ELECTRONICS DESIGN FOR BIOMEDICAL INSTRUMENTATION

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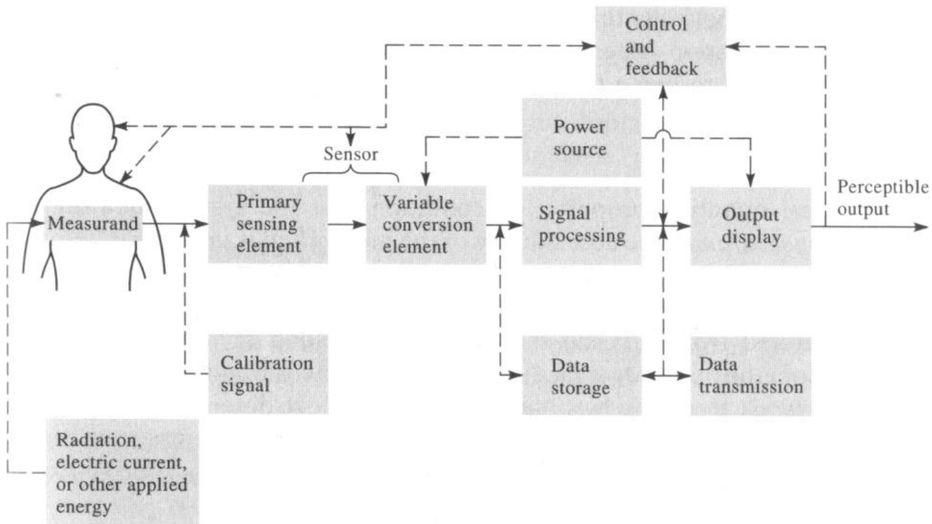
INTRODUCTION

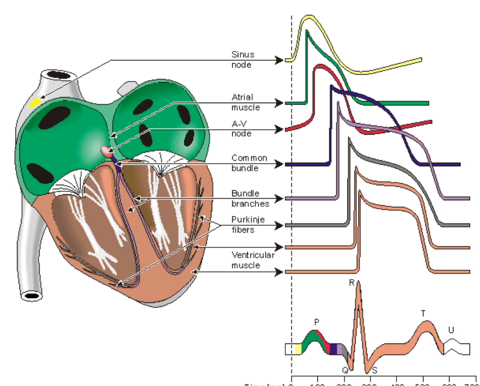


General architecture of a biomedical instrumentation. The signal can be generated spontaneously or by applying an external radiation (medical imaging).
We have a sensing element that senses the physical quantity and converts it in an electrical signal. Then the signal is amplified, filtered and then it goes in a system that extracts the variable of interest.

BIOLOGICAL SIGNALS

If we measure the potential across the wall of a cell, we have a resting potential of -70mV. It is the result of an electrochemical exchange through the membrane of the cell. The imbalance of ions across the membrane causes a potential difference.
If then a cell is stimulated externally, the voltage is modified and a pulse (action potential) is produced and measured.

A particular organ of interest is the heart, in which we have tissues in which an AP (action potential) is taking place. In green we have the atria, in brown the ventricles.
Other nodes are producing AP, and we can see AP of the various tissues. What we see is that the pulses are synchronized in a sequence from top to bottom. This is why atria contracts before ventricles (delays of corresponding AP). We have not access to these local Aps, we can only sense the effects. The overall waveform is the result of the combination of the various AP.



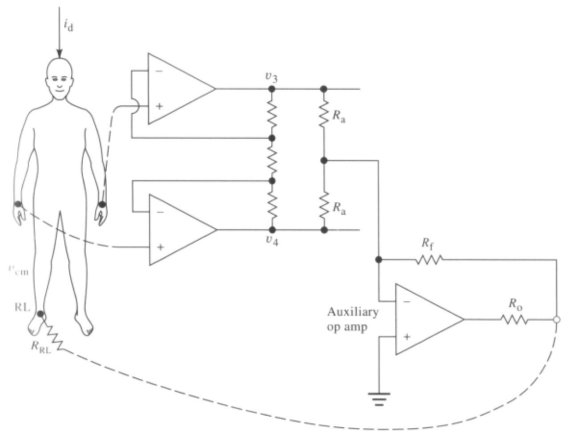
Measurement

A medical instrument must measure Aps, done connecting the patient with electrodes and using a differential amplifier. We don't measure the absolute V (i.e. referred to ground), but the difference. This because superposed to the useful info we have a very large common voltage on the body. With a unipolar measurement the spike would be superposed to this. With a differential one we discard this common voltage.

Effects that work against us:

- Common mode: INA is commonly used because it has a high input impedance, so it is less sensitive to the impedance of the source and good rejection of the common mode → common voltage is cancelled and not amplified.

To reduce the common mode we can use a circuit where the patient is put in a loop in a way that common mode is measured, amplified, inverted and applied to the right leg of the patient → common mode (product between injected current and resistance of the body) is reduced by close loop gain.



PACEMAKER

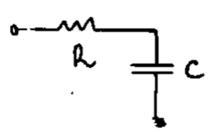
Implanted in the thorax and connected to the internal walls of the heart. It must provide pulses to the heart since the heart cannot do it autonomously.

In pacemakers batteries are very important → must last for several years. Moreover, the pacemaker can also record waveforms → used also for diagnostics. The main challenge is the power consumption. Indeed, the battery capacity is 2000mA/h, similar to the one of the smartphone → challenging to provide the power and don't waste it.

The PM has a sensing amplifier that works with 1,5V and 100nA of current consumption → transistor with small current 'I' to keep them open.

The technology used is the CMOS for the transistor.

To filter the signal, we can use the RC:



The pole is 1/RC. To have a small pole, the RC must be large. C cannot be > 10pF, so R must be in MOhm. But this is a too big value for a resistor in integrated circuits (ICs) → technologies required to implement equivalent large resistors without physically use it. We need a kind of replacement, equivalent resistor → switched capacitors.

Switched capacitors amplifiers

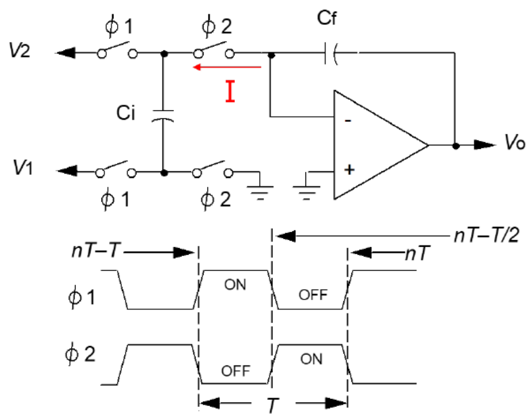
It is a technique. We have an integrator (low pass filter). The problem with R and C is still the same. The R is replaced with an equivalent ohm law.

We have no resistors, but capacitors and switches, that are very small → can be used in ICs. Firstly phi1 is closed and C charged. Open it and then close phi2. C is discharged. This is done iteratively and, if done quickly, it is similar to have an average current 'I' in the C, obtained as:

We have created an artificial average current. If we take the voltage and divide it by the artificial current, we get an equivalent resistance:

$$R_{eff} = \Delta Vi/I$$

So we have created an artificial resistance.



$\Delta Vi = V1\text{-}V2$

$I = \Delta Vi \times f \times Ci$

$Reff = \Delta Vi/I = 1/f \times Ci$

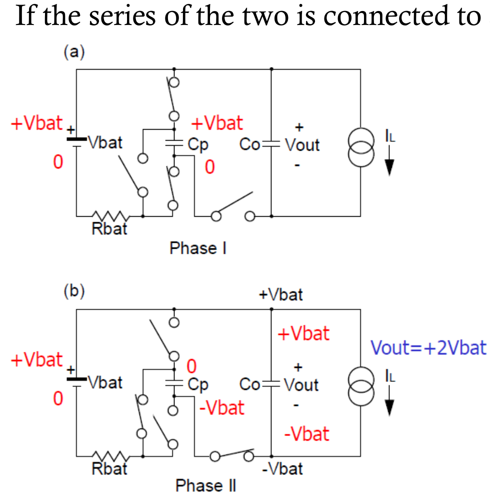
$Vo = I \times 1/j\omega C_f$
$= \Delta Vi \times f \times Ci/j\omega C_f$

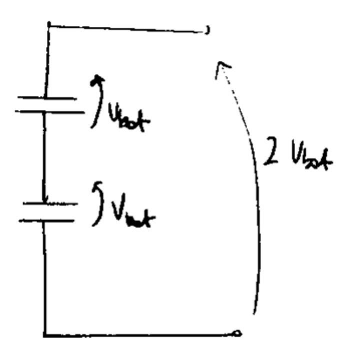
$$\frac{V_o}{\Delta V_i} = \frac{C_i}{C_f}\frac{1}{j\omega T}$$

Another problem is the pacemaker discharge of the battery along the years. Typical battery in pacemakers is of 2,8V, but recently also <2V.
This makes pulsing quite problematic → not providing the sufficient voltage pulse to trigger the AP if too small.
There are circuits able to duplicate the voltage of the power supply → charge pumps, and we can compensate the drop of voltage due to aging.
The principle of these circuits is:

1. Charge a C in parallel to the battery.
2. Disconnect the switch and put the C in series with the battery. So at least for short time the voltage is doubled.

a) Cp is charged through the battery.
b) Switches change and Cp is connected to the bottom of the battery → series between the two. If the series of the two is connected to a load (tissue resistance), the voltage is doubled.





ARTIFICIAL RETINA

Prothesis (array of electrodes) implanted in the retina for patients that have lost the work of the receptors. If photoreceptors are damaged, retina can be still stimulated with an array of electrodes which collects the image, and the image is recorded by external camera and transmitted into the internal prothesis by a radiofrequency link → visual stimulator.

It is an example of microelectronics. Problems:

1. We need to transmit through a radiofrequency link
2. Battery-less system: the prothesis hasn't got a battery → the power supply must be recovered form the radiofrequency link → energy harvesting technique. Energy is retrieved from radiofrequency waves carrying the info.

COCHLEAR IMPLANT

Used when the cochlea is not working. The implant has the role of stimulating the various regions of the cochlea with an electrical signal. It is difficult since the cochlea is frequency-sensitive. The internal sections of the cochlea are sensitive to low frequencies, going outside to high frequencies. We need to mimic the frequency partition; the sections of the cochlea must be stimulated with the correct bandwidth.

For example, for the sound "s-a", since s = high frequency and a = low frequency, we make a selective filtering and stimulate the part of the cochlea according to the frequency (only some channels are selected according to the frequency).

MEDICAL IMAGING

Radiography: X-rays are shined through an object and the image is recorded on a panel. It is a very quick and easy but provides only 2D images → reason why CT is important.

In CT we have high intensity of X-rays that are absorbed by high-density tissues, conversely if we have voids (cavities). With 1D projections along different angles we can reconstruct a 2D image. With CT we are sampling the subject with a frequency given by the Shannon theorem. Similarly for PET and SPECT.

PET: positron emission CT. the positron is the anti-electron. Positrons are exploited since if we inject in the patient a positron emitter and this emitter is veiculated in the patient toward an organ to be imaged, then the positrons are emitted locally, they annihilate with electrons (they disappear); but by Einstein's equation E = mc^2, when a mass m disappears from the universe, an equivalent energy E must be released in the universe.

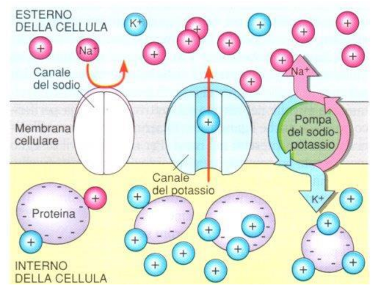
The energy consists of 2 gamma rays back-to-back (to fulfill the momentum). If we trace in a detector the directions of the two gamma rays, we can determine the emitter positron inside the body of the patient. Timing is very important because if we are measuring the time occurrence between the two gamma rays, one photon reaches the detector earlier than the other → the delay can be used to recover the position. This is the Time Of Flight idea.

ORIGINS OF BIOPOTENTIALS

IONS TRANSPORTATION

Let's consider a neuron and its axon. We have a membrane and a voltage difference in-out that is negative: -70mV (up to -90mV) → resting potential. But why this potential difference exists?
It is caused by the occurrence of 3 mechanisms:

1. Na-K pump
2. Diffusion process of ions (consequence of 1)
3. Development of an electrical gradient (consequence of 2)

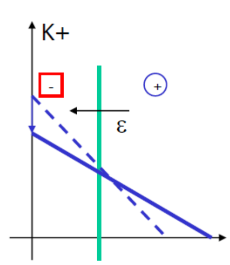


We have an internal part with ions and proteins, and the membrane has transport channels. Ions can travel through when channels are open, both in-out and out-in. In the image, the K-channels allow to go outside. At resting, channels are in equilibrium and relatively closed. The permeability is the capability of the membrane to let ions pass through it. Normally, permeability is small for ions, just a little bit more for K and Na.
In this equilibrium situation (ions moving slowly inside-out) we have a brake in the equilibrium due to the Na-K pump. It moves Na outside (Na is already abundant outside) and K inside (already abundant).

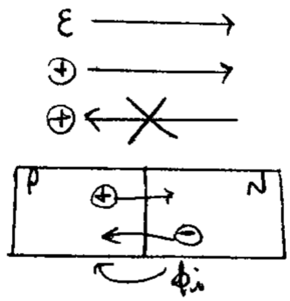
Na-K pump

The process needs energy, retrieved by a molecule, ATP. When ATP breaks it supplies the energy to allow for transportation.
Na channels are open, Na ions enter and exit outside the cell. At the same time, K-ions are moved inside through their channels. These transfers happen simultaneously with a precise quantity: every 3 Na going outside, 2 K enter.

Secondly, we can see we have abundance of Na outside and K inside. Since we have this in a gas volume, diffusion phenomena start to take place. The gradient is created by the NaK pump. What happens is that K starts to cross the membrane and also Na starts to diffuse. K is electrically balanced by the presence of negative charges. There is not an imbalance between + and – charges due to the NaK pump. So if we have a certain concentration of K, we must have a negative counterpart → proteins.
So there is a charge neutrality inside the cell. The negative charges are important. In fact, if K exits due to permeability, negative charges are left inside the cell. So we are creating an electric field. The green line is the membrane, the dashed line the imbalance in the concentration of K. The trend is to move from dashed line to solid line to balance K inside-out. But if we move K outside, we have negative charges accumulated inside → electric field and potential difference.



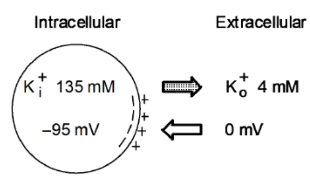
This potential difference stops the iteration of the process. The + charges move as the electric field. So, does K continue to diffuse? No, at a given time it stops, since it has no reason to diffuse against the electric field. It is the same of a pn junction in electronics.
So the diffusion of K is stopped by the growth of voltage difference that is the resting potential (RP). RP is the final effect that stops diffusion in the mechanism created by the NaK pump. It is negative because positive from outside to inside.



In the resting potential, Na doesn't play any role, since Na permeability is very low in this condition. It is so small that diffusion of Na is impaired, it stays outside.

Determination of the electrical field

The voltage difference is computed according to the Nerst equation:



$$E_K = \frac{RT}{zF} \ln \frac{[K_o^+]}{[K_i^+]} = 0.027 \ln \frac{[4]}{[135]} = -0.095 \qquad \text{(V)}$$

Nernst equation
R: universal gas constant
F: Faraday constant
Z: element valence

The Nerst equation tells us that resting potential is as above. For Na and K the valence Z is 1, for Ca2++ is 2.

In reality, the situation across the membrane must consider also other ions, not only K → GHK equation. The difference with the Nerst equation is that ions play a role according to their permeability.

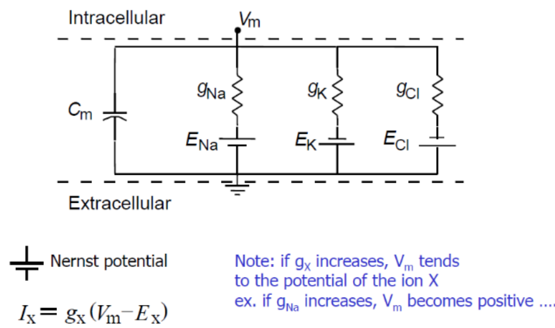
$$E_m = \frac{RT}{F} \ln \frac{P_k[K_o^+] + P_{na}[Na_o^+] + P_{cl}[Cl_i^-]}{P_k[K_i^+] + P_{na}[Na_i^+] + P_{cl}[Cl_o^-]}$$

equation of Goldman-Hodgkin-Katz (GHK)

P_X: membrane permeability to ion X
(es. $P_{na+} = 2\times10^{-8}$ cm/s, $P_{K+} = 2\times10^{-6}$ cm/s)

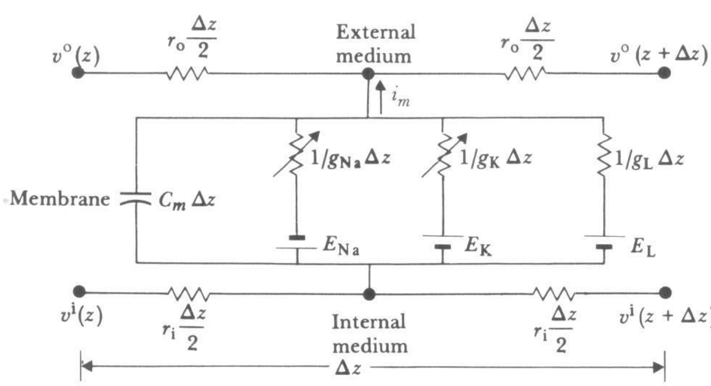
Ion	Extracellular concentration (mM)	Intracellular concentration (mM)	Equilibrium potential (mV)
Na+	145	12	67
K+	4	155	−99
Cl-	120	4	−92

Pna is 2 orders of magnitude smaller than K → permeability of Na and Cl can be neglected → we retrieve the Nerst equation.
In the table we have the RP if only one specific ion at a time is considered. We can build a general electrical model of the cell, with one voltage generator for each ion (Nerst potential). The orientation of the generator is accordingly to the concentration of the species inside-out.



These generators play a role on the overall RP according to the conductances in series with them, that represent the permeabilities. For K, gk is like a shortcircuit (high permeability), while for Na is like an open circuit → each generator plays a role depending on the conductance.

Another model is the distributed parameters model: we can consider small segments, slices of axon of width Δz.

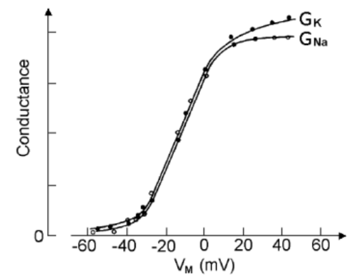


The elements are proportional to Δz. In the model we have also capacitance Cm, since we have a capacity effect due to a separation of positive and negative charges in a volume. Moreover, we have also a conduction in the transfer direction of cells, characterized by a resistance of the medium (ri*Δz/2: distributed resistance on the internal volume). Similarly for outside (ro*Δz/2).

THE ACTION POTENTIAL

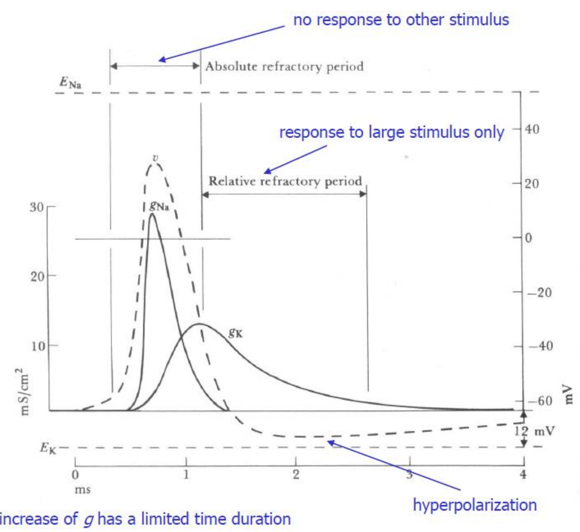
The cell is stimulated → superposed to the resting potential we have an external application of a voltage step. So the consequence is the rise of an action potential.
As for the sign of this stimulation, the polarity is aimed at providing a positive voltage on the internal part of the cell with respect to the external one. So the polarity is used to reduce the voltage across the cell. We are rising the voltage internally.
If the stimulus is strong enough to overcome a given threshold (10-20mV), then we have the start of an AP positive pulse.
The reason for this rise can be seen in the behaviour of the conductances of ions channels.

In the plot we can see the conductance as a function of the potential across the membrane. We see that if it is negative and around -70mV, G is small, but if the applied pulse is able to make the voltage less negative, G starts to rise, in particular it rises earlier for Na ions than K ions.



So if voltage gets less negative or less positive, channels open, in particular Na → Na enters in the cell (this thanks to the application of a pulse).
But if they start to enter, the voltage internally rises even more. If it increases, also G increases → more Na ions enter. It is a kind of positive feedback.

This process however then ends. At a given point, the Na channels close and the K channels open. There is a delay between the flow of Na ions via channels and the K ions' flow.
If they would open simultaneously, there would not be a significant effect.
When K starts, voltage decreases, because the internal part of the cell becomes more negative by the same token of equilibrium. Hence AP ends, voltage drops down → phase of repolarization (AP occurs when the cell was depolarized). We can even have an undershoot (hyperpolarization).



This plot shows rise and fall of the potential (also hyperpolarization). We see gna rises first and then drops down, while the conductivity (gk) of K rises afterwards and then falls down. This is the reason why voltage rises and then drops down to the resting potential.

In the plot we can also see two intervals. The first, between start and early fall of AP is called absolute refractory period: it is a period where the cell is insensitive to further stimulation. If we provide here an additional pulse, the cell doesn't respond.
The next interval is the relative refractory period, in which the cell is slowly restoring the capability of accepting of accepting a new pulse. It responds only to particularly large stimuli. After this period the cell is ready to any type of pulse.

The shapes of APs can be different, depending if we have fast or slow APs; it depends on the occurrence of various ions in the stimulation. For example, the AP on the heart has an initial rise justified by the step rise of gna, but then the flat region is due to the competitive action of 2 ions, K exiting and Ca (present in cardiac cells) entering. The two effects counterbalance for a while, then the K restores the voltage across the membrane to the resting potential.

What are the numbers? Absolute refractory period, which is the minimum period in which the cell cannot be triggered, lasts ~1ms; this means that the repetition rate to stimulate a cell has a max frequency of 1000Hz.

THE CONDUCTION

So far we have seen the rise of an AP in a segment Δx of the axon. How can an AP be transferred along the axon?
We must consider that one segment of the axon is not electrically insulated from the adjacent one, but segments are continuous.
So let's suppose an AP occurs in a certain segment; Na enters in the cell. But if so, Na ions are also spreading laterally (to the right and left segments). But if they spread, they are triggering a new AP on the next segment (but not by external stimulus), this mechanism triggers a new AP, while the 1st segment is repolarizing. This depolarization propagates segment by segment.
This is why AP is triggered segment by segment. The AP is not triggering to the left because the left segment is in the absolute refractory period → unidirectional flow of the AP.

The synapsis

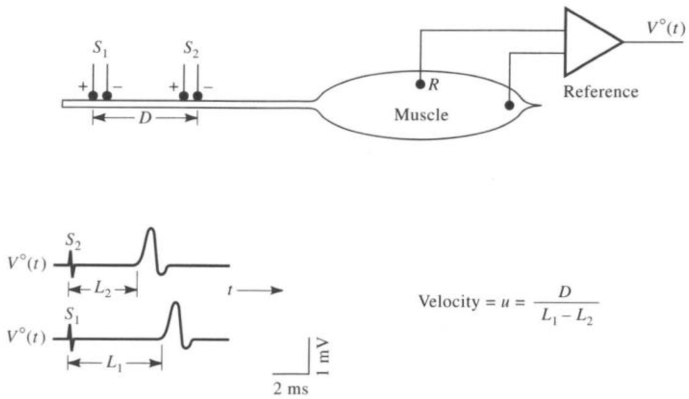
Mechanism that allows a biosignal to be transmitted from a neuron to the other. These are 2 types of synapsis:

1. Electrical synapsis: the 2 neurons are so close to each other that the ion mechanism just described is able to continue from a neuron to the consequent one. The flow of ions is able to perturbate the subsequent neuron just by electrical modification of voltage.
2. Chemical synapsis: if 1) is not possible, there must be a chemical transfer. The electrical transmission of signal is interrupted and AP is generated again via chemical mechanisms.

Let's suppose an AP arrives at the end of a neuron. Here we have containers (vesicles) that contain neurotransmitters, molecules able to open the gate of the incoming neuron. So the arriving AP pushes vesicles to the membrane, they open into the intra-neuronal liquid (or medium) and the transmitters are able to move toward the incoming neuron. The neurotransmitter, by chemical action, is able to open the gates an ion channels → gate of the incoming neuron opened not by means of electrical action but by means of neurotransmitters, which open for a while the channels (so ions flow) and so we have the generation of a new AP. After a while the action of neurotransmitters is over, they are dismantled, the channels close and the chemical synapsis is over.
So there is not a direct ion transfer between the proceeding neuron and the incoming one, but there is a mediation via neurotransmitters.

Conduction velocity of signals in the neurons

We move to the macroscopic level. The velocity of biosignals can be detected and measured across a connective tissue.
We apply a stimulus in a given point of a tissue and we will record the transmitted signal in another point of the tissue or a muscle, and we use an amplifier to detect it.
So if we apply the pulse at S1 (further than S2), L1>L2 with respect to the recording amplifier.
But if we apply the pulse in S1 and S2 and we measure the distance D in between, by knowing L1 and L2 we get the speed.

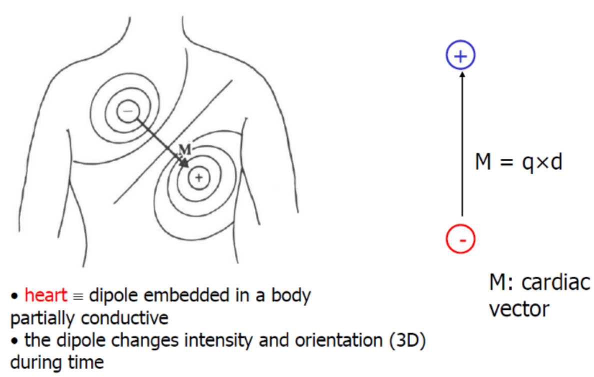


Conduction of signal in the volume

In the body volume. We are interested because it is important in ECG, since the heart must be analyzed from the surface of the body, not with a direct access.
We must analyze the electrical activity in the heart before. Heart is a combination of tissues contracting in different time slots.

First of all, <u>heart walls (atria and ventricles) contract due to the action of Ca ions</u>. When Ca ions are quickly moving in the tissues, in the surface tissues of the heart or in the heart walls, they produce depolarization, AP, and moreover contraction → chemical and mechanical action. They determine a contraction of the surface where they are rushing.
From the point of view of modelling, although this is a macroscopic effect (we are talking about a massive transfer of Ca ions), we must not forget that even in this case, when you have a positive charge or a massive amount of positive charges leaving a volume to move to a contiguous one, they are leaving behind some uncovered negative counterparts in the previous volume (the negative charge doesn't move) → we find a similar effect already seen in AP in cells.

This bring to a model of this behaviour that in physics is the **electrical dipole**.



We have a dipole if we have a separation between positive and negative charges. A dipole is a vector with direction towards positive charges, whose intensity is M = q x d, where q is the charge and d is the distance.
In the heart we have the creation of a dipole, the cardiac vector. It can be considered as a dipole because we have Ca ions rushing away and negative counterparts remaining in the previous volume.

The orientation of the cardiac vector is not static, because depending on the walls of the heart that are contracting the direction and intensity of Ca rushing will be different. Hence M changes continuously in time the orientation and the intensity, in 3D.
Ventricles are the largest tissues in heart → during ventricle contraction M reaches the highest intensity.

In this image, the arrows show the localized depolarization of the tissues of the heart. So we start at 80ms with the atrial depolarization. Ca rushes away from atria → M is as in the pic, and reflects the localized rushing of ions. Then at 230ms I start to have a depolarization of LV, so the M will be very large → largest synchronized rushing of ions which produces the largest intensity of M.

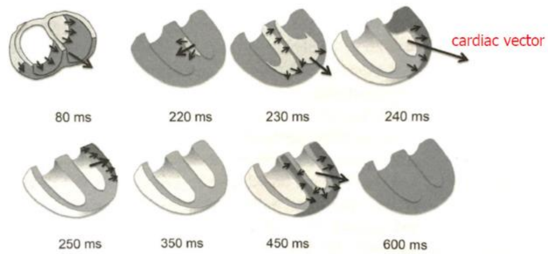


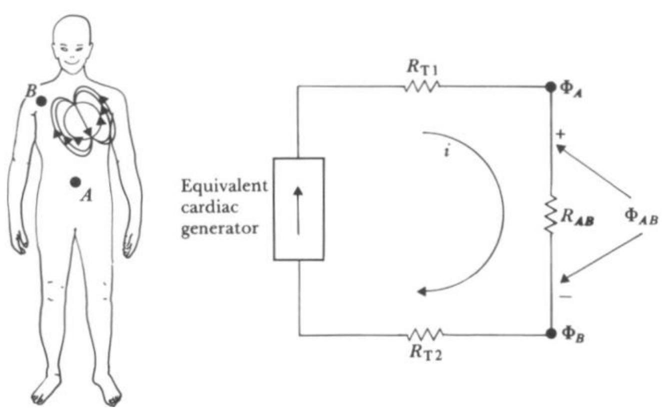
Figure 5.3 Successive steps in depolarization (light areas) and repolarization (dark areas) of the heart. Atrial depolarization ~80 ms, septal depolarization ~220 ms, apical depolarization ~230 ms, left ventricular depolarization ~240 ms, late left ventricular depolarization ~250 ms, ventricles completely depolarized ~350 ms, beginning of ventricular repolarization ~450 ms and ventricles completely repolarized ~600 ms. The thick black arrow shows the direction of the cardiac vector.

Now we must understand how to measure M. this is the purpose of electrocardiography (ECG).

ECG

The ECG does a sampling externally of the heart activity, by measuring potential differences, thanks to electrodes and amplifiers. The voltage difference reflect the flipping and changing of intensity of the cardiac vector (the scalar components of the vector). So in ECG the potential differences are sampled within different parts of our body because the combination of these measurements should provide a representative sampling of the cardiac vector.

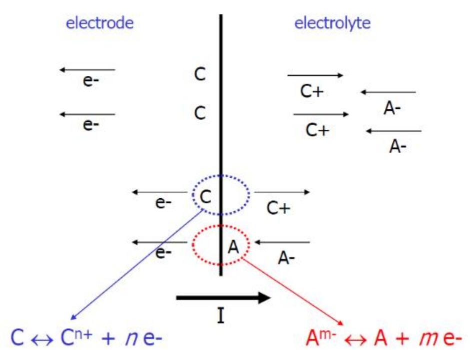
This measurement is difficult since the heart is embedded in the body, while we take measurements on the surface. Of course, electrical activity must be transmitted between the location of the heart and the surface, so the measurement is affected by the internal resistance of the body, that is the resistance of tissues, muscles, bones that are placed in between the heart and the point where I do the measurement.
Simplified electrical scheme of the measurement:



We have a current generator that changes intensity because of time (i change depending on the orientation of cardiac vector), then we have the resistances between heart and point A, point B and we measure the voltage difference between A and B and record it.
What makes the measurement even difficult is the surface resistance between A and B, and it is the skin resistance between A and B. It creates a partition together with Rt1 and Rt2. To have Rab small A and B must be very close.

ELECTRODES

How to readout biosignals?
Electrodes are transducers that allow to record the electrical activity occurring into the body and transfer this activity to the amplifier.
They are important because the electrical activity in tissues is transmitted through ions, flow of them, while in electronic components through electrons → we need an interface between ions in tissues and electrons in amplifiers → electrodes (they are transducers).



When we measure a biosignal we usually use metal → interface between electrode (metal) and electrolyte (either gel or tissue). So metal is in contact with the electrolyte.
If we have such an interface, how are signals transmitted?
The electrode is defined by the metal element C (C are the metal atoms). When we put the metal in contact with electrolyte, the metal experiences a redox. If the metal looses electrons we have oxidation, otherwise reduction. When oxidation takes place, the cations C+ may spread into the electrolyte. At the same time, in the electrolyte we may have anions, that oxidate or reduce.

The interface is complicate. We have neutral atoms in the metal, cations already in the electrolyte and for charge balance also negative counterparts are needed in the electrolyte.
We cannot have electrolytes with just positive charges.

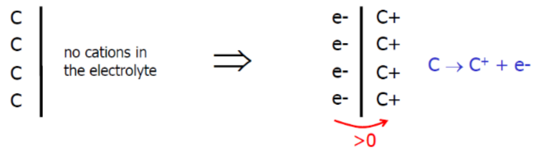
How may a current flow from electrode to electrolyte? It is justified by electrons created by oxidation moving on the left and anions going to left and oxidizing at the interface. So current can flow if C atoms are oxidizing and electrons moving to left. Anions are also oxidizing and moving to the left the electrons.

How may a current run in an opposite direction? If electrons move to the right and produce a reduction, also A atoms get electrons and producing anions that are moving to the right. So current flows between electrode and tissues depending if we have oxidation or reduction.

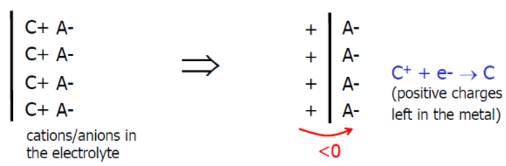
What if we are at the beginning, when we put electrode in contact with electrolyte? Depending on the initial condition, we can have the start of oxidation and reduction in one way or the other. Extreme cases:

- Case 1: meta in contact with the electrolyte, and electrolyte free of preexisting cations. If no cation exist in the electrolyte, oxidation is the most spontaneous reaction → metal oxidates and as a result cations are spread in the electrolyte and electrons are going into the metal. But as soon as this reaction takes place, we start to have the formation of a double layer of charges.

A layer of negative charges on the metal and one of positive charges on the electrolyte. This double layer creates a voltage difference, called semielement potential (or half cell potential). We have a voltage just as a consequence of the initial oxidation. This voltage difference will grow until the oxidation process stops, when it is not convenient that new cations will spread into the electrolyte (positive charges don't move where we have a positive potential).



- Case 2: now in the starting condition we have created in advance cations and anions in the electrolyte. Due to the large abundance of C+ in the electrolyte, the metal doesn't oxidate. Now it is more likely that cations are going to be reduced (collecting electrons from the metal). Again, we have a double layer of charges but now is formed by a layer of anions non more counterbalanced by cations and a layer of positive charges that are atoms of metal that have electrons for the reduction. We have again a semielement potential, but with opposite polarity. So when a metal is in contact with an electrolyte, something is going to happen, and it is the creation of a static semielement potential, but we cannot predict its orientation and intensity, unless we know the preexisting conditions → semielement potential depends on the starting concentrations of ions in the electrolyte.



The formula that quantifies the semielement potential is again the Nerst equation.

$C \leftrightarrow C^{n+} + n\,e^-$ C in the metal, C^{n+} in the electrolyte close to the metal

$$\boxed{E = E_0 + RT/nF \ln (a^{C^{n+}}/a^{C}) = E_0 + 0{,}027/n \ln (a^{C^{n+}})}$$

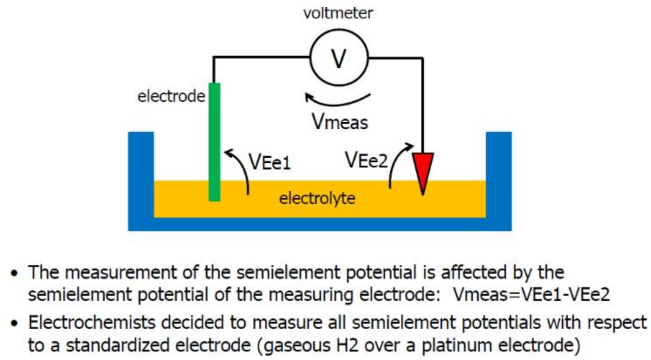
Nernst equation

E: semi-element potential
a: activity (≃ concentration, in slighly diluted solutions)
$a^C = 1$ in the metal
when $a^{C^{n+}}=1$, $E=E_0$ (standard semi-element potential)

E0 = E when aCn+ = 1: close to the interface we have created a standard concentration of 100% existing cations.

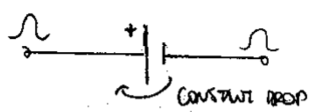
Leaving the chemistry, we need an electrical model. The components are a capacitor (since we have a double layer of charges), a resistor, since the motion of cations experiences resistances and also if the electrolyte is extended to the body, a Rely (that represents the difficulty of charges to move where there is an electric field).

The measurement of E0 is not as easy as we think, there is indeed a problem. We have the electrode, e.g. made of silver, and we want to measure VEe1.

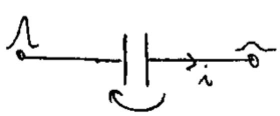


The problem is that Vmeas on the voltmeter is not the absolute voltage, because it is affected by the E of the red electrode used for the measurement, and so it doesn't represent the potential difference between electrode and electrolyte VEe1.
So we have Vmeas = VEe1 – VEe2. It is not a crucial problem, it is sufficient to define a standard for the red electrode used for the measurement.
All the E0 will be measured with the hydrogen electrode as a reference. So E0,H = 0 (but in reality, physically, it is not 0).

Another aspect of the electrode is the polarization: capability of the electrode to keep fixed its voltage while a current tis flowing during the measurement.
It could be that during the measurement done by electrodes there is a current and if it flows in the measurement, it is desirable that the current doesn't change the voltage.
The desirable electrode is the electrode perfectly non polarizable (EPNP): it maintains the semielement potential.



In principle, the EPNP is the voltage generator. If we have a spike before, we record it also later. This happens for example in ECG with silver electrodes. The signal component on the other side is the same, besides the constant drop.



The electrode perfectly polarizable has instead a completely capacitor behaviour. The current in a capacitor changes the voltage drop across the capacitor, indeed. When current flows (i) in the electrode, it changes the potential across the electrode.

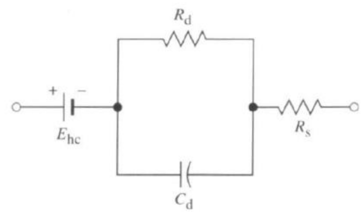
Here the useful component of the signal is lost at the interface (before not).

Equivalent circuit

Electrical model for electrode-electrolyte interface.
Ehc: DC component that represents the Nerst generator, that provides the DC voltage drop across the electrodes (semielement potential). In small signal analysis this generator is put to zero.

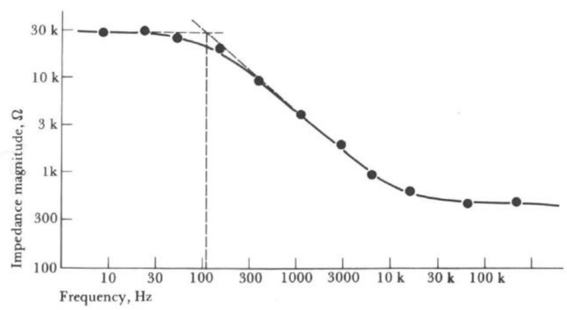
If we have electrode on right hand and left hand, indeed, we expect to have the same semielement potential drop (DC values), but it can be that they are not equal on the right and left end → DC drop.

So for the t.f. of the signal the Ehc can be discarded, for the DC analysis it cannot → these differences can saturate the amplifier.



E_{hc}: semi-element potential
Cd: capacity due to the charge distribution at the interface
Rd: represents the presence of a leakage current
Rs: series resistance in the electrolyte

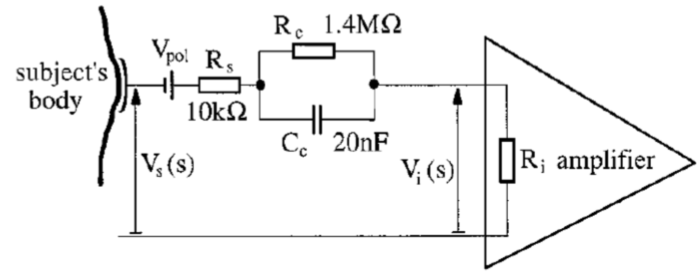
So if we calculate the t.f. of the circuit (small signal) we have the plot below.



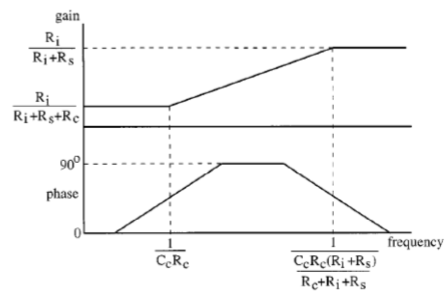
What is the impact of this pole-zero pair in the readout of the electrode signal and how to get rid of it?

We take the upper circuit, we switch off the Nerst generator (DC component) and we compute the t.f., that has a pole and a zero. at LF we have the series of Rs and Rd, at HF only Rs (Rd shorted).
Our transductor has not a flat constant t.f., but it depends on the frequency → **electrodes doesn't have constant impedance**. Order of magnitude? KOhms.
This is important in the pacemaker, because the impedance of the electrodes creates a drop that dissipates the voltage we are applying to tissues.

EXAMPLE OF TISSUE-ELECTRODE-AMPLIFIER COUPLING



$$\frac{V_i(s)}{V_s(s)} = \frac{R_i}{R_i + R_s} \times \frac{\left(s + \frac{1}{C_c R_c}\right)}{\left(s + \frac{1}{C_c[R_c//(R_i+R_s)]}\right)}$$



with $R_i > 60R_c \approx 80\ \mathrm{M\Omega}$

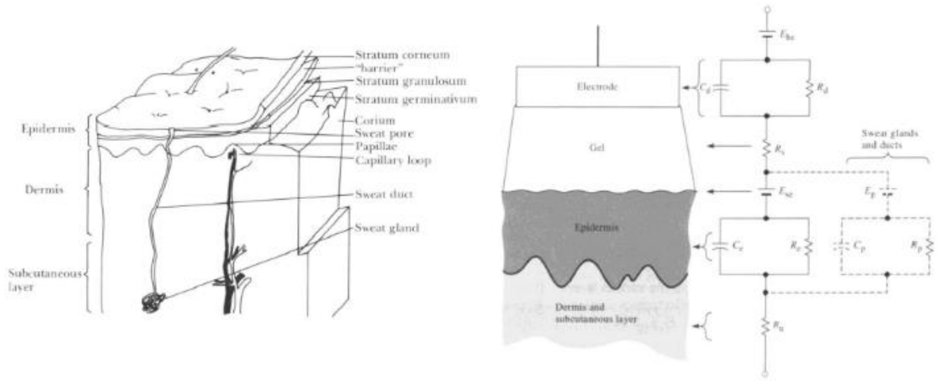
$\Rightarrow$ Gain $\cong$ 1, phase $\cong$ 1°

So we have the electrode on the body, we have the model in which the DC component is in this case to the right, but it doesn't matter since in our signal analysis we neglect how it is oriented. So this is the model with typical value of the components.
The voltage is measured with an amplifier with input impedance Ri. Here it is shown as a unipolar measurement for simplicity, but in reality in ECG we are doing a differential measurement → we should duplicate our model.
Now, if we have a signal Vs, the impedances and the impedance Ri, the voltage at the input of the amplifier is given by the t.f.. We can see the gain is variable due to the impedance Ri.
Moreover, in the phase plot, we can see the capacitor introduces a phase delay in the measurement.

The quicker technique to get rid of the variable gain is to use a high input impedance of the amplifier. How much high? If it is 60 times the highest impedance we have in the model (80 MOhm), the poles and the zeros in the diagram squeeze and they compensate. So the two gains get close to each other and the phase instead of being 90° is about 1°. So this is the best way to be insensitive to the tricky modelling of the electrodes.
So a high impedance amplifier is the best way to be insensitive to the tricky modelling of the electrode. We can also use the poles-zero compensation.

Interface electrode-tissue

On the left we have a detailed representation of the skin. We put an electrode not directly in contact with skin, but we use a gel → model between electrode and gel and then between gel and epidermis.

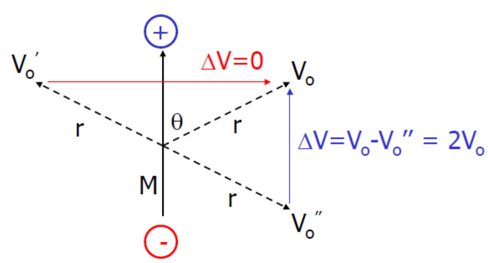


The coupling between electrode and skin is made by means of an electrolyte (transparent gel) containing Cl-

ELECTROCARDIOGRAPHY

The heart in a really simplified way can be considered as a big cardiac vector, dipole vector, because during each operation of the heart we have a sudden rush of Ca ions away from a region and some uncovered negative charges are left behind. M = q x d.

In the electrocardiography we are measuring the components of this vector by means of a series of voltage measurement on the surface of the body. We need more measurements because the voltage difference we measure is a scalar quantity, while we need to sample different components of the vector.



$$V_o = \frac{M \bullet r}{4\pi\varepsilon_0 r^3} = \frac{M \cancel{r} \cos\theta}{4\pi\varepsilon_0 r^{\cancel{3}}} \quad \text{(r large)}$$

Components of the cardial vector can be determined by measuring voltage differences between different reference points.

If I have a cardiac vector M and I consider the origin in between the vector and I measure the voltage where I have V0, we have the formula as in the image.

In ECG we don't measure one unipolar voltage vs ground, but we measure a set of couples of voltage differences; in the drawing, we see that depending on the orientation of the point we use for the voltage difference, we sample different components of the cardiac vector. For instance, if we take the voltage difference between V0 and the symmetric point on the bottom V0'', this voltage difference is V0-V0'', where V0'' that is equal to V0 in modulus but with negative value, since they are symmetric, but one close to the positive charge, the other to the negative.

The result of this measurement is twice V0. So I can say that I have measured in this way the component of the vector that connects + and -. If on the contrary I take V0 – V0’, by symmetry, V0 = V0’, because the two points are symmetric with respect to the cardiac vector. So this tells me that the direction connecting the two voltages must be perpendicular to the cardiac vector. So measuring 0 is equal to say being perpendicular to the cardiac vector.

So when I measure in the body different couples of voltage differences, I measure different components of the cardiac vector. They are ways to sample the various orientation of the cardiac vector. Measuring different component by taking different orientations of the voltage differences.

There are more than 3 measurements that we can use (because of the 3 components of the vector), but there is a redundancy in the measurements → 12 voltage differences.

SET OF VOLTAGE MEASUREMENTS

We must distinguish the **frontal plane** (the plane where the patient lays on the bed) and a **transversal plane**. We will have 6 measurements per plane. Among the 6 of the frontal plane, 3 of them belongs to the Eindhoven triangle; it is composed by direction:

- **I**: given by the voltage difference between left and right arm (voltage measurement = scalar, sampling of the vector component in a given direction).
- **II**: left leg – right arm → forming an angle of 60° with respect to the previous one.
- **III**: right leg – left arm → again an angle on 60° with the previous ones.

Having done these set of measurements it is like sampling the cardiac vector in these three directions. I should expect on III zero in the trace if the cardiac vector is oriented as in the pic, because the component is perpendicular.

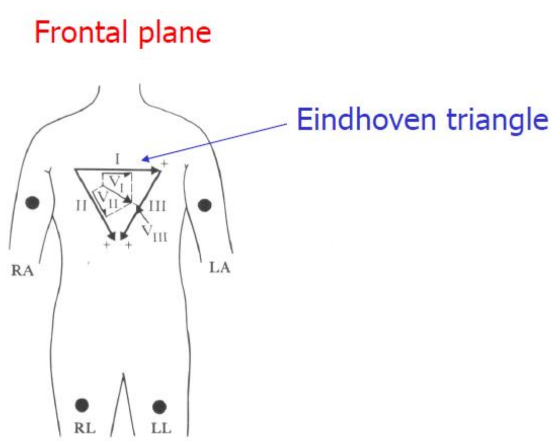
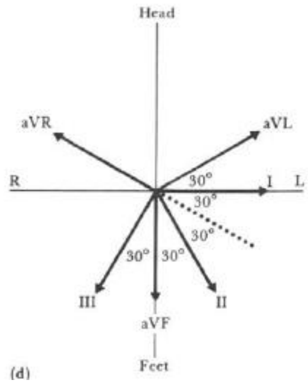


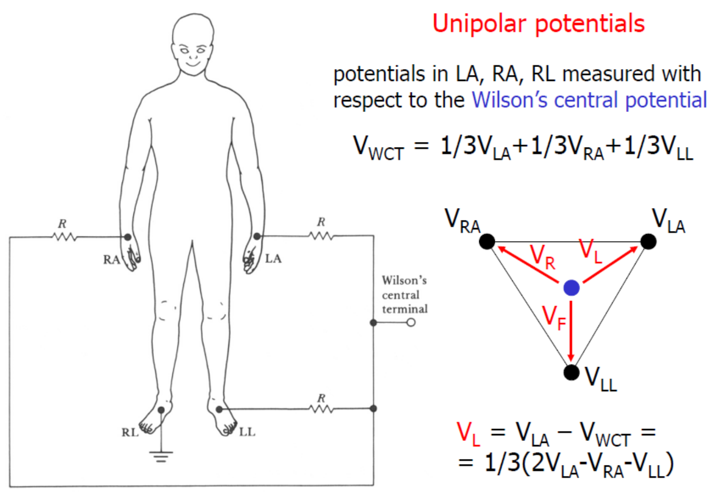
Figure 6.3 Cardiologists use a standard notation such that the direction of the lead vector for lead I is 0°, that of lead II is 60°, and that of lead III is 120°. An example of a cardiac vector at 30° with its scalar components seen for each lead is shown.

The followings are the recordings when the heart is contracting in different tissues. Then we have the depolarization of atria, the cardiac vector is directed as in the image. When we have the relaxation of the atria, it is in the opposite direction, so the plot goes back to zero. The plot has then a steep rise when the ventricles contract, since they compose the great amount of heart tissues.

The other components are 3 sampling now with respect to an orientation shifted by 30° with respect to the previous ones (red arrows are the new sampling).
The end of the story is the plot:



We see the previous measurements I, II, III and the new set of measurements (that is flipped, the polarity is inverted). Overall, in the front plane, we have a sampling every 30°. So we have made a quite complete oversampling.



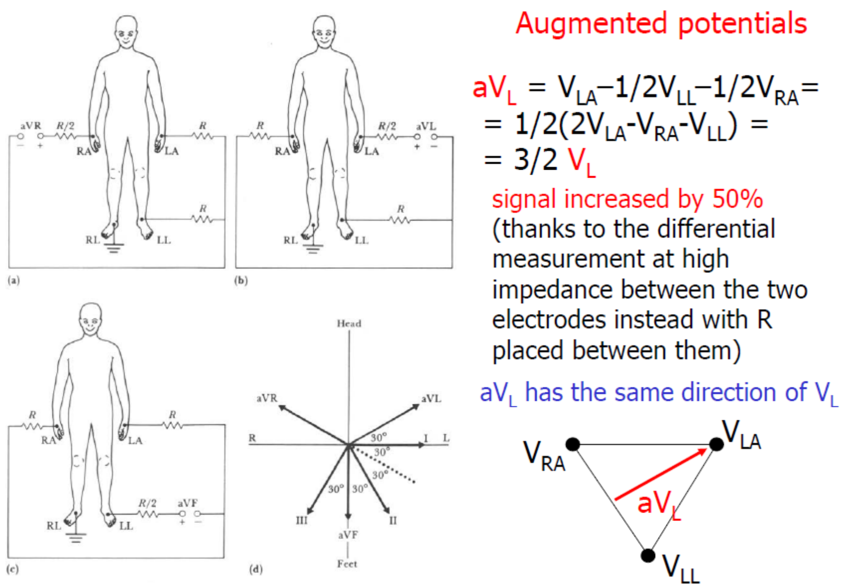
Coming back to the details, I need to measure the other 3 components. So I elect a reference point, the Wilson's central potential, that is in the center of the body and it is given by the average sum of the three potential; it is like a central ground. In this measurement the right leg is put to ground, and the Wilson point I assume is a terminal reference. Then the unipolar potentials are simply the voltage difference between each of the electrodes and the Wilson central potential. So V_L is the voltage difference between V_{LA} and V_{WCT} and so on.

There is an improvement, a smarter way to carry on the measurements that starts from the observation that the measurement of the voltage difference between two terminals connected by a resistor is not so smart. To measure the ddp between two point, a resistor in the middle is a penalizing element (in extreme case if R=0, V=0) → augmented potential.

Augmented potential

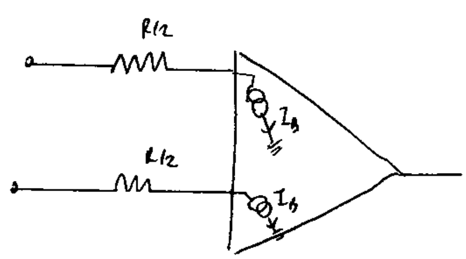
We cut the connection between the point of interest and WTC.
Each measurement is not measured between the point of interest and WCT, but between the point of interest and the average of the other two electrodes.
So we measure aV_L minus the average of the other two voltages. In terms of vector, I'm not measuring V_L, but aV_L, which has however the same orientation. In terms of sampling I'm doing the same thing, but better. Indeed, this aV_L is 50% larger than V_L, measured with respect to the original WTC, because I have removed the resistor in between the two measurement points (I eliminate a voltage partition).

However, in the measurement I have a resistor R/2 on the point of interest line. This is due to the compensation of the bias current of the amplifier.



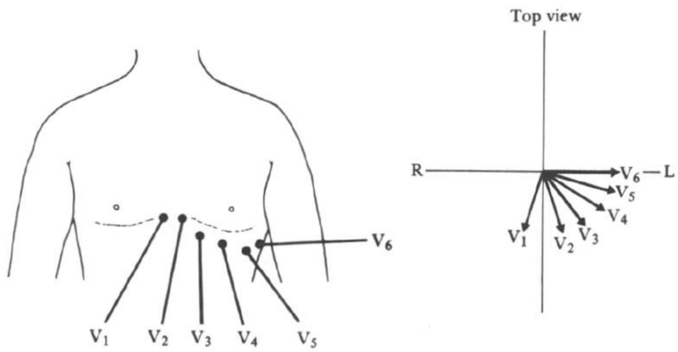
We have the differential amplifier, the electrode and one resistance that is the parallel of the other two resistances on the path connecting the other branch, so I had artificially R/2 also on the other electrode. This because OpAmp may have input bias current. Supposing that input bias currents have the same amplitude, if we let one bias current to go through R/2, it creates a drop, and if we do nothing, we have a drop on one input and no drop on the other. By adding artificially the resistor R/2 we create artificially a drop and so the DC drop at the input of the amplifier is zero, since we have compensated the drop created by a bias current on one node by a corresponding drop to the other node.

1.



TRANSVERSAL PLANE

I need to sample the cardiac vector while it has components on the transversal plane. Electrodes are placed around our chest to sample voltage up to 6 with respect to WCT. So all are potential differences between a point and WTC. The plane is like if we are looking the patient from the head.
On the other image we have all the connectivity. We see that when we have the spike due to ventricular contraction, it is visible in several measurements but not in all, since in that sampling it corresponds to the perpendicular sampling with respect to the cardiac vector.



potentials measured with respect to the Wilson's central terminal

The following is the block diagram of the electrocardiograph. There are some block for amplifier protection circuit; it is either protection of the patient or protection of the instrument from the patient. For instance, when the patient experience defibrillation, this is a huge voltage that may destroy the amplifier. Then we have lead selector and the preamplifier, that is the more sensitive stage of the ECG since it has to detect the signal against the noise.

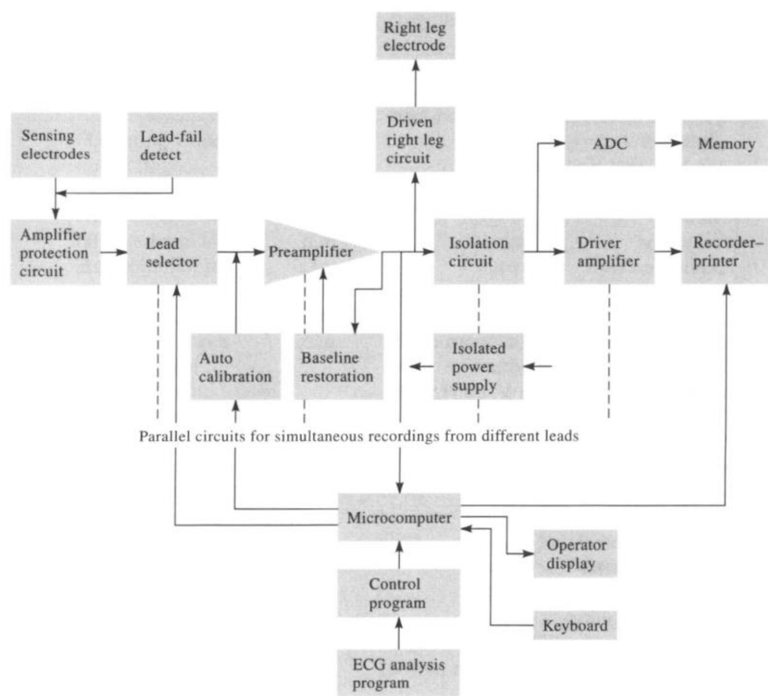


Figure 6.7 Block diagram of an electrocardiograph

Then we have the isolation circuit: it could be that is convenient to refer the ground of the system not to the ground of the building, because if the patient is floating with a high common mode, it is better not to connect the patient with respect to the ground. So very often the instruments are floating, they have a local ground. It is like in the satellite, the amplifiers are not referred to ground, but to a common electrode of the satellite.

Parameters of interest

- **Input dynamic range**: +- 5 mV. The amplifier dynamic range +- 5mV
- **Accuracy**: +- 5% of the range or +- 40uV. The order of magnitudes so are tens of uV
- **Frequency**: 150 Hz. Very small bandwidth, the ECG doesn't exceed hundreds of frequency. However in between this bandwidth we have the 50Hz of the power line, and it is in the middle of the frequency we are interested into.
- **Common mode rejection**: we can have +- 300mV voltage difference on the electrode. The semielement potential, due to the inhomogeneity in placing the electrodes, may have an imbalance of 300 mV in DC, so if we do an AC coupling we cut out such difference. But from right hand to left hand we have a voltage difference of 300 mV. If we have an amplifier with a beautiful gain, because we have to amplify 1mV of signal, we saturate the amplifier, because 300mV of voltage difference applied in DC to the amplifier saturate it.
  The common mode voltage on the patient is not in DC, but at 50Hz; the common mode occurs on the patient since we are irradiated by lights and power line or by other instruments. So the 50Hz in common mode can move the voltage, but the ECG must be insensible to this, and measure voltage differences on the top of which there is a common mode (NB: in USA we have 60Hz).
  We have also 50 KOhm imbalance. This means that when we plug the electrodes into the body, we cannot bet on the semi-element potential value, but neither on the impedance of the electrode. So we have to design the input stage of the amplifier in a way that if an electrode has 50 KOhm difference in impedance with respect to the other, we have not bad surprises in the measurement.
- **System noise**: about 30uV.

ECG SIGNAL CHARACTERISTICS

The signal we have to amplify has the following properties. First of all we have a common mode almost uniform on the body, whose characteristic is to be few volts above zero and it is usually caused by the coupling of patient body to the power line, so it has a sinusoidal shape of 50 or 60 Hz and it is of several volts (common mode that sums up to the voltage difference). The second problem is that we may have a DC (continuous) voltage difference between the electrodes, in the order of +-300mV, due to the electrodes' half-cell potentials difference, since the physical connections of the electrodes to the body are not perfectly equal, so we will have a DC difference but also due to difference in the input impedance. In addition, if we have some input currents on the preamplifier, like the one caused by the OpAmp, this current goes through the impedance of the electrode and can create a voltage drop.

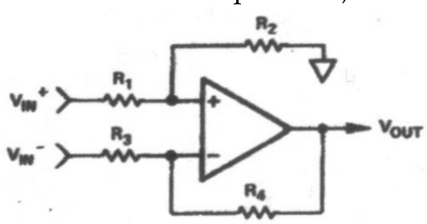
Then I have my ECG signal and the signal is separated to two DC levels corresponding to the offset of the electrode and the frequency range is from fraction of Hz to 150 Hz, with an amplitude of +-2.5mV.

To measure ECG signal, we usually use and instrumentation amplifier. It is a differential amplifier, so it measure the voltage difference between two points, it has a high input impedance (beneficial because the pole-zero couple of the electrode can be neglected. If we have a different electrodes impedance but this impedance is in series with the input impedance of the amplifier, the larger the input of the amplifier, the smaller the difference. In the limit case of having an infinite impedance in input, we have insensitivity to a mismatch impedance in differential measurement), and a high CMRR. In ECG measurement we have a large CM so it is important the amplifiers rejects as much as possible the CM.

Differential amplifier

The instrumentation amplifier has the t.f. that is computed with a superposition of effect, by switching one at a time the inputs. If the ratio R4/R3 is equal to R2/R1, there are some simplification. If the R are all the same, the output voltage is equal to the voltage difference in input.

The CMRR of this amplifier is mainly due to the mismatch of the resistors. So if the resistors are no exactly respecting that R4/R3 is equal to R2/R1, the overall formula is not true. This can be seen if the same voltage is applied to the pins, same common mode voltage. So if in the overall formula all the resistors would be equal to R, the term inside the brachets [ ] is 0 → $V_{CM} = 0$.



$$V_{OUT} = V_{IN}^{+}\left(\frac{R_2}{R_1+R_2}\right)\left(\frac{R_3+R_4}{R_3}\right) - V_{IN}^{-}\left(\frac{R_4}{R_3}\right)$$

if $R_4/R_3 = R_2/R_1$ $\boxed{V_O = \left(V_{IN}^{+} - V_{IN}^{-}\right)\frac{R_4}{R_3}}$

$V_{OUT\,CM} = V_{OUT}$ for $V_{IN}^{+} = V_{IN}^{-}$

$$= V_{IN}\left[\left(\frac{R_2}{R_1+R_2}\right)\left(\frac{R_3+R_4}{R_3}\right) - \left(\frac{R_4}{R_3}\right)\right]$$

$R_1 = R_3 = R_4 = R$

$R_2 = 0.999R$

$$V_{O\,CM} = V_{IN}\left[\left(\frac{0.999R}{1.999R}\right)\left(\frac{2R}{R}\right) - \left(\frac{R}{R}\right)\right]$$

$$= 0.0005V_{IN}$$

CMRR=1/0.0005 → 66dB

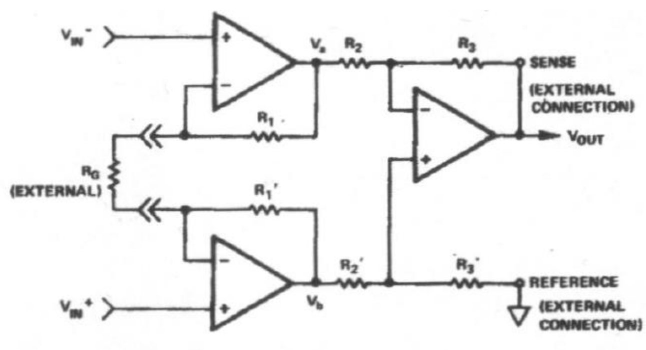
if source impedances are low and/or unbalance, CMRR worsens further

On the contrary, if the resistor are not the same, or three resistors are the same and only one has a mismatch, of a 1 part per 1000 (0.999R, which is very small), we have now that the factor doesn't cancel out → the output voltage is $0.0005V_{IN}$.

By definition, CMRR is V_{diff}/V_{CM}. When all the resistance are equal, the Vdiff is 0, while the V_{CM} is $0.0005V_{IN}$. In the end we have 66dB, which is not good at all.
NB: the CMRR is given by the imbalance of the resistors inside the network. Then if we have imbalance of the sources, considering that the input impedance is not infinite, we are worsening the situation even more.

However, the solution to this is the INA (instrumentation amplifier), made out 3 OpAmp.

INA
A first stage with 2 OpAmp with a common resistor Rg and a second stage with the classical differential amplifier and a first stage with a differential-to-differential amplifier (we sill have a difference at the exit).



$$\boxed{V_{OUT} = (V_{IN}^+ - V_{IN}^-)\left(\frac{2R_1}{R_G}+1\right)\left(\frac{R_3}{R_2}\right)}$$

The input impedance is the one of an operational amplifier, that if it is done with MOSFET or JFET, is a very high input impedance. If we have a BJT it is not so large.

CMRR of INA

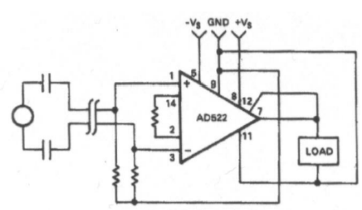
$V_a = V_b = V_{CM} \rightarrow G_{CM1} = 1 \rightarrow G_{CMtot} = G_{CM1} \cdot G_{CM2} = 1 \cdot G_{CM2}$

$G_D = G_{D1} \cdot G_{D2}$

$CMRR_{tot} = (G_{D1} \cdot G_{D2})/1 \cdot G_{CM2} = G_{D1} \cdot CMRR_2$

⇒ CMRRtot increases with G_{D1}
(and does not worsen if R1 e R1' are different)

notes:
- IA with FET inputs (vs. BJT) have larger input impedances and very low input bias currents
- remember to provide a DC path to discharge input IBias, in particular in case with sources AC coupled

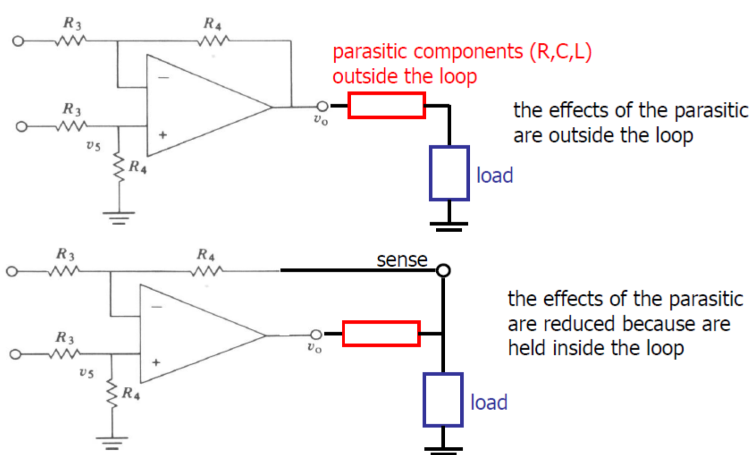


c). AC Coupled

Is given by CMRR of the second amplifier multiplied by the gain stage of the first stage (2R1/Rg + 1). It means that if we consider to use just the differential stage with 66 dB of CMRR (only the second stage), we multiply the 66dB by the gain of the first stage. So we improve the CMRR of the second stage by the gain of the first stage.
As for the demonstration, CMRR is the ratio between differential and CM gain. We have two stages, so the formula has at the numerator the product of the differential gain of the two stages, divided by the CM gain of the two stages.
But the CM gain of the first stage is 1, because if we apply the same CM voltage at the two inputs, it is applied the same voltage across Rg, so it will be Vrg=0 → no current, so output = input, and CM is transferred unchanged to the next stage.

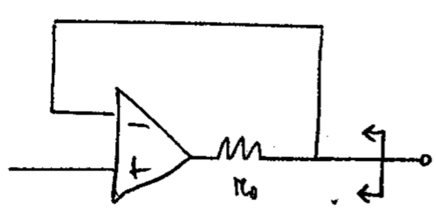
Now, if I take CMRR formula, we have that $CMRR_{tot} = G_1 * CMRR_2$. So I have all the reason to make the first stage with a good amplification, since the more I amplify, the more I increase the CMRR. Once I reached the physical limitation of the component in fact, the CMRR cannot be improved with only a differential one → I use an INA in order to increase it with a gain.

PRACTICAL DETAILS



In a INA, the manufacturer provides an independent connection to the *sense pin*. So the connection is no wired internally, but we can close it externally.
If we have a very long cable connection between the amplifier and the next connecting stage, the cabling determines a lot of parasitic component (both R,L,C, summarized in the red box). These component, together with the load, generates problems. However, if the INA is closed at the beginning, the effect of the parasitic component is outside the close loop. But if we have access to the sense, we can close the loop directly on the load, so to reduce the effect of parasite components, because the loop gain reduces the impedance of an output node of an amplifier.

2.



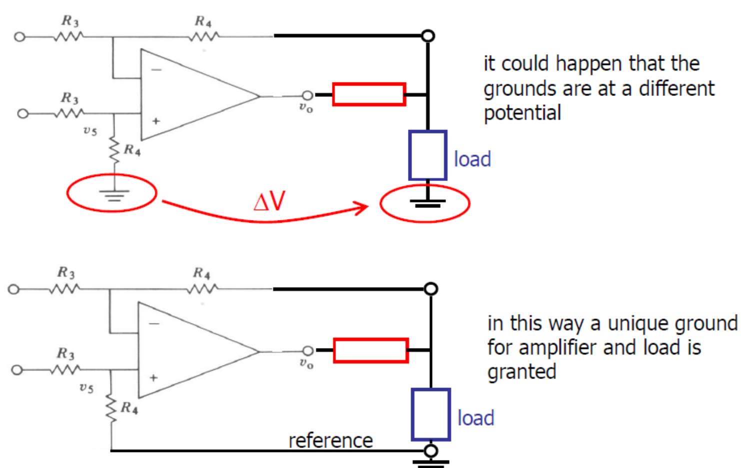
$$\frac{R_o}{1-G_{loop}}$$

In any type of amplifier, the output impedance, thanks to the loop, is equal to the physical impedance of the output divided by the loop gain. The output node is kept at low impedance because the physical impedance is reduced by the loop gain. So if r0 is composed by dirty components, they are nasty, but their effect is reduced by the loop gain → this is the reason why is smarter to close the loop after and not before the dirty components.
The cons are that we are duplicating the cables.

Another smart use of the sense pin is when we need to drive current-hungry loads. When the load has a low impedance, for a given voltage, the amplifier has to provide a large current. If Vout = 1V and the load is 100Ohm, the amplifier has to supply 10mA. INA may be not able to provide large currents, they may have a limited maximal current.

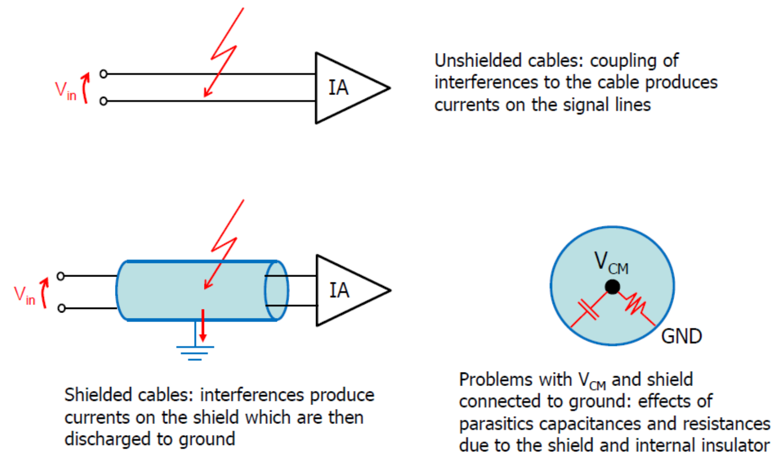
So to fix this, we introduce an additional stage, a buffer, but this time we choose freely this amplifier with high-driving capability (able to provide high currents). But if I place the buffer outside the loop, I would have the high-drive capability but not the loop effect → if I place it in the loop I have both the property, the feedback is closed to the load. No problem to supply current to the load and we keep the precision of the loop since the sense is closed directly to the load.

Moreover, INA can also have the possibility to supply an external ground to the amplifier. If the INA is too far from the load, since the ground are not equal in the same room, if we put INA and load to their own ground, we have a voltage difference, but if we use the pin of the INA and connect it to the ground of the load, we have a common ground → good practice in instrumentation design.



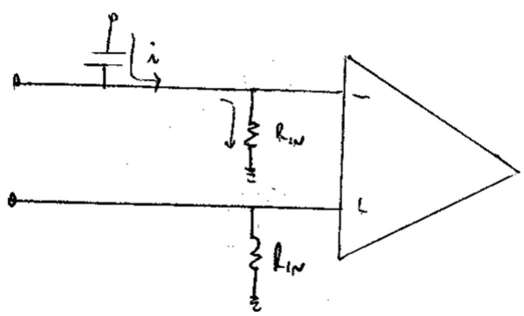
Then there is Vref that can be used to supply a different voltage, so that we have a DC shift on the output.

SHIELDING INPUT AGAINST EXTERNAL INTERFERENCE



It is usually a good practice to shield the cables. Let's imagine to have the patient on the bed, the INA of the ECG 2 meters away. If we have any interference in the room, unshielded cables works as antennas, so we have noise on the line. An antenna is a floating resistor. So we have our cable, the INA with its own impedance Rin. If we have current flowing into the cables (current coupled capacitively by whatever sources of disturbances), this current flows into the high impedance of the cables and we have an antenna effect, noise on the line. So the cables of the ECG are parasitic antennas.

3.

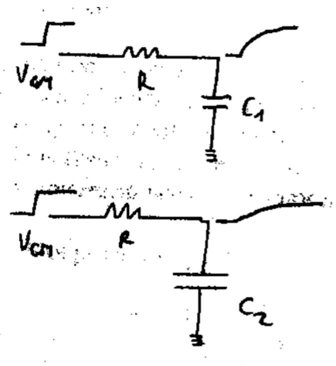


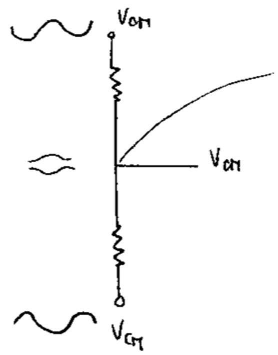
So the cables unshielded collect radiofrequency waves (from instruments already present in the ambient), act like antennas.
Hence we had shielding, which is a Faraday cage across the shield. We embed the cables into a conductor, we ground the conductor, the electromagnetic waves interact with the conductor, they create a current in it, but the current is discharged to ground instead of going through the cables.
Often we shield the cable to ground, but this may be not a good solution for ECG, because in the cables we have internally, if we make a cross-section, the common mode, which is a sinusoidal wave at 50Hz.
So there are two problems if we shield the cables to ground:

- The cables have an insulator between the external conductor and internal wire, but the insulator is not perfect, it has an internal parasitic resistance. So if we have the external shielding at ground and the internal cable at V_{CM}, due to the internal resistance we have some parasitic current, not good at the input at the amplifier.

- With an external metal and an internal wire we have a cylindrical capacitance → parasitic capacitance for the common mode.

4.





superposition of effects here results in a zero pulse → I pick up only Vcm without any effect of Vdiff.

If the capacitance of the cables is not equal, the transfer function of V_{CM} is different. Let's suppose the CM is the same; if the time constants R*C are different, the signal which is supposed to be cancelled it is not cancelled at all, because the same signal is modified on the two branches. So event a mismatch in the shielding of the two cables may result in a catastrophic effect because on common mode we provide two different signal paths.

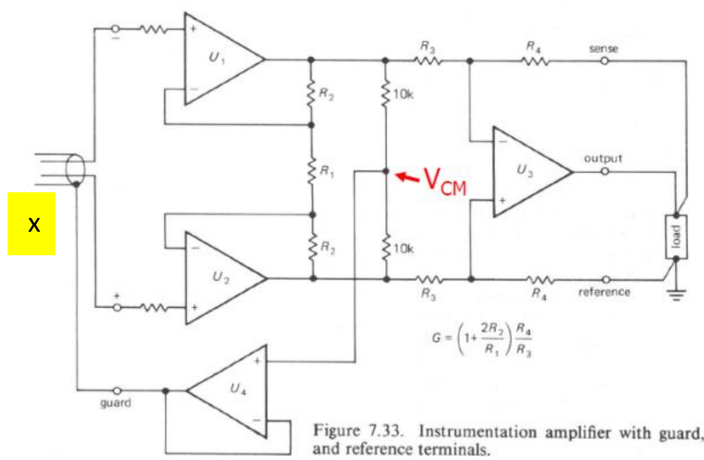


Figure 7.33. Instrumentation amplifier with guard, sense, and reference terminals.

The INA provides a smart solution for cable shielding. In between the two stages, we extract the common mode thanks to 2 resistors. We extract it like this, because, superimposed to CM, we have Vdiff. So thus we pick up the Vcm without the Vdiff. Once we pick it up, we buffer it and we use the CM to drive the shielding. So the shielding is no more driven put to ground but by the common mode. This trick solves the previous problem because now the resistor of the insulator has CM inside and also outside, an also the capacitor. Hence no current flow, the resistor and the capacitor don't exist.
So even if we have to different capacitors, if they are moving with the same CM, they don't play any role.

We must anyway remember that the role of the shield is to collect the current at low impedance. Is the common mode still providing a low impedance pathway for the current (since we have no more the ground)? Yes, because if I calculate the impedance in x, it is the output impedance of the buffer.

So it is not a physical ground, but it is a low impedance → any shaking of the external shield due to the interference will be discharged to the output impedance of the buffer (I have to guarantee the low impedance anyway).

NOTE: in the cables we have V_{diff}, but the capacitors and the resistors aren't affected by the previous considerations due to V_{diff} since it is very small, while the CM is very high.

The common mode is transmitted with gain 1 in the two point at the top and bottom of the two R2, but it is sensed where indicated since there the overall effect of the differential voltage is zero (point of symmetry). We spent two resistors more but thus we extract a clean measurement of the CM mode only. This common mode can be used to drive the guard for shielding the cables, so to bypass the effect of any resistance or capacitor in between the internal wire and the outside shield.

Bootstrap of power supplies

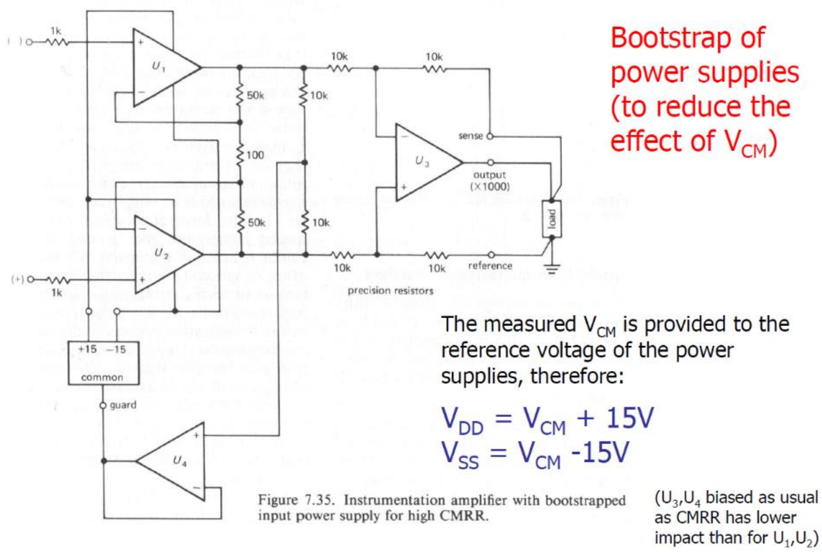
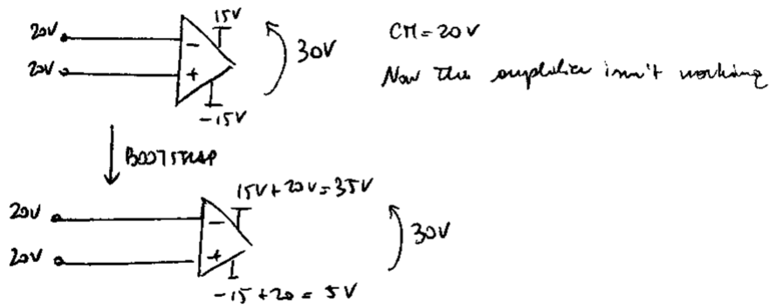


Figure 7.35. Instrumentation amplifier with bootstrapped input power supply for high CMRR.

Moreover, the CM can be used for boostrap of the power supply. We bias the power supply of the OpAmp through the common mode. It is like a rigid translation of the voltage reference in a way that the amplifier doesn't feel any more the CM.
We extract the CM and again we drive the reference electrode of the power supply (the reference electrode of the power supply). Whenever we need to bias a circuit, indeed, we use power supply, to provide like +- 15 voltage with respect to another electrode called common. Often the common is put to ground, so that +- 15V are referred to the ground of the building. But we don't have to use always the ground, we can apply to the common any voltage we like. If we apply the CM, we refer the +-15V to the common mode. This we translate actively the voltages we apply to the OpAmp in a way that they are following the CM, but the CM at the input is no more a surprise, since the voltage is flowing with the common mode.

5.



The amplifier is saturated because it is greater than the power supply. But thank to the boostrap, we refer our power supply to 20V reference → no saturation. The voltage difference is still kept, but the 20V at the input are no more critical, since they are inside the range of the power supply.

Boostrapping is used to eliminate the problem of CM, to have less critical requirement for the CMRR of the first stage. The CMRR of the input amplifier is indeed the one dominating the overall INA; the one of the second stage is less critical. Bootstrapping the power supply of the two input is not needed for the second stage.

FREQUENT PROBLEMS IN ECG INSTRUMENTATION

This is the trace of an ECG. One problem is when the ECG is saturated, when we have strong defibrillation on the patient. The defibrillation is an enormous voltage at the input of the amplifier. We must understand hence how much time the amplifier requires to return to normal behavior.

A second problem are interferences, the ECG is embedded inside a room, so we have other instruments, power lines, so the traces may be affected by some coupling, typically from the power line (produced at 50 or 60Hz).

