ANALYSIS OF A CASCODE - COMPENSATED OTA

The following is an idea to compensate a two-stage OTA in an Ahuja-like technique but without using too much current.

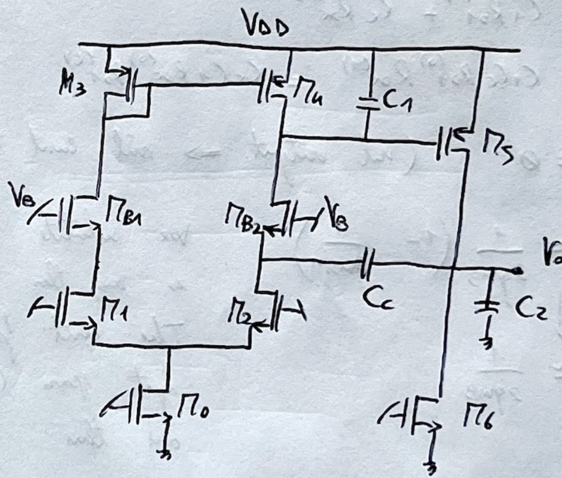
IDEA: substitute the current buffer with a cascode in the first stage.

The T.F. will have this general form:

$$T(s) = G(0) \cdot \frac{a_2 s^2 + a_1 s + 1}{b_3 s^3 + b_2 s^2 + b_1 s + 1}$$

because we have three independent capacitors and one of them is directly attached to the output node from ground.

We find that:



$$G(0) = g_{m1} R_{out,1}\, g_{m5} R_{out,2}$$

$$b_1 = C_1 R_1^{(0)} + C_2 R_2^{(0)} + C_C R_C^{(0)}$$

$$b_2 = C_1 C_2 R_1^{(0)} R_2^{(1)} + C_1 C_C R_1^{(0)} R_C^{(1)} + C_2 C_C R_2^{(0)} R_C^{(2)}$$

$$b_3 = C_1 C_2 C_C R_1^{(0)} R_2^{(1)} R_C^{(1,2)}$$

so for the denominator we need

$$R_1^{(0)} = R_1$$

$$R_2^{(0)} = R_2$$

$$R_2^{(1)} = R_2$$

$$R_C^{(1)} = \frac{1}{g_{mB}} + R_2 \simeq R_2$$

$$R_C^{(2)} = \frac{1}{g_{mB}}$$

in order to compute $R_C^{(0)}$ we may substitute C_C with a current generator i_S. This current will flow basically entirely through MB2, thus generating:

$$v_{gs} = i_S R_1$$

so we get the following balance at the output node:

$$i_S + g_{m5} R_1 i_S + \frac{v_2}{R_2} = 0$$

$$\Downarrow$$

$$v_2 = -i_S R_2 (1 + g_{m5} R_1)$$

Then we have that

$$v_1 \simeq \frac{i_S}{g_{mB}}$$

so we get

$$v_S = v_1 - v_2 = \left(\frac{1}{g_{mB}} + R_2 + g_{m5} R_1 R_2\right) i_S$$ **MILLER EFFECT**

Therefore we get

$$R_C^{(0)} \simeq g_{mS} R_1 R_2$$

and we can safely say that $b_1 \simeq C_c\, g_{mS} R_1 R_2$

Thus, we have that

$$b_2 \simeq C_1 C_2 R_1 R_2 + C_1 C_c R_1 R_2 + C_2 C_c R_2 \cdot \frac{1}{g_{mB}} \simeq C_1 R_1 R_2 (C_2 + C_c)$$

$$b_3 = C_1 C_2 C_c\, R_1 R_2 \frac{1}{g_{mB}}$$

Let's consider now the numerator of the T.F.:

$$a_1 = C_1 R_{01}^{(0)} + C_c R_{0C}^{(0)}$$

$$a_2 = C_1 C_c R_{01}^{(0)} R_{0C}^{(1)} = C_c C_1 R_{0C}^{(0)} R_{01}^{(c)}$$

$R_{01}^{(0)} = 0$ (nil output $\Rightarrow$ nil current through M_S $\Rightarrow$ nil $v_{g,S} = v_S$)

$$R_{0C}^{(0)} = \frac{1}{2g_{mB}}\left(1 - \frac{1}{g_{mS}R_1}\right)$$
$$\downarrow$$
$$\simeq \frac{1}{2g_{mB}}$$

