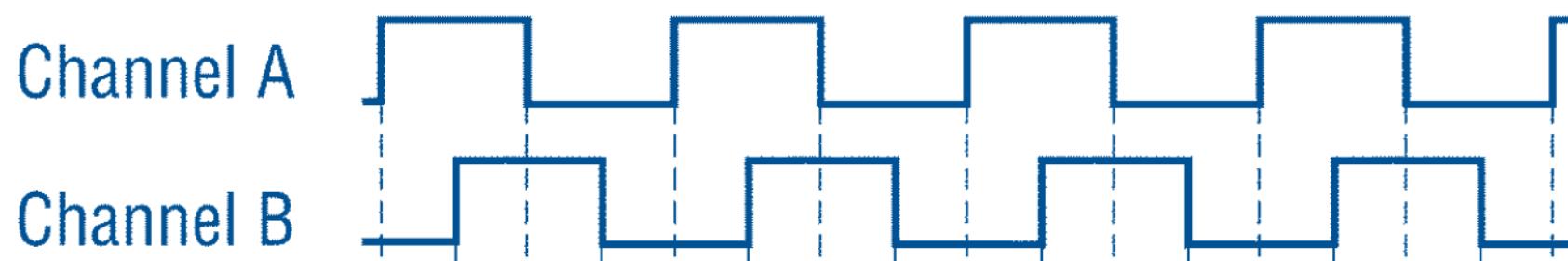


Provide a specific number of pulses per revolution (PPR) in rotary motion, or per inch or millimeter in linear motion.

- **single channel output** → don't provide direction of movement
- **quadrature output** → provide direction sensing  
(two channels 90° out of phase)

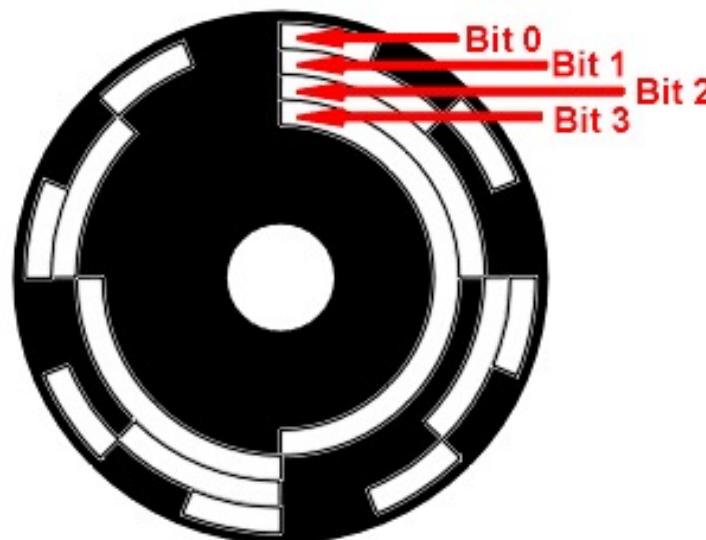
To determine position, its pulses must be accumulated by a counter.

When starting up, the equipment must be driven to a reference or home position to initialize the position counters. Some incremental encoders also produce another signal, the “marker,” produced once per revolution.



Generate digital words that represent the encoder's actual position, as well as its speed and direction of motion:

- if power is lost, its output will be correct whenever power is restored
- it is not necessary to move to a reference position



000  
001  
011  
010  
110  
100  
101  
100  
000

Binary or Gray code

| Binary | Gray |
|--------|------|
| 000    | 000  |
| 001    | 001  |
| 010    | 011  |
| 011    | 010  |
| 100    | 110  |
| 101    | 111  |
| 110    | 101  |
| 111    | 100  |

## Proximity sensors

Commercial and industrial applications:

- Detection of obstructions in the path of garage doors
- Detection of objects on industrial conveyors
- Public washroom sinks

## Distance sensors

Automotive and military applications:

- Autonomous driving
- Pre-crash systems
- LiDAR (Light Detection and Ranging or Laser Imaging Detection and Ranging)