DRAWBACKS

While being inside a wired ambient, with a lot of power lines, the power lines capacitively couple to the body of the patient and also to the instruments.
We have a power line, that represent one plate of the capacitor, and our body or the instrument the other plate. So in the corresponding electrode we have an injection of current. We separate the instrument from the patient.
Usually, the instrument is protected by a shielding, the Faraday cage (it is embedded into a grounded external conductor, so that the current flows in the cage and not in the instrument). Then we have the wires; they are usually also shielded. If we don't shield them, like in the slide, we have a current Id1 coupled to one wire, Id2 to the other one. The current doesn't flow into the instrument, since it is a high input impedance instrumentation amplifier → not collecting current. So the path at low resistance is through the body of the patient, so it moves into the patient and it is drawn into the patient thanks to the electrodes and it is discharged through the nearest ground connection, that usually is the right leg of the patient. So the right leg is connected to ground through a resistance and the current flows. If the two impedance Z1 and Z2 of the electrodes would be equal, this would not be a problem. We will have the same currents in the wires and the same voltage drops → a common voltage on the patient but not a Vdiff. Unfortunately, Z1 not always is equal to Z2.

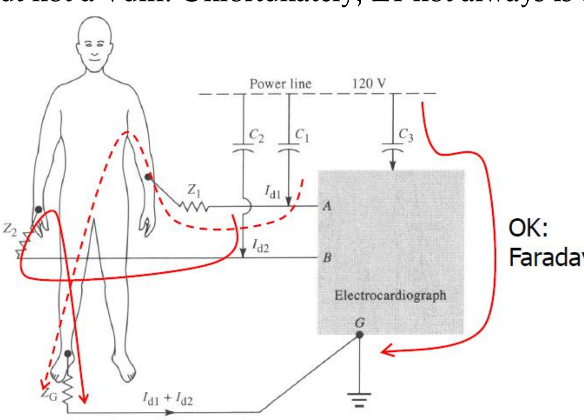
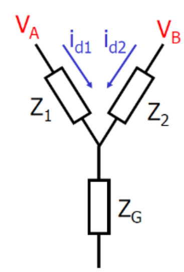


Figure 6.10 A mechanism of electric-field pickup of an electrocardiograph resulting from the power line. Coupling capacitance between the hot side of the power line and lead wires causes current to flow through skin–electrode impedances on its way to ground.



$V_A - V_B = i_{d1}Z_1 - i_{d2}Z_2$

if $i_{d1} \approx i_{d2}$

$V_A - V_B = i_{d1}(Z_1 - Z_2)$

a differential signal appears at the amplifier input

e.g. $i_{d1} \approx 6nA$

$Z_1 - Z_2 \approx 20\ k\Omega$

$\Rightarrow\ V_A - V_B = 120\mu V$

hints:
- shield the inputs
- reduce the electrodes impedance

Hence if we have two currents (Id1 and Id2) in the two electrodes' impedances, due to the Z1 != Z2, we measure a voltage difference. Even if we assume that the two currents are the same (perfect symmetry in the coupling, hence the wires are very close one to the other), we have a Vdiff that is collected by the differential amplifier → bad news.

The impedance difference is in the order of thousands of Ohm.
To prevent, we can do a shielding of the cable or we can reduce the electrodes impedance, so to reduce the relative differences.

Common mode induced by coupling of the power lines

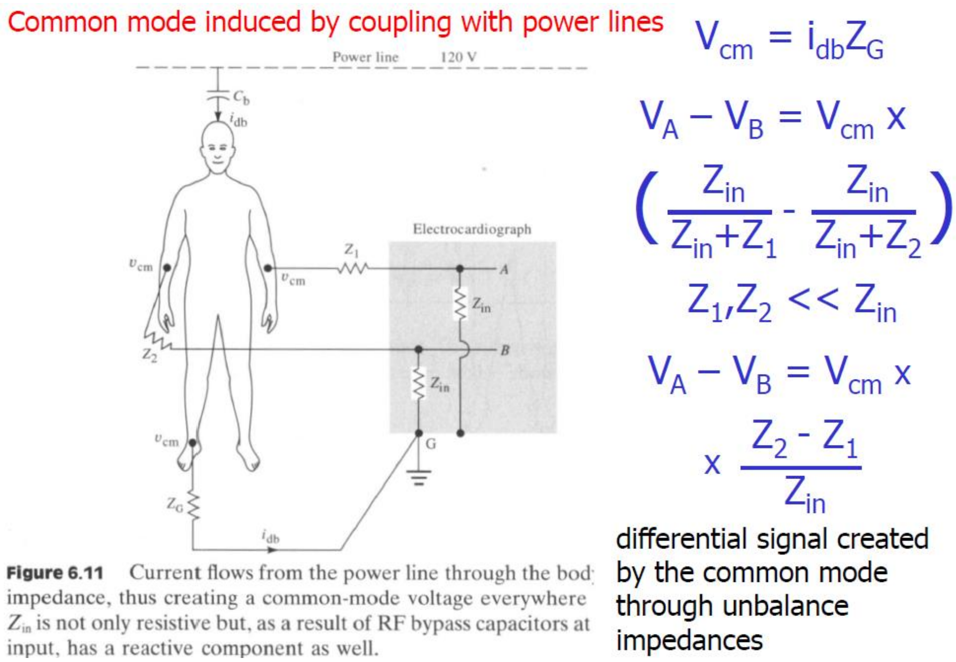
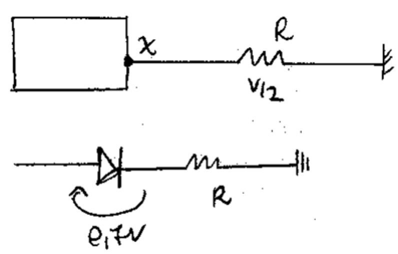


Figure 6.11 Current flows from the power line through the bod impedance, thus creating a common-mode voltage everywhere Z_{in} is not only resistive but, as a result of RF bypass capacitors at input, has a reactive component as well.

The current is injected by the upper capacitors, we assume it is given, not depending on Zg.
If we have a coupling of the PL in the body of the patient, we have the rise of a CM voltage. So the patient is coupled with the PL and we have a current Idb flowing into the body the patient. Again it is collected through the right leg of the patient and collected to ground through the impedance Zg. We don't ground directly the patient, since if we use Zg, Idb creates a CM: Vcm=Idb*Zg.
The direct ground is not done for safety reasons: if by chance the body of the patient touches another instrument or a power supply, if we have a temporary voltage on the body of the patient, this voltage will produce a current, flowing in the path. If we ground the patient the current is enormous (Amperes), so we place a safety resistor Zg because thanks to this impedance, if a current flow into the patient, it creates a voltage drop on the Zg and the voltage on the patient is separated from ground.

6.

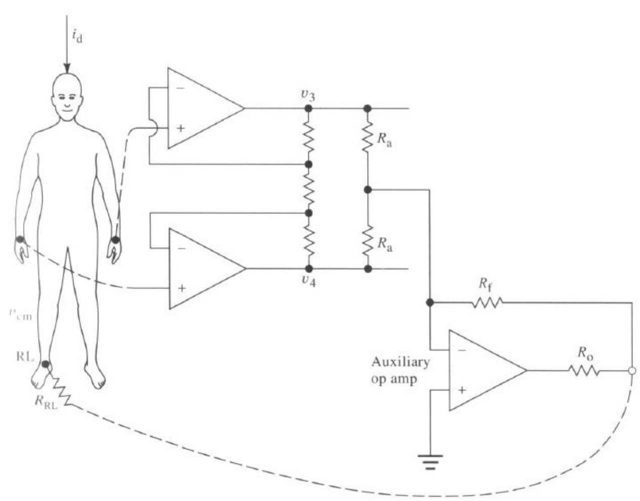


So if I have a very delicate point and a voltage can be applied to this point, to protect it we place a safety resistor. The current will be V/R, and it creates a drop so that the point x is able to float and it is not strongly connected to ground. The larger R, the larger the voltage separation between the ground and the point we want to protect. This is typically done with diodes. If we apply more than 0.7V across it by mistake, we destroy the diode. So usually to protect the diode we put a safety resistor in series. So if the diode starts to draw an extra-large current, it creates a drop on the resistor and overall we reduce the voltage across the diode. Hence **resistors in series are a very simple protection device**; if we have an accidentally large current flowing in the device, the resistor protect it because it creates a voltage drop. In our case the device is the patient, and we want to prevent that large current into the body of the patient occurs by dropping the voltage across the patient.

So the common mode in the patient is created by the current into the impedance Zg. If we have a common mode and a beautiful INA, we are fine. We have a perfect rejection of the CM. However, there is an additional problem. We must consider that the **CM is transmitted to the amplifier to through the electrodes' impedances**. If the two impedances are not equal, assuming the amplifier has an input impedance Zin, the CM is transformed into a differential voltage (Va-Vb) at the input of the amplifier. In conclusion, **the mismatch between the electrodes' impedance transforms the common mode into a differential signal at the input of the amplifier**. If they are the same there is no problem. What we can do is to select the amplifier with the highest input as possible (the largest Zin, the smallest the effect).

Another way to to deal with it is to reduce the reason of the problem, hence the CM. It is given by Idb*Zg. On Idb we cannot do anything, but we can work on the Zg.

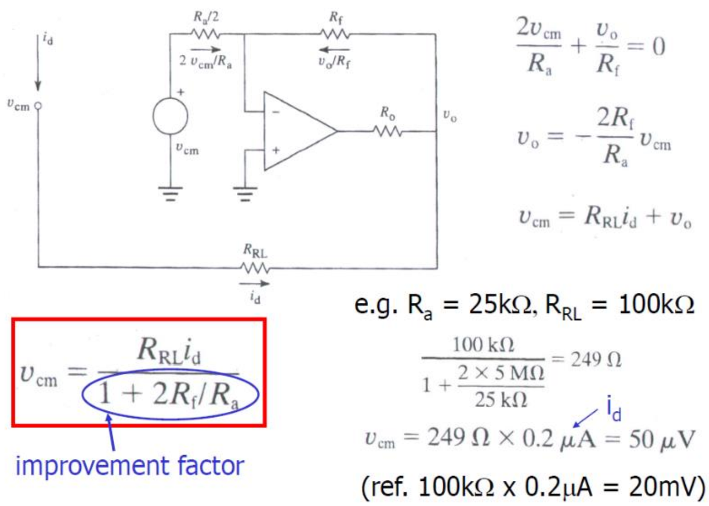
Right-leg driver circuit to reduce the common-mode voltage



We introduce the **right leg driver circuit**, where instead of connecting passively the RL (right leg) of the patient to ground, we connect it to the output of an amplifier so to create a negative loop formed by the patient, a first stage, another amplifier and then we close the loop; so thus we drive actively the RL (we don't put it passively to ground) in a way to reduce the common mode.
We do so: we take the CM, we measure it thanks to the resistance Ra; in reality we put that node at the virtual ground of the amplifier and the output is driven back to the patient. This creates a negative loop.

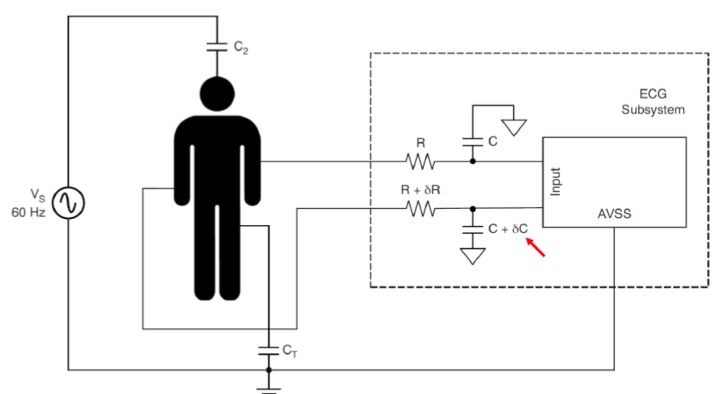
In the equivalent circuit, on the left we have the CM. We create a Thevenin equivalent circuit: the Thevenin generator is the voltage in between the two Ra resistances, so the CM, the Thevenin

resistance is the parallel between the two Ra (we are neglecting the differential signal). So we have an inverting amplifier with v_{cm} as input voltage. Then we have the Ohm law across the resistor R_{RL}.
In the end we have the final formula in which we see that v_{cm} is equal to the product Idb*Zg (R_{RL}*Id) which was the old CM, divided by a factor. So we are killing the CM not by killing the source of it (the current exists), but by killing the conversion factor between the current and voltage. The improvement factor is 1 minus the loop gain. R_{RL} is high for safety reasons, but they are reduced by the loop gain to a small value.
So the overall vcm in open loop is of 20mV, but if we use the loop is of 50uV (much more reduced). Thus, even if we have mismatch in the electrodes' impedance, we have an improvement in reducing the CM actively.



Moreover, the resistor R_0 exists for safety reasons. We can demonstrate that it doesn't play any role in what described so far, since v_0 is determined by the loop of the amplifier. It plays a role only in the cases where we have for example a defibrillation; in that case the loop is open (amplifiers saturated), so we would have the node v0 put to low impedance, so a relatively large current flowing in the path from the patient, R_L, output impedance of the amplifier. So if we want to reduce the voltage drop and the current we put a safety resistor R_0 so that the series in between R_L and R_0 reduces the voltage drop (only when the loop is open).

Beside it is shown why the CM may be transformed in a differential signal. It can be due to the mismatch int the impedance but also in the capacitors of the shielding. The shielding of the cables can be different, so we have a C in series with the resistor R. This capacitor creates an integrator and so we may have different frequency responses for the CM which translate in a different path, so into a differential signal. This was the reason why it was so smart to drive the shielding of the cables not with



Common-mode to differential mode conversion can be due also to mismatches of cable impedances, as different capacitances conductor-shield
(⇒ see positive effect of driving the shield with V_{CM})

ground but with a CM. So the capacitor doesn't exist, and it doesn't play any role.

In green we have what is good for us: the differential biosignal collected from the amplifier amplified by the differential gain. In red we have the effect we dislike. First, the CM amplified by the CM gain of the amplifier (CMRR = Gd/Gcm, we have to select an INA with the highest CMRR possible), then we still have the other factor that is the differential signal created by imbalanced impedances (cannot be reduced with CMRR high). So out amplifier is good in amplifying this difference as it was for the true signal.
Take home message: **be careful about CMRR and imbalanced impedances**.

$$V_{out} = \boxed{G_D V_{biol}} + \boxed{\frac{G_D V_c}{CMRR}} + \boxed{G_D V_c \left(1 - \frac{Z_{in}}{Z_{in} + Z_1 - Z_2}\right)}$$

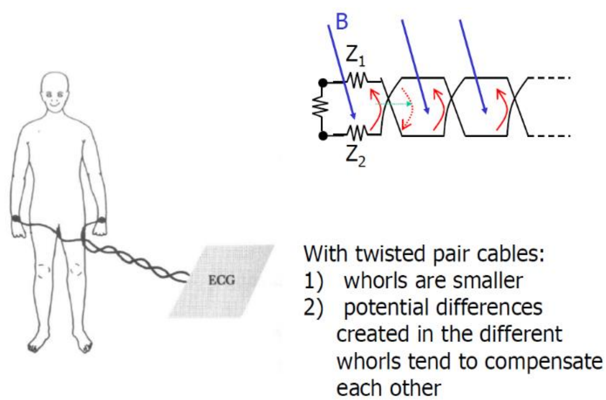
biosignal

limited CMRR of the amplifier

differential signal created by the common mode through unbalance impedances

MAGNETIC COUPLING

So we have not only capacitive coupling from the PL, but they create also a magnetic field (a current flows in a conductor indeed, so we create in the space around a variable magnetic field). By the Faraday law, when we have a variable **B** and it is coupling with a loop, we create an electromotive force (voltage), that of course is at the input of INA. The loop where B is coupled is created by the body of the patient, the arms and wires of ECG. So in the equivalent circuit we have impedances of body and wires.

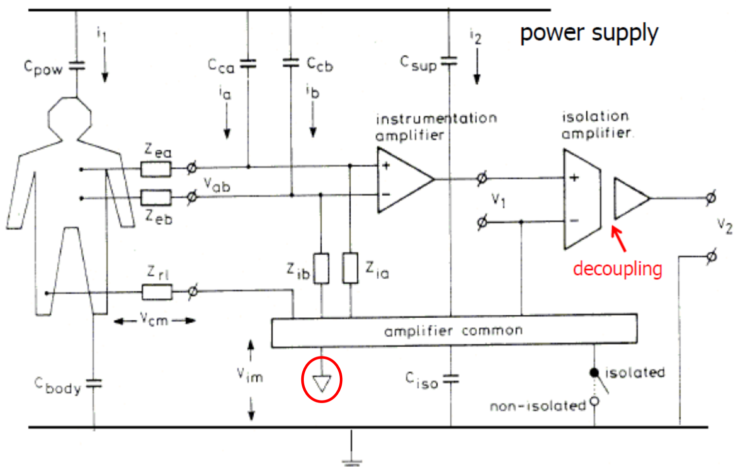


To reduce it, we use a very old trick: twisted pair cable. We twist the wires, because if we do so, we experience 2 advantages:

- The more we twist the wires, the smaller the loop, the surface crossed by the field is reduced. However there is always still a minimal area.
- **Compensating effect**: let's consider the first loop (whorl). If we take it and couple the B, we have at the end a resulting voltage. But because of the twist, the voltage is transmitted flipped just after (dotted arrow). In the second whorl, the geometry is the same, so we create a new voltage, which is equal to the previous one. But if we sum the effects, the new voltage is summed to the old one transmitted by flipping → overall voltage is null. So we are compensating the voltage each even number of loops

Not only we must protect the patient hence, but also the amplifier, typically when the patient experiences large voltages due to defibrillation, so to the wires we apply some protection stages so that the voltage on the wire doesn't exceed a given value. The typical protection mechanism are diodes in antiparallel, or zener diodes.

INSULATED AMPLIFIER



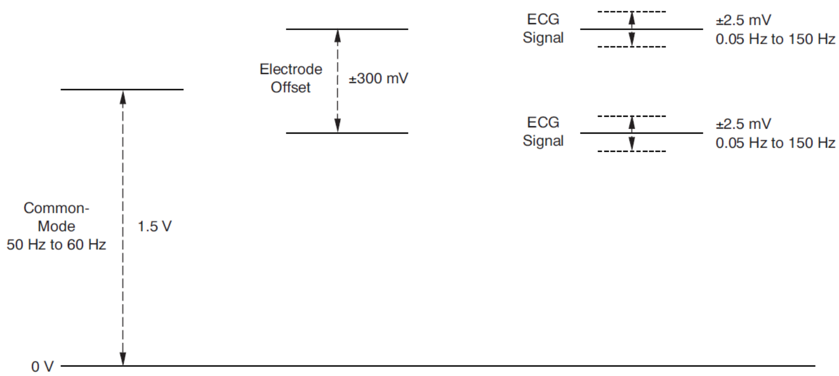
In the scheme we have the patient, all the electrical connections, the sources for interference due to the power supply. We can insulate the amplifier from the resto of the building; we may define an **amplifier common** (shown by the white triangle), a common electrode where all the circuits are referred to. The advantage is that this is an electrically insulated world where we don't use any building ground. This is a useful idea especially with a patient that may occasionally get in contact with other instruments, so it may not be a good idea that the patient or the circuit measuring the patient are heavily grounded (to the ground of the building that is then connected to the real heart). So **we leave the patient to float together with its instrumentation** (the instrumentation is totally differential, since we are interested in measuring a differential signal. I don't care if the differential signal is referred to the building ground or to another ground, since it is a differential measurement (see satellite example, where we define a common electrode to all the amplifiers of the satellite)). In this way, at the end we need to transfer the signal to a monitor or a printer, we need to get the information referred to real ground. This is the role of the insulation amplifier.

It takes the differential information referred to the amplifier common that may be floating (but it is differential) and transforms it into a unipolar voltage referred to the laboratory ground. Then is up to us; if we want to connect the other amplifier common to the ground we close the switch below, if not we keep it open.
In practical terms, we transfer the information from the amplifier reference to the laboratory by means of:

- **Transformers**: transform a voltage difference in another one.
- **AC coupling** (a signal superposed to a certain DC voltage on the left, and a voltage referred to ground on the right. We must check that the capacitor is able to handle the voltages.
- **Optocoupler**: it is a device composed by a light emitting diode (LED) and a photodetector, that collects light. So the information is transmitted through photons, and photons doesn't care about ground. For instance, LED is grounded to 50V, the photodetector is grounded to laboratory. Anyway, any type of non-galvanic connection (no transfer of charge) is fine.

INA NUMERICAL EXAMPLE

So we may have a CM, a DC offset between the electrodes.



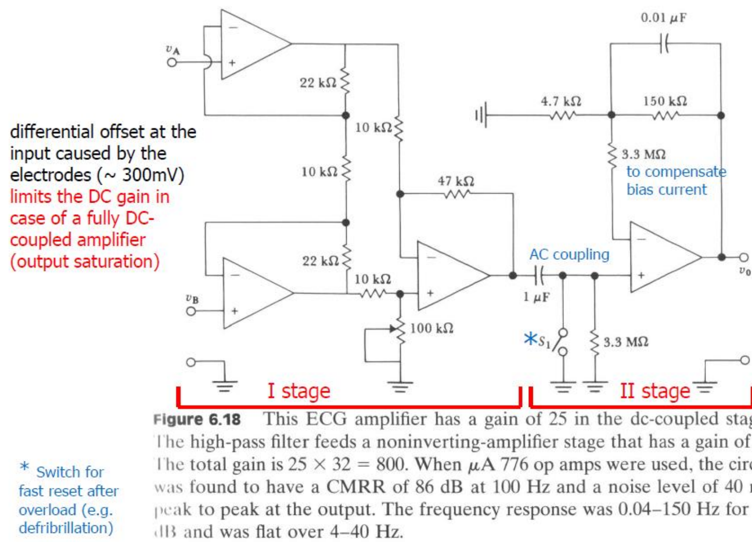


Figure 6.18 This ECG amplifier has a gain of 25 in the dc-coupled stages. The high-pass filter feeds a noninverting-amplifier stage that has a gain of 32. The total gain is 25 × 32 = 800. When μA 776 op amps were used, the circuit was found to have a CMRR of 86 dB at 100 Hz and a noise level of 40 mV peak to peak at the output. The frequency response was 0.04–150 Hz for ±3 dB and was flat over 4–40 Hz.

It is a two stages INA. So INA + non inverting amplifier. We need two stages since we need to gain about 800, because the ECG signal is very small (fraction of mV). Why can't we use only the INA? In principle, we can; but we have a DC offset at the input of +-300mV. If we take it and multiply by 800, we saturate the amplifier → we cannot put all the gain in one amplifier, otherwise we saturate. Hence we split the gain into two stages: in the first (INA) we amplify of 25, the second of 32 → 25*32=800. I have also a decoupling in between, so the 300mV are amplified by 25 but not 32, so I don't have the saturation in the second stage.

Then we have an AC coupling, we decouple the DC component, then the second amplifier is referred to ground and we have the amplification by 32.

So for the signal path we have 25*32, since the capacitor (a large capacitor) is like a short circuit, so it is like if the output of the INA is directly coupled to the input of the non inverting (in AC). But in DC the two nodes are decoupled.

In the first stage we have a potentiometer below to improve the CMRR of the differential amplifier (2^nd^ stage of INA). If we recall the calculation of done in the case if one resistor was different from the others, well the potentiometer can be used to try to compensate for this. With it, we adjust any possible mismatch between resistors. Moreover, the resistor of 3.3MOhm is necessary to define the DC voltage of the corresponding node, otherwise the amplifier is floating. Thanks to that resistor we apply ground

there, but only in DC, since in AC the capacitor is a short circuit and the node il linked with the output of the INA.
The problem is that the AC coupling with the huge resistor, creates a big RC constant. So during defibrillation, the amplifier (INA) saturates, the capacitor is charges and it will take forever to be restored to normal operation because of the 3.3MOhm (3 sec). Hence we use a switch that is closed when we need to discharge fast the capacitor → reduced time constant. Then we release the switch and we still have the normal tau. Since we have an AC coupling, we need to establish a voltage at the input of the amplifier of the second stage, which is zero for instance. To do it, we need to establish it without affecting the signal path. The best way to bias one node without affecting the impedance of the node for signal transmission is to bias it with a large resistor, because the large resistor supplies the voltage (no current flowing, 0V supplied to the input of the amplifier) but the signal path is not affected by the large resistor.

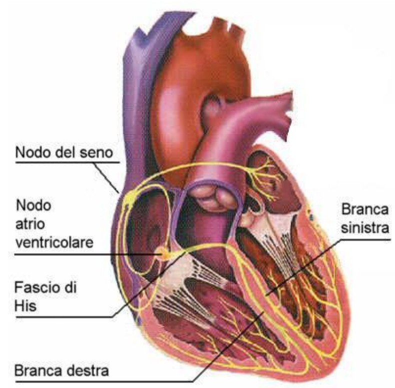
On the second stage we have also the capacitor to have a pole at 150Hz (low pass filter). So in the end it is a bandpass amplifier, from 0.04Hz to 150Hz. The low frequency pole is given by the AC coupling, that is like a high pass filter. The resistor on the minus pin is to compensate bias current, so that the drop due to bias current is the same on the two branches.

Finally the noise analysis is not done, but the noise level is of 40mV at the output. If we want to bring back to the input we divide it by 800 → 50uV, which is right the specification for the noise at the input of the amplifier, so we have to be careful about that.

Moreover, we can theoretically remove the dc offset by setting the reference pin of the INA. However this is good if the offset is constant, but since it changes patient by patient, it is not a perfect strategy. AC coupling kills the differential offset at the output of the first stage.

THE PACEMAKER

The role is to supply pulses, stimuli in the walls of the heart, and also the capability to access the walls allows to make diagnostic (detecting pulses).

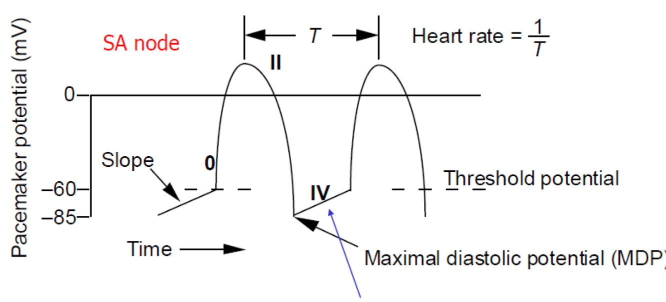


The heart has four chambers: the RA collects blood from the circulus, it is sent to RV to be oxygenated. Then the oxygenated blood is collected to LA and then LV to be pumped out. We are interest in electrical connectivity terms. In green color we can see the extended connectivity able to create the contraction of atria and ventricle and also to transmit the action potential to the various part of the heart. We can identify the main nodes responsible for the electrical activity. The upper node is the sinoatrial node, where the electrical activity starts, then we see the atria (L and R) where the electrical activity allows contraction; then we have the atrioventricular node, that represents the transmission of the signal from the previous contraction of the atria to the upcoming contraction of the ventricle. Then there is the long interconnection in the central part (septum) and then the left and right branches.

Intrinsic properties of the heart

- **Automaticity**: intrinsic capability of generate an AP. The cells that create the pulse are called pacemakers cells.
- **Ritmicity**: the stimulation are quite regular.

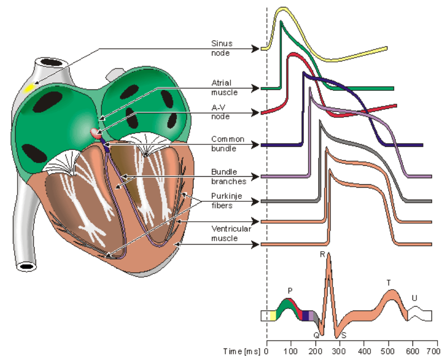
They are represented by the sequence of Aps, which are represented schematically for the SA node.



We recognize the crossing of a threshold; we still have this, and when the potential in the cells is overcame, we have the firing of an AP, then the AP extinguishes and the potential goes back to a negative value called maximal diastolic potential. The key difference is that in standard cells, after the extinguish of an AP, nothing happens anymore, the voltage remains constant to the resting potential as long as a new stimulation occurs. Here in the heart the potential is not fixed to a low value, but starts to drift, to slowly rise. This because of a different situation regarding the ions; the permeability for the ions is not 0, there is a leakage for Na and K ions. They are still able to slowly move in the internal region of the cell. After some drift, the potential reaches the threshold levels, the channels heavily open and we fire an AP. This is the reason for automaticity.

We have now to take the evidence that across the heart, several cells may beat spontaneously in a different way, due to the self-mechanism. What is evident is that the SA node is spontaneously beating at around 70 beats/min. the AV node is instead slower (50/60), and Purkinje fibers 25-40. So how to synchronize these beats?

The conclusion is that the faster dominates the overall beating. This because of course pacemaker cells are self-beating, but this doesn't prevent the cell to be stimulated from another one. So although the cell in the heart would stimulate at different time, once the SA node derives the pulse, in a sort of cascade all the other consequent cells will be triggered by the previous cells. This is why SA node determines the frequency of beating of all the other tissues.



So this plot shows the APs occurring in the different regions of the lung. The signals are one after the other, because the SA node is the first one to start. Then there is just a physiological delay. The other tissues have no time to self-beat because they receive the outside stimulation.

External to the body, we see a superposition of the effects, of these waves: the ECG signal is a t-dependent waveform to which the tissues contribute differently. The P peak is dominated by the contraction of atria (green), while the R peak by the depolarization of the ventricles, and then the T peak by the repolarization of the ventricles.
By measuring the peaks and the distances between peaks, we see if everything is physiological.
Different factors affect the cardiac rhythm:
- Threshold level: the higher, more time is taken by the slope to cross the threshold.
- Slope
- Diastolic potential (starting point)

Arrhythmias and ischemia

We have a block of the AV node. The atria contract but nothing is transmitted to the ventricles, so the two structures beat independently. The pacemaker is sensing the atrium and is firing the ventricle to prevent this, so it substitute in this case the AV node.

Another arrhythmia is when there is a delay between the P-R interval (AV is partially active). When we have a delay, the distance can be abnormal.

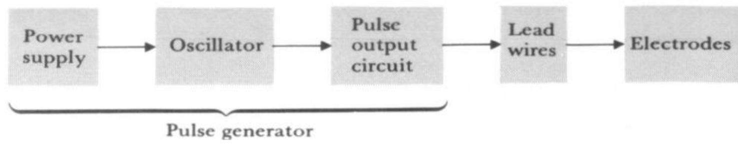
Ischemia is a reduction of oxygenation in the tissues of the heart due to the occlusion of the coronary arteries which change the equilibrium in the heart beating. There is an effect on the external recorded ECG signal.

THE PACEMAKER

It is a device which aims to restore the normal physiological beating activity of the heart. So it provides stimulus to the various chamber of the heart where the cardiac activity is not regular. We have 3 classes:
- **Asynchronous**: operates with a fixed rhythm, independently from what happens in the heart
- **Synchronous**: the pacing is synchronous to something that is recorded.
- **Rate adaptive**: most modern type. Able to provide a regular pacing but also sensible to the status of the patient, since it can identify if the patient is running and so intensify the rhythm.

It can provide pacing to different chambers, it can sense different chambers and it can respond by stimulating or inhibit the beating.

ASYNCHROUNOUS PACEMAKER



It is like a clock. It provides a pulse through a pulsing circuit according to the period given by an oscillator. The stimulus travels through the wires towards the elctrodes. There is a particular type of asynchronous pacemaker called demand-type.

Demand-type

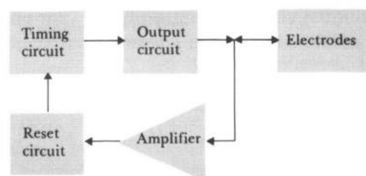


Figure 13.3 A demand-type synchronous pacemaker Electrodes serve as a means of both applying the stimulus pulse and detecting the electric signal from spontaneously occurring ventricular contractions that are used to inhibit the pacemaker's timing circuit.

It is a pacemaker ready to pulse, but it is not providing necessary a pacing if it records a spontaneous pulse by the heart. It is used for those patients that suffer from dangerous slowing of the heart beating. Again, it is composed by a timing circuit and electrodes, to supply any given seconds a pulse. But at the same times the electrodes, that are also sensors, collect a spontaneous beating of the heart, the timer is restarted. So if we exceed a given time the pacemaker provides the pulse. But if a spontaneous pulse is received, the pacemaker doesn't work and timer is restored.

ATRIAL SYNCHRONOUS PACEMAKER

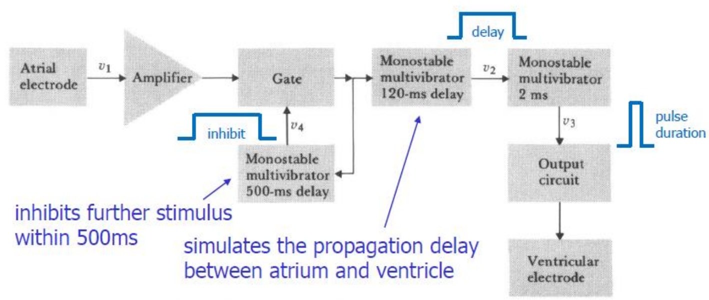


Figure 13.4 An atrial-synchronous cardiac pacemaker, which detects electric signals corresponding to the contraction of the atria and uses appropriate delays to activate a stimulus pulse to the ventricles. Figure 13.5 shows the waveforms corresponding to the voltages noted.

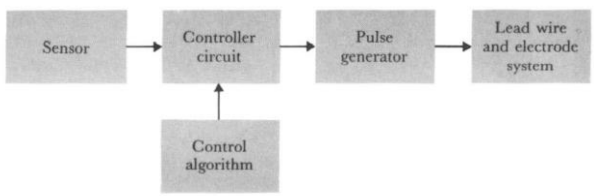
it substitutes a not correct transmission of the stimulus from the atrium to the ventricle by means of the AV node

It is a synchronous one. They are pacemakers that supply a pulse after having received a signal; the pacemaker is recording the contraction of the atria, the AV node is not able to transmit this information to the ventricles and so the pacemaker senses the pulse in the atria and provide pulse in the ventricle. First of all, the pacemaker collects the self-stimulus from an electrode implanted in the atrium and we have and amplifier that is amplifying and providing a trigger. Then we have a gate (transmission state), then a monostable that provides a 120ms delay. The delay is needed to emulate the heart; the monostable is a stage where you have a digital input and a digital output only after a while. Then we have the stimulator that provides a pulse of 2ms. We use 2ms because it is what is needed to really trigger an AP in the heart.

Gate

As the pacemaker is a quite invasive device in the heart, it could be that when we are stimulating the ventricle in a very powerful way, we have a parasite pulse that may be connected by the atrial electrode or other electrodes. So we would have a very bad retriggering modality. Hence we introduce this gate that doesn't let a new pulse to pass through at least within a physiological interval of half second. This because we know that within this period we cannot have a beat physiologically. So we kill the network in a way that even if a crosstalk is collected by the atrium electrode, it is not able to restart the triggering phase. So if we have a crosstalk, instead of transmitting it into the regular path, we open the gate for a while.

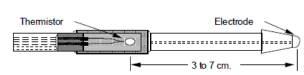
RATE ADAPTIVE PACEMAKER



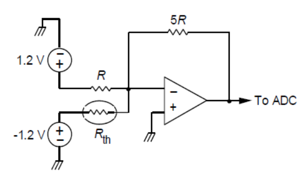
It includes a microcomputer. The pulse generation is done according to some physiological parameters recorded by the pacemaker. So it is equipped with sensors, and according to their outputs, the computer takes appropriate decisions.
The sensors for example measure temperature thanks to a thermistor, waveforms, pH of the blood, body vibration (by means of accelerometers).

The temperature measurement is e.g. placed into the tip of the electrode. The thermistor is a resistor that changes its value according to the temperature. So we can put the thermistor in a circuit where we have also resistors; if the resistances are equal, the two path are the same (R and Rth), so no current flows into the feedback. If the Rth changes, there is no more balance between the currents and we have a current in the feedback, the info is collected by an ADC and sent to a microcomputer.

negative temperature coefficient of – 4%/°C, assumed linear in the temperature range between 35–40°C



CIRCUIT EMPLOYED TO DETERMINE THE VARIATION OF TEMPERATURE TO CHANGE THE STIMULATION FREQUENCY



PACEMAKERS: ELECTRODES

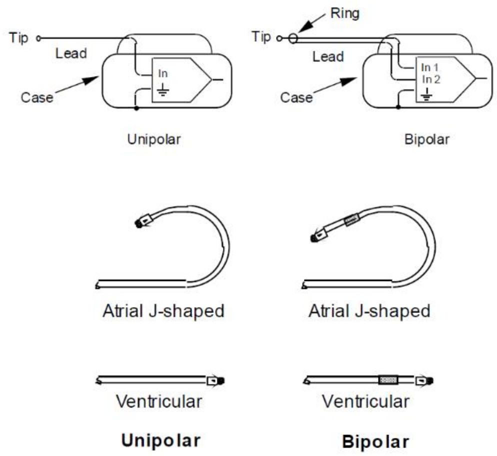


Figure 6.13 Unipolar and bipolar implementations of both J-shaped and nonpreshaped leads. All models have distal cathode. Bipolar designs typically have a ring anode proximal 10–15 mm on the lead.

They have to supply the electrical stimulus to the tissues and also to sense the biosignal. They must fulfill safety properties, they should be effective in transmitting the energy to the tissues by minimizing the possible loss.
There is as always an electrode electrolyte interface (as previously shown). The peculiarity is that the tissues is the endocardium (the one in contact with the electrodes).

Classification

They are divided into **unipolar** and **bipolar**. They have to supply pulses, hence a voltage difference to the point of the tissue of interest, but with respect to what?

Unipolar

We elect the case of the pacemakers as local common. The pacemaker needs to have a common reference, indeed. We apply a voltage to the tip of the electrode with respect to this common reference. Somehow the pacemaker is embedded in the body (like the satellite travels in the space), so we define a local ground and we supply a voltage elsewhere with respect to the reference

Bipolar

The alternative is to provide a local voltage difference, a bipolar voltage difference between the tip as before but with respect to another metal connection (that has typically a ring shape) close to the tip so we supply a local voltage difference between the tip and the ring. Thus we still have the common reference in the pacemaker case, but the voltage is supplied in a differential way between to nearby electrodes.

In the atrial J-shaped we have the unipolar version with the tip, in the bipolar we still have the tip but at a distance of 10mm we have a ring → local voltage difference. The same for the ventricular electrode, which has instead a straight shape (see webster book for more reference).

BATTERY

Battery in the pacemaker are a crucial component, since when we implant the pacemaker by surgery, the pacemaker has to operate for several years (up to 10) without the replacement of the battery. The only thing done is the periodic riprogramming, we have the possibility to reprogram the pacemaker to new physiological parameters, but this is done internally through a radiofrequency link, we have no real access to the hardware. Each battery is characterized by a figure of merit that is its capacity, defined in mAh, the unit of a charge.

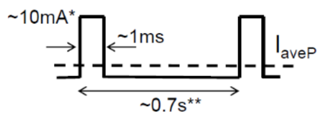
Basic numbers:

- Battery duration: 10 years
- Battery capacity: ~ 2000mAh (vs. iPhone 11: ~ 3000mAh)

⇒ How much current (average) is allowed for the pacemaker in 10Y?

$$I_{ave} = \frac{\text{Capacity}}{\text{Duration}} = \frac{2000\text{mAh}}{10\text{Y}} = \frac{2000\text{mA}\cdot\text{h}}{10\cdot 365\cdot 24\cdot\text{h}} \sim 23\mu\text{A}$$

pulse generation:



* ~10mV/1Ω ** ~ pulse periode

$$I_{aveP} \sim 10\text{mA}\,\frac{1\text{ms}}{0.7\text{s}} \sim 14\mu\text{A}$$

⇒ ~ 9μA (max) available for the remaining electronics

The battery has a capacity of around 2000mAh. The battery of our smartphone is similar for example. The question is which is the best use we can do of such capacity. We can calculate in average which is the current consumption we may allow to be drawn by our pacemaker considering we cannot substitute the battery for 10 years.

The average current is the capacity divided by the time duration. The result is 23uA; overall the pacemaker must draw no more than 23uA. It seems small, but we can also estimate something else. Let’s estimate the current for the main job of the pacemaker, which is to supply pulse, the rest is sensing, telemetry, controlling electronic, but we cannot skip the energy to supply pulses.
The calculation allows us to say that the drawn current to supply pulses is of 14uA. They are supplied about every 0.7s (reversed of the average beating) and they last about 1ms (duration needed for the tissue to generate an AP) and the intensity of the pulse is around 10mA (order of magnitude). 10mA because in order to trigger an AP the order of magnitude of the pulse was about of tens of mV, so we generate such a step if we are able to inject sufficient current into the impedance of the tissue. Since the impedance is in the order of 1Ohm, the current needed to trigger an AP is about 10mA (10mV/1Ohm). The conclusion is that in the pacemaker we have to supply 10mA for 1ms every 0.7s.

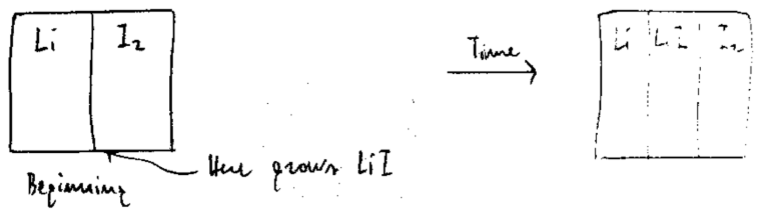
The average current if we have to supply 10mA each 0.7s is obtained by considering the duty cycle → 14uA. This average current is mandatory, it is indeed small than the maximal one.

So once we are around 14uA for pulse generation, for all the remaining electronics we have only 9uA, so the budget is really small → the electronics for pacemakers is a highly low power electronics (few hundreds of nA). All the transistors work in weak inversion.

LITHIUM IODIDE BATTERY

The battery is a solid-state battery, a lithium battery, made with the matching of two solid materials: lithium and iodine. When put in contact, they create a chemical reaction, a redox where the lithium oxidates (looses electrons) and iodine is reduces (gains electrons). They are exchanging electrones through the load. The result of such a redox formula is the lithium iodide, which is a solid, and is the electrolyte created between these two elements.

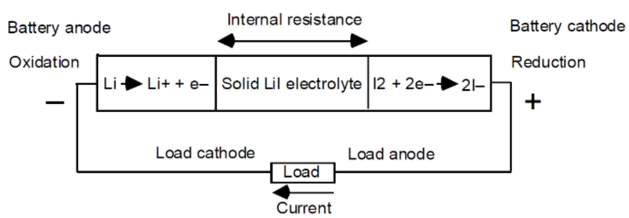
7.



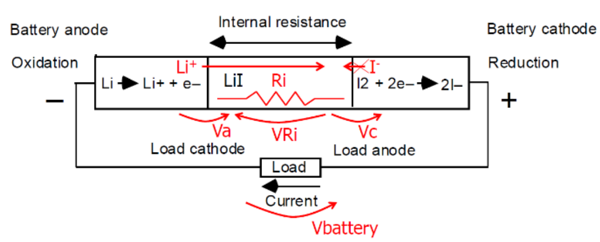
At the beginning we put in contact Li and I2, we have just the interface between them, but then the reaction takes place and we have the electrolyte growing between them.
After sometimes, the LiI_2 becomes a significant separation between the electrodes. At the beginning it is not present, it forms with time.
The result is of 2,8V of battery.

The LiI_2 electrolyte grows during the lifetime of the battery and this is a problem since it creates an internal resistance, that is a penalizing factor for the battery. When the battery gets old, it means that it is growing internally a resistance, so this create the voltage drop (voltage generator + resistance).
It is an internal resistance since it prevents or at least it makes more difficult, the continuation of the reaction. Indeed, to continue the reaction, Li atoms have to reach I2 ions. I2 ions are not able to travel penetrating the electrolyte, Li yes. However, the more the resistance is increased, the more is difficult for Li ions to travel through the resistance.

The growing of the resistance can provide a drop on the voltage for the following reason.

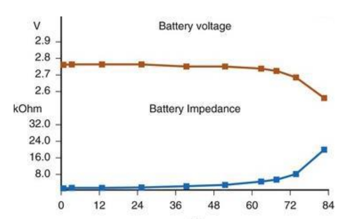


- solid-state battery
- $2Li + I_2 \rightarrow 2LiI$ (solid electrolyte)
- V = 2.8V



- increase of internal resistance Ri (typ. 30kΩ after 1800mAh)
- battery voltage reduction

First of all we consider the voltage Va which is the half cell potential of Li, created when Li is in contact with the electrolyte. Similarly for Vc. In conclusion, the battery voltage is Va+Vc (=2.8). However, during the growing of the internal resistance, we have an internal voltage drop that has the opposite sign. It is the drop occurring when the current of the Li is flowing into the internal resistance. So Li ions moving is a current, but passing in a resistance creates a voltage. So over time the increase of the internal resistance increases the internal voltage drop to the battery itself, so the resulting voltage at the terminals of the battery is no more the initial voltage drop, but it without the drop to the internal resistance. Over time we have an increase of the internal resistance in the order of 30kOhm and a battery voltage reduction.



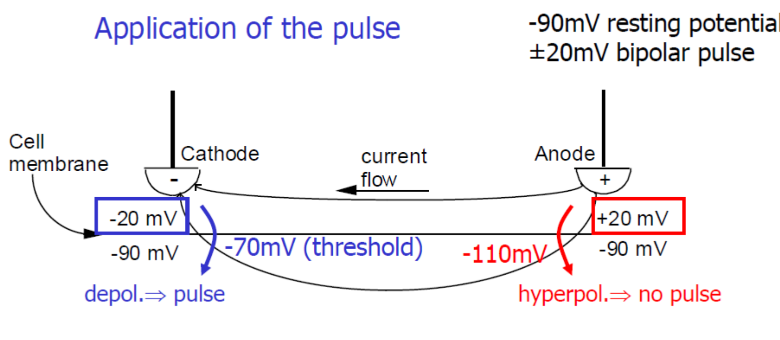
On one side of the plot in blue we see how the internal impedance reaches values of kOhm and hence the voltage of the battery drops. We must take this into account, the amplifier cannot work only at 2.8 as Vdd, but it must be biased also at smaller value of voltage, verifying it is still working.

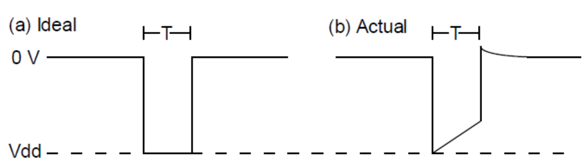
At the same time, the circuits that have to supply the pulses, must work even when the voltage drops.

PULSES GENERATION

The driving parameter to obtain the best pulse is energy conservation (the lower the energy consumption, the better). Moreover, supplying just the battery voltage is not sufficient to stimulate the tissues, so we need to introduce some circuits that allows to supply a pulse 2 or 3 times the battery voltage.

In the most general configuration we have the bipolar pulsing below (we can provide it also unipolar).





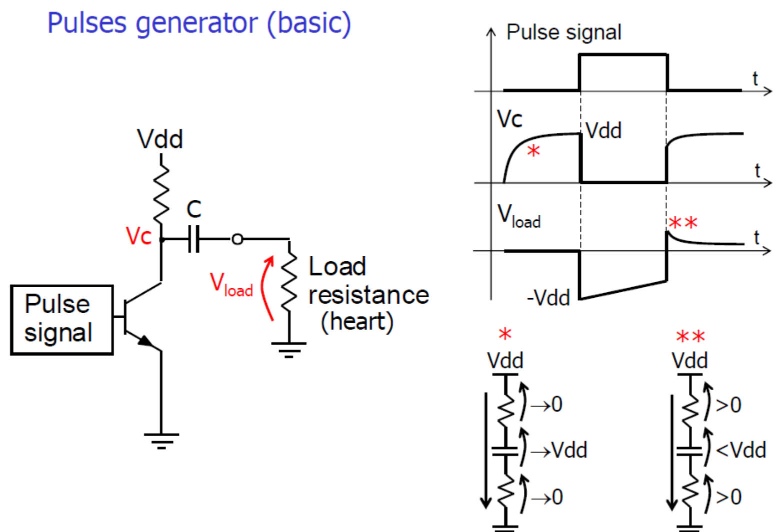
When we put two electrodes in contact with the tissues, I supply a negative voltage on one electrode (cathode) with respect to the other, where is supplied a positive voltage (anode). So I supply a voltage difference. In the image, I imagine to supply 40mV (+-20mV) between the two electrodes. If I put them in contact with tissues, I expect to trigger the AP in the contact point of the cathode. This because we know that depolarization means reducing the voltage difference between the internal and external part of the cell, so it means to decrease the voltage externally to the cell. So we drop the voltage difference thanks to a negative voltage externally. So for example if I have -90mV inside and externally I apply -20mV, the overall voltage across the cell is of 70mV, so I'm going above the threshold, I'm triggering a pulse. Conversely, at the anode we are doing the opposite, we are increasing the voltage, so we are applying 20mV positive. So overall the voltage difference is increasing at the anode → hyperpolarization, we are inhibiting a pulse.

So we are interested to negative pulses because of this. Ideally, the amplitude of the negative pulse is -Vdd, so -2.8V (the one of the battery). So we need to create -20mV and we are able to generate a negative pulse of -Vdd. However the step down is not ideal as in the image.

PULSES GENERATOR

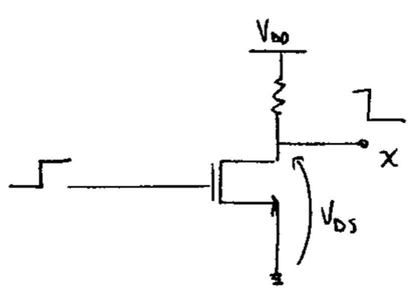
The simpler stage is the unipolar basic stage, we ground the anode so to concentrate in supplying just a negative pulse. The following is the simplest stage to supply a pulse. It is a common source stage (also called common emitter). In the image there is a BJT, but we can also put a nMOSFET.
To switch on a common source mosfet we apply a positive step of duration t. Then we have the tissue that is represented as a resistive load with respect to ground.

At the beginning the pulse is off, the gate is off, so the transistor is of. If so, we have to consider the R-C-R network (below, right). It is an RC network, so if e.g. the C is zero, we have a non zero current flowing into the resistors. Hence we have a rising voltage over the capacitor; C charges up to when the capacitor holds the complete voltage Vdd. Hence Vc at the beginning is 0, since the C is zero, then start to rise with the tau and then we have a steady condition for Vdd, and no more current flows in the network.



Then the pulse occurs. The transistor switches on heavily if it occurs in the steady condition where we have Vdd. It is a common source, so if we have a tension on the gate, the drain gets down (it is an inverter), the voltage on the drain goes down to zero. So we have a heavy inverter with the drain going down to the source.

8.

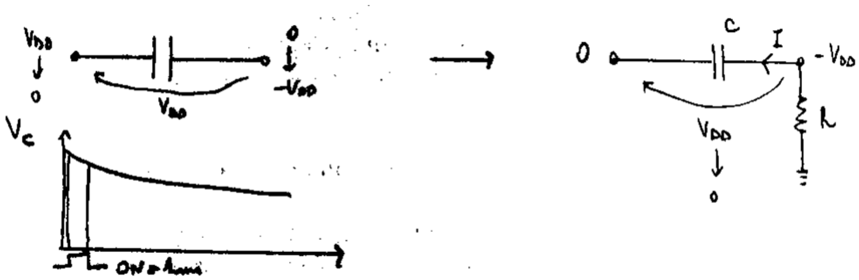


The drain goes down, but if a very large current flows in the transistor, the node goes so slow that the Vds tends to 0 (if we switch on the gate, the drain goes down).
So the Vc voltage drops down. However, the capacitor was charged to Vdd. So if the left hand of the capacitor goes to 0, the right hand goes to -Vdd, since we cannot discharge immediately the capacitor, the voltage drop on the capacitor remains Vdd overall → rigid translation of the capacitor.

This is how we have applied to the load (heart tissue) a negative voltage, that in principle is Vdd.
So we pre-charge the C, then we have heavily pulled down one hand of it and the C pulls down the other hand of the same quantity.

The idea is the following:

9.

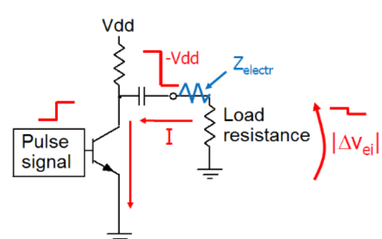


So we do what above. Then we have to consider that the capacitor, for the total duration of the pulse, so during the application of the pulse, is in a RC network. So the voltage won't remain -Vdd, since the C starts to drop its voltage across (Vdd). At the end the Vdd across C tends to zero, since we discharge the capacitor.
If I take the voltage on the capacitor, it is going to be discharged with a long time constant. However, I will not let the C to discharge, since the switch on time for the circuit is very short, so we see only the initial start of discharge of the capacitor → the voltage across the C won't reach 0, but a value a little bit smaller than Vdd.

This explains why in the Vload plot we reach -Vdd and then we have the starts of the exponential decay. It seems linear, but it is exponential: then after a while the transistor is switched off again. Since the C is not fully discharged, the voltage on the load goes up again → we restore the previous condition. In reality, we have also a little positive spike (**), explained by the right circuit below.
The transistor is off, and since the C has been discharged a little, we cannot say that the voltage across C is still Vdd, but a bit lower. But if we take the network, and the Kirchhoff law, the two remaining voltages must be a little bit positive to sum up to Vdd, so we have a bit of positive bounce in the load.

CURRENT VS PULSE DURATION

We first observe that:



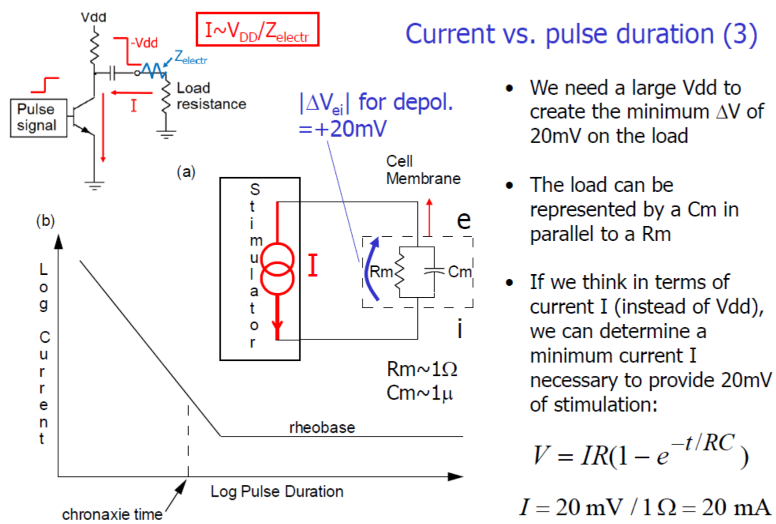
1) Due to $Z_{electr} >> R_{load}$, $\Rightarrow |\Delta V_{ei}| << Vdd$
2) $I = Vdd/(R_L+Z_{electr}) \sim Vdd/Z_{electr}$ (it does not depend on the load and it can be increased by increasing – or multiplying - Vdd)

The previous one was an ideal condition, with the tissue in direct contact with the electrode. But we forgot the electrode impedance. Its value is in the order of tens of kOhms. So in the model everything remains the same, but we are not applying -Vdd to the heart, since we have a very penalizing voltage divider, because we have the series between the electrode impedance and the 1Ohm impedance of the tissue. This is also why in the time constant we don't have only the 1Ohm but also the impedance of the electrode.

So we apply -Vdd but in reality on the load we apply only a small fraction of that. If the load is in the order of 1Ohm and electrode's impedance of 1kOhm, we have a penalization factor of 1000. This explains why it is not easy from a 2.8V battery to achieve 10mV on the load.
First, we have to minimize the electrode's impedance as much as we can; moreover we have also to increase the voltage we really apply in the circuit. Considering now the model, we can also calculate the current taken in the circuit by the load (we assume the impedance of the load is negligible).
The current is I=Vdd/Zelectrode.

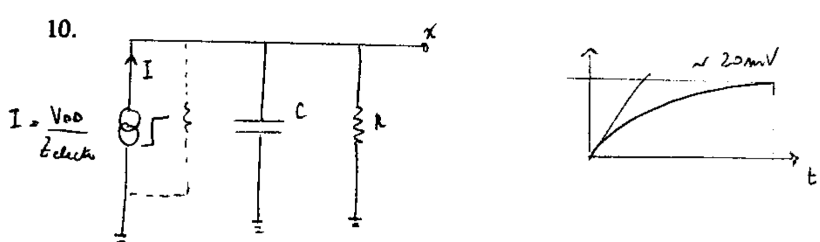
So in order to increase the current flowing I need to multiply the voltage on this stage.

This brings to an important question: considering all these difficulties and that we need to supply a pulse with sufficient intensity, which is the best compromise between the amplitude of the pulse and the duration?



On the upper left we have the modelling introduced (pulser to supply -Vdd, the Zelectr and also the current to be supplied in reality). In principle I can now represent the circuit as an equivalent Norton of my pulser. This because I also introduce an additional penalizing element that is the capacitor across the cell. So the impedance of the cell is not only a resistor, but also a capacitor that creates the overall impedance (e stands for external, i for internal). The good news is that R is 1Ohm ca., the C is 1 uF ca.
Hence my stimulator is represented by a current generator that provides a step in current given by Vdd/Zelectr, the load is seen as a parallel between the resistor and the capacitor.

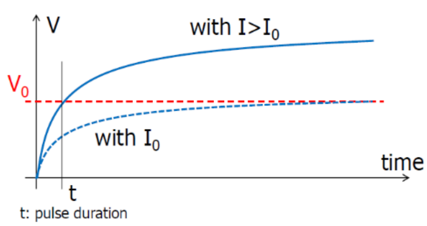
10.



An equivalent way to describe our circuit. We have a current given by Vdd/Zelectr, and we need to charge a capacitor in parallel to a resistor. The one on x will be the voltage that activates the AP. But if we have a step by the generator, on the x we have an RC network output. (we don't' care about the Norton equivalent impedance in parallel). If the voltage Vdd is not sufficient, we will apply some multiplier. Anyway, thanks to Vdd we control the current injected into the tissues. However, due to the R and C parallel, we don't get immediately the voltage to trigger the AP, we have to wait for the RC network.
The minimum current needed to reach at least the 20mV that are the minimum to activate the AP is 20mA.

So is it better to have an intense pulse or a short pulse? If I look at the graph, the answer would be a pulse that lasts forever to reach the stimulation. We need to do something better than forever.

However if we supply just 20mA (I_0) we have to wait forever. In practice, we apply a larger current than I_0, a current I. thus the voltage has still an exponential shape but we apply a larger current. If we apply a larger current we reach the desired voltage V_0 at the time t, which is called pulse duration.
From this plot we can imagine that larger is the current we apply, shorter is the time needed to reach V_0.



$V_0 = \alpha I_0 R(1 - e^{-t/RC})$

V_0 can be reached
with $I=I_0$ ($\alpha=1$), but with a pulse duration $t\to\infty$
or
with $I>I_0$ and a finite pulse duration t

$\Rightarrow$ which is the best compromise for I and t?

$I = \alpha I_0$

$V_0 = \alpha I_0 R(1 - e^{-t/RC})$

$\frac{V_0}{\alpha I_0 R} = (1 - e^{-t/RC}) \qquad -\frac{t}{RC} = \ln(1 - \frac{V_0}{\alpha I_0 R})$

$\ln(1+x) \approx x \qquad \frac{t}{RC} \approx \frac{V_0}{\alpha I_0 R} \qquad t \approx RC\frac{V_0}{\alpha I_0 R}$ inverse proportionality between I and t (for large I)

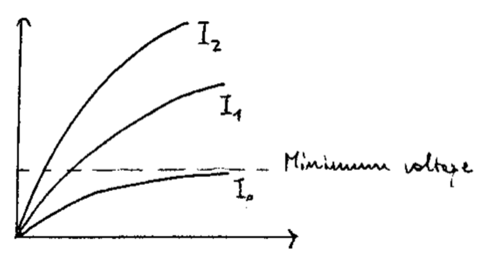
$t = C\frac{V_0}{I} \qquad I = C\frac{V_0}{t}\frac{I_0}{I_0} \qquad I = I_0\frac{t_c}{t} \qquad (t_c = CV_0/I_0)$

$I = I_0\frac{t_c}{t}$ OK for t small; no OK for $t\to\infty$ (does not go to I_0) $\Rightarrow \boxed{I = I_0(1 + t_c/t)}$ ($I \to I_0$ for $t\to\infty$)

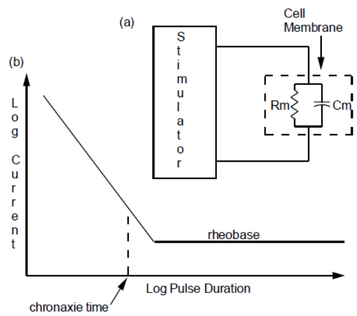
So we now understand why the two plots below are equivalent: they both allow to achieved V_0, but in different ways. There are also calculations to get them.
We have the exponential formula, then we divide and apply logarithm. Then supposing that V0/alpha*I_0*R is sufficiently small, we apply the Taylor and we obtain that t ~ RC* V0/alpha*I_0*R.

The last formula tells us that given the minimum current I_0, which we need to apply, the larger is the current we apply with respect to I_0, the smaller is the time t we need to apply it. This is ok for small tau.

11.



Larger is the current, smaller is the time we have to apply the pulse. For small tau is ok, but what is not working in the formula is what happens for t→inf. If t→inf, from the formula the current should be zero, but in reality it should approach I_0. So we artificially manipulate it to obtain the formula in the red square by summing 1. Note that for small t, 1 is negligible.



$$\boxed{I = I_0(1 + t_c / t)}$$

which is the best compromise for I and t?

⇒ minimize the energy consumption

$$E = I^2 Z t$$

$$\frac{dE}{dt} = I_0^2 Z\left(1 - \frac{t_c^2}{t^2}\right)$$

$\frac{dE}{dt} = 0$ for $t = t_c \Rightarrow I = 2I_0$

$$E = (2I_0)^2 Z t_c = \underline{4} I_0^2 Z t_c$$

$$E = \underline{7.2} I_0^2 Z t_c \quad (t = 0.2 t_c)$$

$$E = \underline{4.5} I_0^2 Z t_c \quad (t = 2 t_c)$$

Once we have this formula, we need to find the best compromise between duration of the pulse and current. In a log-log plot the formula tells us that for small t we have larger current, for long t we have I_0. The best compromise is the one that minimize the energy consumption. The energy consumption, when we charge with a current a resistor, an impedance Z, is equal to the power times the duration of the pulse.

I should find the minimum → I take the derivative of the energy with respect to time duration. The derivative is set to zero. so there is an optimum time and in correspondence of it there is an optimum current, that is twice I_0. We have a reverse proportionality between duration and intensity of the current.

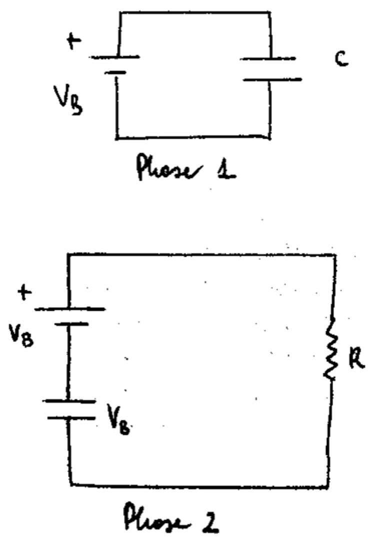
So the best current we can apply to minimize the battery consumption, once I_0 has been defined, is $2*I_0$, no more, no less. Thus we reach the AP but with the optimized use of the battery.

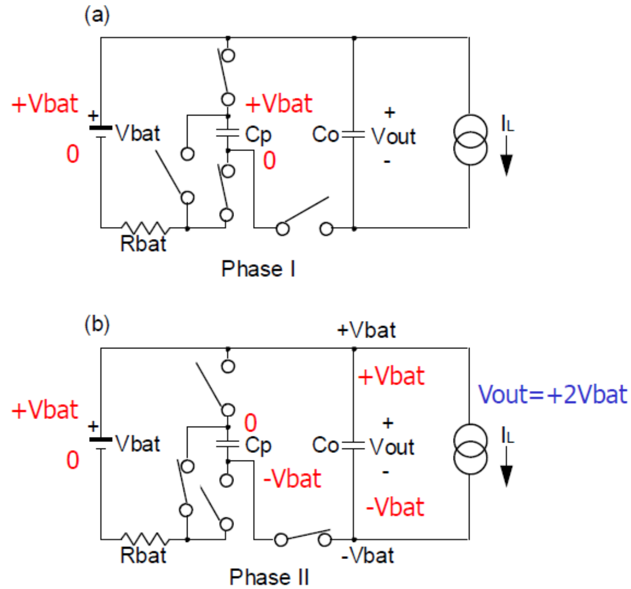
If we are not doing our best, for instance taking a too short pulse duration (current to large), we are duplicating the energy consumption → wasting of battery. Conversely, if we are using a pulse with too much time, we are also dissipating a lot of energy. The power consumption has been computed considering a constant current on the load.

The next topic is the possibility to supply a voltage on the load larger than the power supply.

GENERATION OF A V>Vsupply

12.



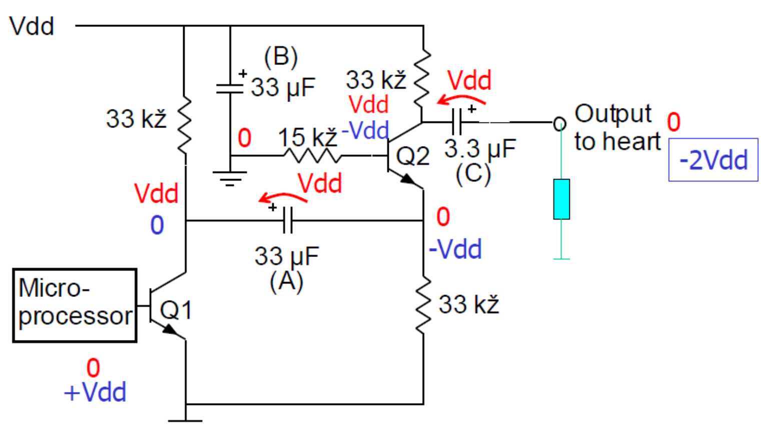


In the first phase we take the battery generator and we charge a capacitor that is in parallel to the power supply, so we accumulate the battery voltage in the capacitor. Then the second step is to put in series the battery with the just charged capacitor. So we have two V_B, V battery. So overall if I apply the voltage difference to a load, I have twice the battery voltage. Of course the capacitor is not an ideal voltage generator, it will discharge, it won't hold the voltage forever, but if we switch between the two phases we continue to store charge on it → the phases need to be repeated so that the C can be considered a voltage generator, at least for a small period of time.

In the two circuits above what just described is shown. Overall on the load we apply the double of the battery voltage

UNIPOLAR PULSE GENERATOR

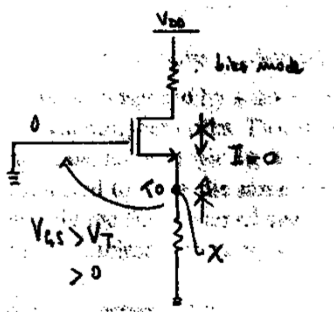
We provide a pulse to the load supposing that the other side of the load is at reference (ground).



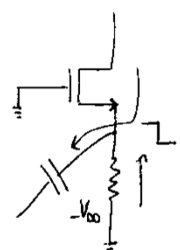
We have a common source transistor (with a BJT or a nMOSFET), the gate is connected to a microprocessor that activates the transistor. Then we have the 33uF, a capacitor in the middle that will be charged to Vdd and then a second stage with a transistor whose gate is grounded (the capacitor B is for filtering the noise). When the transistor Q1 is off we have the red set of voltages, when it is on the blue one.

Let's analyze the circuit, starting from the red color. At the beginning the transistor is off, so we have only to analyze the network made by Vdd, the two resistances and the capacitor A (also the other transistor is off). It is an R-C-R network, so the C is charged to Vdd and no drop on the resistors at the end of the transient. The other transistor is off because the gate is at zero and the source is also at zero. Note that to have this second transistor on, we must have a positive voltage between the gate-source voltage. But if we have a positive voltage Vgs, if Vg is 0, Vs must be lower than zero. But if the resistor is grounded, the current should flow going up. But how is it possible if the current of the mosfet should go down? It is not possible, the Kirchhoff law at node x is not satisfied, so the only possiblecondition is that the current is zero everywhere.

13.



14.



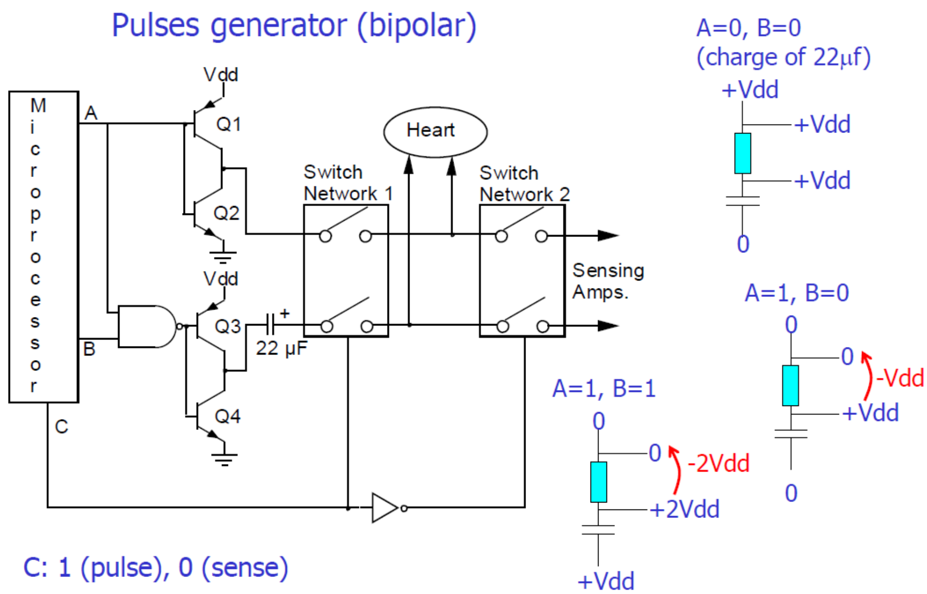
So the only possible resulting voltage during the red phase is 0. So there is no voltage drop on the resistor upper right and so the node C is at Vdd. We have another RCR network → also the capacitor C is charged to Vdd. It is another replica of the first stage with the transistor Q2 off.

Now we deliver the pulse (blue values). If we step up the input, the output goes down heavily, to the minimum voltage that is 0 → the node of the capacitor A drops to 0. However, the C cannot change instantaneously the charge on it, so the right hand drops to -Vdd. Hence since the gate of Q2 is to 0, this transistor switches on. If it is on, also the node left hand to the capacitor (C) goes down, since the drain voltage is going down to the source voltage (we have like a shortcircuit). So this node drops down from +Vdd to -Vdd, because we have to consider the transistor Q2 as a shortcircuit. But the capacitor C has accumulated Vdd across the plate and so the right hand goes down to –2Vdd (the voltage across it cannot vanish).

Have we still the problem of the current as before? Now we are in transient mode, which means that we have to consider the presence of the capacitor. A current can flows from the top and bottom? Yes since they both go to the left, because there is the capacitor because it is in transient mode (14) the capacitor (before it was in bias mode, 13).

BIPOLAR PULSE GENERATOR

The load here is supplied with two electrodes, two tips. Between them we supply twice Vdd.



The circuit is composed by a microprocessor, then we have two inverters (Q1 and Q2), then we have the two switching networks. This means that the two tips can be either stimulated by the pulse circuit (if to line C we have a logic 1), or can be (if the line C has a logic 0) connected to the sensing amplifier (tips connected to it) → the same electrodes can be connected to the pulser or to the amplifier. From now on only the first series of switches will be considered close. In the second inverter (Q3 and Q4) we have a capacitor in the line, so to accumulate Vdd.

The heart is represented by the cyan load on the right. We are representing the various voltage drops across the heart when we have different configuration af A and B by the microprocessor.

CASE 1: A=0, B=0

Initial situation. Output of the inverter is 1, so the upper tip is charged to Vdd. On the other side we have an and, so if A=0, the output is 1, so we have a zero outside the inverter. So the other node on the right hand of the capacitor is 0. So then we wait till the end of the RC transient (load and capacitor) and then the C is charged to Vdd and we will have no more drop on the load. Moreover, the capacitor has been precharged to Vdd.

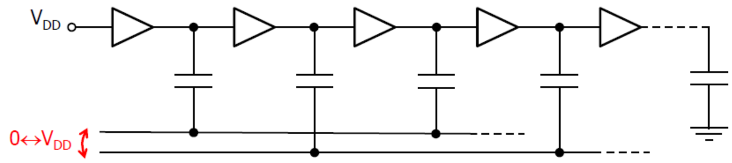
CASE 2: A=1, B=0

Depending on the combination of A and B we can stimulate with -Vdd or –2Vdd, depending on where we are in the life of the battery (the closer to death, provide -2Vdd).
In the first situation B remains 0, we move A to 1. If A goes to 1, the output of the inverter is 0 so on the top side of the tip we move from Vdd to zero. The bottom part remains the same since the inverter 2 (Q3 + Q4) doesn't change. So overall on the load we have applied -Vdd, and it is a bipolar application of the voltage.

CASE 3: A=1, B=1

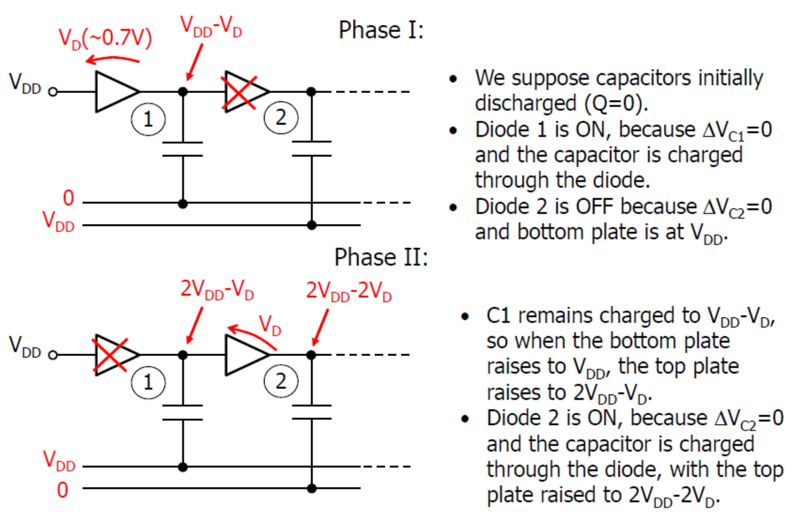
To apply –2Vdd. We move A to 1, so that the top tip changes from Vdd to 0, but now also the bottom tip is changing, since outside the NAND we have a zero. So the output of the inverter is 1 and we are boosting the bottom plate of the capacitor from 0 to Vdd. The capacitor cannot change its charge instantaneously, so the upper plate shifts to +2Vdd → overall voltage difference on the load of -2Vdd.

CHARGE PUMPS – DICKINSON CHARGE PUMP



Chain of diodes and capacitors, with bottom plates of capacitors connected to two bus lines (in common every two stages), alternately switching between 0 and V_{DD}.

We can design a circuit to multiply the power supply of a voltage by a number N.
One of the most popular is the Dickson charge pump, composed by a chain of diode and capacitors. The first diode is connected to Vdd, the last one to the load where we want to have a multiplication of the voltage.
The capacitors are alternatively connected to the same bus, depending on being even or odd. On the two busses we switch continuously the voltage from 0 to Vdd. Thanks to this at the end after several switches we will accumulate on the last capacitor a voltage equal to N*Vdd. We analyze first two phases, in which the voltages on the bus are alternated.

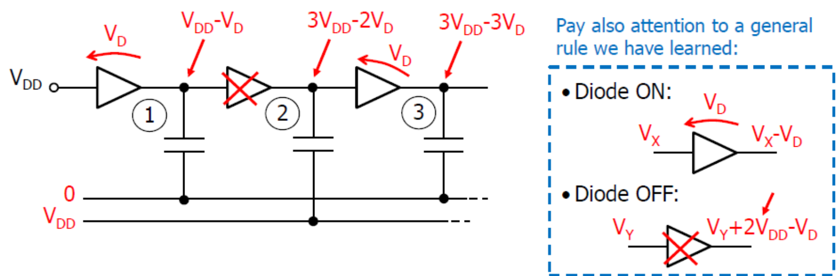


PHASE 1

We have the first capacitor connected to 0 and the second to Vdd. Suppose that there is no previous voltage drop on the C, they are discharged at the beginning. If we assume that the down node of C1 is at zero, C1 is discharged and the up node is at Vdd, the only possibility is that the diode 1 gets on. If it turns on, we have a voltage drop of 0.7V across it (called V_D), and with a diode on the voltage on the top plate of the capacitor 1 is Vdd-V_D; so the capacitor at the beginning was discharged but thanks to the diode turning on the node has been charged to Vdd - V_D.
About the stage 2, if we assume the C2 is discharged at the beginning, if we don't have initial drop on the capacitor, also the upper node is at Vdd of C2, so the diode 2 is off (in between Vdd-V_D and Vdd), because the downstream voltage is higher than the upstream.
So at the end the diode of stage 1 is on, the one of stage 2 is off.

PHASE 2

The bottom plate of the first capacitor is moved up from 0 to Vdd. As usual, the charge on the C1 has not been removed, and so again as in previous circuits, we have a rigid translation of the voltages on the C1 → top voltage moves to 2Vdd – V_D. The first diode has however the input still to Vdd → it switches off. Concerning the second stage, we have that the C2 has the bottom plate down to 0; the C2 was initially discharged, so the top plate was at 0. But due to the very large difference across the diode 2, it turns on and if so, we have a drop V_D, so the upper node of C2 has a voltage. We have accumulated a voltage on C2 but with a penalization, that is 2*V_D.



- Now, as C2 was charged to $2V_{DD}-2V_D$, when the bottom plate of C2 is raised to V_{DD}, to top plate is raised to $3V_{DD}-2V_D$.
- Diode 3 is ON, because $\Delta V_{C3}=0$, and the capacitor is charged through the diode.
- The top electrode of C3 is raised to $3V_{DD}-3V_D$.

Now we go back to phase 1, but the upper node 2, if the bottom line goes up again to Vdd, the top node 2 will be the previous voltage plus another Vdd.

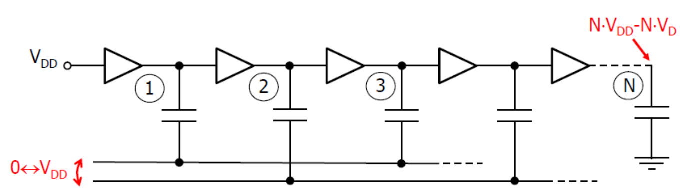
In the figure below we have again phase 1 but due to the charge accumulated on the capacitor, the node is at 3Vdd-2V_D. If I consider a 3rd stage, the C3 has the bottom plate at zero and the upper at 3Vdd-3V_D.

Thanks to this multiple switching, stage after stage we are multiplying the battery voltage, and it is the goal of the charge pump. The more stage we add, the more we accumulate on a node the power supply voltage (but with a penalty, V_D).

In conclusion, if now I extend everything adding N stages, at the output at stage N I will have N*Vdd – N*V_D. So we have multiplied N time the battery voltage.

Besides learning how the network works, we can generalize a **rule**: when the diode is on, wherever it is, if we have an Vx voltage on the left, we have an Vx – V_D on the right. When the diode is off, if we have a voltage Vy on the left, the one on the right is Vy + 2Vdd – V_D. The twice Vdd is the important thing to be remembered.

After several switching phases:



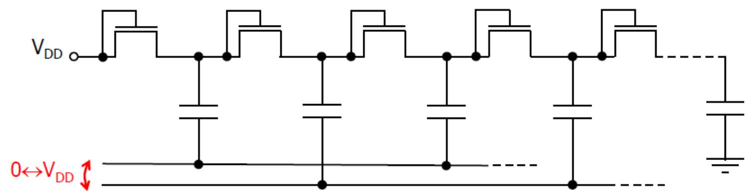
- Following the same charging mechanism for all the capacitors, at the end of the chain, the N_{th} capacitor is charged to $N{\cdot}V_{DD}-N{\cdot}V_D$. A multiplication of the power supply by N has been reached (but reduced by N-times the diode ON-voltage).
- At the beginning, as all capacitors are discharged (Q=0), several clock cycles are needed to charge the capacitors along the chain to achieve the multiplied voltage, as the charge transferred to the following capacitor is taken from the previous one..
- The charge drawn by the load, connected to the last capacitor, is restored again through the chain of the capacitors.

Remarks

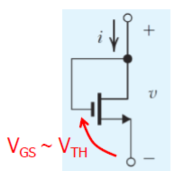
At the beginning, all the C are discharged, so at the beginning, for instance then the 3rd diode turns on, who supplies the charge to charge the C3? The previous capacitor (C2). Who is restoring the charge on C2? The C1 in phase 2. The C1 is the restored by the initial power supply. Hence I need several clock cycles to charge the capacitors up to the end. Moreover, if I have a load profiting of this voltage N*Vdd

– N*V$_D$, the last capacitor is losing charges. But who restore them? The previous chain. So the last voltage is not going to stay forever at this value if it is not continuously richarged by the previous capacitor and so on.

Use of MOSFETs instead of diodes



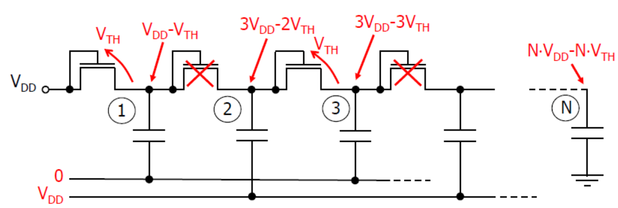
The Transdiode:



- The pn diodes can be substituted in CMOS technology by transdiode placed in the same position.
- The ON voltage drop of the diode (~0.7V) is substituted with the V_{GS} associated to the ON operation of the MOSFET, which is close to the threshold voltage V_{TH}.

