

CAMPPI ELETROMAGNETICI

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Campi Scalari

Un campo scalare è una funzione che associa ad ogni punto dello spazio n-dimensionale un valore scalare (es: la temperatura in una stanza $T(x, y, z)$)

Campi Vettoriali

Un campo vettoriale associa ad ogni punto dello spazio n-dimensionale un valore vettoriale (m valori scalari) (es: il campo elettrostatico $\vec{E}(x, y, z) = E_x(x, y, z)\hat{u}_x + E_y(x, y, z)\hat{u}_y + E_z(x, y, z)\hat{u}_z$)

Operatori sui campi · Gradiente

Divergenza

Rotore

- Il Gradiente opera su un campo scalare e restituisce un campo vettoriale

$$\vec{\nabla} \phi(x, y, z) = \frac{\partial \phi(x, y, z)}{\partial x} \hat{u}_x + \frac{\partial \phi(x, y, z)}{\partial y} \hat{u}_y + \frac{\partial \phi(x, y, z)}{\partial z} \hat{u}_z$$

Significato fisico: direzione di massima variazione

- La Divergenza opera su un campo vettoriale e restituisce un campo scalare

$$\vec{\nabla} \cdot \vec{F}(x, y, z) = \frac{\partial F_x(x, y, z)}{\partial x} + \frac{\partial F_y(x, y, z)}{\partial y} + \frac{\partial F_z(x, y, z)}{\partial z}$$

Significato fisico: sorgenti "pozzo" del campo

- Il Rotore opera su un campo vettoriale (3D) e restituisce un altro campo vettoriale

$$\vec{\nabla} \times \vec{F}(x, y, z) = \det \begin{bmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{bmatrix} =$$

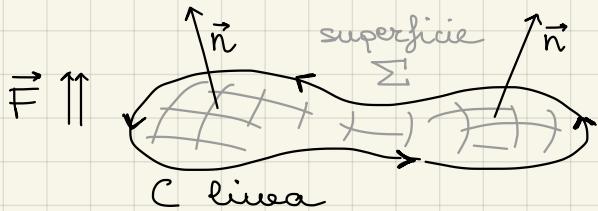
$$= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{u}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{u}_z$$

Significato fisico: sorgenti vorticosi del campo

Il campo elettrico (statico) è un campo irrotazionale, cioè ammette solo sorgenti di tipo pozzo ($\vec{\nabla} \times \vec{E} = 0$)

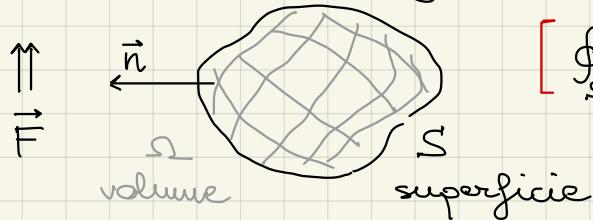
Il campo magnetico (statico) è un campo solenoideale, cioè ammette solo sorgenti di tipo vorticoso ($\vec{\nabla} \cdot \vec{H} = 0$)

Teorema di Stokes



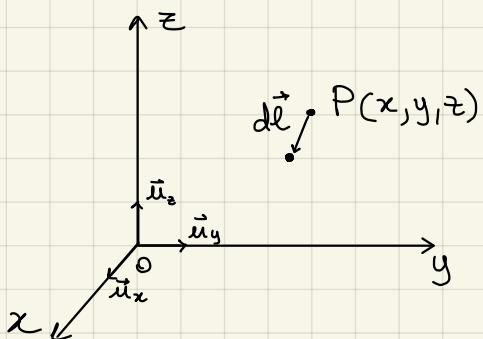
$$\left[\oint_C \vec{F} \cdot d\vec{l} = \int_{\Sigma} \vec{\nabla} \times \vec{F} \cdot d\vec{\Sigma} \right] \text{ (rotore)}$$

Teorema di Gauss



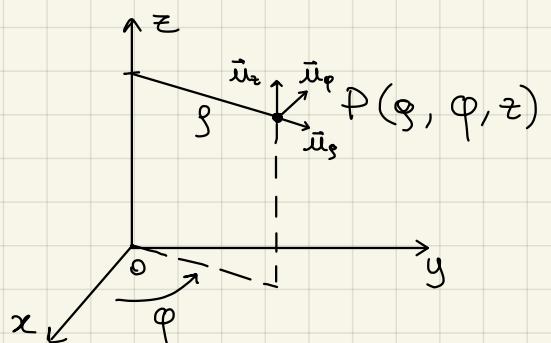
$$\left[\oint_S \vec{F} \cdot d\vec{s} = \int_{\Omega} \vec{\nabla} \cdot \vec{F} d\Omega \right] \text{ (divergenza)}$$

Coordinate cartesiane



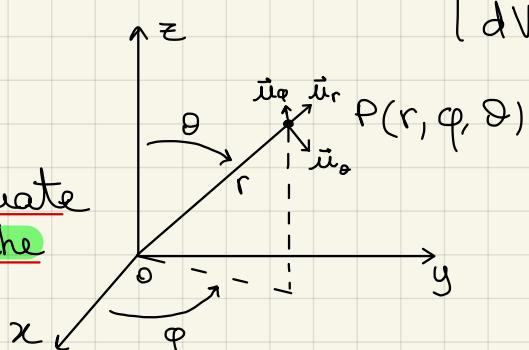
$$\begin{cases} d\vec{l} = dx \vec{u}_x + dy \vec{u}_y + dz \vec{u}_z \\ dV = dx dy dz \end{cases}$$

Coordinate cilindriche



$$\begin{cases} d\vec{l} = dr \vec{u}_r + r d\phi \vec{u}_\phi + dz \vec{u}_z \\ dV = r dr d\phi dz \end{cases}$$

Coordinate sferiche



$$\begin{cases} d\vec{l} = dr \vec{u}_r + r d\theta \vec{u}_\theta + r \sin\theta d\phi \vec{u}_\phi \\ dV = r^2 \sin\theta d\theta d\phi dr \end{cases}$$

\vec{E} [$\frac{V}{m}$] campo elettrico

ϵ [$\frac{F}{m}$] costante dielettrica

\vec{H} [$\frac{A}{m}$] campo magnetico

μ [$\frac{H}{m}$] permeabilità magnetica

\vec{B} [T] densità di flusso magnetico

\vec{D} [$\frac{C}{m^2}$] densità di flusso elettrico

CANPO ELETROSTATICO

Carica elettrica ($q_e = 1.6 \cdot 10^{-19} C$)

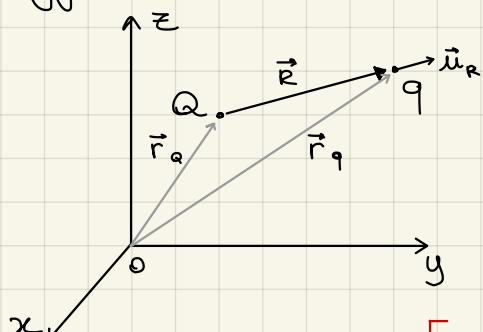
Carica puntiforme Q [C]

Densità di carica lineare g_l [$\frac{C}{m}$]

" " " superficiale g_s [$\frac{C}{m^2}$]

" " " volumetrica g_v [$\frac{C}{m^3}$]

Legge di Coulomb (1785)



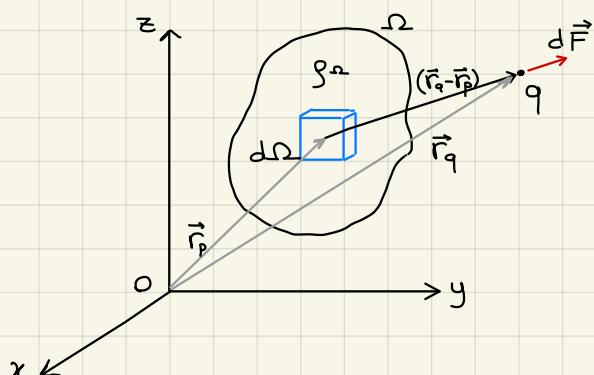
$$\vec{F} = \frac{Q \cdot q}{4\pi\epsilon_0 R^2} \vec{u}_r [N]$$

$$\epsilon_0 = 8,854 \cdot 10^{-12} \frac{F}{N \cdot m^2} \quad (\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{A})$$

$$[\vec{F} = \frac{Q \cdot q}{4\pi\epsilon_0} \frac{(\vec{r}_q - \vec{r}_Q)}{|\vec{r}_q - \vec{r}_Q|^3}] \text{ forza elettrostatica}$$

$$[\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{(\vec{r}_q - \vec{r}_Q)}{|\vec{r}_q - \vec{r}_Q|^3} = \frac{Q}{4\pi\epsilon_0 R^2} \vec{u}_r \text{ campo elettrostatico}]$$

Case di un corpo rigido (carica non puntiforme):

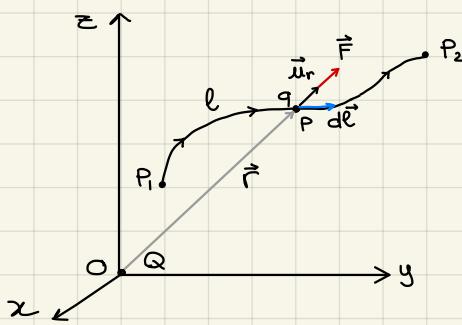


$$d\vec{F} = \frac{q}{4\pi\epsilon_0} \frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3} g_s(\vec{r}_p) d\Omega$$

$$\vec{F} = \int_{\Omega} d\vec{F} = \int_{\Omega} \frac{q}{4\pi\epsilon_0} \frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3} g_s(\vec{r}_p) d\Omega$$

$$\vec{E} = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} = \int_{\Omega} \frac{1}{4\pi\epsilon_0} \frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3} g_s(\vec{r}_p) d\Omega$$

POTENZIALE ELETROSTATICO



$$\vec{F} = q \vec{E} = \frac{qQ}{4\pi\epsilon_0 r^2} \vec{u}_r$$

$V = \frac{W}{q}$ potenziale
(lavoro per unità di carica)

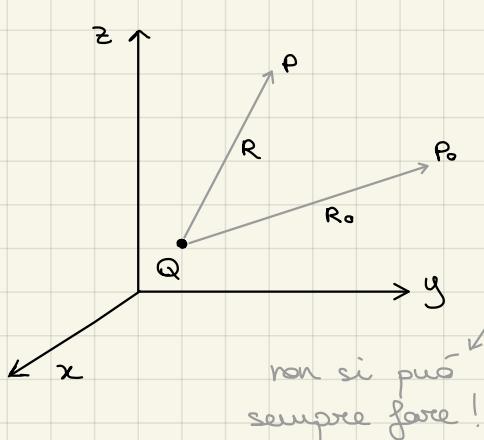
$$-\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} = V(P_2) - V(P_1)$$

$$V(P) = - \int_{P_0}^P \vec{E} \cdot d\vec{l} + V_0 \rightarrow V(P_0) \text{ costante additiva}$$

$$\oint_C \vec{E} \cdot d\vec{l} = 0 \longrightarrow \lim_{\delta \rightarrow 0} \oint_C \vec{E} \cdot d\vec{l} = 0 \longrightarrow [\vec{\nabla} \times \vec{E} = 0]$$

$$dV = - \vec{E} \cdot d\vec{l} \longrightarrow [\vec{E} = -\vec{\nabla} V]$$

il campo elettrostatico è irrotazionale
(non esistono sorgenti oorticose)



$$V(P) = \frac{Q}{4\pi\epsilon_0 R} \quad \left[\frac{Q}{4\pi\epsilon_0 R_0} \right] \text{ costante additiva}$$

$R_0 \rightarrow +\infty, V(P) = \frac{Q}{4\pi\epsilon_0 R}$

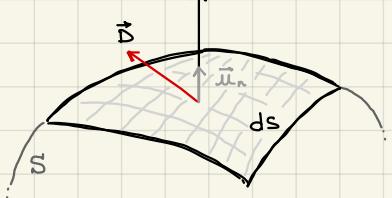
poiché il potenziale dipende solo dalla distanza radiale è conveniente usare un sistema di coordinate polari

$$\vec{\nabla} f = \frac{\partial f}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \vec{u}_\phi$$

$$\Rightarrow \vec{E} = - \frac{\partial V}{\partial r} \vec{u}_r = \frac{Q}{4\pi\epsilon_0 R^2} \vec{u}_r \quad \checkmark$$

$$\vec{D} = \epsilon_0 \vec{E} \quad \left[\frac{C}{m^2} \right]$$

$$\vec{D} = \frac{Q}{4\pi} \frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3} = \frac{Q}{4\pi R^2} \vec{u}_R$$



$$d\phi = \vec{D} \cdot d\vec{s}$$

$$\phi = \oint_S \frac{Q}{4\pi R^2} \vec{u}_R \cdot d\vec{s} = \oint_S \frac{Q}{4\pi} \frac{\vec{u}_R \cdot d\vec{s}}{R^2}$$

$$\oint_S d\Theta = \frac{\vec{u}_R \cdot d\vec{s}}{R^2} \quad \text{seu} \Omega d\phi d\Theta$$

$$\Phi = \oint_S \frac{Q}{4\pi} d\Theta = \frac{Q}{4\pi} \oint_S d\Theta = Q \quad (\text{il flusso di } \vec{D} \text{ è uguale alla carica che genera})$$

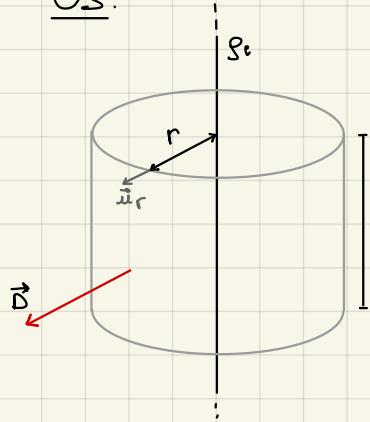
$\oint_S d\Theta = 4\pi$ angolo giro solido

$$\phi = \oint_s \vec{D} \cdot d\vec{s} = \oint_s \vec{D}_1 + \vec{D}_2 \cdot d\vec{s} = \oint_s \vec{D}_1 \cdot d\vec{s} + \oint_s \vec{D}_2 \cdot d\vec{s} = Q_1 + Q_2$$

$$\oint_s \vec{D} \cdot d\vec{s} = Q = \int_{\Omega} g_e d\Omega$$

$$\lim_{\epsilon \rightarrow 0} \oint_s \vec{D} \cdot d\vec{s} = \lim_{\epsilon \rightarrow 0} \int_{\Omega} g_e d\Omega = \lim_{\epsilon \rightarrow 0} g_e \cdot \Omega \longrightarrow [\vec{\nabla} \cdot \vec{D} = g_e]$$

Ese:



\vec{D} è esclusivamente radiale per ragioni di simmetria (considerando il file infinitamente esteso)

$$\phi = \oint_s \vec{D}_r \cdot d\vec{s} = D_r(r) \cdot 2\pi r \cdot h = Q$$

$$Q = \int_{\Omega} g_e d\Omega = \int_h g_e dl = g_e \cdot h$$

$$\vec{D}(r) = \frac{g_e}{2\pi r} \vec{u}_r$$

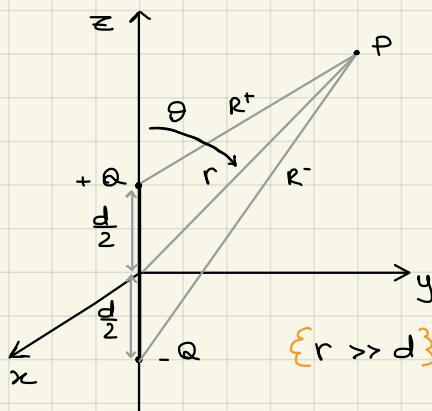
$$\vec{E}(R) = \frac{\vec{D}(R)}{\epsilon_0} = \frac{g_e}{2\pi \epsilon_0 R} \vec{u}_r$$

$$[V(R) = - \int_R^{\infty} \vec{E} \cdot d\vec{l} = - \int_{R_0}^R \frac{g_e}{2\pi \epsilon_0 r} dr = \frac{g_e}{2\pi \epsilon_0} \ln\left(\frac{R_0}{R}\right)]$$

la dipendenza da r non è più quadratica inversa

non posso più stabilire un potenziale per $R_0 \rightarrow \infty$ in quanto divergerebbe

Ese (dipolo elettrostatico):



$$V(P) = \frac{Q}{4\pi \epsilon_0 R^+} - \frac{Q}{4\pi \epsilon_0 R^-}$$

$$R^+ = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 - 2r \frac{d}{2} \cos\theta} \approx r - \frac{d}{2} \cos\theta$$

↑
tes. di Carnot

$$R^- = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 + 2r \frac{d}{2} \cos\theta} \approx r + \frac{d}{2} \cos\theta$$

$$\lim_{\substack{d \rightarrow 0 \\ (Q \rightarrow \infty)}} V(P) \approx \lim_{\substack{d \rightarrow 0 \\ (Q \rightarrow \infty)}} \frac{Q}{4\pi \epsilon_0} \left(\frac{1}{r - \frac{d}{2} \cos\theta} - \frac{1}{r + \frac{d}{2} \cos\theta} \right) = \frac{Q d \cos\theta}{4\pi \epsilon_0 r^2}$$

per mantenere il prodotto $Q \cdot d$ (dipolo) costante

il potenziale decresce più rapidamente che nel caso della singola carica ($\frac{1}{r^2}$ vs $\frac{1}{r}$) a causa dell'azione distruttiva dei campi delle due cariche

$$\vec{p} = Q \cdot \vec{d} \quad \text{dipolo elettrico}$$

$$[V(P) = \frac{Q d \cos\theta}{4\pi \epsilon_0 r^2}] \quad \text{potenziale del dipolo elettrico}$$

$$\vec{\nabla} f = \frac{\partial f}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \vec{u}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \vec{u}_\phi$$

$$\vec{E}(P) = -\vec{\nabla} V = -\left(\frac{\partial V(P)}{\partial r} \vec{u}_r + \frac{1}{r} \frac{\partial V(P)}{\partial \theta} \vec{u}_\theta\right)$$

$$\lim_{d \rightarrow 0} [\vec{E}(P) = \frac{Qd}{4\pi\epsilon_0 r^3} (2 \cos\theta \vec{u}_r + \sin\theta \vec{u}_\theta)]$$

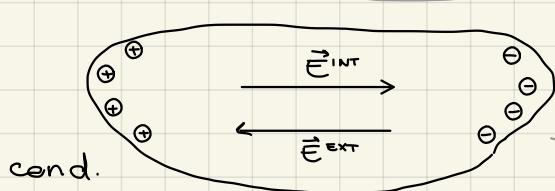
il potenziale del dipolo NON dipende da ϕ !

In generale, per calcolare il campo generato da una particolare distribuzione di carica, lo ricavo attraverso il potenziale secondo l'uguaglianza $\vec{E} = -\vec{\nabla} V$

Campo Elettostatico nella Materia

→ Conduttori

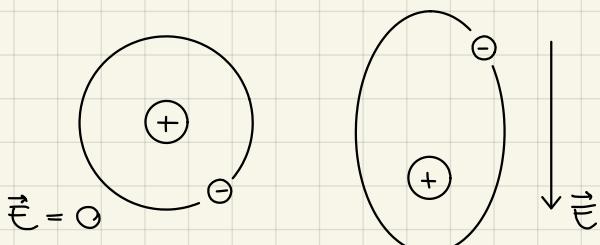
- campo \vec{E} sempre nullo all'interno
- campo \vec{E} tangente alla superficie nullo (sup. equipotenziale)



non campo lavoro a scorrere le cariche lungo la superficie

all'equilibrio
(dopo un certo tempo di rilassamento)

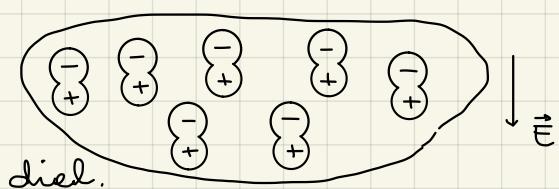
→ Dielettrici



Il campo \vec{E} induce nella materia una distribuzione volumetrica di dipoli.

Si definisce la densità di momento di dipolo:

$$\vec{P} = \lim_{\Delta \Omega \rightarrow 0} \sum_i \frac{\vec{p}_i}{\Delta \Omega}$$



Si dimostra che risulta:

$$\vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = g_e \text{ di carica reale}$$

campus densità momento di dipolo

campus densità di flusso elettrico $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$$\vec{\nabla} \cdot \vec{D} = g_e$$

$$\vec{P} = \epsilon_0 X_e \vec{E} \text{ (mezzi lineari)} \\ \text{suscettività elettrica}$$

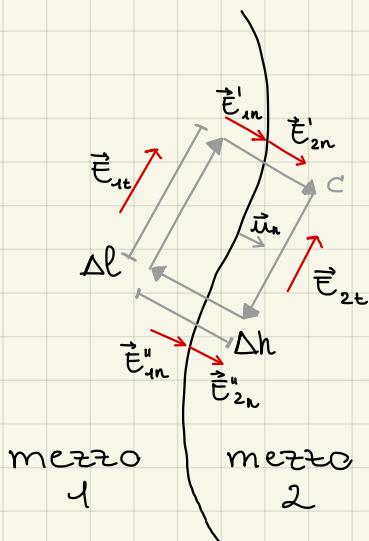
$$\vec{E} = \vec{E}_e + \vec{E}_i; \quad \Sigma = \Sigma_0(1+\chi_e) = \Sigma_0 \Sigma_r \quad \Sigma_r = 1 + \chi_e$$

$$\vec{D} = \Sigma \vec{E} = \Sigma_0 \Sigma_r \vec{E} = \Sigma_0 (1+\chi_e) \vec{E} = \Sigma_0 \vec{E} + \vec{P}$$

\emptyset nel vuoto

χ_e è un numero puro; uno scalare (> 0) nei mezzi isotropi (che mantengono le stesse proprietà in tutte le direzioni), una matrice (3×3) in quelli non isotropi

Condizioni al contorno per \vec{E}



$$\oint_C \vec{E} \cdot d\vec{l} = 0 \quad (\text{considerando } \Delta l \text{ e } \Delta h \text{ piccoli})$$

$$= E_{1t} \Delta l + E_{1n} \frac{\Delta h}{2} + E_{2n} \frac{\Delta h}{2} - E_{2t} \Delta l - E_{2n} \frac{\Delta h}{2} - E_{1n} \frac{\Delta h}{2}$$

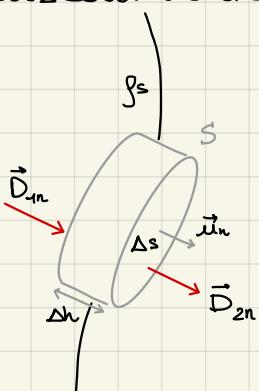
$$\Delta h \rightarrow 0, \quad (E_{1t} - E_{2t}) \Delta l = 0 \Rightarrow [E_{1t} = E_{2t}]$$

$$\vec{n}_n \times (\vec{E}_{1t} + \vec{E}_{2t}) = 0$$

$$\left[\frac{D_{1t}}{D_{2t}} = \frac{\Sigma_1}{\Sigma_2} \right]$$

il campo elettrostatico tangente alla superficie di frontiera a due sostanze diverse si conserva sempre

superficie di discontinuità



$$\oint_S \vec{D} \cdot d\vec{s} = D_{2n} \Delta s - D_{1n} \Delta s = \rho_s \Delta s \Rightarrow [D_{2n} - D_{1n} = \rho_s]$$

$$\vec{n}_n \cdot (D_{2n} - D_{1n}) = \rho_s$$

$$[\epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \rho_s]$$

la densità di flusso di campo elettrostatico normale alla superficie di frontiera si conserva sempre

Dielettrico - Dielettrico: $E_{2t} = E_{1t}$ $D_{2t} = \frac{\epsilon_2}{\epsilon_1} D_{1t}$
 (normalmente $\rho_s = 0$)

$$E_{2n} = \frac{\epsilon_1}{\epsilon_2} E_{1n} \quad D_{2n} = D_{1n}$$

Conduttore - Dielettrico: $E_{2t} = 0$ $D_{2t} = 0$ $E_{1t} = 0$

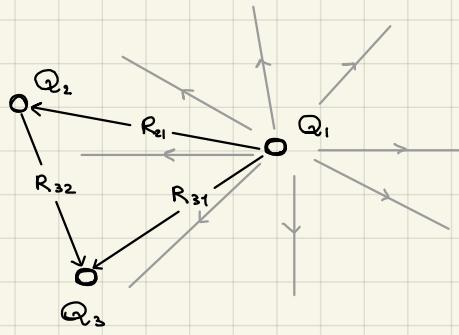
$$E_{2n} = \frac{\rho_s}{\epsilon_2} \quad D_{2n} = \rho_s \quad E_{1n} = 0$$

N.B.: se la componente tangenziale o normale cambia e l'altra no, sia il modulo che il verso del vettore cambiano

Energia del Campo Elettostatico

$$W_{21} = Q_2 \frac{Q_1}{4\pi\epsilon R_{21}} = Q_2 V_{21} = Q_1 V_{12}$$

$$\begin{aligned} W_{31} + W_{32} &= Q_3 \frac{Q_1}{4\pi\epsilon R_{31}} + Q_3 \frac{Q_2}{4\pi\epsilon R_{32}} \\ &= Q_3 V_{31} + Q_3 V_{32} \end{aligned}$$



$$W_e = W_{21} + W_{31} + W_{32} = Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32}$$

oppure equivalentemente

$$W_e = W_{12} + W_{13} + W_{23} = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23}$$

$$\rightarrow W_e = \frac{1}{2} [Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32})]$$

$$\Rightarrow [W_e = \frac{1}{2} \sum_{i=1}^3 Q_i V_i] \quad [W_e = \frac{1}{2} \int_S g_e V d\Omega]$$

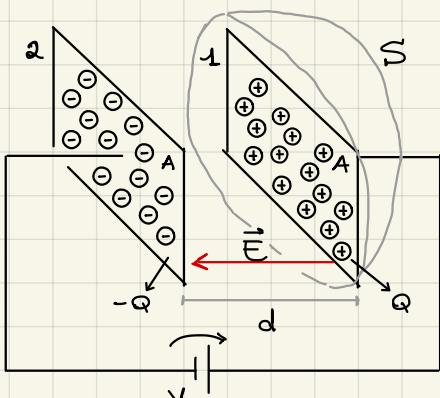
$$\text{Si pu  dimostrare anche che } W_e = \frac{1}{2} \int_{\text{spazio}} \vec{D} \cdot \vec{E} d\Omega$$

$$= \frac{1}{2} \int_{\text{spazio}} \epsilon |\vec{E}|^2 d\Omega$$

$$[W_e = \frac{1}{2} \vec{D} \cdot \vec{E} \text{ oppure } W_e = \frac{1}{2} \epsilon |\vec{E}|^2]$$

energia puntuale o densit  di energia per unit  di volume del campo elettostatico

CAPACIT  ELETTRICA

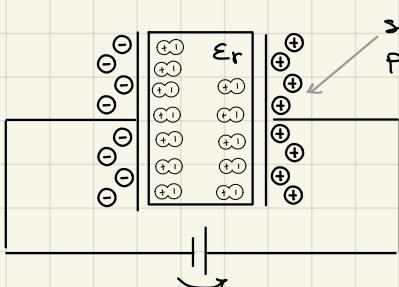


$$\oint_S \vec{D} \cdot d\vec{s} = \epsilon_0 E A = Q \quad (E = \frac{V}{d})$$

$$[C = \frac{Q}{V} = \frac{\epsilon_0 E A}{V} = \frac{\epsilon_0 A}{d}]$$

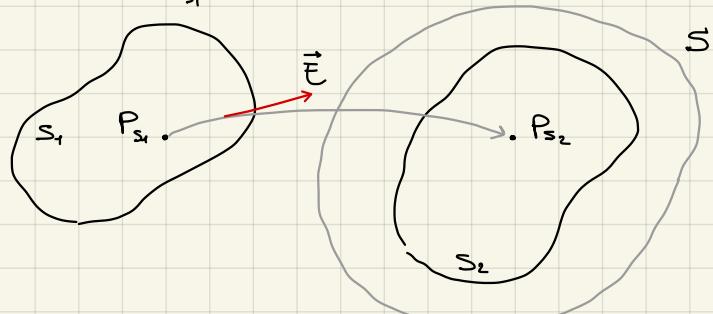
$$C = \frac{Q}{V} = \frac{\oint_S \vec{D} \cdot d\vec{s}}{- \int_{P_{S_1}}^{P_{S_2}} \vec{E} \cdot d\vec{e}}$$

d piccolo per poter trascurare gli effetti di bordo



$$C = \frac{\epsilon_r \epsilon_0 A}{d}$$

L'inserimento di un dielettrico fra le armature ne aumenta la capacit .



Energia immagazzinata nel condensatore

$$\begin{aligned} W_e &= \frac{1}{2} \int_{S_1} g_{S_1} V dS_1 + \frac{1}{2} \int_{S_2} g_{S_2} V dS_2 \\ &= \frac{V_1}{2} \int_{S_1} g_{S_1} dS_1 + \frac{V_2}{2} \int_{S_2} g_{S_2} dS_2 \\ &= \frac{1}{2} Q (\Delta V) \quad \Rightarrow \quad [W_e = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV] \end{aligned}$$

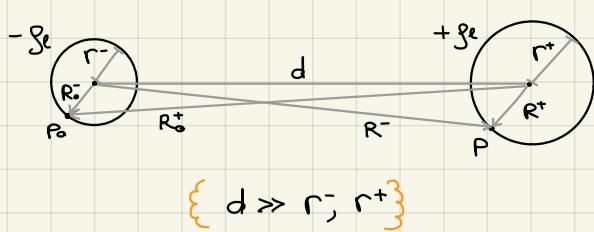
ΔV

ΔV : differenza di potenziale fra le armature del condensatore

$$W_e = \frac{1}{2} \int_{\text{spazio}} \epsilon_0 |\vec{E}|^2 d\Omega = \frac{1}{2} \epsilon_0 \int_{\substack{\text{spazio} \\ \text{tra le armature}}} \frac{V^2}{d^2} d\Omega = \frac{1}{2} \epsilon_0 \frac{V^2}{d} A = \frac{1}{2} CV^2$$

↓
ricavata dall'energia del campo elettrostatico

Es (capacità (per u.l.) di una linea bifilare):

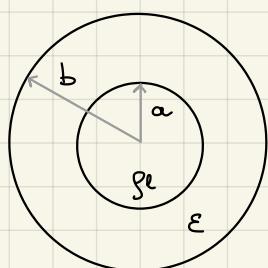


$$\left[C = \frac{g_e}{V} = g_e \frac{2\pi\epsilon_0}{\rho_e \ln(\frac{d^2}{r^+ r^-})} = \frac{2\pi\epsilon_0}{\ln(\frac{d^2}{r^+ r^-})} \right]$$

$$\begin{aligned} V(P) &= \frac{\rho_e}{2\pi\epsilon_0} \ln\left(\frac{R^+}{R^-}\right) - \frac{\rho_e}{2\pi\epsilon_0} \ln\left(\frac{R^-}{R^+}\right) \\ &= \frac{\rho_e}{2\pi\epsilon_0} \ln\left(\frac{R^+ R^-}{R^- R^+}\right) \\ V &\approx \frac{\rho_e}{2\pi\epsilon_0} \ln\left(\frac{d^2}{r^+ r^-}\right) \end{aligned}$$

potenziale generato
da un caro con
densità lineare
di carica

Es (capacità per u.l. di un caro coassiale):



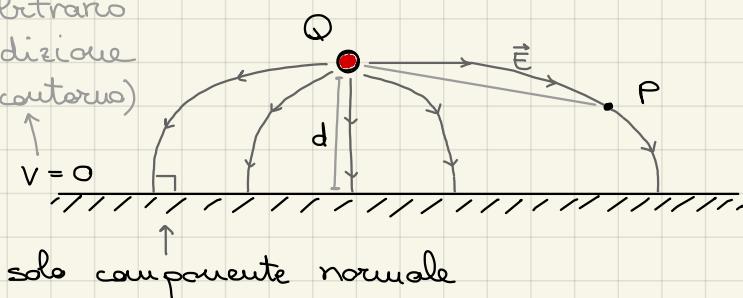
$$E(r) = \frac{\rho_e}{2\pi\epsilon\epsilon_0}$$

$$V = - \int_b^a E(r) dr = - \int_b^a \frac{\rho_e}{2\pi\epsilon\epsilon_0 r} dr = \frac{\rho_e}{2\pi\epsilon\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$\left[C = \frac{\rho_e}{2\pi\epsilon V} = \frac{2\pi\epsilon}{\ln(b/a)} \right]$$

Metodo delle immagini

arbitrario
(condizione
al contorno)



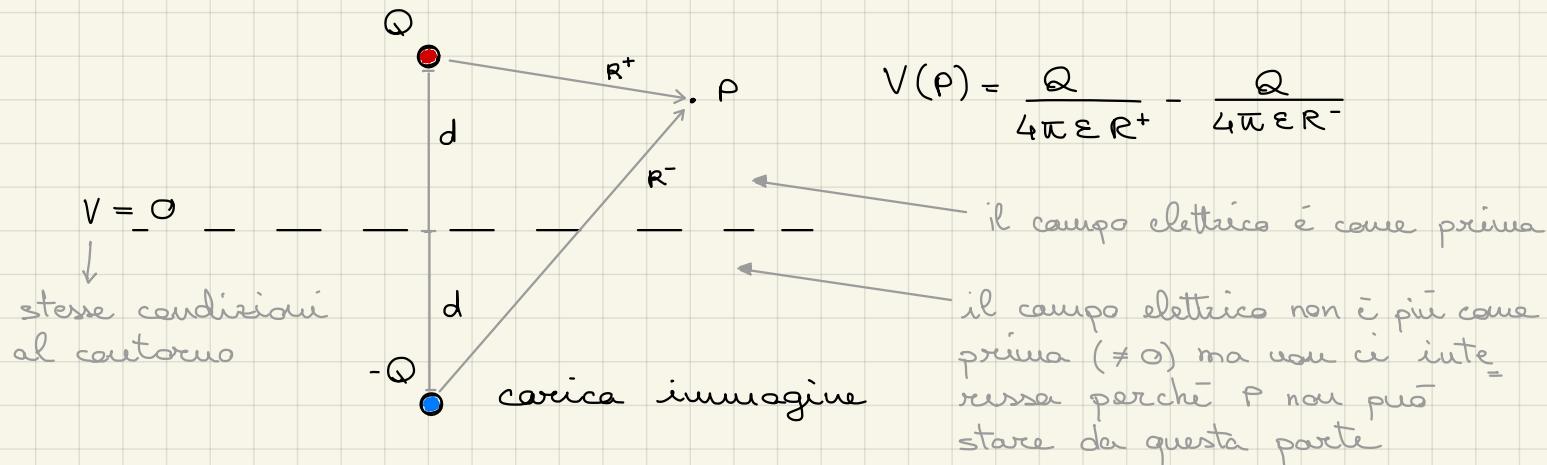
$$V(P) = ?$$

$$\vec{E}(P) = ?$$

piano conduttore

→ Teorema di Unicità

Data una regione di spazio, delimitata da una superficie chiusa, sulla quale siano assicurate le condizioni al contorno (potenziale o sua derivata normale), la soluzione del problema elettostatico (che soddisfa le condizioni al contorno) è unica.



Corrente elettrica e Legge di Ohm

$$\vec{J}_d = \mu q \vec{E} \quad \vec{J} = q N \vec{J}_d = q N \mu q \vec{E} = \sigma \vec{E}$$

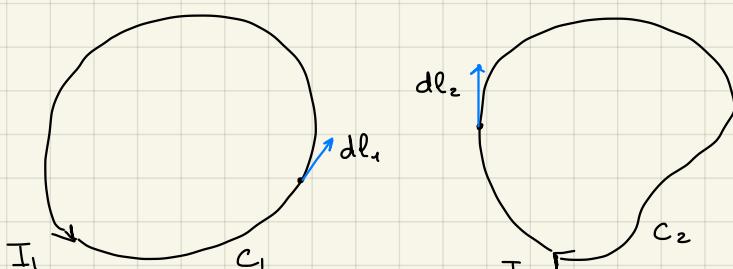
mobilità elettronica

conducibilità $[\frac{\text{S}}{\text{m}}]$

densità volumetrica di portatori

La velocità di deriva v_s è generalmente dell'ordine di solo qualche millimetro al secondo, perché misura lo spostamento medio delle cariche nel conduttore, che è molto inferiore rispetto all'effettiva velocità delle cariche (10^6 m/s) e alla propagazione della corrente e dell'energia nel circuito (10^8 m/s).

CAMPO MAGNETOSTATICO



Legge di Biot-Savart

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi} \frac{I_1 d\vec{l}_1 \times (\vec{d}\vec{l}_2 \times \vec{\mu}_{12})}{R^2}$$

$$\begin{aligned} \vec{F}_{12} &= \frac{\mu_0 I_1 I_2}{4\pi} \iint_{C_1 C_2} \frac{d\vec{l}_1 \times (\vec{d}\vec{l}_2 \times \vec{\mu}_{12})}{R^2} \\ &= \frac{\mu_0 I_1 I_2}{4\pi} \iint_{C_1 C_2} \frac{\vec{\mu}_{12} (\vec{d}\vec{l}_1 \cdot \vec{d}\vec{l}_2)}{R^2} = -\vec{F}_{21} \end{aligned}$$

$$d\vec{F}_2 = I_1 d\vec{l}_1 \times \vec{B}_2 \quad d\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}_2 \times \vec{\mu}_2}{R^2}$$

$$\vec{B}_2 = \oint_{C_1} \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}_2 \times \vec{\mu}_2}{R^2} = \int_S \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{\mu}_2}{R^2} d\Omega$$

Legge di Gauss

$$\oint_S \vec{B} \cdot d\vec{s} = 0 \quad \lim_{S \rightarrow 0} \oint_S \vec{B} \cdot d\vec{s} \quad \nabla \cdot \vec{B} = 0$$

Legge di Ampere

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

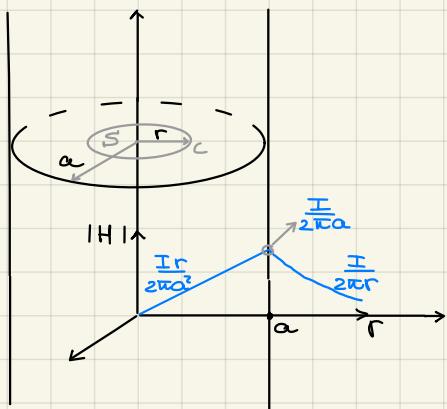
$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s}$$



$$\left(\oint_C \vec{H} \cdot d\vec{l} = \int_S \nabla \times \vec{H} \cdot d\vec{s} = \int_S \vec{J} \cdot d\vec{s} \text{ teo. di Stokes} \right)$$

$$\lim_{S \rightarrow 0} \oint_C \vec{H} \cdot d\vec{l} = \lim_{S \rightarrow 0} \int_S \vec{J} \cdot d\vec{s} = \vec{J} \cdot \vec{S} = I$$

E_s (filo percorso da corrente):



$$J = I / (\pi a^2)$$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s}$$

r < a \vec{H} interno

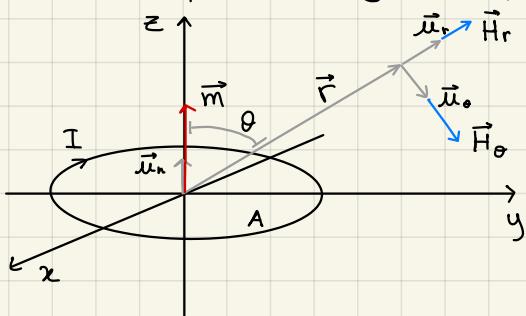
$$\oint_C \vec{H} \cdot d\vec{l} = H_\varphi 2\pi r = \int_S \vec{J} \cdot d\vec{s} = \frac{I}{\pi a^2} \pi r^2$$

$$[\vec{H}(r) = H_\varphi \vec{\mu}_q = \frac{Ir}{2\pi a^2} \vec{\mu}_q]$$

r > a \vec{H} esterno

$$\oint_C \vec{H} \cdot d\vec{l} = H_\varphi 2\pi r = \int_S \vec{J} \cdot d\vec{s} = I \Rightarrow [\vec{H}(r) = \frac{I}{2\pi r} \vec{\mu}_q]$$

E_s (dipolo magnetico):

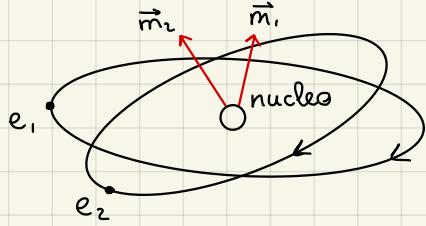


$$\vec{m} = A \cdot I \cdot \vec{\mu}_n$$

$$[\vec{B} = \frac{\mu_0 |\vec{m}|}{4\pi r^3} (2 \cos \theta \vec{\mu}_r + \sin \theta \vec{\mu}_\theta)]$$

$$(\vec{E} = \frac{Qd}{4\pi \epsilon_0 r^3} (2 \cos \theta \vec{\mu}_r + \sin \theta \vec{\mu}_\theta)) \text{ dipolare}$$

Campo magnetico nei materiali

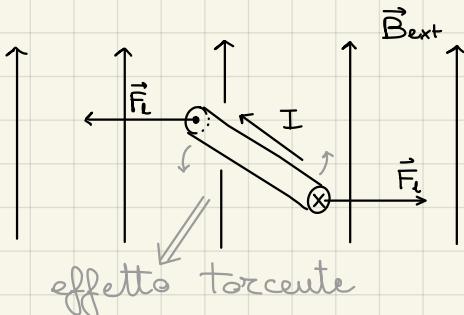


$$\vec{M} = \lim_{\Delta\Omega \rightarrow 0} \frac{\sum \vec{m}_i}{\Delta\Omega}$$

un campo magnetico esterno allinea i momenti di doppio degli atomi

$$\vec{M} = \chi_m \vec{H}$$

χ_m è detta suscettività magnetica



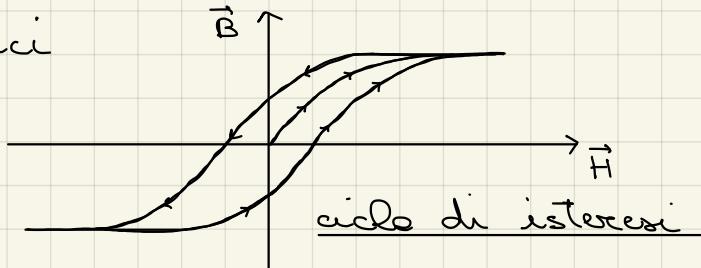
$$M = M_0 \mu_r = \mu_0 (1 + \chi_m)$$

$\mu_r = 1 + \chi_m$ è la permeabilità relativa

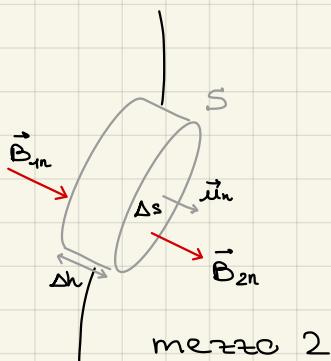
$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 (\vec{H} + \vec{M})$$

Classificazione dei materiali:

- Diamagnetici ($\chi_m \approx -10^{-5}$)
- Paramagnetici ($\chi_m \approx 10^{-3}$)
- Ferromagnetici



Condizione al centro per \vec{H} e \vec{B}

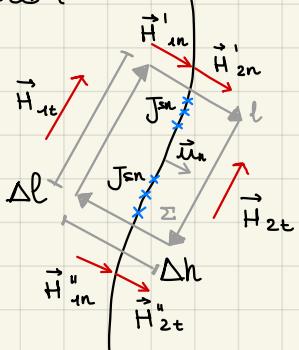


$$\oint_S \vec{B} \cdot d\vec{s} = B_{2n} \Delta S - B_{1n} \Delta S = 0 \implies [B_{2n} = B_{1n}]$$

$$[H_{2n} = \frac{\mu_1}{\mu_2} H_{1n}] \text{ mezzi lineari e isotropi}$$

$$\vec{\mu}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

mezzo 1



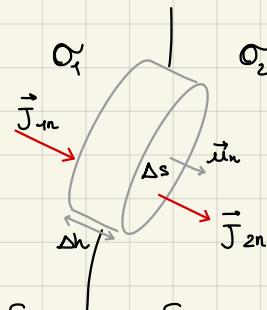
$$\oint_L \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$$

$$\oint_L \vec{H} \cdot d\vec{l} = (H_{2t} - H_{1t}) \Delta l = J_{sn} \Delta l \implies [H_{2t} - H_{1t} = J_{sn}]$$

$$\vec{\mu}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

densità di corrente superficiale [A/m]

Condizioni al contorno per \vec{J}



condutibilità

$$E_{1t} = E_{2t} \quad \vec{J} = \sigma \vec{E}$$

$$\frac{J_{2t}}{\sigma_2} = \frac{J_{1t}}{\sigma_1}$$

$$\oint_S \vec{J} \cdot d\vec{s} = 0 \implies J_{2n} = J_{1n}$$

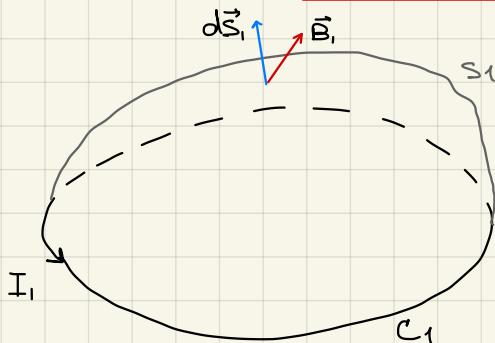
$$\vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$$

$$[\sigma_2 E_{2n} = \sigma_1 E_{1n}]$$

$$\epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \rho_s \implies \rho_s = \left(\epsilon_2 - \epsilon_1 \frac{\sigma_2}{\sigma_1} \right) E_{1n} = \left(\epsilon_2 \frac{\sigma_1}{\sigma_2} - \epsilon_1 \right) E_{1n}$$

densità superficiale
di carica $\left[\frac{C}{m^2} \right]$

Autoinduttanza Magnetica



$$L_{||} = \frac{\Phi_{m,||}}{I_1} = \frac{\int_{S_1} \vec{B}_1 \cdot d\vec{s}_1}{I_1}$$

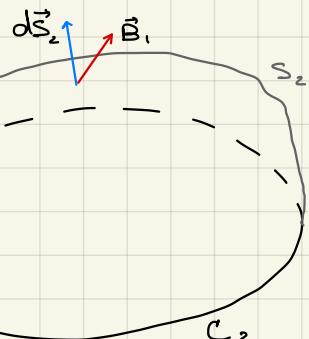
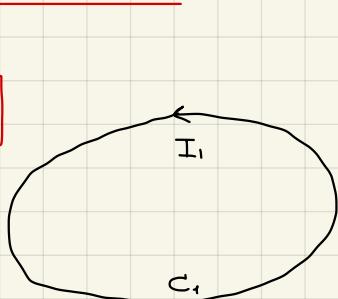
$$\vec{B}_1 = \frac{\mu I_1}{4\pi} \oint_{C_1} \frac{d\vec{l} \times \vec{\mu}_r}{r^2}$$

$$\Phi_{m,||} = \frac{\mu I_1}{4\pi} \int_{S_1} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{\mu}_r}{r^2} \right) \cdot d\vec{s}$$

$$[L_{||} = \frac{\mu}{4\pi} \int_{S_1} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{\mu}_r}{r^2} \right) \cdot d\vec{s}]$$

Mutualinduttanza

$$[L_{21} = \frac{\mu}{4\pi} \int_{S_2} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{\mu}_r}{r^2} \right) \cdot d\vec{s}]$$

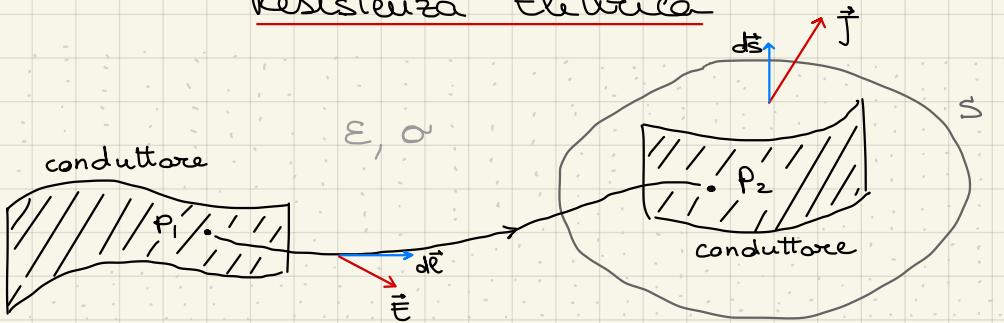


Energia del campo magnetostatico

$$W_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu |\vec{H}|^2$$

$$W_m = \frac{1}{2} L_u I^2$$

Resistenza Elettrica



$$R = \frac{V}{I} = - \frac{\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}}{\oint_S \vec{J} \cdot d\vec{s}} = \frac{1}{\sigma} \frac{\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}}{\oint_S \vec{J} \cdot d\vec{s}}$$

$$C = \frac{Q}{V} = \frac{\oint_S \vec{D} \cdot d\vec{s}}{- \int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}}$$

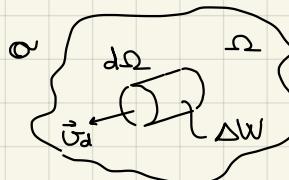
$$R \cdot C = \frac{E}{\sigma}$$

formula generale
della resistenza
elettrica (valida
per qualunque
geometria)

costante di tempo
del materiale

Legge di Joule

N per m³ (N · m⁻³)



$$N \cdot d\Omega$$

$$\vec{J} = N q \vec{U}_d$$

$$\Delta W = N d\Omega \vec{F} \cdot d\vec{l}$$

$$\vec{F} = q \vec{E}$$

$$dP = \frac{\Delta W}{\Delta t} = N d\Omega q \vec{E} \cdot \frac{\vec{dl}}{\Delta t}$$

$$\left[\frac{dP}{d\Omega} = N q \vec{E} \cdot \vec{U}_d = \vec{E} \cdot \vec{J} \right] \left[\frac{W}{m^3} \right]$$

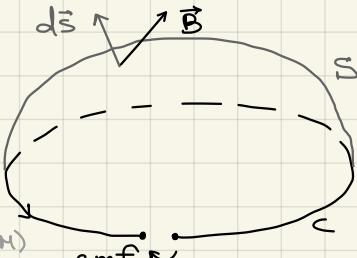
$$P = \int_{\Omega} \vec{E} \cdot \vec{J} d\Omega$$

Regime Dinamico

STATICO: \vec{E} e \vec{H} indipendenti $\vec{\nabla} \cdot \vec{E} = \frac{q}{\epsilon}$ $\vec{\nabla} \times \vec{H} = \vec{J}$

DINAMICO: \vec{E} e \vec{H} legati fra loro (eq. di Maxwell)

→ Equazione sulla circolazione di \vec{E} : Legge di Faraday



$$\rightarrow \text{emf} = - \frac{d \Phi_m}{dt}$$

$$\rightarrow \text{emf} = \oint_C \vec{E} \cdot d\vec{l} [V]$$

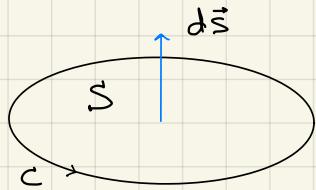
$$\rightarrow \phi_m = \int_s \vec{B} \cdot d\vec{s} \Rightarrow \oint_c \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_s \vec{B} \cdot d\vec{s}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \underbrace{\int_A^B \vec{E} \cdot d\vec{l}}_{\sim 0} + \boxed{\int_B^A \vec{E} \cdot d\vec{l}} = \text{emf}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \text{emf} \neq 0$$

$$E_t = 0 \\ \vec{E} \cdot d\vec{l} = E_t$$

$$\boxed{\oint_c \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_s \vec{B} \cdot d\vec{s} =}$$



$$= - \int_s \frac{\partial}{\partial t} (\vec{B} \cdot d\vec{s}) = - \int_s \left(\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \vec{B} \cdot \frac{\partial d\vec{s}}{\partial t} \right) = \\ = - \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

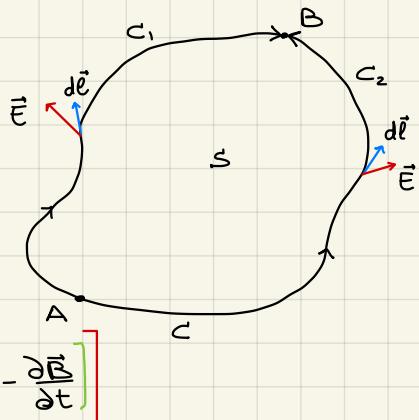
se consideriamo una superficie S che non varia nel tempo

$$\oint_c \vec{E} \cdot d\vec{l} = \int_{c_1} \vec{E} \cdot d\vec{l} - \int_{c_2} \vec{E} \cdot d\vec{l} = - \frac{d\phi_m}{dt}$$

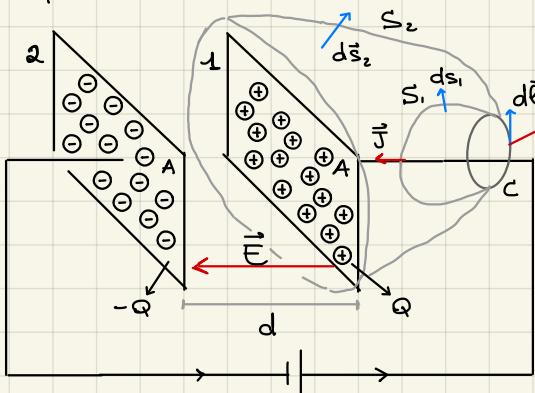
$$\int_{c_1} \vec{E} \cdot d\vec{l} \neq \int_{c_2} \vec{E} \cdot d\vec{l} \quad (\text{se } \phi_m \text{ varia nel tempo})$$

$$\lim_{S \rightarrow 0} \oint_c \vec{E} \cdot d\vec{l} = \lim_{S \rightarrow 0} \left[- \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \right] = - \frac{\partial \vec{B}}{\partial t} \cdot \vec{s}$$

$$\lim_{S \rightarrow 0} \oint_c \vec{E} \cdot d\vec{l} = (\vec{\nabla} \times \vec{E}) \cdot \vec{s} = - \frac{\partial \vec{B}}{\partial t} \cdot \vec{s} \quad \boxed{[\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}]}$$



→ Equazioni sulla circuitazione di \vec{H} e corrente di spostamento



$$\oint_c \vec{H} \cdot d\vec{l} = ? \quad \begin{cases} \int_s \vec{J} \cdot d\vec{s} \neq 0 \\ \int_s \vec{J} \cdot d\vec{s}_2 = 0 \end{cases}$$

$$\oint_c \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

termine stazionario e dinamico
corrente di conduzione e spostamento

$$\oint_c \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_s \vec{D} \cdot d\vec{s} \rightarrow \text{se } S \text{ non varia nel tempo}$$

$$[I_c = \int_s \vec{J} \cdot d\vec{s}] \quad [I_s = \frac{d}{dt} \int_s \vec{D} \cdot d\vec{s}]$$

$$\lim_{s \rightarrow 0} \oint_C \vec{H} \cdot d\vec{l} = \lim_{s \rightarrow 0} \left[\int_s \vec{J} \cdot d\vec{s} + \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \right] = \vec{J} \cdot \vec{s} + \frac{\partial \vec{B}}{\partial t} \cdot \vec{s}$$

$$\lim_{s \rightarrow 0} \oint_C \vec{H} \cdot d\vec{l} = \vec{\nabla} \times \vec{H} \cdot \vec{s} \quad \Rightarrow \quad \left[[\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}] \right]$$

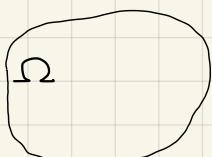
→ Equazioni sui flussi di \vec{D} e \vec{B} : Leggi di Gauss

$$\oint_S \vec{D} \cdot d\vec{s} = \int_{\Omega} \rho_e d\Omega \quad \rightarrow \quad [\vec{\nabla} \cdot \vec{D} = \rho_e]$$

$$\oint_B \vec{B} \cdot d\vec{s} = 0 \quad \rightarrow \quad [\vec{\nabla} \cdot \vec{B} = 0]$$

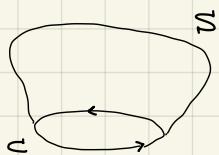
(Sono ricavabili dalle altre equazioni di Maxwell)

Legge di conservazione della carica



$$\oint_S \vec{J} \cdot d\vec{s} = - \frac{d}{dt} \int_{\Omega} \rho_e d\Omega$$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_{\Omega} \vec{D} \cdot d\vec{s} = 0$$



$$\begin{aligned} \oint_S \vec{J} \cdot d\vec{s} &= - \frac{d}{dt} \oint_{\Omega} \vec{D} \cdot d\vec{s} \\ &= - \frac{d}{dt} \int_{\Omega} \vec{\nabla} \cdot \vec{D} d\Omega \\ &= - \frac{d}{dt} \int_{\Omega} \rho_e d\Omega \end{aligned}$$

$$\oint_S \vec{J} \cdot d\vec{s} = \int_{\Omega} \vec{\nabla} \cdot \vec{J} d\Omega = - \frac{d}{dt} \int_{\Omega} \rho_e d\Omega$$

$$[\vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho_e}{\partial t}] \quad (\vec{\nabla} \cdot \vec{J} = 0 \text{ stazionario})$$

$$\vec{J} = \sigma \vec{E}$$

Equazioni di Maxwell (integrali):

$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_S \vec{D} \cdot d\vec{s}$$

$$\oint_S \vec{D} \cdot d\vec{s} = \int_{\Omega} \rho_e d\Omega$$

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\oint_S \vec{J} \cdot d\vec{s} = - \frac{d}{dt} \int_{\Omega} \rho_e d\Omega$$

Equazioni di Maxwell (differentiali)

$$\left(\begin{array}{l} \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{array} \right)$$

$$\vec{\nabla} \cdot \vec{D} = \rho_s$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

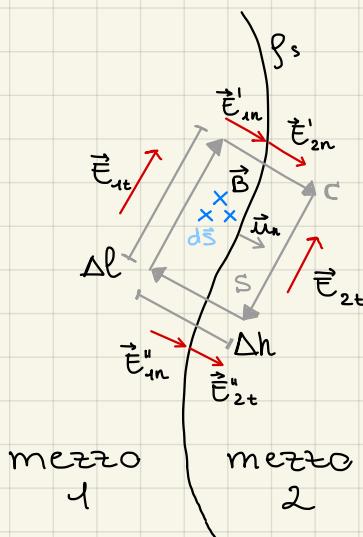
$$\left(\vec{\nabla} \cdot \vec{J} = - \frac{d\rho_s}{dt} \right)$$

Relazioni costitutive dei materiali

- Ci sono 15 incognite (nello spazio 3D):
- Abbiamo 6 equazioni scalari (rotore)
- Le altre 9 equazioni dalle 3 relazioni costitutive:

$$\begin{aligned} \vec{D} &= \vec{f}_0(\vec{E}, \vec{H}) & \vec{D} &= \epsilon \vec{E} = \epsilon \epsilon_r \vec{E} = \epsilon_0 (1 + \chi_e) \vec{E} \\ \vec{B} &= \vec{f}_0(\vec{H}, \vec{E}) & \vec{B} &= \mu \vec{H} = \mu_0 \mu_r \vec{H} = \mu_0 (1 + \chi_m) \vec{H} \\ \vec{J}_c &= \vec{f}_J(\vec{E}, \vec{H}) & \vec{J}_c &= \sigma \vec{E} \end{aligned}$$

Condizioni al contorno per \vec{E} (regime dinamico)



$$\oint_c \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$\Delta h \rightarrow 0 \quad (S \rightarrow O)$$

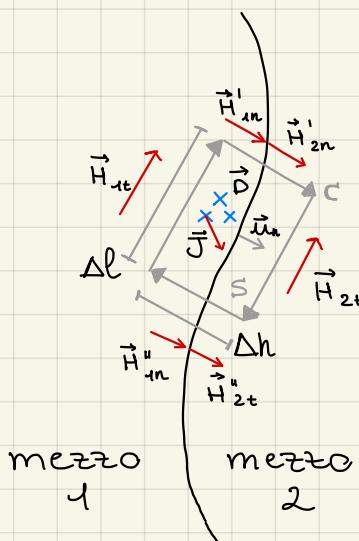
$$\oint_c \vec{E} \cdot d\vec{l} = 0 \implies E_{2t} = E_{1t}$$

uguale al regime
stazionario

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

Condizioni al contorno per \vec{H} (regime dinamico)



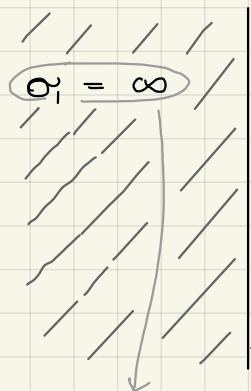
$$\oint_C \vec{H} \cdot d\vec{\ell} = \int_S \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_S \vec{D} \cdot d\vec{s}$$

$$\Delta h \rightarrow 0 \quad (S \rightarrow 0)$$

$$\Rightarrow [H_{2n} - H_{1t}] = J_{sn} \quad \begin{array}{l} \text{componente delle densità di corrente} \\ \text{perpendicolare alla superficie di discontinuità} \\ \text{come nel caso statico} \end{array}$$

$$\vec{n}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{n}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0$$



$$\sigma_1 = \infty$$

$$\epsilon_2 \quad \mu_2$$

$$\sigma_2 = 0$$

nel conduttore

ideale

$$H_{1t} = 0 \quad H_{1n} = 0$$

(I)

$$E_{1t} = 0$$

(II)

$$E_{2t} = 0 \leftrightarrow \vec{n}_n \times \vec{E}_2 = 0$$

$$H_{2t} = J_s$$

$$\vec{n}_n \times \vec{H}_2 = \vec{J}_s$$

$$D_{2n} = f_s$$

$$\vec{n}_n \cdot \vec{D}_2 = f_s$$

$$B_{2n} = 0 = \mu_2 H_{2n} \quad \vec{n}_n \cdot \vec{B}_2 = 0$$

significa che il conduttore ha tempo di rilassamento nullo

Se il conduttore non è ideale ($\sigma < \infty$) il tempo di rilassamento non è più nullo e il materiale ci impiega del tempo a raggiungere questi valori

➡ in regime dinamico queste uguaglianze non sono più vere

Onde Elettromagnetiche

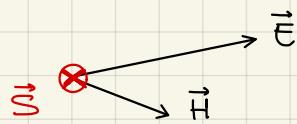
- 1) Teorema (e vettore) di Poynting (dominio del tempo)
- 2) Equazione di Helmholtz (dominio del tempo)
- 3) Caso particolare: onde piane in mezzi senza perdite
- 4) Passaggio al dominio dei fasci (regime sinusoidale)

Teorema di Poynting

Si definisce densità di potenza e si misura in $\frac{W}{m^2}$, la grandezza vettoriale \vec{S}

$$[\vec{S} = \vec{E} \times \vec{H}]$$

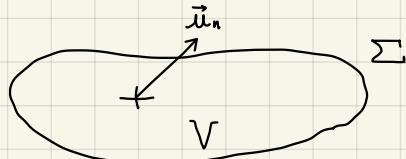
vettore di Poynting



$$\boxed{- \int_S \vec{S} \cdot d\vec{S} = \int_{\Sigma} \vec{E} \cdot \vec{J} d\Sigma + \int_{\Omega} (\vec{E} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{E}}{\partial t}) d\Omega} [W]$$

Dimostrazione:

$$\vec{S} = \vec{E} \times \vec{H} \quad [\frac{W}{m^2}]$$



$$\oint_{\Sigma} \vec{S} \cdot d\vec{\Sigma} = -(\text{variazione istantanea dell'energia in } V)$$

$$\oint_{\Sigma} \vec{S} \cdot d\vec{\Sigma} = \int_V \vec{\nabla} \cdot \vec{S} dV = \int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dV$$

$$\begin{aligned} \vec{\nabla} \cdot (\vec{E} \times \vec{H}) &= \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \\ &= \vec{H} \cdot \left(-\frac{\partial \vec{B}}{\partial t} \right) - \vec{E} \cdot \left(\frac{\partial \vec{B}}{\partial t} + \vec{J} \right) = \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} - \sigma |\vec{E}|^2 = \\ &= \frac{\mu}{2} \frac{\partial |\vec{H}|^2}{\partial t} - \frac{\epsilon}{2} \frac{\partial |\vec{E}|^2}{\partial t} - \sigma |\vec{E}|^2 \end{aligned}$$

$$\begin{aligned} \int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dV &= -\frac{\partial}{\partial t} \left[\int_V \left(\frac{\mu |\vec{H}|^2}{2} + \frac{\epsilon |\vec{E}|^2}{2} \right) dV \right] - \int_V \sigma |\vec{E}|^2 dV \\ &= \oint_{\Sigma} \vec{S} \cdot d\vec{\Sigma} \end{aligned}$$

ENERGIA E.M.
IMMAGAZZINATA
(IN V)

POTENZA
DISSIPATA

Equazione di Helmholtz (delle onde)

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

A sergenti

$$\begin{aligned} \vec{J} &= \vec{J}_c + \vec{J}_e \\ \vec{J} &= \sigma \vec{E} \end{aligned}$$

in assenza di cariche

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\vec{\nabla} \times (\vec{E} \times \vec{E}) = -\mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \quad \text{ma} \quad \vec{\nabla} \times \vec{\nabla} \times \vec{F} = \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) = -\nabla^2 F$$

$$\vec{\nabla} (\cancel{\vec{E} \times \vec{E}}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} (\epsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E})$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$$

e analogamente $\boxed{\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}}$ con la (2).

Queste 2 equazioni differenziali vettoriali (6 equazioni differenziali scalari) non sono di solito risolvibili in forma chiusa, tranne che in alcuni caso particolari.