We substitute C_c with a current source. If the output is zero, then i_S flows into M_S, so $v_{g,S} = -\frac{i_S}{g_{mS}}$. The current from i_S merges with the one due to the input pair (splits), so we have $i_S - i_d$ through M_{B2} and thus

$$i_S - i_d - (i_d) = i_S - 2i_d$$

through R_1. So we get

$$-\frac{i_S}{g_{mS}R_1} = i_S - 2i_d$$
$$\Downarrow$$
$$i_d = \frac{i_S}{2}\left[1 + \frac{1}{g_{mS}R_1}\right]$$

so the voltage at the source of M_{2B} will be

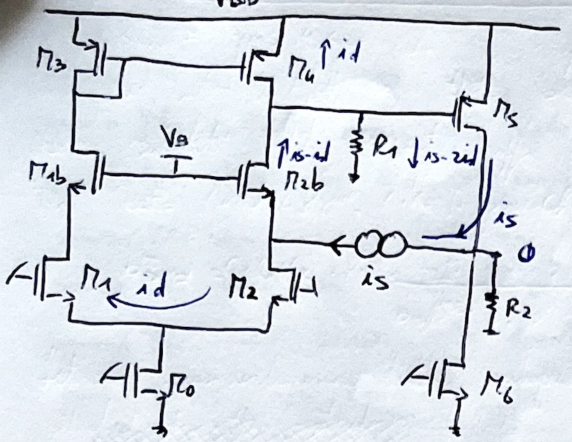
$$\frac{(i_S - i_d)}{g_{mB}} = \frac{1}{g_{mB}}\left[\frac{i_S}{2} - \frac{1}{g_{mS}R_1}\frac{i_S}{2}\right] = v_S$$

$$\Rightarrow \frac{v_S}{i_S} = \frac{1}{2g_{mB}}\left[1 - \frac{1}{g_{mS}R_1}\right]$$

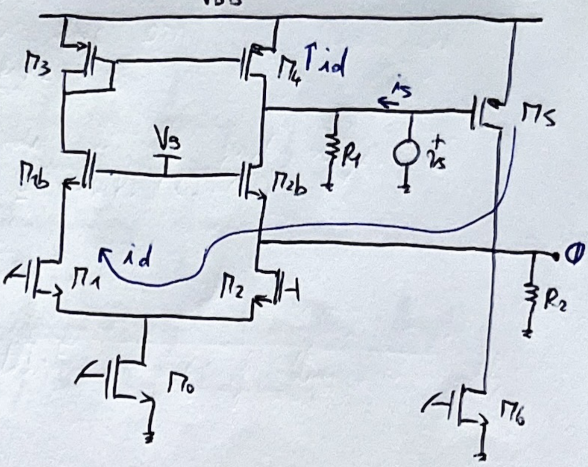
Now, since $R_{01}^{(0)} = 0$, it's better to use the second form, in order not to get indeterminate results:

$$i_d = i_S = -g_{mS} v_S$$

COMPUTATION OF $R_{o2}^{(c)}$



COMPUTATION OF $R_{o1}^{(c)}$



Then $i_s = i_d + \frac{v_s}{R_1} \simeq i_d = -g_{m5} v_s$

so

$$R_{o1}^{(c)} \simeq -\frac{1}{g_{m5}}$$

in the end we have

$$a_1 \simeq \frac{C_c}{2g_{mB}}$$

$$a_2 = C_c C_1 \frac{1}{2g_{mB}} \left(-\frac{1}{g_{m5}}\right)$$

and

$$T(s) \simeq g_{m1} g_{m5} R_1 R_2 \cdot \frac{-s^2 \frac{C_c C_1}{2g_{mB}g_{m5}} + s\frac{C_c}{2g_{mB}} + 1}{C_1 C_2 C_c \frac{R_1 R_2}{g_{mB}} s^3 + C_1 R_1 R_2 (C_2 + C_c) s^2 + g_{m5} C_c R_1 R_2 s + 1}$$

DESIGNERS RULES OF THUMB

B1

→ The common mode swing sets overdrive constraints for the input stage
→ The output swing sets constraints for the overdrive of the output stage
→ SR, GBWP and E^2_{in} set constraints on the current of the input stage
→ The gain sets constraints on the output transistors length
→ The offset sets constraints on the input transistors length
→ The noise corner sets constraints on the NMOS input transistors length
→ CMRR sets constraints on the length of the tail transistor

ROOT LOCUS RULES

- # BRANCHES = # POLES OF $G_{loop}(s)$;
- Branches proceed from poles to zeroes (finite or infinite);
- $\gamma < 0$ regions on the $\mathbb{Re}[\,]$ axis on the left of an odd number of singularities;
- $\gamma > 0$ regions of the $\mathbb{Re}[\,]$ axis on the left side of an even number of singularities;
- # ASYMPTOTES = (#POLES) - (# ZEROES @ FINITE FREQUENCY)
- ASYMPTOTES = branches going to ∞ that always split the Gauss plane into even parts

Write the T.F. in the Evans form.

INSIGHTS FROM SWITCHED CAPACITORS DESIGN

The switched capacitor (SC) mimics the behaviour of a large resistance, so it is widely used in filter design in audio range. We can make integrators based on a SC, who basically integrates current pulses, hence providing voltage steps at the output. The result is a staircase output waveform with an equivalent ramp-rate

$$\frac{\Delta V}{T} = \frac{E \cdot C_1}{C} \cdot \frac{1}{T} = \frac{E}{C \cdot R_{eq}}$$

$$R_{eq} = \frac{T}{C_1}$$

The discrete approximation is good enough provided that the clock frequency is much larger than the BW of the input signal, hence it respects Shannon theorem. This can be easily done, since typical f_{CK} are $\simeq 1\,MHz$, while the audio range is $\simeq 20\,kHz$ at the most.