In CMOS circuitry it is more convenient to use the MOSFET version of a diode than the pn junction itself → transdiode. The transdiode is a transistor with a drain-gate shortcircuit and so we have a bipole which is very similar to a diode. When the transdiode is on, we have a current flowing and a Vgs voltage (gate-source) equal to the threshold voltage.
So we have a chain of transdiode and when the transdiode is on, we have a voltage drop associated to the Vgs that is equal to the threshold voltage. So if I substitute the previous charge pump with transiode, the reasoning is the same with them in place of the diode, and the diode drop is substituted by the threshold voltage. At the end of the chain, we have N*Vdd – N*Vth.

Example of voltage distribution across the chain (Phase I):



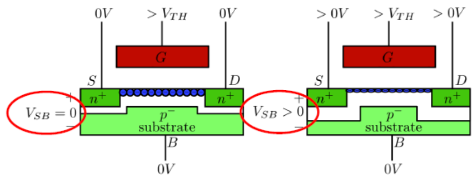
- The charging mechanism is identical to the one using diodes. At the end of the chain, the N_{th} capacitor is charged to $N \cdot V_{DD} - N \cdot V_{TH}$.
- The ON voltage drop across each active transdiode is ~ equal to the threshold voltage V_{TH}.

However, this implementation is not identical to the one of the diode, since we have the problem of the body effect.

Body effect in mosfet

It is a change in the threshold voltage when the transistor electrodes are not at the same voltage of the body or the substrate but they have a more positive voltage. Let's consider a mosfet with the substrate to 0, source and drain to 0 and we have created an inversion layer by applying a threshold on the gate. However, if we now bias source and drain to positive voltages, the layer of electrons is also biased to the same positive voltage, so we have a kind of pn junction because the channel n and the p substrate create a pn junction and having raised the voltage of the n component with respect to the p component is like a diode in reverse biasing.

The Body Effect:



When V_S>V_B, the depletion width of the pn junction increases. That makes it more difficult to create a channel with the same V_{GS}, effectively reducing the channel depth. In order to return to the same channel depth, V_{GS} needs to increase accordingly.
The Body Effect can be seen as a change in threshold voltage and it is modeled as:

$$V_{TH} = V_{T0_n} + \gamma\left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right)$$

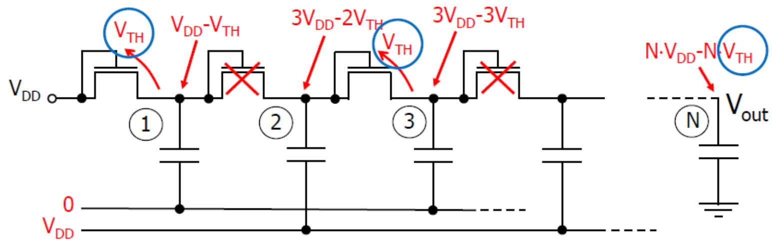
14.

When we have a pn junction we have a depletion layer. But if we apply a more positive voltage on the n semiconductor with respect to the p one we are extending the depletion layer, the junction is getting larger, we are depleting more the junction. Depleting means that electrons in the previous n semiconductor are no more existing, they have left the corresponding volume. So when we are biasing a pn junction or we are applying a more positive voltage on the n side of a semiconductor with respect to the p side, the electrons are escaping, they are moving away.

So if we look at the mosfet on the right, when we provide a positive voltage on the source and on the drain, so we are increasing the voltage difference between the n layer of the channel to the p one, we are depleting the layer, we are loosing electrons. How can we restore the same amount of electrons? We have to increase the voltage on the gate.
<u>So the body effect is an increase of the threshold voltage Vth given by the formula when we change the voltage of the source with respect to the substrate. Because if we change this voltage we have to apply an extra voltage on the gate to restore the same amount of electrons. Hence when we increase the voltage on the source and on the drain we have also to increase the gate voltage to provide the same degree of inversion</u>

This is important for us because the result of the body effect is that for each transistor in the chain we cannot trust on the same Vth, since they are biased at increasing voltages. The different transistors have the source and drain at increasing voltage → also Vth increases.
So N is not multiplying the same threshold voltage, but the output voltage is given by a sum of differences.

The goal is that if moving on in the chain Vth increases too much, the incremental difference Vdd - Vth becomes negligible, we are spending stages adding zero to the final sum.

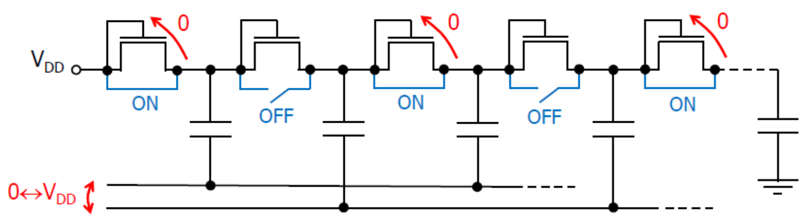


- As the source voltages increase along the chain, the MOSFET threshold voltage V_{TH} increases by the Body Effect and the ON drop increases, reducing the benefit of the multiplication through N stages:

$$V_{out}= (V_{DD}-V_{TH1})+(V_{DD}-V_{TH2})+(V_{DD}-V_{TH3})\ldots = \sum (V_{DD}-V_{THi})$$

- Adding several stages, the terms $(V_{DD}-V_{THi})$ added in the sum are less and less useful when V_{THi} increases along the chain.

How to solve the problem? We eliminate the problem drastically.
We place switches (to be driven in some way) across the transdiodes to be driven in a way that if the transdiode is on the switch is close (shortcircuit, no more voltage drop), when the transdiode must be off, the corresponding switch is off. So we eliminate the problem of threshold voltage, but we need to drive accordingly the switches.



- The switches must be driven suitably so that they are closed (ON) when the corresponding transdiode is ON and open (OFF) when the corresponding transdiode is OFF.
- The switches can be implemented by MOSFETs.

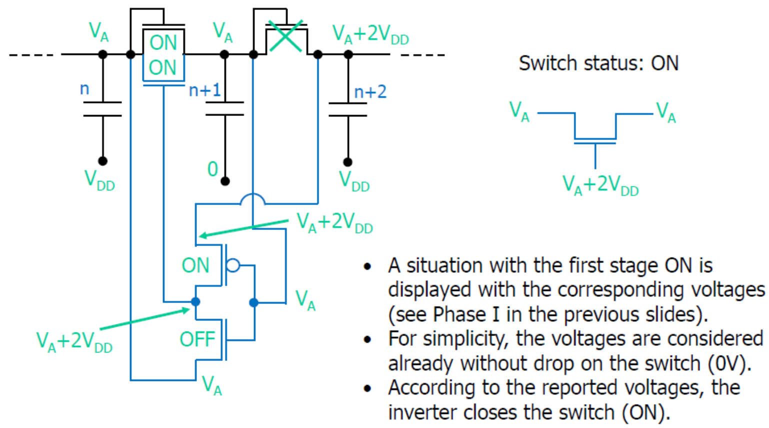
Driving of the switches

We implement switches with nMOSFET. I’m focusing on the switch of the n stage, only this switch is shown in the figure. **The driver is an inverter (at the center)**. Its output is the gate of the switch we have to close and open. The input is the node n+1, the top voltage is the one of the stage n+2 and the bottom voltage is the one of the stage n.

ON state

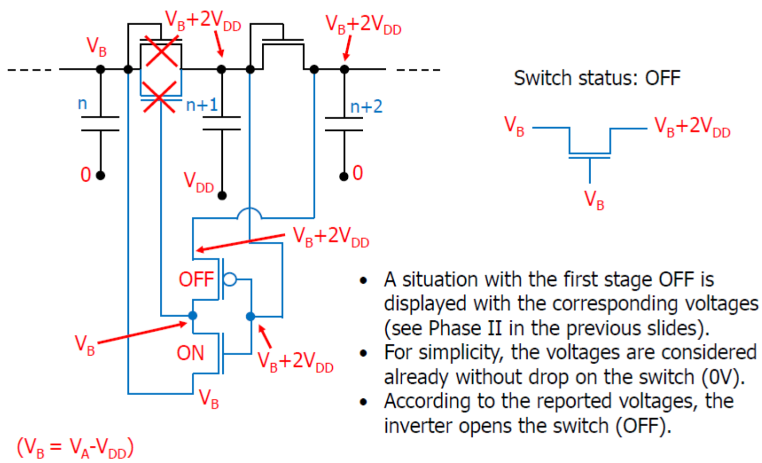
I have to close the switch when the corresponding diode is on. I have a given voltage Va somewhere in the chain; when the transdiode is in ON stage, the bottom of the C of the stage is at Vdd. The voltage on the right is Va, if the transdiode is ON. It should be Va – Vth, but let’s forget Vth, since we are assuming the switch is a shortcircuit.

The next transdiode is OFF. So the voltage at the node n+2 is the previous voltage Va + 2*Vdd, rule of the diode seen above.
The top voltage of the inverter is Va+2*Vdd, the input is Va, so the pMOS is on (shortcircuit), the nMOS is off. So the output voltage is the top voltage, so Va+2*Vdd.
As for the situation on the switch (right image), it has Va on left and right, and the gate has Va+2*Vdd → the switch is closed since the gate voltage is much larger than the source and drain voltage.
So I have demonstrated the switch is closed. But what is the benefit of having a diode now if we use a switch to carry the current? *(Transdiodes are necessary to keep preexisting charges)*

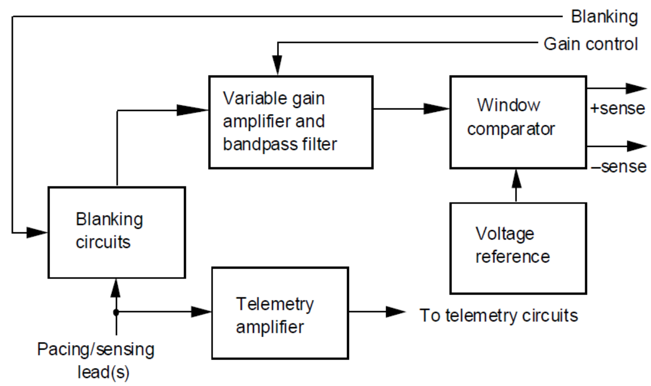


OFF state

When the transdiode has to be off, also the switch has to be off. Now I have a voltage Vb and the other plate of the capacitor is at 0. The relationship is that Vb = Va – Vdd. By the rule, if the transdiode is off, on the right I have Vb + 2*Vdd.
By analyzing the inverter, it as the pMOS off and the nMOS on. So the output voltage of the inverter is Vb. Regarding the situation of the switch, the gate voltage is Vb; it cannot be on since the Vgate is smaller than the voltage on the drain and on the source. So when the transdiode is off, also the corresponding switch is driven to be off.

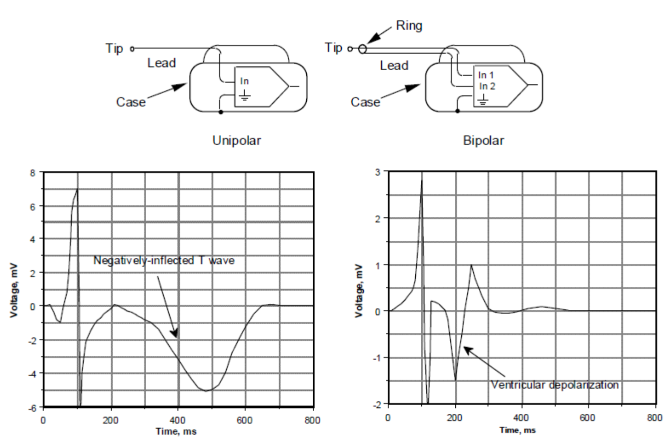


BASIC COMPONENTS OF THE ANALOG SECTION OF THE PACEMAKER



In the diagram we see the main blocks of the sensing circuitry. The same electrodes (bottom left) are used for pacing and sensing. Now the electrodes are connected to a band pass filter (we need to record an process the signal from a fraction of Hz up to few hundreds of Hz). Once filtered and amplified, a comparator must be useful to use a comparator, which simply is used to provide trigger signal, for instance for the microcomputer, to highlight a pulse as occurred, so for example to inhibit pacemaker's pulsing or to trigger a pulse to be given to the ventricle.
Blanking circuits means that while we are pacing the electrodes, we have to disconnect (or blank) the amplifiers. They are circuit make to disconnect the amplifier from the electrode when we are supplying a pulse. The telemetry amplifier is specifically devoted to record waveforms ad to send them externally thanks to the telemetry.

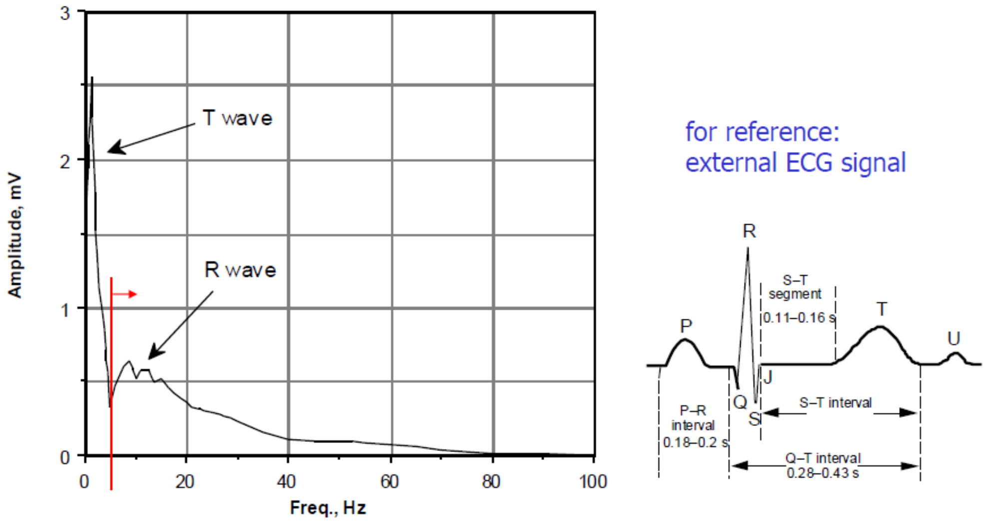
The following are two examples of waveforms provided by the amplifiers. On the left we have an intracardial ventricular pulse. We have a very fast pulse when we have the depolarization, and when we have the repolarization we have a smoother pulse. On the right, we have an intracardiac atrial pulse; we still have a bipolar quick pulse which is the indication that in the atrium there has been a pulse. What is interesting is that in the waveform recording the atrial pulse we see an artifact that corresponds to the ventricular depolarization. So the spike the tip is connected to the atrium to record the atrium pulse, due to the distributed electrical activity, the tip is able to sense an artifact when the ventricle is depolarizing.



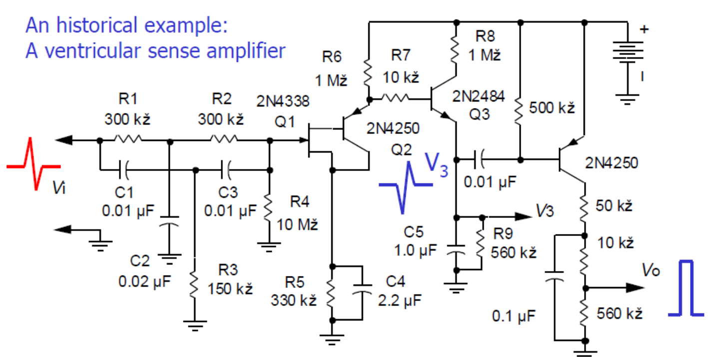
ECG intracardiac ventricular ECG intracardiac atrial

This explains why we need to introduce a gating. When we recorded an atrial pulse, before suppling a consequent pulse we need to be gated by 500ms. This because when we supply the ventricle depolarization, if this is peaken up, it can be considered as a stimulation pulse, which is not true. The message is to be aware of possible artifact, in the wave we can have the signal we are looking for plus other artifacts due to the other tissues.

In this plot we see the frequency spectrum and in particular, if we associate the frequency spectrum to the reference ECG signal recorded externally, the fast R spike provides the components at higher frequencies, while the T wave provides components at low frequencies.



Example of a circuit (not to be learned)



Notch

$$\frac{V_3}{V_i} = T(s) \times \frac{-g_m(1+h_{fe2})(1+h_{fe3})R_6(s-z_1)}{(R_6+R_7+h_{ie2})C_5(s-p_1)(s-p_2)}$$

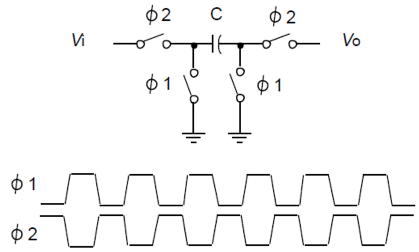
$$z_1 = -1/R_5C_4$$

$$p_1 = -\left(\frac{1+g_m R_5(1+h_{fe2})}{R_5C_4}\right)$$

$$p_2 = -\left(\frac{R_6+R_7+h_{ie2}+R_9(1+h_{fe3})}{R_9(R_6+R_7+h_{ie2})C_5}\right)$$

It is composed by a notch filter that cuts a precise frequency, the 50Hz. At low frequency the C of the notch filter are open and the signal passes through the resistors at the top, while at high frequency the C2 shunts the node and the C1 and C3 are shortcircuit → we have a single zero in the transfer function. Then we have a transistor amplifier that inverts and amplify the signal and then we have a discriminator, a common source stage that acts as a discriminator that provides a pulse..
Overall this is a band pass amplifier plus a discriminator.

SWITCHED CAPACITORS AMPLIFIERS



hypotheses:

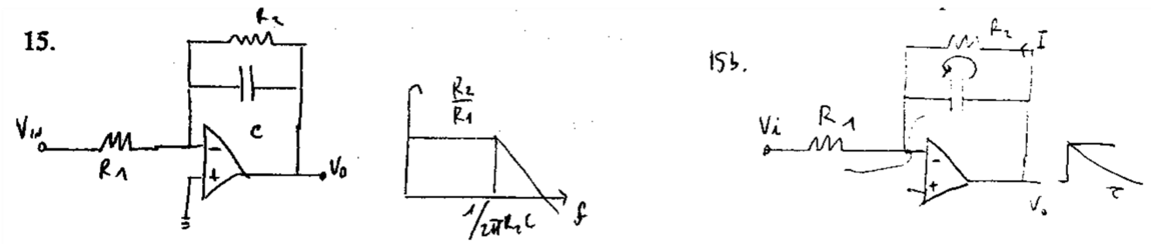
1) clocks never overlapped in the high value (switches never closed contemporary)

2) Vi and Vo change slowly with respect to the clock

$$R_{\text{eff}} = \frac{(V_i - V_o)}{I} = \frac{(V_i - V_o)}{fC(V_i - V_o)} = \frac{1}{fC}$$

ex. C=2pF, f=10KHz
⇒ Reff=50MΩ

We have to face a basic problem if we need to design a filter, which is the size of the component. For instance, we can design an active band pass filter. The following is a prototype of a low pass filter.



The problem is that for example want the fcut of ca. 1kHz. This means that tau ~ 1ms. However, in CMOS technology the largest capacitor we may implement is in the order of 10pF. Hence the resulting resistance must be in the order of 100MOhm → ultra large resistors. However, also these are very large in CMOS technology. Hence in CMOS technology, in particular for amplifiers for pacemaker where the fcut is very small, we have to implement very large resistor.
This brings us to the **switched capacitors technique**; it is a technique able to implement an equivalent resistor of very large value using much smaller devices.

In the corresponding network we have the voltage we want to apply across the resistor (Vin – Vout), a capacitor C and a set of switches that cannot be closed simultaneously. Suppose phi 2 is closed. We accumulate a charge in C equal to C*Voltage difference. Then we open phi2 and close phi1, the charge is lost. Then we open phi1, we close phi 2 and we charge again the capacitor to the same amount as before. If I do it many times, I can imagine to have an average current 'I' flowing into my equivalent bipole (everything in between Vi and Vo), an equivalent resistor, that is given by the charge divided by the period used for charge and discharge. Or equivalently, the current is given by the charge multiplied by the frequency of the clock.
I continue to charge and discharge the capacitor so to have an average (not istantaneous) a current.

Equivalent ohm law

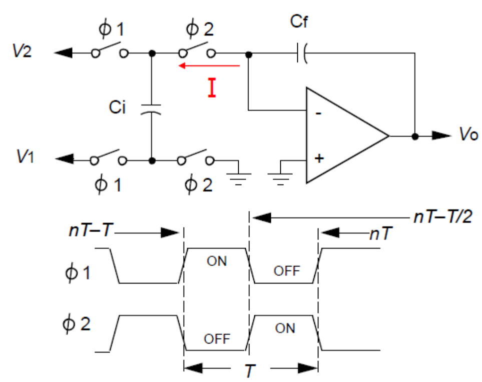
But if I have drawn an average current, what is the equivalent ohm law of the bipole? It is given by the ratio of the voltage across my load (Vi-Vo) divided my average current; the voltage difference cancels and in conclusion the equivalent ohm law, the effective resistor given by this mechanism is 1/f*C. So if I put in the formula a small capacitor and a totally reasonable frequency of 1kHz, I create an equivalent resistor of 100MOhm that can be used in the filter shown before.

If I look at the formula 1/f*C, I will be tempted to use small capacitor, since the smaller the capacitor, the larger the resistance. This is correct, but we don't have to exaggerate since the smaller, the more the network is affected by parasitic capacitor of the switches. In real switches made with mosfet, indeed, the parasitic capacitors play a role together with the C.

Regarding the frequency, may I use a smaller frequency? I cannot exceed a minimum value since the fclock must be larger than the frequency of variation of the voltage I'm sampling, hence the voltage difference. If the voltage is constant is not problematic, but if the voltage across the C changes in time, I need to be fast in clocking because my assumption of having an average current is true as long as the voltage difference is constant in a few clock cycle. If it changes every clock cycle, we cannot say that the current is on average constant.
This is better explained by the Nyquist theorem. This clocking system is nothing more than a sampling system; I'm sampling a voltage over a capacitor, so my reproducibility is good as long as the clock frequency satisfies the Nyquist theorem, that is: the clocking frequency must be twice the maximum frequency of the voltage in my system. So it is important that the clocking must be faster than the frequency of the signal.

This is the reason why at the denominator we cannot put a frequency going to zero. If too small, soon or later we will violate the Nyquist theorem.

EXAMPLE – IDEAL INTEGRATOR



$\Delta Vi = V1 - V2$

$I = \Delta Vi \times f \times Ci$

$Reff = \Delta Vi / I = 1/f \times Ci$

$Vo = I \times 1/j\omega C_f$
$= \Delta Vi \times f \times Ci / j\omega C_f$

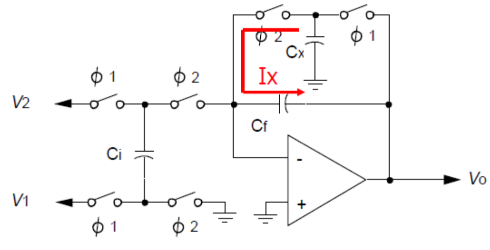
$$\frac{V_o}{\Delta V_i} = \frac{C_i}{C_f}\frac{1}{j\omega T}$$

The network at the input is a resistor, used to convert the input voltage (that in this case can be differential) into a current I that is sent to the feedback capacity. So when I close phi 1 I store the voltage across the capacitor, when I open it and close phi 2 the capacitor is discharged via two grounds, because phi2 below has the bottom plate to a real ground, phi 2 on the top has the plate connected to a virtual ground. So when I close phi 2 the charge moves to the virtual ground and it is transferred to the feedback capacitor. So the capacitor Ci together with the switches creates an effective resistor given by the previous formula. The output voltage is shown in the formula.

REAL INTEGRATOR

I want to implement the (15) integrator, with the feedback resistor. Hence I imagine to have a switched capacitor implementation for the two resistors R1 and R2 of the filter.
Which is the role of the resistor R2? Its physical role is to discharge the capacitor C. if we have a step at the output, so if we have collected some charge on the C, the resistor R2 will discharge the capacitor, it will provide an RC discharge of the capacitor. In fact, if the output voltage steps up, the resistor will

create a current and the current will go to discharge the capacitor. So we have a circulation of the current that is responsible for the exponential decay of the response with a corresponding RC time constant.This is happening also in this circuit, where the capacitor Cx with the switches phi1 and phi2



φ1: Cx is charged to Vo(t)
φ2: Cx is discharged on the virtual ground and the lost charge goes to discharge Cf

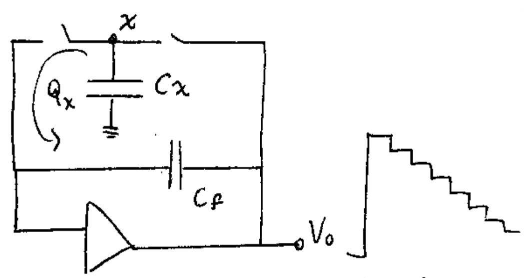
$Qx = Vo(t)\times Cx$
$Ix = f\times Qx = f\times Vo(t)\times Cx$
$Ix = Vo(t)/Rf$
$\Rightarrow Rf = 1/(f\times Cx)$

$$\frac{Vo}{Vi} = \frac{\frac{Rf}{1+j\omega CfRf}}{Ri} = \frac{\frac{\frac{T}{Cx}}{1+j\omega Cf\, T/Cx}}{T/Ci} = \frac{Ci}{Cx}\,\frac{1}{1+j\omega Cf\, T/Cx}$$

$\boxed{G_{DC} = Ci/Cx \quad \omega_T = Cx/(Cf\, T)}$

implements the resistor I have in parallel in (15). The Cx is charged when phi1 is closed. Let's suppose we have accumulated a charge on Cf. The voltage is sampled on Cx by closing phi1; when we open phi1, we close phi2 and the charge accumulated on Cx is flowing into the capacitor Cf. The charge accumulated on Cx flows in the direction given by the arrow, but if so, we are reducing the charge and the voltage. Hence we also continue to discharge Cf. When the current flows in the feedback network, it is not flowing in the input network. The current, if we close phi2, doesn't flow there since the capacitor Ci is grounded in this case. So the current cannot flow in a capacitor grounded on both sides.

16.

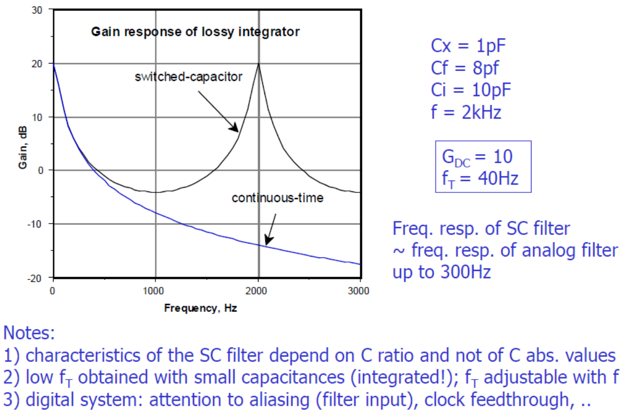


When I have a positive step at the output, the voltage is programmed on the capacitor Cx (node x). when then the ph1 is opened and phi2 closed, charge Qx accumulated on Cx is going to discharge by one step Cf. Then the new Vout is programmed again on the node x of Cx and then a new charge will be discharged on Cf. I have like a staircase.
By charging and discharging Cx I have a continuous removal of charge from the Cf capacitor. Hence more or less we are implementing the same resistor.

The formal equations are the one in the image. The capacitor Cx with the switches is equivalent to a resistor Rf. We can now build our real integrator whose gain is the ratio between the impedance on the feedback and the impedance at the input. In the end we have the final formula of a low pass filter. The DC gain is given by the ratio of the two capacitors. Both DC gain and the cutoff frequency depend on the ratio of the capacitors, which is nice, since in CMOS technology you cannot trust the absolute value of the capacitor because it changes with the technology but you can trust the ratio.
The other good news is that we can tune the frequency of the amplifier using the clock frequency. If we change the fclock we can modify the cutoff frequency.

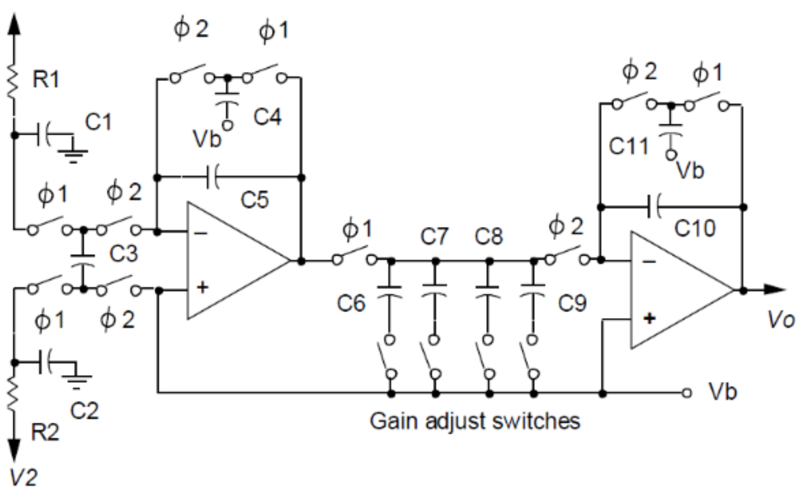
Filter response

It is not a Bode plot. We select some values. Note that if we compare the switched capacitor filter with the classical representation, the two filters are equivalent only at low frequency. At HF, the switched capacitor filter is different. This because at HF the filter is no more satisfying the Nyquist theorem. For instance, if we have 2kHz of the signal, we should use a fclock at 4kHz at least. So if we use a fclock of 2kHz we are not satisfying the theorem. So the switched capacitor technique is a good implementation for filter but only in the region where Nyquist theorem applies.



This is the example of a transfer function, a semi Bode plot. We have the range of frequencies for biosignals, up to 1kHz. We see in case with the value on the right (order of magnitude for the capacitor is maximum 10pF, still reasonable in CMOS implementation, and fclock is 2kHz). The clock frequency should be at least twice the f signal, so f signal should be of 1kHz. The two implementation indeed are good up to 1kHz, but then the switched capacitor deviates → the interest is in the low frequency range.

Example – a low pass filter of 2nd order



(a) Differential–input amplifier and low–pass filter

It is given by a cascade of 2 low pass filter of 1st order. We recognize a real integrator, then a network to transfer the charge and then a second filter with a second pole. So the overall t.f. is a low pass filter with a roll off of – 40db/dec.

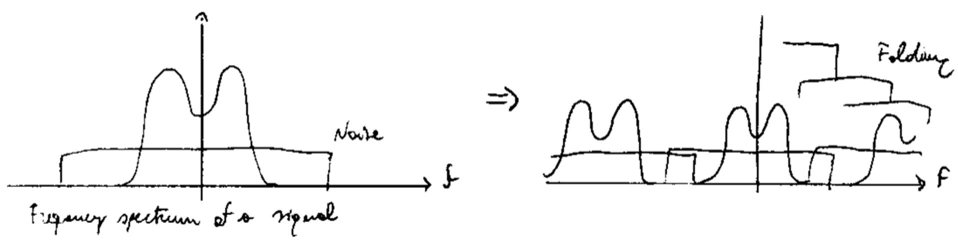
The connecting network is easy. Let's imagine that all the switches are close in the middle, they are only for gain adjust, they are **static switches**, they don't participate to clocking, only phi1 and phi2 participate to clocking. If we close phi1 and phi 2 is open, the voltage accumulated on the first integrator is sampled on the capacitor in the middle, on which we accumulate charge given by Vout1*C. Then we open phi1 and close phi2 and the charge accumulated on the capacitor flows through the virtual ground and then in the second integrator. So we have a temporary accumulation on the capacitors.

The switches in the middle on the capacitors have the role to put one or more C in parallel. It means that for the same output voltage, the total charge accumulated in the battery of the capacitor increases, the more are in parallel, the larger the value of the integrated charge, still given by the voltage multiplied by the capacitance. Depending on the number of closed switches we increase the accumulated charge, we increase the equivalent capacitor.

Inputs

Let's imagine the inputs are the arrows and that they are the tips connected to heart. It is a differential amplifier. But the tips are not directly connected to the input, but we have two additional passive low pass filter. We have them to filter noise and prevent folding.

17.

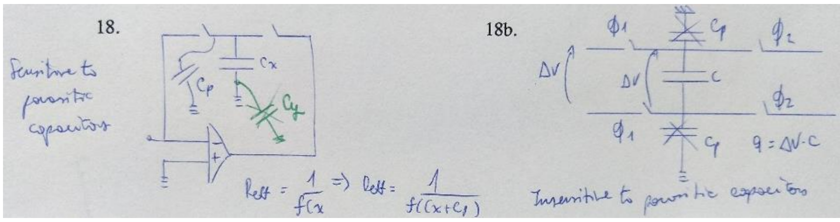


If the one above is the frequency spectrum of a signal, sampling means that we basically multiply the spectrum and we create replicas of the spectrum over the delta in the frequency position placed on the clock frequency multiple. Then if we do a low pass filtering we restore the spectrum at the starting bandwidth.
In fact, if we have unfortunately noise and it is extended over a wide bandwidth, this means we are replicating also the noise when sampling. Hence we have the so called folding, a huge superposition of the noise spectra. This is the reason why in sampling system we first limit by some way the noise, we make a preliminary and rough low pass filtering. If we do it before sampling, the spectrum of the signal is not affected and we limit the bandwidth of the noise, so avoiding the folding.

This is why we have the low pass passive filters at the input, it has the role of anti-aliasing filters.

The use of the switches introduces parasitics. For instance, when we have the capacitor as in the figure (Cx), Cx have a parasitic capacitor in parallel, the smaller Cx (to implement a high $R_{eff} = 1/f*C_x$), the higher the parasitic capacitor Cp → $R_{eff} = 1/f*(C_x+C_p)$. So we have to be careful since the parameter of interest of the circuit may depend also on the parasitic capacitor.

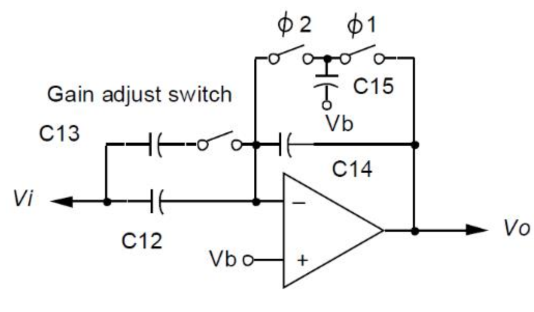
This is not true for the other implementation of the effective resistor (18b).



Usually the parasitic capacitors are between one plate and ground. Then we have also a parasitic capacitor in between the ground plate and ground (Cy), but since the plate is already grounded, it doesn't play any role.

In 18b, the capacitors Cp doesn't play any role because when we close phi1 we apply the voltage across C, so the charge is given by delta V*C. The only capacitor that plays a role is C, because what matters is the voltage across the C. in 18 was critical because delta V was applied both to Cx and Cp, here only C applies. Then when we close phi 2 the charge is discharged to ground, and the parasitic capacitor doesn't play any role → <u>parasitic capacitors play a role depending on the configuration of the effective resistor</u>.

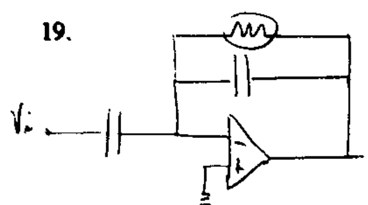
Example – high pass filter

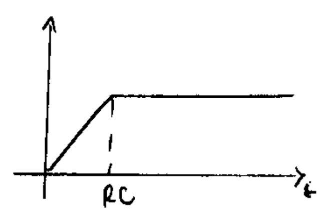


(b) Adjustable–gain high–pass filter

High pass filter of the first order, the derivator.

19.





In terms of switched capacitor, the one we have to implement is on the feedback resistor, since at the input we have no more a resistor but a capacitor.
So we have the feedback network as in the real integrator. Also the input capacitor is switchable for gain adjustment. If we close the switch at the input we have a larger input capacitor, is it changes the gain of the system.

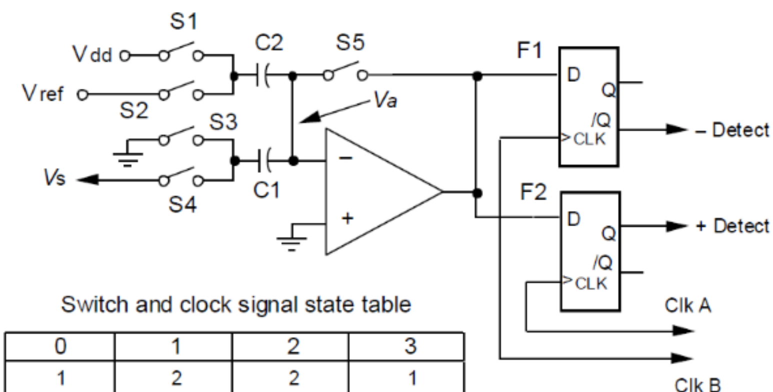
COMPARATOR

A comparator is a circuit which provides to the central unit of the pacemaker a digital signal corresponding to the arrival of a signal. For instance, if we are recording and atrial pulse, we would

like, thanks to the comparator, to provide a timing information that we can use in the synchronous pacemaker to stimulate the ventricle.
It is quite convenient to make a **comparison with two thresholds, one positive and the one negative**. We do so because we explore the bipolar nature of the signal. Typically, both ventricular and atrial signals show indeed a bipolar shape. So we pretend the trigger is generated only if the signal overcome a positive and a negative threshold.

Having a true comparation only if the signal exceeds a positive and a negative threshold makes the detection more robust against noise. Noise can of course trigger occasionally by crossing the threshold, but it is rare that the noise may simultaneously fire the positive threshold and the negative one, only the signal does so. So the double comparator is a solution useful and robust against noise.

The other starting information is that also for this comparator we use a kind of switched capacitors topology. It is a comparator based on the redistribution of charge over two capacitors and the redistribution is determined thanks to the switches that connect the plates of the capacitor to the reference voltage Vref, Vdd, ground and the signal we want to discriminate. Thanks to the alternate activations of the switches we change the distribution of charge, the voltage in the middle point and then we have an opamp that when S5 is closed it is a buffer configuration, but if we open S5 we have an open loop opamp that senses the input and supplies a positive or negative voltage depending if the input voltage is above or below zero (if input<0, output positive).



Switch and clock signal state table

0	1	2	3
1	2	2	1
4	3	4	3
5	Clk B	5	Clk A

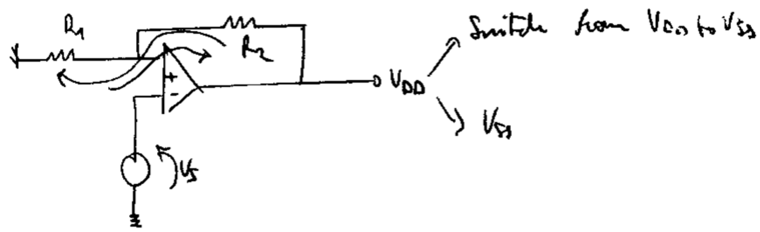
The information is recorded in the two flipflops and when we want to check if we have detected a pulse below the negative threshold Qnegato (Q inverted) should be high, conversely if we check if we have crossed the positive threshold the info will be stored in the logical value Q.

So the circuit is composed by a slot of switches and capacitors which depending on the value of Vs store on the (-) node a value either positive or negative and then the info is provided by the flipflop if the signal is above the negative or positive threshold.

We use this complicate structure because it is based just on dynamic charge transfer of the capacitor, there is no static current flowing anywhere (except for the one in the internal stages of opamp, but it is kept very small).

If on the contrary we take a Schmidt trigger, we have a loop closed on the positive input and we compare the voltage of my signal with the node at +. In this case the output can be Vdd or -Vdd, and we have a static current flowing in the feedback network.

20.

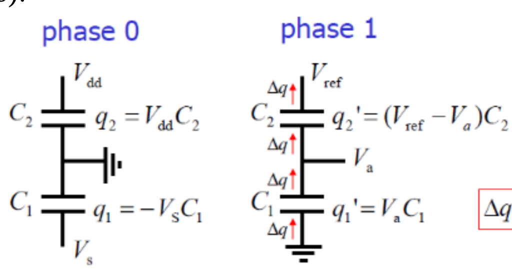


The other comparator is more complicated but not characterized by any static current → good if our goal is to save power for a lot of time.

Analysis

The switches are close according to the phases of the circuit. At **phase 0** we have S1 closed, S4 closed (S2 and S3 open) and S5 closed. If S5 is closed, the opamp is a buffer and the node (-) will be at 0V (virtual ground).

If we take the resulting network made by Vdd, C1, C2 and Vs, the simplified representation is the following (phase 0):



$\boxed{\Delta q = (V_{ref} - V_a)C_2 - V_{dd}C_2}$

$\boxed{\Delta q = V_a C_1 + V_s C_1}$

after phase 0 and 1 $\quad \boxed{V_a = C_2 \dfrac{(V_{ref} - V_{dd})}{(C_1 + C_2)} - C_1 \dfrac{V_s}{(C_1 + C_2)}}$

/Q FF1 high if: $\quad V_a > 0 \quad \boxed{V_s < -\dfrac{C_2(V_{dd} - V_{ref})}{C_1}}$ negative threshold

The top plate of C2 is at Vdd, the top plate of C1 is at 0V. We can now compute the charge stored in the capacitors (voltage difference*capacitance); for C1 it is 0 - Vs. This is the situation at the end of phase 0. The output of the opamp at phase 0 is 0V. Hence the role of the opamp is to set the node to a virtual ground.

In **phase 1**, all the previous switches are released, including S5, so the node (-) is able to float, the loop has been open → open loop comparator.

Now the voltages applied on the capacitors at the end of phase 1 are like in the picture. Especially, the middle point, the common plate has been released, no more fixed to ground but it is floating → we called this voltage (unknown to us) Va.

This transformation of the series of capacitor affects also the charge, I'm expecting that new values of charge over the capacitors have been generated. The new charges are q2' and q1', due to the changes of voltages across the capacitors. Again, the two charges can be computed as voltage*capacitance.

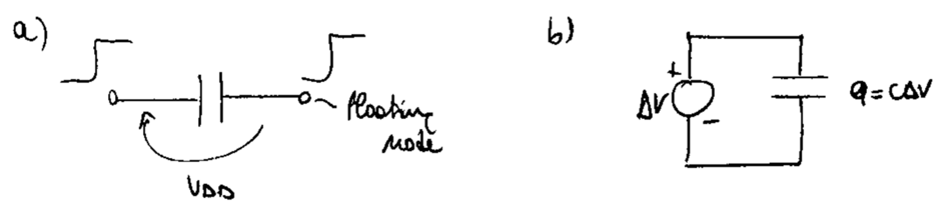
So I compute the new charges, and now I calculate the charge variation starting from the top capacitor C2. I can imagine a delta q charge has been charged over the capacitor, and it is given by the difference between charges. The same reasoning for C1.
Now I make a strong assumption: it is true that the charges over the two capacitors have changed, but is there any relationship between this changes across the two capacitors? Yes, since if the node (-) has been released (no more connected to ground), any electron leaving the bottom capacitor can only flow in the upper capacitor. There is no an alternative way; this is the reason why delta q is assumed to be equal everywhere, because there is no charge that can leave the C1 and not go in the C2.

This assumption that delta q on the top is equal to the one on the bottom allows me to make an equation between the two relationships. Hence I can compute Va. In the expression I have the values of the capacitors, the difference between reference voltage and Vdd (by changing Vref we change the threshold) and the signal.

When Va is larger than 0?
Using the equation, it is when Vs is lower than a certain negative quantity given by the formula (because Vdd>Vref). So since the right part of the inequation is negative, we have a negative threshold. The output of the opamp will be zero and so Qnegato = 1. I have detected the signal being lower than a negative threshold. Hence if Vs is negative and below a given threshold, Va is larger than 1 but the output of the amplifier will be at the logical level 0 and Qnegato = 1.

NB: when we release the node (-), we have the two capacitors C1 and C2 in series and so they are equivalent to a single capacitor. It is a different situation then when one plate of the capacitor is able to float, because in that case we don't change the voltage across the capacitor, we simply shift the voltage on the plates (a). But here by changing the voltage across the capacitors we are changing the charge (b).

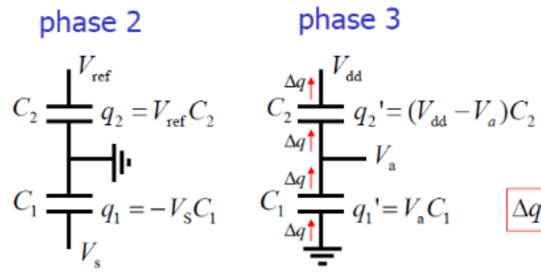
21.



IMPORTANT: the changes of the switches from 0 to 1 seems simultaneous, but it is not so. It is important the switch S5 is released before the changes of the other switches. Indeed, if we still have the node (-) grounded while the other are changing, it is no more true the hypothesis that delta q is equal everywhere, because if for one nanosecond the node is still grounded, we have by the Kirchhoff law, some charge that can take the direction of the ground (virtual ground, indeed). So it is of paramount importance that slightly before changing the voltage across the C we release the node (-).

The result is that after phase 0 and 1 we have eventually tested the negative threshold. If by monitoring the pin Qnegato after phase 1, if it is 1, we now the signal is below the threshold.

In phase 2 and 3 we do the same, but we monitor the logical output Q.



$$\boxed{\Delta q = (V_{dd} - V_a)C_2 - V_{ref}C_2}$$

$$\boxed{\Delta q = V_a C_1 + V_S C_1}$$

after phase 2 and 3 $$\boxed{V_a = C_2 \frac{(V_{dd} - V_{ref})}{(C_1 + C_2)} - C_1 \frac{V_s}{(C_1 + C_2)}}$$

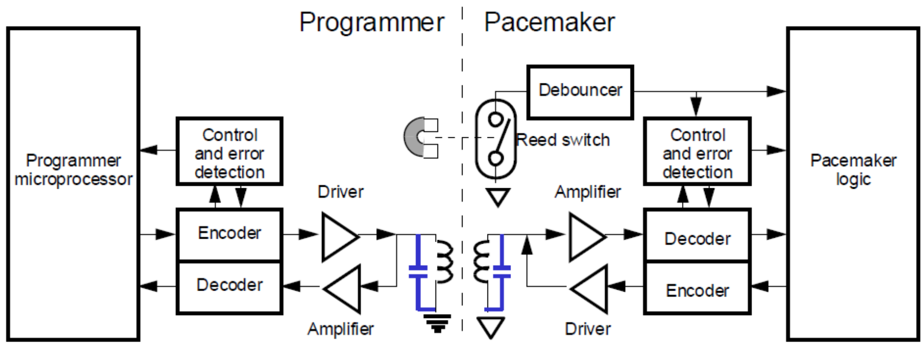
Q FF2 high if: $V_a < 0$ $$\boxed{V_s > \frac{C_2 (V_{dd} - V_{ref})}{C_1}}$$ positive threshold

Phase 2 is not like phase 0, the voltages are different, the only thing in common is the grounded voltage in the middle. The same reasoning for phase 3.
We compute a new formula for Va. Now the question is: when Va is negative? If it is negative, the output of the opamp is 1, D is 1 and Q is 1. This is true when Vs is larger than a positive threshold. If so the comparator will exhibit Q = 1.

NB: both positive and negative threshold can be changed with Vref. Its role is to narrow or enlarge the threshold window, depending on the noise level. This can be done since Vref is inside both the formulas for the thresholds.

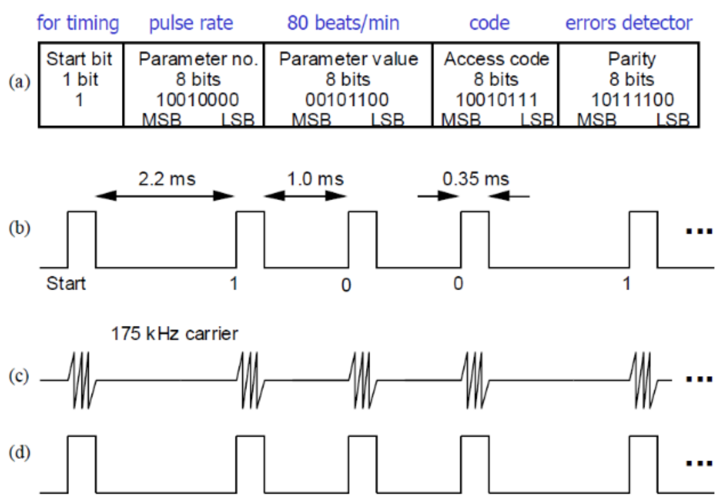
The two flipflops are investigated separately, with two different clock sources, for this reason we have two of them and not only one. We have to investigate the flipflop only when we apply the corresponding sequence of switches corresponding to a certain question.

TELEMETRY FOR EXTERNAL PROGRAMMING



Telemetry means that it is true that the pacemaker is placed in the body of the patient and no more accessible, but we may need to reprogram it, and this is done externally by communicating through a programmer, and programmer and pacemaker uses a radiofrequency link. This means we have an oscillator on the programmer (composed by a capacitor in parallel to an inductor) and the radiofrequency wave is coupled to another resonating LC circuit. So we apply a carrier on the programmer side and it is detected by the other circuit inside the pacemaker.

The information is a binary information, they are encoded and a driver is supplying the data. The carrier generates a sinusoidal wave that is not permanently established, we have intervals when it is on and others when it is off → burst of oscillations. The information about the code we want to deliver, it is not an amplitude modulation, but a time modulation. This means that if the separation between two consecutive bursts is of a certain duration, it means we are transmitting a 1 or a 0.

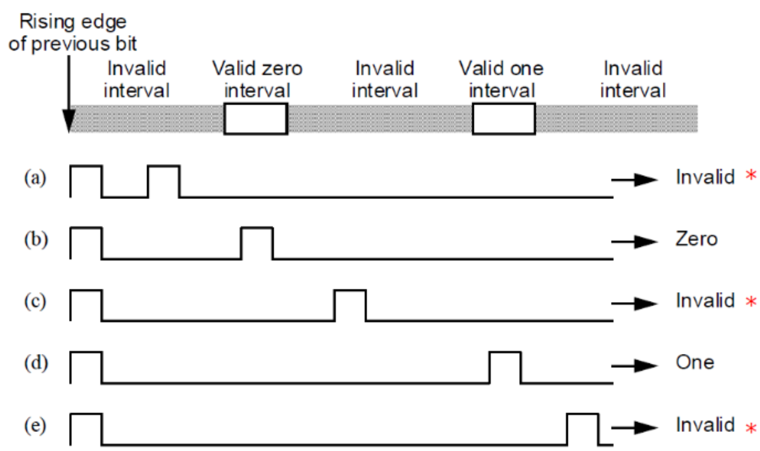


A sequence of 1 and 0 is coded as relative time distances between consecutive pulses. This is encoded in the encoder, and the driver let the oscillator to transmit this sequence of burst. Then the receiver has a collecting antenna, collects the carrier, converts the carrier into logical pulses from 0 to 1, the distance between pulses is identified by a decoder and so the internal unit of the pacemaker is reprogrammed.

Of course this path for the information can be provided also in the other way, from the pacemaker to the programmer. The information is converted in a stream of digital data, the digital data are encode, the antenna is driven in order to provide the bursts and on the other side the programmer detects the arrival of the info, they are amplified and decoded.

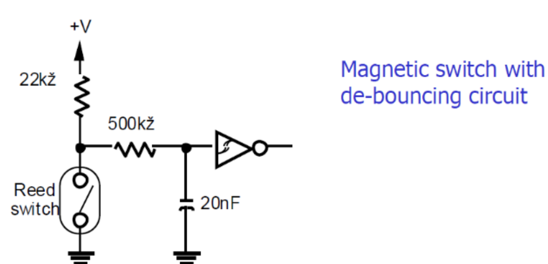
We have a starting bit. Then we may have a sequence of 8 bits that corresponds to a given parameter, for instance to the ‘pulse rate’. Then we want to transmit the next 8 bits on how much to change the pulse rate. Then we may have a code, a key because when the system acknowledges the key it changes the parameter and then we have a parity detector, to assess I something went wrong in the transmission.

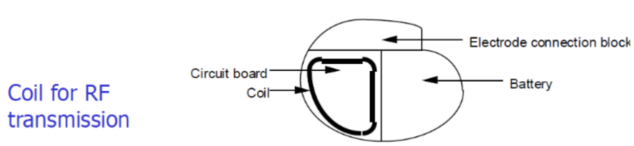
In reception, we detect the rising edge of the previous bit and so the receiver is opening some monitoring interval. If the next pulse arrives in the interval set 1ms after the previous one I’m detecting a 0, if a pulse arrives in the valid window set after 2.2ms from the previous one I’m getting a valid 1, erratic pulses everywhere else are discarded.



* The whole message is discarded

Of course, the pacemaker is not always in the programming mode, we have to switch it into the programming mode thanks to a **relay**, that is closed thanks to a magnet. We have the so-called debouncing circuit (filtro antirimbalzo): when we have a mechanical switch and we close it, the metal wire can bounce multiple times, so we don’t have a single strong transition but a bouncing → we use a low pass filter so that only the steady closure of the switch is recorded.
On the bottom right we see the case of the pacemaker. It is composed mainly by the battery and the coil and the electrode connection. So the coil for the radiofrequency link is a relevant volume.



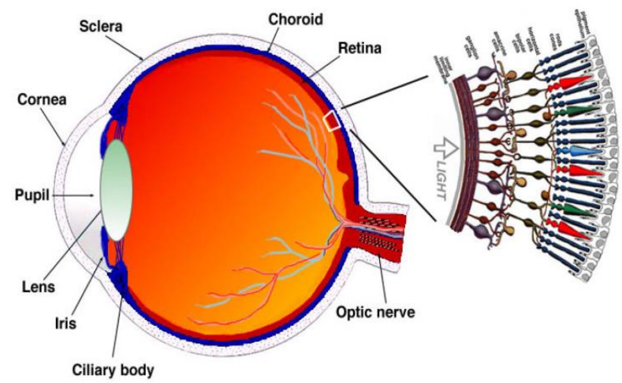


ELECTRONICS FOR ARTIFICIAL VISION

There can be a damage of the photoreceptors in the retina → I implant in the surface of the retain (the more easily accessible) an array of microelectrodes that can be suitably stimulated. The dimensions of such electrodes are limited.
The image which is supplied to the electrodes is recorded externally by a camera and then the image is processed by the camera and transmitted to an internal chip for instance by a radiofrequency link.

Eye anatomy

The image is collected by the pupil and focused on the bottom surface of the eye where the retina is placed. The retina converts the photon image into an electrical signal. As long as the retina is damaged, we can apply prosthesis on the retina region. If it is the optical nerve that is damaged, we have to bypass the nerve.



The retina is composed by different layers of photoreceptors that are embedded in the deeper region of the retina and closer to the surface there are layers of processing cells → light needs to penetrate through the initial layer. The internal layers are composed by two types of photoreceptor, the cones (that are more involved with high intensity of light and colour visualization) and the rods that are involved in the intensity light level.
Once the light is converted into an electrical signal there is a number of different cells which are processing the signal and veiculating such signals to the optical nerve. This is not just transmission, but already elaboration.
A prosthesis which could be implanted in the photreceptor area would profit of the intrinsic capability of the layer to process the signal, but if we implant it directly on the retina we are bypassing the cells and also the processing of the image.

CLASSIFICATION OF PROSTHESIS

Epiretinal prosthesis

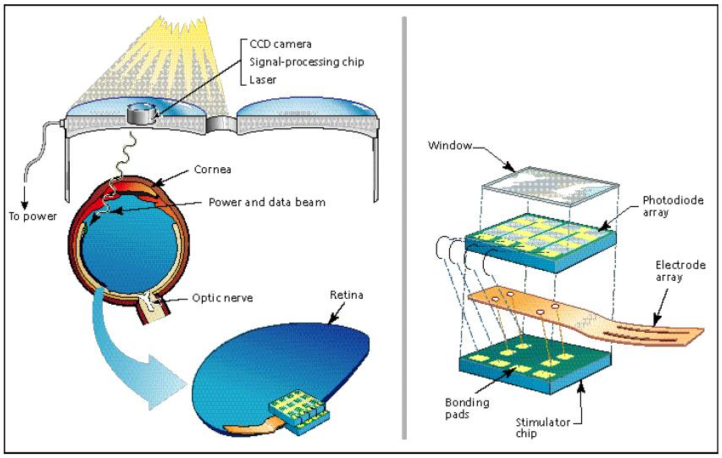
Electrical stimulation on the layer of ganglion cells

- Limits: loss of signal pre-elaboration carried out by the previous cellular layers .
- Need of an external image processing device (DSP).
- Examples of realized prototypes: MIT-Harvard device, MARC project.

Implanted just on the surface of the retina, not touching any of the layers in the depth → the processing of the image should be already done. Examples: MIT-Harvard device and MARC project.

The prosthesis has a video camera that receives the images (for instance the camera is applied to glasses), there is an external video processor that processes the images, and the signal is transmitted to the internal chip, either by a radiofrequency link or by an optical transmission. Once the information is received, a stimulator generates pulses sent to an array of microelectrodes.

The external unit includes also a laser that transmits for example the stream of bits with which the image is encoded and then it transmits through the pupil and there is an internal receiver (shown on the left) composed by an array of photodetectors, because they have to convert the information sent by the optical link into electrical signal. Then the stimulator chips provide the stimuli through a flat cable through an array of electrodes placed in the retain and stimulating the surface of the retina.



The difference with the MARK system is that the information is transmitted via a radiofrequency link in the MARK.

Subretinal prosthesis

- The prosthesis substitutes the layer of degenerated photoreceptors.
- The pre-processing role of the following cellular layers is maintained.
- The prosthesis is made out of an internal unit (relative design simplicity).
- A photodiodes array converts the light into electrical pulses which are transmitted to the healthy cells through gold microelectrodes.

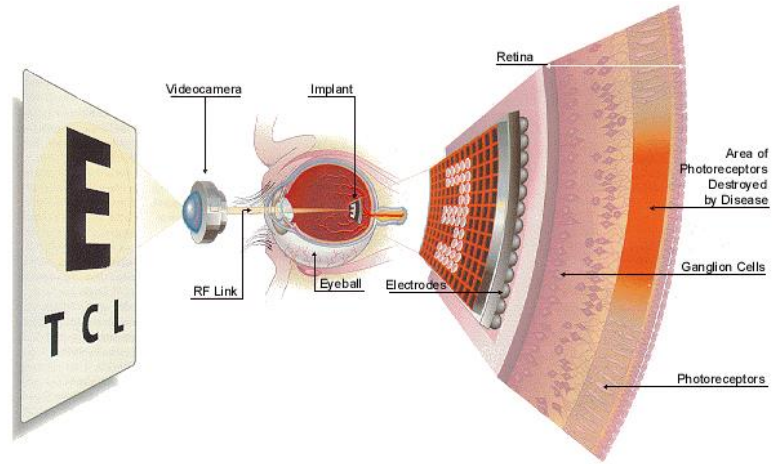
Implanted in the region of the photoreceptors (logical if the pathology affects the photoreceptors). It is composed by photodiodes that convert the light into electrical pulses.

Cortical prosthesis

- The prosthesis is made by an array of microelectrodes which is placed in contact with the cortical tissue.
- A portable computer processes the images and correct them for the non-linearity of the retina-cortex map.
- The external unit is connected to the internal one by a RF system or by a transcranial interconnection.
- These prostheses offer the advantage to cope with pathologies which affect the optic nerve (glaucoma).

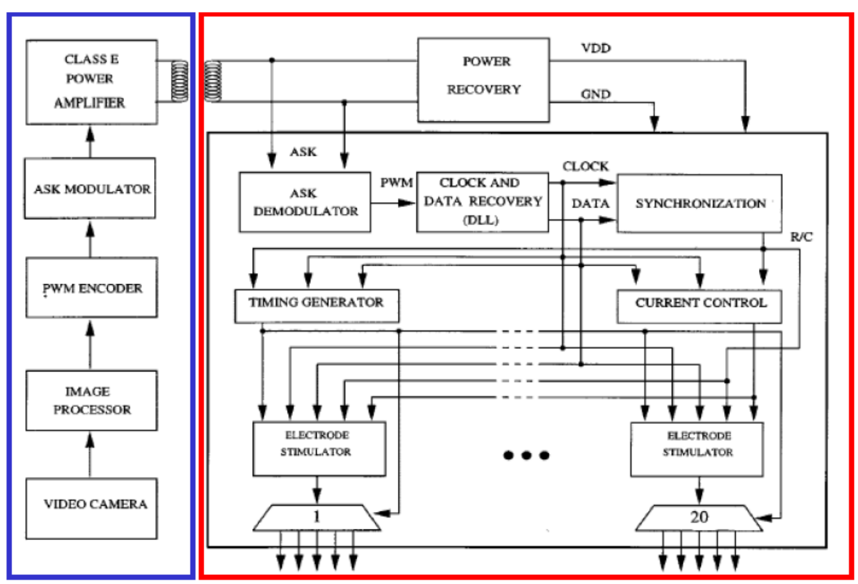
It is a prosthesis implanted not in the eye but directly in the cortical tissue. In terms of the processing chain it is a prosthesis much ahead in the chain it bypass also the optical nerves, not only the retina. It is mandatory if we have a pathology affecting the nerves such as glaucoma. There is the need to communicate externally, either via transcranial interconnection or by means of a radiofrequency link.

MULTIPLE UNIT ARTIFICIAL RETINA CHIPSET (MARC)



It is an epiretinal implant, composed by an array of electrodes implanted on the surface of the retina → they are directly stimulating the surface of the retina bypassing the photoreceptors. The peculiarity is that the image recorded by a video camera is transmitted into the implanted chip through a radiofrequency link (we have a receiving antenna in the chip) and this link is transmitting the power → powerless system, the energy is provided via the link.

The following is the structure.



IMPLANTED STIMULATOR

We have the external components: video camera, image processor, the encoder; the encoder is used because the digital information of the image is encoded in the PWM coding system, then the info is amplitude-modulated, then we have the external transmitter and an internal receiver.
Internally (region marked in red) we have a unit recovering the information and the power. The radiofrequency link allows to record the info but also the power; we have a diode bridge that is rectifying the carrier and is producing Vdd and ground then supplied to the rest of the electronics. It is a complete self-standing system, with no internal battery.

The information is demodulated, the PWM waveform is recovered and then from the PWM waveform the classical stream of 0 and 1 are decoded and then supplied to some units that extract the amplitude and time information and provide them to some blocks, the electrodes stimulator, and then the electrodes stimulator supply the array of electrodes.

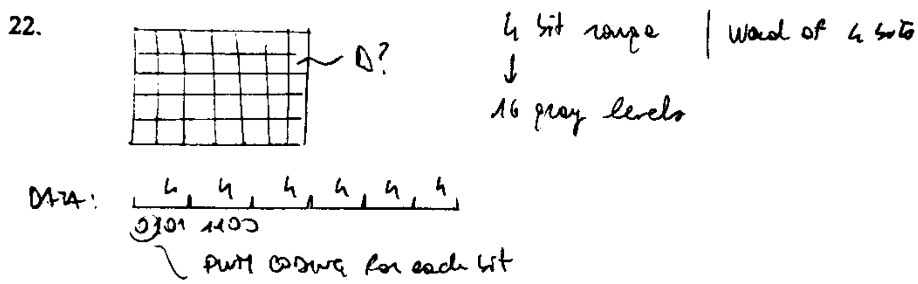
In this example we have 100 electrodes (10x10 array). The electrodes are grouped in 20 groups of 5 since the stimulator is not stimulating all the electrodes simultaneously but only 1 electrode in a group of 5. Of course, this should be done sufficiently fast to provide a regular sensation of motion of the picture.
The carrier frequency is in the order of MHz.

PWM coding

The idea is not to transmit the data as a regular sequence of 0 and 1 with some amplitude modulation, but the data, bits, are encoded as duty cycle of a PWM square wave. We have a square wave where each period the DC encodes the 0 or the 1. A 0 is encoded with DC 50% and 1 with a DC < 50% and then if a 1 has been encoded with e.g. 40%, the next one with a DC larger than 50%, for example 60%. So the 1 are encoded as alternating 40 and 60%.
Moreover, the clock is detected by the risetime of the waveform. Indeed, independently form the DC, the rise time follows the duration of the period. The device must be rise-time sensitive, such as a flipflop sensitive to the rise.

We are considering the bits of an image already processed.
The goal is to provide a stimulation to a number of pixels. The question is: which is the data to be provided? The image is in black and white and provided with an intensity of 4 bits range, so 16 gray levels. So the data to be supplied to each pixel will be a word of 4 bits.
The conclusion is that we have to supply to each pixel these bits, so the data will be composed by a sequence of 4 bits, serially provided to the matrix. We deal with the sequence of bits and each bit is encoded with a PWM encoding.

22.



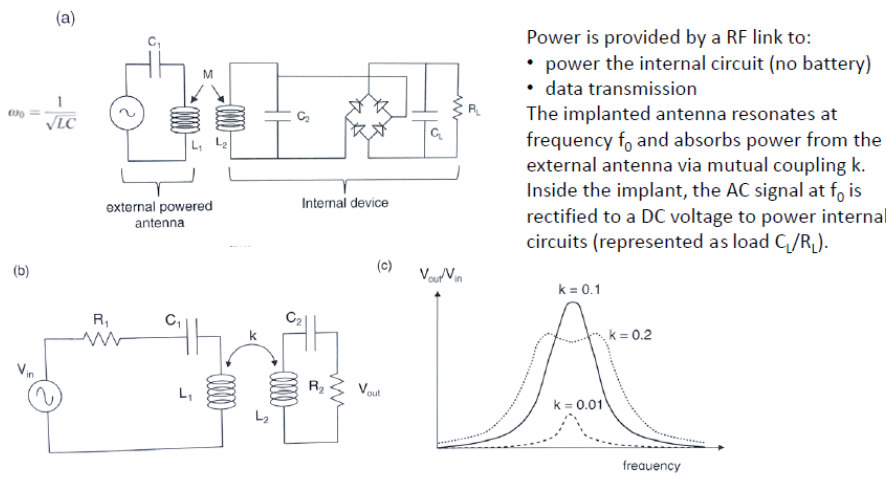
Once we have encoded the stream of data into a PWM encoding, we have to modulate it. From the PWM I modulate a carrier; I take a carrier, for instance a 1MHz carrier and I modulate the amplitude according to the PWM encoding. This carrier is then transmitted inside the eye.
The primary coil is the antenna and the secondary coil is collecting the carrier. The internal chip has to extract from the carrier the Vdd and ground by means of a rectifying circuit.

Let's suppose we are rectifying the power supply from a carrier modulated directly from 0 and 1 and not by a PWM. If we have e.g. a long sequence of 1 and then a long sequence of 0 and we have an envelope detector that is taking the Vdd as envelope of the carrier, if we have just sequence of 1 and

0, the Vdd is not regular, but it changes according to the sequence of bits. On the contrary, if each bit is encoded in the PWM encoding, in each period we have a high amplitude and a low amplitude, so the average value we may extract from the envelope is more constant, since the effective voltage is the same.
This is the reason why alternating 60% and 40% means that in average even the 1 carry the 50% in average of effective voltage, as the 0. Hence even if we have a sequence of 0 we have a sequence of 50% DC, the same → more stable extraction of Vdd when we convert the carrier into a constant voltage.

How is the carrier generated and coupled?

The following is a general architecture. We have an external oscillator, and LC oscillator where the L works as an antenna. It oscillates at a frequency omega-0. Then if nearby we have the inductor belonging to another LC oscillator, there is an electromagnetic coupling of the two oscillators and so the carrier is received by the second unit.

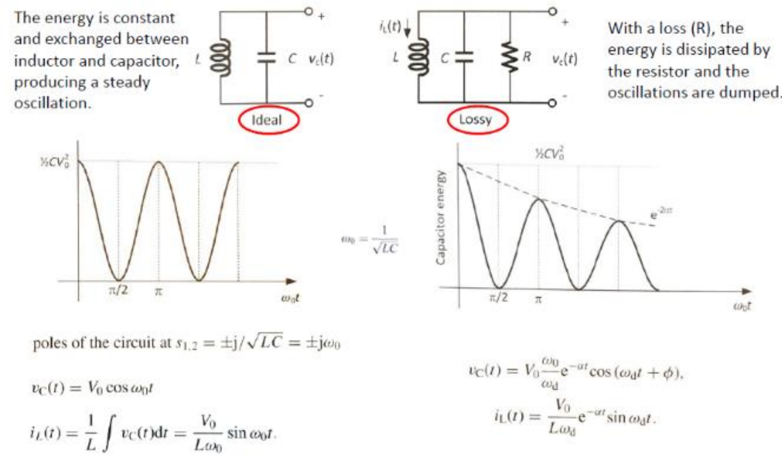


(a) An illustration of an external antenna coupling energy into a wireless implanted device, with diode-based rectification to produce a DC voltage. (b) Circuit diagram representing two coupled resonant circuits. (c) The voltage gain transfer function (and therefore the efficiency of power transfer) depends upon the value of the coupling coefficient, k, between the transmit and receive circuits. The maximum value corresponds to 'critical coupling', in this case k = 0.1.

Since the two antennas are coupled, it is possible to measure a voltage Vout which is the result of the input voltage provided to the oscillator that thanks to the coupling is transmitted to the second one. On the right plot we see the ratio of transfer. It is maximal at the frequency for both the oscillators. If the two networks have an intrinsic oscillation frequency, it is where the transfer i/o is maximum. The transfer is more effective if the coupling between the two antennas is good, and coupling depends on distance (the closer the better) and orientation (the two coils should be parallel).
So the k factor represent the effectiveness of the coupling between the two antennas.

The carrier carries the power and the information, the amplitude modulation carries also the information (two roles). Regarding the power, one typical way to extract a continuous voltage to be supplied to internal circuits is to use a diode bridge with a rectifier done with capacitors.
So on the output load R_L we have a voltage with some residual ripple that we define according to the time constant $C*R_L$.

THE BASIC LC CIRCUIT

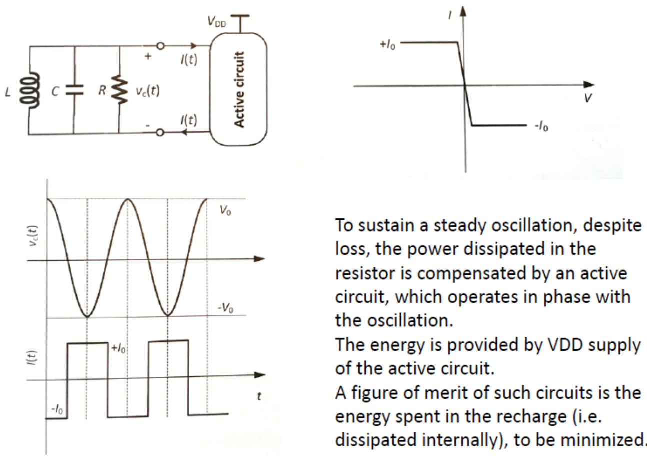


The basic LC circuit is the one on the left, in its ideal form. If we charge the C at the beginning and we do nothing and we measure the voltage at the output port, we see the voltage oscillates. It oscillates because the energy/charge stored initially on the C is transferred to the inductor L that when has a current flowing in it stores energy (of a different type than C9 and then the energy is given back to C). They continue to exchange energy and we have an oscillation at a specific frequency on the output. We can see that if we compute the poles of the network, we have two imaginary poles at omega-0 given by 1/sqrt(L*C).

However, in reality any oscillator, either if we desire it or because of parasitic components, it has a resistive component. While C and L are exchanging each other energy without dissipation, if a current flows in a R we have dissipation of energy due to Joule effect → no more true that R and C exchange the full amount of energy, since it is lost with a time constant alpha related to the electrical parameters of the circuit. The smaller the resistor, the more damped the oscillations. This because if we have a voltage on the R, we have a larger current and so the power is i^2*R → more dissipation.
If R is going to be infinite, we are close to the idea oscillator, if it is a shortcircuit all the charge on the C is lost in the shortcircuit.
So on the output voltage we have no more only a sinusoidal oscillation, but a damped one.

Hence the oscillator looses its energy into the resistor, and if I do nothing I cannot maintain the oscillation for a long time. So we put in parallel to the circuit an active circuit which refill the energy into the oscillator that is dissipated through the resistor.

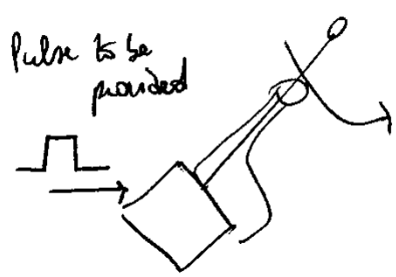
Active compensation for the loss of RLC circuit



The active circuit has generally the t.f. on the right. If the voltage is positive, the network supplies a negative current. When the voltage is negative, the circuit supplies a positive current (see diagram below).
This because thanks to this delivery of the current, the circuit is providing the energy lost in the resistor. Indeed, if we have a positive voltage in the resistor, the current flows from the top to the bottom, and this occurs if the current flows from the right to the left in the branch connected to the active circuit.

Hence this active circuit cannot provide the current at its wish, but it should supply the current exactly at the intrinsic frequency of oscillation of the oscillator.

23.

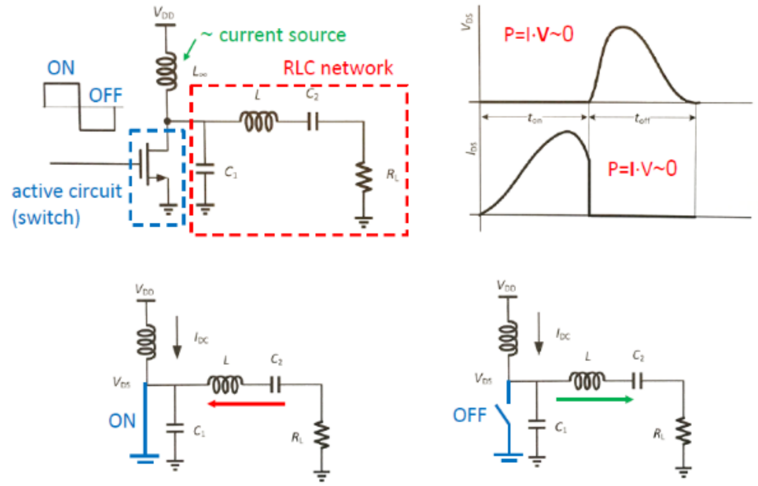


It is the principle of a swing. If we have a swing for children, we push the child because we want to restore the energy lost in the oscillation, otherwise the child will continue forever to swing. So we provide a pulse, but we supply a push when the swing is going in the right direction. It is the same principle for the active circuit, we supply with the network the current in phase with respect to the intrinsic oscillation of the circuit.

In some way, we have to drive the circuit in the right way and supply the energy at the right time. The energy lost in the network is given by the power supply of the active circuit. We have simply to take care to operate carefully.

In this prosthesis is used the **class E amplifier**.

CLASS E AMPLIFIER



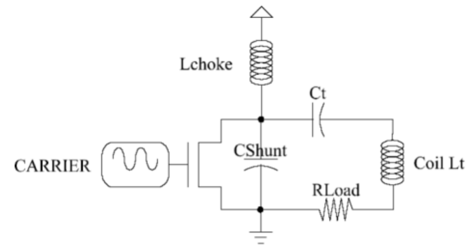
The higher the letter, the best. The figure of merit is the energy spent in the active circuit itself. Indeed, the circuit veiculates and energy in the network, but the class is better if it minimizes the energy lost by the active circuit itself. The class E are the best one, almost with zero dissipation.

The active element which transfers the energy is a transistor that is switched on and off, and the frequency to do so is the intrinsic frequency of the network (we assume the RLC network has an intrinsic frequency). On the top we have a large coil (inductor) that acts as a current source.
So the three elements are: the intrinsic oscillator, an inductor connected to Vdd that acts as a current supply and a switch that works like a red and green street light, letting the current to go to ground or to charge the network.

Below we have the two operating points. On the right we see in green the direction of the current. Since it flows from left to right, it is the moment when the switch is opened and we let the current generator to refill of energy the network. The other time is when, intrinsically in the network, the current flows in the other direction (red arrow, left), because it is not the right time to give a push. So the current is directed to the ground and not to the RLC.

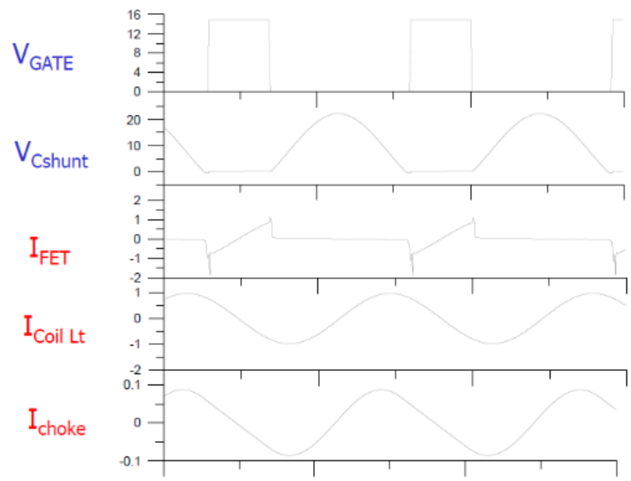
If now we take the diagrams of the voltage Vds across the switch vs time and we plot the corresponding current, in the two phases when the transistor is on we have current in the transistor but the voltage is zero, because drain source voltage goes to zero, so we have ideally zero voltage. Hence the power (V*I) dissipated in the transistor is 0 because the voltage is 0. In the other phase when the transistor is off, Ids is 0, we have a voltage swing across Vds, but still the product is zero (power). Hence we have redirected the current but with zero power dissipated in the transistor.

Class E amplifier in the MARC system



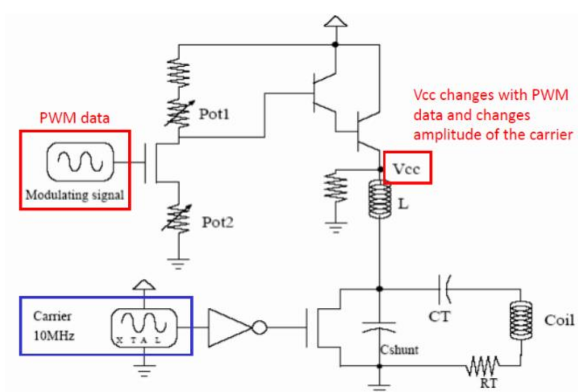
The coil Lt is the one used for transmission (the horizontal one in the previous scheme); we are interested in the current passing in this coil because it creates the electromagnetic field.

The transistor is modulated by the carrier, that is the one we use for transmitting information.
As for the relevant waveforms, we have as follow.



The gate is switched on and off regularly, then Icoil is the current produced on the coil and we see that the current is in phase with the switching on and off of the gate.
The relevant voltages to evaluate the power are the one across the transistor and the current in the transistor. The voltage across the transistor is Vshunt; it is zero if the transistor is one, while when the transistor is off the current is zero. there are also some point in the transition between the two phases where we may have some V*I != 0, but overall when the current is 0 the voltage is != 0 and viceversa.

So we have created the oscillations, but we need to modulate them with a PWM. This is done by driving the top voltage of the inductor. So up to now we have created the carrier frequency on the oscillator, while the change in amplitude is obtained by modulating the Vcc with a PWM signal.

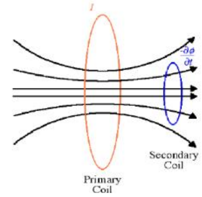


Thus we change the energy the inductor is supplying to the network. When Vcc is high, we have high amplitude in the oscillations, vice versa if it is low. So we modulate the amplitudes according to the

PWM data, and in particular to the DC. There also are potentiometers to change the gain of variation of Vcc.

As for the coupling of the coils, we can see that effectiveness of the magnetic field to couple between one coil (the primary one) depends on how much closer the two coils, are and how they are oriented.

The absolute amount of coupling depends also on the dimension of the coils.



Coupling between coils

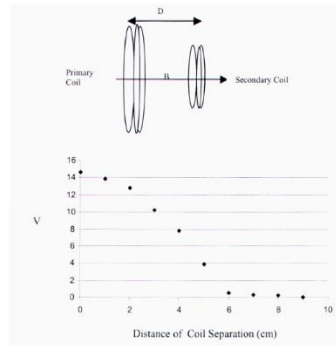


Figure 13B. Voltage vs. axial coil displacement.

ENVELOPE DETECTOR

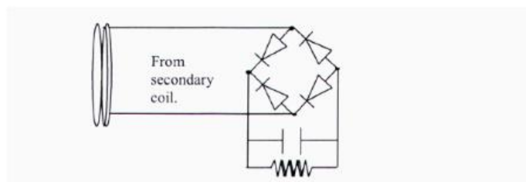
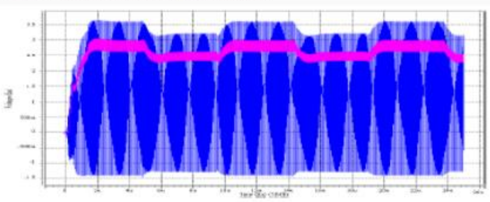


Figure 13A: Full-wave rectification of MARC2 telemetry signal. V_R is measured across a 1K resistor.



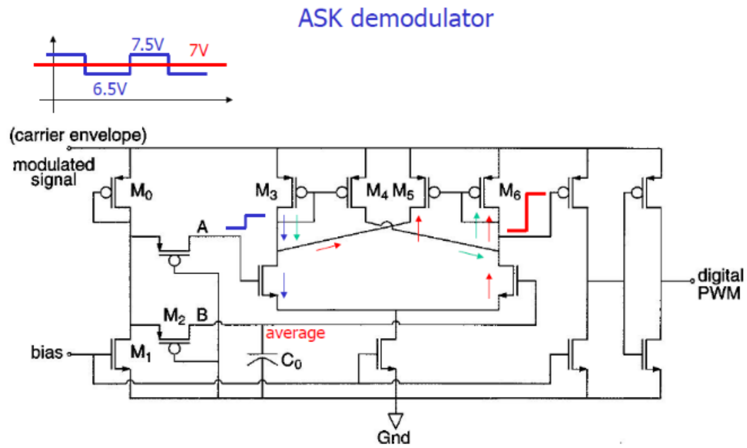
We are after that the carrier has been coupled to the internal coil. How to recover the information and the power?

