POLITECNICO
MILANO 1863

ELECTRONIC SYSTEMS

2021-22 academic year

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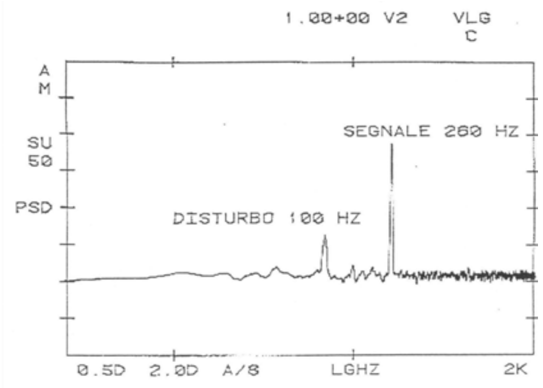
Summary

- Noise, power, rms value
- Types: thermal, shot, flicker, burst, …
- Components' noises
- Equivalent Bandwidth and generators

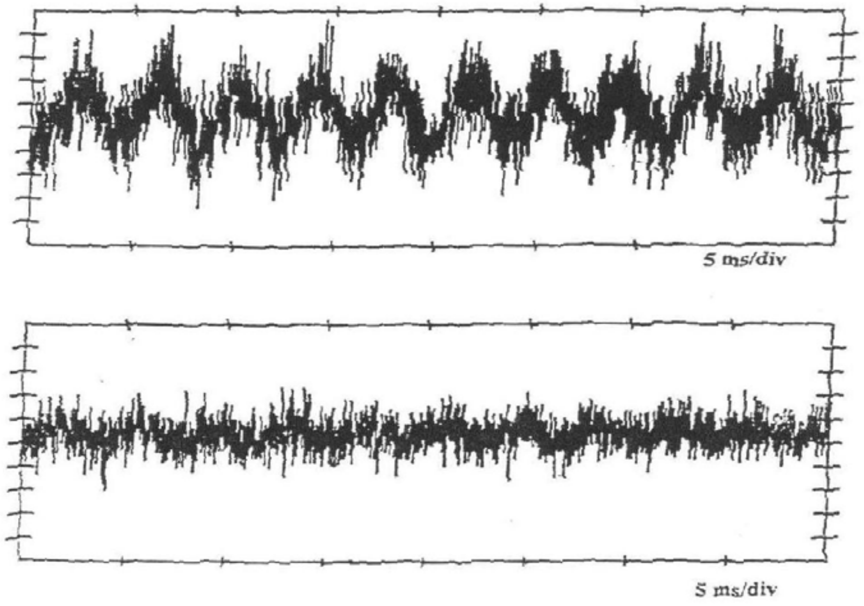
Signal, Noise, Distortion

$$x(t) = s(t) + n(t) + d(t)$$

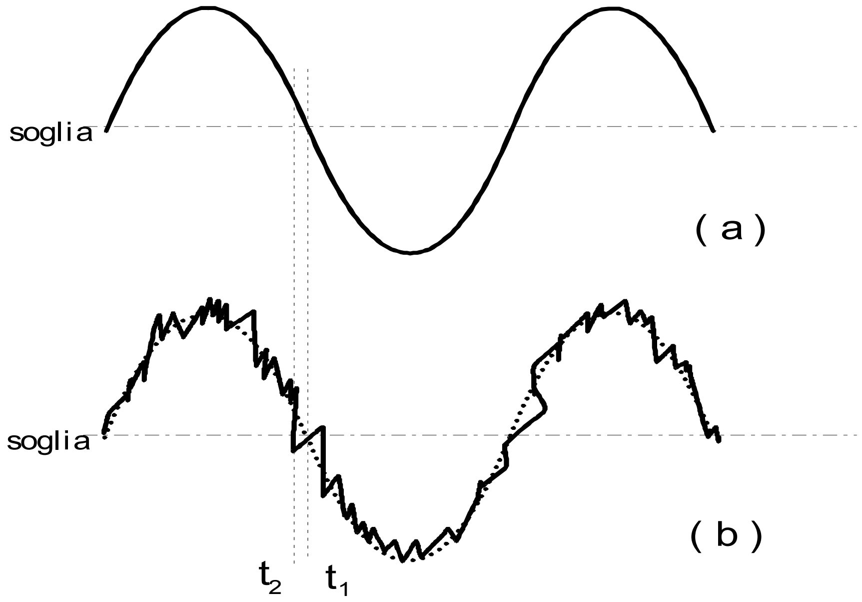


DISTURBO: TONO AD UNA SPECIFICA FREQUENZA

"analog" signal issues



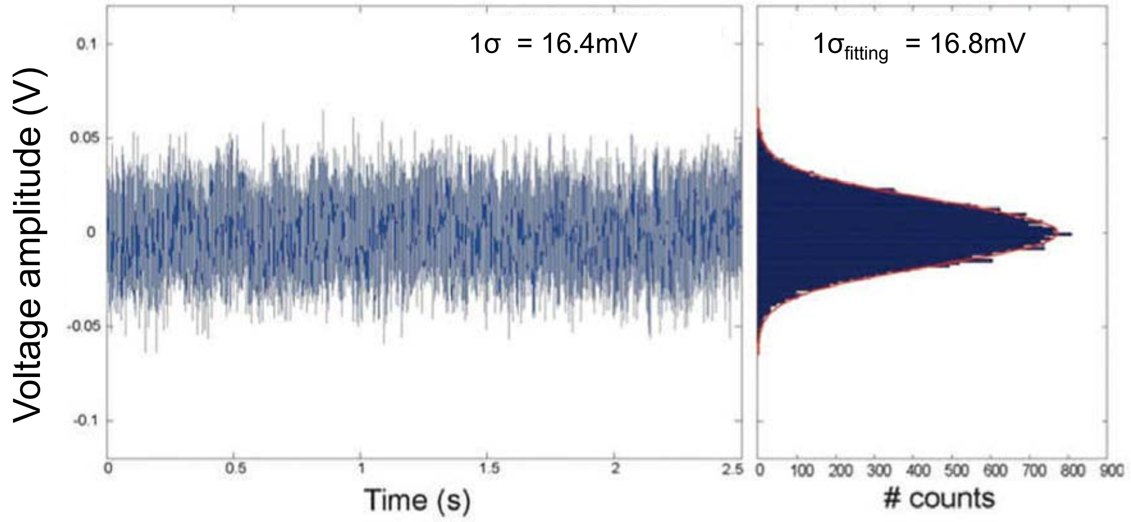
"timing" issues



Noise

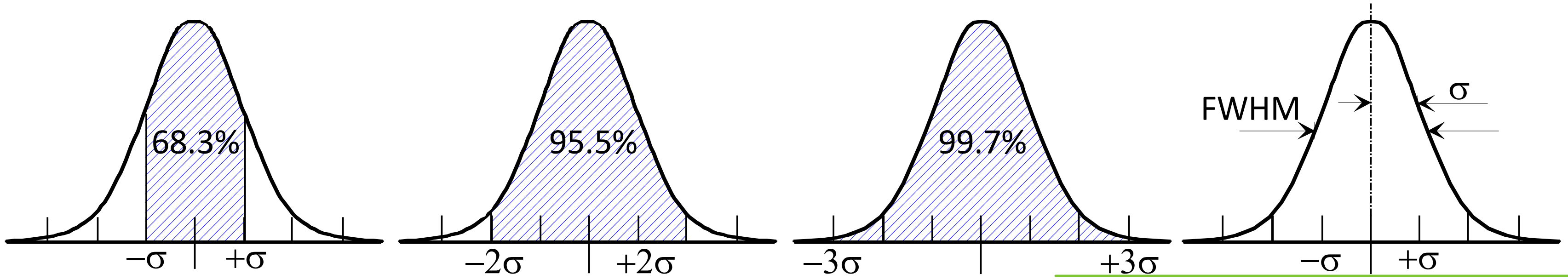
IL RUMORE È UN PROCESSO STATISTICO, NON POSSIAMO DEFINIRE UN VALORE ISTANTANEO.

DEFINIAMO IL RUMORE COME UNA DISTRIBUZIONE GAUSSIANA.



Distribution: $$p(x) = \frac{1}{\sqrt{2\pi\sigma_x}} e^{-\frac{(x)^2}{2\sigma_x^2}}$$

Note that: $$\int_{-\infty}^{+\infty} p(x)dx = 1$$



~~Peak-to-peak~~ value: $x_{99.9\%} = \pm 3 \cdot \sigma$

Full-Width at Half Maximum:

$FWHM = 2.35 \cdot \sigma$

Noise

HA SENSO CALCOLARE LA POTENZA DEL RUMORE

Instantaneous value is nonsense, better to measure the power:

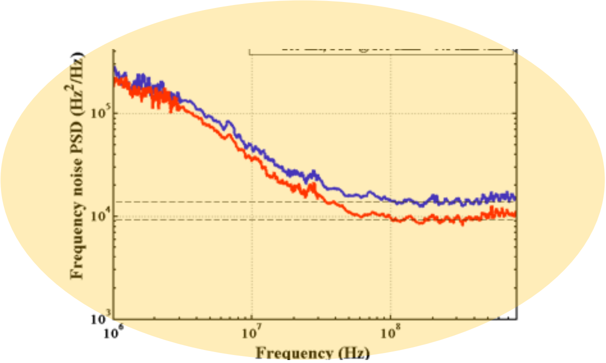
$$Power = \frac{1}{T} \cdot \int_0^T |x(t)|^2 dt$$

"Ergodic" process (time average = samples average), Gaussian, with nil mean value

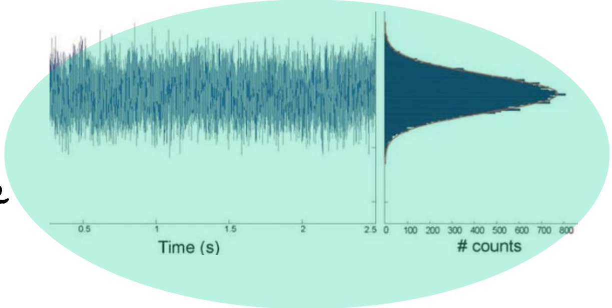
Nel senso che per ogni tempo i sample hanno lo stesso peso, perciò posso scrivere $\frac{1}{T}\int_0^t x(t)dt$

Poi se prendo più campioni nello stesso istante e faccio la media mi viene uguale a quella del tempo (perché processo Ergodico)

Variance, "power", mean squared value: $< x^2(t) >= \sigma^2$



Parceval's theorem: $\sigma^2 = < x^2(t) > = \int_0^\infty S(f)df$

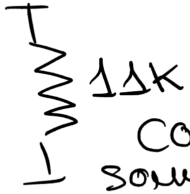


Root mean square, *rms*: $x_{rms} = \sqrt{\sigma^2} = \sigma$ che è la base della gaussiana, noi vogliamo calcolare questa.

