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ELECTRONIC SYSTEMS

2021-22 academic year

prof. Franco ZAPPA

Summary

Tutoring
- Palazzo 25
  Aula 2.3
  14.15 - 18.15

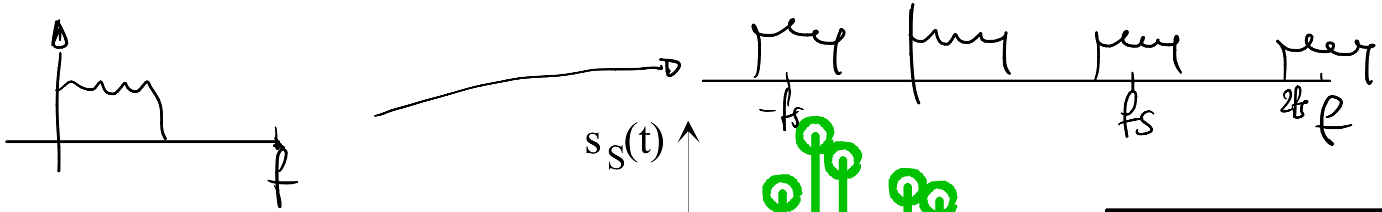
- Basics on sampling theory
- Aliasing issues
- Basic Sample&Hold circuit
- Static and dynamic errors

Sampling theory

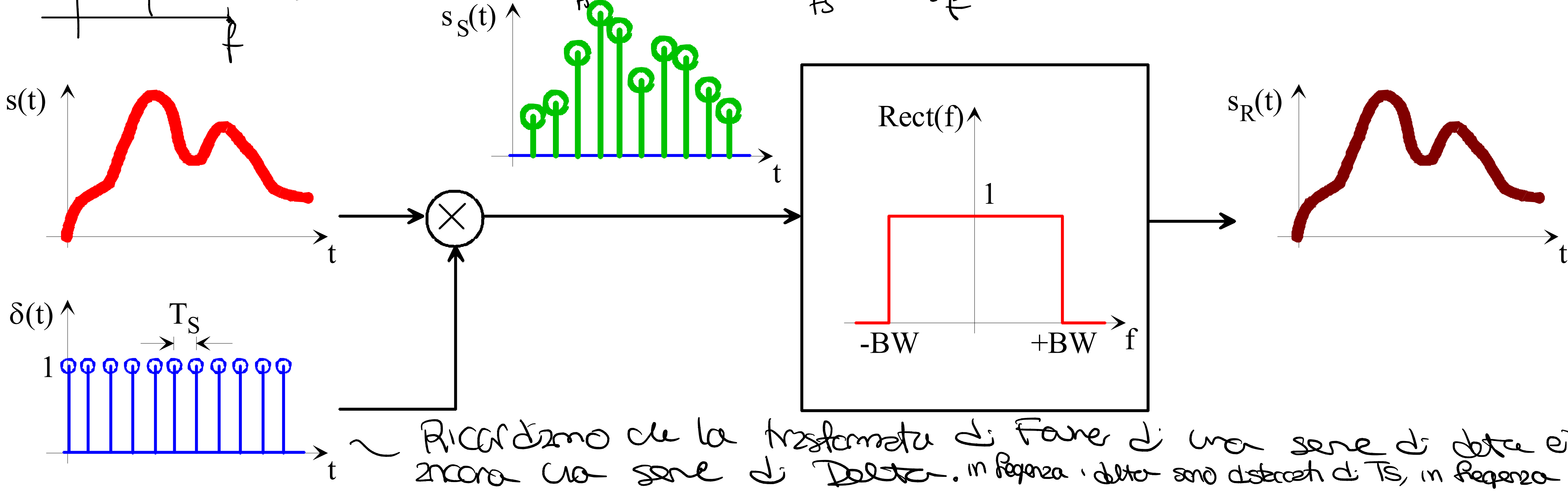
Se un segnale non è simmetrico rispetto all'asse delle tensioni (es solo positivo stile onda quadra), allora no che le componenti in freq sono solo quelle dispari)

Shannon theorem, 1949:

"if a ***function s(t)*** *has no frequency components above BW Hz,*
then it is fully ***determined*** *by its* ***values****, sampled every* $T_S={}^1/_{2BW}$"



lo spettro del segnale filtrato tende a ∞.



~ Ricordiamo che la trasformata di Fourier di una serie di delta è ancora una serie di Delta. in frequenza i delta sono distanziati di Ts, in frequenza di fs

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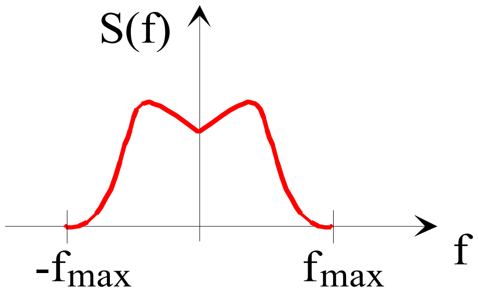
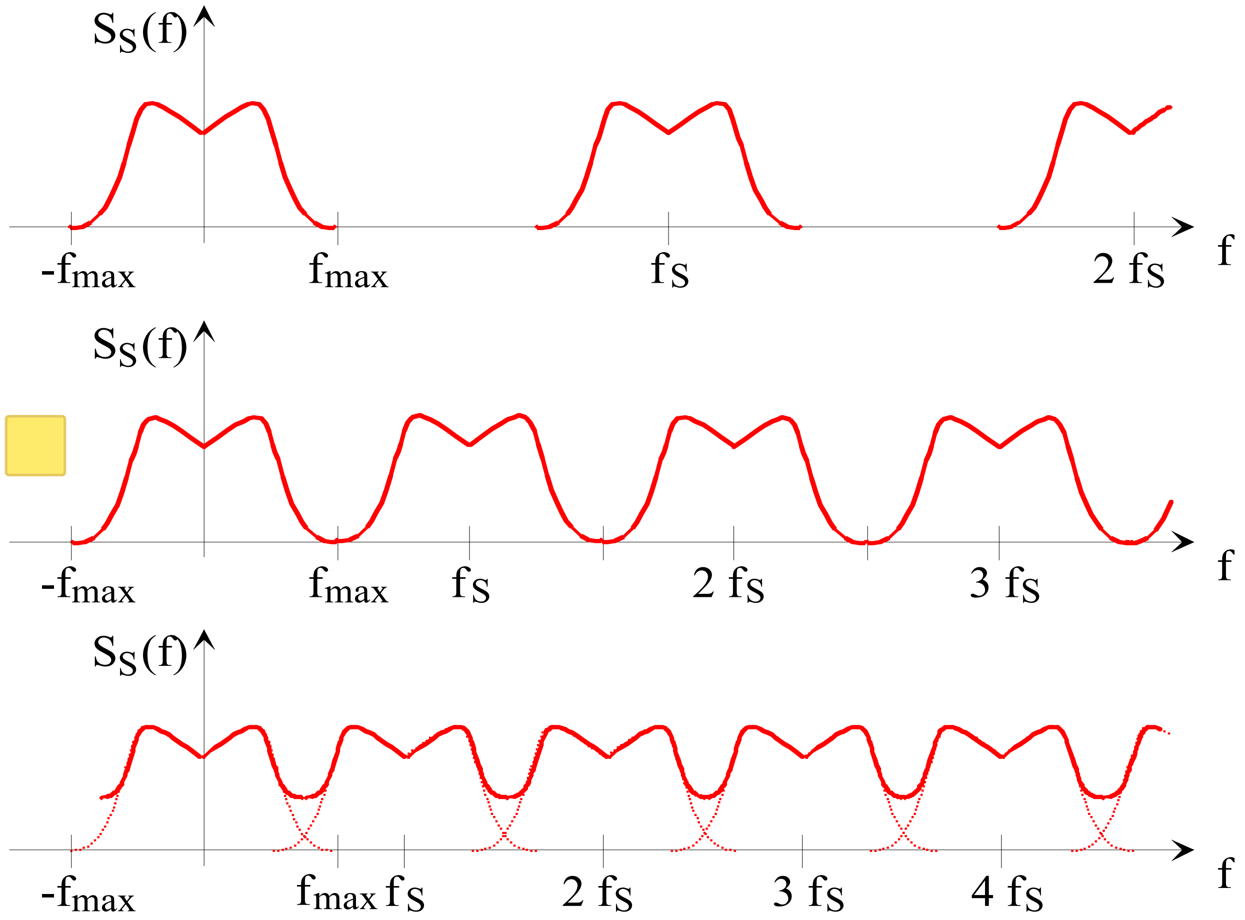
Sampling:



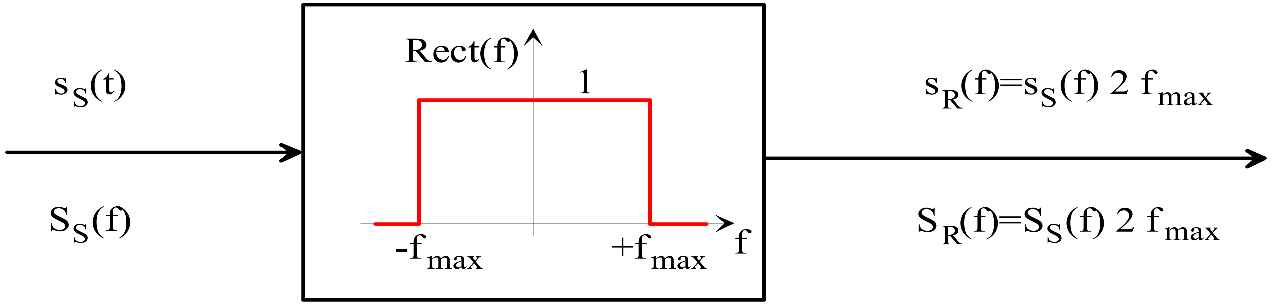
$f_S > 2\ f_{max}$

$f_S = 2\ f_{max}$

$f_S < 2\ f_{max}$



Reconstruction:

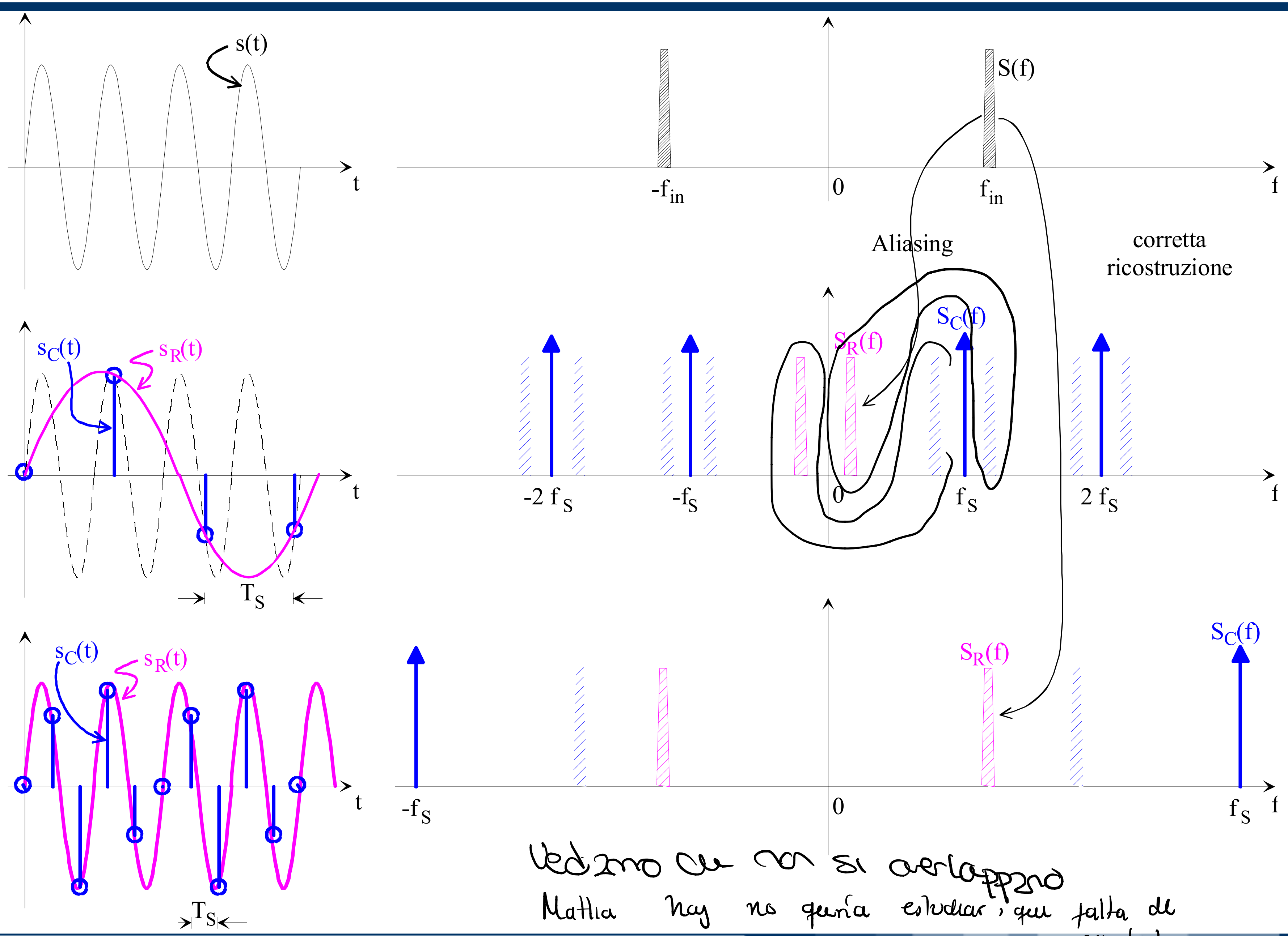


Aliasing: frequency-domain analysis

original

Sampled at $f_S<2\cdot f_{in}$

Sampled at $f_S>2\cdot f_{in}$



Vediamo che non si overlappano

Mattia hoy no quería estudiar, que falta de respeto!

Aliasing: time-domain analysis

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Misinterpretation (aliasing)

when $f > 0.5\ f_S$

Avoid !

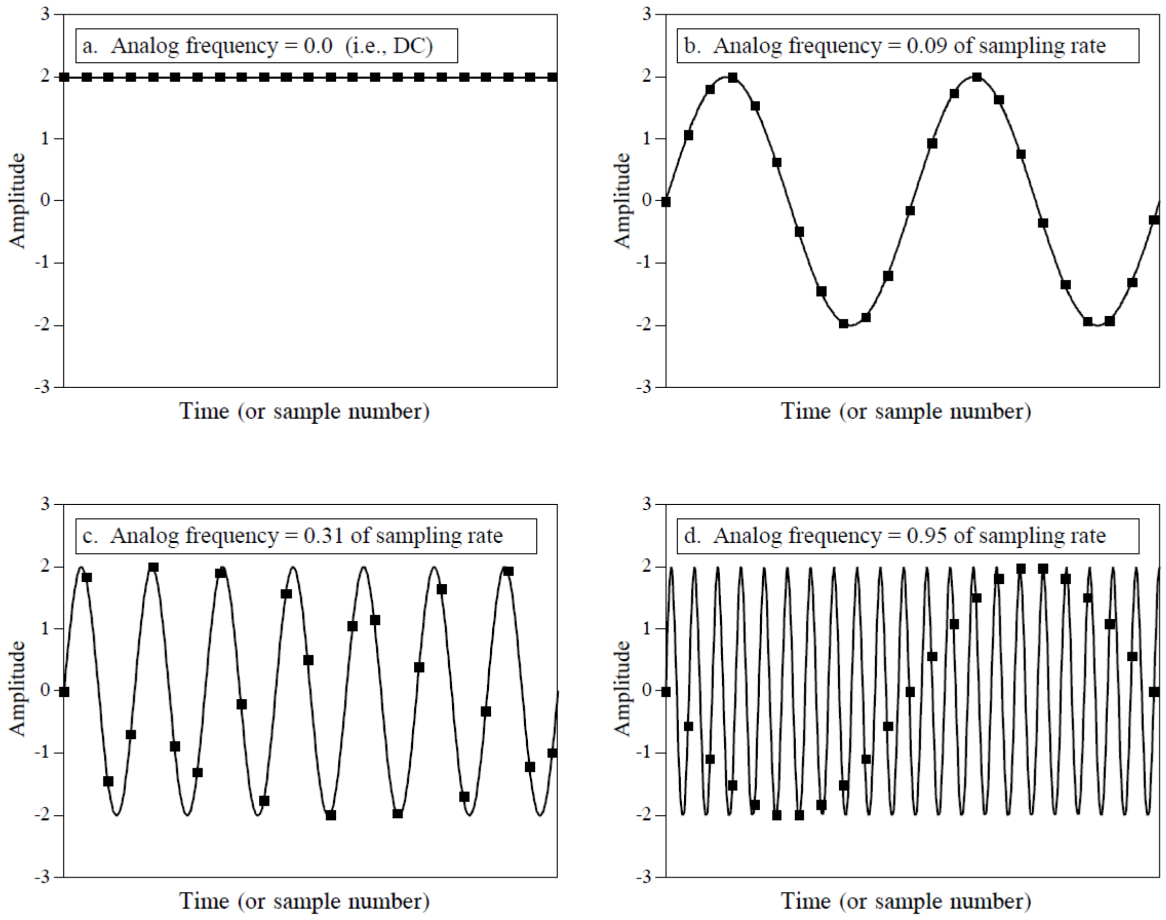


FIGURE 3-3
Illustration of proper and improper sampling. A continuous signal is sampled *properly* if the samples contain all the information needed to recreated the original waveform. Figures (a), (b), and (c) illustrate *proper sampling* of three sinusoidal waves. This is certainly not obvious, since the samples in (c) do not even appear to capture the shape of the waveform. Nevertheless, each of these continuous signals forms a unique one-to-one pair with its pattern of samples. This guarantees that reconstruction can take place. In (d), the frequency of the analog sine wave is greater than the Nyquist frequency (one-half of the sampling rate). This results in *aliasing*, where the frequency of the sampled data is different from the frequency of the continuous signal. Since aliasing has corrupted the information, the original signal cannot be reconstructed from the samples.

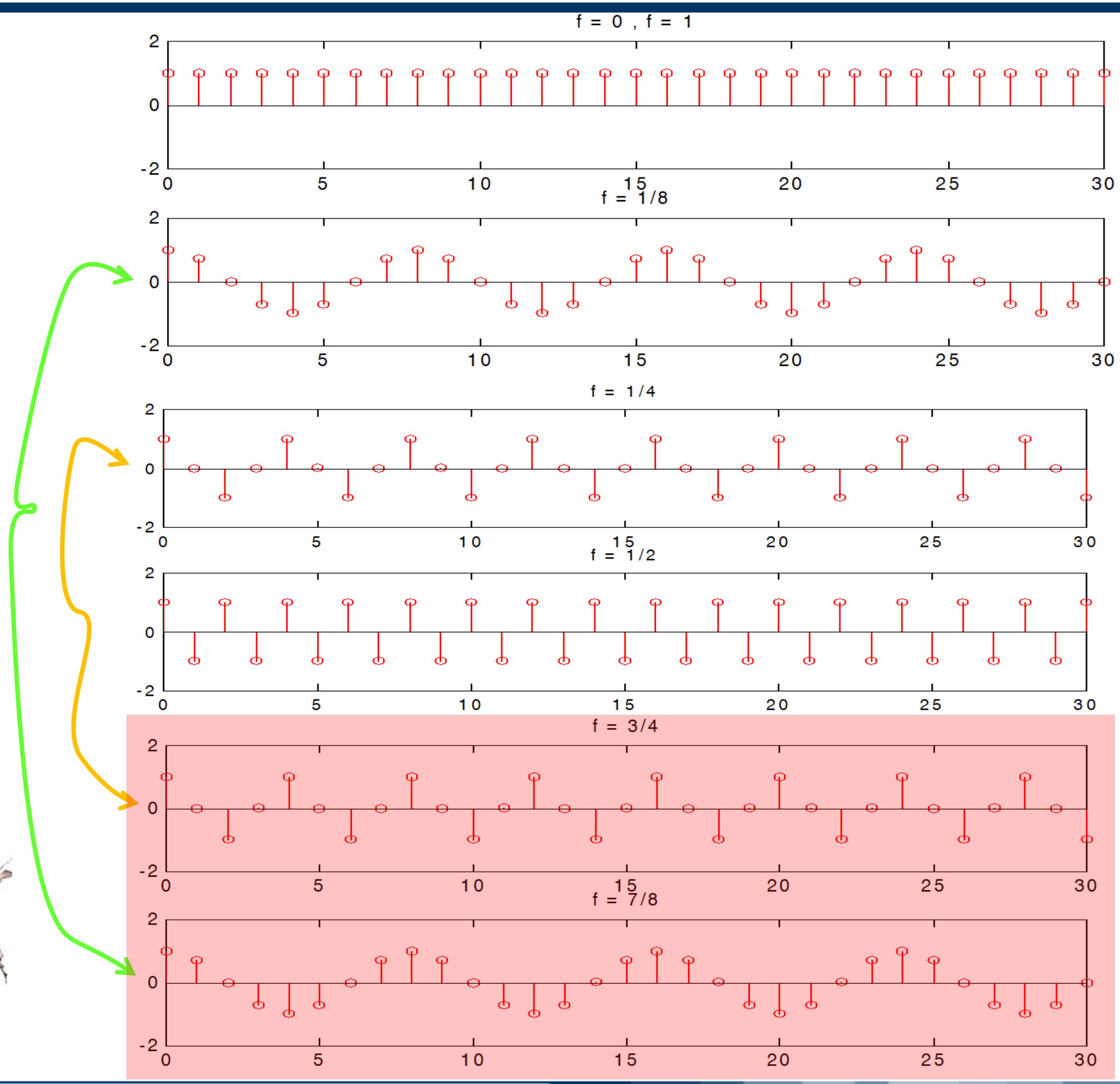
Stephen W. Smith, www.DSPguide.com
"The Scientist and Engineer's Guide to DSP"

