Variability and Matching

CMRR and offset performances of a differential amplifier are heavily affected by components' tollerances. They depend on material properties variability and on dimensional parameters variability.

⟶ **Resistors**

$$R = \rho \frac{L}{W\Delta} = R_\square \cdot \frac{L}{W} \qquad R_\square \in [10\ \Omega/\square\ ;\ 1\ k\Omega/\square]$$

$$\Delta R = \frac{L}{W}\Delta R_\square + \frac{R_\square}{W}\Delta L + R_\square L \frac{\Delta W}{W^2}$$

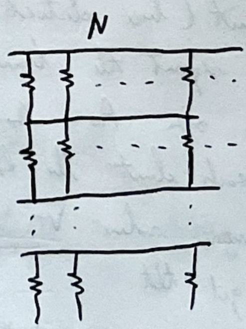
(We sum them up in order to account for the worst case)

$$\downarrow$$

$$\frac{\Delta R}{R} = \frac{L}{W}\frac{\Delta R_\square}{R_\square} + \underbrace{\frac{R_\square}{W}\frac{\Delta L}{L} + R_\square \cdot L \frac{\Delta W}{W}}_{\text{Usually negligible}}$$

We assume that ρ and Δ are subjected to variability over a characteristic spatial length $\Lambda << W, L$. From this conclusion, we can divide the resistor in elementary volumes of height Δ and area $A_0 = \Lambda \cdot \Lambda$. Each elementary volume will have a normal distribution of $R_\square$ with $\overline{R_\square}$ mean value and $\sigma(\overline{R_\square})$ standard deviation.

We can see the resistor as a matrix of $m \times n$ resistors, $m = \frac{L}{\Lambda}$ and $n = \frac{W}{\Lambda}$, each of them being a sample of a Gaussian distribution.



① **Rows** $G_1 = \sum_{i=1}^{N} G_{1,i}$

$$E[G_1] = \sum_{i=1}^{N} E[G_{1,i}] = N\overline{G_\square}$$

Now, being $G = \frac{1}{R}$ we may write

$$\frac{dG}{G} = \frac{dR}{R}, \quad \text{so} \quad \frac{\sigma^2(G)}{G^2} = \frac{\sigma^2(R)}{R^2}$$

assuming to deal with small variances, in order not to have <u>PERCOLATIVE PATHS</u>. (higher total resistance, higher variability)

$$\sigma^2(G_1) = \sum_{i=1}^{N} \sigma^2(G_{1,i}) = N\sigma^2(\overline{G_\square})$$

$$\frac{\sigma(G_1)}{G_1} = \frac{\sqrt{N}\,\sigma(\overline{G_\square})}{N\overline{G_\square}} = \frac{\sigma(\overline{G_\square})}{\overline{G_\square}} \cdot \frac{1}{\sqrt{N}} \simeq \frac{\sigma(\overline{R_\square})}{\overline{R_\square}} \cdot \frac{1}{\sqrt{N}} = \frac{\sigma(R_1)}{R_1}$$

② **Column**

$$R_{ROW} = \frac{1}{N\overline{G_\square}} = \frac{\overline{R_\square}}{N}$$

$$\frac{\sigma(R_{ROW})}{R_{ROW}} = \frac{\sigma(\overline{R_\square})}{\overline{R_\square}} \cdot \frac{1}{\sqrt{N}}$$

(34) So we get that the mean value of the total resistance is

$$\overline{R} = M \frac{\overline{R_\square}}{N}$$

and the standard deviation is

$$\frac{\sigma(\overline{R})}{\overline{R}} = \frac{\sqrt{M\,\sigma^2(R_{ROW})}}{M \frac{\overline{R_\square}}{N}} = = \frac{\sigma(R_{ROW})}{\sqrt{M}\,R_{ROW}} = \frac{\sigma(R_\square)}{R_\square}\cdot\frac{1}{\sqrt{MN}} = \frac{\sigma(R_\square)}{R_\square}\cdot\frac{\sqrt{A_0}}{\sqrt{WL}}$$

$$\Longrightarrow \quad \frac{\sigma(R)}{R} = \frac{K'_{\Delta R/R}}{\sqrt{WL}}$$

