

Course notes by Lorenzo Perlo. These are free, so if you've paid for them, you've been scammed.

I've studied this subject by reading the book provided by the professor on WeBeep before attending the lessons. Many topics are just summarized here because the book explains them well, while others are covered in more detail. Rare topics that are not in the book can be found here (though not fully covered, as the professors slides were clearer).

My advice: use these notes alongside the book before the lesson, and you'll get more out of the class (be sure to attend, the professor is really good at what he does).

Please excuse any possible grammatical mistakes.

SLAB WAVEGUIDE

$$\vec{E}(x, y, z, t) = E(x, y) e^{-j\beta z} e^{j\omega t}$$

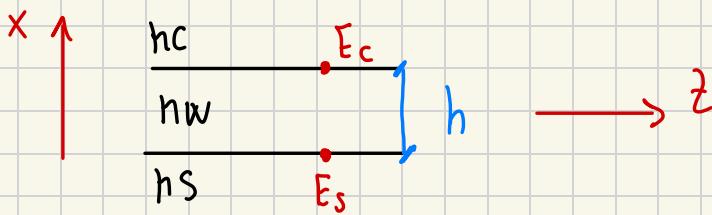
Families of solution :

$$(TE) \rightarrow H_y = 0 = E_x = E_z$$

$$(TM) \rightarrow H_x = 0 = E_y = H_z$$

A waveguide usually carries both, maybe one better than the other.

$$\frac{\partial^2 E_y}{\partial x^2} = \left(\beta^2 - n^2(x) k_0^2 \right) E_y$$
$$\frac{2\pi}{\lambda}$$



The possible solutions for TE :

$$\left. \begin{array}{ll} \textcircled{1} & E_{y_s} = E_s e^{\gamma_s x} & x < 0 \\ \textcircled{2} & E_{y_q} = E_c e^{\gamma_c (x-h)} & x > h \\ \textcircled{3} & E_{y_c} = E_q \cos(K_q x - \phi_s) & 0 < x < h \end{array} \right\}$$

↓
If the peak is NOT at the
center of the core

Now if you substitute the solutions in the wave equation, you find β for each. So put
 ① in eq., find β or γ_s or γ_c , repeat for
 ② and ③.

$$\left\{ \begin{array}{l} \gamma_s^2 = \beta^2 - k_0^2 h_s^2 \\ \gamma_c^2 = \beta^2 - k_0^2 n_c^2 \\ k_q^2 = k_0^2 n_x^2 - \beta^2 \end{array} \right. \Rightarrow \text{Related to the solutions}$$

The boundary condition:

$$\left\{ \begin{array}{ll} x = 0 & E_{y_s} = E_{y_q} \\ x = h & E_{y_c} = E_{y_q} \end{array} \right. \rightarrow \text{From here find}$$

$$H_x = -\frac{\beta}{\omega \mu} E_y$$

//

$$t_q (k_q h) = \frac{k_q (\gamma_c + \gamma_s)}{k_q^2 - \gamma_c \gamma_s}$$

+ 2Nπ

↳

↳ Integer number of solutions,
solved numerically

E_g , E_c and E_s share the same β ! They move at the same speed.

Cannot find $\beta = \text{something}$, because it's the eigen solution of the N equation.

↳ After find n_w, n_c, n_s, h and w I find β , change w or the other, change β .

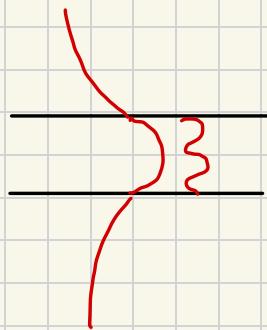
FINALLY



$$\beta = \frac{2\pi}{\lambda} \underbrace{n_{\text{eff}}}_{\text{Neff}}$$

↳ Vary with w and material and shape

Change β , change shape of \vec{E} (eigen vector).



The slab is the only one to have both TE and TM modes, the others are hybrid (so also quasi TE/TM).

↳ 6 components ($E_x, E_y, E_z, H_x, H_y, H_z$
not null)

Special cases for $n_c = n_s$ (symmetric)

- $\omega \rightarrow 0 : \beta = \frac{2\pi}{\lambda} n_s$

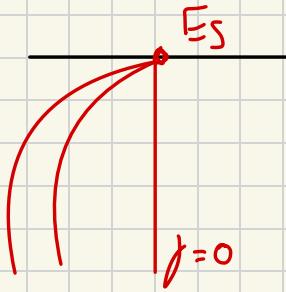
\parallel
 $k_0 \rightarrow 0$

- $\omega \rightarrow \infty : \beta = \frac{2\pi}{\lambda} n_s \omega$

$$k_0 h_s < \beta < k_0 n_w$$

MODE WITH THIS β
ARE GUIDED

$\beta_s \rightarrow 0$: cut-off , the mode
is NOT confined in
the wave guide , but
it extend everywhere



$$k_q (k_q h + 2N\pi) = 0$$

$$k_q h + 2N\pi = N\pi$$

$$\frac{h}{\lambda_{c_{UR,FF}}} = \frac{N}{2\sqrt{n_w^2 - n_s^2}}$$

I have a cut-off for every mode.

• $\sigma \rightarrow \text{Im} \rightarrow$ The field is radiate away



RADIATIVE MODE (plane wave)

• IF WAVEGUIDE SYMMETRIC

↳ NO CUT-OFF

AT $\omega = 0$ for
1st mode

• IF ASYMMETRIC

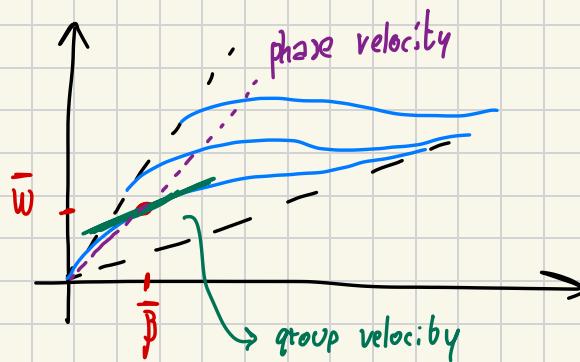
↳ CUT-OFF also for low frequency and 1st mode

IF YOU DON'T EXCITE A MODE
 (RADIATIVE OR GUIDED) AT $\bar{\omega}$,
 THAT MODE DOESN'T PROPAGATE
 EVEN IF AT THAT ω IS A
 SOLUTION FOR THAT GUIDE

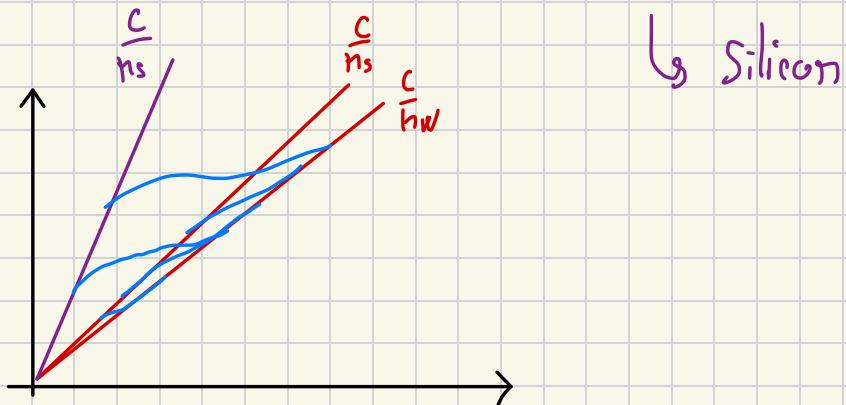
↳ MISMATCH AND ROUGHNESS
 CAN BRING β TO RADIATIVE
 (NOT SEEN IN PROJECT,
 BUT AFTER MANUFACTURING)

- $V_p = \frac{w}{\beta} = \frac{c}{h_{eff}}$

- $V_q = \frac{\delta w}{\delta \beta} = \frac{c}{h_q} = \frac{c}{h_{eff} + w \frac{\partial h_{eff}}{\partial w}}$



For real ω_q with low Δn and high Δn



↳ Silicon

low $\Delta n \rightarrow n_{eff} \approx n_g$

high $\Delta n \rightarrow n_g \gg n_{eff}$

In free space $\Delta n = 0$, $n_{eff} = n_p$, because

you have only TE or TM, not other. If you have also Ez or Hz (z direction of prop.), you have high energy (with high Δh) or low energy (low Δh) that is reactive in the z direction, so slow down the wave

↳ So in Mg (where Et and Hz can be $\neq 0$) the field is slower.

For silicon $\eta_g = 4.2$

Special case

Launch TE₀ in a dual mode waveguide of fixed w, but shifted along the X position, cause the excitation of two modes (TE₀ and TE₁) and they have different neff (so $B_0 \neq B_1$).

↳ Both are guided, but there is a periodicity in the total field due to the interference.

↳ If the x-offset is big, I excite more TE₀ than TE₁. Depending on the size, the X-offset is more or less degrading.



Modes don't change shape and don't exchange power, but the total field has l_B

↳ And, the x-offset count less and the guide is more mono-modal and after some $z \mu m$ the total field is like the one of only 1st mode.

THE ALIGNEMENT IS IMPORTANT, BUT IF THERE IS A X-OFFSET, THE PHOTONIC DEVICE MUST NOT BE PUT TOO NEAR THE ENTRANCE !

↳ More mode propagating is possible, but for most application only mono mode is used and separate strongly excited mode is difficult.

$$L_B = \frac{\lambda}{\Delta \text{heff}_{\text{of modes}}} = \frac{\lambda}{n_{\text{eff},1} - n_{\text{eff},2}}$$

$n_{\text{fundamental}} > n_{\text{modo superiore}}$

LEAKY MODE \Rightarrow LOOSE POWER WHILE PROPAGATION AND AFTER SOME μm IN Z DIRECTION THEY ARE RADIATED AWAY CHANGING THE SHAPE.

No plane wave like radiative mode, but packet of wave.

Birefringence

Different n_{eff} between different polarization (TE and TM)

↳ If wq sym, TE and TM have same λ_{cutoff} , but $n_{\text{eff,TE}} \neq n_{\text{eff,TM}}$

SO THE BEHAVIOUR OF THE DEVICE VARY WITH THE POLARIZATION (EXCITE WHAT YOU NEED!).

↳ The fiber never conserve the fiber polarization, so most of the time I don't know what arrive.

I want polarization independent device. Two solution

$$\textcircled{1} \quad \hookrightarrow B = 0 \Rightarrow \frac{n_{\text{eff}}}{n_E} = \frac{n_{\text{eff}}}{n_H}$$

$$\Delta \phi_{\frac{n_E}{n_H}} = \frac{2\pi}{\lambda} B L$$

$10^{-2} / 10^{-3}$
 10^{-6}

Typically $\sim 10^{-2}$

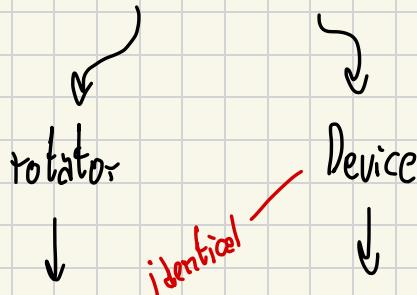
OK for other material

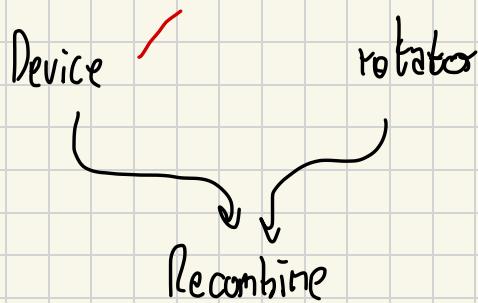
\hookrightarrow So $B < 10^{-6}$, so
n_{eff} must be controlled
to the sixth digit

IMPOSSIBLE FOR SILICON PHOTONICS

$\textcircled{2}$ POLARIZATION DIVERSITY SCHEME

\hookrightarrow Polarization beam splitter





Difficult but possible, ✓ silicon photonics.

Caused by δw , but with same $S(w)$, they are greater on material with high Δn ($n_g \neq n_{eff}$) w.r.t. b. low Δn ($n_g \approx n_{eff}$).

Losses

Depend on roughness leaved by etching ($W_{real} = W_{ideal} \pm \delta w$)

- ↳ I can excite radiative modes
- ↳ Some of fundamental mode is coupled with the counter propagating

mode fundamental.



Important for laser, they are sensible to light that comes back.

For the same wq, TE and TM experience different losses and backscatter

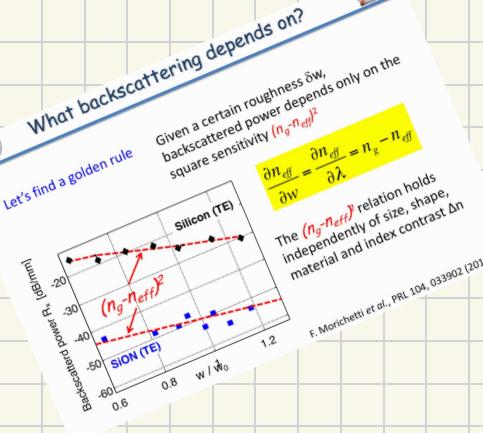
↳ IMPORTANT EVERY TIME
YOU ARE SENSIBLE
TO RADIATION THAT
COMES BACK

LiDAR

(I measure what comes back, backscatter creates errors)



Laser (light comes back, laser works differently)



↙

EVERY LASER IN WAVEGUIDE HAS ISOLATOR BEFORE COUPLING WITH WAVEGUIDE.

BENT WAVEGUIDE

Conformal transform \rightarrow transform the problem to one system of coordinate to one other, so I can see a bend as like a straight one and every value must be transformed like $n(u, v) \rightarrow n(x, y)$



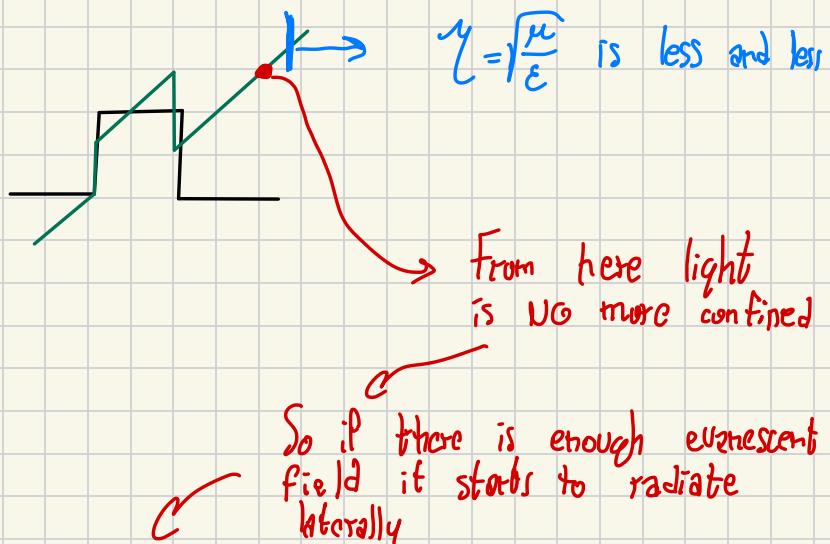
refractive index
in bend guide
equivalent
as if it
was straight

$$R = R \ln \left(\frac{W}{R} \right) \xrightarrow{U+jV}$$

$$n(U, V) e^{\frac{2X}{R}} = n\left(1 + \frac{2X}{R}\right)$$

If straight $R \rightarrow \infty$, otherwise this plays a role

The index profile is tilted a linear term that increase going away from the waveguide due to bend.



The light is pushed away from the center as $R \downarrow$

↳ Different modes with more bend, leaky and not guided

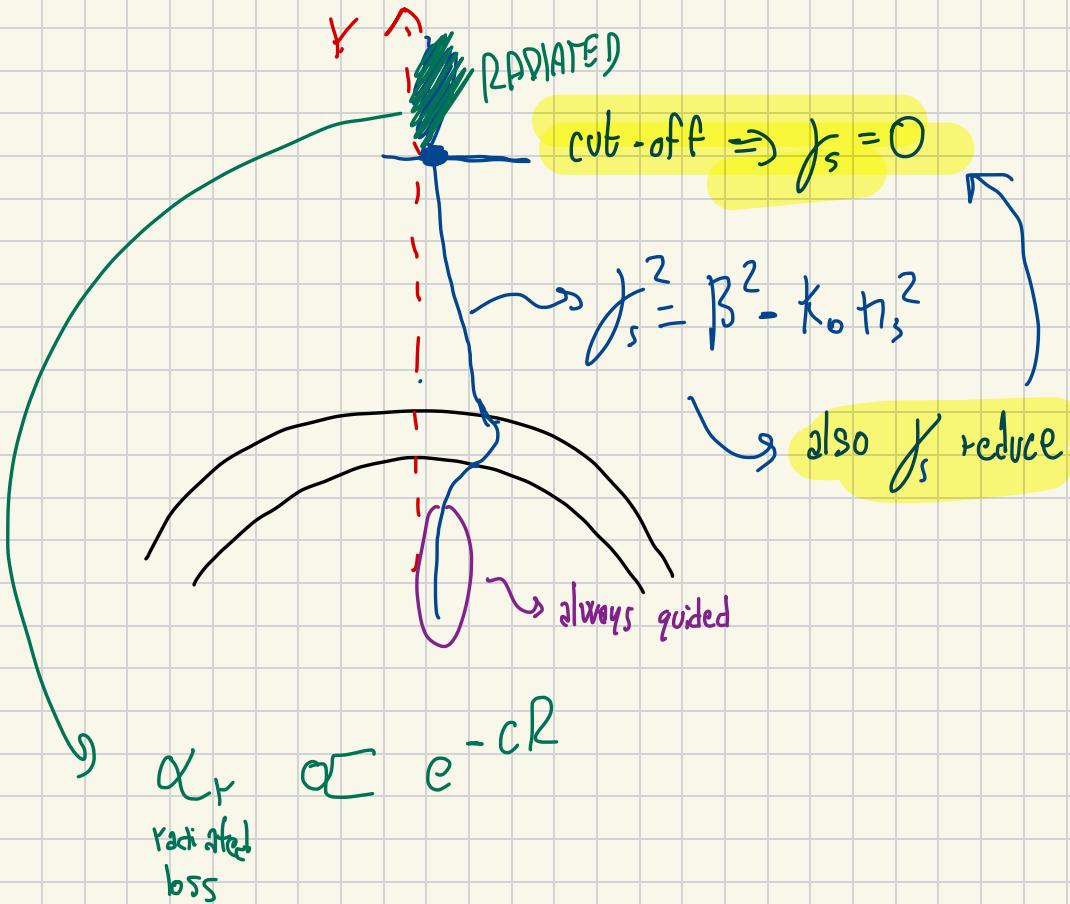
With the same β , in a straight guide every point of the mode is at the same speed, but in a bent one, the phase front move slower as $r \rightarrow R$, so β vary with r :

$$r \frac{d\theta}{dt} = \frac{w}{\beta(r)}$$

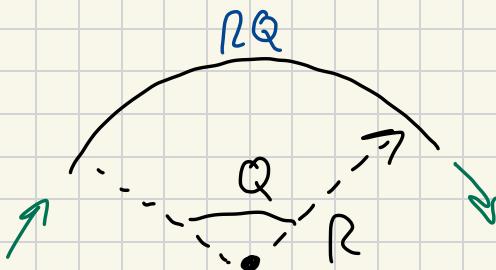
$$r=R \Rightarrow \beta_0 = \beta(R)$$

$$\beta(r) = \beta_0 \cdot \frac{R}{r}$$

With $r \rightarrow R$, $\beta \downarrow$ reaching the cut off at some point were the tail of the modes is dictated.



So the insertion loss (from input to output):



$$I_L = e^{-\alpha L} \stackrel{QR}{=} e^{-c - \frac{cL}{RQ}}$$

↳ SE INIZIO A VEDERE
 UNA PERDITA, LA RELAZIONE
 È COSÌ FORTE CHE PASSANDO
 DA Dritto A Curvo tutto
 È LEAKY.

Con β variabile con γ , più mi allontano e
 più il campo è lento, cioè perdo il campo.

Bending \rightarrow the fundamental mode is more and more
 asymmetrical and distorted, so as cut-
 off approach the cut-off, all is leaky

EVERY TIME I START TO BEND

I LOOSE SOMETHING!

— RADIATED OR EXCITE HIGHER ORDER MODES

↳ Shape of the mode +

↳ n_{eff} vs of a bend
waveguide (little bit)

The efficiency of coupling between straight and bent is lower than 1

COUPLED MODE THEORY

Use the info of one wq to understand how the field change with two wq next to each other. It's a guiding structure and we want a simple method

to study it.

The structure is at least bimodal :

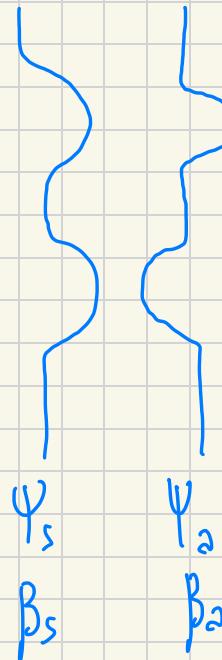
$$\psi_1 \approx \frac{\psi_a + \psi_s}{2}$$

mode of
single wav
Without coupling

$$\psi_2 \approx \frac{\psi_a - \psi_s}{2}$$

ψ_a and ψ_s real mod in
coupled guide From the
Maxwell equations.

h_s	even.
n_{w_1}	guided
n_s	$\sin + \cos,$
n_{w_2}	guided
n_s	even



Coupled mode theory = combine real solution as linear
combination of simple modes

Ψ_1 \ coupled mode
 Ψ_2 /

Ψ_a \ NOR coupled, real mode of
 Ψ_s / the structure

$$\Psi_a \approx \Psi_1 - \Psi_2$$

$$\Psi_s \approx \Psi_1 + \Psi_2$$

$$\beta_{s,a} \propto \beta_1, \beta_2$$

I know how to obtain Ψ_1 and Ψ_2 , from here
 I find Ψ_a and Ψ_s .

$$\nabla_t^2 \Psi + \frac{\partial^2}{\partial z^2} \Psi + k_0^2 n^2(x, y) \Psi = 0$$

$$\Psi \approx A(z) \Psi_1(x, y) e^{-j \beta_1 z} + B(z) \Psi_2(x, y) e^{-j \beta_2 z}$$

ψ_1 and ψ_2 are NOT modes because their amplitude depends on z , but they are solution of the single Wq.

Real solution (field seen in simulation and propagation)

$$\Psi = A \psi_a e^{-j\beta_a z} + B \psi_s e^{-j\beta_s z}$$

const.

$$\left\{ \begin{array}{l} \Delta n_1^2 = n^2(x, y) - n_1^2(x, y) \\ \Delta n_2^2 = n^2(x, y) - n_2(x, y) \end{array} \right.$$

Δn_1 is the perturbation that the second Wq gives to the first and viceversa.

From wave equation substitute Ψ :

$$k_0^2 \Delta n_1^2 A \psi_1 + k_0^2 \Delta n_2 B \psi_2 e^{j \Delta \beta z} - 2 j \beta_1 \psi_1 \frac{\partial A}{\partial z} - 2 j \beta_2 \psi_2 \frac{\partial B}{\partial z} e^{j \Delta \beta z}$$

Multiply for ψ_1^* or ψ_2^* and integrate along X, Y :

$$\frac{dA}{dz} = -j K_{11} A(z) - j K_{12} B(z) e^{j \Delta \beta z}$$

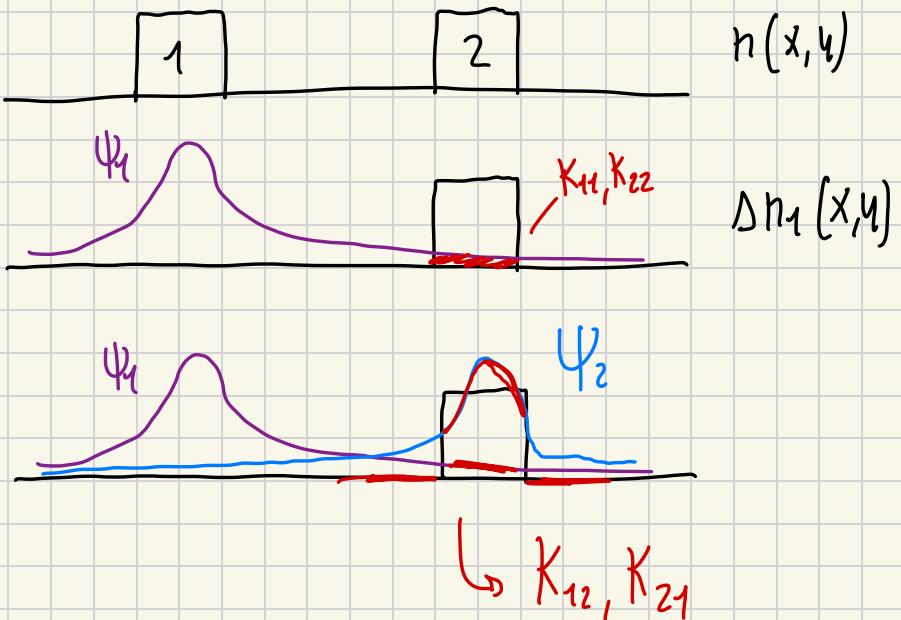
$$\frac{dB}{dz} = -j K_{22} B(z) - j K_{21} A(z) e^{j \Delta \beta z}$$

Remember ψ_1 and ψ_2 are orthogonal [they do not interact]:

$$\int \psi_1 \psi_2 = 0$$

$$K_{11} = \frac{k_0^2}{2 \beta_1} \iint \psi_1 \Delta n_1^2 \psi_1^* , K_{22}$$

$$K_{21} = \frac{k_0}{2\beta_2} \iint \psi_1 \Delta n_2^2 \psi_2 , K_{12}$$



$K_{11}, K_{22} \rightarrow \infty$, K_{12}, K_{21} much larger.

FIELD COUPLING COEFFICIENTS

What if the guides are far away?

• $k_{12}, k_{11} \rightarrow 0$

• $\frac{dA}{dz} \rightarrow 0 = \text{const.}$, Ψ_1 is a mode (appunto della guida singola)

for coupling the guide must be close

Re define variables :

$$\left. \begin{aligned} a(z) &= A(z) e^{-j\beta_1 z} \\ b(z) &= B(z) e^{-j\beta_2 z} \end{aligned} \right] \begin{array}{l} \text{propagating} \\ \text{amplitude of} \\ \Psi_1 \text{ and } \Psi_2 \end{array}$$

Substitute :

$$\left\{ \begin{aligned} \frac{da}{dz} &= -j(\beta_1 + \cancel{k_{11}}^o) a(z) - j k_{12} b(z) \\ \frac{db}{dz} &= -j(\beta_2 + \cancel{k_{22}}^o) b(z) - j k_{21} a(z) \end{aligned} \right.$$



Now I know the exact solution ($\alpha_a e^{-j\beta_a z}$, as $e^{-j\beta_s z}$), I substitute and find:

$$\begin{cases} \alpha_s (\beta - \beta_1) - K_{12} \alpha_a = 0 \\ -\alpha_s K_{21} + \alpha_a (\beta - \beta_2) = 0 \end{cases}$$

The determinant must be 0 to have solutions:

$$\beta_{s,a} = \frac{\beta_1 + \beta_2}{2} \pm \sqrt{\frac{(\beta_1 - \beta_2)^2}{4} + K_{21} K_{12}}$$

$$\downarrow \quad \beta_1 = \beta_2 = \beta_0$$

$$\beta_{s,a} = \beta_0 \pm \sqrt{K_{21} K_{12}}$$

C = coupling coefficient

So Ψ_a and Ψ_s have different β , so different speed. They are the real modes.