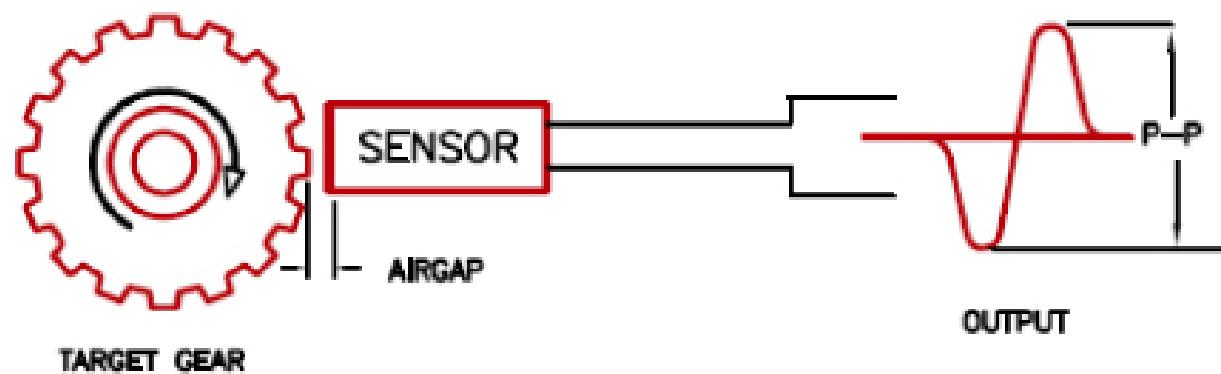
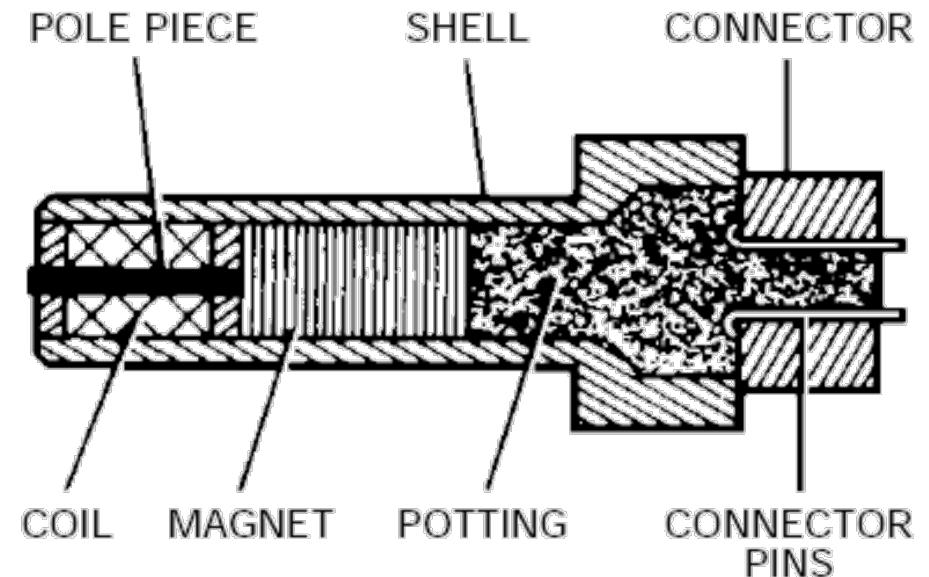
## Optical encoders

High resolution applications:

