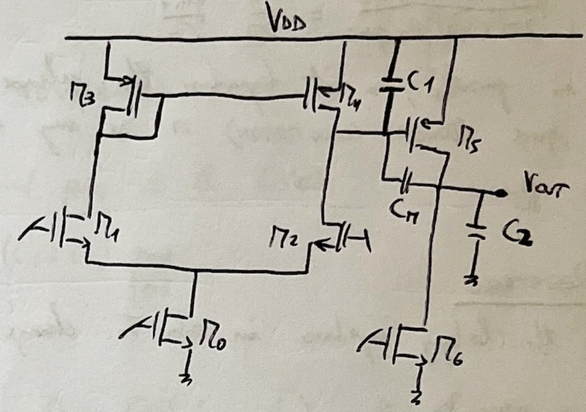
COMPENSATION (TWO-STAGE)

- **MILLER**

Split the two poles due to the two high impedance nodes by placing a C_M bridging the gate-drain nodes of a transistor.

The three capacitors are dependent (they form a loop) so we expect only two poles in the T.F.:



$$b_1 = C_1 \cdot R_1^0 + C_2 R_2^0 + C_M R_M^0$$

$$\begin{cases} R_1^0 = R_{OUT,1} = R_1 \\ R_2^0 = R_{OUT,2} = R_2 \\ R_M^0 = R_1 + g_{m5} R_2 R_1 + R_2 \quad \text{(Miller effect)} \end{cases}$$

$$\Rightarrow \quad b_1 = C_1 R_{OUT,1} + R_2 (C_2 + C_M) + (1 + g_{m5} R_2) C_M R_1 = \tau_1 + \tau_2 + \tau_3$$

$$b_2 = C_1 C_2 R_1^0 R_2^1 + C_1 C_M R_1^0 R_M^1 + C_2 C_M R_2^0 R_M^2$$

$$\begin{cases} R_2^1 = R_2 \\ R_M^1 = R_2 \\ R_M^2 = R_1 \end{cases}$$

$$\Rightarrow b_2 = C_1 R_1 R_2 (C_2 + C_M) + R_1 R_2 C_2 C_M = \tau_1 \tau_2 + \tau_4$$

$$b_3 = C_1 C_2 C_M R_1^0 R_2^1 R_M^{1,2} = \emptyset \quad \text{(as expected)}$$

We expect also a zero when

$$s C_M v_s = g_{m5} v_s$$

$$\downarrow$$

$$s = \frac{g_{m5}}{C_M} \quad \text{(POSITIVE)}$$

If the roots of the denominator are split of more than a decade, we get

$$p_L \simeq -\frac{1}{b_1} \simeq -\frac{1}{R_1 g_{m5} R_2 C_M}$$

$$p_H \simeq -\frac{b_1}{b_2} \simeq -\frac{R_1 g_{m5} R_2 C_M}{R_1 R_2 C_1 (C_2 + C_M) + R_1 R_2 C_2 C_M} \simeq -\frac{g_{m5} C_M}{C_1 C_2 + C_1 C_M + C_2 C_M}$$

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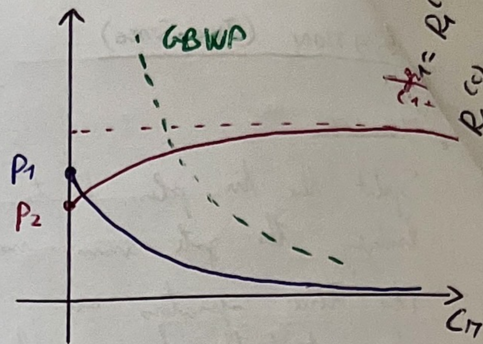
For large C_M values we get

$$p_H \simeq -\frac{g_{m5}}{C_1 + C_2}$$

and p_L moves to lower frequencies.
Finally, we have

$$GBWP = \frac{g_{m1} R_1 g_{m5} R_2}{2\pi C_M R_1 R_2 g_{m5}} = \frac{1}{2\pi} \frac{g_{m1}}{C_M}$$



The zero is positive, so it degrades the phase margin! We must shift it to HF by increasing g_{m5} (POWER CONSUMPTION) or we may increase C_M in order to move p_L away from p_H.