\hookrightarrow So at different z , ψ_1 and ψ_2 vary

$$\psi_s = \frac{\psi_1 + \psi_2}{2} \text{ con } \beta_s$$

$$\psi_a = \frac{\psi_1 - \psi_2}{2} \text{ con } \beta_a$$

After $\beta_s z - \beta_a z = \pi$ all the field from one wave is gone in the second. It seems there is a power exchange between the two, but it's not true, it's just the field that has a max in one and then in another as travelling along z .

$$\text{Periodicity} \rightarrow (\beta_s - \beta_a) L = \pi$$

if $\beta_1 = \beta_2$ (identical wq)

$$(\beta_0 + c - \beta_0 + c)L_c = \pi$$



Se different

$$L_c = \frac{\pi}{2\delta} = \frac{\pi}{\beta_s - \beta_a}$$

$$L_c = \frac{\pi}{2C}$$

So if "coupled" Wg is long one L_c , all the power given in one of the two Wg "pass" on the other.

Real modes have different η_{eff} .

The coupled mode theory is an approx, for $d \gg 0$ is NO more correct and remember that

$$C \propto e^{-bX}$$

So if $X \rightarrow 0$, $C \uparrow \uparrow$ and viceversa, also L_c .

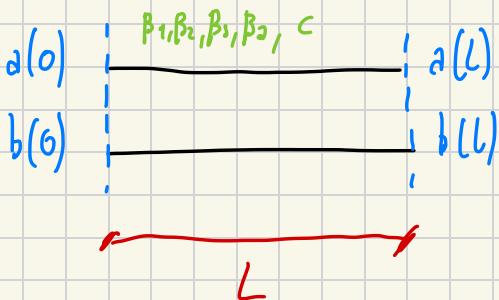
If $\beta_1 \neq \beta_2$ (different Wg), it's not possible to couple all the power between the two. So power remain more in a Wg.

$\hookrightarrow \Delta \beta = 0 \Rightarrow$ control n_{eff}
 to the sixth digit
 to make sure
 is $\Delta \beta = \frac{2\pi}{\lambda} (n_{\text{eff},1} - n_{\text{eff},2})$

Directional coupler \rightarrow solve the system $\textcircled{*} \uparrow$

Solution :

$$\left\{ \begin{array}{l} a(z) = a_s e^{-j\beta_s z} + a_o e^{-j\beta_o z} \\ b(z) = b_{oK} \end{array} \right.$$



In $z = 0$:

$$a(0) = a_1 + a_2$$

$$b(0) = \dots$$

"Put this in a TC" and you find $b(L)$ and $a(L)$:

$$T_C = \begin{bmatrix} \cos(\delta z) - j R \sin(\delta z) & -j S \sin(\delta z) \\ -j S \sin(\delta z) & \cos(\delta z) - j R \sin(\delta z) \end{bmatrix}$$

$$R = \frac{\Delta P}{2\delta} \quad S = \frac{C}{\delta}$$

$$\delta = \sqrt{\frac{\Delta P^2}{4} + C^2}$$

$$\det(T_C) = 1$$



$$R^2 + S^2 = 1$$

If identical Wq $\Rightarrow \beta_1 = \beta_2 = \beta_0$

$$R=0, S=1, \delta=c$$

$$\Gamma_c = \begin{bmatrix} \cos c z & -j \sin c z \\ -j \sin c z & \cos c z \end{bmatrix} e^{-\alpha L}$$

No loss here
yes loss (in power)

Special case ($a(0)=1, b(0)=0$):

$$|a(z)|^2 = 1 - \frac{c^2}{\delta^2} \sin^2 \delta z$$

$$|b(z)|^2 = \frac{c^2}{\delta^2} \sin^2 \delta z$$

$$\delta \geq c$$

$$\text{If } \Delta \beta = 0 \Rightarrow |a(z)|^2 = \cos^2 cz$$

$$|b(z)|^2 = \sin^2 cz$$

It seems like a change of power, but it's not, it's just a phase mismatch between the two field modes that sum with different result in the total field.

The max power exchangeable:

$$P_{\text{MAX}} = \frac{C^2}{\delta^2} = k$$

The relative phase of the two device

The two field in output are in quadrature, for synchronous coupler.

For asynchronous couplers the mismatch depends on the length of the coupler.

↳ NON CIRCULAR CIRCUIT CHE

$$b(L) = 1 \quad e \quad a(L) = 1$$



NON POSSIBILE SE ASINCRONO,
SOLI SE SINCRONO



Mentre se asincrono posso ancora fare un -3dB coupler.



Extreme case $\Delta B = 2C$

$$\delta = \sqrt{\frac{4C^2}{4} + C^2} = C\sqrt{2}$$

$$L = \frac{\pi}{2\sqrt{2}C} \quad \text{to have a splitter}\text{async}$$

Se $\Delta B > 2K$, two wq too different

so I cannot couple all the power from one to another, but it stays more in one of the two (the one that I excite).

Coupling efficiency:

$$K = \frac{P_2}{P_1 + P_2}$$

(=)

IF NO LOSS

$$\frac{P_2}{P_0} = \sin^2(CL)$$

SYNC

$K = 1$ ONLY IN SYNC ONES

If I don't want coupling is useful to have $\Delta B \neq 0$

$$TC = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

Splitter

$$\Delta B = 0$$

$$CL = \frac{\pi}{4}$$

$$TC = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

ASYNC
 $\delta L = \frac{\pi}{4}$

$\Delta \beta = 2K$

The difference is the phase

→ SYNC : quadrature
 → ASYNC : both

So it's preferable SYNC because I know the outputs will be in quadrature.

Dependence of δ to λ in couplers

$$k_{ij} = \frac{k_0^2}{2\beta} \int \varphi_i \varphi_j \Delta h^2$$

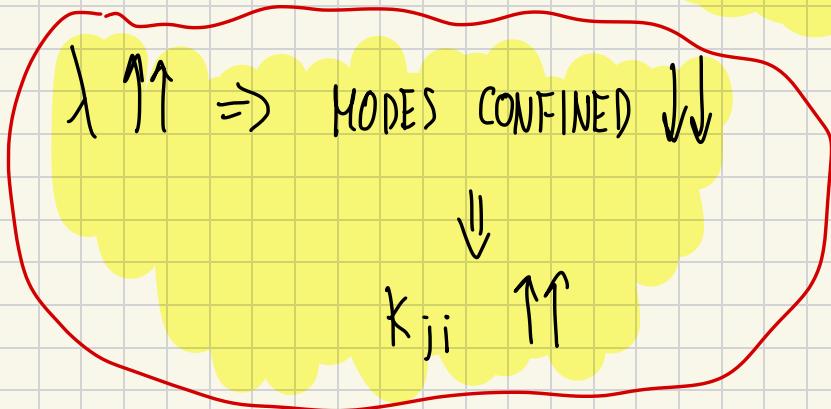
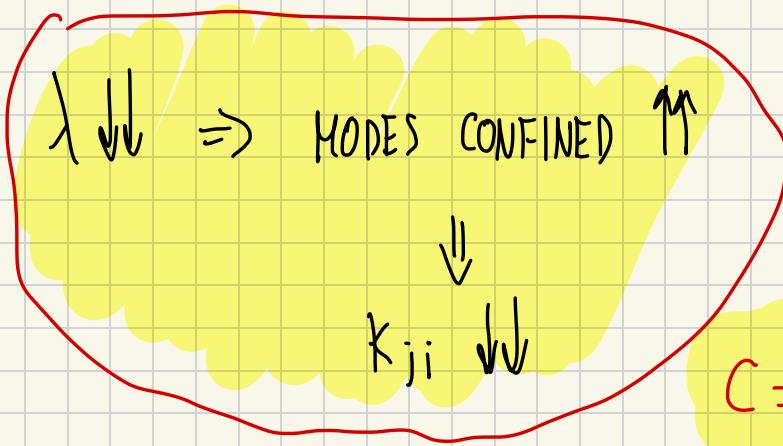
$\propto \lambda^{-1}$

The shape of the modes $\propto \lambda$

Change λ , change shape modes

More confined or less

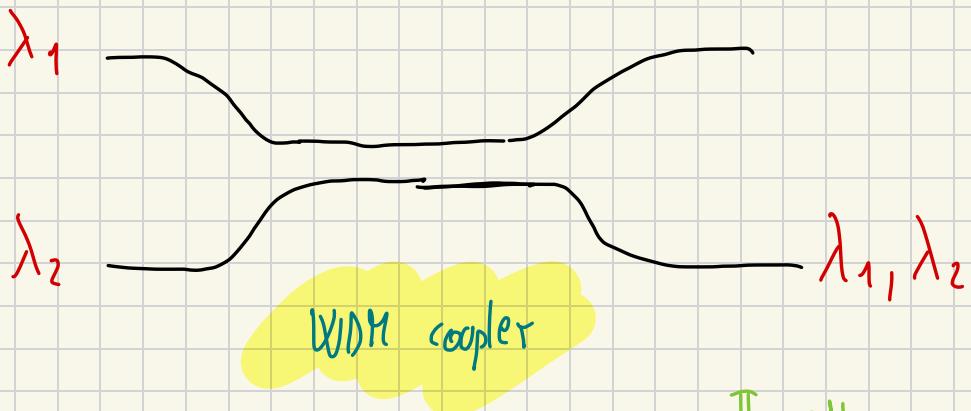
↓
Change overlap \rightarrow so k_{ji}



So low Δh nor so variation for different λ
($C_0 \downarrow\downarrow$), but with $\Delta h \uparrow\uparrow$, $C_0 \uparrow\uparrow$, great

Variation of C with λ .

So use it to split λ in input to do MUX or DEMUX



$$\left\{ \begin{array}{l} P_{\text{cross}}(\lambda_1) = \sin^2(c_0 \lambda_1 L) = 1 \\ P_{\text{BAR}}(\lambda_2) = \cos^2(c_0 \lambda_2 L) = 1 \end{array} \right.$$

$$k = \sin^2(cL)$$

$$c_0 L (\lambda_1 - \lambda_2) = \frac{\pi}{2}$$

$$N = \frac{1}{2} \frac{\lambda_1}{|\lambda_1 - \lambda_2|}$$

N must be an integer, if it's not I cannot have solutions, so it works only for λ_1 far away from λ_2 , like pump + signal in laser and amplifier.

Precision required



$$P_{B\bar{A}R} = \cos^2 C'L_c = 3,16 \cdot 10^{-3}$$

$$C'L_c = \cos^{-1} \left(\sqrt{3,16 \cdot 10^{-3}} \right)$$

$$\begin{array}{ccc} / & | & \backslash \\ 1,5195 & \frac{\pi}{2} & 1,6270 \\ \text{1,57 nominal} \\ \text{value} \\ (\bar{P}_{B\bar{A}R}=0) \end{array}$$

Accuracy required on the process : $\epsilon = \pm 3, 7\%$ w.r.t. nominal one

between 1,5195 and 1,6270 $P_{bar} < -25 \text{ dB}$.

$$C \propto e^{-\gamma g}$$

$$C' \propto e^{-\gamma(g + \delta g)}$$

So error on L are linear, while error on the gap cause great exp variation on C' so error greater on directional coupler.

If I want $CL = \frac{\pi}{2} + \pi$ I need $\epsilon = \pm 1,2\%$

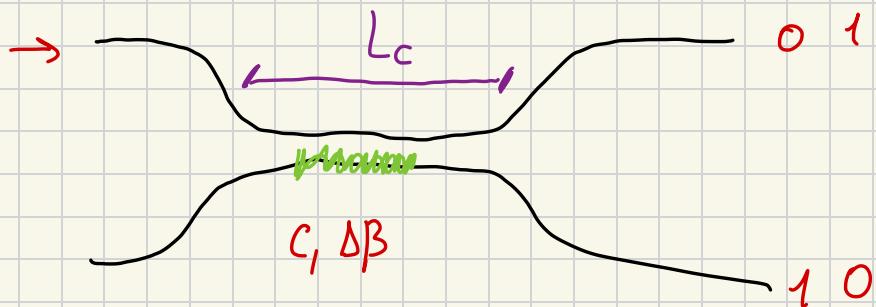


$CL \uparrow \Rightarrow$ more stress on the accuracy of the process

Small error on gap and $\epsilon\%$ is not achievable

SO CHOOSE ALWAYS THE FIRST CL

Switch

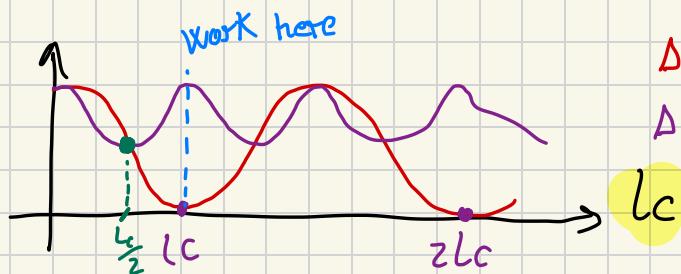


$\Delta\beta = 0$ $L_c = \frac{\pi}{2C} \rightarrow P_{bar} = 0$, but with a particular $\Delta\beta$:

$$P_{bar} = \cos^2 \left(\sqrt{\frac{\Delta\beta^2}{4} + C^2} L_c \right) = 1$$

Induce a change on $\Delta\beta$ (with electro-optic or temperature or UV) to have $P_{bar} = 1$ or 0 , but changing the characteristic:

Phase



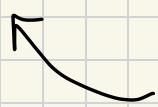
$$Y_1 \text{ mode } \Rightarrow \cos^2(cL_c) \text{ and } \cos^2\left(\sqrt{\frac{\Delta\beta^2}{4} + c^2} L_c'\right)$$

$$\sqrt{\frac{\Delta\beta^2}{4} + c^2} L_c' = \frac{\pi}{2} \rightarrow L_c' = \frac{\pi}{2} \cdot \frac{1}{\sqrt{\frac{4\Delta\beta^2}{9} + c^2}} = \frac{L_c}{2}$$

$$\frac{\pi}{2} \cdot \frac{1}{\sqrt{\frac{\Delta\beta^2}{4} + c^2}} = \frac{\pi}{2c} \cdot \frac{1}{2}$$

$$\frac{\Delta\beta^2}{4} + c^2 = 9c^2$$

$$\Delta\beta = \frac{\sqrt{3}\pi}{L_c}$$



$$\Delta\beta^2 = 9 \cdot (3c^2)$$

$$\Delta\beta = 2 \cdot \sqrt{3}c$$

Difficult to achieve, better other structure!

Dependence on q_{pp}

q_{pp} large \rightarrow

- low loss
- large BIL
- no excitation of higher order modes, low distortion
- large coupler
- "easy" fabrication

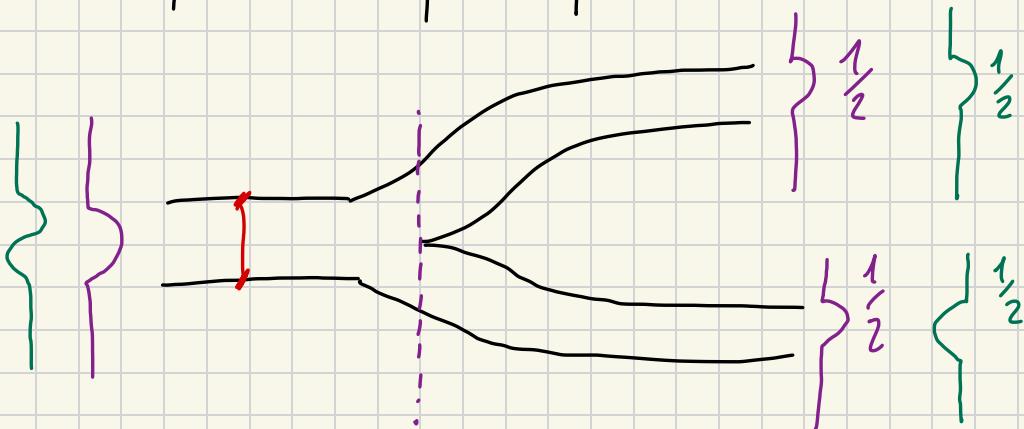
I cannot go too low, I'm limited by the process

$$C \propto \int f_1 f_2 \Delta h \propto e^{-q}$$

Y-BRANCH

Always two input and two output, for
more MUX or Star coupler.

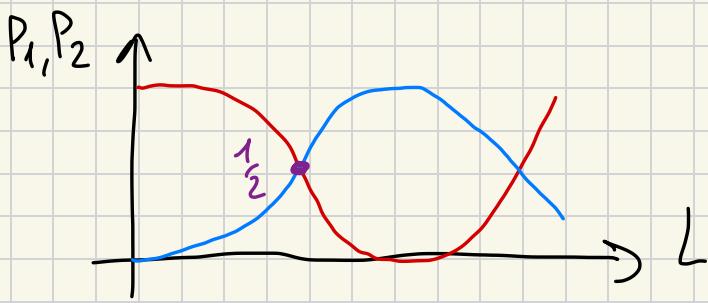
Binodal WG that split apart :



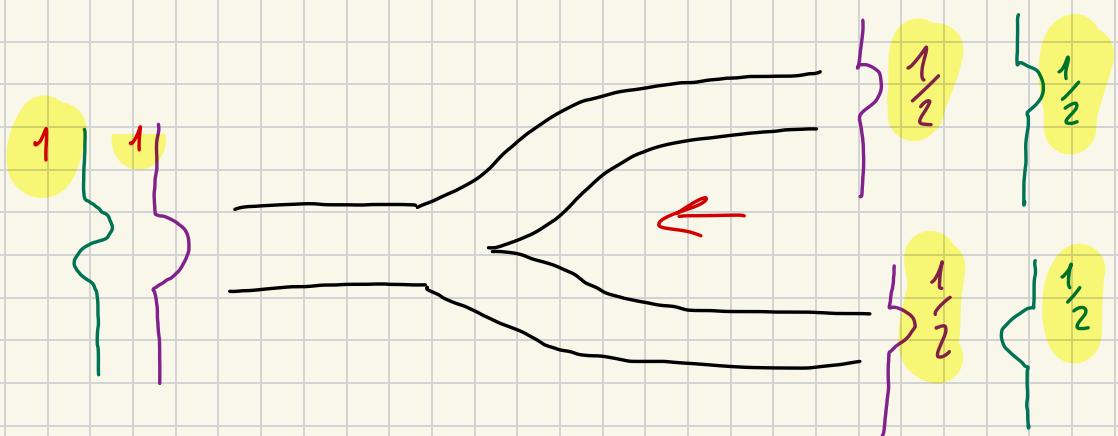
It's a power splitter and with zero phase mismatch
for the fundamental mode and with 180° phase
for the second mode of the single WG :

$$T_C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

If adiabatic the bifurcation, T_C does NOT
depend on λ , extremely robust.



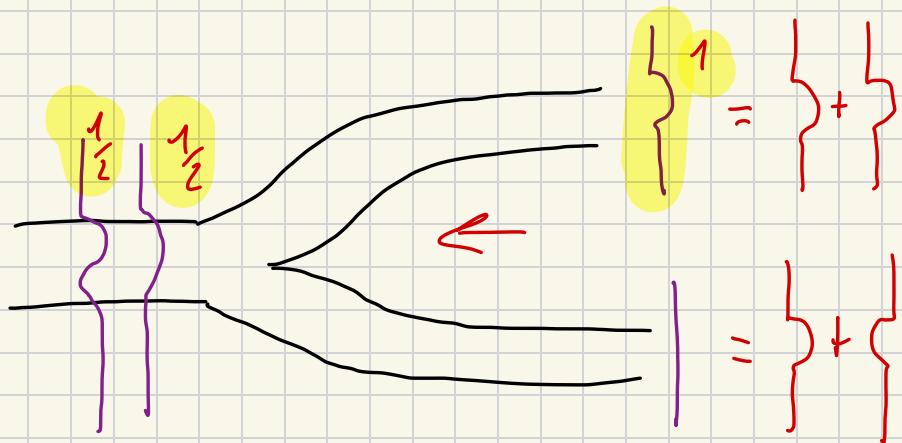
On the reverse direction:



It's a power combiner

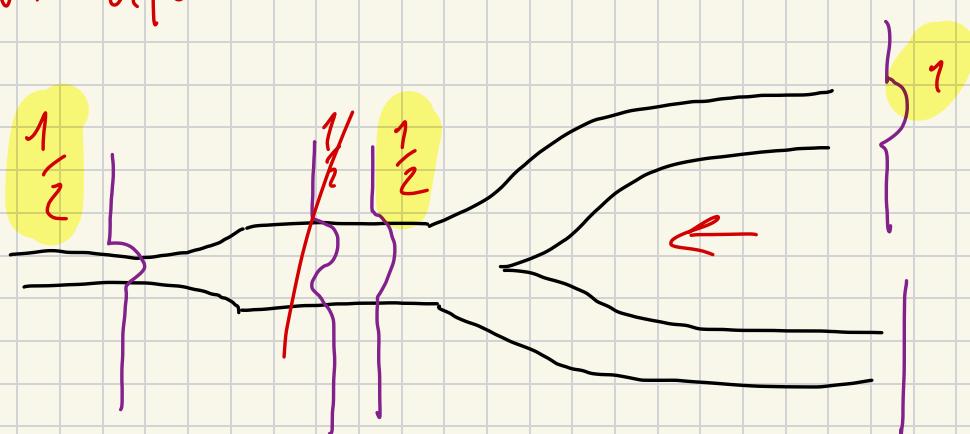
Special case

Field only on one



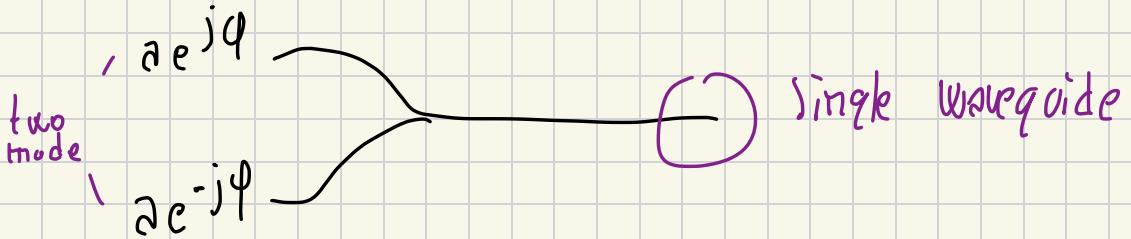
I excite the two modes.

With taper



Only the fundamental there is, but I lost half of the power because I lost the higher order mode.

General case with two wave



$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a e^{j\phi} \\ a e^{-j\phi} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} a (e^{j\phi} + e^{-j\phi}) \\ a (e^{j\phi} - e^{-j\phi}) \end{bmatrix}$$

$$= \begin{bmatrix} a \cos \phi \\ j a \sin \phi \end{bmatrix}$$

1° mode

2° mode (irradiate x monomodal)

So the output amplitude depend on the phase difference between the two input

impossible with wave

IT'S NOT A SUM OF POWER,
BUT AN INTERFERENCE BETWEEN
THE FIELD OF THE TWO.

So with two field with $\varphi = \frac{\pi}{2}$ I can have NO power on the fundamental mode, while with $f=0$ only the fundamental has power.

$$\begin{cases} b_0^2 = a^2 \cos^2 \varphi \\ b_1^2 = a^2 \sin^2 \varphi \end{cases}$$

SO I CAN CONTROL THE PHASE OF THE INPUT TO CONTROL THE AMPLITUDE OF THE OUTPUT

↳ making waves interfere

FILTERS

Characteristic and periodicity (depends on phase so are periodic)

I can also have destructive/constructive interference, so same wave in phase or not.

General TDF:

$$H(f) = \sum_m C_m e^{j\phi_m} e^{-j(2m\pi f T)}$$

interference

FIR

Fourier expansion of the function that I want

$$H(f) = [H_{FIR}(f)]^{-1}$$

IIR

with $T = \frac{\Delta L \cdot n g}{c}$

$L_1 - L_2 \rightarrow$ due path diversi causano uno sfasamento fra le λ ,

The phase delay that I accumulate depends on the λ , so I see the phase shift between two λ :

$$\Delta\phi = \frac{2\pi}{\lambda} (\eta_{\text{eff},1}(\lambda) L_1 - \eta_{\text{eff},2}(\lambda) L_2)$$

$$\Delta\phi(f_m) = \frac{2\pi f_m}{c} \underbrace{\Delta\eta_{\text{eff}}(f_m) L}_{\Delta L_{\text{eff}}(f_m)}$$

f with same $\Delta\phi$

$$\eta_{\text{eff}}(f_m) = \eta_{\text{eff}}(f_0) \pm \frac{\text{FSR}}{2} \frac{\partial \eta_{\text{eff}}}{\partial f}$$

$$FSQ = \frac{C}{n_q DL}$$

The phase shift depends on n_{eff} , while the periodicity on n_q .

↳ Always true for interference filter!

FDF

$$N=1$$

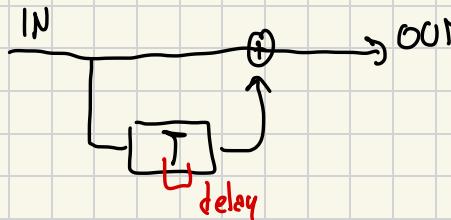
$$H(w) = 1 + \underbrace{C_1 e^{-jw}}$$

if 1

↳ $e^{-j\frac{\omega r}{2}} \left(e^{j\frac{\omega r}{2}} + e^{-j\frac{\omega r}{2}} \right)$

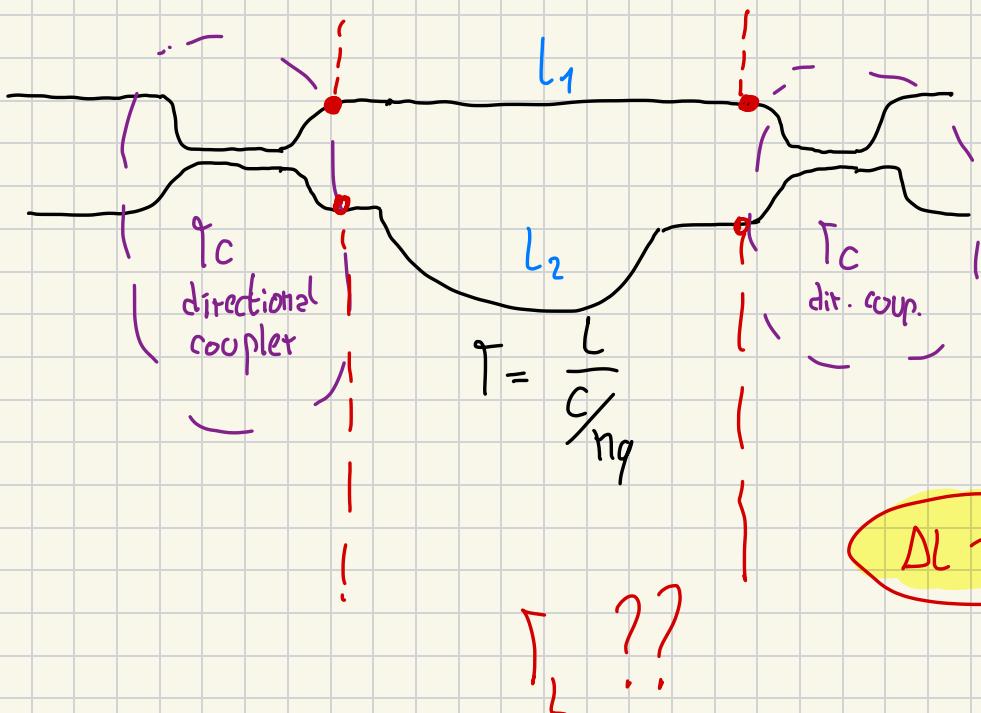
$\cos\left(\frac{\omega r}{2}\right)$

It's a cosine, so a periodic filter, not
fantastic, but it's the building block for FIR
filter of higher order ($N > 1$).



