QUANTITATIVE DESCRIPTION OF NOISE

Noise sets constraints on the minimum current value. We can split any signal into

$$y(t) = \underbrace{s(t)}_{\text{INFORMATION}} + \underbrace{n(t)}_{\text{NOISE}} + \underbrace{d(t)}_{\text{DISTURB}}$$

INFORMATION ⇓ USEFUL — NOISE ⇓ UNAVOIDABLE — DISTURB ⇓ AVOIDABLE

We'll consider Gaussian noise (gaussian amplitude distribution), with zero mean value and parameters being time-invariant. We consider σ as a reference value to study noise fluctuations. Moreover, we consider ERGODIC noise, whose time average is indeed equal to the ensemble average.
In general, noise, like signals, can be described as the superposition of orthogonal harmonics with suitable amplitudes. Let's consider a signal made of the superposition of two harmonics, its mean square value is:

$$\langle x(t)^2 \rangle = \langle A^2 \sin^2(\omega_1 t + \varphi) + B^2 \sin^2(\omega_2 t + \varphi) + 2AB \sin(\omega_1 t + \varphi)\sin(\omega_2 t + \varphi) \rangle$$

$$\downarrow$$

$$= \frac{A^2}{2} + \frac{B^2}{2}$$

and it is also equal to the variance, being $\langle x(t) \rangle = 0$. Therefore the variance is equal to the sum of the variances of each harmonic contributing to the noise $x(t)$.

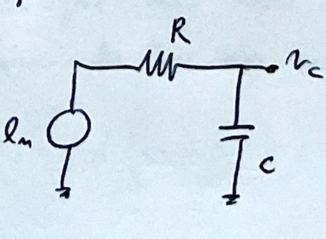
$$\sigma^2 = \sum_i \sigma_{f_i}^2 = \int_0^{+\infty} S_n(f)\,df$$

being $S_n(f)$ the POWER SPECTRAL DENSITY.
Let's consider a resistor, whose noise is due to thermal fluctuations of carriers. We may assume that the potential fluctuations have a very broad spectrum, being them made of sequential spikes in the time domain.
It can be concluded that each component in an electronic circuit can be modelled as an equivalent PSD generator (voltage or current), whose effect is transferred to the output and is summed to the other contributions to noise, being them uncorrelated.

→ RESISTORS

As already said, to first order we may assume $S_n(f) = W$ constant (WHITE)
Let's consider this circuit

$$\frac{v_c}{e_n}(s) = \frac{1/sC}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$



So

$$\langle v_c^2\rangle = \sigma^2 = \int_0^{+\infty} S_v(f)\,|T(j\omega)|^2\,df = W\cdot\int_0^{+\infty}|T(j\omega)|^2\,df$$

$$= \frac{W}{2\pi\tau}\int_0^{+\infty}\frac{2\pi\tau\,df}{1+(2\pi f\tau)^2} = \frac{W}{2\pi\tau}\left[\arctan(2\pi f\tau)\right]_0^{+\infty} = \frac{W}{2\pi\tau}\cdot\frac{\pi}{2} = \frac{W}{4\tau}$$

we call $\frac{1}{4\tau}$ = ENBW, <u>EQUIVALENT NOISE BAND-WIDTH</u>.

The energy inside the capacitor is

$$\frac{1}{2}C\langle v_c^2\rangle = \frac{kT}{2} \quad \text{[EQUIPARTITION PRINCIPLE]}$$

$$\langle v_c^2\rangle = \frac{kT}{C} = \frac{W}{4RC} \implies \boxed{W = 4kTR}$$

→ <u>MOSFETs</u>

- Ohmic region

$$I_{DS} = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right] \simeq \mu_n C'_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)V_{DS}$$

$$R_{ch} = \frac{1}{G_{ch}} = \frac{1}{\mu_n C'_{ox}\frac{W}{L}V_{ov}} = \frac{1}{g_m} \quad \text{like a resistive channel}$$

$$S_I(f) = \frac{4kT}{R_{ch}} = 4kT g_m$$

- Saturation

$$S_I(f) = 4kT\gamma g_m \quad (\gamma \text{ accounts for the non-uniform channel})$$

$$\left(\frac{S}{N}\right)^2 = \frac{\frac{\cancel{g_m^2}\, v_s^2}{2}}{4kT\gamma \cancel{g_m} BW} = \frac{g_m}{8kT\gamma\, BW}\cdot v_s^2$$

⟹ By increasing g_m (current) we increase $\left(\frac{S}{N}\right)^2$, so NOISE sets constraints on current.

