FILTERS

→ Filter synthesis procedure:

1. FILTER MASK
2. FILTER TRANSFER FUNCTION
3. NETWORK IMPLEMENTATION (ideal)
4. NON-IDEALITIES OPTIMIZATION

→ Filters are divided (classified) into

LOW PASS ; HIGH PASS ; BAND PASS ; STOP-BAND ; EQUALIZERS (all pass)

The ideal filter should have
1) constant in band amplitude
2) linear phase shift proportional to ω

⇓

DEM: Let's consider a signal falling in the pass-band of a filter. It is affected by a finite propagation delay τ, so

$$x(t) \longrightarrow \boxed{\text{FILTER}} \longrightarrow y(t) = Ax(t-\tau)$$

τ must affect any harmonic component, so each harmonic suffers from a

$$\Delta\varphi = -\frac{2\pi}{T}\cdot\tau = -\omega\tau \quad \text{shift}$$

These conditions however cannot be fulfilled by a real filter, since

DEM: Let's consider an ideal LP on a bilateral frequency axis, so a rectangle. Its inverse Fourier transform is a

$$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H_0 e^{j\omega t}\, d\omega = \frac{H_0}{2\pi j t}\left[e^{j\omega t}\right]_{-\omega_c}^{\omega_c} = \frac{H_0}{\pi t}\sin(\omega_c t)$$

$$\downarrow$$

$$= \frac{\omega_c H_0}{\pi}\,\frac{\sin(\omega_c t)}{\omega_c t} = h_0 \cdot \text{sinc}(\omega_c t)$$

$h(t) \neq 0$ for $t < 0$ ⟹ ANTI-CAUSAL, NOT FEASIBLE!

→ Important definitions for the FILTER MASK are

A_{BP} ATTENUATION IN BAND-PASS (maximum)

A_{SB} ATTENUATION IN STOP-BAND (minimum)

ω_{BP} CUT-OFF FREQUENCY

ω_{SB} LOWER LIMIT OF THE STOP-BAND

$\omega_0 = \sqrt{\omega_{BP}^+ \omega_{BP}^-}$ CENTRAL FREQUENCY OF A BAND-PASS FILTER

$Q = \frac{\omega_0}{\Delta\omega}$ Q FACTOR OF A BAND-PASS FILTER

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$K = \frac{\omega_{BP}}{\omega_{SB}} < 1$ SECTIVITY INDEX (lower = worst)

$\varepsilon_{BP} = \sqrt{10^{\frac{A_{BP}}{10}} - 1}$ MAXIMUM ATTENUATION INDEX IN BAND-PASS

$\varepsilon_{SB} = \sqrt{10^{\frac{A_{SB}}{10}} - 1}$ MINIMUM ATTENUATION INDEX IN STOP-BAND

$K_\varepsilon = \frac{\varepsilon_{BP}}{\varepsilon_{SB}} < 1$ DISCRIMINATION INDEX (lower = better)

$\tau_{GD} = - \frac{d\varphi}{d\omega}$ GROUP DELAY (ideally constant)

→ ALL POLES FUNCTIONS

- BUTTERWORTH

They require maximum band-pass and stop-band flatness

$$\frac{d^k}{d\omega^k} |H(j\omega)|^2 \Big|_{\omega=0} = 0 \quad \text{for } k = 1, 2, \dots, 2n-1$$

$$\frac{d^k}{d\omega^k} |H(j\omega)|^2 \Big|_{\omega=\infty} = 0 \quad \text{for } k = 1, 2, \dots, 2n-1$$

For a reference $\Omega_0 = 1$ rad/s angular frequency, these conditions are fulfilled by

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}$$

All poles are placed on a circle of radius $\Omega_0 = 1$ rad/s on the Gauss plane, while

$$Q = \frac{1}{2\xi} = \frac{|p|}{2\mathrm{Re}[p]} = \frac{1}{2} \text{ if REAL}$$

Poles are equally spread one from the next on the circle by $\frac{\pi}{n}$ and the ones with higher Q factor are $\frac{\pi}{2n}$ far from the $\mathrm{Im}[\,]$ axis.