GENERAL

↓
In photonic



$$\Gamma_L = \begin{bmatrix} e^{-j\frac{2\pi}{\lambda}n_{eff_1}l_1}e^{-\alpha L_1} & \underbrace{0}_{\Psi_1} \\ 0 & e^{-j\frac{2\pi}{\lambda}n_{eff_2}l_2}e^{-\alpha L_2} \end{bmatrix}$$

$$= e^{-j\varphi_1} \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Delta\varphi} \end{bmatrix}$$

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \underbrace{\Gamma_{c_2} \Gamma_2 \Gamma_{c_1}}_{\text{Opposite direction}} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Different n_{eff} , but with the same $\Delta\varphi$ in the end behave equally.

$$P_{bar} = |\Gamma_{11}|^2 = \cos^2 \frac{\Delta\varphi}{2} \cos^2 2c_L + \sin^2 \frac{\Delta\varphi}{2}$$

Same length for couplers

$$P_{\text{cross}} = |T_{21}|^2 = \cos^2 \frac{\Delta\phi}{2} \sin^2 2cl$$

$$\text{If } K_1 = K_2 = \frac{1}{2} \Rightarrow cl = \frac{\pi}{4} \text{ (SYNC)}$$

$$\left\{ \begin{array}{l} P_{\text{bar}} = \sin^2 \frac{\Delta\phi}{2} \\ P_{\text{cross}} = \cos^2 \frac{\Delta\phi}{2} \end{array} \right.$$

NO loss
considered

$\Delta\phi$ is periodic in w , NOT in λ

↓
So also
 $H(w)$

↳ Channel of WDM are equally spaced in frequency NOT in λ .

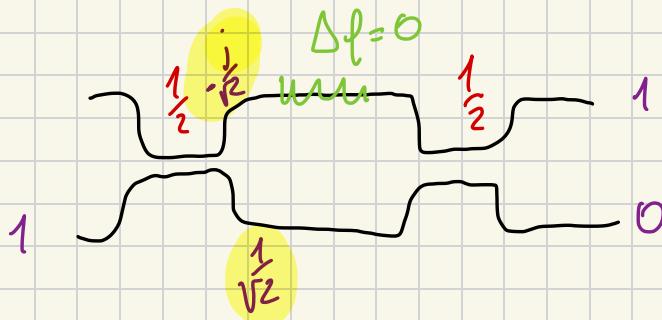
FSR [Hz]

$$\Delta\phi \propto w \text{ or } \frac{2\pi}{\lambda}$$

$w \propto \lambda$, $\Delta\phi$
is periodic

$\lambda \propto \Delta\phi \rightarrow 0$

Esercizi Visivi



$$\Gamma_{C_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

$$b_0 = \frac{1}{2} \begin{bmatrix} a_0 - j a_1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -j - j \end{bmatrix} = -j$$

$$b_1 = \frac{1}{2} \begin{bmatrix} -j a_0 + a_1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -1 + 1 \end{bmatrix} = 0$$

$$\text{If } \Delta f = \pi \Rightarrow |b_0|^2 = 0$$

$$|b_1|^2 = 1$$

Max and min of $P_{\text{bar}}/P_{\text{cross}}$

$$\hookrightarrow \frac{\Delta f}{2} = \pi + 2N\pi + \frac{\pi}{2}$$

↳ if I want P_{bar}
|| 1

Periodicity \rightarrow

$$\text{FSR} = \frac{c}{n_q \Delta L} = \frac{1}{\gamma_q}$$

always because $n_{\text{eff}} \propto w$

$$n_q = n_{\text{eff},0} - w \frac{\partial n_{\text{eff}}}{\partial w}$$

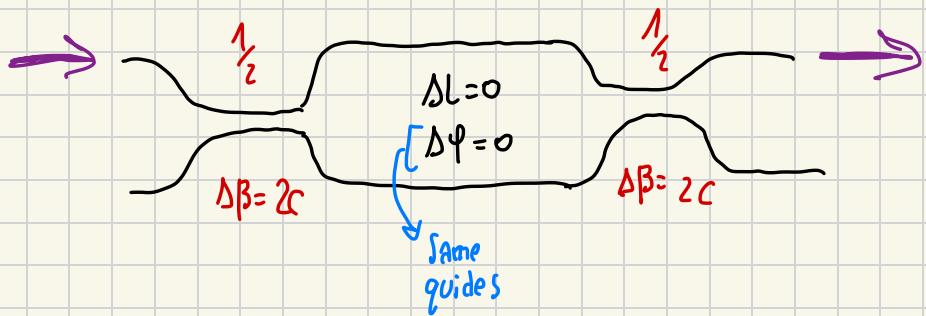
$$\frac{2\pi}{\lambda} n_{\text{eff}} \Delta L = 2N\pi$$

Interested in phase = n_{eff}

What if -3dB coupler is $\Delta\beta = 2c$?

$$\Delta\beta=0 \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

$$\Delta\beta=2c \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



What if two λg different with $\Delta L = 0$?

$$\Delta\varphi = \frac{2\pi}{\lambda} \left(h_{\text{eff},1} l_1 - h_{\text{eff},2} l_2 \right)$$

But h or Temperature :

$$h = h_0 + k_r \Delta T$$

$$k_r = 10^{-5} \quad \left(\begin{array}{l} SiO_2, SiN \\ Si, InP, \end{array} \right)$$
$$k_r = 10^{-4} \quad \left(\begin{array}{l} polymers, TiO_2 \\ \downarrow \\ k_r > 0 \end{array} \right)$$

$k_r < 0$

In Mz I want to switch from cross to bar ($\Delta L = 0$), with an heater :

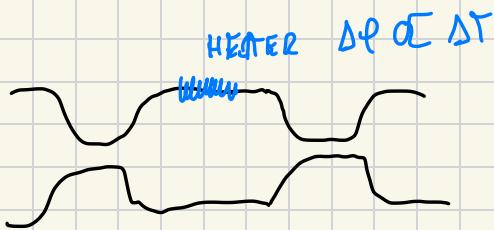
$$\frac{\Delta y}{2} = \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda} \Delta n_{eff} L = \frac{2\pi}{\lambda} k_r \Delta T L = \pi$$

$$\Delta T = \frac{\lambda}{2K + L}$$

For SiO_2 with $L = 1\text{mm}$ $\Delta T = 100^\circ\text{C}$

If $\text{Si} \quad K_r = 10^{-4} \Rightarrow \Delta T = 10^\circ\text{C}$.



Recap MZ with $\Delta\beta = 0$ splitter:

$$\Gamma_{MZ} = \begin{bmatrix} \sin\left(\frac{\Delta\Phi}{2}\right) & \cos\left(\frac{\Delta\Phi}{2}\right) \\ \cos\left(\frac{\Delta\Phi}{2}\right) & \sin\left(\frac{\Delta\Phi}{2}\right) \end{bmatrix}$$

Useful For switch, modulators and variable optical attenuator.

Filter γ_2

$$H(w) = 1 - C e^{-jwT}$$

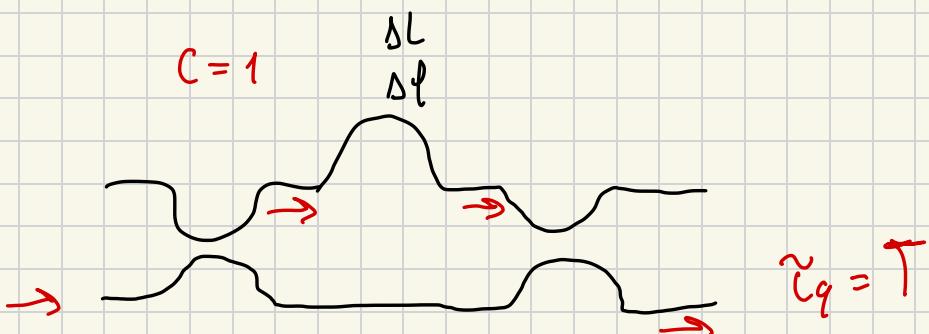
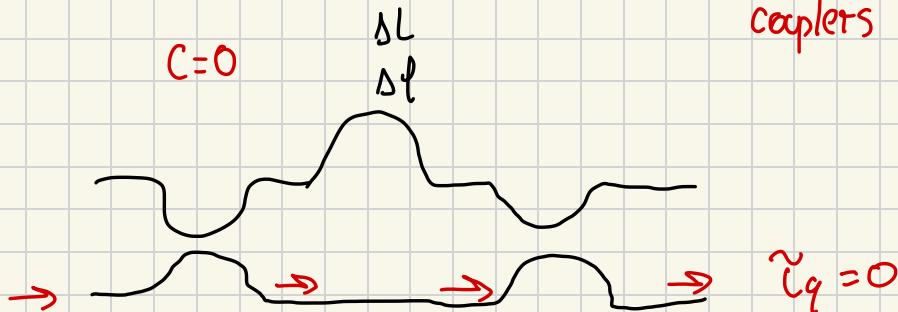
At max/min

$$\omega T = \pi + 2N\pi$$

$$\gamma_q = - \frac{\partial \phi(w)}{\partial w} = \frac{2\pi C}{1+C}$$

speed of light

↳ coupling of the
couplers



$$\text{For } C = \frac{1}{2} \rightarrow \gamma_q = \frac{T}{2}$$

S_0 Hz can be used as tunable delay line.

↳ S_0 in the max/min

$$\gamma_q = c\tau$$

And with bifurcations?

In this case a_2 is the second mode, if monomode guides are used is radiated (power loss if excited).

Particular case



Excited only 1st mode?

At exit only the 1st will exit

General

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Delta t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ e^{-j\Delta f} & -e^{-j\Delta f} \end{bmatrix}$$

monotonic

wg

$$\begin{cases} a_1 = 1 \\ a_2 = 0 \end{cases}$$

$$\frac{1}{2} \begin{bmatrix} 1+e^{-j\Delta f} & 1-e^{-j\Delta f} \\ 1-e^{-j\Delta f} & 1+e^{-j\Delta f} \end{bmatrix} \rightarrow b_1 = 1 + e^{-j\Delta f} \cdot \frac{1}{2}$$

$$= e^{-j\frac{\Delta f}{2}} \left(e^{j\frac{\Delta f}{2}} + e^{-j\frac{\Delta f}{2}} \right) \cdot \frac{1}{2}$$

$$|b_1|^2 = \cos^2 \left(\frac{\Delta f}{2} \right)$$

$$\int_0 - \Delta f = 0 \rightarrow |b_1|^2 = 1$$

$$\Delta f = \pi \rightarrow |b_1|^2 = 0$$

opposite of -3dB
coupler

How to design?

$$1) FSR = 2\Delta f = \frac{C}{n_q D_L} \rightarrow DL$$

$$2) P_{\text{cross}}(\lambda_1) = 1 \rightarrow \frac{2\pi}{\lambda} n_{\text{eff}} \Delta L = N\pi$$

3) Find ΔL

$$N \rightarrow \text{int}(N)$$

FSR'

$\Delta L'$

Right ones

So I optimize P_{cross} for λ_1 , but accepting xtalk $P_{\text{cross}}(\lambda_2) \neq 0$ (low, depend on the application what is acceptable).

$$\Delta L_{\text{eff}} = \lambda \cdot N$$

~ 1000

Now the order of magnitude:

$$\Delta f = \frac{2\pi}{\lambda} h_{\text{eff}} \Delta L \sim 10^{-3}$$

h_{eff} ΔL
 $\sim 10^{-6}$ $h_{\text{eff}} + \delta h_{\text{eff}}$

due to aging,
process error, change
of temperature,
stiction, ...

$$\lambda_0' = \frac{h_{\text{eff}} \Delta L}{N} + \frac{\partial h_{\text{eff}} \Delta L}{N} \frac{\lambda_0}{h_{\text{eff}}}$$

$$\delta \lambda_0 = \lambda_0' - \lambda_0 = \delta h_{\text{eff}} \frac{\lambda_0}{h_{\text{eff}}}$$

$$\frac{\partial \lambda}{\lambda} = \frac{\partial h_{\text{eff}}}{h_{\text{eff}}} = \frac{df}{f}$$

↓

WHATEVER IS THE DEVICE,
 IF THERE IS A SHIFT OF
 THE Neff DUE TO SOMETHING
 CAUSE A SHIFT OF THE TDF
 OF $\delta\lambda$

For example with $\text{FSR} = 200 \text{ GHz}$:

$$\frac{200 \cdot 10^9}{200 \cdot 10^{12}} \cdot 1,46 = \underbrace{\delta_{\text{neff}} = 1,46 \cdot 10^{-3}}$$

A variation of
 neff bigger cause a
 shift the FSR

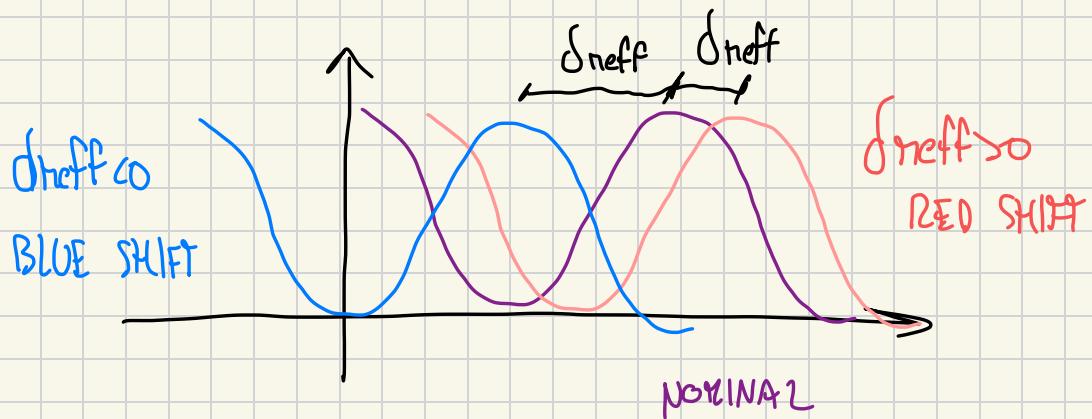
Suppose that

$$\text{Acceptable } \delta F \approx \frac{\text{FSR}}{100} \rightarrow \delta_{\text{neff}} = 1,46 \cdot 10^{-6}$$

$$\text{Then } \delta_{\text{neff}} = k_F \Delta T$$

10^{-5}

Control the temperature
to max change of $1,46^\circ$
otherwise I shift the
FSR of 2 GHz in the
example above



$$P_{bar} = \sin^2\left(\frac{\Delta\phi}{2}\right) = 0$$

$$\frac{1}{2} \frac{\Delta\phi^2}{4} = 0 = \left(\frac{\pi \delta_{\text{neff}} \Delta L}{\lambda_0} \right)$$

$$\delta_{\text{neff}} = \frac{\lambda_0 \sqrt{P_{\text{BAR}}}}{\pi D L}$$

FOR $P_{\text{BAR}} \leq -30 \text{ dB}$

↳ $\delta_{\text{neff}} = 1,5 \cdot 10^{-5}$

Now if $\delta_{\text{neff}} = 0$:

$\underbrace{D L}_{=} = \frac{\lambda_0 \sqrt{P_{\text{BAR}}}}{\pi \delta_{\text{neff}}} = 23 \text{ nm} = 16 \text{ atoms}$

↳ Also this shift the transfer function



NEED FOR PRECISE PROCESSES

Is better to control $\delta_{\text{eff}} / \delta L$ of M₂ or the $ZCL_c = \frac{\pi}{2}$ of the -3dB coupler?

$L \sim 10^{-3} \text{ m}$ so δL so small is very difficult to control

Better spend money to control better ZCL_c .

↳ MORE CRITICAL TO CONTROL ZCL_c , SO THE COUPLER

M₂ is extreme sensible to parameter

↳ It's good also for sensor!

CASCADED M₂'s

↳ SIMPLE

↳ I NEED PERFECTLY ALIGNMENT
OF THE STAGES

↳ DIFFICULTY

LATTICE MZI

↳ LESS SIMPLE

↳ SHADE TDF MORE CONTROLLABLE

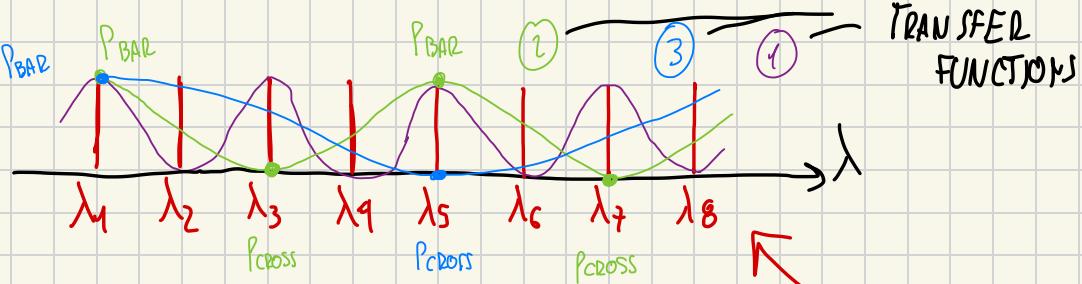
↳ LESS SENSITIVITY ON MISALIGN



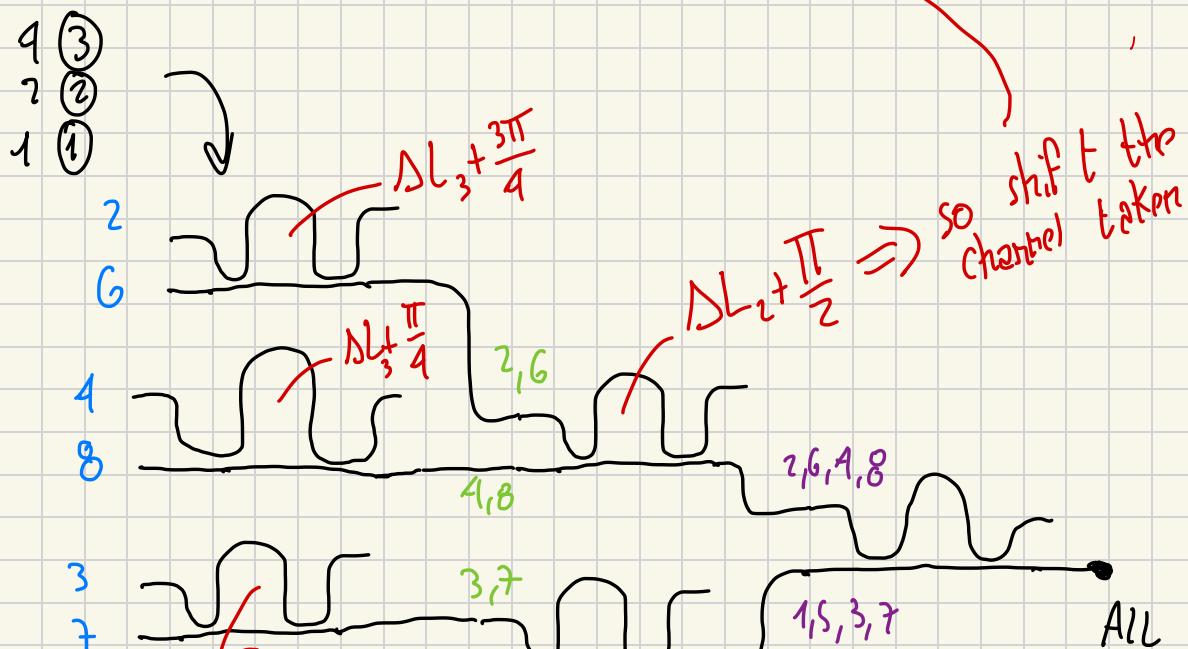
MUX

↳ Combine 8 λ in 1 output

I need 7 MZ

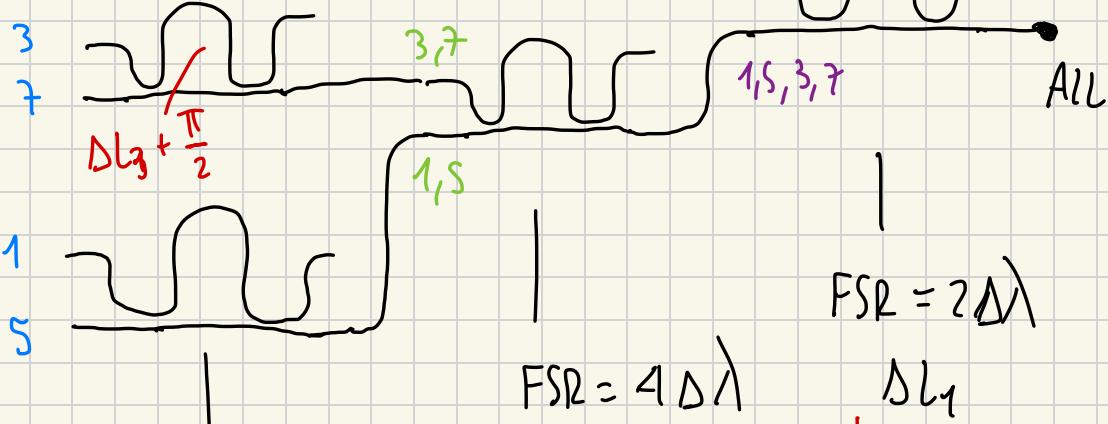


TRANSFER FUNCTIONS



\Rightarrow so shift taken
channel

All



$$FSR = 2\Delta\lambda$$

$$FSR = 4\Delta\lambda$$

$$\Delta L_2 = \frac{\Delta L_1}{2}$$

ΔL_1
Largest one
with FSR
lowest

$$\Delta L_3 = \frac{\Delta L_1}{4}$$

$$FSR = 8\Delta\lambda$$

How to design every MZI

↳ $F_{SR} \rightarrow DL$

↳ $P_{cross}(\lambda_i) = 1$

↳ $N \rightarrow DL'$

There are losses (exits from unused output) but the overall function is preserved.

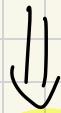
DEMOX

It's the reversed of the MUX

↳ Now, consider losses, they are cross talk (the two output are used both, so leakage is not radiated)



NO WAY TO SEPARATE
THE CHANNELS AFTER X-TALK



MORE DIFFICULT TO DESIGN
A DEMUX

For more channel ?

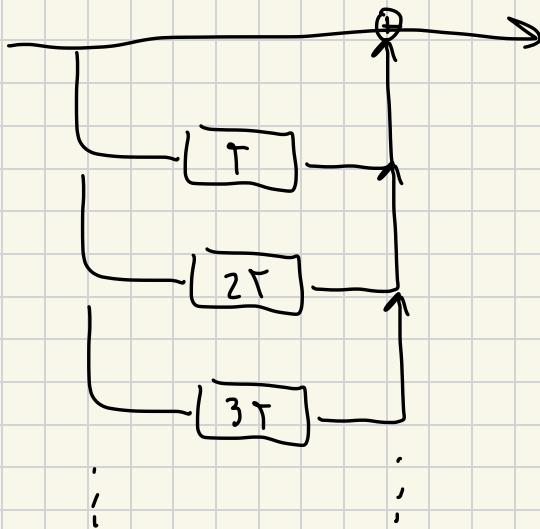
No, it becomes huge and difficult to design



A W G

Arrayed Waveguide grating

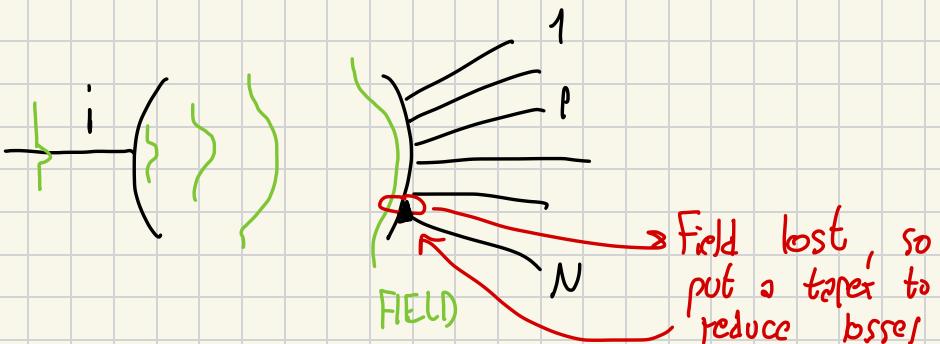
How to implement higher order FIR?



MZ is difficult

STAR COUPLER

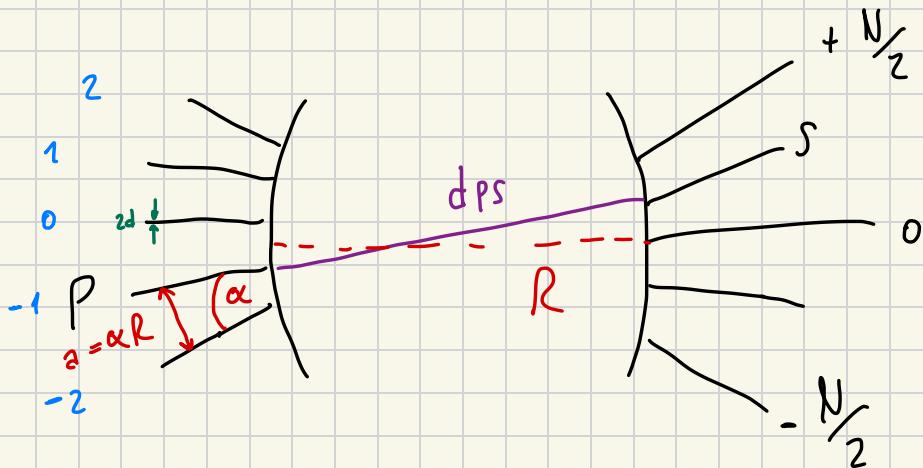
In the FPR the field is confined vertically but NOT laterally.



The field exiting the input has a Gaussian shape and the TDF:

$$T_{ip} = \frac{e^{-\frac{(y-y_0)^2}{2N}}}{\sqrt{N}} e^{j\phi_{ip}}$$

I can have M input :



With this convention:

$$dps = R \left(1 - ps \alpha^2 \right)$$

Note if $p=0 \rightarrow$ d_{ps} equal for all output

The width of the (enlarging while bisecting) beam is :

$$W(R) = \frac{d}{2} \sqrt{1 + \left(\frac{4\lambda R}{\pi n_{eff} d^2} \right)^2}$$

The beam is spreaded reaching the outputs, so choose :

$$R = \frac{N \pi n_{eff} a^2}{2\lambda} \quad \text{to have } \gamma = 0,5$$



FEW OUTPUT (MOLTO VICINI)

THE FIELD IS UNIFORM (THEY SEE THE SAME POWER APPROXIMATELY) BUT I LOSE A LOT

↓
OR VICEVERSA

$\alpha \uparrow\uparrow$, uniform ↑↑
 $\alpha \downarrow\downarrow$, uniform ↓↓

↓
Depend on the application

How to choose ω ?