INPUT REFERRED NOISE SOURCES

A theorem states that for a <u>TWO-PORT LINEAR NETWORK</u> (characterized by two pairs of input and output terminals) we can substitute the real network with an ideal noiseless one with two equivalent noise sources at the input, whose values are <u>INDEPENDENT OF SOURCE AND LOAD IMPEDANCES</u>.

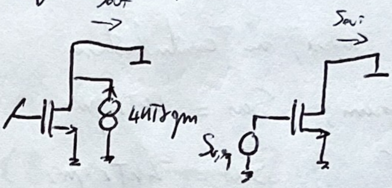
Therefore, to compute $S_{I,eq}$ and $S_{V,eq}$, we may short the output port and take the noise of the short circuit current as output value.

$$\begin{cases} S_{I,eq} \Rightarrow \text{open at the input} \\ S_{V,eq} \Rightarrow \text{short at the input} \end{cases}$$

→ **MOSFETs**

It's a 2-port network, the source being a common terminal for both input and output port.

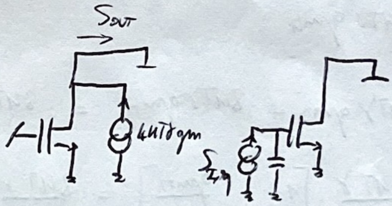
$$\begin{cases} S_{OUT}(I) = 4kT\gamma g_m \\ S_{OUT}(I) = S_{V,eq}\, g_m^2 \end{cases} \quad \text{(input shorted)}$$



$$\Downarrow$$

$$S_{V,eq} = \frac{4kT\gamma}{g_m}$$

$$\begin{cases} S_{OUT}(I) = 4kT\gamma g_m \\ S_{OUT}(I) = S_{I,eq} \cdot \dfrac{g_m^2}{\omega^2 C_G^2} = S_{I,eq} \cdot \left(\dfrac{\omega_T}{\omega}\right)^2 \end{cases}$$



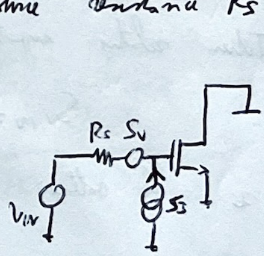
(input open)

$$\Downarrow$$

$$S_{I,eq} = 4kT\gamma g_m \left(\frac{\omega}{\omega_T}\right)^2$$

The relative importance of the two generators is set by the source resistance R_S

$$S_{V,IN} = S_{V,eq} + S_{I,eq} \cdot R_S^2 = \frac{4kT\gamma}{g_m}\left[1 + (g_m R_S)^2 \left(\frac{\omega}{\omega_T}\right)^2\right]$$



so for $f \leq \dfrac{f_T}{g_m R_S}$ the current contribution is negligible.

→ **DIFFERENTIAL STAGE**

Strictly speaking, the differential stage is not a two-port linear network, since its output is sensitive to signals on both input terminals. We may write

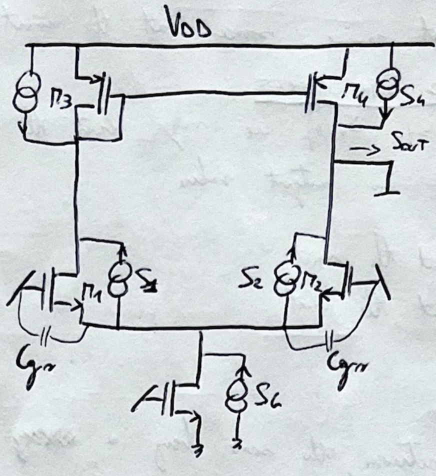
$$\begin{cases} v_1 = v_{cm} + \dfrac{v_d}{2} \\ v_2 = v_{cm} - \dfrac{v_d}{2} \end{cases} \qquad i_{OUT} = G_{m1} v_1 - G_{m2} v_2 = v_d \left(\frac{G_{m1} + G_{m2}}{2}\right) + v_{cm}\,(G_{m2} - G_{m1})$$

(7) If we assume $G_{m2} = G_{m1} = G_m$ then $i_{OUT} = G_m v_d$ and the output is only proportional to the input differential voltage, so the OTA becomes a 2-port network.