Let's consider now two nominally identical resistors R_1 and R_2: if they are manufactured close to each other, the mismatch is mainly due to statistical variability, so we can write

$$\Delta R = \sigma(\Delta R) = \sqrt{\sigma^2(R_1) + \sigma^2(R_2)}$$

$$\Downarrow$$

$$\frac{\sigma(\Delta R)}{\overline{R}} = \sqrt{\frac{\sigma^2(R_1)}{R^2} + \frac{\sigma^2(R_2)}{R^2}} = \sqrt{\frac{2K'^2_{\Delta R/R}}{WL}} = \frac{\sqrt{2}\,K'_{\Delta R/R}}{\sqrt{WL}} = \frac{K_{\Delta R/R}}{\sqrt{WL}} \quad \checkmark$$

$\longrightarrow$ TRANSISTORS

As for resistors, also for transistors the variability due to threshold and conductivity coefficient (hence electrical parameters) are dominant with respect to dimensional parameters.
Let's segment the transistor area into a matrix of $M \times N$ elementary transistors with area $A_0 = \Lambda \times \Lambda$ over which the variability affects the V_T value.
For each element the local V_T is a sample extracted from a gaussian distribution of mean value $\overline{V_T}$ and standard deviation $\sigma(\overline{V_T})$

We get that

[The overall threshold is the average of all the average contributions of each elementary volume]

$$\overline{V_T} = E\left[E[V_{T,0}]\right] = \overline{V_T} = \frac{1}{M\cdot N}\sum_{i=1}^{N}\sum_{j=1}^{M} V_{T,0}$$

$$\sigma^2(\overline{V_T}) = E\left[\left(E[V_{T,0}]\right)^2\right] = \frac{1}{(M\cdot N)^2}\cdot\sigma^2\left(\sum_{i=1}^{m} V_{T,i}\right) = \frac{1}{(M\cdot N)^2}\cdot \cancel{M\cdot N}\cdot\sigma^2(V_{T,0})$$

$$\Downarrow$$

$$\sigma(\overline{V_T}) = \sqrt{\frac{A_0}{W\cdot L}\,\sigma^2(V_{T,0})} = \frac{K'_{\Delta V_T}}{\sqrt{W\cdot L}}$$

{ $V_{T,i} \triangleq$ threshold of an elementary volume
$V_{T,0} \triangleq$ average threshold of an elementary volume
$\sigma(V_{T,0}) \triangleq$ standard deviation of an elementary volume }

and, considering two nominally equal transistors,

$$\sigma(\Delta V_T) = \frac{\sqrt{2}\,K'_{\Delta V_T}}{\sqrt{WL}} = \frac{K_{\Delta V_T}}{\sqrt{WL}}$$

CMRR

We can write, in general,

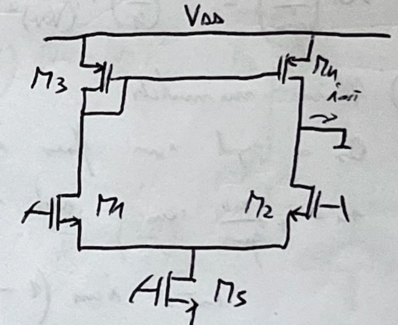
$$v_{OUT} = G_d v_d + G_{cm} v_{cm} = G_d \left[v_d + \frac{G_{cm}}{G_d} v_{cm} \right] = G_d \left[v_d + \frac{v_{cm}}{CMRR} \right]$$

so the common mode signal, together with the CMRR, is responsible for an input referred offset. Since CMRR depends on frequency, @ HF this offset contribution may be relevant.