$\omega \rightarrow 0$: coupling \rightarrow AVOID IT !

$\omega \rightarrow \infty$: losses ↑↑ \rightarrow AVOID IT !

LEZIONE 16 1:36:00

↳ DI CIO CHE ESCE E ARRIVA
NELLA ZONA DI FAR FIELD,
SUGLI OUTPUT NE ARRIVA
LA TRASFORMATA DI FOURIER

SE VUO) CAMPO UNIFORME SU TUTTI
GLI OUT, QUINDI UN RECT
DEVI PARTIRE DA UN SINC E
SI PUÓ ACCOPPIANDO PIÙ GUIDE
D'ONDA IN INPUT.

b) Start gaussian \rightarrow end
gaussian (transform to itself)

AWG o WGR

grating Youtor

Less common name

It's like phase array. It's better to have
 $a = (2 \div 3) d$ and the w_g on the extren more
next to each other.

1° star coupler :

$$T_{PS} = \frac{1}{\sqrt{M}} e^{j \varphi_{PS}}$$

$$\varphi_{PS} = \frac{2\pi}{\lambda} h_{eff} d_{PS}$$

$$\simeq R(1 - ps \alpha^2)$$

R must be a certain value
to have an exact phase
front on the outputs

$$\frac{M h_{eff} \alpha^2}{2\lambda}$$

2° Star coupler

↳ It's a phase array, depending on the
phase of the input focalize the beam
on an output leg

↳ This depends on the λ (on which depends also $\Delta\phi$)

↳ So it's a phase array on a circle that focus the beam on a circle

Change the phase \rightarrow change the phase front

Change the point of focus

SLIDE 12-13-14

(3) gratings

$$T_s = e^{j\psi_s}$$

Total :

$$E_{\text{ASP}} = \frac{1}{\sqrt{M}} e^{j\varphi_{ps}} e^{j\varphi_s} \frac{1}{\sqrt{M}} e^{j\varphi_{sq}}$$

\downarrow

$$\frac{2\pi}{\lambda} \underbrace{h_{\text{eff}}}_{\text{heff}} (L_0 + s \Delta L)$$

\downarrow

Assume that h_{eff} is equal to star coupler



NOR true, but NOR so wrong,
make simple the GDF

$$\Delta\varphi_{pq} = \varphi_{psq} - \varphi_{p,s-1,q} = \frac{2\pi}{\lambda} h_{\text{eff}} (\Delta L - R(p+q) \alpha^2)$$

Now sum the contribution of all the waveguide in the grating :

$$E_{qp} = \sum_{s=0}^{M-1} E_{psq} = \frac{1}{M} \sum_{s=0}^{M-1} e^{j\varphi_{psq}}$$

$|\Gamma_{pq}|^2 = \frac{1}{M^2} \frac{\sin^2(M \frac{\Delta\varphi_{pq}}{2})}{\sin^2(\frac{\Delta\varphi_{pq}}{2})}$

oscillate M times faster than

If $\Delta\varphi_{pq} \rightarrow Q 2\pi : \sin^2(\cdot) \rightarrow 0$

$$|\Gamma_{pq}|^2 = \frac{1}{M^2} \frac{M^2 \frac{\Delta\varphi_{ps}}{4}^2}{\frac{\Delta\varphi_{ps}}{4}} \rightarrow 1$$

Special case $M=2$

$\hookrightarrow |\Gamma_{op}|^2$ is the TDF of a MZ.

How λ , ΔL , R are related to the TDF?

The TDF is periodical, I'm interested in the position of the max, the band and the out of band rejection.

The max are when :

$$\Delta \varphi_{pq} = \frac{2\pi}{\lambda} n_{eff} (\Delta L - R(p+q)\alpha^2) = 2Q\pi$$

$$\lambda_{pq} =$$

$$\frac{n_{eff} (\Delta L - R(p+q)\alpha^2)}{Q}$$

$$\lambda_{pq} = \frac{n_{eff} \Delta L}{Q} - \frac{n_{eff} R(p+q)\alpha^2}{Q}$$

$$= \lambda_{00} - (p+q) \Delta \lambda$$

Doesn't depend

on the star coupler

(from port 0 to 0)

Channel spacing

What is the TDF of p and q-1?

It's the same but shifted of $\Delta\lambda$. The other ports have max where the others have zeros.

M-1 zeros

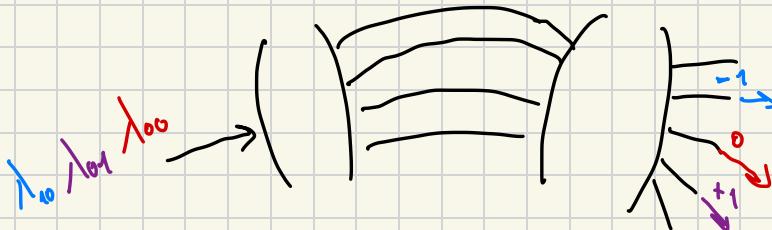
Each TDF is shifted of $\Delta\lambda$

$$FSR = \frac{C}{nq \Delta L}$$

$$\beta = \frac{FSR}{M}$$

↳ MUX/DEMUX
AT THE SAME

TIME



Design ?

(1) FSR $\rightarrow \Delta L$

(2) $\Delta L \rightarrow Q \leftarrow \frac{\lambda_0}{\Delta L'}$

(3) $Q + \underline{\Delta \lambda} \text{ given} \rightarrow R \cdot \alpha^2 \rightarrow \text{Then play with the star coupler}$

(4) Find M depending on B

It's an approx !

$$\hookrightarrow M_{\text{eff}}^{\text{sc}} + M_{\text{eff}}^{\text{gratings}}$$

Because in the SC I can have reflected waves that bounce back and forth, changing phase

↙

Do not change too much the TDF,
but this is the reason why star
couplers have strange shape (to
radiate away the reflected power)

↓

So in the star coupler I have different λ

↓

$$\lambda_{\text{pos}} = \lambda_0 - (\text{pt} + \alpha) \Delta \lambda$$

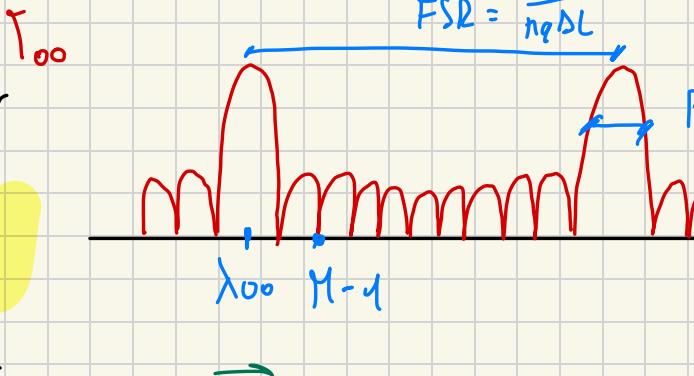
The right one

$$\frac{\text{heft } \Delta L}{Q}$$

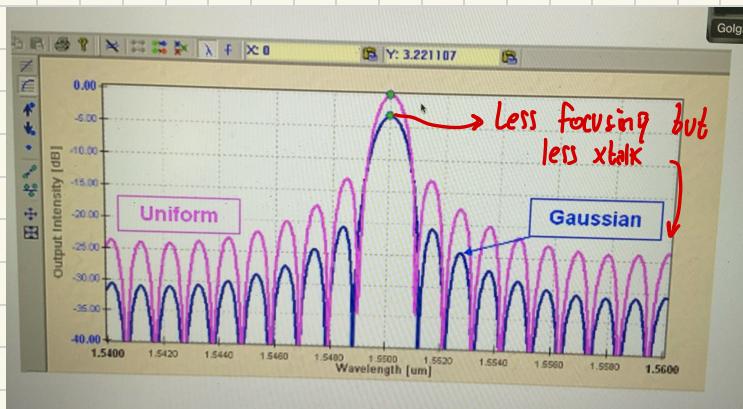
$$! \quad \frac{n g R \alpha}{Q}$$

(FIR)

→ Every time the number of channel increase,
increase the number of ZEO and $B \downarrow \downarrow$.



Then there are AWG with different shape, like gaussian where you pay a little attenuation in band while rejecting more



REMEMBER TO USE TAPER BETWEEN
STAR COUPLER AND INPUT/OUTPUT WAVEGUIDES

↳ Catch the max possible of field
and avoid reflections reducing losses.



MORE SIMPLE DESIGN AWG
BUT REALIZING AND OPTIMIZING
IT IS MORE DIFFICULT

Example of design

" $\Delta\lambda$ " = 100 GHz

8 channels

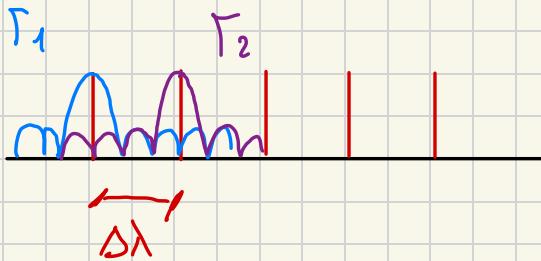
25 Gbit/s NRZ \simeq 20 GHz BW of signal

$n_g = 1,5$

$n_{eff} = 1,15$

↳ PHOTONIC CIRCUITS ON WAVEGUIDE

$$FSR = \frac{c}{n_q \Delta L} = N \cdot "D\lambda" = 800 \text{ GHz}$$



$$\Delta L = 250 \mu\text{m}$$

$$\lambda_0 = \frac{\text{neff } \Delta L}{Q}$$

$$= 1550 \text{ nm}$$

$$Q = 233,8$$

$\hookrightarrow Q = 234 \rightarrow \Delta L' = 250,14 \mu\text{m}$

For SC:

$$\Delta\lambda = \frac{n_q R \alpha^2}{Q} = "100 \text{ GHz}" = 0,75 \text{ nm}$$



$$\text{From here S-C } \hookrightarrow R \alpha^2 = \frac{\Delta\lambda Q}{n_q} = 129,8 \cdot 10^{-9}$$

Glass on silicon \rightarrow $W \approx 5 \mu\text{m}$

$$R \alpha^2 = \frac{a^2}{R} \rightarrow \text{scelgo is } 15 \mu\text{m}$$

$R = 1,8 \text{ mm}$

Ahd then how many wq in the array?

$$M = \frac{FSR}{B} = \frac{800 \text{ GHz}}{20 \text{ GHz}} = 40$$

If $M > 40$ | filter the signal, so M always less or equal to $\frac{FSR}{B}$.

↳ But $M \downarrow \downarrow$, number of zero $\downarrow \downarrow$, $\uparrow \uparrow$ xtalk (out of band lobes)

Now I need to keep aligned the "filter" with the signal

↳

HOW WELL I HAVE TO CONTROL
TEMPERATURE, δn_{eff} , $\delta \Delta L$?

$$\Delta T = 0,1^\circ\text{C} \text{ acceptable for } \underline{\text{Glass on Si}}$$
$$\hookrightarrow \frac{\delta \lambda}{\lambda} = \frac{\delta n_{\text{eff}}}{nq} = \frac{K \Delta T}{nq} \quad (10^{-9} \text{ on Si})$$

With this ΔT depending on the material I have FSR of :

$$\Delta T = 1^\circ\text{C} \left(\begin{array}{l} \text{SiO}_2 \rightarrow 1 \text{ GHz} \\ \text{Si, InP} \rightarrow 10 \text{ GHz} \end{array} \right) \underbrace{\text{shift}}_{B=20 \text{ GHz, small impact}}$$

$$\hookrightarrow \text{Impact } \frac{t}{nq} \rightarrow B = 20 \text{ GHz}$$

$$\hookrightarrow$$

$$\Delta T = 0.1^\circ C$$

$\hookrightarrow Si \rightarrow 1 GHz$

shift

For these materials $M \downarrow$ most, like $M=38$, so $B=27 GHz$ and I can attenuate the effect of $0.1^\circ C$ in Si. For $\Delta T \uparrow\uparrow$, greater complexity!

Acceptable shift are $\frac{1}{10} \frac{FSR}{M} = " \delta \lambda "$

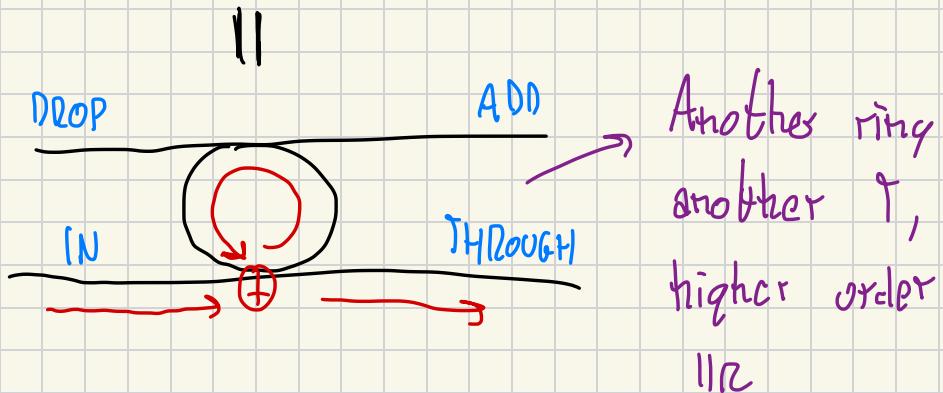
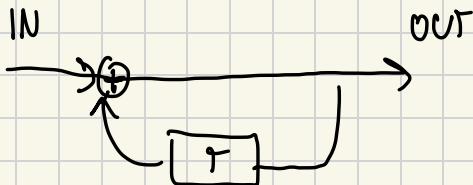
But δ_{ncff} can be also given by other phenomena.

\hookrightarrow In a datacenter every 6 months change something.

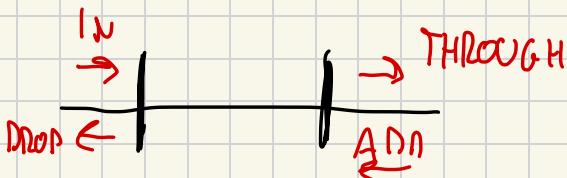
AWG usually are on Glass on Silicon

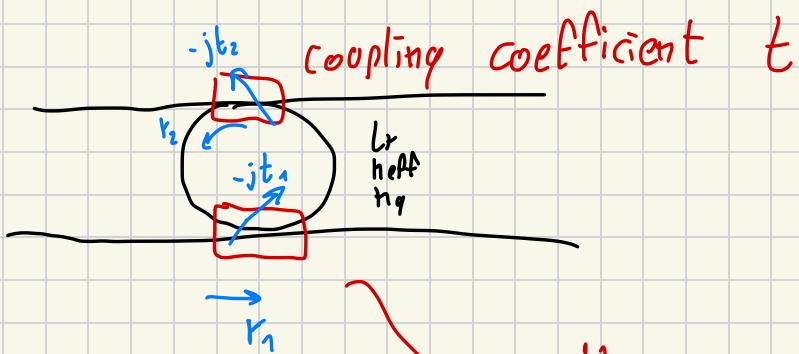
DING RESONATOR

↳ $\text{IIR} \rightarrow \text{opposite of FIR}$

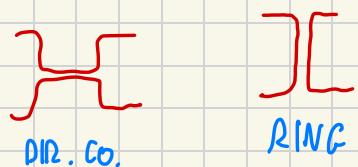


The ring is a resonator and it's a fancy
Peyot cavity, in fact another mode to imple-
ment a resonator is to use partial reflector
mirror:





like a coupler, but
different matrix:



$$\gamma = e^{-\alpha L_r} \leq 1 \quad (\text{LOSSES})$$

For couplers:

$$T_C = \begin{bmatrix} \gamma_1 & -jt_1 \\ -jt_1 & \gamma_1 \end{bmatrix} \quad r_1^2 + t_1^2 = 1$$

For ring, on the through port:

$$H_t(\omega) = r_1 + \underbrace{(-j t_1 \gamma r_2) e^{-j \beta l_r}}_{\text{Light with one round}} (-j t_1) +$$

Light with one round

+ ... +

second round

+ ... +

INFINITE RESPONSE

FILTER

$$H_t = \frac{r_1 - j r_2 e^{-j \beta l_r}}{1 - j r_1 r_2 e^{-j \beta l_r}}$$

$$\beta = \frac{2\pi}{\lambda} n_{eff}$$

Depending on λ H_t can be null or different from 0.

$$H_d = 1 - H_t = \frac{-t_1 t_2 \sqrt{\gamma} e^{-j \beta \frac{l_r}{2}}}{1 - j r_1 r_2 e^{-j \beta l_r}}$$

H_d has not zero but only a pole (can go to ∞) while H_t has also a zero

$H_d \neq 0$ always

When $H_t = 0$?

together

$$\left\{ \begin{array}{l} e^{-j\beta l_r} = 1 \\ r_1 = j r_2 \end{array} \right.$$

$$\rightarrow \beta l_r = N \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda} h_{\text{eff}} l_r = N \frac{2\pi}{\lambda}$$

$H_d = 1$
AT λ_0

RELEVANT λ
OF THE
RING

$$\lambda_0 = \frac{h_{\text{eff}} l_r}{N}$$

Add in phase with the light arriving from
the ring

$$k_1 = k_2 \gamma$$

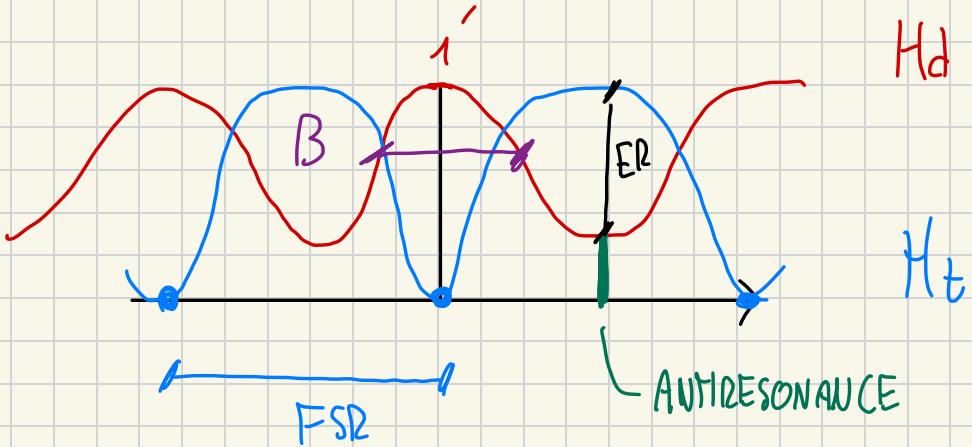
If the two directional coupler are identical ($\gamma_1 = \gamma_2$) apart for the losses, I always have $H_t = 0$ at resonance.

light enter \rightarrow couple in the ring \rightarrow some goes in the H_t , some in the ring \rightarrow

Keeps recirculating in the ring and at λ_0 all exit from drop

If no reflection nothing exit from ADD port.

$$\gamma = 1 \text{ supposed } (\gamma \leq 1)$$



H_d cannot go to zero!

$$FSR = \frac{C}{\pi g L_r}$$

$$\beta = \frac{FSR}{\pi} \frac{k}{\sqrt{1-k}}$$

assuming identical couplers
and $\gamma = 1$

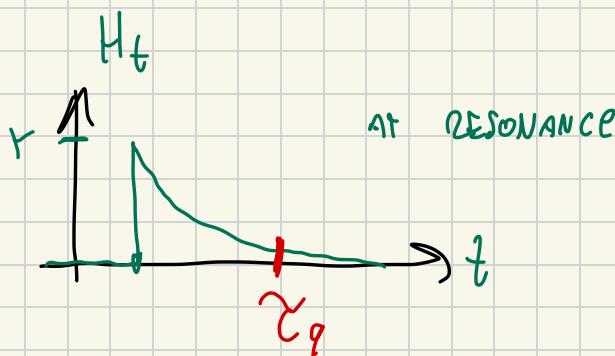
Antiresonance (between the two resonance), when the light that enters the tiny arrive π shifted to the first coupler, adding in antiphase.

Here $H_t - H_d = \text{Extinction ratio}$

$$ER = \frac{(k-2)^2}{k^2}$$

What happens during transient?

Not seen here, all off this is at steady state.



Design?

Given χ_r , τ_{eff} , χ_q , γ and k find B , FSR
 λ_0 and ESR

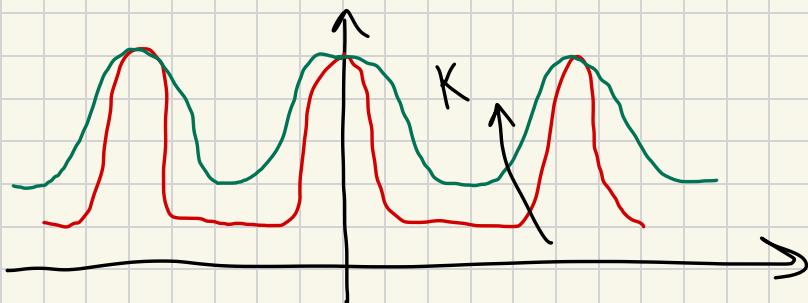
Difference with Hz

Mz is a sine like TDF and BW cannot be changed

↳ With ring, increasing k) can change the BW and ER



Difficulty : have large BW
with low ER



Other important parameter to see how selective is the ring with respect to FSR :

$$F = \frac{FSR}{B}$$

FINESSE

And the quality factor:

$$Q = \frac{f_0}{B} = N \cdot F \frac{h_{eff}}{h_q}$$

from λ_0

$$Q = \omega \cdot \frac{\text{energy}}{\text{power loss}}$$

↳ For any resonant structure

- Power loss ↗
- BAD power loss due to imperfection
 - WANTED ↘ power loss by loading the cavity
(drop port extract power from the resonant ring to work).

$$\frac{1}{Q} = \frac{1}{Q_L} + \frac{1}{Q_Y} \approx \frac{1}{Q_L} \text{ if } Q_Y \gg Q_L$$

Due
to the
coupler

attenuation

(INTRINSIC Q)

$$\hookrightarrow \frac{1}{Q_Y} = 0 \text{ if } \gamma = 1$$

Given Q I can design the ring

\hookrightarrow MORE SELECTIVE ? KM
UNTIL ATTENUATION $(\frac{1}{Q_L})$
BECOMES COMPARABLE WITH
 $\frac{1}{Q_Y}$.

But in photonics we don't work with first order resonance ($N=1$), but higher order, so the most important parameter is F, not Q.

What does F tell me?

How much at resonance | enhancing all the other phenomena.

↳ Time to one round trip:

$$T = \frac{L_r \sqrt{\tau_{tg}}}{C}$$

$$FSR = \frac{1}{T}$$

in reality infinite
with decreasing
Amplitude

q

in average

The finesse is the number of round trip, related to the photon life time

So the group delay:

$$\tau_q = F \cdot T$$

Don't see chromatic dispersion here

I stay in the ring for that much before going to drop port, so the insertion losses approx is (at resonance) :

$$IL \approx F \cdot \gamma$$

How well do I have to control ΔT ?

$$\frac{\Delta\lambda}{\lambda} = \frac{\delta n_{eff}}{n_q} = \frac{k \Delta T}{n_q}$$

F times better than MZ, because as AWG and MZ with a shift of 2π in the phase I shift of one FSR the spectrum.

$$\beta = \frac{FSR}{F}$$

If 1 shift of $\frac{2\pi}{F}$, 1 shift of one β (not acceptable):

$$F = \pi \frac{\sqrt{1-k}}{k}$$

Without loss
↓

Change k to change F ,

valid until $Q_J > Q_L$,

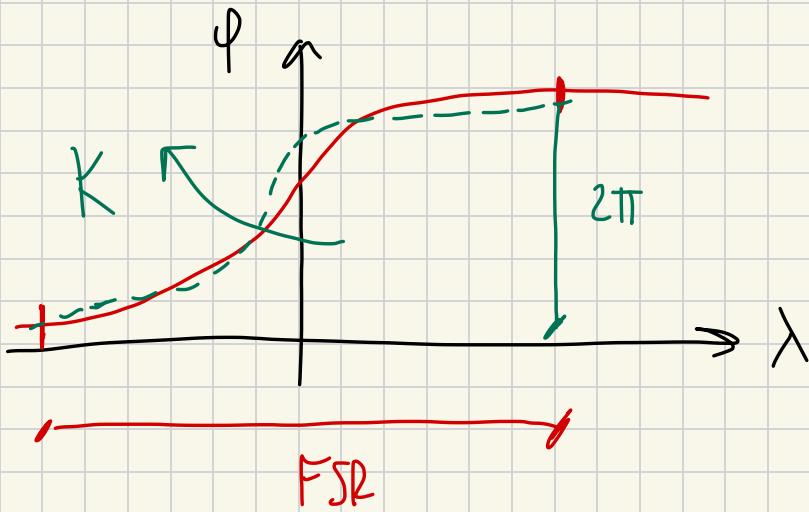
so $\gamma \rightarrow 1$

How much time takes the transient?

$$\tau_q = F T = \frac{1}{B}$$

The phase response

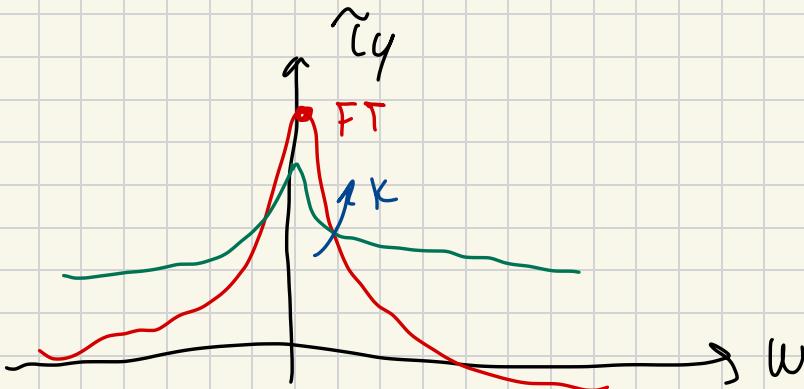
In MZ the phase is linear, here NO!



jump of 2π between the two resonance over one FSR

↳ increase B increase the phase jump around resonance

$$\tilde{\tau}_q = \frac{d\varphi}{d\omega} = \text{slope of } \varphi$$



Selective filter $\rightarrow K \downarrow \downarrow, B \downarrow \downarrow, \tilde{\tau}_q \uparrow \uparrow$
at resonance

CHROMATIC DISPERSION = $\frac{d\tilde{\tau}_q}{d\omega}$

DETERRIMENTIVE IMPACT
ON THE SHAPE OF THE PULSE

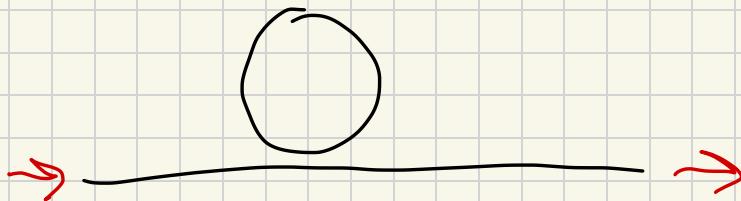
Phase shifter

$r_2 = 1$, $b_2 = 0$, ALL PASS FILTER



Only phase shift

$$T(\omega) = \frac{1 - r e^{-j\beta L_r}}{1 - r^* e^{-j\beta L_r}}$$



All the light going IN go in OUT without losses so $T = 1$ for every λ . But if I have losses I can have notch in the TDF, because I enhance losses during round trip at λ_0 .

