Prototypical Differential Stage

Differential amplifiers ⇒ amplify the potential difference, regardless of the average potential

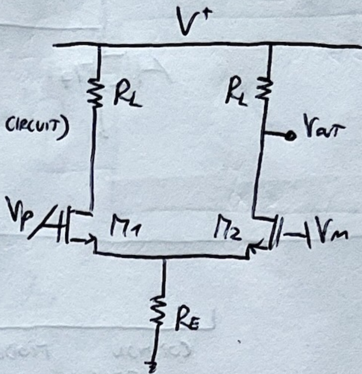
$CMRR \triangleq \left|\frac{G_d}{G_{cm}}\right|$

① Resistances

$G_d = \frac{g_m R_L}{2} = \frac{I R_L}{V_{OV}}$

$G_{cm} = \frac{-v_{cm}}{2R_E + \frac{1}{g_m}} \cdot R_L \simeq -v_{cm}\frac{R_L}{2R_E}$ (HALF CIRCUIT)

$CMRR = \frac{g_m R_L}{2} \cdot \frac{2R_E}{R_L} = g_m R_E$



(SINGLE ENDED OUTPUT)

{ $G_{d,MAX}$ limited by $I \cdot R_L = V_{L\_MAX}$ (M_1 & M_2 may become ohmic)

$CMRR|_{MAX}$ limited by the maximum current I, which is limited by the maximum V_{LOAD}.

② Current Generators

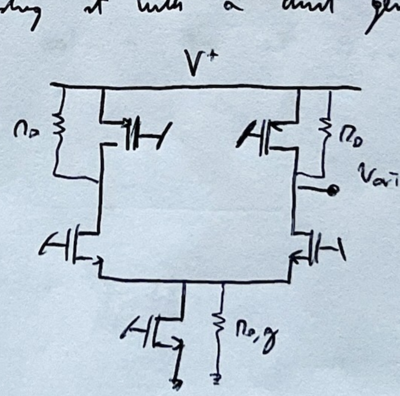
We may think to improve R_{OUT} without having to deal with high voltage drops by replacing R_L with current generators.
Similarly, we should also increase R_E by substituting it with a current generator.

$G_{cm} = \frac{r_0}{2r_{0,g}}$

$G_d = \frac{g_m r_0}{2}$

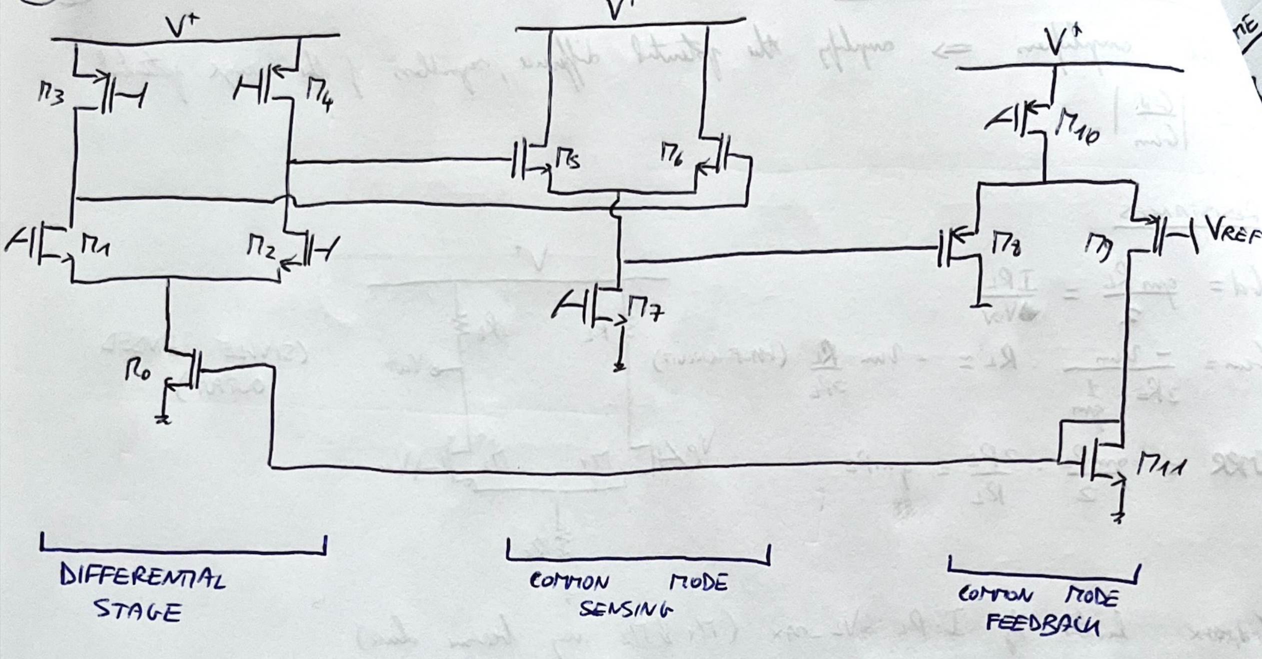
$CMRR = \frac{g_m r_0}{2} \cdot \frac{2r_{0,g}}{r_0} = g_m r_{0,g}$