Let's assume to have an input common mode signal at the input, it is almost entirely copied at this node, being thus followers:

$$i_{cm} \simeq \frac{v_{cm}}{2 r_{o,g}}$$ [common mode current flowing in the two branches]

Due to mismatches we may have that $i_{OUT} = \varepsilon i_{cm}$, so



$$G_{cm} = \frac{\varepsilon R_{OUT}}{2 r_{o,g}}$$

$$CMRR = \frac{g_{mD} R_{OUT}}{\frac{\varepsilon R_{OUT}}{2 r_{o,g}}} = \frac{2 g_{mD} r_{o,g}}{\varepsilon}$$

$\longrightarrow$ DETERMINISTIC ε

① Mirroring error

i_{cm} in the left-hand branch is partially lost into $r_{o,M}$ and not mirrored, thus causing

$$\varepsilon = i_{cm} \frac{1/g_{mM}}{\frac{1}{g_{mM}} + r_{o,M}} = i_{cm} \frac{1}{1 + g_{mM} r_{o,M}} \simeq \frac{i_{cm}}{g_{mM} r_{o,M}}$$

② Unbalanced resistances at the input pair's drain nodes

$$R_{S,1} = \frac{\frac{1}{g_{mM}} + r_{o,D}}{1 + g_{mD} r_{oD}} =$$

$$R_{S,2} = \frac{\frac{r_{oD}}{g_{mD}}}{\frac{1}{g_{mD}} + r_{o,D}} = \frac{r_{o,D}}{1 + g_{mD} r_{oD}}$$

$$\varepsilon i_{cm} = 2 i_{cm} \frac{R_2}{R_1 + R_2} - 2 i_{cm} \frac{R_1}{R_1 + R_2} = \frac{R_2 - R_1}{R_2 + R_1} 2 i_{cm} = \frac{\frac{1}{g_{mM}}}{2 r_{o,D} + \frac{1}{g_{mM}}} 2 i_{cm}$$

$$R_2 - R_1 = \frac{-\frac{1}{g_{mM}}}{1 + g_{mD} r_{o,D}}$$

$$R_2 + R_1 = \frac{2 r_{o,D} + \frac{1}{g_{mM}}}{1 + g_{mD} r_{o,D}}$$

(36)

$$i_{cm}\,\varepsilon \simeq \frac{1}{2r_{o,D}\, g_{mM}} \cdot 2 i_{cm} = \frac{i_{cm}}{r_{o,D}\cdot g_{mM}}$$

$$\Rightarrow \left[\varepsilon_{DET} \simeq \frac{1}{r_{o,D}\, g_{mM}} + \frac{1}{r_{o,M}\, g_{mM}}\right]$$

$\longrightarrow$ STATISTICAL CMRR

$$dg_m = d(2kV_{ov}) = 2dk \cdot V_{ov} + 2k\, dV_T$$

$$\frac{dg_m}{g_m} = \frac{dk}{k} + \frac{dV_T}{V_{ov}}$$

Assuming that Δk and ΔV_T are statistically independent (they are not) we get

[Because for example a variation of t_{ox} modifies C'_{ox}, which is present both in k and V_T formulas]

$$\sigma^2\left(\frac{\Delta g_m}{g_m}\right) = \sigma^2\left(\frac{\Delta k}{k}\right) + \frac{1}{(V_{ov})^2}\,\sigma^2(\Delta V_T)$$

① Mirror mismatch

As a signal i_{cm} flows into M_3, $v_{g,3} = -\frac{i_{cm}}{g_{m3}}$, then we get

$i_4 = +\frac{g_{m4}}{g_{m3}}\, i_{cm}$ and so

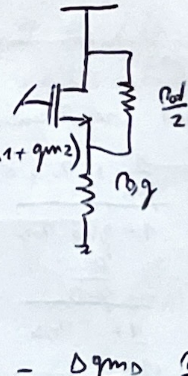
$$i_{out} = i_{cm}\left(1 - \frac{g_{m4}}{g_{m3}}\right) = \frac{g_{m3}-g_{m4}}{g_{m3}}\, i_{cm} \simeq \frac{\Delta g_{mM}}{g_{mM}}\, i_{cm}$$

so

$$\varepsilon_M^{STAT} \simeq \frac{\Delta g_{mM}}{g_{mM}}$$

② Input mismatch

We need v_S, so we FOLD one transistor on the other



$$v_S = v_{cm} \cdot \frac{r_{o,g} /\!/ \frac{r_{o,D}}{2}}{r_{o,g}/\!/\frac{r_{o,D}}{2} + \frac{1}{2g_m}}$$