At λ_0 , $e^{-\beta b^2} = 1$ so if $\gamma = 1 - t^2$, $\Gamma \rightarrow 0$



Critical coupling
(notch at λ_0)

The notch goes to zero only when $\gamma = 1 - t^2$,
otherwise with $\gamma \neq 1 - t^2$ there are notch at λ_0
but not zero.



Only with critical coupling I
can switch off the light at λ_0
on the output



Change phase of ring |
shift Γ

modulator

No γ , No notch!

Easier to design but more sensitive
to tolerance).



Field intensity can be big in the ring causing greater losses depending on material used and change in μ_{eff}

↳ So the filter shift



We try to
work with
 $y \rightarrow 1$

But can go out of resonance
and field intensity ↓↓



So the signal can cause self modulation
with itself



Only in material where neff GC field intensity (most dielectric). I need high power to see these effects.

$$\eta = \eta_0 + \eta_1 I$$

Kerr effect

But also losses depends on I , so also IL .

When happen resonance?

When $l_r = N\lambda_0$, without phase added on the ring by something. In this case I have to add this phase shift when finding resonance:

$$\frac{2\pi}{\lambda} \text{neff } l_r + \varphi_{TH} + \varphi_R \dots = 2N\pi$$

RING CAN SLOW DOWN THE LIGHT

$$V = \frac{C}{\hbar q L} \quad (\text{no } \uparrow \text{ and } L \text{ const, } V \downarrow)$$

$$\Gamma = \frac{L}{C} \hbar q$$

but $\Gamma \uparrow$, also losses increase

$$I_L = e^{-\alpha L \hbar q}$$

NO MEMORY

WITH PHOTONICS

Backscatter



$$\tau_b = F^2 \cdot \tau_{b'}$$

(channelling of losses)

MZ - RING FILTERS

The enhancement of loss are on the transition of the TDF, NOT in the flight BAND.

TEST COST A LOT



BETTER SEND DEVICE TO TEST
ONCE THE DEVICE WORK



ELECTRONIC FEEDBACK
(heater, UV trimmable, electro-optic...)

Why slow light?

To make shorter device.

↳ But for memory instead?

Not possible
in photonic

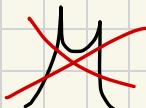
[- Must be small]

[- loss must not corrupt data

Not possible
in photonic

PHOTONIC IS GOOD TO MOVE
THINGS AROUND

Oscillation in group delay are bad,
Keep it as smooth as possible!



FOR ANY STRUCTURE

MAGNETO-OPTIC

gg% of lasers has an isolator to protect it from reflections coming from illuminated wq.

$$\Gamma_c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \text{NOT SYMMETRIC}$$

Until now all devices were symmetric, what break symmetry here?

↳ Only if device is NOT reciprocal,
NOT asymmetric

NON RECIPROCITY

Some materials in presence of a magnetic field change the ϵ tensor that is no more symmetric.

↳ But the material must sensitive to \vec{H} or also in time varying or non linear material.

I'm interested in STEM and RECIPR material with $\vec{H} = 0$, and ASYM and NON RECIPR. with $\vec{H} \neq 0$

↳ So there's optical activity

$$\epsilon = \begin{bmatrix} \epsilon_{\perp} & j\delta\epsilon & x \\ j\delta\epsilon & \epsilon_{\perp} & x \\ x & x & \epsilon_{||} \end{bmatrix}$$

$$\mu = 1$$

Transparent

So αH

What happens to a plane wave inside this material?

Wave equation in form of matrix:

$$\begin{bmatrix} -\beta^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_{\perp} & -j\omega^2 \mu_0 \epsilon_0 \delta\epsilon \\ -j\omega^2 \mu_0 \epsilon_0 \delta\epsilon & -\beta^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_{\perp} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0$$

Now E_x and E_y are coupled. With $\delta\epsilon=0$, the solution is $e^{-j\beta z}$. Now in order to have a solution the determinant must be null.

$$\beta_{R,L} = \frac{2\pi}{\lambda} \sqrt{\epsilon_{\perp} \pm \delta\epsilon}$$

EIGENVALUE

$$E_x = \pm j E_y$$

EIGENVECTOR

Right
and
left
because

$E_z = 0$ (plane wave) and E_x in that way
is a circular polarization wave.

↳ linear polarized wave cannot
be a solution in a material
like that.

So the circular right pol. wave and left wave
have :

$$n_R = \sqrt{\epsilon_{\perp} + \delta\epsilon}$$

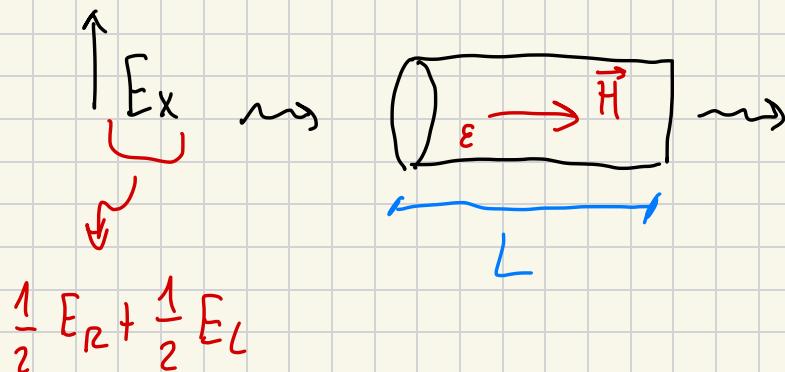
$$n_L = \sqrt{\epsilon_{\perp} - \delta\epsilon}$$

$$B_c = n_R - n_L \approx \frac{\delta\epsilon}{\sqrt{\epsilon_{\perp}}}$$

Circular Birefringence

Exercise

Linear vertical polarized wave in input (NOT a solution, but it's a combination of two circular polarized) :



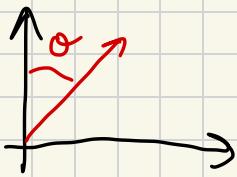
$$E_R e^{-j\beta_R L} + E_L e^{-j\beta_L L}$$

Now come back to x, y and the output is:

$$e^{j \frac{1}{2} (\beta_R - \beta_L) L}$$

$$\sigma = \frac{\omega}{2} \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_L}} \delta \epsilon L$$

θ is the rotation of the output field w.r.t.
the input one. So at output I have E_x
but rotate of θ :



OPTICAL ACTIVITY = ROTATE POLARIZATION
KEEP ELLIPTICITY

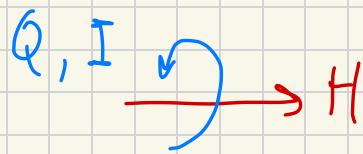
θ can be seen as $\delta\epsilon$ of \vec{H} :

$$\theta = V \cdot H \cdot L$$

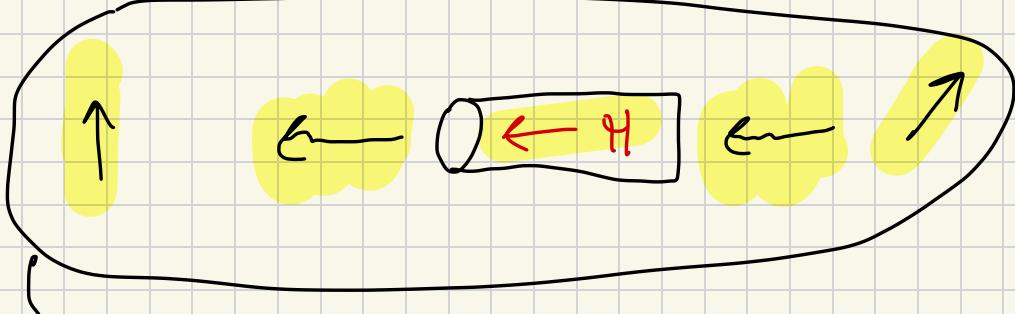
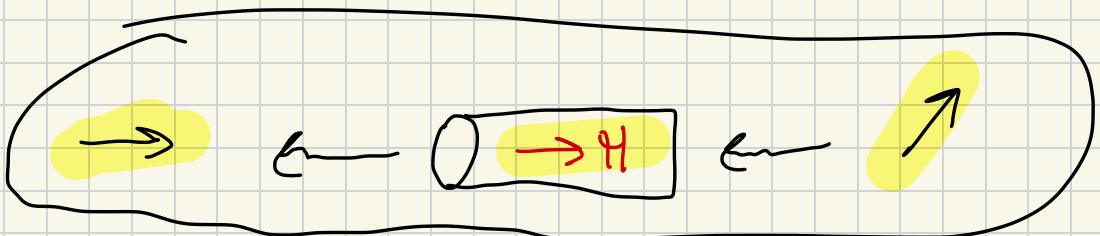
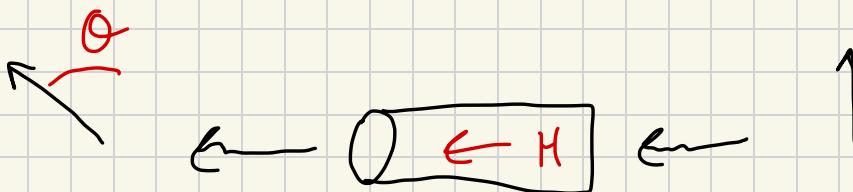
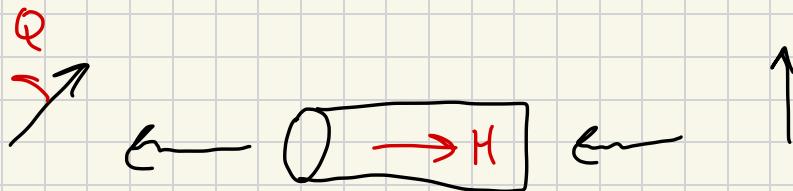
Voigt constant

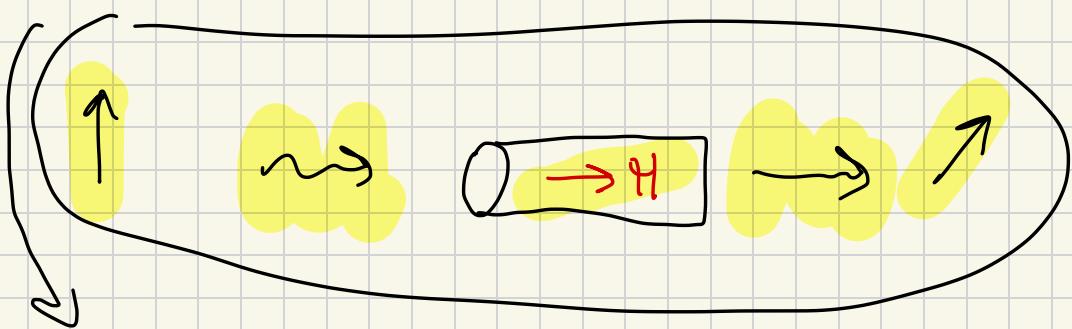
The rotation depends on H so has direction.

↳ same direction of the current



What if I inverted the direction?





To come back as before I need to change \vec{H} (NON RECIPROCAL), changing only the direction is not sufficient.

**ISOLATOR \rightarrow BULKY (NOT POSSIBLE IN
INTEGRATED OPTICS)**

CIRCULATOR \rightarrow 3/4 PORTS CIRCUIT

BRAGG GRATINGS

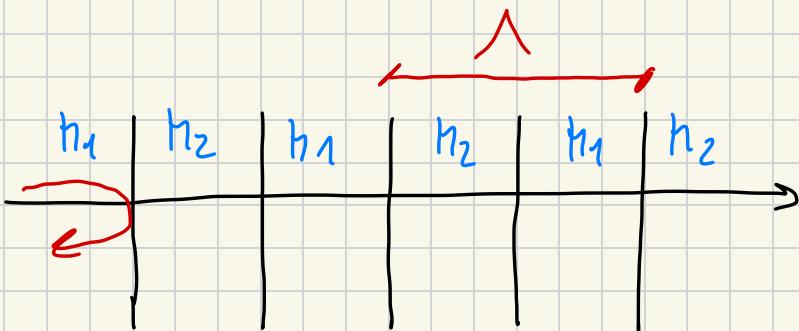
Periodical perturbation of something



Here in the direction of prop.



Like periodic variation of Δn to induce
a periodic variation of $\Delta \phi$.



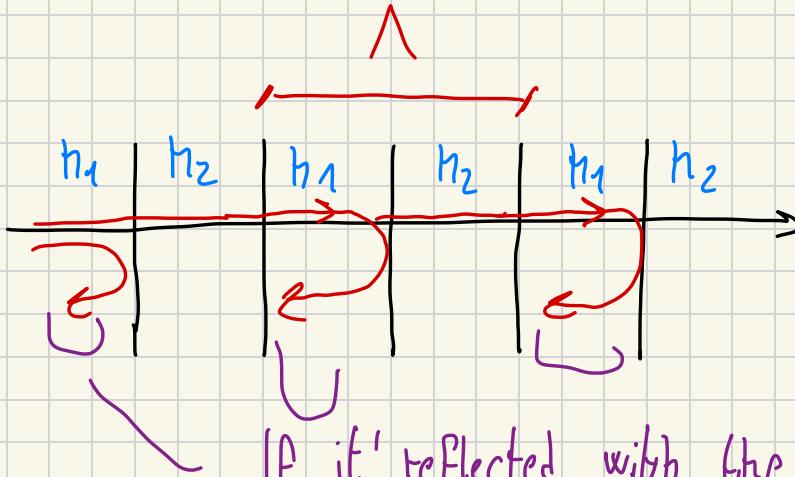
$$\gamma = \frac{n_2 - n_1}{n_1 + n_2}$$

due to the discontinuity
of n

If $h_2 > h_1 \Rightarrow$ phase of reflected wave is positive

If $h_2 < h_1 \Rightarrow$ " " " negative

If $h_2 \approx h_1$, γ is small, most of the wave is transmitted:



If it's reflected with the same phase they interfere, the three γ "sum" in field

So the phase of the second must be:

$$\frac{2\pi}{\lambda} n \Delta \cdot 2 = 2\pi$$

$$n_B = 2 n \Delta$$

BRAGG WAVELENGTH

$$\bar{n} = \frac{n_1 + n_2}{2}$$

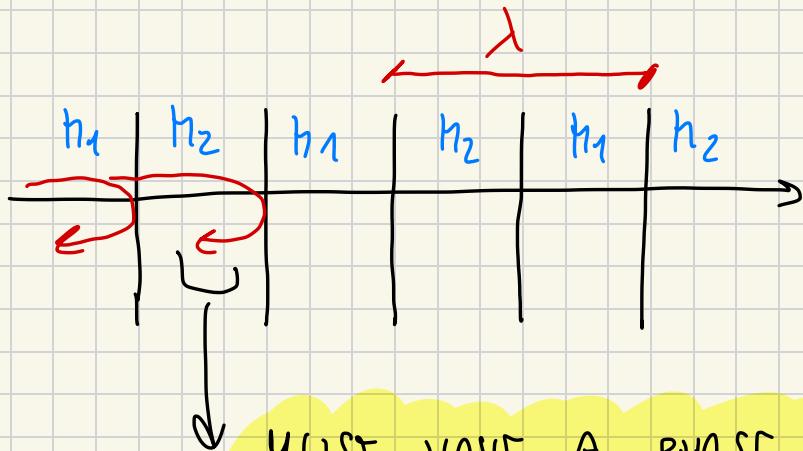
The third reflection must have a phase:

$$\frac{2\pi}{\lambda} \bar{n} \Delta \cdot 4 = 4\pi$$

So the total reflection can be very large even with small γ ($n_2 \approx n_1$), because they pile up.

↳ It's a reflector wavelength dependent.

And reflections between h_2/h_1 ?



MUST HAVE A PHASE OF π ($\gamma < 0$), IT'S CORRECT BECAUSE IT'S HALF A PERIOD λ

$$\frac{2\pi}{\lambda} - \frac{\lambda}{2} + \pi = 2\pi$$

SAME!

Every small reflection add up!

Order of Δ

$$\lambda = 1,5 \mu\text{m}, \quad n = 1,5$$

$$\Rightarrow \Delta = \frac{\lambda_B}{2n} = 0,5 \mu\text{m}$$

$$\Delta n = n_2 - n_1 = 10^{-5} \div 10^{-3}$$



$$r = \frac{10^{-9}}{3} \rightarrow R = 10^{-9}$$

in power

I NEED MANY OF
THEM

to integrate lasers

Bragg reflector are used to create the mirror of Laser, filters, dispersion compensation, equalization filter after amplifier . . .

In a Fabry Perot the mirror (100% almost and 95%) play the role of K in the ring, at resonance all the light exit, but the BW is decided on mirrors

↳ In a Fabry Perot the finesse depends on the mirror reflectivity M_F , if F , the light jumps a lot inside it, but at λ_0 everything exit.

$$\hookrightarrow \text{FSR} = \frac{C}{\pi q \frac{2L}{\text{CAVITY}}}$$

Add Ge in glass on SiO_2 , perturb the lattice

↳ Wanted defect $\rightarrow \delta n = n_2 - n_1$

↙
Not stable, bonding can be broken with UV light to change property

Illuminating Ge- SiO_2 creates $\text{Ge}e^-$ that is more stable than before

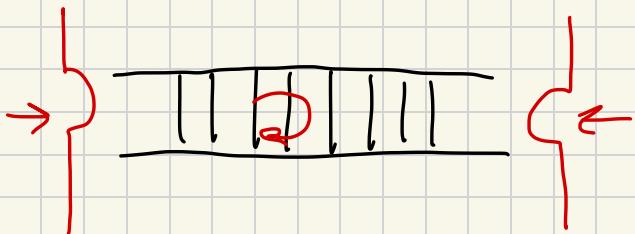
↳ SO PERMANENT VARIATION OF n_{eff}

You notice because you change the absorption spectrum of the material

→ SINONIMO DI CAMBIO DI fase

Now you have produce a long chain of this variation.

I want the transmission matrix



I know that the device reflect, so can I find a relation between the two mode, one propagating in τ and the other counter propagating?

↳ (coupled) mode theory

$$\Psi = A e^{-j\beta_+ z} + \beta e^{-j\beta_- z}$$

Real modes

$$\approx A(z) e^{-j\beta_0 z} + \beta(z) e^{-j\beta_0 z}$$

Approximated modes coupled

Modes of single
wg unperturbed

Not real mode (they
exchange power) but
approximate with A and
 β varying in z

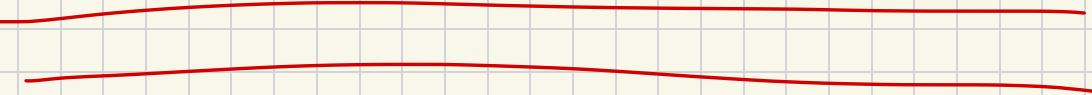
Difference with couplers

$$\Delta \beta \rightarrow 2\beta_0$$

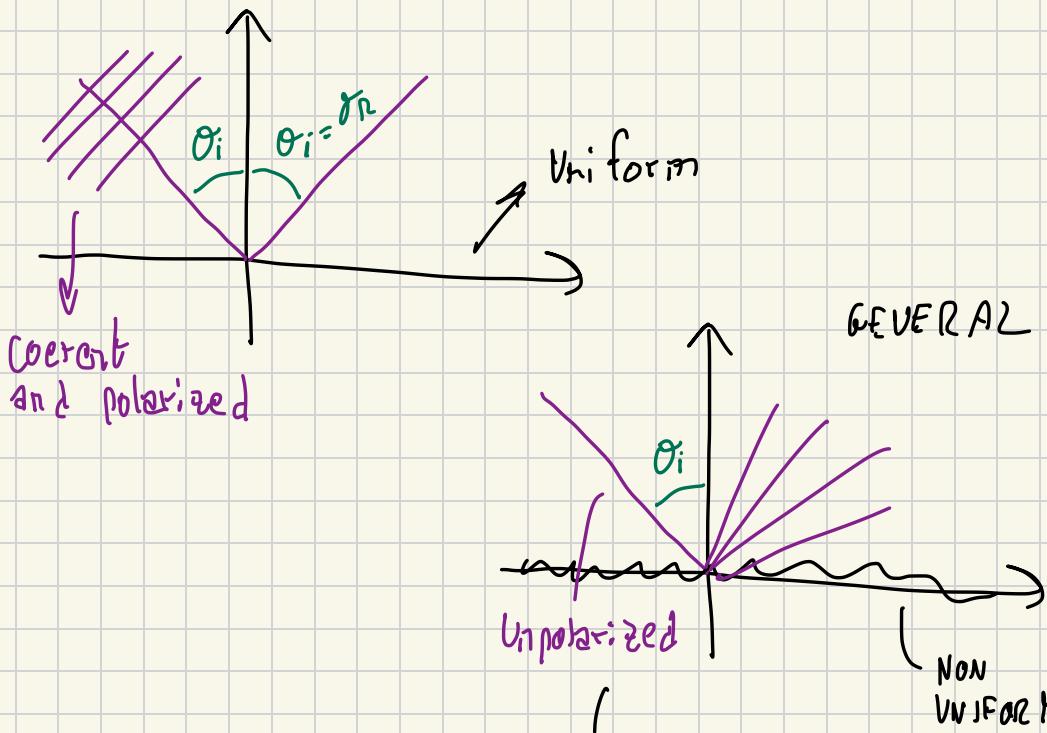
$C \rightarrow$ something $\propto z$, periodical

$\Delta\beta$ is huge compared to c and c is periodical in z .

↳ There will be difference!



Reflection on a mirror



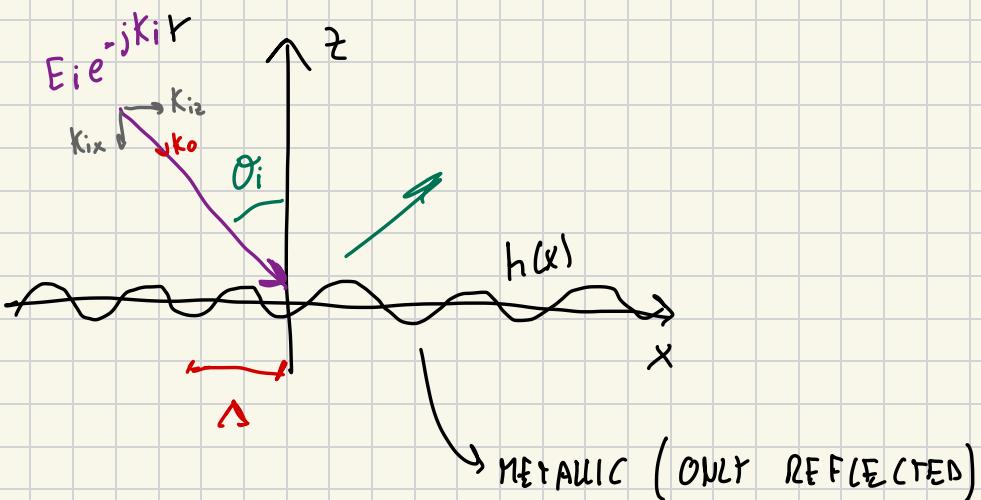
↓ SCATTERING

The scattered field can be represented as:

$$E_R = \int e^{-jk_R \vec{r}}$$

INPUT PLANE WAVE

Now the surface is periodical



The reflected one:

$$\vec{E}_i e^{-jk_i x} \hat{x} e^{-jk_i z} \hat{z} + \underbrace{\vec{E}_R(x, z)}_{\begin{cases} 0 & \text{metal} \\ \frac{1}{E_R} & \text{other} \end{cases}} =$$

$\vec{E} e^{-jk_i x} \hat{x}$ $q(x, z)$

$\underbrace{h(x)}$,
PERIODIC

Also \vec{E}_R will be periodic in Δ , so I can represent with a Fourier expansion (Not integration)

$$E_R, E_T \propto e^{-jk_i x} \sum_m R_m e^{-j \frac{2\pi}{\lambda_m} x} \cdot e^{-jk_z^{(m)} z}$$

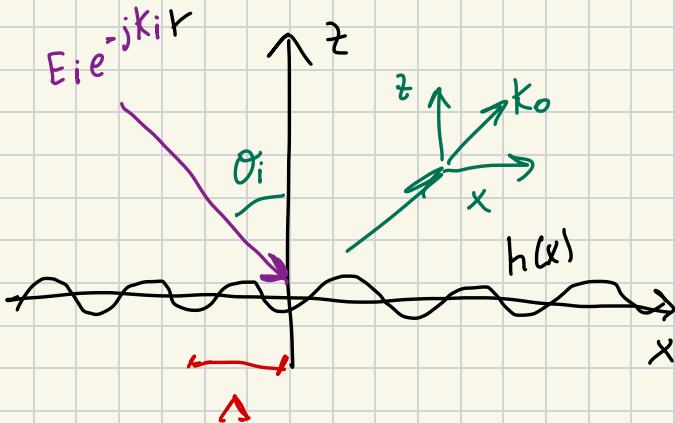
m
order of harmonic

$\underbrace{\quad \quad \quad}_{\text{Harmonic in } x \text{ or } z}$

For the m harmonic the phase:

$$\left[k_{ix} + \frac{2\pi m}{\lambda} \right]^2 + k_{Rz}^{(m)2} = k_0^2$$

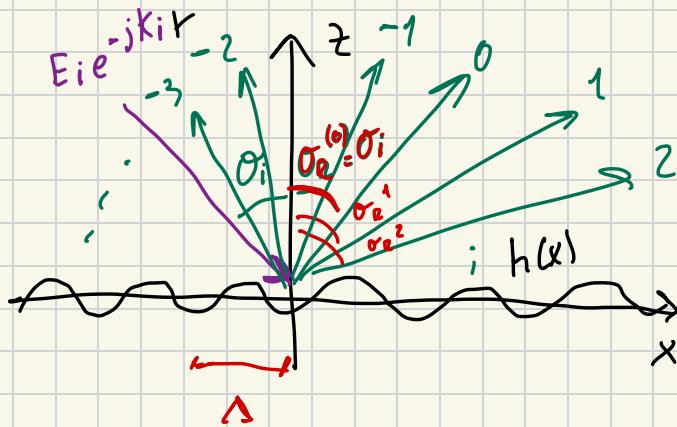
of the reflected



$$\left\{ \begin{array}{l} k_i = \sin \theta_i \\ k_{Rz} = k_0 \sin \theta_R^{(m)} \\ k_0 = \frac{2\pi}{\lambda} \end{array} \right.$$

(metals)

$$\sin \theta_R^{(m)} = \sin \theta_i + \frac{m\lambda}{\Delta}$$



If $\sin \theta_i + \frac{m\lambda}{d} > 1$, θ_R is imm
 Doesn't carry real power

I_m is the intensity, depends on the shape of surface

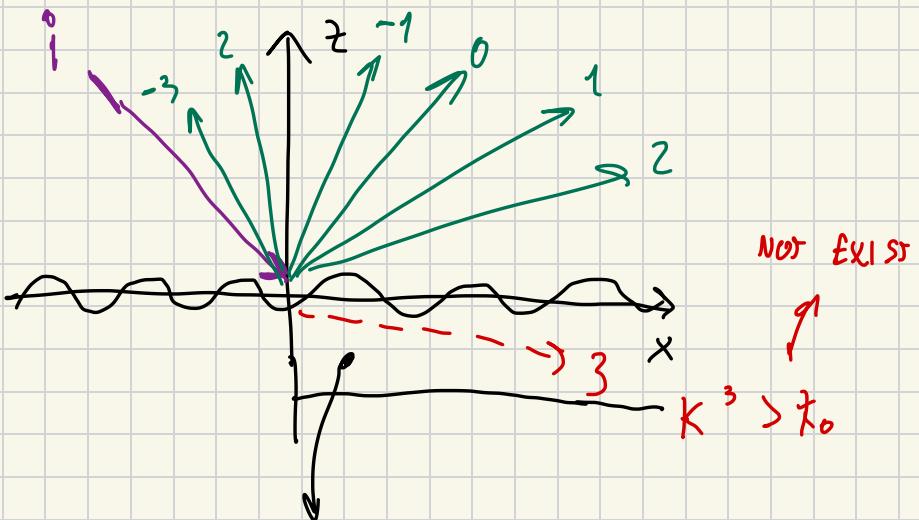
QUESTION 26 AUDIO 29 : 00

THIS IS A DIFFRACTION
GRATINGS

↳ LIKE DVD OR CD

Applications

Look at this :

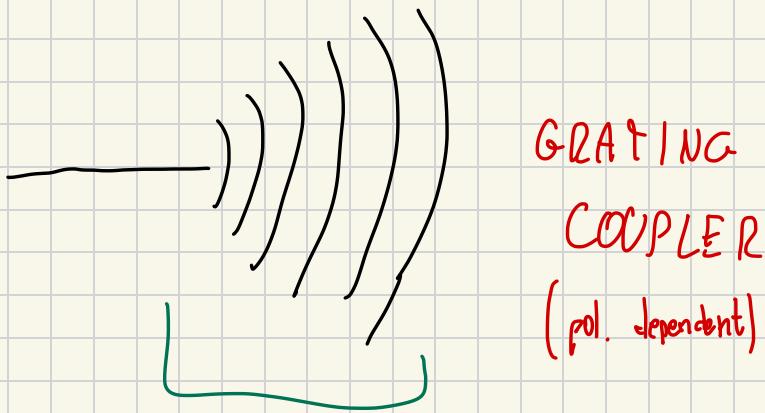


But if this is a λg width
 $\beta = k^3$

↓

I can have coupling in the
waveguide of the modulator.