Onde piane in mezzi ideali (senza perdite)

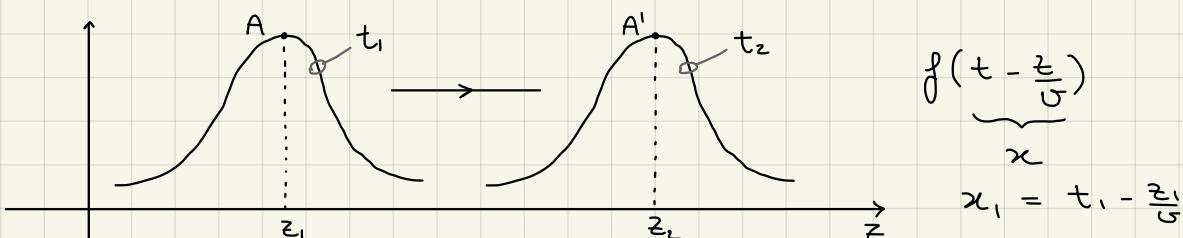
$$\nabla^2 \vec{E} = \mu \sigma \cancel{\frac{\partial \vec{E}}{\partial t}} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Dimostreremo che l'onda piana uniforme è una soluzione dell'equazione delle onde

Onda piana: i piani in cui giacciono i campi \vec{E} ed \vec{H} sono paralleli tra loro in ogni punto dello spazio

Onda piana uniforme: i campi \vec{E} ed \vec{H} non variano su tali piani (in un dato istante)

$f(z) \rightsquigarrow f(t \pm \frac{z}{v})$ ← perturbazione che viaggia con velocità v nel verso delle z positive/negative (onda progressiva/regressiva)



$$t_1 - \frac{z_1}{v} = t_2 - \frac{z_2}{v}$$

$$\underbrace{f(t - \frac{z}{v})}_x$$

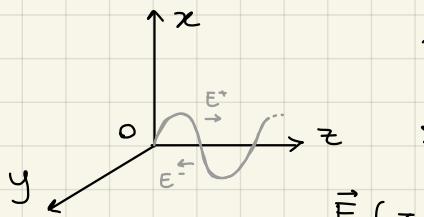
$$x_1 = t_1 - \frac{z_1}{v}$$

$$x_2 = t_2 - \frac{z_2}{v}$$

$$\underbrace{\frac{z_2 - z_1}{\Delta z}}_{\Delta t} = \underbrace{(t_2 - t_1)}_{\Delta t}$$

$$\frac{\Delta z}{\Delta t} = v \quad (\text{velocità})$$

$$\vec{E}(x, y, z, t) = E_x(x, y, z, t) \vec{u}_x + E_y(\dots) \vec{u}_y + E_z(\dots) \vec{u}_z$$



xy piano "trasverso"

su xy \vec{E} ed \vec{H} costanti

independenti
(non interferiscono)

$$\vec{E}(z, t) = \vec{E}^+(t - \frac{z}{c}) + \vec{E}^-(t + \frac{z}{c}) \text{ onda piana unif.}$$

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

equazione di Helmholtz in mezzo senza perdite

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{operatore laplaciano}$$

$$\rightarrow \begin{cases} \frac{\partial^2 E_x(z, t)}{\partial z^2} + \frac{\partial^2 E_x(z, t)}{\partial y^2} + \frac{\partial^2 E_x(z, t)}{\partial x^2} - \mu \epsilon \frac{\partial^2 E_x(z, t)}{\partial t^2} = 0 \\ \frac{\partial^2 E_y(z, t)}{\partial z^2} + \frac{\partial^2 E_y(z, t)}{\partial x^2} - \mu \epsilon \frac{\partial^2 E_y(z, t)}{\partial t^2} = 0 \\ \dots \quad \dots \quad \dots \quad \frac{\partial^2 E_z(z, t)}{\partial z^2} - \mu \epsilon \frac{\partial^2 E_z(z, t)}{\partial t^2} = 0 \end{cases}$$

$$\Rightarrow \frac{\partial^2 \vec{E}(z, t)}{\partial z^2} - \mu \epsilon \frac{\partial^2 \vec{E}(z, t)}{\partial t^2} = 0$$

$$\vec{E}^+(t - \frac{z}{c}), \quad \frac{1}{c^2} \vec{E}^+(z, t) = \mu \epsilon \vec{E}^+(z, t) \quad \text{sse. } \frac{1}{c^2} = \mu \epsilon \rightarrow c = \frac{1}{\sqrt{\mu \epsilon}}$$

Nello spazio vuoto

$$\epsilon = \epsilon_0 = 8,854 \cdot 10^{-12} \text{ F/m} \quad \mu = \mu_0 = 4 \cdot \pi \cdot 10^{-7} \text{ H/m}$$

$$\Rightarrow c \approx 3 \cdot 10^8 \text{ m/s}$$

Proprietà dell'onda piana (dal ROTORE)

$$\vec{\nabla} \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \vec{u}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \vec{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \vec{u}_z$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

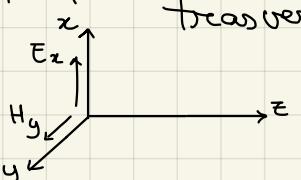
$$\vec{\nabla} \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\begin{cases} -\frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \\ \frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \\ 0 = -\mu \frac{\partial H_z}{\partial t} \end{cases}$$

$$\begin{cases} -\frac{\partial H_y}{\partial z} = \epsilon \frac{\partial E_x}{\partial t} \\ \frac{\partial H_x}{\partial z} = \epsilon \frac{\partial E_y}{\partial t} \\ 0 = \epsilon \frac{\partial E_z}{\partial t} \end{cases}$$

- E_z e H_z costanti (tempo) $\rightarrow E_z$ e H_z componenti nella direzione perpendicolare al campo
- $E_y \xrightarrow{\text{dipend.}} H_z$ e $E_z \xrightarrow{\text{dipend.}} H_y$

ovvero l'onda piano uniforme non ammette componenti nella direzione trasversale

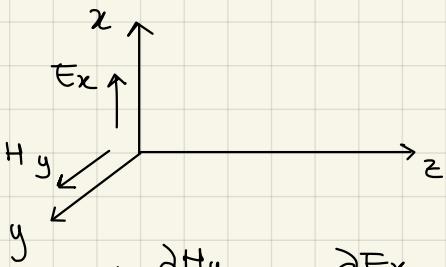


$$E_x = E_x^+(t - \frac{z}{c}) \rightarrow -\frac{1}{c} (E_x^+)' = -\mu \frac{\partial H_y}{\partial t}$$

$$H_y(z, t) = \frac{1}{c\mu} E_x^+(t - \frac{z}{c}) + C = H_y^+(t - \frac{z}{c})$$

$$\frac{E_x^+(t - \frac{z}{c})}{H_y^+(t - \frac{z}{c})} = \sigma \mu = \frac{\mu}{\sqrt{\mu \epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{nel vuoto} \quad \boxed{\sqrt{\frac{\mu}{\epsilon}} = 377 \Omega}$$

Caso sinusoidale

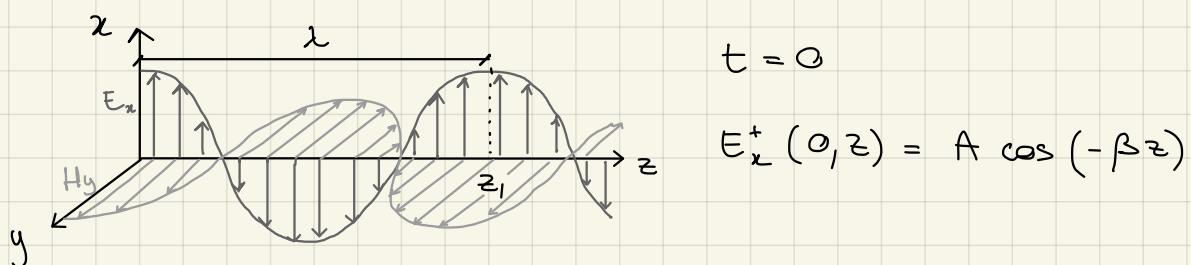


$$E_z^+(t, z) = A \cos \left[\omega \left(t - \frac{z}{c} \right) \right] = A \cos (\omega t - \beta z) \quad \text{con } \beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$H_y^+(t, z) = \frac{A \beta}{\mu \omega} \cos (\omega t - \beta z) = \frac{A}{\eta_0} \cos (\omega t - \beta z)$$

$$\eta_0 = \frac{\mu \omega}{\beta} = \sqrt{\frac{\mu}{\epsilon}} \quad \frac{E_x^+}{H_y^+} = \eta_0 [\Omega] \quad \text{impedenza intrinseca}$$

siamo nel vuoto



$$\beta z' = 2\pi \quad \beta z_n = 2n\pi$$

$$\begin{aligned} n &= 0 & \beta z_0 &= 0 \\ n &= 1 & \beta z_1 &= 2\pi \\ &\vdots &&\vdots \end{aligned}$$

$$\lambda = z_n - z_{n-1} = \frac{2\pi}{\beta} = \frac{2\pi c}{\omega} = \frac{c}{f}$$

λ : lunghezza d'onda f : frequenza c : velocità dell'onda elettromagnetica

Vettore di Poynting

$$\vec{S}_{\text{ist}} = \vec{E} \times \vec{H} = A \cos(\omega t) \frac{A}{\eta_0} \cos(\omega t) \vec{u}_z = \frac{A^2}{\eta_0} \cos^2(\omega t) \vec{u}_z$$

$$\vec{S}_{\text{med}} = \frac{1}{\frac{2\pi}{\omega}} \int_0^{\frac{2\pi}{\omega}} \frac{A^2}{\eta_0} \cos^2(\omega t) dt \cdot \vec{u}_z$$

(z = 0 per semplicità di calcolo)

$$(\text{da } t=0 \text{ a } t=T = \frac{1}{f} = \frac{2\pi}{\omega}) \quad \text{posto } \omega t = \xi \quad t = \frac{\xi}{\omega}$$

$$\vec{S}_{\text{med}} = \frac{A^2 \omega}{2\pi \eta_0 \omega} \underbrace{\int_0^{2\pi} \cos^2 \xi d\xi}_{\pi} \cdot \vec{u}_z = \frac{A^2}{2\eta_0} \vec{u}_z \quad \left[\frac{W}{m^2} \right]$$

Si poteva ricavare lo stesso risultato osservando che:

$$\vec{S}_{\text{(ist)}} = \frac{A^2}{\eta_0} \left[\frac{1}{2} + \frac{1}{2} \cos 2\omega t \right]$$

\uparrow

o su un periodo di sinusoidale
componente in continua

Equazioni di Maxwell (fusorei) val. di picco |E_i|

$$E_i(x, y, z, t) = |E_i(x, y, z)| \cos(\omega t + \theta_{E_i}(x, y, z)) \quad i = x, y, z$$

$|E_i(x, y, z)|$ e $|\theta_{E_i}(x, y, z)|$ sono rispettivamente modulo e fase di E_i .

$$E_i(x, y, z, t) = \operatorname{Re}[E_i e^{j\omega t}] = \operatorname{Re}[|E_i| e^{j\theta_{E_i}} e^{j\omega t}]$$

$$\vec{E} = \operatorname{Re}[(E_x \vec{u}_x + E_y \vec{u}_y + E_z \vec{u}_z) e^{j\omega t}] \rightarrow \text{dipende da } t$$

$$\vec{E} = E_x \vec{u}_x + E_y \vec{u}_y + E_z \vec{u}_z \rightarrow \text{NON dipende da } t \quad (\text{fusore})$$

$$\begin{aligned} \vec{E} &= \operatorname{Re}[\vec{E} e^{j\omega t}] \\ \vec{H} &= \operatorname{Re}[\vec{H} e^{j\omega t}] \\ \vec{D} &= \operatorname{Re}[\vec{D} e^{j\omega t}] \\ \vec{B} &= \operatorname{Re}[\vec{B} e^{j\omega t}] \end{aligned}$$

$$\frac{\partial}{\partial t} (\vec{E} e^{j\omega t}) = j\omega \vec{E} e^{j\omega t}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -j\omega \int_S \vec{B} \cdot d\vec{s}$$

$$\bar{\nabla} \times \vec{E} = -j\omega \vec{B}$$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \bar{J} \cdot d\bar{s} + j\omega \int_S \vec{D} \cdot d\vec{s}$$

$$\bar{\nabla} \times \vec{H} = \bar{J} + j\omega \vec{D}$$

$$\oint_S \vec{D} \cdot d\vec{s} = \int_\Omega \bar{g}_\sigma d\Omega$$

$$\bar{\nabla} \cdot \vec{D} = \bar{g}_\sigma$$

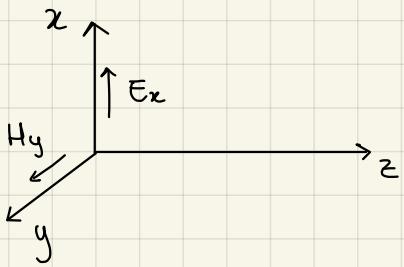
$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\bar{\nabla} \cdot \vec{B} = 0$$

$$\nabla^2 \vec{E} = \mu \sigma (j\omega \vec{E}) + \mu \epsilon (j\omega)^2 \vec{E} = j\omega \mu (\sigma + j\omega \epsilon) \vec{E} = \gamma^2 \vec{E}$$

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

Onde piane uniformi (fasori)



\vec{E} costante su xy

$$\vec{E} = \vec{E}(z)$$

$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0$$

$$\vec{E} = E_x \vec{u}_x + E_y \vec{u}_y + E_z \vec{u}_z$$

↓
vettore
di fasori

$$E_x = E_x(z) = E_x^+(0) e^{-\gamma z} + E_x^-(0) e^{+\gamma z}$$

$$E_x^+(0) = |E_x^+(0)| e^{j\phi_0}$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} - \gamma^2 \vec{E} = 0 \rightarrow \vec{E}^+(0) e^{-\gamma z} + \vec{E}^-(0) e^{+\gamma z}$$

$$\vec{H} \rightarrow \vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H} = \frac{\partial E_x}{\partial z} \vec{u}_y - \frac{\partial E_y}{\partial z} \vec{u}_x$$

$$H_z = 0 \quad \text{e} \quad E_z = 0$$

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z} \quad \text{e} \quad H_y = -\frac{1}{j\omega\mu} \frac{\partial E_x}{\partial z}$$

$$E_x(z) = E_x^+(0) e^{-\gamma z} + E_x^-(0) e^{+\gamma z}; \quad \text{il campo } \vec{H}?$$

$$H_y(z) = -\frac{1}{j\omega\mu} [E_x^+(0) e^{-\gamma z} (-\gamma) + E_x^-(0) e^{+\gamma z} \cdot \gamma]$$

$$\rightarrow H_y(z) = \frac{\gamma}{j\omega\mu} E_x^+(0) e^{-\gamma z} - \frac{\gamma}{j\omega\mu} E_x^-(0) e^{+\gamma z}$$

$$\frac{E_x^+(z)}{H_y^+(z)} = \frac{j\omega\mu}{\gamma} = \eta$$

$$\frac{E_x^-(z)}{H_y^-(z)} = -\frac{j\omega\mu}{\gamma} = -\eta$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = \alpha + j\beta$$

α : costante di attenuazione

$$[\frac{N_p}{m}]$$

"Neper" adimensionale

β : costante di fase $[\frac{\text{rad}}{\text{m}}]$

γ : costante di propagazione $[\text{m}^{-1}]$

$$E_x^+(z) = E_x^+(0) e^{-\gamma z} = E_x^+(0) e^{-\alpha z} e^{-j\beta z} \quad (\text{FASORI})$$

$$\downarrow$$

$$E_x^+(z, t) = \operatorname{Re} \left\{ E_x^+(0) e^{-\alpha z} e^{-j\beta z} e^{j\omega t} \right\} = \quad (\text{TEMPO})$$

$$= |E_x^+(0)| e^{-\alpha z} \cos(\omega t - \beta z + \phi_0) \quad \text{con} \quad \phi_0 = \angle E_x^+(0)$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\alpha > 0 \quad \text{e} \quad \beta > 0$$

(altrimenti non hanno significato fisico)

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} \quad [2]$$

se $\sigma \neq 0$ allora $\eta \in \mathbb{C}$

→ Mezzo ideale (senza perdite)

$$\sigma = 0 \quad \mu \text{ ed } \varepsilon \text{ REALI}$$

$$\gamma^2 = -\omega^2\mu\varepsilon \quad \gamma = j\omega\sqrt{\mu\varepsilon} = j\beta \quad [\frac{\text{rad}}{\text{m}}]$$

$$\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{j} = \omega\sqrt{\mu\varepsilon}} \quad \alpha = 0$$

$$E_x^+(z) = E_x^+(0) e^{-j\beta z} \quad \eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{\mu}{\varepsilon}} \quad (\eta \text{ è REALE})$$

l'onda ritarda ma non s'attenua

(stesso risultato ricavato nel dominio del tempo)

→ Buon conduttore

$$\sigma > 0 \quad \gamma^2 = -\omega^2\mu\varepsilon + j\omega\mu\sigma \quad \omega\mu\sigma \gg \omega^2\mu\varepsilon$$

$\boxed{\sigma \gg \omega\varepsilon}$ affinché il conduttore sia buono

$$\gamma \approx \sqrt{j\omega\mu\sigma} = \sqrt{j} \sqrt{\omega\mu\sigma}$$

→ $\frac{1+j}{\sqrt{2}}$ α e β devono essere re positivi

la condizione non dipende solo dal materiale, ma anche dalla frequenza della perturbazione elettromagnetica

$$= \frac{1+j}{\sqrt{2}} \sqrt{\omega\mu\sigma} = (1+j) \sqrt{\pi f \mu \sigma} = \alpha + j\beta$$

$$(\omega = 2\pi f) \quad \alpha = \beta = \sqrt{\pi f \mu \sigma}$$

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j}\sqrt{\omega\mu\sigma}} = \frac{(1+j)}{\sqrt{2}} \sqrt{\frac{\omega\mu}{\sigma}} =$$

$$= (1+j) \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{E_x^+(z)}{H_y^+(z)}$$

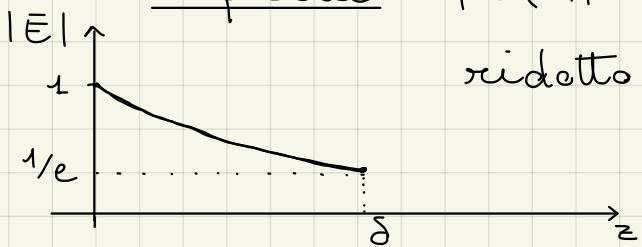
⇒ campo elettrico e magnetico sono sfasati di 45°

Ampiezza

$$|\bar{E}(z)| = |\bar{E}(0)| e^{-\alpha z}$$

ridotto e^{-z} per $z = \frac{1}{\alpha} =$

$$= \frac{1}{\sqrt{\pi f \mu \sigma}} = \delta \quad \text{soppresso pelle}$$



Lunghezza d'onda

\propto
!!

$$\beta = \frac{2\pi}{\lambda} = \frac{1}{\delta} \Rightarrow \lambda = 2\pi\delta$$

$$\lambda = \frac{\omega}{f}$$

Se nel vuoto:

In un buco conduttore

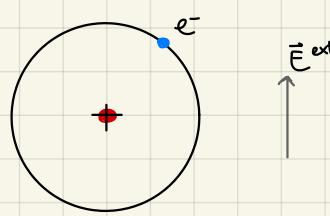
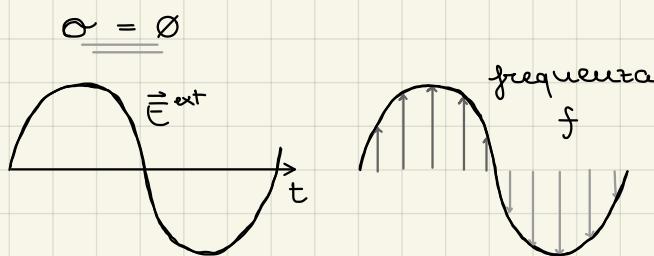
$$\lambda_0 = \frac{c}{f} \quad \lambda \ll \lambda_0$$

$$v = \frac{\omega}{\beta} = \omega\delta$$

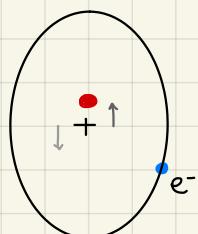
In un buco conduttore, la velocità di propagazione dell'onda elettromagnetica è di molti ordini di grandezza inferiore che nel vuoto.

Se $\sigma \rightarrow \infty, \delta \rightarrow 0$

Perdite nei dielettrici



come una molla



$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0\epsilon_r \quad \bar{\epsilon} = \epsilon \bar{E} \quad \epsilon, \epsilon_r \in \mathbb{C} \quad \epsilon', \epsilon'' \in \mathbb{R}$$

\uparrow
RITARDO

Per tener conto dell'effetto "molla" che le onde elettromagnetiche provocaano in materiali dielettrici, la costante dielettrica non è più reale ma complessa

$$\gamma = \sqrt{-\omega^2 \mu \epsilon} = \sqrt{-\omega^2 \mu \epsilon' + j\omega^2 \mu \epsilon''} \quad (\text{anche } \mu = \mu' - j\mu'')$$

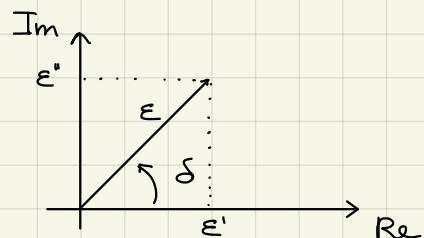
$$= \sqrt{-\omega^2 \mu \epsilon' + j\omega \mu (\omega \epsilon'')} \quad \text{se } \sigma = 0$$

$$\gamma = \sqrt{-\omega^2 \mu \epsilon + j\omega \mu \sigma} \quad \text{se } \sigma \neq 0$$

\Rightarrow La parte complessa della costante dielettrica si comporta come la costante di condutibilità in un conduttore ($\omega \epsilon'' = \sigma_{eq}$) tuttavia se $\omega \rightarrow 0$ $\sigma_{eq} \rightarrow 0$ mentre σ non varia

$$\frac{\epsilon''}{\epsilon} = \tan \delta \quad \text{tangente di perdita}$$

non è lo spessore delle



$$\sigma > 0 \quad \varepsilon = \varepsilon' - j\varepsilon'' \quad \mu = \mu' - j\mu''$$

$$\gamma = \sqrt{-\omega^2 \mu \varepsilon + j\omega \mu \sigma} = \alpha + j\beta$$

$$\eta = j \frac{\omega \mu}{\gamma}$$

$\left. \begin{array}{l} \\ \end{array} \right\} \text{sempre valide}$

Teorema di Poynting (fasori)

Nel dominio del tempo: $\vec{S} = \vec{E} \times \vec{H}$

$$\begin{cases} f_1 = 0 & f_0 & f_0 \\ f_2 = 2f_0 & & \end{cases}$$

1) Nuova definizione di $[\bar{S} = \frac{1}{2} \bar{E} \times \bar{H}^*]$

Stesso approccio della potenza in elettrotecnica

fasori \bar{V} \bar{I} $P = \frac{1}{2} \bar{V} \cdot \bar{I}$

tempo $v(t)$ $i(t)$ $p(t) = v(t) i(t)$

$$P_{\text{med}} = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{2} \text{Re} \{ \bar{V} \cdot \bar{I} \}$$

2) Dimostreremo che $\bar{S}_{\text{med}} = \frac{1}{T} \int_0^T \bar{S}(t) dt = \frac{1}{2} \text{Re} \{ \bar{E} \times \bar{H}^* \}$

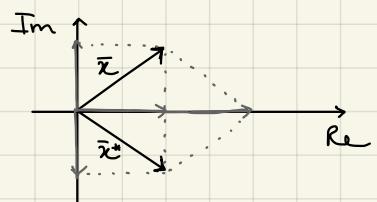
3) Calcoleremo il flusso di potenza trasportato da un'onda TEM piana in un mezzo generico

Nel dominio del tempo:

$$\vec{E} = \text{Re} \{ \bar{E} e^{j\omega t} \} = \frac{1}{2} (\bar{E} e^{j\omega t} + \bar{E}^* e^{-j\omega t})$$

$$\vec{H} = \text{Re} \{ \bar{H} e^{j\omega t} \} = \frac{1}{2} (\bar{H} e^{j\omega t} + \bar{H}^* e^{-j\omega t})$$

$$\vec{S} = \vec{E} \times \vec{H}$$



$$\text{Re} \{ \bar{z} \} = \frac{\bar{z} + \bar{z}^*}{2}$$

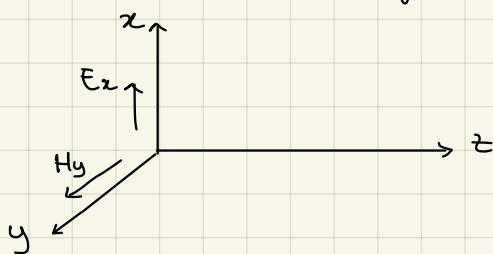
Nel dominio dei fasori:

$$\bar{S} = \frac{1}{4} (\bar{E} \times \bar{H}^* + \bar{E}^* \times \bar{H}) + \frac{1}{4} (\bar{E} \times \bar{H} e^{2j\omega t} + \bar{E}^* \times \bar{H}^* e^{-2j\omega t})$$

$$\text{chiamiamo } \bar{A} = \bar{E} \times \bar{H}^* \text{ e } \bar{B} = \bar{E} \times \bar{H}$$

$$\begin{aligned}\vec{S} &= \frac{1}{4} (\bar{A} + \bar{A}^*) + \frac{1}{4} (\bar{B} e^{j2\omega t} + \bar{B}^* e^{-j2\omega t}) = \\ &= \frac{1}{2} \operatorname{Re}\{\bar{A}\} + \frac{1}{2} \operatorname{Re}\{\bar{B} e^{j2\omega t}\} = \underbrace{\cos 2\omega t}_{\text{integriato sul periodo } T} \text{ è pari a } 0 \\ &= \frac{1}{2} \operatorname{Re}\{\bar{E} \times \bar{H}^*\} + \frac{1}{2} \operatorname{Re}\{(\bar{E} \times \bar{H}) e^{-j2\omega t}\} \\ \vec{S}_{\text{med}} &= \frac{1}{T} \int_0^T \vec{S}(t) dt = \frac{1}{2} \operatorname{Re}\{\bar{E} \times \bar{H}^*\} \text{ c.v.d. (2)}\end{aligned}$$

Onde piane: flusso di densità di potenza (fascio)



$$E_x(z) = E_x^+(0) e^{-\alpha z} e^{-j\beta z} + E_x^-(0) e^{+\alpha z} e^{+j\beta z}$$

$$H_y(z) = \frac{E_x^+(0)}{\eta} e^{-\alpha z} e^{-j\beta z} - \frac{E_x^-(0)}{\eta} e^{+\alpha z} e^{+j\beta z}$$

$$\begin{aligned}\vec{S}_m &= \frac{1}{2} \operatorname{Re}\{\bar{E} \times \bar{H}^*\} = \frac{1}{2} \operatorname{Re}\{E_x(z) \cdot H_y^*(z)\} \vec{u}_z = \\ &\quad \left| H_y^*(z) = \frac{E_x^{+*}(0)}{\eta^*} e^{-\alpha z} e^{+j\beta z} - \frac{E_x^{-*}(0)}{\eta^*} e^{+\alpha z} e^{-j\beta z} \right. \\ (3) &= \frac{1}{2} \left[\frac{|E_x^+(0)|^2}{|\eta|} e^{-2\alpha z} \cos \varphi_\eta - \frac{|E_x^-(0)|^2}{|\eta|} e^{2\alpha z} \cos \varphi_\eta - \right. \\ &\quad \left. - \frac{2 |E_x^+(0)| |E_x^-(0)|}{|\eta|} \sin(2\beta z + \varphi_{E^-(0)} - \varphi_{E^+(0)}) \cdot \sin \varphi_\eta \right] \vec{u}_z\end{aligned}$$

$$\vec{S}_m^+ = \frac{1}{2} \frac{|E_x^+(0)|^2}{|\eta|} e^{-2\alpha z} \cos \varphi_\eta \vec{u}_z \text{ densità di potenza trasportata dall'onda progressiva}$$

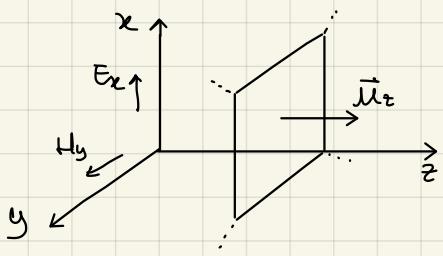
$$\vec{S}_m^- = - \frac{1}{2} \frac{|E_x^-(0)|^2}{|\eta|} e^{+2\alpha z} \cos \varphi_\eta \vec{u}_z \text{ densità di potenza trasportata dall'onda regressiva}$$

$$\vec{S}_m^a = \frac{1 |E_x^+(0)| |E_x^-(0)|}{|\eta|} \sin(2\beta z + \varphi_{E^-(0)} - \varphi_{E^+(0)}) \cdot \sin \varphi_\eta \vec{u}_z \text{ termine di accoppiamento}$$

\hookrightarrow se c'è solo un'onda

\hookrightarrow se il mezzo non ha perdite: η reale $\rightarrow \varphi_\eta = 0$

Come era già stato detto, onda progressiva e regressiva di solito non si "parlano". Questo non è più vero in mezzi con perdite.



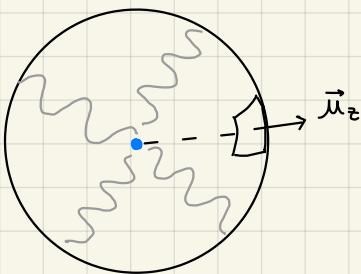
$$E_x(z) = E_x^+(0) e^{-\alpha z} e^{-j \beta z}$$

$$H_y(z) = \frac{E_x^+(0)}{\eta} e^{-\alpha z} e^{-j \beta z}$$

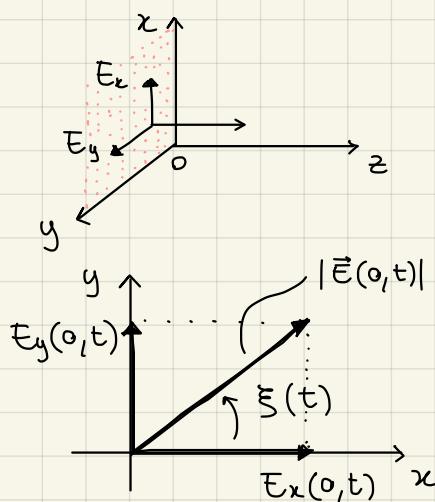
$\vec{S}_m^+ = \frac{1}{2} \operatorname{Re} \{ E_x \cdot H_y^* \} \vec{\mu}_z \left[\frac{W}{m^2} \right]^2$ densità di potenza
costante sul piano $z = \text{cost.}$

⇒ Se l'onda piana uniforme esistesse davvero tra sporterebbe potenza infinita! (l'onda è estesa su tutto il piano xy).
Perciò nella realtà fisica non esiste!

Tuttavia è comunque utile studiarla in quanto le onde sferiche sono approssimabili su un elemento infinitesimo del loro fronte, a delle onde piane uniformi.



Polarizzazione (tempo)



$$\begin{cases} E_x(z, t) = E_x \cos(\omega t - \beta z) \\ E_y(z, t) = E_y \cos(\omega t - \beta z + \varphi_0) \end{cases}$$

Sul piano trasverso ($z = 0$)

$$\vec{E}(0, t) = E_x \cos \omega t \vec{\mu}_x + E_y \cos(\omega t + \varphi_0) \vec{\mu}_y$$

Due casi notevoli:

- $\varphi_0 = 0 (\text{o } \pi)$ E_x, E_y qualsiasi → POLARIZZAZIONE LINEARE

$$\xi(t) = \arctg \frac{E_y(0,t)}{E_x(0,t)} = \arctg \frac{E_y \cos \omega t}{E_x \cos \omega t} = \xi_0 \text{ costante}$$

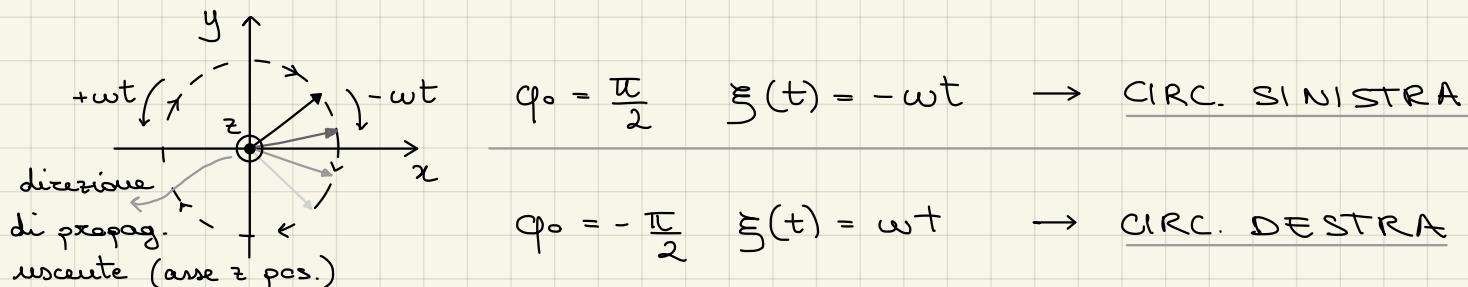
$$|\vec{E}(0,t)|^2 = E_x^2 \cos^2 \omega t + E_y^2 \cos^2 \omega t = (E_x^2 + E_y^2) \cos^2 \omega t$$

$\hookrightarrow |\vec{E}(z,t)|$ varia con il $\cos \omega t$

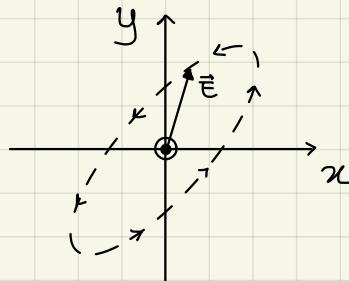
- $\phi_0 = \pm \frac{\pi}{2}$ $E_x = E_y = E \longrightarrow \text{POLARIZZAZIONE CIRCOLARE}$

$$\xi(t) = \arctg \frac{E_y \cos(\omega t \pm \frac{\pi}{2})}{E_x \cos \omega t} = \arctg \left(\frac{\mp \sin \omega t}{\cos \omega t} \right) = \mp \omega t$$

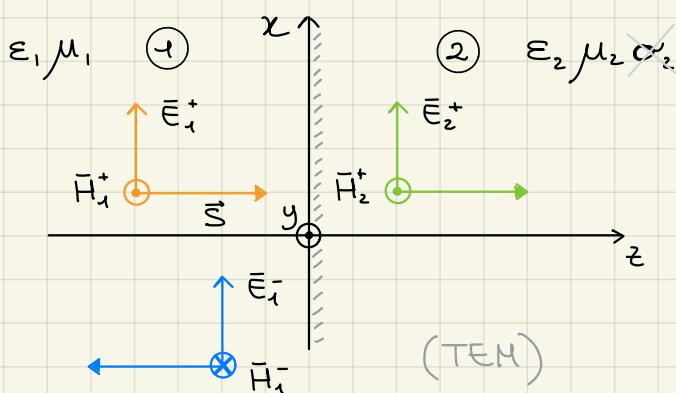
$$|\vec{E}(0,t)|^2 = E^2 \cos^2 \omega t + E^2 \cos^2(\omega t \pm \frac{\pi}{2}) = E^2 \cos^2 \omega t + E^2 \sin^2 \omega t = E^2$$



- Invece $\phi_0 \neq \frac{\pi}{2}$ e $E_x, E_y \neq ? \longrightarrow \text{POLARIZZAZIONE ELLITTICA}$



Incidenza normale su discontinuità piana



Cond. al contorno ($z=0$):

$$\begin{cases} E_{1t} = E_{2t} \\ H_{1t} = H_{2t} \end{cases}$$

Onda incidente (nota) \hookrightarrow ci dobbiamo ricordare a questa che è nota

$$\bar{E}_i^+(z) = E_i^+(0) e^{-k_1 z} \hat{u}_x \quad \text{e} \quad \bar{H}_i^+(z) = \frac{E_i^+(0)}{\eta_1} e^{-k_1 z} \hat{u}_y$$

Onda riflessa

$$\bar{E}_1^-(z) = E_1^-(0) e^{+j\gamma_1 z} \vec{u}_x \quad \text{e} \quad \bar{H}_1^-(z) = -\frac{E_1^-(0)}{\eta_1} e^{+j\gamma_1 z} \vec{u}_y$$

Onda trasmessa

$$\bar{E}_2^+(z) = E_2^+(0) e^{-j\gamma_2 z} \vec{u}_x \quad \text{e} \quad \bar{H}_2^+ = \frac{E_2^+(0)}{\eta_2} e^{-j\gamma_2 z} \vec{u}_y$$

conservazione dei campi (cond. al cont.)

$$\begin{cases} E_1^+(0) + E_1^-(0) = E_2^+(0) \\ H_1^+(0) + H_1^-(0) = H_2^+(0) \end{cases} \rightarrow \begin{cases} E_1^+(0) + E_1^-(0) = E_2^+(0) \\ \frac{E_1^+(0)}{\eta_1} - \frac{E_1^-(0)}{\eta_1} = \frac{E_2^+(0)}{\eta_2} \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} E_1^-(0) = E_1^+(0) \cdot \Gamma(0) \quad \text{con} \quad [\Gamma(0) = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}] \quad |\Gamma(0)| \leq 1 \\ E_2^+(0) = E_1^+(0) \cdot T(0) \quad \text{con} \quad [T(0) = \frac{2\eta_2}{\eta_2 + \eta_1}] = 1 + \Gamma(0) \quad |T(0)| \leq 2 \end{cases}$$

Γ COEFFICIENTE DI RIFLESSIONE

T COEFFICIENTE DI TRASMISSIONE

Campi totali nel mezzo 1

$$\begin{aligned} \bar{E}_1(z) &= \bar{E}_1^+(z) + \bar{E}_1^-(z) = (E_1^+(0) e^{-j\gamma_1 z} + E_1^-(0) e^{+j\gamma_1 z}) \vec{u}_x = \\ &= E_1^+(0) (e^{-j\gamma_1 z} + \Gamma(0) e^{+j\gamma_1 z}) \vec{u}_x \end{aligned}$$

$$\begin{aligned} \bar{H}_1(z) &= \bar{H}_1^+(z) + \bar{H}_1^-(z) = \\ &= \frac{E_1^+(0)}{\eta_1} (e^{-j\gamma_1 z} - \Gamma(0) e^{+j\gamma_1 z}) \vec{u}_y \end{aligned}$$

Campi totali nel mezzo 2

$$\bar{E}_2(z) = \bar{E}_2^+(z) = E_2^+(0) e^{-j\gamma_2 z} \vec{u}_x = E_1^+(0) \cdot T(0) e^{-j\gamma_2 z} \vec{u}_x$$

$$\bar{H}_2(z) = \bar{H}_2^+(z) = \frac{E_2^+(0)}{\eta_2} e^{-j\gamma_2 z} \vec{u}_y = \frac{E_1^+(0)}{\eta_2} \cdot T(0) e^{-j\gamma_2 z} \vec{u}_y$$

Ipotesi: mezzo 1 e mezzo 2 IDEALI $\left\{ \begin{array}{l} \sigma = 0 \\ \epsilon = \mu \end{array} \right.$ REAUX

$$\gamma_1 = j\beta_1 \quad \gamma_2 = j\beta_2 \quad (\alpha_1 = 0, \alpha_2 = 0)$$

COEFFICIENTE DI RIFLESSIONE per qualsiasi z

$$[\Gamma(z) = \frac{E_i^+(z)}{E_i^-(z)} = \frac{E_i^-(0)e^{+j\beta z}}{E_i^+(0)e^{-j\beta z}} = \Gamma(0)e^{2j\beta z}]$$

Definiamo IMPEDENZA D'ONDA (nella sezione z)

$$Z(z) = \frac{E_i(z)}{H_i(z)} = \frac{E_i^+(z) + E_i^-(z)}{H_i^+(z) + H_i^-(z)} = \eta_1 \cdot \frac{e^{-j\beta z} + \Gamma(0)e^{+j\beta z}}{e^{-j\beta z} - \Gamma(0)e^{+j\beta z}}$$

$$[Z(z) = \eta_1 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)]$$

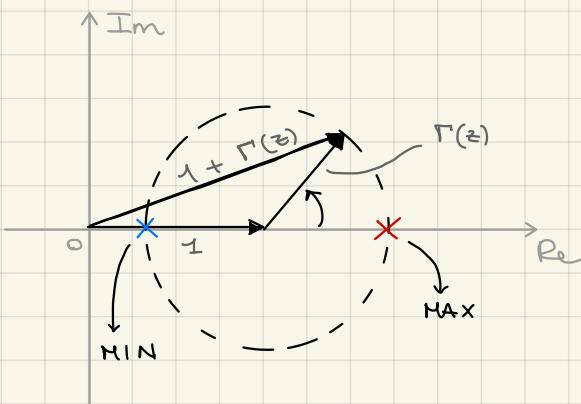
(da non confondere con l'IMPEDENZA INTRINSECA)

$$\eta_1 = \frac{E_i^+(z)}{H_i^+(z)} = - \frac{E_i^-(z)}{H_i^-(z)}$$

$$\rightarrow [E_i(z) = E_i^+(z) \underbrace{\left(1 + \Gamma(0)e^{+2j\beta z} \right)}_{\text{costante ?}}] \quad |E_i(z)| = |E_i^+(z)| \cdot |1 + \Gamma(z)| = |E_i^+(0)| \cdot |1 + \Gamma(z)|$$

↑
se il mezzo è ideale (senza perdite)

Piano dei fasci



Distanza fasciale fra due minimi (o due massimi):

$$2\beta\Delta z = 2\pi$$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \Delta z = 2\pi \rightarrow [\Delta z = \frac{\lambda}{2}]$$

$$|E_i|_{\text{MAX}} = |E_i^+(0)| \cdot (1 + |\Gamma(0)|)$$

$$|E_i|_{\text{MIN}} = |E_i^+(0)| \cdot (1 - |\Gamma(0)|)$$

$$|H_i(z)| = \frac{|E_i^+(0)|}{\eta_1} \cdot |1 - \Gamma(0)e^{2j\beta z}|$$

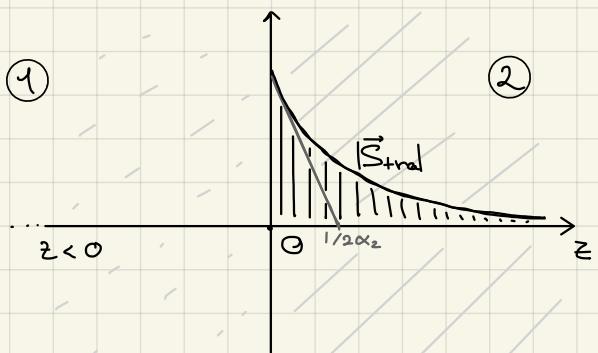
$$\vec{S}_m^+ = \vec{S}_{\text{inc}} = \frac{1}{2} \frac{|E_i^+(0)|^2}{\eta_1} \vec{u}_z \quad \vec{S}_m^- = \vec{S}_{\text{rif}} = - \frac{1}{2} \frac{|E_i^-(0)|^2}{\eta_2} \vec{u}_z \quad \left. \right\} \text{mezzo } \textcircled{1}$$

$$\vec{S}_{\text{tra}} = \frac{1}{2} \frac{|E_i^+(0)|^2}{\eta_2} \vec{u}_z = \frac{1}{2} \frac{|E_i^+(0)|^2}{\eta_2} |\Gamma|^2 \vec{u}_z \quad \left. \right\} \text{mezzo } \textcircled{2}$$

$$\vec{S}_{\text{rif}} = -\frac{1}{2} \frac{|\vec{E}_1^+(0)|^2}{\eta_1} |\Gamma(0)|^2 \vec{\mu}_z = -\vec{S}_{\text{inc}} |\Gamma(0)|^2 \quad \left. \right\} \text{mezzo } 1$$

$$\Rightarrow \text{Risulta } \vec{S}_{\text{inc}} + \vec{S}_{\text{rif}} = \vec{S}_{\text{tra}} \rightarrow \boxed{\vec{S}_{\text{tra}} = \vec{S}_{\text{inc}} (1 - |\Gamma(0)|^2)}$$

Perdite (mezzo 2) :



$$\begin{cases} \varepsilon_2 = \varepsilon_2' - j\varepsilon_2'' \\ \mu_2 = \mu_2' - j\mu_2'' \\ \alpha_2 \\ \beta_2 \end{cases}$$

$$\vec{E}_1^+(z) = E_1^+(0) e^{-k_1 z} = E_1^+(0) e^{-\alpha_1 z} e^{-j\beta_1 z}$$

$$\eta_2 = \sqrt{\frac{j\omega(\mu_2' - j\mu_2'')}{\alpha_2 + j\omega(\varepsilon_2' - j\varepsilon_2'')}} \in \mathbb{C}$$

$$\gamma_2 = \sqrt{j\omega(\mu_2' - j\mu_2'')[\alpha_2 + j\omega(\varepsilon_2' - j\varepsilon_2'')]}} = \alpha_2 + j\beta_2$$

$$\Gamma(0) = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \in \mathbb{C} \quad \Gamma(z) = 1 + \Gamma(0)$$

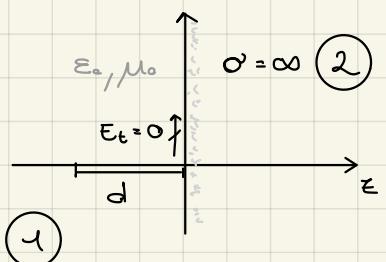
$$\begin{cases} \vec{E}_2(z) = E_1^+(0) \cdot \Gamma \cdot e^{-\alpha_2 z} e^{-j\beta_2 z} \vec{\mu}_x \\ \vec{H}_2(z) = \frac{E_1^+(0)}{\eta_2} \cdot \Gamma \cdot e^{-\alpha_2 z} e^{-j\beta_2 z} \vec{\mu}_y \end{cases}$$

$$\begin{aligned} \vec{S}_{\text{tra}} &= \frac{1}{2} \operatorname{Re} \left\{ \vec{E} \times \vec{H}^* \right\} \vec{\mu}_z = \text{nel mezzo 2} \\ &= \frac{1}{2} \operatorname{Re} \left\{ \Gamma \cdot E_1^+(0) e^{-\alpha_2 z} e^{-j\beta_2 z} \cdot \Gamma^* \frac{E_1^+(0)^*}{\eta_2^*} e^{-\alpha_2 z} e^{+j\beta_2 z} \right\} \vec{\mu}_z = \\ &= \frac{1}{2} \frac{|\Gamma|^2}{|\eta_2|} \cdot |E_1^+(0)|^2 e^{-2\alpha_2 z} \cos \varphi_\eta \vec{\mu}_z \quad (\text{con } \varphi_\eta = \Delta \eta_2) \end{aligned}$$

$$\boxed{\vec{S}_{\text{tra}}(z) = S_{\text{tra}}(0) e^{-2\alpha_2 z} \vec{\mu}_z}$$

Mezzo 2: conduttore perfetto ($\sigma = \infty$)

$$\eta_2 = 0 \Omega \quad \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1}$$



($\eta_1 = 377 \Omega$ mezzo 1 = vuoto)

$$T = 1 + \Gamma = 0 \rightarrow \vec{S}_{tra} = 0 \quad \vec{S}_{rif} = -\vec{S}_{inc}$$

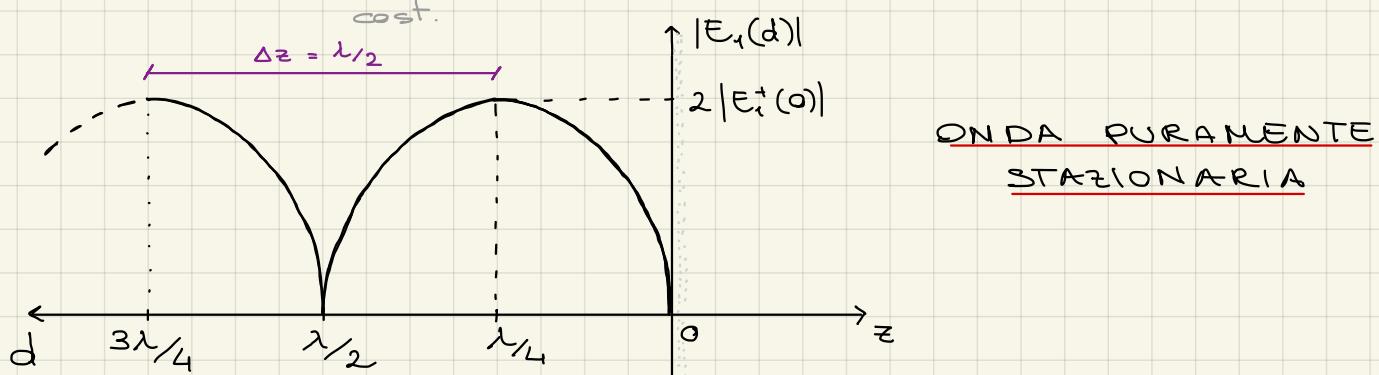
$$\begin{cases} E_i^+(z) = E_i^+(0) e^{-j\beta_1 z} \\ E_i^-(z) = -E_i^+(0) e^{+j\beta_1 z} \end{cases} \quad E_i^-(z) = E_i^+(0) e^{+j\beta_1 z}$$

$$E_i(z) = E_i^+(z) + E_i^-(z) = E_i^+(0) (e^{-j\beta_1 z} - e^{+j\beta_1 z})$$

$d = -z$ distanza dall'interfaccia

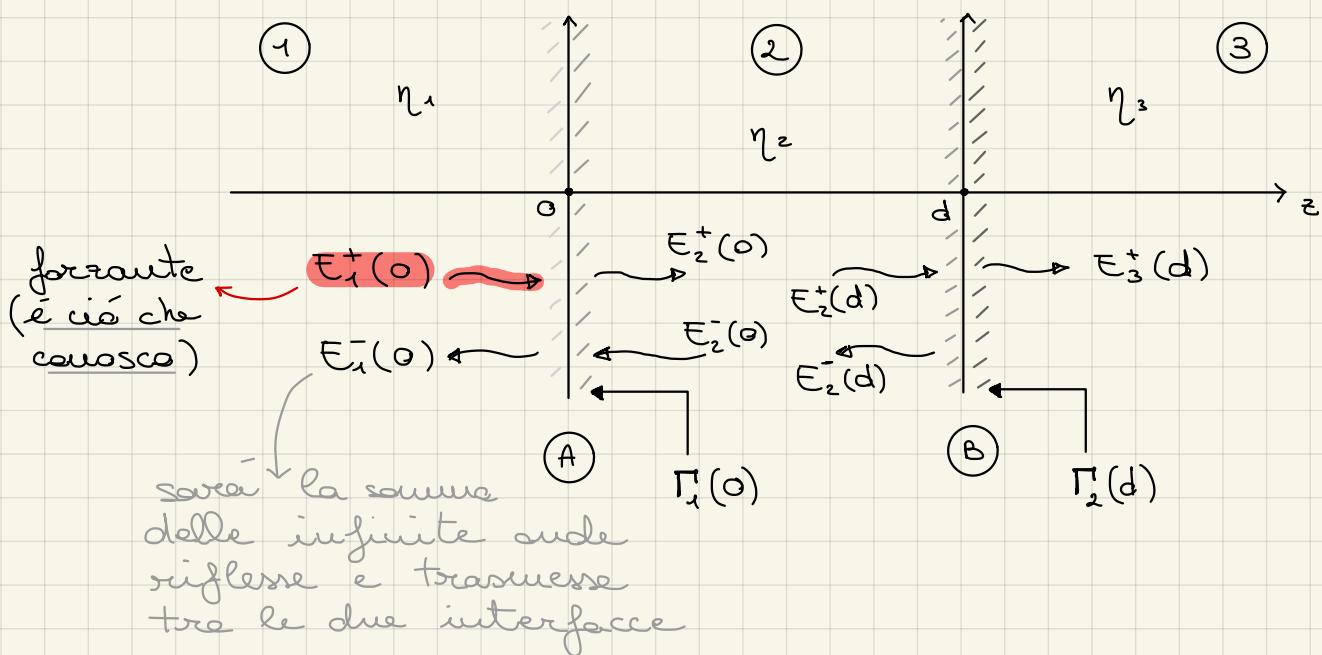
$$E_i(d) = E_i^+(0) \cdot [e^{+j\beta_1 d} - e^{-j\beta_1 d}] = E_i^+(0) \cdot 2j \operatorname{sen}(\beta_1 d)$$

$$|E_i(d)| = 2 \underbrace{|E_i^+(0)|}_{\text{cost.}} |\operatorname{sen}(\beta_1 d)|$$



$$\text{MAX: } \beta d = \frac{\pi}{2} \quad \frac{2\pi}{\lambda} d = \frac{\pi}{2} \quad \boxed{d = \frac{\lambda}{4}} \quad |E_i(d)| = 2 |E_i^+(0)|$$

Incidenza normale su multistrato (piana)



All' interfaccia (B): ($\varepsilon = d$)

$$\left\{ \begin{array}{l} E_2^-(d) = E_2^+(d) \Gamma_2^-(d) \\ E_3^+(d) = E_2^+(d) \Gamma_2^+(d) \end{array} \right. \text{ con } \left\{ \begin{array}{l} \Gamma_2^-(d) = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \\ \Gamma_2^+(d) = 1 + \Gamma_2^-(d) = \frac{2\eta_3}{\eta_3 + \eta_2} \end{array} \right.$$

(é come se fosse il caso già studiato di una superficie singola piana)

All' interfaccia (A): ($z = 0$)

$$\left\{ \begin{array}{l} E_1^+(0) + E_1^-(0) = E_2^+(0) + E_2^-(0) \\ H_1^+(0) + H_1^-(0) = H_2^+(0) + H_2^-(0) \end{array} \right.$$

A dx: $E_2^+(0) = E_2^+(d) e^{k_2 d} \leftarrow E_2^+(d) = E_2^+(0) e^{-k_2 d}$

$$(\varepsilon = d) \quad E_2^-(0) = E_2^-(d) e^{-k_2 d} = E_2^+(d) \Gamma_2^-(d) e^{-k_2 d} \\ = E_2^+(0) e^{-k_2 d} \Gamma_2^-(d) e^{-k_2 d}$$

A sx: $E_1^+(0), \quad E_1^-(0) = E_1^+(0) \Gamma_1^-(0) \quad \text{con} \quad \Gamma_1^-(0) \neq \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

conservazione dei campi $\left\{ \begin{array}{l} E_1^+(0) (\gamma + \Gamma_1^-(0)) = E_2^+(0) + E_2^-(0) e^{-k_2 d} \Gamma_2^-(d) e^{-k_2 d} \\ \frac{E_1^+(0)}{\eta_1} (\gamma - \Gamma_1^-(0)) = \frac{E_2^+(0)}{\eta_2} (\gamma - \Gamma_2^-(0)) \end{array} \right.$

$$\rightarrow \eta_1 \frac{\gamma + \Gamma_1^-(0)}{\gamma - \Gamma_1^-(0)} = \frac{\gamma + \Gamma_2^-(0)}{\gamma - \Gamma_2^-(0)} \eta_2 \rightarrow \Gamma_1^-(0) = \frac{Z_{2,A} - \eta_1}{Z_{2,A} + \eta_1}$$

$Z_{2,A}$ impedenza d'onda nel mezzo (2) in $z = 0$

$$E_1^-(0) = E_1^+(0) \Gamma_1^-(0) \quad \text{e} \quad E_2^+(0) = E_1^+(0) \cdot \frac{1 + \Gamma_1^-(0)}{1 + \Gamma_2^-(0)}$$

Nel mezzo (1): ($\varepsilon < 0$)

$$\left\{ \begin{array}{l} E_1(z) = E_1^+(0) e^{-j\beta_1 z} + E_1^+(0) \Gamma_1^-(0) e^{+j\beta_1 z} \\ H_1(z) = \frac{E_1^+(0)}{\eta_1} e^{-j\beta_1 z} - \frac{E_1^+(0) \Gamma_1^-(0)}{\eta_1} e^{+j\beta_1 z} \end{array} \right.$$

Nel mezzo ② : ($0 < z < d$)

$$\left\{ \begin{array}{l} E_2(z) = E_2^+(0) e^{-\gamma_2 z} + E_2^-(0) e^{+\delta_2 z} \\ H_2(z) = \frac{E_2^+(0)}{\eta_2} e^{-\gamma_2 z} - \frac{E_2^-(0)}{\eta_2} e^{+\delta_2 z} \end{array} \right.$$

Nel mezzo ③ : ($z > d$)

$$\left\{ \begin{array}{l} E_3(z) = E_3^+(d) e^{-\gamma_3(z-d)} \\ H_3(z) = \frac{E_3^+(d)}{\eta_3} e^{-\gamma_3(z-d)} \end{array} \right.$$

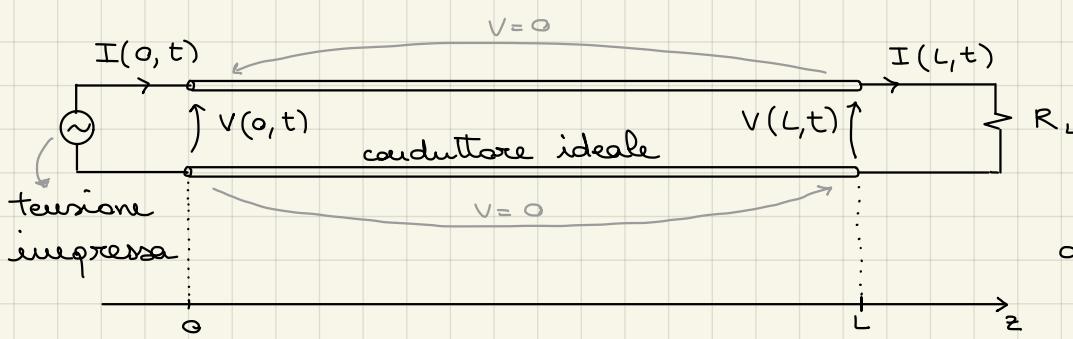
$$\eta = \frac{E^+}{H^+} = - \frac{E^-}{H^-} \text{ impedenza intrinseca}$$

$$Z = \frac{E}{H} = \frac{E^+ + E^-}{H^+ + H^-} \text{ impedenza d'onda}$$

in particolare nel mezzo ③ : $\eta_3 = Z_{3,0}$

Linee di Trasmissione

[TEM]



$$\left\{ \begin{array}{l} V(0,t) \neq V(L,t) \\ I(0,t) \neq I(L,t) \end{array} \right.$$

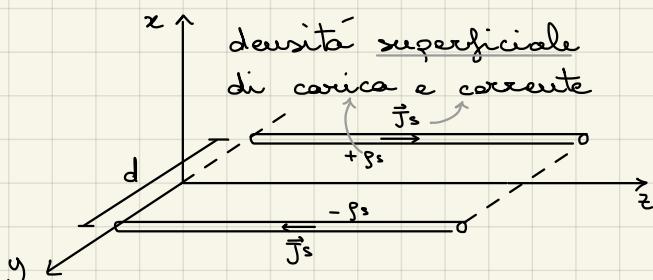
$$\text{se } L \geq \lambda$$

a causa dell'induzione elettromagnetica

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt} \neq 0$$

Come procederemo :

- 1) Dimostreremo che \vec{H} e \vec{E} sono "quasi-stazionari" sul piano trasverso ($\perp z$) \rightarrow definizione di V e I
- 2) Utilizzando Maxwell otterremo due equazioni per V e I \rightarrow eq. delle onde
- 3) Analisi nel dominio del tempo
- 4) " " " dei fasori



Hp: conduttori ideali
dielettrico ideale

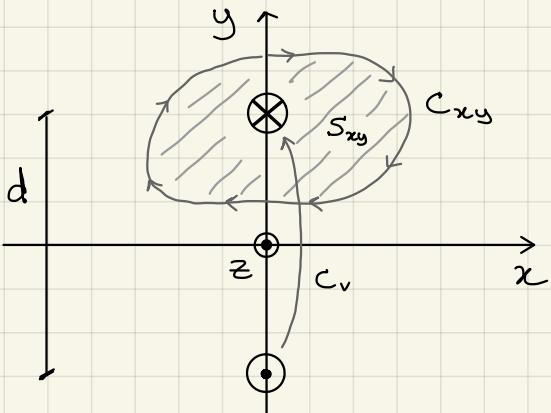
$$d \ll \lambda$$

In statica e se \mathbf{g}_s e $\vec{\mathbf{j}}_s$ fossero costanti con z :

$\vec{\mathbf{E}}$ e $\vec{\mathbf{H}}$ \perp asse z $\rightarrow \nabla E_z, H_z$

(TEM)

\mathbf{g}_s e $\vec{\mathbf{j}}_s$ variano "lentamente" $\rightarrow \vec{\mathbf{E}}_z, H_z$ sono trascurabili



$$\oint_{C_{xy}} \vec{\mathbf{E}} \cdot d\vec{l} = - \frac{\partial}{\partial t} \int_{S_{xy}} \vec{\mathbf{B}} \cdot d\vec{s}$$

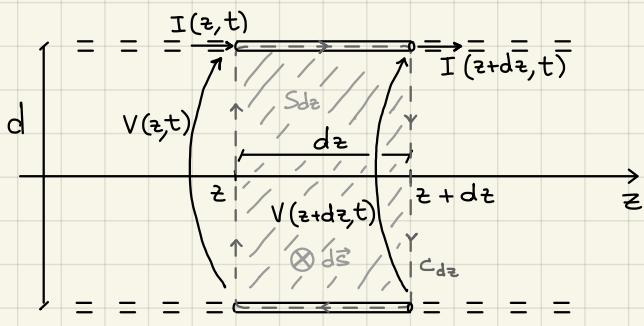
diretto come \vec{n}

$$\oint_{C_{xy}} \vec{\mathbf{H}} \cdot d\vec{l} = \frac{\partial}{\partial t} \int_{S_{xy}} \vec{\mathbf{D}} \cdot d\vec{s} + \int_{S_{xy}} \vec{\mathbf{j}} \cdot d\vec{s}$$

ma $\vec{\mathbf{B}} \cdot d\vec{s} = B_z \approx 0$
 $\vec{\mathbf{D}} \cdot d\vec{s} = D_z \approx 0$

$$\Rightarrow \oint_{C_{xy}} \vec{\mathbf{E}} \cdot d\vec{l} = 0 \quad \boxed{\oint_{C_{xy}} \vec{\mathbf{H}} \cdot d\vec{l} = \int_{S_{xy}} \vec{\mathbf{j}} \cdot d\vec{s} = I(z, t)}$$

$$\boxed{V(z, t) = - \int_{C_v} \vec{\mathbf{E}} \cdot d\vec{l}}$$



dz "piccole"

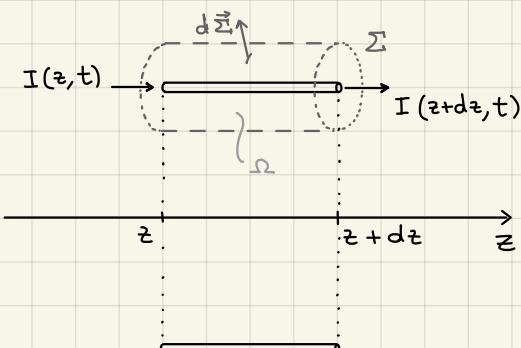
$$V(z + dz, t) \approx V(z, t) + \frac{\partial V(z, t)}{\partial z} dz + \dots$$

$$\oint_{C_{dz}} \vec{\mathbf{E}} \cdot d\vec{l} = - V(z, t) + 0 + V(z + dz, t) + 0 = \frac{\partial V(z, t)}{\partial z} dz$$

ma $\oint_{C_{dz}} \vec{\mathbf{E}} \cdot d\vec{l} = - \frac{\partial}{\partial t} \int_{S_{dz}} \vec{\mathbf{B}} \cdot d\vec{s} = - \frac{\partial}{\partial t} \Phi_u(z, t)$

ma $\Phi_u(z, t) \approx I(z, t) L dz$ (L : induttanza specifica del circuito $\left[\frac{H}{m}\right]$)

$\rightarrow \boxed{\frac{\partial V(z, t)}{\partial z} = - L \frac{\partial I(z, t)}{\partial t}} \quad (1)$



$$\oint_{\Sigma} \vec{\mathbf{j}} \cdot d\vec{\Sigma} = - \frac{\partial}{\partial t} \int_{\Omega} g_s d\Omega$$

$$I(z + dz, t) = I(z, t) + \frac{\partial I(z, t)}{\partial z} dz + \dots$$

$$\oint_{\Sigma} \vec{J} \cdot d\vec{\Sigma} = I(z + dz, t) - I(z, t) = \frac{\partial I(z, t)}{\partial z} \cdot dz = -\frac{\partial}{\partial t} [q(z, t) dz]$$

Ricordando che: $q(z, t) = \epsilon \cdot V(z, t)$ carica per u.l. $\left[\frac{C}{m}\right]$

$$\rightarrow \boxed{\frac{\partial I(z, t)}{\partial z} = -\epsilon \frac{\partial V(z, t)}{\partial t}} \quad (2) \quad (\epsilon: \text{capacità specifica} \left[\frac{F}{m}\right])$$

Derivando (1) rispetto a z e (2) rispetto a t :

$$\frac{\partial^2 V(z, t)}{\partial z^2} = -L \frac{\partial}{\partial z} \frac{\partial}{\partial t} I(z, t)$$

$$\frac{\partial}{\partial t} \frac{\partial I(z, t)}{\partial z} = -\epsilon \frac{\partial^2 V(z, t)}{\partial t^2}$$

Sostituendo:

$$\boxed{\frac{\partial^2 V(z, t)}{\partial z^2} = LC \frac{\partial^2 V(z, t)}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 I(z, t)}{\partial z^2} = LC \frac{\partial^2 V(z, t)}{\partial t^2}}$$

\rightarrow Ricordano le equazioni delle onde di \vec{E} e \vec{H} !

$$\begin{cases} V(z, t) = V^+(t - \frac{z}{v}) + V^-(t + \frac{z}{v}) \\ I(z, t) = I^+(t - \frac{z}{v}) + I^-(t + \frac{z}{v}) \end{cases} \quad \text{con } v = \frac{1}{\sqrt{LC}} \left[\frac{m}{s} \right]$$

$$LC = \frac{1}{v^2} \rightarrow \frac{H}{m} \cdot \frac{F}{m} = \frac{V \cdot S}{A} \frac{1}{m} \cdot \frac{As}{v} \frac{1}{m} = \frac{s^2}{m^2} \quad \checkmark$$

IMPEDENZA CARATTERISTICA Z_c

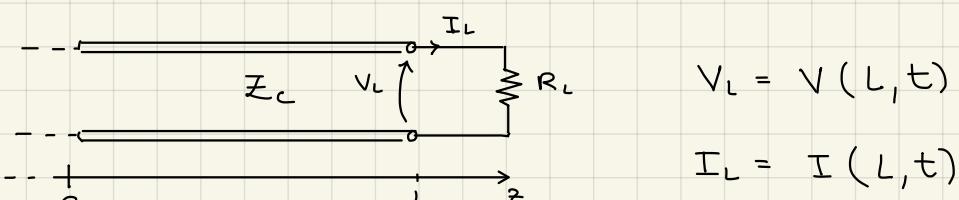
$$Z_c = \frac{V^+(z, t)}{I^+(z, t)} = -\frac{V^-(z, t)}{I^-(z, t)} \quad [\Omega]$$

$$\frac{\partial}{\partial z} \left[V^+(t - \frac{z}{v}) + V^-(t + \frac{z}{v}) \right] = -L \frac{\partial}{\partial t} \left[I^+(t - \frac{z}{v}) + I^-(t + \frac{z}{v}) \right]$$

$$-\frac{1}{v} V^+(t - \frac{z}{v}) + \frac{1}{v} V^-(t + \frac{z}{v}) = \quad " \quad " \quad " \quad "$$

$$\frac{1}{vL} V^+(t - \frac{z}{v}) - \frac{1}{vL} V^-(t + \frac{z}{v}) = I^+(t - \frac{z}{v}) + I^-(t + \frac{z}{v}) + \cancel{C} \quad \text{statica, non ci interessa}$$

$$Z_c = \frac{V^+}{I^+} = -\frac{V^-}{I^-} = v \cdot L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} \quad (\underline{\text{reale}}), \quad V^+ e I^+ \text{ in fase}$$



$$\left\{ \begin{array}{l} V(L,t) = V^+(L,t) + V^-(L,t) \\ I(L,t) = I^+(L,t) + I^-(L,t) \end{array} \right.$$

ma

$$\frac{V^+}{I^+} = Z_c = - \frac{V^-}{I^-}$$

$$V(L,t) = R_L \cdot I(L,t)$$

$$\rightarrow \left\{ \begin{array}{l} V^+(L,t) + V^-(L,t) = Z_c (I^+(L,t) - I^-(L,t)) \\ V^+(L,t) + V^-(L,t) = R_L (I^+(L,t) + I^-(L,t)) \end{array} \right.$$

simultaneamente valide

- $R_L = Z_c \rightarrow I^-(L,t) = 0 \rightarrow V^-(L,t) = 0$ (carico adattato)
 ↪ NO onda riflessa ↪

- $R_L \neq Z_c \rightarrow V^+(L,t) + V^-(L,t) = R_L \left[\frac{V^+(L,t)}{Z_c} - \frac{V^-(L,t)}{Z_c} \right]$

Si definisce COEFFICIENTE DI RIFLESSIONE:

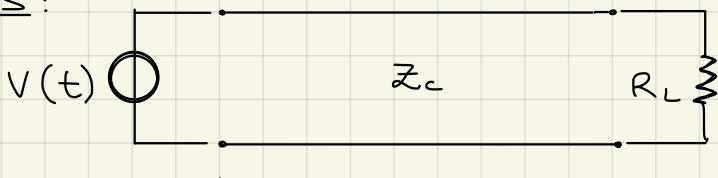
$$\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)}$$

$$\rightarrow V^+(L,t) \left(\frac{R_L}{Z_c} - 1 \right) = V^-(L,t) \left(1 + \frac{R_L}{Z_c} \right)$$

$$\left[\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)} = \frac{R_L - Z_c}{R_L + Z_c} \right] \quad (-1 \leq \Gamma_L \leq 1)$$

$$\Gamma_L = 0 \text{ se } R_L = Z_c$$

Ese:

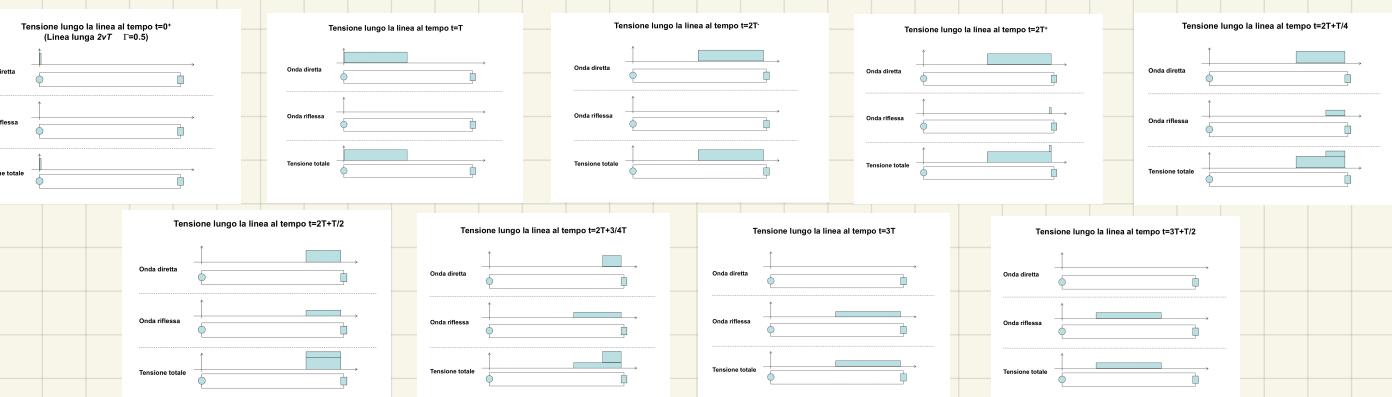


$$Z_c = 50 \Omega$$

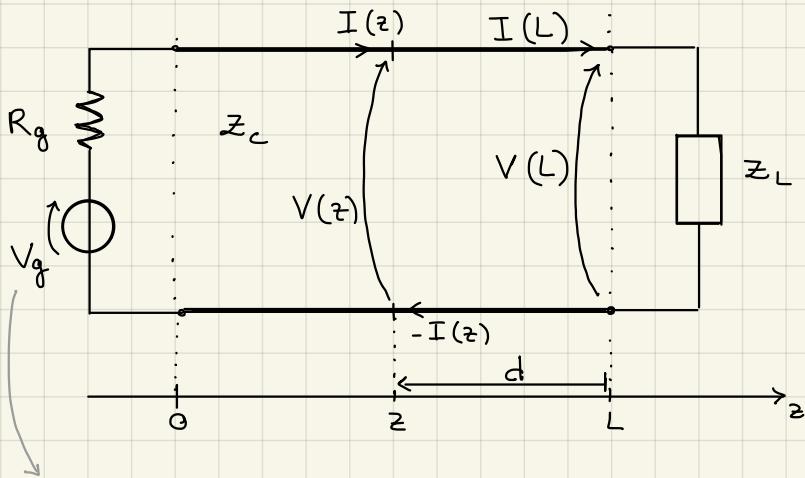
$$R_L = 150 \Omega$$

$$L = 2 \Omega \cdot T$$

$$\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)} = \frac{150 - 50}{150 + 50} = \frac{1}{2}$$



Regime sinusoidale stazionario (fasi)



$$\frac{d^2 V(z)}{dz^2} = -\frac{\omega^2 L C}{\beta^2} V(z)$$

$$\frac{d^2 I(z)}{dz^2} = -\frac{\omega^2 L C}{\beta^2} I(z)$$

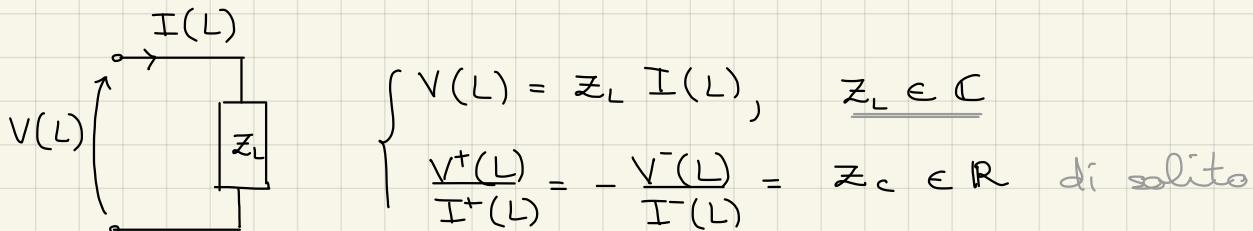
con $LC = \frac{1}{\omega^2}$ e $\frac{\omega^2}{\omega^2} = \beta^2$
 $(\beta = \frac{2\pi}{\lambda}$ costante di fase)

fase: $V_g(t) = |V_g| \cos(\omega t + \varphi) = \underbrace{\text{Re}\{|V_g| e^{j\varphi} e^{j\omega t}\}}_{V_g}$

$$\left\{ \begin{array}{l} V(z) = V^+(0) e^{-j\beta z} + V^-(0) e^{+j\beta z} \\ I(z) = I^+(0) e^{-j\beta z} + I^-(0) e^{+j\beta z} \end{array} \right. \quad (\text{fasi})$$

$$\left\{ \begin{array}{l} V(z, t) = |V^+(0)| \cos(\omega t - \beta z + \varphi_{V^+}) + |V^-(0)| \cos(\omega t + \beta z + \varphi_{V^-}) \\ I(z, t) = |I^+(0)| \cos(\omega t - \beta z + \varphi_{I^+}) + |I^-(0)| \cos(\omega t + \beta z + \varphi_{I^-}) \end{array} \right. \quad (\text{tempo})$$

Usare i fasi ci permette di studiare carichi diversi dalla sola resistenza, come induttori e capacità



$$V^+(L) + V^-(L) = Z_L [I^+(L) + I^-(L)]$$

$$\text{e poi } V^+(L) - V^-(L) = \frac{Z_L}{Z_C} [V^+(L) - V^-(L)]$$

Definiamo $\Gamma_L = \frac{V^-(L)}{V^+(L)} = \frac{Z_L - Z_C}{Z_L + Z_C}$ (complesso)

$$|\Gamma_L| \leq 1 \quad (\text{con carichi } Z_L \text{ passivi})$$

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{V^-(0) e^{+j\beta z}}{V^+(0) e^{-j\beta z}} = \Gamma(0) e^{2j\beta z} = \Gamma_L e^{-2j\beta(L-z)}$$

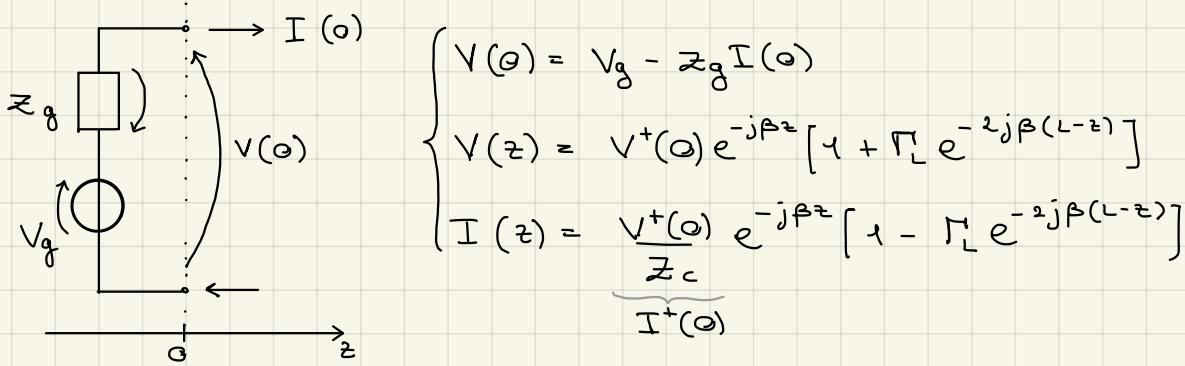
↑
 essendo $\Gamma_L = \frac{V^-(L)}{V^+(L)} = \frac{V^-(0)}{V^+(0)} e^{j2\beta L}$

$$\rightarrow \Gamma(d) = \Gamma_L e^{-2j\beta d} \quad (d = L - z)$$

Tensione (e corrente) lungo la linea:

$$V(z) = V^+(z) + V^-(z) = V^+(z) + V^+(z)\Gamma(z) = V^+(z) [1 + \Gamma(z)]$$

$$= V^+(0) e^{-j\beta z} [1 + \Gamma(z)]$$



$$\Rightarrow V^+(0) (1 + \Gamma_L e^{-2j\beta L}) = V_g - Z_g \frac{V^+(0)}{Z_c} (1 - \Gamma_L e^{-2j\beta L})$$

Hp: $Z_g = Z_c$ (generatore adattato)

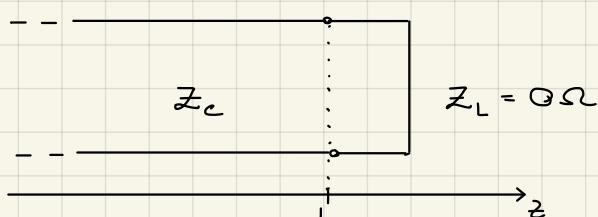
$$V^+(0) = \frac{V_g}{2}$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z} [1 + \Gamma_L e^{-2j\beta(L-z)}] =$$

$$= \frac{V_g}{2} e^{-j\beta z} + \underbrace{\left(\frac{V_g}{2} \Gamma_L e^{-2j\beta L} \right)}_{\text{ONDÀ DIRETTA}} e^{+j\beta z} + \underbrace{\left(\frac{V_g}{2} \Gamma_L e^{-2j\beta L} \right)}_{\text{ONDÀ RIFLESSA}} e^{-j\beta z}$$

$$= V^+(0) e^{-j\beta z} + V^-(0) e^{+j\beta z}$$

Inveluppo



Cortocircuito

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = -1$$

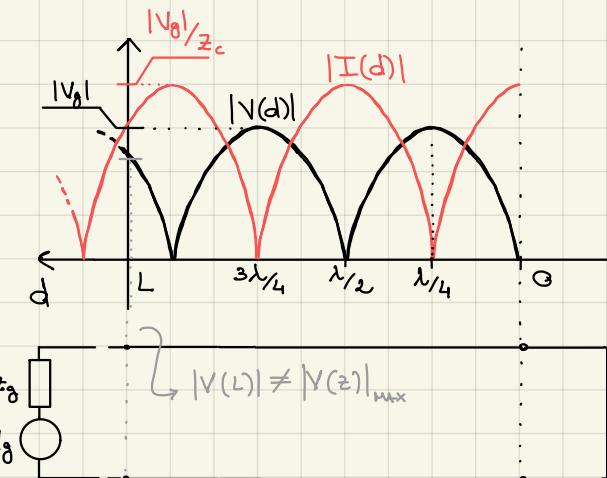
$$V(z) = \frac{V_g}{2} e^{-j\beta z} [1 - e^{-j2\beta(L-z)}]$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z} [e^{j\beta(L-z)} - e^{-j\beta(L-z)}]$$

$$|V(z)| = \left| \frac{V_g}{2} \right| \cdot |j2 \sin[\beta(L-z)]| = |V_g| \cdot |\sin[\beta(L-z)]|$$

$$\rightarrow |V(d)| = |V_g| \cdot |\sin \beta d|$$

in $z = L$ ($d = 0$) $\rightarrow |V(L)| = 0$
(come ci si aspetta da un cortocircuito)



$$\text{In } z = L \rightarrow I(L) = \frac{|V_g|}{Z_c}$$

$$\text{MAX: } \beta d_n = (2n+1) \frac{\pi}{2} = \frac{2\pi}{\lambda} d_n \rightarrow d_0 = \frac{\lambda}{4}$$

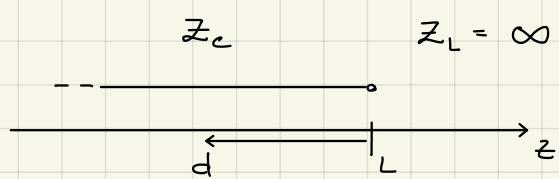
$$\text{MIN: } \beta d_n = n\pi \rightarrow d_1 = \frac{\lambda}{2}$$

$$|I(z)| = \frac{|V_g|}{Z_c} \cdot |\cos [\beta(L-z)]|$$

$$\rightarrow |I(d)| = \frac{|V_g|}{Z_c} |\cos \beta d|$$

Circuito aperto

$$\Gamma_L = 1$$



$$V(z) = \frac{V_g}{2} e^{-j\beta z} [1 + e^{-j2\beta(L-z)}] =$$

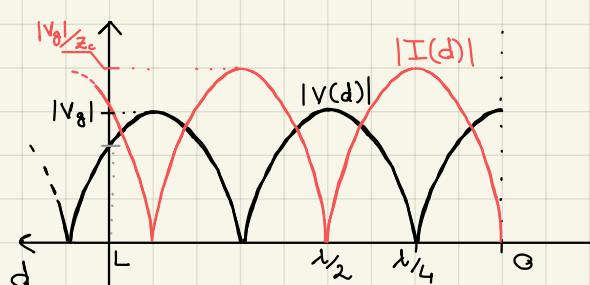
$$= \frac{V_g}{2} e^{-j\beta L} [e^{+j\beta(L-z)} + e^{-j\beta(L-z)}] =$$

$$= \frac{V_g}{2} e^{-j\beta L} \cdot 2 \cos [\beta(L-z)]$$

$$|V(z)| = |V_g| \cdot |\cos [\beta(L-z)]| \rightarrow |V(d)| = |V_g| |\cos \beta d|$$

$$|I(d)| = \frac{|V_g|}{Z_c} |\sin \beta d|$$

$$\text{in } d = \frac{\lambda}{4}: \begin{cases} |V(d)| = 0 \\ |I(d)| = \frac{|V_g|}{Z_c} \end{cases}$$



Carico adattato

$$Z_L = Z_c$$

$$\Gamma_L = 0$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z}$$

$$|V(z)| = \frac{|V_g|}{2}$$

Tensione e corrente costanti

Carichi puramente reattivi

$$Z_L = jX_L \quad X_L = \omega L$$

induttori

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{jX_L - Z_c}{jX_L + Z_c} \rightarrow |\Gamma_L| = 1$$

$$X_L = \frac{1}{\omega C}$$

capacità

→ onda puramente stazionaria

Rapporto di Onda Stazionaria (ROS)
(Standing Wave Ratio - SWR)

$$ROS = \frac{|V(d)|_{\max}}{|V(d)|_{\min}} = \frac{|V^+(0)| [1 + |\Gamma(z)|]}{|V^+(0)| [1 - |\Gamma(z)|]} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \gg 1$$

carico adattato: $ROS = 1$ c.c., c.a. & reattivi: $ROS = \infty$

Rimossa l'ipotesi precedente: $Z_g \neq Z_c$

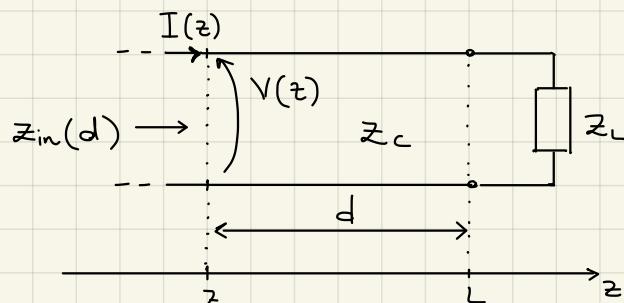
$$V^+(0) (1 + \Gamma_L e^{-2j\beta L}) = V_g - Z_g \cdot \frac{V^+(0)}{Z_c} (1 - \Gamma_L e^{-2j\beta L})$$

$$V^+(0) = \frac{V_g}{[1 + \Gamma_L e^{-2j\beta L} + \frac{Z_g}{Z_c} (1 - \Gamma_L e^{-2j\beta L})]}$$

$V^+(0)$ complessa (V_g e $V^+(0)$ non in fase)

$V^+(0)$ dipende dal carico Z_L (vs. Z_c)

Impedenza lungo la linea



Impedenza d'ingresso: $Z_{in}(d) = \frac{V(d)}{I(d)}$

$$Z_{in}(z) = \frac{V(z)}{I(z)} = \frac{V^+(0) e^{-j\beta z} (1 + \Gamma(z)}}{I^+(0) e^{-j\beta z} (1 - \Gamma(z))} = Z_c \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

$$Z_{in}(z) = Z_c \frac{1 + \Gamma_L e^{-2j\beta(L-z)}}{1 - \Gamma_L e^{-2j\beta(L-z)}}$$

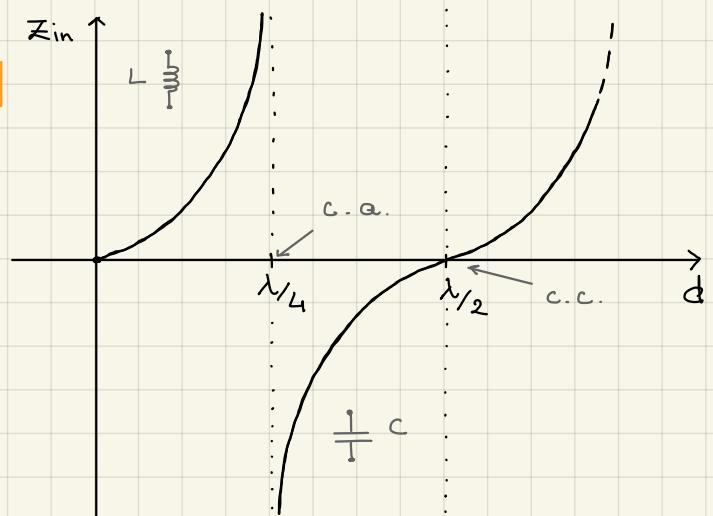
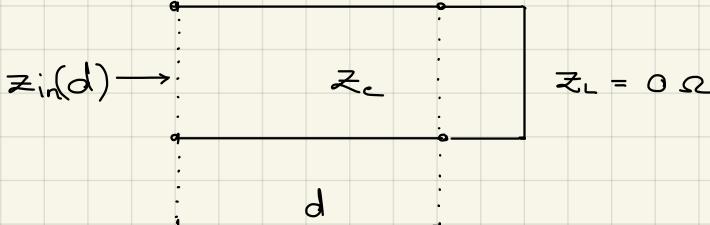
$$\Rightarrow Z_{in}(d) = Z_c \frac{1 + \Gamma_L e^{-2j\beta d}}{1 - \Gamma_L e^{-2j\beta d}}$$

$$(sappendo che e^{-j2\beta d} = \cos(2\beta d) - \sin(2\beta d)) \Rightarrow Z_{in}(d) = Z_c \frac{Z_L + jZ_c \tan(\beta d)}{Z_c + jZ_L \tan(\beta d)}$$

Nel caso di un cortocircuito:

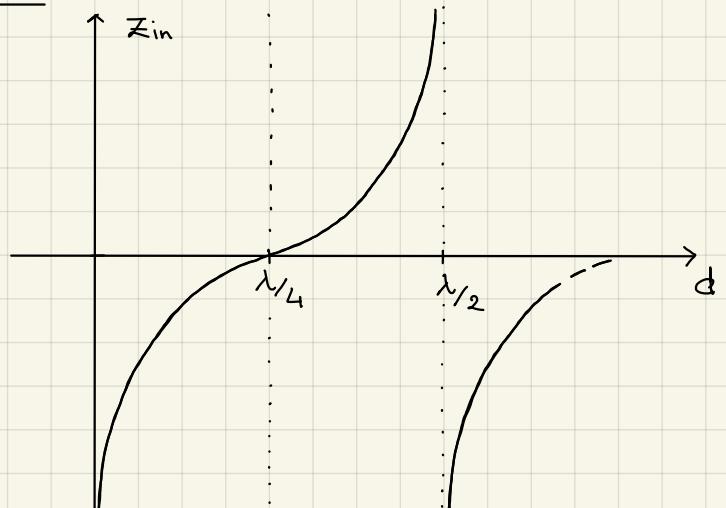
$$Z_{in}(d) = j Z_c \operatorname{tg}(\beta d) = j Z_c \operatorname{tg}\left(\frac{2\pi}{\lambda} d\right)$$

è totalmente reattiva!



Nel caso di un circuito aperto:

$$\begin{aligned} Z_{in}(d) &= -j \frac{Z_c}{\operatorname{tg}(\beta d)} = \\ &= -j Z_c \operatorname{cotg}(\beta d) \end{aligned}$$



Nel caso di carico adattato:

$$R_L^t = 0$$

$$Z_{in}(d) = Z_c$$

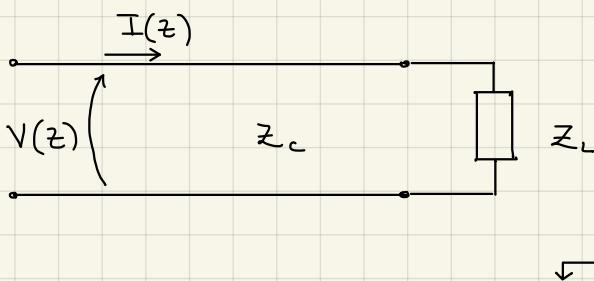


Trasformatore $\frac{\lambda}{4}$ o Invertitore di impedenza

$$d = \frac{\lambda}{4} \quad Z_L \neq \quad \rightarrow$$

$$Z_{in}(d) = \frac{Z_c^2}{Z_L}$$

Flusso di potenza lungo la linea



Poiché conosco tensione e corrente di una linea TEM, posso usarli per calcolare la densità di potenza (invece che usare il teorema di Poynting)

$$P_m = \frac{1}{2} \operatorname{Re} \{ \bar{V} \cdot \bar{I}^* \}$$

Ricordando che:

$$V(z) = V^+(0) e^{-j\beta z} [1 + \Gamma(z)] \quad \text{e} \quad I(z) = \frac{V^+(0)}{Z_c} e^{-j\beta z} [1 - \Gamma(z)]$$

$$\begin{aligned} \rightarrow P_m(z) &= \frac{1}{2} \operatorname{Re} \left\{ \frac{V^+(0) \cdot V^+(0)^*}{Z_c} [1 + \Gamma(z)][1 - \Gamma(z)^*] \right\} = \\ &= \frac{1}{2} \left| \frac{V^+(0)}{Z_c} \right|^2 \operatorname{Re} \left\{ 1 + \underbrace{\Gamma(z) + \Gamma(z)^* - |\Gamma(z)|^2}_{\text{immag.}} \right\} = \\ &= \frac{1}{2} \left| \frac{V^+(0)}{Z_c} \right|^2 (1 - |\Gamma_L|^2) \quad (\text{essendo } |\Gamma(z)| = |\Gamma_L|) \end{aligned}$$

$$\begin{aligned} P_m^+(z) &= \frac{1}{2} \operatorname{Re} \{ V^+(z) I^+(z)^* \} = \frac{1}{2} \operatorname{Re} \{ V^+(0) e^{-j\beta z} \frac{V^+(0)^*}{Z_c} e^{+j\beta z} \} \\ &= \frac{1}{2} \left| \frac{V^+(0)}{Z_c} \right|^2 \end{aligned}$$

$$P_m^-(z) = \frac{1}{2} \operatorname{Re} \{ V^-(z) I^-(z)^* \} = -\frac{1}{2} \left| \frac{V^+(0)}{Z_c} \right|^2 \cdot |\Gamma_L|^2$$

$$P_m(z) = P_m^+(z) + P_m^-(z)$$

Con dei carichi reattivi (C, L, c.c., c.a.):

$$|\Gamma_L| = 1 \rightarrow P_m(z) = 0$$

non posso trasferire potenza.

Carta di Smith

$$\begin{aligned} Z_{in}(d) &= Z_c \frac{1 + \Gamma(d)}{1 - \Gamma(d)} & \xrightarrow{\text{normalizzata}} \bar{Z}_{in} = \frac{Z_{in}}{Z_c} \\ \text{assume} \quad \bar{Z}_c &= 1 \rightarrow \bar{Z}_{in}(d) = \frac{1 + \Gamma(d)}{1 - \Gamma(d)} = \pi + j \times \end{aligned}$$

$$\Gamma(d) = p + jq$$

$$r + jx = \frac{1 + p + jq}{1 - p - jq} \quad r \text{ costante, } x \text{ costante}$$

$$\rightarrow \left(p - \frac{r}{r+1} \right)^2 + q^2 = \frac{1}{(r+1)^2} \quad (\text{eq. parte reale (1)})$$

$$\rightarrow \left(p - 1 \right)^2 + \left(q - \frac{1}{x} \right)^2 = \frac{1}{x^2} \quad (\text{eq. parte immaginaria (2)})$$

\rightarrow r costante \rightarrow la (1) è una circonferenza di
raggio $\frac{1}{r+1}$ e centro $(\frac{r}{r+1}, 0)$

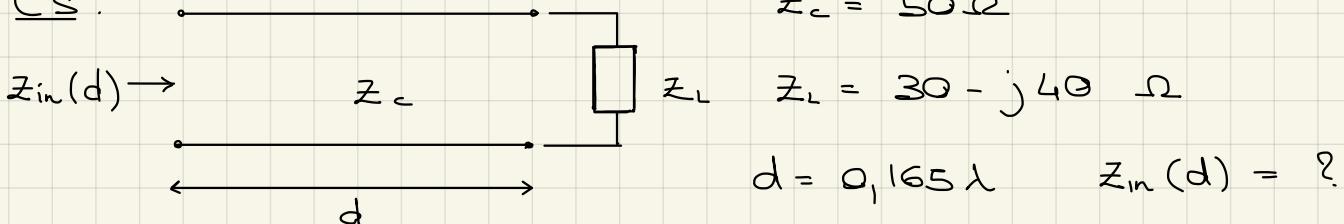
\rightarrow x costante \rightarrow la (2) è una circonferenza di
raggio $\frac{1}{x}$ e centro $(1, \frac{1}{x})$

\rightarrow Casi notevoli :

$$r = 0 \quad \bar{z} = jx \quad |\Gamma| = 1 \rightarrow \text{circonferenza esterna}$$

$$r = 1 \quad x = 0 \rightarrow \text{centro delle carte (cerchio adattato)}$$

Ese:



1) Normalizzare a Z_c

$$\bar{z}_L = \frac{z_L}{z_c} = \frac{30 - 40j}{50} = 0,6 - 0,8j$$

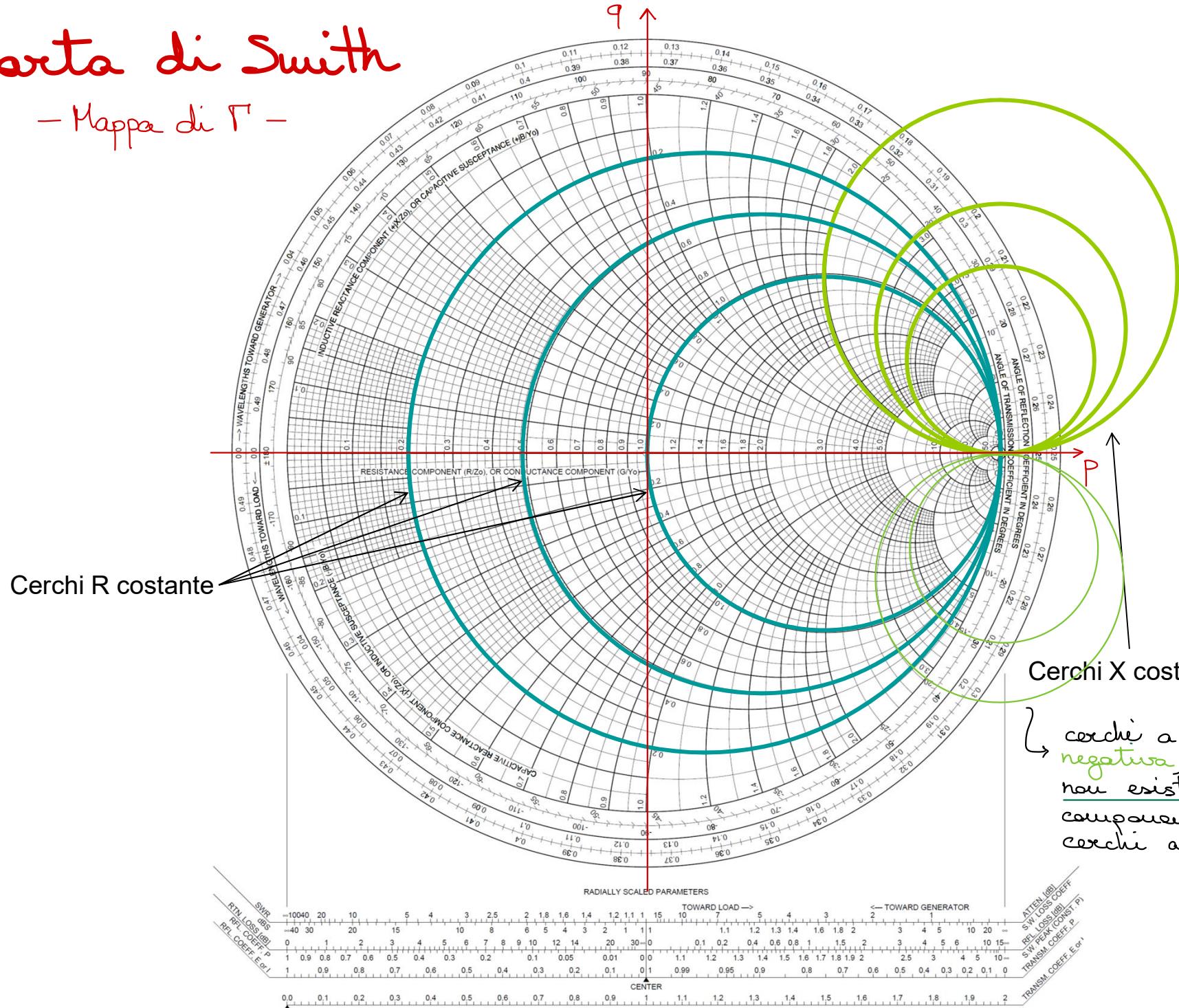
$$\Gamma_L = \frac{z_L - z_c}{z_L + z_c} = -j0,5$$

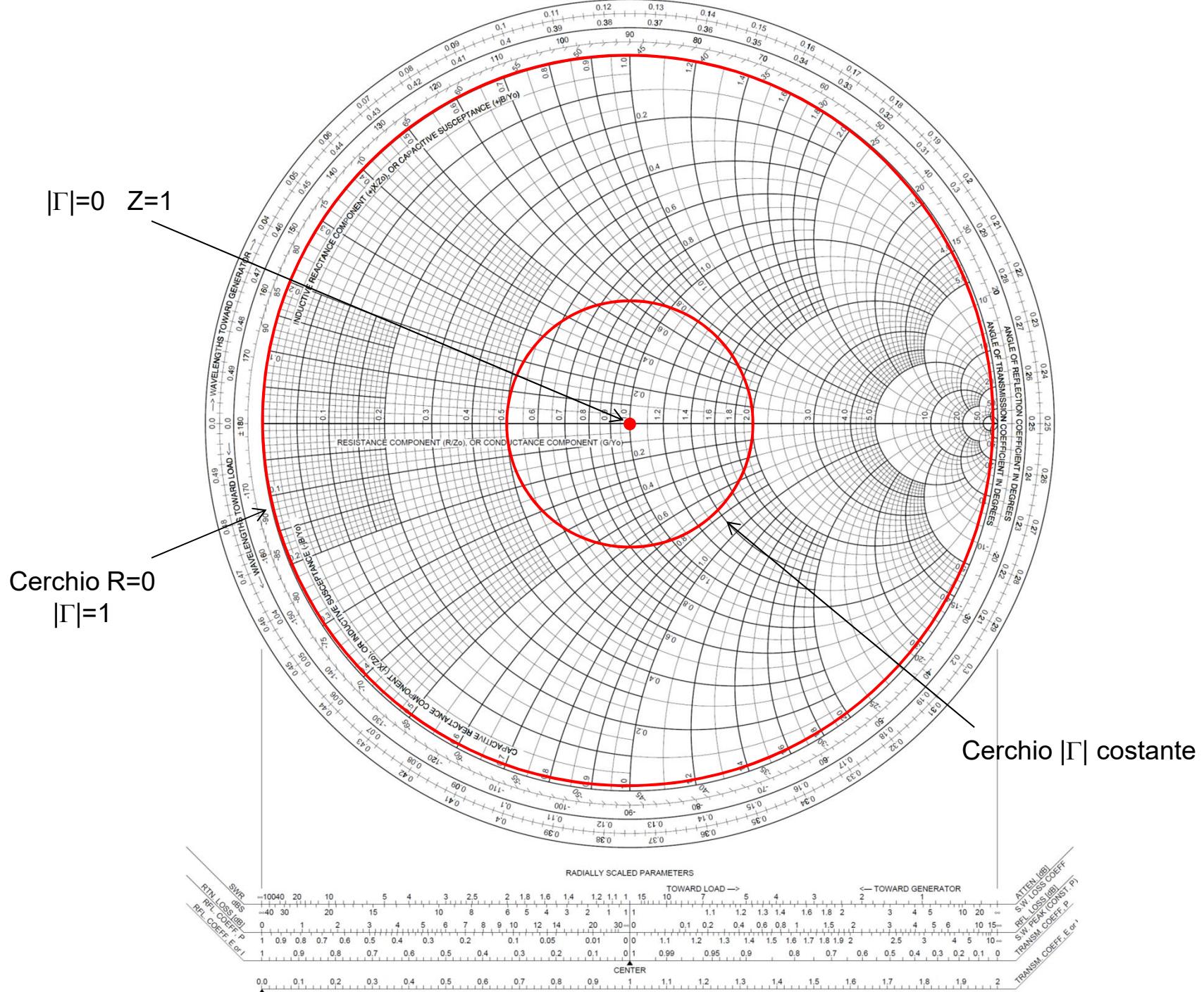
$$\Gamma(d) = \Gamma_L e^{-2j\beta d} \quad \text{ruota in senso orario}$$

$\text{di } \underline{2\beta d} = 2 \cdot \frac{2\pi}{\lambda} \cdot d = 0,66\pi$

Carta di Smith

- Mappa di Γ -

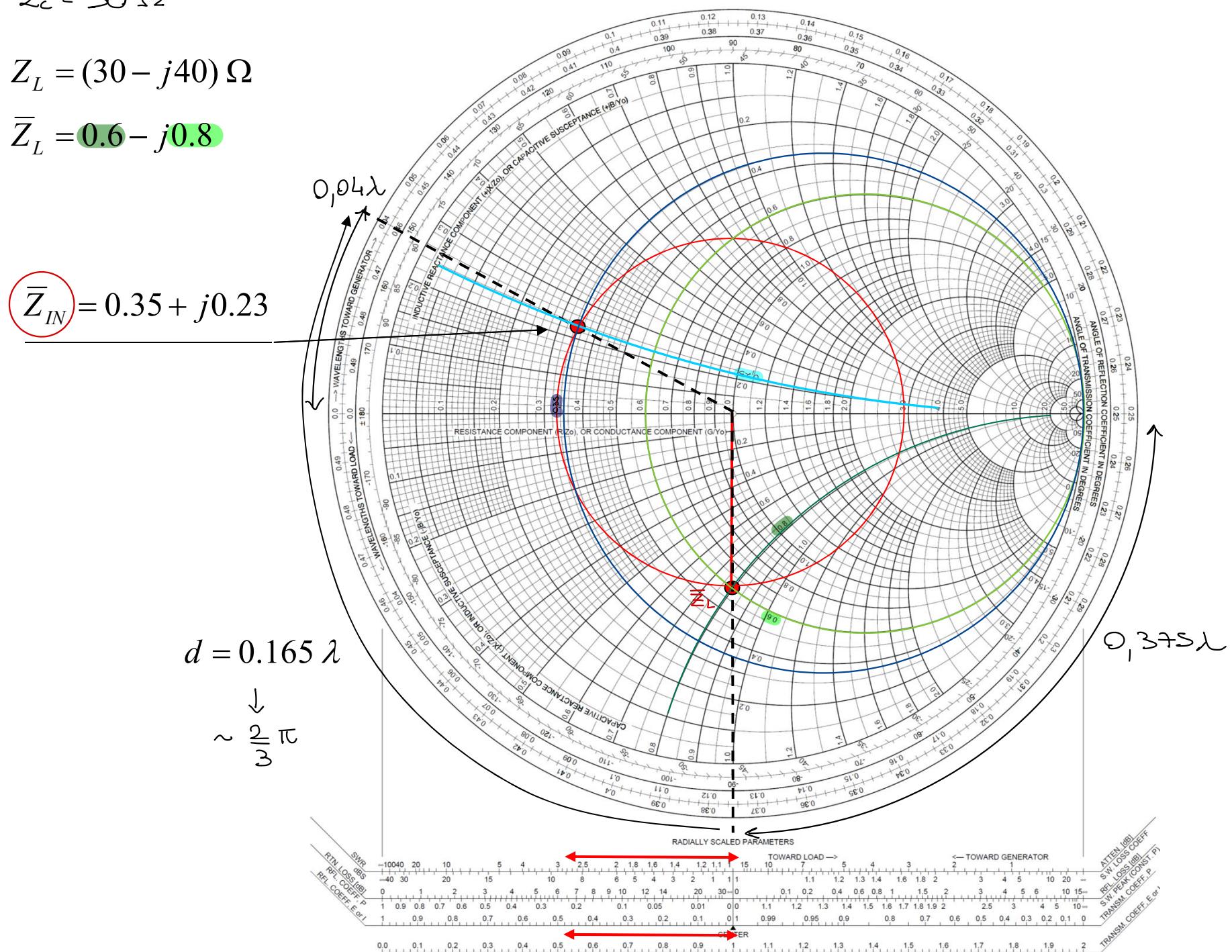




$$Z_c = 50 \Omega$$

$$Z_L = (30 - j40) \Omega$$

$$\bar{Z}_L = 0.6 - j0.8$$



Partiamo da $0,375\lambda$

$$\text{Finiamo in } 0,375\lambda + 0,165\lambda = 0,54\lambda \xrightarrow{d} 0,04\lambda$$

$$\bar{Z}_m = 0,35 + j0,23 \implies Z_m = \bar{Z}_m \cdot Z_c = 17,6 + j11,3 \Omega$$

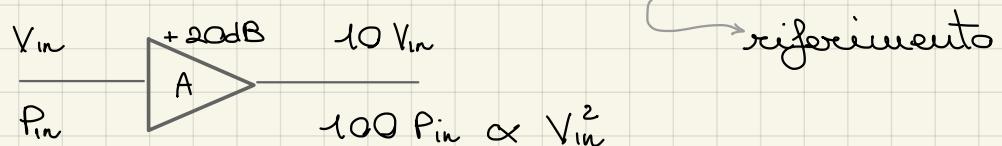
Approfondimenti:

dB e Np

Sono numeri pure - adimensionali - usati per esprimere rapporti.

Tensioni, correnti, campi: $20 \log_{10} \frac{V}{V_0}$ $20 \log_{10} \frac{E}{E_0}$

Potenza, densità di potenza: $10 \log_{10} \frac{P}{P_0}$ $10 \log_{10} \frac{S}{S_0}$



$$1 \text{ Np} \longleftrightarrow 8,686 \text{ dB}$$

$$20 \log_{10} \frac{V}{V_0} = \alpha_{dB} \quad \& \quad \frac{V}{V_0} = e^{\alpha_{Np}}$$

$$\implies \frac{V}{V_0} = 10^{\frac{\alpha_{dB}}{20}} = e^{\alpha_{Np}} \implies \ln 10^{\frac{\alpha_{dB}}{20}} = \alpha_{Np} = \underbrace{\frac{\alpha_{dB}}{20} \ln 10}_{2,302}$$
$$\implies \alpha_{Np} = 8,686 \alpha_{dB}$$

Potenza

$$\text{dB}_W \quad 30 \text{ dB}_W \longrightarrow 1000 \text{ mW}$$

$$\begin{aligned} \text{dB}_m &\quad 10 \text{ dB}_m \longrightarrow 10 \text{ mW} \\ &\quad -10 \text{ dBm} \longrightarrow 0,1 \text{ mW} \end{aligned}$$

Tensione

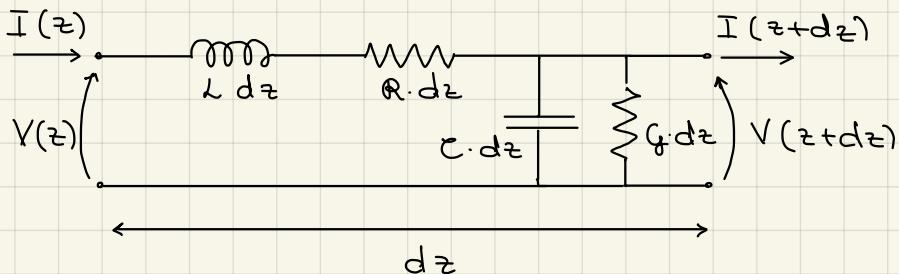
$$\text{dB}_V \quad 20 \text{ dB}_V \longrightarrow 10 \text{ V}$$

$$\text{dB}_{\mu V} \quad 0 \text{ dB}_{\mu V} \longrightarrow 1 \mu \text{V} (10^{-6} \text{ V})$$

Perdite nelle linee

- Perdite nei conduttori ($\sigma \neq \infty$)
 - Perdite nel dielettrico ($\epsilon = \epsilon' - j\epsilon''$ o $\mu = \mu' - j\mu''$)
- correnti
campo elettrico

Modello delle perdite:



L, R, C e G
sono grandezze
fisiche per
unità di
lunghezza

$$\begin{cases} V(z+dz) + I(z)(R + j\omega L)dz = V(z) \\ I(z) = V(z)(G + j\omega C)dz + I(z+dz) \end{cases}$$

chiamando $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\begin{cases} V(z) = V^+(0)e^{-\gamma z} + V^-(0)e^{+\gamma z} \\ I(z) = I^+(0)e^{-\gamma z} + I^-(0)e^{+\gamma z} \end{cases} \quad (\gamma = \alpha + j\beta)$$

$$Z_c = \frac{V^+(z)}{I^+(z)} = \frac{V^+(0)}{I^+(0)} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (\text{complessa})$$

Hp: piccole perdite $\rightarrow R \ll j\omega L$ e $G \ll j\omega C$

$$Z_c \approx \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \quad (\text{reale})$$

(buoni conduttori,
buone dielectrico)

$$\gamma = \sqrt{-\omega^2 LC + j\omega RC + j\omega L G + RG}$$

$$\approx \sqrt{-\omega^2 LC + j\omega(RC + LG)} = j\omega\sqrt{LC} \cdot \sqrt{1 - \frac{j(RC + LG)}{\omega LC}}$$

$$\sqrt{1+x} \quad (\text{con } x \ll 1) \approx 1 + \frac{x}{2}$$

$$\approx j\omega\sqrt{LC} \left(1 - j \frac{RC + LG}{2\omega LC} \right) = j\omega\sqrt{LC} + \underbrace{\frac{CR + LG}{2\sqrt{LC}}}_{j\beta + \alpha}$$

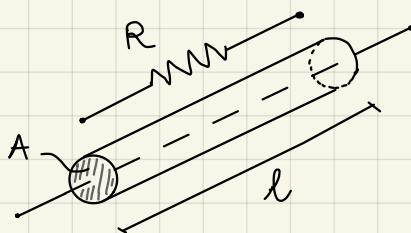
$$j\beta + \alpha$$

$$\beta = \omega \sqrt{Lc}$$

$$\alpha = \frac{R}{2Z_c} + \frac{\gamma Z_c}{2} \quad (\text{dove } Z_c = \sqrt{\frac{L}{C}})$$

attenuazione
legata alle perdite α_c
nel conduttore

attenuazione
 α_0 legata alle perdite
nel dielettrico



$$\sigma = \frac{1}{\tau}$$

In corrente continua ($f = 0$)

$$R = \frac{l}{\sigma \cdot A}$$

non è applicabile
se $f \neq 0$

Effetto pelle

$$\sigma \vec{E} = \vec{J}$$

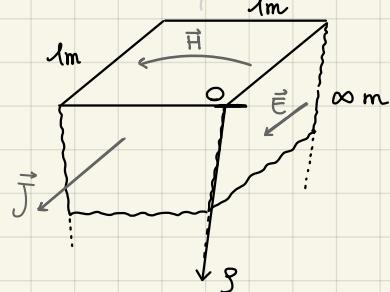
$$\vec{J}(g) = \sigma \vec{E}(g)$$

$$E(g) = E(0) e^{-\gamma g} = E(0) e^{-\alpha g} e^{-j\beta g}$$

$$\text{con } \alpha = \beta = \frac{1}{\delta} \quad (\text{nel metallo})$$

Calcoliamo la $P_{diss} = \frac{1}{2} |I|^2 R$ (blocco di

1m x 1m x ∞ m

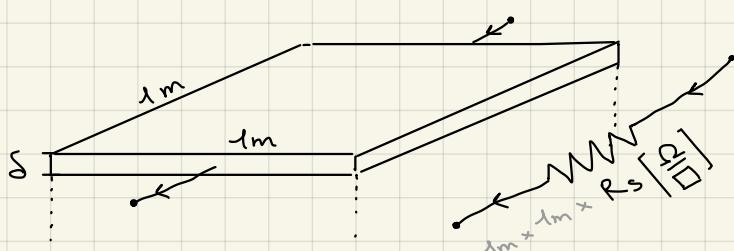


$$\begin{aligned} P_{diss} &= \frac{1}{2} \int_{\Omega} \sigma |\vec{E}|^2 d\Omega = \frac{1}{2} \int_{\Omega} |\vec{J}| d\Omega \quad (\text{con } \vec{J}(g) = J(0) e^{-j\beta g}) \\ &= \frac{1}{2\sigma} \int_0^{+\infty} |J(0)|^2 e^{-2\alpha g} dg = \frac{|J(0)|^2}{2\sigma} \int_0^{+\infty} e^{-2\alpha g} dg = \\ &= \frac{|J(0)|^2}{4\alpha\sigma} = \frac{|J(0)|^2 \delta}{4\sigma} \end{aligned}$$

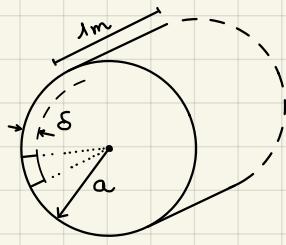
$$I = \int_{\Sigma} J(g) d\Sigma = \left[\int_0^{+\infty} J(g) dg \right] \cdot (1m) = \int_0^{+\infty} J(0) e^{-j\beta g} dg = \frac{J(0)}{\beta} = \frac{J(0) \cdot \delta}{(1+j)}$$

$$\rightarrow R = \frac{2 P_{diss}}{|I|^2} = \frac{2 |J(0)|^2 \delta}{4\sigma} \cdot \frac{2}{|J(0)|^2 \delta^2} = \frac{1}{\sigma \cdot \delta} [\Omega]$$

R è anche la resistenza in corrente continua di
 $1m \times 1m \times \delta m$



Resistenza superficiale
(di un quadro) $\frac{\Omega}{\square}$



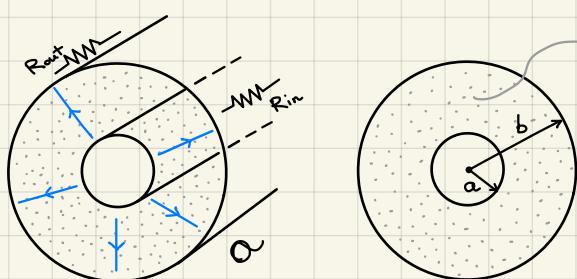
$\delta \ll a$ in un normale conduttore

P è il perimetro ($2\pi a$)

$$R = \frac{1}{\sigma \delta P} = R_s \cdot \frac{1}{P} \left[\frac{\Omega}{m} \right]$$

Linee TEM - parametri fisici (R, L, C, G)

Cavo coassiale



$$\epsilon = \epsilon' - j\epsilon'' \quad (\text{piccole perdite } \epsilon'' \ll \epsilon')$$

$$C = \frac{2\pi \epsilon'}{\ln(b/a)} \quad L = \frac{\mu_0}{2\pi} \ln(b/a)$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\omega \mu}{2\sigma}} = \frac{1}{\sigma \delta} \left[\frac{\Omega}{m} \right]$$

$$R = R_{out} + R_{in} = \frac{R_s}{2\pi b} + \frac{R_s}{2\pi a} = \frac{R_s}{2\pi} \left(\frac{1}{b} + \frac{1}{a} \right)$$

$$\text{Ricordando che } \frac{G}{C} = \frac{\sigma}{\epsilon'} \rightarrow G = C \frac{\omega \epsilon''}{\epsilon'} = \frac{2\pi \omega \epsilon''}{\ln(b/a)}$$

$$Z_c = \sqrt{\frac{L}{C}} = \frac{\eta}{2\pi} \log(b/a) \quad \alpha_c = \frac{R}{2Z_c} \quad \boxed{\alpha_0 = \frac{G Z_c}{2} = \frac{\pi}{\lambda} \frac{\epsilon''}{\epsilon'}}$$

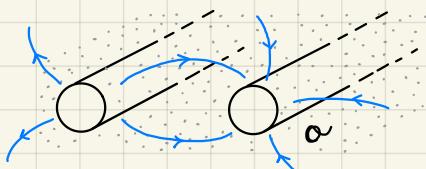
vera x tutte le linee TEM

$$\underline{L \cdot C = \mu \cdot \epsilon} \quad \text{per linee TEM}$$

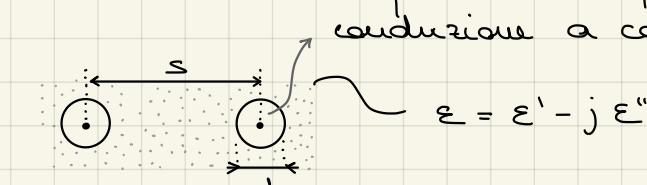
$$L = \frac{\mu_0 \epsilon'}{C} = \frac{\mu_0 \epsilon'}{2\pi \epsilon'} \ln(b/a) = \frac{\mu_0}{2\pi} \ln(b/a)$$

$$Z_c = \sqrt{\frac{\mu_0 \epsilon'}{C^2}} = \sqrt{\frac{\mu_0 \epsilon'}{C}}$$

Linea bifilare



$$C = \frac{\pi \epsilon'}{\operatorname{arccosh}(\frac{s}{d})}$$



$$L = \frac{\mu_0 \epsilon'}{C} = \frac{\mu_0}{\pi} \operatorname{arccosh}(\frac{s}{d})$$

l'interno dei conduttori è solitamente vuoto in quanto non parteciperrebbe alla conduzione a causa dell'effetto pelle

$$G = \frac{\pi w \epsilon''}{\operatorname{arccosh}\left(\frac{s}{d}\right)}$$

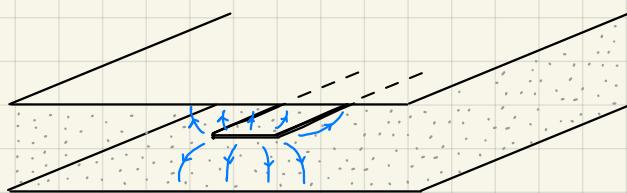
$$R = \frac{2 R_s}{\pi d} \left[\frac{s/d}{\sqrt{(s/d)^2 - 1}} \right]$$

$$Z_c = \frac{\eta}{\pi} \operatorname{arccosh}\left(\frac{s}{d}\right)$$

$$\alpha_c = \frac{R}{2 Z_c} \quad \alpha_d = \frac{\pi}{\lambda} \cdot \frac{\epsilon''}{\epsilon'} = \frac{C_d Z_c}{2}$$

La linea bifilare è poco utilizzata perché le sue linee di campo elettrico e magnetico non sono contenute e interagiscono con i corpi circostanti.

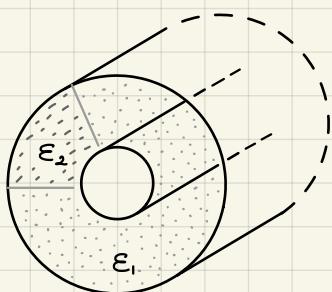
Stripline



Formule dei parametri complicate, non esatte ma approssimanti

Linee quasi-TEM - parametri fisici

Realizzate con dielettrico non omogeneo



$v = \frac{1}{\sqrt{\mu \epsilon}} = ? \rightarrow$ il voto dell'onda non può essere TEM puro perché la v sarebbe diversa nei due dielettrici

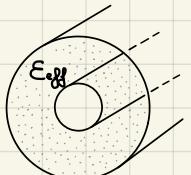
1) Calcolo ϵ

2) Calcolo $L = L_0$ (nel vuoto $\epsilon = \epsilon_0$)

$$v = \frac{1}{\sqrt{L \epsilon}} = \frac{1}{\sqrt{L_0 \epsilon}}$$

non dipende dal dielettrico

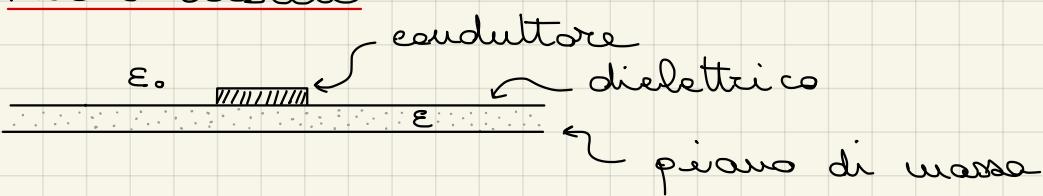
$$\epsilon_{eff} = \frac{L \epsilon}{\mu_0}$$



$$v = \frac{1}{\sqrt{L \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_{eff}}}$$

$L \epsilon \neq \mu \epsilon$
ma
 $L \epsilon = \mu \epsilon_{eff}$

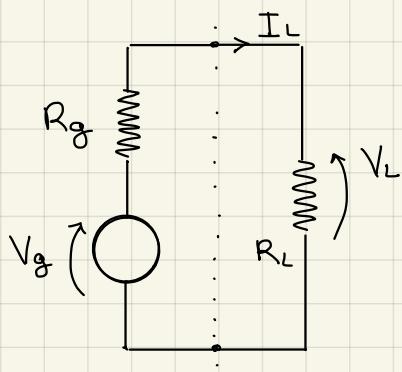
Microstiscia



Anche per la microstrip le formule dei parametri sono complicate e non esatte ma approssimate.

Adattamento (di impedenza) → per evitare che ci sia onda riflessa

Trasferimento di potenza ad un carico



- Potenza massima al carico?
- In che condizioni si verifica?

$$P_L = \frac{1}{2} \operatorname{Re} \{ V_L I_L^* \} = \frac{1}{2} |I_L|^2 R_L$$

$$I_L = \frac{V_g}{R_g + R_L} \rightarrow P_L = \frac{1}{2} |V_g|^2 \frac{R_L}{(R_g + R_L)^2}$$

$$\left\{ \begin{array}{l} R_L \rightarrow 0 \quad P_L \rightarrow 0 \\ R_L \rightarrow \infty \quad P_L \rightarrow 0 \end{array} \right.$$

Ricerca del massimo:

$$\frac{dP_L}{dR_L} = 0 \rightarrow \frac{1}{(R_g + R_L)^2} - 2 \frac{R_L}{(R_g + R_L)^3} = 0 \rightarrow R_g = R_L$$

adattamento

$$P_{L,\max} = \frac{|V_g|^2}{8 R_g} = P_0 \text{ (potenza disponibile)}$$

dipende solo dai parametri del generatore

Ricordando che: $P_m^+(z) = \frac{1}{2} \frac{|V^+(z)|^2}{Z_c}$ con $V^+(z) = V_g/2$ (se $R_g = Z_c$)

$$P_m^+(z) = \frac{|V_g|}{8 R_g}$$

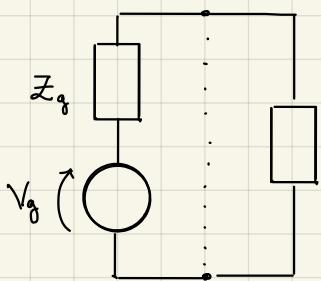
Quindi deve anche valere: $P_L = P_0 (1 - |\Gamma_L|^2)$. Verifichiamolo:

$$V_L = V_g \frac{R_L}{R_L + R_g} \quad P_L = \frac{1}{2} \frac{|V_L|^2}{R_L} = \frac{1}{2} \frac{|V_g|^2 \frac{R_L^2}{(R_L + R_g)^2}}{(R_L + R_g)^2} \cdot \frac{1}{R_L} = \frac{1}{2} \frac{|V_g|^2 R_L}{(R_L + R_g)^2}$$

linee di trasmiss.:

$$\Gamma_L = \frac{R_L - R_g}{R_L + R_g} \quad P_0 = \frac{|V_g|^2}{8 R_g} \quad P_L = P_0 (1 - |\Gamma_L|^2)$$

$$P_L = \frac{|V_g|^2}{8 R_g} \left(1 - \frac{(R_L - R_g)^2}{(R_L + R_g)^2} \right) = \frac{1}{2} \frac{|V_g|^2 R_L}{(R_L + R_g)^2}$$

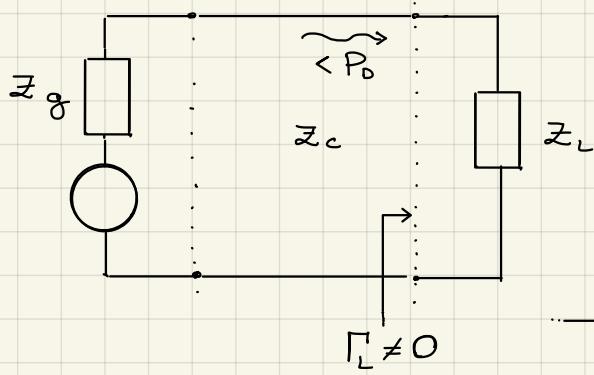


carico adattato $Z_L = Z_g^*$

(max. trasferimento di potenza)

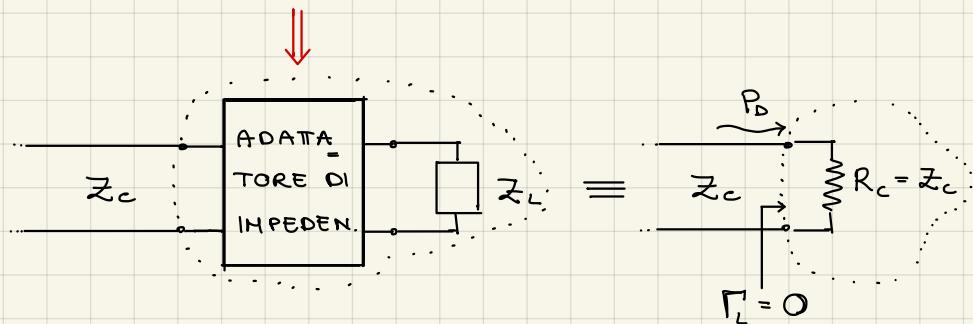
$$Z_g = R_g + j X_g \quad P_d = \frac{|V_g|^2}{8 R_g}$$

la parte immag.
non impatta sulla
potenza disponibile



Z_c reale

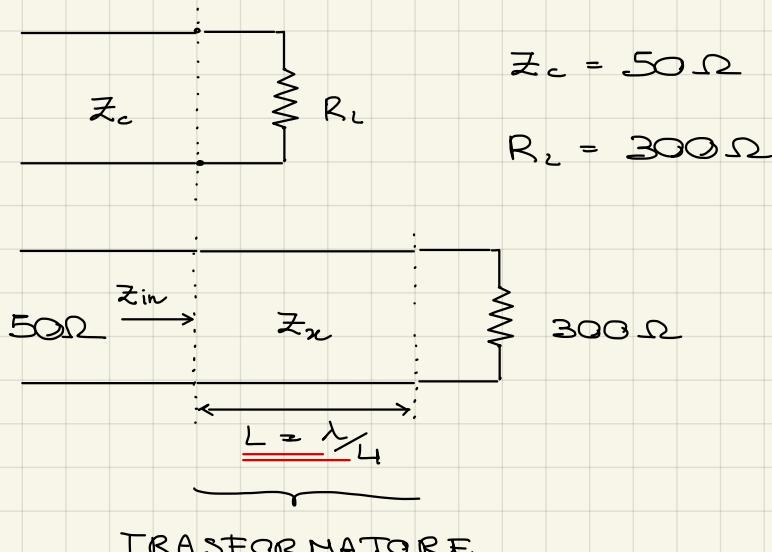
Se $Z_L \neq Z_c$ → $\Gamma_L \neq 0$ $P_L < P_d$
(disadattamento)



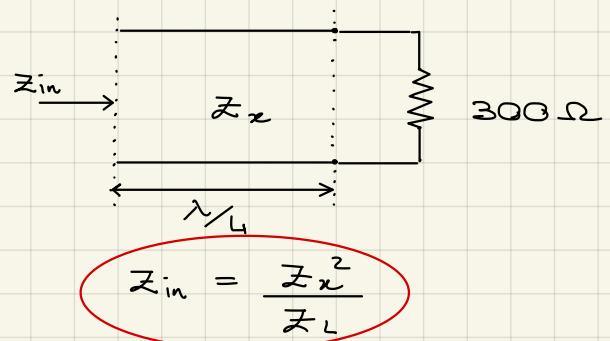
Strutture adattanti:

- trasformatore $\lambda/4$ (solo carichi reali)
- trasformatore $\lambda/4$ con neutrale (solo carichi complessi)
- stub semplice
- doppio stub

Trasformatore $\lambda/4$

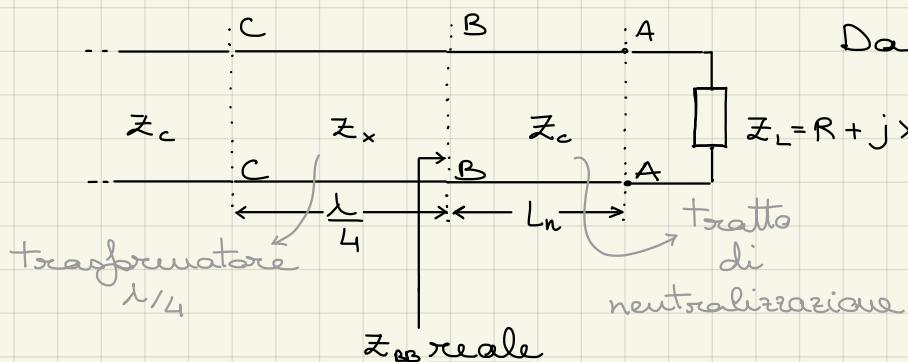


Voglio che $Z_{in} = Z_L$
(carico adattato)



$$Z_x = \sqrt{300 \cdot 50} = 122,5 \Omega$$

Trasformatore $\lambda/4$ con neutralizzazione



Da determinare: L_n e Z_x .

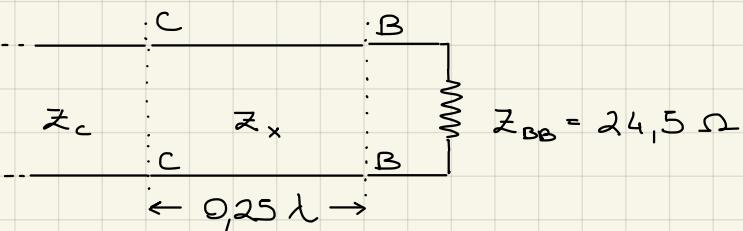
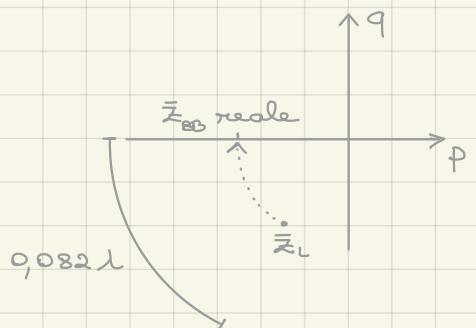
$$Z_L = 30 - j 20 \text{ } [\Omega]$$

$$Z_c = 50 \Omega$$

$$\bar{Z}_L = \frac{Z_L}{Z_c} = \frac{30 - j 20}{50} = 0,6 - j 0,4$$

$$\rightarrow \bar{Z}_{BB} = 0,49 \quad L_n = 0,082 \lambda$$

$$Z_{BB} = \bar{Z}_{BB} Z_c = 24,5 \Omega$$

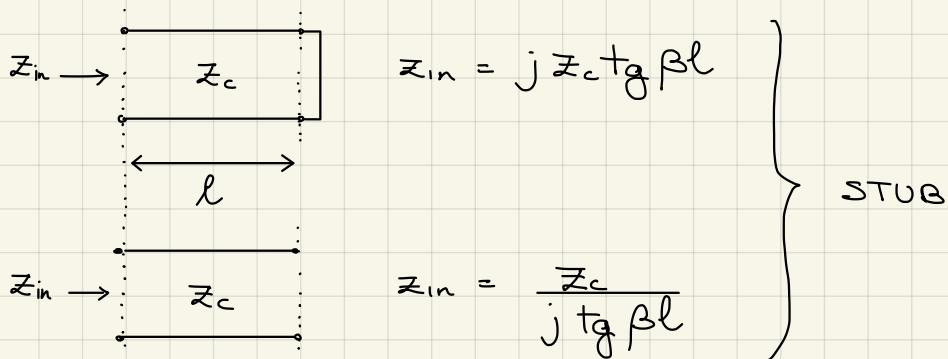


$$\bar{Z}_x = \sqrt{Z_{BB} Z_c} = 35 \Omega$$

come un
trasformatore $\lambda/4$

Stub singolo (semplice)

Stub: elemento reattivo realizzato con linea di trasmissione

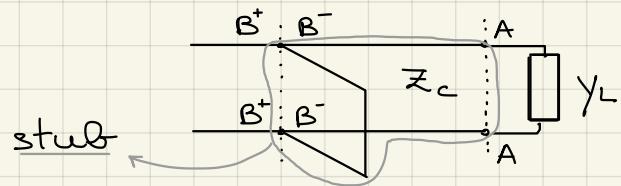


Gli stub sono collocati in un punto opportuno della linea di trasmissione in serie o in parallelo

Stub parallelo

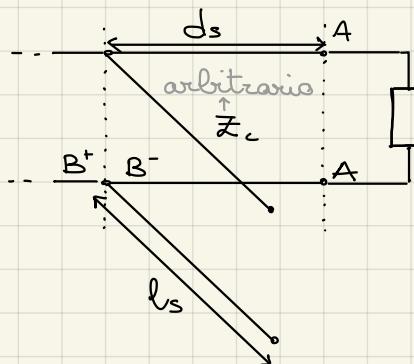
(lavoriamo con le ammettenze)

$$\bar{Y}_L = \frac{Y_L}{Y_c} = \frac{Z_c}{Z_L}$$



esso essere
la stessa
carta di
Smith

Con un tratto di linea si trasforma $\bar{Y}_L = g_L + j b_L$ in $1 + jb = \bar{Y}_{BB^-}$, connettiamo in parallelo $\bar{Y}_S = -jb$; in $\bar{Z}_{BB^+} = 1$ adattamento



$$Z_L = 30 + j 20 \Omega$$

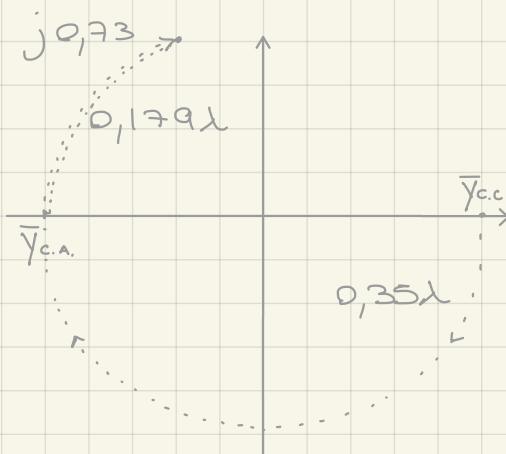
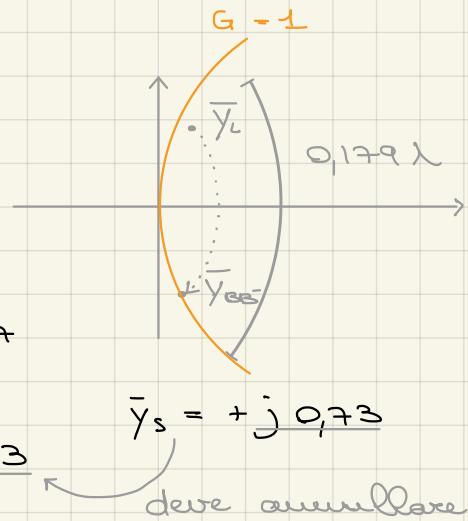
$$Z_c = 50 \Omega$$

$$\bar{Y}_L = \frac{Z_c}{Z_L} = 1,15 + j 0,77$$

stub parallelo in
circuito aperto

$$\bar{Y}_{BB^-} = 1 - j 0,73$$

$$d_s = 0,179\lambda$$



Stub circuito aperto

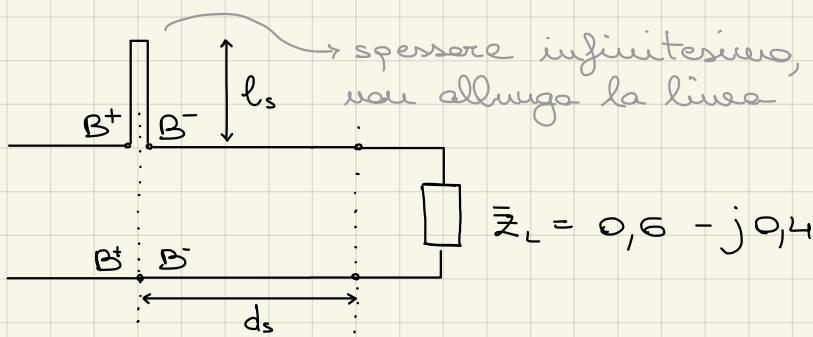
$$Y_S \rightarrow \text{c.a. } \bar{Y}_{c.a.} = 0 \Omega^{-1} \quad l_s = 0,1\lambda$$

Stub corto circuito

$$Y_S \rightarrow \text{c.c. } \bar{Y}_{c.c.} = \infty \Omega^{-1} \quad l_s = 0,35\lambda$$

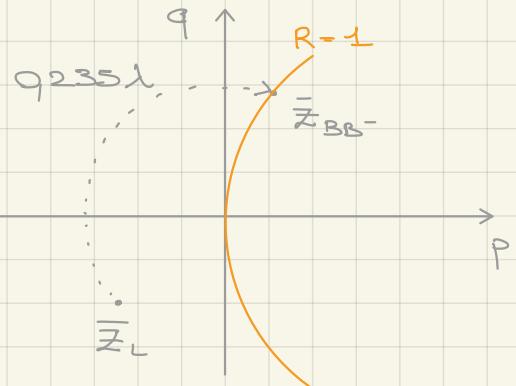
Stub serie

- Con un tratto di linea si trasforma Z_L in $\bar{Z}_{BB^-} = 1 + j x$
- Si introduce in serie uno stub $X_S = -j x$
- $\bar{Z}_{BB^+} = 1 \rightarrow$ adattamento

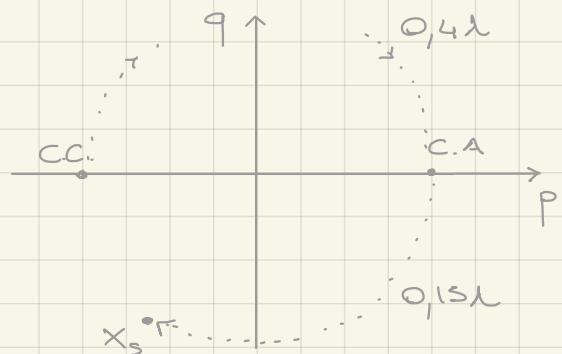


$$\bar{Z}_{BB^-} = 1 + 0.73j \quad X_s = -j0.73$$

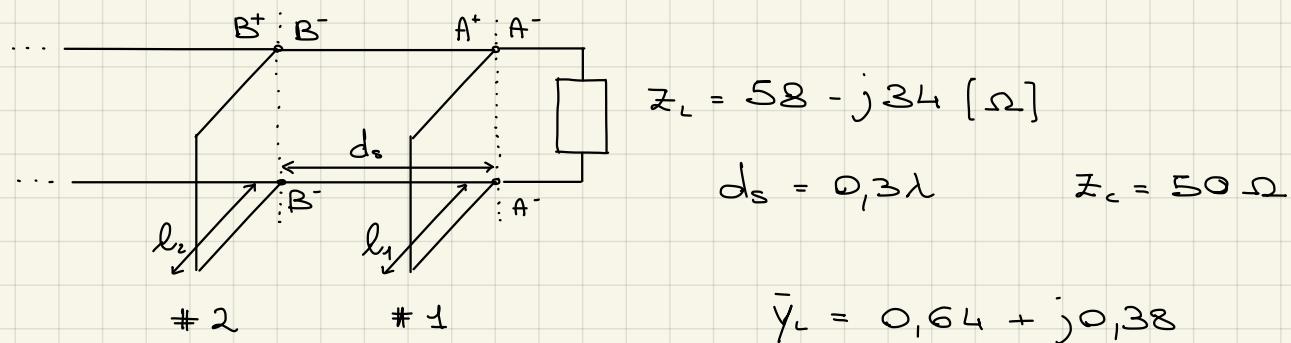
$$d_s = 0.235\lambda$$



$$\begin{array}{ll} \text{---} & Z_{cc} = 0 \Omega \quad l_s = 0.4\lambda \\ \text{---} & Z_{c.A.} = \infty \Omega \quad l_s = 0.15\lambda \end{array}$$



Doppio Stub (parallelo)



$$\bar{Y}_L = 0.64 + j0.38$$

NB: gli stub sono reattivi \rightarrow modificano solo la parte immaginaria (dell'ammittenza); il tratto d_s serveva a modificare anche la parte reale.