In order to achieve a lower A_{BP} than 3dB we may choose a different Ω_0, thus having

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\Omega_0}\right)^{2n}}$$

Let's fulfill the specs:

$$\frac{1}{1 + \left(\frac{\Omega_{BP}}{\Omega_0}\right)^{2n}} \geq \frac{1}{1 + \varepsilon_{BP}^2} \implies \Omega_0 \geq \frac{\Omega_{BP}}{\varepsilon_{BP}^{1/n}}$$

$$\frac{1}{1 + \left(\frac{\Omega_{SB}}{\Omega_0}\right)^{2n}} \leq \frac{1}{1 + \varepsilon_{SB}^2} \implies \Omega_0 \leq \frac{\Omega_{SB}}{\varepsilon_{SB}^{1/n}}$$

DEM: $\Omega_0 \geq \dfrac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \wedge \Omega_0 \leq \dfrac{\Omega_{SB}}{\varepsilon_{SB}^{1/m}}$, by multiplying the first term for $\varepsilon_{SB}^{1/m}$ we get

$$\frac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \cdot \varepsilon_{SB}^{1/m} \leq \Omega_0 \, \varepsilon_{SB}^{1/m} \leq \Omega_{SB}$$

$$\Downarrow$$

$$\Omega_{BP} \cdot k_\varepsilon^{-1/m} \leq \Omega_{SB}$$

$$\downarrow$$

$$k_\varepsilon^{-1/m} \leq k^{-1}$$

$$\downarrow$$

$$\frac{1}{m} \ln (k_\varepsilon^{-1}) \leq \ln (k^{-1})$$

$$\downarrow$$

$$m \geq \frac{\ln (k_\varepsilon^{-1})}{\ln (k^{-1})}$$

And $\dfrac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \leq \Omega_0 \leq \dfrac{\Omega_{SB}}{\varepsilon_{SB}^{1/m}}$

- **BESSEL**

  They require the maximum phase linearity, so

  $$\left.\frac{d^n \tau_{GD}}{d\omega^n}\right|_{\omega=0} = 0 \quad \text{for } n = 1, 2, \dots, m$$

  They have poles placed outside the Ω_0 characteristic frequency circle, in order to reach a better group delay, thus also a lower selectivity.
  In order to select them, we use tables referred to $\Omega_0 = 1$ rad/s and we try to see if a specific order with a specific Ω_0 fulfills the specs.

- **CHEBYSHEV - I**

  They require the maximum selectivity, but with an in-band ripple.

  $$\max\left\{1 - |H(j\omega)|^2\right\}_{\omega \leq 1} \leq \varepsilon_{BP}^2 \quad \text{maximum ripple requirement}$$

  $$\left.\frac{d^n |H(j\omega)|^2}{d\omega^n}\right|_{\omega \to \infty} = 0 \quad \text{for } n = 1, 2, \dots, 2m-1 \quad \text{stop-band flatness}$$

  The DC value must be set to 1 for odd order T.F. and to $\dfrac{1}{\sqrt{1+\varepsilon_{BP}^2}}$ for even order T.F., in order to have $|H(j\omega)| \leq 1$ in band.

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USEFUL CHEBYSHEV FORMULAS

$$m \ge \frac{\mathrm{arcCh}(k\varepsilon^{-1})}{\mathrm{arcCh}(k^{-1})}$$

$$\Gamma = \left(\frac{1+\sqrt{1+\varepsilon_{Bp}^2}}{\varepsilon_{Bp}}\right)^{1/m}$$

Consider only the ones with $\mathrm{Re}[\ ] < 0$

$$s_k = -\sin\left[(2k-1)\frac{\pi}{2m}\right]\frac{\Gamma^2-1}{2\Gamma} + j\cos\left[(2k-1)\frac{\pi}{2m}\right]\frac{\Gamma^2+1}{2\Gamma}$$

(for $k = 1, \dots, 2m$)

m = # of half oscillations in band-pass

→ POLES AND ZEROES

- **CHEBYSHEV II**

  They have imaginary conjugated zeroes in stop band in order to have a ripple there and kill HF disturbances.

  $$\max\left\{1-|H(j\omega)|^2\right\}_{\omega \ge 1} \ge \varepsilon_{SB}^2 \quad \text{— min attenuation in stop band} \quad \left(\varepsilon = \frac{1}{\varepsilon_{SB}}\right)$$

  $$\frac{d^k}{d\omega^k}|H(j\omega)|^2\Big|_{\omega=0} = 0 \quad \text{for } k = 1, 2, \dots, 2m-1$$

- **CAUER**

  They are a constant ripple both in stop-band and in band-pass. Once k and the in-band ripple are set, the stop-band ripple is determined

- **GENERALIZED ELLIPTICAL**

  They give an additional degree of freedom, so we can independently set k and both ripples, but we get a bad τ_{GD}.