$$= v_{cm} \cdot \frac{2g_{mD}\,(r_{o,g} /\!/ r_{o,D}/2)}{1 + 2g_m\,(r_{o,g}/\!/ r_{o,D}/2)}$$

$$i_2 - i_1 = i_{out} = v_{cm}\,\frac{(g_{m1} - g_{m2})}{1 + 2g_m\,(r_{o,g} /\!/ r_{o,D}/2)}$$

$$\simeq \frac{\Delta g_{mD}}{2g_{mD}} \cdot \frac{r_{o,g} + \frac{r_{oD}}{2}}{\frac{r_{o,g}\, r_{o,D}}{2}}\, v_{cm} = \frac{\Delta g_{mD}}{g_{mD}}\,\frac{v_{cm}}{2r_{o,g}}\left(\frac{2r_{o,g} + r_{oD}}{r_{o,D}}\right)$$

$$\Rightarrow \varepsilon_D^{STAT} \simeq \frac{\Delta g_{mD}}{g_{mD}}\left(\frac{2r_{o,g}}{r_{o,D}} + 1\right)$$

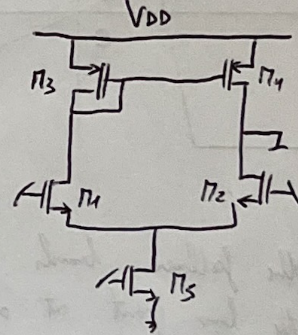
For $r_{o,g} \to +\infty$ we get $CMRR \to \dfrac{2g_{mD}\, r_{o,g}}{\left(\frac{\Delta g_{mD}}{g_{mD}}\right)\frac{2r_{o,g}}{r_{o,D}}} = \dfrac{g_{mD}\, r_{o,D}}{\left(\frac{\Delta g_{mD}}{g_{mD}}\right)}$

OFFSET

In a single-ended OTA, in principle, the output voltage should be at $\frac{V_{DD}-V_{EE}}{2}$ if no differential signal drives the two inputs. If the parameters involved deviate from nominal values, we may have that $V_{OUT} \neq \frac{V_{DD}-V_{EE}}{2}$, thus causing an output voltage offset. We usually describe the component as an ideal one with an offset equivalent generator placed between the input terminals, where

$$V_{OS} = \frac{V_{OUT,OS}}{L(0)}$$

Mismatches between transistors of the input stage are more critical, being thus amplified by the total gain of the stage.



→ **K MISMATCH M_1 & M_2**

$$I_1 = \left(K + \frac{\Delta K}{2}\right)(V_{GS}-V_T)^2 \quad ; \quad I_2 = \left(K - \frac{\Delta K}{2}\right)(V_{GS}-V_T)^2$$

$$|\Delta I| = |\Delta K (V_{GS}-V_T)^2| = |I_1 - I_2|$$

$$V_{OS}\big|_{\Delta K} = \frac{|\Delta I|}{g_m} = \frac{V_{OV}^2 \Delta K}{g_m} = \frac{V_{OV}^2 \cdot \Delta K}{2K V_{OV}} = \frac{\Delta K}{K}\cdot\left(\frac{V_{OV}}{2}\right)$$

→ **V_T MISMATCH M_1 & M_2**

$$I_1 = K\left(V_{GS} - V_T - \frac{\Delta V_T}{2}\right)^2 \quad ; \quad I_2 = K\left(V_{GS} - V_T + \frac{\Delta V_T}{2}\right)^2$$

$$|\Delta I| = \left| K\left[\cancel{(V_{GS}-V_T)^2} + \cancel{\frac{\Delta V_T^2}{4}} - 2V_{OV}\cdot\frac{\Delta V_T}{2} - \cancel{(V_{GS}-V_T)^2} + \left(-\cancel{\frac{\Delta V_T^2}{4}}\right) - 2V_{OV}\frac{\Delta V_T}{2}\right]\right| =$$

$$\downarrow$$

$$= 2K V_{OV}\cdot \Delta V_T$$

$$V_{OS}\big|_{\Delta V_T} = \frac{|\Delta I|}{g_m} = \frac{2K V_{OV}}{2K V_{OV}}\cdot \Delta V_T = \Delta V_T$$

