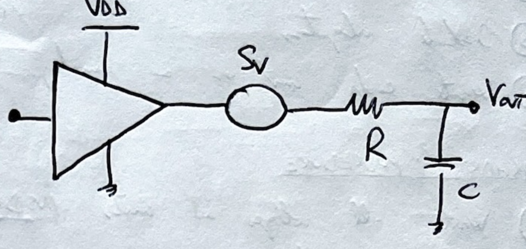
Noise and Op-Amp Non-Idealities

Dynamic Range $\triangleq$ ratio between maximum and minimum signal levels handled by the circuit.

DEM: Let's consider this circuit



The maximum output signal is αV_{DD}, so a sinusoid with $V_{PEAK} = \frac{\alpha V_{DD}}{2}$, so

$$\langle V_{out}^2 \rangle = \frac{\alpha^2 V_{DD}^2}{8}$$

While the RMS value of the noise due to just the resistor R is

$$4kTR \cdot \frac{1}{4RC} = \frac{kT}{C}$$

and the additional noise due to the amplifier is $F \cdot \frac{kT}{C}$, so we get

$$DR = \sqrt{\frac{\alpha^2 V_{DD}^2/8}{(1+F)\,kT/C}} = \alpha V_{DD} \sqrt{\frac{C}{8kT(1+F)}}$$

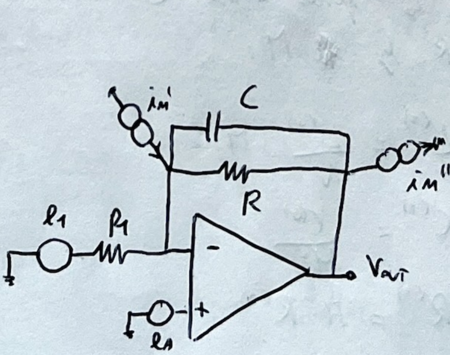
So, to increase the DR (which means to increase the number of bits of an A/D converter, being $DR = \frac{FSR}{LSB}$) we need to increase both C (more area) and VDD (more power dissipation).

On each half-period the capacitance is charged up to αV_{DD} and then discharged, so it dissipates

$$\underbrace{\frac{\alpha V_{DD} \cdot C}{T}}_{\frac{\Delta Q}{\Delta t}} \cdot V_{DD}$$

→ **Noise**

Let's consider this amplifier



① $e_{out,1} = e_1 \cdot \frac{R}{R_1} \frac{1}{1+sRC}$

$\Downarrow$

$$S_{out,R_1} = S_{v,R_1} \left(\frac{R}{R_1}\right)^2 \frac{1}{4RC} = \frac{kT}{C} \frac{R}{R_1}$$

2 $v_{out,2} \Rightarrow S_{I,R2} \cdot R^2 \cdot \frac{1}{4RC} = \frac{kT}{C}$

3 $v_{out,A} \Rightarrow S_A \cdot \frac{\omega_M}{4} + S_A \left(1+\frac{R}{R_1}\right)^2 \frac{\omega_0}{4}$

we take into account also the finite GBWP of the amplifier, so that we consider the HF transfer to be limited by the ENBW $= \frac{\omega_M}{4}$

To be more accurate we should consider that the finite GBWP of the amplifier affects the overall T.F., so

$$e_{out,A} = e_A \left[1 + \frac{R}{R_1}\,\frac{1}{1+sRC}\right] \cdot \frac{1}{\left(1+\frac{s}{\omega_M}\right)} = e_A\left(1+\frac{R}{R_1}\right)\frac{\left(1+\frac{s}{\omega_Z}\right)}{\left(1+\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_M}\right)}$$