→ ADDITIONAL DEMONSTRATION

It can be shown that for a Chebyshev-type-I filter it is

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_m^2(\Omega)} \qquad \begin{cases} \varepsilon \triangleq \text{RIPPLE FACTOR} \\ C_m = \text{Chebyshev polynomial} \end{cases}$$

where

$$C_m(\Omega) = \begin{cases} \cos\left[m \arccos\left(\frac{\Omega}{\Omega_{Bp}}\right)\right] & \left(\frac{\Omega}{\Omega_{Bp}} \le 1\right) \\ \mathrm{Ch}\left[m\, \mathrm{settCh}\left(\frac{\Omega}{\Omega_{Bp}}\right)\right] & \left(\frac{\Omega}{\Omega_{Bp}} \ge 1\right) \end{cases} \qquad (=1 \text{ for } \Omega = \Omega_{Bp})$$

so

$$|H(j\Omega_{Bp})|^2 = \frac{1}{1+\varepsilon^2} \implies \varepsilon = \sqrt{10^{\frac{A_{Bp}}{10}} - 1} = \varepsilon_{Bp}$$

In order to derive $m = \dfrac{\mathrm{settCh}(k\varepsilon^{-1})}{\mathrm{settCh}(k^{-1})}$ write $|H(j\Omega_{SB})|^2 = \dfrac{1}{1+\varepsilon_{SB}^2 C_m^2(\Omega_{SB})} \dots$

ORMALIZATION AND TRANSFORMATION

Filter synthesis is performed in a domain Ω referred to a low pass filter with $\Omega_{sp} = 1$ rad/s. Therefore we need suitable transformations.

$\longrightarrow$ LOW PASS

$\Omega = \dfrac{\omega}{\omega_{sp}}$ in order to get $\Omega_{sp} = 1$ rad/s

Starting from the normalized Butterworth polynomials referred to $\Omega_0 = 1$ rad/s we need first to set

$\hat{s} = \Omega_0 \, p$ $\qquad$ p normalized low-pass with $\Omega_0 = 1$ rad/s

$\qquad\qquad\qquad\quad$ $\hat{s}$ " " " " $\Omega_0 \neq 1$ rad/s

Then, we get

$s = \omega_{sp} \cdot \hat{s}$ $\qquad$ s low-pass

$\longrightarrow$ HIGH PASS

$\Omega = \dfrac{\omega_{sp}}{\omega}$ in order to have $\Omega_{sp} = 1$ rad/s and to flip the axis

$\hat{s} = \dfrac{\omega_{sp}}{s}$

$\longrightarrow$ BAND-PASS

DEM: let's consider this transformation

$$\hat{s} = \hat{p} + \frac{1}{\hat{p}} \qquad \begin{cases} \hat{s} = \Lambda + j\Omega \\ \hat{p} = \alpha + j\omega \end{cases}$$

$$\downarrow$$

$$\hat{p}^2 - \hat{s}\hat{p} + 1 = 0$$

$$\downarrow$$

$$\hat{p} = \frac{\hat{s} \pm \sqrt{\hat{s} - 4}}{2} \implies \alpha + j\omega = \frac{\Lambda + j\Omega \pm \sqrt{\Lambda^2 - \Omega^2 + 2j\Lambda\Omega - 4}}{2}$$

Now, the Im[] axis of $\hat{s}$ $(\Lambda = 0)$ is mapped into

$$\alpha + j\omega = \frac{j\Omega \pm j\sqrt{\Omega^2 + 4}}{2} = j\,\frac{\Omega}{2} \pm j\sqrt{1 + \left(\frac{\Omega}{2}\right)^2}$$

so $\alpha = 0$ and the Im[] axis of $\hat{s}$ is mapped into two points on the Im[] axis of $\hat{p}$.

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In particular,

$$\Omega = 0 \implies w_{1,2} = \pm 1$$

$$\Omega = 1 \implies w_{1,2} = +1{,}62\ ;\ -0{,}62$$

Being $\Omega = 1$ the cut-off of the low pass filter, this is mapped into Ω_{bp}^{+} and Ω_{bp}^{-} of the band pass, so it sets the BW to 1. So, to have a BW smaller than 1, we should apply this transformation to a LP with $\Omega_{lp} = \frac{1}{Q}$. Finally, to shift the central frequency, we should expand $\hat{p}$, this being $s = \hat{p} \cdot w_0$

$$\Downarrow$$

$$\hat{s} = Q\tilde{s} = Q\left[\hat{p} + \frac{1}{\hat{p}}\right] = Q\left[\frac{s}{w_0} + \frac{w_0}{s}\right] = Q\left[\frac{s^2 + w_0^2}{s w_0}\right]$$

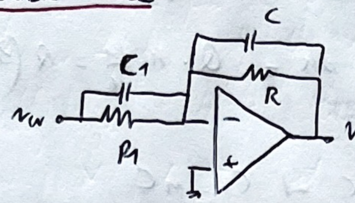
which in terms of radial frequency they correspond to

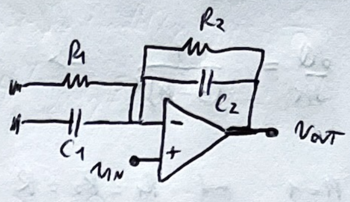
$$\Omega = Q\left(\frac{w^2 - w_0^2}{w w_0}\right)$$

ACTIVE CELLS

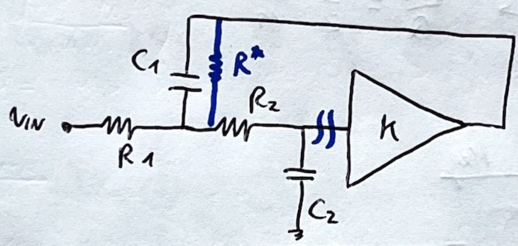
We can always split the T.F. in the product of 1^{st} and 2^{nd} order terms, so we can realize each of them as a block and then cascade all of them (sufficiently decoupled) in order to make the final T.F.. We can use OPAMPS!