- Computer mouse
- Copiers
- Medical instrumentation

# Variable reluctance sensors (VR)

60



Displacement encoder

Magnetic reluctance  $R$   
of a magnetic circuit



Resistance R  
of an electrical circuit

$$R = \frac{F}{\Phi(\vec{B})}$$

$R$  = magnetic reluctance

$F$  = magneto-motive force:  $F = \phi H \cdot dl$

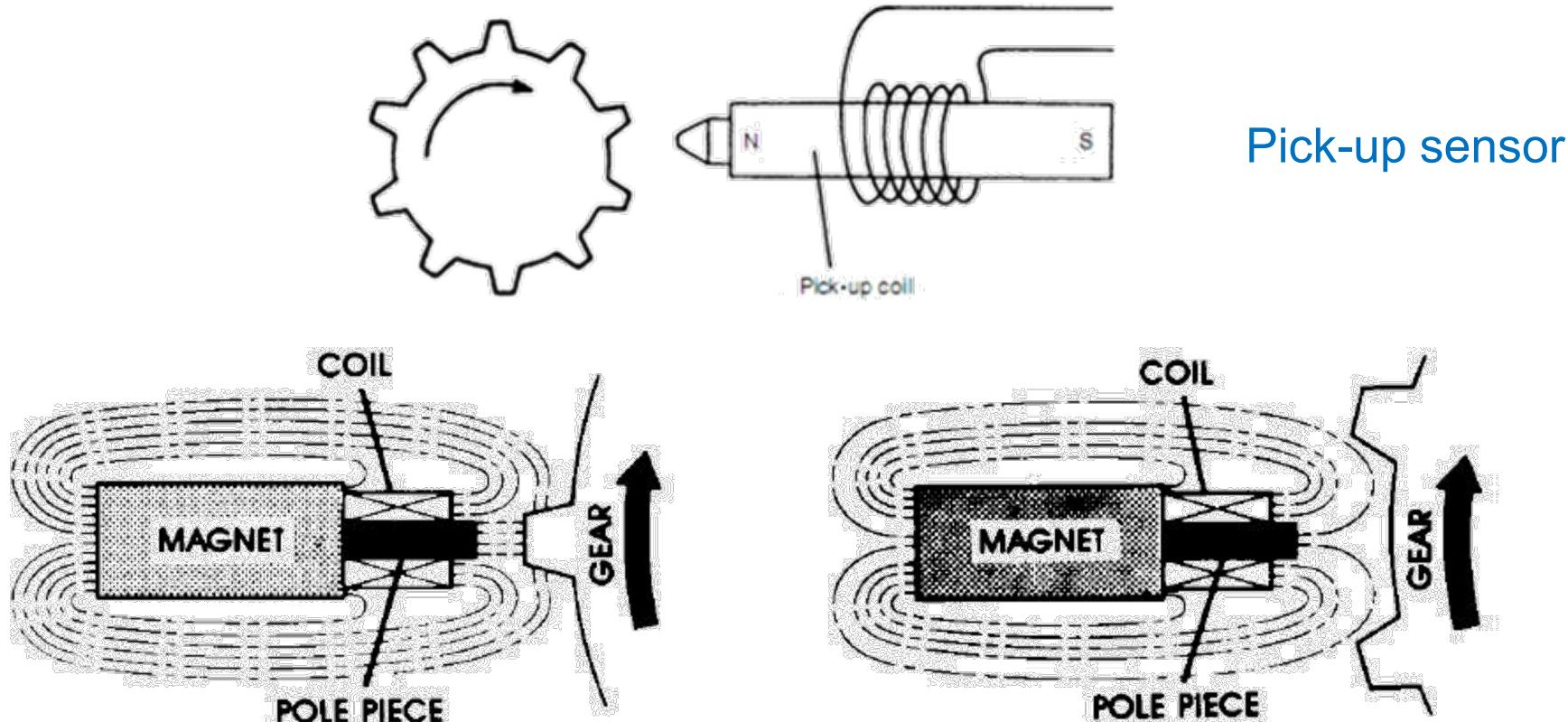
$\Phi(\vec{B})$  = magnetic flux:  $\Phi(\vec{B}) = \oint \vec{B} \cdot \vec{n} dS$ ,

for fixed surface normal to  $B$ :  $\Phi(\vec{B}) = B \cdot A = \mu_0 \mu_r H A$