So it's a way of coupling with fibers, not
entering horizontally, but shining at α_i to
the integrated WG.



GRATING
COUPLER
(pol. dependent)

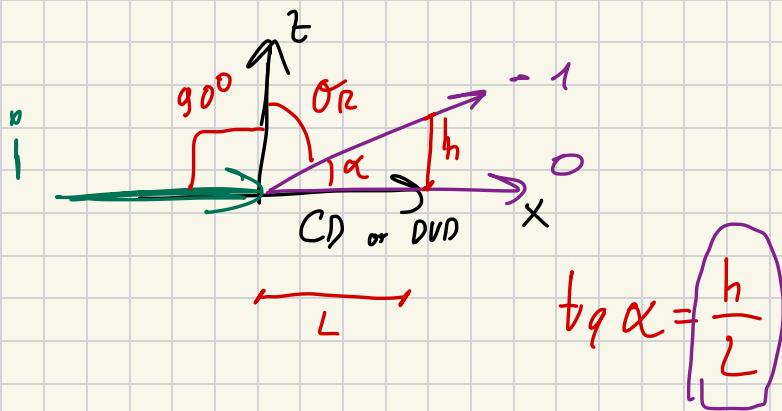
Grating used in Si photonic
to couple with fibers
(same idea of diff. grat.)

Used for besting also and good efficiency

in Si (2 dB/cm of losses, 30 nm of BN
for good coupling).

Last application

$\lambda = 532 \text{ nm}$ (green laser) arrive with a beam tangent to surface



$$\sin \theta_R^{(m)} = 1 + \frac{m\lambda}{\Delta}$$

le misuro

$$\sin \theta_R^{-1} = 1 - \frac{\lambda}{\Delta}$$

$$\Delta = \frac{\lambda}{1 - \sin(\frac{\pi}{2} - \alpha)}$$

Now I measure Ω_R and I can find Δ

\cup

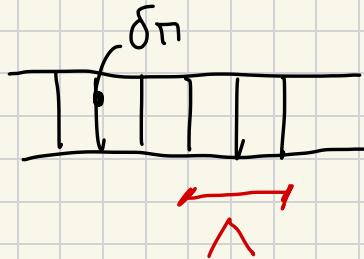
OPTICAL SPECTRUM

ANALYZER

↳ A diff. grat. with light goes around, I place many photodiode around the plating and depending on where light go you know the angle, so Δ . Or the opposite: I know Δ and I "read" what λ are inside the optical source.

Bragg gratings come back

The modes have different sign of β :



Every field inside this structure can be described by backward and forward mode (but they are not mode of the structure):

$$A(z) e^{-j\beta_0 z} + B(z) e^{+j\beta_0 z}$$

Put this assumption in the wave equation:

$$\left\{ \begin{array}{l} \frac{dA}{dz} = -j C_{11} A(z) - j C_{12} B(z) e^{-j(\beta_1 - \beta_2)z} \\ \frac{dB}{dz} = \dots \end{array} \right.$$

$$C_{ij} = \frac{k_0^2}{\beta_i} q(z) \iint \psi_i \psi_j \delta_n(x, y)$$

|

$$= C_{ij}^{-1} q(z)$$

|

$$= C_{ij}^{-1} \sum_m q_m e^{j m \frac{2\pi}{\lambda} z}$$

|P sinusoidal perturbation

$$m=1$$

Solve the system :

$$A(z) = A(0) - j C_{11} \sum_m q_m \int_0^z e^{j m \frac{2\pi}{\lambda} v} A(v) dv$$

$$- j C_{12} \sum_m q_m \int_0^z e^{j \left(m \frac{2\pi}{\lambda} - (\beta_1 - \beta_2) \right) v} B(v) dv$$

So $A(z)$ is a variation of the 1st "mode" due to the second.

Now $e^{jm\frac{2\pi}{\lambda}v}$ rotate every period, so every round trip it comes back, so it cancel itself (it continues to change phase).

Also the second is a changing phase term summed up, but if the phase is null, it doesn't cancel anymore ($e^{j0}=1$, NOT changing phase) and the integral is NOT negligible.

$$m \frac{2\pi}{\lambda} - (\beta_1 - \beta_2) = 0$$

$m=1$

$2\beta_0$

if sinusoidal perturbation

PHASE
MATCHING
CONDITION

$$2\beta_0 = 2 \frac{2\pi}{\lambda} h_{\text{eff}}$$

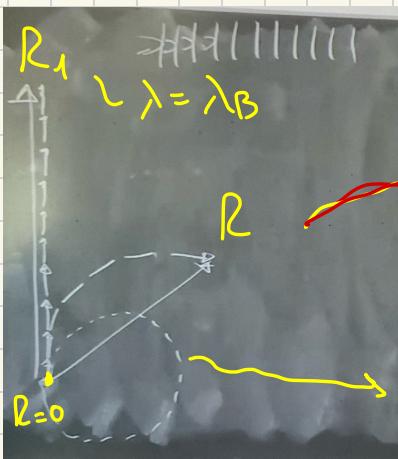
$$m \frac{2\pi}{\lambda} - 2 \frac{2\pi}{\lambda} n = 0$$

refractive index

$$\lambda_B = 2n \Delta / m$$

Bragg λ
(same as above)

So the second integral is important:



$$\left(m \frac{2\pi}{\lambda} - 2\beta_0 = \frac{\pi}{2} \right)$$

For $\lambda \neq \lambda_B$

R become lower

$$\text{if } m \frac{2\pi}{\lambda} - 2\beta_0 = 2\pi$$

$$R=0$$

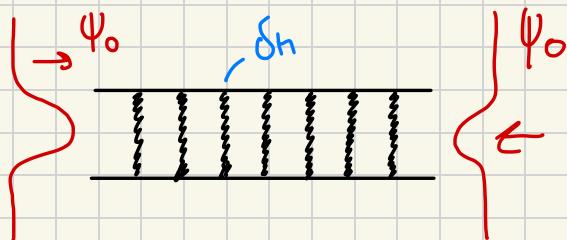


Inside I have reflectivity, but outer is

cancelled by all contribution along z

Now C_{12} has $q(z)$ inside.

It's possible to demonstrate for even δn structure:

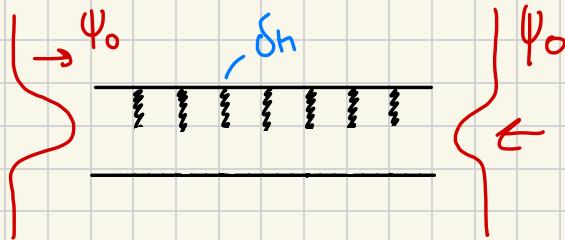


$$C_{ij} = \frac{k_0^2}{\beta_i} q(z) \delta n(x,y) \underbrace{\int \psi_i \psi_j}_{1}$$

$C_{ij} \neq 0$ \Rightarrow The two mode couple.

$$C_{ij} = \frac{\pi \delta n}{\lambda} \text{ if } q(z) \text{ sinusoidal}$$

IA $\int n$ (odd) :



$$C_{12} \rightarrow \iint \underbrace{\Psi_0}_{\text{Even}} \underbrace{\Psi_0}_{\text{Even}} \underbrace{\delta n}_{\text{Odd}} = 0$$

So condition to have reflections:

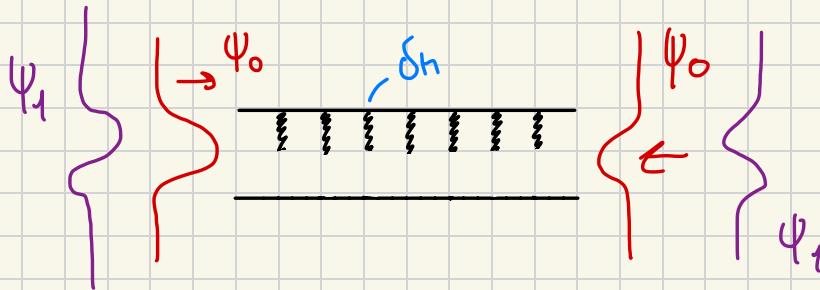
$$C_{12} \neq 0 \quad \lambda_B = 2\pi \frac{\Lambda}{m}$$

How much is the total R?

Depends on ?

What if the guide is bimodal?

Now with δn odd:

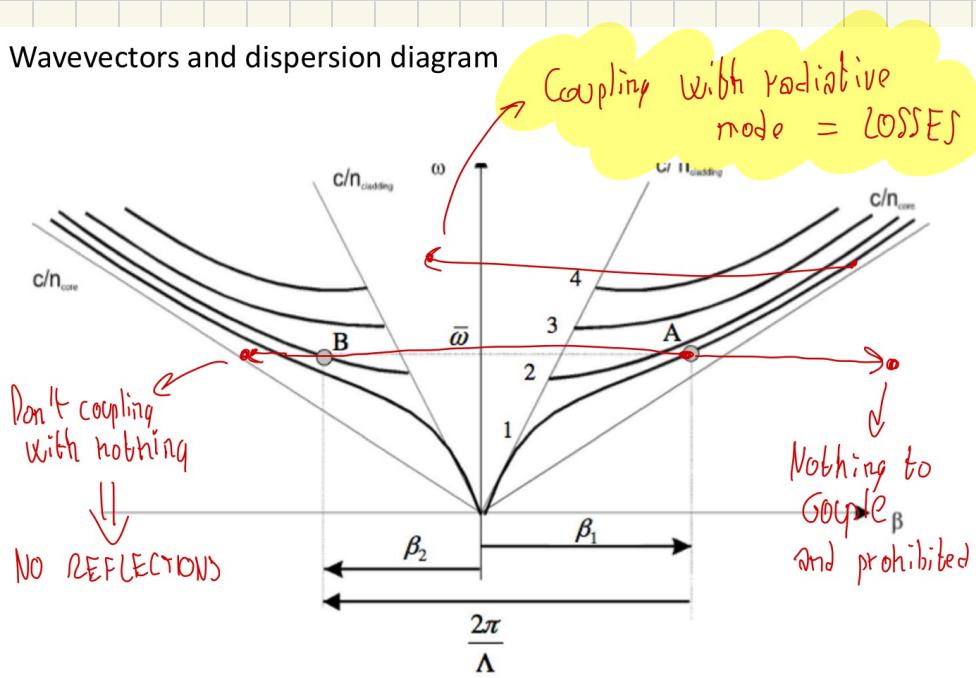


$$c_{12} \rightarrow \iint \underbrace{\Psi_0}_{\text{Even}} \underbrace{\Psi_0}_{\text{Even}} \underbrace{\delta n}_{\text{Odd}} = 0$$
$$\iint \Psi_1 \Psi_1 \delta n \neq 0$$

Only valid for δn small.

$W - \beta$ diagram ↳ let you see coupling for
a given \bar{w}

Wavevectors and dispersion diagram



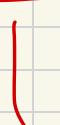
Also valid for multimode Wg (fundamental forward and backward second order), also between two forward modes

↳ LPG.

The phase matching above is not enough ($c_{12} \neq 0$)!

$q(z)$?

If sinusoidal c_{12} has a precise value, if there are higher harmonics, I will couple with higher order modes



KEEP TRANSITION
OF δn SMOOTH
TO AVOID DISCONTI-
NUITIES

The matrix

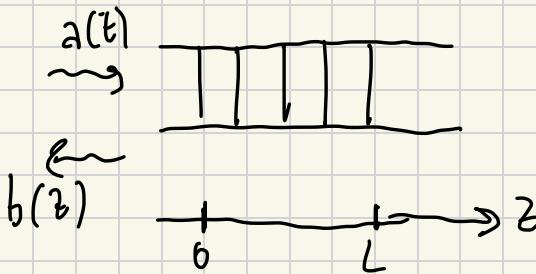
Connect the amplitude of forward and backward

$$\begin{bmatrix} a(0) \\ b(0) \end{bmatrix} = \Gamma_{01} \begin{bmatrix} a(L) \\ b(L) \end{bmatrix}$$

Uni form BG

↳ δn const.

↳ Δ const



Assume to have forward-backward wave:

$$\begin{cases} \frac{da}{dz} = -j \sigma_a(z) - j K a(z) \\ \frac{db}{dz} = +j \sigma_b(z) + j K b(z) \end{cases}$$

$$K = \frac{\pi \delta n}{\lambda} \quad \sigma = K_{11} + \beta - \frac{\pi}{\lambda} = \frac{2\pi}{\lambda} n_{\text{eff}} - \frac{\pi}{\lambda}$$

very
small

At λ_B $\sigma = 0$ and $\frac{d\alpha}{dz}$ depends only on $B(z)$ and vice versa.

From this:

$$T_G = \begin{bmatrix} \cosh(\delta L) - jR \sinh(\delta L) & -jS \sinh(\delta L) \\ jS \sinh(\delta L) & \cosh(\delta L) - jR \sinh(\delta L) \end{bmatrix}$$

$$R = \frac{\sigma}{\delta}, \quad S = \frac{k}{\delta}, \quad \delta = \sqrt{k^2 - \sigma^2}$$

$$S^2 - R^2 = 1 \quad \det(T_G) = 1$$

[hyperbolic functions] are NOT periodical, so you don't have periodical transfer of energy between the two, once reflected is reflected.

At λ_B $\delta = K$, $s = 1$ and $R = 0$

↳ But for $\lambda > \lambda_B$: it changes a lot,
 δ is imaginary.

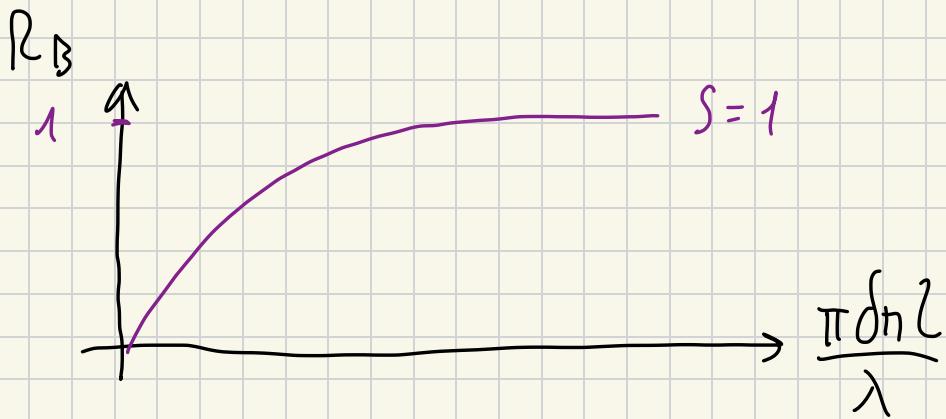
$$\begin{bmatrix} a(L) \\ b(L) \end{bmatrix} = \Gamma_G \begin{bmatrix} a(0) \\ b(0) \end{bmatrix}$$

$$R = \left| \frac{b(0)}{a(0)} \right|^2 = \left| \frac{\Gamma_{G21}}{\Gamma_{G22}} \right|^2 = \frac{\sinh^2 \delta L}{\cosh^2 \delta L - \left(\frac{R}{s}\right)^2}$$

↳ At λ_B :

$$R_B = \text{tanh}^2 \left(\frac{\pi \delta n}{\lambda} L \right)$$

Perfect phase matching
 $c_{12} \neq 0$



The limit is 1 so :

$$R_B \leftarrow 1$$

$$\frac{\pi d_n}{\lambda} L \quad L < \infty$$

K

So for BG R_B never arrive to 1, but can be high, for the right λ_B .

What if $\lambda \gg \lambda_B$ or $\lambda \ll \lambda_B$?

- δ is imaginary
- $R \rightarrow 1$
- $S \rightarrow 0$

$$S_0 \quad \Gamma_C = \begin{bmatrix} e^{-j\delta L} & 0 \\ 0 & e^{-j\delta L} \end{bmatrix}$$

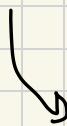
So the wave are not couple anymore and they acquire phase while propagating, like normal wq.

↳ So only phase shift.

$$\operatorname{sech}(x) = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sen}(x) = \frac{e^{ix} + e^{-ix}}{2}$$

So strong BG is for $kL = 2 \div 3$.



L change R
but not the BW

change k to
change R and
BW

BW of reflectivity

From max to the first zero, so for strong grating:

$$BW = \frac{\Delta\lambda}{\lambda_B} = \frac{\Delta n}{nq}$$

Approx

$$\lambda_B = 2 n_{eff} \Lambda$$

WEAK

$$\Delta\lambda = \frac{\lambda_B^2}{n_{eff} L}$$

→ NOT SO USED!

$$\hookrightarrow \text{BW} \propto L$$

Design

- ① $\lambda_B \rightarrow$ given by technology and Δ
- ② BW \rightarrow change δn
- ③ R \rightarrow choose L that maximise it

All of this is from approx of coupled theory
(only two mode inside it)

\hookrightarrow Works well for fibers optics, in integrated optics is a little more difficult (because we change λ to reach δn)

$$\text{FSR of spurious reflection} = \frac{c}{nqL}$$

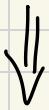


group delay

Chromatic dispersion cause the change of the shape of a pulse, where λ_{IR} are less confined and λ_{V} more confined.

↳ You see it in time domain.

And in simulation 27 1:15:00 you see light trapped in the BG bouncing up and down. It's like a Fabry Perot, quasi.



It means huge group delay

The pulse maintain the shape when reflected (at λ_B), what is transmitted is distorted.

The time that it takes to the light to exit from BG once it's entered is the group delay and I can define the penetration length :

$$L_p = \frac{c}{n_g} \cdot \frac{\gamma_g}{2} \rightarrow v_p \text{ and down}$$

$$= \frac{\sqrt{R_b} \lambda_B}{2\pi \delta n} \rightarrow \frac{\lambda_B}{2\pi \delta n}$$

$\downarrow R \rightarrow 1$

(S; O₂)

For optical fiber $\delta n \approx 10^{-4}$:

$$L_p = 2,5 \text{ mm}$$

$\circ \lambda = 1,55 \mu\text{m}$

In Si $\delta n \approx 0,1$:

$$L_p = 0,25 \mu\text{m}$$

→ In Si the reflection happens in very short distance compared to glass (fiber).

So Now I can design a Fabry Perot in Si, with given finesse and FSR and the design of mirror, knowing the right δn to not have l_p big and modify the resonance of the cavity.



Transmission at λ_B

$$T_B = 1 - R_B = \operatorname{sech}^2 \left(\frac{\delta n \pi L}{\lambda} \right) \quad [\text{power}]$$

l_p and γ_q

① At λ_B I SEE REFLECTIONS



• l_p is more important

$$\gamma_q = \frac{l_p}{2c/nq}$$

• D_G is 0 (good)

② At $\lambda \neq \lambda_B$ I see TRANSMISSION



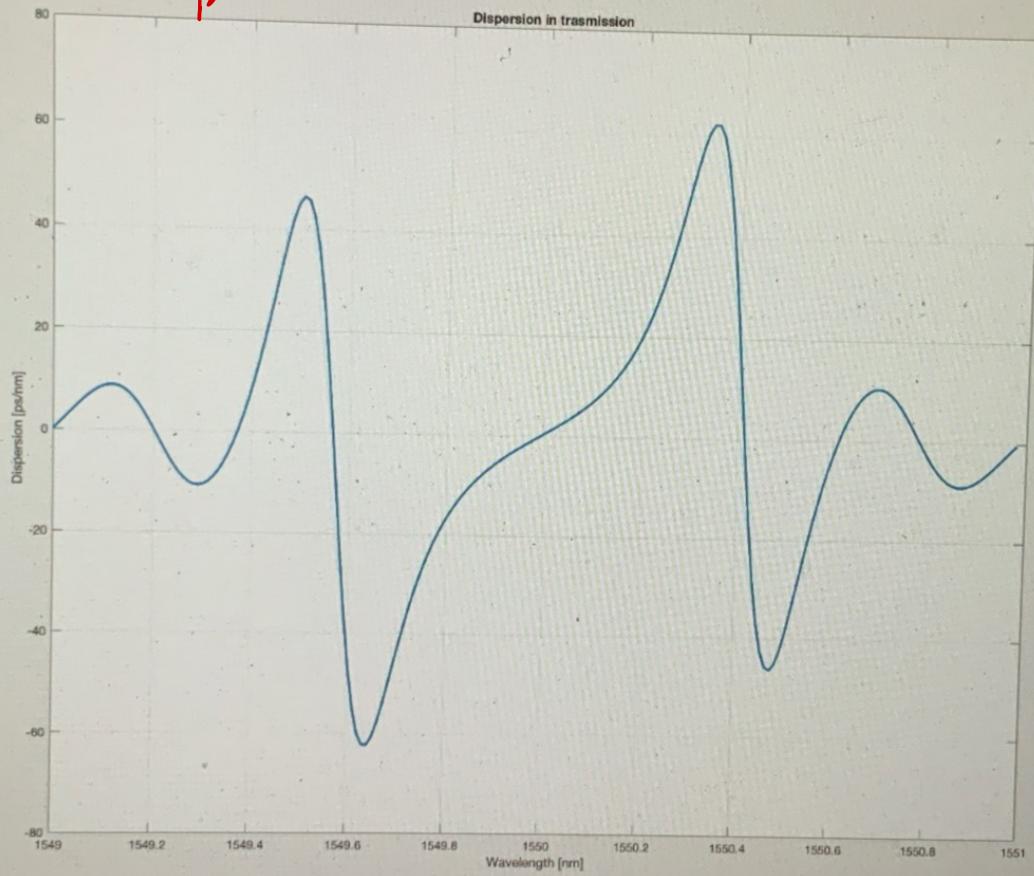
$\overbrace{L}^{\rightarrow \text{length of the grafting}}$

$$\gamma_q = \frac{L}{c/nq} \quad (\text{bigger})$$

• l_p play NO ROLE

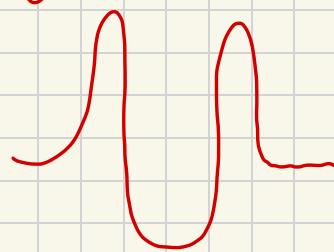
• D_G has peak near λ_B , then $\rightarrow 0$

DG



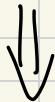
$\tilde{\tau}_q$

\sim



NON UNIFORM BG

Uniform gives a BAD out of band response

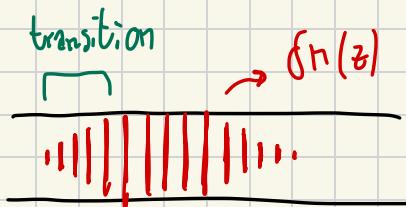


Remove discontinuity in δn

Match the impedance of the incoming wave with that of the wp

Apodized grating

δn NOT const. in z , there are transition zone



The light won't bounce up and down anymore, there isn't anymore the spurious FP like uniform.

↓

But L_p qgq

→ D_G similar to uniform, but less peak!

Every time I have a pick in $\tilde{\gamma}_q$, the field intensity is stronger (accumulation of energy)

↳ Apogized help here.

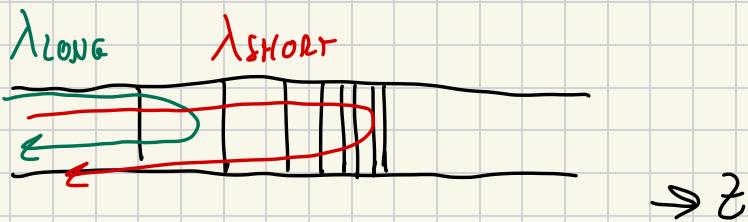
We don't have closed form formula, but rule of thumb for gaussian apogized:

$$\underline{L} = 3L$$

$\left\{ \begin{array}{l} 2 \cdot \frac{1}{3} \text{ transition} \\ \frac{1}{3} \text{ uniform part} \end{array} \right.$

Chirped grating

$\delta(n)$ const. but Λ depends on z



Used for very large \mathcal{R} BW,

If Δ change also λ_B change, cascading it reflect more λ

↳ light reflect where λ match Δ

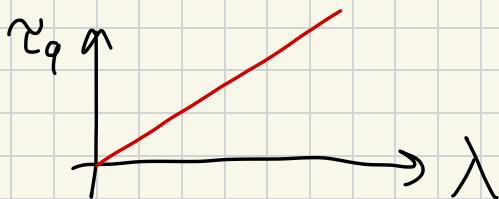
↳ short λ , big L_p , big $\tilde{\gamma}_g$

↳ long λ , low L_p , low $\tilde{\gamma}_g$



Chromatic dispersion compensation

$$\Delta(z) = \Delta_0 \pm Cz \rightarrow \lambda_B(z)$$



So the BW:

$$B = \lambda_{B_L} - \lambda_{B_S} = 2\pi n_{eff} CL$$

NO more related
only to δn

$$\gamma_q = \frac{z}{c/n_q} = \frac{\lambda - \lambda_{short}}{CL}$$

or λ_{long}

So the dispersion:

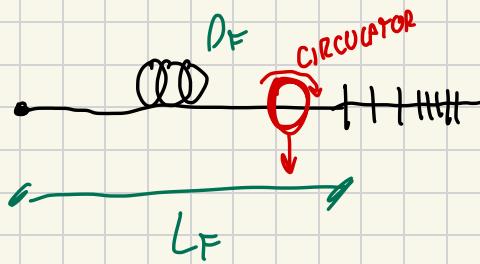
$$\frac{\partial \chi_g}{\partial w, \lambda} = + \frac{1}{cC} = D_g$$

\downarrow giro al contrario il chirped BG, cambia segno

BG, cambia segno

Example in fiber

Mode NOT well confined, quasi a plane wave



Total ch. disp. :

$$D_F \cdot L_F$$

$$\propto$$

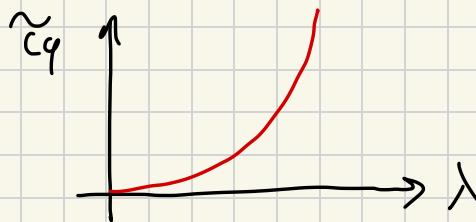
$$D_g$$

$$D_F \sim - \frac{20 \text{ ps}}{\text{nm}}$$

$$L_F \sim \text{km (more)}$$

GCHIO CHE | D_g, D_F
KAVUNO SEGUO

This is only useful for first order D, while higher order i need different chirp:



The BG in fibers + circulator are POL. INDP.