$\longrightarrow$ 1^{ST} ORDER CELL





$\longrightarrow$ 2^{ND} ORDER CELL (Sallen - Key)



DEM:

$$G_{loop}(s) = +K \cdot \frac{R_1}{R_1 + R^*}$$

$$a_1 = R^* C_1$$

$$b_1 = C_2\left(R_2 + R_1 // R^*\right) + C_1 \cdot \left(R^* // R_1\right)$$

$$b_2 = C_1 C_2 \left(R^* // R_1\right) R_2$$

$$G_{loop}(s) = +K \cdot \frac{R_1}{R_1+R^*} \cdot \frac{1 + sC_1R^*}{1 + \left[C_2(R_2 + R_1//R^*) + C_1(R^*//R_1)\right]s + s^2C_1C_2R_2(R^*//R_1)}$$

$$\Big\downarrow R^* \to +\infty$$

$$= +K \frac{sC_1R_1}{s^2 C_1C_2R_1R_2 + s\left[C_1R_1 + C_2(R_1+R_2)\right] + 1}$$

By solving $G_{loop}(s) - 1 = 0$ we get the closed loop denominator:

$$s^2(R_1R_2C_1C_2) + s\left[R_1(1-K)C_1 + (R_1+R_2)C_2\right] + 1 = 0$$

so $\omega_0 = \frac{1}{\sqrt{C_1C_2R_1R_2}}$ and

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_1R_1 + C_2(R_1+R_2) - KC_1R_1}$$

$$\Big\downarrow$$

$$= \frac{(R_1C_1)^{-1}\sqrt{R_1R_2C_1C_2}}{(1-K) + \frac{C_2}{R_1C_1}(R_1+R_2)} = \frac{\sqrt{\frac{R_2C_2}{R_1C_1}}}{(1-K) + \frac{C_2}{C_1}\left(1+\frac{R_2}{R_1}\right)}$$

(55) Q depends on ratios $\Rightarrow$ affected by on chip variations, mitigated by mirroring

ω_0 depends on absolute values $\Rightarrow$ affected by chip-to-chip variations

(1) $K \neq 1$ and $R_1 = R_2 = R$ and $C_1 = C_2 = C$

$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{1}{3-K} \quad \Rightarrow \quad K = 3 - \frac{1}{Q} \quad \text{dependent on } Q\,!$$

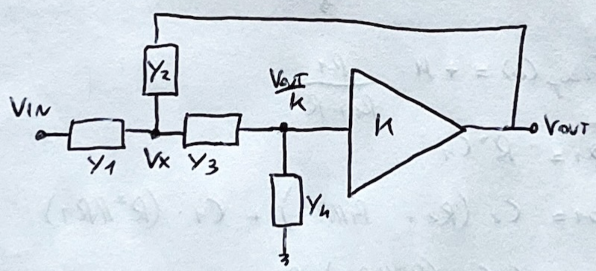
$$\frac{dQ}{dK} = \frac{1}{(3-K)^2} \Rightarrow \frac{dQ}{Q} = \frac{1}{(3-K)}\,dK = Q \cdot K \frac{dK}{K} = Q\left(3 - \frac{1}{Q}\right)\frac{dK}{K}$$

(2) $K = 1$ and $R_1 = R_2 = R$ and $C_2 = C$ and $C_1 = mC$

$$\omega_0 = \frac{1}{\sqrt{m}\,RC} \qquad Q = \frac{\sqrt{m}}{2} \quad \Rightarrow \quad m = 4Q^2$$

$$\frac{dQ}{dm} = \frac{1}{4\sqrt{m}} \Rightarrow \frac{dQ}{Q} = \frac{1}{2}\frac{dm}{m}$$

$\longrightarrow$ HIGH-PASS Sallen-Key



DEM:

$$\begin{cases} (v_{IN} - v_x) Y_1 + (v_{OUT} - v_x) Y_2 = \left(v_x - \frac{v_{OUT}}{K}\right) Y_3 \\ v_x = \frac{v_{OUT}}{K}\left(1 + \frac{Y_4}{Y_3}\right) \end{cases}$$