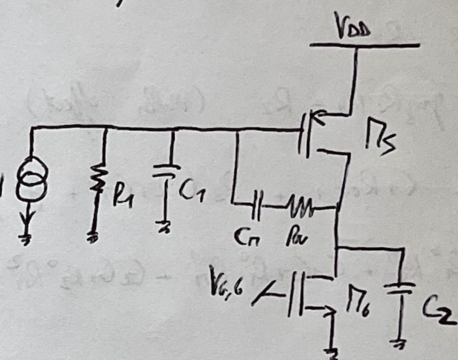
- NULLING RESISTANCE

We modify the bridging impedance in order to change the zero:

$$\frac{v_5}{R_N + \frac{1}{sC_M}} = g_{m5} v_5$$

$$\downarrow$$

$$z = -\frac{1}{C_M\left[R_N - \frac{1}{g_{m5}}\right]}$$



We may size R_N in order to have a negative zero cancelling out with the second pole.
Due to R_N we have three poles

in series to the resistance across gate and drain of M5

$$b_1 = C_1 R_1 + C_2 R_2 + C_C\left(R_1 + R_2 + g_{m5} R_1 R_2 + R_N\right)$$

being $R_N \simeq \frac{1}{g_{m5}}$ we expect

$$b_1 \simeq C_M g_{m5} R_2 R_1 \qquad \text{(UNCHANGED)}$$

$$\Rightarrow \tau_L \simeq b_1 \; ; \; p_L = -\frac{1}{b_1}$$

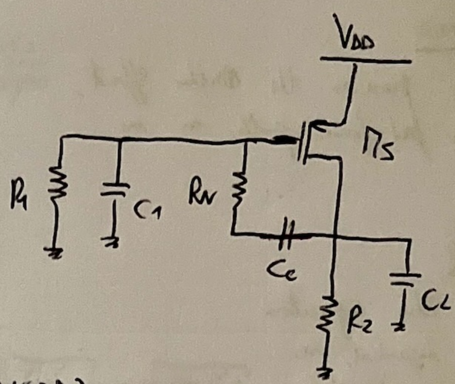
We can compute p_H using the time constant method:

$$\begin{cases} \tau_1^\infty = (R_1 // R_N) C_1 \simeq R_N C_1 \\ \tau_2^\infty = (R_2 // R_N) C_2 \simeq R_N C_2 \\ \tau_C^\infty = R_N \end{cases}$$

$$p_H = -\left(\frac{1}{\tau_1^\infty} + \frac{1}{\tau_2^\infty} + \frac{1}{\tau_3^\infty}\right) \simeq -\frac{1}{R_N (C_1 // C_2 // C_C)}$$

We can compute the second pole assuming that C_C is a short
(this approximation is clearly justified in "APPROXIMATIONS AND CIRCUIT INSIGHTS")

$$b_1 = R_1^{(c)} C_1 + R_2^{(c)} C_2$$



$$R_1^{(c)} = R_1 // \left[\frac{R_V + R_2}{1 + g_{m5} R_2}\right] \simeq \frac{1}{g_{m5}}$$

$$R_2^{(c)} = R_2 // \left[\frac{R_V + R_1}{1 + g_{m5} R_1}\right] \simeq \frac{1}{g_{m5}}$$

so

$$p_2 \simeq - \frac{1}{(C_1 + C_2)\frac{1}{g_{m5}}} = - \frac{g_{m5}}{C_1 + C_2} \quad \text{(UNCHANGED)}$$