"Correlation" among noise sources

SUPPONIAMO 2 RESISTORI IN SERIE

POTREMMO CONSIDERARE



COSA DOBBIAMO SOMMARE QUI PER AVERE IL RUMORE?

Two noise sources: $$v_t(t) = v_1(t) + v_2(t)$$

Mean total value: $$< v_t(t) >= 0$$

Total variance: $$< v_t^2(t) >=< \left[v_1(t) + v_2(t) \right]^2 >=< v_1^2(t) > + < v_2^2(t) > + \underbrace{2 < v_1(t) v_2(t) >}_{\text{C'È ANCHE QUESTA COMPONENTE.}}$$

POSSIAMO AVERE 2 CASI

… in case of NO correlation : $$< v_t^2(t) >=< v_1^2(t) > + < v_2^2(t) >$$

IL RUMORE DI UN RESISTORE NON DIPENDE DALL'ALTRO

Such as the effect overposition principle

… in case of TOTAL correlation $v_1(t) = v_2(t)$: $$< v_t^2(t) >= 4 < v_1^2(t) >$$

NOI VORREMMO CHE TUTTI I COMPONENTI NON SIANO CORRELATI. MA SE FOSSERO CORRELATI NOI COMMETTEREMMO UN ERRORE (SOTTOSTIMIAMO IL RUMORE) MA! L'ERRORE SEMBRA GRANDE MA QUANDO CONSIDERIAMO IL VALORE RMS ABBIAMO "SOLO" UN ERRORE DEL 41% PERCIÒ NOI CONSIDERIAMO TUTTO NON CORRELATO

100% ERROR ! but … NEGLIGIBLE ! ("only" 41% on rms value)

Therefore… let's consider all noise sources as uncorrelated

SHOT (granular) noise

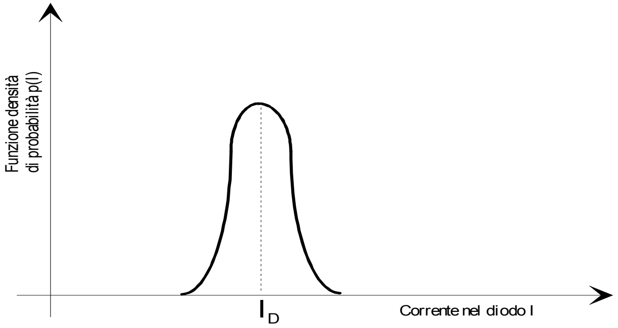
Due to the "granularity" of charge crossing a junction

(like rolling marbles instead of a "liquid" flux)

- shows up only if there is a current flow
- the rms value increases with $\sqrt{I}$
- depends on f… but… almost "white" noise

$$\sigma^2 = \langle i^2 \rangle = 2 \cdot q \cdot I \cdot \Delta f$$

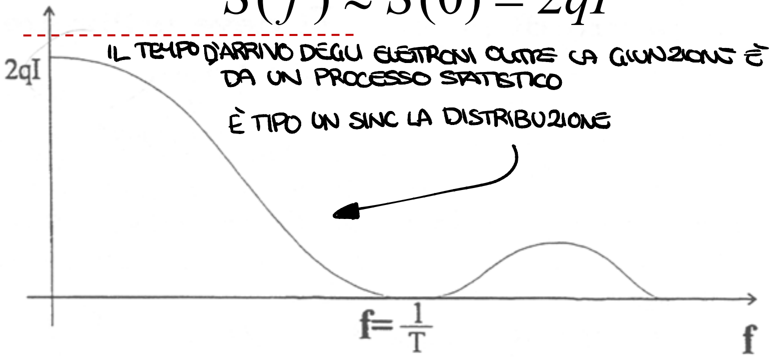
BANDA DELLO STRUMENTO CHE USIAMO PER MISURARE IL RUMORE

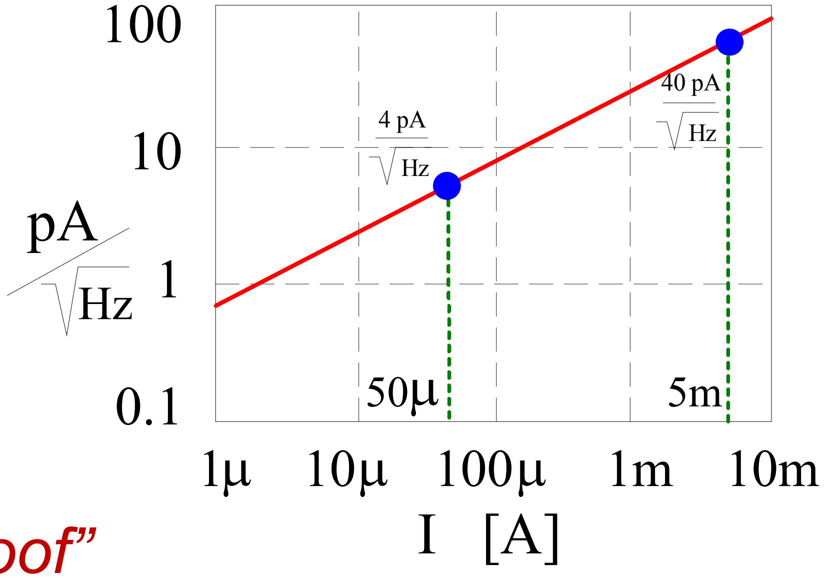


$$S(f) \approx S(0) = 2qI$$

IL TEMPO D'ARRIVO DEGLI ELETTRONI OLTRE LA GIUNZIONE È DATO DA UN PROCESSO STATISTICO

È TIPO UN SINC LA DISTRIBUZIONE

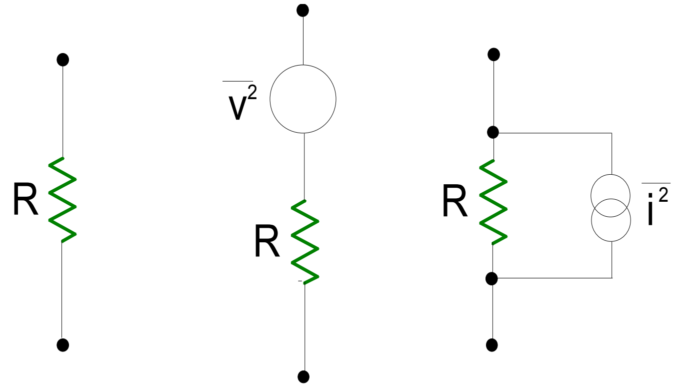




"Like a rain on a thin roof"

THERMAL (Johnson) noise

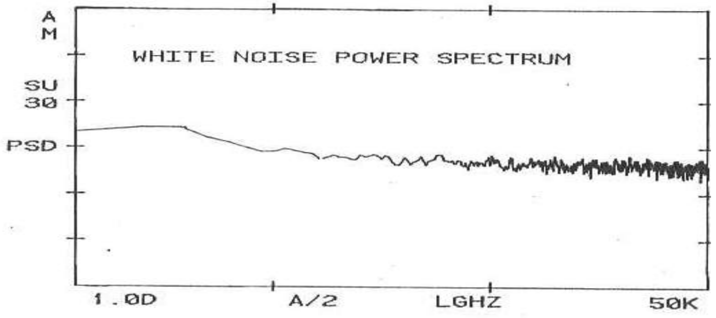
Due to the granular and never fixed nature of charge