$\downarrow$

$$v_{OUT}\left(Y_2 + \frac{Y_3}{K}\right) - v_x\left[Y_1 + Y_2 + Y_3\right] = -v_{IN} Y_1$$

$\downarrow$

$$v_{OUT}\left(Y_2 + \frac{Y_3}{K}\right) - \frac{v_{OUT}}{K}\left[1 + \frac{Y_4}{Y_3}\right]\left[Y_1 + Y_2 + Y_3\right] = -v_{IN} Y_1$$

$\downarrow$

$$v_{OUT}\left[Y_2 + \frac{Y_3}{K} - \frac{Y_1}{K} - \frac{Y_2}{K} - \frac{Y_3}{K} - \frac{Y_1 Y_4}{K Y_3} - \frac{Y_2 Y_4}{K Y_3} - \frac{Y_4}{K}\right] = -v_{IN} Y_1$$

$\downarrow$

$$\frac{v_{OUT}}{v_{IN}} = \frac{Y_1}{\frac{1}{K}\left[Y_1 + Y_2 + Y_3 + \frac{Y_1 Y_4}{Y_3} + \frac{Y_2 Y_4}{Y_3} + Y_4 - KY_2\right]} = \frac{K Y_1 Y_3}{Y_4\left[Y_1 + Y_2 + Y_3\right] + Y_1 Y_3 + Y_2 Y_3 (1-K)}$$

→ **UNIVERSAL CELL**

Also called Kerwin - Huelsman - Newcomb (KHN)

DEM: let's consider a high-pass T.F.

$$\frac{v_{HP}}{v_{IN}}(s) = \frac{\gamma s^2}{\left(\omega_0^2 + s\frac{\omega_0}{Q} + s^2\right)}$$

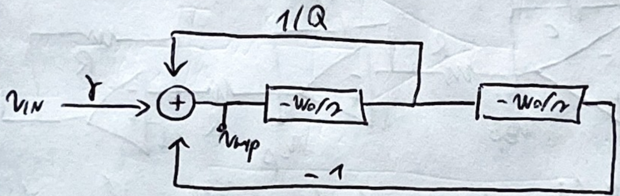
In order to let the integrators appear, let's divide for the highest power of s:

$$v_{HP}\left[1 + \frac{\omega_0}{Qs} + \frac{\omega_0^2}{s^2}\right] = \gamma v_{IN}$$

↓

$$v_{HP} = \gamma v_{IN} - \frac{\omega_0}{s}\frac{v_{HP}}{Q} - \frac{\omega_0^2}{s^2} v_{HP}$$

BLOCK DIAGRAM:

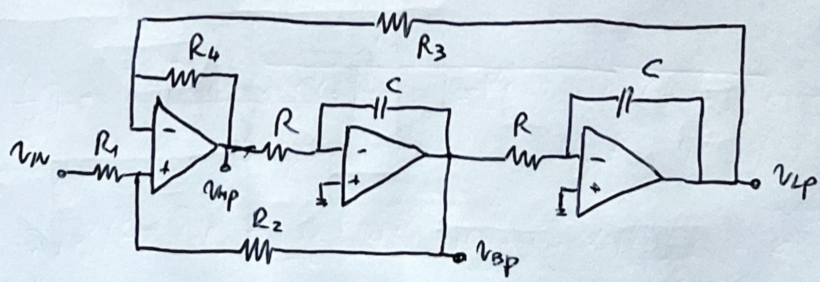


After the first integrator we get a BAND-PASS

$$\frac{v_{BP}(s)}{v_{IN}(s)} = -\frac{\gamma \omega_0 s}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}$$

and after the second we get a LOW-PASS

$$\frac{v_{LP}(s)}{v_{IN}(s)} = \frac{\gamma \omega_0^2}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}$$



$$\Rightarrow v_{HP}(s) = \frac{R_2}{R_1+R_2}\cdot\frac{R_3+R_4}{R_3}\cdot v_{IN}(s) + \frac{R_1}{R_1+R_2}\cdot\frac{R_3+R_4}{R_3} v_{BP}(s) + \left(-\frac{R_4}{R_3}\right) v_{LP}(s)$$

So we get

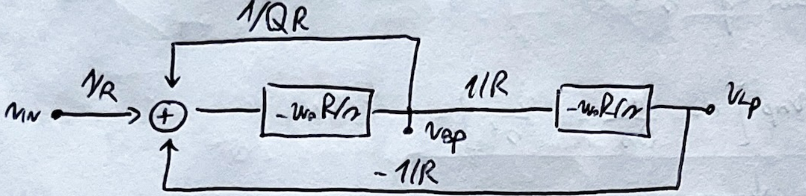
$R_4 = R_3$ ([redacted] -1) $\quad \gamma = \frac{2R_2}{R_1+R_2}$ and $\frac{1}{Q} = \frac{2R_1}{R_1+R_2}$