Maybe improve $r_{0,g}$ using CASCODES!



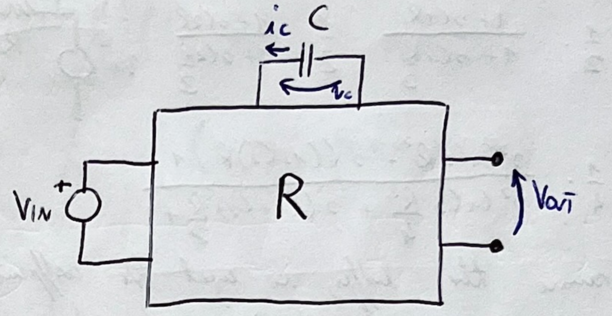
③ Common Mode Feedback

The previous circuit indeed needs to have the PMOS transistors perfectly matching the NMOS current, otherwise the intermediate drain nodes will act in order to push the upper or lower pair into ohmic region, until the currents match. Ohmic transistors though have lower r_0, so lower G_d ⇒ BAD!
We need feedback:


V+
M3
M4
M1
M2
M0
V+
M5
M6
M7
V+
M10
M8
M9
VREF
M11
DIFFERENTIAL STAGE
COMMON MODE SENSING
COMMON MODE FEEDBACK

TIME CONSTANTS METHOD

Let's consider a linear time-invariant network with a resistive core and one capacitor:



$\begin{cases} \text{INPUT VARIABLES: } i_C \ \& \ v_{IN} \\ \text{OUTPUT VARIABLES: } V_{OUT} \ \& \ V_C \end{cases}$

$\Downarrow$

$$\begin{cases} v_{OUT} = A_0 v_{IN} + R_m i_C \\ v_C = B_0 v_{IN} + R_1 \cdot i_C \end{cases}$$

$(A_0, B_0, R_m, R_1) \in \mathbb{R}$ being the core resistive.

Let's consider that $v_C = -\dfrac{i_C}{sC}$

$\Downarrow$

$$\begin{cases} v_{OUT} = A_0 v_{IN} - sCR_m v_C \\ v_C = B_0 v_{IN} - sCR_1 v_C \end{cases} \implies v_C = \frac{B_0 v_{IN}}{1+sCR_1} \ ; \ v_{OUT} = A_0 v_{IN}\left[1 - sC\,\frac{R_m B_0/A_0}{1+sCR_1}\right]$$

$$v_{OUT} = A_0 v_{IN}\,\frac{1+sC(R_1 - R_m B_0/A_0)}{1+sCR_1}$$

So we have a pole at $s = -\dfrac{1}{CR_1}$ and a zero at $s = -\dfrac{1}{C(R_1 - R_m B_0/A_0)}$

For a zero to exist it must be

$$A_0 v_{IN} + R_m i_C = 0$$

$\downarrow$

$$i_C = -\frac{A_0}{R_m} v_{IN} \quad \text{or} \quad v_{IN} = -\frac{R_m}{A_0} i_C$$