ω_Z being $= \frac{1}{C(R \| R_1)}$

If we write it in this way, we can use the tabulated integral

$$\int_0^{+\infty} \left| \frac{1+\frac{s}{\omega_Z}}{\frac{s^2}{\omega_0\omega_M} + s\,\frac{(\omega_0+\omega_M)}{\omega_0\omega_M} + 1} \right|^2 df = \frac{\omega_0\omega_M}{(\omega_0+\omega_M)}\,\frac{1}{4}\left[1+\frac{\omega_0\omega_M}{\omega_Z^2}\right] = \text{ENBW}$$

so

$$e_{out,A} = S_A \cdot \left(1+\frac{R}{R_1}\right)^2 \cdot \text{ENBW} \xrightarrow{\omega_M \gg \omega_0} S_A \cdot \left(1+\frac{R}{R_1}\right)^2 \cdot \frac{\omega_0}{4}\left[1+\frac{\omega_0\omega_M}{\omega_Z^2}\right]$$

$\downarrow$

which corresponds to the approximated result. $= S_A\left(1+\frac{R}{R_1}\right)^2 \cdot \left[\frac{\omega_0}{4} + \frac{\omega_M}{4}\left(\frac{R_1}{R+R_1}\right)^2\right]$

4 $$S_{out} = \overbrace{S_{V\_IN} G^2 \cdot \frac{\omega_0}{4}}^{\text{INPUT}} + \overbrace{4kTR\,\frac{\omega_0}{4}}^{\text{R}} + \overbrace{S_{V\_A} \cdot \left[\eta + (1+G)^2\right]\frac{\omega_0}{4}}^{\text{AMPLIFIER}} + \overbrace{4kTR\,G\,\frac{\omega_0}{4}}^{\text{R1}}$$

$\downarrow$

$$= S_{V\_IN} G^2 \frac{\omega_0}{4}\left[1 + \frac{4kTR}{S_{V,IN}}\left(\frac{1+G}{G^2}\right) + \frac{S_{V\_A}}{S_{V\_IN}}\,\frac{\eta+(1+G)^2}{G^2}\right] = S_{V\_IN} \cdot G^2 \cdot \frac{\omega_0}{4}\left[1+F\right]$$

So, in order to minimize F, we need to

1) have $G > 1$
2) resistors whose noise is $< S_{V\_IN}$
3) amplifier whose noise is $< S_{V\_IN}$
4) limit the broadband noise by a suitable GBWP

$$\begin{bmatrix} G = \frac{R}{R_1} \\ \eta = \frac{\omega_M}{\omega_0} \end{bmatrix}$$

LADDER NETWORKS SUMMARY

→ KL PROCEDURE

1. Use KL in order to write relations among STATE VARIABLES (i_L, v_C)
2. Divide each member by the highest power of s
3. Isolate integrators
4. Write each current as an auxiliary voltage divided by R^*

→ FLOWCHART PROCEDURE

1. Write all voltages and currents INSIDE COMPONENTS
2. Draw integrators
3. Use KL to derive the inputs of integrators (i_C, v_L)
4. Divide the branches going into V.C. by R^* and multiply the integrators T.F. for R^*
5. Multiply currents on the down side for R^*, divide branches going up for R^* and multiply branches going down for R^*
6. Multiply for (-1) inputs of integrators and integrators T.F.

→ DENORMALIZATION

Remember $w_0 \propto \frac{1}{\sqrt{LC}}$ and $Q \propto w_0 RC$, so to shift w_0 to a value N times higher we need to set

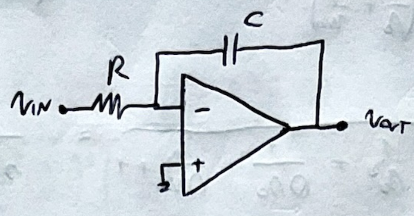
$$\begin{cases} L^{(1)} = \dfrac{L^{(0)}}{N} \\ C^{(1)} = \dfrac{C^{(0)}}{N} \end{cases}$$

This doesn't change Q.
Moreover, to further decrease C without changing w_0 and Q we need to multiply L and R by M:

$$\begin{cases} L^{(1)} = \dfrac{L^{(0)}}{N} \cdot M \\ C^{(1)} = \dfrac{C^{(0)}}{MN} \\ R^{(1)} = M \cdot R^{(0)} \end{cases}$$