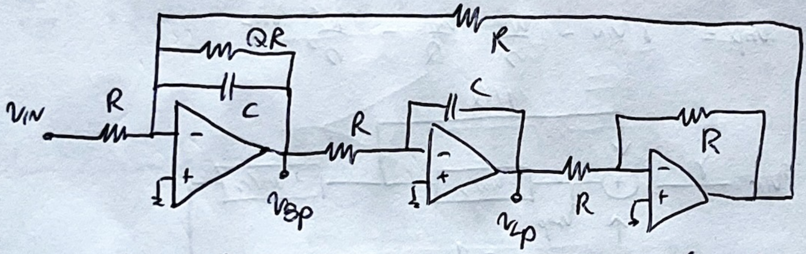
(57) $\frac{R_2}{R_1} = 2Q - 1$ and $\gamma = \left(2 - \frac{1}{Q}\right)$ dependent of each other!

If we combine the three outputs in a suitable good we may get a T.F. with also zeroes

→ **Tow-Thomas and Fleischer-Thomas**

We may sum current signals instead of voltages by using an integrator V.C.. To this aim we need to divide ~~[illegible]~~ entering the integrator by $\frac{1}{R}$ and to multiply integrator T.F. by R.





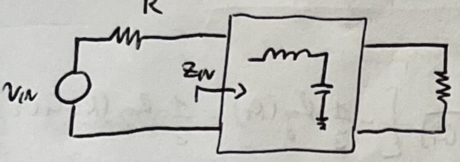
In a fully differential mode we don't need the inverter, it's enough to flip the cables.

By entering with v_{IN} in all three V.C. we get a Fleischer-Thomas cell with also two zeroes.

LADDER NETWORKS

The idea is to exploit interactions between different components in the same cell in order to compensate sensitivities. In active cells instead, each cell contributes independently to sensitivity.

DEM: These networks are <u>PASSIVE COMPONENTS</u>, like this



DOUBLY TERMINATED NETWORK

At the maximum of the T.F. $Z_{IN} = R$. At the nominal frequency we the power delivered to the load can be written as

$$P_L = \frac{\frac{V_{OUT}^2}{2}}{R} = \frac{V_{IN}^2}{2R}\,|T(j\omega, X_0)|^2 \qquad X_0 \triangleq \text{nominal values for } L \text{ and } C$$

Let's assume to have a maximum @ $\omega = \omega^*$, then

$$\left.\frac{\partial P_L}{\partial \omega}\right|_{\omega=\omega^*} = \frac{V_{IN}^2}{2R} \cdot 2\,|T(j\omega^*, X_0)| \cdot \left.\frac{\partial}{\partial \omega}|T(j\omega, X_0)|\right|_{\omega=\omega^*} = 0$$

$$\downarrow$$

$$\left.\frac{\partial}{\partial \omega}|T(j\omega, X_0)|\right|_{\omega=\omega^*} = 0 = \left.\frac{\partial}{\partial X}|T(j\omega^*, X)|\right|_{X=X_0} \quad \text{being an absolute maximum}$$

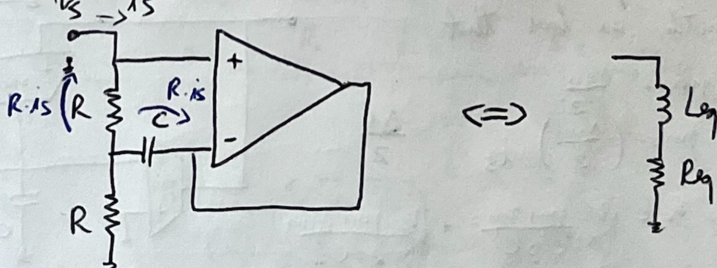
So the sensitivity of the T.F. to variations of L and C is nil at the maximum and remains nil around it due to continuity (ORCHARD THEOREM)

How to implement inductors?

→ <u>GYRATORS</u>

They are active circuits to generate an inductive impedance

(1)



$$v_S = R \cdot i_S + (i_S + R \cdot i_S\, s C) \cdot R$$

$$\downarrow$$

$$= i_S\,[2R + s C R^2]$$

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SENSITIVITY ANALYSIS

$$S_X^Y = \frac{\frac{\partial Y}{Y}}{\frac{\partial X}{X}} = \frac{\partial \log(Y)}{\partial \log(X)} \qquad \begin{cases} X = \text{circuit parameter} \\ Y = \text{filter parameter} \\ \log = \text{natural log } \ln \end{cases}$$