So

$$v_C = -\frac{B_0}{A_0} R_m i_C + R_1 i_C$$

$\downarrow$

$$\frac{v_C}{i_C} = \left(R_1 - \frac{R_m B_0}{A_0}\right) = R_{01}$$

[RESISTANCE SEEN ACROSS C when $V_{OUT} = 0$]

$$\implies \frac{v_{OUT}}{v_{IN}} = A_0\,\frac{1+sCR_{01}}{1+sCR_1}$$

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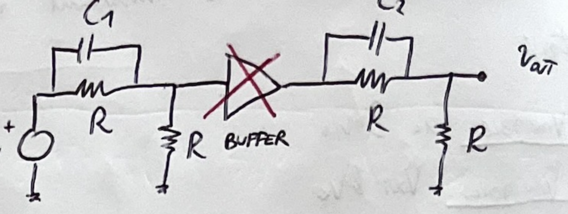
Let's consider now two RC networks decoupled by a buffer, the overall T.F. will be the cascade of the two:



$$T(s) = \frac{1}{2} \cdot \frac{1+sC_1R}{1+sC_1\frac{R}{2}} \cdot \frac{1}{2} \cdot \frac{1+sC_2R}{1+sC_2\frac{R}{2}}$$

$$\downarrow$$

$$= \frac{1}{4} \cdot \frac{s^2C_1C_2R^2 + s(C_1+C_2)R + 1}{s^2C_1C_2\frac{R^2}{4} + s(C_1+C_2)\frac{R}{2} + 1}$$

If we remove the buffer, we expect the coefficients to change, but just for the resistive part. Moreover, also the DC gain changes!

[$A_0 \triangleq$ GAIN WITH ALL THE CAPACITORS OPEN]

$$T(s) = A_0 \frac{s^2C_1C_2\alpha_{12} + s(C_1\alpha_1 + C_2\alpha_2) + 1}{s^2C_1C_2\beta_{12} + s(C_1\beta_1 + C_2\beta_2) + 1}$$

① $C_2 \to \emptyset$

$$T(s) = A_0 \frac{sC_1\alpha_1 + 1}{sC_1\beta_1 + 1}$$

and we know that $\alpha_1 = R_{1,0}^{(0)}$ $\quad \beta_1 = R_1^{(0)}$

② $C_1 \to \emptyset$

$$T(s) = A_0 \frac{sC_2\alpha_2 + 1}{sC_2\beta_2 + 1}$$

and we know that $\alpha_2 = R_{2,0}^{(0)}$ $\quad \beta_2 = R_2^{(0)}$

③ $C_1 \to \infty$ (a short)

$$T(s) = A_0 \frac{s^2C_1C_2\alpha_{12} + sC_1R_{01}^{(0)}}{s^2C_1C_2\beta_{12} + sC_1R_1^{(0)}} = A_0 \frac{sC_1C_2\alpha_{12} + C_1R_{01}^{(0)}}{sC_1C_2\beta_{12} + C_1R_1^{(0)}} =$$

$$= A_0 \frac{R_{01}^{(0)}}{R_1^{(0)}} \frac{sC_2\alpha_{12}/R_{01}^{(0)} + 1}{sC_2\beta_{12}/R_1^{(0)} + 1}$$

and since the network can be seen as the one of a single capacitor C_2 and C_1 as a short, we may write

$$\begin{cases} \frac{\alpha_{12}}{R_{01}^{(0)}} = R_{02}^{(1)} \\ \frac{\beta_{12}}{R_1^{(0)}} = R_2^{(1)} \end{cases} \Rightarrow \begin{cases} \alpha_{12} = R_{01}^{(0)} R_{02}^{(1)} \\ \beta_{12} = R_1^{(0)} R_2^{(1)} \end{cases}$$

If we chose $C_2 \to \infty$ instead of C_1, we'd get

$$\begin{cases} \alpha_{12} = R_{02}^{(0)} R_{01}^{(1)} \\ \beta_{12} = R_2^{(0)} R_1^{(2)} \end{cases}$$

which must be equal to the one obtained for $C_1 \to \infty$, since the T.F. is unique!

→ Popular Results

Starting from a 3^{rd} order T.F., we may write

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1 = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\left(1 - \frac{s}{p_3}\right)$$

$$b_1 = -\left(\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}\right) \simeq -\frac{1}{p_1} \qquad (\text{assuming } p_1 << p_2, p_3)$$