Therefore if we place the second pole at the GBWP:

$$\frac{C_T}{g_{m1}} = \frac{C_1 + C_2}{g_{m5}} \implies C_T = (C_1 + C_2)\frac{g_{m1}}{g_{m5}}$$

then

$$\left[R_V - \frac{1}{g_{m5}}\right] \cancel{sC_T} = \frac{\cancel{C_T}}{g_{m1}} \implies R_V = \frac{\cancel{C_T}}{\cancel{C_T}}\left(\frac{1}{g_{m1}} + \frac{1}{g_{m5}}\right) = \frac{2}{g_{m5}} \quad (\text{if } g_{m5} = g_{m1})$$

We may implement the resistor by using a MOSFET in deep triode region

$$I_{DS} = \frac{1}{2}\mu_n C'_{ox}\frac{W}{L}\left[(V_{GS} - V_T)V_{DS} \cdot 2 - V_{DS}^2\right]$$

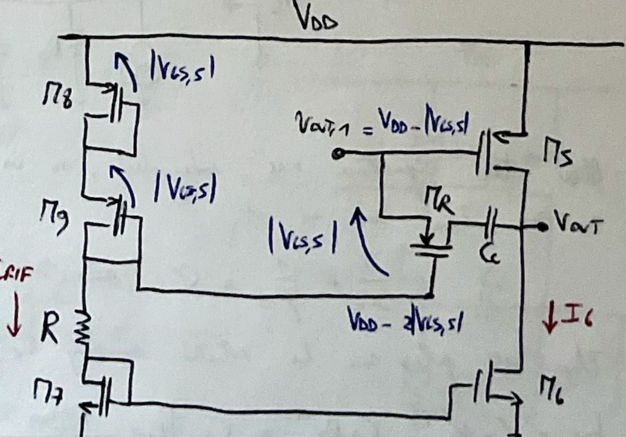
$$g_0 = \frac{\partial I_{DS}}{\partial V_{DS}}\bigg|_{V_{DS}=0} = \frac{1}{2}\mu_n C'_{ox}\frac{W}{L}(2 V_{ov})$$

[Just by chance is equal to the TRANSCONDUCTANCE of a MOSFET in saturation]

therefore we need

$$\frac{g_{m5}}{g_0} = 2 \implies \frac{(W/L)_5}{(W/L)_R}\frac{V_{ov,5}}{V_{ov,R}} = 2$$

If we set carefully $V_{ov,5} = V_{ov,R}$, then the accuracy depends on a ratio, which is optimal in integrated design. In order to achieve this result we need to REPLICATE the bias condition of M_5:



M_5 and M_6 are sized in order to carry the same current with $V_{out} = \frac{V_{DD}}{2}$. This current is set by the reference branch of M_7, which can be set to be $\frac{1}{M}$ of I_6 by sizing the $\left(\frac{W}{L}\right)_7 = \frac{1}{M}\left(\frac{W}{L}\right)_6$. If now we choose $\left(\frac{W}{L}\right)_8 = \left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_7$, then $|V_{GS,8}| = |V_{GS,9}| = |V_{GS,5}|$

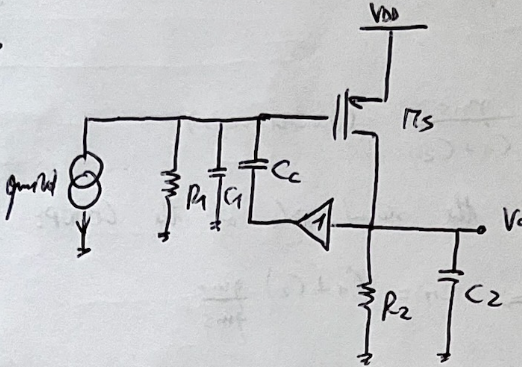
Therefore the position of the zero depends only on the ratio of $\frac{(W/L)_5}{(W/L)_R}$.

- **VOLTAGE BUFFER**

This solution preserves the Miller effect, but nulls the feed-forward path, so no zero!