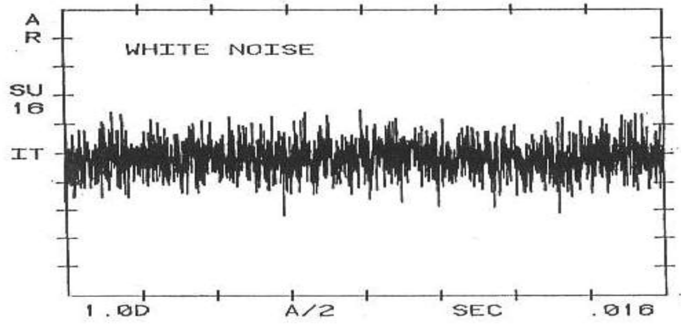


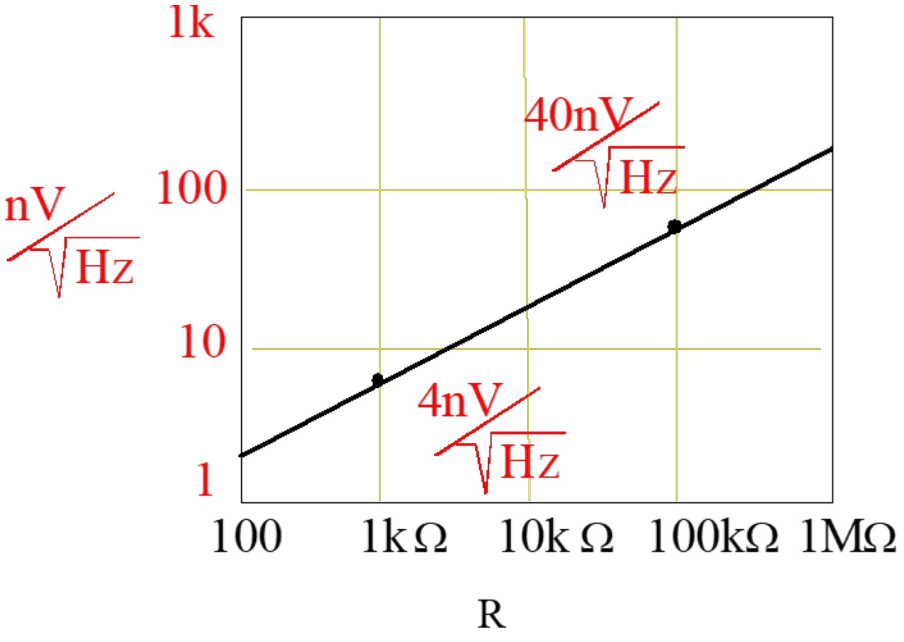
- independent of current flow
- rms value depends on $\sqrt{T}$ and $\sqrt{R}$
- independent of frequency ("white" noise)

$$<v^2> = 4kT \cdot R \cdot \Delta f$$

$$<i^2> = \frac{4kT}{R} \cdot \Delta f$$





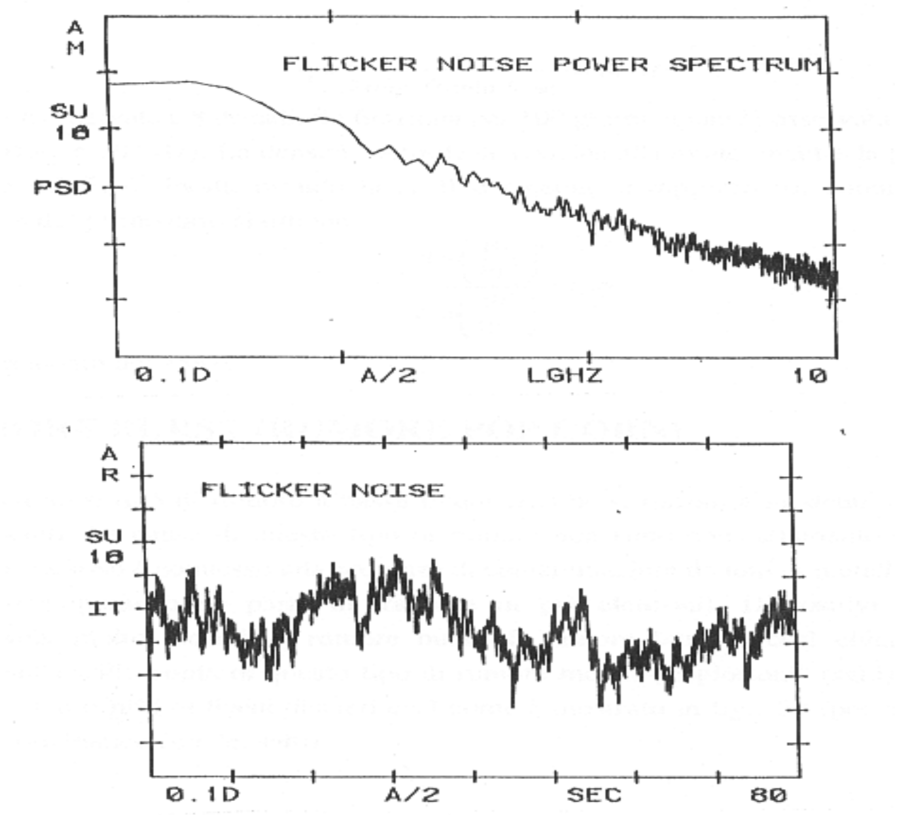


$$4kT = 1.66 \cdot 10^{-20} \frac{V^2}{Hz \cdot \Omega}$$

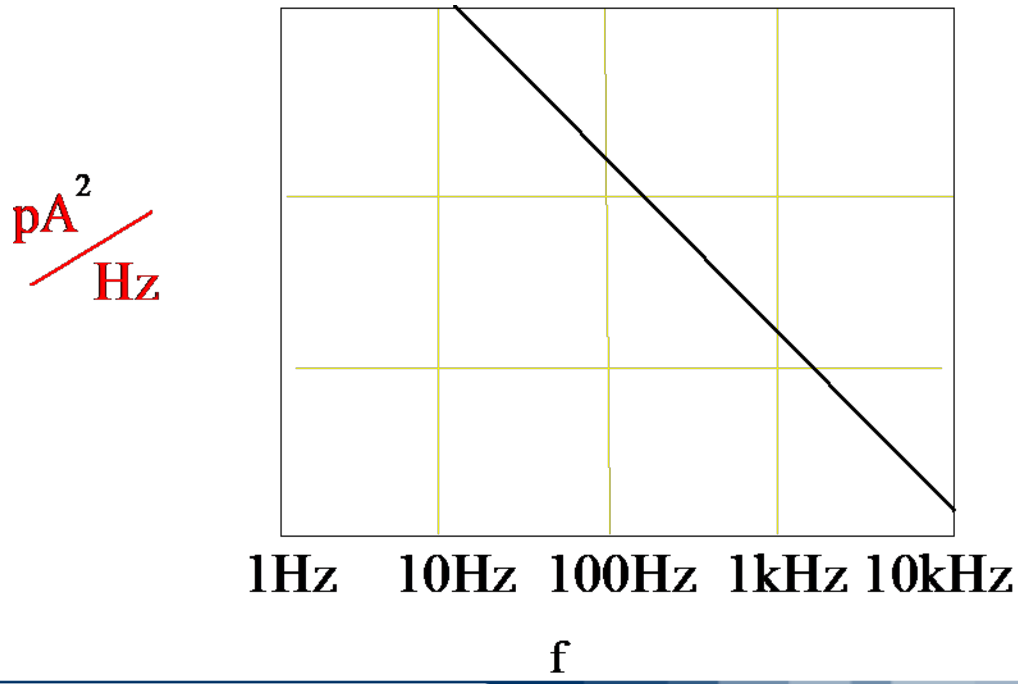
FLICKER (1/f) noise

Many (unknown) origins

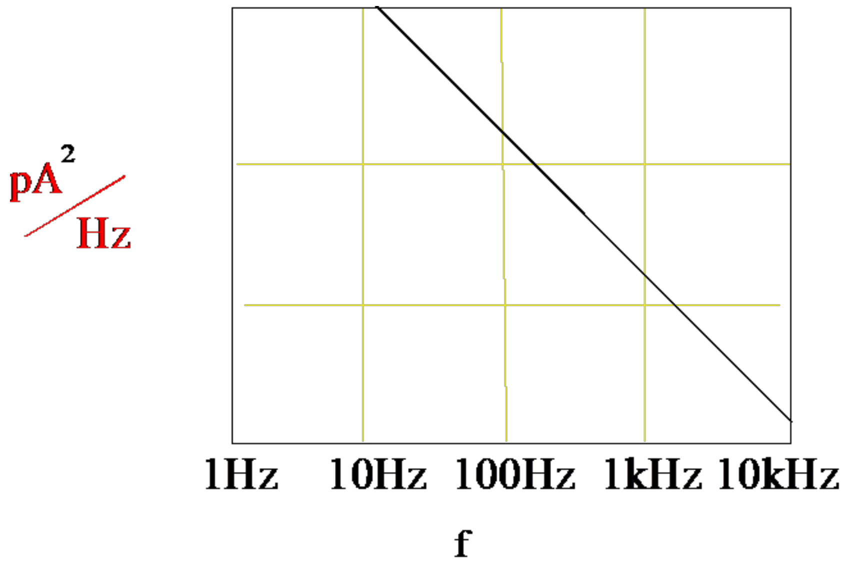
- proportional to current flow
- power spectrum depends on 1/f
- hence is a "coloured" noise



$$\langle i^2 \rangle = k \frac{I^a}{f^b} \Delta f$$



FLICKER (1/f) noise



- constant power for each decade

Coeff di proporzionalità del rumore 1/f

$$P = \int_{f_1}^{f_2} \frac{k \cdot I}{f} df = k \cdot I \cdot \ln\left(\frac{f_2}{f_1}\right)$$

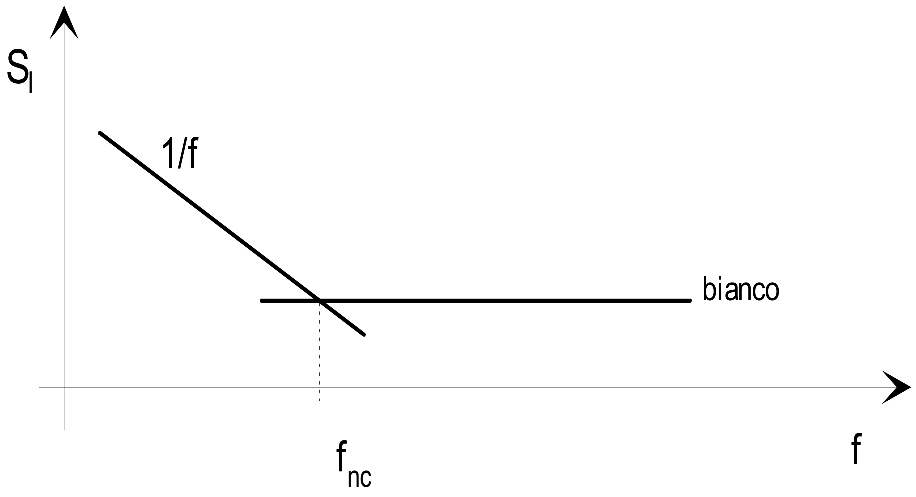
$$P_{onedecade} = k \cdot I \cdot 2.3$$

- does it diverge? $f_{lower} = 0$?