DEM :

$$S_{OUT} = S_{V,eq} \cdot g_{m1}^2$$

$$S_{OUT} = 4\, S_{I,eq} \left(\frac{\omega_T}{\omega}\right)^2$$



i_m flows into the capacitive path of the input, then it causes a differential current in the input pair equal to $\frac{i_m}{sC_{gs}} g_m$, so

$$i_{OUT} = \frac{2 g_m}{s C_{gs}} i_m$$

M6 : common mode noise, no contribution

M1 : splitting theorem, $S_{OUT} = 4kT\gamma g_{m1}$

M2 : " " , $S_{OUT} = 4kT\gamma g_{m2}$

M3 : $S_{OUT} = 4kT\gamma g_{m3}$

M4 : $S_{OUT} = 4kT\gamma g_{m4}$

$$\Rightarrow S_{OUT} = 8kT\gamma g_{mD} + 8kT\gamma g_{mM} = 8kT\gamma g_{mD}\left[1 + \frac{g_{mM}}{g_{mD}}\right] = S_{V,eq}\, g_{mD}^2$$

$$\Rightarrow S_{V,eq} = \frac{8kT\gamma}{g_{mD}}\left[1 + \frac{g_{mM}}{g_{mD}}\right] = \frac{8kT\gamma}{g_{mD}}\left[1 + \frac{V_{OV,D}}{V_{OV,M}}\right]$$

$$\Rightarrow S_{I,eq} = S_{V,eq} \cdot \frac{g_{mD}^2}{4} \left(\frac{\omega}{\omega_T}\right)^2$$

Their relative importance again depends on the source resistance

$$S_{V,IN} = S_{V,eq} + S_{V,eq} \cdot \frac{g_{mD}^2}{4} \cdot R_S^2 \cdot \left(\frac{\omega}{\omega_T}\right)^2$$

so the voltage noise is dominant up to $\omega \sim \frac{2\,\omega_T}{g_{mD} R_S}$

NOISE MODELS

→ THERMAL NOISE (DEM.)

Let's consider two resistors R_0 connected through a lossless coaxial cable, supposed to be isothermal. The cable is sized to match R_0.
The left-hand side resistor causes fluctuations that reach the resistor on the right with no reflections, being them properly matched. The same happens on the opposite direction, and these two flows are equal because of the 2nd principle of thermodynamics. (NO NET TRANSFER OF ENERGY AT TH. EQ.)
Now, let's short the two ends, thus trapping the energy inside. This energy is distributed along the characteristic modes inside the cable, solution of the wave equation (with boundary conditions on the voltage):

$$\lambda_k = \frac{2L}{k} \qquad f_k = \frac{c}{2L}k \qquad f_k \lambda_k = c \qquad \text{[WAVE FUNCTION]}$$

so, in a frequency interval df we have

$$\frac{df}{\frac{c}{2L}} \qquad (\bar{B} \text{ and } \bar{E} \text{ degrees of freedom})$$

modes, each of them having two degrees of freedom, so for the equipartition principle, kT, so

$$dE = \frac{2L}{c} kT \cdot df$$

The two energy flows are equal to

$$E = 2 \cdot P \cdot T = 2 \cdot \underbrace{\left(\frac{V_m}{2}\right)^2 \cdot \frac{1}{2R_0}}_{P} \cdot \frac{L}{c} \qquad \left[P \triangleq \text{average power due to one resistor}\right]$$

Considering the harmonics in the df interval, we get

$$dE = S_v \cdot \cancel{df} \, \frac{1}{2R_0} \cdot \frac{\cancel{L}}{\cancel{c}} = \frac{2\cancel{L}}{\cancel{c}} kT \, \cancel{df} \qquad \underbrace{\left[\frac{V_m^2}{2} = \langle V_m^2 \rangle = S_v \cdot df\right]}_{\text{Harmonic signal}}$$

$$\downarrow$$

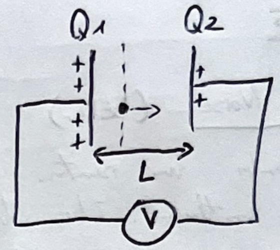
$$S_v = 4kT R_0$$

→ JUNCTION SHOT NOISE

The average number of electrons crossing a junction in a pn-junction is represented by the average current, while the actual number per unit time is subjected to fluctuations. It can be compared to an electron being ejected from one plate of a capacitor and reaching the other. This charge causes an opposed charge on the two plates proportional to its distance from it:

(9)

$$\begin{cases} Q_1 = q\,\dfrac{(L-x)}{L} \\ Q_2 = q\,\dfrac{x}{L} \end{cases} \qquad Q_1 + Q_2 = q$$



This induced charge causes a current

$$\frac{dQ_2}{dt} = \left|\frac{dQ_1}{dt}\right| = \frac{q}{L}\cdot v(t) \Rightarrow \text{instantaneous carrier speed}$$

This velocity is

- increasing proportionally to the constant electric field between the plates in the case of the charge between the capacitor's plates;
- constant if we assume the carriers to move in a semiconductor at $v = v_{SAT}$;

in both cases, we get $\int_0^T i(t)\,dt = q$, being it a discrete pulse of a constant duration T.

We can describe the current in a junction as the superposition of many discrete current pulses starting at random time instants, namely

$$q\,h(t) \qquad [h(t) \triangleq \text{normalised shape of the pulse}]$$

We call

$$\lambda = \frac{I}{q} \qquad \left[\begin{array}{l}\text{average number of carriers crossing the} \\ \text{junction per-unit time}\end{array}\right]$$

Let's consider a current measured at $t = \bar{t}$, we can say that it is given by the superposition of all the pulses started at $t < \bar{t}$, so

$$i(t) = q\,h(t_1) + q\,h(t_2) + \dots$$

To switch from discrete to continuous we must weight every contribution for the probability for a pulse to occur between t_1 and $t_1 + dt$, so $\lambda\,dt$

$$\langle i(t)\rangle = \int_0^{+\infty} h(t)\,q\,\lambda\,dt = q\lambda = I \;\checkmark$$

Similarly we can compute the average square value:

$$i^2(t) = [q\,h(t_1) + q\,h(t_2) + \dots]^2 = \underbrace{q^2 h^2(t_1) + q^2 h^2(t_2) + \dots}_{\text{SQUARED VALUES}} + \underbrace{q\,h(t_1)\,q\,h(t_2) + \dots}_{\text{CROSS PRODUCTS}}$$

$\downarrow$

$$\langle i^2(t)\rangle = \int_0^{+\infty} \lambda\,q^2 h^2(t)\,dt + \int_0^{+\infty}\!\!\int_0^{+\infty} (q\,h(x)\,\lambda\,dx)(q\,h(y)\,\lambda\,dy) = q^2\lambda \int_0^{+\infty} h^2(t)\,dt + (q\lambda)^2$$

knowing that the variance of a casual variable is

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

we may write

$$\sigma_I^2 = \langle i^2 \rangle - \langle i \rangle^2 = \int_0^{+\infty} h^2(t)\,dt\,(q^2)\lambda + (q\lambda)^2 - (q\lambda)^2 = q^2\lambda \int_0^{+\infty} h^2(t)\,dt$$

$$= q I \int_0^{+\infty} h^2(t)\,dt$$

Now, using the Parseval theorem, we can write

$$\int_0^{+\infty} |h(t)|^2\,dt = \int_{-\infty}^{+\infty} |H(f)|^2\,df = 2\int_0^{+\infty} |H(f)|^2\,df$$

$h(t)$ real $\iff$ $|H(f)|$ even function

So, in the end, we get

$$\sigma_I^2 = 2qI \int_0^{+\infty} |H(f)|^2\,df = \int_0^{+\infty} S_I(f)\,df = 2q^2\lambda \int_0^{+\infty} |H(f)|^2\,df$$

and from the equation we derive

$$S_I(f) = 2qI\,|H(f)|^2$$

Finally, since our pulses have a duration T of the order of ps, then the amplitude of their Fourier transform will be $\simeq 100$ GHz, so to first order we can consider $|H(f)|^2 \simeq 1$ in the BW of interest. ✓