To the secondary coil we have a full wave rectifier connected, that is a circuit that starting from a carrier determines the envelope of the oscillations (pink). It is the average voltage across the capacitor; it is not a fully stable voltage because we need to extract Vdd but also the information. If we have a hard rectification, we would have a constant voltage, and we cannot extract clock and data.

However, from the pink one we can there will be also circuit to further low pass for power supply, but we still have to keep it with oscillations for the information.

Now we leave apart the Vdd business and we concentrate on getting the data.

ASK DEMODULATOR



The pink voltage (upper left, blue plot) is quite inconvenient, because with respect to a standard PWM it is not a Vdd to 0 signal; it holds the info on the duty cycle, but it ranges between two close values → we need to extend the swing of this waveform, to let it go from 0 to Vdd. This is done by **the ASK demodulator**.

It receives at the input the modulated signal; it makes a comparation. It takes the signal in blue and passes it through a transistor in triode regime (as if it was a resistance). Then the same signal in blue is passed through an identical MOS M2 also in triode regime, but in addition we have a capacitor. When we have a C after a resistor we have a RC network, so a low pass filter that gets the average of the voltage.

The concept is the following: I take the signal that ranges e.g. from 6.5 and 7.5 and pass it through a resistor. Then the same signal passes through an RC network (below). Now I have a low pass version of the signal in output. If the time constant is large enough, at the output of B I have the average value of the signal (out of A we have the same signal).
Now I give this low pass signal to a comparator made by a single differential amplifier with a load (DISCRIMINATOR). An input goes up, the other input goes down, we have a current mirror in the upper part, so it is a stage that mirrors the current from the left equal to the right so in node x we have the sum of the current flowing from the bottom and the one flowing from the top. If we have a load r0 we have an output voltage.

That was the basic structure for a discriminator; if we have a differential voltage at the input, the two transistors imbalance in the lower part, one carries more current than the other and then the mirror flips the current and at the output we have an amplified voltage.
Thanks to the ASK demodulator, that is a comparator between the blue envelope signal and its average that has been low pass filtered in the channel below. It is a positive comparator with a positive loop to speed up the pulse.

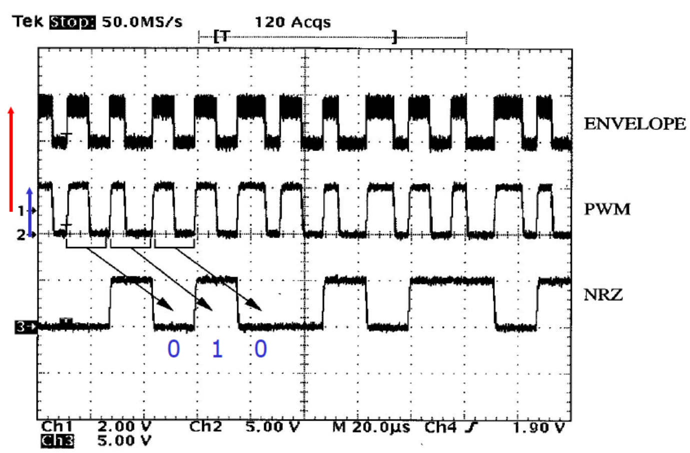
The goal of this stage is to give an output voltage when we have a signal going up with respect to a signal going down. So I feed this stage in the hand-picture with the two signal out of A and B. If A is larger than B, the output of the discriminator will go up. When A is lower than B, the discriminator will go down.

In conclusion, I've taken the waveform, split it into two pathways (path B in extreme case is a constant average voltage), then feed to a discriminator.

The result is that if the input voltage ranges e.g. from 6.5 to 7.5, the output ranges from 0 to Vdd, because the amplifier is biased from 0 to Vdd. The waveform however still carries the PWM information, but with an extended swing.

Coming back to the real circuit, we have the path A made by a resistor implemented with a pMOS in triode regime (it behaves as a resistor). The transdiode M0 before is just a voltage translation, it is used to transfer at lower voltage the signal. In the middle we have the **comparator** and the output now is the voltage (in red) that is almost full swing and then we have a common source and an inverter (another amplifier stage) so that we are sure the output voltage is a rail-to-rail one.

It the upper plot we have the 'pink voltage', the envelope, pushed to high value. This trace is a trace where the 0 in the oscilloscope is one, the 0V is where I have the 1. The arrow in red marks that the envelope is displaced from zero. Hence the 1 is centered with respect to a zero level indicated by the blue 1. Then we have the PWM, that is the one at the output of the previous circuit. It moves from 0 to a certain voltage (Vdd). The average voltage of the envelope is equal to the average voltage of the PWM signal because the scales of 1 and 2 are different. The amplitudes are the same.



We see that the duty cycle is unchanged. We have changed the set of voltages at the output.

NB: in the middle we don't have the single mirror as in the drawing done by me, but it is a more sophisticated set of mirrors that creates a positive loop in the response of the discriminator.

Let's suppose now for instance we have a positive step on the left with respect to the average (in blue); so on the right we have the average, on the left a positive step. Consequently, we have the signal currents in blue and red. On the top, we have a pMOS mirror. So the red current enters in the mirror and it is mirrored as the red entering in M5. So in the node below M3 we have an exiting red current. This exiting current cannot come from the bottom, because we have the drain impedance of a MOSFET, so it comes from the top.
In green we have the current taken from the top mirror that is flowing and becoming red in the network of M5. The current in green is just the additional contribution of the current in red. The green current

is mirrored, so the transistor M4 supplies the current in green on the node below M6. Here the current in green again cannot go down but only up, enters into M6 and it is flipped again (as if it was red)

This loop can be continued forever. It is a positive loop because once the red current has started to enter into M6, after a full turn of the loop we have an additional red current in M6. So the current in red has stimulated a huge amount of current flowing in M6.

In the end, the voltage transition in the output node below M6 is the total current flowing in that node multiplied by the impedance of that node. If I would have only the red current, I would have a given transition. Since we have also the green one, with more current we have a steeper output voltage.

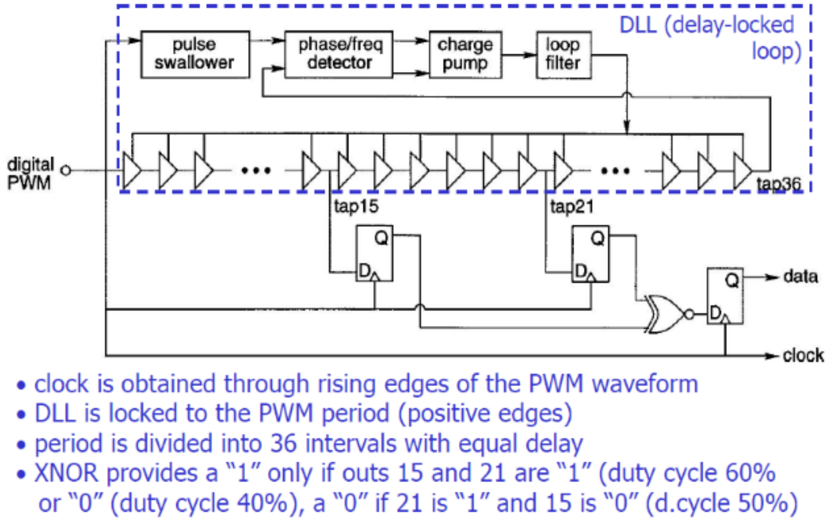
Hence the goal of the extra mirror in positive loop is to make faster the transition, because the positive loop pushes more current to enter in the output node. But is this a fast circuit? No, I don't need very steep pulses, but steeper is relative instead to the power consumption of a circuit.
If we want to have a discriminator and a pulse, the speed of the pulse depends on the current consumption of the stage, on how much current we have in I0, because the transconductance is proportional to the current. So we can have a faster discriminator If we spend more power in the circuit

We are talking of an implanted circuit, so I want the lowest possible power consumption, so to have the comparator to operate with the lowest possible current. If I reduce the current, the output of the transition would be very slow, so I boost up back the transition using this positive loop. So the goal of the positive loop is to speed up the transition and to keep low the power consumption, because I can reduce the current in the current generator of the differential stage (mosfet attached to ground in the image).

NB: in the ASK demodulator, the same blue envelop is used also as Vdd. We don't have a clean Vdd, but it is not a problem because the bouncing of Vdd is synchronous with the transition we are investigating. If we need a clean Vdd we need a further rectifier.

CLOCK AND DATA RECOVERING CIRCUIT

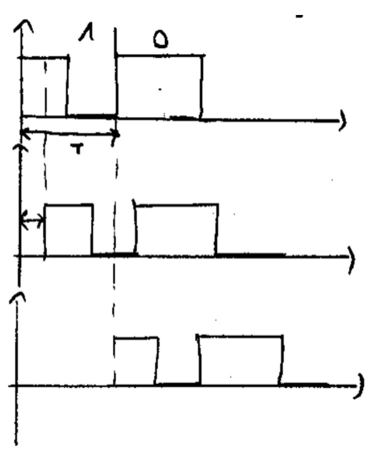
I want to convert the PWM signal into a serial stream of 1 and 0.



I need a decoder from the PWM waveform, and it is the role of the upper circuit. As for the clock, the clock is indeed the PWM. The rising edge of the PWM is totally synchronized with the clock. So I take the PWM and make all the digital block rise-time sensitive, so that all the logical transition will occur at the rising edge of the PWM.

To extract the stream, I use a battery of **36 buffers** (or **digital inverter**). Each inverter will introduce its own delay, and the delay can be controlled by controlling the power supply (the transition time of the response depends on the power supply). At the tap 36 we will have the start of PWM, but delayed. We take the delayed waveform and put it at the input of a phase detector, a circuit that is sensitive to the rising edge difference between input and output.

25.



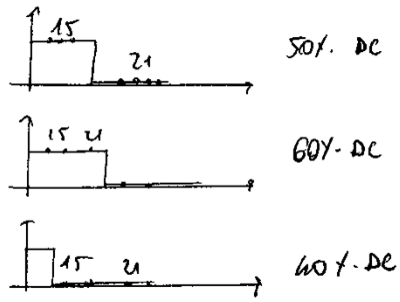
I consider 2 period (one for the 0 and one for 1). Thanks to the delayer, we created a delayed replica of the PWM. The phase detector is sensible to the original rise signal and to the rise of the delayed signal.

The inverter delay changes in a way that it is so big that we have achieved with a delay a full period. So at a given point the delay is one period. But if so, the difference between the delayed waveform and the next one of the original one is zero.
In this way the delay is stopped, we have locked a full period of delay.
Hence the delay is increased as long as the delay reaches a full period, we lock the delay up to one single period.

The delay is changed because the phase detector is sensitive to the difference in the rise of the original signal and the rise of the delayed one, and it changes the Vdd of the inverter.

Now, if I have locked a full period of delay, in each inverter I will have a kind of picture of the delay with 1/36 quantization. The period-delay is cut in 36 pieces.
Then I take two notable pieces, tap 15 and tap 21 to understand the duty cycle of the waveform.

26.



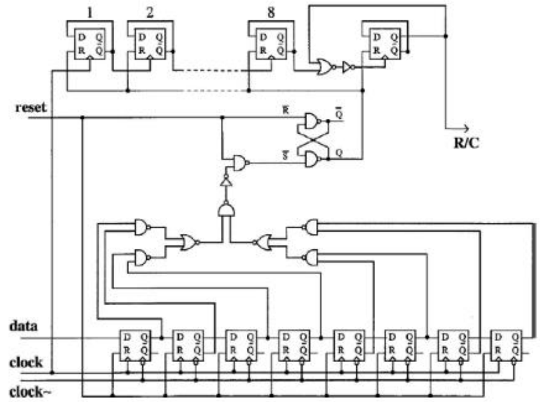
In particular, I have available in the inverters the tap 15 and tap 21. I compare the two with and EXNOR and if I am in the 50% of DC, the two will be different, so the result of the exnor will be 0.
If I have 40%, the tap 15 and 21 are after the edge, so the result of the EXNOR will be 1.
Instead, if I have 60% of DC, both the taps are high, so the result of the EXNOR will be one. The ‘1’ is detected hence both if I have 40% or 60% DC.

I need flipflops everywhere for synchronization, because I have to make the decision when I’m sure that a full period is stored in the set of inverters, I need to sense the output of tap 15 and 21 only at the rising of the clock.

Now we understand why the circuit provides my 0 when I have 50% of DC and 1 if I have 60% or 40%. We have latency by 2 clock cycles, so the 0 comes two clock cycles after the real zero because I have the latency introduced by flipflops, but it doesn’t matter.

We need the delay of 1 period because if not, there is no more correspondence between the information extracted from pins 15 and 21 and the period itself, because the DC is a period-dependent information.

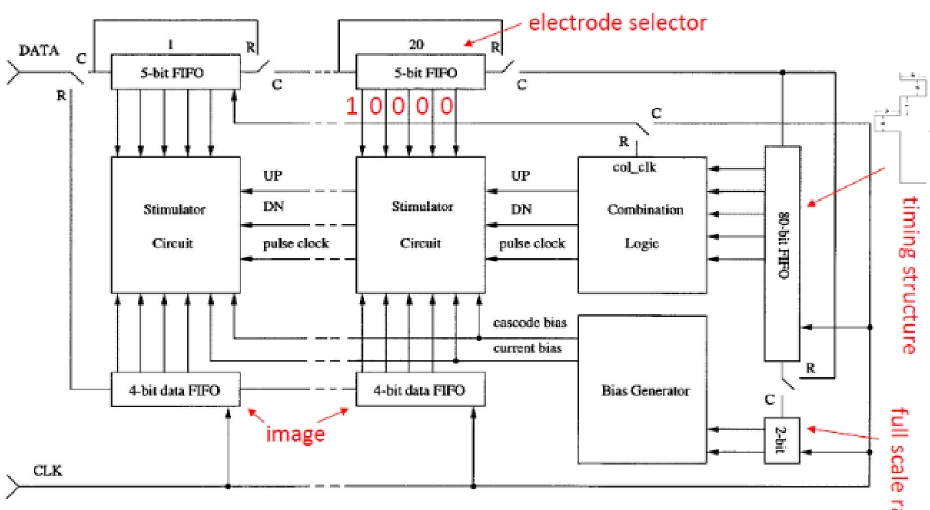
SYNCHRONIZATION CIRCUIT



It acknowledges a known sequence (key) and switches the chip from the configuration state (C) to the running state (R)

According to a known sequence (called key) of 1 and 0, this network recognizes as output a bit which is acknowledge as configuration state C or running state R. In correspondence of a specific key (word), the system switches to a configuration state labelled as C, in all the other cases the system is running in the running state.

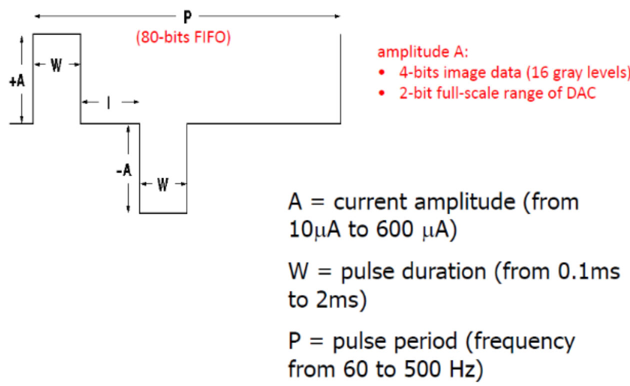
The following is the architecture of the stimulator. The data comes from the left, and we have a unique stream of data arriving to the block, but depending on being in configuration or running state, these bits take different path.
On the top we see a sequence of 20 FIFOs of 5 bits each one and in the configuration state (when we are feeding with data, the FIFO are all connected and programmed, because in configuration stage we are programming the chip, in running state we are providing the pulses) and the FIFO are connected serially (the switch C is closed, R is open).
<u>When we are configuring the bits, each one of the FIFO is an electrode selector</u>. The stimulator circuit stimulates only one electrode at a times basically, and this is done by configurating the FIFO. So if in the configuration stage we program the FIFO with 10000 (all the FIFOs), this means that electrode n°



The circuit purpose is to set the stimulating pulse parameters (timing and current intensity).
By changing the switches status (R/C) it switches from the configuration to the stimulation (RUN) state.
An additional 2 bit register specifies the used current range (200, 400 or 600 μA).
Frame delivery time = 80 clock cycles x 5 (electrode selections) = 400ck cycles (to be <perception time, 1/50s)

1 will be the first one to be stimulated. Then when the FIFO is closed in running mode, its output is connected to the input → the so-called **ring counter**: it is a FIFO that when a clock is applied, one bit is transferred to the following position. When the 1 has shifted to the last position it is fed to the first position again.
Hence the electrode selector has to provide the number of electrodes to be stimulated by the stimulation circuit (block in the middle). In the example, the stimulator will provide a pulse to electrode 1 and then to the others.
Of course, all the 20 FIFO needs to be programmed with 1 and all the other bits 0 (we cannot pulse two or more electrodes simoultaneously).

The 80-bit FIFO on the left, in configuration phase, is connected to the previous one (C is closed) and it is programmed with the time structure of the pulse. The time structure of the pulse is shown below.

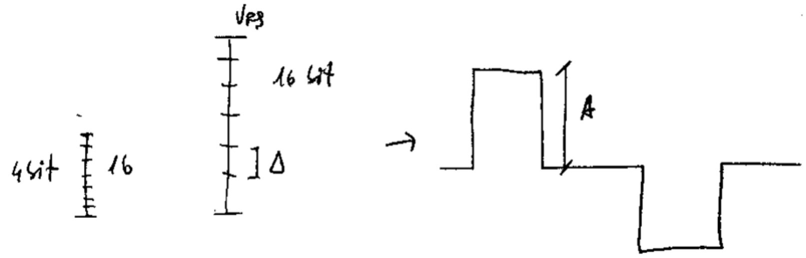


We see an entire period of the pulse and what happens in the period, which is described by a sequence of 80 bits. First of all, the first info is that the **pulse is provided through a biphasic pulse**. **Only one of the two will provide depolarization**, so the firing of an AP. The other pulse of the biphasic has the role that by injecting charge, we inject charge in one polarity and then in the other, so that **the net amount of charge injected in the tissues is 0**. So among the two pulses only one is strictly needed to generate depolarization, but in terms of charging the tissues, since the stimulator is providing a charge, the net amount of charge delivered is 0, otherwise the tissues will continue to grow a net amount of charge. This explains why in the period we have a positive and a negative pulse.

With the **80 bits we define the time structure**: the width 'w' of the pulse (how long we want the w to be, the system already know that it has to provide a positive pulse and then a negative one), for example we have among the 80 as many 1 as the pulse is long (e.g. 10 ones, 20 zeros, 10 ones and the rest zeros). So with a sequence of bits we define the time structure of the pulse.

What about the amplitude of the pulse? **The 'A' amplitude is supplied by the 4 bits defining the 16-gray levels**. These 4 bits are delivered in a different way.
They are taken from the path below, that is working only in the running configuration.
Moreover, we have to remember that when we supply 16 grey levels with 4 bits, we need to define a reference scale. So the full scale range of the pulses need to be specified.

27.



When we define the 16 gray levels with 4 bit, who defines the real amplitude of each step in the scale of possible 16 gray levels? The full scale range. Hence if we want to specify the pulse amplitude A, it is coded with 4 bit word, but we need also the define the full scale before, otherwise we are specifying one level but not the absolute value. This is the reason why we need to define during the configuration phase we need additional bits to specify the full scale range. They are 2 bits, in the last register → only 4 options as FSR (from 200 to 600 uA). We define it once forever.

The scale range is defined as current range, since stimulation is identified with current.

In conclusion, in configuration phase all the 4 FIFOs are serially connected and we sent into the chip a word of 182 bits. 100 bits are in the data FIFO, 80 are in the other one and 2 are for the FSR.
Now, when we switch from the configuration phase to the running phase, we see that the upper FIFOs are closed as if they were ring current (bits continuously shifted). The bits of the FIFOs are read by a logical block that reads the sequence and provides to the stimulation circuit two main command: go up or go down. It is not needed to provide a much finer information, just to give a pulse up or down.

Finally, the stimulator circuit receives the information together with a bias generator that tells the FSR, and so the stimulator circuit receives:

- Time information (go up or down)
- Addresses of electrodes to be stimulated
- FSR to be given (maximum current to be given)
- Image (in the form of a 4-bit word)

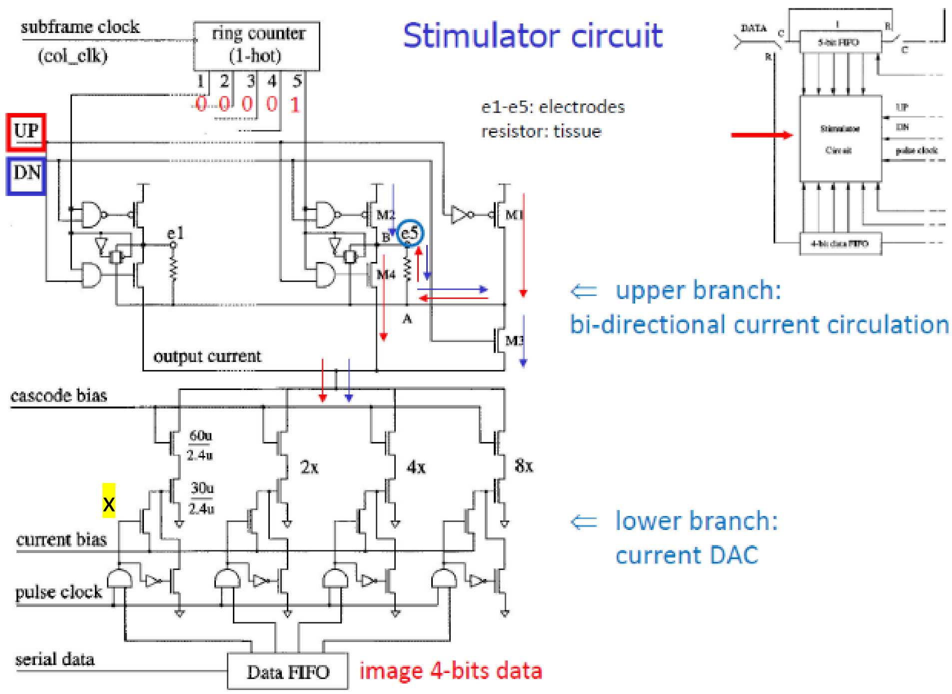
How long does it take a pulse to be delivered?
It needs to be delivered in 80 ck cycles, because the 80-bit FIFO has made a full turn, so a full period has been read. Is this the total image? No, because with 80 ck cycles we have stimulated just one electrode. Then we have to move to the other electrodes (80 ck cycles each).
I have finished my job when all the 5 electrodes have been stimulated: 80 ck cycles * 5 = **400 ck cycles to deliver the pulse to all the 100 electrodes** (20 FIFO for each 5 group of stimulators, 1 stimulator activated per time).

Which is the speed needed for the ck cycle?
It is important the t the total time is lower than the patient perception time, around 1/50 s. If the 400ck happen within 1/50s, the patient feels like he has received a full image.

NB: during the 80 bit scrolling, 4*20 bits are provided to the stimulator. So during the period of 80 bits, the new set of data to be delivered to the electrodes are 4 bits * 20, so 80 bits. In a period in the run line a new set of 80 bits are transmitted because they will be the one given to the stimulator at the next turn. For each period, we need to upload the 4-bit information. How much time I have available for it? 80 bits time. In 80 bits we upload a sequence of 4*20, the new data for the stimulator.

NB: We must have a buffer between the FIFOs below and the stimulation circuit, because the data of the image must be stable during the stimulation. When the bottom sequence of FIFOs is uploaded, the previous word must be stable, so we need some flipflops working as a buffer → data cannot scramble while the stimulator uses them.

STIMULATOR CIRCUIT



It is a circuit that takes the information of the electrode to be stimulated, the commands up and down, the clock and the word regarding the amplitude to be given and the FSR. It is composed by an upper branch and a lower one.

Lower branch

The lower branch is a current DAC. According to the 4-bit word, we need to provide a current within 16 gray levels and for a given FSR. So we have simply 4 branches in parallel. The transistor 30u/2.4u is a common source transistor and its gate can be connected through the switch to a line called **current bias**, but it is not a current, but a line coming from the full scale range. So according to the chosen FSR in the stimulator, we put the gate to a given value or another, when the switch x is closed. Hence it is simply a current generator with a transistor biased at a given voltage.

The switch is closed when the corresponding bit in the word data is 1. So if in the FIFO the bit is 1, we have an AND and so the switch x is close and we connect the current bias line to the gate, so we have a given current. The other transistor in parallel are simply connected to the other bits of the word, but the only difference is that each one of the transistor is sized twice than the previous one. So we have created 4 currents, one doubled than the previous one; if we out the currents all together at the output of the DAC we have created 16 levels of possible values.

The transistor in the top of this stage (60u) is a cascode: a cascode over a common source improves the output impedance of the current generator.
So it is a current generator to the current generated according to the 4 bits word in the data line. The cascode bias signal is a constant voltage for the cascode (that is a common gate stage), it is not changing.
Differently, the current bias line changes with the FSR.

Top branch

It is simply a **recirculation circuit**. It takes the current and let it go through the load (represented by resistors, that are the tissues) and electrode e1 and e5 are the stimulator. Hence in the array, we have a common electrode A, a specific electrode and in between the two we have the tissues. So this circuit, depending if the command is up or down, let the current prepared on the bottom DAC to flow into the resistor in one direction when we have a pulse up and in the other direction when we have a pulse down.
The inputs of this upper branch are:

- Electrode selector (ring counter aka FIFO): in this specific case the electrode n°5 has to be stimulated, so the line 5 has the bit 1, all the other lines bit 0. We don't have to learn all the circuitry, but we can verify that if we have 0 on line 1 and we consider all the ports (where we have the pass transistor), the resistor e1 (electrode 1) is shorted, because the pass transistor (little square in the middle, between A and e1) is closed and so the resistor is shorted. All the network with 0 are not operational, hence current doesn't pass through the tissues.
- Pulse up or down

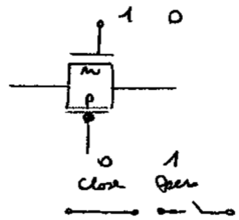
This is not true for the electrode n°5. If we put 1, the switch (little square) is open, but we need the additional information on up or down.
The situation corresponding to up and down are the colors red and blue. In the bottom line, independently on having up or down, the current flows in the same direction, the DAC delivers the current always in the same direction.

Let's start with the red 'up'. If I have up 1, thanks to the inverter, M1 is operational, because the gate is low (if we have a 0 on the pMOS it is on), while M3 is off, because it is controlled by down signal. Similarly, if up is 1 the M4 is on (also because the ring counter is 1), while the M2 is off.
In conclusion, among these 4 transistors, the transistors in red are on, the transistors M2 and M3 are off. So which is the only possible path for the current? It is the one arising from the DAC, taking the direction of M4, in the resistor, and then up in M1.

When I have down, instead, it is viceversa: the transistors in blue are on, and so the same current is taking the path of M3, the resistor (in the opposite direction) and exiting through M2. It is hence a bidirectional current circulation circuit.

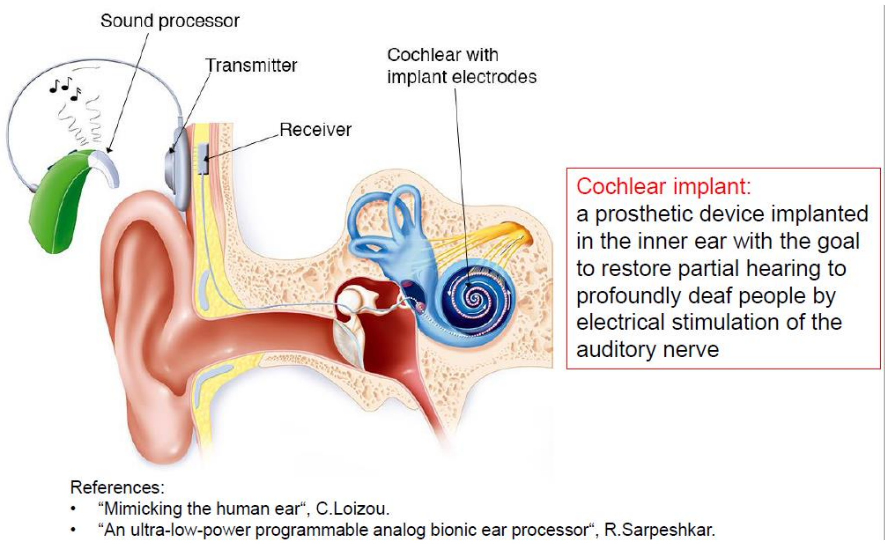
Pass transistor

28.



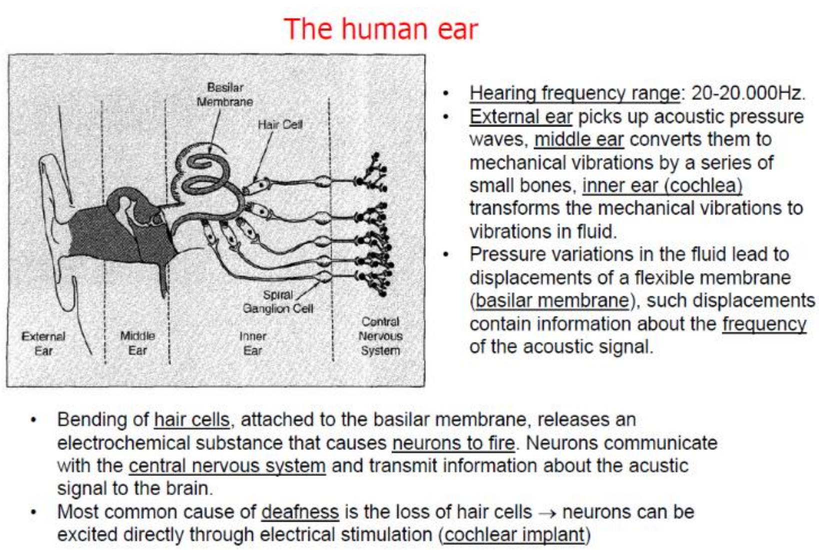
It is composed by a pMOS in parallel to a nMOS. Hence the switch is closed (shortcitcuit) if the n and p transistor are on, while is open when both are open.

COCHLEAR IMPLANTS



The prosthetic device is implanted in a patient with a partial or complete incapability to hear. The cochlear implant has the **electrodes implanted directly in the cochlea**.
The device collects the sounds from ant external microphone and transmit it through a radiofrequency link and the receiver provides information and stimulation to a series of electrode implanted in the cochlea. In this way we bypass the non-operational cochlea and provide a stimulation directly to the auditory nerve.

THE HUMAN HEAR



The frequency range is in between 20 and 20.000 Hz, but most of the hearing capability is concentrated over few kHz. The various stage of our hear are:

- **Externa ear**: has the role to pick up the sound waves.
- **Middle ear**: converts the sound waves in mechanical vibrations.

- **Inner ear**: there is the conversion of the vibrations into electrical signals. First the vibrations are transformed in mechanical vibration in a fluid, and then the pressure variation in the fluid are responsible for displacement of the basilar membrane, and this displacement are taken by hair cells that release electrochemical substances that causes the neuron to fire. Once the neurons are fired, we are in the field of electrical signal, provided to CNS.

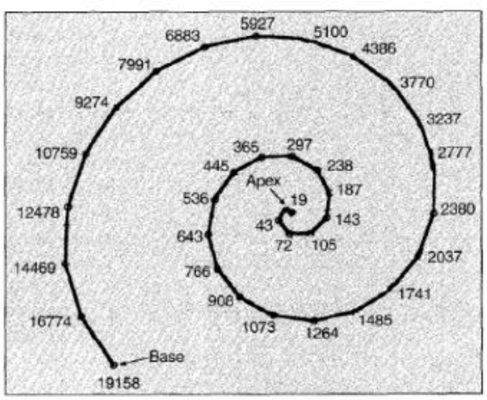
The severe deafness is at the level of hair cells, so if we are missing the step of transformation of spatial displacement into firing the neuron, it is there that some electrical stimulations need to be supplied, because we need to fire the neuron that cannot be fired anymore by the hair cells.

The bridge between the external unit and the internal tissues is in the cochlea. In the cochlea the sound is transformed externally in electrical signal and electrodes are placed in the inner ear to restore the firing of the neuron.

THE BASILAR MEMBRANE

The basilar membrane, embedded in the fluid, is able to supply a displacement along its extensions and the displacement is sensed by hair cells and so it is converted into an electrical signal. We need to implant electrodes on the basilar membrane so that, instead of detecting the displacement, by detecting the external sounds the electrodes can stimulate the neurons, that is something the hair cells are no more able to do.

There is another aspect. The **basilar membrane is operating as a spectrum analyzer**. The membrane is not sensitive everywhere to every frequency, but it is frequency sensitive → different points of the basilar membrane are sensitive mainly to a certain frequency range. Hence the vibrating of the membrane is happening only at specific frequencies.
The internal segment (apex) is sensitive to about 20 Hz and then moving towards more external segments we sense higher frequencies, up to the base (external segment) which is the segment specifically sensitive to the 20 kHz.



▲ *3. Diagram of the basilar membrane showing the base and the apex. The position of maximum displacement in response to sinusoids of different frequency (in Hz) is indicated.*

- Basilar membrane is responsible for analyzing the input signal into different frequencies, because different frequencies cause maximum vibration amplitude at different points along the basilar membrane: low-frequencies at the apex, high-frequencies at the base.
- The corresponding hair cells, bent by the displacements, stimulate adjacent nerve fibers therefore to specific frequencies.
- An electrode array can be used so that different auditory nerve fibers can be stimulated at different places in the cochlea

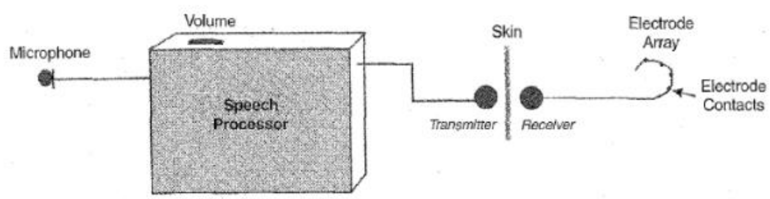
If we now plan to cover the membrane with electrodes all over it, it is important that each electrode receives the power only in its specific frequency range. We need first to operate a band selection of the frequencies, because each electrode needs to be stimulated by a specific bandwidth of the sound.

General scheme of the cochlear implant
Consist of:

- Microphone
- Speech processor: needs to perform the frequency selection.
- Transmitter and receiver through the skin
- Electrodes implanted all over the basilar membrane each one receiving the specific power in the specific rage.
-

We have two main approaches to the cochlear implant:

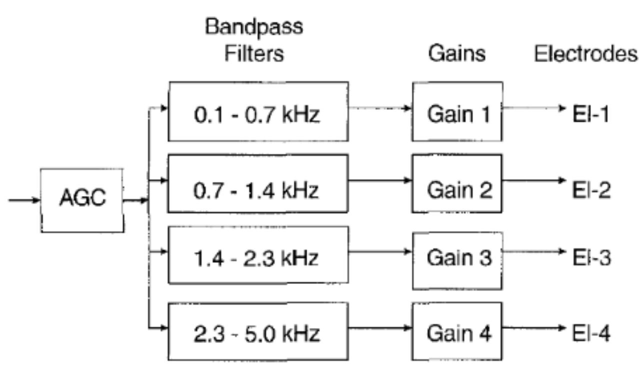
- Compressed analog (CA)
- Continuous interleaved sampling (CIS)



Approaches:

- Compressed Analog (CA)
- Continuous Interleaved Sampling (CIS)

COMPRESSED ANALOG APPROACH



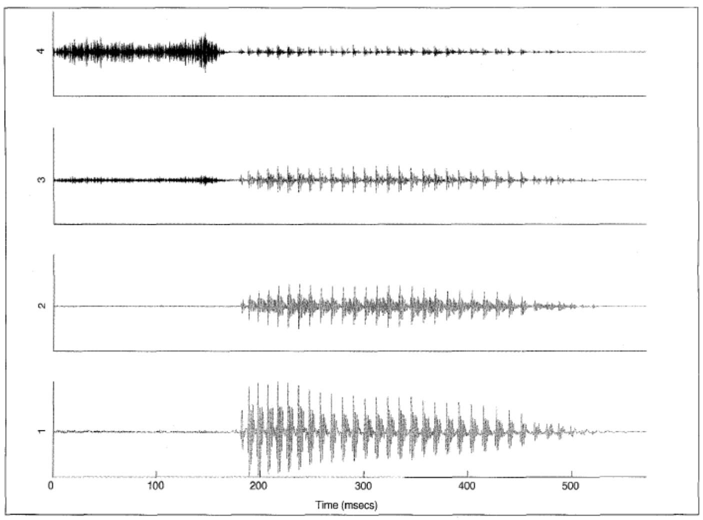
- AGC: automatic gain control (for signal compression).
- The filtered waveforms are delivered simultaneously to the electrodes in analog form.

For simplicity there are only 4 electrodes. The first block, **AGC, performs a first signal compression**; the human earing is indeed extended over several decades of capability (several tens of dB) but our human ear produces a compression, so the electrical stimulation is in a lower range. Our earing system performs a quasi-logarithmic compression between the very large sound extension into the electrical stimulation of the nerves. This is the reason why we need to have at the beginning a signal compression, that is performed by AGC, a circuit that automatically changes its gain depending on the signal itself.

Then we have **a bank of bandpass filters**. **Then** we have a **gain stage and the stimulation of the electrodes**.

The peculiarity of this approach is that once we have detected the intensity recorded in a given bandwidth, the electrode is analogously stimulated exactly with the same signal, there is no a translation between the recorded intensity and the electrode, we have a direct stimulation of the electrode directly with the analog signal coming out from the filter. This is done also simultaneously, each electrode is stimulated by its own bandwidth but the electrical stimulation happens simultaneously.

Syllable "s-a"

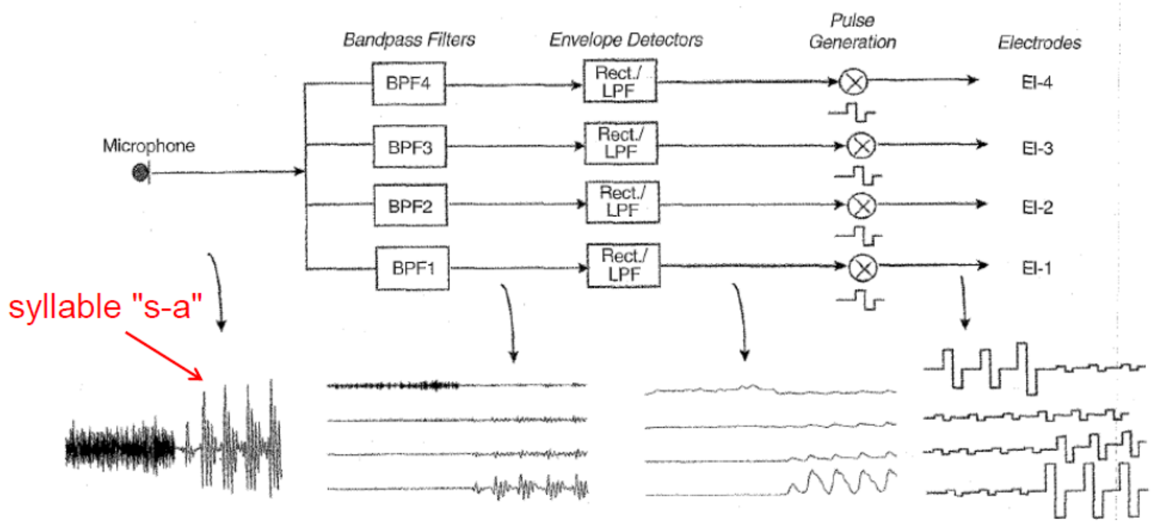


syllable "s-a"

▲ *13. Bandpassed waveforms of the syllable "sa" produced by a simplified implementation of the compressed analog approach. The waveforms are numbered by channel, with channel 4 being the high-frequency channel (2.3-5 kHz), and channel 1 being the low frequency channel (0.1-0.7 kHz).*

Recorded waveforms at the output of the filter. The letter 's' includes high frequencies (channel 4 is from 2.3 to 5 kHz), while in correspondence to the letter 'a' we have power in the low frequency range. We see that the content on the four channel is different. These analog waveforms are supplied contemporaneously to the electrodes.

CONTINUOUS INTERLEAVED SAMPLING (CIS)



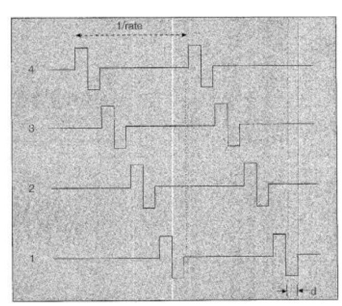
▲ 4. Diagram showing the operation of a four-channel cochlear implant. Sound is picked up by a microphone and sent to a speech processor box worn by the patient. The sound is then processed, and electrical stimuli are delivered to the electrodes through a radio-frequency link. Bottom figure shows a simplified implementation of the CIS signal processing strategy using the syllable "sa" as an input signal. The signal first goes through a set of four bandpass filters that divide the acoustic waveform into four channels. The envelopes of the bandpassed waveforms are then detected by rectification and low-pass filtering. Current pulses are generated with amplitudes proportional to the envelopes of each channel and transmitted to the four electrodes through a radio-frequency link. Note that in the actual implementation the envelopes are compressed to fit the patient's electrical dynamic range.

The early stages are similar to the previous one, we have a microphone, an automatic control, the bandpass filter to split the signal among the different frequency ranges. The differences start from the envelope detectors on, because we have some envelope detectors, which are circuits that take just the envelope of the signal, and then we have specific pulse generation. For each channel we have the generation of biphasic pulses. They are biphasic for the same reason in the MARK system: we want to supply a stimulation but with a 0 net charge delivered to the tissue.

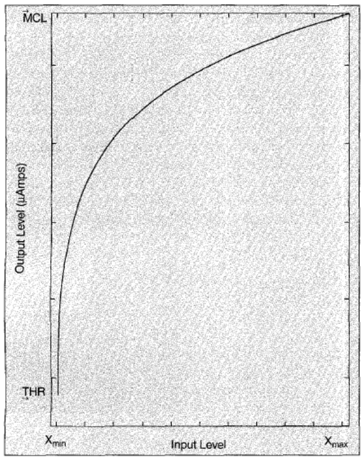
So we generate biphasic pulses and their amplitude is proportional to the envelope recorded.

What is the main difference with the previous strategy?
In the previous one, the stimulation of the electrodes is simultaneous, electrodes 1,2,3,4 are stimulated contemporaneously. Since the electrodes are not perfectly isolated (planted nearby), we can have some crosstalk of one stimulation to the closer electrode, and this can introduce some nonlinearity → a portion of the tissue is stimulated also by the neighbours, and not only by the specific waveform. And this accumulation of crosstalk may change the response of the tissue, that should be the response of a specific waveform, but on the contrary is the response to the waveform plus the crosstalk.

The CIS is called *interleaved* because the **biphasic pulses are** not simultaneous, but **alternated** (or interleaved). When we have a stimulation on electrode 4, there is no stimulation on electrode 1, 2, 3. Each pulse is delivered to its tissue without being affected by the others. So we preserve the electrodes from the crosstalk. At any instant of time we have only one channel fired.



COMPRESSION (acoustic amplitudes → electrical amplitudes)



▲ *19. Example of a logarithmic compression map commonly used in the CIS strategy. The compression function maps the input acoustic range [x_{min}, x_{max}] to the electrical range [THR, MCL]. X_{min} and x_{max}, are the minimum and maximum input levels respectively, THR is the threshold level, and MCL is the most comfortable level.*

- Compression is introduced while transforming acoustical amplitudes into electrical amplitudes.
- It is necessary because the range in acoustic amplitudes is larger than the implant patient's dynamic range, defined as the range in electrical amplitudes between *threshold* and *loudness uncomfortable level*.
- Logaritmic function (Y=A log(x)+B) is commonly used for compression as it matches the loudness between acoustic and electrical amplitudes.

I need it because the sound extension is very large (our hearing capability is to collect sounds over more than 70 dB dynamic range) and if we would linearly stimulated the electrodes with the amplitude detected from the sound without any compression, we will create in the patient a very uncomfortable sensation → important to perform an "**acoustic into electrical compression**". It is represented by the plot

In the plot, on the horizontal axis, we have the distribution of the sounds levels; xmin represents the minimum threshold each of us is able to hear and xmax represents the loudness uncomfortable level, above this level the sound is uncomfortable.
If we derivate a linear relationship between sounds and electrical signal, we would supply a very large current at xmax for the stimulatio. On the contrary, our natural earing system performs a compression in a way that the maximum sound level corresponds to a maximal electrical stimulation still within the comfortable level. So we **need to provide internally a compression curve**, transforming the acoustic amplitudes into electrical currents, where threshold corresponds to threshold and maximum amplitude corresponds to a level not exceeding what our brain detects as uncomfortable.

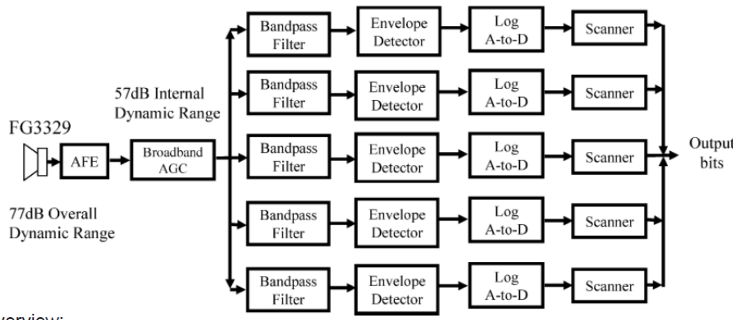
So we need to perform such a compression and we can do it with a logarithmic function → we need a logarithmic compression in the system.
A diode has a voltage to current logarithmic compression → we will have a diode somewhere.

EXAMPLE OF CIS COCHLEAR IMPLANT PROCESSOR

We have a microphone that takes the sound and converts them into an electrical signal, then we have a JFET-based audio amplifier, then we will have an AGC which starts to compress the audio 77dB input range into a 57 dB dynamic range. It is not a complete full compression, but a preliminary one so that the other stages don't saturate. We will have another compression at the end with a logarithmic amplitude to digital conversion. Hence the compression is in two stages.
We don't use it just at the beginning because otherwise we reduce to much the signal-to-noise ratio because we compress the signal but not the noise.

It is not all at the end because if the dynamic range is too high, we cannot cover the compete dynamic range with one single stage. So the AGC is linear in most of its range, but when the signal reaches the high intensity level of the range it introduces a nonlinear compression to remain in the dynamic range.

Then we have the bandpass filters and then the envelope detectors (the waveforms are still sinusoidal), and analog to digital conversion, the bits are taken by the scanner (that is a multiplexer to serialize the bits) and the bits are converted by a DAC into analog signal for stimulation.



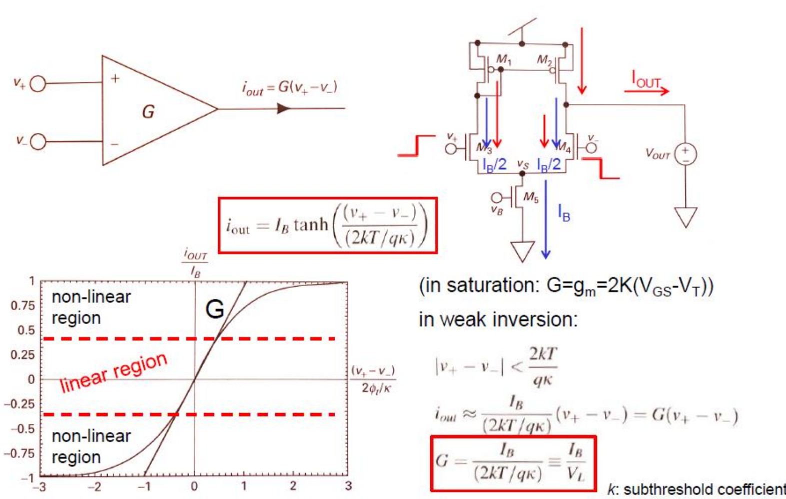
Overview:
- 2.8V supply, 211µW power consumption
- 16 channels, CIS approach
- JFET-buffered electret microphone + audio front-end (AFE)
- Automatic-gain-control (AGC) compresses 77-dB input dynamic range to 57-dB internal dynamic range
- Gm-C bandpass filter + envelope detector
- Logaritmic dual-slope ADC to provide further compression
- ADC bits are sequentially scanned by a CIS clock and latched into DACs that control electrode stimulation currents (not here described)

What is not described is the generation of the biphasic pulses starting from the digital information. Similarly for the MARK, we take the bits, we transform them into an analog current through a DAC and then we stimulate the electrodes.

BUILDING BLOCKS

- Simple OTA
- gm-R amplifier
- gm-C filter

Operational Transconductance amplifier (OTA)



It is a differential amplifier that takes a voltage difference at the input and provides a current at the output that is proportional to such voltage difference through a parameter that has the dimension of a transconductance (the gain). On the upper left we see the simplest configuration of the OTA. We have a differential pair at the input (M3 and M4) and a pMOS current mirror as a load. In this way the imbalance of the current at the input due to the differential input produces a current at the output. In blue we see the current flowing in the branches at the equilibrium stage, when the voltage difference at the input is 0. The tail current Ib is shared equally in half component in each branch. Thus we have the same current on the left and on the right, the current on the left is mirrored matching the one on the right so the output current is zero.

On the contrary, when we apply for instance a positive signal on the left input with respect to a negative one on the right, we imbalance the stage, we increase the current on the left, we decrease the one on the right, the sum is still Ib. however we have more current on the left and less on the right; the one on the left is flipped with the same intensity and so inf we make a Kirchhoff law, we have a lot of current on the top, less current from the bottom and so we have a net current Iout exiting from the stage.

Instead of looking the current on the left and right increasing and decreasing, we can alternatively think to have a positive signal going down on the left given by the difference between the red and blue current and correspondingly we may think to have a current signal going up given by the difference between the smaller current in red and the one in blue. The currents in the image are the absolute values. If we think at the differences, the current on the left points down, the one on the right points up and so the result is that the double of the contribution (summed) is exiting.

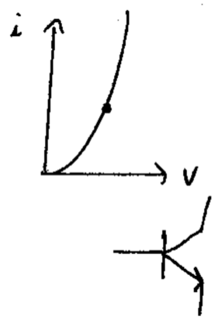
In the middle we have the analytical formula for the exiting current. It has a hyperbolic tangent relationship with respect to the input voltage because it is a peculiar relationship for the weak inversion operation of the transistor.
Hence this stage is not operating at high current, so with a transistor in strong inversion; this because we are talking about implants, circuits that are implanted and need to work at low power. So the best way to make them work at low power is to operate them just at the limit of their operational condition, in the so called weak inversion. It is a regime where the Vgs is below the threshold voltage, hence the equation for the transistor in saturation are no more valid and we need to use equation based on modelling of the transistor in weak inversion → hyperbolic tangent behaviour.

In particular, if we work in the small signal regime, that is the input voltage (V+ - V-) is smaller than 2*k*T/q*csi, we have that kT/q is the thermal voltage. Csi is a particular coefficient related to the subthreshold regime.
Besides 2 and csi, what matters is that the small signal regime means having a voltage difference lower than something proportional to the thermal voltage, that is around 25mV at room temperature. Csi in general ranges in between 1 and 2.
Hence if we operate in such regime, the hyperbolic tangent may be approximated linearly and so we can derive as usual for the small signal regime a linear relationship between the voltage difference at the input of the amplifier and the output current.
This parameter of proportionality is the transconductance G.

The G is given by the formula. It is proportional to the tail current Ib.

This proportionality should not be strange because it is typical of transistors working in weak inversion.

29.



$g_m = \frac{I}{V_T}$; $V_T = \frac{kT}{q} \simeq 25mV$

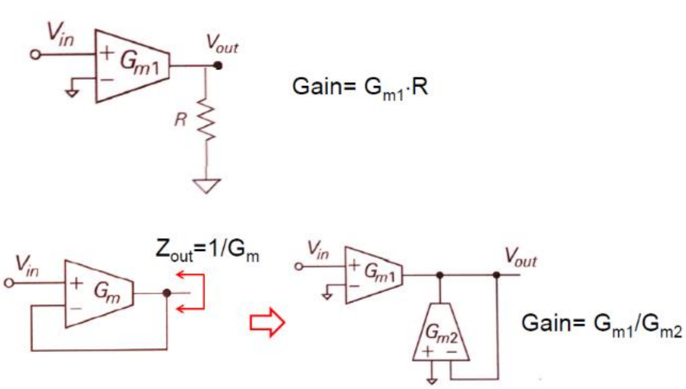
Weak inversion: $I = I_b/2$

In transistor working in weak inversion, the transconductance is similar to the transconductance of the pn junction, so the transconductance of diodes or BJT.
The transconductance of a BJT is gm = I/Vt, where Vt is the thermal voltage. So we have direct proportionality between gm and the current.

In the transistor in weak inversion, the relationship is similar. In fact, I = Ib/2 and gm is still proportional to the current.
Hence the formula seen for G is of course the linear approximation of the transfer function having an hyperbolic behaviour, but it is practically equal to the transconductance of a BJT.
So it is the ratio between the current (each transistor has biasing current Ib/2, so we have to put Ib/2 and not Ib at the numerator) and at the denominator we have the thermal voltage with the subthreshold coefficient (because the MOSFET is not identical to a BJT).

In OTA, when working in the linear region, we can use this relationship for G, hence knowing that the gain is proportional with the biasing current, and by change it we can change the gain.

Gm-R amplifiers

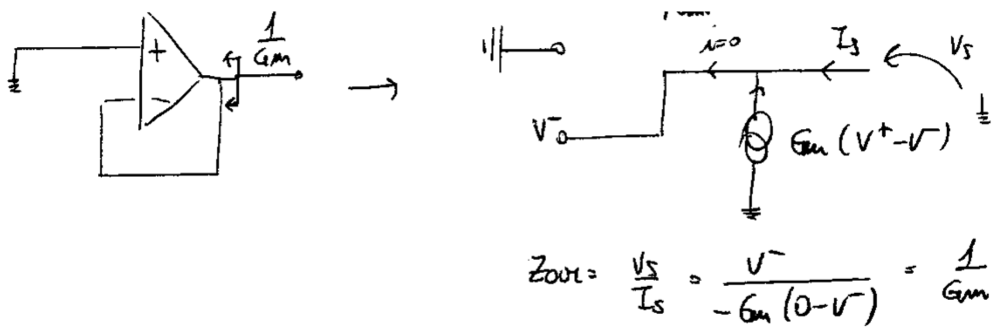


- Gain can be set by means of the currents I_b in the transconductors ($G=I_b/V_L$)
- Solution affected by non-linearity of G_m for large input signals.

On the upper left we have an OTA with a Vin applied to the non-inverting input versus ground; the current is then sent to a resistor and hence we have created a voltage amplifier, because we have a conversion of the current gm*Vdiff (Vdiff = voltage difference at the input, Vin – 0), multiplied by R. this is the gain. This is the prototype of a Gm-R amplifier.

We can do more; resistor may be tricky to be integrated in CMOS technology, especially if we want large resistor because we want to boost the gain, so we may be interested to look for alternative implementation for the resistors.
An alternative implementation (bottom left) is a transdiode, realized with an OTA with output shorted to the negative input. It can be demonstrated that if we look the output impedance of this bipole (it is a bipole indeed because we have output pin vs input pin) it is 1/Gm, where Gm is the gain of the OTA.

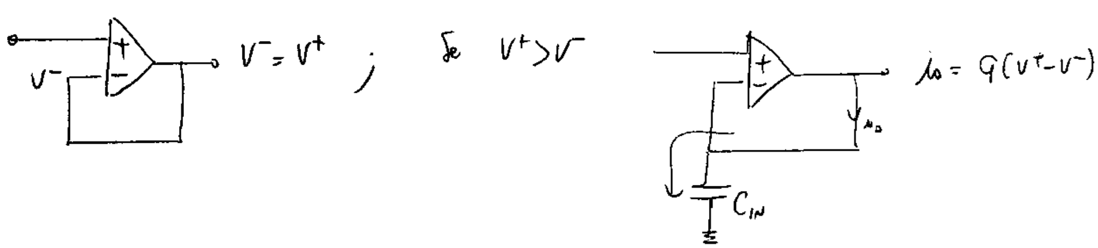
30.



To look at the output impedance, we have to ground the input. The input is floating, the OTA is by definition providing a current that is Gm*(V+ - V-) and we are testing the impedance. So we are applying a testing voltage Vs and recording the absorbed current Is. The impedance Z is Vs/Is.
But Vs = V- and the current Is = - Gm*(V+ - V-), where V+ = 0.

Whenever we take and OTA and short the output with the input, the impedance we see is 1/Gm.
Moreover, there is another property of the OTA with the output shorted to the negative input.

31.



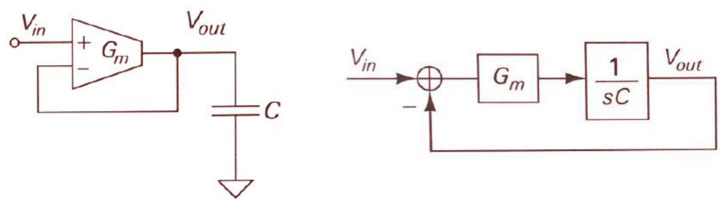
The property is that V- = V+, because this is the only operative condition.
Let's suppose for instance that V+ > V-. In this case, the output current would be exiting, because the current is i = Gm*(V+ - V-), so we would have an exiting current. Is this current going somewhere? Ideally no, because we have the infinite input impedance, but in reality we can assume to have a capacitance because we have a MOSFET. In this case, the current flows into the capacitor and the node where we have V- is rising up as long as it matches the other node voltage V+ and the current is 0 so we stop charging the capacitor. So the voltage difference between the two inputs is 0. If not, we would have an exiting current that, integrated somewhere on the V- input, would restore equality.

Hence the only conclusion is that the current is 0, condition satisfied if also the voltage at the input is 0.
In the end the output impedance of the network on the bottom left is 1/Gm.

On the bottom right we have a smart implementation of the configuration on the bottom left to have the resistance needed for the configuration upper left.
We have the open loop OTA that is Gm1, while the Gm2 is the close loop OTA → Gm2 is simply a resistor with resistance 1/Gm.
The final gain is Gm1/Gm2. Now since the transconductance is proportional to the tail current Ib, I can choose Ib in a way to maximize Gm1 (by increasing Ib) and decrease Gm2 (by reducing Ib) → gain larger than 1.

NB: these approximation of the gain with a constant transconductance depending on Ib are valid only for small signals. If we go to the large signals regime these approximations are no more valid → if we use Gm-R amplifier we must be sure that the dynamic range of the input signal is small enough. Hence one problem arises, since CIS is a circuit with a large dynamic range.

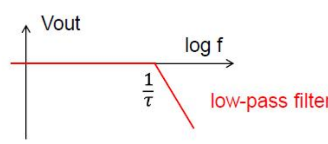
Gm-C filter



$G_m(V_{in}\text{-}V_{out}) = sCV_{out}$ $\quad \tau = C/G_m \quad$ $V_{in}\text{-}V_{out} = \tau s V_{out}$

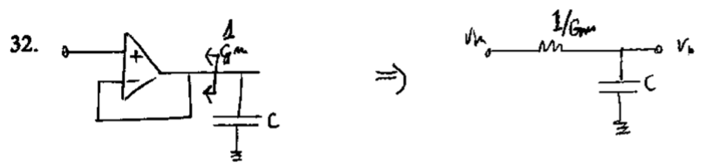
$V_{in} = (1 + \tau s)V_{out}$

$$\boxed{V_{out}(s) = \frac{1}{1+\tau s}V_{in}(s)}$$

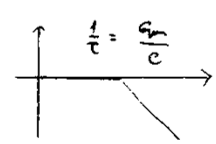


It is a low pass or high pass filter. The one in the figure is a low pass, composed by a transdiode (OTA with closed feedback loop) in series with a capacitor. We have the analytical calculation of the transfer function.

32.



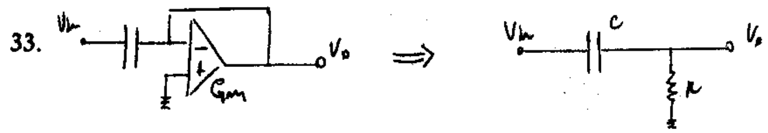
$$V_o = V_{in}\frac{\frac{1}{sC}}{\frac{1}{sC}+\frac{1}{G_m}} = V_{in}\frac{\frac{1}{sC}\cdot sC\,G_m}{G_m + sC} = V_{in}\frac{G_m}{G_m}\frac{1}{1+s\frac{C}{G_m}}$$



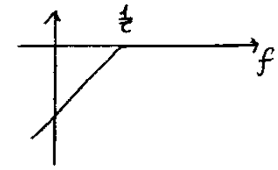
If we have a transdiode in series with a capacitor and the output impedance is 1/Gm, it is like to have a resistor in series with a capacitor → classical low pass filter whose tau of the pole is C/Gm.
The exiting current is Gm*(Vin-Vout) that is equal to the current flowing in the capacitor.

As for the high pass filter see next page.

33.

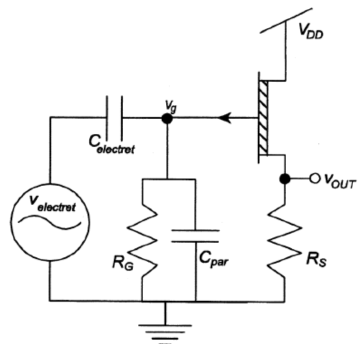


$$V_o = V_{in}\cdot\frac{\frac{1}{G_m}}{\frac{1}{G_m}+\frac{1}{sC}} = V_{in}\cdot\frac{s\tau}{1+s\tau} \quad \text{where } \tau = \frac{C}{G_m}$$

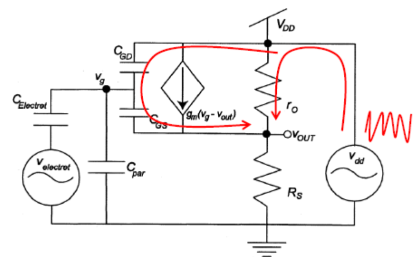


This is equivalent to a CR network. These filters will be used in the band pass filters of the cochlear implant.

THE MICROPHONE PREAMPLIFIER (AFE)



- Electret microphone: a condenser microphone with a permanent charge built into it.
- A self-biased JFET source-follower provides low-impedance voltage output (G=1).
- JFET output resistance and gate capacitances contribute to couple power-supply (VDD) noise to V_{out}:



A microphone is a capacitor where the sound waves are able to change the distance between the plates of the capacitor.

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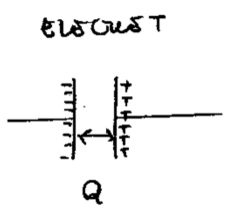
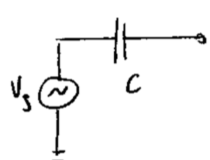


$$C = \frac{Q}{V}$$

$$V = \frac{Q}{C}\updownarrow$$

$\rightarrow$



When there are sound waves arriving to the capacitor, the waves change the distance of the plate of the capacitor and if we have a circuit sensitive to the value of the capacitor, we can transform the sound waves into an electrical signal. We are dealing with a particular type of microphone called **electret**, that is a capacitor where the charge is already fixed and generated at manufacturer site. So we have a preexisting charge integrated on the capacitor. This means that when the sound changes the distance between the plates of the capacitor, as the charge is fixed, we change the voltage.
If the C is changing, we change the voltage.

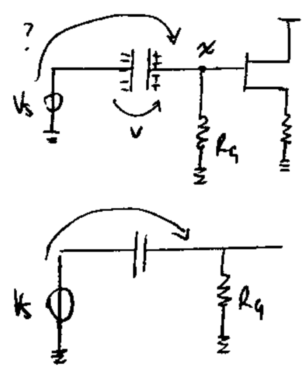
This electret microphone provides a voltage signal when the microphone receives the sounds.

The equivalent circuit we may consider is the variable capacitor but with a signal, that is given by the variation of the capacitance for a constant charge.

This clarifies why in the modelling we have that the electret microphone is represented by a voltage to be read out. So Vin is the voltage of the electret capacitor.
The basic read out stage we have in the system is a source follower stage based on a JFET, that is a very low noise transistor.
Since it is a source follower, we take the gate and read out the signal at the source, where we have an impedance Rs. The gain is 1.

The advantage of this read out is that is very simple, made out of a JFET and a resistor, so very often this is integrated, we buy the microphone with already embedded the JFET, so to have an output pin from the device.
Moreover, the gate of the JFET is biased through a large resistor Rg. We bias the transistor with a 0 DC voltage through this very large resistor. Is this a mandatory choice? No, in principle we could have done as below:

35.



We have an electret, so we have already a bias voltage and we could have use it to bias the gate, without any resistor. However, we have to remember that the JFET shows a gate current, because it is made out with a pn junction, so we have a reverse bias pn junction and a gate current. But this gate current can go only into the capacitor and if so, the voltage across the electret capacitor is drifting and so the node x has an unknown voltage. So in this case the biasing of the transistor depends on the gate current.

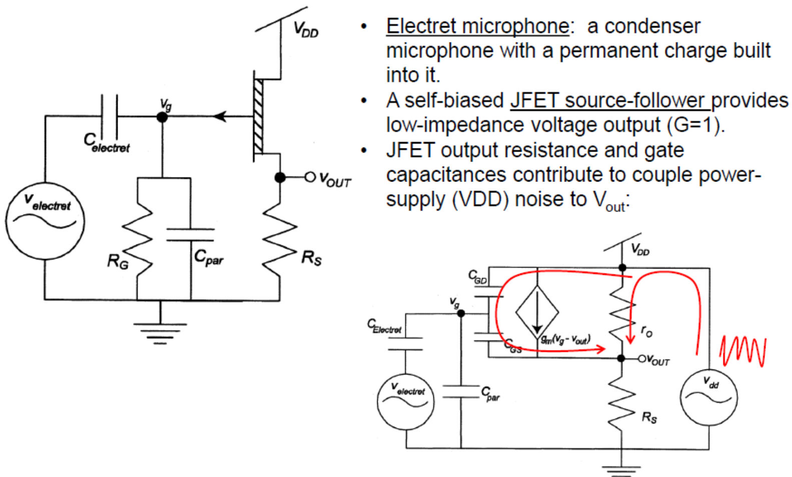
What we can do is to connect the gate with a large resistor to ground. Thus in DC, because the gate current is a DC current, the gate current takes preferably the path through the resistor and not the one through the capacitor. Thus the voltage is biased close to zero by the large resistor (if the gate current is small the drop on the resistor is almost zero) so we are sure to give a steady biasing of the gate of the transistor to 0. The resistor Rg helps in the biasing of the JFET.

Is the Rg affecting the signal transfer function (from Vs to x)?
No, because if the resistor is sufficiently large, we have a high pass filter and the signal is transferred easily to the gate of the JFET.
Hence the resistor helps in biasing because it provides an escape path for the gate current, but on the other side it is not affecting the transfer from the electret microphone.

The other capacitor Cpar represent the gate capacitance of the JFET but doesn't play a big role.

This source follower solution is used but it has few problems:

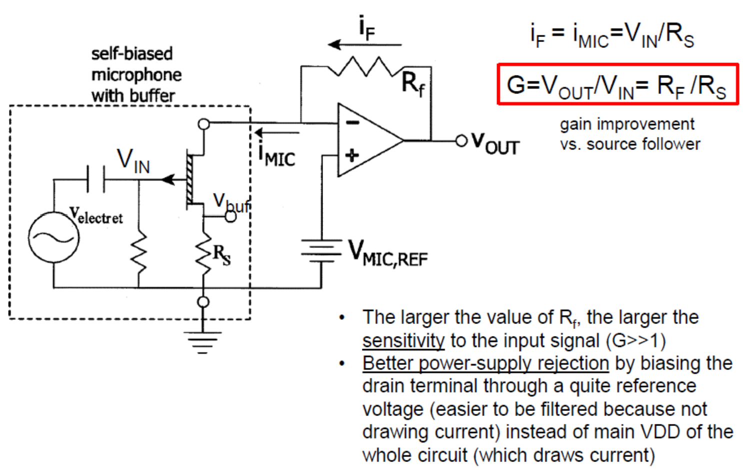
- It is not amplifying, we have a gain 1.
- If the Vdd voltage is the same shared with all the other circuits of the cochlear implant, this Vdd voltage is going to be noisy, and the noise is represented by the other generator on the bottom left connected to Vdd.



NB: when we have a single power supply in common with all the amplifiers of the implant and also in common with digital blocks (that introduce digital spikes), the Vdd is going to be very dirty.

The spikes are transferred to the output node of the follower by means of both the drain-source r0 impedance of the JFET and also through its gate drain and gate source capacitances.
So in the image we have with the 'rombo' the small signal model of the JFET, with the output resistor r0 and the two capacitors. All these components create a parasitic path from the noisy Vdd to Vout.

How to solve these problems



We may use an alternative front-end, still based on source follower, but instead of reading the voltage at the output of the follower, we read out the current. So the current of the transistor is read out by the transimpedance amplifier. (the pole of the high pass must be at lower frequency than the electret signal)

But I need to bias the transistor, the transistor drain cannot state to zero, it needs to be biased. It is biased by putting the non-inverting input of the transimpedance amplifier not to 0, otherwise also the voltage V- would be 0, but to a given reference voltage we want to apply to the gate of the transistor so that the transistor is operating in a suitable biasing condition. By biasing V+ to Vref, since we have a virtual shortcircuit in the amplifier, also V- is Vref.

The readout is done not reading Vbuf, but by reading i_{mic} (i_{mic} = Vbuf*Rs), which is readout and converted in voltage through the large resistor Rf ($i_f = i_{mic}$). Moreover, Vbuf = Vin, because it is a follower. In conclusion the overall gain between Vout and Vin is Rf/Rs.
Hence we simply choose Rf >> Rs and we have obtained a very good voltage amplifier → reading the voltage of the microphone with a larger gain than before.

NB: when we have delicate sensor, we can use transimpedance amplifier. We still bias the sensor but we readout the current of the sensor and not the voltage.

What about the noise?
We have no more Vdd, but the drain is biased through Vref. If Vref is noisy, also the drain is noisy and I still have the problem. However this is a better solution because Vref is a separate and custom power supply with respect to Vdd in common to all the circuit. Moreover, while the Vdd of the initial configuration is receiving the current of the JFET, the new power supply is not passed by by any current, because the voltage is supplied at the high impedance pin of the amplifier → we can filter better the power supply.

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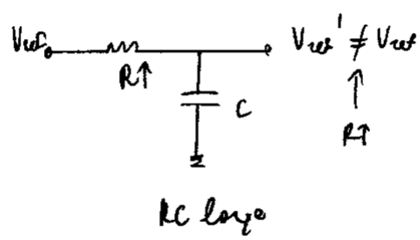


When we have a noisy power supply (e.g. a noisy Vref or a noisy Vdd), I use RC filters. In this way the noise of the power supply is low pass filtered. We put in the boards an RC network if we need to filter the noise of whatever power supply, because we are doing a low pass filtering.
The noise il filtered better the larger the RC network, so that the pole is at lower frequency. Of course, we cannot exceed to much with the values, in particular of the resistor.

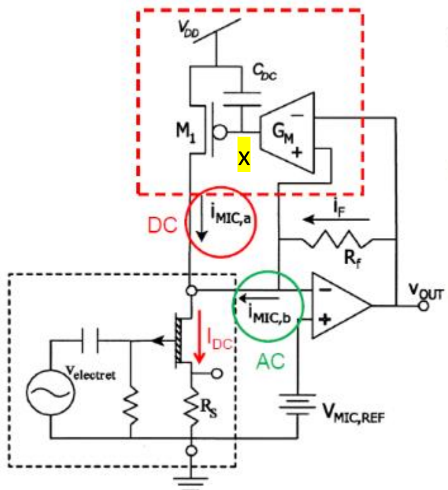
Indeed, if we make the resistor to large and my power supply is flown (attraversato) by a current, so the power supply has to provide not only a voltage but also a current, larger the R, larger is the drop across R, so Vref we get after the filter is dropped with respect to the initial Vref.
Hence there is a difference in filtering Vdd in the initial configuration with respect to filtering Vref in the second one, because with Vref I have 0 current, so I can put a very large resistor in the filter, hence even if it is very noisy, I can filter it very well.

Conversely, Vdd has the current flowing in the JFET, so if I put large resistor for filtering I have a big drop over the resistor → big problems.

In conclusion Vref is a better voltage to be filtered than Vdd, because it is without current.

Subtraction of the DC current

The gain is larger the larger Rf. However, the current of the JFET flowing in the feedback is not only due to the signal, but also due to the bias current of the JFET. Idc is the bias current, the current flowing in the JFET with no signal (DC component). Imic,b is the AC component, the one to be amplified, while Idc is annoying, because is going to saturate the amplifier if Rf is too large.
Hence it would be nice to subtract the DC component elsewhere and let only the AC component go through the transimpedance amplifier (the AC component is a small current, so I can choose large resistor before the saturation of the amplifier).



- A large R_f is desirable to achieve large gain and low thermal noise of the resistor (microphone DC current I_{DC} would saturate amplifier (2.8V supply).
- A low-frequency negative feedback by G_M, C_{DC} and M_1 subtracts the bias current of the microphone $I_{MIC,a}$ before amplification by the sense amplifier.
- Feedback is active only at low freq. (<100Hz), while AC microphone current $I_{MIC,b}$ is amplified. (pole given by C_{DC} and the output impedance of G_M)

The block in red is used to subtract the DC component of the JFET current. It is an additional feedback that senses the voltage drop across the resistor Rf and produces a current Imic,a that is going to match the Idc current on the bottom.

How does it work?

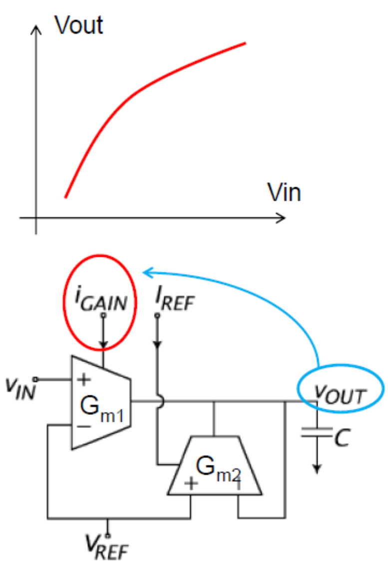
Let's suppose that the current, whatever it is the DC or AC, flows first of all in Rf. The current, with the orientation of If, creates a voltage drop across Rf.
In the upper part we have an OTA. Since the voltage on the right of Rf is larger than the one on the left because of the previous considerations, we have current flowing inward to the OTA (if + > - the current is exiting, otherwise it is entering, as in our case). Then we have the capacitor Cdc. If we are extracting current from the capacitor, the node x is going down.
But we have also a pMOS. If the gate of the pMOS is going down, the pMOS current is increasing, because a pMOS increases the current if the gate goes down. Hence we have a current increasing going downward in M1. This current is going to compensate the JFET current.

The red one is hence a negative loop. More current in Rf, more current is subtracted by the negative loop.
Since we have a loop, the error signal (zero signal), we have a perfect negative loop when the feedback current Imic,a is perfectly compensating the Idc, so no current is flowing in the branch where we have Imic,b.

By this component we have compensated Idc. But this compensation must be active only for the DC component, not also for the AC component → I need to introduce a low frequency pole to kill the loop at low frequency.
The pole is in the x node. It is given by the time constant given by Cdc and the output impedance of the transconductor. The **output impedance of the OTA** is not 1/Gm, that occurse only when we have the short, while **here it is r0/2**. So the tau is of a low pass filter so that the loop is functional only when we need to subtract the DC current, but for the AC current due to the microphone the loop is killed, the capacitor is shortening the node and the pMOS cannot change the current Imic,a → Imic,b flows in the resistor and the amplification of the signal is granted.

AUTOMATIC GAIN CONTROL (ACG) CIRCUIT

It is an amplifier before the bandpass filters. This amplifier has to face a problem: with 77dB input dynamic range, so with a very large dynamic range of the microphone signal (the sound covers a very large dynamic range), if I would have used a standard linear amplifier, I could have not cover this range.
Hence I use an amplifier that compresses the dynamic range form 77 to 57dB.



- Automatic-gain-control (AGC) compresses the 77-dB input dynamic range to 57-dB internal dynamic range, by varying its gain such that soft sounds are amplified by large gain while loud sounds are amplified weakly.
- AGC is based on sensing the output envelope of the amplifier and using it to control the gain of the amplifier itself (feedback AGC).
- It is implemented by a variable gain amplifier (VGA) based on regulating the transconductance G_{m1} of a G_{m1}/G_{m2} configuration.

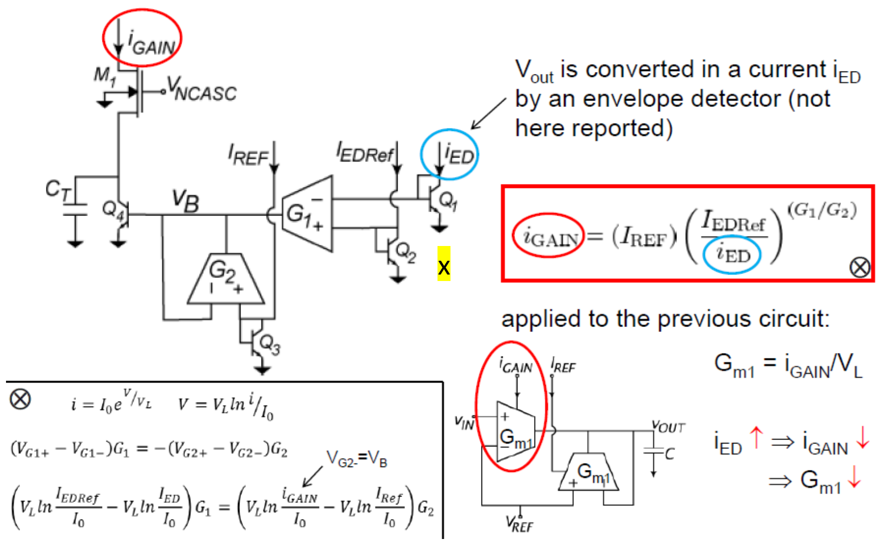
We use a circuit with a nonlinear transfer function. When Vin is small (low intensity acoustic wave) we have linear relationship between output and input. But when the input signal is getting too loud, so it is going to saturate the bandpass filters (that are linear circuits), I need to introduce a compression in the gain.

So this is a compression amplifier and works on the principle of automatic gain control.
Automatic gain control means that the circuit, while sensing the signal, is automatically changing the gain from high gain to low gain.

The circuit is based on Gm-R configuration. We recognize the configuration with a transconductor Gm and a load which is the 1/Gm (another OTA). Gm1 is the gain of the first stage and 1/Gm2 is the load, and we put them to reference or ground voltage.
If we use a standard configuration, we have a constant gain amplifier, a constant Gm-R amplifier of gain Gm1/Gm2.

However, the transconductance Gm1 is proportional to the tail current Ib. so if I found a way to control the current Ib (that is Igain) with the output voltage of the amplifier (I take the output voltage and when it is too large I reduce the current Igain), I reduce the transconductance Gm1.

So the principle of this circuit called variable gain amplifier (VGA) is to have a standard Gm-R amplifier but with an additional loop so that I record the voltage at the output and I control the current of the input amplifier. The gain of Gm1 is compressed by means of Igain according to the sensed output voltage.
This is done with an additional block that takes the output voltage of the main amplifier, converts it into a current and, using this current, we change the current on the main amplifier.
We see that in the formula the gain current is modulated with the term at the denominator. If i_{ED} increases, Igain decreases.



By controlling iED we control the current of the gain stage which changes the transconductance Gm1. Hence we need a voltage-current coverter between Vout and iED. The circuit on the top is connected to the one in the bottom with a voltage to current converter, e.g. by means of a resistor connected to the node Vout. Then Igain is connected to Gm1. Igain is going to change the tail current of the OTA then.
Thanks to the dependence of the transconductance by means of iED, if voltage increases, iED increases and Igain decreases → **reversed proportionality**.

The circuit on the top is composed by a Gm-R (or Gm-Gm) circuit (G1 and G2). It is not the same of the one in the bottom, but of the same type, in fact we have the ratio of the two transconductance in the formula.
What is more tricky are the inputs. The voltages at the inputs of the blocks are created by diodes (x) (they are BJT with a collector shorted to the base → it is just a junction, a base emitter junction, hence Q1, Q2 and Q3 work as diodes). So if we push a current in a diode (like iED, Iref, Iedref), we get a voltage across the diode that is represented by a logarithmic relationship with the current. The current in a diode has the exponential dependence with the voltage, and if we flip the relationship, the voltage at the input of G1 and G2 are related to the current we are pushing in the diodes by the classical logarithmic relationship.

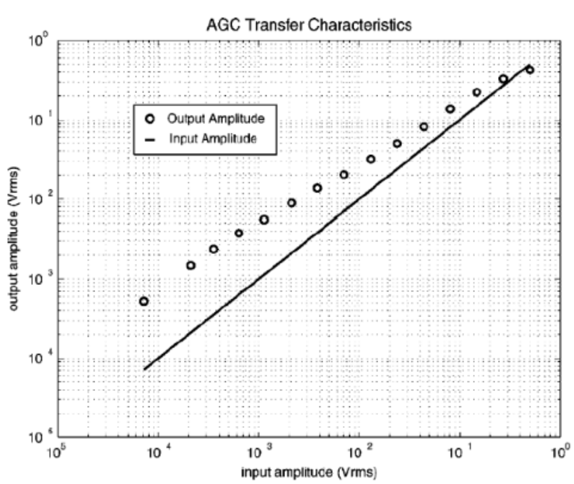
Calculation

We have a **Gm-R amplifier with voltages that are obtained through a logarithmic relationship with the two currents Iref and Iedref**, that are fixed, while **iED is proportional to the output voltage**. The second equation is simply the Kirchhoff law in the node where we have Vb, where the output current of G1 must be equal to the one of G2 (the one in Q4 is negligible and the other is in the inverting pin of G2). The exiting current of G1 is equal to minus the exiting current from G2.