Aliasing: time-domain analysis

Misinterpretation (aliasing)

when $f_S<2f$

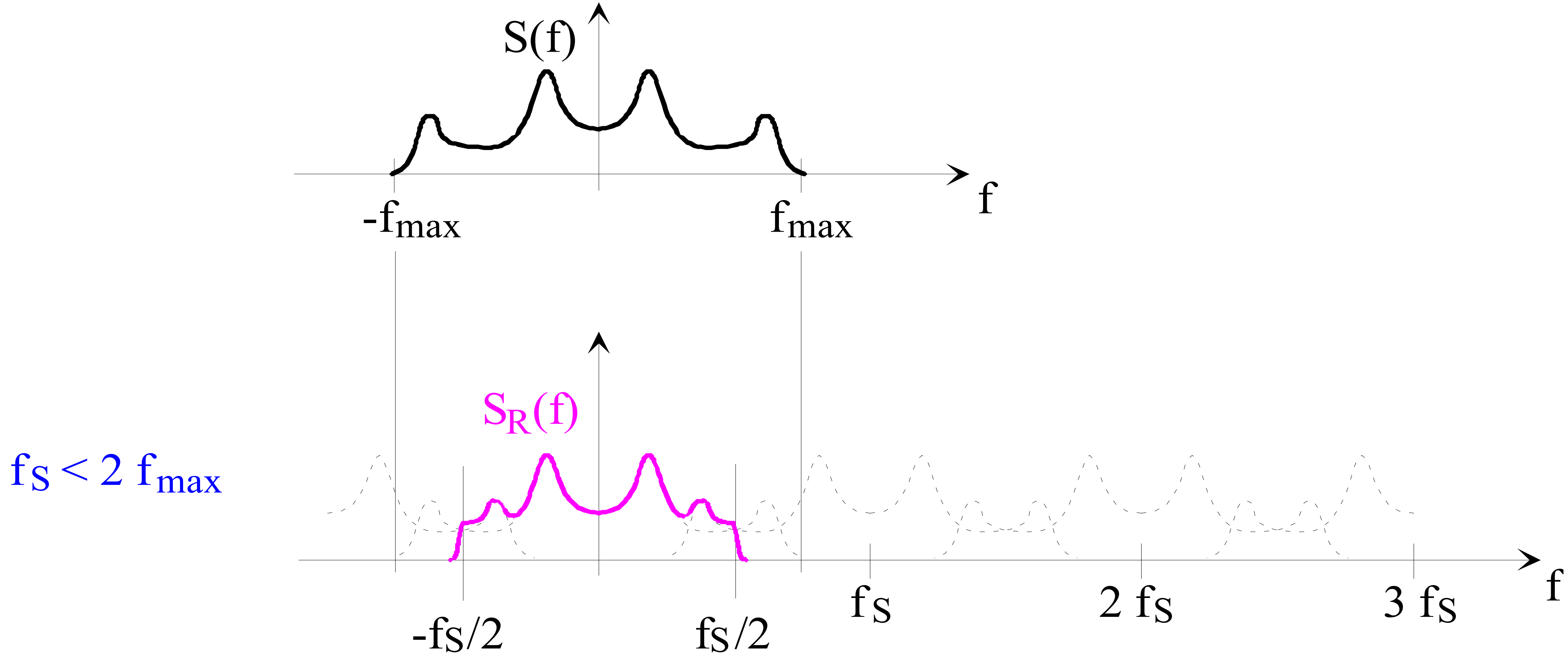
Avoid !



LIKE IN A WESTERN MOVIE.
WHEEL SPINNING BACKWARD

Aliasing: time-domain analysis

Therefore aliasing drastically deforms the original spectrum

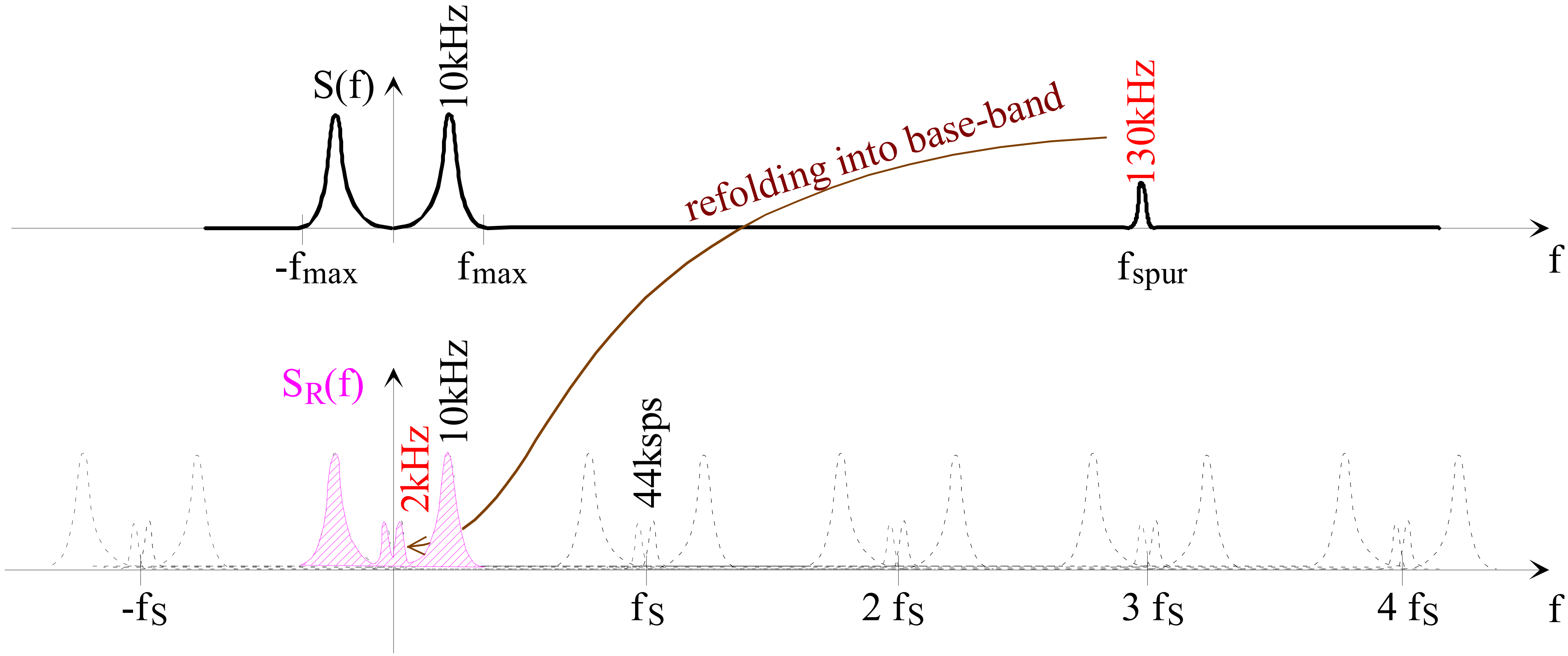


Avoid aliasing BEFORE it comes into play, otherwise no way to remove it !

Anti-aliasing filtering

: è essenziale perché sono un segnale ad alta frequenza manda tutto a puttane

Aliasing stems out also from unexpected spurious disturbances with $f_{spur}>f_S/2$

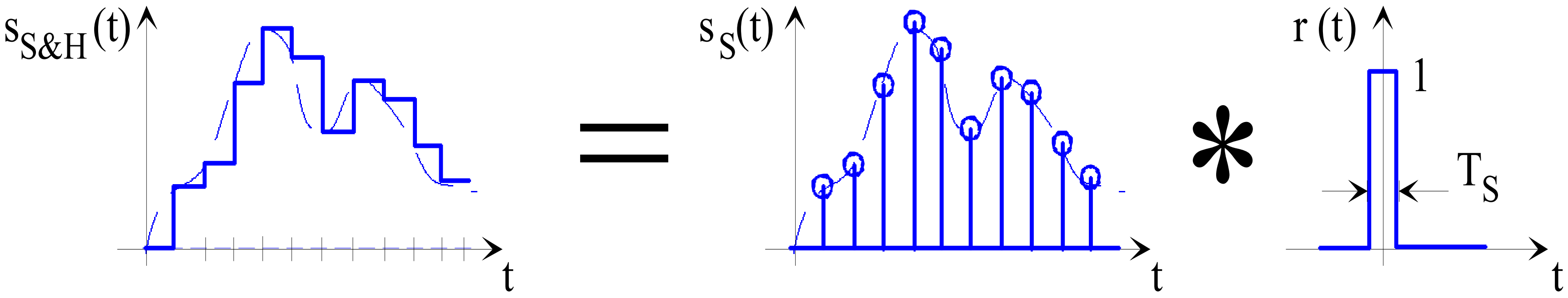


Never trust just on the bandwidth self-limitations of the input welcome signal …

… anyway, bandpass filtering is a must !

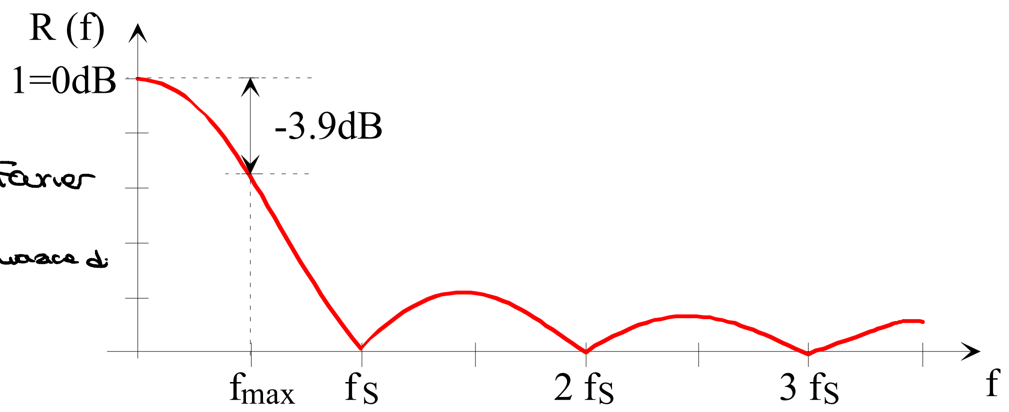
Spectral distortion and equalization

It is easier to employ "rectangular" samples instead of delta-like ones



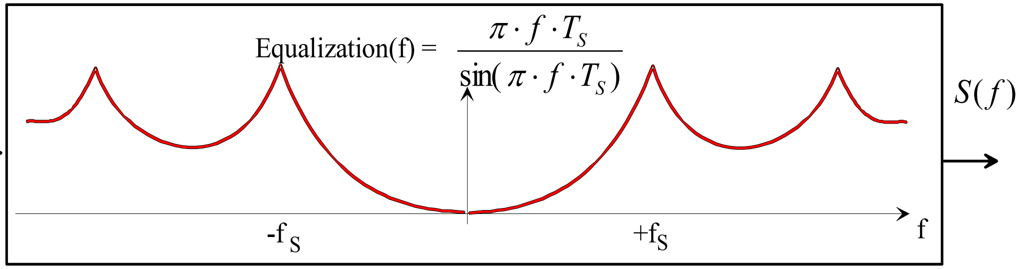
… but this causes spectral distortion:

Questo perché sappiamo bene che il segnale rettangolare ha trasformata di Fourier che è un sinc. Questo rompe il cazzo perché vado a moltiplicare anche le alte componenti spettrali con i lobi del sinc.
Ho quindi della distorsione che fa sì che il mio segnale abbia attenuazioni delle alte frequenze. Ho un attenuazione di circa 4dB per le alte frequenze. Devo usare un filtro equalizzatore che abbia guadagno 1 in DC e 4dB ad alte frequenze.