→ Ideal

The three capacitors are dependent, so we expect just two poles. The dominant pole is still due to the Miller effect

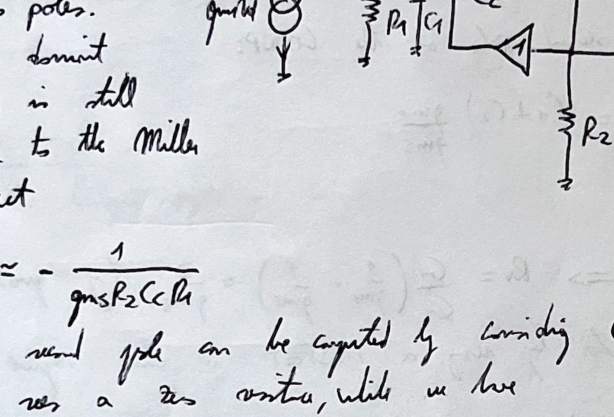


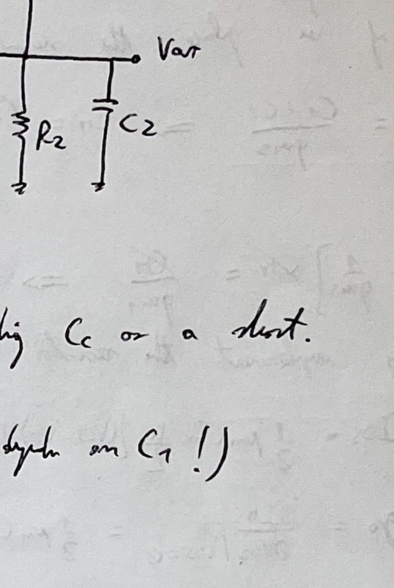
$$P_L \simeq -\frac{1}{g_{mS}R_2C_cR_1}$$

The second pole can be computed by considering C_c as a short. C_1 sees a zero resistance, while we have

$$P_H \simeq -\frac{1}{C_2 \cdot \frac{1}{g_{mS}}}$$ (higher, it doesn't depend on C_1 !)

→ Real





Now the capacitors are independent, so we have an additional HF pole and a zero. The zero is at

$$\frac{1}{sC_c} + \frac{1}{g_{mB}} = 0 \implies z = -\frac{1}{C_c \cdot \frac{1}{g_{mB}}}$$ (output resist. of 1st stage toward ground)

The HF poles can be estimated considering C_c as a short

$$b_1 = C_1R_1^{(c)} + C_2R_2^{(c)} = C_1 \cdot \left[R_1 // \frac{1/g_{mB}}{1+g_{mS}R_2}\right] + C_2 \cdot \left[\frac{R_2}{1+g_{mS}R_2}\right] \simeq \frac{C_2}{g_{mS}}$$

$b_2 = C_1 C$

$$b_2 = C_1 C_2 R_1^{(0)} R_2^{(1)(C)} = C_1 C_2 \frac{R_2}{g_{mB}\, g_{mS} R_2} = \frac{C_1 C_2}{g_{mB}\, g_{mS}}$$

So we get

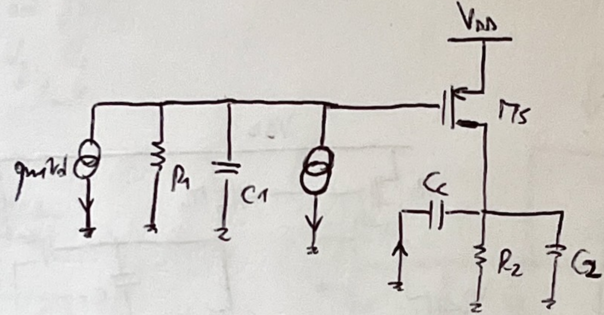
$$\left[s^2 \frac{C_1 C_2}{g_{mB}\, g_{mS}} + s \frac{C_2}{g_{mS}} + 1\right] = 0$$

$\downarrow$

$$\omega_0 = \sqrt{\frac{g_{mB}\, g_{mS}}{C_1 C_2}} \qquad Q = \frac{g_{mS}}{C_2}\sqrt{\frac{C_1 C_2}{g_{mB}\, g_{mS}}} = \sqrt{\frac{g_{mS} C_1}{g_{mB} C_2}} \qquad \text{(POLE PAIR)}$$