OP-AMP NON IDEALITIES

Let's consider an integrator: we have that the forward gain is the opAMP, while the $\beta(s)$ is



$$\beta(s) = \frac{R}{R + \frac{1}{sC}} = \frac{sCR}{1 + sCR} \qquad \begin{cases} CR = \tau_0 \\ \frac{1}{CR} = \omega_0 \end{cases}$$

$$\Downarrow$$

$$\frac{1}{\beta}(s) = \frac{1 + \frac{s}{\omega_0}}{\frac{s}{\omega_0}} = 1 + \frac{\omega_0}{s}$$

The $G_{loop}(s)$ is equal to $G_{loop}(s) = -\frac{A_0}{(1+s\tau)} \frac{s\tau_0}{(1+s\tau_0)}$, it has a low frequency pole due to the opAMP, a zero in DC and a pole at ω_0, so the loop gain @ middle frequency is

$$|G_{loop}(MF)| = A_0 \frac{\tau_0}{\tau} = \frac{GBWP}{f_0}$$ (we need $GBWP \geq 100 f_0$ to have a suitable loop gain)

and the closed loop transfer function has a pole at

$$\omega_L = \frac{\omega_0}{A_0} = \frac{1}{A_0 RC}$$ (MILLER POLE)

and another pole at

$$\omega_H = GBWP$$

So, in the end,

$$H_{int}(s) = \frac{-A_0}{\left(1 + \frac{s}{\omega_L}\right)\left(1 + \frac{s}{\omega_H}\right)} \qquad \begin{cases} \text{DC gain} \\ \text{LOW FREQUENCY POLE} \\ \text{HIGH FREQUENCY POLE} \end{cases}$$

→ DC GAIN AND LF POLE EFFECT

$$T(s) = \frac{\omega_0^2 \cdot \frac{1}{s^2}}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right) \cdot \frac{1}{s^2}} = \frac{\left(-\frac{\omega_0}{s}\right)^2}{\left(-\frac{\omega_0}{s}\right)^2 - \frac{1}{Q}\left(-\frac{\omega_0}{s}\right) + 1} = \frac{H_{int}^2(s)}{H_{int}^2(s) - \frac{1}{Q} H_{int}(s) + 1}$$

Let's consider now $H_{int}(s) = \frac{-A_0}{1 + \frac{s}{\omega_L}}$ at LOW FREQUENCY, so we can write it as

$$H_{int}(s) = -\frac{1}{\left(\frac{1}{A_0} + \frac{s}{\omega_0}\right)}$$

and replace it into $T(s)$

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$$T(s) = \frac{\dfrac{1}{\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}}{\dfrac{1}{\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}+\dfrac{1}{Q\left(\frac{1}{A_0}+\frac{s}{w_0}\right)}+1} = \frac{1}{1+\frac{1}{Q}\left(\frac{1}{A_0}+\frac{s}{w_0}\right)+\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}$$

$$= \frac{1}{\frac{s^2}{w_0^2}+\frac{2s}{w_0A_0}+\frac{1}{A_0^2}+\frac{1}{QA_0}+\frac{s}{Qw_0}+1} = \frac{w_0^2}{s^2+\frac{2sw_0}{A_0}+\frac{w_0^2}{A_0^2}+\frac{w_0^2}{QA_0}+w_0^2+\frac{sw_0}{Q}}$$

$$= \frac{w_0^2}{s^2+sw_0\left(\frac{1}{Q}+\frac{2}{A_0}\right)+w_0^2\left(1+\frac{1}{QA_0}+\frac{1}{A_0^2}\right)}$$

$$\Rightarrow w_0' = w_0\sqrt{1+\frac{1}{A_0Q}+\frac{1}{A_0^2}}$$ RADIAL FREQUENCY SHIFT

$$\Rightarrow \frac{1}{Q'} \simeq \left(\frac{1}{Q}+\frac{2}{A_0}\right) \longrightarrow Q\cdot\left(\frac{1}{Q}-\frac{1}{Q'}\right) = \frac{Q'-Q}{Q'Q}\cdot Q \simeq \frac{\Delta Q}{Q} = -\frac{2Q}{A_0}$$ IN-BAND DROOP