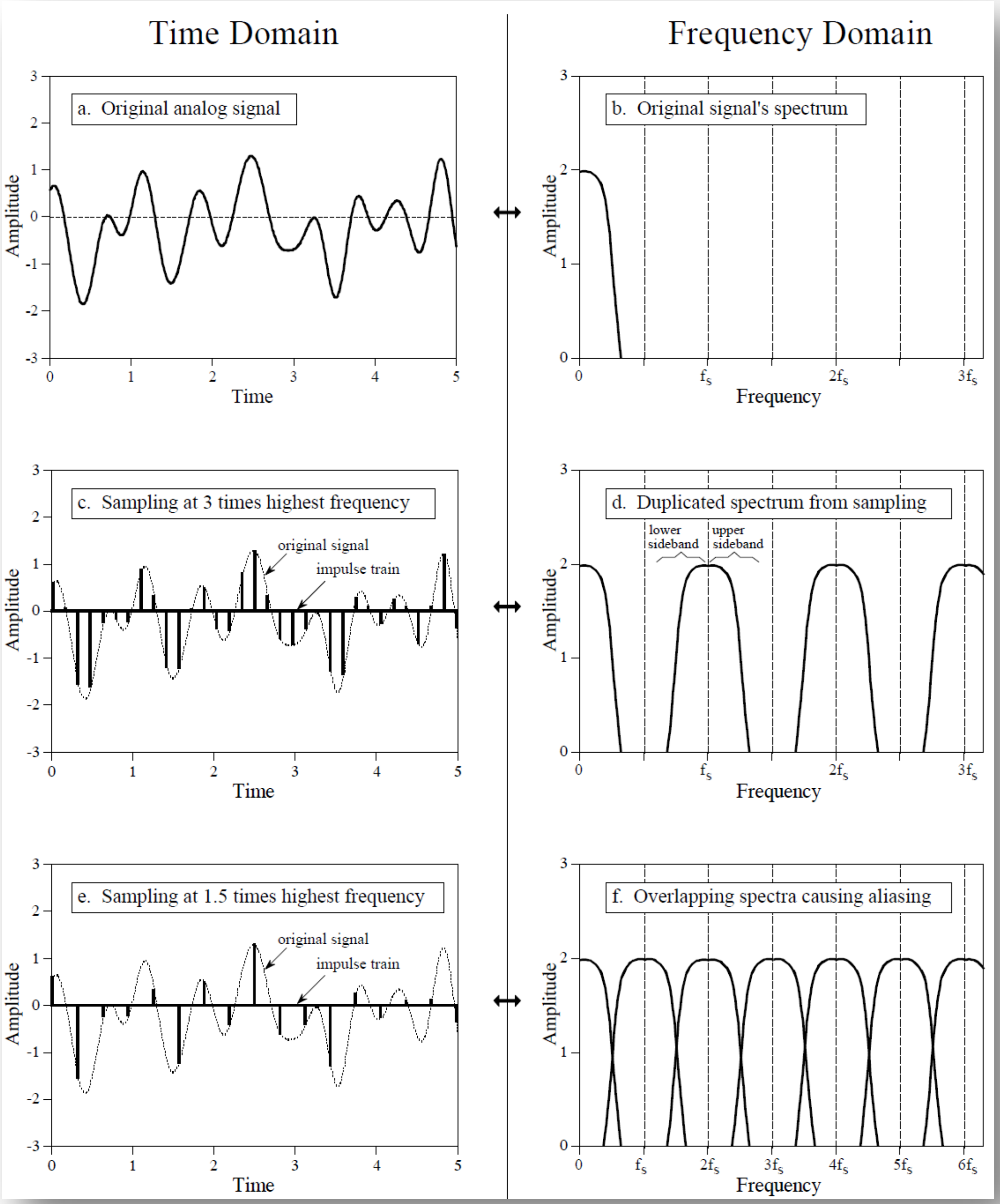


Therefore **equalization** is compulsory:

$$S_R(f) = \frac{\sin(\pi \cdot f \cdot T_S)}{\pi \cdot f \cdot T_S} \cdot S(f)$$



Recap



Filtro ricostruttivo ideale

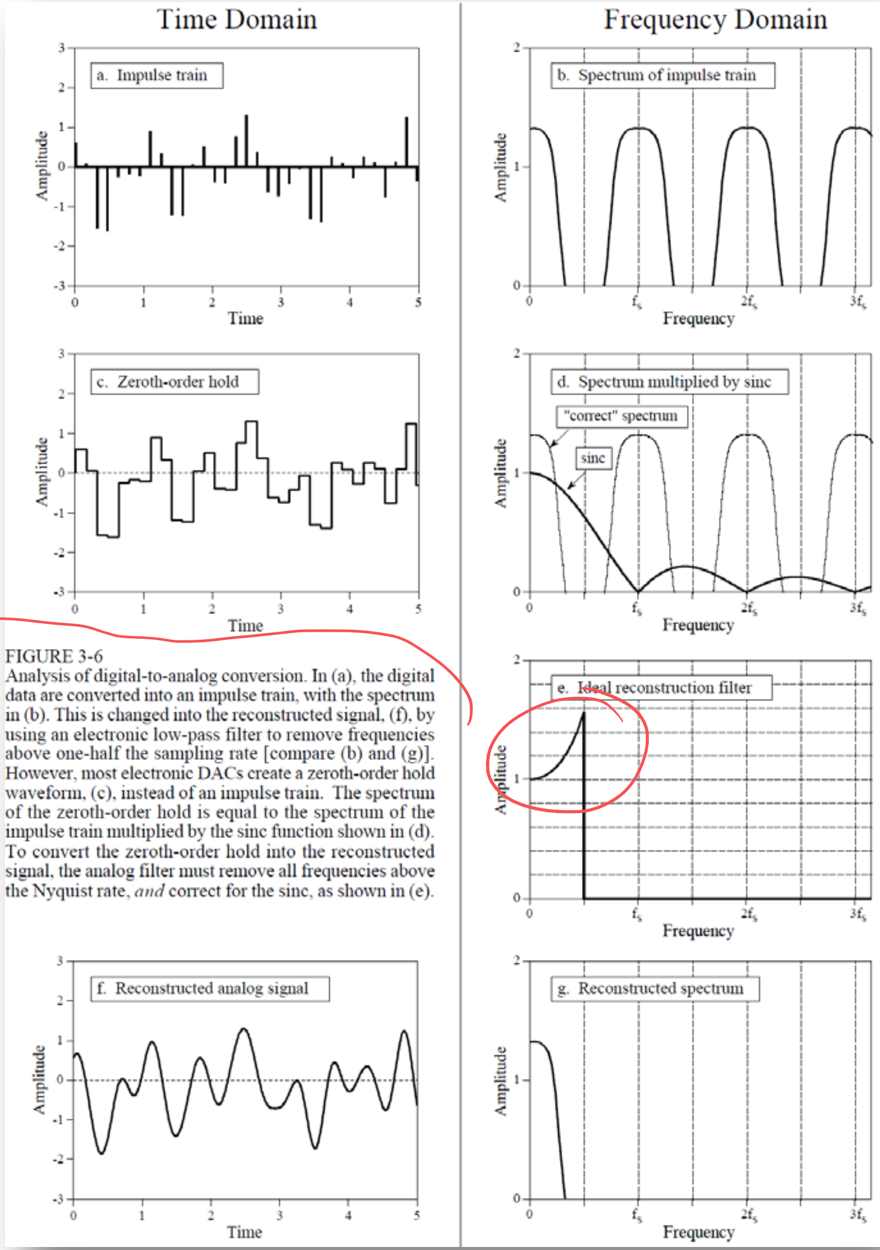


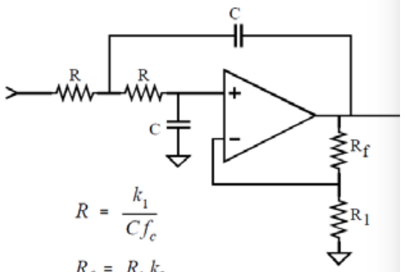
FIGURE 3-6
Analysis of digital-to-analog conversion. In (a), the digital data are converted into an impulse train, with the spectrum in (b). This is changed into the reconstructed signal, (f), by using an electronic low-pass filter to remove frequencies above one-half the sampling rate [compare (b) and (g)]. However, most electronic DACs create a zeroth-order hold waveform, (c), instead of an impulse train. The spectrum of the zeroth-order hold is equal to the spectrum of the impulse train multiplied by the sinc function shown in (d). To convert the zeroth-order hold into the reconstructed signal, the analog filter must remove all frequencies above the Nyquist rate, *and* correct for the sinc, as shown in (e).

Stephen W. Smith, www.DSPguide.com
"The Scientist and Engineer's Guide to DSP"

Analog Filtering

Noi vogliamo un filtro molto selettivo nel dominio del tempo (e non log-log) quindi mi serve un filtro con decisamente più di un polo (tipo 6 poli)

FIGURE 3-8
The modified Sallen-Key circuit, a building block for active filter design. The circuit shown implements a 2 pole low-pass filter. Higher order filters (more poles) can be formed by cascading stages. Find k_1 and k_2 from Table 3-1, arbitrarily select R_1 and C (try 10K and 0.01µF), and then calculate R and R_f from the equations in the figure. The parameter, f_c, is the cutoff frequency of the filter, in hertz.



$$R = \frac{k_1}{C f_c}$$

$$R_f = R_1 k_2$$

TABLE 3-1
Parameters for designing Bessel, Butterworth, and Chebyshev (6% ripple) filters.

# poles		Bessel k_1	Bessel k_2	Butterworth k_1	Butterworth k_2	Chebyshev k_1	Chebyshev k_2
2	stage 1	0.1251	0.268	0.1592	0.586	0.1293	0.842
4	stage 1	0.1111	0.084	0.1592	0.152	0.2666	0.582
	stage 2	0.0991	0.759	0.1592	1.235	0.1544	1.660
6	stage 1	0.0990	0.040	0.1592	0.068	0.4019	0.537
	stage 2	0.0941	0.364	0.1592	0.586	0.2072	1.448
	stage 3	0.0834	1.023	0.1592	1.483	0.1574	1.846
8	stage 1	0.0894	0.024	0.1592	0.038	0.5359	0.522
	stage 2	0.0867	0.213	0.1592	0.337	0.2657	1.379
	stage 3	0.0814	0.593	0.1592	0.889	0.1848	1.711
	stage 4	0.0726	1.184	0.1592	1.610	0.1582	1.913

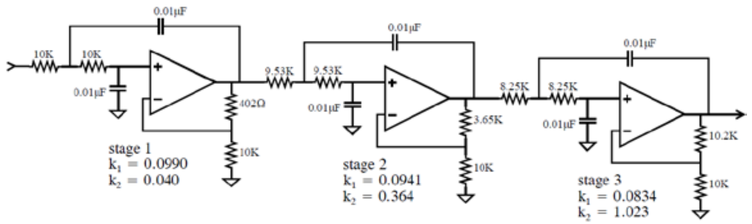


FIGURE 3-9
A six pole Bessel filter formed by cascading three Sallen-Key circuits. This is a low-pass filter with a cutoff frequency of 1 kHz.