Now the equation in the bottom line is the same Kirchhoff law simply by substituting the voltages with the corresponding equations according to the logarithmic dependence of such voltages with the corresponding currents into the nodes connected with Q1, Q2 and Q3. For instance, Vg1+ is given by the logarithmic relationship of the transistor Q2. So I take the equation in Q2 and I found to be equal to Vl*ln(Iedref/I0) (I0 is the diode saturation current). Similarly, the voltage on the negative input is given by Q1, and the voltage across Q1 is given by the other relationship. The same for the term on the right.
The remaining one is the voltage Vg2-. This voltage is the voltage Vb at the input of the transistor Q4; hence I can involve the transistor Q4 in the equation, and the transistor Q4 is responsible for the current Igain → current Igain is present in the logarithmic formula.

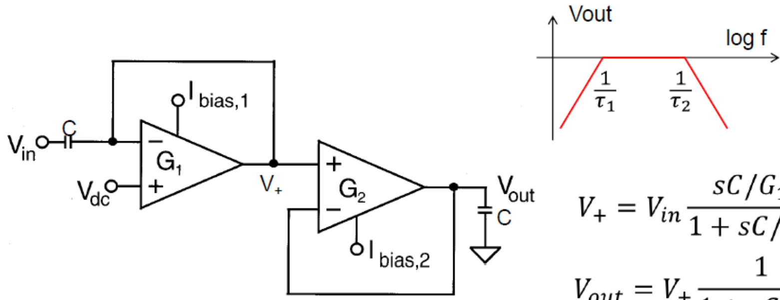
What is missing is the solution of the bottom equation, that is based on canceling Vl everywhere, making the difference of logarithm, and then eliminate them by making the exponential. The result is the one in the red box.
In this equation **Igain is reversely proportional to iED** with some reference constants and also by the ratio of the gain of the transconducotrs at the exponents. But what matters is that we have demonstrated that if Vout changes iED we change Igain and hence the transconductance Gm1.

AGC input-output compression



Log-log transfer function between the input voltage and output voltage. If the amplifier would have been linear, we would have a perfect straight line in the t.f, but as the amplifier introduce compression, we see that the t.f. is not linear, maybe linear at the beginning and then we have the compression.

BANDPASS FILTER



$$V_+ = V_{in}\frac{sC/G_1}{1+sC/G_1}$$

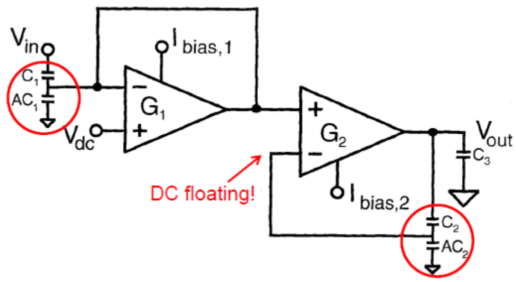
$$V_{out} = V_+\frac{1}{1+sC/G_2}$$

- The basic bandpass topology is a cascade of first-order high-pass and first-order low-pass filters based on G_m-C stages. The lower pole is proportional to the bias current of G_1, while the higher pole is proportional to the bias current of G_2.
- The major limitation of this design is its small linear range as the signal amplitude is limited to the linear range of the transconductors.

We introduce a low pass filter (G2) and also the corresponding derivator (highpass, G1). The bandpass filters are done by using a cascade of the two filters, one is the high pass (G1) with its time constant that defines the low frequency pole that is given by the ratio between C and transconductance (by change the transconductance we can change the pole) and a low pass (G2).
So by providing the suitable tail current Ibias (1 and 2) to the corresponding OTA we set the position of the poles and so we set the bandpass. Of course for each channel we define a different bandpass location.

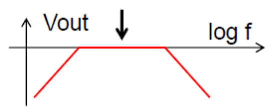
There is a problem: the usage of a compressive amplifier before, the level of amplitude of the signal, especially in the louder amplitudes of the sounds, is high, and in the OTA the assumption of a linear behaviour is true only if the input signal is small, otherwise if it is large, the response of the OTA becomes highly nonlinear. The problem is that we have a circuit devoted to amplification and filtering of signals in a very high dynamic range and so the assumption of having very nice bandpass filter is questionable, because the input voltage may be so large that the OTA operates in nonlinear conditions, and if so we cannot use the normal equation of the filters, that rely on the linear operation of the transconductor.

TOPOLOGY WITH IMPROVED LINEAR RANGE



- The linear range is improved by attenuating the output in the feedback in order to limit the voltage swing at the input of the transconductors.
- In the high-pass stage, the signal is attenuated by a factor of A+1, thanks to the partition of capacitances impedances. In the low-pass stage, a gain of A+1 is applied to signals in the passband, therefore the overall gain is unchanged!

$$V_{out} = V_{in}\frac{sC_1}{G_1 + sC_1(A+1)}\frac{(A+1)G_2}{G_2 + sC_2A}$$



$$V_{out} \sim V_{in}\frac{sC_1}{sC_1(A+1)}\frac{(A+1)}{1} \sim V_{in}$$

as in the basic filter of the previous slide

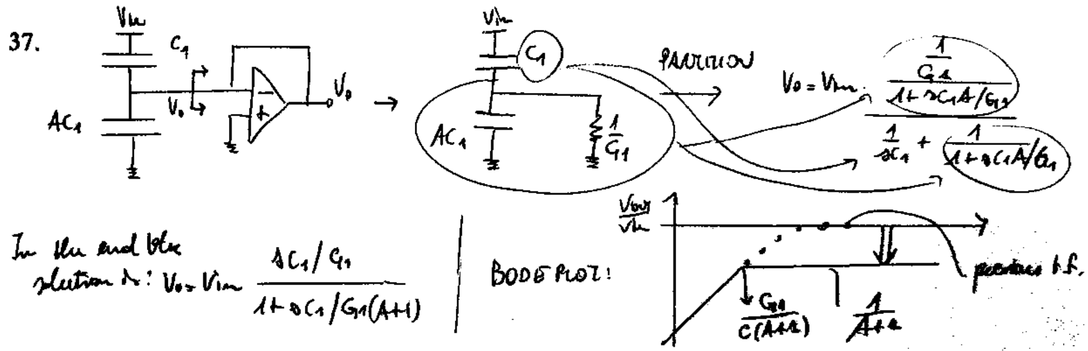
The problem is solved keeping the cascade of the two high pass and low pass blocks (not changing the topology) but we introduce a voltage partition (at the input of the derivator and at the output of the integrator) because if I have a voltage applied across two impedances and the AC1 is 8 times larger than C1, the impedance that is 1/s*AC1 is going to be much smaller than the other 1/sC. So we have introduced a penalizing voltage divider.
So to the OTA we applied a penalized voltage with respect to the input, so we bring at the input a more friendly voltage, still in the region where the OTA is linear enough. → we introduce a linear partition, a linear voltage divider.

In the low pass filter we introduce the same partition directly in the loop, so we take the output and before closing the loop, we close it with the voltage partition → at the negative input of the amplifier we introduce an attenuated signal, and if we do so we operate the OTA in a linear regime.

The one in the middle is the overall t.f., that is different from the previous product of t.f., because we have the terms depending on A. But if we compute the gain of the formula in the **bandcenter**, that it where we have our signal, in the center of the bandwidth the first pole has already been passed, so the term on s is dominating in the denominator with respect to G1. On the right term (the low pass), we are still below the pole, hence G2 dominates over sC2*A. In the end we have that Vout = Vin.

This result means that we haven't changed the overall gain of the stage, we have simply changed the application of the signal on the OTAs, because in the first G1 the signal is attenuate by a factor A+1, in G2 we amplify by a factor A+1 so they compensate and the overall gain is preserved.
In this way, we have achieved the same gain as before so we don't need to change the gain of AGC amplifier, because the **gain remains the same**, but the **OTA of the bandpass works with a smaller signal at the input**.
I cannot use only a partition because it would reduce the gain (if at the beginning) and I need to recover the gain then.

Calculations



Firstly, I compute the first partition on G1; the impedance on the right of a transconductor is 1/G1. In the Bode plot, the central frequency gain is attenuated by a factor A+1, that will be recovered by the integrator. Now we understand why the OTA G1 works better, that is because the voltage at its input has been reduced with respect to the previous transfer function.

Similarly for the integrator. Vin' is the output of the previous stage and Vo' is the intermediate node. If we do the calculation, we have the formula for Vo'. The final formula tells us that the voltage at the input of the OTA is not fully the voltage in the full range of the circuit, but it is attenuated by a factor A+1. Once again the OTA is happy because the signal at its input is an attenuated replica of the voltage at the output.

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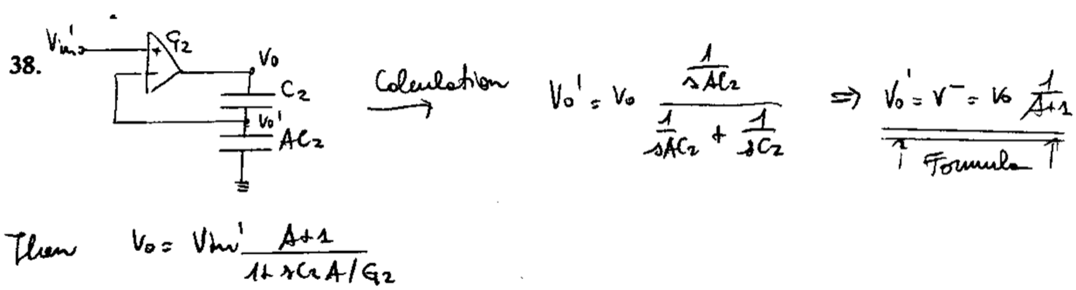
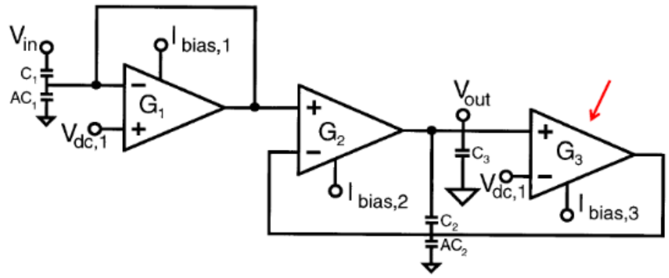


In conclusion the main concept is that we have attenuated the signals at the input of both stages, but at the same time there has been no penalty on the overall gain of the filter, because in the center bando of the filter the gain is still 1 as before.

Moreover, this circuit is not going to work, there is a basic problem, that is that the node V- of G2 is floating, there is no DC voltage given to this node, because it is connected only to capacitor, and in DC capacitor are open circuits, hence the circuit with G2 is floating.

TOPOLOGY WITH DC STABILIZATION

We have an additional loop given by G3 which is simply fixing the output voltage. So the output voltage Vout which would be floating because everything in the G2 loop is floating, is fixed by the additional loop, and it is fixed to the voltage Vdc. Why Vout equals Vdc?



- The transconductor G_3 adds a weak low-frequency path between V_{out} and G_2 input and constrains DC operating point of the circuit.
- A capacitance C_3 is added in parallel with the attenuating capacitances to increase the filtering capacitance.

$$H(s) = \frac{sC_1(A+1)G_2}{[sC_1(1+A)+G_1][G_2+s(C_3(A+1)+AC_2)]}$$

(2 stages of this 2nd-order filter are finally used in the system)

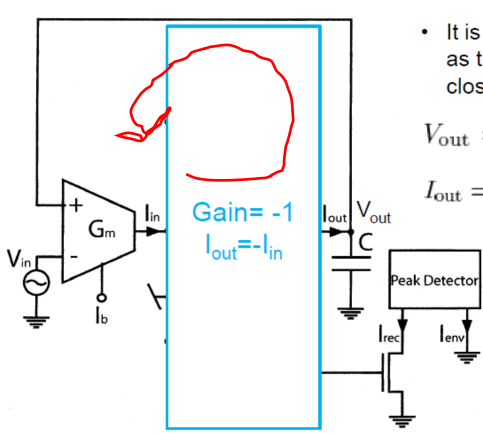
Let's assume that this is not true, that for example Vout > Vdc. If so, we would have an imbalance input at G3. We would have an exiting current that would integrate on the capacitor C2 and AC2 and so the node V2- would rise, but if so, we have an entering current in G2 from the out, but if so, C3 is discharging and so Vout is decreasing. So G3 and G2 perform a negative feedback so that the result of the negative feedback is to fix the voltage Vout to Vdc. If this would not be the case, we would have an exiting current which would restore things on the node of Vout.

If we do the calculation again, we still get a t.f. with one zero and two poles, simply we have in addition the effect of C3, that helps in defining the pole in addition to AC2. So we have not only AC2 but A(C2+C3), but the t.f. is the same.

In conclusion, in the real circuit we have a cascade of this structure, that is replicated twice, so we have 4 poles and the roll off is of 40dB and not 20dB.

ENVELOPE DETECTOR

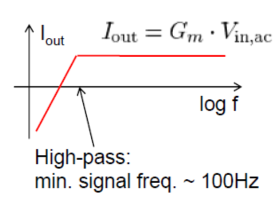
It is a circuit that takes the peak of the amplitude after the bandpass filter. The output of the filter is still the sum of sinusoidal waves, but filtered in a given bandwidth and we need to pick up the peaks of these sinusoids.



- It is based on a G_m-C filter, but as the Gain = -1 the loop is closed on + and not on -.

$$V_{out} = (V_{in})/(1+s(C/G_m))$$

$$I_{out} = (sC \cdot V_{in})/(1+s(C/G_m))$$

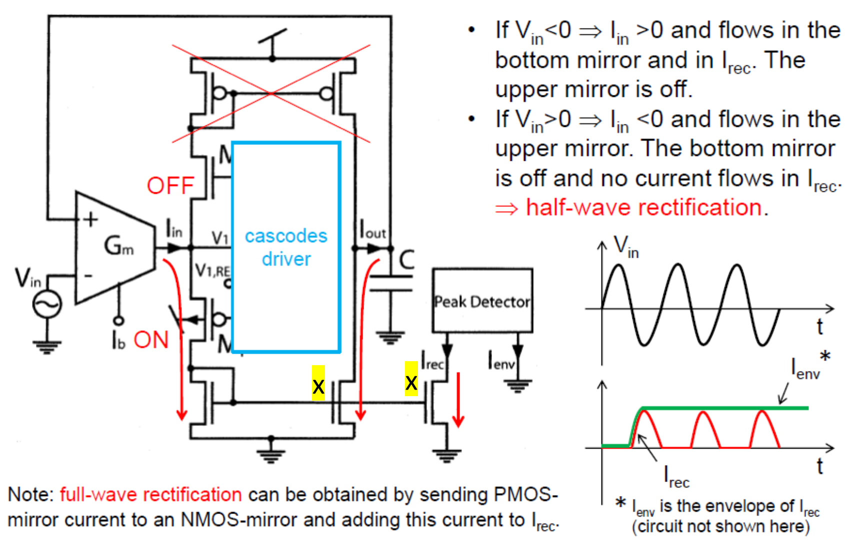


The enveloper detector is composed by several blocks. For now the light blue is a mystery box, we concentrate on the loop now (in red). The loop is a Gm-C filter, the only difference is **the block in the middle that is a gain = -1 buffer**. So if we have a current in the input, the current at the output is flipped by sign. So if we insert an inversion, we will notice that the loop of the Gm-C is closed at the non inverting input. In the classical Gm-C it is closed to the non inverting input, but since n the middle we have inserted an invertion, to keep negative the loop we need to close it on the non inverting input. Besides this, the t.f. is the same of Gm-C.

However, we are not interest in the voltage output, but to the current flowing in the capacitor C. This current is equal to the voltage divided by the impedance of the capacitor 1/sC. **The relationship voltage vs current is** no more a low pass filter, but a **high pass filter**, because we have a voltage to current conversion through the capacitor. This current is mirrored to the transistor where we have Irec, so Iout = Irec. If so, we have obtained a high pass filter.
<u>The goal of the high pass filter is to filter the possible noise sources below 100Hz</u>. (if we change the capacitor C for each channel, we can set a specific high pass frequency for each channel)

In conclusion, we have transformed the input voltage into a current in the transistor (Irec) but we have also add a high pass filter because we want to cut the frequencies that are not interesting and filter out the noise.

Let's now disclose the box in the middle.



It is a cascode plus a current mirror. It is a pMOS cascode and a current mirror on the bottom and on the top we have an nMOS cascode and a current mirror. **The cascode and current mirror in the bottom implement the gain = -1 t.f**, because when a current is entering in the pMOS cascode, we have a flipped current at the output, thanks to the mirror. So it is indeed a current buffer with gain = -1, because we have flipped the sign of the current.

When does the bottom cascode and mirror work?

When Iin is positive with the direction of the red arrow, because only a current larger than zero (positive) can enter in the pMOS cascode, the top cascode is off. **Iin is positive only when the input Vin is negative**, because in this case the transconductor produces an output current.

So negative input, in correspondence of the negative part of the wave, we have a positive exiting current Iin and it is flipped. Then the transistor with Irec is simply a replica of transistor of the mirror (x).
If I plot Irec as a positive current, Irec exists and it is positive only in correspondence of the negative lobes of the waves (red plot on the left). The negative lobes are hence producing positive currents on the output of this circuit. Then if I take a stage (peak detector, a rectifying stage) that is sensitive to the peaks, the current Ienv is just the peak of my rectified halved wave.

This is the so-called **half wave rectifier**. It is half wave because we have rectified only the negative half wave. Infact, the positive lobes produce an inward Iin in the OTA. But if we have Iin as so, this current cannot go in the cascode below, because it can accept only current from the top → hence the n-cascode and the mirror are switched on, but this cascode and mirror are going nowhere, Irec is no connected → positive lobe is not connecter to Irec.
How can we modify it to have a full wave rectification?

Positive lobe

It means Iin inward in the OTA.

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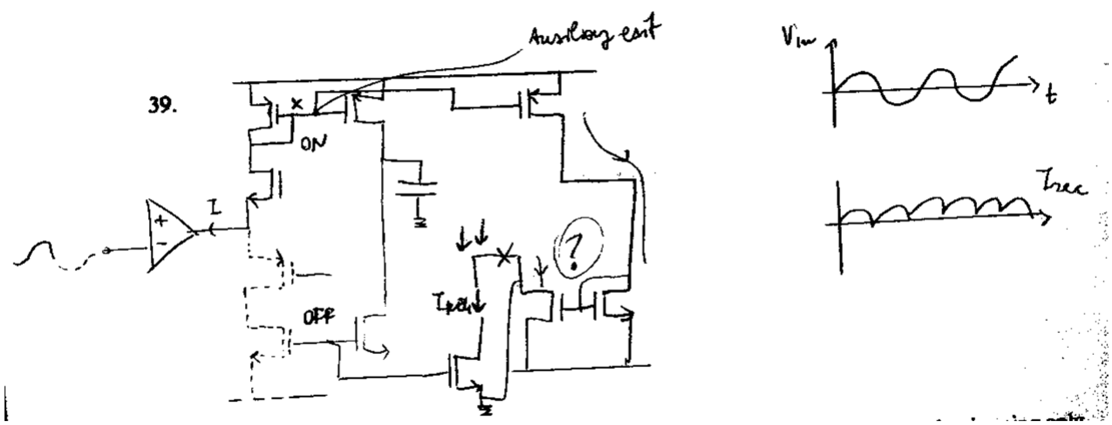
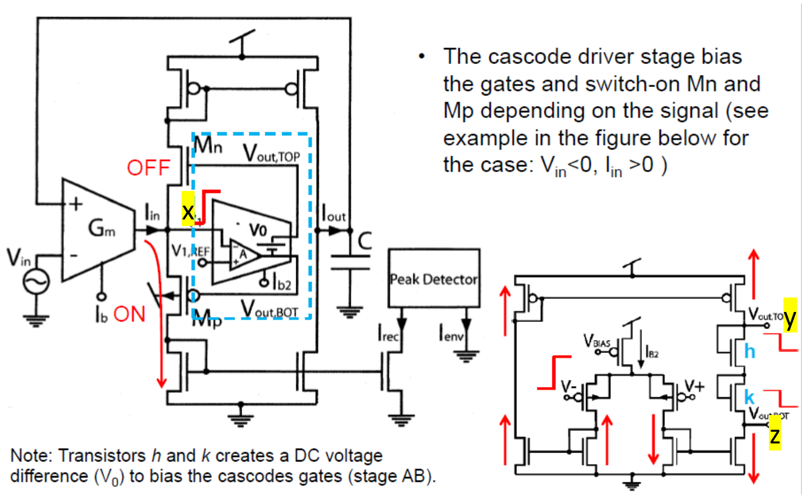


This inward current is compatible only with a n-cascode, then we have a p-mirror that is going only on the capacitor, while the p-cascode in the bottom is off.

However, I have the Irec transistor, which is now connected an off block (blocco spento). To have a full wave rectification, it is sufficient to take an auxiliary exit from the top mirror (x), I connect to a current mirror (the current flowing in the top is hence mirrored) and this current can exiting from the transistor and join Irec.
So normally Irec is only connected to the mirror below, but now it is off; when we have positive lobes, the mirror at the top is taking the current. But if I take an auxiliary exit and use a nMOS mirror, I have a current in the same direction than the current that was collected during the negative lobe.
In this way we, in correspondence of positive and negative lobes, a full wave rectification.

Cascode driver

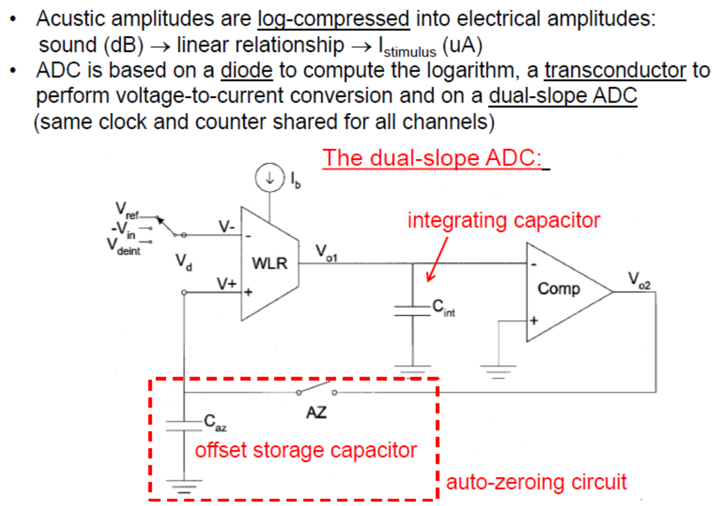


It is a service circuit that switches on and off the cascodes. It senses the direction of the current, because according to the direction of the current the input node x goes up or down. For instance, if we have an exiting current Iin the node x will step up (if we inject current in a node, the node goes up). The amplifier in the middle (bigger one), when we have a positive input, switches on the pMOS and off the nMOS. It is done following the circuit on the right.

The output are connected, respectively, to the top transistor and to the bottom, so when we have a positive input of the amplifier, we have a signal current going up on the left, a signal current going down on the right, the two currents are mirrored and on the left extreme branch the current goes up, it is again mirrored, extracted from y and to se point y that is the gate of the nMOS transistor is going down (if we extract the current from that node, the voltage of that node goes down).
In this way we are switching off the nMOS because the gate is going down. Conversely, the current mirrored on the right side is also extracted from node z, so also the node z is going down, but it is the gate of a pMOS → if the gate of the pMOS goes down, it switches on.

The two transistors h and k implement the AB-stage, are two transdiode that keep the voltage difference between the two nodes, of the transistors Mn and Mp, positive in a way so that the two transistors are not completely off but in weak inversion

LOGARITHMIC AMPLITUDE-TO-DIGITAL CONVERTER (ADC)



We want to convert the envelope in a digital word. It is a logarithmic ADC because we need to compress further the signal so not to exceed the comfortable hearing level. In practical terms, the sound already in dB is linearly converted into a stimulation current. To make a logarithmic compression we use a diode.

We start with the ADC topology. It is a classical dual slope ADC (Wilkinson ADC). It consists of a transconductor (OTA), we provide different input voltage according to the phase at which we are (input analog signal, Vref, negative replica of input signal for polarity reason) and then the output current of the OTA will be integrated on the capacitor and then we have a comparator that is monitoring if the voltage across the capacitor is crossing zero. when it will cross zero, so become lower than 0, the comparator will switch, and we will have a digital-like signal indicating the end of the conversion.
These blocks (OTA, Cint and comparator) represent the typical dual slope ADC.

We have an additional block on the bottom, which is just a service block called auto-zeroing circuit, that doesn't cover a functional role, but it is used to compensate the offset at the input of both the OTA and the comparator. The offset of the OTA and comparator introduces errors in the conversion, so this circuit allows for compensation of them.

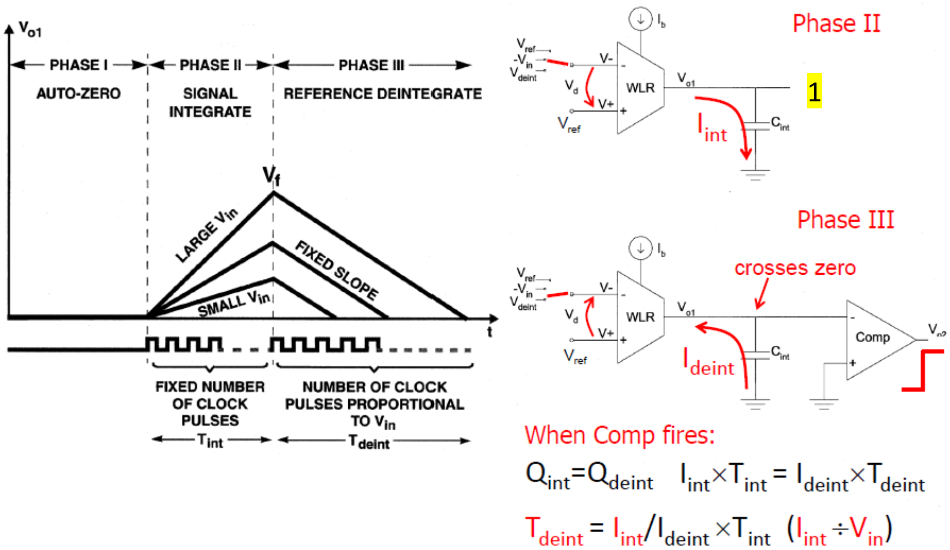
ADC OPERATION

It is divided in 3 phases:

- Autozero (phase I)
- Signal integrate (phase II)
- Reference deintegrate (phase III): conversion takes place in phase II and III

Phase II: Signal integrate

In phase II one input of the OTA is kept at a reference voltage (V+), while the negative one is connected to -Vin, so we should have recovered somewhere Vin from the previous stage and we make a negative replica so that in this phase the voltage across the input of the OTA is positive as indicated by the arrow in picture 1. If we have a positive voltage, the current is going out and it is integrating on the capacitor Cint. So we have a **ramp**, and the slope of the ramp is proportional to the input signal to be converted (larger the voltage, larger the current, steeper the ramp).



We have a clock and we allow this integration lasts a fixed time, Tint. Then we reach a final voltage Vf and we move to phase III.

Phase III

The input now is changed to a specific voltage that is called Vdeint, the voltage at the input of the OTA flips and changes sign and hence the current at the output of the OTA changes sign and of course we start to remove the charge accumulated on the C on the previous phase.

The peculiarity of this phase is that, while in previous phase the slope was proportional to the signal, in this phase III; since we are applying always the same Vdeint voltage, the Ideint current will be always the same, so the slope will be constant.
Hence we have change in slope in phase II and constant slope in phase III (plot on the left).

In this phase we start the counter again, the clock, and we reach a 0 charge on the capacitor when the voltage on the capacitor is 0. In this case the arrival to 0 is detected by the comparator, because when the voltage crosses zero and becomes negative the comparator rises the output. When this happens (C completely discharged and voltage reaches 0) the clock is stopped.

The ADC conversion consists in the calculation or counting of the number of clock pulses. By counting them we compute the integration time that is proportional to the input voltage. Indeed, the larger the input voltage, steeper the slope and so longer will be the time to reach zero.
The conversion consists in counting the ck cycles from a minimum of 0 (if Vin is 0) up to a full scale range that is the maximum voltage allowed to be integrated on the C and the resolution is the ck cycle and the maximum range is the maximum number of ck cycles we are able to count for the maximum voltage.

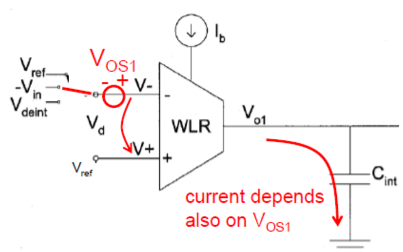
In the bottom we have the calculation of what just described. In both integration and deintegration the amount of charge accumulated on the C will be the same and so the product current * time is constant and so by counting the deintegration time as the deintegration current is constant and integration time is constant, the only variable is the integration current that is proportional to the input signal and so we have a perfect proportionality between the number of clock counted and the amplitude of the signal.

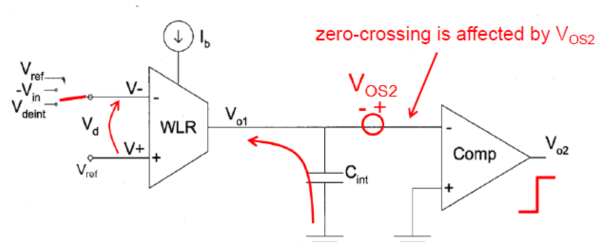
The advantage of this ADC with respect for example to the single ramp is that single ramp are based on a ramp proportional to Vin, so we place a reference voltage and we count how many ck cycles has occurred before reaching a given threshold. The advantage of this dual slope circuit is that integration and deintegration happens on the same capacitor. If we would have a conversion relying on the absolute voltage value across the capacitor, this current to voltage conversion would depend on the value of the capacitor, and if the value of C changes prototype by prototype, we would have a conversion factor depending on the value of the capacitor.

Since in this case the capacitor is the same in both phases, if C changes, of course we change the slope in phase 2, but we also change correspondingly the slope in phase 3 → any change of the capacitor is irrelevant with respect to the final conversion → insensitivity to the value of the capacitor.

Now we move to some details. The first is the sensitivity of this conversion with offset of both OTA and comparator and how the auto-zeroing circuit solves the problem.

Effect of OTA offsets (if auto-zeroing is not used)





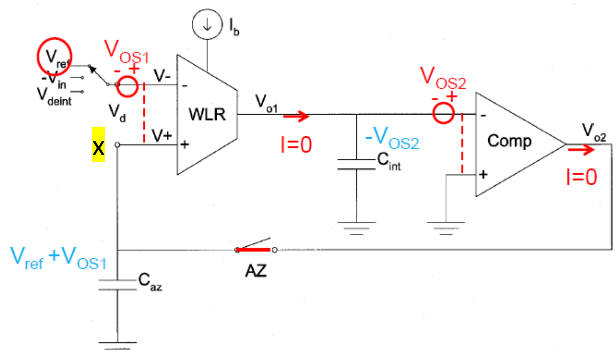
Let’s consider the OTA and that we don’t have the auto-zeroing circuit. When we connect the OTA to an input voltage we have a voltage Vos1 that depends from prototype from prototype that is usually represented as a voltage generator at the input of the circuit. If we have this voltage generator, the overall voltage across the input is not just given by the difference between input and Vref, but it is given by Vin, Vref + Vos1 (offset value). As the output current depends on the overall voltage at the input of the OTA, the current we are integrating on the capacitor is sensitive with respect to this offset.

A similar problem arises even at the input of the comparator. If we have such a voltage, the comparator is not fired when the voltage on the capacitor Cint reaches 0, because having 0 on the capacitor doesn’t mean to have a zero on V- of the comparator, but we have 0 + Vos2. If the offset is represented with the polarity as in the image, we need to have a more negative value on the capacitor, so we need the ramp to continue up to negative values on the capacitor before the overall input voltage is 0, and so before the comparator is fired.

Hence the offset in the OTA produces a mistake in the slope of the ramp, because it acts on the input signal, while the offset on the comparator produces a mistake on the time when the ramp is supposed to cross 0.
Of course, if the offset would be the same forever, these mistakes could be calibrated, but if they change with time, temperature or other non-deterministic factors, we introduce and error in the conversion.

This bring us to the auto-zeroing circuit.

Phase I: auto-zeroing circuit



At the end of this phase, all capacitors are charged (I=0). Virtual short circuits at the inputs of OTAs transfer offset voltages on the capacitors.
⇒ $V_{ref}+V_{OS1}$ is stored on C_{AZ}. As offset it is stored on V+, it will be then applied to both inputs and will not affect the measurement.
⇒ $-V_{OS2}$ is stored on C_{int}. It will not count anymore in the zero-crossing.

It is the starting phase, before we apply the input voltage at the input, but we apply a reference voltage that we want to store on the input of V+, on the capacitor Caz. By the way in this phase we also explain how the reference voltage in phase 2 is given, since it is programmed in this phase on the capacitor Caz. Then this voltage will remain for the rest of the conversion.

When we close the loop in this auto-zeroing circuit, we connect the output of the comparator and we create a negative loop. In this negative loop, as long as the voltages at the inputs + and - of both OTAs (the first and the comparator, also the comparator is an OTA) are not the same, by definition we have a nonzero current at the output of the OTAs.
In the autozeroing phase the voltage on pin x is not equal to the reference on the other pin V+, and so we have current starting to flowing outward, changing the voltage on the – of the comparator and so when – changes, the current outward from the comparator changes.
The negative loop stops when we have a virtual shortcircuit at the input of both amplifiers, otherwise we would continue to integrate current on Cint and Caz.

Hence the final situation is when we have the virtual shortcircuit.
But if we have this virtual shortcircuit, which corresponds to a static condition of the loop (with 0 current everywhere), by the K. law, in the network at the input of WLR, in the node x we have the reference voltage plus the offset (Vos1).

So we have accumulated on node x not only the reference voltage that was needed to start phase 2, but as a bonus, **we have programmed on Caz also the content of the offset generator**. By the K. law, Vref + Vos is equal to the voltage in x.

This is very nice because when we will have in phase 2 the input voltage, in reality we will have the input voltage plus the offset (on the – pin) but on the other pin (+) we have also the offset, which has been previously programmed, so offset and offset cancel out (we have offset voltage on both inputs) → the new voltage difference will depend just on the voltage difference between Vref and Vin, because the offset will be cancel out.
Hence during the autozeroing phase the offset is cancelled because already programmed in this phase.

The same applies for the second OTA, the comparator, because if the current is 0 in output, we have a virtual shortcircuit and for the K. law the voltage accumulated on Cint is -Vos2. So when at the end of the second integration we have completed the complete disintegration, we will have a remaining content of charge on the capacitor which will compensate the offset of the comparator so that the comparator is fired at the right time (the two offset Vos2 cancel out).

Recap: phase I is the autozeroing, phase II is the integration of the signal (offset is no more a problem, it is self compensating), phase III is the removal of the charge on the capacitor, so a fixed slope and in conclusion we have that the same charge added has been removed, the capacitor Cint doesn't play any role because has been integrated and deintegrated and in the end by counting the ck cycles I get an information about Vin with a quantization error given by the ck period.

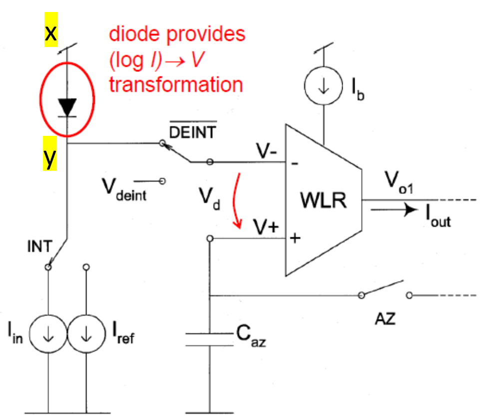
$$Q_{added} = Q_{removed}$$ (independent from $Q_{offset2}$ pre-charged on C_{int})

$$I_{int} \times T_{int} = I_{deint} \times T_{deint}$$ (independent from V_{OS1} applied to both inputs)

depends on V_{in}

$$T_{deint} = \frac{I_{int}}{I_{deint}} \times T_{int}$$

Adding logarithmic conversion to the ADC



- During I_{ref}, V+/V- charges to:

  $-\phi_T \ln \frac{I_{ref}}{I_s}$ (plus offset)

- During I_{in}, V- charges to:

  $-\phi_T \ln \frac{I_{in}}{I_s}$

- The differential voltage at the input is:

$$V_d = \phi_T \ln \frac{I_{in}}{I_s} - \phi_T \ln \frac{I_{ref}}{I_s}$$

$$= \phi_T \ln \frac{I_{in}}{I_{ref}}$$ log dependence on I_{in}

(ϕ_T=KT/q)

We need to add a logarithmic conversion → we add a diode. Nothing is touching the ADC; that remains the same we have introduced so far, we simply add at the input a logarithmic conversion between the signal and the voltage at the input of the ADC.

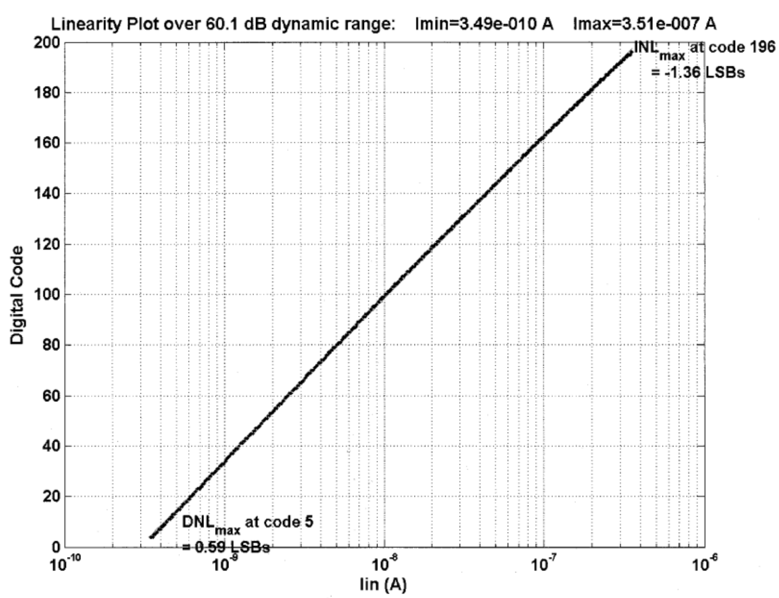
How is the Vin signal is generated?
It is generated through a diode and the diode has the input current that is the current coming out from the previous circuit (envelope detector, Ienv), and we have a logarithmic conversion between the current and the voltage.

During the Iref phase, so the autozeroing, we send to the diode the Iref current (not Iin) and the input of WLR are shorted, we have on + exactly what we have on -, including the offset. Hence on the two inputs V+ and V- that are shortcircuited we have the voltage across the diode, that is given by: -phiT*ln(Iref/Is) + offset, where phiT is the thermal voltage and we have an additional term in case we have an offset. Hence the voltage at V+ and V- is equal and equal to the voltage across the diode and the minus sign is because x is considered ground, and the other y a lower voltage.

During the first integration phase, we now connect the diode to the generator Iin as in the image, so we generate a voltage that is now applied only to the – pin, because the + pin is now fixed to the previous voltage (-phiT*ln(Iref/Is)). Instead, during Iin integration on the other pin (-) we have the voltage written in the picture, that depends on the input current. So in this phase the differential voltage at the input of the OTA, with the sign marked in the image, is equal to V+ - V-, so we have the last formula.

In conclusion we get that the voltage across is proportional to the logarithm of the ration between Iin and Iref → we get the desired logarithmic compression.
Once we have a logarithmic representation of Vd, for sure the slope (during deintegration) will just depend on the transonductance of the WLR OTA.

The following is the result.



The input scale is logarithmic (log on the input current) and the vertical scale is already the linear representation of the digital code → straight line.

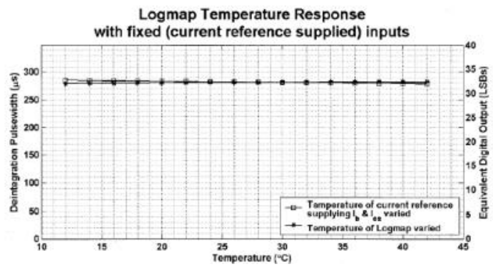
Independency of the temperature

$$I_{\text{out}} = G_m \cdot V_d \quad (\text{linear } V \rightarrow I \text{ transformation})$$
$$= \frac{I_b}{\cancel{V_L}} \cdot \cancel{\phi_T} \ln \frac{I_{\text{in}}}{I_{\text{ref}}}$$

as in subthreshold transconductors V_L depends on T as ϕ_T (ϕ_T =KT/q, $V_L=2\phi_T/k$)

⇒ dependency on temperature remains only on I_b.

⇒ if we assume I_b constant during both integration and de-integration, then long term variation of I_b with T is cancelled out by the dual-slope topology



The logarithmic conversion is independent from temperature. This is important because the cochlear implant is inside the body, so if we have that some parameters like the OTA gain or the current generators Iin and Iref depend on the temperature, we may have a conversion temperature-dependent. This is not true because the current Iout at the output of the OTA that is used during integration is given by the transconductance Gm times the differential voltage Vd just computed before.

The Gm is given by the tail current Ib of the OTA divided by Vl. Vl is then twice the thermal voltage divided by the subthreshold coefficient. So we have thermal voltage at the denominator and at the numerator of Iout → cancels out, so we have independency of the integration current on the temperature.

In the plot we see that by changing the temperature over a very large range, the conversion remains constant. The only remaining dependency on the temperature can be the generator Ib.
If Ib changes with temperature, of course we will have a change in Gm, but it is sufficient that the generator Ib doesn't change during the two last phases.
Hence if during phase II and III the generator doesn't change (quite obvious because the two phases last for a very short time), if we have a temperature variation on long time, the temperature dependence of Ib doesn't matter, because the important thing is that Ib remains the same during the two integration2 (hence the two phases).

INA FOR ECG

The ECG is generated by the AP of the heart cells. There is a small amount in the top part of the atrium that is generating a trigger signal that depolarizers the other cells so to have a propagation of the signal through the heart. This will generate an electric filed that propagates through the chest of the patient and that can be recovered from the skin. It is unipolar.

Usually, the vector that defines the electric field will propagate in space with a direction, so we will find a negative and positive values of voltage on the two sides → we want to find a differential signal that gives us the amount of E generated.

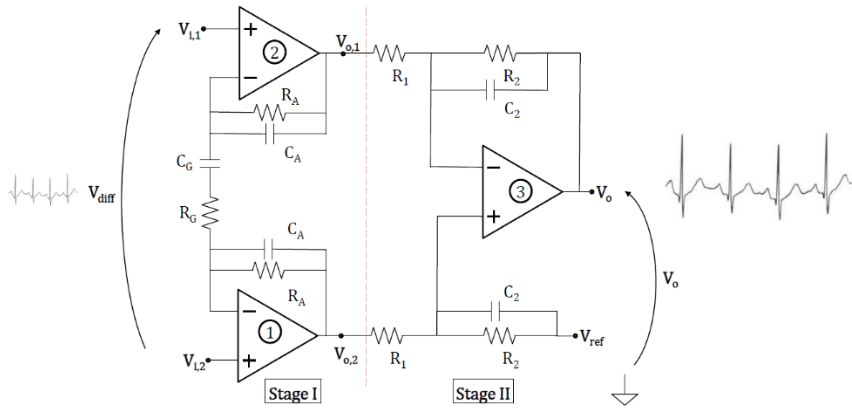
This differential signal to be recovered is in the order of 100uV to 1mV upon a CM signal that can span over 100mV to 1V. These CM fluctuation in the voltage space are usually given by interference signals.
Hence our project needs requirements: we want a differential amplifier that takes as input a differential signal and amplifies it so to have a scale of the differential output in the order of 1, 2 or 3 volt → differential gain in the order of 10^3, and we mustn't amplify any common signal with the same gain. Indeed, we use a few volts biasing for our circuit, we can image that 1V signal amplified through the output will saturate the amplifier → we need a CMRR that is higher than 10^3 or 4, something around 90dB.

A_0	10^3
BW	$\left(f_{low} = 0.16[Hz]\right) \div \left(f_{high} = 25[Hz]\right)$
CMRR	$> 90[dB]$
$\sigma_{v_{n,in}}$	$< 10[\mu V]$

Moreover, the noise cannot fluctuate more than few microvolts. The noise superimposed to the signal must be at least one tenth of out signal → 10uV.
To keep the noise low, we need a suitable filtering for our circuit to eliminate all the noise in the spectral frequency → our ECG recovering instrumentation must be built so to have a bandpass filter, because our signal is between 0.16 and 25 Hz.

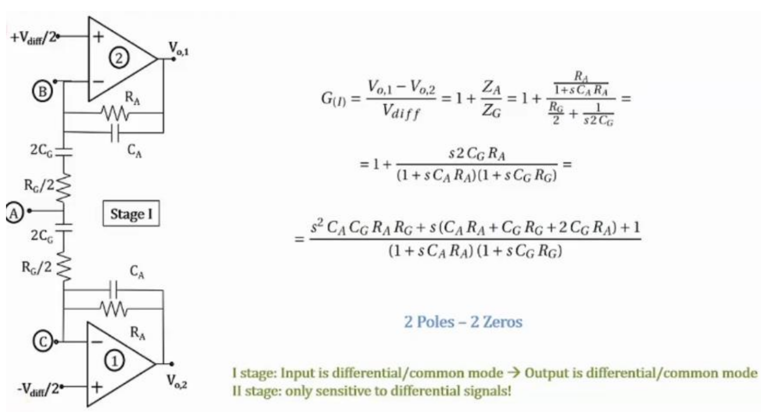
In order to get to a successful design of our instrumentation we have to:
- Compute the frequency response of the INA.
- Size the components to fulfill the design requirements.
- Study the CMRR of the INA and the equivalent noise.
- Design an equivalent electrical model for the electrodes, to get if there are physical reason to change our configuration.
- Check if the design complies with the requirements.

INA FREQUENCY RESPONSE



This is a typical INA made of 3 OpAmp. We will divide the schematic in two stages. The first stage has a very large gain so to amplify the signal as soon as we can and in the second stage we have just a differential stage (classical differential amplifier).

1st stage



Node A under a differential signal can be considered at ground. Indeed, if we apply a differential signal at the input, we can model it as a positive differential signal on one node of the amplifier and an equal but negative differential signal on the other input. Because of the feedback loop of Ca and Ra, the voltage in C is at -Vd/2. The same happens in the other part of the stage, the positive differential half for B.

Hence the node A sees exactly the same impedance through B and C, so it can be considered as grounded, doesn't change its voltage. If node **A is always at ground**, then the circuit is split in two halves, two noninverting amplifiers.

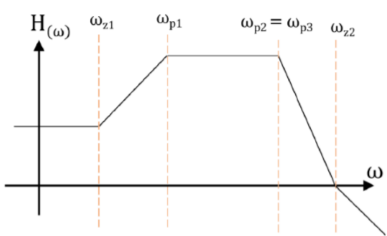
Hence the differential voltage is propagated towards Vo2 and Vo1 propagating hence a differential voltage.

NB: the transfer function of this stage has 2 poles and 2 zeros.

Moreover, if we apply a CM signal with respect to a differential one, the two B and C are kept at the same voltage, so we will have the branch with Rg and Cg with no potential drop → no current flows in this part. Because of this, when we apply a CM there is also no current flowing in the feedback branches of the first stage amplifiers, so the CM input is directly transferred to the output, because there is no current flowing through the feedback branches.

On the second stage, the CM signal, since we have a differential amplifier, in the ideal case there will be no Vout generated due to the CM. This is good for our application, there is amplification only for the Vd and not Vcm (Vcm is not amplified in the 1st stage and cancelled in the second one).

TRANSFER FUNCTION



$$H_{(s)} = -\frac{R_2}{R_1}\frac{\left(1 + s\,C_G\,R_G\left(1 + \frac{2R_A}{R_G}\right)\right)\left(1 + s\,\frac{C_A\,R_A}{1+\frac{2R_A}{R_G}}\right)}{(1 + s\,C_G\,R_G)(1 + s\,C_A\,R_A)(1 + s\,C_2\,R_2)}$$

$$H_{(s),FB} \approx -\frac{R_2}{R_1}\frac{\left(1 + s\,C_G\,R_G\left(1 + \frac{2R_A}{R_G}\right)\right)}{1 + s\,C_G\,R_G}$$

In the end we have 3 poles and 2 zeros in the overall t.f.. We want to get a bandpass filter in the region where our signal is (0.16-25Hz), with a large gain in this region and also a very large CMRR.

We want to cut the low frequencies → a zero very close to the DC frequencies. Moreover, we want to sharply cut the higher frequency. Poles P1 and P2 define the width of our band pass filter.

We can build a t.f as in the image, where the poles p1 and p2 define the band.
A good practice in electronics is that we must try to have as much gain as possible close to the source of our signal → INA must be designed so to have all the gain in the first stage of the amplifier and then we will use a second stage with gain 1 to kill Vcm.

Moreover, in 1st stage, to reject the low freq, we can use the capacitance Cg so that the differential signal at the input in the low freq domain is decoupled, so that there is no current in the Rg-Cg brand and if no current flows, then for the CM signal we will have the output voltage that is simply the copy of the input voltage (no amplification), and the same in DC when the C are open circuit.

If we see this from the t.f. point of view, we see that the gain of stage I is inversely proportional to the impedance Zg, so if Zg is very large, then there is no amplification in the signal that propagates from the input to the output → amplifiers 1 and 2 are in buffer configuration

We need to keep the amplification close to the source of the signal for 2 main reasons:

- If we amplify the signal closely to the source, we get a large signal that propagates through the next circuits. Hence the noise will be superposed to a large signal and we will have a large signal to noise ratio. Moreover, if we try to compute the equivalent input noise, we will find that the noise generated from sources after the first stage has a smaller weight on the signal.
- If we have a CMRR in the second stage that is affected by some components, having a larger gain in the first stage allows us to neglect the contribution of the CMRR in the second stage.

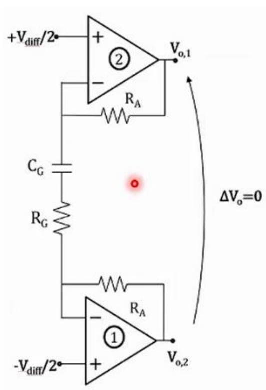
POLES AND ZEROS

The first pole is due to the feedback resistance of the differential amplifier in the second stage. Hence the first pole is identified by the couple R2-C2.
In the first stage, similar to what happens in the second stage, that is when C2 is comparable with R2 we have a pole, when Ca has an impedance similar to Ra, the impedance in the feedback branch decreases and we have a decrease in the gain. So in proximity to the frequency where this happens, that is when Ca is similar to Ra at least in absolute value, the gain reduces and we find another pole.

As for the 3rd pole, at 0Hz the capacitance Cg is completely open and the two feedback branches see and infinite impedance on the g-path. Hence this pole says that when Cg is starting to decrease its impedance, the g-branch has an equivalent impedance that gets smaller, so the gain (of the noninverting amplifier) increases.
However, the gain doesn't increase up to infinity, but there is a frequency at which the Cg becomes comparable to Rg and so we will get the maximum of our gain → Rg*Cg is our 3rd pole.

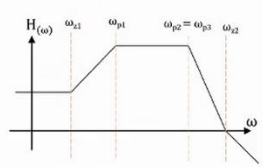
To identify the position of the zeros, I need to build an equivalent circuit.
The first pole is with a tau of Ca*Ra. Similarly, the third pole is at C2*R2. If in the differential stage the impedance of C2 gets smaller than R2 impedance, then we don't build an output voltage and we don't get any gain (high freq).
The third pole is given by the fact that in DC we have a complete decoupling if the feedback branches of the opamp 1 and 2 up to a point where Cg becomes negligible with respect to Rg.

Computation of the zeros



$$i_{R_A}\left(2R_A + R_G + \frac{1}{s_{z,1}C_G}\right) = 0 \Rightarrow$$

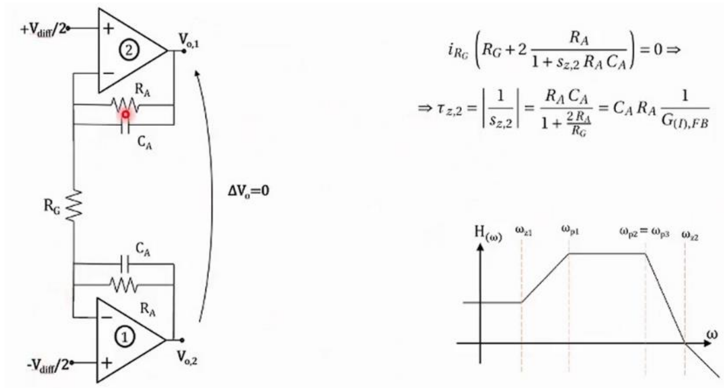
$$\Rightarrow \tau_{z,1} = \left|\frac{1}{s_{z,1}}\right| = C_G\,(2R_A + R_G) = C_G R_G\left(1 + \frac{2R_A}{R_G}\right) = C_G R_G G_{(I),FB}$$



We know there is a very large space between wz1 and wz2 because of how the t.f. is build. So we can consider the feedback capacitance Ca as an open circuit where we are below its pole and Cg as a shortcircuit above its pole.

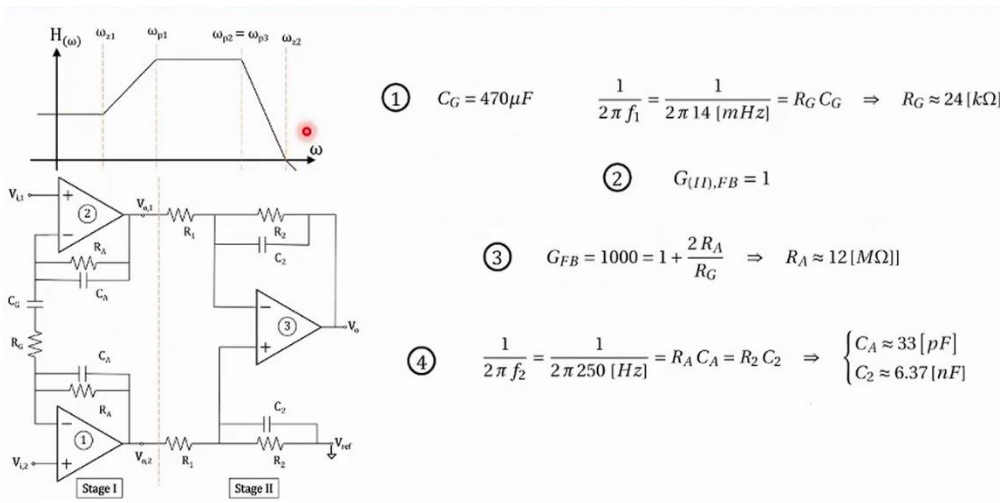
Let's focus on wz1 and let's see if there is a frequency at which I have a differential signal at the input but still I cannot have any differential output. If I use K. law, I see that the output is 0 if there is no drop on the series of Ra, Zg and Ra. If I suppose there is a current iRa is not zero, is there any wz1 that is leading me to a null drop at the output of the amplifier? I'm supposing there is a current in that branch but no voltage between Vo1 and Vo2 (this is the definition of a zero).
If I do so, then I have to impose the term in the parenthesis to 0; I find that the time costant of my first zero is equal to the time constant of the pole multiplied by the gain of the first stage.

I can now apply the same reasoning to the other zero (in the HF part). Cg can be considered as a shourtcircuit. Again, I can suppose that there is a current flowing in the Rg with a null output voltage.



We get the result as in the image. It is at very high frequency, hence another property of this circuit is that while the low freq zero is put at very low freq by our architecture, the HF zero is delayed in frequency by the same gain of the first stage → I can get a very large bandwidth to operate and properly filter the noise in my device with these zeros.

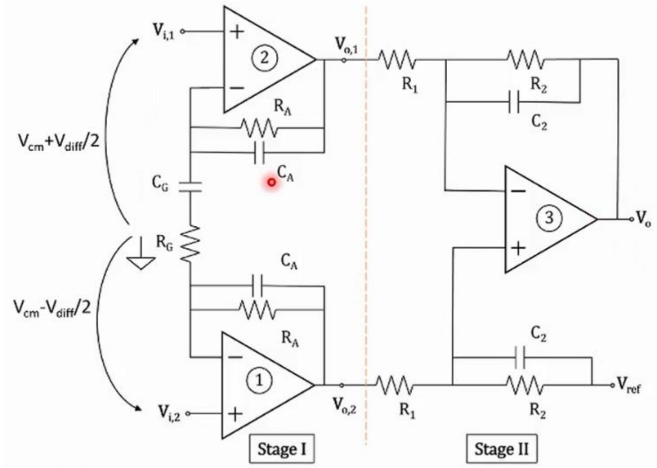
DISCRETE COMPONENTS SIZING



We know that the LF zero of our t.f. is at a frequency defined by the Cg. So we take Cg very large (470uF). We want it very large because even if we choose the frequency of the pole as the product Rg*Cg, we don't have Rg very large, because Rg is at the denominator of the gain of the first stage, so we want it relatively small. Moreover, we can select Rg so to have the Rg at the frequency of our high pass filter to target our flat bandwidth.

We want moreover to keep all the gain in the first stage of the INA, so in the second stage we choose a gain = 1 → R1 = R2 (in the order of 100kOhm, not so large to have noise propagating in the second part of the circuit, but big enough so not to have too large currents). We also want to have a flat band gain in the order of 1000 → Ra = 12MOhm.
Moreover, to properly filter the HF part of our signal, we want to pose wp2 = wp3 at 250Hz, ant in this way, because we know Ra and R2 we can size Ca and C2.

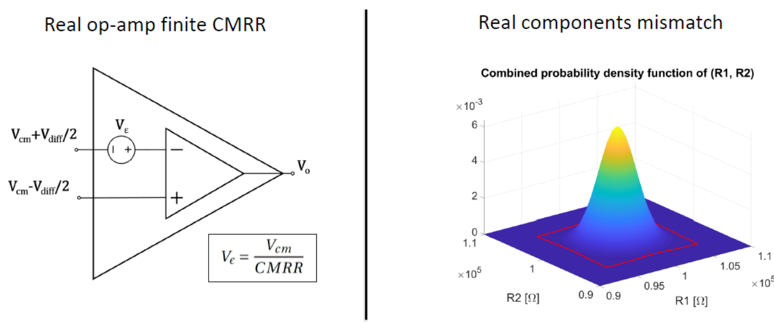
CMRR: IDEAL CASE



$$CMRR = \frac{G}{G_{cm}} = \frac{\left(\frac{V_{out}}{V_{diff}}\right)}{\left(\frac{V_{cm,out}}{V_{cm}}\right)}$$

It is the ratio between the differential gain of the INA and the CM gain. If we don't have a sufficiently large CMRR, then our circuit won't operate in the correct biasing point, but it will just be subjected to the very large input voltage fluctuation and so won't be able to recover our signal.

CMRR: NON-IDEALITIES



$$CMRR = \frac{G_{(I)} \cdot G_{(II)}}{G_{cm,(I)} \cdot G_{cm,(II)}} = CMRR_{(I)} \cdot CMRR_{(II)}$$

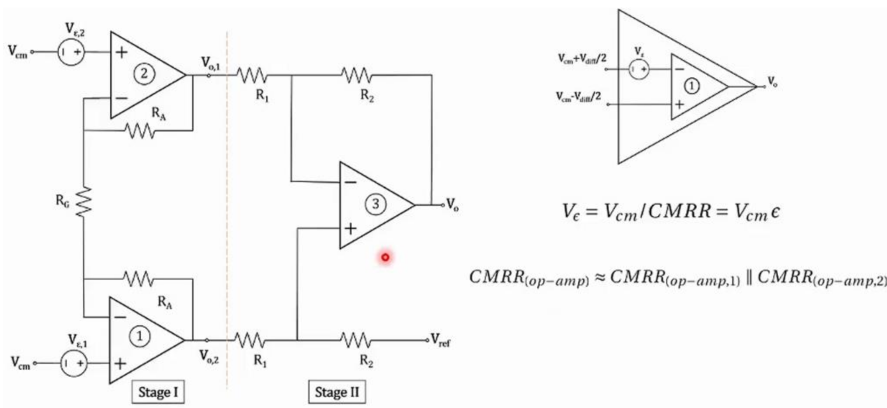
Theoretically, the CMRR is infinite in the ideal case, because the first stage has a propagation of the CM signal that is 1, while the second stage has a gain for CM input signal theoretically equal to 0. But what is affecting the CMRR in the non-ideal case?

What we want to have in our INA is a net 0 gain for the CM from input to output. This is mined by the modelling of our circuit. We usually think about the small signal behaviour of our electronics, but our electronics is working in a certain biasing condition and with non-ideal components. Its opamp is working with a biasing condition that changes in time when I change the CM of the input and all the components have not the ideal value we expect, but a small fluctuation in their values.

A very easy way to model the nonideal behaviour of an opamp is to take an ideal opamp and put in series to that a voltage generator that models the non-ideal behavior. V-epsilon generates a differential input for the opamp and consequently an output that is proportional to the CM at the input of the amplifier. V-epsilon is proportional to the CM gain and CM input.

Regarding instead the discrete components such as R and C, we cannot use a modeling as for the opamp. We need to take into account that the values of these components are statistical values and then check what happens to the t.f. if we go at the extreme boundaries of the probability distribution of the distribution of our components' values.
For example the ideal value for R1 and R2 is 100 kOhm, but the probability that I'm picking this value is 0, and I will get instead different combination of the resistance R1 and R2. The worst case is when the two resistance are very imbalanced, because the t.f. won't be anymore the one of an ideal differential amplifier and we will have some CM gain in the second stage.

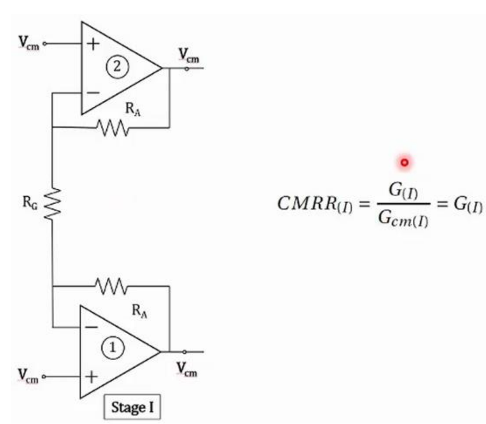
Let's now model the opamp as a finite CMRR amplifiers → we put equivalent differential signal generators. Let's imagine we are applying a CM at node + of OA1. The generator in series with Vcm generates a differential input at the OA1. For OA1, we have that V-eps1 (that models the differential signal at the input of the amplifier that is generated by CM voltages at the input) is propagated at the output exactly as a differential signal.
So we have to choose OA1 and OA2 very carefully, they must have a very high CMRR.



Moreover, if we try to check what is the effect on the input of CM in the amplifier 3, we will see that the CMRR limitation given by OA3 is decreased by the gain of the stage I. This is another reason why we want to keep the gain of the first stage very large, because not only it helps to reduce the effect of the noise on the second stage, but also helps to reduce the influence of the 2nd stage on the CMRR.

Component mismatch

If we imagine to have a mismatch between the resistances Ra, then we will see that there is no effect on the CMRR. This because the gain of the 1st stage for a CM signal is 1, and so there is no current flowing in Rg for an ideal behavior of the OA1 and OA2. So because there is no current flowing in Rg in presence of a CM signal, the CM is just copied at the input of 2nd stage. Hence even if there is an imbalance between the resistances in the feedback branches, this won't affect the CMRR.

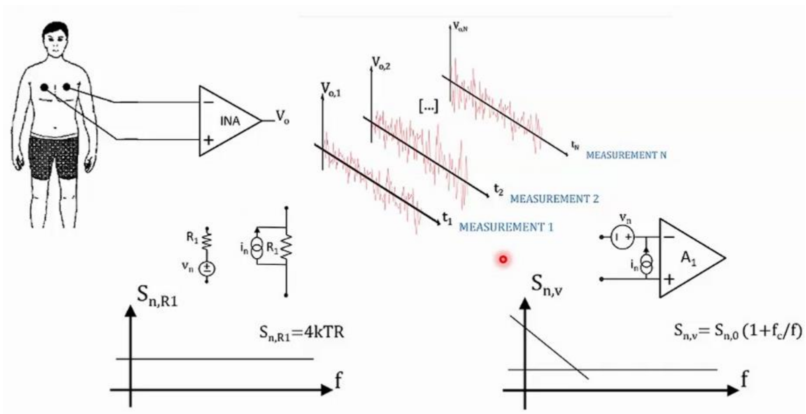


In this case, the CMRR given by the component mismatch is just the gain of the first stage, because the gain of the CM is 1, so the ratio will be G(I).

As for the component mismatch consequences on the 2nd stage, the CMRR is changed.

In conclusion, what is limiting the CMRR of the INA is the mismatch of the resistances in the 2nd stage and the CMRR of the two input opamp.
The only option we have is to limit the CMRR by selecting correctly the opamp at the input of the INA, because they have a big influence on the CMRR of the INA.

NOISE



As seen for the CMRR, also the noise is modeled in electronics using ideal components that we add to our circuit to mimic the physical effect of the noise. For example, if we want to take into account the thermal noise that is generated into resistors, we have to add a parallel current generator to the resistor. Similarly, if we want to take into account the equivalent input noise at the input of an opamp, we need to add a current generator in parallel to the inputs and a voltage generator in series.

Anytime we make a measurement of our voltage amplitude, there is a fluctuation superposed to this voltage that is considered as a statistical variable that is the noise. So we don't know a priori what is the amplitude of this fluctuation and its punctual value, but we know that we can model the noise in electronic circuits using the noise spectral density.

We start from anywhere in the circuit, we compute the effect that each noise source has on the node we want to analyze and then we use the computation of the variance to check what is the effect of the noise source on that specific node of the circuit. So anytime we need to compute the contribution of the noise, we add the noise sources to our schematic, then we compute how much each source influences the node and then to compute the fluctuation over this node we have to compute the integral of the noise spectrum.
The integral of the noise spectrum is equal to the variance of the signal that we are seeking.

Wiener-Kintchin Theorem:

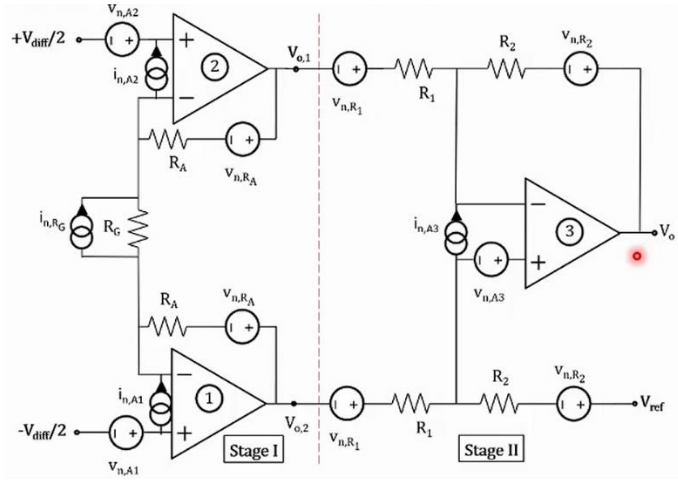
$$S_{n(f)} = \int_{-\infty}^{+\infty} e^{-2\pi i f \tau} R_{nn(\tau)}\, d\tau$$

Signal variance calculation:

$$\int_{-\infty}^{+\infty} S_{n(f)}\, df = \sigma_n^2$$

$$\sigma_{out}^2 = \sum_{i=1}^{N} \sigma_{out,i}^2 = \sum_{i=1}^{N} \left\{ \int_{-\infty}^{+\infty} S_{n,i(f)} \cdot \left|H_{i,(f)}\right|^2 df \right\}$$

INA equivalent output noise



Because we imbalanced the gain in our INA, we have that the influence of the noise over the output node of the generator that are on the second stage is negligible, so we will focus only on the noise generator of the input stage, that are the noise generators associated to OA1 and OA2 and the noise generators associated to the resistances Ra and Rg. Once their effect has been computed, we will integrate the spectral density that we have, we will compute the fluctuation at the output node and then we can compute the equivalent input that corresponds to these fluctuations just by dividing by the gain of the INA.

(see notes for analytical computations)

If then we take into account all the contributions together we get the following.

$$\begin{cases} S^{A}_{n,out(f)} = \left(S^{A1}_{n,v(f)} + S^{A2}_{n,v(f)}\right)\left(1 + \dfrac{2R_A}{R_G}\right)^2 \left(\dfrac{R_2}{R_1}\right)^2 + \left(S^{A1}_{n,i(f)} + S^{A2}_{n,i(f)}\right) R_A^2 \left(\dfrac{R_2}{R_1}\right)^2 \\ \\ S^{R}_{n,out(f)} = S^{R_G}_{n,i(f)}\,(R_A + R_A)^2 \left(\dfrac{R_2}{R_1}\right)^2 + \left(S^{R_A}_{n,v(f)} + S^{R_A}_{n,v(f)}\right)\left(\dfrac{R_2}{R_1}\right)^2 \end{cases}$$

$$\boxed{\text{LTC-2057}}$$

$$\boxed{\dfrac{S^{R_G}_{n,i(f)}\,(R_A + R_A)^2 \left(\dfrac{R_2}{R_1}\right)^2}{\left(S^{R_A}_{n,v(f)} + S^{R_A}_{n,v(f)}\right)\left(\dfrac{R_2}{R_1}\right)^2} = \dfrac{\dfrac{4kT}{R_G}\,(2R_A)^2}{2\cdot 4kTR_A} = \dfrac{2R_A}{R_G}}$$

$$\begin{cases} S^{A1}_{n,v(f)} = \left(11\,[nV/\sqrt{Hz}]\right)^2 \\ S^{A1}_{n,v(f)} = \left(0.15\,[pA/\sqrt{Hz}]\right)^2 \\ 4kTR_G = \left(19{,}9\,[nV/\sqrt{Hz}]\right)^2 \end{cases}$$

The first equation of the system is the contribution given by the opamp, the second one given by the resistances. The input noise generated by the amplifiers can be divided into two contributions. The first contribution is given by the voltage noise at the input of the amplifier, the other is the current noise at the input.

As for the resistances, we have the Rg and Ra contributions.

The overall noise at the input of the INA is given by twice the noise of the opamp, and the noise associated to the resistance Rg (4kTRg). The middle term is the input current noise of the opamp that has been transferred towards the output and then back to the input.

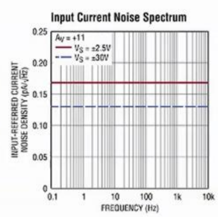
We can also compute the ration between the noise of Rg and Ra (in the rectangle). We find that the noise associated to Rg is higher than Ra, because the current noise of Rg is amplified by the feedback. Hence we can neglect the contribution of Ra to the noise. Secondly, we can also neglect the contribution of R2/R1 because we fixed the gain of the second stage to 1.
If we know compute the equivalent noise, that is the equivalent fluctuation at the input that generates the output noise, we have to take the output noise spectral density (that is the sum of each spectral density) and divide it by the gain of the INA.
Once we do that, we find the expression in the upper part of the image below. We need to compare this with the signal we want to retrieve then. Since we have signal at the input of the electrodes that is, differential, and between 100uV and 1mV, we have to keep the noise below 10uV → we have to verify that the input equivalent fluctuations are smaller than 10uV.

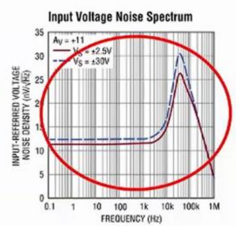
$$S^{tot}_{n,in(f)} = 2\,S^{A1}_{n,v(f)} + 2\,S^{A1}_{n,i(f)}\left(\frac{R_G}{2}\right)^2 + 4\,k\,T\,R_G$$

$$\sigma^2_{in} \approx \left(2\,S^{A1}_{n,v(f)} + 2\,S^{A1}_{n,i(f)}\left(\frac{R_G}{2}\right)^2 + 4\,k\,T\,R_G\right)\cdot f_2 \qquad \boxed{\sigma_{in} \approx 0{,}5\mu V}$$

LTC-2057

$$\begin{cases} S^{A1}_{n,v(f)} = \left(11\ [nV/\sqrt{Hz}]\right)^2 \\ S^{A1}_{n,v(f)} = \left(0.15\ [pA/\sqrt{Hz}]\right)^2 \\ 4\,k\,T\,R_G = \left(19{,}9\ [nV/\sqrt{Hz}]\right)^2 \end{cases}$$



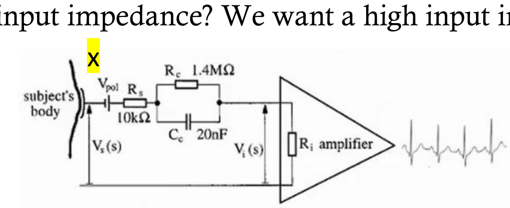


We want to keep the equivalent input noise smaller than 0.5uV and if we use the expression computed for the equivalent input noise, the LTC-2057 is an opamp that fulfill our requirements.

We should have also considered the 1/f noise, since we are at low frequency (considered in the additional notes). However, in the opamp selected it is negligible, this is why we don't see any contribution in the t.f..

ELECTRODES MODELING

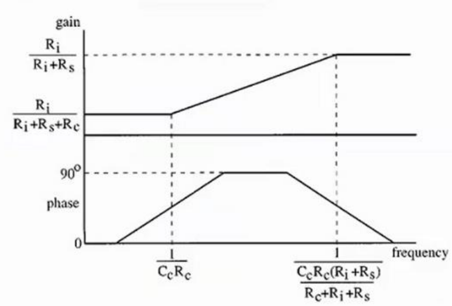
If we translate the physical model into an electrical one, we can understand if there is some physical reason for which we can change the input stage. Do we have any requirements on the input stage in terms of input impedance? We want a high input impedance to have a better SNR.



Frequency:	0.1	1.0	10	50	100	1000	Hz.
Electrode Impedance:	1360	1360	416	152	101	17	kΩ
Model Impedance:	1360	1200	508	152	85	17	kΩ

$$\frac{V_{i(s)}}{V_{s(s)}} = \frac{R_i}{R_i + R_s + \dfrac{R_c}{1 + s\,R_c\,C_c}} =$$

$$= \frac{R_i(1 + s\,R_c\,C_c)}{(R_i + R_s + R_c)\left(1 + s\,C_c\,\dfrac{R_c(R_i + R_s)}{R_i + R_s + R_c}\right)}$$



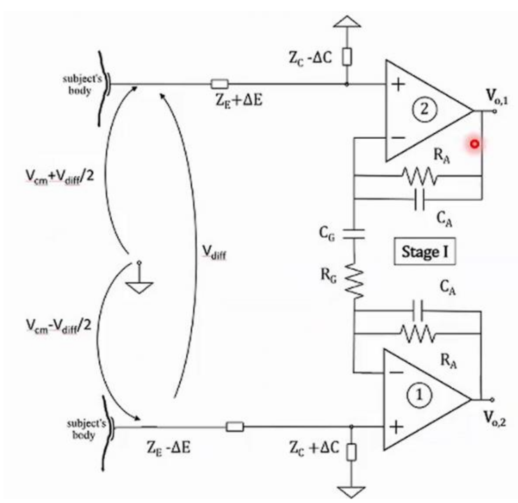
There is a biasing generator x that models the polarization voltage of the electrodes that is in series with a complex impedance given by the series of 3 component. Then on the right we have a list of experimentally computed impedances over the electrodes as a function of frequency and corresponding to the model we are considering. By experimental modeling, the equivalent impedance of the electrodes can be described with the configuration of the image.

In the end, after the calculation, we found a t.f. from the patient to the input of the amplifier that has 1 pole and 1 zero (high pass filter). This is important because what we want to have in the response of our circuit is not only a large bandwidth to avoid the damping of my input signal (we don't want that the signal voltage that propagates from Vs to Vi degrades because of the frequency response of our circuit, but we don't want also the degrade the phase).
Indeed, every singularity in the t.f. introduces a phase shift. We don't want to have these phase shift in the bandwidth of our signal because otherwise we would need some filter after the INA to recover the correct phase.

The easiest way to avoid that the phase shift is in the signal bandwidth is to have zero and pole very distant, or to make them match.
If we have a very high input impedance, we see that the zero and the pole match in frequency and so the phase shift of the response is preserved.

Hence we want a very high input impedance to avoid distortion of the signal and no phase shift in the band.

The fact that we are modeling the input impedance as very high doesn't mean that we are degrading the CMRR of the circuit. To verify this, we introduce a small variation of the impedances of the ideal components (electrodes) that I'm modeling to check whether there is a degrading in the CMRR or not due to the variation of these components.



$$CMRR_{electr} = \frac{G_{diff}}{G_{cm}} \approx \frac{\frac{Z_C}{Z_C + Z_E}}{G_{cm}}$$

$$G_{cm} = \frac{v_{cm}\left(\frac{Z_C - \Delta_C}{Z_E + \Delta_E + Z_C - \Delta_C} - \frac{Z_C + \Delta_C}{Z_C + \Delta_C + Z_E - \Delta_E}\right)}{v_{cm}} \approx$$

$$\approx \frac{Z_C\,(2\,\Delta_C - 2\,\Delta_E)}{(Z_E + Z_C)^2}$$

In the model, I imagine to have Ze and Zc as impedance of the electrodes and input impedance of the INA. What happens if I have a mismatch between these two in the two branches?
In the most unfortunate case, we have opposite changes of impedance, as in the figure. We imagine there is a delta-e for the electrode impedance that is different for the two branches and the same for delta-c of the INA.

If we go through the calculation, this is a limiting factor for the CMRR, and in particular the CM signal that we develop at the input of the electrodes (if it is partitioned in the two branches) becomes a differential signal at the input of the INA. We need to avoid this, otherwise there would be no way to retrieve a good CMRR.

We can **increase as much as we can Zc**. If we do that, then small variation of the electrodes impedance will not affect the patition over the series Zc and Ze, because all the voltage will fall on Zc. So an high impedance input stage allows us to have a small decay of the distortion and moreover it allows us to neglect the contribution to the CMRR of the two electrodes impedance.
This is because even if I do have a mismatch on electrodes impedance, then because the input impedance is high, all the voltage will be on Zc.

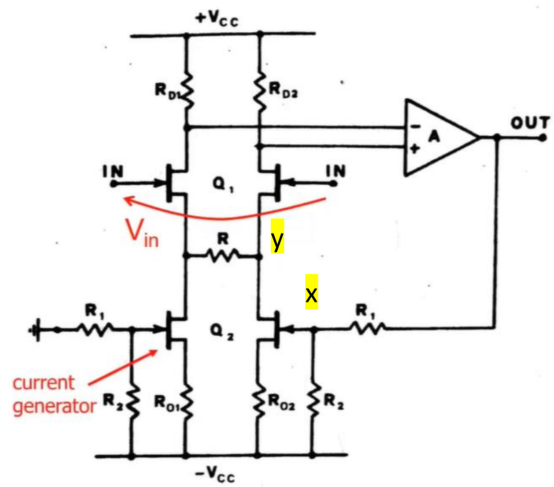
BERTOLACCINI'S INA – A LOW NOISE INSTRUMENTATION AMPLIFIER

Features:

- Custom solution than 3 OPA INA
- Lower noise, also thanks to the input JFETs
- Zin still large, due to the input JFETs
- Example of a 'current feedback' amplifier

It is an example of the current-feedback amplifier.

SIMPLE VERSION



It is an INA, so it has a differential input and the goal is to amplify the differential voltage signal. We have two pairs of transistors but it is not the classical differential stage. Indeed, a differential stage is composed by two input transistors and a tail current generator, so the two sources are connected together in the classical differential stage. Here it is different, because the resistor R connects the two sources of the transistor. It is hence a symmetric stage but not a differential pair configuration.

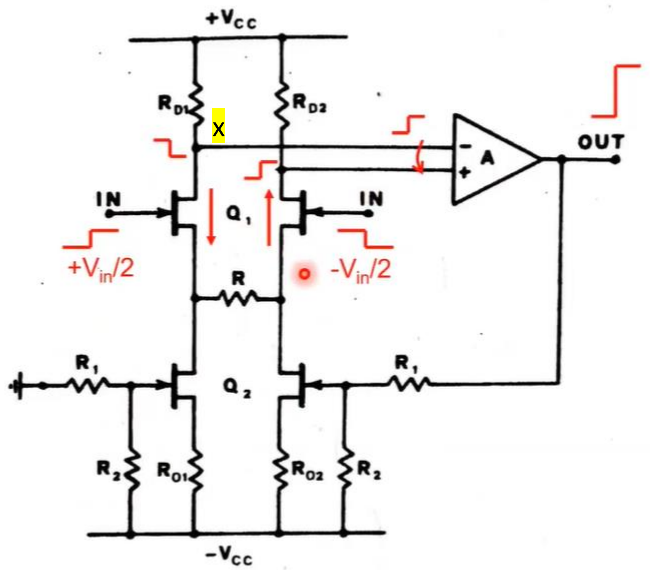
On the top of the drain of n-JFET we have 2 resistors and the voltage developed as differential on the node + and – is further amplified by the differential amplifier A. Then the output voltage is taken back in the loop (it is a negative feedback amplifier), we have the resistive divider R1 and R2 and the voltage on node x is converted in current by the transistor Q2 that is a source follower with a resistor on the source. Hence the voltage is taken to the source (from the gate to the source it works as a source follower with a unitary gain) but what matter is that the voltage on the source is converted into current through the resistor R02. So we have a current that is flowing on the top and is coming in the node y where we have a resistor on the middle and the low impedance 1/gm on the top.

On the left side we have a replica of the stage on the right, but from the point of view of the functionality it is a current generator, because there is no signal neither primary or feedback signal applied to the left branch → from the schematic point of view can be represented as a current generator made with the JFET Q2 with a degeneration of the resistor R01 on the source and so the bias current is provided at high impedance.