Accounting for both physical mechanism involved in diode's current (DRIFT + DIFFUSION) we get that

$$S_I = 2q\,(I + 2I_0) \qquad \begin{cases} I_{DIFF} = I + I_0 \\ I_{DRIFT} = -I_0 \end{cases}$$

So, we get

<u>REVERSE BIAS</u> $\Rightarrow$ $S_I = 2qI_0$

<u>FORWARD BIAS</u> $\Rightarrow$ $S_I \simeq 2qI$

<u>ZERO BIAS</u> $\Rightarrow$ $S_I = 4qI_0 = 4qKT\dfrac{I_0}{KT} = 4KT\,g_{m,0}$

(11)

Finally, we can say that also the current in weak-inversion MOSFETS is affected by shot noise, being it composed by the superposition of current pulses generated by carriers surfing above the barrier.

$$S_I = 2q I_D = 2q \, I_D \cdot \frac{nV_{TH}}{nV_{TH}} = 2q n \frac{kT}{q} \cdot g_m = 4kT \frac{n}{2} g_m = 4kT\gamma \, g_m$$

where $\boxed{\gamma = \frac{n}{2}}$

→ RTN

Let's consider a resistor made of n-Si. The average current flowing through it is

$$I = \frac{V}{R} = V \frac{W \cdot \Delta}{L} q \mu_n n = q\mu_n \cdot \frac{N}{L^2} V \quad (I \propto N)$$

being N the total number of free carriers in a volume $\mathcal{V} = W \cdot \Delta \cdot L$

If a carrier is trapped or emitted by a trap, this causes a reduction or an increase of the total current. Each event causes a variation of N equal to ΔN, and so

$$\frac{\Delta I}{I} = \frac{\Delta N}{N} \Rightarrow \Delta I = \frac{I}{N} \quad (\text{for } \Delta N = 1)$$

Each trapping or emission event causes current variations that happen to recover the steady state value exponentially, since the time needed to recover it is a statistical variable. So we can write each pulse like:

$$i(t) = \Delta I \cdot e^{-\frac{t}{\tau}} = (\Delta I \cdot \tau) \cdot \frac{1}{\tau} e^{-\frac{t}{\tau}} = Q \cdot h(t)$$

At steady state λ (# of emission events per unit time) is equal to # of capture events per unit time. We expect

$$\lambda \propto N_T \quad (\text{number of traps in the volume})$$

$$\downarrow$$

$$\lambda = \beta \frac{N_T}{\tau}$$

As already shown for the pn-junction, a noise characterized by the superposition of well-defined current pulses of area Q can be written as

$$S_I(f) = 2\lambda Q^2 |H(f)|^2 = 2\lambda Q^2 \frac{1}{(1+\omega^2\tau^2)}$$

$$\downarrow$$

$$= 2\beta \frac{N_T}{\tau} \Delta I^2 \cdot \tau^2 \cdot \frac{1}{1+\omega^2\tau^2} = 2\beta N_T \Delta I^2 \frac{\tau}{1+\omega^2\tau^2}$$

$$\downarrow$$

$$= 2\beta N_T \left(\frac{I}{N}\right)^2 \frac{\tau}{1+\omega^2\tau^2}$$

This contribution is summed to the one due to emission events, being them statistically independent from the capture events.

so we get

$$S_I(f) = 4\beta N_T \left(\frac{I}{N}\right)^2 \underbrace{\frac{\tau}{(1+w^2\tau^2)}}_{\text{Lorentzian shape}}$$

where

$$\underbrace{\beta = \frac{\tau_E \tau_C}{(\tau_E + \tau_C)^2}} \qquad \tau = \frac{\tau_E \tau_C}{\tau_E + \tau_C}$$

asymmetry coefficient,
maximum for $\tau_E = \tau_C$
and equal to $\frac{1}{4}$
given by states at the Fermi level

$$S_I(f) = N_T \cdot \left(\frac{I}{N}\right)^2 \frac{\tau}{1+w^2\tau^2}$$

→ **1/f NOISE**

Defects with different time constants may exist, so let's introduce the function $g(\tau)$ which is a distribution of defects over the τ axis, so that

$$dN_T(\tau) = N_T\, g(\tau)\, d\tau \qquad (g(\tau) \text{ is normalized})$$

therefore we can write

$$S_I(f) = N_T \cdot \left(\frac{I}{N}\right)^2 \int_{\tau_{MIN}}^{\tau_{MAX}} \frac{g(\tau)\,\tau}{1+w^2\tau^2}\, d\tau$$

$$\begin{cases} \tau_{MIN} = \text{shortest } \tau \text{ that can be measured, set by the BW of the apparatus} \\ \tau_{MAX} = \text{set by the experiment duration} \end{cases}$$