EX: $w_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$

$$S_{R_1}^{w_0} = \frac{\partial \log(w_0)}{\partial \log(R_1)} = \frac{\partial}{\partial \log(R_1)}\left[-\frac{1}{2}\log(R_1) - \frac{1}{2}\log(R_2C_1C_2)\right] = -\frac{1}{2}$$

same for $S_{R_2}^{w_0}$ $S_{C_1}^{w_0}$ $S_{C_2}^{w_0}$, and for $S_K^{w_0} = 0$

EX: $Q = \frac{1}{[R_1(1-K)C_1 + (R_1+R_2)C_2]w_0}$

$$S_{R_1}^{Q} = \frac{\partial}{\partial \log(R_1)}\left[-\log(w_0) - \log(R_1(1-K)C_1 + (R_1+R_2)C_2)\right] =$$

$$= \frac{1}{2} - \frac{R_1}{\partial R_1}\left[\log(R_1(1-K)C_1 + (R_1+R_2)C_2)\right] =$$

$$= \frac{1}{2} - R_1 \frac{(1-K)C_1 + C_2}{R_1(1-K)C_1 + (R_1+R_2)C_2} =$$

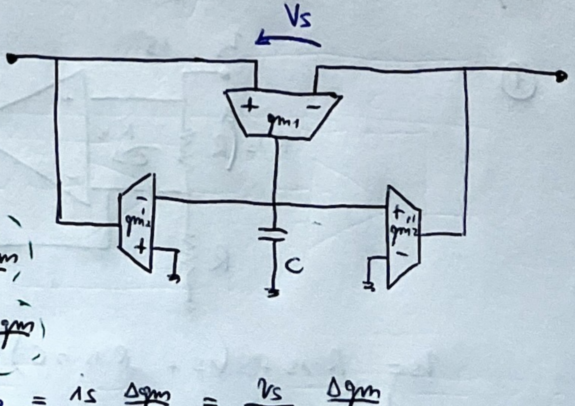
$$= \frac{R_1[1-K]C_1 + (R_1+R_2)C_2 - 2C_1R_1(1-K) - 2C_2R_1}{2[R_1(1-K)C_1 + (R_1+R_2)C_2]} =$$

$$= -\frac{C_1R_1(1-K) + (R_1-R_2)C_2}{2[R_1(1-K)C_1 + (R_1+R_2)C_2]}$$

The others are performed basically in the same way...

MISMATCH ISSUE IN GYRATORS

If the two g_{m2} transconductances do not match, we have that



$g_{m2}' = g_{m2} + \frac{\Delta g_m}{2}$

$g_{m2}'' = g_{m2} + \left(-\frac{\Delta g_m}{2}\right)$ $\quad i_s \quad \frac{\Delta i}{2}$

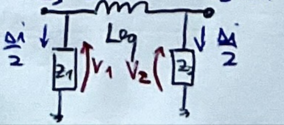
$$i_{IN} = g_{m2}' v_c = \frac{v_s g_{m1} g_{m2}}{sC} + \frac{v_s g_{m1} \Delta g_m}{2sC}$$

$$i_{OUT} = g_{m2}'' v_c = \frac{v_s g_{m1} g_{m2}}{sC} - \frac{v_s g_{m1} \Delta g_m}{2sC}$$

$$i_{IN} - i_{OUT} = \Delta i = \frac{v_s g_{m1}}{sC} \cdot \frac{\Delta g_m}{g_{m2}} g_{m2} = i_s \frac{\Delta g_m}{g_{m2}} = \frac{v_s}{sL_{eq}} \frac{\Delta g_m}{g_{m2}}$$

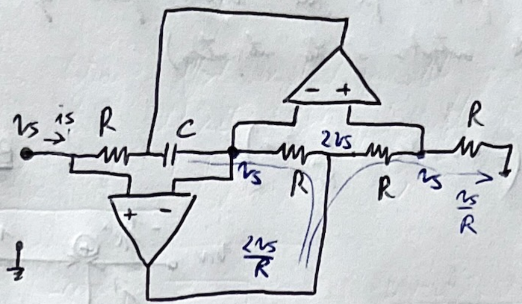
We basically get this network:

$z_2 = \frac{v_2}{\frac{\Delta i}{2}}$; $z_1 = \frac{v_1}{\frac{\Delta i}{2}}$



$$\begin{cases} v_1 = \frac{v_s}{2} + v_{cm} \\ v_2 = -\frac{v_s}{2} + v_{cm} \end{cases}$$