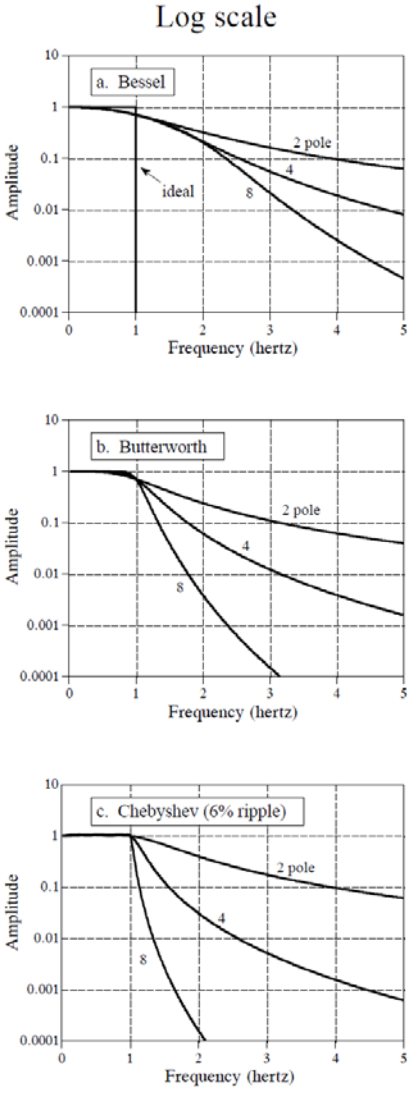
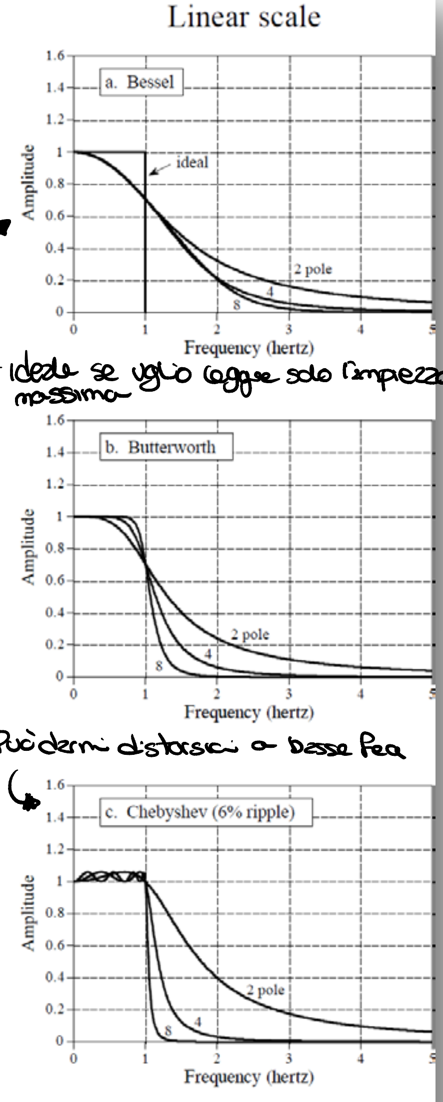


FIGURE 3-11
Frequency response of the three filters on a *logarithmic* scale. The Chebyshev filter has the sharpest roll-off.



Ideale se voglio leggere solo l'ampiezza massima

Può dare distorsioni a basse freq

FIGURE 3-12
Frequency response of the three filters on a *linear* scale. The Butterworth filter provides the flattest passband.

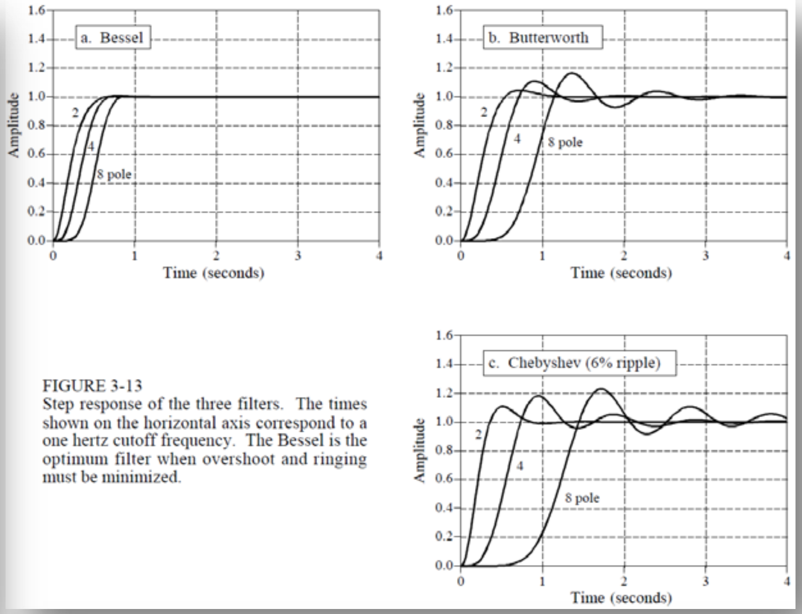


FIGURE 3-13
Step response of the three filters. The times shown on the horizontal axis correspond to a one hertz cutoff frequency. The Bessel is the optimum filter when overshoot and ringing must be minimized.

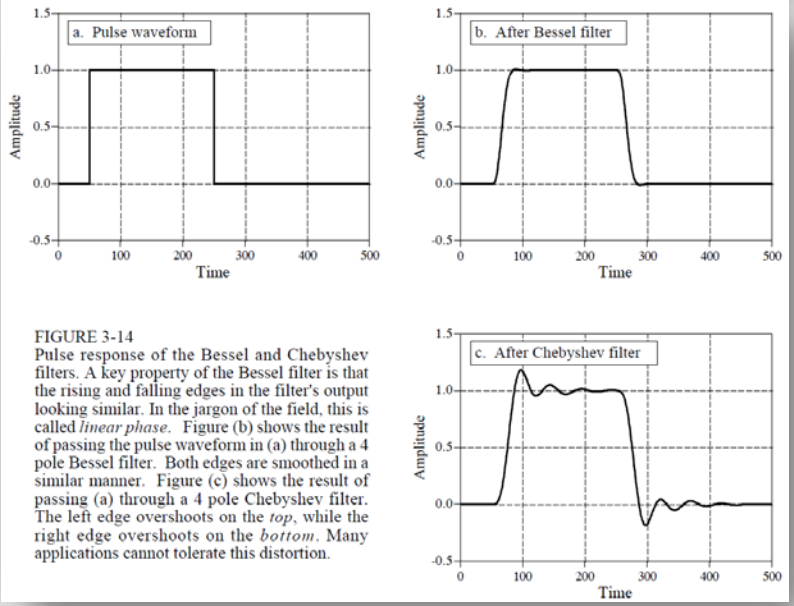


FIGURE 3-14
Pulse response of the Bessel and Chebyshev filters. A key property of the Bessel filter is that the rising and falling edges in the filter's output looking similar. In the jargon of the field, this is called *linear phase*. Figure (b) shows the result of passing the pulse waveform in (a) through a 4 pole Bessel filter. Both edges are smoothed in a similar manner. Figure (c) shows the result of passing (a) through a 4 pole Chebyshev filter. The left edge overshoots on the *top*, while the right edge overshoots on the *bottom*. Many applications cannot tolerate this distortion.

Stephen W. Smith, www.DSPguide.com
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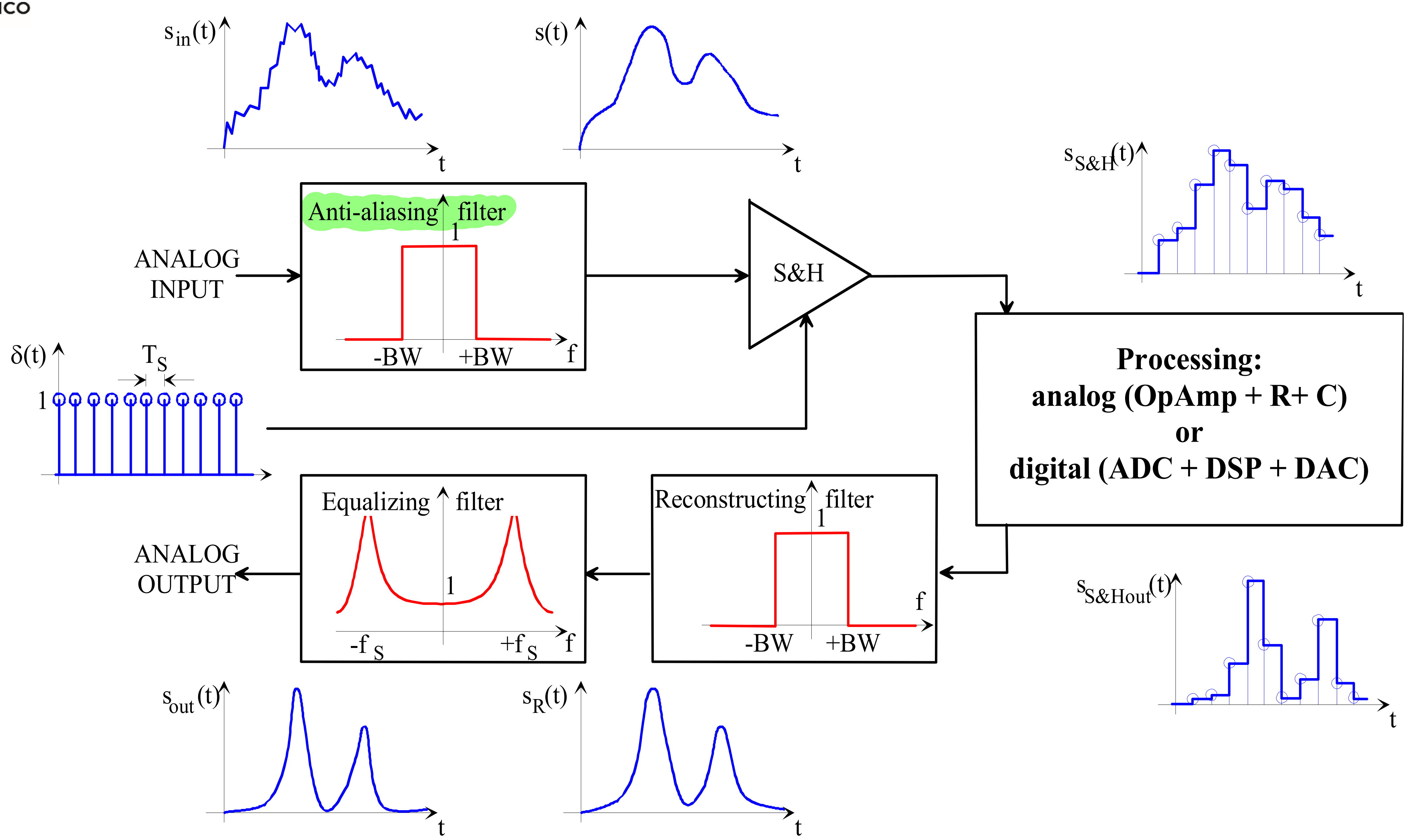
Signal Processing chain