In AA^- : siamo in $\bar{Y}_L = 0.64 + j0.38$

stub #1

In AA^+ : siamo sul cerchio a parte reale 0.64 ($0.64 + jb_{AA^+}$)

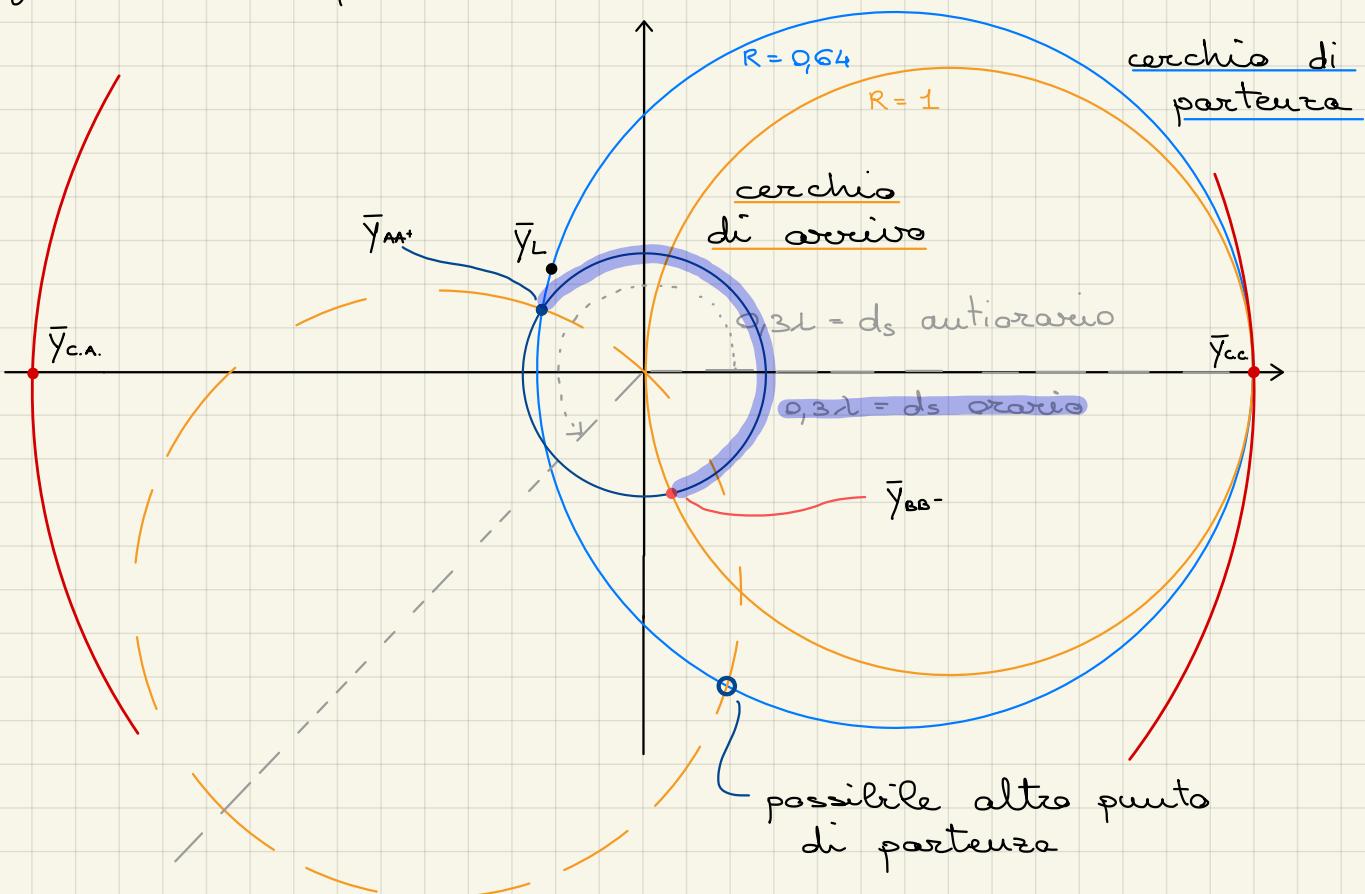
(d_s)

In BB^- : " " " " " " 1 ($1 + jb_{BB^-}$)

stub #2

In BB^+ : siamo nel centro della carta di Smith ($\bar{Y}_{BB^+} = 1$)

Dobbiamo identificare il "punto di partenza" in AA⁺ ($\bar{Y}_{AA^+} = 0,64 + j0,225$) che, ruotato in senso orario di ds , giunge nel "punto di arrivo" in BB⁻ ($\bar{Y}_{BB^-} = -1 + j0,5$)



$$\bar{Y}_{AA^+} = 0,64 + j0,225$$

$$\bar{Y}_{BB^-} = -1 - j0,5$$

$$\bar{Y}_{S_1} = j(0,225 - 0,38) = j0,155$$

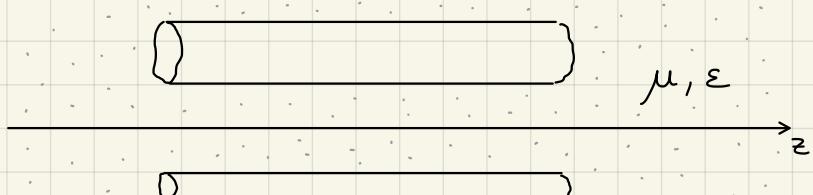
$$\bar{Y}_{S_2} = j0,5$$

$$\text{in c.c. } \begin{cases} l_1 = 0,226\lambda \\ l_2 = 0,324\lambda \end{cases}$$

$$\text{in C.A. } \begin{cases} l_1 = 0,476\lambda \\ l_2 = 0,074\lambda \end{cases}$$

Strutture guidanti μ -TEM (modi)

Strutture metalliche, cilindriche (asse diretto come \vec{e}_z), uniformi (lungo la direzione \vec{e}_z)



Hp: mezzo senza perdite (conduttori ideali)

$$\bar{E}(x, y, z) = \bar{E}(x, y) e^{\pm j \gamma z} = \bar{E}(x, y) e^{\pm (\alpha + j \beta) z}$$

il campo rispetto
a z cambia solo in
modulo e fase

Equazioni di Helmholtz:

$$\nabla^2 \bar{E} = -k^2 \bar{E} \quad \nabla^2 \bar{H} = -k^2 \bar{H}$$

$$K^2 = \omega^2 \mu \epsilon$$

(deve essere soddisfatta nella regione esterna ai conduttori)

$$\nabla^2 = \underbrace{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}_{\nabla_t^2} = \nabla_t^2 + \frac{\partial^2}{\partial z^2} \quad (\text{ess } \frac{\partial^2}{\partial z^2} \bar{E} = \gamma^2 \bar{E})$$

$$\rightarrow \boxed{\nabla_t^2 \bar{E} = -(\gamma^2 + K^2) \bar{E}}$$

$$\boxed{\nabla_t^2 \bar{H} = -(\gamma^2 + K^2) \bar{H}}$$

1) Eq. del rotore ($\bar{\nabla} \times \bar{E}, \bar{\nabla} \times \bar{H}$)

2) Esprimere tutte le componenti di \bar{E} e \bar{H} in
funzione delle due componenti E_z e H_z

$$\bar{\nabla} \times \bar{E} = -j \omega \mu \bar{H}$$

$$\bar{\nabla} \times \bar{H} = j \omega \epsilon \bar{E}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j \omega \mu H_x$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j \omega \epsilon E_x$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j \omega \mu H_y$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j \omega \epsilon E_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} = -j \omega \mu H_z$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} = j \omega \epsilon E_z$$

$$E_x = -\frac{1}{\gamma^2 + K^2} \left(\gamma \frac{\partial E_z}{\partial x} + j \omega \mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = \frac{1}{\gamma^2 + K^2} \left(-\gamma \frac{\partial E_z}{\partial y} + j \omega \mu \frac{\partial H_z}{\partial x} \right)$$

$$H_x = \frac{1}{\gamma^2 + K^2} \left(j \omega \epsilon \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x} \right)$$

$$H_y = -\frac{1}{\gamma^2 + K^2} \left(j \omega \epsilon \frac{\partial E_z}{\partial x} + \gamma \frac{\partial H_z}{\partial y} \right)$$

Si definisce $k_c^2 = \gamma^2 + k^2$ (senza perdite $\gamma = j\beta$)

$$\left[\nabla_t^2 E_z = -k_c^2 E_z \right] \quad \text{e} \quad \left[\nabla_t^2 H_z = -k_c^2 H_z \right]$$

Classificazione (dei modi)

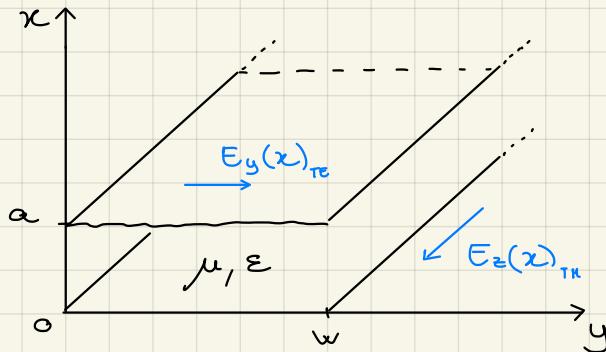
$$E_z = H_z = 0 \quad \text{TEM}$$

$$H_z = 0, \quad E_z \neq 0 \quad \text{TM}$$

$$H_z \neq 0, \quad E_z = 0 \quad \text{TE}$$

$$E_z \neq 0 \quad \text{e} \quad H_z \neq 0 \quad \text{ibridi (TM+TE)}$$

Linea a striscia (trascurare gli effetti di bordo):



- Modo TEM: $E_z = 0, H_z = 0 \quad \gamma^2 = -k^2$ cioè $\gamma = \pm jk = \pm j\omega\sqrt{\mu\epsilon}$

$$H_y = \frac{\gamma}{j\omega\mu} E_x = \pm \sqrt{\frac{\epsilon}{\mu}} E_x \dots \quad (\text{come già visto})$$

- Modo TM: $H_z = 0, E_z \neq 0 \quad \nabla_t^2 E_z = -k_c^2 E_z$

$$\frac{\partial}{\partial y} = 0$$

$$\boxed{\frac{\partial^2}{\partial x^2} E_z = -k_c^2 E_z}$$

$$\text{con } k_c^2 = \gamma^2 + k^2$$

perché
trascuriamo
gli effetti di bordo $E_z(x) = A \operatorname{seu}(k_c x) + B \cos(k_c x)$

A e B vanno determinate con condizioni al contorno

→ conduttori ideali (campi tangenti nulli)

$$\left\{ \begin{array}{l} E_z(0) = 0 \implies B = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} E_z(a) = 0 \implies A \operatorname{seu}(k_c a) = 0 \quad k_c a = m\pi \quad m = 1, 2, \dots \end{array} \right.$$

$$\left\{ \begin{array}{l} E_z(x) = A \sin \left(m \frac{\pi x}{a} \right) \\ E_x(x) = - \frac{\gamma}{K_c^2} \frac{dE_z}{dx} = - \frac{\gamma a}{m\pi} A \cos \left(m \frac{\pi x}{a} \right) \\ H_y(x) = - \frac{j\omega \epsilon}{K_c^2} \frac{dE_z}{dx} = - \frac{j\omega \epsilon a}{m\pi} A \cos \left(m \frac{\pi x}{a} \right) \\ H_x = 0, \quad E_y = 0, \quad H_z = 0 \end{array} \right.$$

Analizziamo γ :

$$K_c^2 = K^2 + \gamma^2 = \left(\frac{m\pi}{a} \right)^2 \quad \gamma = \sqrt{K_c^2 - K^2} = \sqrt{\left(\frac{m\pi}{a} \right)^2 - \omega^2 \mu \epsilon}$$

affinché il modo TM_m sia "in propagazione"

$$\gamma = j\beta_z \text{ (immag. pura)} \rightarrow \left(\frac{m\pi}{a} \right)^2 - \omega^2 \mu \epsilon < 0$$

$$\Rightarrow \omega > \frac{m\pi \nu}{a}$$

ω_c (pulsazione di taglio / cut-off)

$$\left\{ \begin{array}{l} \gamma = j \underbrace{\omega \sqrt{\mu \epsilon}}_{K(\beta)} \sqrt{1 - \left(\frac{\omega_c}{\omega} \right)^2} \end{array} \right\} \quad (\omega > \omega_c)$$

$$\left\{ \begin{array}{l} \gamma = \alpha = \frac{m\pi}{a} \sqrt{1 - \left(\frac{\omega}{\omega_c} \right)^2} \end{array} \right\} \quad (\omega < \omega_c)$$

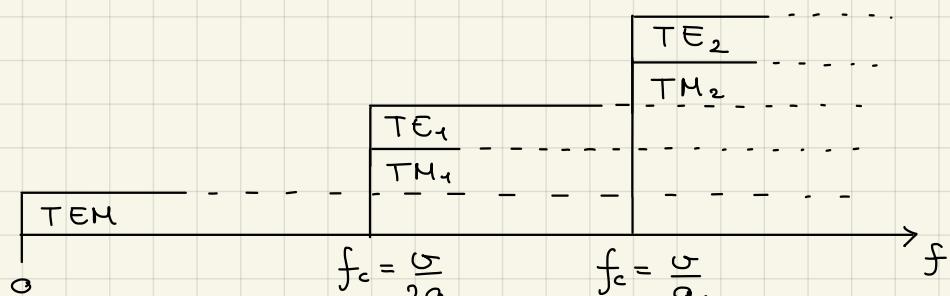
Se $\omega < \omega_c$: $e^{-j\beta_z z}$ (propagazione)

Se $\omega > \omega_c$: $e^{-\alpha_z z}$ (attenuazione)

$$\omega_c = \frac{m\pi \nu}{a} = 2\pi f_c$$

$$f_c = \frac{m \nu}{2a} = \frac{\nu}{\lambda_c}$$

$$\lambda_c = \frac{2a}{m}$$



Velocità di "propagazione" v_f :

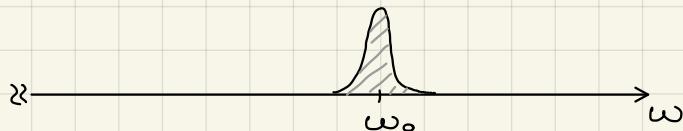
$$\beta_z = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_f} = \frac{\omega}{v_f} \rightarrow v_f = \frac{\omega}{\beta_z} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} \frac{\omega}{\sqrt{1 - (\frac{\omega_c}{\omega})^2}}$$

$$v_f = \frac{\omega}{\sqrt{1 - (\frac{\omega_c}{\omega})^2}} \quad ; \quad \text{nel vuoto} \quad v = c \quad \downarrow \quad v \text{ dell'onda TEM}$$

$v_f > c$! È possibile perché non è una velocità reale (non è la velocità di propagazione dell'energia) ma è una velocità apparente (velocità di fase)

$v_f = v_f(\omega) \rightarrow$ frequenze diverse si muovono a velocità diverse ("dispersione")

Segnale a BANDA STRETTA:



$$s(t) = f(t) \cos \omega_0 t = \operatorname{Re} \{ f(t) e^{j\omega_0 t} \}$$

$f(t) \leftrightarrow F(\omega)$ $s(t) \leftrightarrow S(\omega)$ (trasformata di Fourier)

$$S(\omega) = F(\omega - \omega_0) \quad S(\omega) \xrightarrow{\quad} A e^{-j\beta_z L} \xrightarrow{\quad} S_o(\omega) \quad (|A| \leq 1)$$

$$S_o(\omega) = A S(\omega) e^{-j\beta_z(\omega)L}$$

con $\beta_z = \underbrace{\omega\sqrt{\mu\epsilon}}_{\beta} \sqrt{1 - (\frac{\omega_c}{\omega})^2}$

lunghezza della
guida

Sviluppo in serie di Taylor di β_z :

$$\beta_z(\omega) = \underbrace{\beta_z(\omega_0)}_{\beta_0} + \underbrace{\frac{d\beta_z(\omega)}{d\omega} \Big|_{\omega_0}}_{\beta'_0} \cdot (\omega - \omega_0) + \dots$$

$$S_o(\omega) \approx A S(\omega) e^{-j\beta_0 L} e^{-j\beta'_0 \Delta\omega L}$$

$$S_o(t) \approx A \underbrace{f(t - \beta'_0 L)}_{\text{informazione}} \cos(\omega_0 t - \beta_0 L) \underbrace{\cos(\omega_0 t - \beta_0 L)}_{\text{portante}}$$

$$v_g = \frac{1}{\beta'_0} = \frac{1}{\frac{d\beta_z(\omega)}{d\omega} \Big|_{\omega_0}}$$

informazione

$$v_f = \frac{\omega}{\beta_z}$$

portante

$$v_g = v \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \leq c$$

\nwarrow velocità di gruppo

$$v_f \cdot v_g = v^2$$

Nelle linee TEM:

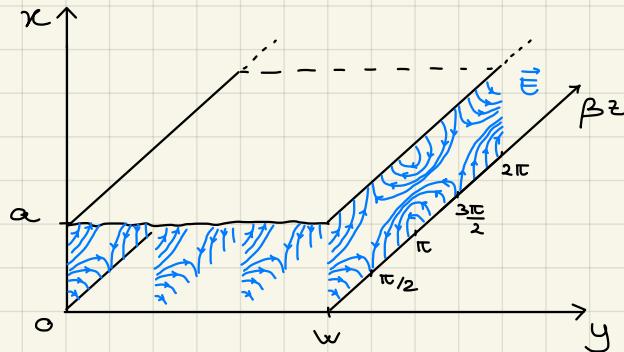
$$\beta = \frac{\omega}{v} \quad v_f = \frac{\omega}{\beta} = v \quad v_g = \frac{1}{\frac{d\beta(\omega)}{d\omega}} = v$$

→ v_f e v_g sono uguali nei modi TEM ←

Impedenza modale

$$[Z_m = \frac{E_t}{H_t} = \frac{E_x}{H_y} = \eta \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \frac{\beta z}{\omega \epsilon}]$$

TM₁ $E_z(x) = A \sin\left(\frac{\pi x}{a}\right)$ $E_x(x) = \frac{\gamma a}{\pi} \cos\left(\frac{\pi x}{a}\right)$



$$\begin{cases} E_z(x, z) = A \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_z z} \\ E_x(x, z) = \frac{\gamma a}{\pi} A \cos\left(\frac{\pi x}{a}\right) e^{-j\beta_z z} \\ E_y = 0 \end{cases}$$

N.B.: le cariche indotte sui conduttori superiore ed inferiore sono dello stesso segno

↓
non è possibile definire un potenziale fra i due conduttori (campo \vec{E} non irrotazionale)!

- Modi TE: $E_z = 0, H_z \neq 0$ $\nabla_t^2 H_z = -K_c^2 H_z$

$$H_z(x) = A \sin(K_c x) + B \cos(K_c x)$$

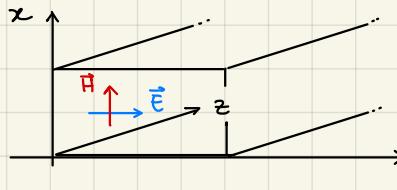
Condizioni al contorno: $E_y \propto \frac{\partial H_z}{\partial x}$ $E_y(0) = 0$ $E_y(a) = 0$

$$A = 0 \quad | K_c = \frac{m\pi}{a}$$

$$\left\{ \begin{array}{l} H_z = B \cos\left(\frac{m\pi x}{a}\right) \\ E_y = -j\omega\mu a B \sin\left(\frac{m\pi}{a}x\right) \\ H_x = j\frac{\beta}{m\pi} a B \sin\left(\frac{m\pi}{a}x\right) \\ E_x = 0, \quad H_y = 0, \quad E_z = 0 \end{array} \right. \quad \text{TEM}_m$$

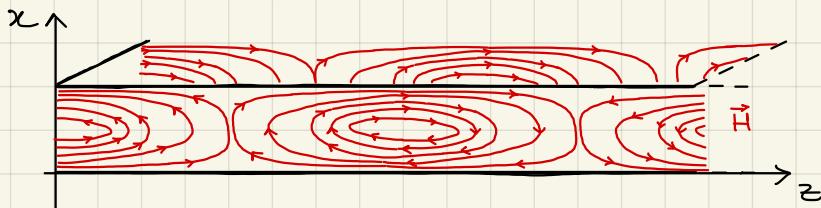
$$\left\{ \begin{array}{l} \gamma = j\beta_z = j\omega\sqrt{\mu\epsilon} \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \\ (\omega > \omega_c) \end{array} \right.$$

$$\left[Z_{TE} = \frac{E_t}{H_t} = -\frac{E_y}{H_x} = \frac{n}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \right]$$



Z_{TE} è riferita
al verso delle
 z positive

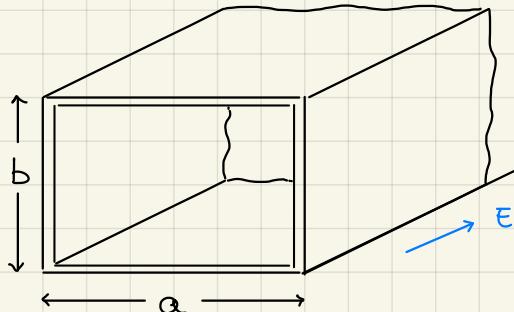
V_f e V_g sono le stesse del modo TM_m .



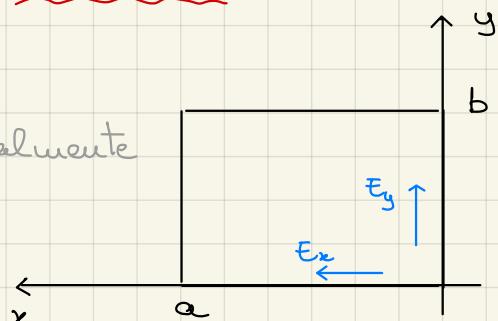
NB: non c'è carica indotta sui 2 conduttori,
inoltre non ci sono componenti di campo magnetico
lungo l'asse y

non è possibile definire una corrente che
scorre lungo i conduttori (verso z , c'è invece
corrente verso y)

Guide d'onda rettangolari (metalliche)



$a > b$
convenzionalmente



non ci sono 2 conduttori

Un solo conduttore \rightarrow 1 modo TEM (\nexists soluzione statica)

Solo onde TM e/o TE.

Onde TM $\nabla_t E_z(x, y) = -k_c^2 E_z(x, y)$ hp: $E_z(x, y) = F(x) G(y)$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = -k_c^2 E_z \rightarrow F''(x) G(y) + F(x) G''(y) = -k_c^2 F(x) G(y)$$

$$\rightarrow \frac{F''(x)}{F(x)} + \frac{G''(y)}{G(y)} = -k_c^2 \implies \frac{F''(x)}{F(x)} = -k_x^2, \frac{G''(y)}{G(y)} = -k_y^2$$

con $k_x^2 + k_y^2 = k_c^2$

Soluzione dell'eq. differenziale:

$$E_z(x, y) = \underbrace{[A' \sin(k_x x) + B' \cos(k_x x)]}_{F(x)} \cdot \underbrace{[C' \sin(k_y y) + D' \cos(k_y y)]}_{G(y)}$$

Determiniamo A', B', C', D' :

$$E_z(0, y) = 0 \rightarrow B' = 0$$

$$E_z(x, 0) = 0 \rightarrow D' = 0$$

\uparrow campo elettrico \uparrow tangente al conduttore nullo

$$\implies \{ E_z(x, y) = A \sin(k_x x) \sin(k_y y) \quad (H_z = 0)$$

$$E_z(a, y) = 0 \rightarrow k_x a = m\pi \quad | \quad k_x = \frac{m\pi}{a} \quad (m = 0, 1, \dots, \infty)$$

$$E_z(x, b) = 0 \rightarrow k_y b = n\pi \quad | \quad k_y = \frac{n\pi}{b} \quad (n = 0, 1, \dots, \infty)$$

TM_{mn}

$$K_c^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{K_c^2 - k^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon < 0 \rightarrow \omega > \omega_c \quad \text{con}$$

no attenuazione

$$\omega_c = \frac{1}{\sqrt{\mu \epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\left\{ \gamma = j\beta_z = j\omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right\} \quad (\omega > \omega_c)$$

σ_f e σ_g hanno la stessa espressione dei modi TM_m

$$\left\{ \begin{array}{l} E_x = -j \frac{\beta K_x}{K_c^2} A \cos(K_x x) \sin(K_y y) e^{-j \frac{\beta z}{2}} \\ E_y = -j \frac{\beta K_y}{K_c^2} A \sin(K_x x) \cos(K_y y) e^{-j \frac{\beta z}{2}} \\ H_x = j \frac{\omega \epsilon K_y}{K_c^2} A \sin(K_x x) \cos(K_y y) e^{-j \frac{\beta z}{2}} \\ H_y = -j \frac{\omega \epsilon K_x}{K_c^2} A \cos(K_x x) \sin(K_y y) e^{-j \frac{\beta z}{2}} \end{array} \right.$$

TM_{mn}

- $E_z = 0$ in $y=0$ e $y=b$ $E_y = 0$ in $x=0$ e $x=a$
- Sia m che n devono essere > 0 (altrimenti $E_z \equiv 0$)
- Il modo a frequenza più bassa è TM₁₁

Onde TE $E_z = 0$

$$\nabla_t H_z(x, y) = -K_c^2 H_z(x, y) \quad \text{hp: } E_z(x, y) = M(x) N(y)$$

Analogamente al modo TM si ricava:

$$H_z(x, y) = [A'' \sin(K_x x) + B'' \cos(K_x x)] [C'' \sin(K_y y) + D'' \cos(K_y y)]$$

$$\begin{aligned} E_x &= -j \frac{\omega \mu}{K_c^2} \frac{\partial H_z}{\partial y} = \\ &= -j \frac{\omega \mu}{K_c^2} K_y [A'' \sin(K_x x) + B'' \cos(K_x x)] [C'' \cos(K_y y) - D'' \sin(K_y y)] \end{aligned}$$

$$E_y = j \frac{\omega \mu}{K_c^2} K_x [A'' \cos(K_x x) - B'' \sin(K_x x)] [C'' \sin(K_y y) + D'' \cos(K_y y)]$$

$$E_x(x, 0) = 0 \rightarrow C'' = 0 \quad E_y(0, y) = 0 \rightarrow A'' = 0$$

$$\implies \left\{ H_z = \underbrace{B''}_{\substack{\leftarrow B'' \cdot D''}} \cos(K_x x) \cos(K_y y) \right.$$

$$E_x(0, b) = 0 \rightarrow K_y b = n\pi \quad \boxed{K_y = \frac{n\pi}{b}} \quad (n = 0, 1, \dots, \infty)$$

$$E_y(a, 0) = 0 \rightarrow K_x a = m\pi \quad \boxed{K_x = \frac{m\pi}{a}} \quad (m = 0, 1, \dots, \infty)$$

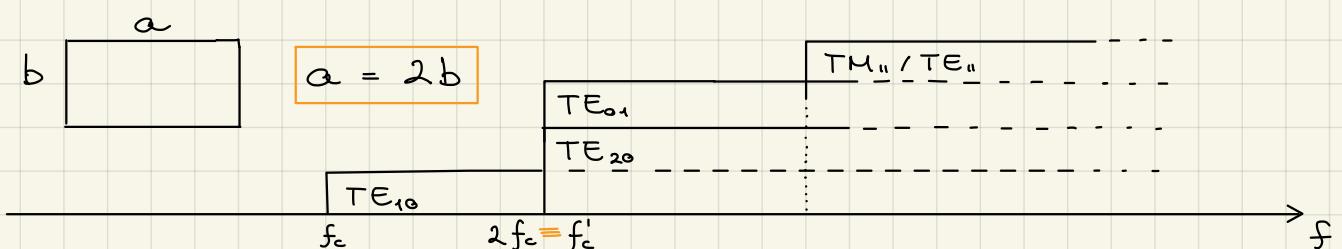
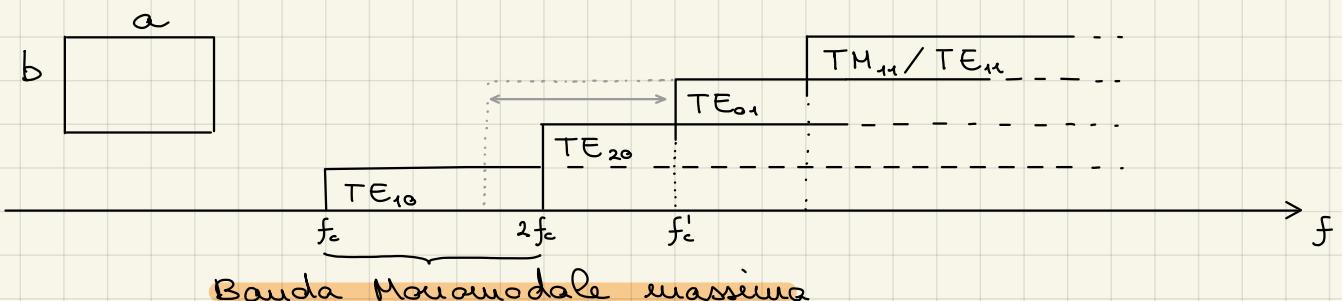
ω_c e γ (σ o β) sono come per il modo TM_{mn}

$$\left\{ \begin{array}{l} E_x = j \frac{\omega \mu k_y}{k_c^2} B \cos(k_x x) \sin(k_y y) e^{-j \beta_z z} \\ E_y = -j \frac{\omega \mu k_y}{k_c^2} B \sin(k_x x) \cos(k_y y) e^{-j \beta_z z} \\ H_x = j \frac{\beta_z k_x}{k_c^2} B \sin(k_x x) \cos(k_y y) e^{-j \beta_z z} \\ H_y = j \frac{\beta_z k_y}{k_c^2} B \cos(k_x x) \sin(k_y y) e^{-j \beta_z z} \end{array} \right.$$

TE_{mn}

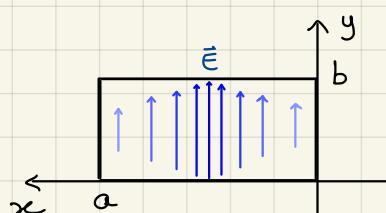
- m o n possono annularsi → TE₁₀ e TE₀₁
- Modo a frequenza più bassa - modo fondamentale

TE₁₀ (se $a > b$) TE₀₁ (se $a < b$)
 per convenzione è
 sempre questo



Modo fondamentale TE₁₀

$$m = 1, \quad n = 0 \quad K_x = \frac{\pi}{a}, \quad K_y = 0 \quad \lambda_c = 2a$$

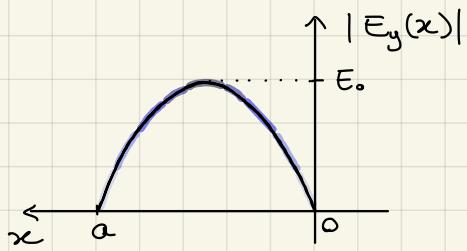


$$H_z = B \cos(k_x x) e^{-j \beta_z z}$$

$$E_y = -j \frac{\omega \mu B}{K_x} \sin(k_x x) e^{-j \beta_z z} = E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j \beta_z z}$$

$$H_x = j \frac{\beta_z B}{K_x} \sin(k_x x) e^{-j \beta_z z} = -\frac{E_0}{Z_{TE}} \sin\left(\frac{\pi}{a} x\right) e^{-j \beta_z z}$$

$$\text{con } [Z_{TE} = -\frac{E_0}{H_x} = \frac{\omega \mu}{\beta} = \frac{\omega \mu}{K \sqrt{1 - (\omega_c/\omega)^2}}]$$



$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad v = \frac{1}{\sqrt{\mu\epsilon}} \quad (\text{di solito } \eta = \eta_0 \quad v = c)$$

$$Z_{TE} = - \frac{E_y}{H_x} = \frac{\eta}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

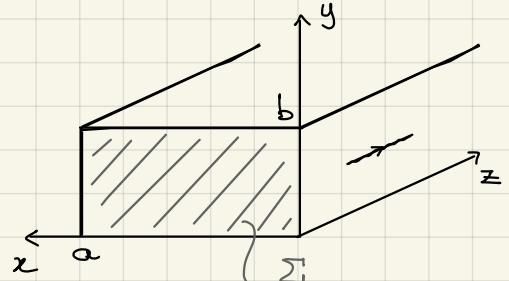
$$f_c = \frac{1}{2a\sqrt{\mu\epsilon}} = \frac{v}{2a} \quad \lambda_c = 2a$$

$$v_f = \frac{v}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \quad v_g = v \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \quad \lambda_g = \frac{v_f}{f} = \frac{2\pi}{\beta}$$

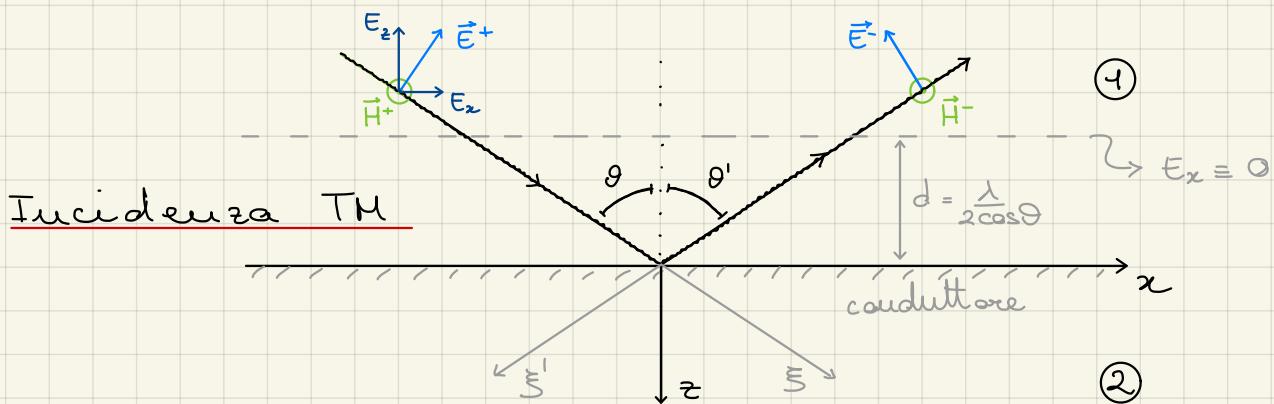
$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \quad \text{con} \quad \lambda = \frac{v}{f}$$

Calcolo della potenza nella guida (lungo Σ)

$$\begin{aligned} P &= \frac{1}{2} \operatorname{Re} \left\{ \int_{\Sigma} (\vec{E} \times \vec{H}^*) \cdot d\vec{\Sigma} \right\} = \\ &= \frac{1}{2} \operatorname{Re} \left\{ \int_0^a \int_0^b (-E_y \cdot H_x^*) dy dx \right\} = \\ &= \frac{1}{2} \frac{|E_0|^2 b}{Z_{TE}} \int_0^a \sin^2 \left(\frac{\pi x}{a} \right) dx = \\ [P &= \frac{|E_0|^2 a b}{4 Z_{TE}}] \end{aligned}$$



I incidente obliqua (di onde TEM piane) su superficie metallica piana (ideale)



$$\vec{E}(x, z) = \vec{E}^+ e^{j\beta\xi} + \vec{E}^- e^{-j\beta\xi'} \quad \text{ma}$$

$$\begin{cases} \xi = x \sin \theta + z \cos \theta \\ \xi' = -x \sin \theta' + z \cos \theta' \end{cases}$$

$$E_x(x, z) = E^+ \cos \theta e^{-j\beta(x \sin \theta + z \cos \theta)} - E^- \cos \theta' e^{j\beta(-x \sin \theta' + z \cos \theta')}$$

$$E_z(x, z) = E^+ \sin \theta e^{-j\beta(x \sin \theta + z \cos \theta)} - E^- \sin \theta' e^{j\beta(-x \sin \theta' + z \cos \theta')}$$

$$H_y(x, z) = H^+ e^{-j\beta(x \sin \theta + z \cos \theta)} + H^- e^{j\beta(-x \sin \theta' + z \cos \theta')}$$

Condizioni al contorno:

piano conduttore ($z = 0$) $E_x(x, 0) = 0 \quad (\forall x)$

$$E^+ \cos \theta e^{-j\beta x \sin \theta} = E^- \cos \theta' e^{j\beta x \sin \theta'} \quad (\forall x)$$

$$\hookrightarrow \boxed{\theta = \theta'} \quad \text{e} \quad \boxed{E^+ = E^-}$$

legge di Snell

$$\Rightarrow \begin{cases} E_x(x, z) = -2j E^+ \cos \theta \sin(\beta z \cos \theta) e^{-j\beta x \sin \theta} \\ E_z(x, z) = -2 E^+ \sin \theta \cos(\beta z \cos \theta) e^{-j\beta x \sin \theta} \\ H_y(x, z) = \frac{2 E^+}{\eta} \cos(\beta z \cos \theta) e^{-j\beta x \sin \theta} \end{cases}$$

L'onda:

- progressiva in direzione \vec{n}_x
- puramente stazionaria in direzione \vec{n}_z

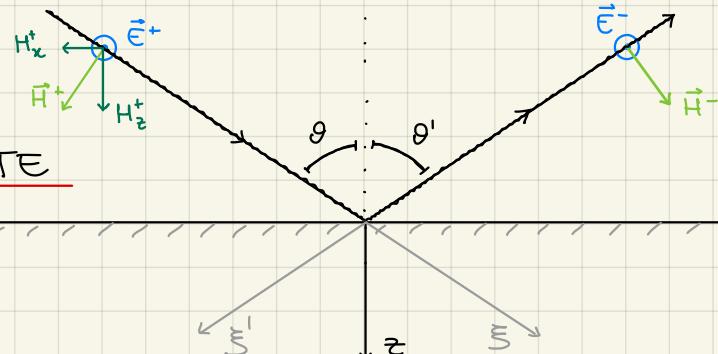
$E_x(x, z) = \dots \sin(\beta z \cos \theta) \dots$ si annulla in $z = 0$
e su tutti i piani paralleli al piano xy tali
che $\beta z \cos \theta = n\pi$

$$\hookrightarrow$$

$$\boxed{z = d_n = \frac{n\lambda}{2 \cos \theta}}$$

$$E_x(x, -d) = 0$$

come se ci fosse
un piano condut.



Incidenza TE

$$E_y = E^+ e^{-j\beta(x \sin \theta + z \cos \theta)} + E^- e^{j\beta(-x \sin \theta' + z \cos \theta')}$$

$$H_x = -\frac{E^+}{\eta} \cos \theta e^{-j\beta(x \sin \theta + z \cos \theta)} + \frac{E^-}{\eta} \cos \theta' e^{j\beta(-x \sin \theta' + z \cos \theta)}$$

$$H_z = \frac{E^+}{\eta} \sin \theta e^{-j\beta(x \sin \theta + z \cos \theta)} + \frac{E^-}{\eta} \sin \theta' e^{j\beta(-x \sin \theta' + z \cos \theta)}$$

Piano conduttore ($z=0$) $E_y(x, 0) = 0$ ($\forall x$)

$$E^+ e^{-j\beta x \sin \theta} = -E^- e^{j\beta x \sin \theta'} \quad (\forall x)$$

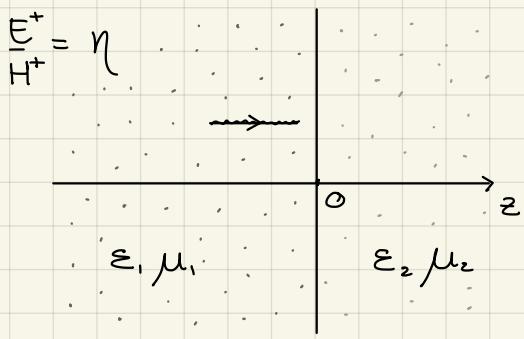
$$\rightarrow \boxed{\theta = \theta'} \quad \text{e} \quad \boxed{E^+ = -E^-}$$

$$\rightarrow \begin{cases} E_y = -2j E^+ \sin(\beta z \cos \theta) e^{-j\beta x \sin \theta} \\ H_x = -2 \frac{E^+}{\eta} \cos \theta \sin(\beta z \cos \theta) e^{-j\beta x \sin \theta} \\ H_z = -2j \frac{E^+}{\eta} \sin \theta \sin(\beta z \cos \theta) e^{-j\beta x \sin \theta} \end{cases}$$

L'onda ha le stesse proprietà dell'incidenza TM.

$$E_y(x, -d) = 0 \quad \text{per} \quad d_n = \frac{n\lambda}{2 \cos \theta}$$

Analogia tra linee di trasmissione e onde TEM

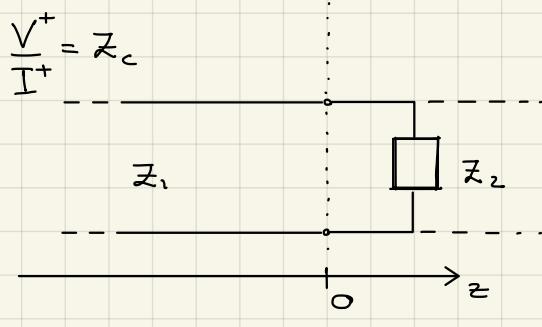


$$E(0) = E_2^+(0) = E_1^+(0) + E_1^-(0) = E^+(0) T(0)$$

$$E_1^-(0) = E_1^+(0) \Gamma(0)$$

$$\Gamma(0) = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{con} \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$E^+(z) = E^+(0) e^{-j\beta z} \quad \vec{S} = \frac{1}{2} \operatorname{Re} \{ \vec{E} \times \vec{H} \}$$

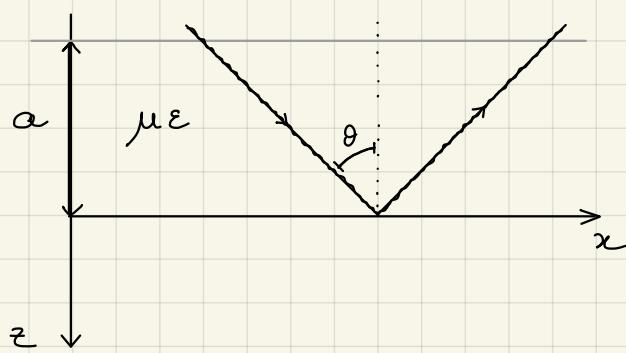


$$V(0) = V^+(0) + V^-(0) = V^+(0) T$$

$$V^-(0) = V^+(0) \Gamma \quad \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

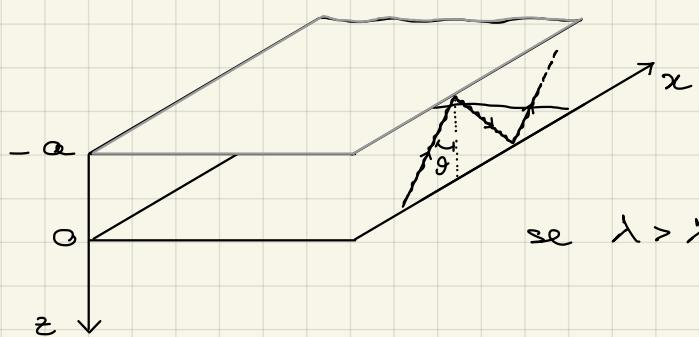
$$V^+(z) = V^+(0) e^{-j\beta z} \quad P = \frac{1}{2} \operatorname{Re} \{ V I^* \}$$

Onde guidate tra conduttori piani paralleli come sovrapposizione di onde piane



E_t (tangente al conduttore) (E_x o E_y) nullo a distanza

$$a = \frac{m\lambda}{2\cos\theta} \quad \text{come se avessi un conduttore}$$



$$\cos\theta = \frac{m\lambda}{2a} = \frac{\lambda}{\lambda_c} = \frac{\omega_c}{\omega}$$

l'angolo di incidenza dell'onda nella guida è fissato dalla frequenza

$$\text{se } \lambda > \lambda_c \quad (f < f_c) \quad \cos\theta > 1 \quad \text{impossib.}$$

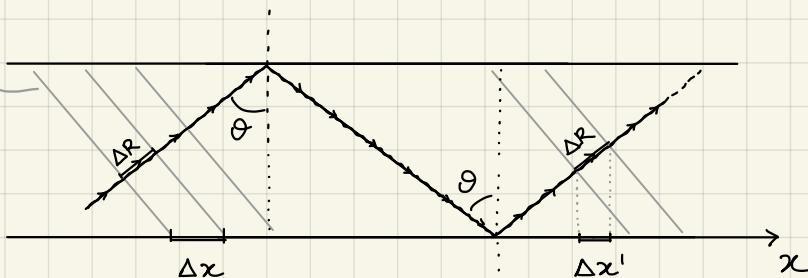
$$\text{se } \lambda = \lambda_c = \frac{2a}{m} \quad \cos\theta = 1 \quad \theta = 0^\circ$$

l'onda rimbomba
ma non va avanti

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

piano
equifase

$\Delta R = v \cdot \Delta t$ spostamento del fronte d'onda lungo la direzione di propagazione



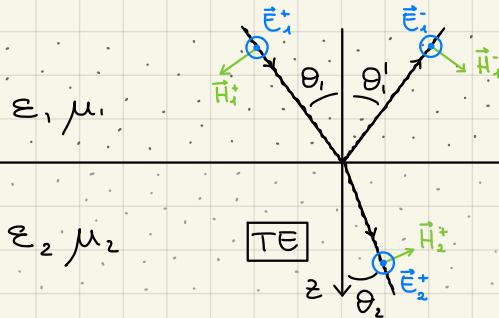
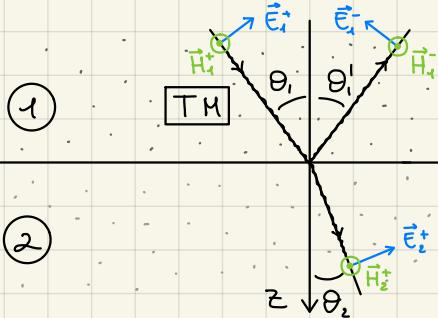
$\Delta x = \frac{\Delta R}{\sin\theta}$ spostamento delle superfici equifase lungo l'asse x (guida)

$$v_f = \frac{\Delta x}{\Delta t} = \frac{\Delta R}{\Delta t} \cdot \frac{1}{\sin\theta} = \frac{v}{\sin\theta} = \frac{v}{\sqrt{1-\cos^2\theta}} = \frac{v}{\sqrt{1-(\frac{\omega_c}{\omega})^2}} \quad \text{velocità di fase}$$

$\Delta x' = \Delta R \operatorname{sen}\theta$ spostamento del fronte d'onda lungo l'asse x (guida)

$$v_g = \frac{\Delta x'}{\Delta t} = v \cdot \sqrt{1 - \cos^2\theta} = v \cdot \sqrt{1 - (\frac{\omega_c}{\omega})^2} \quad \text{velocità di gruppo}$$

Incidenza obliqua su discontinuità tra dielettrici



$$K = \omega \sqrt{\mu \epsilon}$$

$$n = \sqrt{\mu_r \epsilon_r}$$

Condizioni al contorno: $E_{t_1} = E_{t_2}$ $H_{t_1} = H_{t_2}$ in $z = 0$

$$\rightarrow K_1 \sin \theta_1 = K_2 \sin \theta'_1 = K_2 \sin \theta_2$$

$$\theta_1 = \theta'_1$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

leggi di Snell

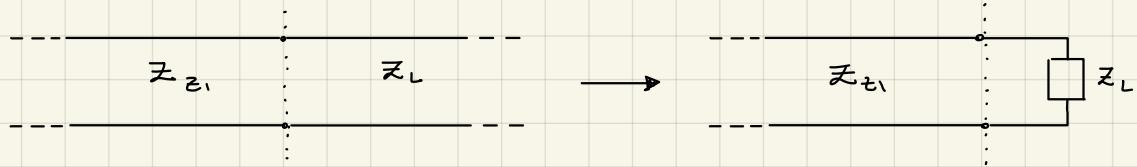
TM Condizioni al contorno ($z = 0$):

$$\begin{cases} E_{x_1}^+ + E_{x_1}^- = E_{x_2}^+ \\ H_{y_1}^+ + H_{y_1}^- = H_{y_2}^+ \end{cases} \quad \begin{cases} E_{x_1}^+ + E_{x_1}^- = E_{x_2}^+ \\ \frac{E_{x_1}^+}{Z_{z_1}} - \frac{E_{x_1}^-}{Z_{z_1}} = \frac{E_{x_2}^+}{Z_L} \end{cases}$$

Definiamo l'IMPEDENZA D'ONDA:

$$\begin{cases} Z_{z_1} = \frac{E_{x_1}^+}{H_{y_1}^+} = \frac{E_1^+ \cos \theta_1}{H_1^+} = \eta_1 \cos \theta_1 = -\frac{E_{x_1}^-}{H_{y_1}^-} \\ Z_L = \frac{E_{x_2}^+}{H_{y_2}^+} = \frac{E_2^+ \cos \theta_2}{H_2^+} = \eta_2 \cos \theta_2 = \eta_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1} \end{cases}$$

$$\begin{cases} E_{x_1}^- = \Gamma \cdot E_{x_1}^+ \text{ con } \Gamma = \frac{Z_L - Z_{z_1}}{Z_L + Z_{z_1}} \\ E_{x_2}^+ = T \cdot E_{x_1}^+ \text{ con } T = \frac{2Z_L}{Z_L + Z_{z_1}} = 1 + \Gamma \end{cases}$$



Campi totali ①

$$\begin{cases} E_x = E_1^+ \cos \theta_1 e^{-j \beta_{x_1} x} [e^{-j \beta_{x_1} z} + \Gamma e^{j \beta_{x_1} z}] \\ E_z = -E_1^+ \sin \theta_1 e^{-j \beta_{x_1} x} [e^{-j \beta_{x_1} z} - \Gamma e^{j \beta_{x_1} z}] \\ H_y = \frac{E_1^+}{\eta_1} e^{-j \beta_{x_1} x} [e^{-j \beta_{x_1} z} - \Gamma e^{j \beta_{x_1} z}] \end{cases}$$

ONDA STAZIONARIA

$$\beta_{x_1} = K_1 \sin \theta_1$$

$$\beta_{z_1} = K_1 \cos \theta_1$$

TE Condizioni al contorno $z=0$:

$$\begin{cases} E_{y_1}^+ + E_{y_1}^- = E_{y_2}^+ \\ H_{x_1}^+ + H_{x_1}^- = H_{x_2}^+ \end{cases}$$

IMPEDIMENTI D'ONDA:

$$\begin{cases} Z_{z_1} = \frac{E_{y_1}^+}{H_{x_1}^+} = \frac{E_i^+}{H_i^+ \cos \theta_i} = \frac{\eta_1}{\cos \theta_i} \\ Z_L = \frac{\eta_2}{\cos \theta_2} = \eta_2 \left[1 - \left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_i \right]^{-1/2} \end{cases}$$

$$\Gamma = \frac{E_y^-}{E_{y_1}^+} = \frac{Z_L - Z_{z_1}}{Z_L + Z_{z_1}} \quad T = \frac{E_{y_2}^+}{E_{y_1}^+} = \frac{2Z_L}{Z_L + Z_{z_1}}$$

Campi totali ①

$$\begin{cases} E_y = E_i^+ e^{-j\beta_{x_1} x} [e^{-j\beta_{z_1} z} + \Gamma e^{j\beta_{z_1} z}] \\ H_x = - \frac{E_i^+}{\eta_1} \cos \theta_i e^{-j\beta_{x_1} x} [e^{-j\beta_{z_1} z} - \Gamma e^{j\beta_{z_1} z}] \\ H_z = \frac{E_i^+ \eta_1}{\eta_1} \sin \theta_i e^{j\beta_{x_1} x} [e^{-j\beta_{z_1} z} + \Gamma e^{j\beta_{z_1} z}] \end{cases}$$

Riflessione totale : $|\Gamma| = 1 \quad Z_L = 0, \infty, j\infty$

Z_L immaginaria se

cand. ideale cand. magnetico
ideale

$$1 - \left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_i \leq 0$$

$\sin \theta_i \geq \frac{n_2}{n_1}$ (si verifica solo se $\frac{n_2}{n_1} \leq 1$)

$$\theta_c = \arccos \left(\frac{n_2}{n_1} \right)$$

θ_c : angolo critico

$|\Gamma| = 1 \rightarrow$ non c'è passaggio di densità di potenza dal mezzo 1 al mezzo 2, ma i campi nel mezzo 2 sono nulli ($T = 1 + \Gamma \neq 0$)

Nel mezzo ② c'è la cosiddetta "onda evanescente"

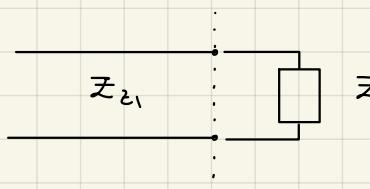
$$\beta_{z_2} = K_2 \cos \theta_2 = K_2 \sqrt{1 - \left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_i} = - j K_2 \underbrace{\sqrt{\left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_i - 1}}_{\alpha_{z_2}}$$

$e^{j\beta_{z_2} z} \Rightarrow e^{-\alpha_{z_2} z}$ l'onda si consuma meno a mano che si propaga

Trasmissione (Rifrazione) Totale: $\Gamma = 0$ $Z_L = Z_{z_1}$

$$\varepsilon_1, \varepsilon_2 \quad \mu_1 = \mu_2 = \mu_0$$

Incidenza TM



$$Z_L \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\eta_1 \cos \theta_1 = \eta_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1}$$

$$\cos \theta_1 = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_1}$$

$$\theta_p = \arccos \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} = \operatorname{arctg} \left(\frac{n_2}{n_1} \right)$$

θ_p : angolo di BREWSTER

Teoria della Radiazione

Obiettivo: un algoritmo SEMPLICE per il calcolo di \vec{E} e \vec{H} .

→ SORGENTI SEMPLICI (da determinare)

- 1) Eq. di Maxwell (incluso le sorgenti)
- 2) Ottieniamo l'eq. $\nabla^2 \vec{E} + \beta^2 \vec{E} = \text{sorgenti}$ ("complesse")
- 3) Troviamo la soluzione per sorgenti puntiformi elementari.
- 4) Introduciamo \vec{A} (potenziale vettore) e ottieniamo $\nabla^2 \vec{A} + \beta^2 \vec{A} = \text{sorgenti}$ ("semplici")
- 5) Ottieniamo \vec{E} ed \vec{H} da \vec{A}
- 6) Sorgente elementare: dipolo hertziano
- 7) Sorgenti composte

Definizione dei fioroni

$$\left\{ \begin{array}{l} \bar{\nabla} \times \bar{E} = -j\omega \mu \bar{H} \\ \bar{\nabla} \times \bar{H} = j\omega \bar{E} + \bar{J} \quad \text{con} \quad \bar{J} = \sigma \bar{E} + \bar{J}_x \\ \bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon} \\ \bar{\nabla} \cdot \bar{H} = 0 \\ \nabla \cdot \bar{J} = -j\omega \rho \end{array} \right. \quad \beta = \frac{\omega}{c} = \omega \sqrt{\mu \epsilon}$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -j\omega \mu \bar{\nabla} \times \bar{H} = -j\omega \mu (j\omega \epsilon \bar{E} + \bar{J})$$

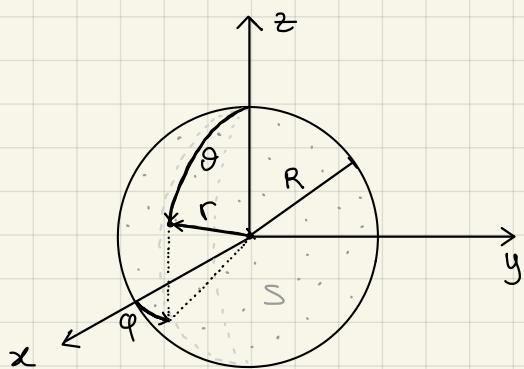
$$\text{ma } \bar{\nabla} \times \bar{\nabla} \times \bar{E} = \bar{\nabla} (\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E}$$

$$\rightarrow \underbrace{\nabla^2 \bar{E} + \beta^2 \bar{E}}_{(\Delta)(\cdot)} = \bar{\nabla} \left(\frac{\rho}{\epsilon} \right) + j\omega \mu \bar{J} = -\underbrace{\frac{1}{j\omega \epsilon} \bar{\nabla} (\bar{\nabla} \cdot \bar{J})}_{\text{sorgenti}} + j\omega \mu \bar{J}$$

$$(\Delta)(\cdot) = \nabla^2(\cdot) + \beta^2(\cdot) \quad \text{d'Alambertiano}$$

$$[\nabla^2 \psi + \beta^2 \psi = \text{sorgente puntiforme}] \quad (\psi: \text{generica componeente di } \bar{E})$$

Caso statico $\omega = 0 \quad \beta = 0 \rightarrow \nabla^2 \psi = \text{sorgente puntiforme}$



$$\nabla^2 \psi(r) = -s \quad r \leq R$$

$$\nabla^2 \psi(r) = 0 \quad r > R \quad (\text{no sorgenti})$$

$$\psi(r, \varphi, \theta) \rightarrow \psi(r) \quad (\text{simmetria sferica})$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right)$$

Soluzione interno ($r \leq R$)

$$\nabla^2 \psi = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = -s \quad d \left(r^2 \frac{d\psi}{dr} \right) = -sr^2 dr$$

$$r^2 \frac{d\psi}{dr} = -s \frac{r^3}{3} + A \quad \frac{d\psi}{dr} = -\frac{sr^2}{3} + \frac{A}{r^2}$$

$$\boxed{\psi(r) = -\frac{sr^2}{6} - \frac{A}{r} + B}$$

Soluzione esterna ($r > R$)

$$\frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = 0 \quad r^2 \frac{d\psi}{dr} = C \quad d\psi = \frac{C}{r^2} dr$$

$$\boxed{\psi(r) = -\frac{C}{r} + D}$$

Condizioni al contorno

- se $r \rightarrow 0$ $\psi(r)$ finito $\Rightarrow A = 0$
- se $r \rightarrow +\infty$ $\psi(r) \rightarrow 0 \Rightarrow D = 0$

Impongo la continuità di $\psi(r)$ e $\frac{d\psi}{dr}(r)$ in $r=R$

$$\Rightarrow \begin{cases} \psi(r) = s \left(\frac{1}{2} R^2 - \frac{1}{6} r^2 \right) & r \leq R \\ \psi(r) = \frac{s R^3}{3r} & r > R \end{cases}$$

VOLUME della sfera vale $V = \frac{4}{3} \pi R^3$

$$\psi(r) = \frac{V \cdot s}{4\pi r} \quad \text{se } V \rightarrow 0 \quad \text{ma } \underbrace{V \cdot s}_{\substack{\text{densità di carica} \\ \text{carica totale}}} = \text{costante}$$

Se ad esempio:

$$\nabla^2 \psi = -\frac{p}{\epsilon_0} \quad \text{cioè} \quad s = \frac{p}{\epsilon_0} \quad \rightarrow \quad \frac{V \cdot s}{4\pi \epsilon_0 r} = \frac{Q}{4\pi \epsilon_0 r}$$

POTENZIALE ELETROSTATICO

Caso dinamico

Soluzione esterna ($r > R$)

$$\nabla^2 \psi + \beta^2 \psi = 0$$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) + \beta^2 \psi = 0 \quad \text{poniamo} \quad X = r \cdot \psi \\ \text{ossia} \quad \psi = \frac{X}{r}$$

$$\frac{d\psi}{dr} = \frac{1}{r} \frac{dX}{dr} - \frac{X}{r^2}$$

$$r^2 \frac{d\psi}{dr} = r \frac{dX}{dr} - X$$

$$\frac{d}{dr} \left(r \frac{dX}{dr} - X \right) = \frac{dX}{dr} + r \frac{d^2X}{dr^2} - \frac{dX}{dr}$$

$$\frac{1}{r} \frac{d^2 X}{dr^2} + \beta^2 \frac{X}{r} = 0$$

$$\frac{d^2 X}{dr^2} + \beta^2 X = 0$$

$$X(r) = M e^{-j\beta r} + N e^{+j\beta r}$$

soluzione "centrifuga" e "centripeta"

Condizioni al contorno

- ✓ soluzione centripeta (è anticausale) $\Rightarrow N = 0$

Caso dinamico tende al caso statico per $f \rightarrow 0$
cioè $\beta \rightarrow 0$

$$X(r) = M e^{-j\beta r} \rightarrow \Psi(r) = \frac{M e^{-j\beta r}}{r}$$

$$\lim_{\beta \rightarrow 0} \Psi(r) = \frac{M}{r} = \frac{sV}{4\pi r} \quad \Rightarrow \quad M = \frac{sV}{4\pi r}$$



$$\Psi(r) = \frac{sV e^{-j\beta r}}{4\pi r}$$

potenziale
ritardato

Onda sferica:

$$* |\Psi(r)| \propto \frac{1}{r}$$

* le sup. equifase sono delle sfere

Condizione di Sommerfeld:

qualsiasi effetto elettromagnetico Ψ generato da una distribuzione di sorgenti di estensione limitata, deve soddisfare la seguente condizione:

$$\lim_{r \rightarrow +\infty} r \cdot \left(\frac{\partial \Psi}{\partial r} + j\beta \Psi \right) = 0$$

La soluzione centripeta NON soddisfa questa condizione.