↳ Unless you need varying R (difficult that L_F change).

Why in reality in \tilde{c}_q there are ripples?

They cause ISI, they born from discontinuity in the single section of the BG chirped.



USE APOGIZED + CHIRPED
GRATINGS

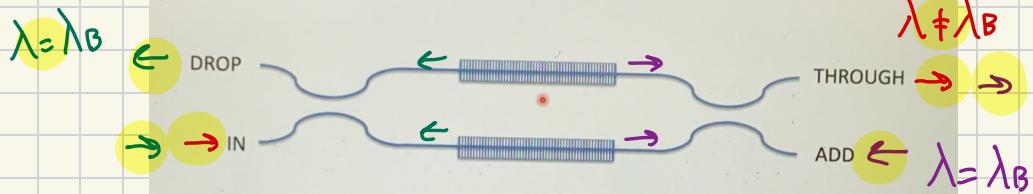
Less ripples, less ISI.

Applications of BG

- ↳ CDC (above)
- ↳ ADD - DROP
- ↳ Sensors
- ↳ Fiber isolator

Add - drop :

Bragg grating based Add-Drop Multiplexer



$$R_B^2 = \tanh^2\left(\frac{\pi\delta n}{\lambda}L\right)$$

$$T_B^2 = \operatorname{sech}^2\left(\frac{\pi\delta n}{\lambda}L\right)$$

Important to match L_p , otherwise the field reflected arrives to the coupler phase shifted to the other and can exit in IN/ADD port.

Only for one λ ($= \lambda_B$)

FOR ADD DROP BETTER AWG-RING

Field inside in BG

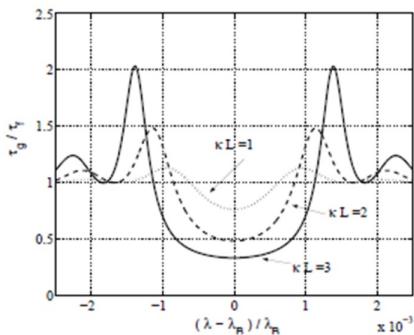


Figure 6.14: Group delay normalized to the uniform Bragg gratings having reflectivity spectrum of Fig. 6.13 (b).

Even if it is totally transmitted, in the BG the intensity is bigger



all the reflection built up inside and cancel after L

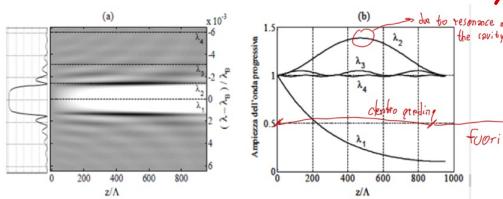


Figure 6.15: Behaviour of the field inside a uniform grating.

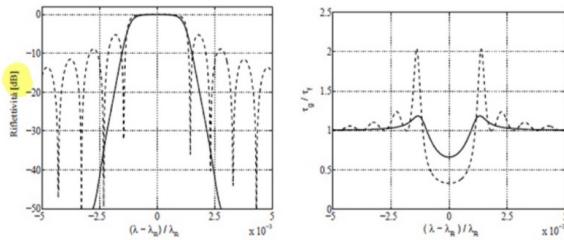


Figure 6.17: Reflectivity spectrum (a) and normalized group delay (b) for a uniform grating (Dotted line) and an apodized grating with Gaussian profile (continued line).

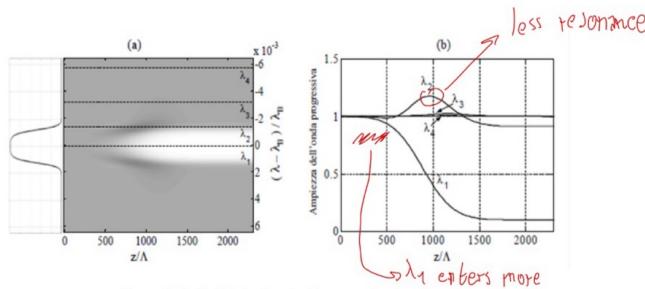
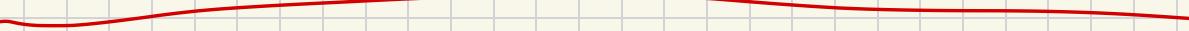


Figure 6.18: Field behaviour inside an apodized grating.

λ_1 enters more

Very small imperfections in the center of the BG (causing it to "divide" in two equivalent, quasi) I have a Fabry Perot, (NOT BG anymore).
cavity



Fake isolator \rightarrow solution for integrated optic ?

↳ NO!



MODULATOR

Receive the info to transmit and add it on the carrier.

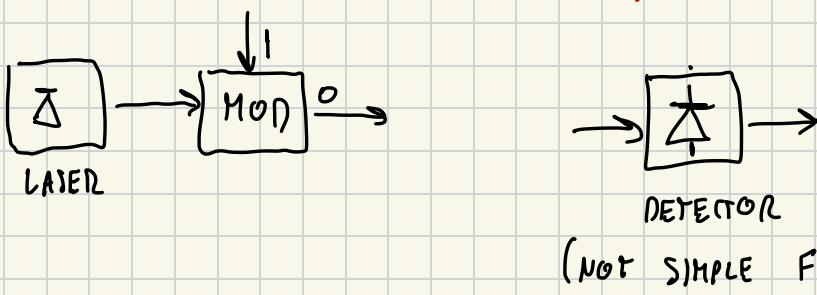
↳ Intensity

↳ Phase
↳ Frequency
↳ Polarization

And then DEMODULATOR (after link with losses, noise and dispersion).

The best? Laser + MODULATOR

↳ Intensity
↳ Phase } QAM
↳ Polarization (abandoned)
↳ But NOT frequency mod.
(λ fixed, difficult to act on laser)

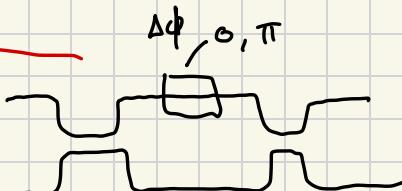


Why only 5 years ago the first photonic QAM?

Because you need the correct modulator.

- ONLY INTENSITY
ON-OFF

→ MHz



- QAM → MORE DIFFICULT

I need very fast $\Delta\phi$ change → HEATER
CANNOT DO THAT

THIS IS DIFFICULT

($1 \div 10 \mu\text{s}$,
few MHz
of modulation)

✓

What material can do this?

The signal is electric, the "medium" is optic



ELECTRO-OPTIC

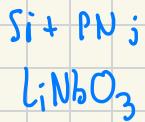


electrorefractive

electroabsorbing

material

material



Δn



$\Delta\phi$

PHASE MODULATOR

$\Delta\alpha$



INTENSITY

MODULATOR

ΔI

Problem

In those materials if you change Δn , you also change $\Delta\alpha$ and viceversa.

Si
InP + PN junction
Better

TO CHANGE
THE NUMBER
OF CARRIER
INSIDE
BY BIAS

↳ So there's NOT the best one

If I want to induce $\Delta\alpha$ to switch on or off the light, it's detrimental to have also a phase shift (modulation) due to Δh . It's difficult to NOT have Δh after $\Delta\alpha$. So these material are used rarely for AM. Only InP is used for short link AM

↳ It's easier to change Δh without change $\Delta\alpha$ too much in the right material

↳ PM is more feasible



The dream material is LiNbO_3 , a crystal where \vec{E} produce Δh without $\Delta\alpha$

Si + PN junction can have Δn (also $\Delta \alpha$, but small) and short w_p are used in Si photonics). In general less efficient than LiNbO₃.

Intensity modulator with PM

↳ With LiNbO₃ + M_z
(or Si+PN junction)

LiNbO₃ is extremely fast \gg 50 - 60 GHz
↓
Si + PN;

It's a microwave problem
how to design electrodes for E
at that frequency.

ELECTRO-OPTIC MATERIAL (LiNbO₃)

$\vec{E} \Rightarrow \Delta n$

↓
Crystal, really fast time response

With \vec{E} , the molecule of the crystal change a little bit the orientation or the polarizability and the molecule are oriented in the same way, overall the little change produce Δh .

↳ glass = Amorphous cannot do this, the effect is at average null.

↳ You need crystal

↳ Also for magneto-optic

w.r.t. b. center

Si is symmetric^V, if you induce a rotation of molecules in the lattice you don't see anything

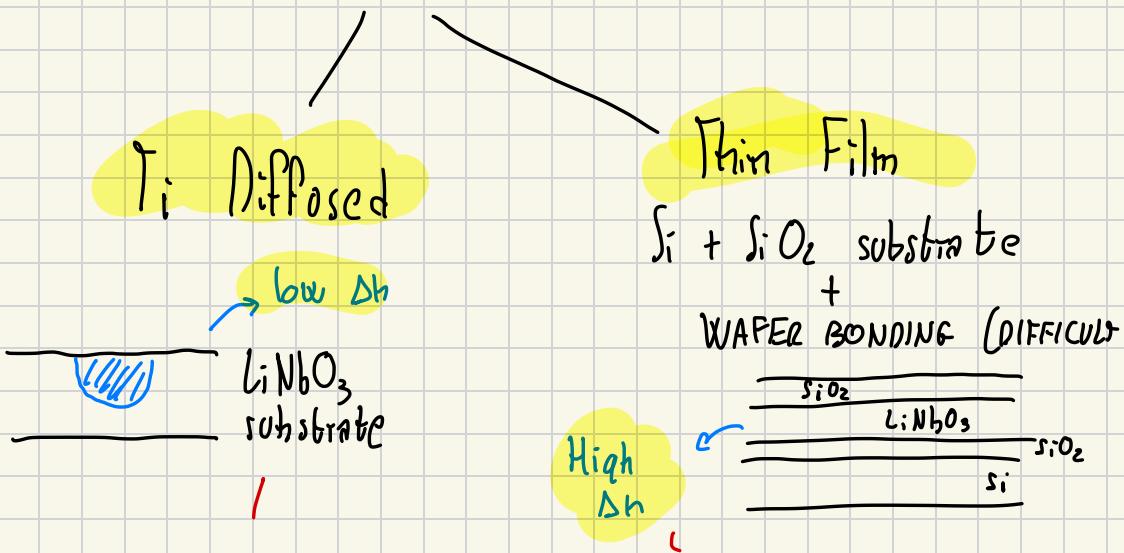
↳ LiNbO_3 is asymmetric (crystal)

EO EFFECT = CRYSTAL ASYMMETRIC

With interferometer + LiNbO_3 you can have AM.

How to build $\Delta\phi$ in LiNbO_3 ?

It's a crystal, cannot be deposited. So I have to start from the crystal (a wafer) and construct on it.



Best move for modulators
(high frequency)

More compact

But high loss
 $R_{min} = 100 \mu m$
 $\Delta h = 2,2$

But I cannot diffuse too much Ti otherwise losses ↑↑ (yes good confinement).

$$\begin{aligned} R_{min} &\geq 5 \text{ cm} \\ \Delta h &= 0,2 \div 0,4 \end{aligned}$$

How it works?

$$P(E) = \epsilon_0 \chi \vec{E}$$

Polarizability

$(\epsilon_r - 1)$

$$D(E) = \epsilon_0 E + P(E)$$

E from now on
is external, not
that of the optical
mode

↳ in general depends

on E , $E^?$, ...

POCKELS EFFECT

I can increase or
decrease Δn

KERR EFFECT

I can only
increase Δn

So ϵ is a tensor:

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

If $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} \rightarrow$ MATERIAL ISOTROPIC

" $\epsilon_{xx} = \epsilon_{yy} \neq \epsilon_{zz} \rightarrow$ " ANISOTROPIC UNIAXIAL

" $\epsilon_{xx} \neq \epsilon_{yy} \neq \epsilon_{zz} \rightarrow$ " BIAXIAL

LiNbO_3 is uniaxial, along X and Y has the same η_{eff} , along Z is different.

How can the wave propagate?

ϵ is NOT scalar in the wave equation. Then with external \vec{E} I change ϵ , so η_{eff} .

But when I change \vec{E} , maybe I change $E_{xx}(E)$ ok? But then I change also the other coeff. in the matrix

↳ I cannot say: apply \vec{E} to change η_{eff} in this direction, it changes in all directions.

Any material has its own Δt_k matrix (dispense),
 γ_{33} is the big in LiNbO_3 .

$$\frac{\text{meter}}{\text{Volts}} = 30.8 \cdot 10^{-12} \frac{\text{m}}{\sqrt{\text{V}}}$$

For strong Δn I have to work with r_{33} .

$E \rightarrow$ low frequency
changing the lattice
 $(r$ is the biggest)

$E \rightarrow$ It's constant only for
modulating at high
frequency

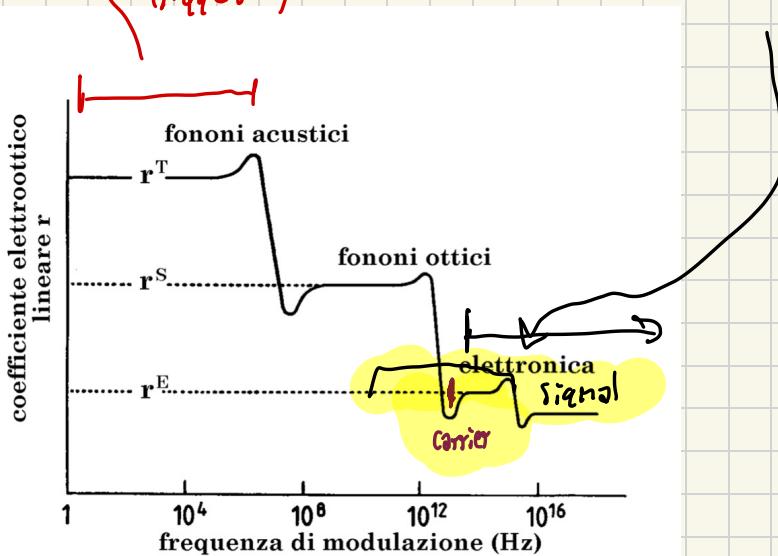


Figure 5.3: Dependence of the electro-optical coefficients on the frequency of the applied electric field.

I can stretch the molecules,
 r is bigger

With electric signal at 10 Gbit/s (~ 10 GHz)

from O

I excite almost all of these three



SO SOME PART OF THE
SPECTRUM IS MODULATED
BETTER THAN
OTHER PART

The carrier is at 200 THz.

Directions

If we propagate on a generic direction, the wave will depends on the entire Δk . But if we propagate along x or y or z , there is only the transverse characteristic of the wg to play any role.

↳ So polarization dependent

↳ Use the \vec{E} along one axis.

Then the \vec{E} applied can (comunque) produce change along other axis, so use an \vec{E} with a direction to exploit γ_{33} .

↳ \vec{E} along z .

LIGHT ALONG X TE POLARIZED
 \vec{E} CONTROL ALONG Z

Now place electrodes to induce E_z and:

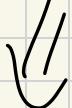
$$\Delta n_z \propto \gamma_3 E_z$$

z -cvt

Δh_y and Δh_x will change also (via γ_{13}) but they don't play any role, because the light is polarized TE (doesn't see $h_x | h_y$).

Other case X-CUT, TE (horizontal pol.)

\vec{E} always along z and wave TE along x
pol. along z



Change position of electrodes

What if I enter with TM (vertical polarized)?

E_z will produce Δh_x and Δh_y with γ_{13} :

$$TM \ll \gamma_{13} \ll \gamma_{33}$$

There will be a (varying) birefringence.

↳ X-CUR

The z-axis is the optical axis (\perp to the substrate).

The phase along the wq in the two case is:

$$\Delta\phi = \frac{2\pi}{\lambda} \frac{\hbar^3}{2} \underbrace{r_{33} E_z}_{{\Delta n}} L$$



Semiconductor

In PN junction you can change the number of carrier to change Δn (also $\Delta\phi$ though), via the biasing.

↳ For every Δn change I pay also $\Delta\chi$, so keep small variation



How much can i reduce the carrier in the core?

↳ 10^{18} cm^{-3}

Remember: this produce a $\Delta\phi$, but it's not electro-optic effect but plasma dispersion.

Slide 15

I have more Δn than $\Delta\chi$ in Si, so it's easier to built PHE than AHE in Si. But it's not possible to reach a pure PHE, always

a residual one (it cause a chirp, distortion of the pulse).

↳ NOT SO EFFICIENT FOR
ON-OFF KEYING

So applying V I can have $\Delta\phi = \pi$, but also $\Delta IL = 20 \text{ dB}$.

↳ I DON'T WANT CHIRP, so
I HAVE TO CHANGE THE
DEVICE.

NO MZ,
BUT RING

Come back to LiNbO_3

Photoresist \rightarrow pattern mask \rightarrow lithography \rightarrow etch

Deposit Ti all over (10 nm thick)

Remove the photoresist + Ti



Increase T and Ti diffuse more and more while time pass

Ti in LiNbO_3
change n_{eff}
for two reason

Substitute Ti
larger than LiNbO_3

DIFFUSE WAVE GUIDE

Ti is larger and induce a stress in the lattice.

smooth DH

graded index WG,
not step index

$\Delta h \approx 0,3\%$

- low losses
- ↳ large mode
- ↳ weak confinement
- ↳ poor bending capability
- ↳ low density (device size large)

$R_{min} = 5\text{cm}$ ←

Now apply \vec{E}

Electrode size $L \sim$ less than mm and
 \vec{E} must be horizontal, or vertical field is
 the wq.

x -cut z -cut

↳ S_0

$$\Delta h = \frac{\hbar^3}{2} \gamma_{33} E_z$$


VOLTAGE

DISTANCE

BETWEEN ELECTRODES

So to have low voltage (high frequency is not so feasible to have high power), so small d , but not too close to not attenuate the optical field.

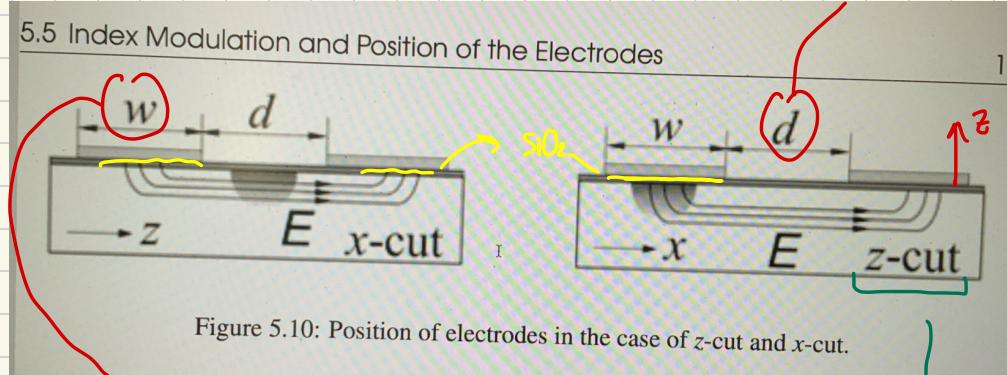


Figure 5.10: Position of electrodes in the case of z-cut and x-cut.

$$w > d$$

Metal - SiO_2 - LiNbO_3

If the thickness of SiO_2 is smaller, the attenuation is larger and E larger

TRADE-OFF

SiO_2 serve only as isolation so not so thick.

Electrode

Are microstrip line because of fast signals.

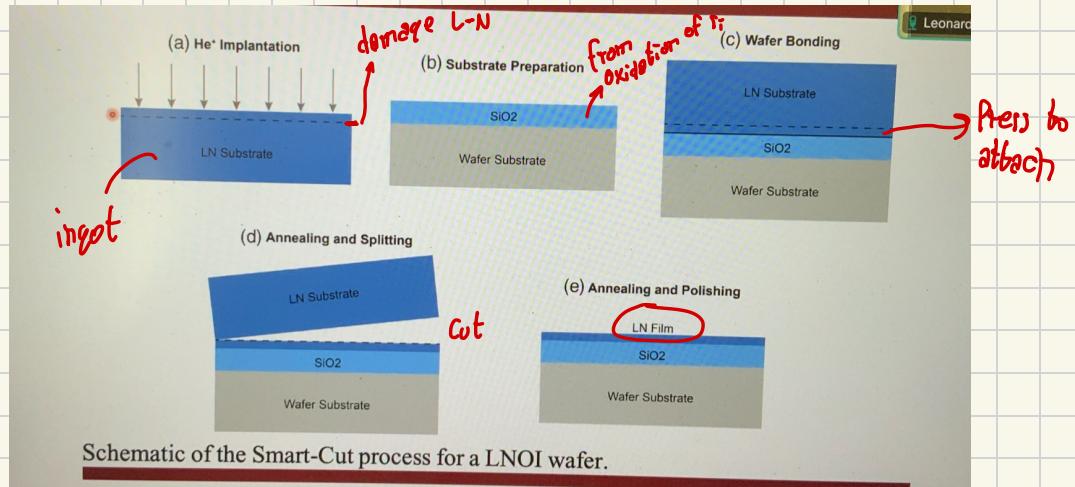
After made the $\text{W} \rightarrow$ glue, so a few nm of Cr are deposited \rightarrow Deposite Au \rightarrow Deposit photoresist \rightarrow Remove ^{lithography} all unless where it's protected
 \rightarrow Enlarge the thickness of Au (small R)

(ion deposition)

At least $3 \div 9 \mu\text{m}$
(1 GHz, skin effect limits).

Cr collo bny Au e LiNbO_3

How to make thin film L-N



Thin film \rightarrow 3 ÷ 700 nm

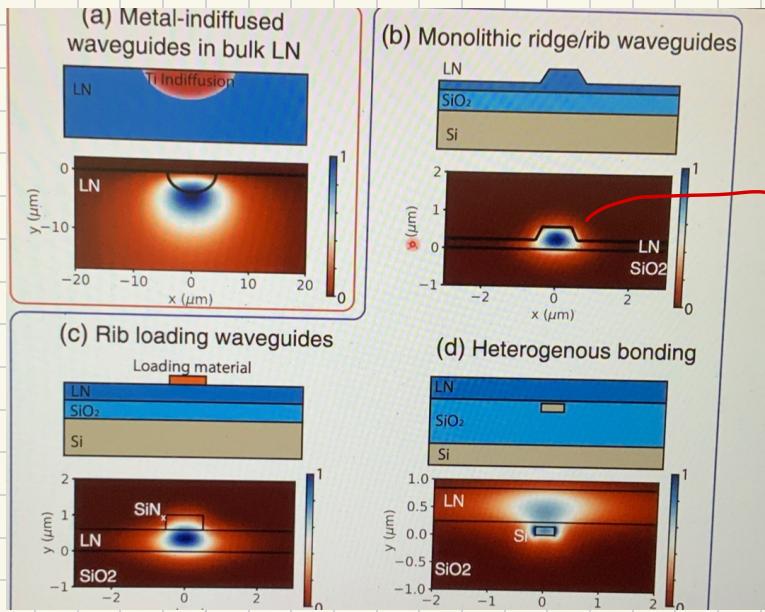
SiO₂ substrate \rightarrow LNOI (LiNbO₃ on Insulator)
(adding air)



With λ_Q smaller, electrodes can be closer to the λ_Q

↳ Volt ↓ → Higher frequency

Everyone is trying with LiNO_3 instead of LiNbO_3 (used because simple to produce in mass scale)



1 order of magnitude smaller

How to apply the high freq. signal?

In the figure of x-cut and z-cut you see that \vec{E} is not exactly oriented in the direction of the optical axis z, so I have to consider the overlap between optical and external field:

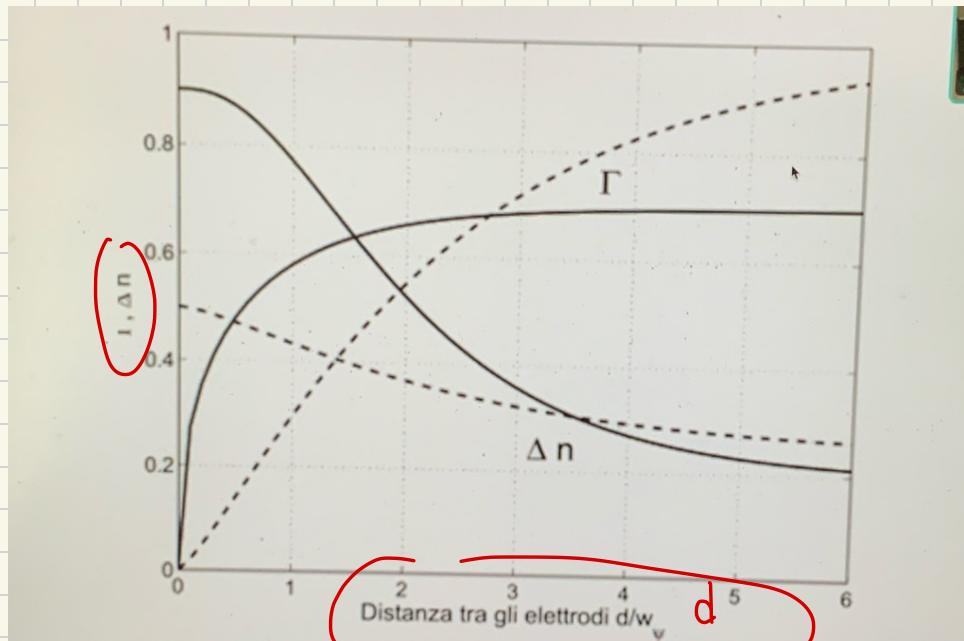
$$\Gamma = \frac{d}{V} \int E |\psi|^2 d\sigma \leq 1$$

Over the cross section

$\Gamma = 1$ in case of perfect overlap, so:

$$\Delta n = \frac{\hbar^3}{2} r_{33} \Gamma E$$

Then I want Δn as large as possible:



-- z cut
— x cut

- $\uparrow\uparrow \rightarrow \Gamma \uparrow\uparrow$, until const
- $d \downarrow\downarrow \rightarrow \Gamma \downarrow\downarrow$, $E \rightarrow$ tends to stay between electrodes and doesn't penetrate the wdg
- $d \uparrow\uparrow \rightarrow \Delta n \downarrow\downarrow$
- $d \downarrow\downarrow \rightarrow \Delta n \uparrow\uparrow$

X-cut gives larger Δh than z-cut until the electrodes are $3 \div 4$ times the dimension of the optical gap (W), then it's better z-cut



X-cut is to prefer if the two electrodes are not too far away from each other

So SiO_2 in z-cut matter to not have $\Delta h \approx 0$.