- not for real… $f_{(real)lower} = \frac{1}{24h \cdot 60\min \cdot 60\sec} \cong 10^{-5} Hz$

Non diverge perché nella realtà noi facciamo sempre uno zero.

so 10^{-5}-10^{-4}=10^{-4}-10^{-3}=…=10^{2}-10^{3}= just 8x

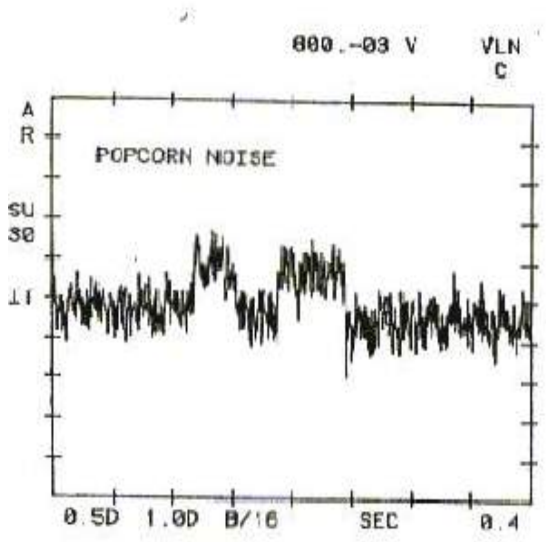


- comparable to white noise at the

Noise Corner Frequency

BURST (POP-CORN) noise

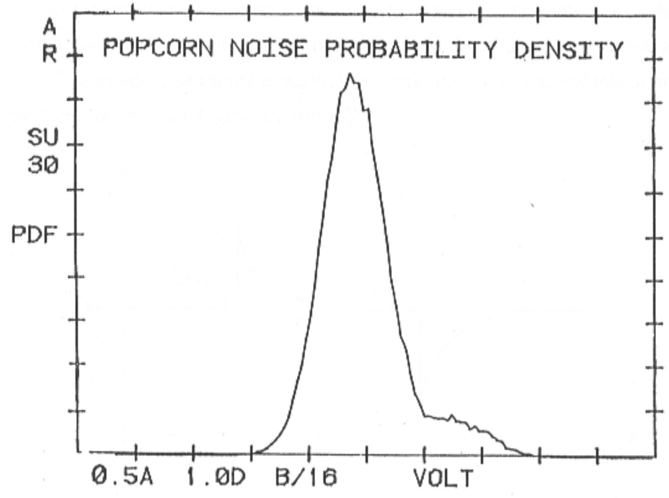
Lattice defects, charge trapping centers …



SEMBRA CHE IL RUMORE SIA SOLO ATTORNO A ZERO MA NELLA RELTÀ OGNI TANTO SCATTA DI LIVELLO.

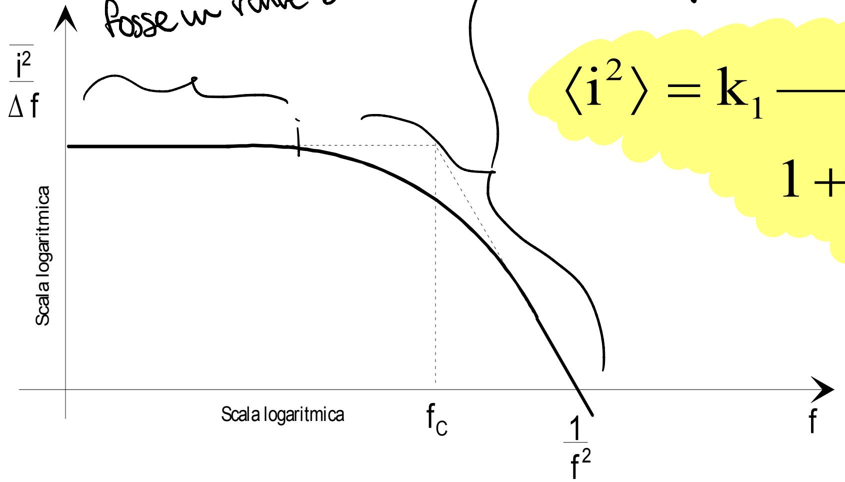
LA DISTRIBUZIONE DI QUESTA FREQ NON È PIÙ GAUSSIANA.

• not Gaussian



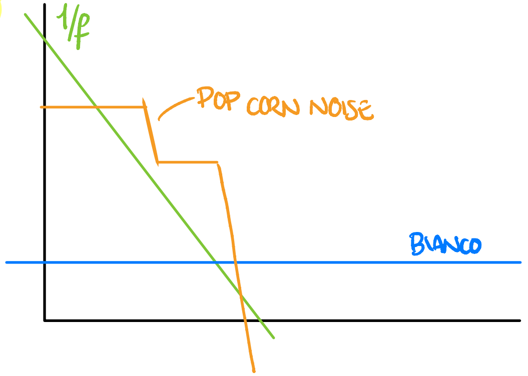
valore costante come fosse un rumore bianco

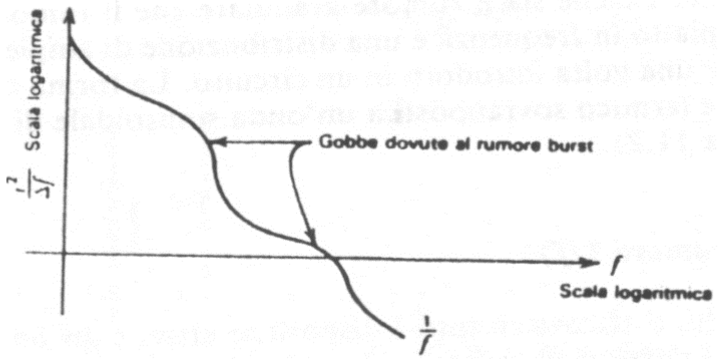
Nella realtà la distribuzione scende di $1/f^2$ ad alte frequenze



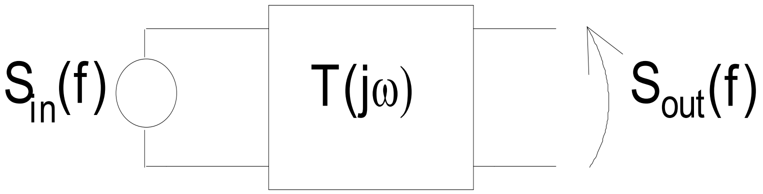
$$\langle i^2 \rangle = k_1 \frac{I^c}{1+\left(\frac{f}{f_c}\right)^2}\Delta f$$

•many different contributions





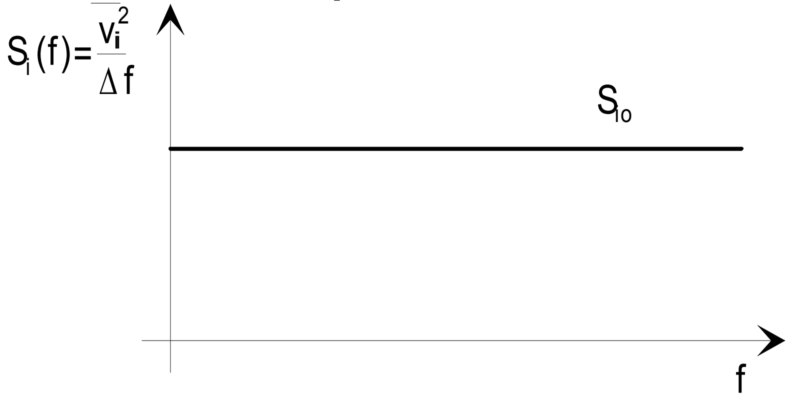
Noise in-out transfer (frequency response)



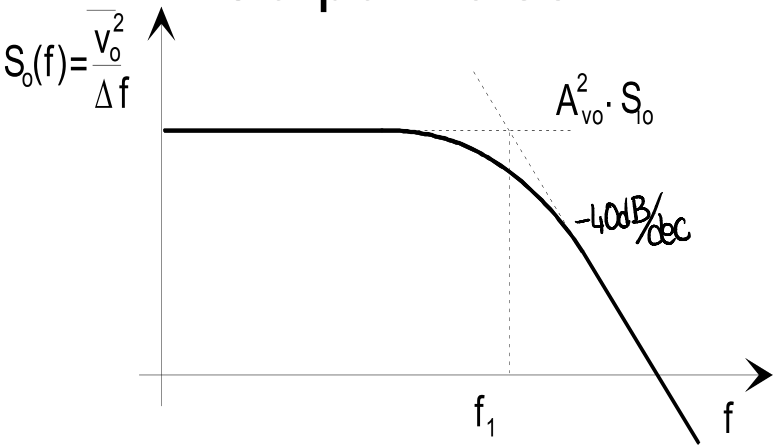
$$S_{out}(f)df = S_{in}(f)\cdot\left|T(j\omega)\right|^2 df$$