$\longrightarrow$ FINITE GBWP EFFECT

Let's write now the real integrator T.F. at high frequency:

$$H_{int}''(s) = -\frac{w_0}{s}\frac{1}{\left(1+\frac{s}{w_M}\right)}$$

so we get

$$T(s) = \frac{\left(\frac{w_0}{s}\right)^2\cdot\dfrac{1}{\left(1+\frac{s}{w_M}\right)^2}}{\left(\frac{w_0}{s}\right)^2\dfrac{1}{\left(1+\frac{s}{w_M}\right)^2}+\dfrac{w_0}{Qs}\dfrac{1}{\left(1+\frac{s}{w_M}\right)}+1}$$

$$= \frac{1}{1+\frac{s}{Qw_0}\left(1+\frac{s}{w_M}\right)+\frac{s^2}{w_0^2}\left(1+\frac{s}{w_M}\right)^2}.$$

This T.F. has two complex conjugate poles + two additional poles due to the integrator. Assuming to work at $|s| << w_M$ we may write

$$\left(1+\frac{s}{w_M}\right)^2\left[\frac{s^2}{w_0^2}+\frac{s}{Qw_0\left(1+\frac{s}{w_M}\right)}+\frac{1}{(1+s/w_M)^2}\right] = 0$$

$$\Downarrow \simeq$$

$$\left(1+\frac{s}{w_M}\right)^2\left[\frac{s^2}{w_0^2}+\frac{s}{Qw_0}\left(1-\frac{s}{w_M}\right)+\left(1-\frac{s}{w_M}\right)^2\right] = 0$$

so we get

$$\frac{s^2}{\omega_0^2}\left[1-\frac{\omega_0}{Q\omega_M}+\left(\frac{\omega_0}{\omega_M}\right)^2\right]+\frac{s}{\omega_0}\left(\frac{1}{Q}-2\frac{\omega_0}{\omega_M}\right)+1=0$$

$$\downarrow$$

$$s^2+s\,\frac{\omega_0}{\left[1-\frac{\omega_0}{Q\omega_M}+\left(\frac{\omega_0}{\omega_M}\right)^2\right]}\,\frac{\left(1-2\frac{\omega_0}{\omega_M}Q\right)}{Q}+\frac{\omega_0^{\,2}}{\left[1-\frac{\omega_0}{Q\omega_M}+\left(\frac{\omega_0}{\omega_M}\right)^2\right]}=0$$

$$\Longrightarrow\quad \omega_0'=\frac{\omega_0}{\sqrt{1-\frac{\omega_0}{Q\omega_M}+\left(\frac{\omega_0}{\omega_M}\right)^2}}\simeq\omega_0\left(1+\frac{\omega_0}{2Q\omega_M}\right)\qquad(\omega_M\gg\omega_0)$$

$$\Longrightarrow\quad \frac{1}{Q'}\simeq\frac{1}{Q}\,\frac{\left(1-\frac{2\omega_0}{\omega_M}Q\right)}{\left[1-\frac{\omega_0}{Q\omega_M}+\left(\frac{\omega_0}{\omega_M}\right)^2\right]}\simeq\left(\frac{1}{Q}-\frac{2\omega_0}{\omega_M}\right)\qquad(\omega_M\gg\omega_0)$$

So
$$\frac{Q'-Q}{Q'}\simeq\frac{\Delta Q}{Q}=\frac{2\omega_0}{\omega_M}\cdot Q$$

<u>CONCLUSION</u>: the real response of the opAMP affects the ω_0 and the Q of the real integrator, so eventually it affects the gain of the filter, which becomes more sensitive if the Q factor is higher! This happens because the $\frac{\Delta Q}{Q}$ terms are proportional to Q.
Although the positive Q shifts due to HF singularities may compensate the negative shift due to the DC gain and the LF pole, we consider the worst case, setting a constraint on the maximum absolute value for both the positive and negative swing.