Metodo dei potenziali

$\bar{H} \cdot \bar{H} = 0$ possiamo scrivere $\bar{H} = \bar{\nabla} \times \bar{A}$ poiché $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0$

$$\bar{\nabla} \times \bar{E} = -j\omega \mu \bar{H} = -j\omega \mu \bar{\nabla} \times \bar{A} \quad \bar{\nabla} \times (\bar{E} + j\omega \mu \bar{A}) = 0$$

sorgenti vettoriali di \bar{A}

\bar{F}

$$\bar{E} + j\omega \mu \bar{A} = -\bar{\nabla} \phi$$

$\bar{\nabla} \times \bar{F} = 0$ possiamo scrivere $\bar{F} = -\bar{\nabla} \phi$ poiché $\bar{\nabla} \times (\bar{\nabla} \phi) = 0$

$$\bar{\nabla} \times \bar{H} = j\omega \varepsilon \bar{E} + \bar{J}$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{A} = j\omega \varepsilon (-j\omega \mu \bar{A} - \bar{\nabla} \phi) + \bar{J}$$

$$\text{ma } \bar{\nabla} \times \bar{\nabla} \times \bar{A} = \bar{\nabla} (\bar{\nabla} \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\rightarrow \bar{\nabla} (\bar{\nabla} \cdot \bar{A}) - \nabla^2 \bar{A} = \beta^2 \bar{A} - j\omega \varepsilon \bar{\nabla} \phi + \bar{J} \quad \xrightarrow{\text{sorgenti puro di } \bar{A}}$$

Possiamo fissare a piacere $\bar{\nabla} \cdot \bar{A}$: $\bar{\nabla} \cdot \bar{A} = -j\omega \varepsilon \phi$

$$\bar{\nabla} (\bar{\nabla} \cdot \bar{A}) = -j\omega \varepsilon \bar{\nabla} \phi$$

$$\boxed{\nabla^2 \bar{A} + \beta^2 \bar{A} = -\bar{J}} \quad \rightarrow \quad \boxed{\bar{A}(r) = \frac{\bar{J} \cdot \sqrt{e^{-j\beta r}}}{4\pi r}}$$

$$\textcircled{*} \begin{cases} \bar{H} = \bar{\nabla} \times \bar{A} \\ \bar{E} = -j\omega \mu \bar{A} - \bar{\nabla} \phi = -j\omega \mu \bar{A} + \frac{1}{j\omega \varepsilon} \bar{\nabla} (\bar{\nabla} \cdot \bar{A}) \end{cases}$$

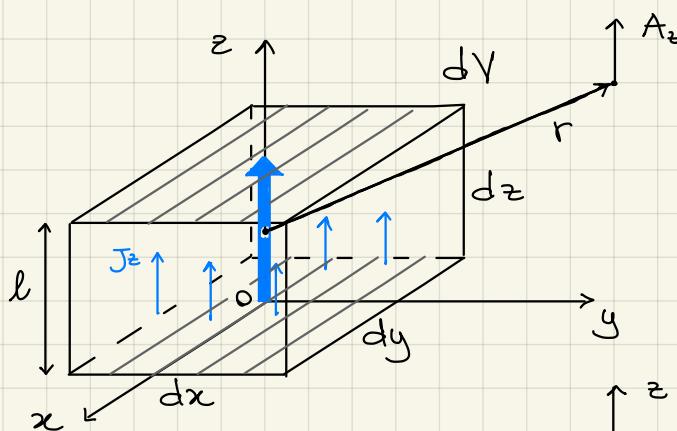
L'altra equazione (che usa la densità di carica invece che la densità di corrente) è:

$$\nabla^2 \phi + \beta^2 \phi = -\frac{J}{\varepsilon}$$

(più difficile da usare perché J è più conosciuto di ϕ)

Sorgente elementare (dipolo elettrico o hertziano)

$$H_p: \bar{J} = J \vec{u}_z$$



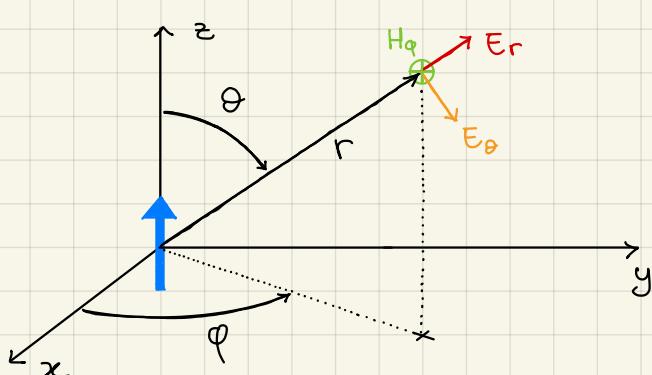
$$\bar{J} dV = J_z dx dy dz \vec{u}_z$$

$$= I dz \vec{u}_z$$

||
l: lunghezza del dipolo

$$\left\{ A_z = \frac{I l e^{-j\beta r}}{4\pi r} \right\}$$

introducendo
in $\textcircled{*}$ ricavo le
componenti di
campo elettrico
e magnetico

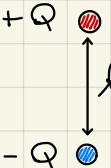


$$\left\{ \begin{array}{l} E_r = \frac{Il}{2\pi} e^{-j\beta r} \left(\frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3} \right) \cos\theta \\ E_\theta = \frac{Il}{4\pi} e^{-j\beta r} \left(j\frac{\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3} \right) \sin\theta \\ H_\phi = \frac{Il}{4\pi} e^{-j\beta r} \left(j\frac{\beta}{r} + \frac{1}{r^2} \right) \sin\theta \end{array} \right.$$

Campo "vicino" $r \rightarrow 0$

$$\left\{ E_r = \frac{Il e^{-j\beta r}}{2\pi j\omega\epsilon r^3} \cos\theta \quad E_\theta = \frac{Il e^{-j\beta r}}{4\pi j\omega\epsilon r^3} \sin\theta \right\}$$

H_θ trascurabile



$$E_r = \frac{Ql \cos\theta}{2\pi\epsilon r^3} \quad E_\theta = \frac{Ql \sin\theta}{4\pi\epsilon r^3} \quad \text{caso statico}$$

$$I = j\omega Q \quad Q = \frac{I}{j\omega} \quad r \rightarrow 0 : \quad \beta r = \frac{2\pi}{\lambda} r \rightarrow 0 \quad e^{-j\beta r} \rightarrow 1$$

$$\left\{ E_r \approx \frac{Ql \cos\theta}{\epsilon 2\pi r^3} \quad E_\theta \approx \frac{Ql \sin\theta}{4\pi\epsilon r^3} \right\}$$

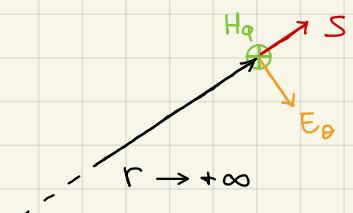
campo \bar{E} quasi-statico

Campo "lontano" $r \rightarrow +\infty$ ($r \gg \lambda$)

$$\left\{ E_\theta = j\frac{\omega\mu Il}{4\pi r} e^{-j\beta r} \sin\theta \quad H_\phi = j\frac{\beta Il}{4\pi r} e^{-j\beta r} \sin\theta \right\}$$

E_r trascurabile

$$E_\theta \perp H_\phi \quad \frac{E_\theta}{H_\phi} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$



Vettore di Poynting : $\bar{S} = \frac{\bar{E} \times \bar{H}^*}{2}$

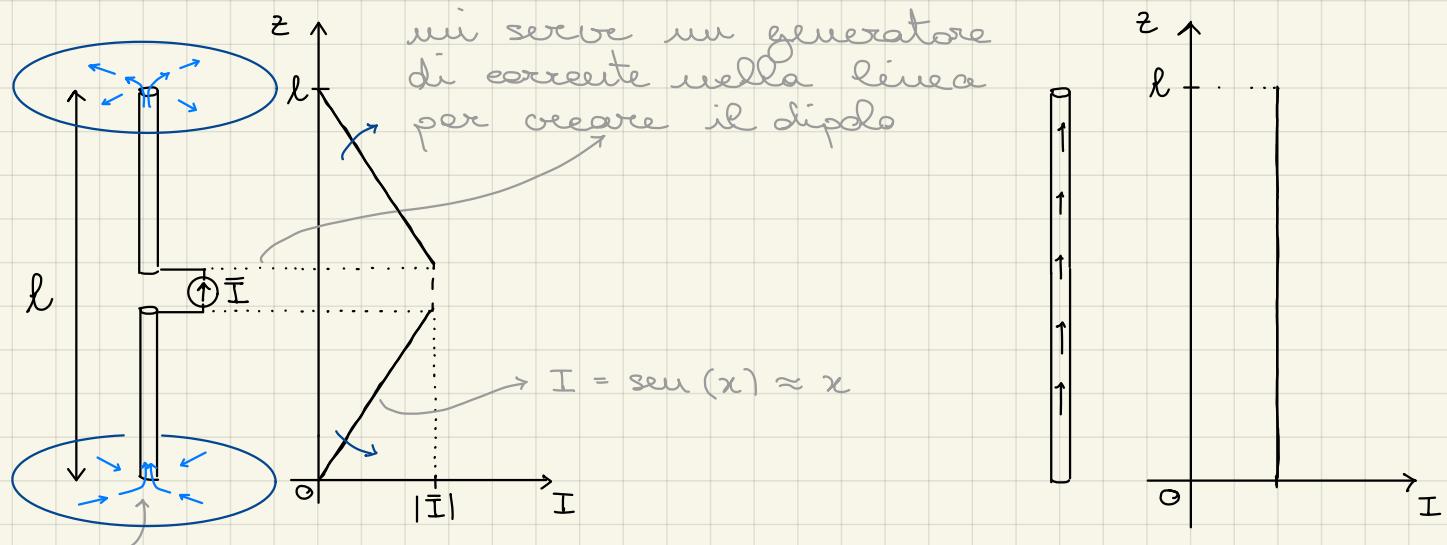
$$\bar{E} \times \bar{H}^* = \det \begin{bmatrix} \vec{\mu}_r & \vec{\mu}_\theta & \vec{\mu}_\phi \\ E_r & E_\theta & 0 \\ 0 & 0 & H_\phi^* \end{bmatrix} = \underbrace{E_\theta H_\phi^*}_{2S_r} \vec{\mu}_r - \underbrace{E_r H_\phi^*}_{2S_\theta} \vec{\mu}_\theta$$

Ricordando che $j\frac{\omega\mu}{4\pi} = j\frac{\eta}{2\lambda}$ risulta:

$$\left\{ \begin{array}{l} S_r = \frac{1}{2} \eta \frac{|Il|^2}{4\lambda^2} \left(\frac{1}{r^2} + \frac{1}{j\beta^3 r^5} \right) \sin^2 \theta \\ S_\theta = \frac{1}{2} \eta \frac{|Il|^2}{8\pi^2} \left(-\frac{j\beta}{r^3} + \frac{1}{j\beta^3 r^5} \right) \sin \theta \cos \theta \end{array} \right. \rightarrow \text{componente solo immaginaria}$$

$$\rightarrow \operatorname{Re}\{\vec{S}\} = \frac{1}{2} \cdot \eta \frac{|Il|^2}{4\lambda^2} \frac{\sin^2 \theta}{r^2} \vec{u}_r = \frac{\epsilon_0 H_0^*}{2} \vec{u}_r$$

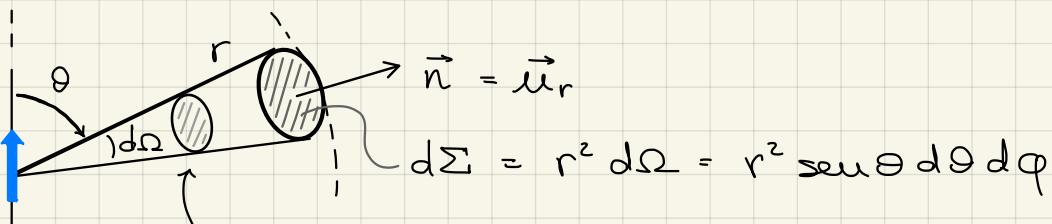
Le uniche componenti che contribuiscono al trasporto di potenza dell'onda sono E_p e H_0 cioè le componenti del campo lontano (che per questo è anche detto "campo di radiazione")



è come
una linea di
trasmissione
dove $l \ll \lambda$

Per rendere il circuito di
sinistra come quello di destra
lo chiudo con un condensatore.

$$[\vec{S}(r, \theta) = \frac{\eta_0}{8} \frac{(Il)^2}{\lambda^2 r^2} \sin^2 \theta \vec{u}_r]$$



$$dP = \vec{S} \cdot d\vec{\Sigma} = \eta_0 \frac{(Il)^2}{8\lambda^2 r^2} \sin^2 \theta r^2 \sin \theta d\theta d\phi$$

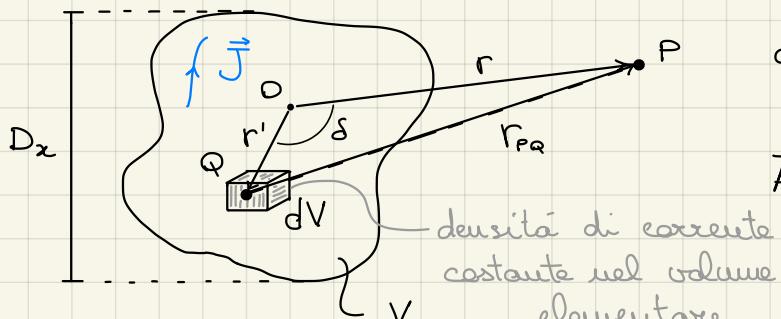
la potenza in un tubo di
flusso è costante

$$P = \oint_{\text{sfera}} \vec{S} \cdot d\vec{\Sigma} = \int_0^{2\pi} d\varphi \int_0^{\pi} \eta_0 \frac{(Il)^2}{8\lambda^2} \sin^3 \theta d\theta = 2\pi \eta_0 \frac{(Il)^2}{8\lambda^2} \int_0^{\pi} \sin^3 \theta d\theta =$$

4/3

$$P = \frac{\pi}{3} \eta_0 I^2 \frac{l^2}{\lambda^2} \quad [\text{W}]$$

Radiazione da un insieme di sorgenti



$$d\bar{A} = \frac{\bar{J}(Q)}{4\pi r_{pq}} dV e^{-j\beta r_{pq}}$$

$$\begin{aligned} \bar{A}(P) &= \int_V d\bar{A} = \text{potenziale vettore totale} = \text{somma} \\ &= \int_V \frac{\bar{J}(Q)}{4\pi r_{pq}} e^{-j\beta r_{pq}} dV \text{ di potenziali vettori element.} \end{aligned}$$

$$\text{Se } r \gg D_z \rightarrow r_{pq} \approx r - \underbrace{r' \cos \delta}_{\ll r} \quad (\text{poiché } r_{pq} \parallel r \text{ circa})$$

$$\begin{aligned} \bar{A}(P) &= \int_V \frac{\bar{J}(Q)}{4\pi r} e^{-j\beta r} e^{+j\beta r' \cos \delta} dV = \underbrace{\frac{e^{-j\beta r}}{4\pi r} \int_V \bar{J}(Q) e^{+j\beta r' \cos \delta} dV}_{(I \cdot l) [\text{A} \cdot \text{m}]} = \\ &= \underbrace{\bar{N}(\theta, \varphi)}_{4\pi r} \frac{e^{-j\beta r}}{4\pi r} \quad \text{momento di dipolo} \end{aligned}$$

vettore di radiazione

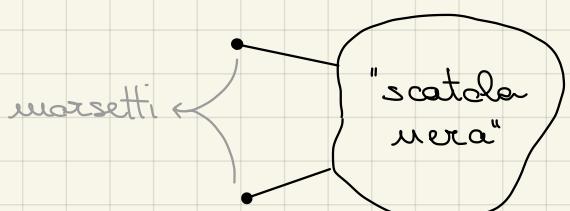
Si dimostra che il campo è di radiazione vale

$$\bar{E} = -j\omega \mu_0 \frac{e^{-j\beta r}}{4\pi r} [N_\theta(\theta, \varphi) \hat{u}_\theta + N_\varphi(\theta, \varphi) \hat{u}_\varphi]$$

$$\bar{E} = \eta_0 (\bar{H} \times \hat{u}_r)$$

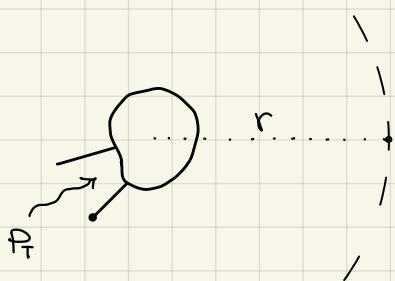
$$\bar{H} = \frac{j\beta}{4\pi r} e^{-j\beta r} [N_\varphi(\theta, \varphi) \hat{u}_\theta - N_\theta(\theta, \varphi) \hat{u}_\varphi]$$

Antenne come TRASDUTTORI → convertono una forma di energia in un'altra



- {
- Passive
- Reciproche

Antenna isotropa (ideale, senza perdite - non esiste)



$$S(r) = \frac{P_T}{4\pi r^2} \left[\frac{W}{m^2} \right] \quad (S \text{ isotropa} \\ \text{ cioè una dipendenza da } \theta \text{ e } \varphi)$$

$$\oint_{\text{sfera}} S(r) \cdot d\Sigma = P_T$$

$$\text{Infatti: } \oint_{\text{sfera}} S(r) \cdot r^2 d\Omega = \oint_{\text{sfera}} \frac{P_T}{4\pi r^2} \cdot r^2 d\Omega = \frac{P_T}{4\pi} \oint_{\text{sfera}} d\Omega = \frac{P_T}{4\pi} \cdot 4\pi = P_T$$

Le antenne reali sono direttive:

$$S(r, \theta, \varphi) = \frac{P_T}{4\pi r^2} f(\theta, \varphi) \cdot D$$

$f(\theta, \varphi)$: funzione di direttività

D : direttività

$$0 \leq f(\theta, \varphi) \leq 1 \quad \exists \text{ (almeno) una direzione } (\bar{\theta}, \bar{\varphi}) \text{ t.c.} \\ f(\bar{\theta}, \bar{\varphi}) = 1 \\ \text{(direzione di massima radiazione)}$$

$$\text{Per } \bar{\theta}, \bar{\varphi}: \quad S_{\text{MAX}}(r, \bar{\theta}, \bar{\varphi}) = \frac{P_T}{4\pi r^2} D \quad S_{\text{iso}} = \frac{P_T}{4\pi r^2}$$

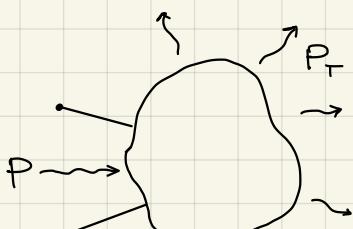
$$\frac{S_{\text{MAX}}}{S_{\text{iso}}} = D (> 1) \text{ ma vale sempre } \oint_{\text{sfera}} S(r, \theta, \varphi) d\Sigma = P_T$$

$$\oint_{\text{sfera}} \frac{P_T}{4\pi r^2} f(\theta, \varphi) D \cdot d\Sigma = P_T$$

$$\left[D = \frac{4\pi}{\oint_{\text{sfera}} f(\theta, \varphi) d\Omega} \right] \leftarrow \frac{D}{4\pi} \oint_{\text{sfera}} f(\theta, \varphi) d\Omega = 1$$

sempre valida

\exists sempre perdite:



$$P_T \leq P$$

$$P_T = \gamma P$$

con $0 \leq \gamma \leq 1$ rendimento

$$S(r, \theta, \varphi) = \underbrace{\frac{P}{4\pi r^2}}_G \gamma D f(\theta, \varphi)$$

$\gamma \cdot D = G \geq 0$ guadagno

Di solito il guadagno è espresso in dB_I rispetto all'isotropo

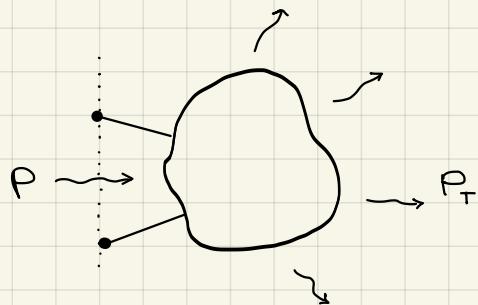
$$10 \log_{10} \frac{G}{G_{\text{iso}}} = 10 \log_{10} G$$

rispetto al dipolo risonante

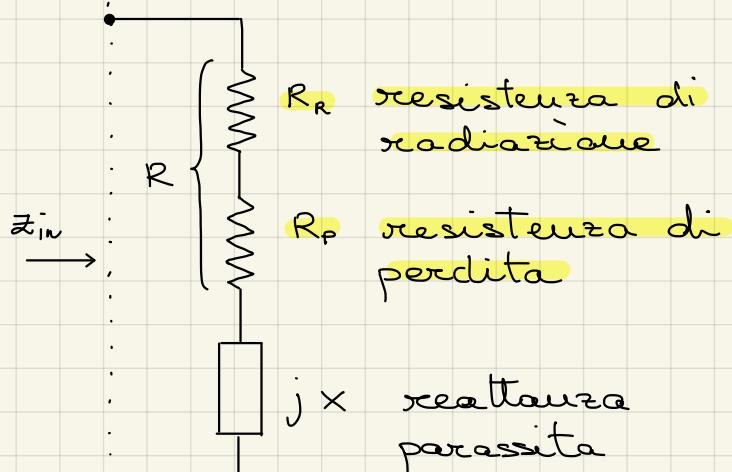
$$A \text{ volte in } dB_d \rightarrow 10 \log_{10} \frac{G}{1,64}$$

Parametri delle antenne: D , $f(\theta, \varphi)$, ν , G , R , A_e , le

antenna
trasmettente



equivalente circuituale



$$\gamma = \frac{P_T}{P} \quad \text{ma} \quad P_T = \frac{|I|^2}{2} R_R$$

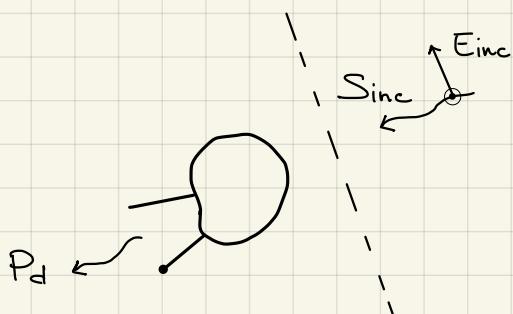
$$P_p = \frac{|I|^2}{2} R_p$$

$$P = P_T + P_p$$

$$Z_{in} = R + jX$$

il rendimento è solitamente maggiore per antenne più grandi poiché R_R cresce più rapidamente di R_p al crescere delle dimensioni

Antenne riceventi



- onda incidente è TEM (localmente) piatta
- c'è adattamento di polarizzazione

$$P_R = P_d = S_{inc} A_e \cdot f_R(\theta, \varphi)$$

$$\left[\frac{W}{m^2} \right] \left[m^2 \right]$$

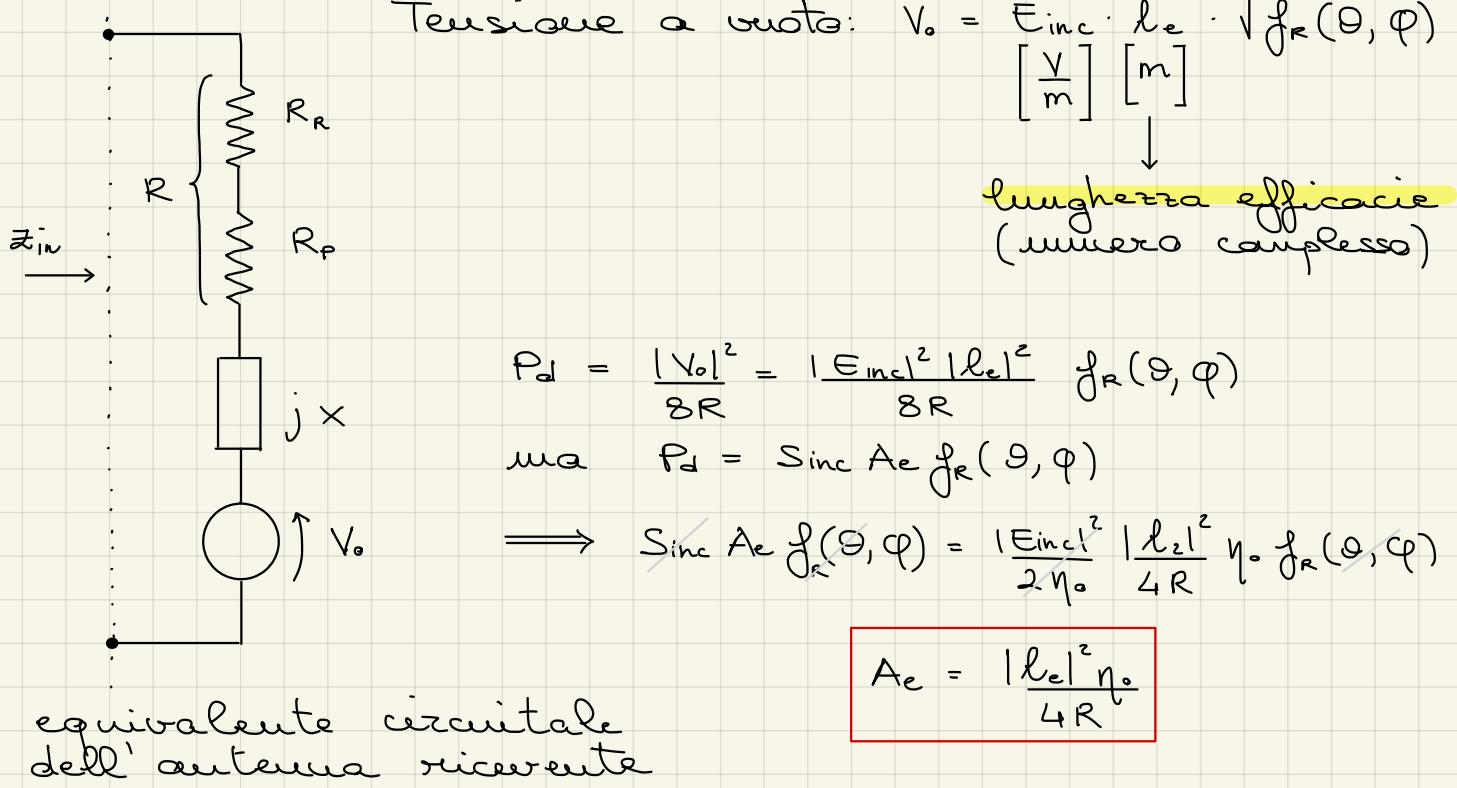
area efficace

Per antenne reciproche si dimostra che:

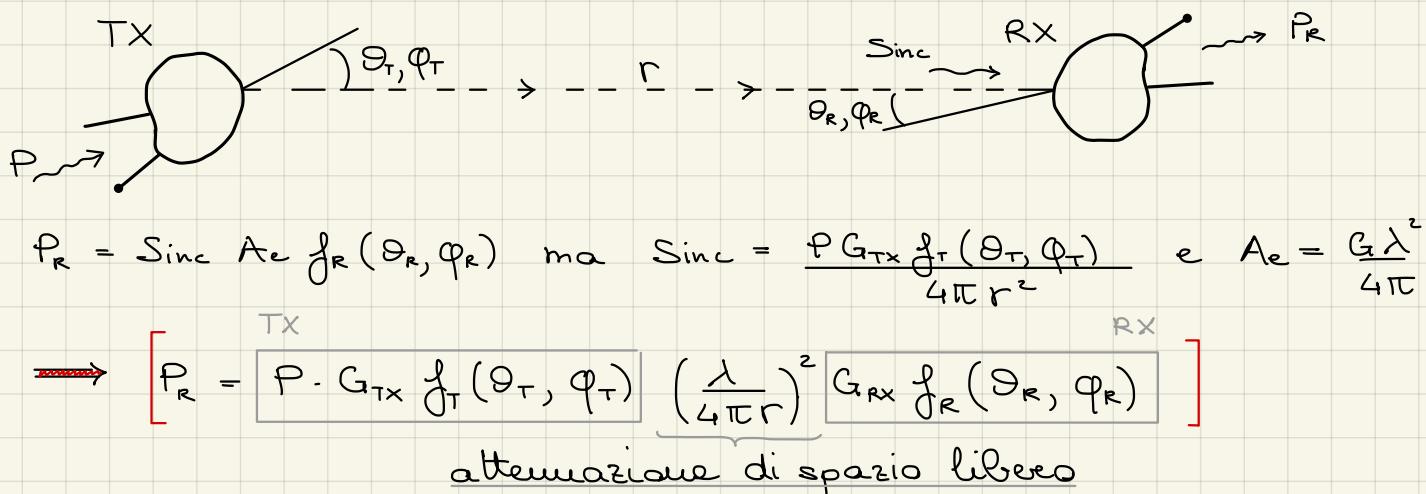
$$f_R(\theta, \varphi) = f_T(\theta, \varphi)$$

Relazione universale:

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2}$$



Link Budget (spazio libero)



Se le antenne sono "ben puntate": $f_T(\theta_T, \varphi_T) = 1$

$$f_R(\theta_R, \varphi_R) = 1$$

$$P_r = \underbrace{P G_{Tx}}_{\text{E.I.R.P.}} \left(\frac{\lambda}{4\pi r} \right)^2 G_{Rx}$$

E.I.R.P. [W] = Equivalent
Isotropic
Radiated
Power

(ad es. WiFi)
EIRP = 0,1 W

Parametri del dipolo hertziano (ideale, $\nu = 1$)

$$S = \frac{\eta_0 (Il)^2}{8\lambda^2 r^2} \sin^2 \theta \quad \text{e} \quad P_T = \frac{\pi}{3} \eta_0 \frac{(Il)^2}{\lambda^2}$$

$$\implies S = \frac{P_T}{4\pi r^2} \cdot \frac{3}{2} \cdot \sin^2 \theta \quad \text{confronto con } S = \frac{P_T}{4\pi r^2} D f(\theta, \varphi)$$

densità di potenza
del dipolo

densità di potenza
di un'antenna

Per il dipolo hertziano :

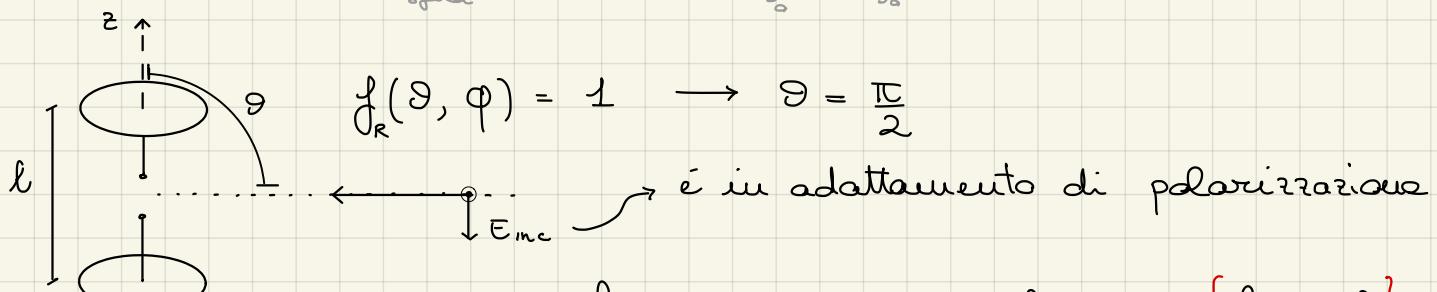
$$\left\{ \begin{array}{l} f(\theta, \varphi) = \sin^2 \theta \\ D = \frac{3}{2} = G \quad (\nu = 1) \end{array} \right.$$

$R_R \propto l$
 $R_R \propto l^2$

$$R_R \rightarrow P_T = \frac{|Il|^2}{2} R_R = \frac{\pi}{3} \eta_0 \frac{(Il)^2}{\lambda^2} \rightarrow \left\{ R_R = \frac{2}{3} \pi \eta_0 \left(\frac{l}{\lambda} \right)^2 \right\}$$

NB: potevo anche ricavare D usando la formula

$$D = \frac{4\pi}{\iint_{\text{sfera}} f(\theta, \varphi) d\Omega} = \frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^\pi \sin^2 \theta \sin \theta d\theta}$$



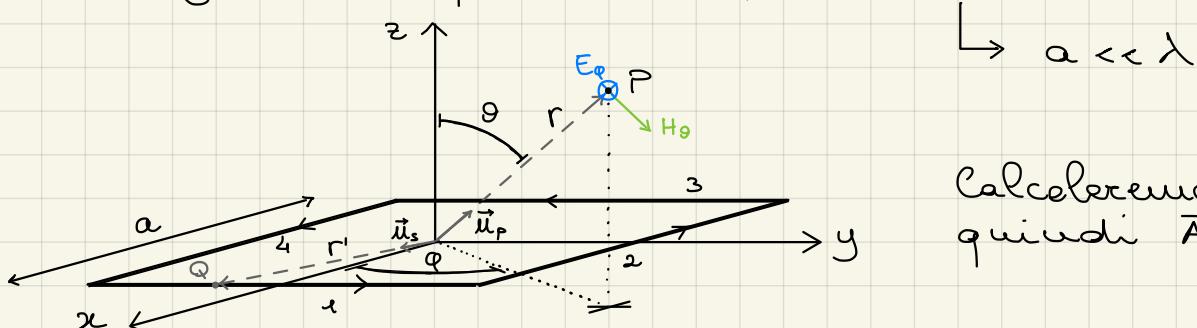
$$V_o = E_{\text{inc}} \cdot l \quad \text{ma} \quad V_o = E_{\text{inc}} \cdot l_e \rightarrow \{ l_e = l \}$$

$$\left\{ A_e = \frac{|l_e|^2 \eta_0}{4 R_R} = \frac{|l|^2 \eta_0}{4 \cdot \frac{2}{3} \pi \eta_0 \left(\frac{l}{\lambda} \right)^2} = \frac{3 \lambda^2}{8 \pi} \right\} \quad [\text{m}^2]$$

Potevo ricavare A_e dalla formula universale

$$\frac{A_e}{G} = \frac{\lambda^2}{4\pi}, \quad G = \frac{3}{2}$$

Sorgente composta (spira piccola)



Calcoleremo $\bar{N}(\theta, \varphi)$ e quindi \bar{A} in P .

$$\bar{N}(\theta, \varphi) = \int_V \bar{j}(Q) e^{+j\beta r' \cos \delta} dV \quad \text{con} \quad \cos \delta = \vec{\mu}_p \cdot \vec{\mu}_s$$

Se la spira è piccola: $r' \ll r$, $r' \ll a \rightarrow r' \ll \lambda$
 $\Rightarrow e^{+j\beta r' \cos \delta} \approx 1 + j\beta r' \cos \delta$ e $r' \approx \frac{a}{2}$

$$\bar{N}(\theta, \varphi) = \sum_{i=1}^4 \underbrace{\bar{I}_i \cdot a}_{\bar{j}(Q) \cdot dV} \left(1 + j\beta \frac{a}{2} \cdot \vec{\mu}_p \cdot \vec{\mu}_{s_i} \right)$$

$$\vec{\mu}_p = \sin \theta \cos \varphi \vec{\mu}_x + \sin \theta \sin \varphi \vec{\mu}_y + \cos \theta \vec{\mu}_z$$

lato	\vec{I}_i	$\vec{\mu}_{s_i}$	$\vec{\mu}_{s_i} \cdot \vec{\mu}_p$
1	$I \vec{\mu}_y$	$\vec{\mu}_x$	$\sin \theta \cos \varphi$
2	$-I \vec{\mu}_x$	$\vec{\mu}_y$	$\sin \theta \sin \varphi$
3	$-I \vec{\mu}_y$	$-\vec{\mu}_x$	$-\sin \theta \cos \varphi$
4	$I \vec{\mu}_x$	$-\vec{\mu}_y$	$-\sin \theta \sin \varphi$

$$\Rightarrow \bar{N}(\theta, \varphi) = j\beta a^2 I \sin \theta \underbrace{(-\sin \varphi \vec{\mu}_x + \cos \varphi \vec{\mu}_y)}_{\vec{\mu}_p} = j\beta a^2 I \sin \theta \vec{\mu}_p$$

$$\left\{ \bar{A}(r, \theta, \varphi) = \bar{N}(\theta, \varphi) \frac{e^{-j\beta r}}{4\pi r} = j\beta a^2 I \sin \theta \frac{e^{-j\beta r}}{4\pi r} \vec{\mu}_p \right\}$$

Sostituendo \bar{A} nelle equazioni $\#$ ricava \bar{E} e \bar{H} :

$$\left\{ \begin{array}{l} H_r = j \frac{\omega \mu I S}{2\pi \eta_0} \left(\frac{1}{r^2} - \frac{j}{\beta r^3} \right) \cos \theta e^{-j\beta r} \\ H_\theta = j \frac{\omega \mu I S}{4\pi \eta_0} \left(\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right) \sin \theta e^{-j\beta r} \\ E_\varphi = -j \frac{\omega \mu I S}{4\pi} \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) \sin \theta e^{-j\beta r} \end{array} \right.$$

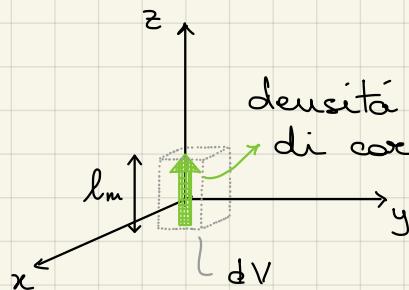
con S: area della spira "piccola"

sotto gli stessi
per una spira
di forma
qualsiasi

I campi elettrico e magnetico sono invertiti rispetto al dipolo hertziano.

Per questo si dice che dipolo e spira sono sorgenti duali.

La spira è equivalente a un dipolo attraversato da corrente magnetica



densità volumetrica

di corrente magnetica $\bar{J}_m \left[\frac{V}{m^2} \right]$

$$\bar{J}_m \cdot dV = V_0 \cdot l_m$$

$\left[\frac{V}{m^2} \cdot m^3 \right] = V \cdot m$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H} (-\bar{J}_m)$$

$$\nabla \times \vec{H} = j\omega \epsilon \vec{E} + \bar{J}$$

momento di dipolo magnetico

Campo lontano

$$\left\{ H_0 = j\frac{\omega \mu I S}{4\pi \eta_0} \left(\frac{j\beta}{r} \right) \sin \theta e^{-j\beta r} \quad E_0 = -j\frac{\omega \mu I S}{4\pi} \left(\frac{j\beta}{r} \right) \sin \theta e^{-j\beta r} \right\}$$

$$\operatorname{Re} \left\{ \frac{\bar{S}}{2} \right\} = \frac{1}{2} E_0 \cdot H_0^*$$

il vettore di Poynting dipende solo dalle componenti di campo di radiazione

$$\frac{E_0}{H_0} = -\eta_0$$

$$\frac{1}{2} \operatorname{Re} \left\{ \frac{\bar{S}}{2} \right\} = \frac{1}{2} \frac{|E_0|^2}{\eta_0} = \frac{1}{2} \eta_0 |H_0|^2 \quad (\text{come nel dipolo})$$

funzione di direttività

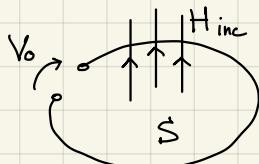
direttività

area efficace

$$\left\{ f(\theta) = \sin^2 \theta \right\} \rightarrow \left\{ D = \frac{3}{2} \right\} \rightarrow \left\{ A_e = D \cdot \frac{\lambda^2}{2\pi} = \frac{3\lambda^2}{8\pi} \right\}$$

$$E_{\text{inc}} \cdot l_e = V_0$$

$$V_0 = j\omega \mu H_{\perp} S$$



Adattamento di polarizzazione
 $H_{\text{inc}} \perp$ spira

$$\text{ma } H_{\perp} = H_{\text{inc}} = \frac{E_{\text{inc}}}{\eta_0}$$

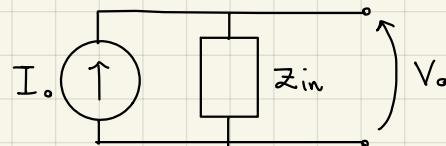
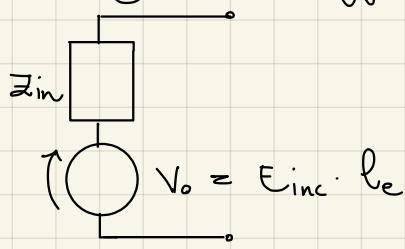
$$|e| = \frac{d\phi(\vec{B})}{dt}$$

$$V_0 = j\omega \mu \frac{E_{\text{inc}}}{\eta_0} S$$

$$\rightarrow \left\{ l_e = j\frac{\omega \mu S}{\eta_0} = j\beta S \right\}$$

lunghezza efficacia (elettrica)

lunghezza efficace magnetica l_m



$$I_o = H_{inc} l_m$$

$$A = \frac{A}{m} \cdot m$$

$$I_o Z_{in} = V_o \quad H_{inc} l_m \cdot Z_{in} = E_{inc} \cdot l_e \quad \text{ma} \quad H_{inc} = \frac{E_{inc}}{\eta_0}$$

$$\frac{l_e}{l_m} = \frac{Z_{in}}{\eta_0}$$

Ora dimostreremo che: $j\omega\mu I S = V l_m$

$$j\omega\mu I S = j\omega\mu \frac{V_o}{Z_{in}} S = j\omega\mu \frac{l_m}{l_e \eta_0} V_o S = j\beta S \frac{l_m}{l_e} V_o = l_m V_o$$

$$\rightarrow l_e = j\beta S$$

$$\left\{ R_R = \frac{l_e l^2 \eta_0}{4 A_e} = \eta_0 \frac{8\pi^3}{3} \left(\frac{S}{\lambda^2} \right)^2 \right\} [\Omega] \quad \text{resistenza di radiazione}$$

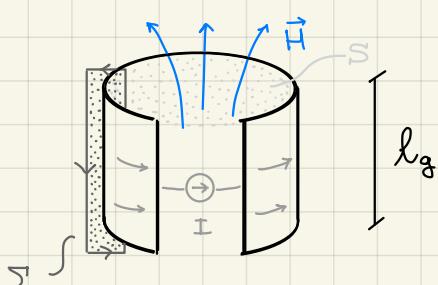
$Z_{in} = R_R + jX \rightarrow X = ?$ Senza delle specifiche sulla geometria delle spire non siamo in grado di determinare la reattanza.

Senza la reattanza non siamo in grado di determinare la lunghezza efficace magnetica poiché

$$l_m = l_e \cdot \frac{\eta_0}{Z_{in}}$$

Nastro di corrente

Verifichiamo che: $\underline{l_g} = \underline{l_m}$



$$\int_{\Sigma} \bar{\nabla} \times \bar{H} \cdot d\bar{\Sigma} = \int_{\Sigma} \bar{J} \cdot d\bar{\Sigma} = I$$

($j\omega\mu \bar{E}$ manca: $E_t = 0$)

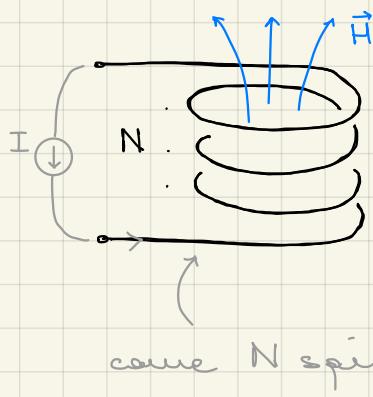
$$H \cdot l_g = I \quad \text{ma} \quad V l_m = j\omega\mu I S = j\omega\mu H l_g S$$

$$j\omega \phi_B l_g = V_{fem} \cdot l_g; \quad \text{se il conduttore è ideale}$$

$$V_{fem} = V \rightarrow l_m = l_g$$

$$L = \mu_0 \frac{S}{l_g}$$

Solenoids



$$-\frac{l_g}{N} = \ell_m$$

in TX: spira solenoide

campo E_0 NE_0

densità
di potenza

$$S_0 = \frac{|E_0|^2}{2\eta_0}$$

$$S_s = \frac{N^2 |E_0|^2}{2\eta_0} = N^2 S_0$$

potenza trasmessa

$$P_0$$

resistenza di radiazione

$$R_{R_0}$$

tensione a vuoto

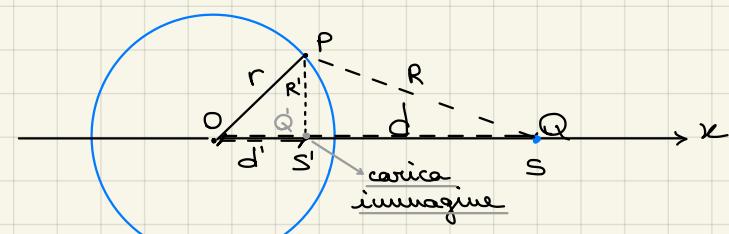
$$V_0$$

potenza disponibile

$$P_{d_0}$$

$$P_{d_s} = \frac{|NV_0|^2}{8N^2R_0} = \frac{N^2|V_0|^2}{8N^2R_0} = \frac{|V_0|^2}{8R_0} = P_{d_0}$$

Esercizi



$V = 0$ sfera metallica conduttrice

$$\begin{aligned}\overline{OS} &= d \\ \overline{OS'} &= d' \\ \overline{SP} &= r\end{aligned}$$

$$V(P) = 0 = \frac{Q}{4\pi\epsilon_0 R} + \frac{Q'}{4\pi\epsilon_0 R'} \quad \forall P \in \text{sfera}$$

$$\frac{\overline{OP}}{\overline{OS}} = \frac{\overline{SP}}{\overline{SP}} = \frac{\overline{OS'}}{\overline{OP}} \Rightarrow \underbrace{\frac{r}{d}}_{\text{costante}} = \frac{R'}{R} = \frac{d'}{r}$$

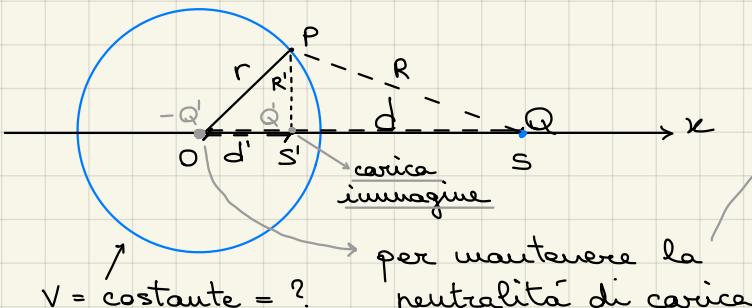
$$\rightarrow d' = \frac{r^2}{d} \quad Q' = -\frac{Qr}{d}$$

posizione e valore della carica per il metodo delle immagini

$$\frac{Q}{R} = -\frac{Q'}{R'}$$

$$\underbrace{\frac{Q}{Q'}}_{\text{costante}} = -\frac{R}{R'} \quad \begin{array}{l} \text{costante se} \\ \text{\overline{OS} e $\overline{OS'}$} \\ \text{sono simili} \end{array}$$

E se la sfera non fosse a potenziale nullo?



$$V(P) = 0$$

$V(P)$ 3 contributi :

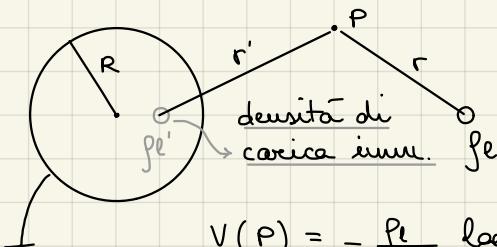
$V(P)$ sulla sfera ?

$$Q_{\text{tot}} = 0 \quad \text{sulla sfera}$$

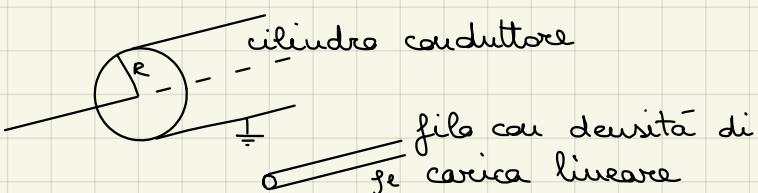
$$Q' = -Q \frac{r}{d} \quad d' = \frac{R^2}{d}$$

$$\left. \begin{array}{l} Q \\ Q' \\ -Q' \end{array} \right\} = 0$$

$$V(P) = \frac{-Q'}{4\pi\epsilon_0 R} = \frac{Q'r/d}{4\pi\epsilon_0 R} = +\frac{Q}{4\pi\epsilon_0 d}$$



$$V(P) = -\frac{pe}{2\pi\epsilon_0} \log r - \frac{pe'}{2\pi\epsilon_0} \log r' + c$$

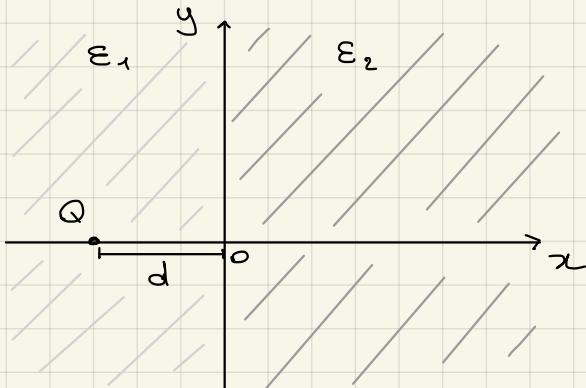


\exists soluzione se $pe = -pe'$

$$V(P) = -\frac{q}{2\pi\epsilon_0} \log\left(\frac{r'}{r}\right) + C = 0 \quad \forall P \in \text{cilindro}$$

$$\frac{r}{r'} = \text{constante} \quad (\forall P \in \text{cilindro}) \Rightarrow d' = \frac{R^2}{d}$$

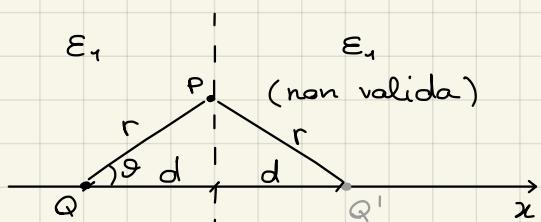
$$\frac{r}{r'} = \frac{d}{R} \Rightarrow -\frac{q}{2\pi\epsilon_0} \log\left(\frac{d}{R}\right) + C = 0$$



$$\rightarrow E_{t1} = E_{t2}$$

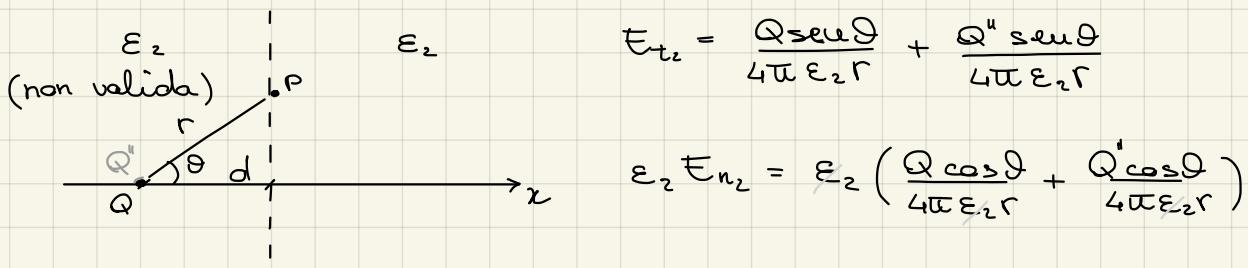
$$(D_{n1} = D_{n2})$$

$$\rightarrow \epsilon_1 E_{n1} = \epsilon_2 E_{n2}$$



$$E_{t1} = \frac{Q \sin \theta}{4\pi\epsilon_1 r} + \frac{Q' \sin \theta}{4\pi\epsilon_1 r}$$

$$\epsilon_1 E_{n1} = \epsilon_1 \left(\frac{Q \cos \theta}{4\pi\epsilon_1 r} - \frac{Q' \cos \theta}{4\pi\epsilon_1 r} \right)$$



$$E_{t2} = \frac{Q \sin \theta}{4\pi\epsilon_2 r} + \frac{Q'' \sin \theta}{4\pi\epsilon_2 r}$$

$$\epsilon_2 E_{n2} = \epsilon_2 \left(\frac{Q \cos \theta}{4\pi\epsilon_2 r} + \frac{Q'' \cos \theta}{4\pi\epsilon_2 r} \right)$$

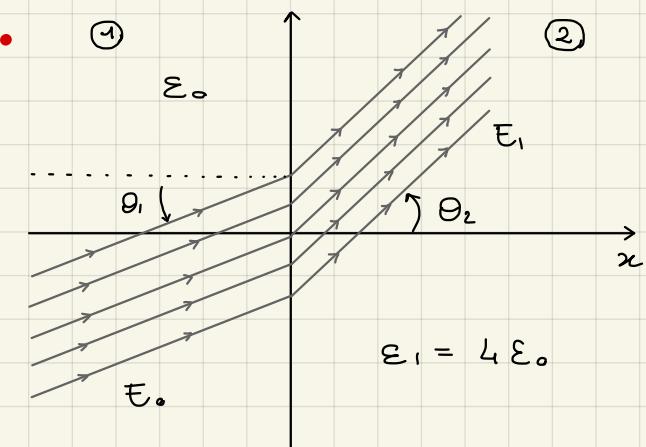
$$\frac{Q \sin \theta}{4\pi\epsilon_1 r} + \frac{Q' \sin \theta}{4\pi\epsilon_1 r} = \frac{Q \sin \theta}{4\pi\epsilon_2 r} + \frac{Q'' \sin \theta}{4\pi\epsilon_2 r}$$

$$\frac{Q \cos \theta}{4\pi r} - \frac{Q' \cos \theta}{4\pi r} = \frac{Q \cos \theta}{4\pi r} + \frac{Q'' \cos \theta}{4\pi r}$$

$$\begin{cases} \frac{Q + Q'}{\epsilon_1} = \frac{Q + Q''}{\epsilon_2} \\ Q - Q' = Q + Q'' \end{cases} \Rightarrow$$

$$\begin{aligned} Q'' &= Q \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1} \\ Q' &= -Q'' \end{aligned}$$

$$V(x, y) = \begin{cases} \frac{Q}{4\pi\epsilon_0 \sqrt{(x+d)^2 + y^2}} - \frac{Q \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}}{4\pi\epsilon_0 \sqrt{(x-d)^2 + y^2}}, & x < 0 \\ \frac{Q}{4\pi\epsilon_0 \sqrt{(x+d)^2 + y^2}} + \frac{Q \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}}{4\pi\epsilon_0 \sqrt{(x+d)^2 + y^2}}, & x > 0 \end{cases}$$



Quanto deve valere θ_1 di modo che $\theta_2 = \frac{\pi}{4}$?

Condizioni al contorno:

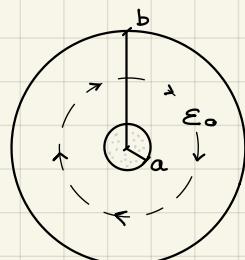
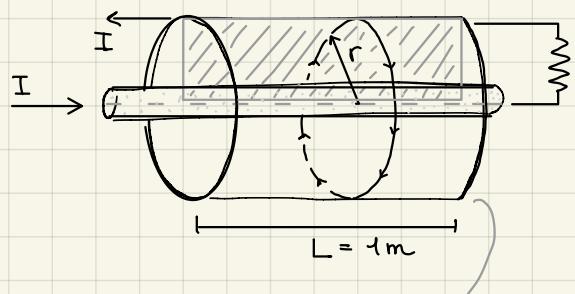
$$\left\{ \begin{array}{l} E_{t1} = E_{t2} \\ D_{n1} = D_{n2} \end{array} \right. \rightarrow \left\{ \begin{array}{l} E_{t1} = E_{t2} \\ \epsilon_0 E_{n1} = \epsilon_1 E_{n2} \end{array} \right.$$

$$\rightarrow \left\{ \begin{array}{l} \frac{E_0 \sin \theta_1}{\epsilon_0 E_0 \cos \theta_1} = \frac{E_1 \sin \theta_2}{\epsilon_1 E_1 \cos \theta_2} \\ \epsilon_0 E_0 \cos \theta_1 = \epsilon_1 E_1 \cos \theta_2 \end{array} \right. \Rightarrow \frac{1}{\epsilon_0} \tan \theta_1 = \frac{1}{\epsilon_1} \tan \theta_2$$

$\uparrow \pi/4$

$$\rightarrow \tan \theta_1 = \frac{1}{4} \Rightarrow \theta_1 \approx 14^\circ$$

Induttanza (esterna) di un coassiale



$$H_\phi = \frac{I}{2\pi r}$$

$$B_\phi = \mu_0 H_\phi$$

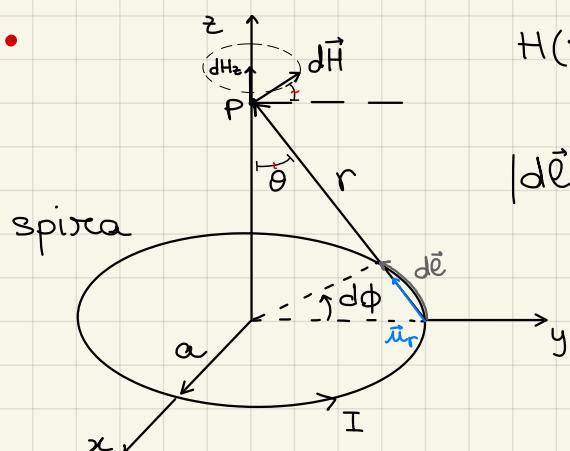
$$L = \frac{\Phi(B)}{I} = \frac{\mu_0}{I} \int_a^b \frac{I}{2\pi r} dr =$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad \left[\frac{H}{m} \right]$$

il conduttore coassiale è infinitamente esteso; ne calcolo l'induttanza su una sezione ($L = 1m$) e poi la generalizzo per unità di lunghezza

induttanza per unità di lunghezza

$H(z)$ lungo l'asse z ?



$|d\vec{H}| = a \cdot d\phi$ è perpendicolare a \vec{dr}

$$d\vec{H} = \frac{I a \vec{d\phi} \times \vec{dr}}{4\pi r^2}$$

$$dH = \frac{I a d\phi}{4\pi r^2} = \frac{I a d\phi}{4\pi (a^2 + z^2)}$$

con $r^2 = a^2 + z^2$ e $0 \leq \phi \leq 2\pi$

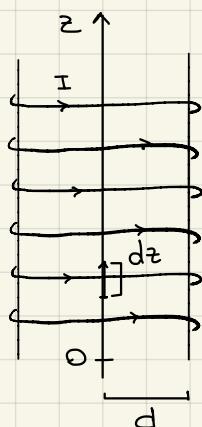
$$dH_z = \frac{I \cdot a \cdot d\phi}{4\pi(a^2 + z^2)} \sin\theta \quad \text{ma} \quad \sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + z^2}}$$

$$H_z = \int_0^{2\pi} dH_z = \frac{Ia^2}{4\pi(a^2 + z^2)^{3/2}} \cdot 2\pi = \frac{Ia^2}{2(a^2 + z^2)^{3/2}} \quad \left[\frac{A}{m} \right]$$

Le componenti radiali di H sono complessivamente nulle

$$\boxed{z=0} \rightarrow H_z = \frac{I}{2a} \text{ sul piano della sfera}$$

- Campo H all'interno di un solenoide infinito



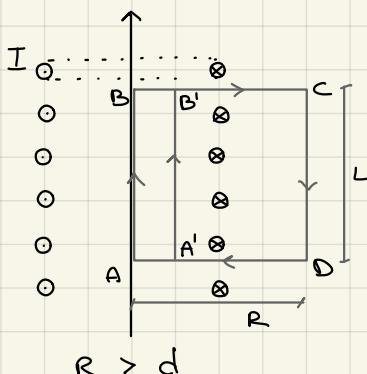
Campo \vec{H} sull'asse è?

$$N \cdot dz \text{ spire}$$

$$dH_z = \frac{Ia^2 N dz}{2(a^2 + z^2)^{3/2}}$$

$$H_z = \int_{-\infty}^{+\infty} \frac{Ia^2 N}{2(a^2 + z^2)^{3/2}} dz = \frac{Ia^2 N}{2} \underbrace{\int_{-\infty}^{\infty} (a^2 + z^2)^{-3/2} dz}_{2/a^2} = N \cdot I$$

dai risultati precedenti



$$\oint_{ABCD} \vec{H} \cdot d\vec{l} = \sum_{\Sigma} \vec{J} \cdot d\vec{S} / N \cdot I$$

$$\oint_{ABCD} \vec{H} \cdot d\vec{l} = \int_A^{B'} \vec{H} \cdot d\vec{l} + \int_{B'}^C \vec{H} \cdot d\vec{l} + \int_C^{D'} \vec{H} \cdot d\vec{l} + \int_{D'}^A \vec{H} \cdot d\vec{l} = H_z \cdot L + \int_c^0 H(R) dz = N \cdot I$$

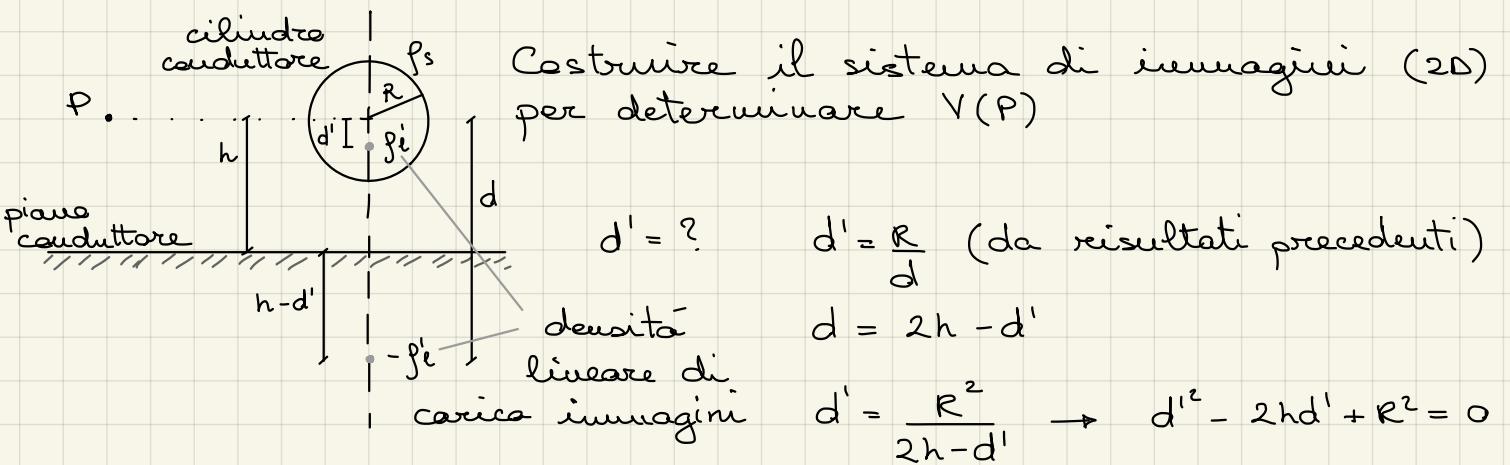
$$\hookrightarrow H(R) = 0$$

il campo magnetico all'esterno del solenoide è nullo

$$\oint_{AB'C'D'} \vec{H} \cdot d\vec{l} = \int_{A'}^{B'} \vec{H} \cdot d\vec{l} + \int_{B'}^C \vec{H} \cdot d\vec{l} + \int_C^{D'} \vec{H} \cdot d\vec{l} + \int_{D'}^{A'} \vec{H} \cdot d\vec{l} = \int_{A'}^{A'} \vec{H} \cdot d\vec{l} + 0 = N \cdot I$$

$$\hookrightarrow H_z = N \cdot I$$

il campo magnetico all'interno del solenoide è uniforme



g' deve essere tale da mantenere invariata la carica totale nel conduttore:

$$g_s \cdot 2\pi R \cdot L = g'_L \cdot L \rightarrow g'_L = 2\pi R g_s$$

\uparrow
lunghezza del cilindro

- Regione di spazio vuoto (μ_0, ϵ_0)

$$\vec{E}(x, y, z, t) \quad t = 0 \rightarrow \vec{E} = 0$$

$$t > 0 \rightarrow \text{si genera: } B_x = -k y$$

$$B_y = k x$$

$$B_z = 0$$

$$\vec{E}(t > 0) = ?$$

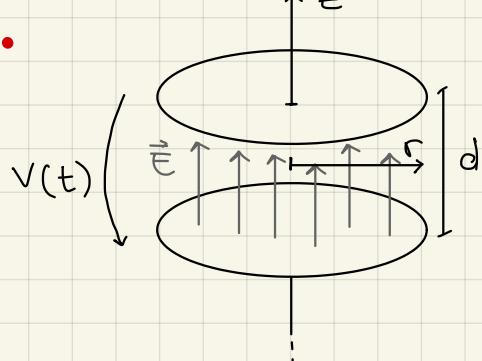
$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \det \begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -ky & kx & 0 \end{bmatrix} \rightarrow \begin{cases} -\frac{\partial(kx)}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} = 0 \\ -\frac{\partial(-ky)}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t} = 0 \\ \frac{\partial(kx)}{\partial x} - \frac{\partial(-ky)}{\partial y} = \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t} = 2k \end{cases}$$

$$\Rightarrow \begin{cases} E_x = C_1 \\ E_y = C_2 \\ E_z = \frac{2k}{\mu_0 \epsilon_0} \cdot t + C_3 \end{cases} \rightarrow \vec{E} = \vec{E}_0 + \frac{2k}{\mu_0 \epsilon_0} \cdot t \vec{u}_z$$

\downarrow
condizione iniziale
 $\vec{E}(t=0) = 0 = \vec{E}_0$

$$\rightarrow \vec{E}(x, y, z, t) = 2k c^2 t \vec{u}_z$$



Trascurare gli effetti di bordo.

$$V(t) = V_0 \cos \omega t$$

$$\vec{E}(t) = \frac{V(t)}{d} \vec{\mu}_z = \frac{V_0 \cos \omega t}{d} \vec{\mu}_z$$

\vec{H} tra le armature?

$$\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{j}_c \rightarrow \text{non c'è corrente di conduzione}$$

$$\oint \vec{H} \cdot d\vec{l} = \epsilon_0 \frac{\partial}{\partial t} \int_{\Sigma} \vec{E} \cdot d\vec{\Sigma} \rightarrow H_\phi 2\pi R = \epsilon_0 \frac{\partial}{\partial t} \left(\frac{V_0 \cos \omega t}{d} \right) \pi R^2$$

$$H_\phi \cdot 2 = \epsilon_0 \left(-\omega \frac{V_0}{d} \sin \omega t \right) \cdot \rho$$

Altro metodo:

$$\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \xrightarrow{\text{diretto solo come } \vec{H}} \text{in coordinate cilindriche}$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{R} \left[\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\phi}{\partial \phi} \right] \vec{\mu}_z = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \vec{\mu}_z, \text{ per simmetria } \frac{\partial H_\phi}{\partial \phi} = 0$$

$$\frac{1}{\rho} \frac{\partial (\rho H_\phi)}{\partial \rho} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow \partial (\rho H_\phi) = -\rho \epsilon_0 \frac{V_0}{d} \omega \sin(\omega t) \partial \rho$$

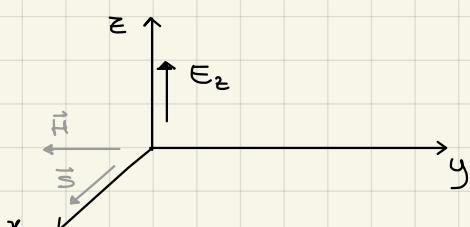
$$\rightarrow \rho H_\phi = -\frac{\rho^2}{2} \epsilon_0 \frac{V_0}{d} \omega \sin(\omega t) \rightarrow H_\phi(\rho) = -\rho \epsilon_0 \frac{V_0}{2d} \omega \sin(\omega t)$$

$$\vec{\nabla} \times \vec{F} = \left(\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \vec{\mu}_\rho + \left(\frac{\partial F_\phi}{\partial z} - \frac{\partial F_z}{\partial \phi} \right) \vec{\mu}_\phi + \frac{1}{\rho} \left(\frac{\partial (\rho F_\phi)}{\partial \rho} - \frac{\partial F_\phi}{\partial \phi} \right) \vec{\mu}_z$$

$$\vec{E}(x, t) = 15 \cdot 10^{-2} \cos(1,7\pi \cdot 10^3 t - \beta x) \vec{\mu}_z \quad \left[\frac{V}{m} \right]$$

Calcolare β , λ , direzione di propagazione (nel verso).

$$\vec{E}(x, t) = A \cos(\omega t - \beta x) \vec{\mu}_z$$



μ_0, ϵ_0

$$\omega = 2\pi f = 1,7\pi \cdot 10^3 \frac{\text{rad}}{\text{s}} \rightarrow$$

la direzione di propagazione è l'asse x , il verso è dato dal segno ("-") per cui è un'onda progressiva che propaga verso le x positive

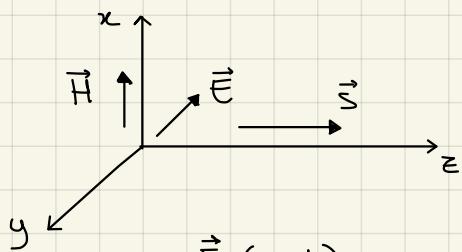
$$\vec{S} = \vec{E} \times \vec{H}$$

$$\left\{ \begin{array}{l} f = 850 \text{ MHz} \\ \lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{8,5 \cdot 10^8} = 0,353 \text{ m} \end{array} \right.$$

$$\beta = \frac{2\pi}{\lambda} = 17,8 \frac{\text{rad}}{\text{m}}$$

$$\vec{H}(z, t) = 0,1 \text{ sen}(\omega t - 9,3z) \vec{u}_x \left[\frac{\text{mA}}{\text{m}} \right]$$

Calcolare f , $\vec{E}(z, t)$ al $t = 10^{-4}$ s in $z = 2$ m, $|\vec{s}|$ (nel vuoto)



$$\beta = \frac{2\pi}{\lambda} = 9,3 \frac{\text{rad}}{\text{m}} \rightarrow \lambda = 0,676 \text{ m}$$

$$f = \frac{c}{\lambda} = 444 \text{ MHz}$$

$$\omega = 2\pi f = 2,79 \cdot 10^9 \frac{\text{rad}}{\text{s}}$$

$$\vec{E}(z, t) = \eta_0 H(z, t) \cdot (-\vec{u}_y) =$$

$$= -37,7 \text{ sen}(\omega t - 9,3z) \vec{u}_y \left[\frac{\text{mV}}{\text{m}} \right]$$

$$\vec{E}(2 \text{ m}, 10^{-4} \text{ s}) = -28,5 \vec{u}_y \left[\frac{\text{mV}}{\text{m}} \right]$$

$$|\vec{s}| = |\vec{E} \times \vec{H}| = \frac{1}{2} \frac{|\vec{E}|^2}{\eta_0} = \frac{1}{2} \eta_0 |\vec{H}|^2 = \frac{0,00142}{2 \cdot 377} = 1,88 \cdot 10^{-6} \frac{\text{W}}{\text{m}^2}$$

- $\vec{E} = 0,5 \cdot 10^8 \text{ sen}[2\pi(10^8 t - 0,5x - 0,125)] \vec{u}_z$

Calcolare f , λ , v , direzione di propagazione, ϵ_r nel mezzo.