Identical Bode diagram, but for noise only “power” transfer matters (independent of phase)

Input noise:



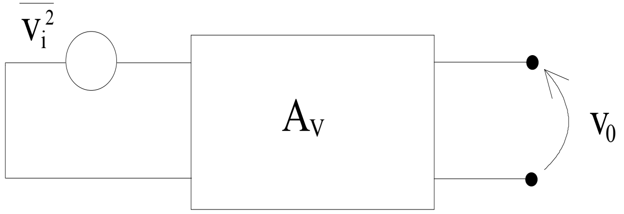
Output noise:



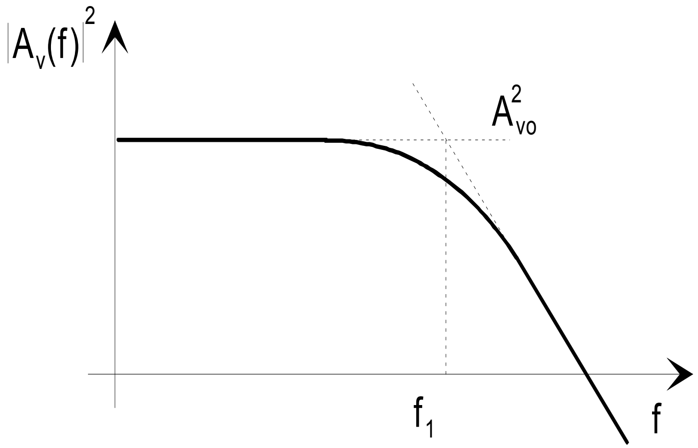
The total output noise power is $< v_{oT}^2 >= \int_0^\infty S_0(f)df = \int_0^\infty \left|A_v(f)\right|^2 \cdot S_{io}df = S_{io}\cdot\int_0^\infty \left|A_v(f)\right|^2 df$

… that's a pity we cannot compute the area (power) as "base x height" …

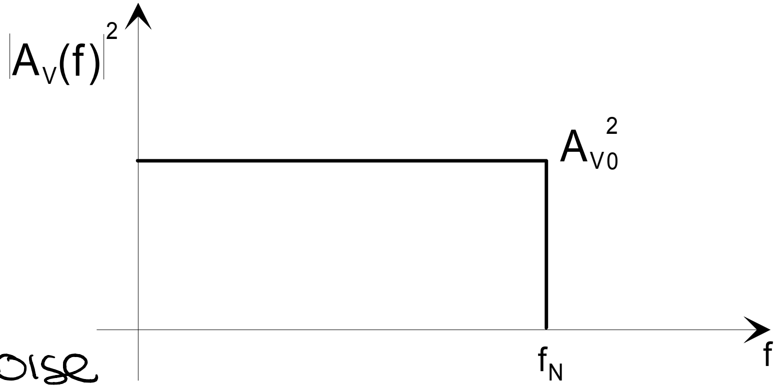
Noise Equivalent Bandwidth



$$< v_{oT}^2 > = S_{io} A_{vo}^2 f_N$$



The pole is NOT the noise bandwidth $f_N = \Delta f$ to consider !



Noise Equivalent Bandwidth: $$f_N = \frac{1}{A_{vo}^2} \int_0^{\infty} \left| A_v(f) \right|^2 df$$

Very simple cases:

1 pole: $$f_N = \int_0^{\infty} \frac{df}{1 + \left(\frac{f}{f_1} \right)^2} = \pi \cdot \frac{f_1}{2} = 1,57 \cdot f_1$$

equivalent noise Bandwith.

2 poles: $$f_N = 1,22 \cdot f_1$$

OK then, the rms value is SQRT("base x height") of such an equivalent rectangle

“Weird” result

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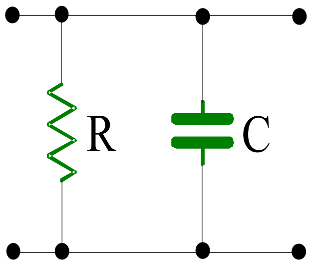
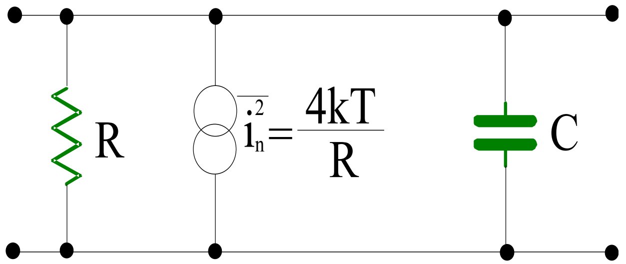
Simple RC network:





Full computation:

$$T(j\omega) = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{1 + sCR}$$

$$S_0(f) = \frac{4kT}{R} \cdot |T(j\omega)|^2 = \frac{4kT}{R} \cdot \frac{R^2}{1 + \omega^2 C^2 R^2}$$

$$< v_{0T}^2 >= \int_0^\infty S_0(f)\,df = \frac{4kT}{R} \cdot \int_0^\infty \frac{R^2}{1 + \omega^2 C^2 R^2} df =$$

$$= 4kTR \cdot \left[\frac{\operatorname{arctg}(\omega CR)}{2\pi RC} \right]_0^\infty = \frac{4kTR}{2\pi RC} \cdot \left(\frac{\pi}{2} \right) = \frac{kT}{C}$$

“Equivalent” computation:

$$< v_{0T}^2 >= 4kTR\Delta f = 4kTR \cdot \frac{1}{4RC} = \boxed{\frac{kT}{C}}$$

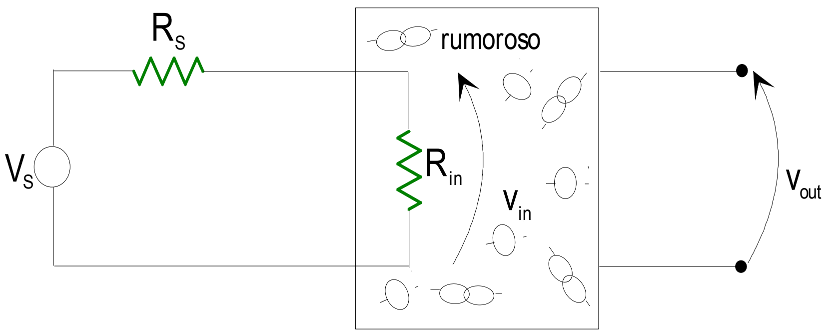
for C=1pF we get 65µV_{rms}

DIVERTENTE! CI VIENE CHE IL RUMORE NON DIPENDE DA R, PERCIÒ SE CAMBIO R MA TENGO IL RESTO INMUTATO ALLORA IL RUMORE NON CAMBIA. SUCCEDE CHE IL PROCESSO È DIPENDENTE DA R

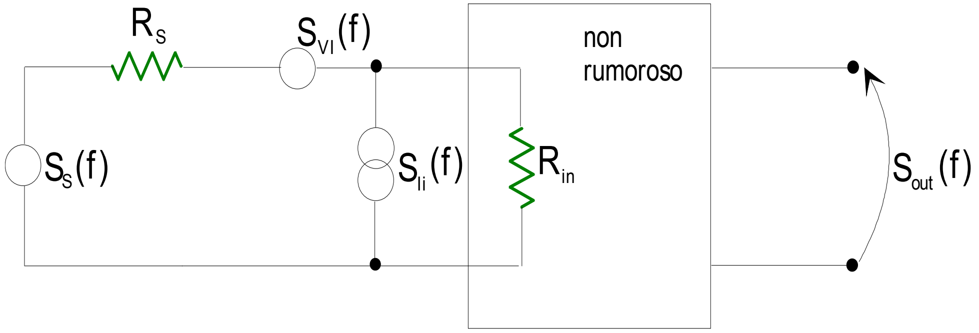
So what? Does rms noise depends only on C? Is C noisy instead of R?

Noise Equivalent Generators

Real circuit:



Equivalent circuit:



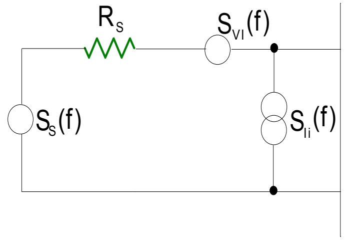
Total input-referred noise (no R_{in}):

$$S_{in}(f) = 4kTR_S + e_V^2 + i_I^2 \cdot R_S^2$$

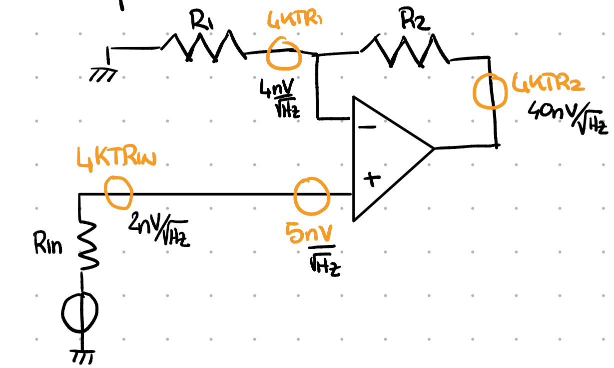
Total output noise:

$$S_{out}(f) = |T(j\omega)|^2 \cdot \left\{ [S_v(f)] \cdot \frac{R_{in}^2}{(R_S + R_{in})^2} + S_I(f) \cdot \frac{R_{in}^2 R_S^2}{(R_S + R_{in})^2} \right\}$$

No need to compute the output noise… let's stop at the input!