McWhorter pointed out that in a MOSFET free carriers can be captured by defects at the interface and inside it, due to tunneling. Defects inside the oxide have a

$$\tau = \tau_0 e^{\gamma x} \qquad \begin{cases} \gamma \propto \text{height of the barrier} \\ \tau_0 \pm \tau \text{ for defects at the interface} \end{cases}$$

Let's consider defects to be distributed uniformly all over the oxide thickness

$$n_T' = \frac{N_T}{t_{ox}} \quad [\text{density per unit length}]$$

so we can write

$$n_T' dx = N_T \frac{dx}{t_{ox}} = N_T\, g(\tau)\, d\tau$$

and

$$d\tau = \gamma \tau_0 e^{\gamma x} dx = \gamma \tau\, dx \implies dx = \frac{d\tau}{\gamma\tau}$$

so

$$g(\tau) = \frac{1}{\gamma \tau t_{ox}} \qquad \text{(we've linked } g(\tau) \text{ to the parameters we know)}$$

$$\Rightarrow S_I(f) = \frac{N_T}{\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{1+w^2\tau^2} = \frac{N_T}{\gamma t_{ox}} \frac{1}{w} \left(\frac{I}{N}\right)^2 \left[\arctan(w\tau)\right]_{\tau_{min}}^{\tau_{max}}$$

$$= \frac{N_T}{\gamma t_{ox}} \frac{1}{w} \left(\frac{I}{N}\right)^2 \left[\arctan(w\tau_{max}) - \arctan(w\tau_{min})\right]$$

Considering $w\tau_{max} >> 1$ and $w\tau_{min} << 1$ we get

$$S_I(f) = \frac{N_T}{\gamma t_{ox} w} \left(\frac{I}{N}\right)^2 \frac{\pi}{2} = \frac{N_T}{4\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \cdot \frac{1}{f}$$

$\longrightarrow$ TSIVIDIS FORMULA

$N = C'_{ox}\,(WL)\,(V_G - V_T) \cdot \frac{1}{q}$ carriers in the channel

$N_T = n_T \cdot WL\, t_{ox}$ ~~traps~~ charges in the oxyde

$$S_I(f) = \frac{n_T \cdot WL \cdot t_{ox}}{\gamma t_{ox} \cdot 4} q^2 \frac{\left[\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)(V_G - V_T)^2\right]^2}{\left[C'_{ox}\,(WL)\,(V_G - V_T)\right]^2} \cdot \frac{1}{f}$$

$$= \frac{n_T \cdot \cancel{WL}\, q^2}{4 \cdot \gamma (\cancel{WL})^2} \cdot \frac{1}{f} \cdot \frac{1}{4}\mu_n^2 \cdot \left(\frac{W}{L}\right)^2 \cdot (V_G - V_T)^2 =$$

$$= \frac{n_T q^2}{8\gamma L^2} \cdot \frac{\mu_n}{C'_{ox}} \cdot \frac{I}{f} = \frac{K_I^{1/f}}{L^2} \frac{I}{f}$$

$$S_V(f) = g_m^{-2} S_I(f) = \frac{n_T \mu_n q^2}{8\gamma C_{ox} L^2} \cdot \frac{I}{f} \cdot \frac{V_{ov}^2}{4 \cdot I^2} = \frac{1}{f} \frac{n_T}{16\gamma C'_{ox}} \cdot \cancel{\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right) V_{ov}^2} \cdot \frac{1}{\cancel{I}} \cdot \frac{1}{C'_{ox} WL}$$

$$= \frac{1}{f} \frac{n_T q^2}{16\gamma C'_{ox}} \cdot \frac{1}{C'_{ox} WL} = \frac{K_V^{1/f}}{C'_{ox} WL} \cdot \frac{1}{f} \checkmark$$