(μ_0 , $\epsilon = \epsilon_0 \epsilon_r$) $\vec{E} = A \text{ sen}(\omega t - \beta x + \varphi_0)$ direzione x pos.

$$\vec{E} = 0,5 \cdot 10^8 \text{ sen}\left(2\pi \cdot 10^8 t - \pi x - \frac{\pi}{4}\right) \vec{u}_z$$

$$\omega = 2\pi \cdot 10^8 \frac{\text{rad}}{\text{s}} \rightarrow f = 100 \text{ MHz} \quad \left. \begin{array}{l} v = \lambda f = 2 \cdot 10^8 \frac{\text{m}}{\text{s}} \\ v = \sqrt{\frac{1}{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = 2 \cdot 10^8 \frac{\text{m}}{\text{s}} \end{array} \right\}$$

$$\beta = \pi \frac{\text{rad}}{\text{m}} \longrightarrow \lambda = \frac{2\pi}{\beta} = 2 \text{ m}$$

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = 2 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

$$\sqrt{\epsilon_r} = \frac{3}{2} \rightarrow \epsilon_r = 2,25$$

- Onda TEM nel vuoto (dominio dei fasci)

$$\vec{E} = (5 - j3) e^{-j2z} \vec{u}_x \left[\frac{\text{V}}{\text{m}} \right] \quad (\text{nel vuoto})$$

Calcolare il valore del campo elettrico in $z = 0,4$ m al tempo $t = 3 \cdot 10^{-9}$ s.

$$\vec{E} = \vec{E}_0(t) e^{-j\beta z} \quad E_0(t) = 5 - j3, \quad \beta = 2 \frac{\text{rad}}{\text{m}} = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \quad \rightarrow \omega = 6 \cdot 10^8 \frac{\text{rad}}{\text{s}}$$

$$\vec{E}(z, t) = \operatorname{Re} \{ \vec{E} e^{j\omega t} \} \vec{u}_x = \operatorname{Re} \{ (5 - j3) e^{-j2z} e^{j6 \cdot 10^8 t} \} \vec{u}_x$$

$$\vec{E}(0, 4m, 3 \cdot 10^9 s) = 5,22 \vec{u}_x \left[\frac{V}{m} \right]$$

- Calcolare la profondità di penetrazione (spessore pelle) di un'onda con $f = 10 \text{ kHz}$ nei seguenti materiali:

a) acqua marina: $\epsilon_r = 81$ $\sigma = 4 \text{ S/m}$

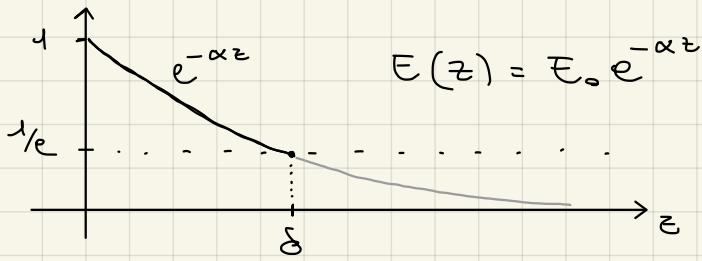
b) terreno umido: $\epsilon_r = 10$ $\sigma = 10^{-2} \text{ S/m}$

c) " asciutto": $\epsilon_r = 3$ $\sigma = 10^{-4} \text{ S/m}$

I materiali sono buoni conduttori se:

minore è la presenza di acqua, minore è la cost. dielettrica

$$\sigma \gg \omega \epsilon \rightarrow \sigma \gg 2\pi f \epsilon_0 \epsilon_r \quad \text{vero per a), b) e c)}$$



$$\text{spessore pelle } \delta = \frac{1}{\alpha}$$

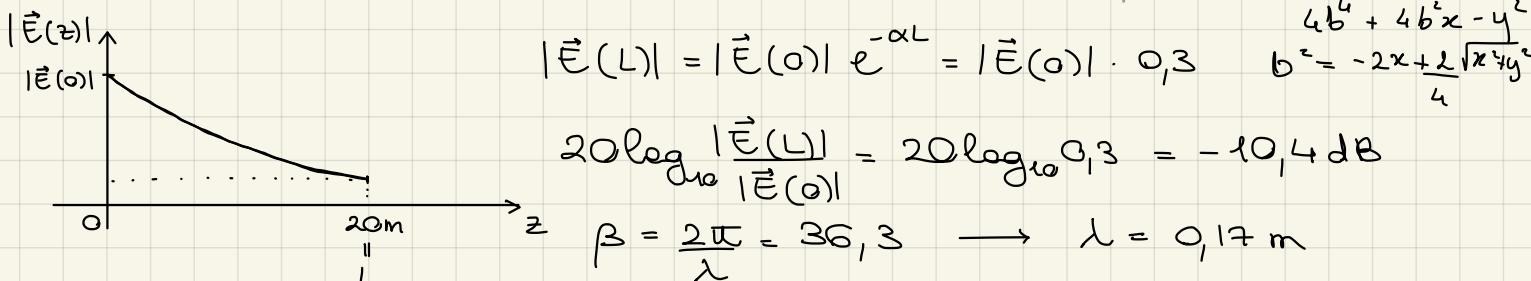
$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \quad \begin{cases} \delta_a = 2,5 \text{ m} \\ \delta_b = 50,3 \text{ m} \\ \delta_c = 503 \text{ m} \end{cases}$$

- Onda TEM piana $f = 1 \text{ GHz}$, si propaga in un mezzo con $\epsilon_r = 3 - j0,01$.

Valutare di quanti dB si è ridotto il campo \vec{E} dopo essersi propagato per 20m.
Valutare λ .

$$\gamma = \sqrt{-\omega^2 \mu_0 \epsilon_0 \epsilon_r} = \sqrt{-1318 + j4,4} = (0,06 + j36,3) \text{ m}^{-1} \quad \begin{matrix} y^2 - b^2 = x \\ 4b^2 \end{matrix}$$

$$\boxed{\begin{aligned} \beta &= \sqrt{\frac{x^2 + y^2}{2}} \\ \alpha &= \frac{y}{\beta} \end{aligned}}$$



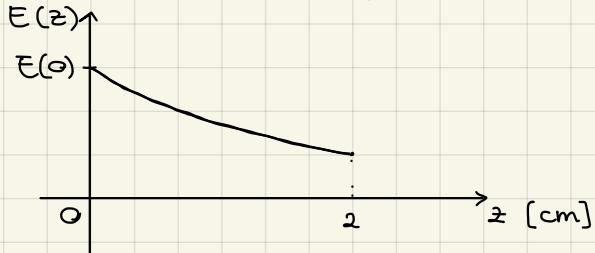
$$|\vec{E}(L)| = |\vec{E}(0)| e^{-\alpha L} = |\vec{E}(0)| \cdot 0,3$$

$$b^2 = -2x + \frac{1}{4} \sqrt{x^2 + y^2}$$

$$20 \log_{10} \frac{|\vec{E}(L)|}{|\vec{E}(0)|} = 20 \log_{10} 0,3 = -10,4 \text{ dB}$$

$$\beta = \frac{2\pi}{\lambda} = 36,3 \rightarrow \lambda = 0,17 \text{ m}$$

Onda TEM $f = 1\text{MHz}$ in mezzo con $\epsilon_r = 4$ e $\sigma = 10^3 \frac{\text{S}}{\text{m}}$



$$E(0) = 1 \frac{\text{V}}{\text{m}} \quad E(z=2\text{cm}) = ?$$

$$\lambda = ?$$

Buon conduttore?

$$2,2 \cdot 10^{-4}$$

$$\overbrace{\omega \epsilon_0 \epsilon_r}^{<< \sigma} \ll \sigma \rightarrow \text{SI}$$

$$\gamma = \frac{1+j}{2} \sqrt{\omega \mu_0 \sigma} = (62,8 + j62,8) \text{ m}^{-1}$$

$$E(z) = E(0) e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z} = 0,284 e^{-j1,256} \left(\frac{\text{V}}{\text{m}}\right) = \\ (z = 2\text{cm}) = (0,088 - j0,27) \frac{\text{V}}{\text{m}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{62,8} = 0,1 \text{ m} = \frac{\lambda}{f} \rightarrow \lambda = 10^5 \frac{\text{m}}{\text{s}}$$

NB: non sarebbe stato corretto calcolare la velocità e la lunghezza d'onda con l'equazione

$$v = \frac{c}{\sqrt{\epsilon_r}} = 1,5 \cdot 10^8 \frac{\text{m}}{\text{s}} \times \text{perché non considera la conducibilità } \sigma$$

- Onda TEM $f = 1\text{GHz}$ in acqua marina ($\epsilon_r = 80$ $\sigma = 4 \frac{\text{S}}{\text{m}}$)

Vedere a quale distanza l'ampiezza di \vec{E} si sarà ridotta di $\frac{1}{10}$ del suo valore. Vediamo λ .

$\omega \epsilon_0 \epsilon_r = 4,45 > \sigma \rightarrow$ NON è un buon conduttore

$$\gamma = \sqrt{-\omega^2 \mu_0 \epsilon_0 \epsilon_r + j\omega \mu_0 \sigma} = \sqrt{-35147 + j31582} = (77,8 + j202) \text{ m}^{-1}$$

$$|E(z)| = |E(0)| e^{-\alpha L} = \frac{1}{10} |E(0)| \rightarrow -\alpha L = -2,306$$

$$L = 0,0286 \text{ m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{202} = 0,0311 \text{ m}$$

$$\text{Se } \sigma = 0 \text{ e } \epsilon_r = 80 \text{ allora } \lambda = \frac{c}{\sqrt{\epsilon_r} \cdot f} = 0,0335 \text{ m}$$

↓
la conducibilità
diverse da σ cambia
(poco) la lunghezza
d'onda

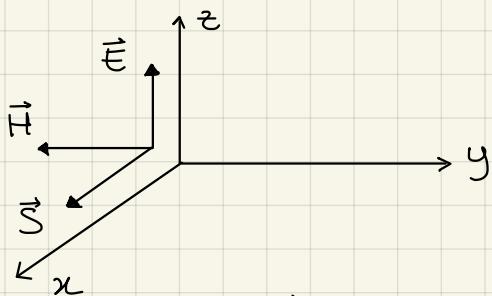
- Onda TEM, mezzo: $\epsilon_r = 36$ $\mu_r = 4$ $\sigma = 1 \frac{S}{m}$

$$\vec{E} = 100 e^{-\alpha z} \cos(\omega t - \beta z) \vec{u}_z [\frac{V}{m}]$$

Determinare α , β , \vec{H} .

$$\gamma = \sqrt{-\omega^2 \mu_0 \mu_r \epsilon_0 \epsilon_r + j \omega \mu_0 \mu_r \sigma} = \sqrt{-15782 + j 15775} = \\ = (57,15 + j 138) \text{ m}^{-1}$$

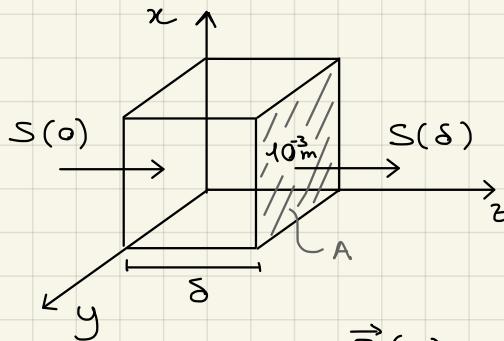
$$\eta = \sqrt{\frac{j \omega \mu_0 \mu_r}{\sigma + j \omega \epsilon_0 \epsilon_r}} = \\ = (97,6 + j 40,4) \Omega \\ = 105,6 e^{+j \frac{\pi}{8}} \Omega$$



$$\vec{H} = \frac{\vec{E}}{\eta} (-\vec{u}_y)$$

$$\vec{H} = -0,85 e^{-57,15 z} \cos(10\pi \cdot 10^8 t - 138z - \frac{\pi}{8}) \vec{u}_y [\frac{V}{m}]$$

- Onda TEM, $|\vec{E}| = 100 \frac{V}{m}$ $f = 7 \text{ GHz}$ mezzo: $\epsilon_r = 81$ $\sigma = 4 \frac{S}{m}$



Calcolare la potenza dissipata in un blocco d'acqua avente una superficie di 10 cm^2 e uno spessore pari a δ (spessore pelle)

$$\vec{S}(z) = \frac{1}{2} \frac{|\vec{E}(z)|^2}{\eta} \cos \varphi_\eta \vec{u}_z = \frac{1}{2} \frac{|\vec{E}(0)|^2}{\eta} e^{-2\alpha z} \cos \varphi_\eta \vec{u}_z$$

$$\eta = \sqrt{\frac{j \omega \mu_0}{\sigma + j \omega \epsilon_0 \epsilon_r}} = \sqrt{1724 + j 219} = (41,6 + j 2,6) \Omega$$

$$|\eta| = 41,7 \Omega \quad \cos \varphi_\eta = \cos \left(\arctg \left(\frac{2,6}{41,6} \right) \right) = 0,998$$

$$\text{Ricordando che } \delta = \frac{1}{\alpha}: \quad P(0) = S(0) \cdot A = \frac{1}{2} \frac{|\vec{E}(0)|^2}{\eta} \cos \varphi_\eta \cdot 10^{-3} \\ = 0,12 \text{ W}$$

$$\Rightarrow P_{\text{dis}} = P(0) - P(\delta) \\ = 0,104 \text{ W}$$

$$P(\delta) = \frac{1}{2} \frac{|\vec{E}(0)|^2}{\eta} e^{-2\alpha \frac{1}{\alpha}} \cos \varphi_\eta \cdot 10^{-3} = \\ = 0,016 \text{ W}$$

- Onda piana TEM che si propaga come \vec{u}_z

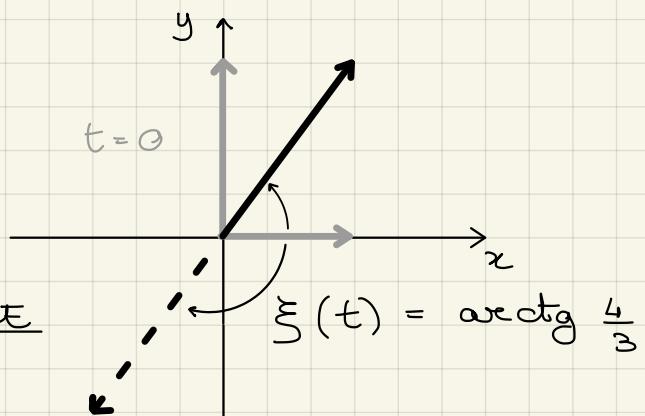
$$\vec{E} = 3 \cos(2\pi f t - \beta z) \vec{u}_x + 4 \cos(2\pi f t - \beta z) \vec{u}_y$$

Determinare la polarizzazione.

Piano xy ($z=0$):

$$\begin{cases} E_x(0,t) = 3 \cos 2\pi f t \\ E_y(0,t) = 4 \cos 2\pi f t \end{cases}$$

$$\varphi_0 = 0 \rightarrow \text{POL. } \underline{\text{LINEARE}}$$



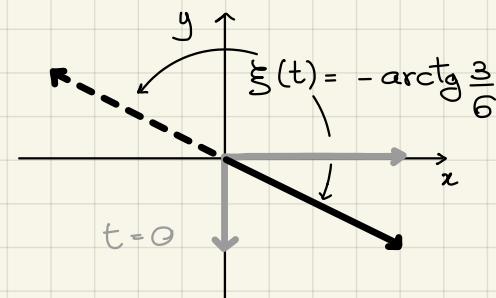
- Onda TEM $\vec{E} = 6 \cos(2\pi f t - \beta z + \frac{\pi}{4}) \vec{u}_x - 3 \cos(2\pi f t - \beta z + \frac{\pi}{4}) \vec{u}_y$

Polarizzazione?

In $z=0$:

$$\begin{cases} E_x(0,t) = 6 \cos(2\pi f t + \frac{\pi}{4}) \\ E_y(0,t) = -3 \cos(2\pi f t + \frac{\pi}{4}) \end{cases}$$

$$\varphi_0 = \pi \rightarrow \text{POL. } \underline{\text{LINEARE}}$$



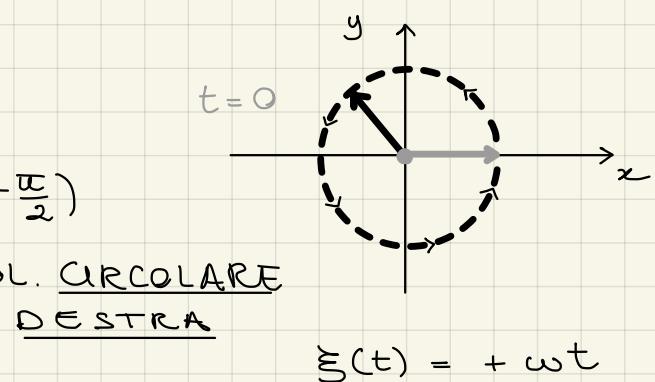
- Onda TEM $\vec{E} = 4 \cos(2\pi f t - \beta z) \vec{u}_x + 4 \cos(2\pi f t - \beta z - \frac{\pi}{2}) \vec{u}_y$

Polarizzazione?

In $z=0$:

$$\begin{cases} E_x(0,t) = 4 \cos 2\pi f t \\ E_y(0,t) = 4 \cos(2\pi f t - \frac{\pi}{2}) \end{cases}$$

$$\varphi_0 = -\frac{\pi}{2} \text{ e } E_x = E_y \rightarrow \text{POL. } \underline{\text{ARCOLARE DESTRA}}$$



- Onda TEM $f = 300 \text{ MHz}$ propaga come \vec{u}_z

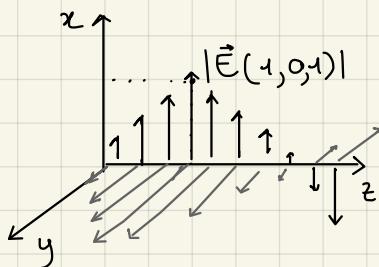
$$\text{mezzo: } \epsilon_r = 6 \text{ e } \sigma = 0,1 \text{ S/m}$$

Sapendo che $|\vec{E}(1,0,1)| = \frac{1}{m} \text{ V}$ calcolare:

$$|\vec{H}(1,0,1)| \text{ e } |\vec{E}(0,0,0)| ?$$

Buon conduttore? $\omega \epsilon_0 \epsilon_r = 1,9 \cdot 10^9 \cdot 8,85 \cdot 10^{-12} \cdot 6 = 0,1 = \sigma$
 \rightarrow non è un buon conduttore

$$\vec{H} = \frac{\vec{E}}{\eta}$$



$$|\eta| = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\epsilon_0\epsilon_r}} =$$

$$\omega = 2\pi f = 1,9 \cdot 10^9 \text{ rad/s}$$

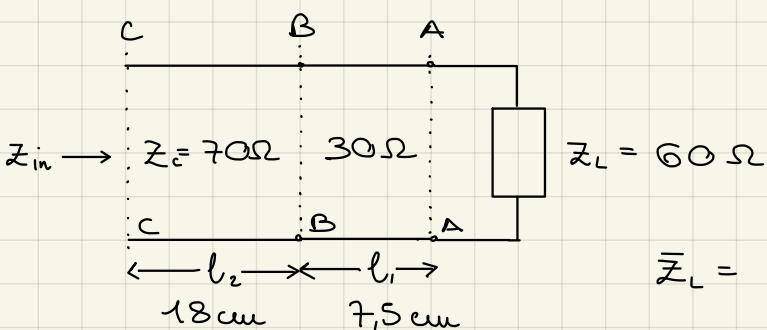
$$\begin{aligned} &= \sqrt{\frac{j \frac{24 \cdot 10^3}{1}}{0,1 + j0,1}} = \\ &= \sqrt{\frac{240 - j240}{0,02}} = \sqrt{1200} \sqrt{1 - j} = \\ &= 109,5 \cdot \sqrt{1 - j} = 130 \Omega \end{aligned}$$

$$|\vec{H}(1,0,1)| = \frac{|\vec{E}(1,0,1)|}{|\eta|} = 7,69 \text{ mH/m}$$

$$\vec{E} = \vec{E}(t, z) = |\vec{E}(1,0,1)| e^{-\alpha(z-1)} e^{-j\beta(z-1)} e^{j\omega t}$$

$$\begin{aligned} \alpha + j\beta = \gamma &= \sqrt{j\omega\mu_0(\sigma + j\omega\epsilon_0\epsilon_r)} = \\ &= \sqrt{j2400 \cdot 0,1(1+j)} = 15,5 \sqrt{-1+j} = \\ &= 18,42 \sqrt{e^{j\frac{\pi}{4}}} = 18,42 (\cos \frac{3}{8}\pi + j \sin \frac{3}{8}\pi) \\ &= 7,05 + j 17,02 \end{aligned}$$

$$|\vec{E}(0,0,0)| = |\vec{E}(1,0,1)| |e^{+\alpha}| = 1,15 \cdot 10^3 \text{ V/m}$$



$$f = 1 \text{ GHz} \rightarrow \lambda = 0,3 \text{ m}$$

$$Z_{in} = ?$$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{60}{30} = 2$$

$$\frac{l_1}{\lambda} = \frac{7,5}{30} = 0,25$$

$$\frac{l_2}{\lambda} = \frac{18}{30} = 0,6 \rightarrow 0,1$$

ruotare di 0,5λ
 significa compiere
 (una rotazione
 completa sulla
 carta di Smith)

$$\Gamma_{AA} = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{1}{3} \text{ parto da } 0,25\lambda$$

$$0,25\lambda + 0,25\lambda$$

$$\bar{Z}'_{BB} = 0,5 + j0$$

$$\bar{Z}'_{BB} = \bar{Z}'_{BB} \cdot 30\Omega = 15\Omega$$

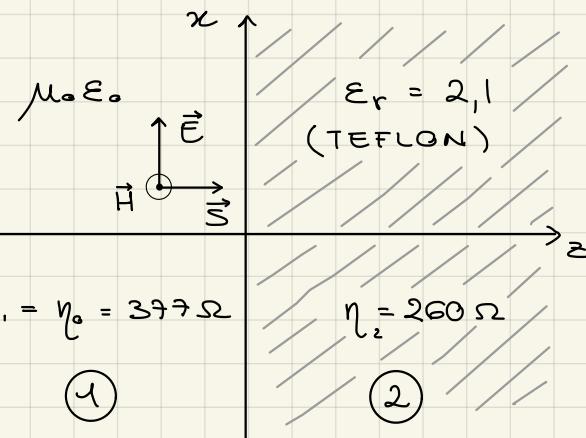
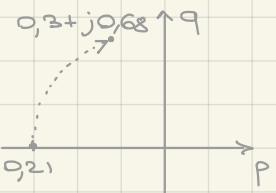


$$\Gamma_{BB} = \frac{\bar{Z}'_{BB} - \bar{Z}_c}{\bar{Z}'_{BB} + \bar{Z}_c} = \frac{15\Omega - 70\Omega}{15\Omega + 70\Omega} = -\frac{11}{17} \text{ parte da } 0\lambda$$

$$\bar{Z}''_{BB} = \frac{\bar{Z}'_{BB}}{70\Omega} = \frac{15}{70} = 0,21$$

$$\bar{Z}_{cc} = 0,3 + j0,68$$

$$Z_{cc} = \bar{Z}_{cc} + 0 = (22 + j47,5)\Omega = Z_{in}$$



Onda TEM $f = 1 \text{ GHz}$

$$|\vec{E}_{inc}| = 1 \frac{\text{V}}{\text{m}}$$

Calcolare S_{inc} , S_{rif} , S_{tra} e i campi \vec{E} e \vec{H} nel mezzo 2.

$$S_{inc} = \frac{1}{2} \frac{|\vec{E}_{inc}|^2}{\eta_1} = 1,327 \cdot 10^{-3} \frac{\text{W}}{\text{m}^2}$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_r}} = 260 \Omega$$

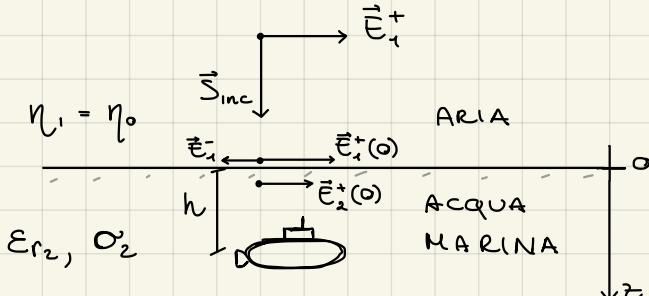
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0,184 \quad \begin{cases} S_{tra} = S_{inc}(1 - |\Gamma|^2) = 1,28 \cdot 10^{-3} \frac{\text{W}}{\text{m}^2} \\ S_{rif} = S_{inc} \cdot |\Gamma|^2 = 4,4 \cdot 10^{-5} \frac{\text{W}}{\text{m}^2} \end{cases}$$

$$\beta_2 = \omega \sqrt{\mu_2 \epsilon_2} = 2\pi f \sqrt{\mu_0 \epsilon_0 \epsilon_r} = 30,4 \frac{\text{rad}}{\text{s}}$$

Assumiamo fase zero per $\vec{E}_{inc}(0)$

$$\begin{aligned} \vec{E}_2^+(z) &= E_2^+(0) e^{-j\beta_2 z} \vec{u}_x = \underbrace{E_{inc}(0) \cdot T}_{E_1^+(0)} e^{-j\beta_2 z} \vec{u}_x \quad \text{con } T = 1 + \Gamma \\ &= 0,816 e^{-j30,4 z} \vec{u}_x \frac{\text{V}}{\text{m}} \end{aligned}$$

$$\vec{H}_2^+(z) = \frac{E_2^+(z)}{\eta_2} \vec{u}_y$$



$$f = 100 \text{ kHz}$$

$$\epsilon_{r2} = 80$$

$$S_{inc} = 1,33 \cdot 10^{-5} \frac{\text{W}}{\text{m}^2}$$

$$\Omega_2 = 4 \frac{\text{S}}{\text{m}}$$

$$|\vec{E}_{min}| = 1 \mu\text{V}/\text{m}$$

Calcolare h_{\max} (massima profondità di immersione).

$\omega \epsilon_0 \epsilon_r \ll \alpha_2 \rightarrow$ buone conduttrice

$$\eta_2 \approx (1+j) \sqrt{\frac{\pi f \mu}{\sigma}} = (1+j) 0,314 \Omega$$

$$S_{inc} = \frac{1}{2} \frac{|E_1^+|^2}{\eta_0} \rightarrow |E_1^+| = \sqrt{2 \cdot S_{inc} \cdot \eta_0} = 0,1 \frac{V}{m}$$

$$T = 1 + \Gamma = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = (1+j) 1,66 \cdot 10^{-3} \rightarrow$$

forte riflessione

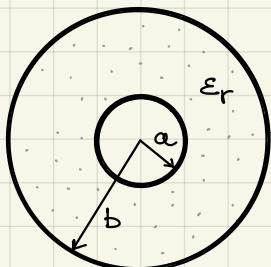
$$|E_2^+(0)| = |E_1^+(0)| \cdot |T| = 235 \frac{\mu V}{m}$$

$$|E_2^+(z)| = |E_2^+(0)| \cdot e^{-\alpha z} \text{ con } \alpha = \frac{1}{\delta} = \sqrt{\frac{\omega \mu \sigma}{2}} = 1,2566 \frac{Np}{m}$$

$$|\vec{E}_{min}| = |E_2^+(0)| e^{-\alpha h_{\max}} \Rightarrow h_{\max} = 4,3 \text{ m}$$

$1,2566$
 $- \alpha \cdot h_{\max}$

- Linea TEM - caavo coassiale



$$a = 2 \text{ mm} \quad \epsilon_r = 2,5$$

$$b = 4,5 \text{ mm}$$

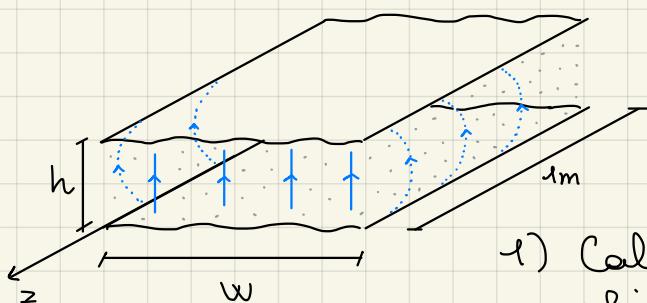
Calcolare C , L , Z_c .

$$C_d = \mu \epsilon$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln(b/a)} = 171 \frac{pF}{m} \quad L = \frac{\mu_0}{2\pi} \ln(b/a) = 1,62 \cdot 10^{-7} \frac{H}{m}$$

$$Z_c = \sqrt{\frac{L}{C}} = 30,8 \Omega$$

Linea a striscia



$$w = 10 \text{ mm}$$

$$h = 4 \text{ mm}$$

$$\epsilon_r \approx 1$$

1) Calcolare C , L , Z_c (trascurando gli effetti di bordo)

$$C = \epsilon_0 \epsilon_r \frac{w \cdot h}{h} = 22 \text{ pF} \quad L = \frac{\mu_0 \epsilon}{C} = 5,06 \cdot 10^{-7} \text{ H}$$

$$Z_c = \sqrt{\frac{L}{C}} = \frac{1}{C} \sqrt{\mu_0 \epsilon} = 151 \Omega$$

NB: C' (capacità reale) > $C \rightarrow Z'_c$ (imped. reale) < Z_c

che considera gli effetti di bordo

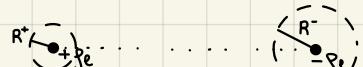
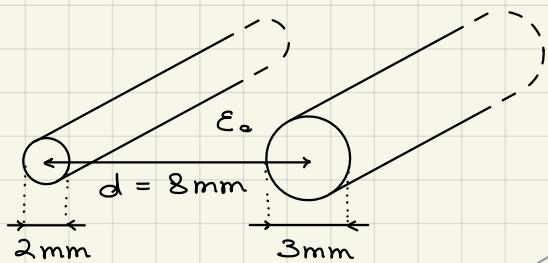
2) Calcolare tensione e $|\vec{E}|$ di picco sapendo che nella linea transita un'onda (progressiva e diretta) $P_m^+ = 100 \text{ W}$

$$P_m^+ = \frac{1}{2} \frac{|V^+(z)|^2}{Z_c} \rightarrow |V^+(z)| = \sqrt{2 Z_c P_m^+} = 174 \text{ V}$$

$$|V| = |\epsilon| \cdot h \rightarrow |\vec{E}| = \frac{|V^+(z)|}{h} = 43,5 \frac{\text{kV}}{\text{m}}$$

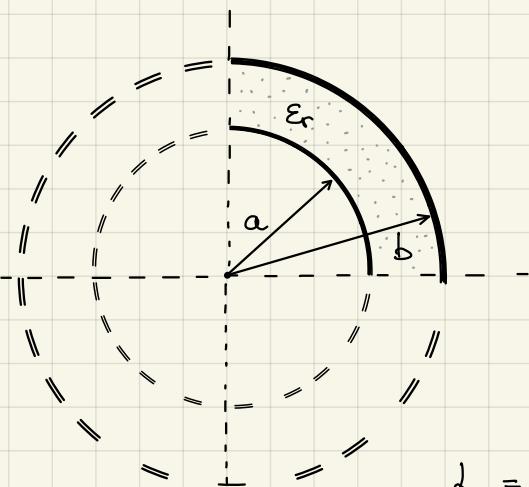
• linea bifilare

Calcolare Z_c con l'approssimazione dei conduttori sottili.



$$C = \frac{\sigma_e}{V} = \frac{2\pi \epsilon_0}{\ln\left(\frac{d^2}{R^+ R^-}\right)} = 14,8 \text{ pF/m}$$

$$Z_c = \sqrt{\frac{\mu_0 \epsilon_0}{C}} = 225 \Omega$$



$$a = 3 \text{ mm}$$

$$b = 5 \text{ mm}$$

$$\epsilon_r = 4$$

Calcolare C, L, Z_c

la capacità è $\frac{1}{4}$ di quella di un caos coassiale

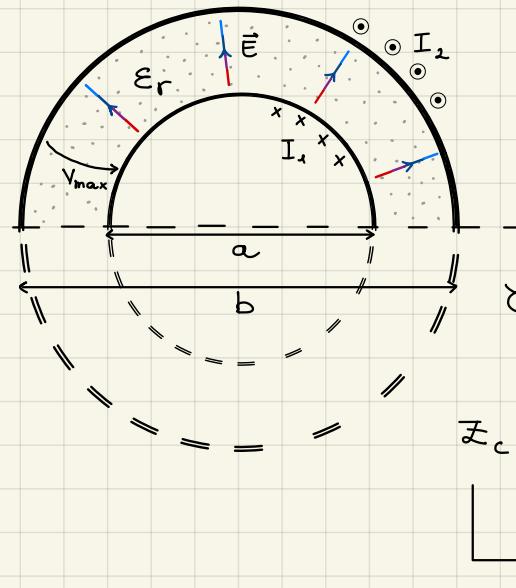
$$C_0 = \frac{2\pi \epsilon_0 \epsilon_r}{\ln\left(\frac{b}{a}\right)}$$

$$C = \frac{C_0}{4} = \frac{\pi \epsilon_0 \epsilon_r}{2 \ln\left(\frac{b}{a}\right)} = 109 \text{ pF/m}$$

$$L = \frac{\mu_0 \epsilon_0 \epsilon_r}{C} = 4,08 \cdot 10^{-7} \frac{\text{H}}{\text{m}}$$

$$Z_c = \sqrt{\frac{L}{C}} = 61,2 \Omega$$

$$(L = 4 L_0)$$



$$a = 1 \text{ cm} \quad \epsilon_r = 2$$

1) Determinare b in modo che $Z_c = 50 \Omega$

$$Z_c = \frac{C_0}{2} = \frac{\pi \epsilon_0 \epsilon_r}{\ln(\frac{b}{a})} \quad Z_c = \sqrt{\frac{\mu_0 \epsilon_0 \epsilon_r}{c}} = 50 \Omega$$

$$Z_c = \frac{\eta}{\pi} \ln\left(\frac{b}{a}\right) = 50 \Omega \quad \text{con } \eta = \sqrt{\frac{\mu}{\epsilon}} = 267 \Omega$$

$$\rightarrow b = 1,8 \text{ cm}$$

2) Calcolare I_1 e I_2 con $P_m^+ = 50 \text{ W}$

$$P_m^+ = \frac{1}{2} |I|^2 Z_c = 50 \text{ W} \rightarrow |I| = 4,41 \text{ A} = |I_1| = |I_2|$$

correnti uguali e opposte

3) Calcolare P_m^+ sapendo che $|\vec{E}_{max}| = 1 \frac{\text{kV}}{\text{cm}}$ non è uniforme nel coassiale!

$$\text{Da } |\vec{E}_{max}| \text{ ricaviamo } V_{max} : V_{max} = E_{max} \ln\left(\frac{b}{a}\right) \frac{a}{2} =$$

$$= 10^5 \frac{\text{V}}{\text{m}} \ln(1,8) \cdot \frac{10^{-2}}{2} =$$

$$= 294 \text{ V}$$

è maggiore
dove il raggio
è minore

4) Come al p.to 3, ma la linea è chiusa su un carico $Z_L = 300 \Omega$ (non in adattamento)

$$V_{max} = 294 \text{ V} \quad V(z) = V^+(z) + V^-(z) \quad |V(z)| = |V^+(z)| |1 + \Gamma(z)|$$

$$|V_{max}| = |V^+(0)| (1 + |\Gamma|)$$

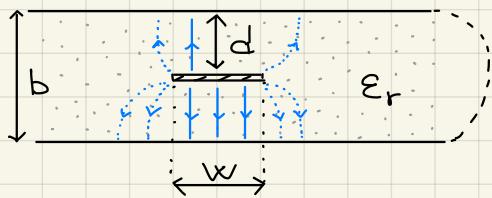
$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{300 - 50}{300 + 50} = 0,71 \quad |V_{max}| = |V^+(0)| 1,71$$

$$\rightarrow |V^+(0)| = \frac{V_{max}}{1,71} = \frac{294}{1,71} = 172 \text{ V}$$

$$P_{max}^+ = \frac{1}{2} \frac{|V^+(0)|^2}{Z_c} = 295 \text{ W}$$

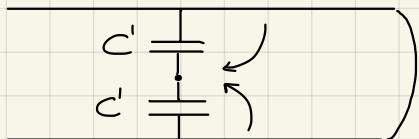
• stripline

Calcolare w in modo che $Z_c = 50 \Omega$.



$\epsilon_r = 2,2$ (adottare l'approx. del condensatore ideale)

$$b = 0,32 \text{ cm}$$



$$C = 2C' \text{ con } C' = \epsilon_0 \epsilon_r \frac{w}{d}$$

$$b = 2d$$

$$\Rightarrow C = \epsilon_0 \epsilon_r \frac{w}{b} \frac{1}{4}$$

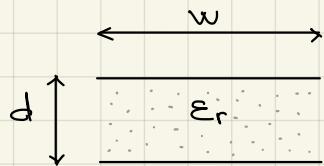
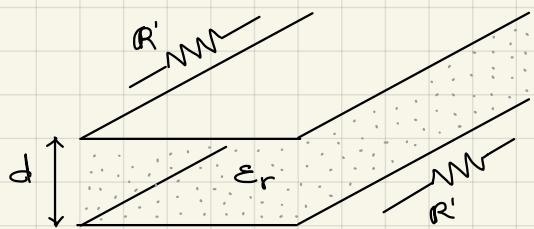
$$Z_c = \sqrt{\mu \epsilon} \frac{c}{v}$$

$$C = \sqrt{\frac{\mu_0 \epsilon_0 \epsilon_r}{Z_c}} = 99 \text{ pF/m}$$

$$w = \frac{bC}{4\epsilon_0 \epsilon_r} = 0,41 \text{ cm} \quad (\text{senza effetti di bordo})$$

Senza trascurare gli effetti di bordo

$$w = 0,26 \text{ cm}$$



$$\alpha_c = 5 \cdot 10^7 S/m$$

$$f = 500 \text{ MHz}$$

$$G = \frac{C}{3}$$

$$\epsilon_r = \epsilon'_r - j\epsilon''_r \quad e \quad \frac{\epsilon''_r}{\epsilon'_r} = 10^{-4} \quad Z_c = 100 \Omega \quad d = 2 \text{ mm}$$

Calcolare w e $\alpha_{tot} = \alpha_c + \alpha_o$ (in N_p/m , dB/m)

$$G = \frac{C}{\sqrt{\epsilon'_r}} = \frac{C}{3} \rightarrow \epsilon'_r = 9$$

$$Z_c = \sqrt{\mu \epsilon} \frac{c}{v}$$

$$C = \frac{\sqrt{\mu \cdot \epsilon_r \epsilon_0}}{Z_c} = 100 \text{ pF/m}$$

$$C = 100 \text{ pF/m} = \epsilon_0 \epsilon_r \frac{w}{d} \rightarrow w = 2,5 \text{ mm}$$

$$\alpha_c = \frac{R}{2Z_c} \text{ con } R = \frac{R_s}{w} \cdot 2, \quad R_s = \frac{1}{\alpha_c \delta} \quad \text{dove } \delta = \sqrt{\frac{1}{\pi f \mu_0 \alpha_c}}$$

$$\rightarrow R = 5,02 \Omega/m \quad e \quad \alpha_c = 0,025 N_p/m$$

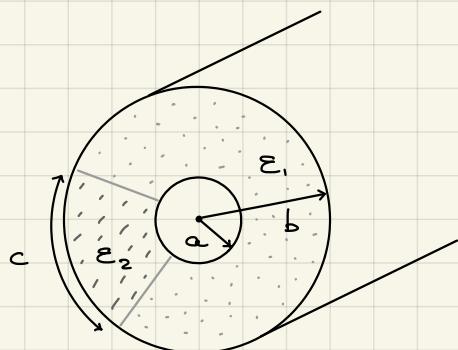
$$\alpha_D = \frac{C \cdot Z_C}{2} \quad G = \underbrace{\omega \epsilon''}_{\alpha_{eq}} \frac{W}{D} = 3,14 \cdot 10^{-5} \frac{S}{m}$$

$$\rightarrow \alpha_D = 1,57 \cdot 10^{-3} \frac{N_p}{m} \quad \text{oppure}$$

$$\alpha_D = \frac{\pi}{\lambda} \frac{\epsilon_r''}{\epsilon_r'} =$$

$$= \frac{\pi f}{5} \frac{\epsilon_r''}{\epsilon_r'} = 1,57 \cdot 10^{-3} \frac{N_p}{m}$$

$$\Rightarrow \alpha_{TOT} = \alpha_c + \alpha_D = 0,0266 \frac{N_p}{m} \quad (= 0,23 \frac{dB}{m})$$

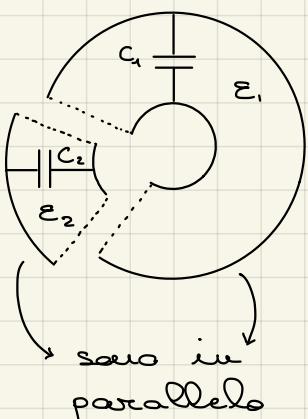


$$\epsilon_1 = 2\epsilon_0 \quad a = 0,5 \text{ cm}$$

$$\epsilon_2 = 2\epsilon_0 \quad b = 1 \text{ cm}$$

$$c = 1 \text{ cm}$$

Calcolare C , L , Z_C , ϵ_{eff} .



$$C = C_1 + C_2 =$$

$$= \frac{2\pi \epsilon_1 \cdot (2\pi b - c)}{\ln \frac{b}{a}} + \frac{2\pi \epsilon_2 \cdot c}{\ln \frac{b}{a}} = 2,62 \cdot 10^{-10} \frac{F}{m}$$

$L = L_0$ (L_0 l'induttanza non viene influenzata dal dielettrico con cui è riempita la linea)

$$C_0 = \frac{2\pi \epsilon_0}{\ln \frac{b}{a}} = 8 \cdot 10^{-11} \frac{F}{m}$$

$$L_0 C_0 = \mu_0 \epsilon_0$$

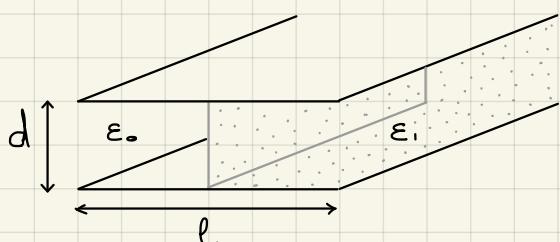
$$\rightarrow L_0 = 1,38 \cdot 10^{-7} \frac{H}{m}$$

$$Z_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{L_0}{C}} = 23 \Omega$$

$$G = \frac{1}{L_0 C} = 1,66 \cdot 10^8 \frac{m}{s}$$

$$C_0 L_0 = \mu \epsilon_{eff}$$

$$\rightarrow \epsilon_{eff} = \frac{C_0 L_0}{\mu_0} = 3,31 \epsilon_0$$



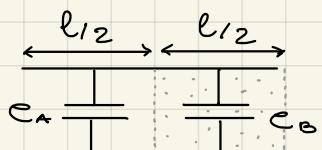
$$d = 0,5 \text{ cm}$$

$$\epsilon_1 = 3\epsilon_0$$

$$l = 3 \text{ cm}$$

$$\sigma = 3 \cdot 10^7 \frac{S}{m}$$

Calcolare Z_c e α a 1GHz.



$$C = C_A + C_B = \frac{\epsilon_0 l}{2d} + \frac{\epsilon_1 l}{2d} = 1,06 \frac{F}{m}$$

$$\lambda = \lambda_0 \quad C_s = \frac{\epsilon_0 l}{d} = 5,32 \cdot 10^{-11} \frac{F}{m} \quad C_0 \lambda_0 = \mu_0 \epsilon_0$$

$$Z_c = \sqrt{\frac{\lambda}{C}} = \sqrt{\frac{\lambda_0}{C}} = 44,4 \Omega$$

$$\rightarrow \lambda_0 = 2,09 \cdot 10^7 \frac{Hz}{m}$$

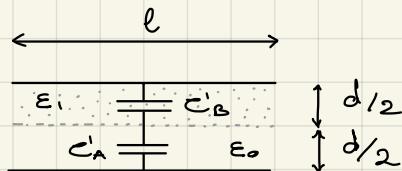
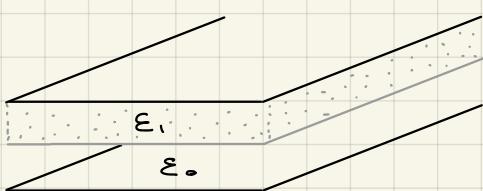
$$U = \frac{1}{\sqrt{2C}} = 2,12 \cdot 10^8 \frac{m}{s} \quad \epsilon_{eff} = 2$$

$$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = 0,0115 \Omega$$

$$R = 2R' = \frac{R_s}{l} \cdot 2 = 0,767 \frac{\Omega}{m}$$

$$\alpha = \frac{R}{2Z_c} = 8,6 \cdot 10^{-3} \frac{N_p}{m}$$

Stessa linea quasi-TEM, ma con il dielettrico disposto diversamente.



$$C' = C'_A + C'_B \text{ con } C'_A = \epsilon_0 \frac{l}{d/2} \text{ e } C'_B = \epsilon_1 \frac{l}{d/2}$$

$$= \frac{C'_A \cdot C'_B}{C'_A + C'_B} = 0,796 \cdot 10^{-10} \frac{F}{m}$$

$$L' = \lambda_0 \text{ come prima.}$$

R e R_s come prima.

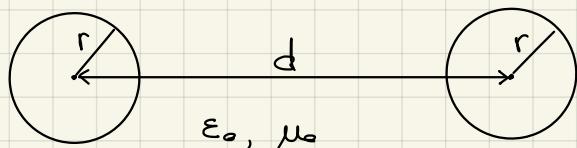
$$Z'_c = \sqrt{\frac{\lambda_0}{C'}} = 51,3 \Omega > Z_c$$

$$\alpha' = \frac{R}{2Z'_c} = 7,5 \cdot 10^{-3} \frac{N_p}{m} < \alpha$$

$$U' = \frac{1}{\sqrt{2C'}} = 2,45 \cdot 10^8 \frac{m}{s}$$

$$\epsilon'_{eff} = 1,5$$

minore perdite



$$d = 3 \text{ cm}$$

$$f = 100 \text{ MHz}$$

$$Z_c = 300 \Omega$$

$$\alpha = 5 \cdot 10^7 \frac{S}{m}$$

Determinare r (approx. conduttori sottili).
Calcolare α_c (N_p/m).

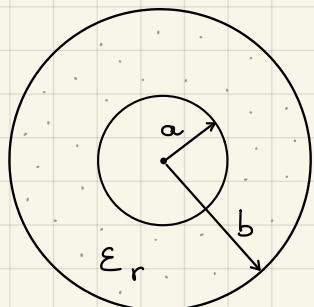
$$Z_c = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\pi} \ln \frac{d}{r} = 300 \Omega \rightarrow \frac{d}{r} = 12,18 \rightarrow r = 0,25 \text{ cm}$$

$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} = 7,12 \mu\text{m}$ → pochissimo dello spessore del conduttore viene effettivamente attraversato da corrente

$$R = 2R' = 2R_s =$$

$$= \frac{1}{\pi r \sigma \delta} = 0,36 \frac{\Omega}{\text{m}}$$

$$\alpha_c = \frac{R}{2Z_c} = 6,05 \cdot 10^{-4} \frac{\text{Np}}{\text{m}}$$



Dimensionare la linea in modo che

$$G = \frac{c}{2} \quad Z_c = 75 \Omega \quad b = 0,5 \text{ cm}$$

Calcolare α_c e α_d a 500 MHz con $\sigma = 5 \cdot 10^6 \frac{\text{S}}{\text{m}}$ e $\epsilon_r'' = 10^{-3} \epsilon_r'$

$$Z_c = \frac{1}{G \tau} \rightarrow \tau = \frac{1}{\frac{c}{2} Z_c} = 89 \text{ pF/m}$$

$$G = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c}{2} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cdot \frac{1}{2} \rightarrow \sqrt{\epsilon_r} = 2 \quad \epsilon_r = 4$$

$$\tau = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \frac{b}{a}} \rightarrow \ln \frac{b}{a} = 2,5 \quad a = 0,041 \text{ cm}$$

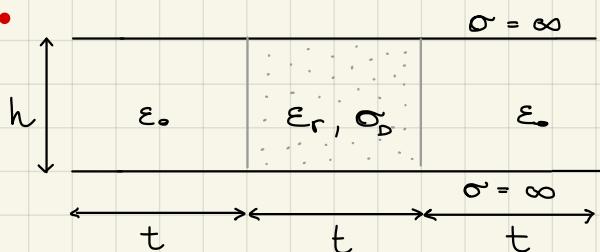
$$R_s = \frac{1}{\sigma \delta} = 6,28 \cdot 10^{-3} \frac{\Omega}{\square} \quad R = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) = 2,64 \frac{\Omega}{\text{m}}$$

$$\alpha_c = \frac{R}{2Z_c} = \frac{0,0176}{m} \frac{\text{Np}}{\text{m}}$$

x le linee TEM

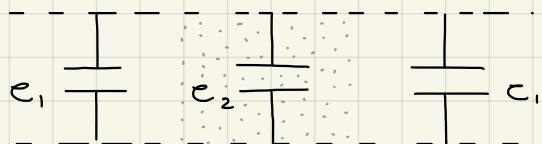
$$\alpha_d = \frac{\pi}{\lambda} \frac{\epsilon_r''}{\epsilon_r'} = \frac{\pi}{\frac{c}{2f}} \frac{\epsilon_r'}{\epsilon_r''} = \frac{10,48 \cdot 10^3}{m}$$

Oppure $\alpha_d = \frac{G Z_c}{2}$
x tutte le linee



$$\epsilon_r = 4 \quad \sigma_d = 5 \cdot 10^{-4} \frac{\text{S}}{\text{m}} \quad h = 1 \text{ mm}$$

Calcolare t affinché $Z_c = 60 \Omega$ (trascurando gli effetti di fondo).



$$C_1 = \frac{\epsilon_0 t}{h}$$

$$C_2 = \epsilon_0 \epsilon_r \frac{t}{h}$$

$$C = 2C_1 + C_2 = 6 \frac{\epsilon_0 t}{h}$$

$$Z_c = 60\Omega = \sqrt{\frac{L_0}{C}} = \sqrt{\frac{\mu_0 h \cdot h}{3t \epsilon_0 \epsilon_r t}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{h}{t} \cdot \frac{1}{\sqrt{18}} \rightarrow t = 1,48 \text{ cm}$$

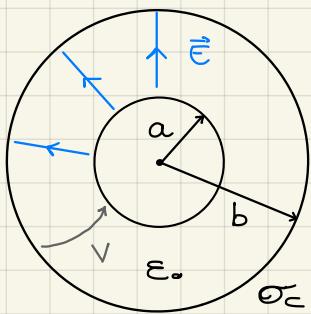
Calcolare α_D ($\frac{dB}{m}$) dovuta alle perdite nel dielettrico.

$$G = \frac{\sigma_D}{\epsilon} C_2 = \sigma_D \frac{t}{h} = 7,4 \cdot 10^{-4} \frac{S}{m}$$

ho perdite solo nel dielettrico

$$\alpha_D = \frac{G Z_c}{2} = 0,022 \frac{N_p}{m}$$

$$= 0,191 \frac{dB}{m}$$



$$b = 0,5 \text{ cm}$$

$$\sigma_c = 5,8 \cdot 10^7 \frac{S}{m}$$

$$\frac{b}{a} = 3,5 \rightarrow a = 0,143 \text{ cm}$$

Calcolare la massima potenza nel cavo coassiale, sapendo che $|E_{max}| = 30 \frac{kV}{cm}$ adottando un fattore di sicurezza 2.

Calcolare la potenza specifica dissipata, nei conduttori esterno ed interno, con $f = 1 \text{ GHz}$.

$$Z_c = \frac{1}{2\pi} \ln \frac{b}{a} = 75 \Omega$$

$$E(r) = \frac{V}{\ln \frac{b}{a}} \cdot \frac{1}{r}$$

$$E_{max} = E(r=a)$$

$$= 30 \frac{kV}{cm} \cdot \frac{1}{2}$$

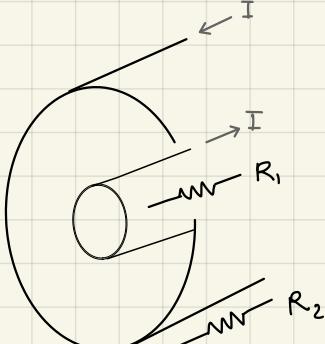
$$V_{max} = E_{max} \cdot \ln \frac{b}{a} = 2690 \text{ V}$$

$$P_{max} = \frac{1}{2} \frac{|V_{max}|^2}{Z_c} = 48 \text{ kW}$$

$$= 1,5 \cdot 10^6 \frac{V}{m}$$

$$I = \sqrt{\frac{2P_{max}}{Z_c}} = 35,8 \text{ A} = \frac{V_{max}}{Z_c}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{1}{\sigma S} = 0,00825 \frac{\Omega}{m}$$



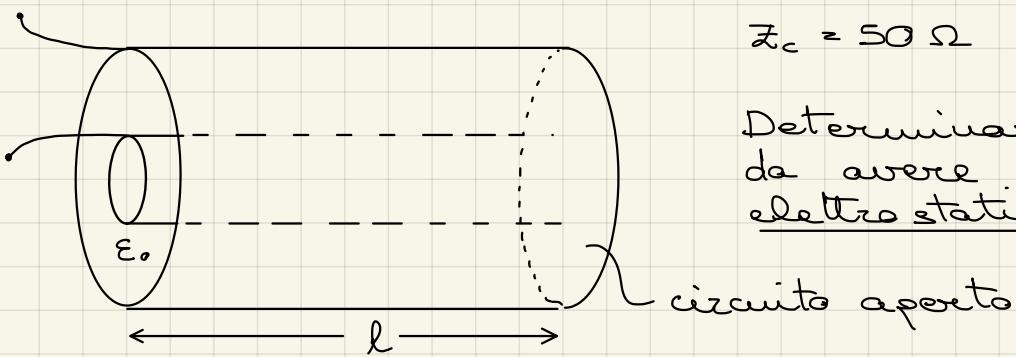
$$R_1 = \frac{R_s}{2\pi a} = 0,918 \frac{\Omega}{m}$$

$$R_2 = \frac{R_s}{2\pi b} = 0,263 \frac{\Omega}{m}$$

$$P_1 = \frac{1}{2} |I|^2 R_1 = 588 \frac{W}{m}$$

$$P_2 = \frac{1}{2} |I|^2 R_2 = 168 \frac{W}{m}$$

→ molto elevata!



Determinare l in modo da avere una capacità elettristica $C_o = 6 \text{ pF}$.

$$Z_c = \sqrt{\frac{\mu_0 \epsilon_0}{C}} \rightarrow C = 66,7 \text{ pF/m}$$

$$C_o = C \cdot l \rightarrow l = 0,09 \text{ m (9 cm)}$$

Determinare la frequenza massima alla quale si può usare il condensatore, per cui la capacità cambia del 5%

$$Y_{in} \rightarrow \frac{Z_c}{l} \quad Y_{in} = j \gamma_c \operatorname{tg} \beta l \quad Y'_{in} \rightarrow \frac{1}{C_o} \quad Y'_{in} = j \omega C_o$$

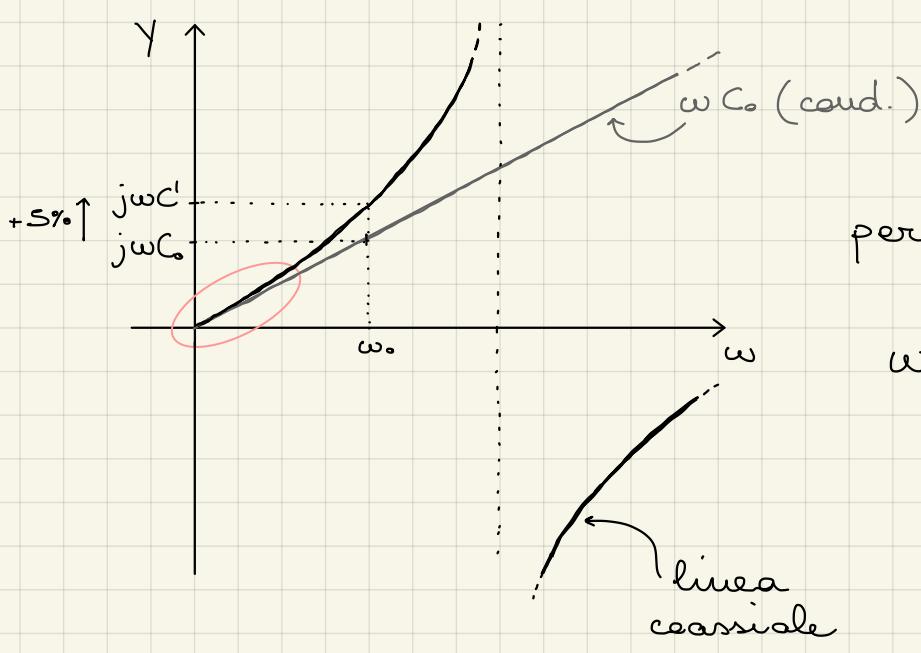
$$Y_{in} = Y'_{in} \quad \omega C_o = \gamma_c \operatorname{tg} \beta l$$

$$\operatorname{cos} \beta l = \frac{2\pi l}{\lambda} \ll 1 \quad (\omega \rightarrow 0, \lambda \rightarrow +\infty)$$

quindi $\operatorname{tg} \beta l \approx \beta l$

$$\omega C_o \approx \gamma_c \frac{2\pi}{\lambda} l = \gamma_c \frac{2\pi}{\lambda} \cdot l \rightarrow \frac{\gamma_c}{C} \cdot l = C_o \rightarrow l = 9 \text{ cm}$$

come prima!
(la soluzione dinamica per $\omega \rightarrow 0$ equivale al
la soluzione statica)



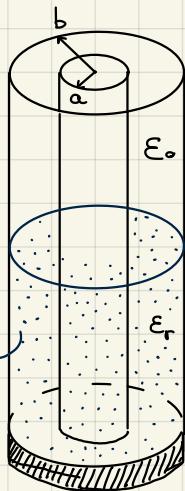
$$\text{per } \omega = \omega_0 \quad C' = C_o (1 + 5\%) \\ = 6,3 \text{ pF}$$

$$\omega_0 C' = \gamma_c \operatorname{tg} \left(\frac{\omega_0}{c} \cdot l \right)$$

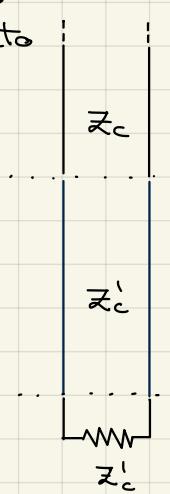
↓ risoluzione numerica

$$\omega_0 = 1,2 \cdot 10^9 \frac{\text{rad}}{\text{s}}$$

$$f_o = \frac{\omega_0}{2\pi} = \underline{190 \text{ MHz}}$$



liquido
carico adattato



$$|V_{max}| = 2V \quad |V_{min}| = 0,6mV$$

Ricavare la εᵣ del liquido

$$\begin{cases} Z_c' = \frac{\eta_0}{2\pi F \epsilon_r} \ln \frac{b}{a} \\ Z_c = \frac{\eta_0}{2\pi} \ln \frac{b}{a} \end{cases}$$

$$ROS = \frac{|V_{max}|}{|V_{min}|} = 3,33$$

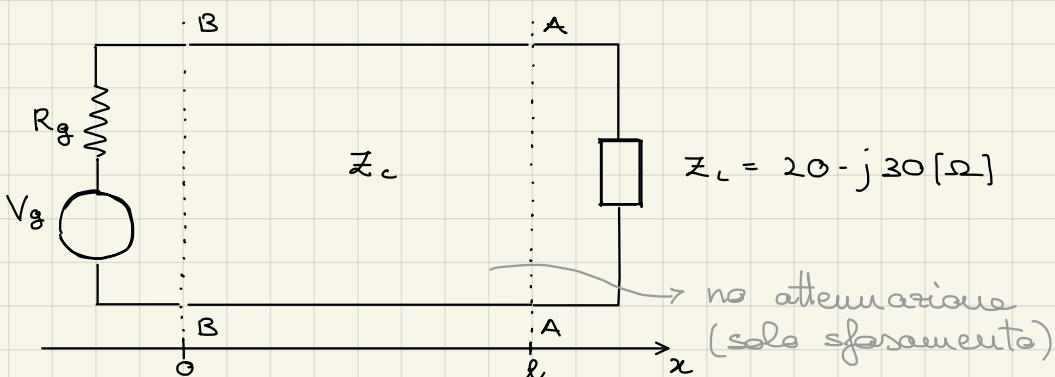
$$|\Gamma| = \frac{ROS - 1}{ROS + 1} = 0,54$$

Z_c' e Z_c reali
 $Z_c' \leq Z_c$ → Γ reale e negativo

$$\Gamma = \frac{Z_c' - Z_c}{Z_c' + Z_c}$$

$$\Rightarrow \frac{Z_c' - Z_c}{Z_c' + Z_c} = -0,54 \rightarrow \sqrt{\epsilon_r} = 3,33$$

$$\underline{\epsilon_r \approx 11}$$



$$V_g = 10V$$

$$R_g = 50\Omega$$

$Z_c = 50\Omega$ } adattata

$$l = 3,5m$$

$$f = 100 \text{ MHz}$$

Calcolare P_L , $|V_{AA}|$, $|I_{AA}|$, $|V_{max}|$, $|V_{min}|$ e V_{BB} .

$$P_d = P_m^+ = \frac{|V_g|^2}{8R_g} = 0,25 \text{ W} \quad \text{potenza disponibile}$$

adattamento tra generatore e linea

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = -0,207 - j0,517 \quad |\Gamma_L| = 0,557$$

$$P_L = P_d (1 - |\Gamma_L|^2) = 0,172 \text{ W}$$

$$P_L = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\{Y_L\} = \frac{1}{2} |I_{AA}|^2 \operatorname{Re}\{Z_L\}$$

$$\hookrightarrow P_L = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\{0,0154 + j0,023\} \quad P_L = \frac{1}{2} |I_{AA}|^2 \operatorname{Re}\{20 - j30\}$$

$$|V_{AA}| = 4,37 \text{ V}$$

$$|I_{AA}| = 0,131 \text{ A}$$

Altro metodo:

$$|V^+(0)| = \frac{V_g}{2} = 5 \text{ V}$$

adattamento

tra generatore
e linea

linea non
attenuativa

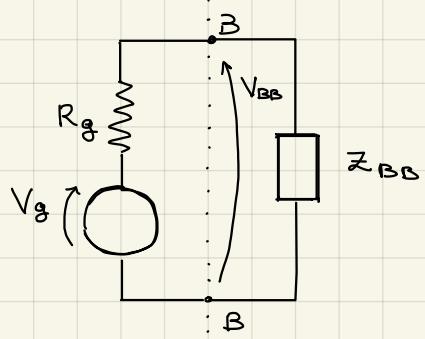
$$|V_L| = |V_{AA}| = |V^+(l) + V^-(l)| = |V^+(l)| |1 + \Gamma_L| = |V^+(0)| |1 + \Gamma_L| = 5 \cdot |1 - 0,207 - j0,517| = 4,73 \text{ V}$$

$$|I_{AA}| = |I^+(l) + I^-(l)| = |I^+(l)| \cdot |1 - \Gamma_L| = \frac{|V^+(0)|}{Z_c} |1 - \Gamma_L| = 0,131 \text{ A}$$

$$\lambda = \frac{c}{f} = 3 \text{ m}$$

$$|V_{MAX}| = |V^+(0)| + |V^-(0)| = |V^+(0)| (1 + |\Gamma_L|) = 7,78 \text{ V}$$

$$|V_{min}| = |V^+(0)| - |V^-(0)| = |V^+(0)| (1 - |\Gamma_L|) = 2,21 \text{ V}$$



$$Z_{BB} = Z_c \frac{Z_c + j Z_c \tan(\beta l)}{Z_c + j Z_c \tan(\beta l)} = 17,2 + j 22 \text{ [Ω]}$$

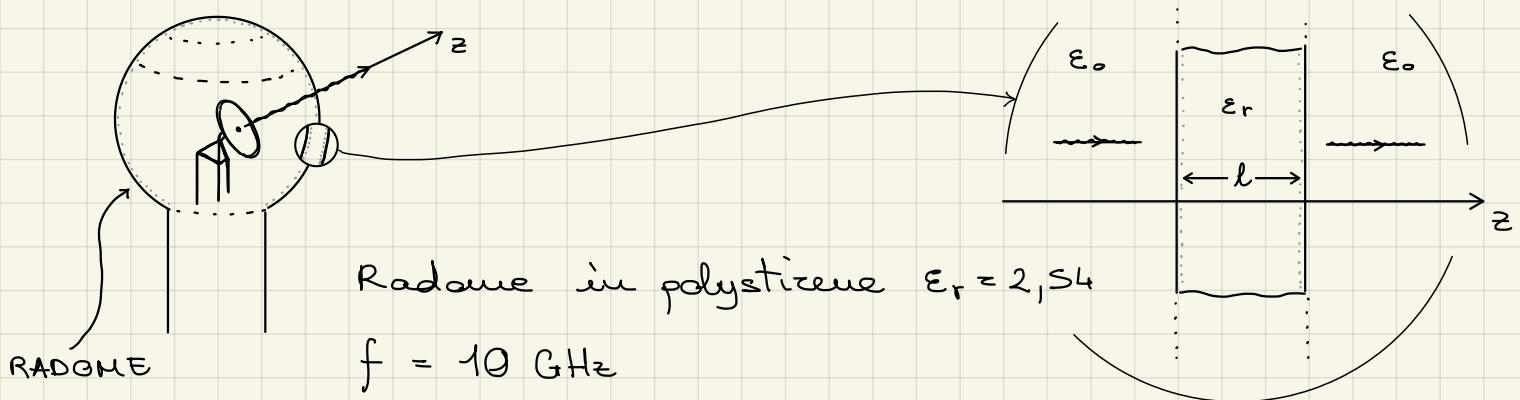
(Altro metodo: usa la carta di Smith)

$$V_{BB} = V_g \frac{Z_{BB}}{R_g + Z_{BB}} = 3,27 + j 2,19 \text{ [V]}$$

Altro metodo:

$$V_{BB} = V(0) = V^+(0) + V^-(0) = V^+(0) (1 + \Gamma(0)) = 3,27 + j 2,19 \text{ [V]}$$

$$\Gamma(0) = \Gamma_L e^{-j\beta l} = -0,344 + j 0,438$$



Calcolare l in modo che il radome sia trasparente per il segnale radar.

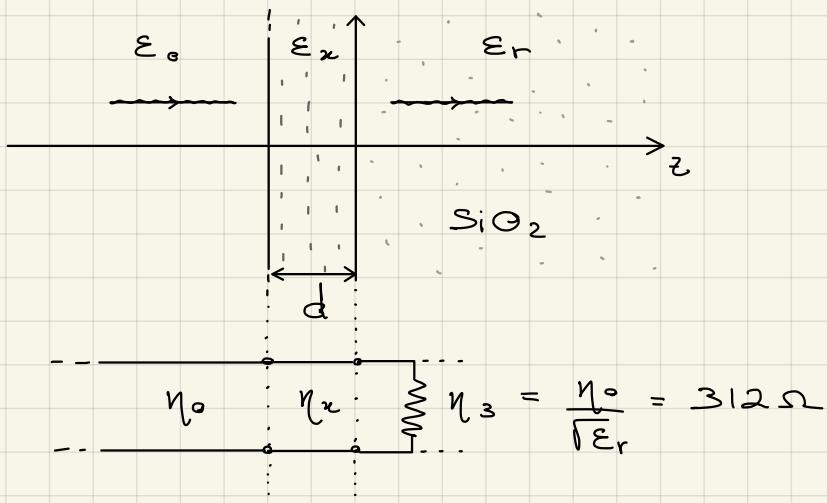


$$l = \frac{\lambda}{2} = \frac{\lambda_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{2} = \frac{c}{\sqrt{\epsilon_{rf}}} \cdot \frac{1}{2} = 9,4 \text{ mm}$$

→ da A a B deve compiere un giro completo ($0,5\lambda$) sulla carta di Smith

$$\eta_0 = 377 \Omega$$

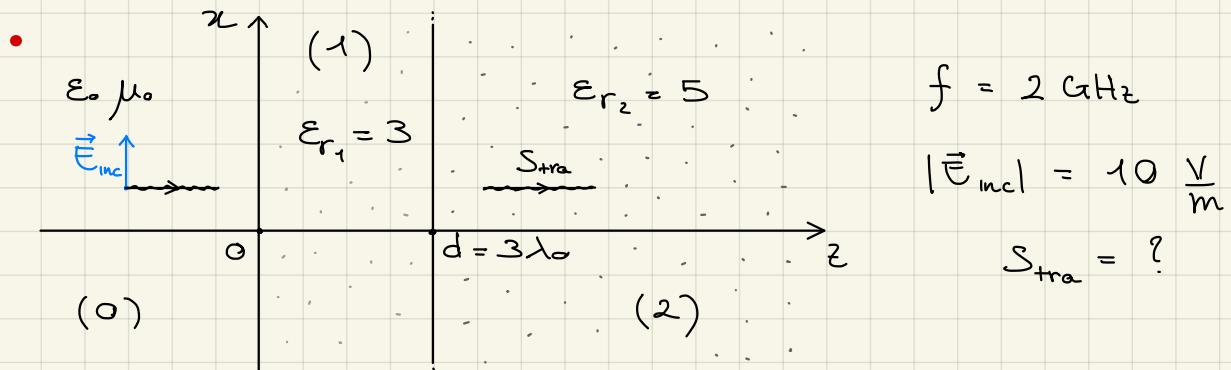
- Laser ad argon ($\lambda_0 = 488 \text{ nm}$) incide su silice fusa ($\epsilon_r = 1,46$)
- Dimensionare un setto antiriflesso (ϵ_x ? d ?)

