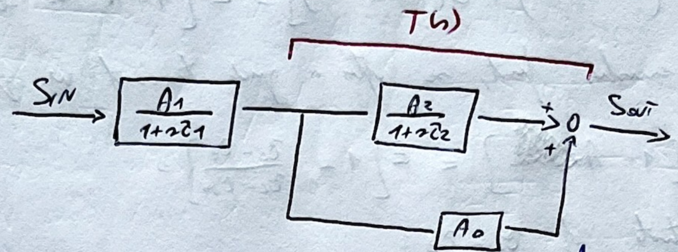
In-Band Zero-Pole Doublets

In-band zero-pole doublets appear any time we need for example to drive a large load capacitance with a nulling-compensated OTA stage, or when we use a FEED-FORWARD compensation technique:



→ low-pass block with broad band and low gain (GBWP trade-off)

$$T(s) = \frac{A_2}{1+s\tau_2} + A_0 = \frac{1 + s\left(\frac{A_0\tau_2}{A_2+A_0}\right)}{1+s\tau_2}(A_2+A_0)$$

so we get a zero @ $\tau_z = \frac{A_0}{A_2+A_0}\tau_2 < \tau_2$ (so $f_z > f_2$).

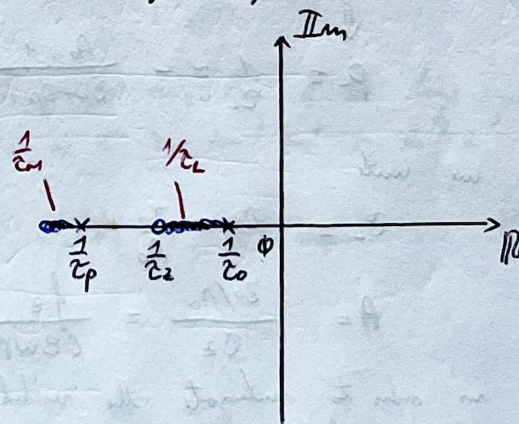
These doublets increase ϕ_m but worsen the transient response of the amplifier.

Let's consider

$$A(s) = \frac{A_0(1+s\tau_z)}{(1+s\tau_0)(1+s\tau_p)} \quad \text{(T.F. of a nulling-compensated OTA)}$$

and $\tau_p < \tau_z < \tau_0$. The closed loop T.F. in a buffer configuration will be

$$H(s) = \frac{(1+s\tau_z)}{(1+s\tau_L)(1+s\tau_H)}$$



Let's consider the response to a step function E

$$V_{OUT}(s) = \frac{E}{s}H(s)$$

$$\downarrow$$

$$= \frac{E}{s}\frac{1+s\tau_z}{(1+s\tau_L)(1+s\tau_H)}$$

$$\downarrow$$

$$= \frac{E}{s}\left[\frac{A}{1+s\tau_L} + \frac{B}{1+s\tau_H}\right]$$

where

$$A = \lim_{s\to -\frac{1}{\tau_L}} H(s)(1+s\tau_L) = \frac{1+\tau_z\left(-\frac{1}{\tau_L}\right)}{1+\tau_H\left(-\frac{1}{\tau_L}\right)} = \frac{\tau_L - \tau_z}{\tau_L - \tau_H}$$

$$B = \lim_{s\to -\frac{1}{\tau_H}} H(s)(1+s\tau_H) = \frac{1+\tau_z\left(-\frac{1}{\tau_H}\right)}{1+\tau_L\left(-\frac{1}{\tau_H}\right)} = \frac{\tau_H - \tau_z}{\tau_H - \tau_L}$$

We get that

$$H(t) = \frac{A}{\tau_L} e^{-\frac{t}{\tau_L}} + \frac{B}{\tau_H} e^{-\frac{t}{\tau_H}}$$

so, by integration, we get:

$$v_{out}(t) = E\left[A\left(1 - e^{-\frac{t}{\tau_L}}\right) + B\left(1 - e^{-\frac{t}{\tau_H}}\right)\right] = E\left[\overset{A+B}{1} - Ae^{-\frac{t}{\tau_L}} - Be^{-\frac{t}{\tau_H}}\right]$$

The fast exponential vanishes first, so after a quick transient we get

$$v_{out}(t) \simeq E\left[1 - A\right] = E\left[1 - \frac{\tau_L - \tau_Z}{\tau_L - \tau_H}\right] =$$

We need an estimate of $\tau_L - \tau_Z$:

$$G_{loop}(s) = -A_0 \frac{1 + s\tau_Z}{(1 + s\tau_0)(1 + s\tau_p)}$$

$$-G_{loop}(s) + 1 = 0$$

$$\Downarrow$$

$$-A_0 - A_0\tau_Z \cdot s = 1 + s\tau_0 + s\tau_p + s^2\tau_0\tau_p$$

$$\downarrow$$

$$s^2\tau_0\tau_p + s(\tau_0 + \tau_p + A_0\tau_Z) + A_0 + 1 = 0$$

$$\Downarrow$$

$$p_L = -\frac{1}{\tau_L} \simeq -\frac{A_0 + 1}{\tau_0 + \tau_p + A_0\tau_Z} \simeq -\frac{1}{\tau_Z + \frac{\tau_0}{A_0}} \quad (\text{being } \tau_0 >> \tau_p)$$