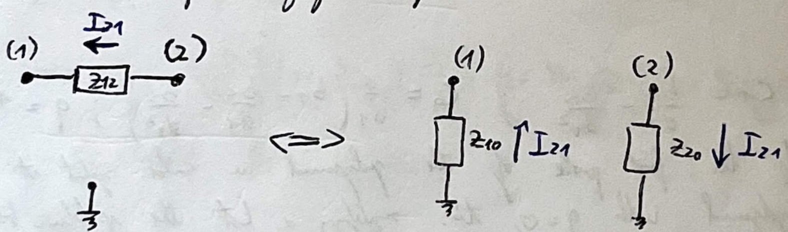
@ HF

$$D(s) \simeq b_3 s^3 + b_2 s^2 = s^2 (b_3 s + b_2) = 0$$

$$p_3 \simeq -\frac{b_2}{b_3} = \text{reshape terms} \ldots$$

→ Miller Theorem

We want to replace Z_{12} with two impedances Z_{10} and Z_{20}, we must set that the current entering or going out of the nodes remains unchanged



$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = -\frac{V_1}{Z_{10}} \qquad \left[\frac{V_2}{V_1} = H(s)\right] \qquad I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{V_2}{Z_{20}}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$V_2 - V_1 = -\frac{Z_{12}}{Z_{10}} V_1 \qquad\qquad Z_{20} = \frac{V_2}{V_2 - V_1} Z_{12}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$Z_{10} = -\frac{V_1}{V_2 - V_1} \cdot Z_{12} = \frac{V_1}{V_1 - V_2} Z_{12} \qquad\qquad Y_{20} = \frac{V_2 - V_1}{V_2} Y_{12}$$

$$\downarrow$$

$$\Rightarrow Y_{10} = \frac{V_1 - V_2}{V_1} Y_{12} = [1 - H(s)] Y_{12} \qquad\qquad \Rightarrow Y_{20} = \frac{H(s) - 1}{H(s)} Y_{12}$$

→ Approximations and Circuit Insights

It's important to derive some estimates of the poles and zeros by using approximations in order to understand which parameters are limiting the stability. Let's consider a third order T.F. for example:

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1$$

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We may divide it for a first order term

$$(b_3 s^3 + b_2 s^2 + b_1 s + 1) \div (d_1 s + 1)$$

$$\begin{array}{l|l}
b_3 s^3 + b_2 s^2 + b_1 s + 1 & d_1 s + 1 \\
\hline
b_3 s^3 + \frac{b_3}{d_1} s^2 & \frac{b_3}{d_1} s^2 + \frac{1}{d_1}\left(b_2 - \frac{b_3}{d_1}\right) s + \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) \\
\hline
// \left(b_2 - \frac{b_3}{d_1}\right) s^2 + b_1 s + 1 & \\
\left(b_2 - \frac{b_3}{d_1}\right) s^2 + \frac{1}{d_1}\left(b_2 - \frac{b_3}{d_1}\right) s & \\
\hline
// \left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) s + 1 & \\
\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) s + \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) &
\end{array}$$

So we get

$$c_2 s^2 + c_1 s + c_0 + \frac{q}{d_1 s + 1}$$

where

$$c_2 = \frac{b_3}{d_1} \quad ; \quad c_1 = \frac{b_2}{d_1} - \frac{b_3}{d_1^2} \quad ; \quad c_0 = \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) \; ; \; q = 1 - \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right)$$

If $-\frac{1}{d_1}$ was exactly the first pole of the polynomial we could split it into a 2^{nd} and a 1^{st} order polynomial with $q = 0$, thus simplifying a lot the problem. However, even if we don't now exactly p_1, we know that it is ≈ set by the Miller capacitance, so we may say

$$p_1 \simeq -\frac{1}{b_1}$$

so we get

$$c_2 = \frac{b_3}{b_1} \; ; \quad c_1 = \frac{b_2}{b_1} - \overbrace{\frac{b_3}{b_1^2}}^{\text{Usually negligible}} \; ; \quad c_0 = \frac{1}{b_1}\left[b_1 - \overbrace{\frac{b_2}{b_1} - \frac{b_3}{b_1^2}}^{\text{Usually negligible}}\right] \; ; \; q = \overbrace{\frac{1}{b_1}\left(\frac{b_2}{b_1} - \frac{b_3}{b_1^2}\right)}^{\text{Usually negligible}}$$

and split the denominator like this

$$(c_2 s^2 + c_1 s + 1)(1 + s b_1)$$

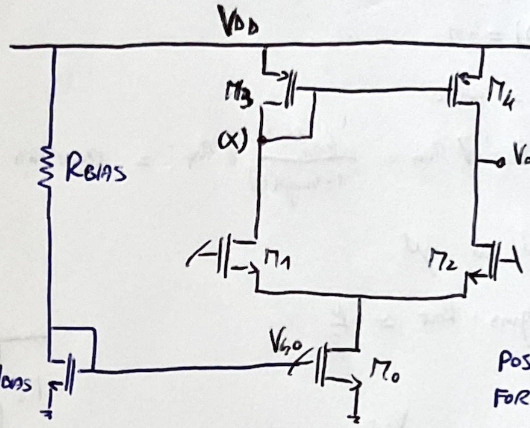
assuming $q \simeq 0$. It can be shown that these results are really close to the ones obtained by shorting C_c and computing the remaining poles using the extended time constant method.