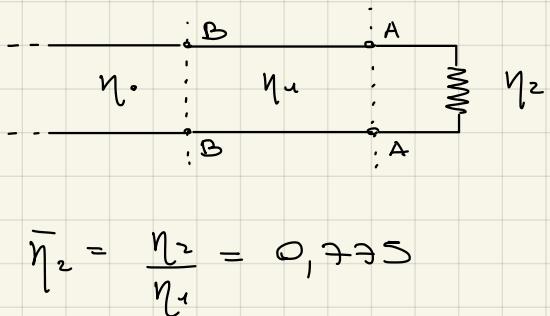


$$\rightarrow \text{Trasformatore } \frac{\lambda}{4}: \eta_x = \sqrt{312 \cdot 377} = 343 \Omega$$

$$\eta_x = \frac{\eta_0}{\sqrt{\epsilon_x}} \rightarrow \epsilon_x = 1,21 \Rightarrow 1 \text{ ok!}$$

$$d = \frac{\lambda}{4} = \frac{\lambda_0}{\sqrt{\epsilon_x}} \cdot \frac{1}{4} = 110 \text{ nm}$$





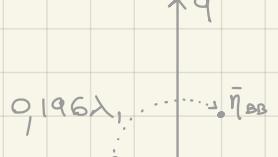
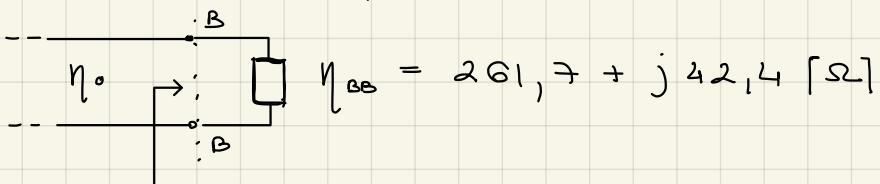
$$\eta_0 = 377 \Omega$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r_1}}} = 217,7 \Omega$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}} = 168,8 \Omega$$

$$\bar{\eta}_2 = \frac{\eta_2}{\eta_1} = 0,775$$

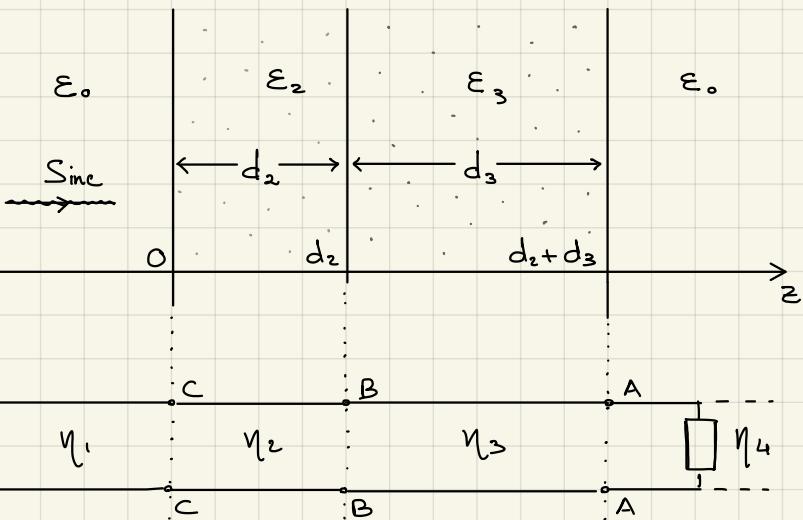
$$\frac{d}{\lambda_1} = 3\lambda_0 \frac{1}{\frac{\lambda_0}{\sqrt{\epsilon_{r_1}}}} = 3\sqrt{\epsilon_{r_1}} = 0,196 \rightarrow \bar{\eta}_{BB} = 1,2 + j0,195$$



$$\Gamma_{BB} = \frac{\eta_{BB} - \eta_0}{\eta_{BB} + \eta_0} = -0,175 + j0,078$$

$$S_{inc} = \frac{1}{2} \frac{|\vec{E}_{inc}|^2}{\eta_0} = 0,133 \frac{W}{m^2}$$

$$S_{tra} = S_{inc} (1 - |\Gamma_{BB}|^2) = 0,128 \frac{W}{m^2}$$



$$\epsilon_2 = 2,54 \epsilon_0$$

$$\epsilon_3 = 4 \cdot \epsilon_0$$

$$d_2 = 2 \text{ mm}$$

$$d_3 = 3 \text{ mm}$$

$$f = 10 \text{ GHz}$$

$$S_{tra} = ?$$

$$\eta_4 = \eta_1 = \eta_0 = 377 \Omega$$

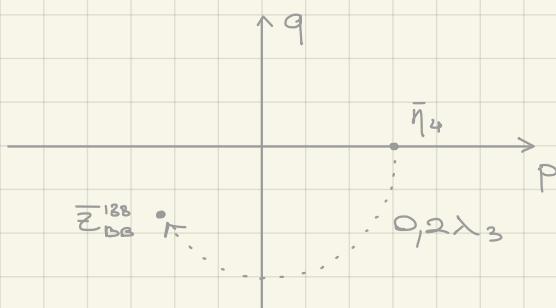
$$\eta_3 = \frac{\eta_0}{\sqrt{\epsilon_{r_3}}} = 188,5 \Omega$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}} = 236,5 \Omega$$

$$\bar{\eta}_4 = \frac{\eta_4}{\eta_3} = 2$$

$$\frac{d_3}{\lambda_3} = \frac{d_3}{\lambda_0} \sqrt{\epsilon_{r_3}} = \frac{3}{30} \sqrt{\epsilon_{r_3}} = 0,2$$

$$\frac{d_2}{\lambda_2} = \frac{d_2}{\lambda_0} \sqrt{\epsilon_{r_2}} = 0,106$$



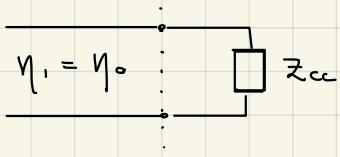
$$\bar{Z}_{BB}^{188} = 0,55 - j0,235$$

$$\bar{Z}_{BB}^{256} = \bar{Z}_{BB}^{188} \cdot \frac{\eta_3}{\eta_2} = 0,438 - j0,187$$

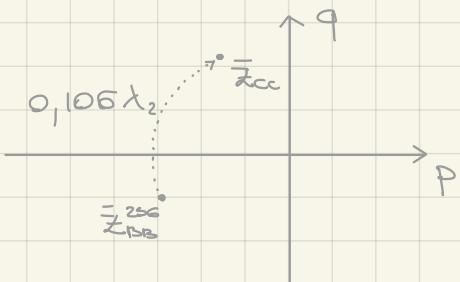
rinormalizzo

$$\bar{Z}_{CC} = 0,49 + j0,38$$

$$Z_{CC} = 166 + j90 \Omega$$



$$|\Gamma_{CC}| = \left| \frac{Z_{CC} - \eta_0}{Z_{CC} + \eta_0} \right| = 0,55$$



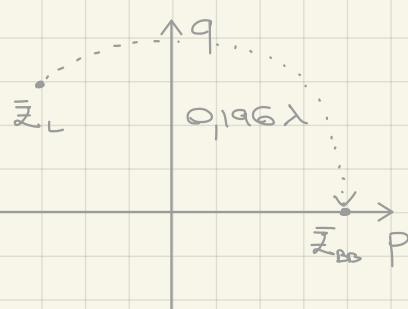
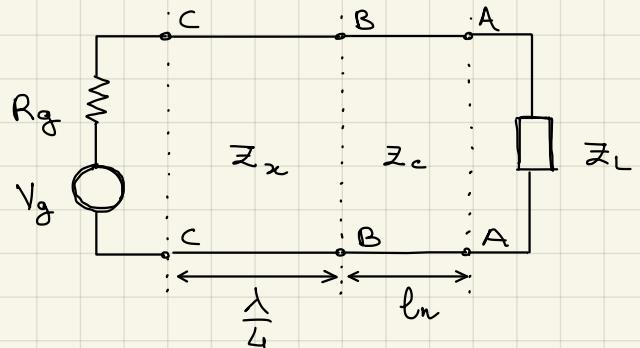
$$S_{tra} = S_{inc} \cdot (1 - |\Gamma_{CC}|^2) = 0,7 S_{inc}$$

$Z_L = 40 + j30 \Omega$ da adattare a un generatore con impedenza interna $R_g = 20 \Omega$ e $V_g = 10V$ tramite trasformatore $\frac{1}{4}$. Linea di neutralizzazione $Z_c = 100 \Omega$

Calcolare $|V_{AA}|$, $|V_{BB}|$ e $|V_{CC}|$.

$$\bar{Z}_L = \frac{Z_L}{Z_c} = 0,4 + j0,3$$

$$l_n = ? \quad Z_x = ?$$



$$\bar{Z}_{BB} = 2,76$$

$$l_n = 0,196 \lambda$$

$$Z_{BB} = \bar{Z}_{BB} Z_c = 276 \Omega$$

$$Z_x = \sqrt{\bar{Z}_{BB} R_g} = 74,3 \Omega$$

$$P_L = P_d = \frac{1}{8} \frac{|V_g|^2}{R_g} = 0,625 W$$

adattamento

$$P_L = \frac{1}{2} |V_{BB}|^2 \operatorname{Re}\{Y_{BB}\}$$

$$\rightarrow |V_{BB}| = 18,5 V$$

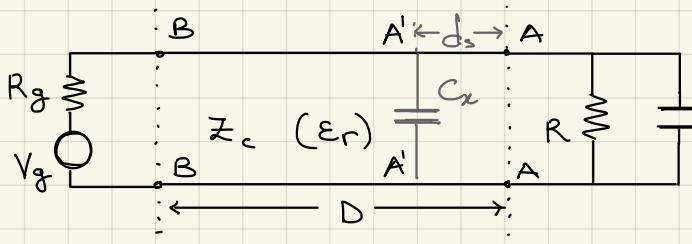
$$P_L = \frac{1}{2} |V_{CC}|^2 \operatorname{Re}\{Y_{CC}\}$$

$$\rightarrow |V_{CC}| = 5 V$$

$$P_L = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\{Y_{AA}\}$$

$$\rightarrow |V_{AA}| = 8,8 V$$

→ è (molto) più alto di V_g !



$$R = 100 \Omega$$

$$D = 46 \text{ m}$$

$$C = 20 \text{ pF}$$

$$\epsilon_r = 4$$

$$R_g = 75 \Omega$$

$$f = 100 \text{ MHz}$$

a) Calcolare la potenza assorbita dal carico

$$P_d = 1 \text{ W}$$

$$Z_c = 50 \Omega$$

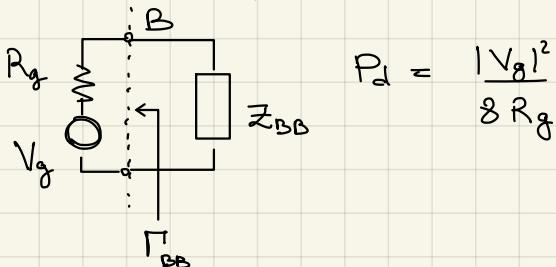
$$Y_{AA} = \frac{1}{R} + j\omega C = \frac{1}{100 \Omega} + j2\pi f C = \\ = 0,01 + 0,0126 [\Omega^{-1}]$$

$$\bar{Y}_{AA} = \frac{Y_{AA}}{Y_c} = Y_{AA} \cdot Z_c = 0,5 + j0,63$$

$$\frac{D}{\lambda} = \frac{D}{\lambda_0} \sqrt{\epsilon_r} = 10,6667 = 0,1667$$

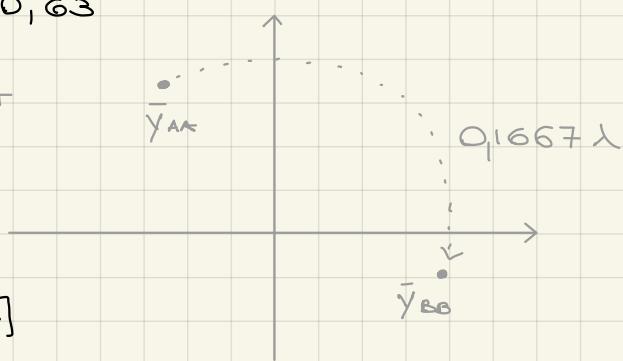
$$\bar{Y}_{BB} = 2,6 - j0,8$$

$$Z_{BB} = \frac{1}{\bar{Y}_{BB}} \cdot Z_c = 17,5 + j5,5 [\Omega]$$



$$P_L = P_d (1 - |\Gamma_{BB}|^2) = 0,611 \text{ W}$$

$$\text{con } |\Gamma_{BB}| = \frac{Z_{BB} - R_g}{Z_{BB} + R_g} = 0,6234$$



b) Adattare il carico alla linea, utilizzando un condensatore posto ad opportuna distanza d_s dalla sezione AA in parallelo alla linea

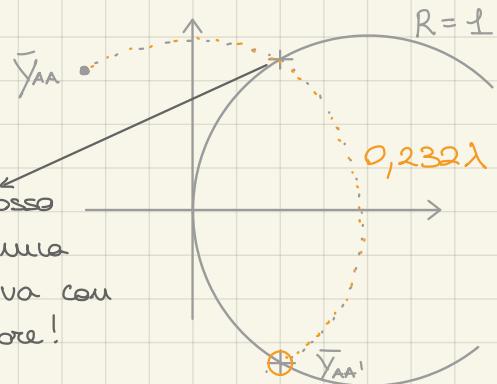
$$Y_{AA} \xrightarrow{d_s} 1 + jB \quad d_s = 0,232 \lambda$$

$$\bar{Y}_{AA} = 1 - j1,1$$

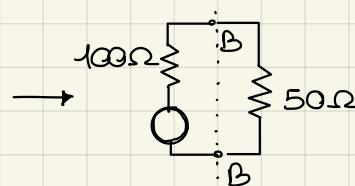
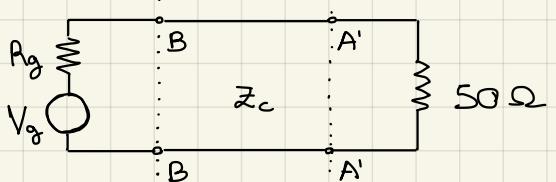
$$Y_s = +j1,1 Y_c \\ = j0,022 [\Omega^{-1}]$$

$$Y_s = j\omega C_x \rightarrow C_x = 35 \text{ pF}$$

non posso
compensare una
susettanza positiva con
un condensatore!

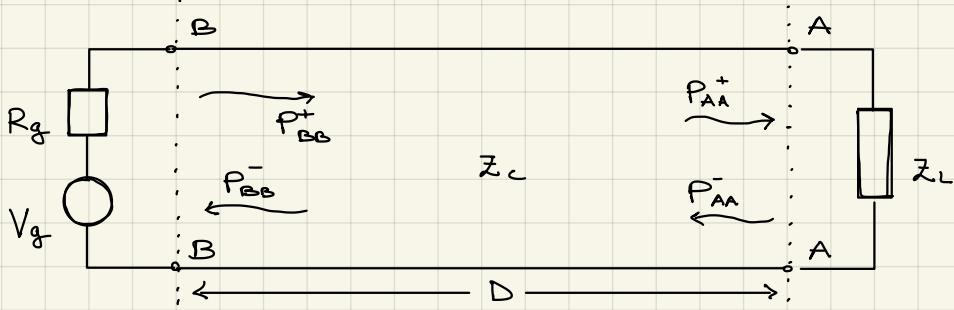


c) Calcolare la potenza assorbita dal carico con la capacità appena inserita.



$$\Gamma_{BB} = \frac{50 \Omega - 75 \Omega}{50 \Omega + 75 \Omega} = 0,2$$

$$P_L = P_d (1 - |\Gamma_{BB}|^2) = 0,96 \text{ W}$$



$$R_g = 50 \Omega \quad \left. \begin{array}{l} \text{linea} \\ \text{adattata} \end{array} \right\} \text{al generatore}$$

$$Z_c = 50 \Omega$$

$$Z_L = 170 \Omega$$

$$D = 25 \Omega$$

Calcolare:

$$P_L \text{ (assorbita dal carico)}$$

$$P_{dis} \text{ (dissipato dalla linea)}$$

$$P_g \text{ (erogata dal generatore).}$$

$$\text{coeff. di attenuazione} \quad \alpha_{dB} = 8 dB / 100m$$

$$P_d = 163 W$$

$$\alpha = \frac{8}{100} \cdot \frac{1}{8,686} = 0,0092 \frac{Np}{m}$$

$$P_{BB}^+ = P_d \quad P_{AA}^+ = P_{BB}^+ e^{-2\alpha \cdot D} = 103 W$$

$$|\Gamma_{AA}| = \left| \frac{Z_L - Z_c}{Z_L + Z_c} \right| = 0,545 \quad P_L = P_{AA}^+ (1 - |\Gamma_{AA}|^2) = 72,3 W$$

$$P_{BB}^- = P_{AA}^+ |\Gamma_{BB}|^2 = 30,7 W \quad P_{AA}^- = P_{AA}^+ e^{-2\alpha D} = 19,3 W$$

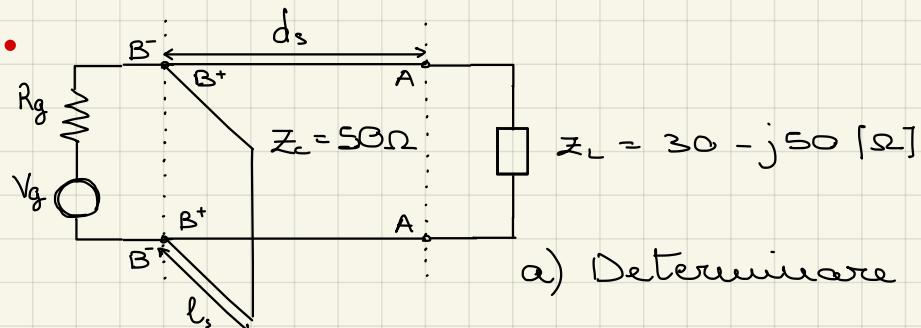
$$P_{dis} = \Delta P^+ + \Delta P^- = (163 - 103) + (30,7 - 19,3) = 71,4 W$$

$$P_g = P_d - P_{BB}^- = 163 - 19,3 = 143,7 W \quad (\text{oppure } P_g = P_d (1 - |\Gamma_{BB}|^2))$$

$$\text{con } |\Gamma_{BB}| = |\Gamma_{AA}| e^{-2\alpha D}$$

Calcolare P_L nel caso di linea non attenuativa ($\alpha = 0$)

$$P_{AA}^+ = P_{BB}^+ = P_d \quad P_L = P_{AA}^+ (1 - |\Gamma_{AA}|^2) = 114,6 W$$



$$R_g = 83 \Omega$$

$$V_g = 1 V$$

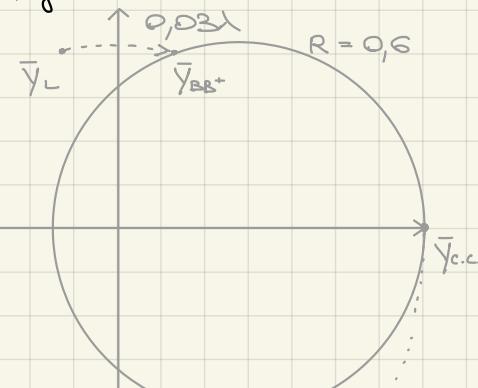
- Determinare d_s , l_s per l'adattamento
- Calcolare P_L , $|V_{AA}|$, $|V_{BB}|$
- Tracciare l'andamento delle tensioni lungo lo stub.

$$a) \bar{Y}_L = \frac{Z_c}{Z_L} = 0,44 + j 0,73$$

Dobbiamo finire nel punto $\bar{Y}_S = \frac{Z_c}{R_g} \approx 0,6$

$$d_s = 0,03\lambda \quad \bar{Y}_{BB^+} = 0,6 + 1j$$

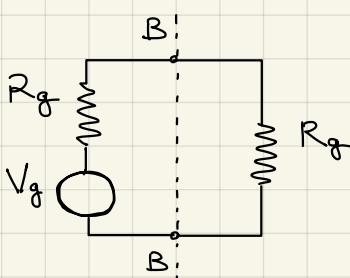
$$\rightarrow \bar{Y}_S = -1j \quad l_s = 0,125\lambda$$



adattato

$$b) \text{ Adattato} \quad P_L = P_d = \frac{|V_g|^2}{8R_g} = 1,5 \cdot 10^{-3} \text{ W}$$

$$|V_{BB}| = \frac{V_g}{2} = 0,5 \text{ V}$$

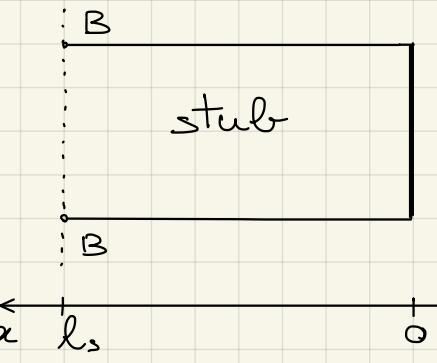


$$P_L = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\{Y_L\}$$

$$\rightarrow |V_{AA}| = 0,584 \text{ V}$$

$$\downarrow 0,0088 + j 0,0147$$

c)



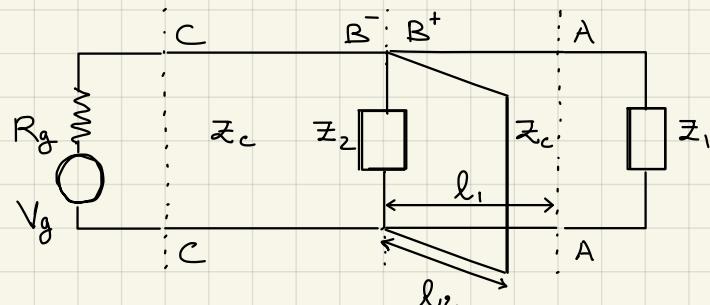
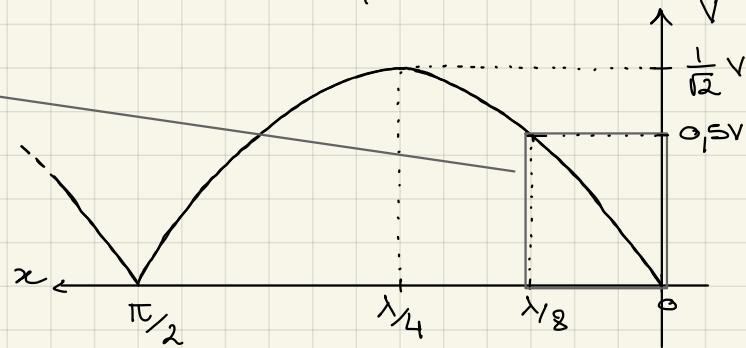
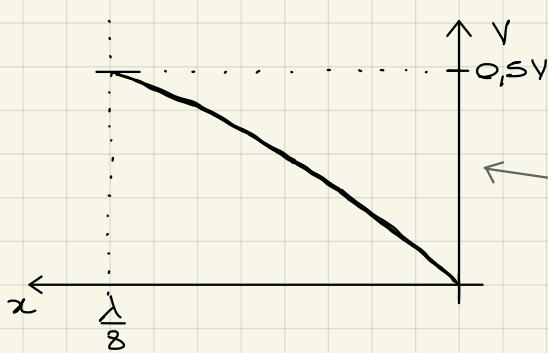
$$|V(x)| = |2V^+| |\sin(\beta x)|$$

Cond. al contorno:

$$|V(l_s)| = |V_{BB}| = 0,5 = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}$$

$$= |2V^+| |\sin(\beta l_s)| \frac{\pi}{4}$$

$$|2V^+| = \frac{0,5}{\sin \frac{\pi}{4}} = \frac{1}{\sqrt{2}}$$



$$R_g = 50 \Omega$$

$$V_g = 100 \text{ V}$$

$$Z_c = 50 \Omega$$

$$Z_2 = (100 - j100) \Omega$$

$$Z_1 = (100 + j50) \Omega$$

a) Determinare λ_1 e λ_2 in modo da avere adattamento in BB

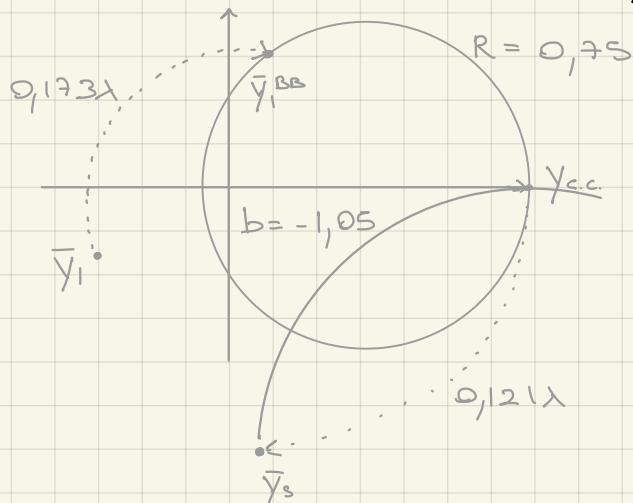
b) Calcolare P_{L1} e P_{L2}

$$a) \bar{y}_1 = \frac{\bar{z}_c}{\bar{z}_1} = 0,4 - j0,2 \quad \bar{y}_2 = \frac{\bar{z}_c}{\bar{z}_2} = 0,25 + j0,25$$

le metto a posto con le stub

$$\bar{y}_2 // \bar{y}_{BB} = 1 + jb \rightarrow 0,25 + j0,25 + \bar{y}_{BB} = 1 + jb$$

$$\Rightarrow \operatorname{Re}\{\bar{y}_{BB}\} = 0,75$$



$$\bar{y}_{BB} = 0,75 + j0,8$$

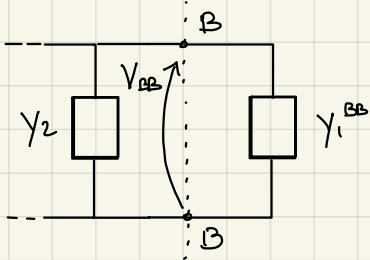
$$\lambda_1 = 0,173\lambda$$

$$\bar{y}_2 // \bar{y}_{BB} = 1 + j(-1.05)$$

$$\rightarrow y_s = -1.05j$$

$$\lambda_2 = 0,121\lambda$$

b) Adattato: $P_{L1} + P_{L2} = P_d$ con $P_d = \frac{|V_{BB}|^2}{8R_g} = 25W$



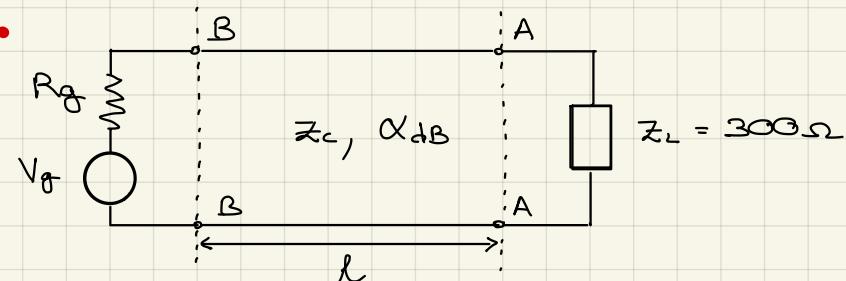
$$\frac{P_{L1}}{P_{L2}} = \frac{\frac{1}{2} |V_{BB}|^2 \operatorname{Re}\{Y_{BB}\}}{\frac{1}{2} |V_{BB}|^2 \operatorname{Re}\{Y_2\}} \Rightarrow \frac{P_{L1}}{P_{L2}} = \frac{\operatorname{Re}\{Y_{BB}\}}{\operatorname{Re}\{Y_2\}}$$

$$P_{L1} = P_d \cdot 0,75$$

$$P_{L2} = P_d \cdot 0,25$$

$$P_{L1} = 18,75W$$

$$P_{L2} = 6,25$$



$$R_g = 75\Omega \quad f = 100MHz$$

$$V_g = 10V \quad l = 100m$$

$$\alpha_{dB} = 2dB/100m$$

Calcolare: P_L , P_{diss} , P_{gen} , V_{AA} .

$$\bar{Z}_L = \frac{\bar{z}_L}{\bar{z}_c} = 6$$

$$\lambda = 3m$$

$$\frac{l}{\lambda} = \cancel{33} + \frac{1}{3}$$

$$|\Gamma_{BB}| = |\Gamma_{AA}| e^{-2\alpha l} \quad \text{con} \quad \alpha = \frac{2}{100} \cdot \frac{1}{8,686} = 2,3 \cdot 10^{-3} \frac{\text{N}_F}{\text{m}}$$

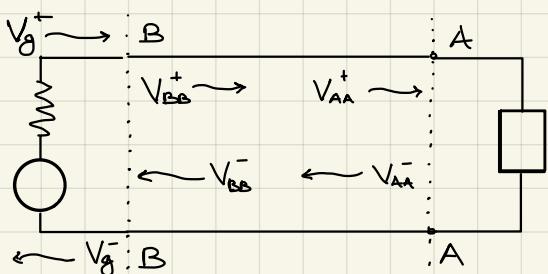
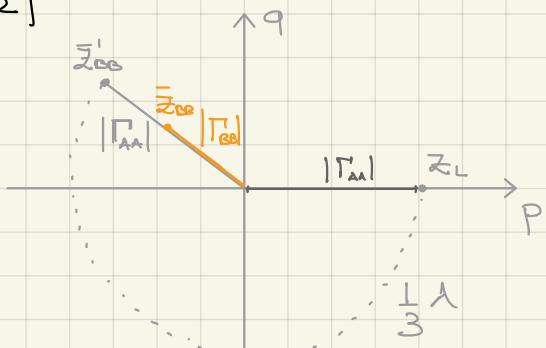
$$|\Gamma_{AA}| = \left| \frac{Z_L - Z_c}{Z_L + Z_c} \right| = 0,714 \quad \rightarrow \quad |\Gamma_{BB}| = 0,45$$

$$\bar{Z}_{BB} = 0,48 + j 0,48 \quad Z_{BB} = 24 + j 24 [\Omega]$$

$$|\Gamma_{BB}^{R_g}| = \left| \frac{Z_{BB} - R_g}{Z_{BB} + R_g} \right| = 0,55$$

$$P_{gen} = P_d (1 - |\Gamma_{BB}^{R_g}|^2) = 0,116 \text{ W}$$

$$\frac{|V_g|^2}{8R_g} = \frac{1}{6} \text{ W}$$



$$V_g^+ = \frac{V_g}{2} = 5 \text{ V}$$

$$V_{AA}^+ = \Gamma_{AA} V_{AA}^+ \quad \text{con} \quad \Gamma_{AA} = 0,714$$

$$V_{BB}^- = V_{AA}^- e^{-rl} \quad \text{con} \quad \gamma = \alpha + j\beta \\ = \alpha + j \frac{2\pi}{\lambda}$$

$$\gamma = 2,3 \cdot 10^{-3} + j 2,094 \left[\frac{1}{\text{m}} \right] \quad V_{BB}^+ = V_{AA}^+ e^{rl}$$

$$\text{In } BB: V_g^+ + V_g^- = V_{BB}^+ + V_{BB}^- \quad \text{con} \quad V_g^- = V_g^+ \cdot \Gamma_{BB}^{R_g} \quad \text{e}$$

$$\Gamma_{BB}^{R_g} = -0,431 + j 0,347$$

$$V_g^+ (1 + \Gamma_{BB}^{R_g}) = V_{AA}^+ (e^{rl} + \Gamma_{AA}^- e^{-rl})$$

$$\hookrightarrow V_{AA}^+ = -1,23 - j 2,93 \text{ [V]}$$

$$V_{AA} = V_{AA}^+ (1 + \Gamma_{AA}) = -2,1 - 5j \text{ [V]}$$

$$P_L = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\{Y_L\} = 0,0496 \text{ W}$$

$$P_{diss} = P_{gen} - P_L = 0,0664 \text{ W}$$

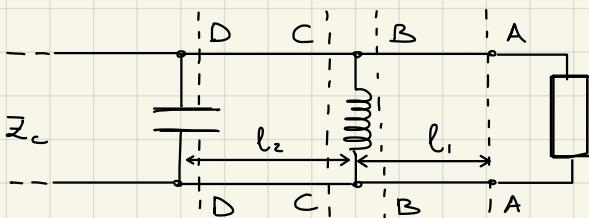
$$Z_c = 50 \Omega$$

$$f = 100 \text{ MHz}$$

$$l_1 = 0,165 \text{ m}$$

$$l_2 = 0,375 \text{ m}$$

Determinare C ed L in modo da adattare il circuito alla linea.



$$Z_L = 9 - j 32,5 \Omega$$

$$\bar{Y}_L = \bar{Y}_{AA} = \frac{\bar{Z}_c}{\bar{Z}_L} = 0,4 + j 1,43 \quad \frac{l_1}{\lambda} = 0,055 \quad \frac{l_2}{\lambda} = 0,125$$

L (

$$\bar{Y}_{BB} = 1,74 + j 3,17$$

$$\bar{Y}_{CC} = 1,74 + j 1,65$$

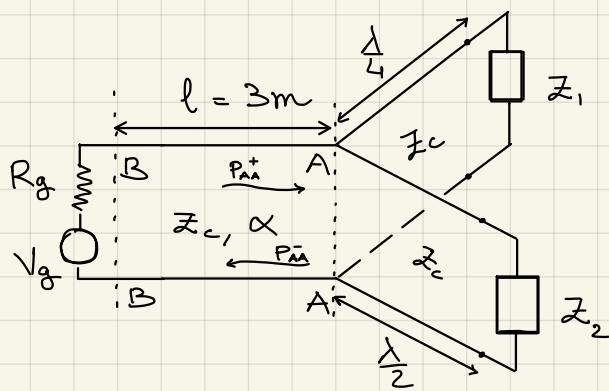
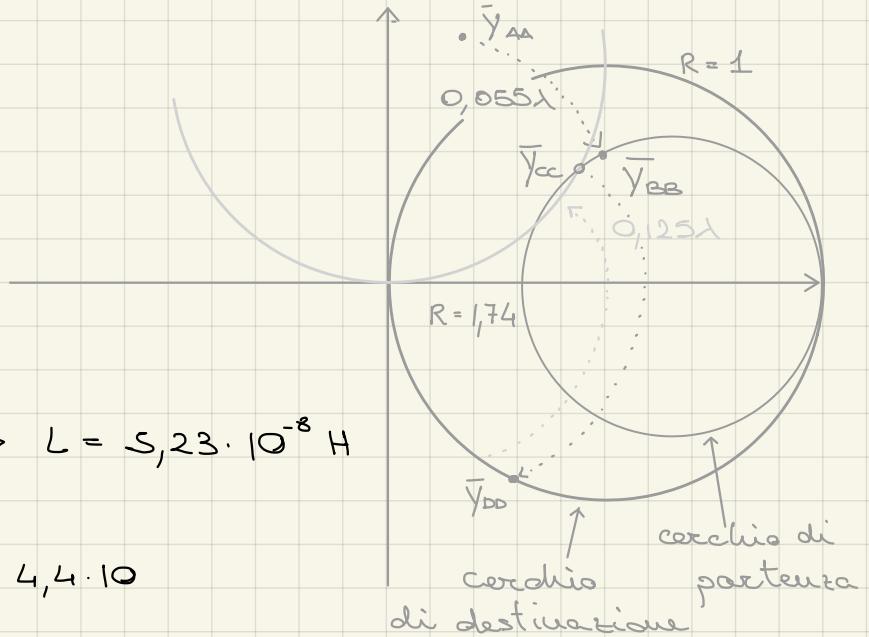
C (

$$\bar{Y}_{DD} = 1 - j 1,38$$

$$\bar{Y}_C$$

$$\frac{1}{j\omega L} = j \left(\frac{1,65 - 3,17}{\bar{Z}_c} \right) \rightarrow L = 5,23 \cdot 10^{-8} \text{ H}$$

$$j\omega C = j \frac{1,38}{\bar{Z}_c} \rightarrow C = 4,4 \cdot 10$$



$$Z_c = 50 \Omega$$

$$R_g = 50 \Omega$$

$$\alpha_{dB} = 50 \text{ dB} / 100m$$

$$Z_1 = (25 - j 25) \Omega$$

$$Z_2 = (50 - j 100) \Omega$$

$$\alpha_{NP} = 0,05756 \frac{Np}{m}$$

Calcolare P_{L1} , P_{L2} , P_{diss} .

$$Y_1^{AA} = Y_2 = \frac{1}{Z_2} = (0,004 + j 0,008) \Omega^{-1} \rightarrow \text{giro di } \frac{\lambda}{2} \rightarrow \text{imped. stessa}$$

$$Y_1^{AA} = \frac{Y_c^2}{Y_1} = \frac{Z_1}{Z_c^2} = (0,01 - j 0,01) \Omega^{-1} \rightarrow \text{trasformatore } \frac{\lambda}{4}$$

$$Y_{AA} = Y_1^{AA} + Y_2^{AA} = (0,014 - j 0,002) \Omega^{-1} \neq Y_c$$

generatore
adattato)

$$|\Gamma_{AA}| = \left| \frac{Y_c - Y_{AA}}{Y_c + Y_{AA}} \right| = \left| \frac{Z_{AA} - Z_c}{Z_{AA} + Z_c} \right| = 0,186$$

$$P_{AA}^+ = P_d e^{-2\alpha l} = 70,8 \text{ W} \quad P_L = P_{AA}^+ (1 - |\Gamma_{AA}|^2) = 68,3 \text{ W}$$

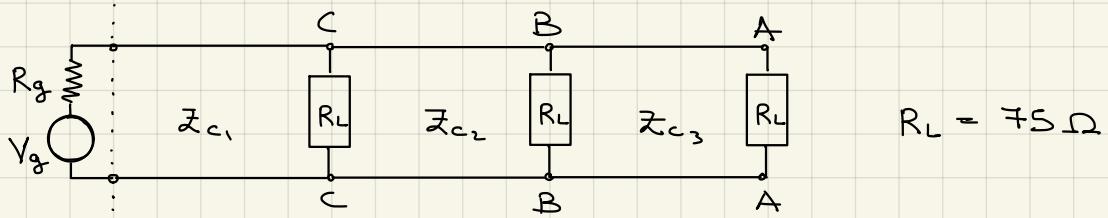
$$P_{AA}^- = P_{AA}^+ |\Gamma_{AA}|^2 = 2,5 \text{ W}$$

$$P_{L1} + P_{L2} = P_L$$

$$P_{L1} = \frac{1}{2} |Y_{AA}|^2 \operatorname{Re}\{Y_1^{AA}\} \quad P_{L2} = \frac{1}{2} |Y_{AA}|^2 \operatorname{Re}\{Y_2^{AA}\}$$

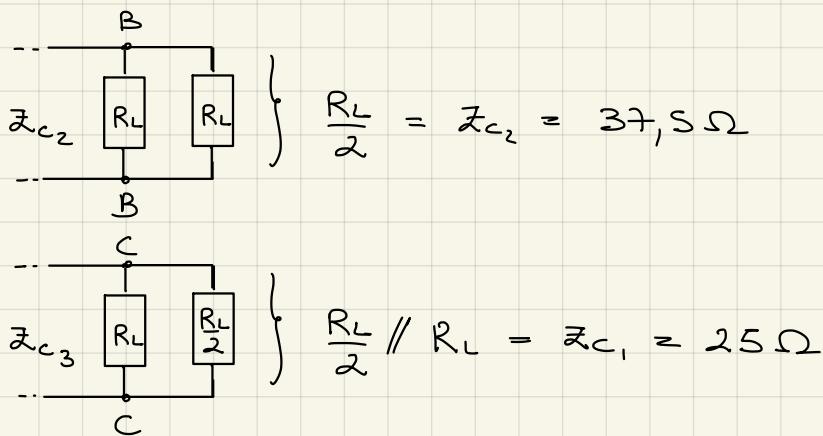
$$\begin{cases} P_{L_1} + P_{L_2} = P_L \\ \frac{P_{L_1}}{P_{L_2}} = \frac{\operatorname{Re}\{Y_1^{AA}\}}{\operatorname{Re}\{Y_2^{AA}\}} \end{cases} \implies P_{L_1} = 48,8 \text{ W} \quad P_{L_2} = 19,5 \text{ W}$$

$$P_{BB}^- = P_{AA}^- e^{-2\alpha l} \quad P_{diss} = (P_d - P_{AA}^+) + (P_{AA}^- - P_{BB}^-) = 29,9 \text{ W}$$

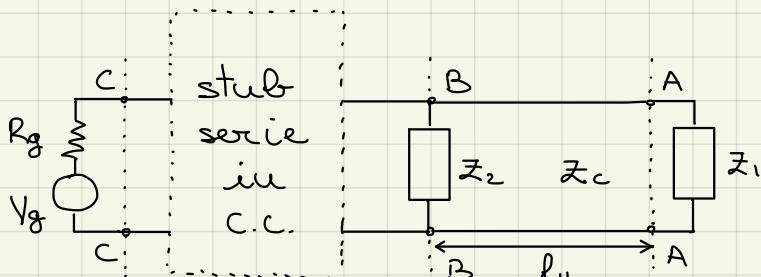


Determinare Z_{c_1} , Z_{c_2} , Z_{c_3} e R_g in modo da non avere riflessione qualunque sia la lunghezza delle linee (= qualunque sia la frequenza del generatore).

$$Z_{c_3} = R_L = 75 \Omega \quad (\text{linea BA adattata a } R_L^{AA})$$



$$R_g = Z_{c_1} = 25 \Omega$$



$$Z_c = 50 \Omega$$

$$V_g = 50 \text{ V} \quad R_g = 50 \Omega$$

$$Z_1 = 25 \Omega$$

$$Z_2 = (50 + j50) \Omega$$

$$f = 600 \text{ MHz}$$

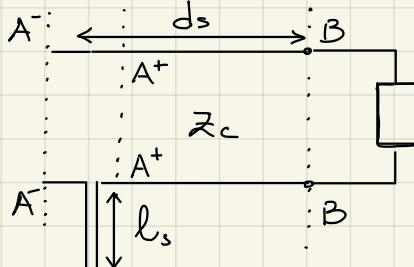
Determinare l_1 in modo che $P_{L_1} = P_{L_2}$ e dimensionare lo stub.

Per avere $P_{L_1} = P_{L_2}$ deve essere $\operatorname{Re}\{Y_{1,2}^{\text{BB}}\} = \operatorname{Re}\{Y_L\}$

$$\bar{Y}_1 = \frac{Z_c}{Z_1} = 2 \quad \bar{Y}_2 = \frac{Z_c}{Z_2} = 0,5 - j0,5$$

$$\bar{Y}_{1,2}^{\text{BB}} = 0,5 \quad l_1 = \frac{\lambda}{4}, \quad \lambda = 0,5 \text{ m}$$

$$\bar{Y}_{BB} = \bar{Y}_2 + \bar{Y}_{1,2}^{\text{BB}} = 1 - j0,5 (\neq Y_c)$$



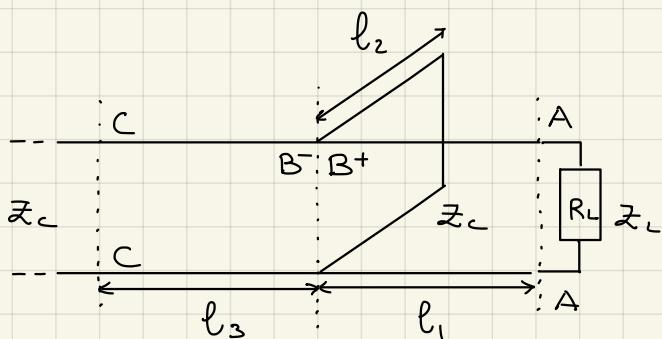
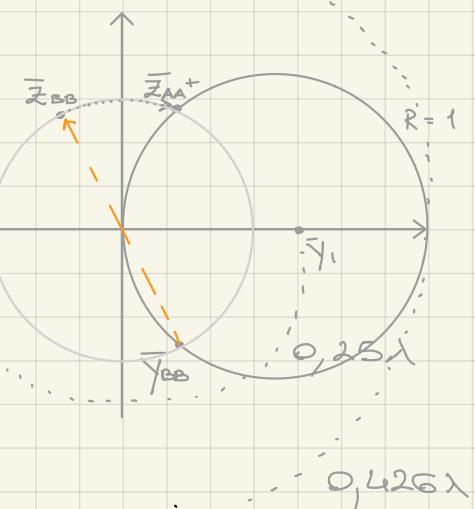
$$\bar{Z}_{BB} = 0,8 + j0,4$$

$$Z_{BB} = 40 + j20 [\Omega]$$

$$d_s = 0,038 \lambda$$

$$\bar{Z}_{AA+} = 1 + j0,5 \rightarrow \bar{z}_s = -0,5j$$

$$l_s = 0,426 \lambda$$



$$l_1 = 1,05 \text{ m}$$

$$l_2 = 0,67 \text{ m}$$

$$l_3 = 1 \text{ m}$$

$$Z_c = 50 \Omega$$

In CC si trova il MINIMO della tensione. Il ROS è 3. Determinare Z_L .

$$\text{ROS} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 3 \rightarrow |\Gamma| = 0,5$$

In CC Γ è reale negativo $\rightarrow \Gamma_c = -0,5$

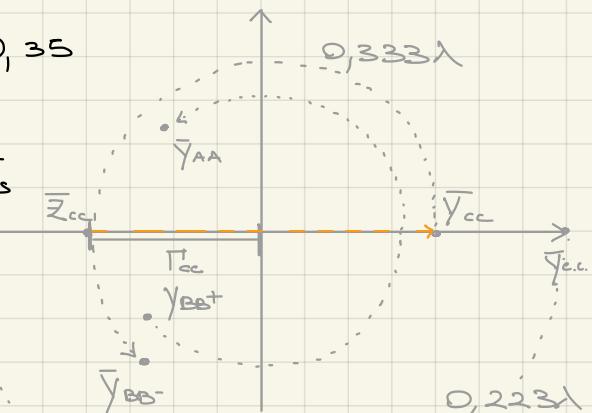
$$\frac{l_3}{\lambda} = 0,333 \quad \frac{l_2}{\lambda} = 0,223 \quad \frac{l_1}{\lambda} = 0,35$$

$\bar{Y}_{BB} = 0,42 - j0,50$ bisogna sottrarre \bar{y}_s

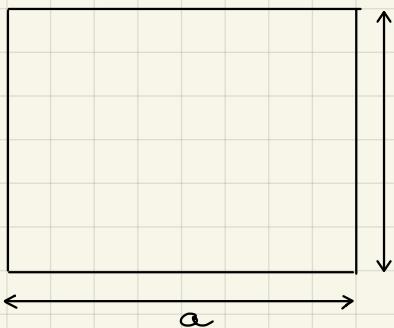
$$\bar{y}_s = -j0,17 \rightarrow \bar{Y}_{BB+} = 0,42 - j0,33$$

$$\bar{Y}_{AA} = \bar{Y}_L = 0,5 + 0,5j$$

$$Z_L = \frac{Z_c}{Y_L} = 50 - j50 [\Omega]$$



NB: passare da impedenza (Z) a suscettanza (y) o viceversa sulla carta di Smith significa trovare il punto speculare al centro.



$$a = 10 \text{ cm}$$

$$b = 7,5 \text{ cm}$$

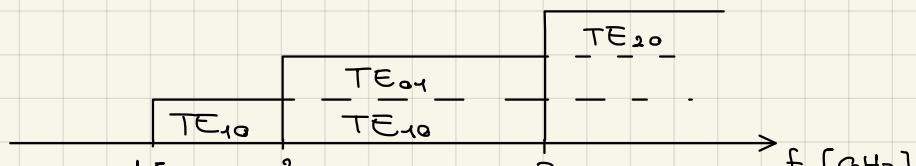
Determinare la banda di funzionamento monomodale

$$\text{TE}_{10}: \lambda_c = 2a = 0,2 \text{ m} \quad f_c = \frac{c}{\lambda_c} = 1,5 \text{ GHz}$$

$$\text{TE}_{01}: \lambda_c = 2b = 0,15 \text{ m} \quad f_c = 2 \text{ GHz}$$

$$\text{TE}_{20}: \lambda_c = a = 0,1 \text{ m} \quad f_c = 3 \text{ GHz}$$

$$\text{TE}_{02}: \lambda_c = b = 0,075 \text{ m} \quad f_c = 4 \text{ GHz}$$

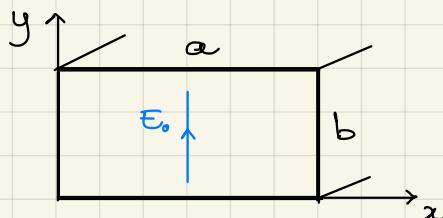


banda monomodale: 1,5 - 2 GHz

Qual è il valore di ϵ_r , in modo che $f_c = 1 \text{ GHz}$?

$$\lambda_c = 0,2 \text{ m} \quad f_c = \frac{c}{\sqrt{\epsilon_r \cdot \lambda_c}} = 1 \text{ GHz} \rightarrow \sqrt{\epsilon_r} = 1,5 \rightarrow \epsilon_r = 2,25 \quad (\text{TE}_{10})$$

$$\lambda_c = 0,15 \text{ m} \quad f_c = \frac{c}{\sqrt{\epsilon_r \cdot \lambda_c}} = 1,33 \text{ GHz} \quad \Rightarrow \quad \begin{array}{l} \text{nuova banda} \\ \text{monomodale} \\ 1 - 1,33 \text{ GHz} \end{array} \quad (\text{TE}_{01})$$



$$a = 3 \text{ cm}$$

$$f_c = 7 \text{ GHz}$$

$$b = 1,5 \text{ cm}$$

Rigidità dielettrica dell'aria: 30 kV/cm

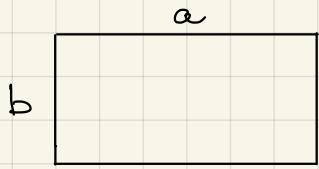
Calcolare la potenza massima, in condizioni di adattamento, con un coefficiente di sicurezza 2.

$$\text{TE}_{10}: \lambda_c = 2a = 0,06 \text{ m} \quad f_c = 5 \text{ GHz} \rightarrow \begin{array}{c} 5 \text{ GHz} \\ 10 \text{ GHz} \end{array} \quad \text{Banda monomod}$$

$$|E_{\max}| = 30 \frac{\text{kV}}{\text{cm}} \cdot \frac{1}{2} = 15 \frac{\text{kV}}{\text{cm}} = 1,5 \frac{\text{MV}}{\text{m}}$$

$$E_y(x) = |E_{\max}| \sin \left(\frac{\pi x}{a} \right) \quad P^+ = \frac{|E_{\max}|^2 a \cdot b}{4 \pi \epsilon_{r0}}$$

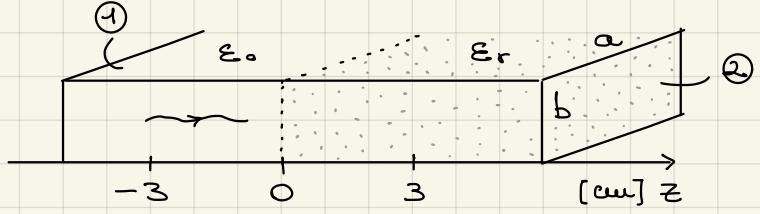
$$Z_{TE_{10}} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} = \frac{377 \Omega}{\sqrt{1 - \left(\frac{5}{7}\right)^2}} = 538,6 \Omega \implies P_{max}^+ = 470 \text{ kW}$$



$$a = 10 \text{ cm}$$

$$b = 5 \text{ cm}$$

$$\epsilon_r = 4$$

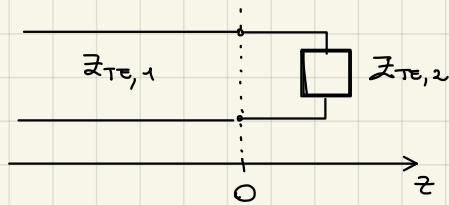


$$f_0 = 2 \text{ GHz} \quad |E_0| = 20 \frac{\text{mV}}{\text{m}} = |\epsilon^+(0)|$$

Calculare $|E|$ în $z = 0$, $z = 3 \text{ cm}$ și $z = -3 \text{ cm}$.

$$\lambda_c = 2a = 0,2 \text{ m} \quad \begin{cases} f_{c,1} = \frac{c}{\lambda_c} = 1,5 \text{ GHz (aria)} & (1,5-3) \text{ GHz} \\ f_{c,2} = \frac{c}{\sqrt{\epsilon_r} \lambda_c} = 0,75 \text{ GHz (dielectrico)} & (0,75-1,5) \text{ GHz} \end{cases}$$

$$Z_{TE_{10}} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_{c,1}}{f_0}\right)^2}} = 570 \Omega \quad Z_{TE_{12}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \frac{1}{\sqrt{1 - \left(\frac{f_{c,2}}{f_0}\right)^2}} = 203 \Omega$$



$$\Gamma = \frac{Z_{TE_{12}} - Z_{TE_{10}}}{Z_{TE_{12}} + Z_{TE_{10}}} = -0,475$$

Tratul ② (z pozitiv): ↗ se o unde progressiva

$$|E(0)| = |\epsilon^+(0)| \underbrace{|1 + \Gamma|}_{\Gamma} = 10,5 \frac{\text{mV}}{\text{m}}$$

$$|E(3 \text{ cm})| = |E(0)| = 10,5 \frac{\text{mV}}{\text{m}}$$

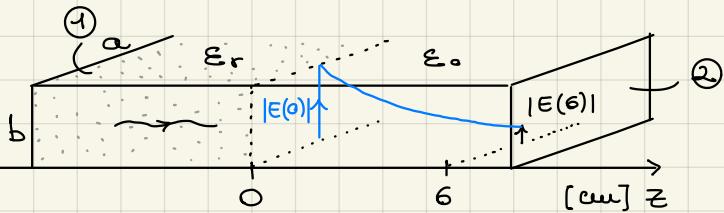
Tratul ① (z negativ):

$$\lambda_{g,1} = \frac{\lambda = \lambda_0}{\sqrt{1 - \left(\frac{f_{c,1}}{f_0}\right)^2}} = 0,227 \text{ m} \quad \beta_z = \frac{2\pi}{\lambda_g} = 27,706 \frac{\text{rad}}{\text{m}}$$

$$\begin{aligned} E(z) &= E^+(z) + E^-(z) = \epsilon^+(0) e^{-j\beta_z z} + \epsilon^-(0) e^{+j\beta_z z} = \\ &= \epsilon^+(0) \left(e^{-j\beta_z z} + \Gamma e^{+j\beta_z z} \right) = (\mp, 0, \mp + j 21, \mp) \frac{\text{mV}}{\text{m}} \end{aligned}$$

$$|E(-3 \text{ cm})| = 23 \frac{\text{mV}}{\text{m}}$$

$$z = -3 \text{ cm}$$



$$a = 10 \text{ cm} \quad |\mathbf{E}_0| = |\mathbf{E}^+(0)| = 10 \frac{\text{mV}}{\text{m}}$$

$$b = 5 \text{ cm} \quad f_0 = 1 \text{ GHz}$$

$$\epsilon_r = 4 \text{ m}$$

Calcolare $|\mathbf{E}(z)|$ in $z=0$ e $z=6 \text{ cm}$.

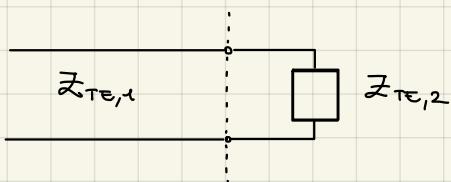
$$f_{c,1} = \omega_0 / 2\pi = 75 \text{ GHz}$$

$$f_{c,2} = 1,5 \text{ GHz} > f_0 !$$

$$Z_{TE,1} = \frac{\eta}{\sqrt{1 - \left(\frac{f_{c,1}}{f_0}\right)^2}} = 285 \Omega$$

$$Z_{TE,2} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_{c,2}}{f_0}\right)^2}} = \frac{377 \Omega}{\sqrt{1 - (1,5)^2}} = j 337 \Omega$$

$$\beta = K \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = j 1,22 \text{ K} \quad \alpha = j \beta = j 1,22 \text{ K}$$



$$\Gamma = \frac{Z_{TE,2} - Z_{TE,1}}{Z_{TE,2} + Z_{TE,1}} = 0,166 + j 0,986$$

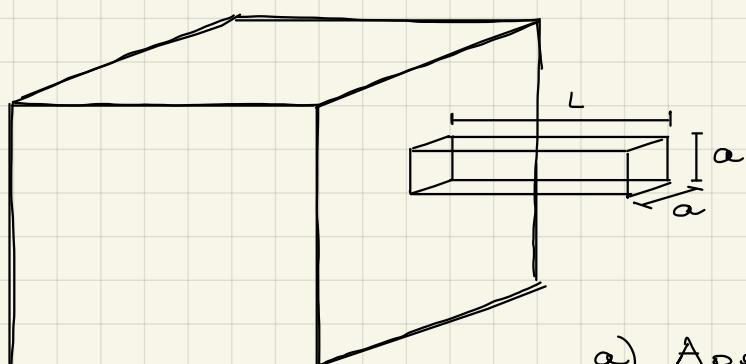
$$|\Gamma| = 1 \quad \mathbf{E}(0) = \mathbf{E}^+(0) + \mathbf{E}^-(0) = \mathbf{E}^+(0) (1 + \Gamma)$$

$$|\mathbf{E}(0)| = |\mathbf{E}^+(0)| / |1 + \Gamma| = 15,3 \frac{\text{mV}}{\text{m}}$$

$$\alpha_{z,2} = \frac{2\pi}{\lambda_c} \sqrt{1 - \left(\frac{f_0}{f_{c,2}}\right)^2} = 23,41 \frac{\text{Np}}{\text{m}} \quad |\mathbf{E}(z)| = |\mathbf{E}(0)| e^{-\alpha_{z,2} z} = 3,75 \frac{\text{mV}}{\text{m}}$$

\uparrow
 $z = 6 \text{ cm}$

gabbia di Faraday



$$f_{MAX} = 100 \text{ MHz}$$

Attenuazione minima di 60 dB

a) Apertura \approx massima consentita per $L = 2 \text{ cm}$

$$|\mathbf{E}(l)| = |\mathbf{E}(0)| e^{-\alpha l} \quad \text{per } l = L$$

$$\frac{|\mathbf{E}(L)|}{|\mathbf{E}(0)|} = \frac{1}{1000} = e^{-\alpha L}$$

$$-\alpha L = -6,9 \quad \alpha = 345 \frac{\text{Np}}{\text{m}}$$

$$-60 \text{ dB}$$

$$\lambda_c = 0,0182 \text{ cm}$$

$$\frac{2\pi}{\lambda_c} \sqrt{1 - \left(\frac{f_{MAX}}{f_c}\right)^2} = \alpha \rightarrow \left(\frac{\alpha \lambda_c}{2\pi}\right)^2 = 1 - \left(\frac{\lambda_c f_{MAX}}{c}\right)^2 \rightarrow$$

$$\alpha = \frac{\lambda_c}{2} = 9,1 \text{ mm}$$

b) Lunghezza L se l'apertura deve essere $10 \times 10 \text{ cm}$

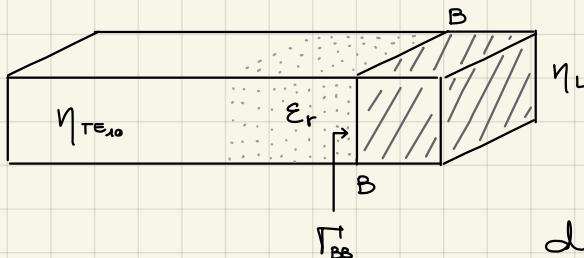
$$\lambda_c = 0,2 \text{ m} = 2a \quad f_c = 1,5 \text{ GHz}$$

$$a = 10 \text{ cm}$$

$$\alpha = \frac{2\pi}{0,2} \sqrt{1 - \left(\frac{0,1}{1,5}\right)^2} = 31,3 \frac{\text{Np}}{\text{m}}$$

$$\alpha \cdot L = 6,9 \quad (\text{per garantire} i - 60 \text{ dB})$$

$$L = 0,22 \text{ m}$$



$$a = 2 \text{ cm}$$

$$f = 91 \text{ GHz}$$

$$b = 1 \text{ cm}$$

$$\Gamma_{BB} = 0,3$$

Dimensionare una struttura dielettrica adattante. ($\epsilon_r = ?$)

$$\Gamma_{BB} = \frac{\eta_L - \eta_{TE_{10}}}{\eta_L + \eta_{TE_{10}}} = 0,3$$

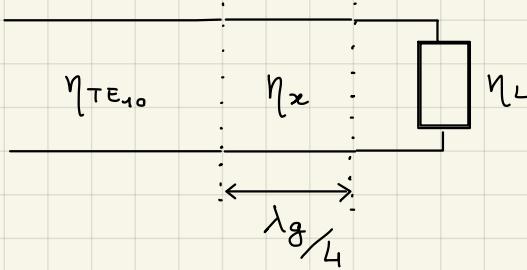
$$\eta_L = \eta_{TE_{10}} \cdot 4,857 \quad (\text{senza adattamento})$$

$$\lambda_c = 2a = 0,04 \text{ m}$$

$$f_c = 7,5 \text{ GHz}$$

$$\eta_{TE_{10}} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = 515 \Omega$$

$$\eta_L = 956 \Omega$$



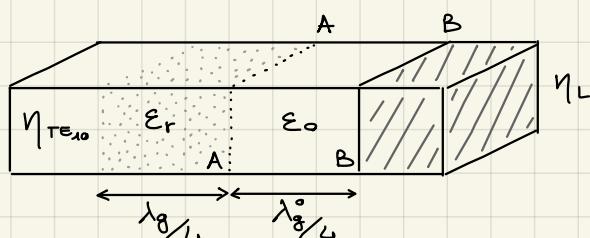
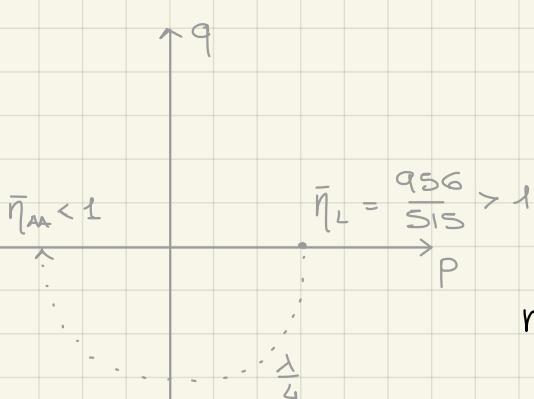
$$\eta_x = \sqrt{\eta_L \cdot \eta_{TE_{10}}} = 400 \Omega$$

$$\eta_x = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\sqrt{1 - \frac{1}{\epsilon_r} \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$f_c = \frac{1}{\lambda_c} \cdot \frac{c}{\sqrt{\epsilon_r}}$$

$$f = \frac{c}{\lambda_0} \cdot \frac{3 \cdot 10^8}{4 \pi \cdot 10^9}$$

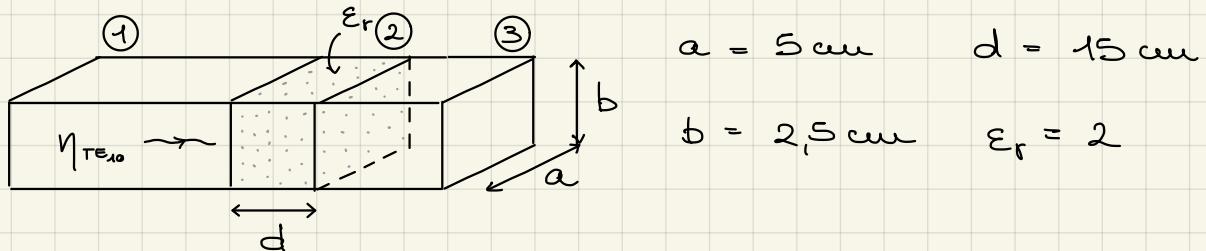
$$\Rightarrow \epsilon_r = \left(\frac{\eta_0}{\eta_x} \right)^2 + \left(\frac{\lambda_0}{\lambda_c} \right)^2 = 0,754 < 1 \rightarrow \text{non va bene}$$



$$\eta_{AA} = \frac{\eta_{TE_{10}}^2}{\eta_L} = 277 \Omega \quad \eta_x = \sqrt{\eta_{TE_{10}} \cdot \eta_{AA}} = 378 \Omega$$

$$\epsilon_r = \left(\frac{377}{378} \right)^2 + \left(\frac{2,73}{4} \right)^2 = 1,46 > 1$$

$$\frac{\lambda_g}{4} = \frac{1}{4} \frac{\lambda_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\sqrt{1 - \frac{\lambda_0^2}{\epsilon_r \lambda_c^2}}} = 0,684 \text{ m} \quad \frac{\lambda_g^\circ}{4} = \frac{\lambda_0}{4} \frac{1}{\sqrt{1 - \frac{\lambda_0^2}{\lambda_c^2}}} = 1,267 \text{ cm}$$



Calcolare la frazione di potenza riflessa alla f_0 (centro banda TE_{10} - guida vuota).

$$\text{TE}_{10}: \lambda_c = 2a = 10 \text{ cm} \quad f_c = 3 \text{ GHz}$$

$$\text{TE}_{01}: \lambda_c = 2b = 5 \text{ cm} \quad f_c = 6 \text{ GHz}$$

$$\text{TE}_{20}: \lambda_c = a = 5 \text{ cm} \quad f_c = 6 \text{ GHz}$$

$$\eta_{\text{TE}_{10}}^{(1)} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} = 506 \Omega = \eta_{\text{TE}_{10}}^{(3)}$$

$$\left. \begin{array}{l} f_0 = 4,5 \text{ GHz} \\ (\lambda_0 = 6,66 \text{ cm}) \end{array} \right\}$$

Setto dielettrico: $\lambda_c = 10 \text{ cm}$ $f_c = 2,12 \text{ GHz}$

$$\eta_{\text{TE}_{10}}^{(2)} = \frac{\eta_0}{\sqrt{\epsilon_r}} \frac{1}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} = 302 \Omega$$



$$\Gamma_{AA} = \frac{\eta_{\text{TE}_{10}}^{(3)} - \eta_{\text{TE}_{10}}^{(2)}}{\eta_{\text{TE}_{10}}^{(3)} + \eta_{\text{TE}_{10}}^{(2)}} = 0,252$$

$$\lambda_g^{(2)} = \frac{\lambda_0}{\sqrt{\epsilon_r}} \frac{1}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} = 5,36 \text{ cm}$$

$$\frac{d}{\lambda_g^{(2)}} = \frac{15}{5,36} = 2,8067 \approx 0,3067$$

$$\bar{\eta}_L = \frac{\eta_{\text{TE}_{10}}^{(3)}}{\eta_{\text{TE}_{10}}^{(2)}} = 1,67 = \bar{\eta}_{AA} \quad \xrightarrow{d} \quad \bar{\eta}_{BB} = 0,649 + j 0,23$$

$$\eta_{BB} = \bar{\eta}_{BB} \cdot \eta_{\text{TE}_{10}}^{(2)} = (196 + j 69,3) \Omega$$

$$\Gamma_{BB} = \frac{\eta_{BB} - \eta_{\text{TE}_{10}}^{(2)}}{\eta_{BB} + \eta_{\text{TE}_{10}}^{(2)}} = -0,43 + j 0,14$$

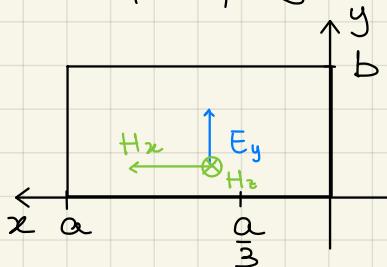
$$P_{\text{rif}} = P^+ |\Gamma_{BB}|^2 = P^+ \cdot \underline{0,203}$$

- Dimensionare una guida d'onda rettangolare con le seguenti caratteristiche:

1) banda monomodale: $f \rightarrow 1,5 \text{ GHz}$

2) dimensione del lato maggiore $a < 10 \text{ cm}$.