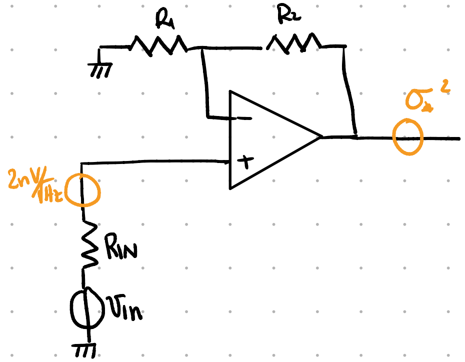


Esempio



$$SNR = \frac{V_{in}^2}{\sigma_{in}^2} \qquad SNR = \frac{V_{out}^2}{\sigma_{out}^2} \qquad F = \frac{SNR_{in}}{SNR_{out}} \geq 1$$

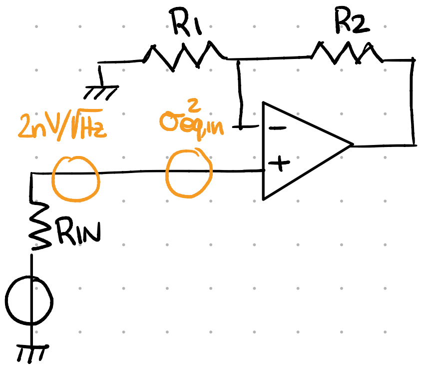
Possiamo considerare unicamente il rumore sull'uscita



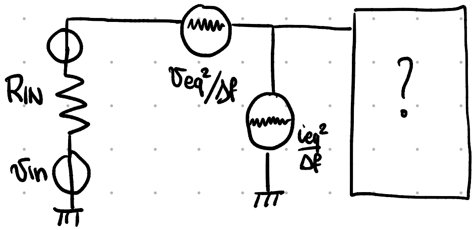
$V_{out} = G V_{in}$

$\sigma_{out}^2 = \sigma_{in}^2 \cdot G + \sigma_*^2$

Però così non è bello perché abbiamo il rumore d'ingresso da un lato e l'altro dall'altro.
Possiamo anche riferire tutto il rumore all'ingresso



Così tutte le comparazioni tra segnale e rumore le possiamo fare all'ingresso.
Se ho un generatore di tensione di input posso calcolare la Veq ma non ieq al contrario se l'in ho un gen di corrente allora non posso usare Veq perché quando ho un gen di corrente spento ho un aperto e quindi non passerebbe tensione. Stesso discorso per ieq quando ho una tensione in ingresso dato che questa va in corto.
Allora per essere easy peasy calcolo sia Veq che ieq e li metto entrambi tanto poi uno si elimina da solo.



$$V_{ineq}^2 = 4KTR\Delta f + \frac{V_{eq}^2}{\Delta f}\Delta f + \frac{i_{eq}^2}{\Delta f} R_i^2 \Delta f$$

$$\frac{V_{ineq}^2}{\Delta f} = 4KTR_s + V_{eq}^2 + i_{eq}^2 R_s^2$$

POSSIAMO DEFINIRE LA NOISE FIGURE COME

$$NF = 10\log_{10}\left(\frac{\text{total input noise}}{\text{rumore della sorgente}}\right) = 10\log_{10}\left(\frac{4KTR_s + V_{eq}^2 + i_{eq}^2 R_s^2}{4KTR_s}\right)$$

Signal-to-Noise Ratio and Noise Factor

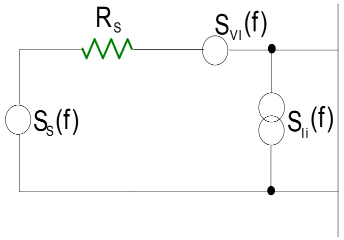
Signal-to-Noise Ratio:

$$\frac{S}{N} = SNR = 10\log_{10}\left(\frac{V_{in}^2}{v_{n,i}^2}\right) = 10\log_{10}\left(\frac{V_{in}^2}{\left(4kTR_S + e_V^2 + i_I^2 R_S^2\right)\cdot\Delta f}\right)$$

Noise Factor:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{S_i}{N_i}\cdot\frac{N_0}{S_0} = \frac{N_0}{G\cdot N_i} = \frac{\text{Total output noise}}{\text{Total output noise just due to } R_S} =$$

$$= \frac{\text{Total input noise}}{4kTR_S}$$



Noise Equivalent Temperature: $T_{eq} = (F-1)\cdot T_0$ just to tell how "hot" the noise is

Noise Figure, NF

Noise Figure:

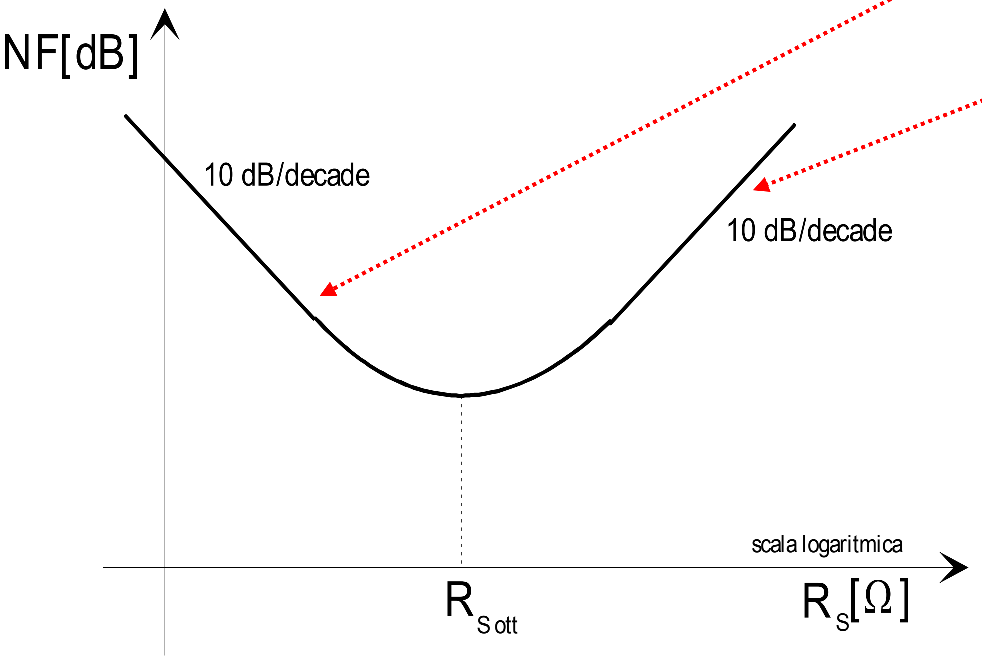
$$NF = 10\log_{10}\left(\frac{4kTR_S\Delta f + \langle v_i^2\rangle + \langle i_i^2\rangle R_S^2}{4kTR_S\Delta f}\right) = 10\log_{10}\left(1 + \frac{\langle v_i^2\rangle}{4kT\Delta f}\cdot\frac{1}{R_S} + \frac{\langle i_i^2\rangle}{4kT\Delta f}\cdot R_S\right)$$

ANDAMENTO DELLA NOISE FIGURE AL VARIARE DI RSOURCE
LA RS OTTIMA SI CALCOLA IMPONENDO

$$\frac{v_{eq}^2}{4kTR_S} = \frac{i_{eq}^2 R_S^2}{4kTR_S}$$

ALLORA

$$R_{Sopt} = \sqrt{\frac{v_{in}^2/\Delta f}{i_{in}^2/\Delta f}} = \sqrt{\frac{v_{in}^2}{i_{in}^2}}$$



Minimum at:

$$R_{S,opt} = \sqrt{\frac{\langle v_i^2\rangle}{\langle i_i^2\rangle}}$$

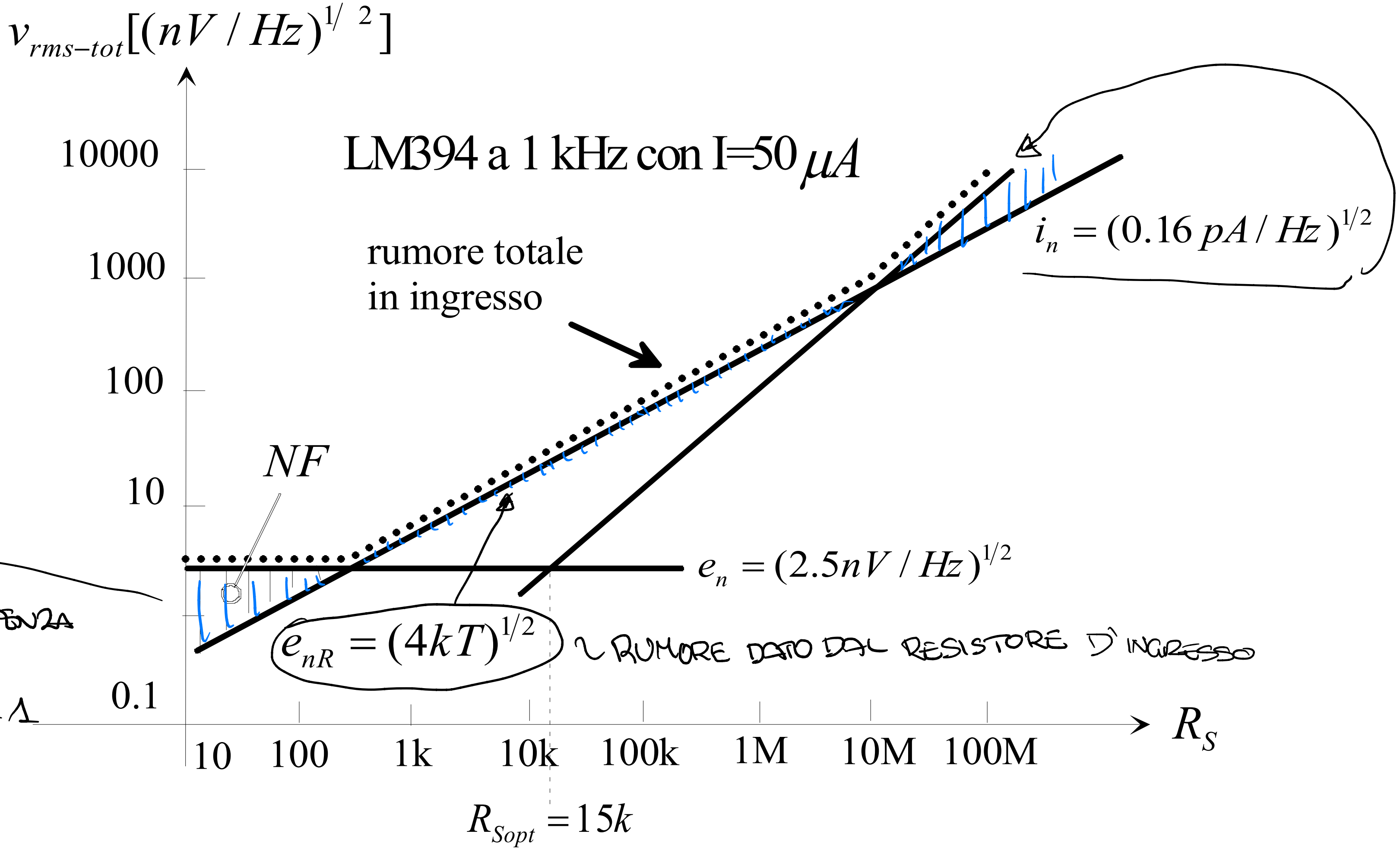
È LA NOISE FIGURE OTTIMA

$$NF_{opt} = 10\log_{10}\left(1 + \frac{\langle v_i^2\rangle}{4kTR_{s,opt}\Delta f} + \frac{\langle i_i^2\rangle R_{s,opt}}{4kT\Delta f}\right) = 10\log_{10}\left(1 + 2\cdot\frac{\sqrt{\langle v_i^2\rangle\cdot\langle i_i^2\rangle}}{4kT\Delta f}\right)$$

Total input Noise

Total input noise (no R_{in}):

$$S_{in}(f) = 4kTR_S + e_V^2 + i_I^2 \cdot R_S^2$$



LA NOISE FIGURE NON È ALTRO CHE LA DISTANZA TRA IL RUMORE DELLA RESISTENZA D'INGRESSO E IL RUMORE TOTALE

NOTIAMO CHE PER VALORI MEDI DI R LA NF=1

MOLTO BENE.

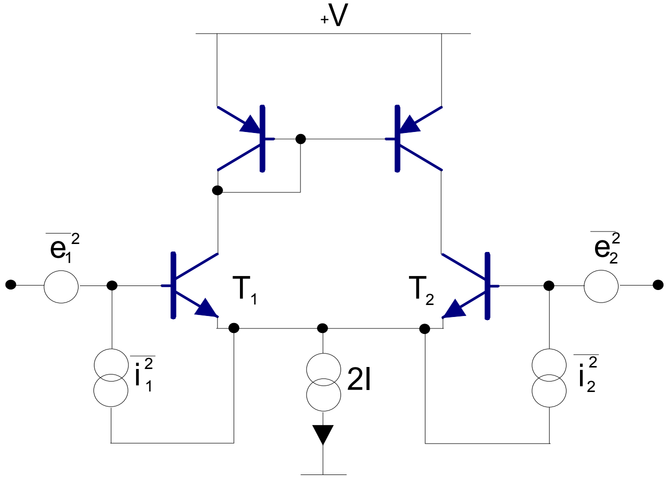
NF shows how electronics worsens the intrinsic noise of the source

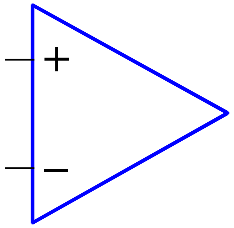
NF is NOT a signature of the total noise !

Noise in OPAMPs

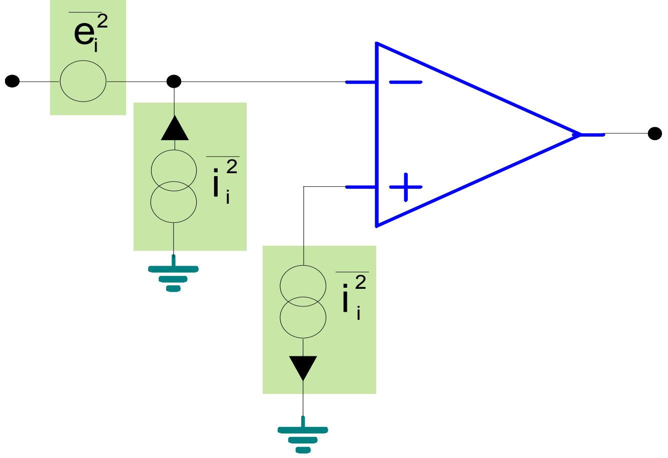
POLITECNICO MILANO 1863

Differential input stage:

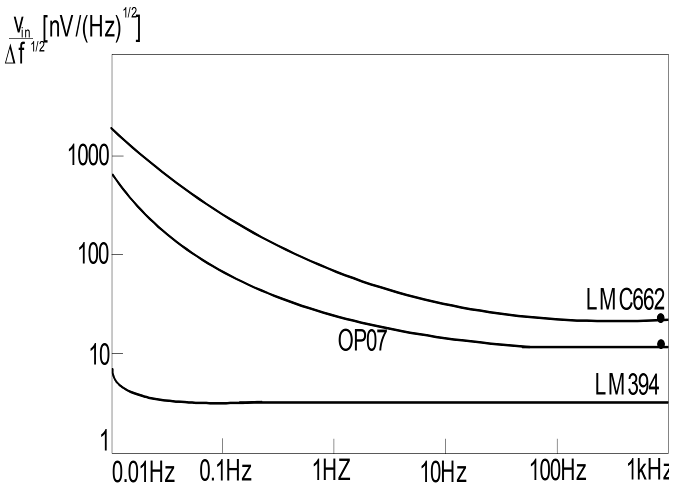


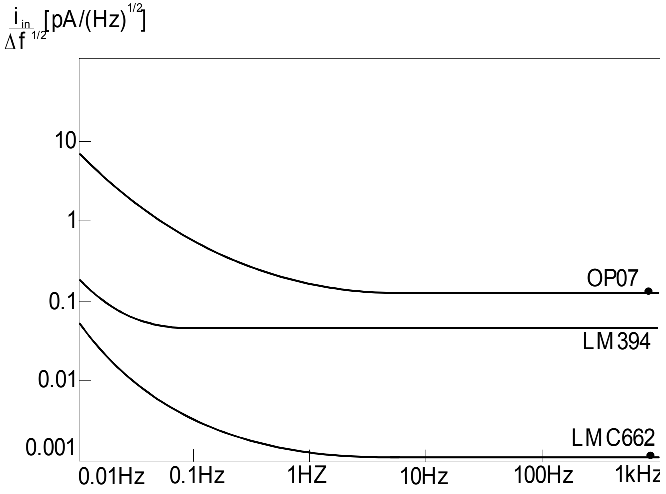


Input equivalent generators:

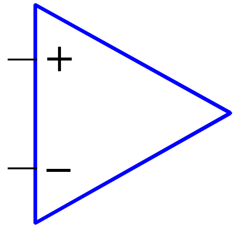


… and both do matter!





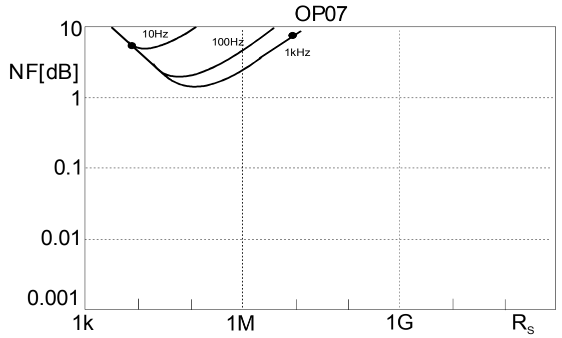
Noise in OPAMPs



Which one is better? Either lower current or voltage noise?