SINGLE - ENDED OTA

The single - ended configuration is another option to guarantee consistent bias of the stage. The adoption of the current mirror links the current from M_4 to the current delivered by the tail, hence implementing the control loop directly in the differential stage.

In order for the bias to be consistent, we need

$$\begin{cases} I_1 = I_3 \\ I_2 = I_4 \end{cases}$$



POSSIBLE CIRCUIT FOR $V_{G,0}$

(neglecting V_{ov} in $(V_{DS}-V_{ov})/V_A$)

$$\begin{cases} \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_1 (V_{ov,D})^2\left[1+\frac{V_X-V_S}{V_A}\right] = I_1 \\ \frac{1}{2}\mu_p C'_{ox}\left(\frac{W}{L}\right)_3 (V_{ov,M})^2\left[1+\frac{V_{DD}-V_X}{V_A}\right] = I_3 \\ \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_2 (V_{ov,D})^2\left[1+\frac{V_{OUT}-V_S}{V_A}\right] = I_2 \\ \frac{1}{2}\mu_p C'_{ox}\left(\frac{W}{L}\right)_4 (V_{ov,M})^2\left[1+\frac{V_{DD}-V_{OUT}}{V_A}\right] = I_4 \end{cases}$$

$$\frac{I_1}{I_2} = \frac{I_3}{I_4} \implies \left(1+\frac{V_X-V_S}{V_A}\right)\left(1+\frac{V_{DD}-V_{OUT}}{V_A}\right) = \left(1+\frac{V_{DD}-V_X}{V_A}\right)\left(1+\frac{V_{OUT}-V_S}{V_A}\right)$$

by inspection it turns out that $V_X = V_{OUT}$

→ VOLTAGE SWINGS

- $V_{CM}|_{MAX} = V_{DD} - V_{SG,3} + |V_T| = V_{DD} - V_{ov,3} - |V_T| + |V_T| = V_{DD} - V_{ov,3}$ (Saturation of M_1)
- $V_{CM}|_{MIN} = V_{ov,0} + V_{GS,1} = V_{ov,0} + V_{ov,1} + |V_T|$ (Saturation of M_0)

Not symmetric! Towards ground it has to accommodate for a V_{GS} in addition to an overdrive voltage.

- $V_{OUT}|_{MIN} = V_{CM\_MIN} - |V_T| = V_{ov,0} + V_{ov,1}$ (Saturation of M_2)
- $V_{OUT}|_{MAX} = V_{DD} - V_{ov,4}$ (Saturation of M_4)

→ DIFFERENTIAL GAIN

Due to the introduction of the mirror, the circuit is not symmetric anymore, since the resistances on the drain of M_1 and M_2 are different! In order to recover symmetry, we use the Norton theorem:

- in order to compute i_{cc} we need to short the drain of M_4. In this way both the drain of M_1 and M_2 are connected to low-Z nodes, we can assume them to be 0 with an error $\sim \frac{1}{g_m R_0}$.

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Now we've restored symmetry and can say that vdiff splits evenly on the two transistors, thus having the same status on small signal operation. Neglecting also the mirroring errors, we get

$$i_{cc} = g_m v_{diff}$$

- R_{OUT} can be computed by noticing that we have a loop acting to fix V_{OUT} to V.G., so we may cut the loop, reconstruct the impedance ($2r_{o2}$) and evaluate

$$G_{loop}(0) = -1$$

$$R_{OUT}'^{(0)} = 2r_{o2}$$

$$R_{OUT} = R_{OUT}' \parallel r_{o4} = \frac{R_{OUT}'^{(0)}}{1 - G_{loop}(0)} \parallel r_{o4} = r_{o2} \parallel r_{o4}$$

In the end we get

$$G_0 = g_{mD} \cdot R_{OUT} \simeq \frac{\mu}{2}$$