- AMUSA

→ Ideal



The Miller effect is still present ($v_C = g_{mS} v_S R_2$, $i_C = s C_c g_{mS} R_2 v_S$)
Same LF pole.
C_c and C_L are in $\parallel$, so only two poles are expected. We can consider C_c as a short and compute the MF pole:

$$b_1 = C_2 \cdot 0 + C_1 \cdot \left(R_1 \parallel \frac{1}{g_{mS}}\right) \simeq \frac{C_1}{g_{mS}}$$

$$P_H \simeq -\frac{1}{C_1 \cdot \frac{1}{g_{mS}}} \quad \text{(Load capacitance plays no role!)}$$

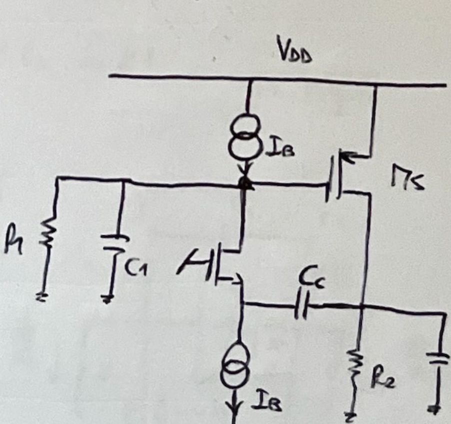
→ Real

We have a LHP zero at

$$\frac{1}{sC_c} + \frac{1}{g_{mB}} = 0$$

$\downarrow$

$$s = -\frac{1}{C_c \cdot \frac{1}{g_{mB}}} \quad \text{(output shorted to ground)}$$



and three poles. The LF pole is still due to the Miller effect, while we can estimate the MF ones by shorting C_c:

$$b_1 = C_1 \cdot \left[R_1 \,//\, \frac{1}{g_{m5}}\right] + C_2 \cdot R_2 \,//\, \left(\frac{1/g_{mB}}{1+g_{m5}R_1}\right) \simeq C_1 \frac{1}{g_{m5}}$$

$$b_2 = C_1 C_2 \cdot \frac{1}{g_{m5}} \cdot \frac{1}{g_{mB}}$$

So we get

$$\frac{C_1 C_2}{g_{m5} g_{mB}} s^2 + s \cdot \frac{C_1}{g_{m5}} + 1 = 0$$

$$w_0 = \sqrt{\frac{g_{m5}\, g_{mB}}{C_1 C_2}} \qquad Q = \frac{g_{m5}}{C_1} \cdot \sqrt{\frac{C_1 C_2}{g_{m5}\, g_{mB}}} = \sqrt{\frac{C_2\, g_{m5}}{C_1\, g_{mB}}}$$

For the same C_2, the Ahuja compensation has a higher Q factor than the voltage buffer compensation, so poles remain close and don't tend to split.

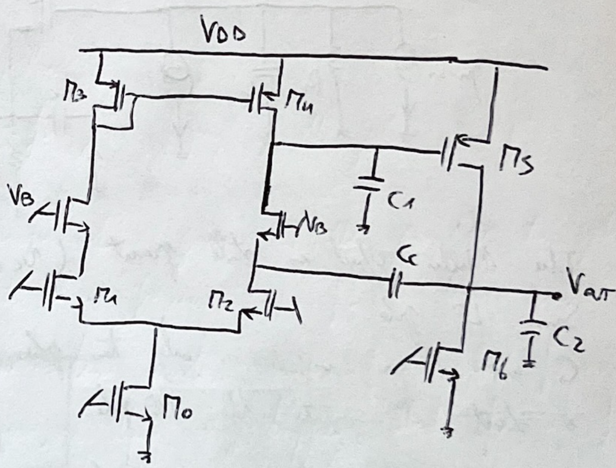
- ALTERNATIVE AHUJA (CASCODE)