→ **K MISMATCH M_3 & M_4**

$$I_3 = \left(K_M + \frac{\Delta K_M}{2}\right)(V_{GS}-V_T)^2 \quad ; \quad I_4 = \left(K_M - \frac{\Delta K_M}{2}\right)(V_{GS}-V_T)^2$$

$$|\Delta I| = \cancel{2}\,\frac{\Delta K_M}{\cancel{2}}(V_{GS}-V_T)^2$$

$$V_{OS}\big|_{\Delta K} = \frac{|\Delta I|}{g_m} = \frac{\Delta K_M}{2K_D V_{OV,D}} V_{OV,M}^2 = \frac{\Delta K_M V_{OV,M}^2}{2I} V_{OV,D} = \frac{V_{OV,D}}{2}\left(\frac{\Delta K_M}{K_M}\right)$$

→ **V_T MISMATCH M_3 & M_4**

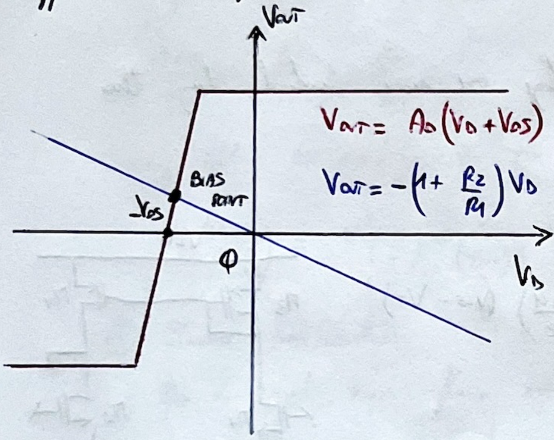
$$I_3 = K_M\left(V_{GS} - V_T - \frac{\Delta V_T}{2}\right)^2 \quad ; \quad I_4 = K_M\left(V_{GS} - V_T + \frac{\Delta V_T}{2}\right)^2$$

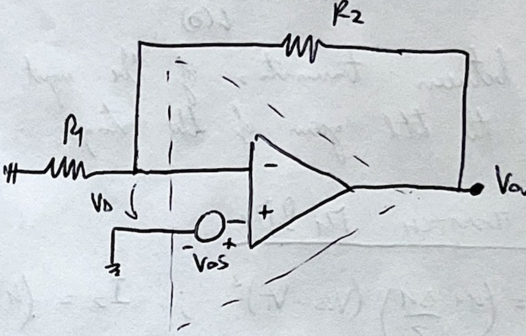
$$|\Delta I| = 2K_M V_{ov,M} \cdot \Delta V_T$$

$$V_{os}\big|_{\Delta V_T} = \frac{2K_M V_{ov,M}}{g_m}\,\Delta V_T = \frac{2K_M V_{ov,M}}{2I} \cdot V_{ov,D} \cdot \Delta V_T = \frac{2I}{V_{ov,M}} \cdot \frac{V_{ov,D}\,\Delta V_T}{2I} = \frac{V_{ov,D}}{V_{ov,M}}\,\Delta V_T$$

$$\Longrightarrow \sigma^2(V_{os}) = \sigma^2(\Delta V_{T,D}) + \sigma^2(\Delta V_{T,M})\left(\frac{V_{ov,D}}{V_{ov,M}}\right)^2 + \left[\sigma^2\left(\frac{\Delta K_D}{K_D}\right) + \sigma^2\left(\frac{\Delta K_M}{K_M}\right)\right]\left(\frac{V_{ov,D}}{2}\right)^2$$

Offset is compensated by **FEEDBACK**:





the feedback branch introduces a new relationship in the system, thus setting the bias point at a value lower than the power supply. In order to have this situation, it must be

$$\left(1 + \frac{R_2}{R_1}\right) << A_D$$

$$\downarrow$$

$$\underbrace{A_D \frac{R_1}{R_1 + R_2}}_{|G_{Loop}(0)|} >> 1$$

[Therefore in order to properly act on the feedback nodes, the loop gain must be properly sized and $>> 1$.]