$$\left|\frac{\Delta Q}{Q}\right|\leq\varepsilon_{ERR}\%$$

As a final result, we may consider the contribution arising from the positive zero in the open loop Op-AMP T.F., so

$$H_{int}(s)=-\frac{\omega_0}{s}\left(1-\frac{s}{\omega_z}\right)$$

$$T(s)=\frac{H_{int}^2(s)}{H_{int}^2(s)-\frac{1}{Q}H_{int}(s)+1}=\frac{\left(\frac{\omega_0}{s}\right)^2\left(1-\frac{s}{\omega_z}\right)^2}{\left(\frac{\omega_0}{s}\right)^2\left(1-\frac{s}{\omega_z}\right)^2+\frac{\omega_0}{Qs}\left(1-\frac{s}{\omega_z}\right)+1}$$

The poles will be given by the roots of

$$\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}\left(1-\frac{s}{\omega_z}\right)+\left(1-\frac{s}{\omega_z}\right)^2=0$$

So we get

$$\frac{s^2}{\omega_0^2}\left[1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2\right]+\frac{s}{Q\omega_0}\left(1-2\frac{\omega_0}{\omega_z}Q\right)+1=0$$

$$\downarrow$$

$$s^2+\frac{s\,\omega_0}{\left[1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2\right]}\,\frac{\left(1-2\frac{\omega_0}{\omega_z}Q\right)}{Q}+\frac{\omega_0^2}{\left[1-\frac{\omega_0}{Q\omega_z}+\frac{\omega_0}{\omega_z}\right]^2}=0$$

So

$$\omega_0'=\frac{\omega_0}{\sqrt{1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2}}\simeq\omega_0\left(1+\frac{2\omega_0}{Q\omega_z}\right)\qquad(\omega_z>>\omega_0)$$

$$\frac{1}{Q'}=\frac{1}{Q}\,\frac{\left(1-2\frac{\omega_0}{\omega_z}Q\right)}{\sqrt{1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2}}\simeq\frac{1}{Q}\left(1-\frac{2\omega_0}{\omega_z}\right)\qquad(\omega_z>>\omega_0)$$

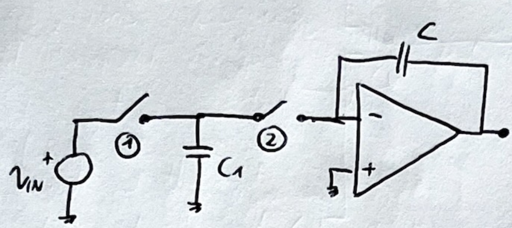
$$\Downarrow$$

$$\left(\frac{1}{Q}-\frac{1}{Q'}\right)Q\simeq\frac{\Delta Q}{Q}=\frac{2\omega_0}{\omega_z}Q$$

POSITIVE SHIFT, like for the GBWP

SWITCHED CAPACITORS

In the low frequency range we'd require very large resistors in order to have suitable values of capacitors, but they are hard to be made in integrated technology, so we use switched capacitors



Phase ① $\Rightarrow$ ① is closed and C_1 stores $Q = C_1 \cdot v_{IN}$

Phase ② $\Rightarrow$ ② is closed and C_1 charges C, so $v_C = \frac{C_1 V_{IN}}{C}$

At the output we get a staircase with slope $\frac{C_1 V_{IN}}{C \cdot T}$

The output of an integrator has a slope of

$$\frac{V_{IN}}{RC}$$

so we can say that the switched capacitor provides $R_{eq} = \frac{T}{C_1}$

ADVANTAGES:
- makes large resistors by very small capacitors
- no need of buffer output stages in ladder networks made of integrators
- A_{v0} depends on the ratio between two capacitors, which can be controlled very well

DISADVANTAGES: we have to deal with a discrete signal, so Shannon must be respected