On the contrary, also R02 provides a current, but it is driven by the output of the amplifier A.

Feedback analysis

We give a perturbation to the input of the signal, like a differential voltage. As usual, we may split such differential voltage as a positive step on the left of Vin/2 and a negative step on the right of amplitude -Vin/2 so that the difference between the two inputs is equivalent to having applied a voltage difference Vin between the two inputs.



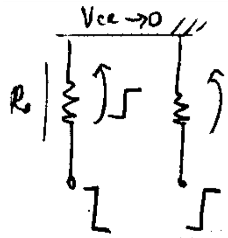
The main driving rule to check if we have a negative feedback is that, upon the application of a perturbation like the one we give, the circuit tries autonomously to minimize the effects of such perturbation. If we have a negative feedback the circuit tries to damp the effect of the perturbation (not the perturbation itself because externally applied).

In is intuitive to see that if we take the left transistor Q1 and we rise the voltage on the gate, we have a signal current (we are reasoning in term of signal current, not absolute current) and so if we have a transistor with a gate jumping up we have an increase of the current in the indicated direction (left side), because we are increasing more Vgs. Conversely, when on the right transistor the gate is jumping down by the same quantity, we are expecting a signal current going up of the same intensity. Again, it is a signal current, so in reality the transistor has still the current going down, but simply we represent on the network just the positive signal that reduces it.

The one explained is the classical response of the differential pair in terms of currents. Now if we move further in the network, a current going down in the left is exiting from the node x and so it is producing a negative pulse (**when current is exiting from a node, the voltage of the node is dropping down**). Symmetrically, if we have on the corresponding right node a current is entering, the voltage is rising by the same quantity.

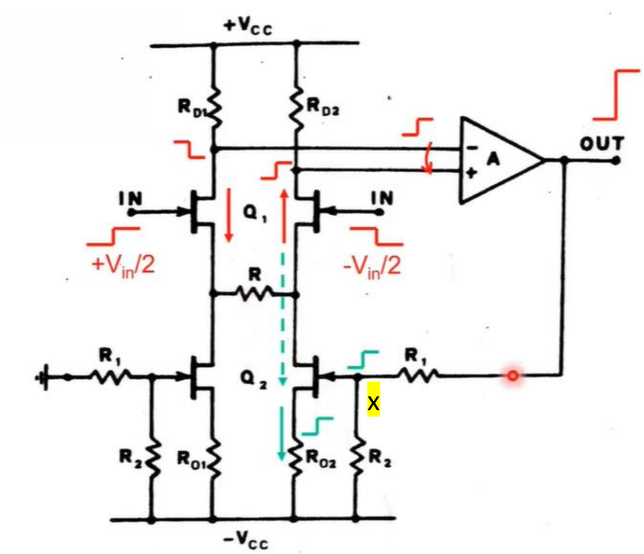
If we now make the overall analysis and compute the voltage difference at the input of the high gain of the amplifier taken as the sign indicated by the curved arrow, it is going to be a positive step. Since it is positive with the indicated sign, the positive voltage of the amplifier is going to be also a large signal with a positive direction.

40.



As for the reasoning on the variations of the nodes, we have our Vcc that in small signal analysis is at ground, then we have the two resistors and extracting current from one of them. But since the upper node is fixed to ground, this means that the bottom node is going to have a jump down. The opposite if the current enters in the node.

Then we have to continue analyzing the following components (green).

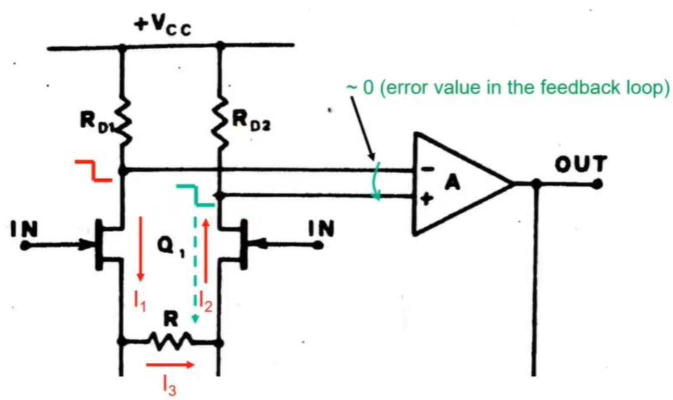


If I start from the large pulse at the output of the amplifier, by a voltage partition given by R1 and R2 I will have a smaller replica of this voltage on the node x. Then the right transistor Q2 is a source follower, so the same step on the gate is reproduced on the source. But if we have a voltage step on the source, the voltage across R02 will increase because the bottom node is grounded → increase of the current in R02.
Again, the block on the left is totally ineffective in this analysis because it will simply supply the constant bias current, it is not involved in anything concerning the signal.

Hence the green current is passing through the transistor itself and then it becomes the dashed current and now the question is: when the green dashed current identical to the one bottom reaches the right node of R, where does it go?

The point is that this dashed current entering in the node could be shared between the left direction (through R and the 1/gm of the left transistor, upper left) or the top path, that is only 1/gm. So on the left the impedance is R + 1/gm, on the upper part only 1/gm.
Of course the current prefers to go in the lower impedance path (1/gm, on the top).

Trying to reach a conclusion, **we supposed that the current in dashed green** (that was created by the feedback), **takes uniquely** (strong approximation) **the top direction**. Hence it is going to oppose with respect to the initial red perturbation I2. And this is the first sign of a negative feedback, because as the perturbation has produced the left current going down and the right going up, the effect of the feedback that is green is apparently doing nothing on the current on the left, but it is flipping or opposing to the current on the right, so it is produces in the (-) node, that was characterized by a positive step, a negative step.

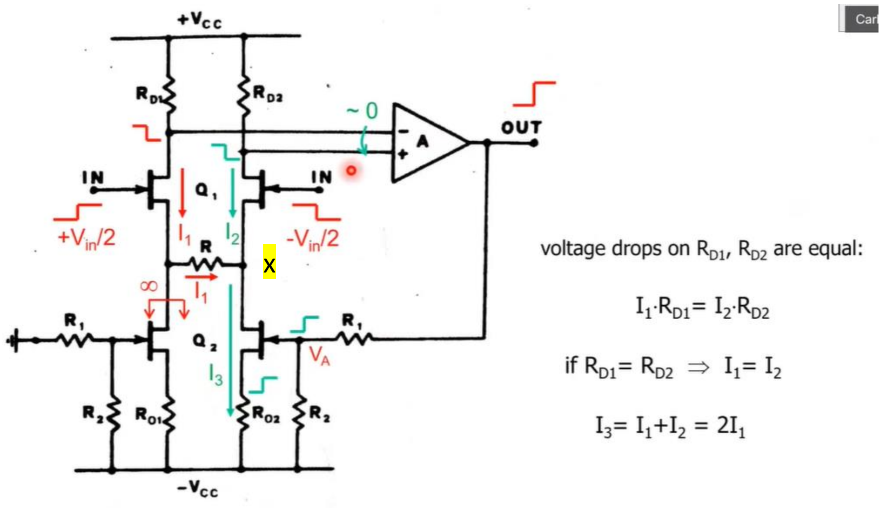


The negative loop minimizes the effect of the input perturbation by 'flipping' the direction of I_2. When feedback reaches the steady state, the result is to nullify the voltage at the input of the main amplifier (error voltage).

In conclusion, the effect of the negative loop is creating an opposing current (in green) that at the extreme end flips the sign of the current marked as I2 (going upwards at the beginning).
If we understand that is a negative feedback, the second thing we must do is to find the error signal.

The error signal in a negative feedback is the signal minimized by the suppression of the perturbation by the negative feedback action. Usually, as in the classical inverting or noninverting amplifier, the error signal is the signal at the input of the highest gain of the circuit, which is the opamp A. Hence we imagine that the suppression of the perturbation by the negative feedback is ideally so effective that the two nodes + and – are equal, they shows both a negative jump (because the green current has flipped the sign of the I2) and the difference between these two step, which is then the input of the amplifier, is 0. So we assume that the feedback is so good, that is when the main amplifier has an infinite gain, so that we have a virtual shortcircuit at its input, the difference between the two step is nullified.
NB: we are not nullifying the step, but they have now the same amplitude and so the overall difference at the input is nullified. So the error signal is the green arrow at the input of the opamp, produced by the nullification of the perturbation inside the input stage.

Gain calculation



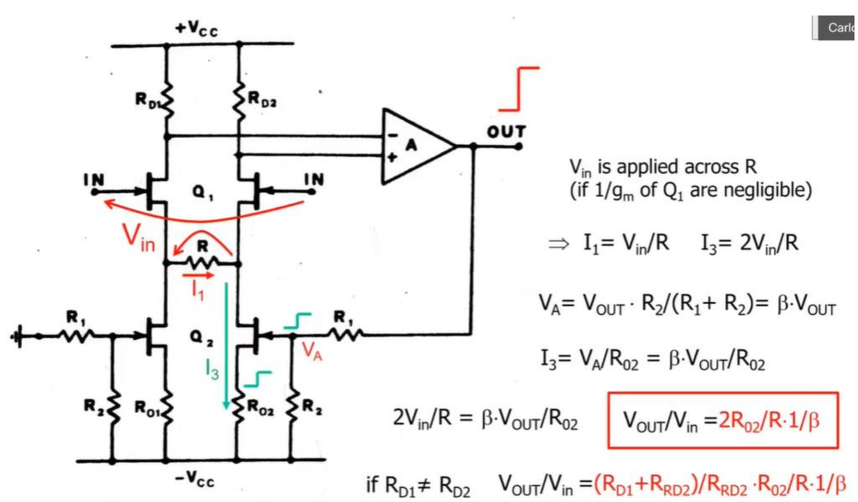
The last conclusion we have drawn is that the feedback current, which is now I3, was so effecting in flowing in the right branch of the circuit that it has flipped the steady state current I2 (now green, and I'm considering the steady state after the perturbation). The current has been flipped so that the two voltages + and – are the same.

As for mathematical conclusion, if the two voltages are the same because the error at the input of the main amplifier is 0, it means that the voltage drops across Rd1 and Rd2 are the same. So I can write the first equation.

If I consider for simplicity the two resistors with the same value, then I1 = I2, with the same sign and intensity.

Then I compute the K. law on the node x, and the exiting current I3 is equal to the current flowing from the top, I2, plus the current flowing from the left. But this current flowing from the left is indeed I1, because in the left bottom branch have a current generator, hence the impedance is infinite if looked from the drain of the left transistor Q2 → I3 = I1 + I2 = 2*I1.

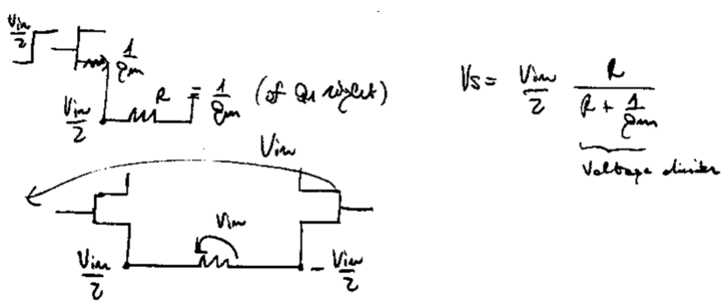
Now I can compute the current I1 considering the application of the input voltage Vin. As the two transistors Q1 are in source follower configuration (we are considering the transistor Q1 on the left with the resistor R as a kind of source follower), the same gate is applied to the source.



Hence Vin/2 = Vs, and the same applies for the transistor Q1 on the right, -Vin/2 = Vs.

So I can conclude that the Vin voltage across the two input is equal to a Vin voltage applied across the resistor R. in reality, **I'm neglecting the 1/gm of the two transistors**.

41.



The transistor on the left is acting as a source follower, the step Vin/2 on the gate is reported on the source. In reality this is not fully true, the true relationship of the source follower is the one in the drawing, but if I neglect the 1/gm of the left transistor, the formula tells me that we have ca. Vin/2.
The same can be done for the other side.
The conclusion is that if the voltage difference is Vin, this voltage will be on the resistor R.

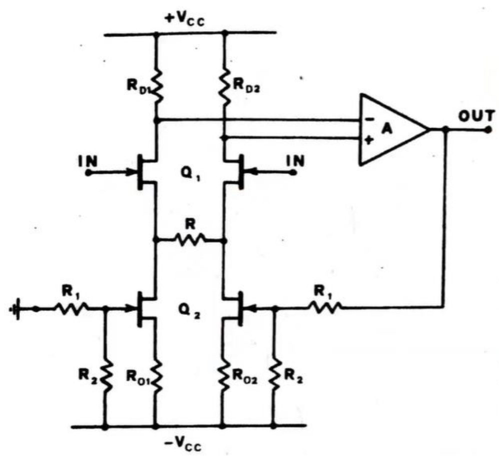
Coming back to the schematic, by the Ohm law I can compute that I1 = Vin/R.
Then by the previous formula I calculate I3 versus Vin. But at the same time I can calculate the current I3 depending on the output voltage, because if I have an output voltage called Vout, I can compute the t.f. between Vout and I3.
If I have a Vout voltage at the output, I firstly compute the voltage Va that is given by Vout time the resistive partition, that from now on will be called beta (it is not the beta of BJT, simply a labelling).
Then I3 flowing in the transistor Q2 right (that remember is a source follower) is beta*Vout/R2.
Also the bottom right transistor Q2 is considered as an ideal source follower where we are neglecting the 1/gm also.

In conclusion, the current I3 can be equalized, and we get the Vout/Vin, the complete t.f., the gain of the amplifier considering an ideal loop (considering the loop gain infinite, as usual).

This relationship was calculated according to the assumption that the two resistors were equal. If not, we have simply to consider the equation I1*Rd1 = I2*Rd2, but the voltages remain equal, we simply cannot say I1 = I2.

At the bottom we have the final equation if the two resistors were different.

LIMITATIONS OF THIS SOLUTION



1) $V_{GS} \neq 0$ because $I \neq 0$ in Q_1
$\Rightarrow$ $1/g_m$ of Q_1 in the Gain
2) Miller effect on Q_1 because of the voltage amplification on the drain
3) effect of $R_{D1} \neq R_{D2}$ on Gain
4) Gain dependent on R_{02}: if R_{02} too large, bias problems for Q_2
5) CMRR of A is important because $V_{CM} \neq 0$ at its inputs

The first limitation is having neglected the 1/gm of the transistor Q1. Another way to look the circuit is that we have two signal currents flowing into the transistors producing the two voltages on + and – and so the two voltages Vgs are different than 0. So equivalently we can say that the Vgs is different from 0 because I != 0 or that the 1/gm cannot be neglected.
This is not good, because if the gain of an amplifier depends on the 1/gm, the 1/gm is going to change with physical parameters of the transistor or the biasing, and so the gain.

The second limitation is the Miller effect on the two transistors Q1, that are affected by the Miller effect, because when we give a signal to the gate, we have a voltage amplification to the drain.
In fact, when in a JFET we have an amplification between gate and drain, the gate to drain capacitance of the transistor is amplified by the Miller effect.
So it is like to have at the input of the amplifier a Miller capacitance, and a Miller capacitance together with the electrodes impedance creates a low pass filtering.
So the voltage steps on + and – are not nice because they introduce a Miller effect on the gate-drain capacitance of the transistor.

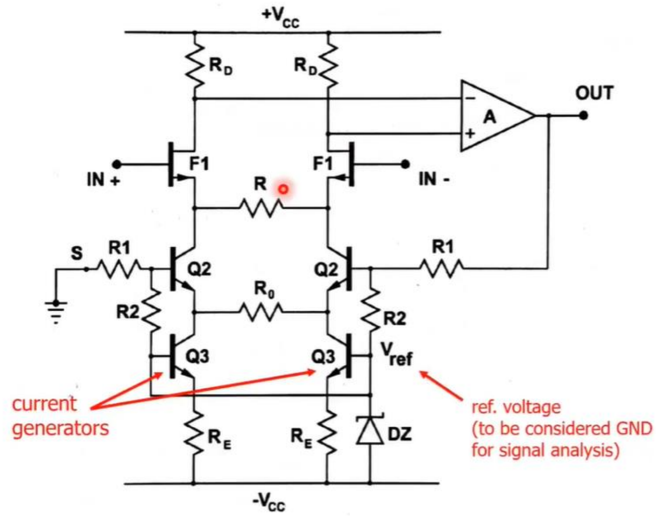
The third limitation is that the gain may be dependent on the mismatch between the drain resistors Rd1 and Rd2. It is not a serious effect, but it may be annoying-

The fourth limitation is that in the gain we have a dependency with the resistor R02. The problem is that as the resistor R02 is at the numerator of the overall gain, we may be tempted to increase it as much as we can to increase the gain. Unfortunately, the resistor R02 is also part of the biasing network in the right branch.
Hence if we increase it too much, for a given current flowing in the branch, we may bring the transistor Q2 left out of the good operating point → the transistor tends to switch off.
So R02 plays a role both in the gain formula (not a problem) but it is also part of the biasing network.
I cannot have total freedom in choosing R02 for the gain, because if it is too large I may have a problem in keeping correct the biasing.

The fifth limitation is that, even if I have nullified the input voltage on the amplifier A, this nullification is obtained not by nullifying the voltages themselves, but by making them equal. This means that **the opamp experiences two input voltages that are indeed a common mode**.

So if I don’t want to have problem, I need to choose an opamp with a good CMRR. This is not a dramatic problem, but it imposes the choice in the catalog of the opamp.

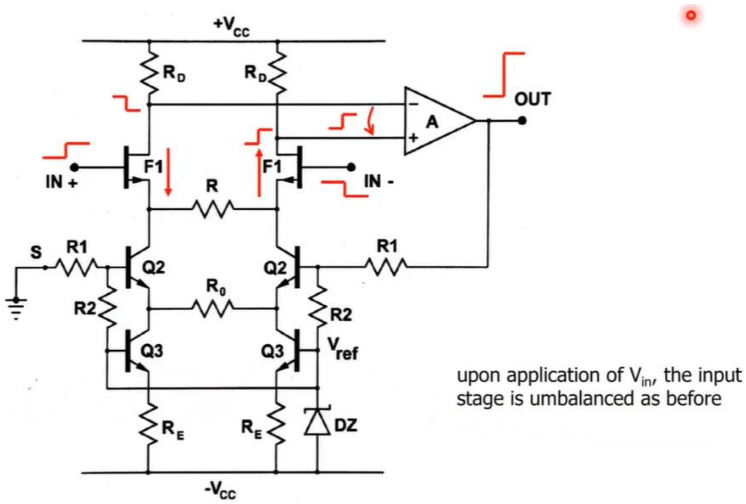
IMPROVED VERSION



The input stage is the same (former Q1). It is the bottom part that is different; now we have two more transistors Q2 and another resistor in between R0. The branch on the left is put to ground, the one on the right is connected to the output of the amplifier. In both cases there is a voltage partition.
Then on the bottom we have Q3, but Q3 and Re composes a current generator. This because they are biased through the Vref voltage that is fixed by a Zener diode. In fact, when a Zener diode is on, it clamps the voltage across → voltage is fixed to a convenient Vref that we have chosen in concordance with the Zener.

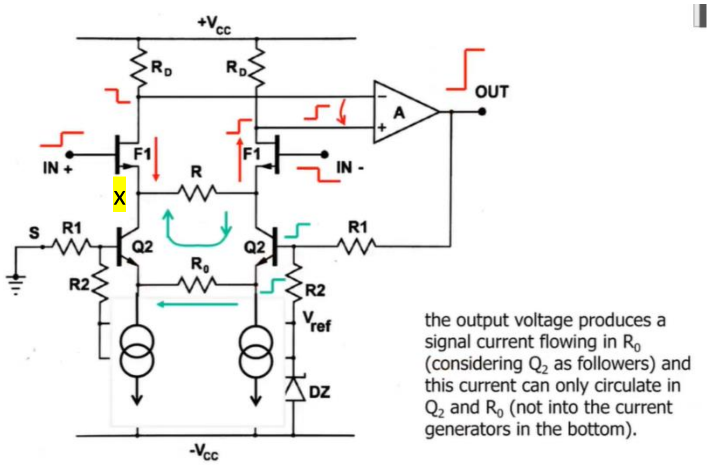
From now on I wont care about Vref, since it is fixed by the Zener. And for the signal analysis, when we have a voltage fixed, it is like to consider it grounded.
Hence R1 and R2 will represent a voltage divider toward ground like in the previous configuration.

Feedback analysis



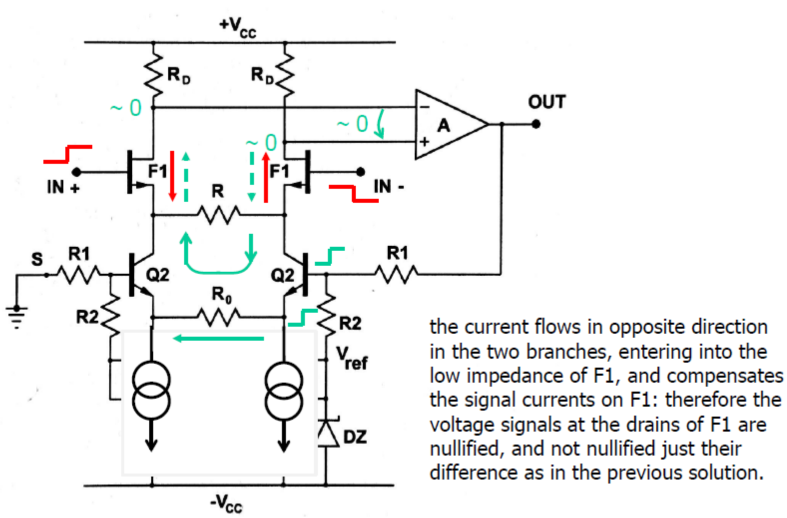
We have the Vin application as before, the currents flowing in the input transistors, the voltages going up and down in the two branches and we have a volage step at the input of the amplifier. So the output voltage is positive (as before, so far).

What is different is the network on the bottom.



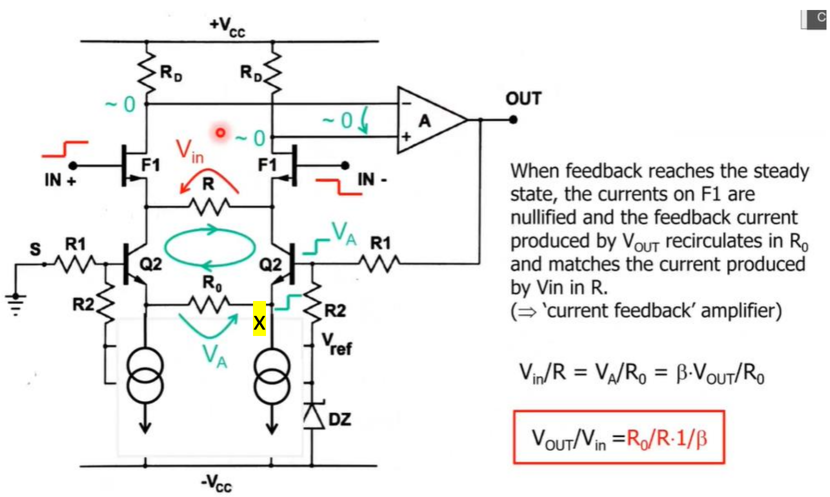
Let's start from the output voltage. Vout, thanks to R1 and R2 produces an attenuated step on the base of the transistor Q2 right, then the transistor is a follower, so it produces a voltage also on the emitter but now we have also the new resistor R0. The left end of R0 is the low impedance of Q2 (from the emitter of Q2 we see the 1/gm of Q2 left). If we take R0 and we step up the right and do nothing on the left, we have, by the Ohm law, a current flowing in green in R0. So the perturbation in green on the right branch has produced by the Ohm law a current in R0.

But now, where does this current go? On the right will go through Q2, but it can also go on the left through the other Q2. Hence the current flowing in R0 with the indicated direction is the current flowing down on the right and going up on the left.
So, the output voltage produces a step voltage that is applied across the resistor, the resistor has a current flowing and this current can only go downwards on the right and upwards on the left, nowhere else.



This current, that is the one created by the feedback, will flow on the top. In fact, in node x the winning impedance is not the R on the right, because we have the 1/gm of F1. Hence the current prefers to go to F1 (dashed green arrow). The same reasoning can be applied for the downward current contribution on the right branch.
In conclusion, the upwards current on the left will go to compensate the initial perturbation; the downward current on the right will go to compensate the red perturbation. Hence I'm still going to have a nullification of the voltage at the input of the amplifier (the result is the same as before), but is the mean to achieve this nullification that is now different.
In fact, in the previous configuration the nullification was obtained by having two voltages of the same sign and value, whereas here the nullification is obtained by nullifying the perturbation current → the voltage on + and – pins are nullified, it is a 0 made out of 'zero minus zero'.

Gain calculation



In the steady state, when the feedback has established after several turns, the conclusion is that we have 0V on the drain of F1 right, no current in the top transistors, and if we have no current in the top transistors, by the K. law, the current on R0 (still existing because created by the feedback) circulates only between R0 and R.

In conclusion, if overall we have no signal current flowing in the transistors F1, the current R0 can only reflow between R0 and R.
If so, we can calculate the t.f. of the amplifier because the current flowing in R is still Vin/R (as previously); then we have a current flowing in R0 which is equal to Vout, transformed in Va by the partition (beta) and then Vout is transferred to node x (because it is an emitter-follower configuration) and so Va is fully applied across R0. Hence the current flowing in R0 is Va/R0.

Hence the recirculation of the current in the circle provides the first equation of the image, that tells us that Vin/R = Va/R0.
Finally, this brings to the red box equation.

In conclusion, this is a current feedback amplifier because the feedback produces a current which is matched to the current produced by the input → the current flowing in R0 matches the current in R.

Improvements of this solution

In the previous configuration we have 5 problems.

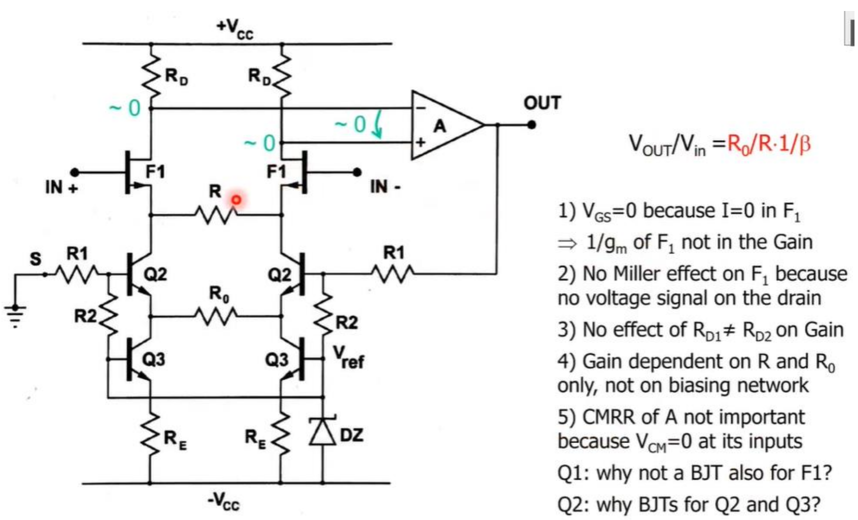
Now, Vgs and current in the transistors at the input (F1) are 0. This because we have no more current in the transistor F1, but if so, Vgs = 0 and 1/gm doesn't play any role, because the transistor has no drop on the Vgs → F1 are perfect followers, because since we have no current in the transistors, the Vgs are equal to 0, and so the input voltage is rigorously transferred to the source, there is no loss → **no dependance on 1/gm**.

As for the Miller effect, **there is no Miller effect** on the drain of the transistors F1, because the drains of the transistors are 0, not equal steps. If we have 0V on the drain, there is no Miller effect.

Moreover, **there is not the effect of the imbalance of Rd1 and Rd2 on the gain**, because we have no currents flowing in them. So if we nullify the currents, we nullify the drain voltages, even if Rd are different.

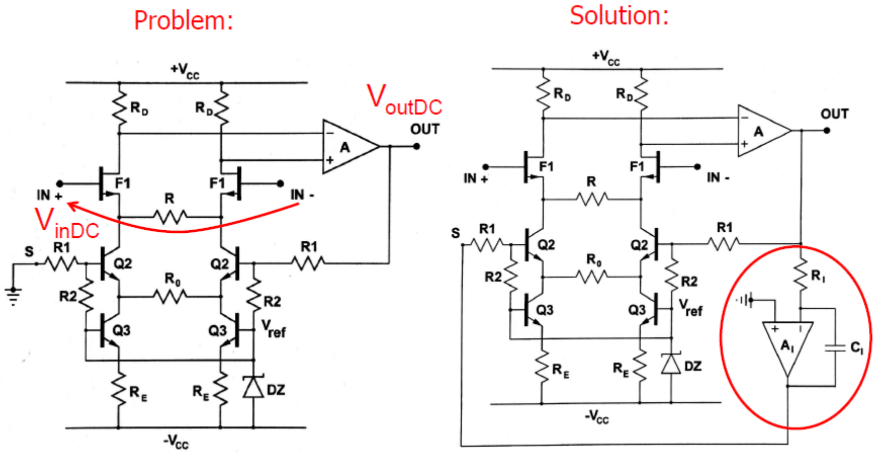
Then, **the gain formula is no more dependent on any resistor participating to the biasing**. It depends just on R and R0, and these two don't participate to the biasing, because when we have no signal, there are no currents flowing into R and R0, because the current on the left branch is identical to the one on the right, and so there in no imbalance in current so that a current flows in R and R0 (architecture is symmetrical). → R ad R0 are free to be chosen for the gate, typically R0 large and R small in order to have a small gain, because no one of them is participating to the biasing network.

Finally, **the CMRR of A is no more important**, because we don't have a CM voltage at the input of the amplifier, because they are rigorously zero → this relaxes the choice of the amplifier.



But why we haven't chose BJT also for F1? Why BJTs for Q2 and Q3?

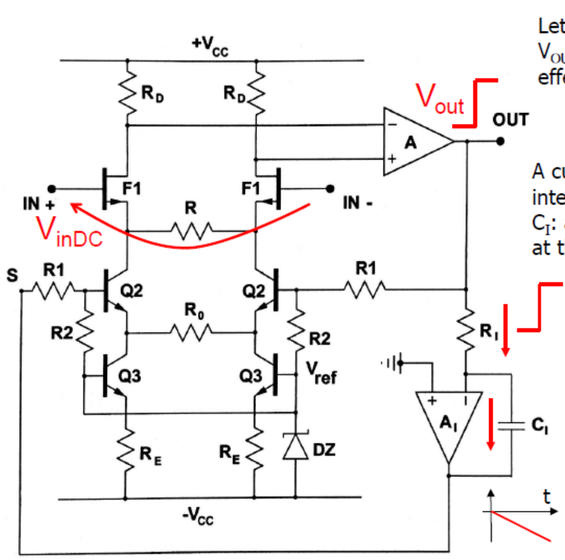
FURTHER IMPROVED VERSION INSENSITIVE TO INPUT DC OFFSET



DC differential voltage at the input (e.g. due to electrodes DC offset) produces a DC output shift.

An integrator senses the output and counterbalances such effect.

The circuit is sensitive to the differential voltage at the input, but unfortunately I may have also a DC component of such differential voltage.
Indeed, we may have at the input, of e.g. the amplifier, a differential voltage due to the mismatch of the electrodes potential. Basically the problem is that the gain here, which is desirable to be large, is also a gain for the DC component of the differential voltage.



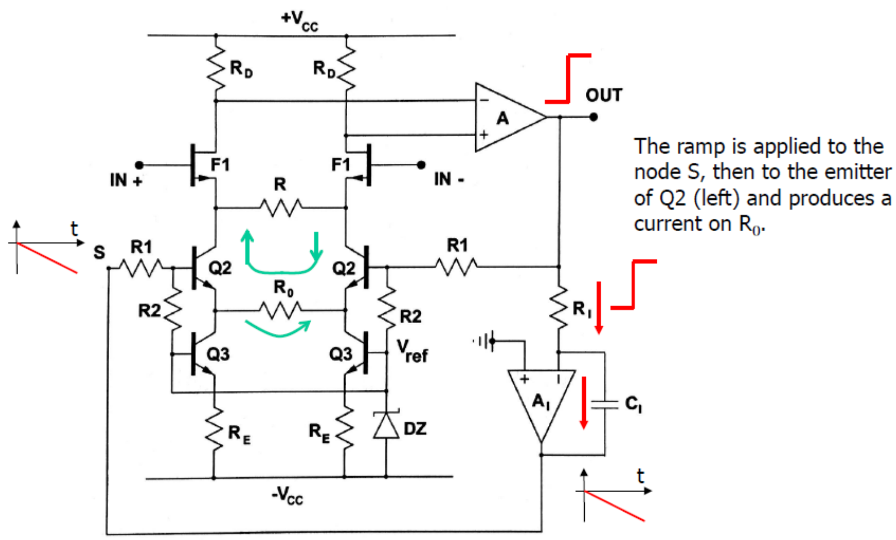
Let's suppose to have V_{inDC}: V_{OUT} raises and we track its effect along the added loop.

A current V_{OUT}/R_I flows in the integrator and it is integrated on C_I: a negative ramp is produced at the output of the integrator.

A possible solution to introduce an additional stage, the classical integrator Ai that's senses the output voltage and it is connected to the left branch of the main amplifier (S). Hence S is no more grounded. The idea is to sense the voltage Vout and provide a new feedback (we have a second feedback loop) aimed at counterbalance the effect of the DC voltage. Of course, we don't want to kill the gain of the amplifier even at the useful AC component of the input signal → the additional loop must operate only at low frequency, and this is the reason why we use an integrator, because it is a low pass filter. Instead at high freq (where we have the biosignal) the integrator is not operational.

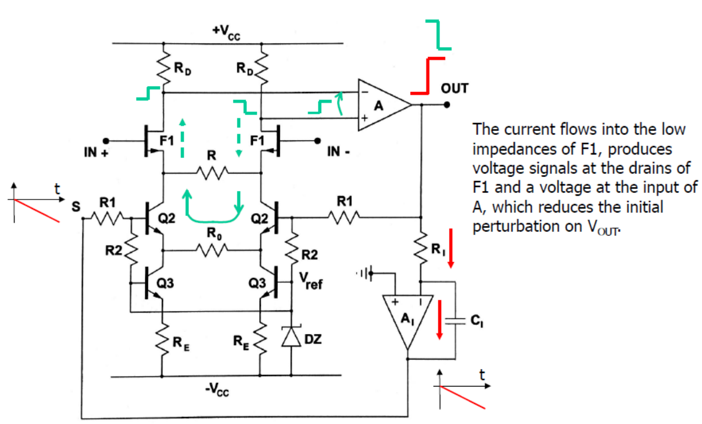
Let's start from the condition of an input differential DC voltage, for example of 300mV. If we do nothing, the differential input will produce an output on the amplifier A represented by a step. This is not really a rigorous representation because we are considering a DC voltage, and should produce a DC voltage at the output, but since we are analyzing a feedback, it is easier if we make an inspection considering signals, but I'm still looking at DC voltages.
This signal on the output will produce on Ri a current step and the current will flow in Ci and produce a negative ramp (we are integrating the step, and the result is a negative ramp).
The output of the integrator is connected to the node S and we have a ramp also on S.

Now, similarly to before when we created a current flowing in R0 that moves upwards on the left, we are applying the same concept. The ramp on node S at the input of Q2 created a voltage drop on R0 and generates a current with the polarity in the image, because the voltage on the emitter of Q2 left is negative.



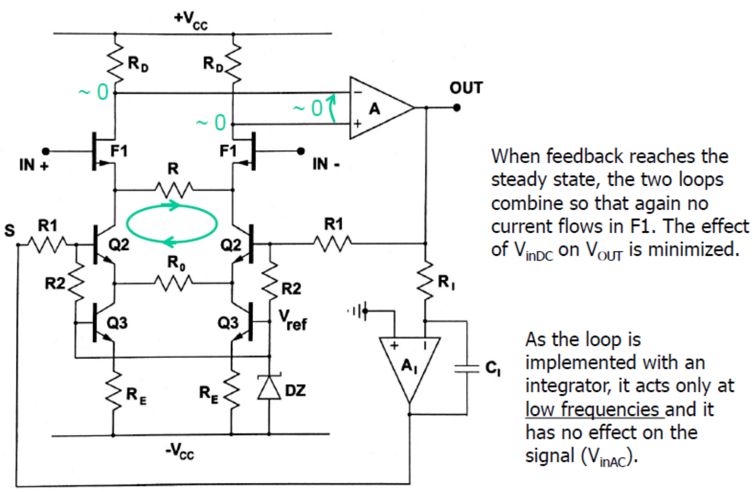
Of course, I have the effect of the previous feedback still, but now we are looking just at the effect of the additional loop.

So the current in green passes through F1 because they show low impedance, the current moving upwards on the left will create a positive step on node + and a negative on node – and so the voltage at the input of the amplifier taken with that arrow is positive.

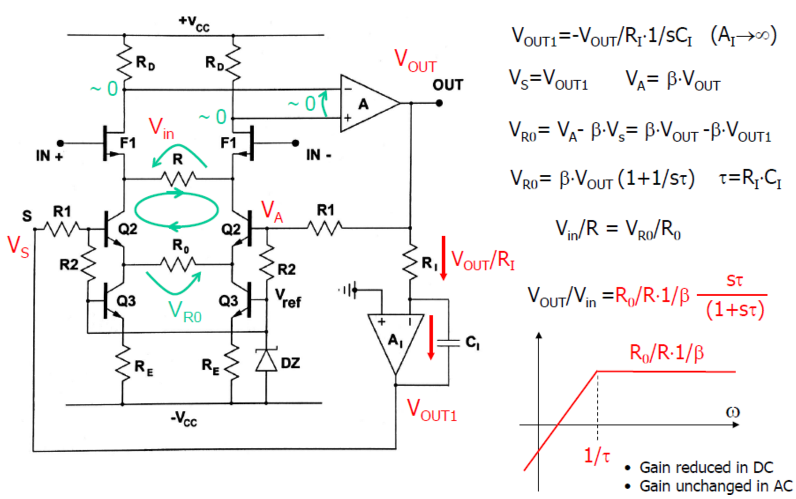


But if we have a positive voltage at the input of the amplifier with that direction, the Vout will be negative. The result is that we have still a negative feedback, because the initial perturbation is compensated by the loop with a negative counterbalancing effect.
In conclusion the integrator adds an additional negative feedback. The conclusions are that we have two simultaneous feedback in action, the previous one and the one given by the integrator to be superposed to the main one.
The overall effect of such feedbacks will be again that the input voltages at the inputs of the amplifier will be nullified.

The combination of the two collaborative feedbacks is that the voltage on the drain of the two transistors will be zero and so the voltage at the input of the main amplifier → the new feedback in combination with the previous one is not devastating everything.
Hence we still have the recirculation of current. Moreover, this situation must exist only at low frequency, when we have the differential component that we want to kill. The feedback given by the integrator mustn't be operational at high frequencies.



Gain calculation



First of all, if we have a Vout voltage, we compute the voltage at the outuput of the integrator, Vout1, that is Vout/Ri, that is the current, and the impedance of the capacitor 1/sCi. The minus sign is due to the direction of the current, if as in the image, the output is negative.
This is the classical formula of the ideal integrator with a perfect feedback, with the pin – at a perfect virtual ground, that occurs if Ai tends to infinite.

Then the voltage is applied to the S node. In the other loop, we have the Va voltage as previously calculated.

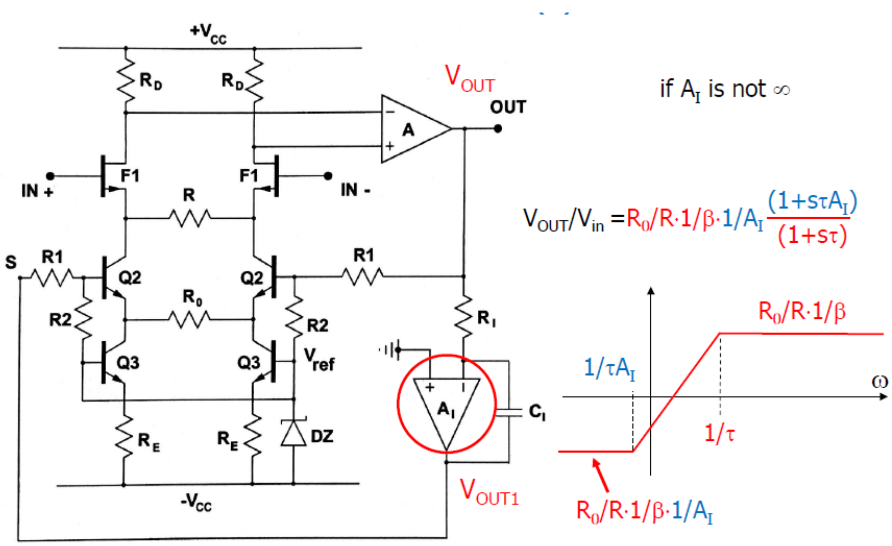
Now we calculate the voltage across R0 and consequently its current. Before, the voltage across R0 was just Va. Now Vr0 is not just Va, but the difference between Va and the corresponding value on the left that is Vs*beta.
Then we substitute and we get that Vr0 = beta*Vout*(1+1/s*tau) where tau is the time constant of the integrator (Ci*Ri).

Then we apply the balance of the currents between R and R0 and we get the final t.f. that is the red one. We have the **same gain factor as before** (AC component of the gain), and we have a zero in the origin and a pole.

From the Bode plot we see that at HF, where I have the biosignal, the gain is the same as the amplifier before, while at LF, in particular going to the DC, the gain is going to 0. This means that **Vout is no more sensitive to a DC component of the input voltage**. Hence the integrator kills the gain at LF but is no more operational at HF. So the integrator is a low pass in the loop, but becomes a high pass in the overall t.f.. In fact, at HF the capacitor is a shortcircuit so we have ground everywhere, we restore the ground on node S → at HF the loop doesn't exist.

Non-ideal integrator

The previous considerations where based on the concept of an ideal integrator (with gain infinite). Hence if the gain is not infinite, we have to recalculate the gain of the integrator.
If we introduce the classical formula A1/pole for the finite gain, we get the following output formula.



The amplifier has a gain not infinite, but equal to Ai and we compute the t.f due to the not infinite gain.

In the updated formula (blue parts) we are considering we have a finite gain of the amplifier. We have a zero and a term 1/Ai. We can clearly see that if Ai goes to infinite again we get the previous ideal formula.

The difference in the Bode plot is that the gain at LF is not going to minus infinite but to a fixed constant value. The zero hence is no more in the origin but at very low frequencies.
In conclusion, the concept is that if Ai is not infinite, the DC gain is not 0 but a very small value.

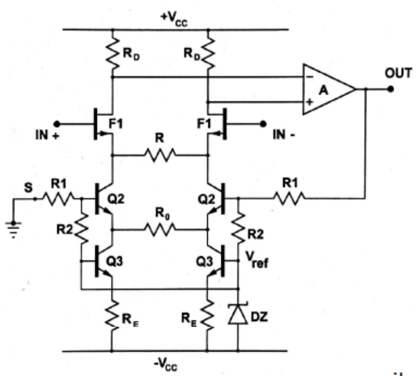
NB: the formula for Vout one is the following

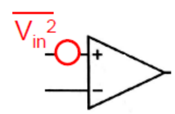
50.

$$V_{OUT} = -\frac{V_{OUT}\cdot A_I}{1+s\tau A_I}$$

Previously it was $V_{OUT_1} = -V_{OUT}\frac{1}{s\tau}$, now it is $V_{OUT_1} = -V_{OUT}\frac{A_I}{(1+s\tau A_I)}$ because if $A_I \to \infty$

ELECTRONIC NOISE ANALYSIS





Contributions from resistors:

$$4kTR\left(1+\frac{R}{R_0}+\frac{R}{2R_D}+\frac{R}{2R_E}+2\frac{R}{R_0}\frac{R_1\|R_2}{R_o}\right)$$

suitable choice of R_0, R_D, R_E, (larger than R) can result in noise dominated by R only

$V_{OUT}/V_{in} = R_0/R\cdot 1/\beta$ (note that large $R_0 >> R$ is good for Gain)

We want to calculate the equivalent input noise generator of this INA, in particular the voltage equivalent input noise generator. **It will be dominated by the shot noise of the input transistors**. This is why we use the JFET and not BJT, because they have a small gate current and so a small shot noise at the input of the amplifier, while if we would have used a BJT, we would have a much larger base-current of the BJT and so the shot noise of the base current would be much larger.

The input current generator is not appearing because the current noise generator is dominated by the shot noise of the gate current, due to the selection of the input transistors.

On the contrary, the equivalent voltage generator is composed by several contributions, and the contributions are summarized in the formula.

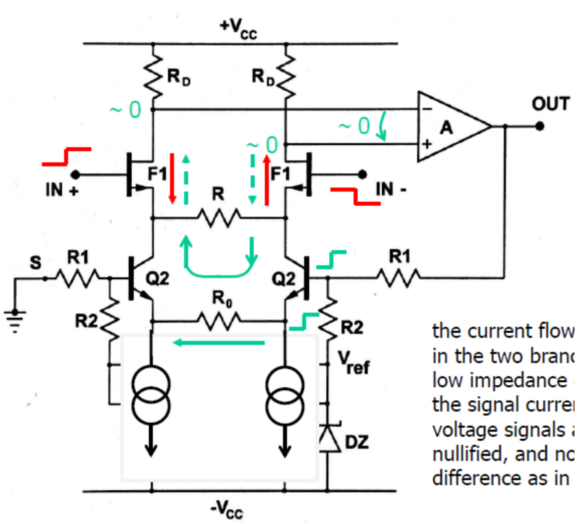
The conclusion of the noise due to all the resistors around is the one in the formula. In particular, we have a contribution 4kTR*1 that is the thermal noise of the resistor R that is at the input of the amplifier (we have that the contribution is multiplied by 1 in the parenthesis, because the t.f. between the input and the nodes of R is 1), then we have the noise due to R0, Rd, Re and R1 and R2. So we have one contribution per resistor or per couple of resistors.

The conclusion is that if we are good designer, we may choose appropriately all the resistors at the denominators so that the ratio R/R0 is much smaller than 1. If R0 > R, the second contribution can be neglected with respect to the 1 (contribution due to 1, related to the resistor R). And I can choose R0 with respect to R, because in the formula of the main gain of the amplifier, it is a convenient choice to choose R0 > R, because it means I will have a gain larger than 1.

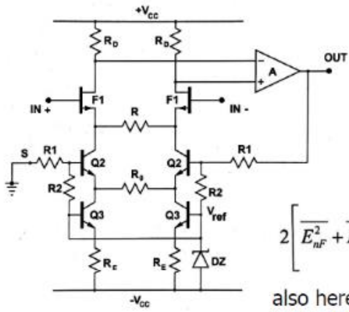
So a good choice of R/R0 is good both for the noise and the gain.

Similarly, I can choose Rd > R, and this is a free choice, because there are no constrains in the circuit (except for not having Rd too big for not having biasing problems in F1). The same then for Re and the last term, so that **all the contributions can be negligible with respect to the 1**.

Hence in terms of resistors, the contribution of the resistor R is unavoidable, because we have the 4kTR and there are no reduction factors. A small R increases the gain and at the same time reduces the noise. Choosing a small value of R increases the gain and reduces the noise.
So should I choose a value of R tending to 0? It is a good thing to choose R as small as possible so that gain tends to infinite and noise to 0? Theoretically yes, practically no.
What is a possible limitation in bringing R going to 0 (i.e. zero noise and infinite gain)?
The fact that the red current instead of going upwards to F1 and its 1/gm would go in R it is too small. But another way to highlight this phenomenon is to say that in this way we are killing the loop gain. All my analysis was based on the fact that the current (green, left) was going upwards, but if not so, all the equations on the gain are inconsistent. If the current doesn't go into F1 the loop gain is not infinite and the formula is no more valid.



Calculation of the contribution of all the generators of the transistor



Contributions from transistors:

$$2\left[\overline{E^2_{nF}}+\overline{E^2_{nQ2}}\frac{R^2}{R_0^2}+\frac{1}{4}\overline{E^2_{nQ3}}\frac{R^2}{R_E^2}+\overline{I^2_{nQ2}}R^2\left(\frac{R_1\|R_2}{R_0}+\frac{1}{2}\right)^2+\frac{1}{4}\overline{I^2_{nQ3}}R^2\right]$$

also here, suitable choice of R_0, R_E, vs. R can result in noise dominated by F1 only

Contributions from amplifier A:

$$\left(\overline{E^2_{nA}}+4\overline{I^2_{nA}}R_D^2\right)\frac{R^2}{4R_D^2}=\frac{1}{4}\overline{E^2_{nA}}\frac{R^2}{R_D^2}+\overline{I^2_{nA}}R^2$$

Also here we have the final results. **The voltage generator of the input transistor F1**, that is duplicated because we have two F1 that are statistically independent, **cannot be neglected** (first term in the square brackets); we can just choose it as small as we can. On the contrary, all the other generators have their effects depending on the ratio between resistors. So if for example we choose R0 >> R, we can reduce a lot the effect of the second generator, and the same for all the other terms (even if we have a current generator, where we choose R small so to damp down the effect of the generators).

The conclusion is that if we are good designer, we choose the resistors in a way that the only surviving contribution is the first one, the one of the transistors F1. The same can be done for the main amplifier A: the main amplifier is characterized by its noise generators and we see that if we make the right choice of the resistors Rd and R we can reduce the contribution.

Our main conclusion is that **the noise has to be dominated by F1 and R only** (and their respective generators). Of course, once we are dominated only by these two, we try to minimize these two as much as we can.

- Noise (to be) dominated by F1 and R
- We have neglected 1/f of R, but 1/f depends on I and I=0 (in DC).
- JFET (F1) provide low input current noise and 1/f noise (ex. below)

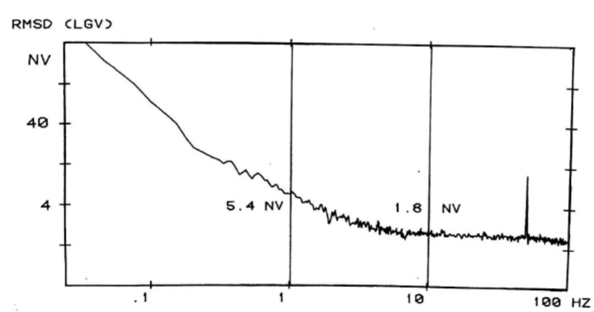


Fig. 6 - Noise spectrum of the amplifier of Fig. 3 with R = 150 Ω and Q_1 = 2SK146.

The second point is that the R resistor is not characterized only by thermal noise, but it is also affected by 1/f noise. However, the 1/f noise is dependent on the current flowing in the resistor.

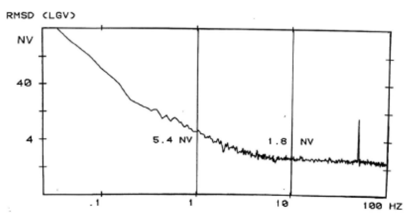
The part in the second and third segments is the white noise part.
But is there any DC current flowing in the resistor R and R0? No, due to the symmetry there is no DC current, bias current, flowing in them. Hence if the current is 0, also the 1/f noise is 0 → **the resistors are affected only by thermal noise, not 1/f noise**.

The third conclusion is that since the noise is dominated by the input transistor and we are talking about LF applications, because we already know that biosignal amplifiers deal with LF applications, we need to choose a JFET because not only it has a small shot noise due to the gate current (that could have been achieved also by choosing a MOSFET), but the JFET is better even than MOSFET because it has a low 1/f noise.

Input noise spectrum of the amplifier – conclusion

- Noise contribution of R=150Ω:

  $4KTR = 4V_T qR = 4 \cdot 25mV \cdot q \cdot 150\Omega \Rightarrow 1.5nV/\sqrt{Hz}$



Subtracting R noise in the plot:
$\Rightarrow$ 1.0nV/√Hz @10Hz
and 5.2nV/√Hz @1Hz
(1/f dominated by F1)

- Integrating the noise in the bandwidth: $\sigma^2_{nTOT} = \int_{\Delta f} S_N(f) \cdot df$

Bandwidth	Noise (rms)
0.025 – 10Hz	40nV
0.025 – 250Hz	50nV
0.025 – 1KHz	185nV

Notes:
- Ultra low noise (e.g. vs. ~ μV rms of 3 OPA conventional ECG amplifiers)
- Noise dominated by 1/f noise (⇒ JFET)

It is the spectrum of the voltage generator on the pin + of the overall INA and this input noise generator is characterized by a small thermal noise (1.9nV) and we see the 1/f noise on the left. The ultra-low noise at the input has been achieved using a resistor of R 150 Ohm and a particularly good JFET.

If we would have used a MOSFET, the 1/f noise would have been much larger.

Let's consider again the overall noise spectrum and let's calculate the components. What about the noise contribution of the resistor R?
If a resistor of 150 Ohm has been chosen, we can calculate its noise generator. 4kTR is equal to 4Vt*q*R, where Vt is the thermal voltage (Vt = kT/q). For a Vt of 25mV at room temperature, q is the charge of electron, R is 150 Ohm and in the end we have that the contribution of R is 1.5nV over square root of Hz.
Hence in the plot, the contribution of the thermal noise of the resistor is a constant line corresponding to 1.5nV.

Now, if we know from measurements that the total noise is 1.9nV/sqrt(Hz), and we know that the noise of the R only is 1.5nV/sqrt(Hz), we make the 'square subtraction', 1.9^2 - 1.5^2 and we make its square root.

51. $n_{JFET1} = \sqrt{1{,}8^2 - 1{,}5^2}$

F1 TOT R

The noise just of the JFET F1 is equal to the square root of the total noise (1.8^2) minus the contribution of the resistor (1.5^2). We are calculating from the measurement the contribution due to the transistor by subtracting the known contribution of the resistor R.

If we make the square subtraction, in the white region (3^{rd} block) the contribution is 1.0nV/sqrt(Hz) @ 10Hz.
The first conclusion is that we have made a very good choice of the transistor and the resistor so that one is not much dominating the other in terms of noise, because the resistor is 1.5, the transistor is 1.0. If we would have chosen a much larger resistor, the noise of the resistor would have been dominating.
Moreover, it doesn't make sense to select a JFET much better than this, because its contribution in the thermal region is already very small.

The same calculation can be done at 1 Hz, where the noise is dominated by the 1/f component. So we make 5.4^2, we subtract 1.5^2 that is the noise of the resistor and we get 5.2, that is the contribution of the transistor, the 1/f is dominated by the transistor because we have assumed that the resistor has not 1/f noise.

Once we have calculated or measured the input noise spectral density of the input voltage generator, it is time to calculate the overall sigma of the noise at the input of my amplifier; which is the rms of my amplifier that represents the minimum signal I can measure with this amplifier? The sigma square, that is the variance of the noise, is given by the integral of the noise spectral density, which is 1 in the 3^{rd} region, over the bandwidth of the amplifier.

Here in the table in the bottom are proposed the calculation of the noise given different bandwidth of the amplifier (we suppose to introduce a pole so to have a bandpass filter where the low frequency pole is given by the integrator).
So let's suppose we introduce a HF pole; in the table we have the final bandwidth. We see we have typical values for biosignal amplification.

What is interesting is to compute the integral of the noise given different upper cut off frequency. If we choose 10Hz, the integral is from 0.025 up to 10Hz, so we integrate the 1^{st} and 2^{nd} parts of the noise spectral density. If we calculate this integral, we get 40nV of rms.
If we extend the integral to 250Hz (even outside the plot), we get a noise of 50nV, despite we have increase the bandwidth of 1 order of magnitude (a factor of 10). Hence even if we have extended the integration bandwidth from 10 Hz to 250, we have just integrated more white noise, but the overall integral is still dominated by the 1/f noise.
The important conclusion is that the overall noise in the integral is so dominated by the 1/f component that even if we extend the upper frequency of my amplifier, we get the same noise. This is a very good news, because we speed up the amplifier by increasing the bandwidth but practically with the same noise.

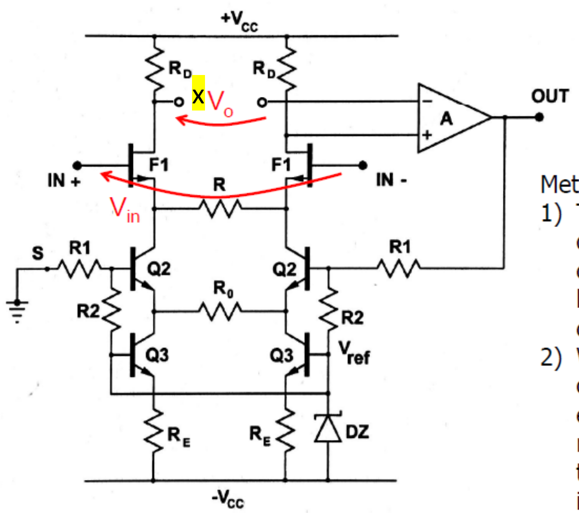
Hence **it is also of a fundamental importance to choose a JFET with a very low 1/f noise**, because the 1/f noise is the one dominating in the integral.

If we extend the bandwidth even further, we see that the noise then increases, but not that much. It becomes almost 200nV, but we have also increased a lot the integration bandwidth.

Final remarks

Be careful about the 1/f noise that is dominating, and moreover the overall rms noise is ultrasmall, in the order of 50 or 100nV. If we go back to the general INA and we check the noise of the classical 3-opamps configuration, the input rms noise was in the order of uV, not nV.
It is hence important to design a custom amplifier, because with a custom amplifier, if we are a good designer, we can reach ultralow noise performance that cannot be achieved with a commercial 3-opamps INA.

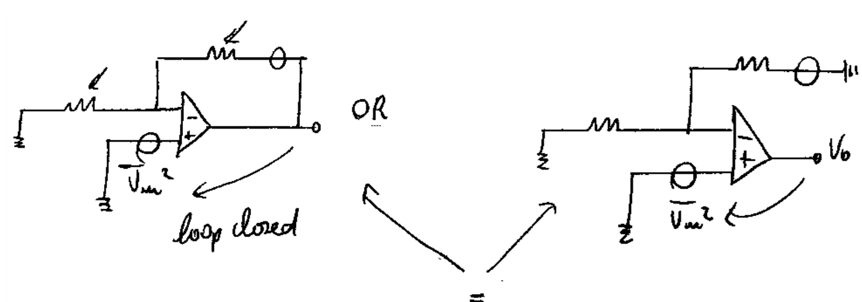
Hints for contributions and calculation of the electronic noise



Method:
1) The input noise generators do not depend if the circuit is considered in open or closed loop, provided that all contributions are considered;
2) We will consider the circuit in open loop and will calculate each contribution at the reference position V_o and then transferred back at the input by the gain V_o/V_{in}.

The calculations are based on two methodologies (rules) listed above. One is a general rule for noise calculations: when we have a feedback amplifier, **we may calculate the input noise generators either considering the loop closed or considering it open**. In the image the loop is open because there is a missing connection between the terminals where we have V0. If we calculate the noise considering the loop open or closed, of course the noise at the output is different, because by closing or opening the loop the gain is different, but the rule states that **the noise at the input is the same**, it cannot change if we are considering the loop closed or open.

52.



We take a non-inverting amplifier and we want to compute the input noise generator at the input of this amplifier. We can calculate it considering the noise contribution of the resistors, keeping close the loop, or we could calculate the equivalent noise generator by opening the loop, for instance by opening it at the output of the main amplifier. If we open the loop, the input output gain is different from the closed loop, so the t.f. of the noise generator to the output if different. But if we go back and compute the noise at the input, it is equal in both the configurations → we can adopt the easier configuration for us.

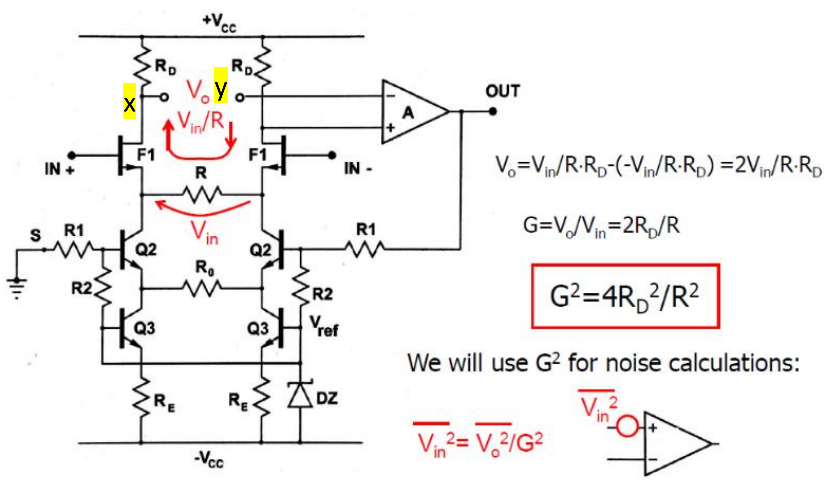
Coming back to our amplifier, it is convenient to open the loop. This doesn't mean that the amplifier A is not to be consider, but it must be considered as connected to the node x (so we connect the two terminals; simply we don't consider the effect of the feedback, but just the open loop gain between the input (in+ and in-) and the node of the amplifier.

Our strategy is to **elect a reference port** (node) that is the voltage between the two electrodes V0. I elect it as a reference port in the OL calculation. I elect as a reference it because I will take the noise of all the component and calculate the t.f. of the noise up to this port, and then go back from this port to the input. I take the port as a reference, I compute all the t.f. in the specific locations of the noise generator up to this port and then bring it back to the input (in+ and in-), in this way I will have the input noise generator.

To do so, **the first thing to do is to compute the OL gain between input and output port**. If I compute it, then I have the factor to be used in the 'returning back' of the noise generators.

Vout/Vin open loop gain calculation

Below there is the computation of the OL i/o gain.

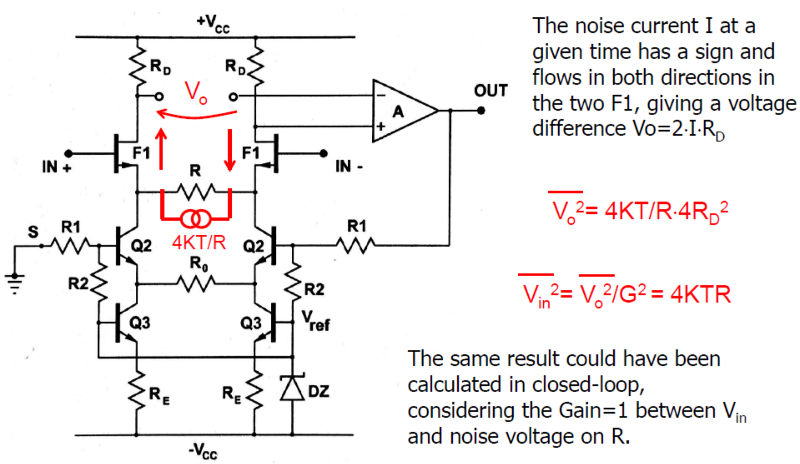


Let's suppose we have an input signal Vin. This Vin differential is applied directly across the resistor R because F1 works as a source follower (even in OL). Then this voltage across the R creates a current, Vin/R. This current flows (**in the OL calculation we have to forget about the closed loop recirculation of the current between R and R0**, because it was a property of the circuit in CL. Now the situation is similar to when we were analyzing the feedback before the loop was closed). Hence the current in R cannot go into the bottom, because we have the high impedance of the collectors of Q2 → can only go in the low impedance 1/gm of the F1. So we have this current ant this current is flowing on the node x, is entering in the resistor Rd and it is producing a voltage on node x that is Vin/R * Rd; this is the voltage across the left Rd. To this voltage must be subtracted the voltage on node y; but the voltage on node y is the opposite than node x, because the current is with a flipped direction → negative voltage with the same modulus as before. Hence the overall Vo is the one in the first formula.

This is the result of Vo/Vin. The t.f. G between Vo and Vin is 2*Rd/R, in OL. However in noise calculation we don't use the linear form for the t.f., but its square. So we have to square G as in the red box.
This means that each time we will calculate a noise contribution on the port (hence noise contribution means sigma squared, a variance), each time we will calculate the variance of the noise in the output position, so Vo^2, the corresponding variance at the input Vin^2, so the equivalent noise generator, will be the output variance divided by G^2.

NB: the current should be in the opposite direction but, whatever the orientation, since we are considering the square of the gain, even if the gain would have a minus, it would be insignificant.
Now we go through the contributions.

Contribution of R



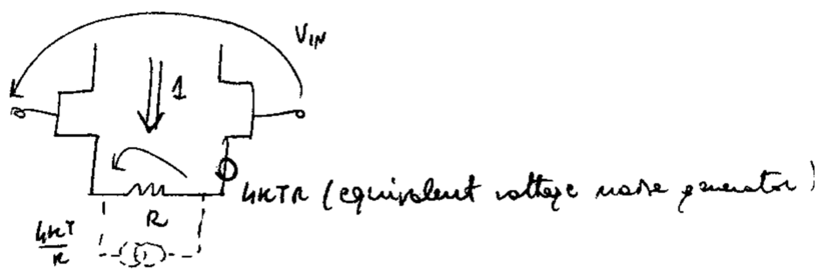
The noise of the resistor R can be represented either as a voltage noise generator or current one (better to consider it as a current one), so we see the current noise generator in parallel to R with a spectral density 4kT/R. Now I have to calculate the t.f. from this generator to Vo. Also in this case, to compute it, I consider a current and how it flows to the top (two arbitrary orientations has been selected, no matter the sign since the t.f. would be the same).
If the generator 4kT/R at a given time has the orientations of the currents in the image (we imagine the generator to have a direction in a frame of second), the current can go only in the resistor Rd, since it cannot go below into the high impedance of Q2.

Hence the t.f. from the 'I' of R to Vo is 2*I*Rd because we would have a contribution on Rd left that is I*Rd minus an opposite contribution on the right Rd. Hence the linear t.f. between a 'I' generator parallel to R and the port is Vo = 2*I*Rd.

If we have this linear t.f., we have to use its square for the noise. Instead of the output voltage we have the variance of the output voltage (sigma squared, the noise variance); so instead of the generator 'I'

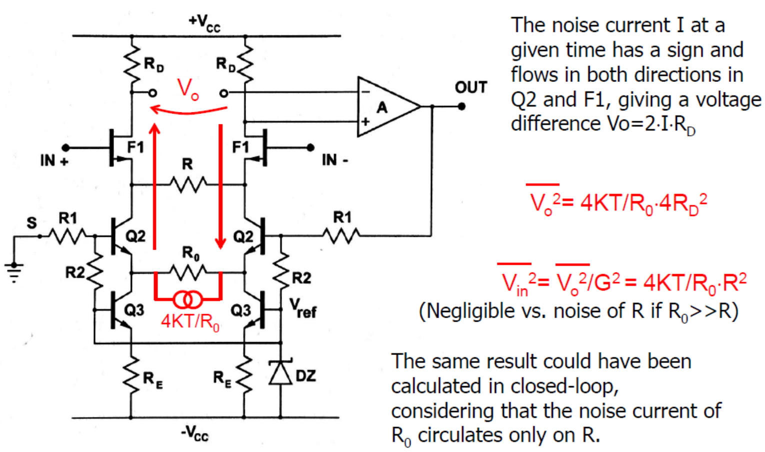
we have the noise spectral density 4kT/R and instead of the t.f. that is 2*I*Rd we have its square. The first red formula is the t.f. of the current generator to the output voltage.
Now if I have the noise, I adopt the previously stated formula (with G^2) between Vin and Vo. In the end we obtain a voltage noise 4kTR, that is the first term we have in the final formula of the overall noise.
The same result could have been calculated in closed loop. In CL, there is a gain 1 between the voltage at the input of the amplifier and the voltage generator on R.

53.



Even in CL, we have a transfer 1 between the voltage at the input of the circuit and the voltage on the network. So if instead of considering the equivalent noise generator in current I would have considered it as in voltage, I could say that this voltage generator (4kTR) is already transferred to the input, because there is a transfer 1 between the voltage at the input and the voltage in the branch where we have R. Because of this, any voltage noise for R is hence equivalent to the noise at the input at the input. In conclusion, the voltage noise I have at the input is the same I have in terms of voltage noise I have on R.

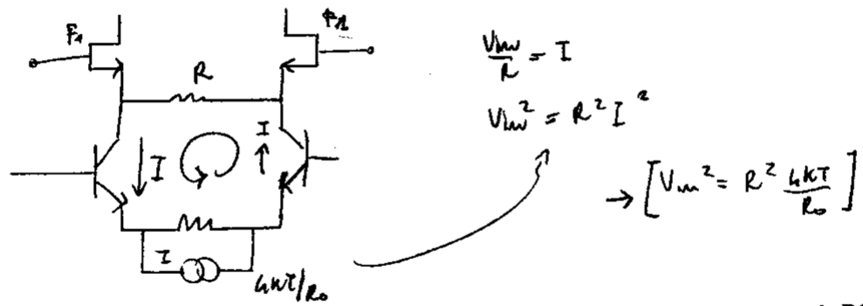
Contribution of R0



Its contribution is very similar to the previous one, because if we calculate the t.f. of a whatever current 'I' in R0, it will enter in the 1/gm of Q1, the 1/gm of F1 (the low impedance path. *Non attraversa R perchè ai suoi capi ho in+ e in- che sono a massa nell'analisi del rumore di R0 → no voltage drop across R and no current*) and finally across the resistor Rd (as for the previous contribution of R).
Hence the sigma squared noise over Vo is the input noise generator 4kT/R0 multiplied by 4*Rd^2, up to the node Vo.
Then the input noise is Vo^2/G^2 → we get the second contribution in the final formula for noise from the resistors.

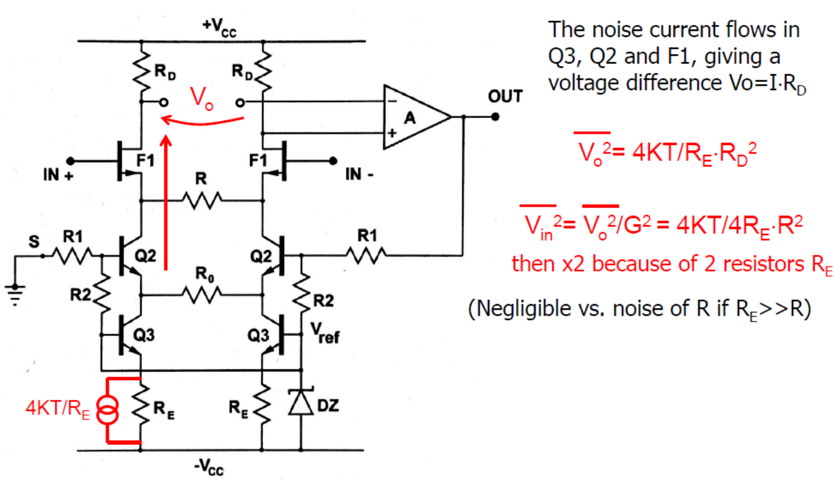
This contribution is negligible with respect to the one of R if R0 >> R. Moreover, this result could have been achieved similarly if we consider the CL gain. In fact, if we have a noise contribution on R0, in CL, it will recirculate in the network composed by Q2, R and R0, so we can calculate it with the CL gain.

54.



In CL, the rule is that the current flowing in R, which is Vin/R, is equal to the current flowing in R0, which is 4kT/R0. Hence the current noise generator will recirculate in the network, so equal to Vin/R. this means that Vin^2 = R^2 * I^2. But in place of I^2 I can put its noise generator → I get the same formula as in OL. So in open loop the noise generator 'I' is going upwards to Rd, now in CL it is recirculating over R → I can use the CL formula, but the result is identical.

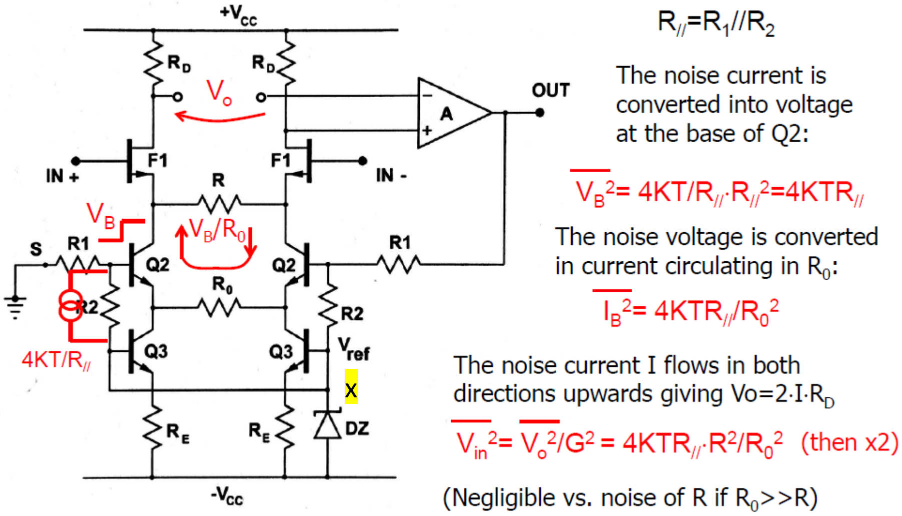
Contribution of Re



The resistor Re has a thermal noise generator, 4kT/Re. Again, OL calculations, so I'm going to see the effect of this generator all over Vo and then go back from Vo to Vin.
As before, this noise current generator can only enter in Q3, Q2, F1 and then Rd and the final voltage is Vo = I*Rd. It is not 2*Rd because the current is only flowing in the left, not in the right as for the contribution of R0. Then the corresponding input voltage is by dividing for G^2. Again, negligible contribution if Re >> R.

However, we have a resistor Re also on the right → total noise calculation must be multiplied by 2, because I have to add the noise of the second resistor, and as the two noises are statistically independent, we can add them → reason why the 4th term in the final formula includes the factor 2 due to the presence of 2 Re resistors (at the denominator).

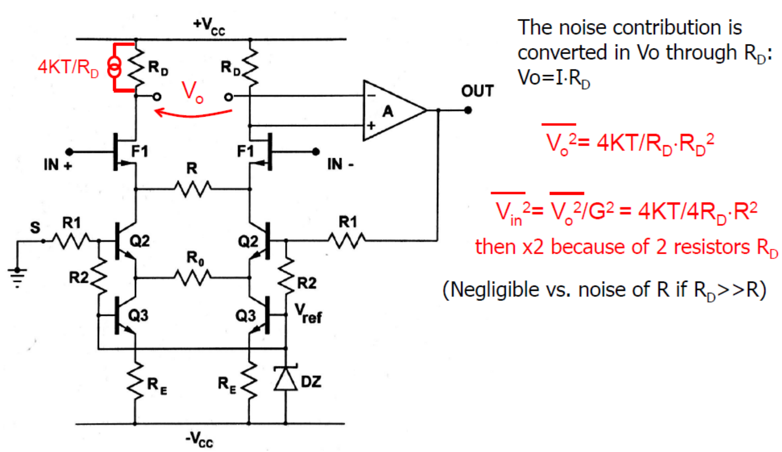
Contribution of R1 // R2



Better to consider them together if we consider the node x at ground, so that we can consider a single resistor that is the parallel of R1 and R2.

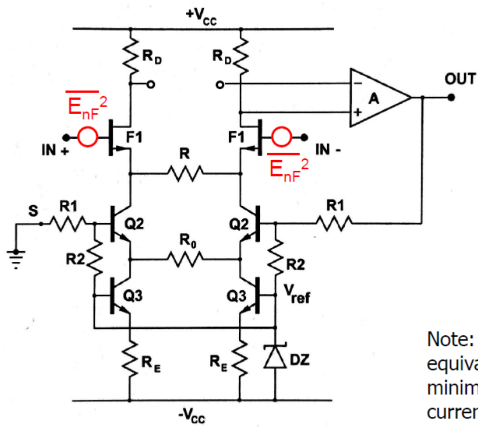
The equivalent resistor has a noise generator 4kT/Rparallel. Now I compute the t.f. between this generator on R2 and Vo. Firstly I transform the current into the voltage Vb (current multiplied by the resistor). Then this voltage creates a current in the resistor R0 that is Vb/R0. Then this current is producing a voltage on the nodes of the port that corresponds to the current multiplied by 2*Rd.
Then I have to square to have the sigma of the noise. In fact, Vb^2 = 4kT/Rparallel * Rparallel^2 and Ib^2 = 4kTRparallel/R0^2 and so on. Again, the final voltage generator must be multiplied by two because I have two parallels, on the left and on the right.

Contribution of Rd



A generator on Rd is already at the output, I simply multiply the current generator for Rd. So the voltage at the output is Vo^2 = 4kT/Rd * Rd^2. Then I bring back the contribution to the input and in the end the contribution can be negligible if Rd >> R.
Also this contribution has to be multiplied by 2 because I have two resistors Rd.

Contribution of F1



Noise generator of F1 already at the input, to be counted x2 because of two JFETs:

$\overline{V_{in}^2} = 2 \cdot \overline{E_{nF}^2}$

Note: input shot noise of F1 (for equivalent current generator) minimized using JFET (low gate current)

Noise generators of the transistors that can be represented as a voltage and a current generator. The voltage generator of the JFET (in the image) includes both the 1/f noise and the thermal noise. These generators are already at the input.

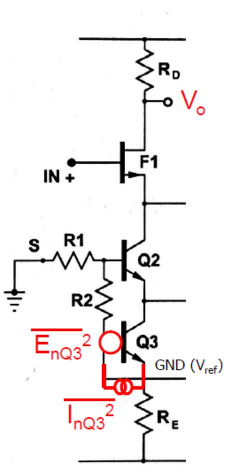
NB: the overall input noise voltage is twice the generator, because we have two noise sources statistically independent.

This is the first unavoidable contribution in the following formula:

$$2\left[\overline{E_{nF}^2} + \overline{E_{nQ2}^2}\frac{R^2}{R_0^2} + \frac{1}{4}\overline{E_{nQ3}^2}\frac{R^2}{R_E^2} + \overline{I_{nQ2}^2}R^2\left(\frac{R_1 \| R_2}{R_0} + \frac{1}{2}\right)^2 + \frac{1}{4}\overline{I_{nQ3}^2}R^2\right]$$

The contribution of the shot noise is not analyzed because it doesn't give contribution to the voltage noise but it gives contribution only to the equivalent current generator. So, since I have JFETs, I have to be careful about the shot noise of the gate current.

Contribution of Q3



$\overline{E_{nQ3}^2}$ is converted in current by R_E and then in Vo through R_D:

$\overline{V_o^2} = \overline{E_{nQ3}^2}/R_E^2 \cdot R_D^2$

$\overline{V_{in}^2} = \overline{V_o^2}/G^2 = \overline{E_{nQ3}^2}/4R_E^2 \cdot R^2$ (then x2)

(Negligible vs. noise F1 if R_E>>R)

$\overline{I_{nQ3}^2}$ flows into Q3, Q2, F1 and is converted in Vo through R_D:

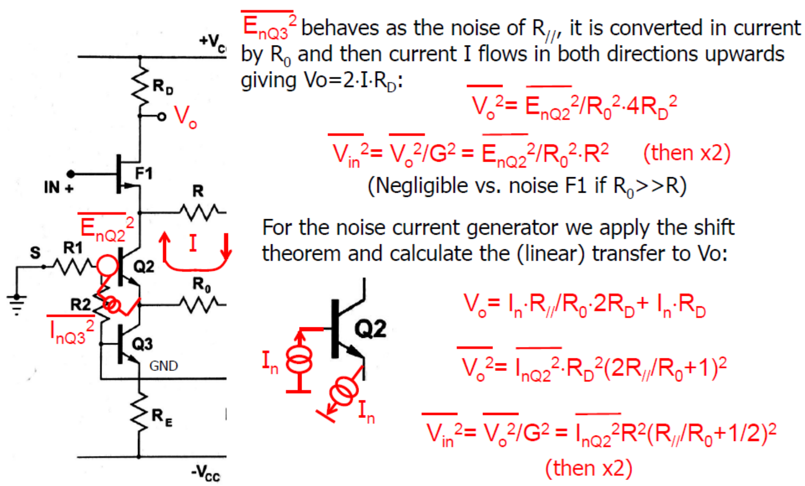
$\overline{V_o^2} = \overline{I_{nQ3}^2} \cdot R_D^2$

$\overline{V_{in}^2} = \overline{V_o^2}/G^2 = \overline{I_{nQ3}^2}/4 \cdot R^2$ (then x2)

The transistor Q3 has a voltage noise generator and a current noise generator. Let's start from the voltage. The voltage is converted into a current (En/Re) through the resistor Re, because a voltage on the base is transferred from the base to the emitter because it is a source follower. Then we already know that a noise current flowing across Re goes to the top and it is converted into voltage by Rd. Again, I have to multiply by two because of the left and right branch. If Re >> R (previously adopted condition for the resistors), this noise can be considered negligible with respect to the one of F1.

Instead, as for the current noise generator InQ3, it is placed between ground (the base is at ground because I have the zener, it is like a low impedance) and emitter. It is converted into noise at the output thanks to Rd.

Contribution of Q2



Voltage generator at the base

It is in the identical position of the voltage noise previously calculated for R1||R2. We can carry on the identical calculations. The noise is converted in current through R0^2 and then is multiplied by Rd. This noise contribution is negligible with respect to F1 if we choose R0 >> R.

Current generator

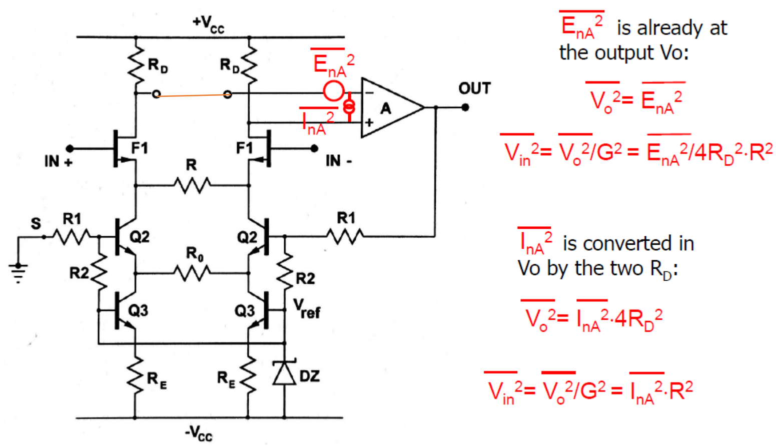
In is in a very bad position for calculation, because it is not plugged between ground and the emitter like InQ3, but it is plugged between the base and the emitter, and the base is not grounded.