So we write

$$\tau_L = \tau_Z + \frac{\tau_0}{A_0}$$

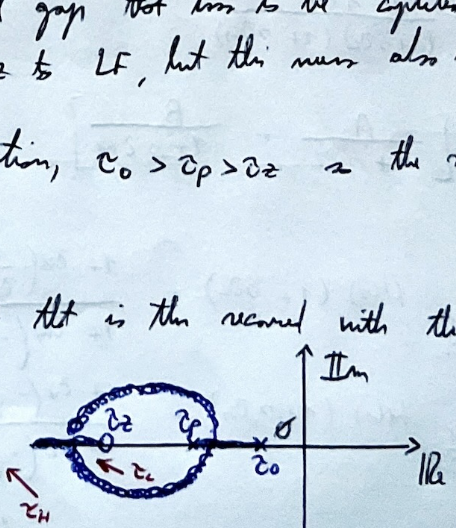
$$A = \frac{\tau_0 / A_0}{\tau_Z} = \frac{f_Z}{GBWP}$$

so in order to mitigate the residual gap that has to be completed with the slow time constant, we must shift f_Z to LF, but this means also to slow τ_L down!

For the case of feed-forward compensation, $\tau_0 > \tau_p > \tau_Z$ so the root locus is different and we expect

$$\tau_L < \tau_Z$$

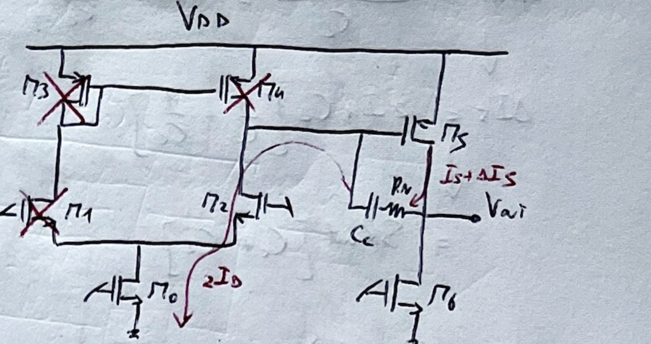
so the response has an overshoot that is then recovered with the slow time constant τ_L



Slew Rate

The slew-rate performance is related to the finite current available to charge capacitors.

If we consider a large signal driving for example M_3 it fully unbalances the differential couple, thus switching M_1-M_3-M_4 off and letting I_{M_0} flow into C_c. As a result, $V_{G,5} \downarrow$ and M_5 carries



$$I_5 + \underbrace{\Delta I_5}_{2I_D}$$

The initial response is a linear ramp with $SR = \frac{I_0}{C_c}$. If we assume to have the OTA in a buffer configuration, then we recover the exponential behavior when

$$\begin{cases} SR = \dfrac{\Delta V}{\tau} \\ \Delta V = V(\infty) - SR \cdot t_{SLEW} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \Delta V = SR \cdot \tau \\ t_{SLEW} = \dfrac{V(\infty)}{SR} - \tau \end{cases} \implies v_{OUT}(t) = \begin{cases} SR \cdot t & (t \le t_{SLEW}) \\ V(\infty) - \Delta V \cdot e^{-\frac{(t - t_{SLEW})}{\tau}} & (t \ge t_{SLEW}) \end{cases}$$

This discussion is valid under the assumption that the rise time of the large input signal is $< \frac{\tau_L}{G(0)} = \frac{1}{GBWP(2\pi)}$ of the OTA, so that V_{OUT} doesn't react to the large signal at first.

If we consider an harmonic signal instead, whose amplitude is the maximum one allowed by the output dynamics, then

$$2\pi A_{max} f_{max} = \left.\frac{dv_{out}}{dt}\right|_{max} \le SR$$

$$\downarrow$$

$$f_{max} \le \frac{SR}{2\pi A_{max}} \triangleq \underline{\text{POWER BANDWIDTH}}$$

[Once GBWP is set, we need to increase $V_{ov,D}$ in order to increase SR, but this degrades $\overline{E_{in}^2}$]

Finally, we can point out that

$$SR = \frac{I_0}{C_c} = \frac{V_{ov,1} \cdot g_{m1}}{C_c} = 2\pi V_{ov,1} \cdot GBWP$$

$\longrightarrow$ **External SR**

Let's consider now a load capacitance. We must distinguish now two cases

1. large V_p (positive)

M_1-M_3-M_4 are off and $2I_1$ flows into C_c. Under the assumption that V_A reaches a steady state, the ramp rate across C_c must be equal to the

(4b) one across C_L, being it equal to $SR_{int} = \frac{2I_1}{C_c}$ imposed by the first stage, so it must be