We've already discussed the advantages, let's see now what's the bad part of dealing with discrete time systems:

we can write the output waveform like a staircase

$$v_{OUT}(t) = \sum_{m=0}^{+\infty} v_{OUT}(mT) * \left\{ rect\left[\frac{t - mT}{T}\right]\right\}$$

where

$$rect(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

and

$$\mathcal{F}[rect(t)](f) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j2\pi f t}\,dt = -\frac{1}{2\pi f j}\left[e^{-j2\pi f t}\right]_{-\frac{1}{2}}^{\frac{1}{2}} =$$

$$= \frac{1}{\pi f} \cdot \frac{1}{2j}\left[e^{j\pi f} - e^{-j\pi f}\right] = \frac{\sin(\pi f)}{\pi f} = \mathrm{sinc}(f)$$

Let's see now what is the link between the Fourier transform of the continuous-time signal and the discrete-time one, from input to output:

$$\mathcal{F}[v_{OUT}(t)] = \sum_{m=0}^{+\infty} v_{OUT}(mT) \cdot \underbrace{e^{-j\omega mT}\,\mathrm{sinc}\left(\frac{f}{T}\right) \cdot T}_{\mathcal{F}\left\{rect\left[\frac{t-mT}{T}\right]\right\}} = \underbrace{\sum_{m=-\infty}^{+\infty} v_{OUT}(mT)\, z^{-m}\Big|_{z=e^{j2\pi f t}}}_{\text{ZETA TRANSFORM}} \cdot T\,\mathrm{sinc}\left(\frac{f}{T}\right)$$

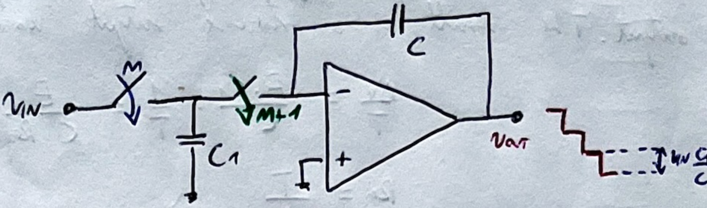
C_2

So we came to this result:

$$V_{OUT}(f) = V_{OUT}(z)\Big|_{z=e^{j2\pi f t}} \cdot T\,\mathrm{sinc}\left(\frac{f}{T}\right)$$

Now, from the circuit we have that

$$v_{OUT}(n+1) = v_{OUT}(n) - v_{IN}(n)\,\frac{C_1}{C}$$



$\Downarrow$ ZETA TRANSFORM

$$V_{OUT}(z)\cdot z = V_{OUT}(z) - V_{IN}(z)\,\frac{C_1}{C}$$

$\downarrow$

$$\frac{V_{OUT}}{V_{IN}}(z) = -\frac{C_1}{C}\,\frac{1}{z-1} = H(z) \Longrightarrow$$ Transfer function of the discrete-time filter

So we can write

$$V_{OUT}(f) = V_{IN}(z)\cdot H(z)\Big|_{z=e^{j2\pi f t}} \cdot T\,\mathrm{sinc}\left(\frac{f}{T}\right)$$

where

$$V_{IN}(z)\Big|_{z=e^{j2\pi f t}} = \sum_{n=0}^{+\infty} v_{IN}(nT)\,e^{-j2\pi f t n} = \int_{-\infty}^{+\infty} v_{IN}(t)\cdot\sum_{n=0}^{+\infty}\delta(t-nT)\,e^{-j2\pi f t}\,dt$$

$$\downarrow$$

$$= \mathcal{F}\left[v_{IN}(t)\sum_{n=0}^{+\infty}\delta(t-nT)\right](f)$$

$$\downarrow$$

$$= V_{IN}(f) * \sum_{k=-\infty}^{+\infty}\frac{1}{T}\,\delta\left(f-\frac{k}{T}\right)$$

and, in the end, we have

$$V_{OUT}(f) = \underbrace{\left[V_{IN}(f) * \sum_{k=-\infty}^{+\infty}\frac{1}{T}\,\delta\left(f-\frac{k}{T}\right)\right]}_{\text{aliasing due to sampling}}\cdot\underbrace{\left[-\frac{C_1}{C}\,\frac{1}{e^{j2\pi f t}-1}\right]}_{\text{action of the integrator, is a periodic filter}}\cdot\underbrace{\left[T\,\mathrm{sinc}\left(\frac{f}{T}\right)\right]}_{\text{spurious cardinal sine due to sampling}}$$

In order to have a reliable output, we need:

→ a RECONSTRUCTION FILTER that kills all the HF harmonics due to aliasing;

→ an ANTI-ALIASING FILTER before the the SC filter in order to kill HF disturbances that may be brought to base-band due to aliasing;

→ an EQUALIZING FILTER that compensates the spectrum shape alteration due to the cardinal sine term.

All these filters can be easily implemented by continuous-time networks