Si calcoli l'ampiezza del campo elettrico e magnetico totale per $x = \frac{a}{3}$, se nelle guide si propaga un'onda a $f_0 = 1,25 \text{ GHz}$, $P^+ = 1 \text{ W}$



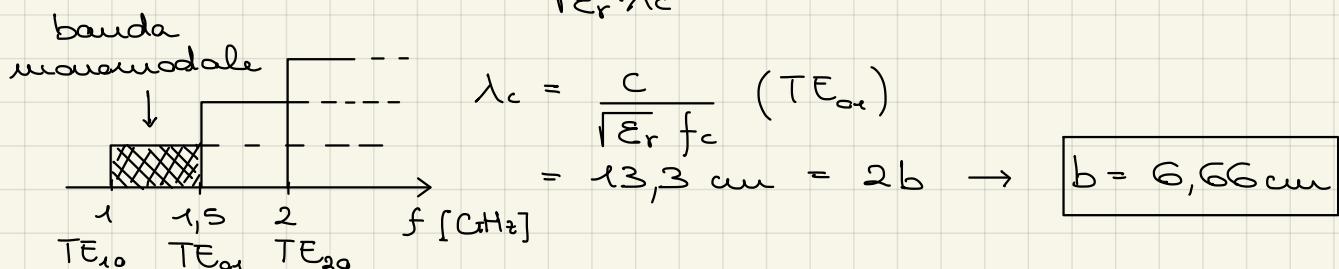
Guida vuota, $a = 10 \text{ cm}$:

$$\lambda_c = 2a = 20 \text{ cm} \quad f_c = 1,5 \text{ GHz} > f_0 !$$

f_0 non può propagarsi in una guida vuota.

Guida con dielettrico, $a = 10 \text{ cm}$:

$$\lambda_c = 20 \text{ cm} \quad f_c = \frac{c}{\sqrt{\epsilon_r \cdot \lambda_c}} = 10^9 \rightarrow \epsilon_r = 2,25$$



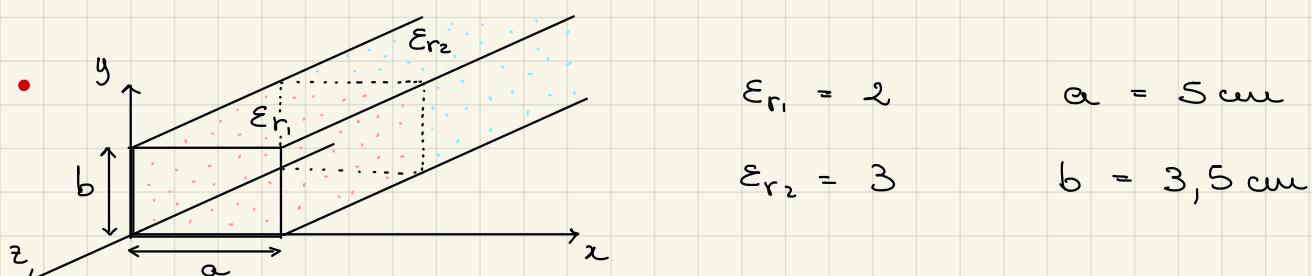
$$P^+ = \frac{1}{4} \frac{|E_0|^2 ab}{\eta_{TE_{10}}} = 1 \text{ W} \quad \text{con} \quad \eta_{TE_{10}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\sqrt{1 - \left(\frac{f_c}{f_0}\right)^2}} = 419 \Omega$$

$$\Rightarrow |E_0| = 500 \text{ V/m}$$

$$E_y(x) = E_0 \sin\left(\frac{\pi x}{a}\right) \quad x = \frac{a}{3}, z = 0 \quad \rightarrow E_y\left(\frac{a}{3}\right) = 433 \text{ V/m}$$

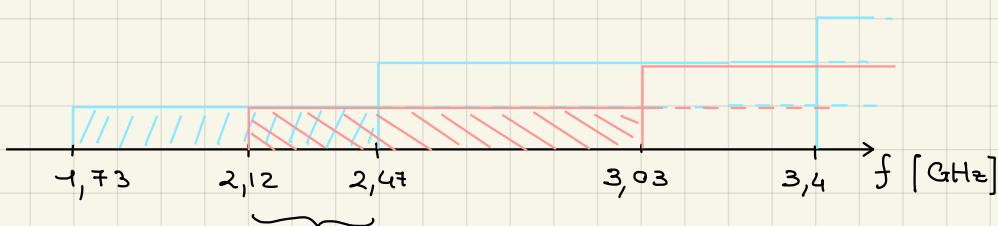
$$H_x(x) = -\frac{E_y(x)}{\eta_{TE_{10}}} \quad \rightarrow H_x\left(\frac{a}{3}\right) = -1 \text{ A/m}$$

$$H_z(x) = j \frac{E_0}{\eta_0} \left(\frac{1}{2a}\right) \cos\left(\frac{\pi x}{a}\right) \quad \rightarrow H_z\left(\frac{a}{3}\right) = j 0,796 \text{ A/m}$$



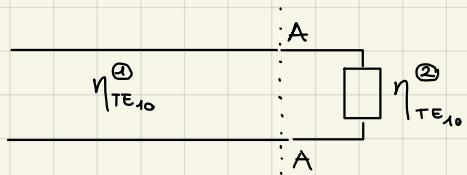
- a) Calcolare la banda monomodale (TE_{10}) per l'intera struttura.
- b) Alla frequenza f_0 (centro banda), si propaga un'onda con $P^+ = 100\text{W}$; Trovare il valore/i di x per cui il modulo del campo elettrico nel secondo dielettrico vale $6,43 \frac{\text{kV}}{\text{m}}$

	Guida 1	Guida 2
TE_{10} : $\lambda_c = 2a = 10\text{cm}$	$f_c^{(1)} = 2,12 \text{ GHz}$	$1,73 \text{ GHz} = f_c^{(2)}$
TE_{01} : $\lambda_c = 2b = 7\text{cm}$	$3,03 \text{ GHz}$	$2,47 \text{ GHz}$
TE_{20} : $\lambda_c = a = 5\text{cm}$	$4,24 \text{ GHz}$	$3,4 \text{ GHz}$



banda monomodale $2,12 \xrightarrow{\downarrow} 2,47 \text{ GHz}$

$$f_0 = 2,295 \text{ GHz}$$



$$\eta_{TE_{10}}^{(1)} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{(1 - (\frac{f_c}{f_0})^2)^{1/2}} = 696 \Omega$$

$$\eta_{TE_{10}}^{(2)} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{(1 - (\frac{f_c}{f_0})^2)^{1/2}} = 331 \Omega$$

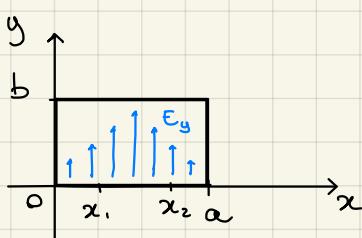
$$\Gamma_{AA} = \frac{\eta_{TE_{10}}^{(2)} - \eta_{TE_{10}}^{(1)}}{\eta_{TE_{10}}^{(2)} + \eta_{TE_{10}}^{(1)}} = -0,355$$

$$T_{AA} = 1 + \Gamma_{AA} = 0,645$$

$$P^+ = \frac{1}{4} \frac{|E_0|^2 a \cdot b}{\eta_{TE_{10}}^{(1)}} = 100 \text{ W} \quad \rightarrow \quad |E_0| = 12,6 \frac{\text{kV}}{\text{m}}$$

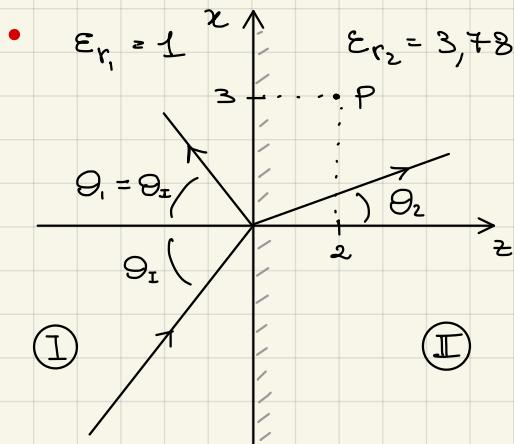
$$E_{II} = E_0 \cdot T_{AA} = 8,13 \frac{\text{kV}}{\text{m}}$$

Nel tratto ②: $E_y(x) = E_{II} \sin(\frac{\pi x}{a}) = 6,43 \frac{\text{kV}}{\text{m}}$



$$\sin\left(\frac{\pi x}{a}\right) = 0,79$$

$$\begin{cases} x_1 = 1,45 \text{ cm} \\ x_2 = 3,55 \text{ cm} \end{cases}$$



(I)

(II)

- Calcolare la frazione di densità di potenza trasmessa e riflessa

Calcolare i campi E e H in $P(2,3)$ [m]

Incidenza TE o TM

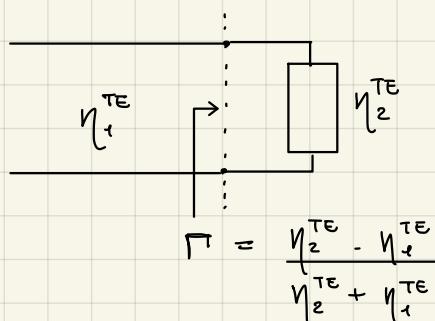
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin 60^\circ = 1,94 \sin \theta_2 \rightarrow \theta_2 = 26,5^\circ$$

$$\eta_1 = \eta_0 = 377 \Omega$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}} = 194 \Omega$$

Incidenza TE



$$\eta_1^{TE} = \eta_1 \frac{1}{\cos \theta_1} = 754 \Omega$$

$$\eta_2^{TE} = \eta_2 \frac{1}{\cos \theta_2} = 217 \Omega$$

$$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0,553$$

$$T = 1 + \Gamma = 0,447$$

$$S_{inc} = \frac{1}{2} \frac{|E_1^+|^2}{\eta_1} = 0,5 \cdot 10^{-3} \frac{W}{m^2} \rightarrow |E_1^+| = 0,614 \frac{V}{m}$$

$$E_2^+ = E_1^+ \cdot T = 0,275 \frac{V}{m} \text{ cui corrisponde } S_{tra} = \frac{1}{2} \frac{|E_2^+|^2}{\eta_2} =$$

$$= 1,95 \cdot 10^{-4} \frac{W}{m^2}$$

$$\gamma_2 = j \omega \sqrt{\mu \epsilon} = j \beta = j 4,0752 \text{ m}^{-1}$$

$$\gamma_{2z} = \gamma_2 \cos \theta_2 = j 1,818 \text{ m}^{-1}$$

$$\gamma_{2x} = \gamma_2 \sin \theta_2 = j 3,647 \text{ m}^{-1}$$

$$E_2^+(x, z) = E_2^+(0, 0) e^{-j \gamma_{2x} x} e^{-j \gamma_{2z} z} = -0,117 - j 0,25 \left[\frac{V}{m} \right] \rightarrow$$

$z = 2, x = 3, \text{ fase in } (0,0) = 0$

$$H_2^+ = \frac{E_2^+}{\eta_2} = \frac{-0,117 - j 0,25}{194} \left[\frac{A}{m} \right] = -0,603 - j 1,289 \left[\frac{mA}{m} \right]$$

Incidenza TM

$$\eta_1^{TM} = \eta_1 \cdot \cos \theta_1 = 188,5 \Omega$$

$$\eta_2^{TM} = \eta_2 \cos \theta_2 = 174 \Omega$$

$$\Gamma = -\frac{\eta_2^{\text{TM}} - \eta_1^{\text{TM}}}{\eta_2^{\text{TM}} + \eta_1^{\text{TM}}} = 0,04$$

$$T = 1 + \Gamma = 1,04$$

$$|H_1^+| = \sqrt{2 \cdot \text{Sinc} \frac{\eta_1}{\eta_2}}$$

$$H_2^+ = H_1^+ \cdot T = 1,69 \cdot 10^{-3} \frac{A}{m}$$

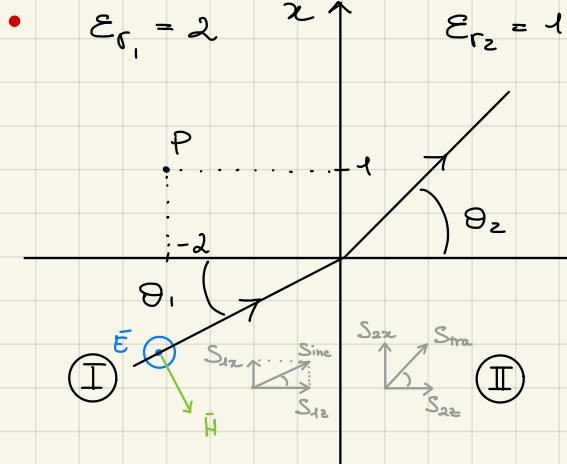
$$S_{\text{tra}} = \frac{1}{2} |H_2^+|^2 \eta_2 = 2,78 \cdot 10^{-4} \frac{W}{m}$$

anche se $T \approx 1$ la potenza trasmessa è $\sim \frac{1}{2}$ la potenza incidente perché solo la componente // all'asse z viene trasmessa

$$H_2^+(x, z) = H_2^+(0, 0) e^{-j\beta_2 x} e^{-j\gamma_2 z} =$$

$$z=2, x=3 \quad = (-7,19 - j15,3) 10^{-4} \left[\frac{A}{m} \right]$$

$$E_2^+(x, z) = H_2^+(x, z) \cdot \eta_2 = -0,14 - j0,29 \left[\frac{V}{m} \right]$$



$$\bar{E}_1^+ = E_0 e^{-j(x + \sqrt{3}z)} \vec{u}_y$$

$$\text{con } E_0 = 3 \frac{V}{m}$$

a) Determinare frequenza, θ₁ e θ₂.

$\bar{E} \parallel \vec{u}_y \rightarrow$ Incidenza TE

$$\bar{E}_1^+ = E_1^+(0, 0) e^{-j\beta_{1x} x - j\beta_{1z} z} \cdot \vec{u}_y$$

$$\begin{cases} \beta_{1z} = \beta_1 \cos \theta_1 = \sqrt{3} \text{ m}^{-1} \\ \beta_{2x} = \beta_1 \sin \theta_1 = 1 \text{ m}^{-1} \end{cases} \implies \frac{\beta_{1x}}{\beta_{2x}} = \tan \theta_1 = \frac{1}{\sqrt{3}} \rightarrow \theta_1 = 30^\circ$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \rightarrow \theta_2 = 45^\circ$$

$$\beta_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi \sqrt{\epsilon_r}}{c} \cdot f \text{ ma } \beta_1 = \frac{\beta_{1z}}{\cos \theta_1} = 2 \text{ m}^{-1} \rightarrow f = 67,6 \text{ MHz}$$

b) Calcolare la densità di potenza trasmessa nel mezzo II

$$\eta_1^{\text{TE}} = \frac{\eta_1}{\cos \theta_1} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\cos \theta_1} = 308 \Omega$$

$$\eta_2^{\text{TE}} = \frac{\eta_0}{\cos \theta_2} = 533 \Omega$$

$$\text{Sinc} = \frac{1}{2} \frac{|E_1^+|^2}{\eta_1} = 0,0169 \frac{W}{m^2}$$

$$\Gamma = \frac{\eta_2^{\text{TE}} - \eta_1^{\text{TE}}}{\eta_2^{\text{TE}} + \eta_1^{\text{TE}}} = 0,267$$

$$\text{dove } \eta_1 = \frac{\eta_0}{\sqrt{\epsilon_r}} = 267 \Omega$$

$$S_{1z} = S_{inc} \cdot \cos \theta_1 = 0,0169 \cdot \cos 30^\circ = 0,0146 \frac{W}{m^2}$$

$$S_{2z} = S_{1z} (1 - |\Gamma|^2) = 0,0136 \frac{W}{m^2}$$

$$S_{tra} = \frac{S_{2z}}{\cos \theta_2} = 0,0192 \frac{W}{m} \rightarrow S_{inc}! \text{ è possibile poiché in II cambia la direzione di propagazione dell'onda}$$

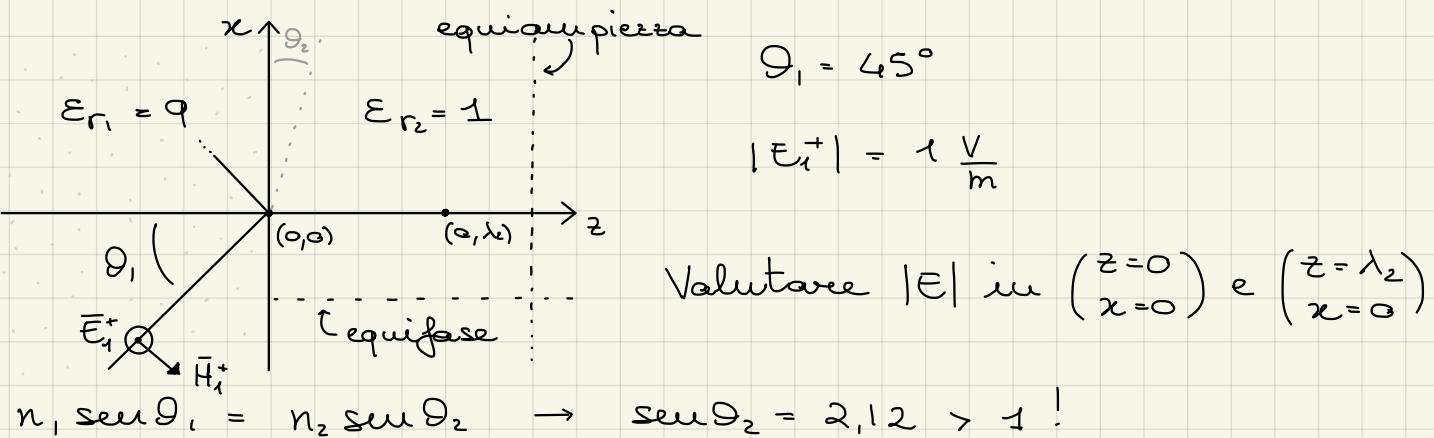
c) Calcolare i campi nel punto P(1, -2)

$$E_1(x, z) = E_1^+(x, z) + E_1^-(x, z) = E_1^+(0, 0) e^{j\beta_1 x - j\beta_1 z} + E_1^-(0, 0) e^{-j\beta_2 x + j\beta_2 z}$$

$$\text{con } E_1^-(0, 0) = E_1^+(0, 0) \Gamma$$

$$\rightarrow E_1(x, z) = 3 [e^{-j(x + \sqrt{3}z)} + 0,267 e^{-jx + j\sqrt{3}z}]$$

$$E_1(1, -2) = (-2,53 + j2,67) \frac{V}{m}$$



Incidenza oltre l'angolo critico \rightarrow riflessione totale

Incidenza TE

$$\eta_1^{TE} = \frac{\eta_1}{\cos \theta_1} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{1}{\cos \theta_1} = 178 \Omega$$

$$\begin{aligned} \eta_2^{TE} &= \frac{\eta_2}{\cos \theta_2} = \frac{\eta}{\sqrt{1 - \sin^2 \theta_2}} = \frac{377 \Omega}{(\pm) j 1,87} \\ &= j 200 \Omega \end{aligned}$$

$$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0,116 + j 0,99$$

$$\Gamma = 1 + \Gamma = 1,116 + j 0,99$$

Il campo E deve attenuarsi al crescere di z:

$$\begin{aligned} \beta_{2z} &= \beta_2 \cos \theta_2 = \frac{2\pi}{\lambda_2} \sqrt{1 - \sin^2 \theta_2} \\ e^{-j\beta_{2z} z} &= e^{-\alpha_{2z} z} = \frac{2\pi}{\lambda_2} (\pm j 1,87) \end{aligned}$$

$$\alpha_{2z} = j \beta_{2z} > 0 \Rightarrow -j 1,87$$

$$E(0, 0) = E^+(0, 0) \Gamma = (1,116 + j 0,99) \frac{V}{m}$$

$$|E(0,0)| = 7,5 \frac{V}{m}$$

varia solo
l'ampiezza

$$E_2^+(0, \lambda_2) = E_2^+(0,0) e^{-\alpha_{22} \lambda_2}$$

$\beta_{2x} = \beta_{2z} \rightarrow$ campi tg. continui

$$E_2^+(x, z) = E_2^+(0,0) e^{-j\beta_{2x} x} e^{-\alpha_{22} z}$$

$x = \text{const.} \leftrightarrow \text{sup. equifase}$ $z = \text{const.} \leftrightarrow \text{sup. equi-ampiezza}$

$$\rightarrow |E_2^+(0, \lambda_2)| = 7,89 \cdot 10^{-6} \frac{V}{m}$$

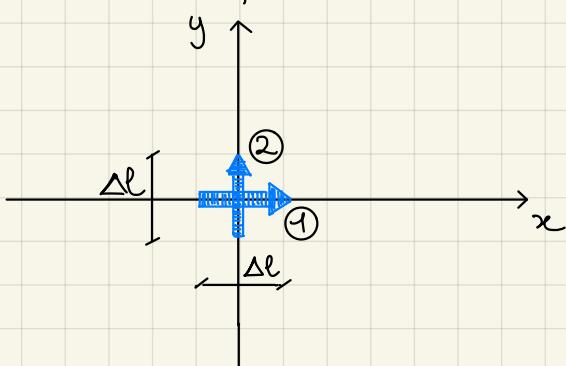
- $$\begin{cases} E_x = 10 e^{-j\beta z} \left[\frac{V}{m} \right] \\ E_y = j10 e^{j\frac{\pi}{2}} e^{-j\beta z} \left[\frac{V}{m} \right] \end{cases}$$
Polarizzazione?

Piano trasverso (ad es. $z = 0$):

$$\begin{cases} E_x = 10 \left[\frac{V}{m} \right] \\ E_y = j10 e^{j\frac{\pi}{2}} \left[\frac{V}{m} \right] \end{cases} \rightarrow E_x = 10 \cos(\omega t) \\ E_y = \operatorname{Re} \{ 10 e^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}} e^{j\omega t} \} = -10 \cos(\omega t + \pi)$$

$\phi = \pi \Rightarrow$ polarizzazione lineare

Due dipoli hertziani trasmettenti di eguale lunghezza Δl

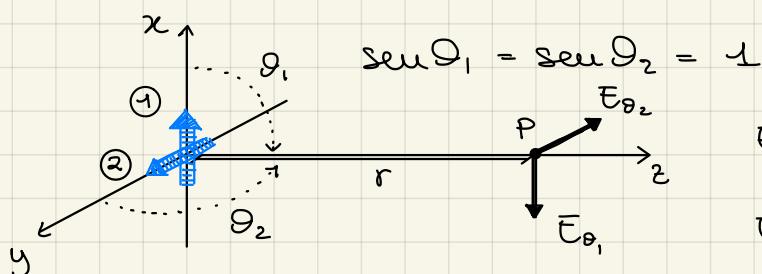
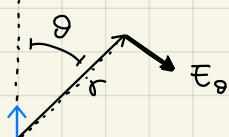


$$I_1 = j2 A$$

$$I_2 = -j4 A$$

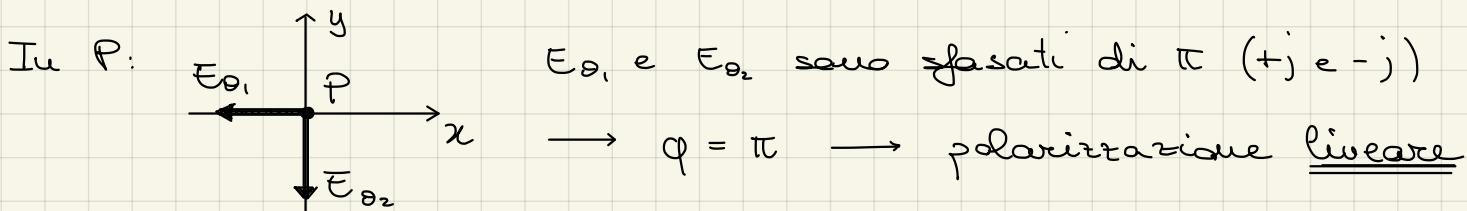
Determinare la polarizzazione dell'onda irradiata in direzione asse z .

$$E_\theta(\theta, r) = j\omega \mu I \cdot l e^{-j\beta r} \sin \theta$$



$$E_{\theta 1} = j\omega \mu I_1 l e^{-j\beta r} = K I_1 = K j2$$

$$E_{\theta 2} = j\omega \mu I_2 l e^{-j\beta r} = K I_2 = -K j4$$

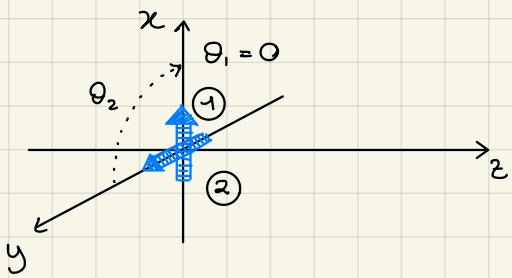


Se $I_1 = 2A$, $I_2 = j2A$. Polarizzazione?

$$E_{\theta_1} = K I_1 = K 2 A \quad E_{\theta_2} = K I_2 = j K 2 A$$

$$\rightarrow \varphi = \frac{\pi}{2}, |E_{\theta_1}| = |E_{\theta_2}| \rightarrow$$
 polarizzazione circolare sinistra

Determinare la polarizzazione dell'onda irradiata in direzione asse x

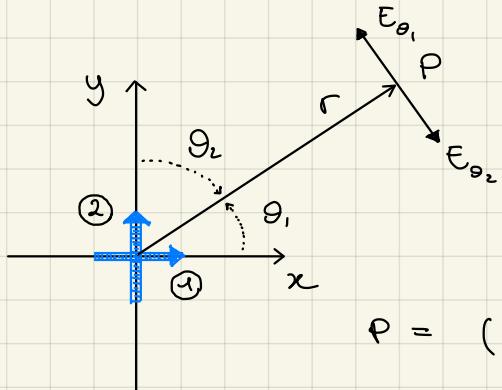


$$\theta_1 = 0^\circ \rightarrow \sin \theta_1 = 0$$

$$\theta_2 = 90^\circ \rightarrow \sin \theta_2 = 1$$

Polarizzazione lineare (c'è solo il dipolo ② che irradia lungo x)

- Due dipoli hertziani trasmettenti di uguale lunghezza e intensità.



$$l = \frac{\lambda}{10} \quad I = 1 A$$

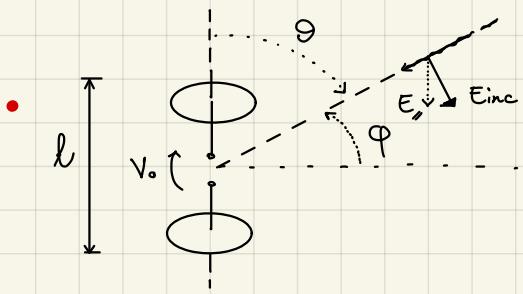
$$f = 600 \text{ MHz} \quad (\lambda = 0,5 \text{ m})$$

Calcolare $\bar{E}(100, 100)$ (di sola radiazione)

$$P = (100, 100) \rightarrow \theta_1 = \theta_2 = \frac{\pi}{4}$$

$$E_{\theta} = j\omega \mu \frac{Il}{4\pi r} e^{-j\beta r} \sin \theta \rightarrow E_{\theta_1} = E_{\theta_2} \text{ in } P$$

$$\bar{E}(100, 100) = 0$$



$$\theta = 60^\circ \quad \phi = 30^\circ \quad f = 100 \text{ MHz} \quad (\lambda = 3 \text{ m})$$

$$|\bar{E}_{\text{inc}}| = 10^{-2} \frac{V}{m} \quad l = \frac{\lambda}{10} = 0,3 \text{ m}$$

$$|V_o| = ?$$

Metodo 1:

$$E_{\text{in}} = E_{\text{inc}} \cdot \cos \varphi = 8,66 \cdot 10^{-3} \frac{\text{V}}{\text{m}} \quad V_o = E_{\text{in}} \cdot l = 2,6 \cdot 10^{-3} \text{ V}$$

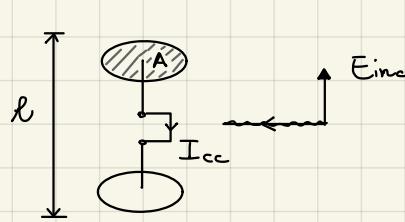
Metodo 2: (hp: adattamento di polarizzazione)

$$f(\theta) = \sin^2 \theta \quad A_e = \frac{\lambda^2}{8\pi} \quad R_R = \frac{2}{3} \pi \eta_0 \left(\frac{l}{\lambda} \right)^2 = 7,9 \Omega$$

$$\text{Sinc} = \frac{|E_{\text{inc}}|^2}{2\eta_0} = 1,326 \cdot 10^{-7} \frac{\text{W}}{\text{m}^2} \quad A_e = 1,074 \text{ m}^2$$

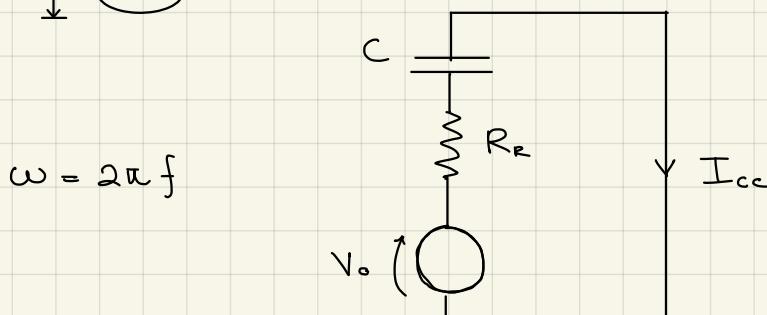
$$P_d = \text{Sinc} \cdot A_e \cdot f(\theta) = 1,06 \cdot 10^{-7} \text{ W} \quad \text{ma} \quad P_d = \frac{|V_o|^2}{8R_R}$$

$$\rightarrow |V_o| = 2,6 \cdot 10^{-3} \text{ V}$$



$$|E_{\text{inc}}| = 1 \frac{\text{V}}{\text{m}} \quad l = 1 \text{ m} \quad A = 10 \text{ m}^2$$

Calcolare $|I_{\text{cc}}|$ a $f = 30 \text{ MHz}$ ($\lambda = 10 \text{ m}$)



$$R_R = \frac{2}{3} \pi \eta_0 \left(\frac{l}{\lambda} \right)^2 = 7,9 \Omega$$

$$C = \epsilon_0 \frac{A}{l} = 88,6 \text{ pF}$$

$$|V_o| = |E_{\text{inc}}| \cdot l_e = 1 \text{ V}$$

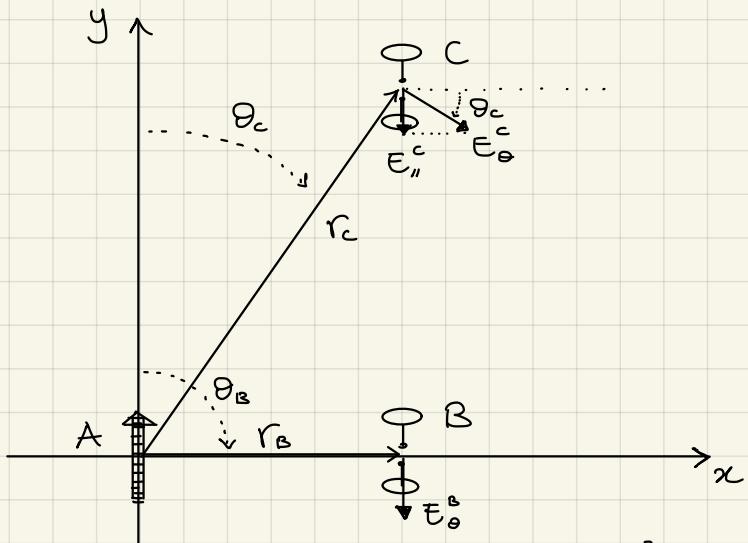
l per il dipolo

$$|I_{\text{cc}}| = \frac{|V_o|}{|R_R + \frac{1}{j\omega C}|} = 16,5 \text{ mA}$$

Si può ottenere una $|I| \gg |I_{\text{cc}}|$ inserendo un componente (passivo) al posto del corto circuito?

Sí, inserendo un induttore tale che $\frac{1}{j\omega C} + j\omega L = 0$

$$\Rightarrow L = \frac{1}{\omega^2 C} = 317 \text{ nH} \rightarrow |I| = \frac{|V_o|}{R_R} = 126,6 \text{ mA}$$



$$P_t = 1 \text{ W}$$

$$f = 1 \text{ GHz}$$

$$\mathbf{B} = (30, 0)$$

$$\mathbf{C} = (30, 60)$$

$$l = \frac{\lambda}{10}$$

Calcolare la tensione a questo di B e C.

$$P_t = \frac{\pi}{3} \eta_0 |I_A|^2 \left(\frac{l}{\lambda} \right)^2 \rightarrow |I_A| = 0,5 \text{ A}$$

$$|E_s^B| = \left| \frac{j\omega \mu I_A l e^{-j\beta r_B}}{4\pi r_B} \sin \theta_B \right| = \frac{2\pi f \mu_0 I_A l}{4\pi r_B} = 0,314 \frac{\text{V}}{\text{m}}$$

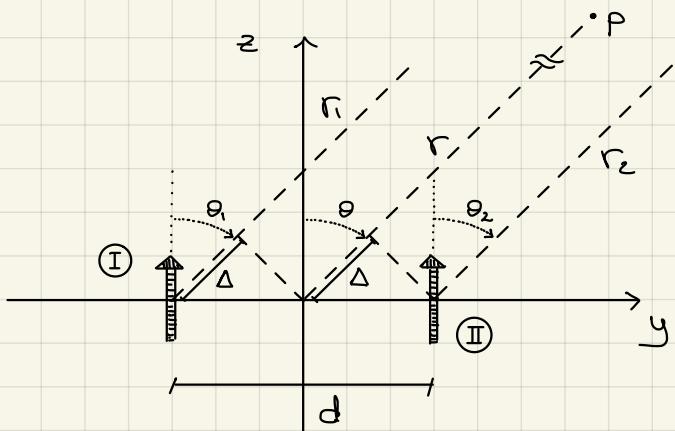
$$|V_B| = |E_s^B| \cdot l = 0,0094 \text{ V}$$

$$\theta_c = 26,6^\circ = \arctg \left(\frac{30}{60} \right) \quad r_c = 67 \text{ m}$$

$$|E_s^C| = \frac{2\pi f \mu_0 I_A l}{4\pi r_c} \sin \theta_c = 0,063 \frac{\text{V}}{\text{m}}$$

$$|V_c| = |E_s^C| \cdot l = |E_s^C| \sin \theta_c \cdot l = 8,4 \cdot 10^{-4} \text{ V}$$

Dipoli hertziani (trasmettenti) in gruppo



Campo totale È irradiato?

$$r \gg d \quad (\gg \lambda)$$

$$I_1 = I_2 = I$$

$$l_1 = l_2 = l$$

$$E_{s_1}^I = \frac{j\omega \mu I_1 l_1 e^{-j\beta r_1}}{4\pi r_1} \sin \theta_1$$

$$E_{s_1}^{II} = \frac{j\omega \mu I_2 l_2 e^{-j\beta r_2}}{4\pi r_2} \sin \theta_2$$

$$\text{Se } r \gg d \rightarrow r_1 \parallel r_2 \parallel r \rightarrow \theta_1 \approx \theta_2 \approx \theta$$

$$r_1 = r + \Delta, \quad r_2 = r - \Delta \quad \text{con} \quad \Delta = \frac{d}{2} \sin\theta \quad (\Delta \ll r)$$

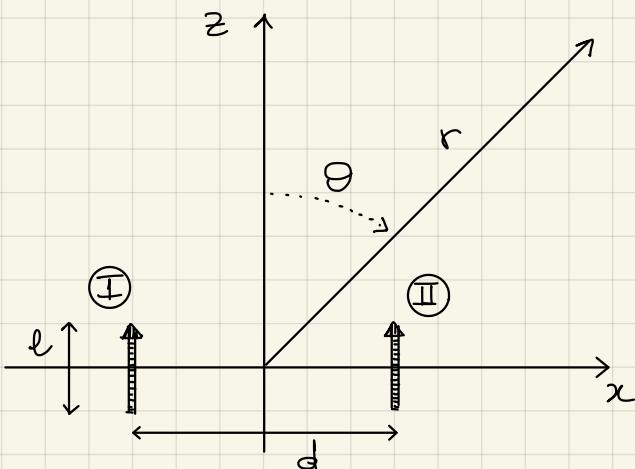
Al fine del denominatore: $r_1 \approx r_2 \approx r$

$$E_s^I = j \frac{\omega \mu I l e^{-j\beta r}}{4\pi r} e^{-j\beta \frac{d}{2} \sin\theta} \sin\theta$$

$$E_s^{II} = j \frac{\omega \mu I l e^{-j\beta r}}{4\pi r} e^{+j\beta \frac{d}{2} \sin\theta} \sin\theta$$

$$\left\{ E_s = E_s^I + E_s^{II} = j \frac{\omega \mu I l e^{-j\beta r}}{4\pi r} \sin\theta \underbrace{\left[e^{-j\beta \frac{d}{2} \sin\theta} + e^{+j\beta \frac{d}{2} \sin\theta} \right]}_{2 \cos\left(\beta \frac{d}{2} \sin\theta\right)} \right\}$$

parallelili



$$l = \frac{\lambda}{20} \quad I_1 = I_2 = I$$

$$f = 100 \text{ MHz}$$

Determinare la minima distanza d in modo che la radiazione in direzione $\theta = 45^\circ$ sia nulla.

$$E_s(\theta = 45^\circ) = 0$$

$$E_s^{tot} \approx j \frac{\omega \mu I l e^{-j\beta r}}{2\pi r} \underbrace{\sin\theta \cos\left(\beta \frac{d}{2} \sin\theta\right)}_{\text{unica termine che si può annullare}}$$

unica termine che si può annullare

$$\cos\left(\beta \frac{d}{2} \sin\theta\right) = 0 \rightarrow \beta \frac{d}{2} \sin\theta = \frac{\pi}{2} \quad \text{con} \quad \beta = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda} \frac{d}{2} \sin\theta = \frac{\pi}{2} \rightarrow d = \frac{\lambda}{2 \sin\theta} = 2,12 \text{ m} \quad \begin{aligned} &(\text{le soluzioni sono infinite, questa} \\ &\text{ha il } d \text{ minore}) \end{aligned}$$

Se invece $I_1 = I$, $I_2 = -I$ determinare il minimo $d (> 0)$.

$$|E_s^I| = |E_s^{II}| \quad \Delta \text{ II} : \beta r_2 + \pi \quad \Delta \text{ I} : \beta r_2 + \beta 2\Delta$$

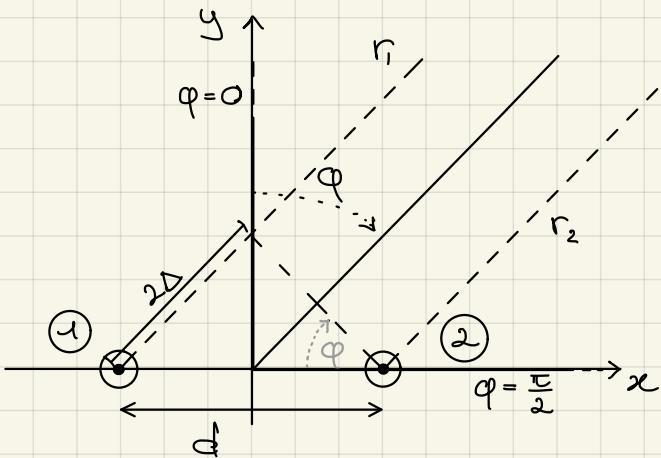
Affinché $E_s^I = -E_s^{II}$ devo avere

$$\Delta \text{ I} - \Delta \text{ II} = \pi$$

$$\beta 2\Delta - \pi = \pi$$

$$\frac{2\pi}{\lambda} \cdot d \sin\theta = 2\pi$$

$$\Rightarrow d = \frac{\lambda}{\sin\theta} = 4,24 \text{ m}$$



$$I_1 = 1A \quad l = \frac{\lambda}{10}$$

$$|I_2| = |I_1| \quad f = 300 \text{ MHz}$$

a) Determinare I_2 e d in modo che il campo in direzione $\varphi = 0$ e $\varphi = \frac{\pi}{2}$ sia nulllo (simultaneamente)

Dipoli // asse z $\rightarrow \theta_1 = \theta_2 = 90^\circ$

Per $\varphi = 0$, d è inutile ($r_1 = r_2$) $\rightarrow I_2 = -I_1 = -1A$

Per $\varphi = \frac{\pi}{2}$, $2\Delta = d$: $\beta 2\Delta = \beta d = 2\pi \rightarrow d = \lambda = 1m$

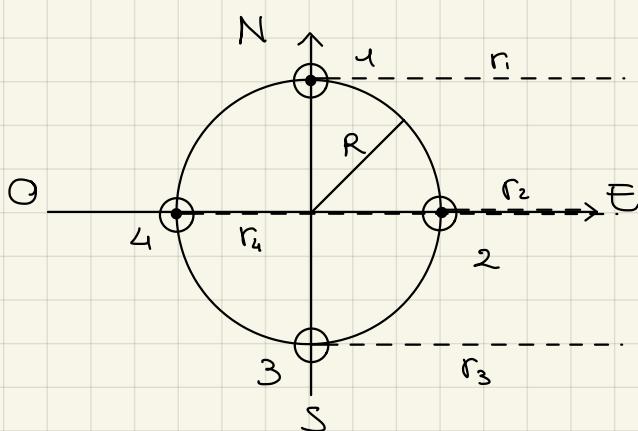
E^1 e E^2 devono essere IN FASE per annullarsi poiché sono opposti.

b) Determinare la direzione di massima radiazione

$$E_s^1 \text{ in fase con } E_s^2 \quad 2\Delta = d \sin q \quad \beta 2\Delta = \pi$$

$$\rightarrow q = \arcsin\left(\frac{\lambda}{2d}\right) = \arcsin\left(\frac{1}{2}\right) \quad \frac{2\pi}{\lambda} d \sin q = \pi$$

$$= 30^\circ$$



$$f = 300 \text{ MHz}$$

Stessa l, stessa I.

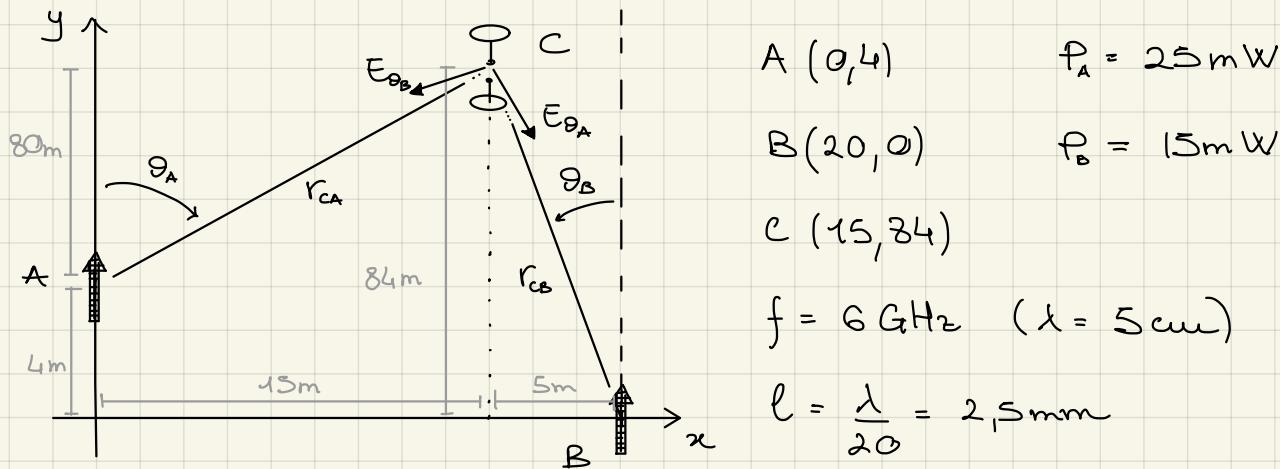
Determinare il raggio R minimo per il quale non c'è radiazione simultaneamente nelle direzioni N, S, E, O.

$$E_{st}: r_1 = r_3 \quad (\forall R) \quad |E_0| = |E_s^1| = |E_s^2| = |E_s^3| = |E_s^4|$$

Il campo totale generato da 1 e 3 vale $2|E_0|$

per simmetria
anche le altre direzioni hanno campo nulla

$$\begin{aligned} \text{La differenza } r_1 - r_2 &\approx R \rightarrow \beta R = \pi \\ r_3 - r_4 &\approx R \rightarrow \beta R = \pi \end{aligned} \quad \left. \begin{array}{l} \text{per amm. } E_s^1 \\ \text{per amm. } E_s^3 \end{array} \right\} R = \frac{\lambda}{2} = 0,5m$$



I_A e I_B in fase. Calcolare $|V_c|$ (tensione a vuoto)

$$P = \frac{\pi}{3} \eta_0 |I|^2 \left(\frac{l}{\lambda}\right)^2 \quad \begin{cases} I_A = 0,159 \text{ A} \\ I_B = 0,123 \text{ A} \end{cases}$$

$$\theta_A = \arccos\left(\frac{80}{r_{CA}}\right) = 10,6^\circ \quad \theta_B = \arccos\left(\frac{84}{r_{CB}}\right) = 3,4^\circ$$

$$r_{CA} = \sqrt{(84-4)^2 + 15^2} = 81,394 \text{ m}$$

($\lambda = 0,05 \text{ m}$ --- 7° per mm
se trovi troppo cifre decimali rischia di commettere un errore notevole)

$$r_{CB} = \sqrt{84^2 + 5^2} = 84,1487 \text{ m}$$

$$E_{\theta_A} = \frac{j \omega \mu I_A l}{4 \pi r_{CA}} e^{-j \beta r_{CA}} \sin \theta_A \quad E_{\theta_B} = \frac{j \omega \mu I_B l}{4 \pi r_{CB}} e^{-j \beta r_{CB}} \sin \theta_B$$

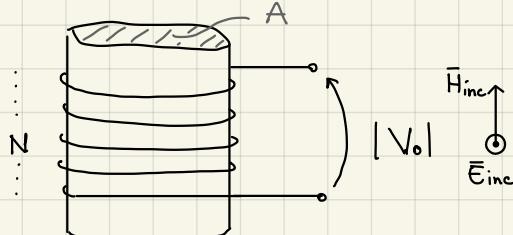
$$E_{yA} = -E_{\theta_A} \sin \theta_A = (4,198 - j 4,585) \cdot 10^{-4} \frac{\text{V}}{\text{m}}$$

$$E_{yB} = E_{\theta_B} \sin \theta_B = (8,01 - j 47,83) \cdot 10^{-6} \frac{\text{V}}{\text{m}}$$

$$|E_{\text{tot}}| = |E_{yA} + E_{yB}| = 6,629 \cdot 10^{-4} \frac{\text{V}}{\text{m}}$$

$$|V_c| = |E_{\text{tot}}| \cdot \frac{R_e}{R} = \underline{1,657 \mu \text{V}}$$

Onde radio (basse frequenze)



$$N = 1000 \text{ spire}$$

$$|E_{\text{ind}}| = 1 \frac{\text{mV}}{\text{m}}$$

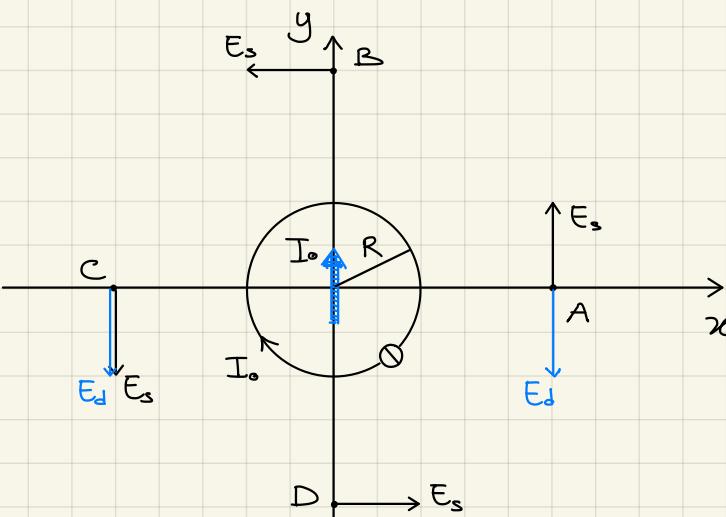
$$(\text{ferrite}) \quad A = 1 \text{ cm}^2 \quad \mu_r = 100$$

$$f = 1 \text{ MHz} \quad (\lambda = 300 \text{ m})$$

$$\text{Determinare } |V_o| \rightarrow |e| = N \frac{d\phi}{dt} = N j \omega \mu H \cdot A$$

$$V_o = N \cdot j \omega \mu H \cdot A \quad \text{con } H = H_{\text{inc}}$$

$$|V_o| = |N \cdot 2\pi f \mu \mu_r A| \underbrace{|H_{\text{inc}}|}_{|E_{\text{inc}}|} = 0,21 \text{ mV}$$



$$l = 1 \text{ m}$$

$$I_0 = 1 \text{ A}$$

$$R = 1 \text{ m}$$

$$f = 10 \text{ MHz } (\lambda = 30 \text{ m})$$

Calcolare $|\vec{E}|$ in A, B, C e D
a distanza $r = 1000 \text{ m}$

$$E_s = -j\beta \frac{j\omega \mu I_0 S}{4\pi r} e^{-j\beta r} \quad \text{con } S = \pi R^2$$

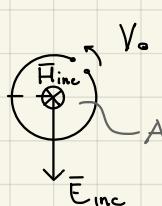
$$E_d = j\omega \mu \frac{I_0 l}{4\pi r} e^{-j\beta r}$$

$$\text{In B e D: } |\vec{E}| = |E_s| = 4,1 \cdot 10^{-3} \frac{\text{V}}{\text{m}} \quad (\text{il doppio non irradia lungo il suo asse})$$

In A e C: E_s e E_d sono in quadratura (sfasati di $\frac{\pi}{2}$)

$$|\vec{E}| = |E_d \pm E_s| = |j6,3 \cdot 10^{-3} \pm 4,1 \cdot 10^{-3}| = \\ = 7,5 \cdot 10^{-3} \frac{\text{V}}{\text{m}}$$

uno è Re e
l'altro è Im



$$l = \frac{\lambda}{10}$$

$$r = 100\lambda$$

$$A = 10^{-3} \lambda^2$$

$$|I_0| = 1 \text{ A}$$

$$\theta = \frac{\pi}{2}$$

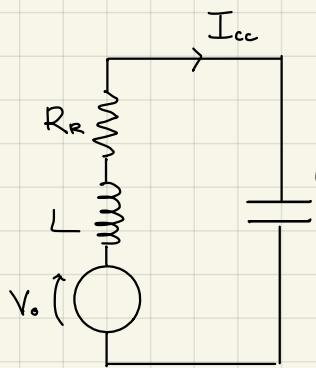
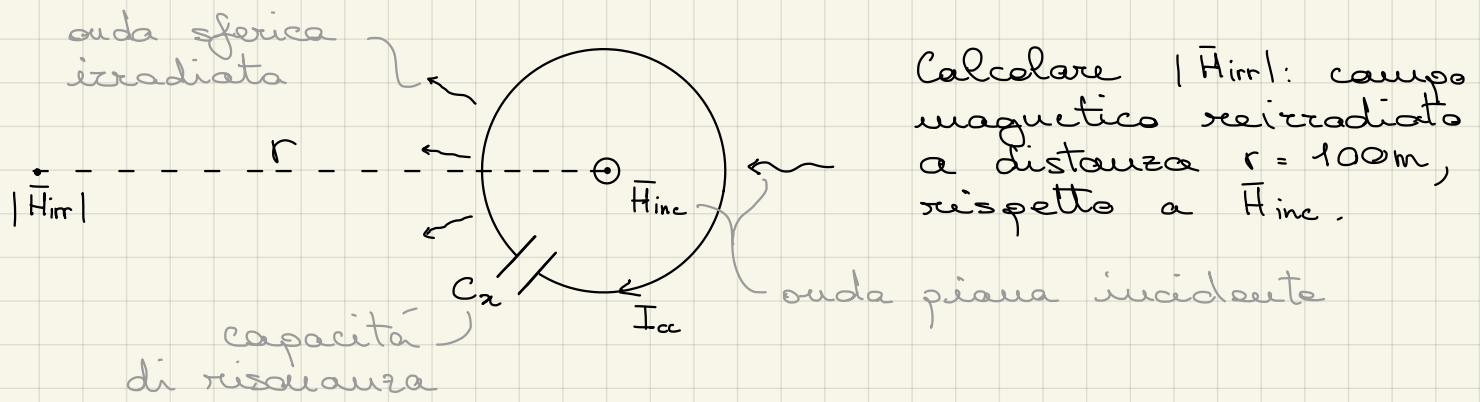
$$V_o = ?$$

$$V_o = \underbrace{j\omega \mu_0 A}_{A_e} H_{\perp} = j\omega \mu_0 A \frac{E_{\text{inc}}}{\eta_0}$$

$$|E_{\text{incl}}| = \frac{\omega \mu |I_{\text{dil}}|}{4\pi r} = 2\pi f \mu_0 \frac{\lambda |I_{\text{dil}}|}{10} \frac{1}{4\pi 100\lambda}$$

$$|V_{\text{dil}}| = 2\pi f \mu_0 10^{-3} \lambda^2 \frac{2\pi f \mu_0 |I_{\text{dil}}|}{1000 \cdot 4\pi} \quad \text{con } \lambda = \frac{c}{f}$$

$$= 1,18 \cdot 10^{-3} \text{ V}$$



$$j\omega L = -\frac{1}{j\omega C_x} \rightarrow |I_{\text{cc}}| = \frac{|V_o|}{R_R}$$

$$|I_{\text{aa}}| = \frac{|j\beta A \cdot E_{\text{incl}}|}{R_R} = \frac{\beta A |H_{\text{inc}}| \eta_0}{R_R}$$

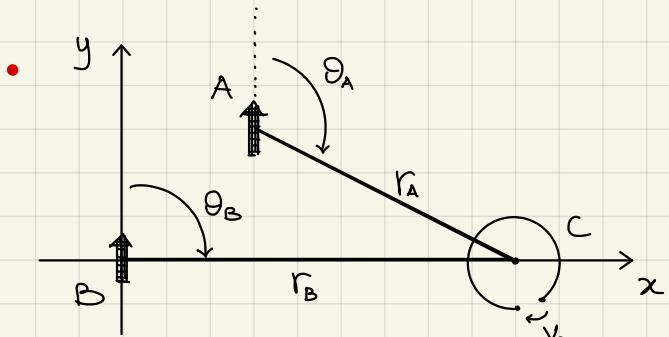
$$|H_{\text{irr}}| = \left| \frac{j\beta}{\eta_0} \frac{j\omega \mu I_{\text{aa}} A}{4\pi r} e^{-j\beta r} \sin \theta \right| \quad \text{con } \theta = \frac{\pi}{2}$$

$$= \frac{\beta}{\eta_0} \frac{\omega \mu A}{4\pi 100\lambda} \cdot \frac{\beta A \eta_0}{R_R} |H_{\text{inc}}|$$

$$\text{dove } R_R = \eta_0 \frac{8}{3} \pi^2 \left(\frac{A}{\lambda} \right)^2, \quad \omega = 2\pi f = \frac{2\pi c}{\lambda}, \quad \beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow |H_{\text{irr}}| = 3 \frac{(2\pi)^3}{\lambda^3} \frac{1}{8\pi^3} \frac{\lambda^4 |H_{\text{inc}}|}{4\pi 100\lambda} = 2,4 \cdot 10^{-3} |H_{\text{inc}}|$$

$$\frac{|H_{\text{irr}}|}{|H_{\text{inc}}|} = 2,4 \cdot 10^{-3} = -52,4 \text{ dB}$$



B(0,0) A(5,5) C(15,0)

$$f = 500 \text{ MHz} \quad P_T = 1 \text{ W} \quad \ell = \frac{\lambda}{10}$$

$a = 3 \text{ au}$ I_A e I_B in fase

Calcolare $|V_o|$.

$$\Theta_B = \frac{\pi}{2} \quad \bar{H}_A \text{ e } \bar{H}_B \text{ sono // tra loro e } \perp \text{ alla spira}$$

$$P_t = \frac{\pi}{3} \eta_0 |I|^2 \left(\frac{e}{\lambda} \right)^2 = 1 \text{ W} \rightarrow |I_A| = |I_B| = 0,503 \text{ A}$$

$$\lambda = 0,6 \text{ m} \quad e = \frac{\lambda}{10} = 0,06 \text{ m}$$

$$|\bar{H}_A| = \left| \frac{j \omega \mu I_A e}{4 \pi r_A \eta_0} \sin \Theta_A \right| \quad \text{con} \quad \Theta_A = \frac{\pi}{2} + \arctg \frac{e}{r_A}$$

$$r_A = \sqrt{10^2 + 5^2} = 11,18034 \text{ m}$$

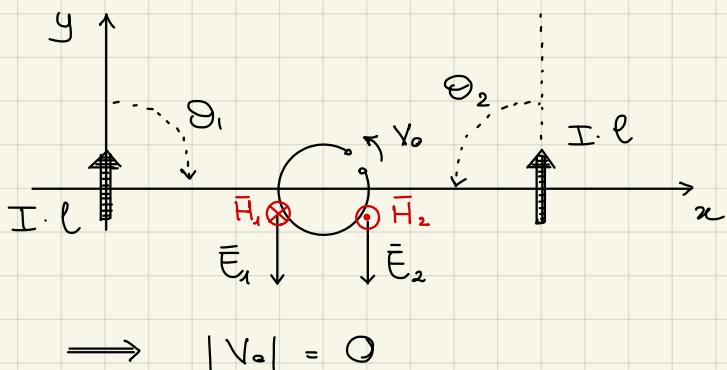
$$\Rightarrow H_A = 0,002012 \frac{\text{A}}{\text{m}} \quad H_B = \left| \frac{j \omega \mu I_B e}{4 \pi r_B \eta_0} \right| = 0,001677 \frac{\text{A}}{\text{m}}$$

$$r_B = 15 \text{ m} = 25 \lambda \rightarrow \bar{H}_B \text{ reale positivo}$$

$$r_A = \cancel{18\lambda} + 0,634\lambda \rightarrow \bar{H}_A \text{ complesso}$$

$$H_{\text{tot}} = |\bar{H}_A + \bar{H}_B| = |H_A e^{-j \beta 0,3804} + H_B| = 1,537 \cdot 10^{-3} \frac{\text{A}}{\text{m}}$$

$$|V_o| = |j \omega \mu A e H_{\text{tot}}| = 0,171 \text{ V} \quad \text{con} \quad A = \pi a^2$$

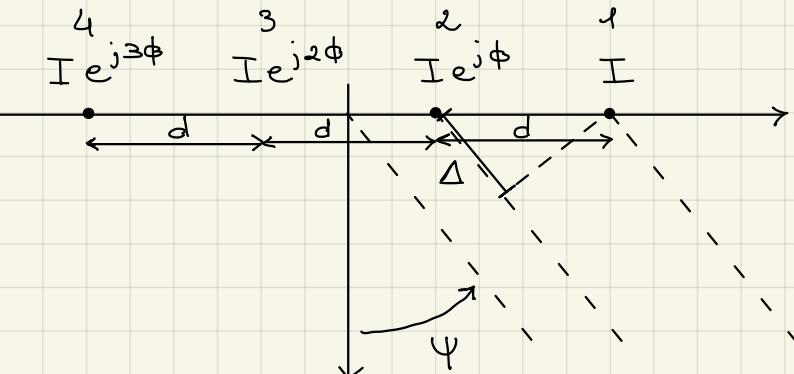


Calcolare $|V_o|$.

$$E_{\text{tot}} = E_1 + E_2 = 2E_1$$

ma

$$H_{\text{tot}} = H_1 - H_2 = 0$$



$$\phi = 20^\circ$$

$$d = \frac{\lambda}{2}$$

a) Determinare Ψ_{\max} di massima radiazione.

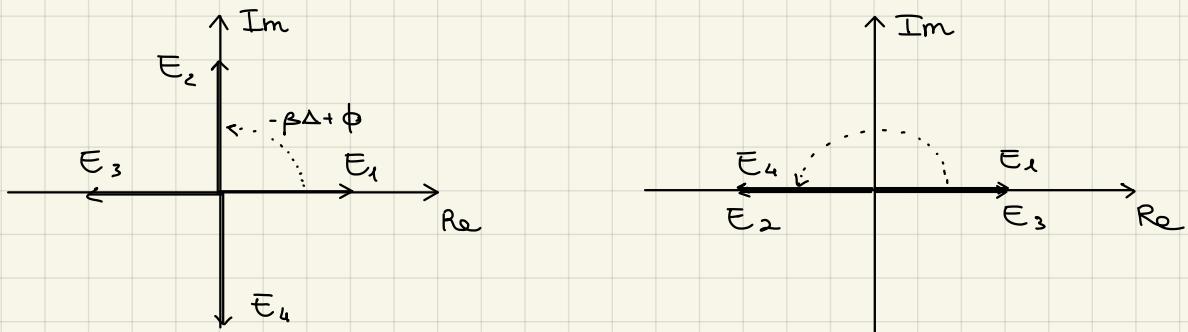
Affinché i vali dipoli si sommino in fase deve imporre:

$$e^{-j\beta\Delta} \cdot e^{j\phi}$$

ritardo di propagazione = anticipo di alimentazione

$$\beta\Delta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \text{ seu } \Psi_{MAX} = \phi \implies \Psi_{MAX} = 64^\circ$$

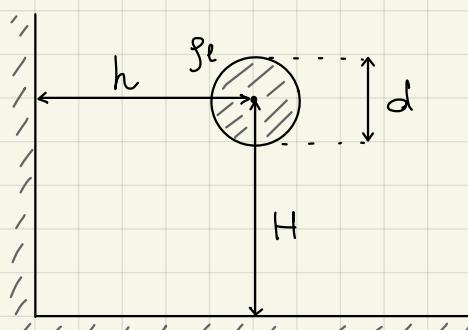
b) Determinare Ψ_0 di radiazione nulla



Entrambe le soluzioni sono valide (l'importante è che le fasi fra i dipoli limitrofi sia le stesse)

Differenza di fase tra elementi limitrofi deve essere $\pm k90^\circ$ ($k = 1, 2$)

$$k=+1: -\beta \frac{\Delta}{2} \text{ seu } \Psi_0 + \phi = 90^\circ \implies \Psi_0 = -22,9^\circ$$



$$h = 5 \text{ cm} = H \quad d = 1 \text{ cm}$$

Calcolare Z_c (approx conduttori sottili)

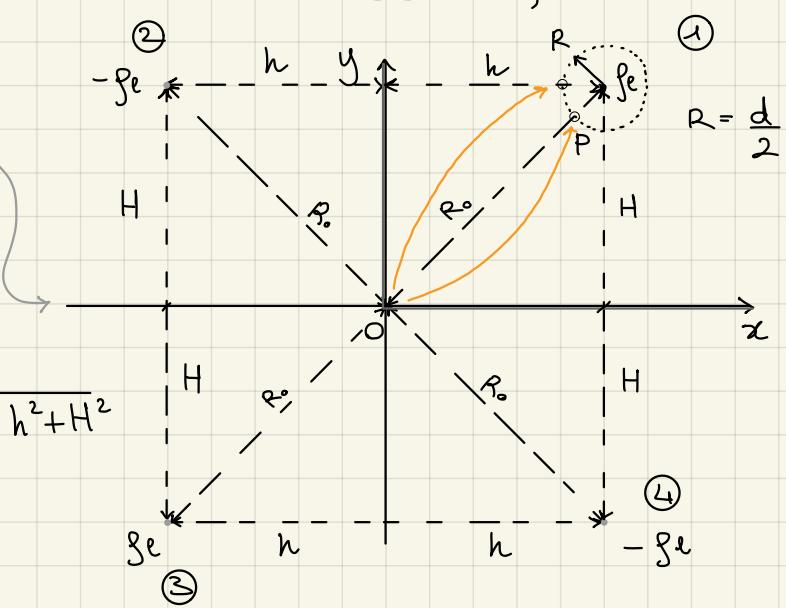
$$V(P) = -\frac{g_e}{2\pi\epsilon_0} \left[\ln\left(\frac{R}{R_o}\right) - \right.$$

$$\left. - \ln\left(\frac{2h-R}{R_o}\right) + \right]$$

$$+ \ln\left(\frac{2R_o-R}{R_o}\right) \quad (3)$$

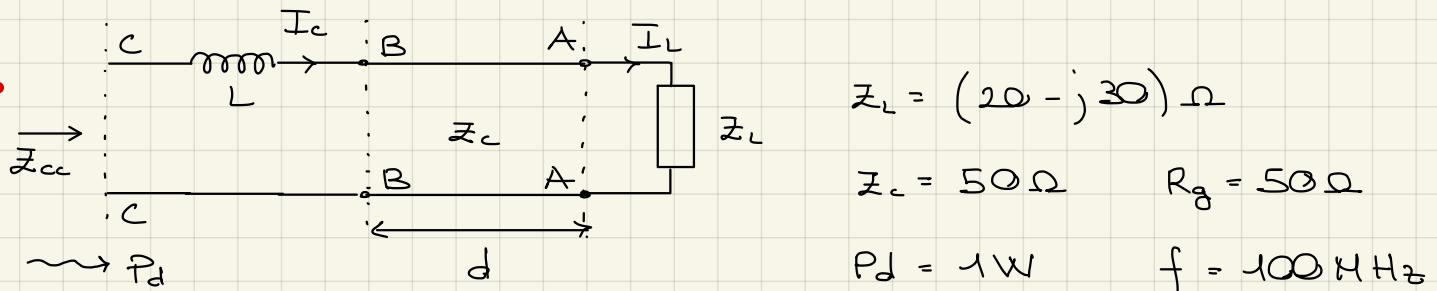
$$- \ln\left(\frac{2H-R}{R_o}\right) \quad (4)$$

$$R_o = \sqrt{h^2 + H^2}$$



$$V(P) = -\frac{f_L}{2\pi\epsilon_0} \ln \left[\frac{R}{R_0} \cdot \frac{R_0}{2h-R} \cdot \frac{2R_0-R}{R_0} \cdot \frac{R_0}{2H-R} \right]$$

$$C = \frac{f_L}{V(P)} = 21,54 \text{ pF} \quad Z_C = \sqrt{\frac{\mu_0 \epsilon_0}{C}} = 154,38 \Omega$$



Determinare L e d per avere adattamento in CC
Determinare $|V_{AA}|$ e $|V_{BB}|$, $|I_L|$ e $|I_c|$

Collegamento serie \rightarrow usiamo le impedenze

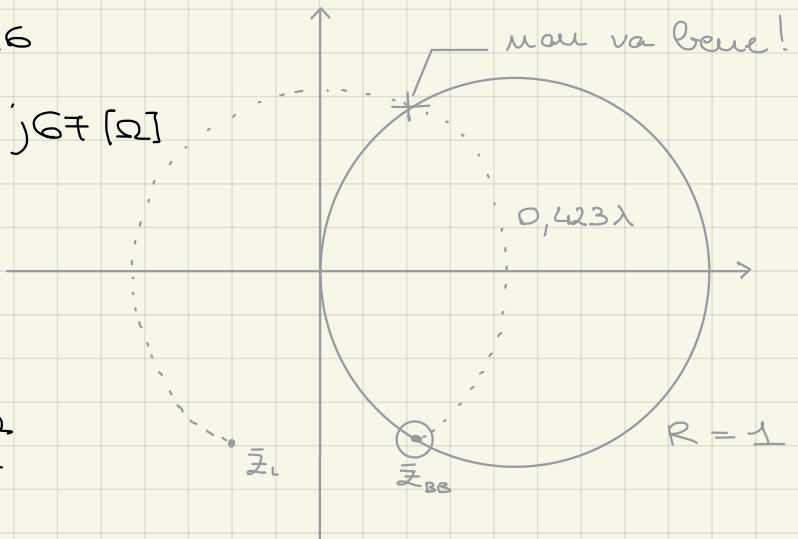
$$\overline{Z_L} = \frac{20 - j30}{50} = 0,4 - j0,6$$

$$\overline{Z_{BB}} = 1 - j1,34 \quad Z_{BB} = 50 - j67 \Omega$$

dove essere
negativa per permettere
all'induttanza di
compensarla

$$\rightarrow L = 1,07 \cdot 10^{-7} \text{ H} = \frac{67 \Omega}{2\pi f}$$

$$\rightarrow d = 0,423 \lambda$$



$$P_d = 1 \text{ W} = \frac{1}{2} |V_{BB}|^2 \operatorname{Re}\left\{\frac{1}{Z_{BB}}\right\} = \frac{1}{2} |V_{AA}|^2 \operatorname{Re}\left\{\frac{1}{Z_L}\right\}$$

$$\rightarrow |V_{BB}| = 16,7 \text{ V}, \quad |V_{AA}| = 11,4 \text{ V}$$

$$P_d = \frac{1}{2} |I_L|^2 \operatorname{Re}\{Z_L\} = 1 \text{ W} \rightarrow |I_L| = 0,316 \text{ A}$$