Compute b_3, b_2, b_1 and a_2, a_1 and write

$$b_3 s^3 + b_2 s^2 + b_1 s + 1 = 0$$

$$\Downarrow \simeq$$

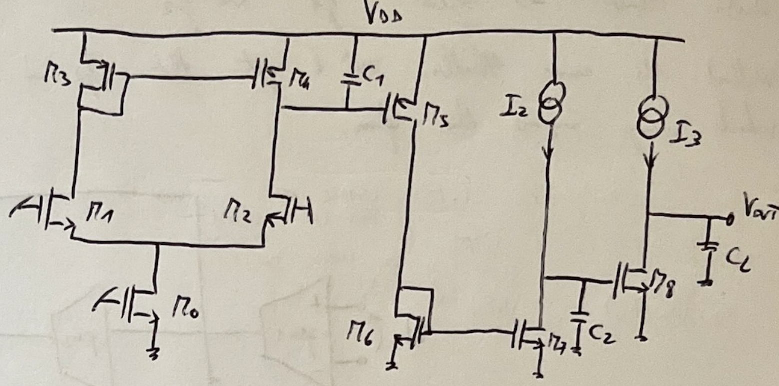
$$(1+b_1 s)\left(1 + \frac{b_2}{b_1} s + \frac{b_3}{b_1} s^2\right) = 0$$



COMPENSATION (THREE-STAGE)

We use three-stage OTAs any time we need a large gain and do not have enough voltage headroom to accomodate cascodes:

Now we have three high impedance nodes, so we need two sequential Miller compensations.



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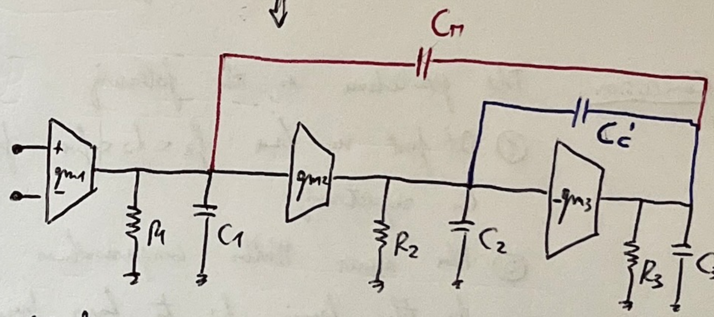


① Place a Miller capacitor across the third stage, in order to split f_2 and f_3. Let's consider only the cascode of the last two stages:

$$GBWP_{23} = \frac{g_{m2}}{2\pi C_c'}$$

$$f_3' = \frac{g_{m3}}{2\pi (C_3 + C_2)}$$

$$f_z = \frac{g_{m3}}{2\pi C_c'} \quad \text{(POSITIVE)}$$

Usually $g_{m3} >> g_{m2}$, so we can neglect f_z.
Set φ_m of the 2-3 stage by placing $f_3' = 2\, GBWP_{23}$

② Place an outer Miller capacitor in order to further lower the frequency of the first pole

$$GBWP = \frac{g_{m1}}{2\pi C_M}$$

The second pole can be estimated by shorting C_M:

$$R_3^{(0)} = \frac{R_1 // R_3}{1 + g_{m2} R_2 g_{m3} (R_1 // R_3)} \quad \text{(seen by } C_1 \text{ and } C_3 \text{ in //)} \simeq \frac{1}{g_{m2} R_2 g_{m3}}$$

$$R_2^{(0)} = \frac{R_2}{1 + g_{m2} R_2 g_{m3} (R_1 // R_3)} \simeq \frac{1}{g_{m2} g_{m3} (R_1 // R_3)}$$