Anti-aliasing Filtering

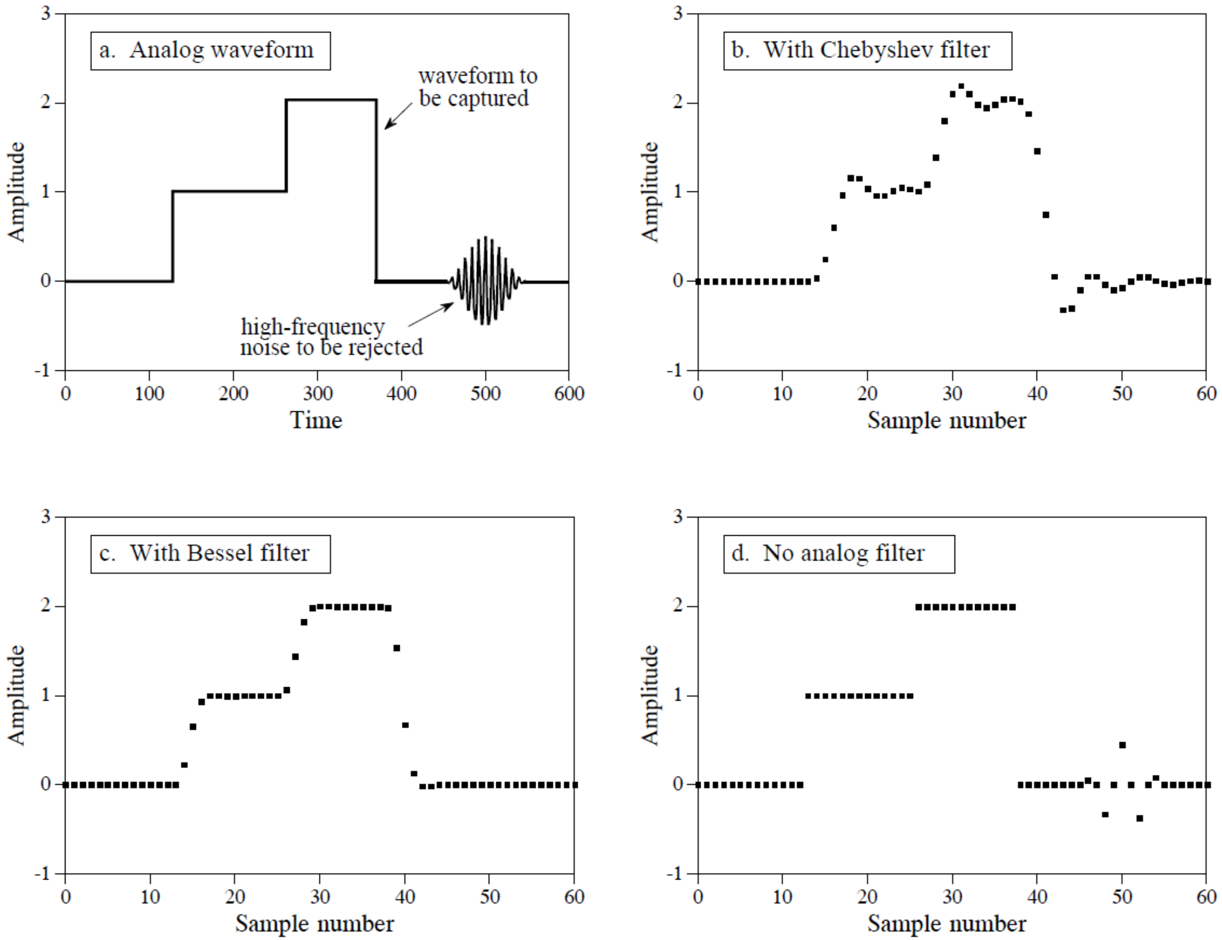


FIGURE 3-15
Three antialias filter options for time domain encoded signals. The goal is to eliminate high frequencies (that will alias during sampling), while simultaneously retaining edge sharpness (that carries information). Figure (a) shows an example analog signal containing both sharp edges and a high frequency noise burst. Figure (b) shows the digitized signal using a *Chebyshev filter*. While the high frequencies have been effectively removed, the edges have been grossly distorted. This is usually a terrible solution. The *Bessel filter*, shown in (c), provides a gentle edge smoothing while removing the high frequencies. Figure (d) shows the digitized signal using *no antialias filter*. In this case, the edges have retained perfect sharpness; however, the high frequency burst has aliased into several meaningless samples.

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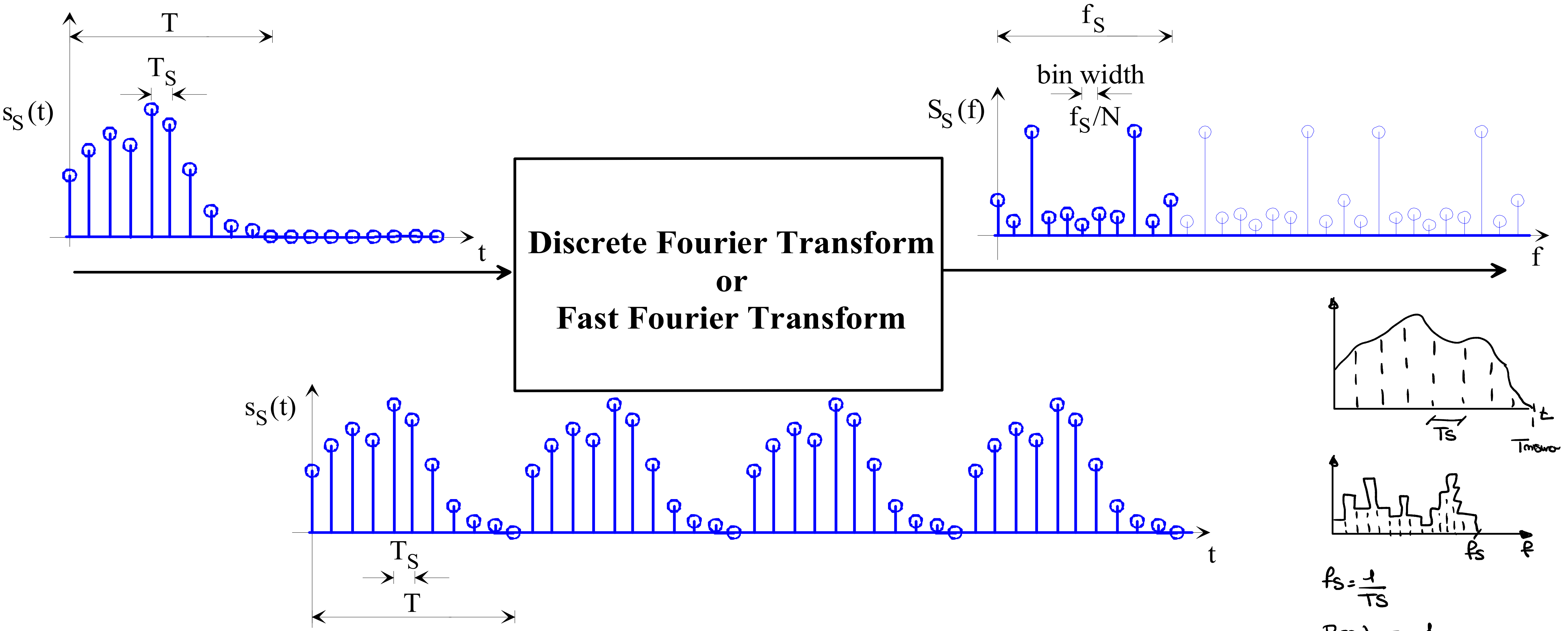
How to compute the spectrum (FFT)



$f_S = \frac{1}{T_S}$

$\text{Banda} = \frac{1}{T_{misra}}$

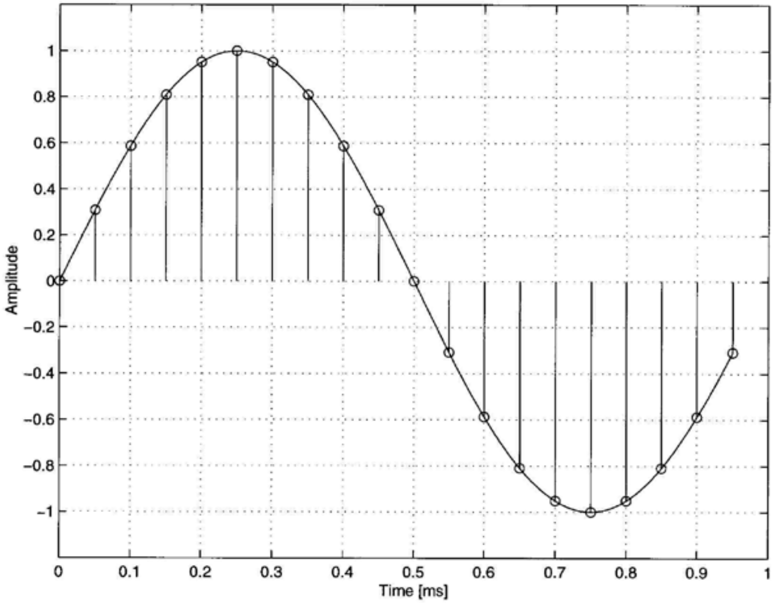
Se il segnale nel tempo è reale allora lo spettro è simmetrico quindi possiamo plottarlo sempre metà.

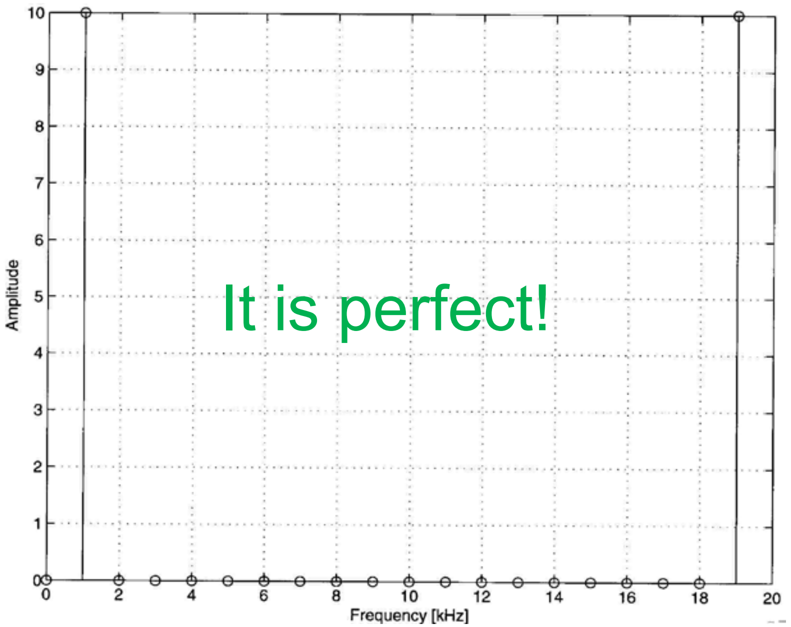
The FFT algorithm trusts on the periodicity of the input sequence

... hence FFT spectrum is for the periodic sequence and not the original one!

Errors in the FFT

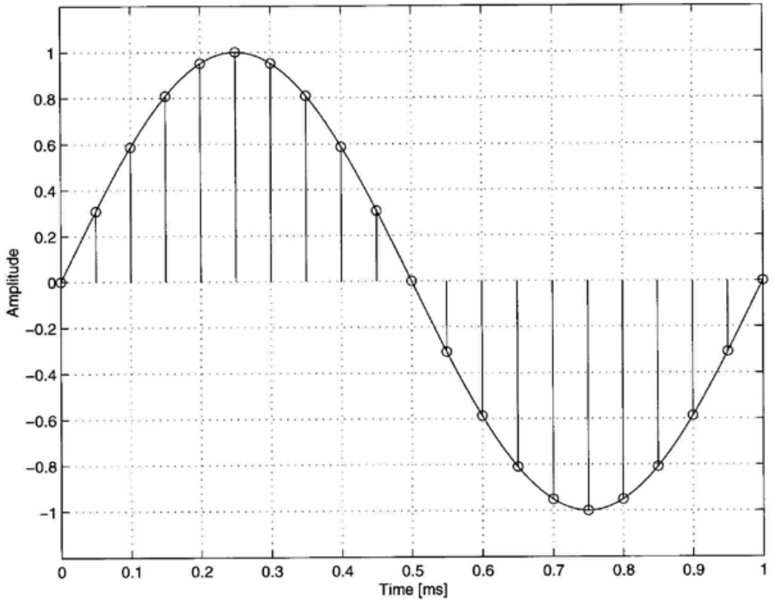
Correct choice:

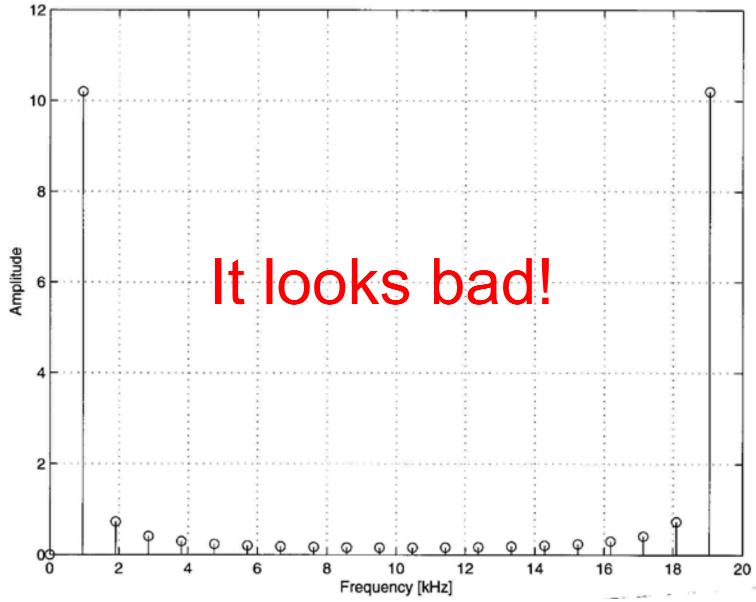




Wrong (unlucky) choice:

Questo perché abbiamo preso 2 volte lo zero e quindi quando metto in serie i vari periodi non avrò un unico ma 2 sample a zero e non è più una sinusoide perfetta.