I don't apply high Voltages [$1 \div 2$ volts] over distances of 10cm , so $|E|$ is huge.

DO NOT REACH BREAKDOWN

Small electrodes

$$C = \epsilon_{\text{eff}} \frac{A}{d} \rightarrow C \downarrow \downarrow, d \uparrow \uparrow$$

HIGH BW

But $d \uparrow \uparrow$, weak E , so small Δn , so I achieve $\Delta \phi$ by increasing the length L :

$$\Delta \phi = \frac{2\pi}{\lambda} \frac{\hbar^3}{2} \gamma_{33} \Gamma \frac{V}{d} L$$

Depend on frequency
(see later)

$$f_{\text{cor off}} = \frac{1}{R C}$$

But increase $L \rightarrow$ no more lumped.

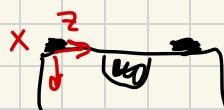
$$\left. \begin{array}{l} \epsilon_{r_{LN}} = 28 \text{ along } x \\ \mid \\ = 43 \text{ along } z \end{array} \right\} \text{LN is anisotropic}$$



$$\left. \begin{array}{l} \epsilon_{ff_{air-LN}} = 20 \text{ (z)} \\ \mid \\ = 15 \text{ (x)} \end{array} \right\} \text{(average)}$$

C DEPENDS ON THE CUT

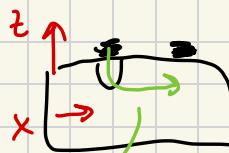
X-cut



$$\epsilon_{r_z} = 13$$

$$C_x = \epsilon_{ff_x} \frac{A}{d}$$

z -cut

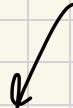


most of the material is crossed along x

$$\epsilon_{r_x} = 20$$

$$C_z = \underbrace{C_{ff,z}}_{15} \frac{A}{d} < C_x$$

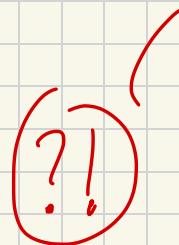
So from the graph I say x is more efficient, but it can be slower.



z -cut is more efficient because C is smaller



x -cut is more efficient because Δh is bigger



L PLAY A ROLE

$$l \sim 1 \text{ cm} \text{ for } V \sim < 10 \text{ V}$$

Speed of modulation

The V voltage has a λ :

$$\lambda = \frac{c}{\sqrt{\epsilon_r} f} \Rightarrow l$$

to use the lumped model

Max BW $\sim 1 \text{ GHz}$

↳ For more you decrease
 l , and increase V (d
cannot be touched anymore)

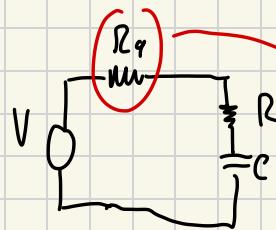
Special case: Si + PN j

The Si-Ring is the way to build modulator
in silicon.

↓

Here I can use the lumped electrodes also for higher BW

The ring is a C (or C with R if losses).



matched with
Z of the electrode
to not have reflections



OTHERWISE ISI

In LiNbO_3

$$L \approx \lambda$$



USE TRAVELLING WAVE ELECTRODES

TO GO FASTER AT

10BW V

I use a coax between generator and electrodes, with matched impedances.

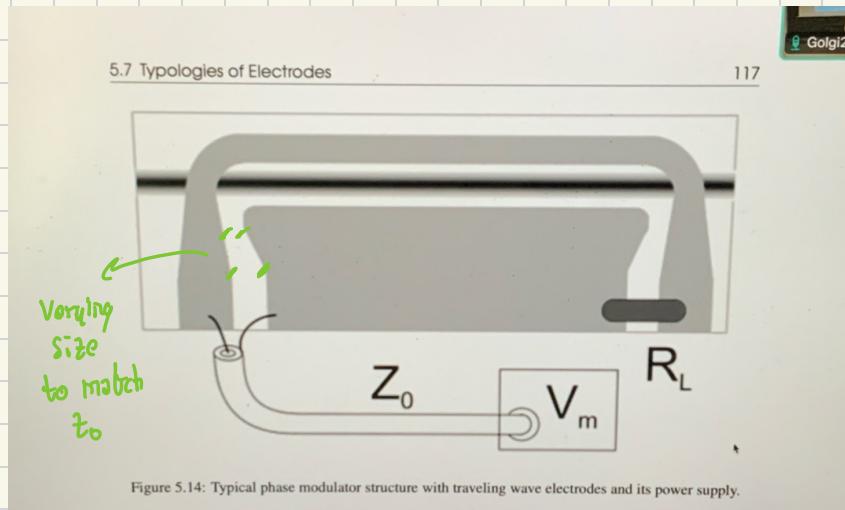


Figure 5.14: Typical phase modulator structure with traveling wave electrodes and its power supply.

$$Z_0 = Z_e = R_L$$

MUST !

But Z_e depends on d , that play a role in Δh and γ !

To have $Z_e = Z_0 = S_0$ you have to set d

NOT so short unless you vary the size of elect.
or charge δq .

Now I have a wave that travel in \mathbf{z} e
and modulate the light that travel in the \mathbf{w} w

↳ A wave perturbs Δn with a
certain time response and light
read Δn at different speed



VELOCITY MISMATCH

$$V_0 = \frac{C}{n_{2N}}$$

|

$$= \frac{C}{2,2}$$

$$V_m = \frac{C}{\sqrt{\epsilon_{eff}}}$$

|

$$= \frac{C}{\sqrt{15}} \approx \frac{C}{4}$$

modulating

There is a factor of 2, the optic is faster than the electrical signal, in LiNbO₃.

For Si, InP:

$$V_0 = \frac{C}{n_{Si}}$$
$$= \frac{C}{3,5}$$

$$V_m = \frac{C}{\sqrt{L}}$$
$$= \frac{C}{\sqrt{4}} = \frac{C}{2}$$

In Si the electrical signal is faster than the optical signal.

↳ Also here a mismatch though.

Velocity mismatch

$$\Delta \varphi = \frac{2\pi}{\lambda} \int_0^L \Delta n \, dL$$

NOT CONST. IN L

\hookrightarrow $\Delta n \propto E, V$

It's interesting to see what voltage sees the light, NOT the electrodes.

The modulating voltage :

NOT OPTICAL AXIS \rightarrow PROPAGATION ALONG X OR Y

$$V_m(z, t) = V_0 \sin\left(\frac{2\pi f_m}{c} \sqrt{\epsilon_{eff}} z - \omega_m t\right)$$

Change the reference (the signal is like not moving, while the WG is moving) :

$$V_m(z, t_0) = V_0 \sin\left(\frac{2\pi f_m}{c} (\sqrt{\epsilon_{eff}} - n_{eff}) z - \omega_m t_0\right)$$

I'm interested in the phase velocity, NOT group one. Now I have V , find E and Δn :

$$\Delta n = \frac{2\pi}{\lambda} \frac{n^3}{2} r_{33} \frac{V_0}{d} \int_0^L \sin\left[\frac{2\pi f_m}{c} (\sqrt{\epsilon_{eff}} - n_{eff}) z\right] dt$$

↓
CHISSENE

$$\cdot e^{-\alpha_m z}$$

Attenuation of
the voltage along
the electrodes

Approx $\alpha_m = 0$, now I find the phase modulation induced due to velocity mismatch and then I don't consider the vel. mismatch, but $\alpha_m \neq 0$, so I have the two limit:

$$\Delta \varphi = \Delta \varphi_0 \cdot \frac{\sin \frac{\pi f_m}{f_0}}{\pi \frac{f_m}{f_0}} = \Delta \varphi_0 \operatorname{sinc}\left(\pi \frac{f_m}{f_0}\right)$$

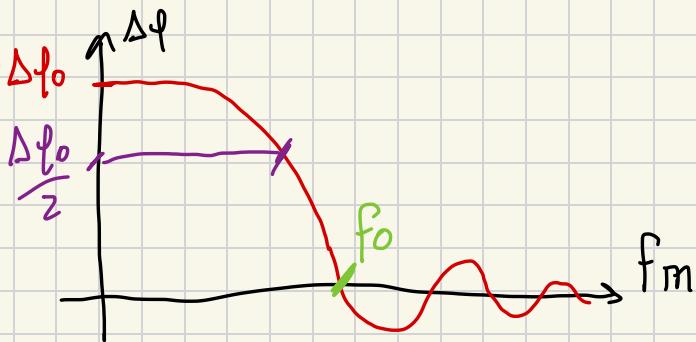
$\alpha_m = 0$

where

$$f_0 = \frac{C}{L(\sqrt{E_{irr}} - h_{eff})}$$

This must be small

$$\Delta \varphi_0 = \frac{2\pi}{\lambda} \frac{n^3}{2} r_{33} \frac{V_0}{d} \Gamma L$$



$$BW = f_o \cdot \frac{2}{\pi}$$

BW of modulator

The light and the electric beam starts together, the e-beam modulates the light, they combine to do this at same point they arrive in opposition of phase and the e-beam cancel the modulation beam of the first part.



THE DIFFERENCE IN VELOCITY IS THE MAX REASON THAT LIMIT THE BW OF MODULATORS

AND THE LENGTH

If $L \uparrow\uparrow$, $f_0 \downarrow\downarrow$, BW $\downarrow\downarrow$



You want to go fast? $L \downarrow\downarrow$, but
then $V \uparrow\uparrow$,
cost $\uparrow\uparrow\uparrow\uparrow$,

In L-N $\rightarrow f_{0\max} = 15 \text{ GHz}$

With attenuation

There is the skin effect increase at high freq,
so increased losses.

Now $\sqrt{\epsilon_r - n_{eff}} = 0 \rightarrow \sin(\omega t) \text{ exit intars}.$

$$\Delta\varphi = \Delta\varphi_0 \int_0^L e^{-\alpha_m z} dz$$

The mod. signal continues to decrease while propagating along electrodes.

$\alpha_m \propto \sqrt{f}$ because of skin effect

	A ₀	16 Hz	10 GHz
thickness of current [δ]		2,4 μm	0,76 μm
α_m	0,5 dB/cm		1,58 dB/cm

$$\alpha_m \sim 0,5 \sqrt{f} \frac{dB}{cm}$$

in GHz

$S_0 :$

$$\Delta\varphi = \Delta\varphi_0 \left(\frac{1 - e^{-\alpha_m L}}{\alpha_m L} \right)$$

//

V

REDUCTION OF BW DUE TO ATTENUATION

For $1 \div 2 \text{ dB/cm} \rightarrow \text{BW}_{\text{dm}} \approx 10 \text{ GHz}$

If you have also velocity mismatch, you combine the two effect.

THE PROBLEM IS L , IF
YOU REDUCE IT, \uparrow POWER

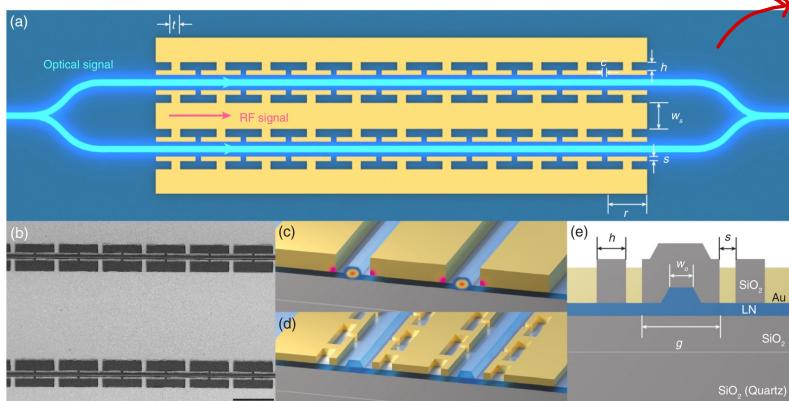
In $L-N \rightarrow$ O faster than E, you cannot due much unless shrink L and $V \uparrow$

In Si, InP \rightarrow E faster than O:

$$V_{in} = \sqrt{\frac{1}{LC}}$$

$$t_0 = \sqrt{\frac{L}{C}}$$

I can decrease E by increasing C,
so use **periodical backed electrode**



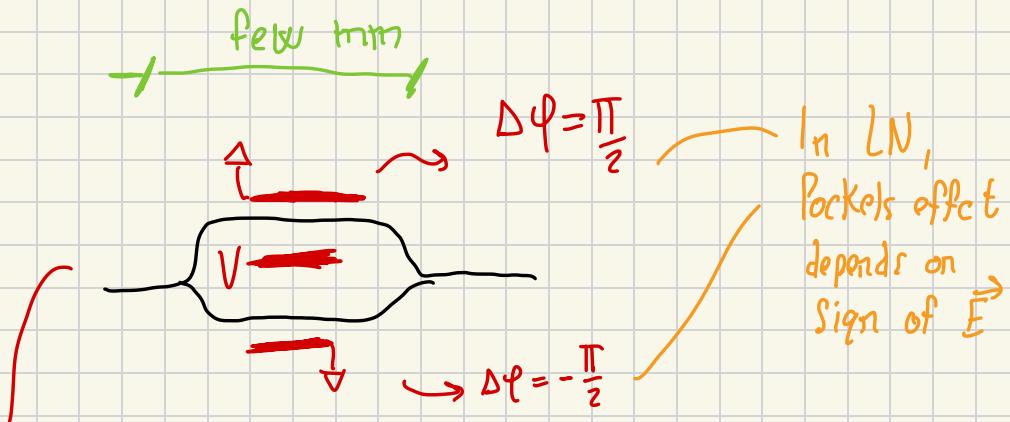
POSSIBILITY FOR VELOCITY MATCHING

IN SEMICONDUCTOR

INTENSITY MODULATION FROM PY



Interferometer \rightarrow MZI



The voltage to produce $\Delta\phi = \pi$ is called V_{π} , with two $\Delta\phi$.

PUSH-PULL CONFIGURATION

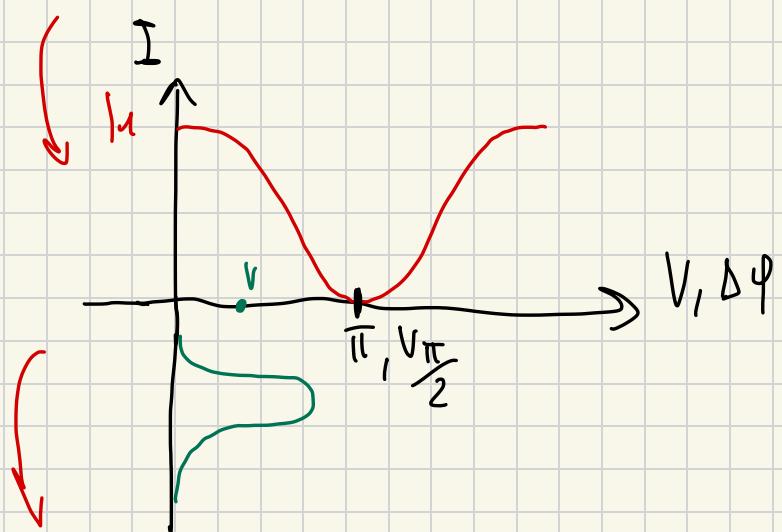
FOR ELECTRODES,

USE LESS POWER ($\frac{1}{4}$)

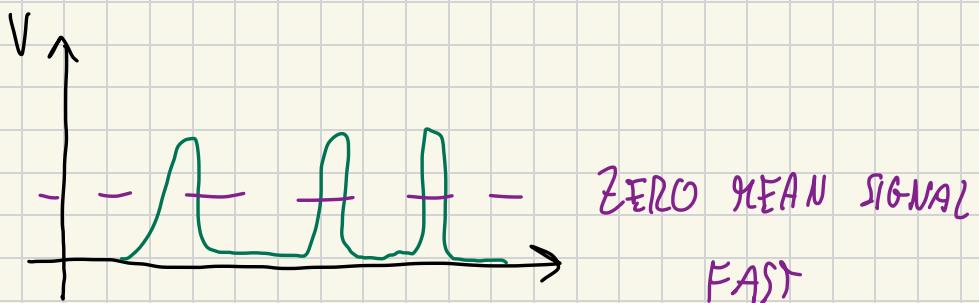
HALF VOLTAGE, SO

COOL DOWN THE DEVICE

$\frac{V_{\pi}}{2}$ produce π



So the RF signal :



ELECTRONIC DOESN'T
LIKE IT

I want to remove the DC component, I have

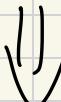
to polarize the modulator to stay at $\frac{\pi}{2}$ at DC



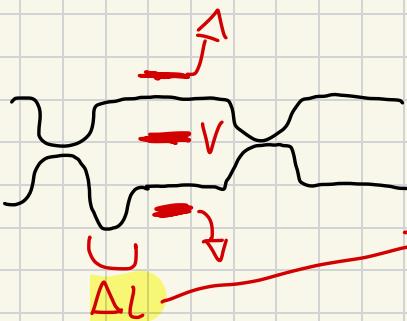
Bias



But a constant V dissipates a lot



Unbalance the MZ

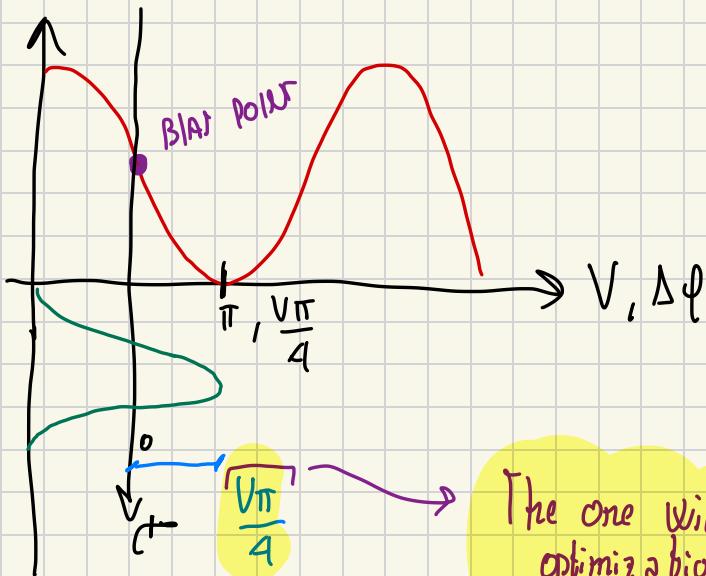


$$\frac{\lambda}{4} = 250 \text{ nm}, 900 \text{ d}!$$

So without signal $\Delta\varphi = \frac{\pi}{2}$.

Now V can be a time less, I need
to move only of $\pm \frac{\pi}{2}$, so $\frac{V}{4}$

Another factor of 2
in POWER



The one without
optimization

$$V\pi = \frac{\lambda}{n^3 r_{33} \pi} \frac{d}{L}$$

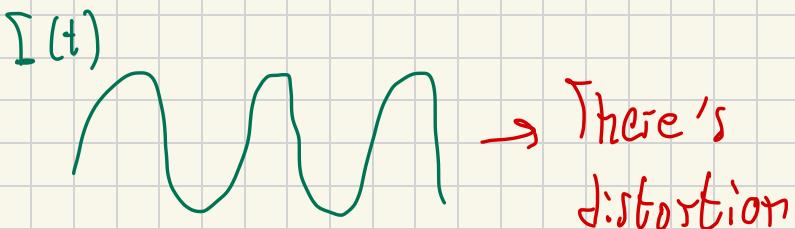
So from simple MT to optimized
is used.

$\frac{1}{16}$ of power

Attention

- If RF signal is $\sin(t)$, $I(t)$ is NOT just a copy of $\sin(t)$, pass through a MT, so

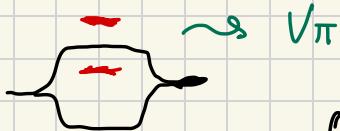
$$I = \cos^2(\sin(t))$$



$S(t)$



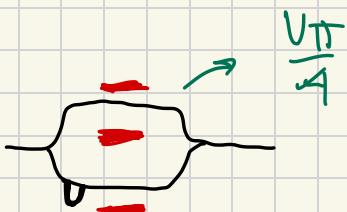
Push-pull vs simple



$$OUT = 1 + e^{-j\Delta\varphi(t)}$$

$$= \boxed{e^{-j\frac{\Delta\varphi(t)}{2}}} \cos\left(\frac{\Delta\varphi(t)}{2}\right)$$

CHIRP

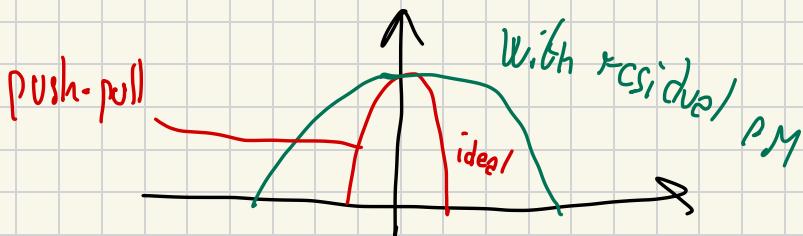


$$OUT = e^{-j\Delta\varphi(t)} + e^{+j\Delta\varphi(t)}$$

$$= \cos\left(\frac{\Delta\varphi(t)}{2}\right)$$

The push-pull generates the correct signal to the output, without a residual phase modulation like the simple one.

↳ PTT cause spectral regrowth



↓

UNWANTED IN WDM AND
NEED FILTER WITH LARGE
BLU (COST ↑↑)

Also the chromatic dispersion $D = \beta_2 L$
is larger, so greater distortion

↓

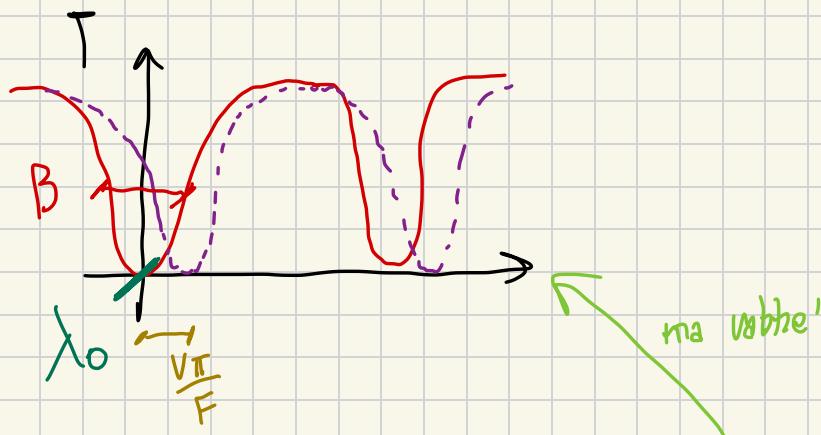
EVERYONE USE PUSH-PULL

RING MODULATOR (Si)

It has output intensity sinusoidal and the extinction ratio is $20 \div 40 \text{ dB}$, so I'm able to switch off the input if it's

enough small (α K).

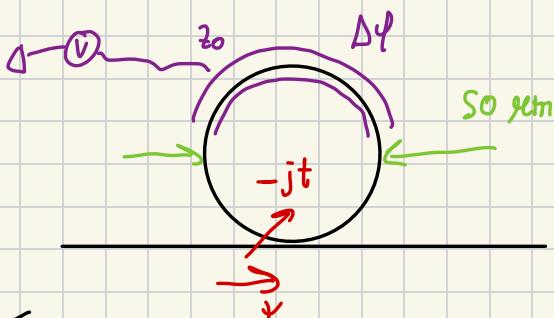
I can make a modulator with ring. If in critical coupling ($r_1 = \gamma r_2$) I can induce a $\Delta\ell$, so shift the ring characteristic:



So I change the output intensity.

$$\Delta n = - \dots \Delta N, \Delta P$$

in f.i.
in realtà verso sx



$$\tilde{\chi}_q = \frac{1}{\beta}$$

Molto più piccolo del Hz.

The shift will be given by $\frac{V_{\text{FMz}}}{F}$, so Blu

Move the notch to cancel or not the signal.



MAN MODULATOR ON CHUP,
LARGE DENSITY AND COMPACT
(W.R.T. Hz), SMALL VOLTAGE

Disadvantage

The electrodes are lumped, but the ring can be a pure C, so there will be reflections, the driving circuit must accept it.

What if $F \uparrow\uparrow$?

$B \downarrow\downarrow$, high selective and $V \downarrow\downarrow$ (FSR const.)



BUT THE LIMIT IS DUE
TO γ , $B \downarrow\downarrow$ CAN'T
HAPPEN FOR TOO MUCH

In a modulator you need $\gamma (= \tau)$.



$$\gamma_q = \frac{1}{B} \uparrow\uparrow \text{ if } B \downarrow\downarrow$$



BAD FOR MODULATOR
INCREASED TIME RESPONSE

$B = 1 \text{ GHz}$, but $\sim \gamma$ too much slow, so the perturbation is wrong.

↳ So I need $B \uparrow\uparrow$ to go faster, but to have V low I need FSR $\uparrow\uparrow \rightarrow$ small $\uparrow\uparrow$

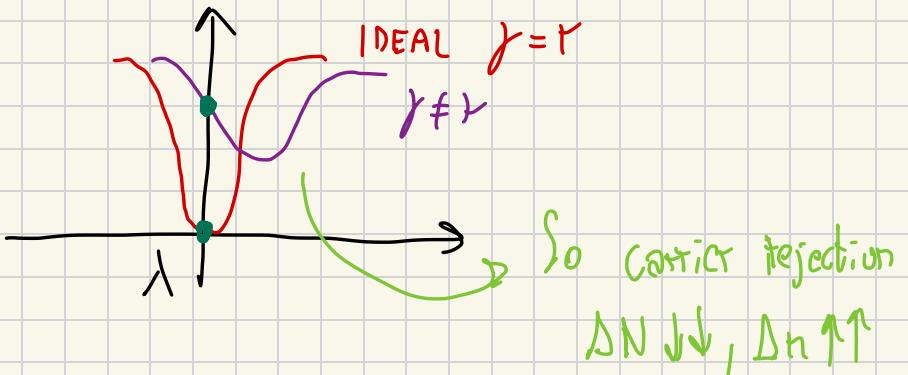
$$\Delta f \propto L \downarrow \curvearrowleft$$

✓

Complexity $\uparrow\uparrow$

What if $\gamma \neq \gamma'$?

In Si, I change the number of carrier (DN or DP) to produce Δf , I shift the spectrum but I change also $\Delta \zeta$ (in Si, InP p-n junction), so γ becomes different.



Here attenuation help, because the shift is as above but losses increases the transmission and here is wanted (switch on), the $V \rightarrow \text{bias}$, $\Delta n \rightarrow 0$, again $T=0$ & λ_0 , so switch off.

Another problem

The phase response is NOT linear

↳ SPURIOUS UNWANTED
PHASE MODULATION

SO MT OR RING? CHOOSE
BASED ON THE APPLICATION



After km of ocean is
better MT without chirp



Close P_F and
spurious phase change



Rotated to the
instantaneous frequency

the spectrum of the
signal has different
frequency for PM



$$if = \frac{d\phi}{dt}$$

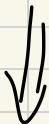
Not possible to control chirp in other modulator



But remember that PM increase
the BW of a signal intensity modulated

Chromatic dispersion

D_F = the difference of time that two harmonics at different λ arrive at destination



My signal in fibers has all the BW, so different λ arrive at different time

Note on Si + PN junction

MZ, push-pull can be implemented in Si photonics, so in Si it's possible to have modulator without chirp.

Ring modulator are good for short distances, like directly on chip or between near chips.