$$R_{C_c}^{(0)} = \frac{1}{g_{m2}} - \frac{1}{g_{m3}}$$

$$\downarrow$$

$$= \frac{g_{m3} - g_{m2}}{g_{m3}\, g_{m2}}$$

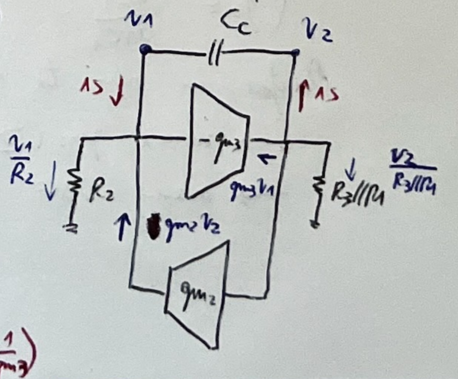
$$\begin{cases} i_s = -g_{m2} v_2 \\ i_s = -g_{m3} v_1 \end{cases}$$

$$\Downarrow$$

$$v_1 = -\frac{i_s}{g_{m3}}$$

$$v_2 = -\frac{i_s}{g_{m2}}$$

$$v_1 - v_2 = i_s \left(\frac{1}{g_{m2}} - \frac{1}{g_{m3}} \right)$$

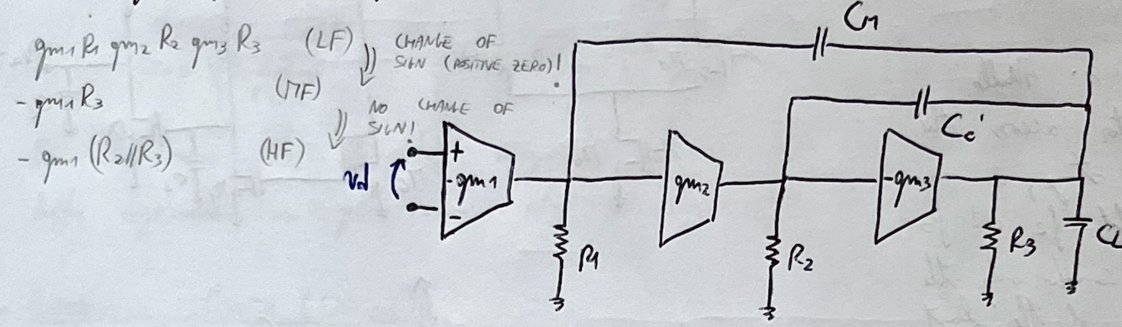


(29)

a better estimate of the singularities can be evaluated by neglecting C_1 and C_2, since the capacitive loads at these nodes are dominated by the Miller capacitors.

$$\begin{cases} \text{Inner Miller} \Rightarrow \text{pre-split } f_2 \text{ and } f_3 \\ \text{Outer Miller} \Rightarrow \text{split } f_1 \text{ and } f_2 \end{cases}$$ (ROOT LOCUS)

Without the inner Miller we'd get that f_2 and f_3 may become a pole pair, which may degrade the φ_m.



CONCLUSION: The procedure is the following:

1. At first we have $f_3 < f_2 < f_1$ frequency of the poles due to C_3, C_2 and C_1 respectively
2. The inner Miller compensation gives us the possibility to split f_3 and f_2, thus bringing f_2 to lower frequencies and f_3 to higher ones
3. The outer Miller compensation instead moves f_1 to lower frequencies and f_2 to higher frequencies, so there may happen that f_2 and f_3 move closer to each other and become a pole pair.

Neglecting the fact that the outer Miller compensation brings f_2 and f_3 closer to each other, we have that

$$\begin{cases} GBWP \simeq \dfrac{g_{m1}}{2\pi C_M} \\ f_2 \simeq \dfrac{g_{m2}}{2\pi C_c'} \quad \left(\text{or } \dfrac{1}{2\pi} \dfrac{1}{C_c\left[\frac{g_{m3}-g_{m2}}{g_{m3}g_{m2}}\right]} \right) \\ f_3 \simeq \dfrac{g_{m3}}{2\pi C_3} \end{cases}$$

Therefore, in order to improve φ_m, we could either

→ increase g_{m2} (MORE POWER DISSIPATION) and g_{m3}

→ decreasing GBWP by increasing C_M (MORE SILICON AREA)