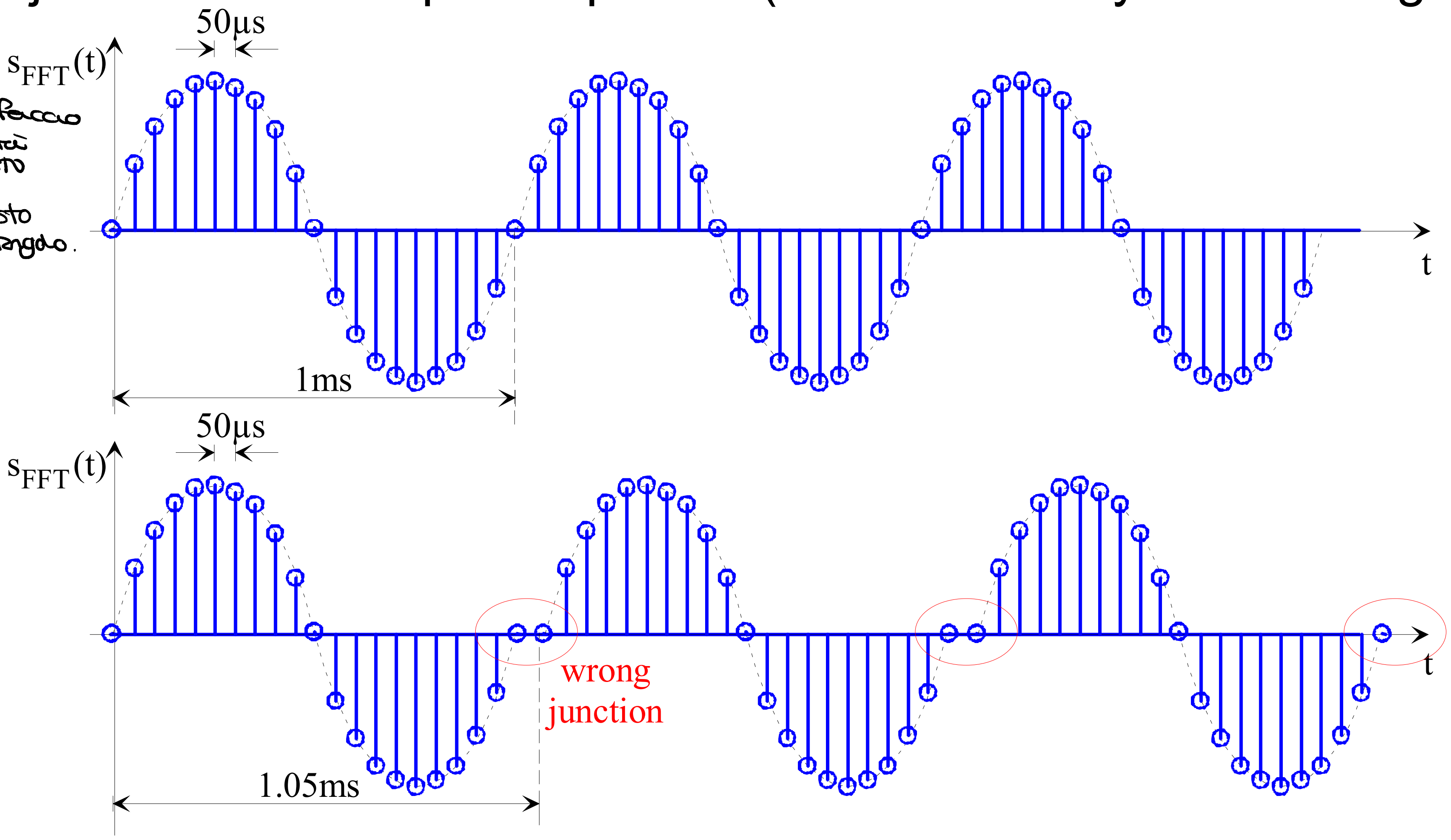


Errors in the FFT

CAZZO !!

Due to uneven junction of the input sequence (as assumed by the FFT algorithm)

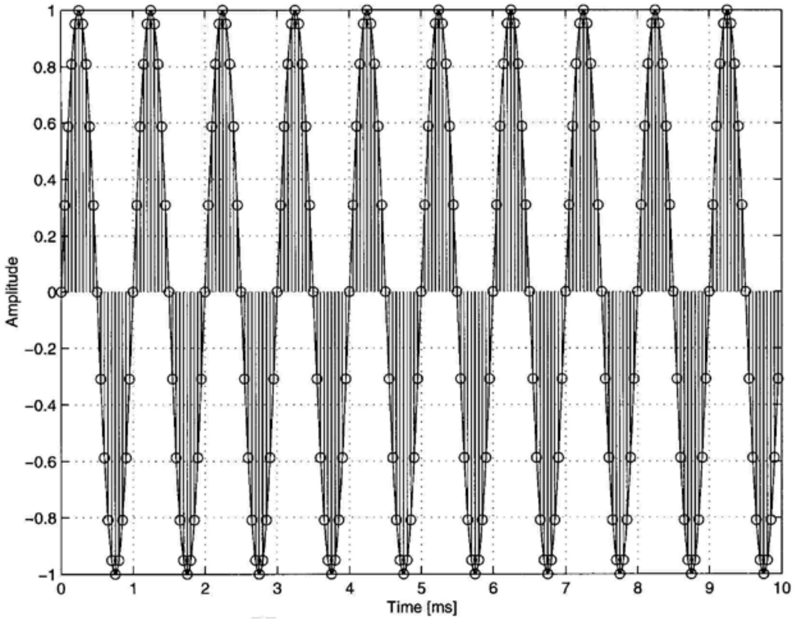
Per poter evitare questo problema
dell'FFT dato a troppi sampling
posso fare un windowing cioè faccio
uno smorzamento delle aperture
in modo da non avere più questo
problema con l'FFT.
Alcuni filtri ideali per fare questo
sono quelli a triangolo o rettangolo.

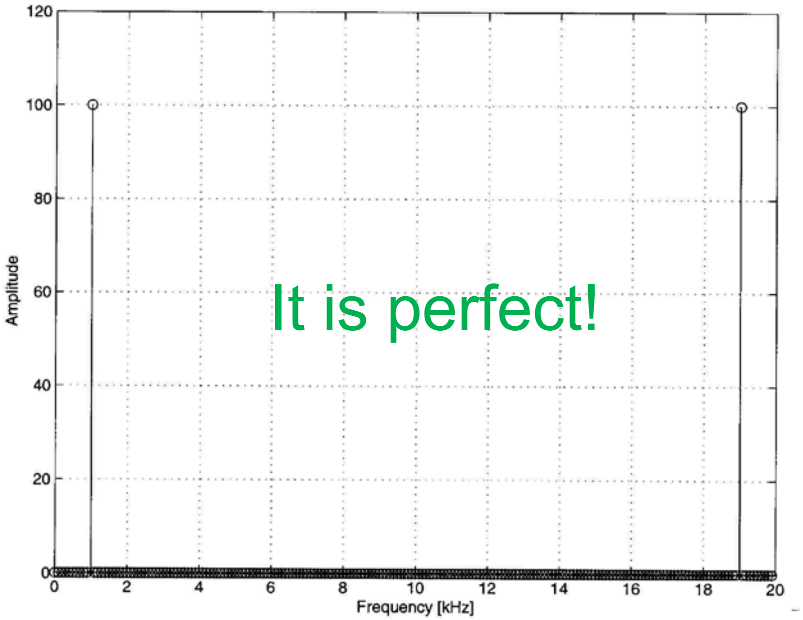


Therefore, let become independent of the junction…

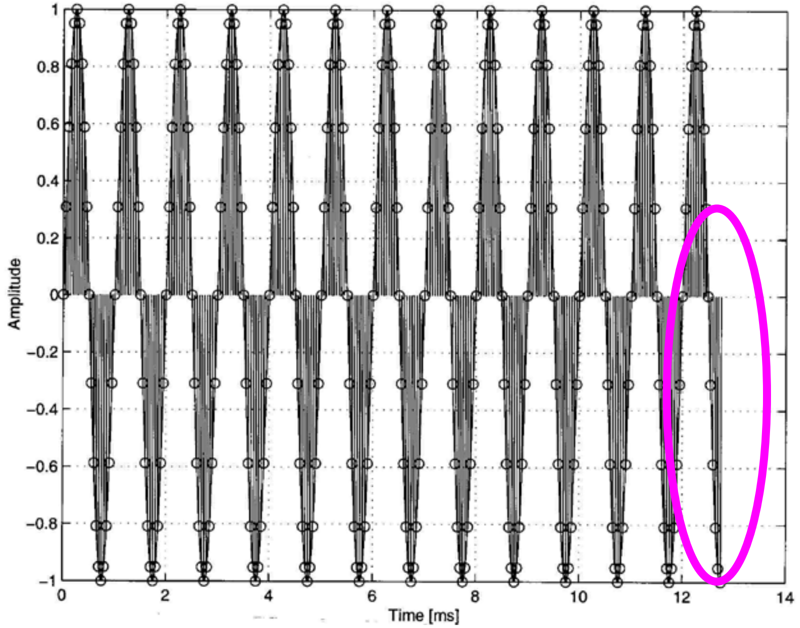
Errors in the FFT

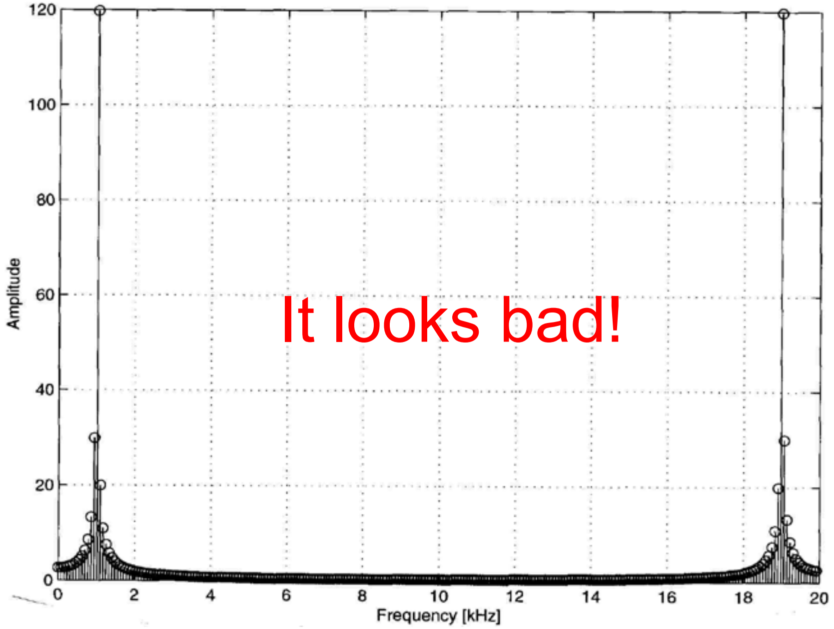
N=200:





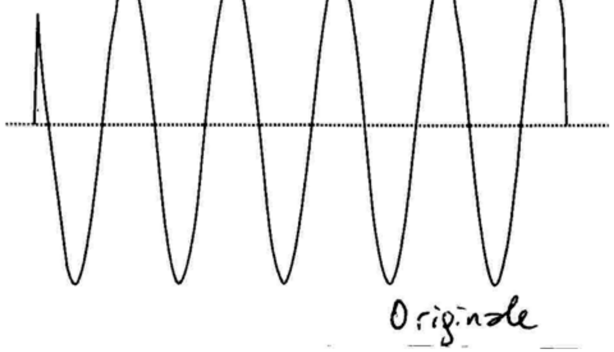
N=256:

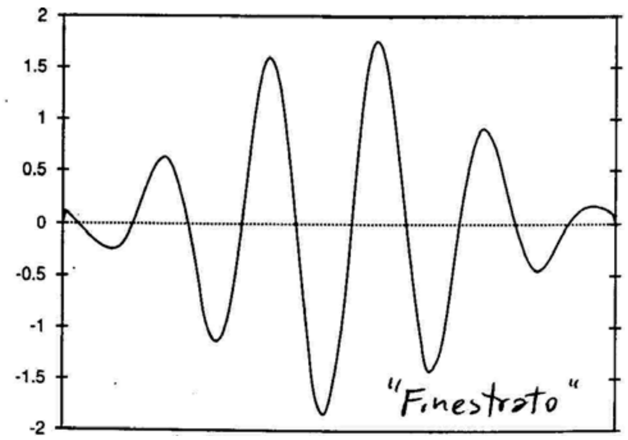


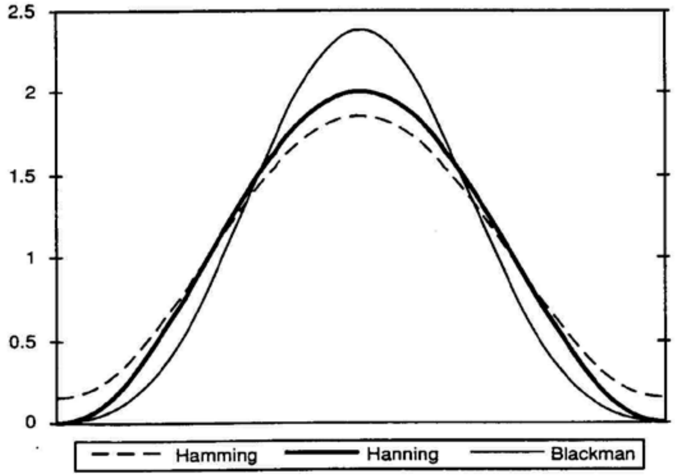


Windowing before FFT

The oddness at the junction can be smoothed out through **windowing**







Rectangular (no windowing): $\text{window}(n) = 1 \quad \text{for } -M \le n \le M \quad \text{and 0 elsewhere}$

Triangular: $$\text{window}(n) = 1 - \frac{|n|}{M} \quad \text{for } -M \le n \le M \quad \text{and 0 elsewhere}$$

Hanning (rised cosin): $$\text{window}(n) = \frac{1}{2} \cdot \left(1 + \cos\frac{2\pi \cdot n}{N}\right) \quad \text{for } -\frac{N-1}{2} \le n \le \frac{N-1}{2} \quad \text{and 0 elsewhere}$$

Hamming: $$\text{window}(n) = 0.54 + 0.46 \cdot \cos\frac{2\pi \cdot n}{N} \quad \text{for } -\frac{N-1}{2} \le n \le \frac{N-1}{2} \quad \text{and 0 elsewhere}$$

Blackman: $$\text{window}(n) = 0.42 + 0.5 \cdot \cos\frac{2\pi \cdot n}{N} + 0.08 \cdot \cos\frac{4\pi \cdot n}{N} \quad \text{for } -\frac{N-1}{2} \le n \le \frac{N-1}{2} \quad \text{and 0 elsewhere}$$

Windowing before FFT

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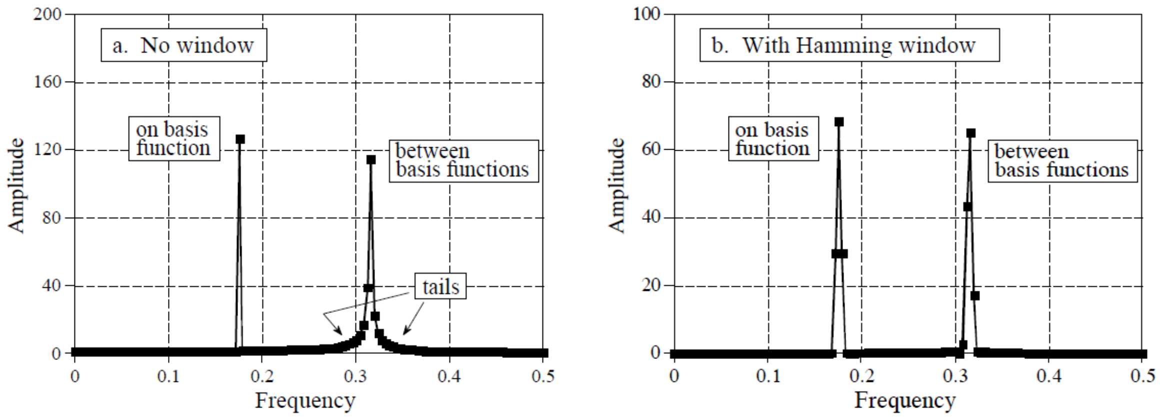
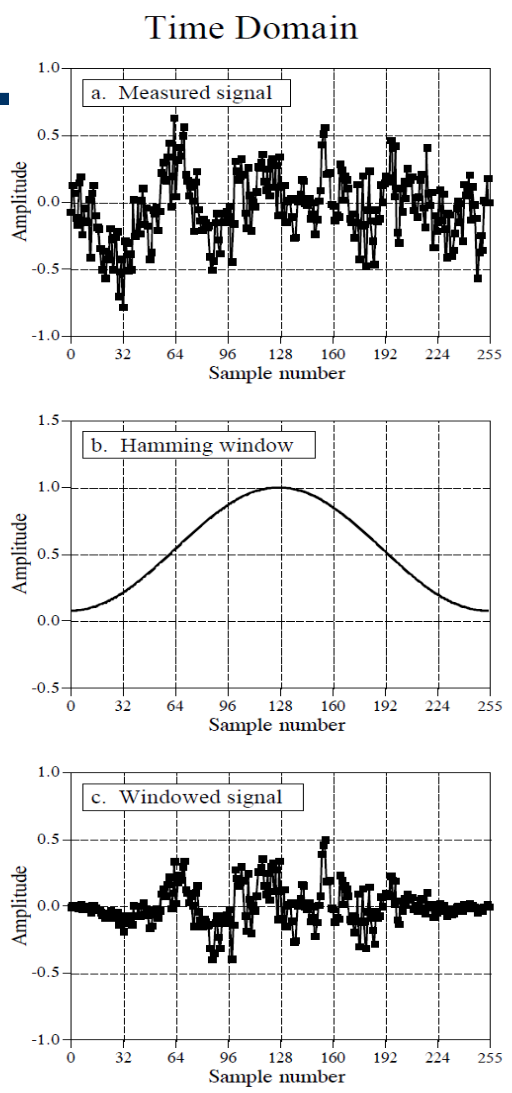


FIGURE 9-4
Example of using a window in spectral analysis. Figure (a) shows the frequency spectrum (magnitude only) of a signal consisting of two sine waves. One sine wave has a frequency exactly equal to a basis function, allowing it to be represented by a single sample. The other sine wave has a frequency *between* two of the basis functions, resulting in *tails* on the peak. Figure (b) shows the frequency spectrum of the same signal, but with a Blackman window applied before taking the DFT. The window makes the peaks look the same and reduces the tails, but broadens the peaks.



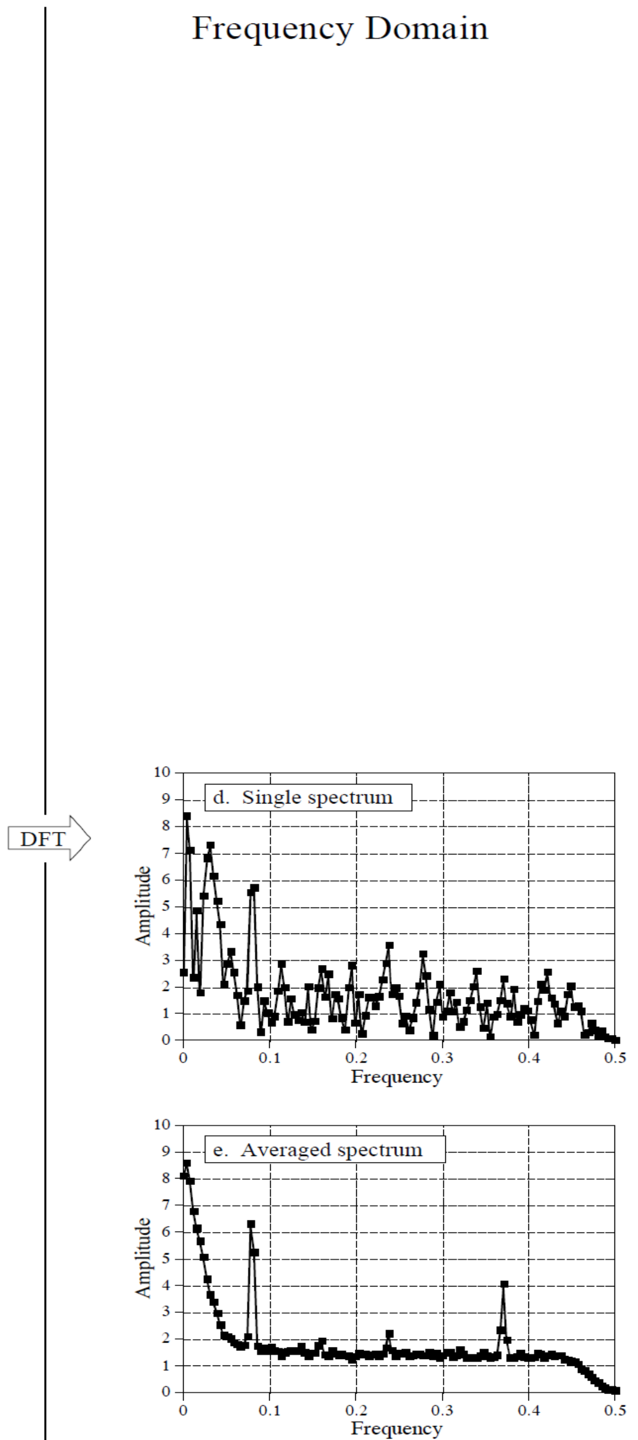


FIGURE 9-1
An example of spectral analysis. Figure (a) shows 256 samples taken from a (simulated) undersea microphone at a rate of 160 samples per second. This signal is multiplied by the Hamming window shown in (b), resulting in the windowed signal in (c). The frequency spectrum of the windowed signal is found using the DFT, and is displayed in (d) (magnitude only). Averaging 100 of these spectra reduces the random noise, resulting in the averaged frequency spectrum shown in (e).

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Conclusions

- “Few” samples instead of whole waveform
- Sampling correctly is fundamental
- Aliasing causes artifacts
- Mind also other issues

Next lesson: **10 – S&H circuits**