② ANTONIOU CIRCUIT

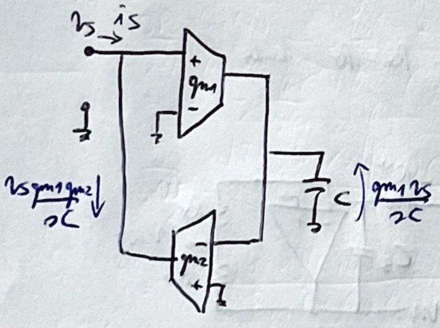


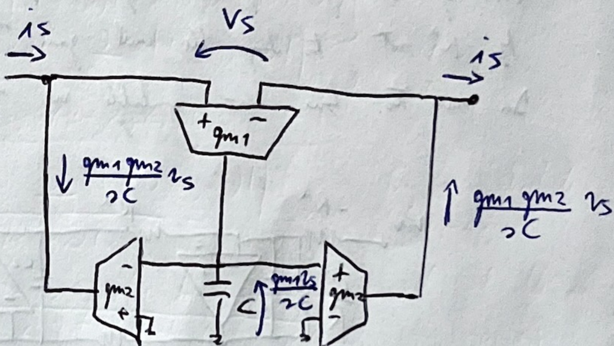
$$\frac{v_s}{R}\cdot\frac{1}{sC} = i_s \cdot R$$

↓

$$\frac{v_s}{i_s} = sCR^2$$

③ Other solutions



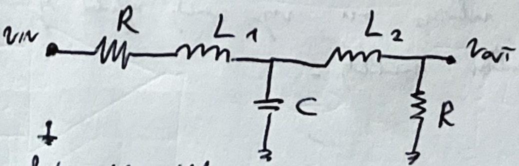


Problem of gyrators are
- limited GBWP of active elements
- noise

→ INTEGRATORS (DEM:)

Let's consider this 3rd order ladder network:

$$\begin{cases} KVL: v_{IN} - v_C = (sL_1 + R)\, i_1 \\ KVL: v_C - v_{OUT} = sL_2\, i_2 \\ KCL: i_1 - i_2 = sC v_C \end{cases}$$



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find state variables
(inductor currents, capacitor voltages)

Now write the currents as auxiliary voltages divided by an auxiliary resistance R^*

$$\begin{cases} v_{IN} - v_C = \frac{sL_1 + R}{R^*}\cdot v_1 \\ v_C - v_{OUT} = \frac{sL_2}{R^*}\, v_2 \\ \frac{v_1 - v_2}{R^*} = sC v_C \\ v_{OUT} = \frac{R}{R^*}\, v_2 \end{cases}$$

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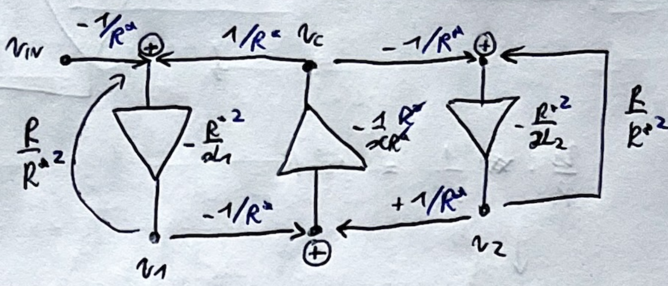
Now divide each equation for the highest power of s in order to let the integrators appear: (which the outputs of the integrators)

$$\Downarrow$$

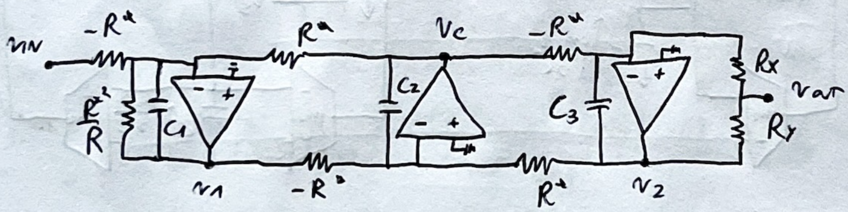
$$\begin{cases} v_1 = -\dfrac{R^*}{sL_1}\left(-v_{IN} + v_C + \dfrac{R}{R^*} v_1\right) \\ [redacted] \\ v_C = -\dfrac{1}{sCR^*}\left(-v_1 + v_2\right) \\ v_2 = -\dfrac{R^*}{sL_2}\left(-v_C + v_{OUT}\right) \\ v_{OUT} = \dfrac{R}{R^*} \cdot v_2 \end{cases}$$

BLOCK DIAGRAM



We may use virtual grounds in order to sum signals, so we need to divide each T.F. of the inputs of the integrators by $\frac{1}{R^*}$ and then to multiply each integrator T.F. by R^*, not to change the overall gain.

In order to have v_{OUT} available we need an $R_X - R_Y$ divider



$$\begin{cases} R_X + R_Y = \dfrac{R^{*2}}{R} \\ v_{OUT} = \dfrac{R_X}{R_X + R_Y} v_2 = \dfrac{R}{R^*} v_2 \end{cases} \Rightarrow R_X = R^* \text{ and } R_Y = \frac{R^{*2}}{R} - R^*$$

These networks are very weakly affected by feedback capacitors' variability, but they are affected by resistor variability.