To perform the calculation, we use the **shift theorem**: we take the generator, we shift it into two generators between ground; one generator between ground and the base, the other between ground and the emitter. In order to have this two generators equivalent with respect to the initial one, we have to consider the directions of the generators, because the two generators are not uncorrelated, they express the same noise.

If we do so, we then compute the t.f. between the generator at the base (In*Rparall/R0 * 2Rd) and the top and we subtract the t.f. between the generator at the emitter and the top. We subtract it because they are correlated, so we cannot add the contributions, but to subtract them. The contribution of the second generator is -In*Rd.

In conclusion, we get the net contribution at Vo, and then we square it to get the output noise. We have to square the final t.f., not the single contributions.

Contribution of A



Considering the OL doesn't means that the amplifier is not physically connected, it is physically connected at the port. We have its noise contribution also, and the one below is the result.

$$\left(\overline{E_{nA}^2}+4\overline{I_{nA}^2}R_D^2\right)\frac{R^2}{4R_D^2}=\frac{1}{4}\overline{E_{nA}^2}\frac{R^2}{R_D^2}+\overline{I_{nA}^2}R^2$$

We have to consider that we have the equivalent voltage and current generators at the input of the amplifier. We simply need to bring back such generators at the input.
Any voltage fluctuation on (-) is a voltage fluctuation on the output port Vo. Hence the corresponding voltage at the output port is EnA. Again, this noise contribution is negligible if Rd >> R.

As for the current contribution InA, a current generator in that position is converted into voltage by the factor 4*Rd^2, because it is a current directly plugged between the two nodes of the port. And so the corresponding voltage is the current times 4Rd^2.
We are optimistic about this contribution that is then bring back at the input because Rd is much greater than R. InA depends on if the amplifier has a BJT input or a JFET input. If it has a BJT input, this contribution can be large.

The current InA is not only on one branch, because it passes in the port, Rd on the left up and then Rd on the right down, because the (-) terminal is still connected to the left branch. In this case we open the loop by cutting the loop at the output of the amplifier A; before it was easier to cut the loop where I have the port, but if I do so now, the amplifier is not working. So I reconnect the loop at the port and cut it at the output of the amplifier.

MEDICAL IMAGING

The goal is to get an image of the patient and to try to exploit the image to identify a spatial distribution of something of interest like:

- Morphology: status of tissues, bones, …
- Regions where pathologies are localized.
- Physiological functionalities.

The imaging modalities are hence divided in morphological one and functional one.

The main techniques used in medical imaging are:

- Xray radiography, Xray CT
- SPECT
- PET: together with SPECT belongs to the field of nuclear medicine
- MRI
- Ultrasound (echography)

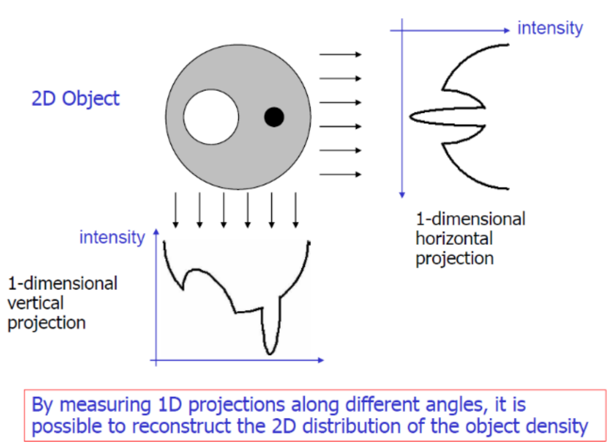
The first three have in common the use of radiations, while the last two are not using radiations.

RADIOGRAPHY

Based on Xrays generated by a Xray tube. They are emitted and shine an object; the survival ones are recorded by a panel that records the intensity of the recorded Xrays.
In radiography, the **source of radiation is external to the body**, and this is different from the nuclear medicine modalities where the source is injected in the patient. The other difference is that we record the Xrays that are not absorbed by the body → in white color we see the part of the image corresponding to bones, because bones are highly absorbing materials, and few rays reach the slab. On the contrary, if rays are absorbed nowhere or in soft tissues, we have black or gray (high intensity recorded).

Moreover, Xrays are emitted in a broad range of energy, but tissues are absorbing more or less depending on the energy of the rays. Of course, since the tissues are shined by the complete energy range, the absorption is a mixture of absorptions at the different energies → absorption not really selective.

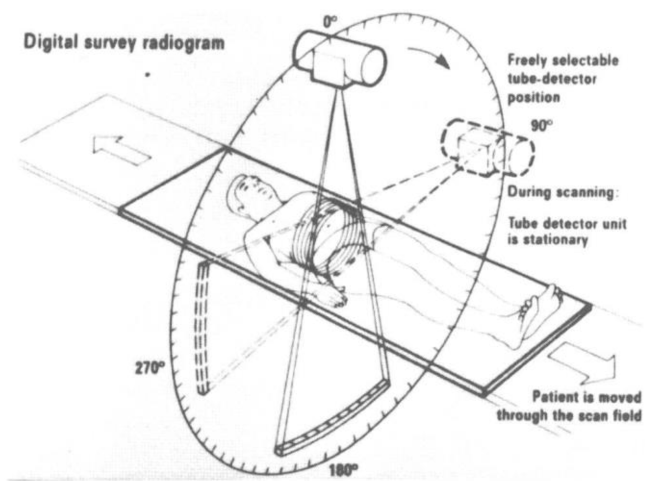
COMPUTED TOMOGRAPHY (CT)



When we irradiate with a normal radiography the patient, we have a 2D image, we cannot identify the depth of the organs in the image, for example in a planar image the chest and the spine would have been superposed. In order to distinguish better the objects inside the body at different depth we use the tomography.
In fact, if we take just a single radiographic image, the resulting intensity depends on the amount of material in gray passed through the Xray (the less material, more material intensity – see image at previous page), but moreover, in correspondence of the white cylinder we have a valley of intensity, because less Xray are absorbed, while in correspondence of the black one we have a peak because more rays are absorbed (plot on the right).
So from this single image we cannot identify if the white cylinder is on the left or on the right → we need to take another image from another point of view.

Hence tomography helps us to understand the 3D nature of the object, and not only the 2D.

This can be extended to a CT scan, where basically we have an xray tube and a detector for the xrays. The detectors aren't in a static configuration but we take an image (that is a slice of the patient, just a projection) all around the patient by turning the detection system.
It can be demonstrated that if we take multiple scans with an angular frequency of, for instance, 1 projection per degree (360 projections) precisely, we can get an information inside the body with a precision that is higher than the spatial frequency of the patient. The final details of the image are as good as the spatial frequency we use.



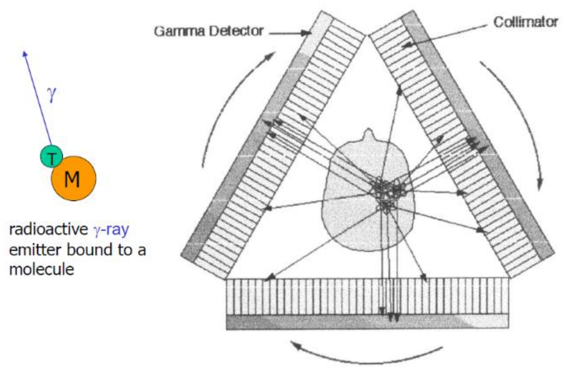
Moreover, to have information also in the longitudinal plane, we have to scan the patient by moving him on the bed.

We can also do better than just making a full turn with the scanner and then move the bed of the patient. We can do a spiral scan: while rotating the tube all over the patient, the bed is continuously moving. Hence we are scanning the patient angularly while he is already moving → spiral recording.
It can be demonstrated that the amount of information provided by this technique is similar to the classical one.

SPECT

The source of radiation is in the body. The principle is to **bound a molecule with a radioactive emitter**. In radioactive atoms we may have the emission of gamma rays that are detected externally by the imaging system to identify the presence of the radioactive atom. The topic of interest is not the atom itself, but where it is fixed the atom. Usually it is fixed in a place where it is veiculated by the molecule. The molecule is driven toward the region of the pathology and as the molecule brings its own radioactive atom, we have then a localized emitter of gamma rays.

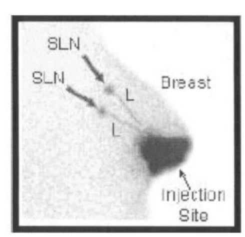
When we take an image of the patient, like in radiography, we have the planar recording or we can have the tomographic recording. If we take a planar recording, we are talking about scintigraphy, that is a planar recording of the emission of the radioactive atoms.
Instead, if the detector is moved across the patient (doing something similar to CT) we are talking about tomography. We are recording single photon emission in a tomographic way.



An important component of the camera is the **collimator**. It is a grid of metals, high absorbing metal, that allow only the gamma rays parallel to the collimator walls to pass through. Gamma rays in a tilted direction are absorbed by the collimator. This selection is made because our radioactive atoms are emitting isotropically, not in a preferrable direction, but everywhere. So if we wouldn't have a collimator but a detector taking a recording normally, how could we associate the recording on the detector to the origin of the emission of the gamma ray? If the emitter is emitting everywhere, how can I recognize from the detected events the location of the emitter?
If I have a preselection thanks to the collimator, we know by a modality called **back-projection**, if we record a signal, by back-projecting such hit of the ray, we can determine the direction of the emitter. Hence the collimator allows to identify the emission along a given direction.

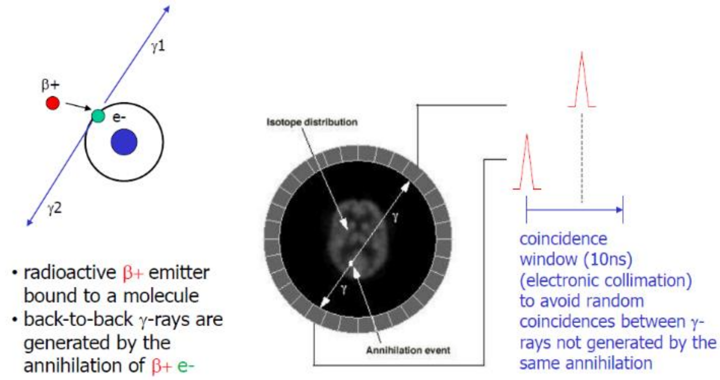
How can I then determine that along a direction the emitter is in a given position? I use the tomography. I take several projections, I make the back-projection and I can identify the position of the emitting atoms, at the crossing between two or more directions of recording.

Scintigraphy



We want to understand for example how far the cancer has developed. One way to do it is, for breast cancer, the sentinel lymph node technique, because cancer cells may circulate through the lymphatic system. We inject a radiotracer in the position of interest and then, when the molecule tight with the radiotracer spreads through the lymphatic system, we look the first lymphonode (sentinel) and if we see a signal like in the image it means that the lymphonde has tumor cells. If instead I see a white image, the cancer has not reached the node. It is an example of functional imaging.

PET – positron emission tomography



PET: higher efficiency with respect to SPECT (10^{-2} vs. 10^{-4})

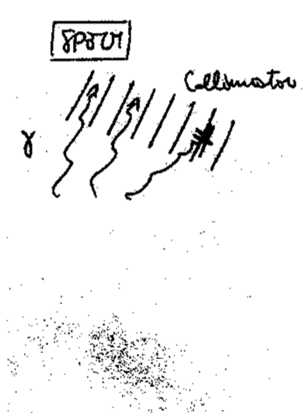
The collimator is needed in SPECT to select the direction of rays but there is a price to be paid, that is that only a small fraction of gamma rays passed through. Most of the gamma rays, the tilted ones, are absorbed. Hence the SPECT is a very inefficient technique.

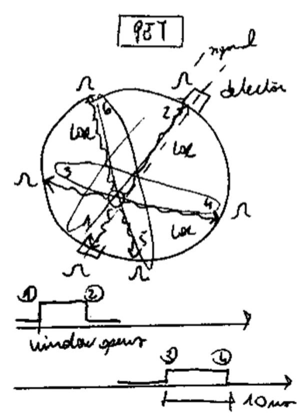
To reach the same goal but without a mechanical collimator we use the PET. In the PET we also bound a radioactive emitter to the molecule, but the difference is that **in PET the emitter is a positron emitter** (positron is the antimatter of electron → PET is an application of antimatter). So we have a producer of beta+ emitter, and since antimatter is rare in the body, the positron travels a short distance and annihilates with an electron (as our body is mostly composed by electrons, it is very probable that a positron in a short range finds an electron). When positron and electron meet, they annihilate and so they disappear. But according to the Einstein formula, if a quantity of mass has disappeared, the corresponding quantity of energy has to show up tin the same location. In our case the amount of energy is generated by 2 gamma rays emitted back to back. They are emitted back to back because they have to satisfy the momentum equation (that at the beginning was zero, and so the final momentum must be zero. If it was not zero, we cannot assume that the two gamma rays are back to back → error in the reconstruction).

The energy of such gamma rays is equal to 511keV because E = mc^2 and if the mass of electron is equal to the mass of positron, the E is 1022keV. Hence the energy emitted in PET is fixed by Einstein equation.

$E = 511\,keV$ because $\underset{\uparrow\, m_{e^-} = m_{\beta^+}}{E = mc^2} = 1022\,keV$ 511 keV • 511 keV

55.





In SPECT, I have only 1 single gamma ray. So how can I determine its direction? I use the collimator. However, in PET I know that the gamma rays belong to the same line, direction. Hence in a PET scanner I don't need the collimator to identify the direction of the emission of the couple of gamma rays, because I simply need to have 2 detectors and if one detector provides a signal and the other provides a signal in a short time, I can say that I have the emission of 2 gamma rays back to back in the direction that connects the two detectors. So I can get the direction where the source is present without the usage of any collimator. This technique is also called electronic collimation (in contrast with the mechanical one).

To determine the source in PET then, I simply wait for the next couple of gamma rays emitted (they are emitted isotropically indeed). Simply by recording another couple of gamma ray I can determine the position of the source by crossing the two directions. With other gamma rays I can then reinforce the information. So the technique of tomographic reconstruction of PET is made out by crossing several lines of response (LOR, that is the direction extracted by the detection of two gamma rays).

Coincidence window

However, we need in PET to use coincidence. It means that when for instance we have our first ray, when we detect it, we need to open a time window, the coincidence window, and wait for the arrival of the corresponding opposite second ray, and then we close the window. It is important to check if events happen inside a suitable time window because later on we can have the emission of other rays, so in PET reconstruction it is important that 1 is associated with 2, 3 with 4 and 5 with 6. It would be completely catastrophic if 2 would be associated with 4, because then we would assume that the LOR would be the connection between the connector taking 2 with the detector recording 4.
Hence it is of paramount importance that the first hit we get opens a time window and we check the photon arriving within the time window. Then of course later on we would have the event 3 and we put in a coincidence window the event 4, and so on.

In order to do so, we need to keep the time windows as small as we need. Typically, a reference value is on **10 ns**.
It is of 10ns because the gamma rays travel at the speed of light, so in 10ns is already exceeding the maximum time for travelling the whole diameter of the patient. If in 10ns a coincidence has not happen, it will never happen. if in 10ns we don't record both photon 1 and photon 2, we will never record.
So it is important to keep the coincidence window small because of course the larger we keep it, the more we allow different photons to be put in coincidence.
If it is too much large, also photon 3 could participate to the coincidence, erroneously. These are called spurious coincidence, that we want to avoid.

If we are so unlucky that the couple 1-2 is absolutely simultaneous to couple 3-4 we can do nothing. But as long as they are not simultaneous, we have to associate the couples with time windows.

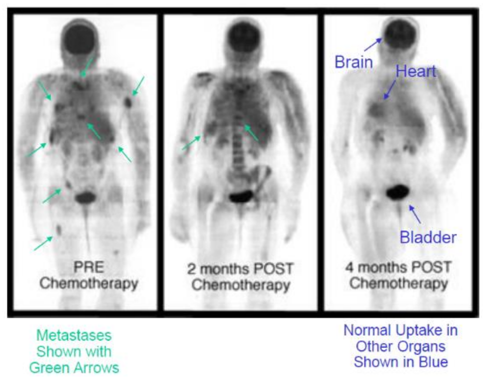
Hence PET is based on coincidence between detectors placed around the patient because if we can record such a coincidence, we can draw the LOR which tells the direction of the source.
When a signal arrives, we will start a coincidence window and we will wait the second gamma ray to be recorded inside the coincidence window. If not so, the second gamma ray is lost, because it is better to close the window than to record it.

If we take the window open for more than 10ns, other couples of gamma rays can be detected and so we would not know to which gamma ray associate the window.

Efficiency

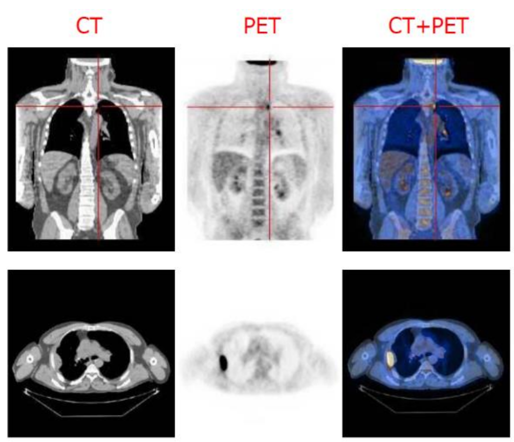
The missing of collimator makes the **PET much more efficient than SPECT**, it is 10^-2 with respect to 10^-4. 10^-2 means that among 100 gamma rays emitted by the patient, only 1 is recorded and provides a point into the image. In SPECT it is 10^-4, we need 10000 gamma rays emitted by the patient to get just one useful for image reconstruction, because almost all the gamma rays are killed by the collimator.

PET – Examples



PET image of a patient with hot spots of accumulation of the radiotracer. It is localized there because the regions of tumors are able to absorb molecules like glucose or other specific molecules (black parts). Hence the scope of PET is not to look at the morphology of the patient, but to get where the pathologies are located. In the image we have the improvement of patient's tumor under chemiotherapy. There is a high accumulation of the radiotracer in the bladder because the radiotracer veiculates through the body but then it is expelled by the bladder.

Differences with other techniques



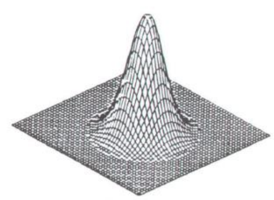
The scope of a CT is the morphology, of PET is the functionality and moreover today we can have also multimodality. On the same recording CT and PET are coupled, the two images are combined, and we have advantages because we can see the location of the pathology superposed to the morphology image, so providing good information for surgery and radiotherapy.

Less popular is the MRI+PET.

The choice of the imaging technique depends on the parameter which has to be visualized.
Figures of merit:

- Sensitivity: efficiency of the imaging system to detect the parameter of interest.
- Selectivity (specificity): capability of the system to distinguish the parameter of interest from other possible signals. Our imaging system may be able to detect a signal, but unfortunately not always this signal is associated with the pathology we are looking for. So it is the capability not only to detect a signal, but to detect the correct one.
- Resolution and contrast: capability of the system to distinguish details of the distribution very close each other and separate regions with different concentrations (to improve contrast, specific contrast agents can be employed). Contrast is the capability of the system to distinguish regions with slightly different concentrations. For instance, we have a good contrast in CT to separate tissues from bones; however, CT has some difficulties in distinguish among the soft tissues themselves. Instead, MRI is particularly able to resolve soft tissues with contrast.

SPATIAL RESOLUTION: the Point Spread Funcion (PSF)



MARGIN FIGURE 17-3
The point spread function PSF (x, y).

We start with resolution, the capability to distinguish closely spaced details. One key parameter is the PSF, that is the response of the imaging system to a delta-like point source. So if we suppose to have in our image a delta-like object, the response of our system will be a peaked image (red, right). Usually the resulting image of the delta is not as much narrow as the delta, and the width of the PSF is actually the spatial resolution of my imaging system. Narrower the PSF, the better is the imaging system.

Let's quantify this a bit better.
We characterize the PSF of our system as a bidimensional function h(x,y); if we have an object defined with O(x,y), the resulting image in the imaging system will be the convolution of the object distribution with the PSF. In fact, in signal processing, the correspondence of the PSF is the pulse response h(t). if our system is characterized by a time response h in the time domain, the response of the system in the time domain will be the input stimulus in the time domain convoluted for the h of the system.

$I(x,y) = O(x,y)*h(x,y)$ $\quad$ $h(x,y)$: PSF

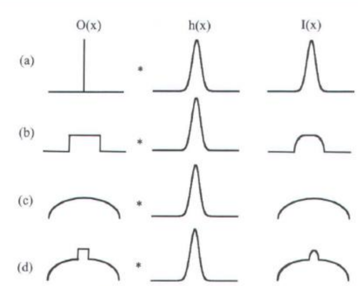


FIGURE 5.3. Projections I(x) resulting from the convolution of different one-dimensional objects O(x) with a one-dimensional Gaussian PSF, h(x). (a) If O(x) is a delta function I(x) and h(x) are identical, and thus the acquired image can be used to estimate h(x). (b) Sharp edges and boundaries in the object are blurred in the image I(x). (c) If the image is very smooth, then the overall effect of h(x) is small, but if, within the smooth structure, there are sharp boundaries, as in (d), then these boundaries appear blurred in the image.

In imaging systems, if we make a convolution of the PSF with a delta-like object (a), by definition the image response will be the PSF, because the PSF is the response of the system to a delta-like object. If we have an object different from the delta but with sharp edges (b), a convolution with a not fully sharp h will be the input object but with the boundaries smoothened by the smoothing of the PSF. In (c) we have an object that is already smoothened, so the convolution with the PSF produces the object identical to the original one, because there are no sharp boundaries filtered out by the PSF. In (d) we have the superposition of a sharp object with some sharp details and, once again, the PSF is smoothing the sharp edges but not the soft ones.

In particular, the width of the PSF provides us the spatial resolution of the system. This is shown in the image below.

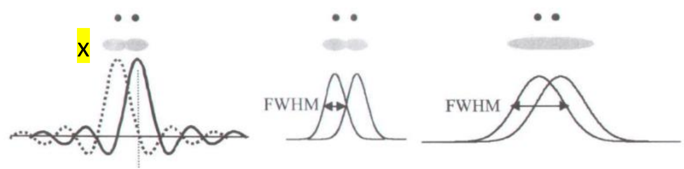


FIGURE 5.4. *(Left) For a sinc PSF, the signals from the two point sources can be resolved when the separation between them is less than half the width of the main lobe of the sinc function. (Center) For an arbitrary form of the PSF, the two point sources can be resolved when their separation is less than the FWHM of the function. (Right) The two point sources can no longer be resolved due to the broad FWHM of the PSF.*

$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-x_0)^2}{\sigma^2}\right)$$ Gaussian PSF (or Gaussian approx. of PSF)

$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\sigma \cong 2.36\sigma$$ Full-Width-at-Half-Maximum

We see two deltas (upper left) and x are the corresponding images produced by the two corresponding PSF. Of course, the larger the width of the PSF, the more becomes impossible to distinguish the two objects, because the two responses of the imaging system merge and overlap. If the PSF is larger than the distance between the two objects, we cannot distinguish them.

This intuitive view provides us a quite useful rule of thumb: *if we quote the full width at half maximum (FWHM), so we take the peak of the PSF and we quote the width of the PSF at half its peak value, we can barely distinguish two points that are placed at a distance equal to FWHM*.

For instance, if FWHM is of 2mm, we can distinguish two points that are separated by 2mm. Of course it is a rule of thumb; if the two points, instead of 2mm, have a distance of 1.8mm, probably we can still distinguish them. So practically the spatial resolution of our imaging system is the FWHM.

A very common way to quote the PSF is the gaussian fitting. It doesn't mean that all the imaging systems have a gaussian PSF. Some of them do, but other one, like the one below the x, don't. However it sill may be convenient to approximate at least the first lobe of the PSF with a gaussian (as done in the central image). If we do so, so we extract the PSF with the fitting of a gaussian curve, the gaussian fit has a sigma and there is a precise mathematical correspondence between the sigma of a gaussian and its FWHM. And the algebraic correspondence is the one in the bottom of the image. In this way we can get the FWHM, that is the 'rule of thumb estimation' of our spatial resolution.

Line Spread Function (LSF) and Edge Spread Function (ESF)

$$\mathrm{LSF}(y) = \int \mathrm{PSF}(x, y)\, dx$$

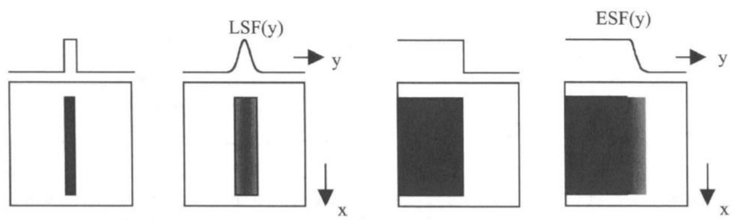


FIGURE 5.5. Illustration of the concept of (left) the line spread function (LSF) and (right) the edge spread function (ESF). For measuring the LSF, the object consists of a thin line, with the one-dimensional projection of the object in the y dimension shown above. The actual image is broadened, with the LSF defined by the one-dimensional y projection of the image. (Right) For measurement of the ESF, a wide object with a sharp edge is used.

LSF is the integration of the PSF along one coordinate. If we do so, we have a response in one coordinate. ESF is how an edge in the image has been reproduced by the imaging system. These two are related to PSF.

Modulation Transfer Function (MTF)

$$\mathrm{MTF}(k_x, k_y, k_z) = \iiint \mathrm{PSF}(x, y, z) e^{-j2\pi k_x x} e^{-j2\pi k_y y} e^{-j2\pi k_z z}\, dx\, dy\, dz$$

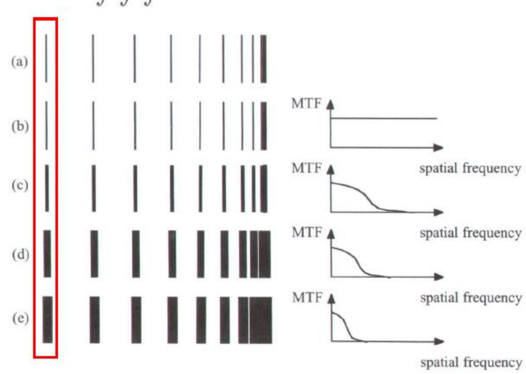


FIGURE 5.6. (a) A schematic of a line phantom used to measure the MTF of an imaging system. (b–e) The images produced from the phantom by imaging systems with the MTF shown on the right. As the MTF becomes progressively narrower (corresponding to a broader LSF), the image becomes more blurred.

In signal processing, we have also a quantity in the frequency domain that is the Fourier transform. The Fourier transform is the property of the response of a system determined in the frequency domain. Also in image processing we have the analogous function that is the MTF. It is nothing more than the Fourier transform of the PSF, but the difference is that the Fourier transform was with respect to a unique variable, that was the time domain. Here we have the spatial domain, that is characterized by three variables, x, y, z, so it is calculated according to them.

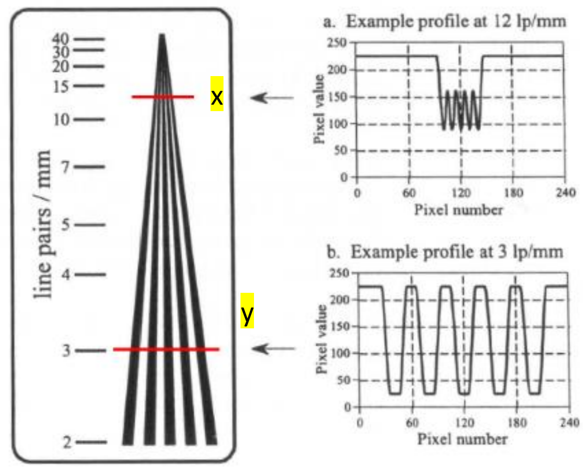
A more intuitive point of view is provided by the drawing below the formula. We see the images of very tiny line (a) (unidimensional example), and we record different images of this line with a system (from b to e) with worst MTF (e) and we intensify the frequency of the lines; so in the image the lines are accumulated more and more one close to the other. From top to bottom, the imaging system worsens.

We can see that moving from the example (a) to the example (e), the system is characterized by worst performances. In fact, moving down, the system is producing a broadened image of the line.
A system with a broadened image (e) is characterized on the right by a lower cut off frequency of the MTF. Indeed, the more the electronic system is low pass filtering, the more the resulting image will be smoothened, and we will have a low cut off frequency.

The conclusion is that having an MTF with lower cut off in spatial frequency means that we are less and less able to distinguish lines at higher spatial frequency. The spatial frequency is associated to the spatial frequency of accumulation of lines at HF; so in the worst system (e) we are no more able to distinguish consecutive lines if they fall at too high spatial frequency.

Spatial resolution – Example

FIGURE 25-2
Line pair gauge. The line pair gauge is a tool used to measure the resolution of imaging systems. A series of black and white ribs move together, creating a continuum of spatial frequencies. The resolution of a system is taken as the frequency where the eye can no longer distinguish the individual ribs. This example line pair gauge is shown several times larger than the calibrated scale indicates.



The one in the middle is an object used to quote the resolution in a radiographic system. The object is unique, but made by 5 diverging lines. These lines are the same, but the spatial density, quoted in line pairs per mm, is not the same, because in y we have 5 lines over a given distance (and the density is 3), while in x the same lines are lying in a much smaller distance (higher density).
These two examples are two spots in two different spatial frequencies, and our system has an MTF that could be not able to cover all the spatial frequencies. For example, in y the spatial frequency is so small that the system is able to measure it, but in x the system is no more able.
On the right of the image we have the profiles. If we cut the image in the middle, we see that the profile in y is a very sharp image, whereas if we cut the profile at x the image starts to be less resolved (imaging system no more able to distinguish any line).

FWHM rule of thumb – specification

Coming back to the rule of thumb, the imaging systems have similar resolving time if they have a similar FWHM. This rule is not rigorous.
In the image we have the PSF of 3 completely different imaging system. The first one (P) is the so-called pillbox. It is the classical pixelated imaging system. In this system, if a photon falls within the pixel dimension we have 100% chance to detect it and elsewhere is 0 (if a photon falls in a neighbor we have 0% chance to have a signal on the main pixel). It is the classical response of a pixelated sensor, so 100% of detection inside the pixel and 0% elsewhere.
This is not true for all the imaging systems; there can be a system characterized by a gaussian PSF (G) or by an error function (E).

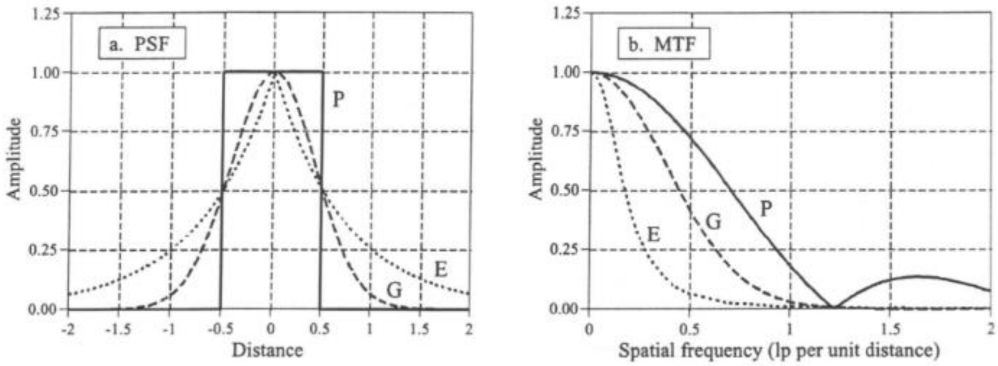
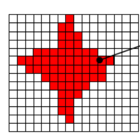


FIGURE 25-1
FWHM versus MTF. Figure (a) shows profiles of three PSFs commonly found in imaging systems: (P) pillbox, (G) Gaussian, and (E) exponential. Each of these has a FWHM of one unit. The corresponding MTFs are shown in (b). Unfortunately, similar values of FWHM do not correspond to similar MTF curves.

All these three imaging systems share the same FWHM, so according to the rule of thumb they should provide the same imaging capability. But this is not true, because if we plot the corresponding MTF, that is the frequency response, the MTFs are different. So if we process the image with MTF we would get a quite different resulting image.
Hence the FWHM is a rule of thumb, but not absolutely true, because the MTF can be different.

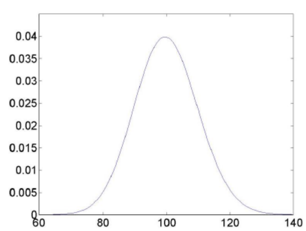
SNR: THE POISSON STATISTICS



counts in a pixel acquired in the time T

$$P(N) = \frac{\mu^N e^{-\mu}}{N!}$$

P(N): probability to have N counts given an average μ of counts acquired in the time T



$\sigma^2 \sim \mu$ variance

$\sigma \sim \sqrt{\mu}$ r.m.s

$\boxed{SNR = N/\sigma \sim \mu/\sigma \sim \sqrt{\mu}}$

Another fundamental parameter is contrast, that is the capability to identify boundaries between regions. In part contrast is related to resolution, but it is mostly related to the statistics of our signal, so how much signal we are recording during our data-take.
For instance, let's suppose we have a pixelated sensor (characterized by the pillbox function); we have pixels and we are collecting xrays in the red region. We count how many photons we have in pixels in the time T, that is the time needed to record the image; for example, in radiography the acquisition time is of 1s (time in which the xray tube is switched on). **This procedure is done pixel by pixel, we have to calculate the statistics for each pixel**.
Moreover, let's suppose that in this 1s a pixel collects an average u of counts. So we push the bottom several times and we are able to get the average number of photons collected in a pixel in the time T. Then the question is: how much does this count fluctuate statistically around u?

u is the average, but N is the recording of the count each time we repeat the measure. So how does N fluctuate around u?
At first approximation, the probability of counting a number N of counts around the average follows the Poisson statistic, given by the formula. The formula tells us, one we have u, the probability to count any kind of N around u.
If for instance u = 100, the highest probability is to count N = 100. However, we have also a probability of counting different numbers from 100.

One of the properties of the Poisson statistic is that the variance of the counts is equal to the average number. Given an average number, the variance is equal to it, and the sigma (std) is equal to its square root. Since we know the average number and the sigma then, in Poisson dominated systems we can easily calculate the SNR.
The SNR is equal to the number of counts in a pixel divided by the corresponding sigma. N can then be approximated with the average amount and since u is sigma-squared, the SNR is given by the square root of the average number.
This result says that in order to increase the SNR we need to increase the average counts in the pixel. This can be done in different way; in radiography the operator can improve the SNR of the image by pushing the button for a prolonged time (more than 1s) → increased acquisition time and so the average number of counts. The drawback is that we are irradiating more the patient.
The other way to increase statistic is to create a better system, to have a more efficient use of the photons, to increase the number of u for the same amount of time.

Example
We have a 4-pixels system and a flat image. For instance, we have in average u = 100 per pixel. The sigma is 10. Hence the SNR is sqrt(100) = 10.
Now we want to have better images → we improve the spatial resolution of our system. If we do so, we have more pixels, but for the same acquisition time of 1s the 100 counts in the previous system are now 25. Hence we have improved the spatial resolution but at a cost of worst statistic and SNR.
Is good to improve the image resolution (pixel number) but worsened the statistics (SNR)? We must find a trade-off.

56.

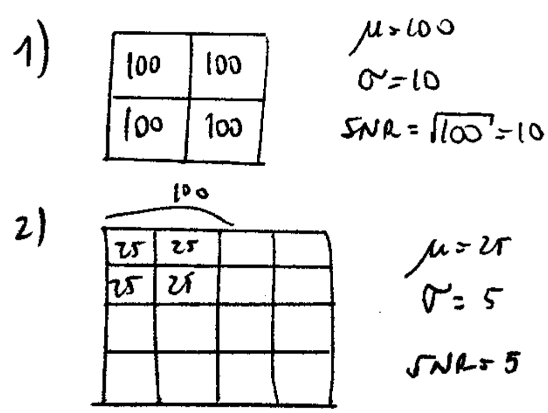


IMAGE CONTRAST

Capability to distinguish in the image the different tissues
(e.g. bones vs. soft tissues, healthy tissues vs. pathologies)

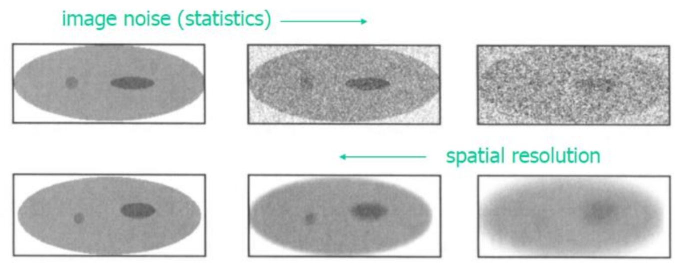
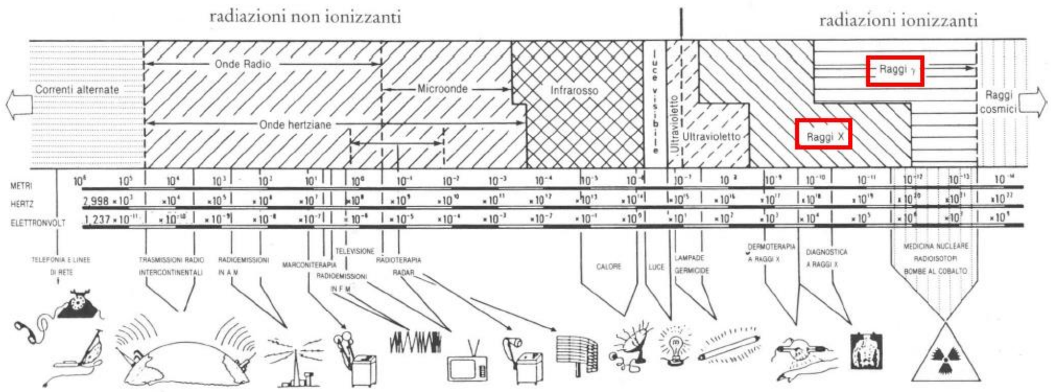


FIGURE 5.9. *(Top left-to-right) As the noise level increases in an image with high intrinsic contrast, the CNR degrades such that structures within the image can no longer be discerned. (Bottom left-to-right) As the spatial resolution of the image decreases, then the image contrast becomes worse, particularly for small objects within the body.*

We have the original image on the bottom left, with two zones of different density. Going toward the right in the bottom set of images we go to system with a worst spatial resolution. When the spatial resolution is too bad, we cannot distinguish anymore the small object, because it is smashed all over the background. So, a very bad spatial resolution doesn't allow to detect contrast well, at least for very small objects.
On the top we have a system with always a perfect resolution, the resolution is kept the same from the left to the right (the borders are as good as in the original object). However, the statistics is worsening, the image starts to have lower statistics. Hence the counting of the pixels is no more the same, even in uniform pixels (middle). At a given point the statistics is so low that we are no more able to distinguish the object.

In conclusion, statistic is as important as resolution, and in the trade off between spatial resolution and statistics we have to find a good compromise.

RADIATIONS IN RADIOGRAPHY, SPECT AND PET



These techniques have in common the usage of x and gamma rays. The x and gamma rays are electromagnetic waves, so they belong to the electromagnetic spectrum, and are the more energetic region of the electromagnetic waves.
In particular, as we can see in the pic, the distinction between x and gamma is not a matter of energy, because we can have an energy where we have both x rays and gamma rays. To distinguish between them, we consider the origin. **Xrays are associated to phenomena involving atoms, they are based on emission from atom, while gamma rays are based on emission from nuclei or to annihilation of positron and electrons**. Hence the distinction is not the energy range.

Moreover, xrays are most present in the low energy range, they may go from 10keV to hundreds of keV, while gamma rays are more pushed to higher energies. But we have also a range where they are in common. In green we have the energy range in medical imaging.

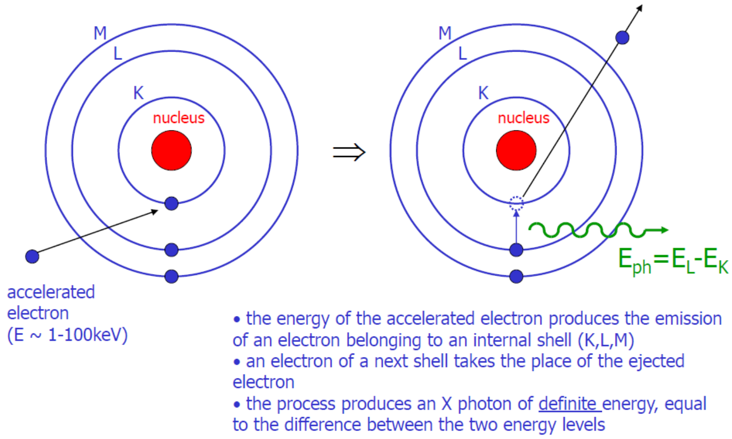
Xrays
- Energy: 10 eV → 1keV → 100-300 keV
- Origin: fluorescence from atoms, Bremsstrahlung
- Applications in medical diagnostics: radiography (conventional, mammography, densitometry, …, CT)

Gamma rays
- Energy: 10 keV → 100keV → 10MeV
- Origin: nuclear emission, annihilation of positrons
- Applications in medical diagnostics: SPECT, PET

ORIGIN OF X-RAYS
There are 3 main phenomena for the origin of x-rays.

1. Fluorescence
It takes place in atoms. In the image we have the Bohr atom representation with electrons in fixed energy orbitals; when we accelerate an electron as in the xray tube, the electron accumulates kinetic energy (from 1 to 100keV), and there is a given probability that the electron is able to kick out the electron staying in one of the three orbitals of the atom.

This contact between electrons doesn't leave the atom in an equilibrium state, because we have an empty slot in the low energy orbital (from where the other electro has been spilled off), and it is possible that the slot is filled by an electron belonging to the second or third nearest orbital.



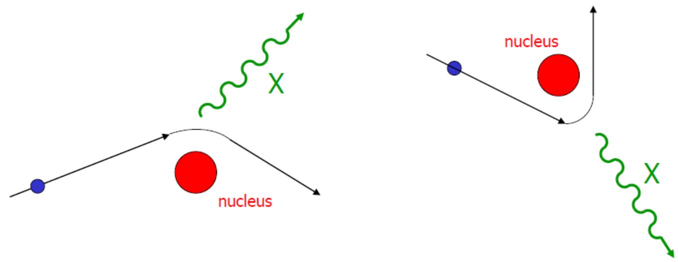
The most probable is the recovering by the nearest orbital.
Since the electron filling the position has an higher energy (because it is further from the nucleus), the final energy state for the electron is at lower energy → some energy must be released in the universe.
One mechanism (not the unique one) that may happen is the emission of xray by fluorescence. It is emitted by the recovering of the slot by an electron in the outer orbitals. This xray has a precise and finite value, that is the energy of the level to which the electron was previously belonging minus the energy of the new level (green formula). This energy difference is positive.
Moreover, we have that also the electron in the third level will move to the second one to fill the slot and so the emission of another photon → we may have a cascade of effects.
However, this outcome Eph, called radiative outcome, so the relaxation of the atom by emission of photons, is not only the possible output. We may have also other non-radiative effect in place of the emission of the photon.
This means also that the cascade of the atom to get equilibrium will always lead to the emission of xrays, there are also other competing effect, like the Oegeau effect.

Of course, we may also have that an electron from a further level would go in an inner slot. In this case the energy of the emitted photon will be larger.

This effect was also called photoelectric effect and it was firstly proposed by Einstein in 1905 and the description of this phenomenon rewarded Einstein with the Nobel price.

While fluorescence is a discrete energy emission modality because the xrays depend on discretized energy levels, in the following Bremsstrahlung we have infinite ways to have possible bendings (from a quasi-zero bending to a very severe one and a total crash).

2. Bremsstrahlung



• the Bremsstrahlung radiation is produced when an electron changes its velocity following the Coulombian interaction with the atomic nucleus
• the emission is larger more energetic is the electron and larger is Z of the absorbing material
• the energy emitted as X rays has a continuous spectrum from 0 to Emax (the kinetic energy of the incoming electron, in the case it is fully absorbed in the interaction)

Bremsstrahlung is a german word that in English corresponds to 'break and loosing velocity'.
This phenomenon occurs when and electron is travelling in the proximity of a nucleus. Due to the charge attraction, the electron changes direction → it is a coulombian effect between a fast-travelling charge and a resting nucleus.
As the electron is bending, so changing trajectory, the deceleration (the electron is changing direction and loosing velocity) implies emission of energy in some ways, and the way is xrays.
Hence we have an emission of xrays when an electron looses energy passing close to a nucleus.

However, we may have different situations, depending on the trajectory. On the left we have a soft bending, on the right we have a hard bending, because the electron is passing particularly close to the nucleus. This means that differently from fluorescence where the emission energy is discretized, here we have a 'continuous' emission of energy, because we may have for instance a very very soft bending because the electron is very far away from the nucleus and so the emission of xray will have a very small energy (left), or a complete opposite situation in which we have a crash, the electron is not even bending, but crashing in the nucleus.
In the former case the energy of the xray will be close to 0 because the bending is very small, in the latter case the energy of the xray will be equal to the kinetic energy of the electron, because it has lost his kinetic energy because it has been completely stopped.

Spectrum of the x-ray tube

It is a histogram of all possible xray energies as a function of the energy, so the number of photons of a given energy. We can see that we have a continuous spectrum, from 0 to 150keV. This because the Bremsstrahlung is producing photons from 0 to total crash. Superposed to this continuoum we have discrete lines, and these lines are the lines due to florescence.
Hence in the xray tube we have fluorescence lines plus bremsstrahlung and in bremsstrahlung we have energies not ranging from 0 to infinite, but up to the kinetic energy of the electron.

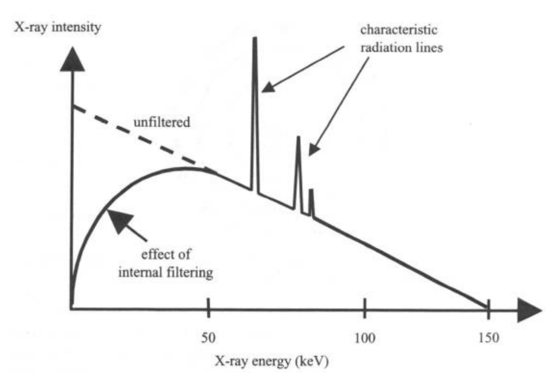
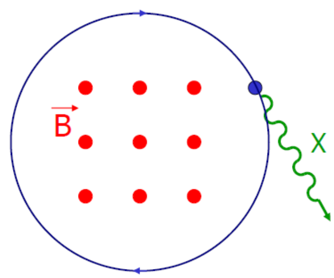


FIGURE 1.5. A typical X-ray energy spectrum produced from a tube with a kV_p value of 150 keV, using a tungsten anode. Low-energy X-rays (dashed line) are absorbed by the components of the X-ray tube itself. Characteristic radiation lines from the anode occur at approximately 60 and 70 keV.

So in the Bremsstrahlung we have a continuous spectrum of xrays ranging from 0 (no bending at all) to the maximum energy of the electron in the field.

3. Synchrotron light



an electron forced to run a circular trajectory loses energy in the form of X rays (synchrotron radiation)

In summary:

an e- <u>decelerated</u> loses part of its energy in the form of X rays in the following ways:

1) the deceleration changes its <u>tangential</u> velocity → bremsstrahlung
2) the deceleration changes its <u>normal</u> velocity → synchrotron light

Mechanism generated only in specific accelerators called synchrotrons. Although it is not used for medical imaging like in radiography, they have the property that can be monochromatized, they can be generated with a very precise energy, that is not the case of Bremsstrahlung.

The synchrotron light it is a complex mechanism. We have an electron travelling in a region where a magnetic field is applied (red sposts) directed exiting from the surface (in the image) and because of the negative charge of the electron, by using the Lorentz force and the 3 fingers rule, we have that the electron is attracted to the center of the circle and so it is experiencing a force that keeps it to the circular trajectory. Since the electron is changing velocity, not in term of tangential speed but in term of velocity vector, because the vector is bending, this phenomenon is also determining an emission of energy in the form of xrays.

Hence when an electron is travelling in a magnetic field, it changes trajectory at the expense of the emission of energy in terms of xrays.
If an electron is moving along a trajectory and it is loosing energy, it should not follow anymore a perfect circular trajectory because it is loosing momentum → the trajectory should be a spiraled one.

58.

Inside the magnetic field, the electron bended is releasing xrays but loosing energy, so if we do nothing the trajectory will be spiral as long as the final energy will be zero. In the real case we have along the trajectory some stages, some linear stages which basically have a linear electric field which restores the kinetic energy of the electron. So if the electron looses energy by means of an xray, then while entering into the stage 1 it is accelerated again so that it recovers the lost energy. Again for stage 2 and so on. Thus the electrons can stay in the ring for hours, they continuously irradiate, they have a time constant of days before they stop to irradiate, because the energy of the irradiation is restored by the linear acceleration stages.

It is a technique not used a lot in medicine, but there are techniques in which it is used for medical imaging.

GENERATION OF XRAYS: XRAY TUBE

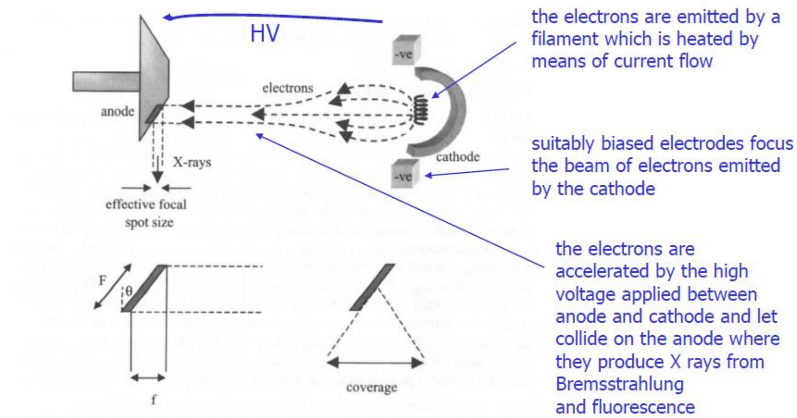


FIGURE 1.4. *(Top) A negatively charged focusing cup within the X-ray cathode produces a tightly focused beam of electrons and increases the electron flux striking the tungsten anode. (Bottom) The effect of the anode bevel angle θ on the effective focal spot size f and the X-ray coverage.*

Main generator of xrays. It is exploiting the phenomena 1 and 2; both phenomena are based on an energetic electron, so we need first to accelerate an electron and this is done by the cathode of the xray tube where there is a circuit which provides current in the order of the ampere to a filament (a filament is a wire which is heated like in lamps). So there is a circuit which heats a filament, the electrons in the filament are emitted by the heat and we have a region with vacuum where we have only electrons emitted by the filament. Then there are some electrodes used to confine the electrons emitted by the filament on a more precise spot and in the second part of the xray tube we have the anode. It is a piece of metal, typically tungsten, and the characteristic of the anode is that it is placed at very high voltage differences form the cathode. So between anode and cathode we have a high positive voltage difference and the electrons emitted by the filament are accelerated within the vacuum chamber and hitting the anode.
When they hit the anode they produce xrays by means of florescence and bremsstrahlung. The emission of xrays is in principle isotropic, but since we are interested in a given direction, the spot of origin of the xray can be considered with a dimension f that is given by the width of the electron hitting the anode. Theoretically the xray tube should be considered as a kind of pointlike source, but in reality it is no, since it has a given dimension.

The energy of the electrons hitting the anode is given by the definition of electron-volt. 1eV is the energy acquired by one electron travelling across a voltage difference of 1V. By definition, if now the voltage difference is now HV (High Voltage), that is for instance 100kV, it means that the electron will acquire a kinetic energy of 100keV. So it is very easy the range of the emitted xrays, it will be from 0 to 100keV.

In fact, if we look to a typical spectrum of an xray tube, if I would have used a tube with 150kV of voltage difference between cathode and anode, this means that the maximum energy of the bremsstrahlung xray will be 150keV.

Hence the operator of the xray tube can change the range of the emitted xrays just by changing tha voltage of the xray tube.
In principle we have a distribution from 0 to 150keV and superposed to this continuum that is the bremsstrahlung we have the fluorescence characteristic xrays (peaks, discretized xrays). The latter are more than one single energy, because we have more than one possibility of recovering of the empty places. For instance, the more intense peak (most probable phenomenon) is referred to an electron moving from the next band to the vacancy in the one below; however, we may have also that other electrons from outer levels would go to the vacancy (peak at larger energy, but with less probability).

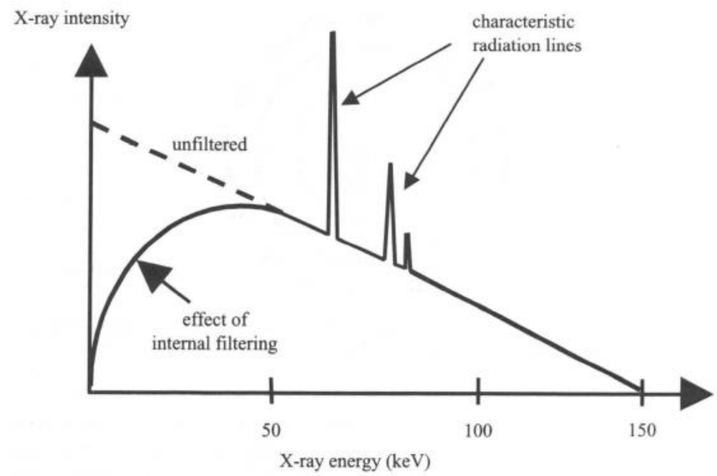


FIGURE 1.5. *A typical X-ray energy spectrum produced from a tube with a kV_p value of 150 keV, using a tungsten anode. Low-energy X-rays (dashed line) are absorbed by the components of the X-ray tube itself. Characteristic radiation lines from the anode occur at approximately 60 and 70 keV.*

So in the real xray spectrum (continuous line, not dashed) there is a drop of xrays at low energies. This drop can be understood if we look at the real drawing of an xray tube.

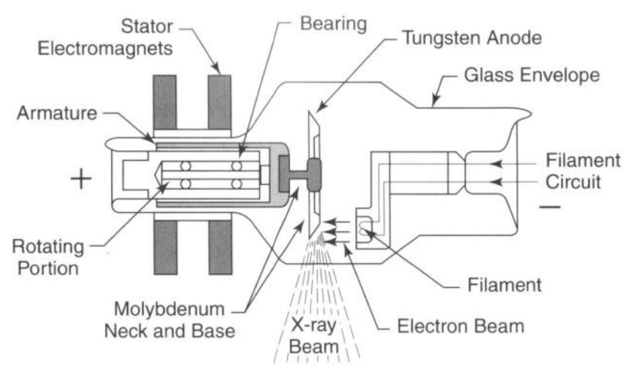


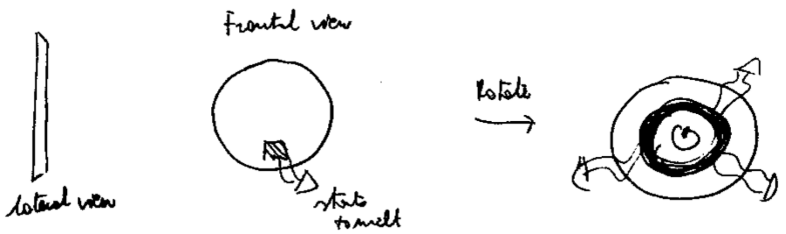
FIGURE 5-1
Simplified x-ray tube with a rotating anode and a heated filament.

We can see the filament and its circuit (heating and emission of xrays), then we have the anode at high voltage and the emission of the beam. Everything is enclosed in vacuum (glass envelope) and all the surface is covered by a protective element so that the irradiation emitted everywhere is shielded, so that the xray tube is not irradiating the operator, for instance.
We have then an exiting window, that is not an opening in the glass, or otherwise we would break the vacuum, but it is an opening in the shielding.
Since we have a layer of glass, **some radiation is absorbed by the glass envelope**, and this is the reason why we have the absorption of radiation in the spectrum. The absorption is not only in the glass, but also in the air. 1 m of air in between the source and the patient can absorb a significant amount of radiations.

Moreover, the anode is not a static piece of metal, but a rotating disc. It is rotating thanks to an electrical motor. This is done in order not to hit with the electrons the metal always at the same point.

If we hit the metal always at the same point, the metal gets hot and can melt. This because xray emission is not the unique product of the hit of the electrons, but a lot of the crash energy is converted in heat (increase in temperature). So we are heating up the anode.

59.



If we are rotating the anode, we heat a coronal region and not always the same piece of anode. In this way the dispersion of heat is better. The energy lost in heat is the complete kinetic energy of the electron; so the energy is either lost in heat or in xrays. Unfortunately, the heat lost is much more probable than the xray generation. → xray emission is a very inefficient phenomenon.

Coming back to the spectrum, it is very broad. What is not nice of such a broad spectrum is that the absorption properties of the body are energy-dependent. A tissue absorbs selectivity one energy of the xray differently from the other. The operator can just tune the end point, the highest energy of the xray, but not the internal ones. Hence it is a quite broad spectrum for irradiation.

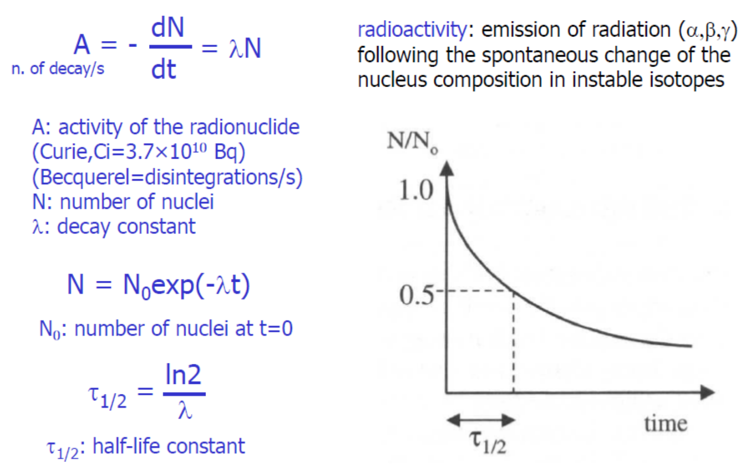
ORIGIN OF GAMMA-RAYS

They are originated by the nuclei, they are following a reaction involving instable nuclei, or nuclei of instable isotopes.

An instable isotope is an isotope that soon or later will change the composition of its nucleus. Most of the isotopes are stable in nature, but some of them can break themselves and in this modification of the nucleus composition they can emit radiation, and in this case we talk about radioactivity.

So the radioactivity is a very general phenomenon occurring in instable isotopes producing 3 types of radiations: alpha, beta and gamma.

Alpha is a nucleus of He (two protons and two neutrons), beta is an electron (generated inside the nucleus, so we distinguish it from the ones travelling across the nucleus) and then gamma.



The rate of emission of such radiation is, in radioactivity, called activity (A) and it is regulated by the equation above.
We have a number N of isotopes, a number N of nuclei. The activity is given by minus the derivative of the number of nuclei in time. This is obvious because if the instable isotopes change, they disappear. So the derivative is a negative number, because the isotopes are transformed in something else, they are no more themselves after a radioactive emission. Hence with a minus in front the term gets positive and it is the number of emission.

Of course the number of emission is equal to the rate of disappearance of the isotopes.
How many decays I have per second? As many as the number of isotopes that disappear per second.

The second part of the formula is that the activity, so the number of emission of radiation is proportional to the number of isotopes themselves by a proportionality factor lambda. So the number of emissions is proportional to the number of atoms.

Half life constant
If we integrate the differential equation we get an exponential formula. **The number of remaining atoms over time is obtained by integrating the equation**. It is given by the initial number of atoms N0 at time t0 multiplied by an exponential decay. The larger lambda, the smaller the time constant in the exponential decay.
In the radioactivity field, more than specify a tau for the decay, it is specified a constant (that is still proportional to the tau by a logarithm of 2, ln(2)) that is called half life constant. It is the tau multiplied by ln(2).
Its physical meaning is that it is the time that passes before the radioactive material has reduced by a factor of 2. After a half life constant, the radioactive material has been divided by 2. Then after another half time constant there is another cut by 2. Hence basically the radioactive isotopes reduce with a power of 2 of the half-life constant.

This is important in the case of nuclear accident, because there are some radioactive isotopes whose half life constant is of seconds or minutes (and after 1 week most of them are gone), but also others in the other of 10 years or even more → the remaining isotopes will half by a factor of 2 only after 10 or 20 years, and this is why contamination is difficult to be extinguished, because we need to wait that all the relevant isotopes has been reduced by waiting several time their half life time.

In medicine, the most common gamma emitter is Technetium 99, used in SPECT or scintigraphy. It is originated by the decay of a parent element, Molibdeno 99 (99 is the atomic mass, sum of protons and neutrons in the nucleus). Mo is an instable isotope with half life constant of 66 hours (if we take 1kg of Mo, after 66h we get 1/2 kg of Mo, because the remaining half kilo has become Tc 99).
Tc is a different element, with one proton more than Mo, but the atomic number is the same, because a proton has been transformed in a neutron. There is a change in the proton number but not in the atomic mass. In order to conserve the charge, we have one proton more but the emission of a beta particle.

Tc is unstable itself, but now with a half life constant of 6 hours and it emits a gamma ray. So, Tc is a radioactive isotope that produces gammas, that are the one used for diagnostic.
When in SPECT we bound a radioactive element to a molecule, this element is Tc. It is bounded to a molecule so that it emits gamma rays that are used for SPECT diagnostic.

These gamma rays have a precise energy that is 140keV.

One popular γ-ray emitter for medicine: Tecnetium 99
(used in 90% of the diagnostic analyses in nuclear medicine)

$$^{99}_{42}\mathrm{Mo} \xrightarrow{\tau_{1/2}\ 66\ \mathrm{h}} \beta + {}^{99\mathrm{m}}_{43}\mathrm{Tc} \xrightarrow{\tau_{1/2}\ 6\ \mathrm{h}} {}^{99\mathrm{g}}_{43}\mathrm{Tc} + \gamma$$

- ^{99m}Tc is metastable state ($\tau_{1/2}$ = 6h, useful for diagnostics)
- Energy γ = 140 keV (good energy for diagnostics)

Few general notes on radionuclides for diagnostics:

- $\tau_{1/2}$ too short $\Rightarrow$ short time between injection and diagnostics
- $\tau_{1/2}$ too long $\Rightarrow$ low activity, patient radioactive for too long
- Eγ too low $\Rightarrow$ few rays reach the detector
- Eγ too high $\Rightarrow$ patient 'transparent', difficult detection

So Tc is generated by Mo and then when injected in the body it emits gamma rays of 140keV collected and measured by the measuring apparatus.

Properties of a gamma emitter

The two properties that characterize a gamma emitter are the half life and the energy. This of the Tc is a particular case, we have several possible emitters. But the question is: is 6h good? Is 140keV good? If the half life is too short, for instance 10s, we don't have enough time between the injection of the radiotracer in the patient, the veiculation of it and the corresponding image recording. So the risk if the half life is too short is that when we take the image of the patient, the radiotracer has already gone.

Conversely, if the half life is too long, firstly the half life is proportional to lambda, so a tau too long means lambda too small which means activity too low, and we need too long to get an image. Moreover, another practical limitation is that the patient is radioactive for too long.
So 6h is good, it is an ideal number. In 6h we have sufficient time to record the image without that the radiotracer has vanished, and on the other hand it is not too long.

As for the energy, if the energy would be too small, the gamma rays could not escape from the body of the patient, they could be absorbed by the body and the external apparatus won't see anything. Instead, energy too high means that the patient is transparent, but also the detector has difficulties in stopping the gamma rays. So we need to find a good compromise, and 140keV can easily escape from the patient but can also be detected by the detector.

There are also other possible phenomena, called electron capture; we can have isotopes that capture an electron and they are transformed in stable isotopes that emits gamma rays. In the table we see other gamma

Other nuclides radioactive for electron capture
(capture by the nucleus of an electron from shell K or L, followed by an emission of a γ ray and possible X-ray fluorescence):

$$^{123}_{53}\mathrm{I} + {}^{0}_{-1}e \xrightarrow{13\,\mathrm{h}} {}^{123}_{52}\mathrm{Te} + \gamma \quad (159\ \mathrm{keV}) \qquad \boxed{e^- + p \rightarrow n}$$

$$^{201}_{81}\mathrm{Tl} + {}^{0}_{-1}e \xrightarrow{73\,\mathrm{h}} {}^{201}_{80}\mathrm{Hg} + \gamma \quad (167\ \mathrm{keV}) + \text{X-rays} \quad (68\text{–}82\ \mathrm{keV})$$

$$^{111}_{49}\mathrm{In} + {}^{0}_{-1}e \xrightarrow{67\,\mathrm{h}} {}^{111}_{48}\mathrm{Cd} + \gamma \quad (171\ \mathrm{keV})$$

TABLE 2.1. Properties of Common Radionuclides Used in Nuclear Medicine

Radionuclide	Half-life	γ-ray Energy (keV)
^{99m}Tc	6.02 h	140
^{67}Ga	3.2 d	93, 185, 300, 394
^{201}Tl	3.0 d	68–82 (X-rays)
^{133}Xe	5.3 d	81
^{111}In	2.8 d	171, 245
^{131}I	8 d	364
^{123}I	13 h	159

rays emitter, but they have different and longer half life. In fact, the duration of the emission is more a nasty problem than the energy.

Generation of Tc in the hospital

Generation of ^{99}Tc in hospital starting from ^{99}Mo

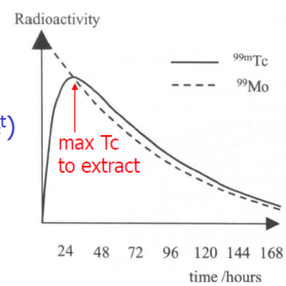
$$\frac{dN_{Mo}}{dt} = -\lambda_{Mo}N_{Mo} \qquad \frac{dN_{Tc}}{dt} = \lambda_{Mo}N_{Mo} - \lambda_{Tc}N_{Tc} \qquad A_{Tc} = +\lambda_{Tc}N_{Tc} \qquad \tau_{1/2\ Mo} = 66h \quad \tau_{1/2\ Tc} = 6h$$

$$N_{Mo}(t) = N_{Mo}(0)e^{-\lambda_{Mo}t} \qquad N_{Tc}(t) = N_{Mo}(0)\lambda_{Mo}/(\lambda_{Tc}-\lambda_{Mo})(e^{-\lambda_{Mo}t} - e^{-\lambda_{Tc}t})$$

$$A_{Mo} = -\frac{dN_{Mo}}{dt} = \lambda_{Mo}N_{Mo}(0)e^{-\lambda_{Mo}t}$$

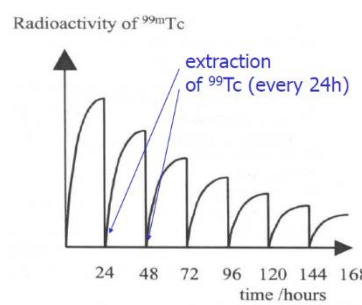
$$A_{Tc} = N_{Mo}(0)\lambda_{Mo}\underbrace{\lambda_{Tc}/(\lambda_{Tc}-\lambda_{Mo})}_{\sim 1}(e^{-\lambda_{Mo}t} - e^{-\lambda_{Tc}t})$$

per t >>0 $\quad A_{Tc} \sim A_{Mo}$



It is the application of the radioactivity formula. The decay of Mo follows the radioactive formula, the derivative of Mo is equal to minus lambda for Mo minus N for Mo.
Tc has two terms, because the derivative of Tc is proportional to the factor due to the decay of Tc, but there is also a positive factor given by the decay of Mo. When one Mo isotope decay, we have one atom of Tc more. One term is associated to the decay of Tc, the other to the decay of Mo.

In the end, supposing that at time t0 we don't have Tc at all, at the beginning we have a quick rise of Tc, with a half life of 6h, but then if we do nothing, the Tc will follow the decay curve of Mo, which have a decay life of 66h, because it becomes the dominating factor in the formula. Hence in the activity of Tc we have a fast and a low exponential.

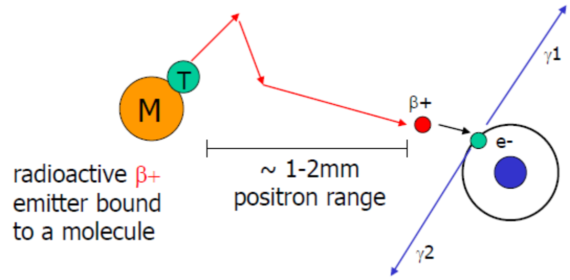


In hospitals, every Monday we have the new fresh Mo. So we start to produce Tc and when Tc reaches its maximum it is chemically separated from Mo. Then Tc rises again and we extract it and so on. So in hospitals every 24h the clinician extracts the Tc used for the diagnostic of the day, then he lets Tc grow again.
Of course, the Tc produced is less and less, because it follows the decay of Mo, that is of about 3 days.
Tc is used in SPECT

RADIONUCLIDES BETA+ EMITTER FOR PET

We need some isotopes also in PET, but in this case are beta plus emitter, so positron emitter. Then we inject it into the patient, it travels a bit (the travelled distance is called positron range) and then it annihilates with an electron and emits two gamma rays.

The figure in the next page explains one source of error, that is indeed the positron range. In fact, in PET reconstruction, we reconstruct the position of annihilation by detecting two gamma rays, not the position of emission of the positron (that is where the molecule is), because we have some distance between the origin of the positron and the annihilation. And this distance is an error that must be modelled.



1) β+ emission:

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + \beta^{+} + \upsilon$$

2) annihilation:

$$\beta^{+} + e^{-} \rightarrow \gamma + \gamma$$

β+ emitter isotopes typically used:
^{11}C, ^{15}O, ^{18}F, ^{13}N

Positron emitters are created by the decay of a parent atom. The parent atom decays in a Z-1 atom with the emission of a positron and a neutrino. Then in the annihilation, the positron is annihilated with an electron. The typical positron emitters are C11, O12, F18 and N13. (nu indicated the neutrino)

If we give a look at the half life (table), it is terribly short, in the order of minutes. This is the drawback of PET, the reason why not all hospitals have PET. Actually, with such short half life, we need to generate fresh positron emitter directly in the hospital. Hence most of the hospitals equipped with PET have an internal cyclotron.
The cyclotron is an accelerator that allows proton of deuterium to hit an atom and create the radioactive isotope that we inject in the patient very quickly.
In cyclotron we bombard an atom with a proton, the atom is transformed in unstable isotope and this latter is injected in the patient and we record the PET image in a very short time.

TABLE 2.3. Properties of the Most Common Radionuclides Used for PET

Radionuclide	Half-life (min)
^{11}C	20.4
^{15}O	2.07
^{13}N	9.96
^{18}F	109.7

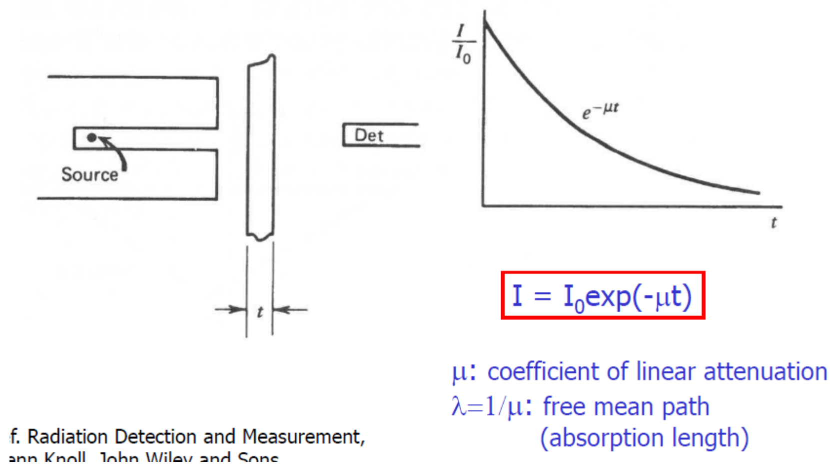
- β+ radionuclides are produced in a cyclotron (directly in hospital) by means of irradiation with protons (~10MeV) or deuterium (~5MeV)
- they are bound to the molecule by means of chemical synthesis

$$^{12}C \xrightarrow{\text{proton}} {}^{11}C + \text{proton} + \text{neutron}$$
$$^{14}N \xrightarrow{\text{proton}} {}^{11}C + {}^{4}_{2}He$$
$$^{16}O \xrightarrow{\text{proton}} {}^{11}N + {}^{4}_{2}He$$
$$^{13}C \xrightarrow{\text{proton}} {}^{13}N + \text{neutron}$$
$$^{15}N \xrightarrow{\text{proton}} {}^{15}O + \text{neutron}$$
$$^{14}N \xrightarrow{\text{deuterium}} {}^{15}O + \text{neutron}$$
$$^{18}O \xrightarrow[\cancel{\text{deuterium}}]{\text{proton}} {}^{18}F + \text{neutron}$$
$$^{20}Ne \xrightarrow{\text{deuterium}} {}^{18}F + {}^{4}_{2}He$$

INTERACTION OF X AND GAMMA RAYS WITH MATTER

There are two reason to be interested in the absorption of radiation in material:

1. **Shielding**: we need to protect against radiation, so we need to know for instance how thick must be a layer of material to protect from radiation.
2. **How effectively one material of a sensor can be sensitive and efficient to absorb gamma rays**. This is the topic of our interest, because we want to detect the gamma rays in a very efficient way, which means that most of the radiations is absorbed in our detector.



In the image we see the typical formula that describes the absorption of gamma rays in a slab of material. If we take a material of a given composition and thickness t and we have a collimated source of X and gamma rays and we count with a detector how many gamma rays pass through the material, we get an exponential behaviour like in the plot.
If I0 is the intensity of the beam of gamma rays before the absorption, the exiting intensity of the gamma rays recorded by the detector is given by the formula.

t is the thickness, so the formula tells us **that the thicker the material, more we exponentially absorb the radiations**, because less radiation I are exiting from the material. So I0 is the intensity of the source before any absorption in the slab, 'I' is the exiting intensity. Of course, I < I0 because the material has absorbed some radiations.
For instance, when we shield a source we usually choose a t so that the exiting intensity is reduced to a safe level.

However, each material is different one from the other. The physical nature of the material is embedded in the coefficient u, also called coefficient of linear attenuation. The larger u, the faster the exponential decay. For instance, lead (piombo) has a larger coefficient than wood. Lambda, that has a different meaning than before, is called free mean path. For a t = lambda, the I0 has reduced of 66%.

There are some cases where we may specify u with respect to the density ro of the material. So we ca have another definition, u' = u/ro, that is called coefficient of mass absorption. If we use u', the formula becomes as in the next page.
In most of the cases, we still use the formula above, because a single material has typically a constant density. ro matters in case of gasses.

$$\boxed{I = I_0 \exp(-\mu t)}$$

μ depends not only on the material composition but also on its density ρ

$\Rightarrow \mu' = \mu/\rho$ coefficient of mass absorption

$$\boxed{I = I_0 \exp(-\mu' \rho t)}$$

μ' depends on three absorption meccanisms:
1) photoelectric absorption
2) Compton absorption
3) production of e-/e+ pairs

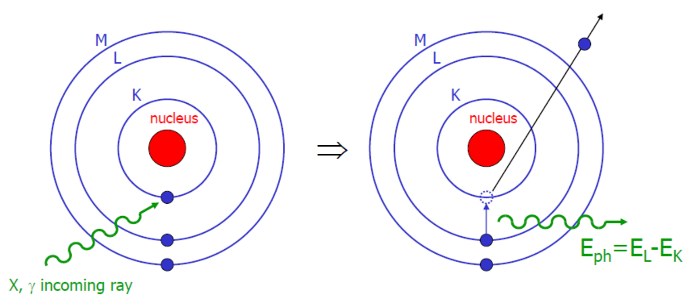
$$\boxed{\mu' = \mu'_{photoelectric} + \mu'_{Compton} + \mu'_{pair}}$$

Since the absorption has an exponential behaviour and u' is specifying the absorption of a material, why a slab of material should absorbe x and gamma rays? Because of the occurrence of 3 physical phenomena:

- Photoelectric absorption
- Compton absorption
- Production of e-/e+ (electron/positron) pairs

They are in principle simoultaneous, so that u' can be split in the sum of the three, but depending on the energy of x and gamma rays, one phenomenon will be more dominating with respect to the others.

Photoelectric absorption/effect



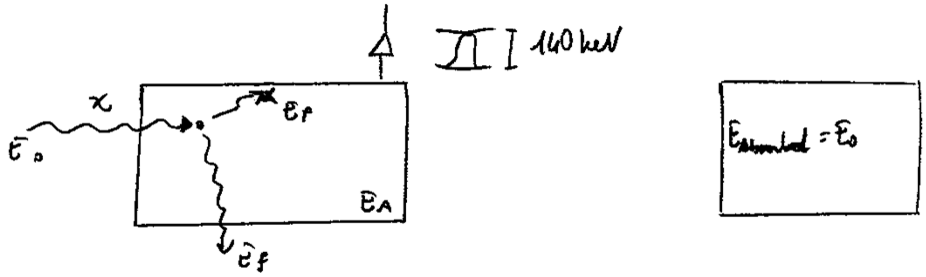
the incoming radiation succeeds to extract an electron belonging to an internal shell (K,L,M), an electron of the next shell fills the empty position and an X-ray is emitted

*already see as mechanism for the production of X rays

It is the photoelectric effect that we have seen for the xray tube, but this time the incoming particle is not the electron, but an x and gamma ray. If x and gamma ray have a sufficient energy, where sufficient means an energy larger than the binding energy of the electron in the orbital, the electron is kicked away and the photon is absorbed; eventually, we have the emission of a fluorescence xray.
This phenomenon must not be thought as a phenomenon generating xrays, but as a phenomenon responsible for the total absorption of xrays and gamma rays in the atom.

We are interested in looking at this phenomenon form a thermodynamic point of view. In this photoelectric effect, has the full energy of the incoming ray being absorbed? The answer is maybe.

60.



I have my material and a x ray of energy E0 has made an absorption by photoelectric effect. Has all the energy E0 been absorbed in the material? Yes, unless the fluorescence photon has exited the material. In the xray tube the exiting fluorescence was collected as xrays shining on the patient, but it may be that the fluorescence energy of the xray is not exiting the material, but it is fully absorbed in the material. <u>In case the energy of the fluorescence photon is not able to travel completely the material and to exit, I can conclude that the complete energy absorbed in the material is equal to the total energy of the xray photon entering in it</u>. If E0 enters and nothing exits, I have absorbed the complete energy in the material.
This is positive because if my material is a detector, a sensor, knowing that the total absorbed energy is equal to the total energy of the xray means that if, for instance, E0 = 140keV of Tc99, I can rely that my measurement of the energy by means of an amplifier (energy absorbed converted in charge and then amplified), the pulse will be proportional to 140keV.

So **if I have a total absorption of energy inside the detector and I get a signal out of it, I can trust that the amplitude of the signal is proportional to the entire energy of the entering photon (ideal detector)**.
Hence photoelectric effect is positive because if there is no fluorescence photon escaping (very rare event that a photon can escape from the detector, usually it is reabsorbed by the detector itself), from the thermodynamic point of view I can say that the total energy I measure in the detector is equal to the energy of the incoming ray.

from the energy point of view:

$$E_{e-} = h\upsilon - E_b$$

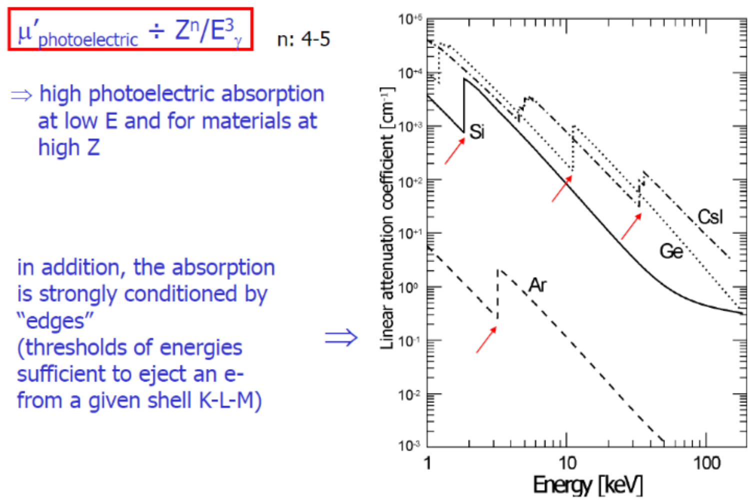
energy of photoelectron (e- emitted)

energy of incoming X ray

binding energy of the e- in the shell

- the energy of the photoelectron is the following absorbed by means of next ionizations and hits in the materials
- if the incoming X ray is sufficiently energetic and is absorbed in depth, the X fluorescence photon (which has low E) is re-absorbed in the material
  $\Rightarrow$ the whole energy of the incoming photon is absorbed in the material

All these phenomena have and energy dependent probability. If we plot the u’ coefficient (linear attenuation coefficient) vs energy of the incoming x or gamma ray, the coefficient is not constant at all, but it shows a **falling behaviour**. This is particular **severe for the photoelectric effect**.
In the photoelectric effect u’ drops with the cube of the gamma ray energy → the higher the energy the lower the probability of absorption.