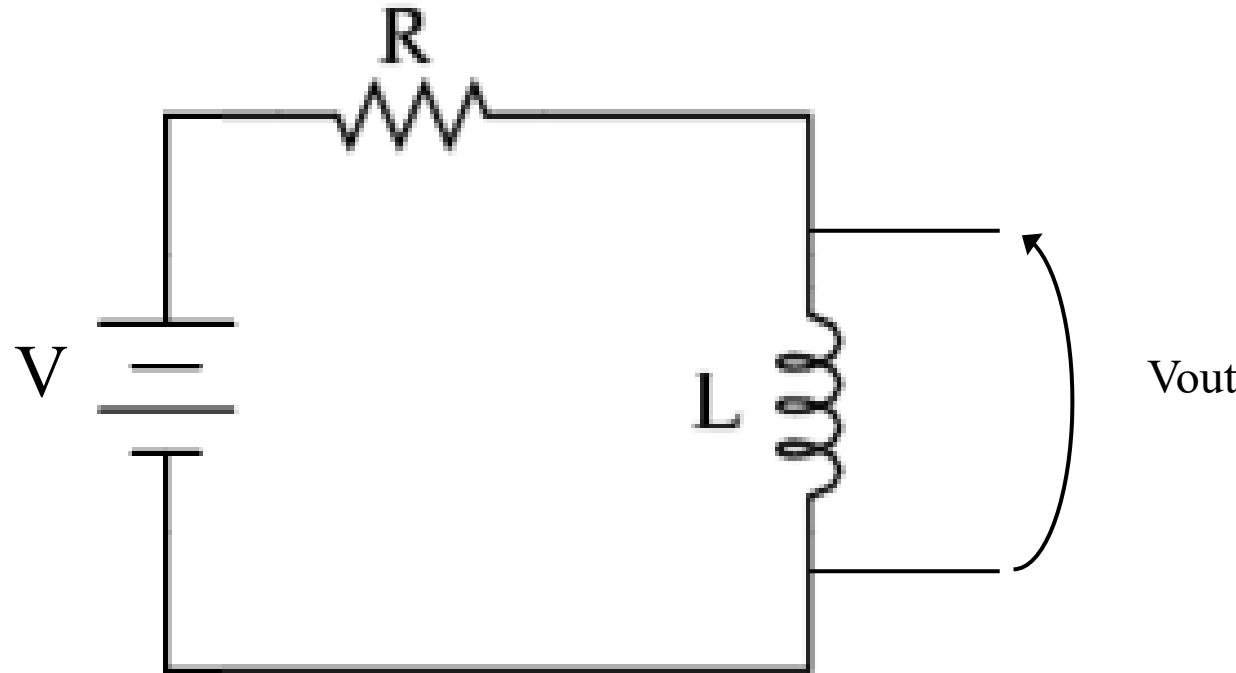
Permanent magnet: establishes a fixed magnetic field ( $H$ )

Ferrous metal target: changes the magnetic reluctance  $\rightarrow$  changes  $\Phi(\vec{B})$

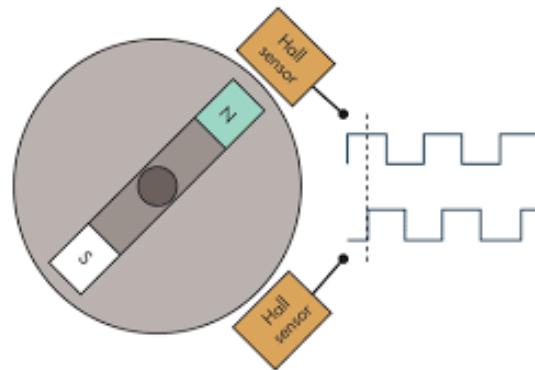
Coil winding: experiences an induced voltage  $V_i = -n \cdot \frac{d\Phi(\vec{B})}{dt}$



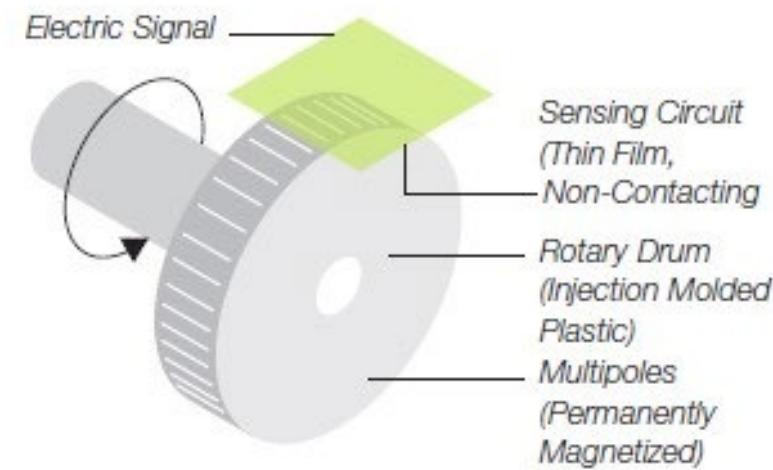
Permanent magnet replaced by small dc-bias current through the coil creating an electromagnetic field.  
Sensor output → AC voltage signal riding on top of the dc bias.



## Rotation frequency and direction



## Rotation speed



- One permanent magnet in the wheel
- Two magnetic field sensors  
(Hall sensors or magneto resistance)

- Many permanent magnets in the wheel
- One magnetic field sensor  
(Hall sensors or magneto resistance)

Non contact sensor, reliable in harsh environments

- Rotation speed
  - motors feedback
  - ABS control
- Rotating gears position
  - high precision positioning
  - human machine interface
  - machine to machine interface