Comparisons:

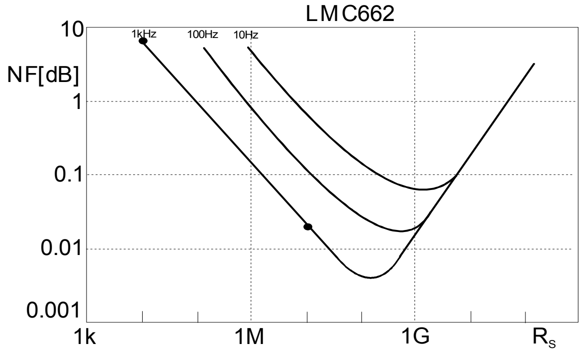
OP07	e_{in}=9.6nV/√Hz	i_{in}=120fA/√Hz
LMC662	e_{in}=22nV/√Hz	i_{in}=0.113fA/√Hz



For R_s=10kΩ: Se Rs è piccolo devo tenere conto di ein principalmente

OP07 $$v_{rms\ totale} = \sqrt{\left(9.6\cdot 10^{-9}\right)^2 + \left(128\cdot 10^{-11}\right)^2} = 9.7\,nV/\sqrt{Hz}$$ NF=4dB

LMC662 $$v_{rms\ totale} = \sqrt{\left(22\cdot 10^{-9}\right)^2 + \left(0.113\cdot 10^{-11}\right)^2} = 22\,nV/\sqrt{Hz}$$ NF=11dB



For R_s=10MΩ: Se Rs è molto grande devo tenere conto del generatore iin principalmente

OP07 $$v_{rms\ totale} = 1.3\,\mu V/\sqrt{Hz}$$ NF=19dB

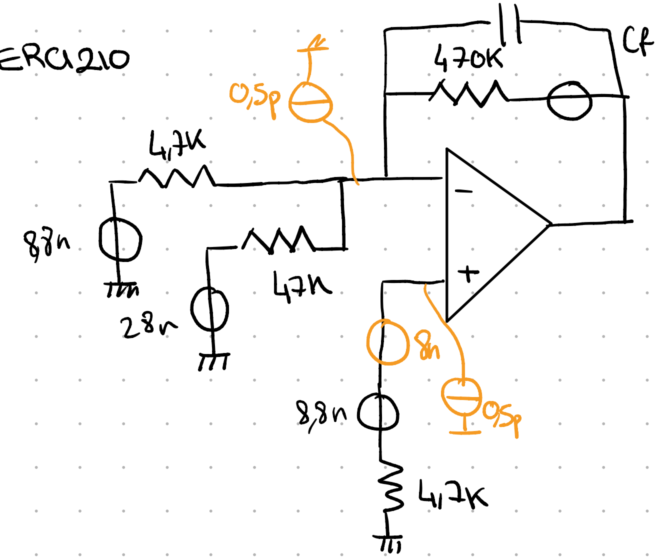
LMC662 $$v_{rms\ totale} = 22\,nV/\sqrt{Hz}$$ NF=0.025dB

Conclusions

- Noise seems easy to understand
- However, tricky computations… $\sqrt{v^2}$ … spectral density, rms…

Next lesson: **06 – INA**

ESERCIZIO

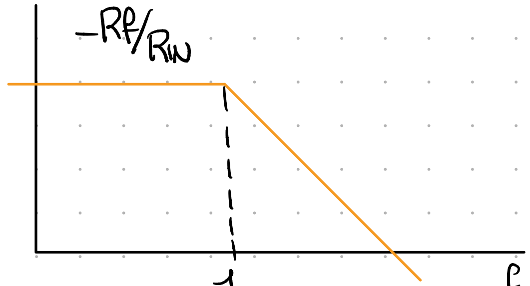


$4KT\,4{,}7K\Omega = 7{,}8\cdot10^{-17}\,V^2/Hz = \left(8{,}8nV/\sqrt{Hz}\right)^2_{\text{valore RMS}}$

$4KT\,47K = \left(28nV/\sqrt{Hz}\right)^2$

$4KT\,470K = \left(88nV/\sqrt{Hz}\right)^2$

Spostiamo ora tutti i generatori all'output
Se l'opamp è ideale ho che



$\dfrac{1}{2\pi C_f 470K} = 339\ Hz$

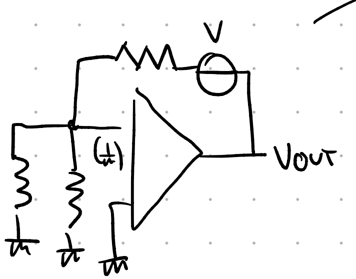
Noi sappiamo che la noise equivalent Bandwith è

$\dfrac{\pi}{2}339 = 532\,Hz$

Perciò

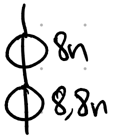
$$\left(8{,}8\frac{nV}{\sqrt{Hz}}\right)^2\cdot\left(-\frac{470K}{4{,}7K}\right)^2+\left(28nV/\sqrt{Hz}\right)^2\left(-\frac{470K}{47K}\right)^2+\left(88nV/\sqrt{Hz}\right)^2(1)^2+\left(11{,}9nV/\sqrt{Hz}\right)^2\underbrace{\left(1+\frac{470K}{4{,}7K//47K}\right)}_{\text{guadagno visto dal pin +}}+\underbrace{\left(0{,}5pA/\sqrt{Hz}\right)^2\cdot(4{,}7K)^2\left(1+\frac{470K}{4{,}7K//47K}\right)^2}_{\text{gen di corrente sul pin +}}$$

$$+\left(0{,}5pA/\sqrt{Hz}\right)\cdot(470K)^2$$



Per terra virtuale non può circolare corrente quindi $V_{out} = V$

è dato dalla serie



$(8nV)^2 + (8{,}8)^2 = 11{,}9$

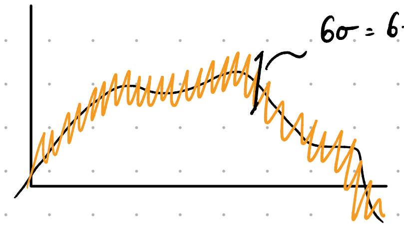
Gen di corrente sul −, sulle altre resistenze non può scorrere corrente perché ho la terra virtuale

$\simeq \left(3{,}09\,\mu V/\sqrt{Hz}\right)^2$

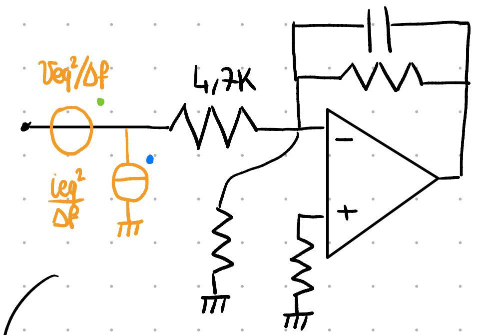
Se vogliamo calcolare

$$V_{OUT_{RMS}} = \sqrt{\left(3{,}09\,\frac{\mu V}{\sqrt{Hz}}\right)^2 \cdot \Delta f} = 3{,}09\,\frac{\mu V}{\sqrt{Hz}}\cdot\sqrt{\frac{\pi}{2}\,339} = 71\,\mu V_{RMS}$$

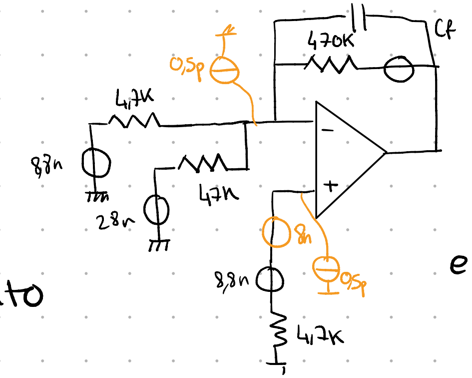
Potremmo quindi dire che



Voglio riportare il rumore d'uscita all'ingresso



Calcoliamo i contributi dei 2 rumori. Per calcolare il contributo di $V_{neq_{in}}$ allora cortocircuito l'ingresso così disabilito il gen di corrente



Nel circuito reale sono in questa situazione, perciò so che la real output noise è $3nV/\sqrt{Hz}$

Questi 2 devono essere uguali

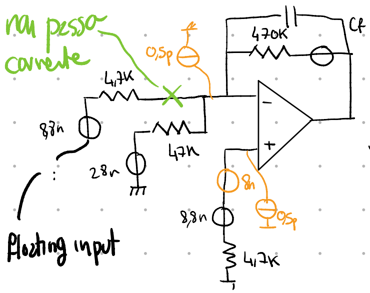
perciò

$$\frac{V_{neq}^2}{\Delta f} = \left(30\,nV/\sqrt{Hz}\right)^2$$

il rumore d'uscita di questo circuito considerando solo $V_{eq}^2/\Delta f$ è:

equivalent output noise = $\frac{V_{eq}^2}{\Delta f}\cdot\left(\frac{470K}{4{,}7K}\right)^2$

Consideriamo ora l'effetto del gen di corrente, per farlo lascio l'input aperto



quella del circuito equivalente viene

- equivalent output noise: $\left(\frac{i_n}{\Delta f}\right)^2\cdot(470K)^2$

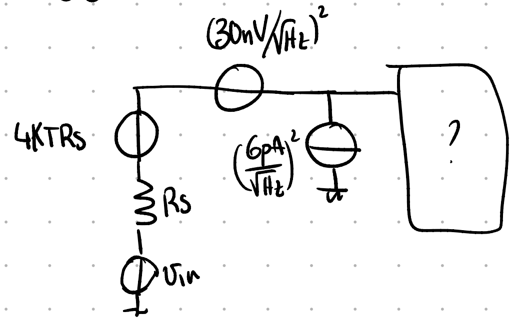
il guadagno di questo al contrario è:

- è lo stesso conto lungo di prima che tolgo la componente 88n e non ho il parallelo 47K//4,7K ma solo 47K.

Ci viene che il guadagno è 11

Ci viene che $i_{eq} = \left(6\,pA/\sqrt{Hz}\right)^2$

Perciò



$$R_{S\,OPT} = \sqrt{\frac{(30nV/\sqrt{Hz})^2}{(6pA/\sqrt{Hz})^2}} = 5K\Omega$$

$$NF_{opt} = 10\log\left(1 + \frac{\overline{v_{in}^2}/\Delta f}{4KTR_{SOPT}} + \frac{\overline{i_{in}^2}/\Delta f}{4KTR_{sopt}} \cdot R_{sopt}^2\right)$$

$$= 10\log\left(1 + 2\,\frac{\sqrt{(30n \cdot 6pA)^2}}{4KT}\right)$$

- non è ottima perché non è Ø non è un ottimo amplificatore