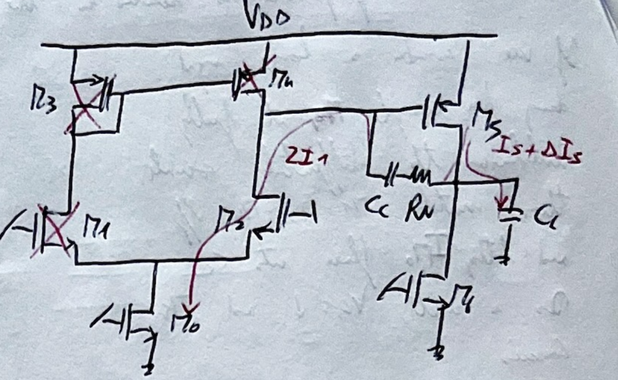
$$SR_{out}^+ = \frac{\Delta I_s - 2I_1}{C_L} = SR_{int} = \frac{2I_1}{C_c}$$

$\downarrow$

$$\Delta I_s = 2 I_1 C_L \left[\frac{1}{C_L} + \frac{1}{C_c}\right]$$

$\downarrow$

$$= SR_{int}\left[C_c + C_L\right]$$



(2) V_D large (negative)

M_2 off, $2I_1$ flows into $M_1 - M_3 - M_4$, M_5's current decreases

$$SR_{out}^- = \frac{\Delta I_s - 2I_1}{C_L} = SR_{int} = \frac{2I_1}{C_L}$$ (if SR_{int} is more limiting than the output load)

$\downarrow$

$$\Delta I_s = 2I_1 C_L \left[\frac{1}{C_L} + \frac{1}{C_c}\right] = SR_{int}(C_L + C_c)$$

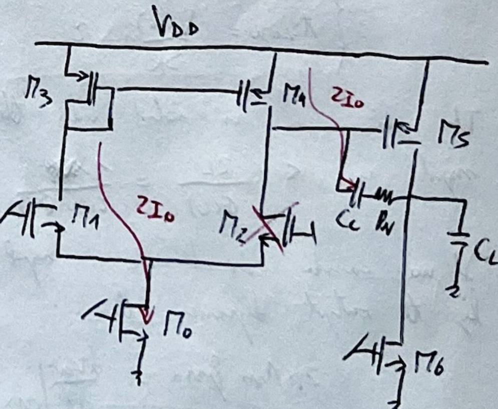
But if $\Delta I_s > I_s$, then M_5 turns off and M_6 has to drive both C_c and C_L. In this case, at steady state the ramp rate across C_c matches the one across C_L, so

$$\frac{I_6 - I_4}{C_L} = \frac{I_4}{C_c} = SR_{out}^-$$

$\downarrow$

$$\begin{cases} I_4 = I_6 \cdot \dfrac{C_c}{C_L + C_c} < 2I_0 \\ SR_{out}^- = \dfrac{I_6}{C_L + C_c} \end{cases}$$

M_4 is ohmic! $\Uparrow$



In the end

$$SR_{out}^- = \min\left\{SR_{,int}\ ;\ \frac{I_6}{C_L + C_c}\right\}$$

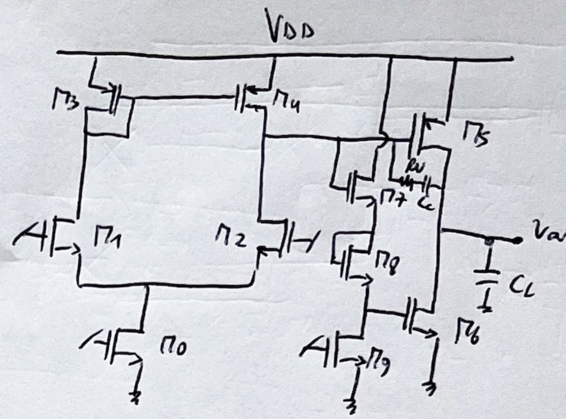
How to have that $SR_{,int}$ is the limiting one on both polarities?

→ increase I_6

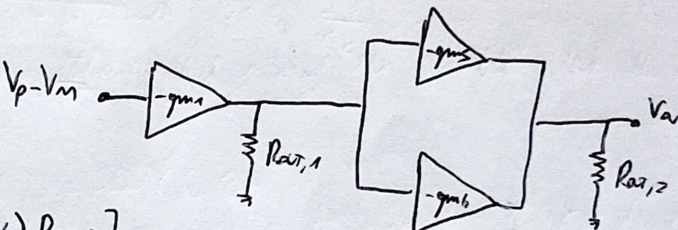
→ use a class A-B, raise M_6 current only when needed!

CLASS AB STAGE FOR SLEW RATE

M7 - M8 act as voltage shifters to properly set M6's bias. We size them in order to provide a shift of 0,8V + 0,8V.
In this way M6 reacts in order to increase its current only when needed, so we do not increase static power dissipation!



NOTE: the gain changes



$$G_d = \underbrace{[g_{m1} \cdot R_{OUT,1}]}_{G_1} \cdot \underbrace{[(g_{m5} + g_{m6})\, R_{OUT,2}]}_{G_2}$$

A possible alternative is the AC coupling: the capacitance C_{ac} makes it possible to properly bias the M6 transistor and also to let it come into action when needed.