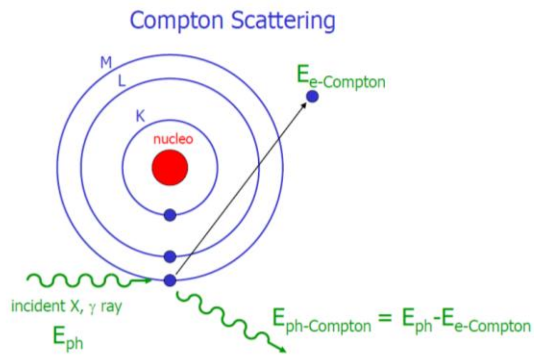
This seems a bit strange, because we are tempted to say that the more energetic the particle, the incoming ray, the higher the probability to kick out an electron from the atom. Unfortunately, this phenomenon (photoelectric absorption) is not described by classical physics but described by quantum physics.
It is more a **resonant phenomenon**; we have a resonance (e.g. in diapason) when the energy is not much smaller or larger than needed, but right equal to what is needed. So the probability to kick out the electron is when the incoming ray has exactly the binding energy of the electron, not less, not more. If the energy is the same, it resonates with the atom and the electron goes away. If the energy of the gamma ray is larger, the probability that the gamma ray kicks out the electron drops with a cubic power.

In addition to this, from the graph we can see that there are some edges (linee di incremento verticale), some energy where the probability steps up again and then drops again. These steps that we have for all the materials are corresponding to the opening of new absorption channels; it means that the energy of the gamma ray is sufficient to kick out an electron from a specific shell.
If we consider for example the more external electron, it is the one where we need the smaller energy to be kicked out. If the gamma ray reaches that energy, we have the opening of a channel for kicking out that electron. This corresponds to a given step in the plot, the energy is sufficient to let an electron to be ejected.
Then, when the energy increases, the probability to kick out the external electron drops with a power of three, but if the energy is sufficient, we have the energy to kick out the second outer electron.
For instance, for Germanium, we have a first step at low energy and then a new step at higher energy → the higher one corresponds to the kicking out of another electron. Then probability drops down again.

So all the edges (only one for Si and Ar) correspond to the capability or sufficient energy to kick out inner and inner electrons.

Finally, the probability (red box) is also dependent with a large power (4 or 5) with the atomic number. In fact, **to enhance the photoelectric absorption we need to choose materials with high atomic number**.

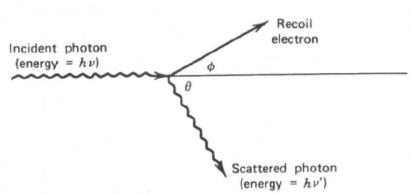
Compton Scattering



Compton scattering takes place when an incoming photon hits an electron weakly bound to the atom and produces a free electron and a photon deflected of lower energy given by the difference between the initial energy and the one of the free electron (note: also momentum is conserved)

It is still an interaction of the incoming ray with an electron belonging to one of the shells, but we have an electron kicked out (like in the photoelectric effect) but **the incoming photon is** not disappearing, but **scattered away**, the initial photon is scattered. The scattered photon of course holds a lower energy then the incoming one → higher wavelength, because part of the energy of the incoming photon has been given to the ejected electron, also called Compton electron.

So we have a classical hit that is described by the classical equation of the momentum.
We have the incoming photon with initial energy h*v (h is the Plank constant and v is the frequency), then after the hit, the electron is going in one direction and the scattered photon in the other (see image). The two directions are described by the angles phi and theta and the scattered photon has an energy h*v', where v' is the frequency of the scattered photon.



$$\boxed{h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)}}$$ x

Energy of emitted photon

$$\boxed{\frac{d\sigma}{d\Omega} = Zr_0^2\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right)}$$

fraction of emitted photons at a given angle θ (Klein-Nishina)

r_0: e- radius
$\alpha = h\upsilon / m_0 c^2$

$$\boxed{\mu'_{Compton} \div Z}$$

(the probability depends on the number of electrons available for the hits and is therefore proportional to Z)

If we use the conservation of the momentum (incoming one equal to the one of the exiting), we can compute the formula in the green upper box (x), that gives the energy of the emitted photon. The energy of the emitted photon depends on the energy of the incoming photon but also on the mass of electron, the speed of light and the angle of the scattered photon. The worst case is when the angle is of 180°, and we have backscattering; in this case the scattered photon holds the lowest possible energy. So in conclusion this formula describes the energy of the scattered photon with the dependency on the scattered angle.

The formula in the middle (not to be known) gives another information; the info is the probability that the scattered photon takes a given direction. Indeed, by the precious equation, we know which is the energy if the photon takes a given angle. But which is the probability that the photon takes one angle or another one? This is described by the Klein-Nishina formula.

The formula is represented in a polar system of coordinates. The polar system is represented with the angles and if we plot a function in the polar system, the distance between a point in the function and the origin (radius) is the independent variable (the dependent variable is the angle). **The radius represents the formula**.

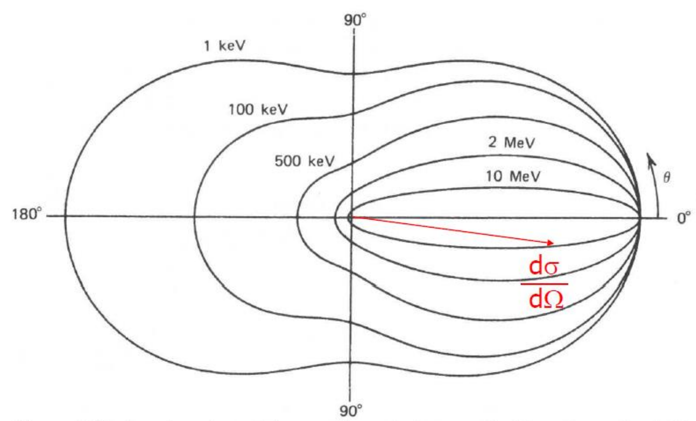


Figure 2.19 A polar plot of the number of photons (incident from the left) Compton scattered into a unit solid angle at the scattering angle θ. The curves are shown for the indicated initial energies.

If we look at the function for different incoming energies, at low incoming energies, hence for small energies of the incident photon, the KN formula provides to us an almost isotropical probability (almost because it has a beam-shape, we have a bit larger probability at 0° and 180° and a bit lower at +-90°). On the contrary, higher and higher the energy of the incoming photons, the probability is more peaked (larger values) for smaller angles. It is larger for angles around 0 while for backscattering the radius is smaller. Hence **the larger the photon energy, the more probable is that the scattered photon will continue towards the same direction**, hence smaller theta angles are the most probable exiting directions.

This information is useful because when we are dominated by Compton Scattering, the preferrable direction of exiting photon is the forward direction. So for instance, if we want to increase the efficiency of our material, we increase the thickness of it toward the direction of the incident photon (small theta) and not in the perpendicular one, because it is less probable that the electron takes this direction.

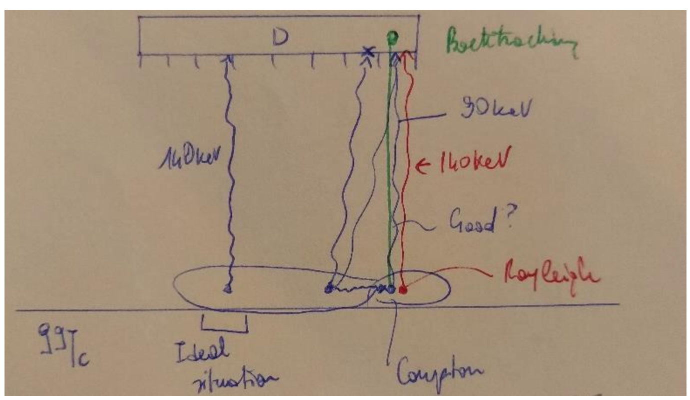
Also in Compton scattering we have a probability increasing with increasing atomic number (the higher the better) but the power of increase is smaller, not 4 or 5 like in photoelectric, but with power 1. If we increase Z we are hence increasing more the photoelectric absorption than the Compton Scattering.

The **C.S**. (Compton Scattering) **is also called anaenelastic scattering**, because it is a scattering, a hit, where part of the energy is given to an electron.
There is another type of scattering that is elastic, which means that the energy of the deflected photon is equal to the incident one, there is no release of energy elsewhere.

Elastic scattering (Rayleigh)
In this phenomenon the material is not absorbing any energy, and this is not a good phenomenon for a detector, because it will never absorb energy. It is also a hit between the incoming photon and an electron weakly bound to the atom.
Differently from the Compton scattering, the hit is elastic, i.e. the photon is deflected without energy transfer to the electron.
Consequently, Rayleigh scattering does not involve energy absorption by the matter but. However, it is important because it can change significantly the flight direction of the photon.
(ex. photons emitted by a region of the patient may be not collected by detectors placed along the flight direction as they are deviated; on the contrary, photons arriving from other regions can be deviated on the perpendicular direction of the detectors and they can be misunderstood as photons originated from a source aligned along this direction).

R.S. is pretty bad in nuclear medicine imaging because it doesn't change the energy.

61.



If we have a patient and we are doing for instance a SPECT, and we have a detector, and for instance the patient has been injected with Tc99 (140keV for photon). Normally, if we have the emission of a photon passing through the collimator, it is detected by the detector, the detector makes the backtracking of the hit and we can relate one point in the image to the corresponding emission. This is good.

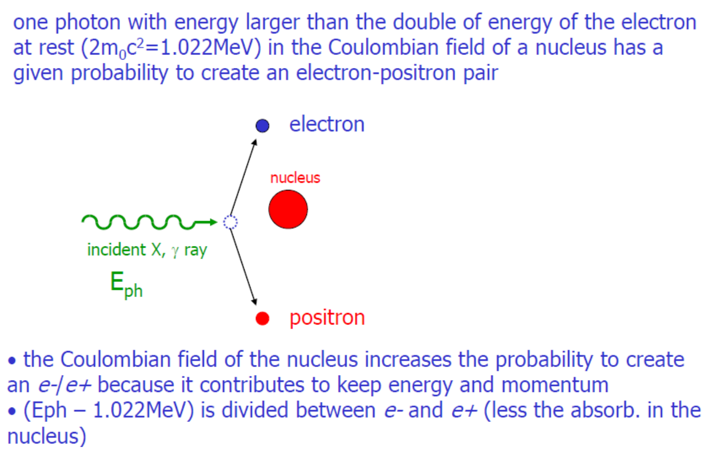
However, we have isotropical emission of gamma ray from the patient, not parallel to the collimator, but also tilted with respect to it. However, there are also a larger number of phenomena like the following: the 140keV is emitted from the patient, it experiences the Compton scattering in the patient, it takes a direction parallel to the collimator and in the detector this can be interpreted as a good event.

So there is a probability that a photon emitted in a direction that should be excluded by the detector, due to the CS, it may take a direction parallel to the collimator and seen as a good event
Since we don't know it comes from a CS, we do the backtracking and get a wrong information regarding the emission point.

To solve this problem, we relate to the fact that the scattering is indeed a Compton one, so the photon taking the direction theta doesn't hold anymore the 140keV energy, but e.g. 90keV, because it is a Compton photon, it has a lower energy. Hence by the prior knowledge that Tc emits 140keV and we measure 90keV, we can exclude the photon and the point in the image, and everything is good → the prior knowledge of the energy of the emitted photon allows us to exclude from the imaging system scattered phenomena, because the CS results in an exiting photon that is no more the initial energy of the Tc99.

The RS is very bad for us because if the scattering would have been done through an elastic scattering, the photon would have 140keV, because we don't lose energy in RS. → **we are taking a wrong point in the image**. When RS happens, we cannot discover it because it is an elastic scattering.

Generation of pairs



Third mechanism of absorption of gamma or X rays. It is the opposite phenomenon with respect to annihilation.
When a photon holds enough energy, it is able to create, close to a nucleus, a couple of electron and positron pairs and to disappear.
Can this phenomenon happen at all possible energies? No, there is a minimum energy below which it cannot happen. This energy is the one corresponding to the masses of electron and positron; if fact, by the Einstein equation E = mc^2, the creation in the universe of 2*mc^2 (where m is the mass of the electron) corresponds to 1.022MeV.
So if the incoming gamma ray doesn't hold such energy, it is impossible that the energy is transformed into twice m, because it is not sufficient. Hence it is a phenomenon that happens only at very high energy, that are not of interest for SPECT and PET, but in radioteraphy this energy could be exceeded.

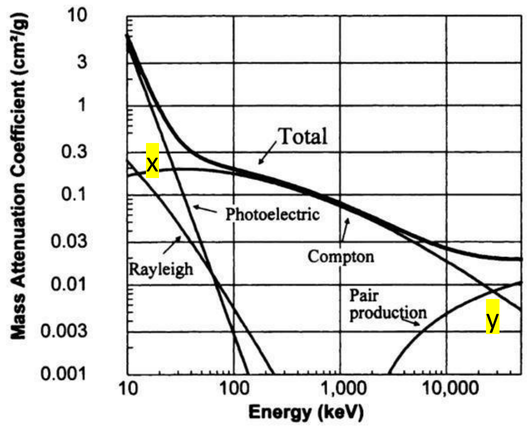
Moreover, **this phenomenon** cannot happen in vacuum, but **happens only if in the vicinity we have a nucleus**; this because we need to make the conservation of the momentum, and for the conservation of the momentum is easier if we have a nucleus because it participates to the conservation of the momentum.

CONCLUSIONS

We have the 4 phenomena (photoelectric, CS, RS, pairs production). How do these phenomena combine to generate the overall absorption?

In the overall attenuation coefficient u (eventually normalized with density, u'), the phenomena contributes with different weights depending on the energy of the gamma rays.

The photoelectric effect is important only at low energy, because it drops for high energies. CS exists in the wide energy range, although it is also dropping a bit at high energy.



Pair production starts at 1022keV (doesn't exist at all energies, but only when the gamma ray is above this energy) and then it becomes significant.

In the black heavy line, we have the superposition of all these effects, it is the overall mass attenuation coefficient u'. At low energy, the mass attenuation coefficient u' is dominated by photoelectric, in the medium energy range by Compton and in the high energy range by Pair production.

There are however different dependencies with Z (atomic number). The PE increases more with Z than the CS. In the plot below we have on the horizontal scale the energy, but in the vertical scale we have the Z of the material. The lines plotted represent the boundaries where the two effects (photoelectric and Compton) have the same probability. So the line in the left is the point x in the previous plot.

The other line is when Compton is equal to pair production (y). But these points are not the same for all energies for all Z. When the Z increases, the points are pushed towards higher energy in the case of photoelectric-Compton, to lower energy in the Compton-pair case.

It is like, if I increase Z, that the x moves to the right and the y to the left, both closer to the central region of the plot.

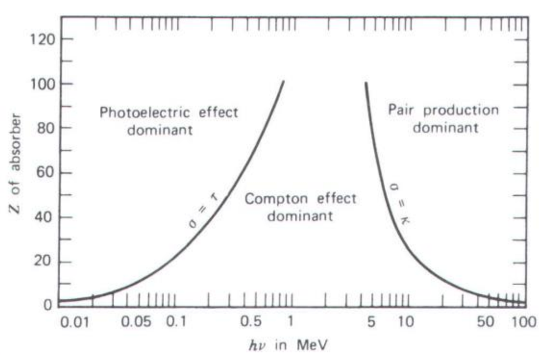


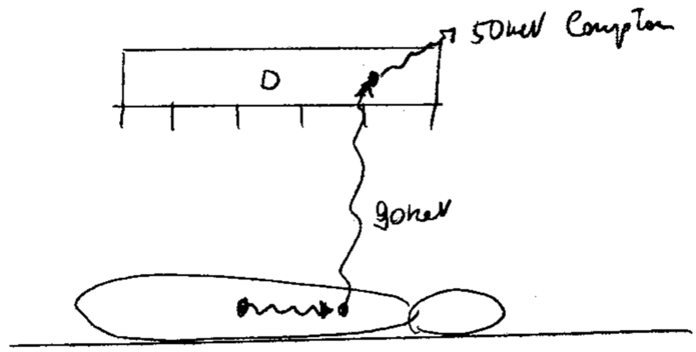
Figure 2.20 The relative importance of the three major types of gamma-ray interaction. The lines show the values of Z and $h\nu$ for which the two neighboring effects are just equal. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

This means that if we take for instance a specific energy, e.g. 140keV of Tc, if we have a low Z material Compton is dominating. But if we increase Z, photoelectric is dominating.
By choosing the Z of the absorber, for a given energy, we can choose if we want to have photoelectric or Compton dominating (fixed an energy).

Photoelectric dominating is good because in the photoelectric effect all the energy is absorbed in the material, with the exception of the fluorescence (that is likely to be reabsorbed). Hence if we choose photoelectric effect by increasing Z, we are getting all the energy on my detector.
On the contrary, if we are making Compton more favorable, part of the energy may escape from the detector due to this effect.

What's wrong with Compton with respect to photoelectric?
Let's consider the example in image 61. I can exclude Compton on the patient because I assume that my detector measures fully the energy of the incoming xray. So if I assume that 90keV are fully absorbed by the detector, I can say I measured 90keV and I can discard the photon.
But this is true only if the 90keV are absorbed by photoelectric effect in the detector (so all the 90keV are absorbed).
But let's suppose that instead of absorbing 90keV, part of them are leaving due to Compton (e.g. 50keV are leaving, 40keV are absorbed).

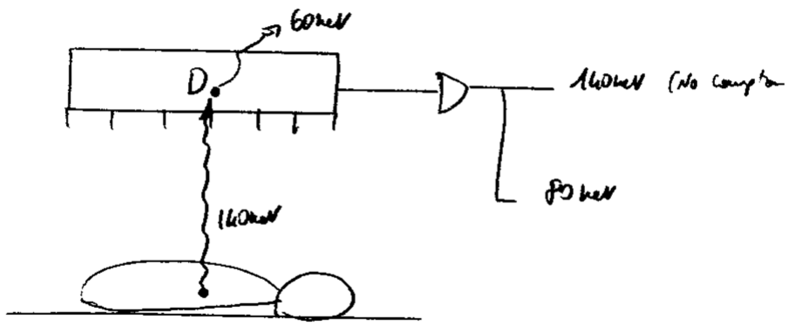
62.



In terms of decision I can perform the same decision, because 40keV are not 140keV. But in terms of precision of the measurement, I didn't measure the full energy, but only a part of it.

Let's suppose now we have a correct 140keV photon arriving in the detector and absorbed in it. If it is fully absorbed by the detector by photoelectric effect, we measure on the amplifier the full energy of 140keV, and we can say it is a good photon and make the back projection.
But if the 140keV photon is not absorbed by the photoelectric effect but makes a Compton interaction in the detector (now I'm talking about Compton interaction in the detector, not in the patient), this interaction leaves e.g. 80keV in the detector and 60keV are escaping.
In this case the detector measures 80keV, that are related (because of my interpretation) to a Compton scattering in the patient, so I discard the point. However, I discard it because it makes a Compton in the detector, not in the patient, and it was conversely a good event to be measured.

63.



Hence if I take a detector with **a large Z**, I may **maximize the probability that I'm measuring in the detector the 140keV and minimize the probability to have CS inside the detector**.

These graphs below represent the linear attenuation coefficient, but plotted in the range of radiography, that is focused on the low energy range, from 0 to 140keV. In these regions, photoelectric and CS are the dominant effects (left graph).
On the right, the graph is the same plot than on the left (however not on u but on u'), and we see the absorption of our body depending on the different types of material, and the materials are the tissues of our body.
The various tissues show a different u' one with respect to the other. For instance, u' of bone is larger than muscle and fat. This is indeed the principle of radiography. In fact, bones can be easily seen because they absorb more xrays, and less xrays are collected on the detector.
As for fat and muscles (soft tissues), the difference among them is very small → reason why it is difficult in radiography to discriminate between them.

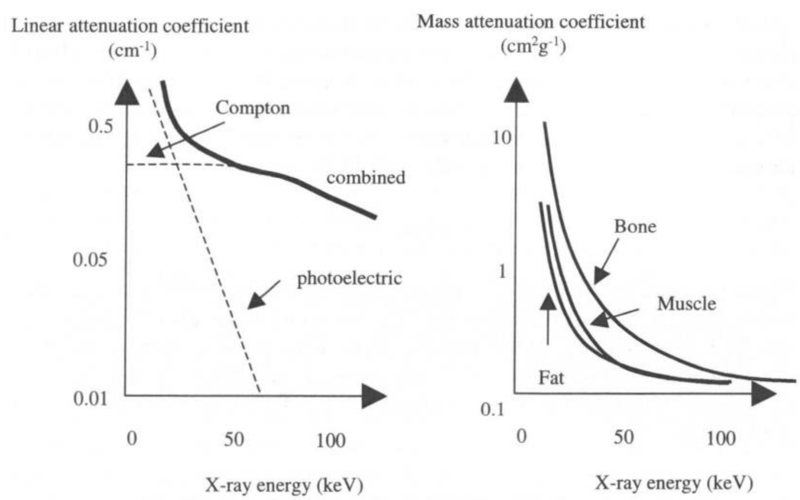


FIGURE 1.10. *(Left) The relative contributions from Compton scattering and photoelectric interactions to the linear attenuation coefficient in soft tissue as a function of the incident X-ray energy. The dashed lines represent straight-line approximations to the relative contributions, with the solid line representing actual experimental data corresponding to the sum of the contributions. (Right) The mass attenuation coefficient in bone, muscle, and fat as a function of X-ray energy.*

Note on Rayleigh
Rayleigh doesn't contribute to absorption, it may occur, but it is an elastic scattering, so no energy is absorbed. If we are interest to know about interactions in the patient, I'm interest to know Rayleigh

and it belongs to an absorption mechanism in the patient because it is able to deflect gamma rays. If we are looking to a detector, so seeing if gamma rays are interacting in the detector, Rayleigh is absolutely irrelevant, because won't leave any energy to the detector, so the output amplifier won't produce any signal occurring due to Rayleigh.

In conclusion, if we are looking to absorption in the patient, Rayleigh is important because it will deflect gamma rays; if we are talking about absorption in the detector, it is irrelevant.

FUNDAMENTALS ON X AND GAMMA RAYS DETECTOR

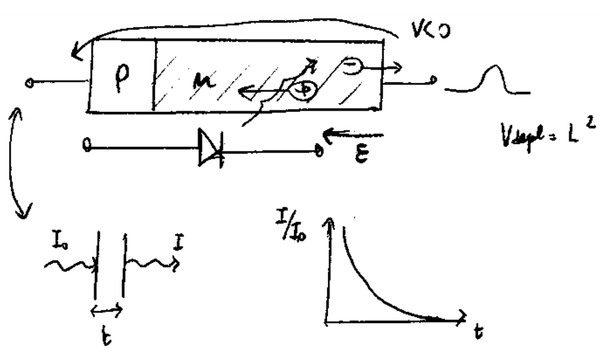
A detector is composed by a material that absorbs x and gamma radiations with the previous mechanisms. The task is to convert energy released by a photon in the material of the detector into an electrical signal. It is a material that absorbs energy and converts it.
Then we can process the electrical signal with suitable electronics, for instance to improve the SNR and then we can measure a number of quantities. In some modalities, we may be interest also to know the time of occurrence of the event, like in PET.
The electric signal is processed by a suitable electronics for the purpose to determine the energy of the photon, the interaction point(to the determine the origin of the emission) and, if required, the time instant in which the interaction has taken place.

There are 2 families of detectors:

- Detectors with direct conversion: detectors where the energy absorbed in the detector material is converted in an electrical signal directly in the material itself. These are ionizing detectors, where the energy is responsible for ionization of charges in the detector material and then the charges are separated and collected at a given electrode on the detector.
- Detectors with indirect conversion: typically, detectors based on a two steps process. In a first step, the energy of the photon is converted in another physical quantity (e.g. visible photons or temperature); the second step is another detector that converts the intermediate physical quantity into an electrical charge.

We don't use all the time direct conversion detectors because in indirect conversion detectors we disentangle the capability of the detector to absorb the x and gamma rays efficiently from the mechanism to generate charges. It doesn't happen always that having in the same material the conversion (direct detector) is the most efficient way.

64.



Let's take the classical pn junction, reversed biased diode. It is a beautiful detector, because when we reverse the bias on the pn junction we create a wide depleted region and, in this region, an xray may interact and we create couples of electrons and holes. This is a typical example of direct conversion detector; the energy of the xray has been converted directly in charge and the charge has been separated and collected and so we have an electrical pulse at the terminals of the diode.
However, the diode may be not really efficient, in the sense of capability to absorb rays. If the thickness of a detector is too small, the radiation has a larger probability to be transmitted across the detector and not being absorbed. So we need a large thickness.

But on one side we want to have an efficient detector with t sufficiently large, but on the other side, the pn diode cannot be as large as we like in some cases because there are limitations in the production of very large devices, and moreover we need to deplete the detector by applying a reversed voltage. We need to deplete because if we want to have signal charges created by the radiation, we need to create a region that is empty of mobile charges (this is why the diode as a sensor must be reversed biased and depleted, and by applying a reversed voltage we deplete it).

The reversed voltage goes with the square of the length of the device → if we double the thickness of a device, we need four time the voltage to deplete it. This provides a limitation

With direct conversion detectors like pin diodes, we are not always free to choose the thickness of the device, which makes the device efficient in terms of absorption of the radiation. A device may be nice in creating the charge but may have difficulties in thickness.

Scintillator

A typical example of indirect detector is instead a scintillator coupled with a photodiode. A scintillator is a passive material where the xray is converted in a flash of visible light. They are not converted in charge like in the diode; they are converted in visible photons. Then we have a photodetector (like pn diode) that converts each photon in an electron-hole pair and then we get the signal.
This is the reason why it is a two step process. First xrays to visible photons, then visible photons to electrical signal.

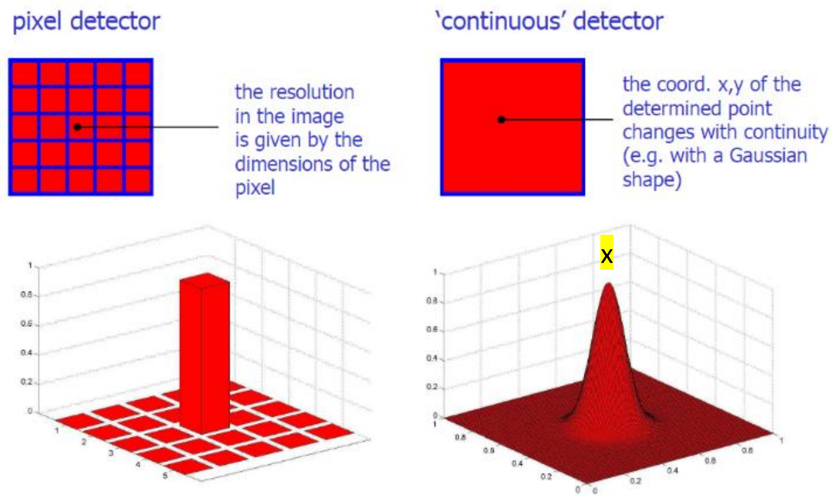
This system is potentially more effective than the direct approach because the thickness of the scintillator can be chosen freely, because it doesn't need to be biased like the diode, it is just a piece of material, so there is no limitation in the thickness of the scintillator. So we can choose 't' good for efficiency.
The problem of the photodiode here is no more present because the photodiode is not converting directly xrays, but visible photons. To convert a visible photon, a photodiode can be relatively thin → no problem for PD to convert visible photons.

However, we don't use indirect methods all the time because there are disadvantages; if fact, it is the conversion cascade of two mechanisms → statistical uncertainty of the two stages adds and so we have **worst performances of the indirect conversion**.

IMAGING DETECTORS

We are interested to imaging detectors, divided in two classes, either direct or indirect, it is a subdivision regarding how the image is recorded:

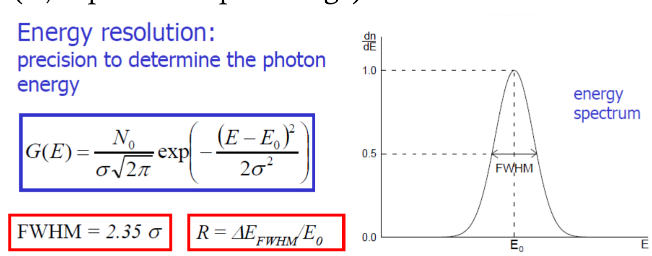
- Pixel detector: like the camera of the phone, the image is collected pixel by pixel. The resolution in the image is given by the dimensions of the pixel. The response is a pillbox function → wherever a photon arrives inside the pixel we have 100% probability that that pixel provides the signal and 0% on the other pixels.
- Continuous detector: the response is represented by a point spread function where we have a highest probability that a photon in point x is indeed attributed to that point, but there is a nonzero probability that a photon arriving in x is attributed elsewhere (due to the exponential decay of the response that is not present in pixel detectors).



Spatial resolution: it may be a pixelated one or a continuous one and the FWHM defines the spatial resolution of our detector. It is the precision to determine the position of interaction.

Energy resolution: precision to determine the photon energy. We are interested also to measure the energy. It is important to reject Compton events in the patient. The detector must be able to provide an energy spectrum (histogram of the energies measured vs energy); in principle we would like to have a delta response of the detector, but unfortunately, the energy response of the detector, like the PSF, it is represented by a spread function that can be approximated with a gaussian one. Also here, the FWHM of the energy response, related to the signa of the gaussian by 2.35, provides the precision of the detector when measuring the energy.

So one parameter is the FWHM, but in detector characterization is often given the ratio between the FWHM and the energy (R, expressed in percentage).



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Resolution for SPECT: $R = 10\%$.

$\frac{140\text{keV}}{^{99}\text{Tc}}$ → 14 keV of resolution (FWHM)

126 keV |——| 154 keV

For instance, the resolution of a typical gamma camera for SPECT is of 10%. 10% of resolution means that if we have 140keV of Tc, we have a resolution of 14keV at FWHM. This means that around 140keV, our detector capability to measure the energy can range from to 154keV to 126keV.

So if we have a detector like this (resolution of 10%), the 140keV can be measured with a precision ranging as in the image.
Since we can reject CS in the patient if we are not measuring precisely the 140keV, this can be a problem. A detector with an energy response with an error down to 126keV we can reject the Compton of 90keV, but conversely if the energy of the photon due to Compton has changed due to scattering from 140keV to 130keV, these 130keV are inside the resolution of the detector. So how can say that they are due to CS? We cannot, we are forced to accept also this value and we cannot reject the photon as a Compton event → **the worst the intrinsic resolution of the detector, the less we are able to make the Compton discrimination**, hence energy resolution is very important.

DETECTION EFFICIENCY – first figure of merit of a detector

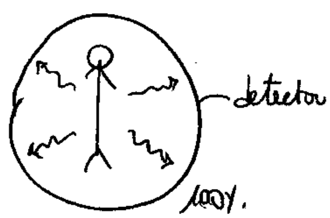
for a given source activity (photon flux), how many (valid) events are generated in the detector?

Detection efficiency = number of detected events/number of events generated by the source

Detection efficiency =
Geometrical efficiency × Absorption efficiency × 'Photopeak' efficiency

The first goal of a detector is to stop radiation, to absorb the radiation. The capability to do so is given by its detection efficiency.
For a given source activity, so given a photon flux by the patient, how many valid events are generated in the detector? What is the efficiency of the detector to absorb events generated by the patient?
The definition of the detection efficiency is given as the number of detected events divided by the number of events generated by the source. So it is in percentage, and in the better case it is 100%.

66.



Detection efficiency should be 100%. The perfect 100% efficient detector is a detector that is a complete sphere around the patient. In fact, if the patient emits 250phs/s, the detector measures 250phs/s. The most efficient detector detects the highest possible number of photons generated in the body of the patient.

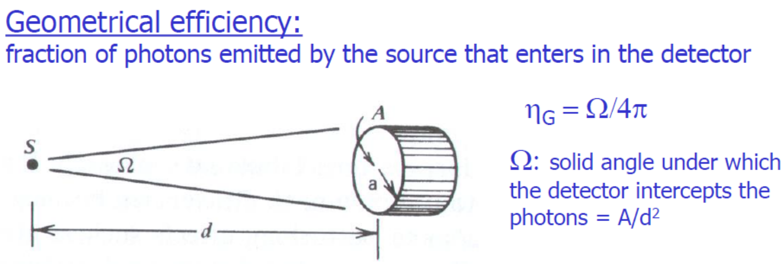
Unfortunately, we never have detectors 100% efficient. We have 10^-4 in SPECT, and 10^-2 in PET → efficiency dramatically far from ideality, because efficiency is given by three contributions (each far from 100%):

- Geometrical efficiency
- Absorption efficiency
- Photopeak efficiency

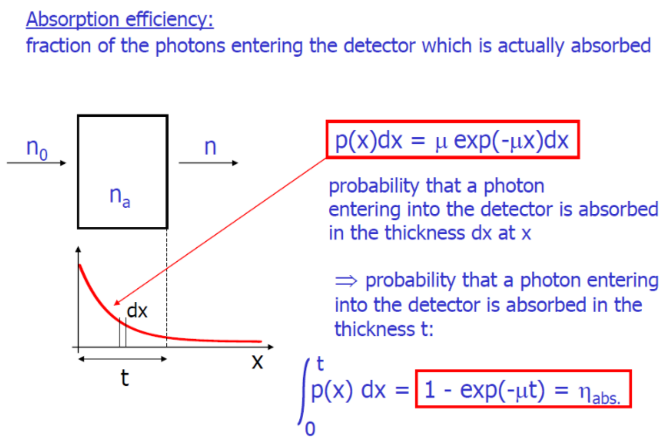
Geometrical efficiency

The patient is not surrounded by a sphere, because the detector has a limited surface. The geometrical efficiency is how many photons, emitted by the patient, are entering into the detector. It is just

geometry, the solid angle coverage (of a sphere it is 2*pi). In real detector we have a limited surface A at a distance d from the patient, so the solid angle is omega, and it can be approximated as A/d^2.
Once we have omega, the geometrical efficiency is omega/4*pi.
This is typically far from 100% because we simply cannot surround the patient with a sphere. To increase this efficiency, we can make the detector larger and place it closer to the patient. **The closer, the better**.



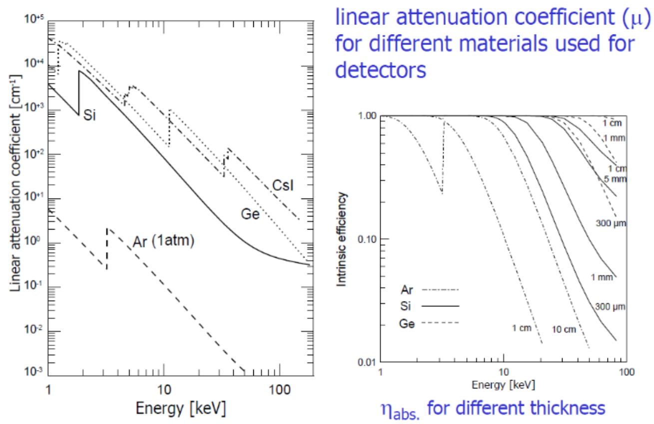
Absorption efficiency



The fact that gamma rays enter in the surface A doesn't mean that the gamma ray is absorbed in the material, because the material has a finite thickness t.
The absorption efficiency is the fraction of photons entering in the detector and not exiting, so being absorbed. The final formula for this efficiency (bottom red box) tells us that the absorption efficiency is 100% minus the probability that the photon is exiting, that is the exponential probability.
In fact, the probability that a photon is transmitted (passes through the material) is exp(-u*t).

We can arrive to this formula considering the probability that a photon entering in the detector is absorbed at the thickness delta(x)/x. And this probability p(x) is given by the formula in the upper red box. We integrate the probability from 0 to t and we get the absorption efficiency.
NB: we have a u in front of the formula of the probability because t theoretically can go to infinite and if so, the p(x) must be one → the u in front allows the formula to reach 1 if the t tends to infinite.

Whereas it is impossible to surround the patient with a sphere and so the geometrical efficiency is very low, it is not impossible to make the absorption efficiency close to 80-90%.
In fact, we can choose the right material for the detector, and hence the right u, and also the right thickness t.



These graphs show how important is the choice of the material. u must be as high as possible.; for instance, for Argon it is catastrophic, because u is very small. Silicon is high but not so high: it is a bad news because it is a beautiful material to manufact a detector, but at the same time not much efficient. Germanium is more efficient (in fact it has a larger Z than Si, and larger the Z, larger the efficiency). Cesium iodine (CsI) is a scintillator.

Conversely, on the right graph, once we have chosen u, we can see for each material the absorption efficiency with respect to the thickness. For instance, silicon has an efficiency that increases with the thickness. It is 1 at larger and larger energy range.
But 1cm of silicon drops inefficiency below 100keV → Si can still be good for radiography, but not for nuclear medicine.
Germanium, thanks to the larger Z, it is better. For instance, 1mm of Ge is much better than 1mm of Si → for the same thickness, Ge is more efficient.
Argon is a gas, hence rather catastrophic, but since it is a gas, it is much easier to build 10cm of a gas enclosure than 1cm of Si. So efficiency of Ar drops down with thickness, but on the other side it is easier to build a thicker detector.

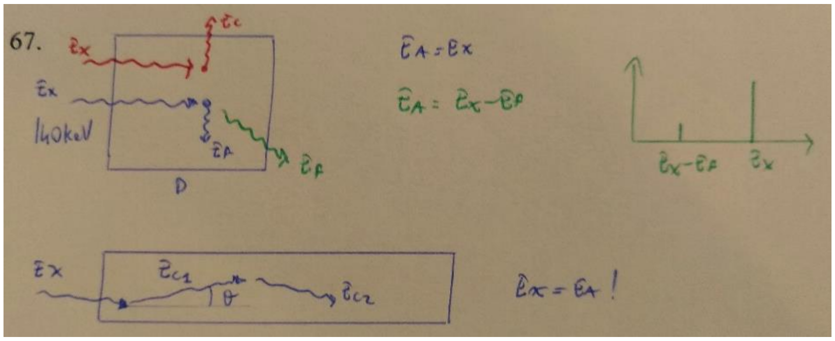
Photopeak efficiency

Fraction of photons that have interact in the detector and that have released completely their energy (and therefore provide a peak in the energy spectrum in correspondence of the energy of the incoming photon).
Photons absorbed by photoelectric effect release completely their energy in the detector (with the exception of those interactions in which the fluorescence photon of the material escapes from the detector: escapepeak).

The interactions occurring by Compton effect imply a partial absorption of the energy of the primary photon in case the secondary photon leaves the detector without further interactions.
To increase the efficiency it is necessary to size suitably the detector in order to maximize the probability that also the Compton photon is also absorbed in the material by photoelectric effect or by a second Compton interaction (note: Compton photons always have less energy than the primary photon).

The photopeak efficiency is an efficiency related to the fact that the photon not only has been absorbed, but it has also released its full energy. So we say that the photon has been fully absorbed only if we absorb fully the energy.

67.



If we have a detector, our problem for this type of efficiency is not the photoelectric effect, as long as also the fluorescence photon is absorbed in the material. If the 140keV is absorbed and also the fluorescence photon (Ef), I can say that the energy absorbed in the detector is equal to the one of the xray and so the photopeak efficiency is 100%.
However, I may have the case in which the fluorescence photon exits from the detector. The absorbed energy is the xray energy (Ex) minus the fluorescence photon energy. So in the energy spectrum we have a peak at Ex and a small one at Ex-Ef. This peak is called escape peak. It is the peak of energy absorbed in the detector if the fluorescence photon escapes from the detector.
Finally, we may have a much more catastrophic situation: a Compton interaction (red). In this case we cannot say we have a full efficiency.

So the photopeak efficiency is related to the capability of the detector to absorb the full energy of the incoming xray.
To build an efficient detector also for Compton interaction, we create a detector much more extended in the forward direction (according to the Klein Nishina formula). If Ex is the energy of the entering photon, Ec1 is the energy of the Compton 1 photon, Ec2 is the possible second interaction, they are all forward because the theta angle is the one more probable and, in the end, also the final Compton photon will be fully absorbed by photoelectric effect, because Compton photon loses energy interaction after interaction, so if they lower the energy, in the end there is a higher probability that they are absorbed by photoelectric effect.
In this case I can say that the total energy absorbed in the material is equal to the initial energy (Ex = Ea), because I designed the detector in a so good way that I could catch also the energies of the various Compton interactions (they are not escaping the material).

In conclusion, photons cannot release 100% of the energy when:

1. A Compton interaction occurs, so only a part of the energy is absorbed in the material because then the Compton photon escapes from the detector.
2. A fluorescence photon escapes after photoelectric effect. It is a very rare case.

In these two situations we don't have a full energy absorption → problems because we cannot determine the original energy of the incoming photon.

CONVERSION FROM ENERGY TO ELECTRICAL CHARGE – second figure of merit

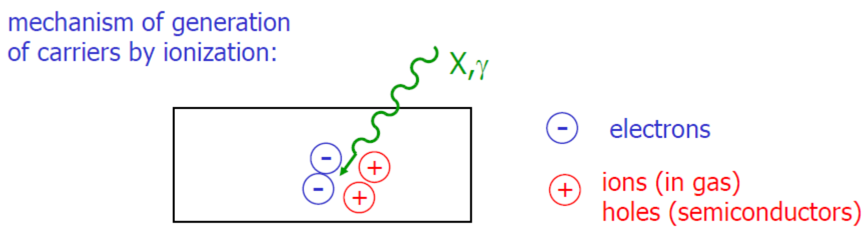
The first task of the detector is to collect photons, to stop them.

Then, the second figure of merit, if we are interested in collecting the charge, is how much charge is created by the energy. We are in the field of the class of ionizing detectors.

An ionizing detector is a detector where the energy absorbed in the material is converted into generation of electron-holes pairs in semiconductor or electron-ions in gas detector.

The figure of merit is: for a given energy, how much charge is created in the detector.

for a given energy released by a detected photon, how much charge is created in the detector?



Following the photon absorption in the material by one of the possible mechanisms (photoelectric, Compton, e-/e+ generation), the energy absorbed by the material causes the creation of electron-ion (hole) pairs. The creation of couples is related to a chain mechanism of further ionizations started by the first electron to which the energy has been released. The ionizations are due also to the re-absorption of possible fluorescence or Compton photons.

There is also the sensitivity, that is how much the energy is converted in electrical charges. We are interested in detectors that deliver an electrical charge.

Observations

1. The conversion of energy in charge is not the only possibility, we may convert energy in temperature rise (not all detectors are producing ionization, but we are interested in the ones that produce ionization).
2. Detection efficiency must not be confound with conversion in charge efficiency. They are two completely separated figure of merit. One is the ability to stop photons independently if then absorbing photon gives rise to a temperature rise or an ionization (I'm not looking at the product of the detection). In the energy-charge conversion I'm instead looking at the product. So absorption was already done and now the focus is how many electron-hole pairs have been produced by the energy.

Our interest is to have as many as possible charges for a given amount of energy. In fact, to have a larger signal against the statistical fluctuation, my goal is to have the largest possible amount of charges. This is given by the following formula.

the generated charge is proportional to the photon energy and this proportionality factor is given by the parameter at the denominator that is the conversion factor epsilon.

The charge Q created in the detector is given by the energy divided by the factor epsilon multiplied by the charge of the electron. The smaller epsilon, the better. In this way we can increase the charge and so the signal registered, hence improving the SNR.

The generated charge si proportional to the photon energy:

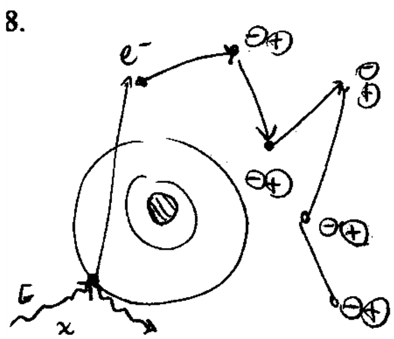
$$\boxed{Q = qE/\varepsilon} \qquad \text{(q: e- charge)}$$

the conversion factor ε is with good approx. independent from the energy and from the mechanism which has provided the absorption
(generated charge amount proportional only to the energy released to the electrons responsible for the ionization)

the conversion factor ε determines the sensitivity of the detector and depends strongly on the type of material used

Moreover, with a good approximation, the factor epsilon is independent from the mechanism that has provided the absorption. If the charge has been created by photoelectric effect or by Compton, no matter of the effect, the **epsilon factor is always the same**.

68.



$$Q = Zq = q\frac{E}{\varepsilon}$$

Basically, when a photon is absorbed either by photoelectric effect or Compton scattering, we have an ejected electron with a very high kinetic energy and this electron is going to hit several time across the material and in each hit it produces electron-hole pairs.
So we have an xray absorbed either by photoelectric effect or Compton and what is in common between photoelectric and Compton is that we have an ejected electron with high kinetic energy that will go around the crystal (or gas), will lose kinetic energy by ionization, I will count the number of charges created, Q will be the sum of all the created charges and it will be given by the previous formula.
Then, epsilon is independent from the fact that the first hit was made by photoelectric effect or Compton, because what matters are the ionizations → **epsilon doesn't depend on the starting physical phenomenon**.

Epsilon is defined as the energy spent per electron/ion or electron-hole pair. So energy E has the dimension of eV, epsilon has dimension of eV/pair.
Moreover, epsilon is an average value. It is the average value of energy to create pairs;

few examples:

Argon	$\varepsilon \sim 26$ eV for e-/ion pair
Silicon	$\varepsilon \sim 3.6$ eV for e-/h pair
Se-amorphous	$\varepsilon \sim 20$ eV for e-/h pair
CsI+PMT	$\varepsilon \sim 25$ eV for e-/h pair (ref. scintillator+photodiode: indirect conversion detector)

the energy E is creating several hits, and then epsilon is the average for each one. So if in the end I have that 140keV creates 1230 pairs, epsilon will be given by 140keV/1230pairs (energy of the incoming ray divided by the number of generated pairs).
As said before, the smaller epsilon, the better. Epsilon is not related only to create a single pair, but several pairs. It is an energetic electron created by photoelectric effect or Compton that moves around

the material and create 1000 ionization. Since it is an average number, it fluctuates, and so the number of charge Q.

For instance, Si is very good, Se-amorphous also, Argon is instead very bed (all the gasses are typically very inefficient).
In a scintillator-photodetector couple (last row) the epsilon is not so high, it is not a strictly speaking an ionizing detector, because at first we have the cascade of gamma rays transformed in scintillation light, and then the scintillation light is transformed in electron by the photodetector. However, we can anyway calculate also for the scintillator which is the average energy spent for the final number of electrons we get at the photodetector, and we get 25eV. It is not very efficient.

Once again, the advantage of indirect detector like scintillator is not in the gain, but in other reasons. Apparently, Si is a good detector, but it is inefficient in terms of detection efficiency, not in terms of this sensitivity epsilon (we can have a good sensitivity but collect few rays).

NOTES ON DETECTOR MATERIALS
Silicon is an optimum material but is efficient only up to 10 –30 keV because of limited thickness that can be depleted in practice.
Germanium has a worse technology and has to be cooled to reduce the dark current. Gas detectors are intrinsecally low efficient but they can be still used for X rays because they can be fabricated of large dimensions (even few tens of cm).
Scintillator materials, like CsI (see later), are not able to create charge by ionization but they are also used (indirect conversion) thanks to their high efficiency. There are semiconductor materials with high Z more efficient than Si (CdTe, HgI2, …), considered with interest in medical imaging. They work fine at room T because of the large energy gap. However, they suffer from charge trapping effects.

Si is not usable in nuclear medicine (we have 140keV of Tc99 for SPECT and 511keV for PET). For radiography is still valuable, but not for chest radiography, since we need usually 100-140keV. There is a special type of radiography that is mammography (radiography of the breast) in which, due to the soft tissue nature of breast, we usually don't exert 30keV → we can have radiographic systems based on Si, but otherwhere else we won't see Si used in nuclear medicine.

Germanium is more efficient than Si **thanks to its higher atomic number (higher Z)**. Unfortunately, the problem of Ge that makes it unusable in medical imaging is that it has too much dark current.

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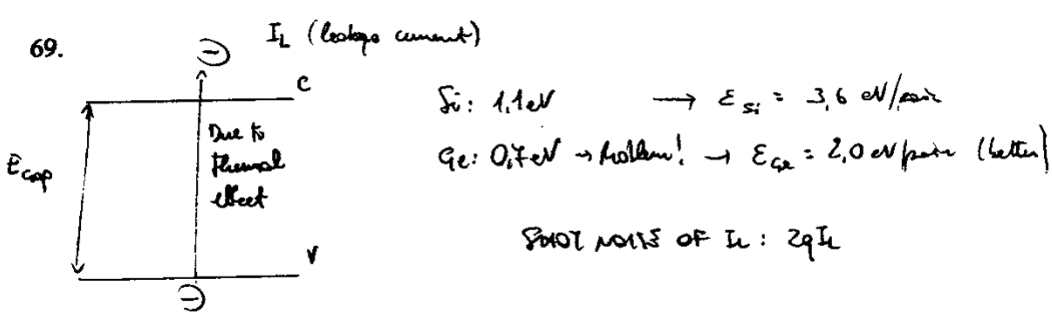


Semiconductor have a band gap called energy gap between the valence band and the conduction band. Usually the mobile charges are in the valence band and very few charges are in the conduction band,

unless we dope it. In undoped semiconductor for detectors, in ideal one, (where we want the volume without conduction charges because we would like to have the charges only created by the radiation, not already existing in the material) the conduction band is empty of mobile charges, so that the mobile charges are created by radiations. Unfortunately, **electrons of VB, due to thermal excitation, are promoted to CB**. This is a problem, because they can mix up with electrons created by radiation, and we want to prevent this promotion of electron due to thermal energy.
We have two tools:

1. Choose the semiconductor with a large energy gap. In fact, the promotion of electron is proportional to the exponential of the energy gap.
2. Cooling the detector. If we cool the detector the thermal energy of the particles decreases and we decrease the probability of promotion. Of course cooling complicates a lot the apparatus.

Let's consider Si, whose energy gap is of 1.1eV. It is quite good, the promotion of electron at room temperature is manageable. In Ge it is 0.7eV, so we have a problem. Ge material has such small energy gap that at room temperature is not usable, because we have so many promoted electrons already.

Of course, **the energy gap has implications also in the scattering**. When an electron goes around and creates the pairs, in Si the average epsilon is 3.6eV/pairs, in Ge is 2.0eV/pairs. We see that Ge has better ionization properties, because epsilon is a smaller number. We have better ionization because the gap is smaller → **if the gap is smaller is easier to have more ionization for the same energy**.

So the **energy gap enhance the ionization but at the same time it facilitates the thermal promotion**. This is a trade off we cannot play much around.
The average current Il that is measured due to electrons in the CB is called leakage current or dark current. It is the average current that we measure at the exit electrode of my detector due to electrons thermal promotion.
It is a problem because in principle, if we have for instance a radiographic system, which is a system where we measure the average current due to the photons (it is a static detector, a radiographic system measures the DC flux of photons), we can in principle measure the leakage current first, in a measure called dark current measure (dark means that we measure the current in dark current for the detector, the detector is not taking photons). Then we switch on the tube, we measure the signal current plus the leakage current (or dark current) and we make an algebraic subtraction.
So in order to get rid of the dark current we do two measurements: the signal plus the Il, then the Il only and we make the subtraction. Apparently we have solved the problem.

The problem is that we have noise on the current. This mechanism of thermal promotion is a statistical one, so we cannot rely on the value of Il measured. We have a shot noise associated to the dark current and we cannot subtract it, otherwise we would had the shot noise twice.
In conclusion, dark current is a relevant contribution to the noise in a detector, and we need to know it. We cannot work directly on the dark current, we can only limit it physically by cooling the detector (impractical due to the large sizes of the cryostat).

Gas detectors

Intrinsically low efficiency. However they can be used due to the size. The gasses are enclosed in chambers, so in principle is not difficult to build a big detector enclosing gasses. So the linear attenuation coefficient is very bad, but the thickness of the detector can be very favorable.

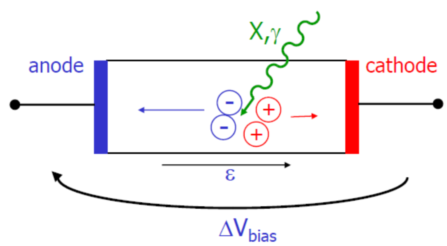
Scintillator

Very efficient because we can choose the material for the scintillator independently, because then the conversion in charge is made with a separated photodetector that has nothing to deal with the capability of stopping gamma rays.

Binary semiconductors

Nowadays there are a lot of systems using high Z material of compounds of semiconductor, like CdTe. There are semiconductors composed by binary elements and they have a very high Z, so can be used in medical imaging. However, since they are composed by two elements, in the material we have several defects. If we have a crystal where we need to have always a precise alternation of Cd and Te, time to time we have a defect in the crystal, a missing atom, and this creates sites where a charge can be trapped → response is not very clean because the signal collected depends on how much distance the charges have to travel.

PRINCIPLE OF A BASIC IONIZING DETECTOR



the electric field ε generated by the application of the potential difference is responsible for the separation of the e- from the positive charges (ions or holes) and it makes the e- drift toward the anode and the positive charges toward the cathode

An ionizing detector is a detector where the energy absorbed from the x or gamma rays creates several couples of charges. How are these charges collected to electrodes in the detectors in order to measure for instance a voltage difference?

The principle of a ionizing detector is the facing of two metal plates like in a capacitor with the volume between the two electrodes responsible for the conversion of the gamma rays into charges. It is like a capacitor where we apply a voltage between two electrodes. We apply voltage because we need to separate charges (first goal).

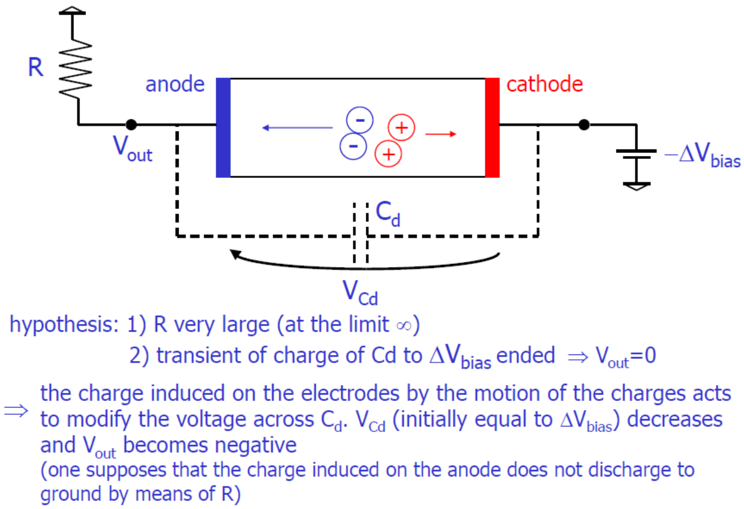
One xray creates an even amount of holes and pair and if we do nothing they recombine → we need to create an electrical field to separate charges. Then we separate charges and we measure ONE of the two charges in one of the two electrodes. We can measure the signal delivered by electrons or delivered by holes, they are identical.

The same principle applies for a gas detector, not with holes and electrons like in semiconductors but with ions and electrons.

How can I get a voltage out of such a detector? – RC NETWORK

We can create a connection to read the detector as below, that is the simplest one.

We connect one of the two electrodes to a bias voltage (e.g. the cathode to a negative bias voltage) and the anode to a large resistance. In DC this is an RC network; once we have distinguished the transient, so no more current flows in the network, we will have a positive voltage across the capacitor (Vbias) and we will have zero voltage drop on the resistor, so no more current.



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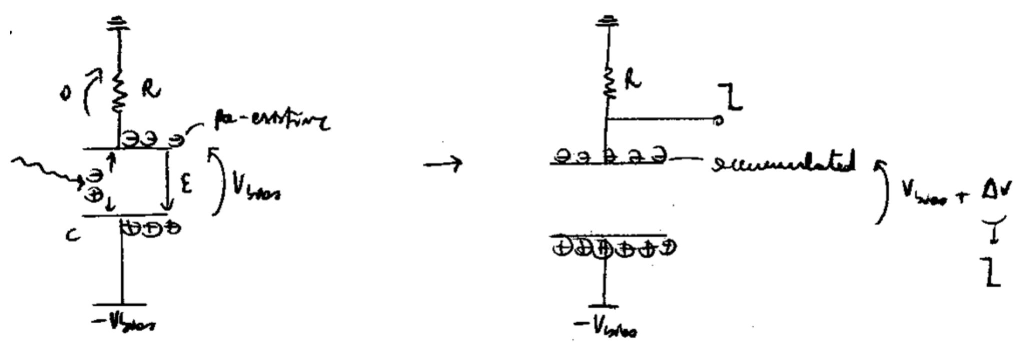
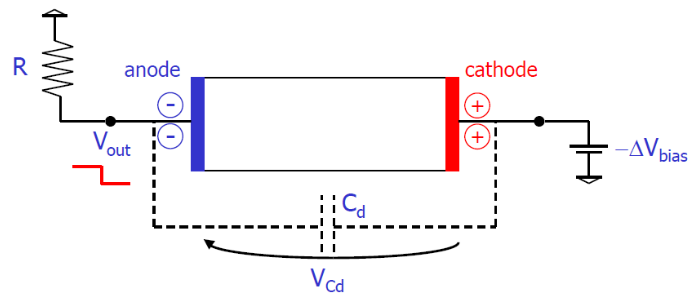


When all the transients are gone, we have created and electric field inside the capacitor, and this electric field allows to separate electrons and holes. So the biasing of the network has created the electric field that will be used to separate the charges.
What will happen then is that at the end of the motion of all charges, the detector won't be anymore the same, because we will have a layer of negative and positive charges on the two plates. However, even when we have the bias we have accumulated charges, but since they are associated to the bias voltage we are not interested in them. We are interested to the charges created by the radiations and moved to the plates.

Hence we have a delta charge accumulated on the plates. Then, If we have a delta charge, we will have no more the Vbias, but the Vbias + delta voltage, and the delta voltage is associated to the accumulation of the new amount charges.
This delta voltage will be measured at the electrode below R. In fact, the -Vbias node is fixed, so if we change the voltage across the C we change the voltage on the electrode. This is our signal.

The resistor R must be very large because the network is an RC. So if we accumulate charges on the capacitor and we have a small resistor, practically the resistor will absorb the charges immediately, the charges have no time to accumulate on the plates of the capacitors and will be immediately discharged but the resistor → R must be sufficiently large to have at least time to collect the charges across the plates of the capacitor, reading the signal and then I will have a transient. But if the resistor is so small that the time constant of the RC network is even smaller than the transit time of the charges, the charges have no time to complete their transfer that will be washed away by the resistor R.

In the image below charges have been separated, collected on the two electrodes and the voltage across the detector, called Vcd (because the detector can be represented as a capacitor Cd), is the initial voltage (Vbias) plus a delta voltage. The responsible of the delta voltage is the delta charge, so the delta voltage is given as below (the minus is because we have a negative charge on the anode, we have a negative step).



when the charges have reached the electrodes:

$V_{Cd} = V_{Cd\ initial} + \Delta V_{Cd}$ $\quad \Delta V_{Cd} = -|\Delta Q|/C_d$

(as the charge $-\Delta Q$ has modified the total amount of charge present on the electrodes of C_d)

$\Delta V_{out} = \Delta V_{Cd} = -|\Delta Q|/C_d$

$|\Delta Q| = qE/\varepsilon$ $\quad \Rightarrow \boxed{\Delta V_{out} = -\, qE/(C_d\varepsilon)}$

The output voltage, which was 0 after the transient, is now only equal to the difference, so the delta voltage Vcd (in this case Vout = delta Vout because Vout was 0 at the beginning).
The amount of charge created in the detector (delta Q) is given by the sensitivity, the amount of charge is proportional to the energy of the radiation (qE/epsilon).

In conclusion, with such a simple ionizing detector we will measure with a voltage amplifier a negative voltage step of amplitude as in the red box (voltage measurement proportional to the energy).
If we know a priori that the detector has absorbed the full energy of the xray, by means of this formula we can have a calibration, we know we have a correspondence in mV of my amplifier to keV of xrays. Through this formula we can make calibration of the detector, but of course we have to rely on the fact that 100% of the energy has been absorbed. If we have a Compton effect we cannot rely on the formula in the red box.

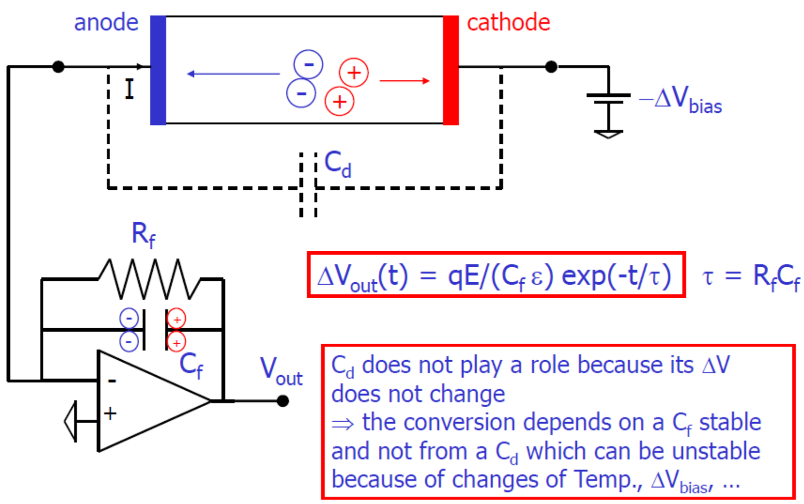
There is also a limit in increasing Vbias, otherwise we start to have a sort of breakdown or braking of the dielectric. We break the dielectric and a current starts to flow between the plates.
The material in the capacitor must be an insulator, and semiconductors are very good because they are intrinsically insulator, but if we have mobile charges created by xrays, they are able to conduct the charge. This is why semiconductors are popular in the field of radiation detectors. In fact, they can stand a reverse bias, without braking the dielectric, and at the same time they can transport charges.

Drawback

The drawback in the formula is the detector capacitance, because the detector capacitance C may depend on specifical status of the detector in terms of temperature, biasing, humidity and so on. Cd is not a real physical capacitor that we ‘purchase’, it is a parasitic capacitor of a physical material. In a semiconductor detector, the one between the plate is a depletion region and the one associated is the depletion capacitance, so it may change if we apply different biasing voltages on the detector or if we change the temperature of the detector.

It is not nice to have in a measuring system a dependence of the output voltage with a physical parameter that may change in time, voltage, humidity, it is not reliable → we need an alternative configuration.

THE CHARGE PREAMPLIFIER



Alternative readout, but now the anode is no more connected to a resistor, but to the virtual ground of an integrator. This anode voltage is no more free to change. Hence the output signal is not taken on the anode, because the detector is completely locked at both extremities: the cathode at -delta Vbias, the anode at virtual ground.

Can the charge still be transported? Yes, because we have locked the two extremities, but we still have the electric field inside the material. Electrons and holes can still be separated and the electrons will move to the anode.

However, the difference now is that the electrons, instead of being collected at the anode, are transferred through the virtual ground to the capacitor Cf. The negative charges are not stopped at the anode of Cd, because the anode cannot change its voltage, it is locked, and so by the K. law the only escaping path is to be accumulated on Cf (field capacitor):

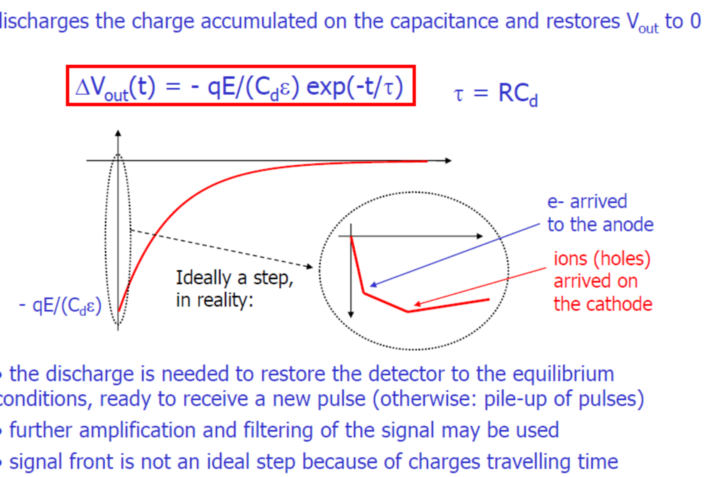
The charge now is measured on Cf, with the same formula as before (if we don’t consider the exponential part). There is a + instead of a – due to the polarity (we have a positive step instead of a negative one), but the big difference is that at the denominator we don’t have anymore the detector capacitance Cd, but the feedback capacitor Cf.

In the end, the conversion formula doesn’t include anymore the detector capacitance that was changing with temperature, humidity, bias, etc…, but depends on the value of the capacitor Cf, a capacitor that we can choose and it is fixed forever. It is indeed a component, not a parasitic parameter of a detector, we can trust its value.

This solution, the most used one nowadays, is the most common readout of a radiation detector.

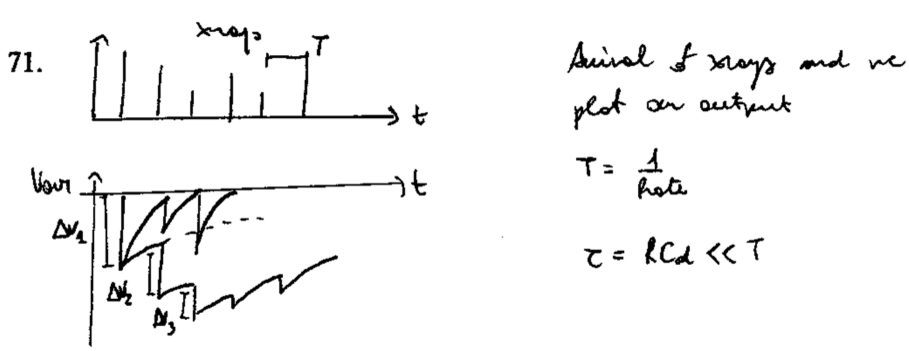
Considerations on R in the RC network

R must be indeed very large, but soon or later it will still discharge exponentially the charge on the capacitor. So the real recorded waveform is not a step, but a step (eventually negative) followed by an exponential decay.



The time constant of the classical exponential transient is R*C.
The best choice of the time constant tau has to be made carefully. In fact, if too small, we discharge the electrons too soon, they don't have enough time to finish to accumulate on the capacitor that the signal is already washed out by the RC transient.
On the other side, a too large tau means having no time to extinguish the previous RC transient that we accumulate already the charge due to the second photon. Then we try to discharge the second step and we have a third one and so on → superpositions of several RC transient, and it is a mess, it is difficult to measure delta V.
This phenomenon is called pile-up, that is the superposition of several pulses which have not concluded their time response and they are piled up together.

71.

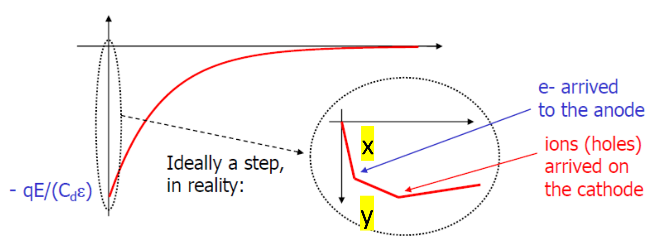


In this case we should choose the RC time constant in a way that we reduce so much the discharge that the amplifier is ready for the arrival of the next pulse (green). The tau is in a way that prior to the arrival of a pulse, the amplifier is restored to the baseline. We have to return to 0 early enough to be able to detect the next pulse.

Of course, the arrival of the rays is statistical, so we have to do some statistical considerations, but as a rule of thumb, if my events arrive with an average period of T = 1/rate, where rate is the arrival of photons (e.g. 100 photons/sec), the tau must be << T.
For instance, if tau is 10% of T, we can say that the probability of pile-up is 10%.

This same reasoning on the RC network readout can be extended to the charge amplifier (that's why we have a feedback resistor in parallel to Cf, we need a resistor to discharge exponentially the capacitor).

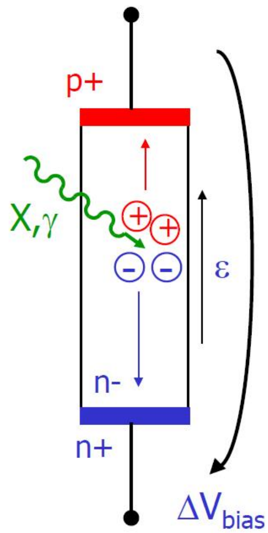
Finally, in reality if we zoom on the front of the signal, we don't have a sharp step, and Heaviside step; this because we need to wait for the travel of the charges. At the beginning when the charges are joint together we have no signal on the anode. Then when the charges start to move away we start to have something, because we have the electrostatic induction → the anode start to have a signal. Then in the final status we have all the charges accumulated.
So in the ideal case charges accumulate and create a step, but we have some time between the initial and final condition → we don't have a perfect Heaviside step, but some 'lines' that correspond to the induction of charges towards the electrodes.
Then, since the electrons typically travels faster than holes in semiconductors, we have first steep ramp due to electron separation (x), then holes will arrive and the remaining part is due to the holes (y). finally when both charges have arrived, we have the full amplitude.



→ **Formation of the signal is not instantaneous**, we have to wait for the ramps.
Moreover, if we have an exponential decay too short, we don't let these ramps to reach their final value that the signal is deleted.

SEMICONDUCTOR DETECTORS

THE PN DIODE



n- : inserted to increase the depletion region

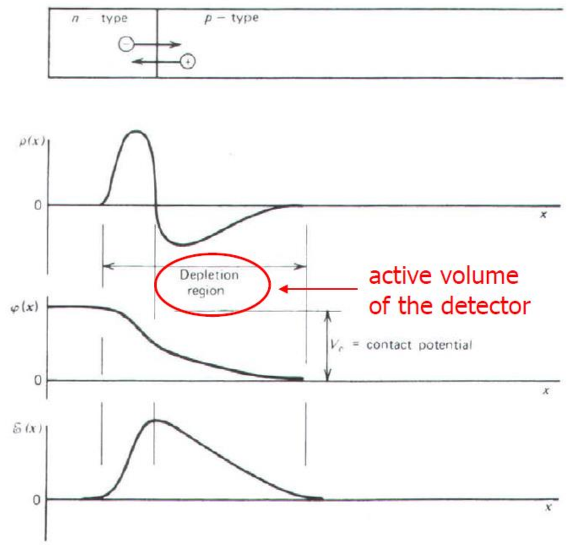


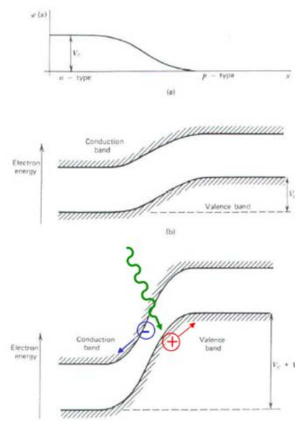
Figure 11.8 The assumed concentration profiles for the *n-p* junction shown at the top are explained in the text. The effects of carrier diffusion across the junction give rise to the illustrated profiles for space charge $\rho(x)$, electric potential $\varphi(x)$, and electric field $\mathcal{E}(x)$.

A basic semiconductor detector is a pn junction. The pn junction becomes a detector only if we exploit the depletion region of the junction. First of all, we are considering pn diodes always reversed bias.
If we take a pn diode and we apply a reverse bias, we enlarge more and more the region across the junction, that is depleted by mobile charges, and so it becomes a nice active volume to stop the radiation. We have to be sure that we have sufficient volume to stop the radiation. And the only volume we have at disposal is the depletion region.
Of course, we can absorb xrays also elsewhere, in the non-depleted region of the semiconductor, but since it is undepleted it is full of charge, so we cannot count single electrons in this enormous region.

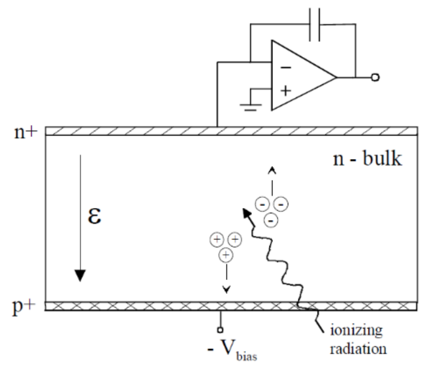
In the graphs we deplete the junction, and in the depletion region we have an electric field, that is the electric field responsible for the separation of charges. The last graph is the shape of the electric field inside the pn junction. We note that is not constant, but it has always the same polarity.
Since it is no constant, the calculation of the transit time for the charges is a little bit trickier, we need to calculate the transient time with an integral of the electrical field.

Very often, the pn diode is created as 'pin', p-intrinsic/n diode. We put between the pn electrodes an almost intrinsic semiconductor. In the image is called n-. n+ is the doped electrode, n- means that is a silicon slightly doped. We still have a pn junction, that is in the contact between p and n-, but as in the middle we have light doped semiconductor, it is easy to be depleted with a relatively low voltage. → in order to enhance the depletion region, the lower the doping of the semiconductor, the larger is the volume of the semiconductor we can place with a reasonable voltage reverse bias applied.

The next image is showing again the reverse bias. In the middle we have the standard semiconductor, and in the bottom the reverse bias.



the diode is reverse biased in order to deplete the entire volume of the detector from mobile charge and to make it active for the creation of the signal charge generated by the absorption of the photon



the same potential applied for the depletion of the detector allows the generated charges to be collected by the electrodes

On the right we have again the same schematization of the previously described pin diode.
What is new is that the charge preamplifier is added in one of the two sides, in order to read charge. In this design, the amplifier is connected to the n+ electrode → we will bias the p side with a negative voltage, the n side is biased with a virtual ground of the integrator and so electrons will move up and holes down. Then electrons will go into the feedback capacitor and we will have a positive signal.

We can have an alternative. We may put the charge preamplifier connected to the p+ electrode, in this case the p+ will be grounded (we impose 0V) and we create the equivalent electric field by biasing the n+ electrode with +Vbias. → we need to have a reverse bias junction always.
In case we flip the readout like this, we are now collecting holes, the signal is based on holes. However it is always better to readout electrons because they are collected faster than holes.

Why is silicon not good in nuclear medicine?

what is limiting in practice the depletion in a Silicon *pn* detector

absorption efficiency for X rays:

thickness	efficiency
1 mm	40% at 30keV
	10% at 50keV
1 cm	90% at 30keV
	40% at 80keV

voltage required for the depletion of a thickness x_d:

$$\boxed{V_{depl} = x_d^2 \frac{qN_{dop}}{2\varepsilon_0\varepsilon_r}} \Rightarrow$$

thickness	V_{depl}
100 μm	7.8 V
300 μm	70 V
1 mm	780 V
1 cm	78 kV (!)

$\varepsilon_0 = 8.8\ 10^{-14}$F/cm
$\varepsilon_r = 11.7$
$N_{dop} \sim 10^{12}$ dopants/cm^3 ⇒ use of Si *pn* detectors limited to E of few tens of keV

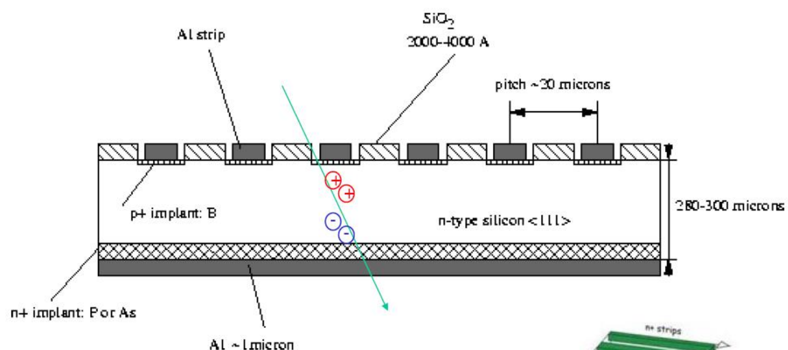
We are trying to understand what is limiting in practice the depletion region of silicon.

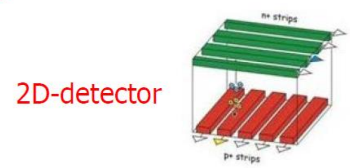
First of all, form the point of view of absorption efficiency (first duty is to stop rays), 1mm of Si is efficient only 40% at 30keV and 10% at 50keV. If we increase the thickness, the efficiency increases. So 1cm can still be good for radiography. However, 40% at 80keV means that we are delivering 60% of xrays to the patient for not getting a signal out of it → the patient is exposed to radiation with a very low efficient radiographic panel.

In semiconductor physics, there is a formula that connects the depletion width x_d with the depletion voltage Vdepl we have to apply to reach such width. The bad news is that the depletion voltage scales with the square of the width of the depletion region, so if we want to duplicate the depletion region to make the detector more effective, we need to apply 4 times the depletion voltage.
The proportionality factors are: charge of electron, doping of Si (the smaller the doping, the better, that's why we add an n- material in the middle. However, it is impossible to reach a doping level smaller than 10^{12} dopants/cm3) and vacuum and Si constant.

In the table thickness vs depletion voltage we can see the issue. In fact, to deplete 1cm of Si, that was already bad in terms of detection efficiency, we need to apply 78kV. This is the problem, because to deplete a Si thickness just useful for radiography, we would need an impossible voltage to be applied → Si must be discarded, better to use high Z semiconductors.
Only in mammography the mammographic system may use 1mm of Si.

Example of Si imaging detector: the Microctrip detector



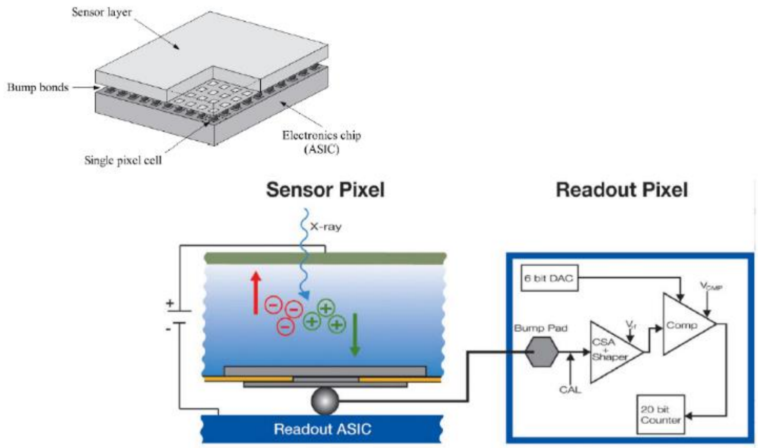


We can make an imager out of a pn diode, because a pn diode is only one imaging element; to build an imager we can use the microstrip detector, that is simply the parallelization of several diodes. Instead of having just one big diode in a Si wafer, we segment one electrode in strips. We get an unidimensional information. So we have a segmentation of the sensor with a spatial resolution given by the separation of the strips.

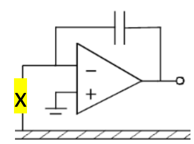
It is a unidimensional detector if we segment just the upper side, but it can become a bidimensional one if we segment also the bottom side.
In this way we have a two dimensional detector, because the charges collected to the top will provide a coordinate along the direction indicated by the green arrow, and the holes collected on the bottom will provide a signal in the other direction.
So we need to have a double readout, two charge preamplifier on the top and on the bottom, to have two parallel readouts.

The coordinate of interaction is then the crossing between one green stripe and one red strip.

Example of a silicon imaging detector: the Pixel detector



A pixel detector is a further application of the pn junction. We haven't a unidimensional pn junction as before, but a matrix of individual small pn junctions, pn detectors. So we have a pixelated detector. The difference is that in the previous example we can apply the readout directly at the end of the strip, while here is more tricky because we have to connect an individual amplifier to an individual pixel. We have an interconnection problem that is solved by a flip chip bonding the creek.



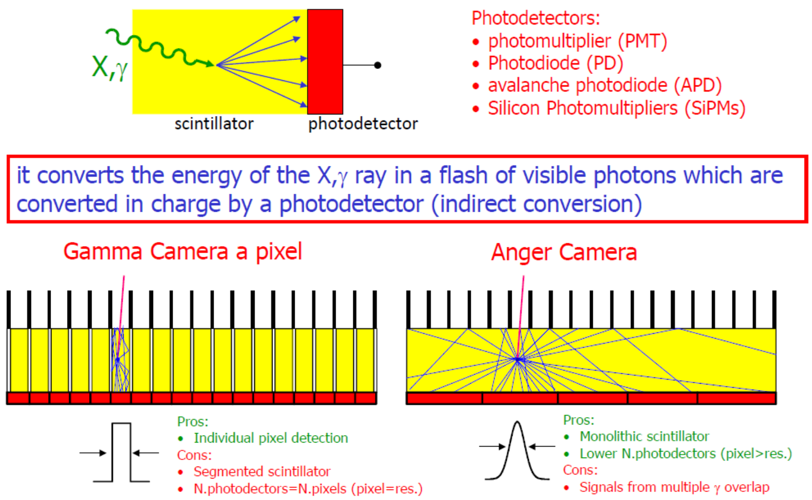
In the chip we have the sensor and then an electronic chip, typically an ASIC, that is 1-to-1 coupled with a sensor. If the sensor has a pitch of 100u, the amplifier chip should have a pitch of 100u. Then we take the two chips, we clip them, we connect with a bowl of gold that connects the sensor plate to the virtual ground of the integrator (connection line x is made with a gold wire).

THE SCINTILLATOR

Detectors based on indirect conversion.
A scintillator is a material where the energy absorbed by x and gamma rays doesn't create charge, but it creates a flash of optical photons, several optical photons. It is not a 1-to-1 conversion.
Then we need to combine optically, to optically couple the scintillator with a standard photodetector in order to convert the optical photons in individual charges. Ideally, out of 5000 photons we would have 5000 electrons. In fact, now the photodetectors work as standard one, that is 1 photon = 1 electron. **There is no more the problem of a wide depletion layer (no direct conversion of high energy photons)**, because if we use a photodetector to convert optical photon into charges, we need only few tens of microns to make the conversion effective.
Hence all the problems seen about the depletion layer are irrelevant here.

Advantage: the material responsible for scintillation has nothing to do with any biasing → all the problems seen for semiconductors are not existing here, we don't have to apply any voltage to a scintillator crystal, it is a piece of material, a passive material.

And because of this we can also build the scintillator as big as we want, we have no constrains in thickness, and so no constrains in efficiency (of course there are limits in manufacturing) → with scintillator the problem of efficiency is decoupled with the problem of charge creation.



In medical imaging, scintillator detectors are used in the two configurations in the bottom part of the image. On the left we have the 'pixelated gamma camera', where we have an individual unity scintillator photodetector, so it is the pillbox detector. In fact, the response is the PSF of a typical pixelated detector.
The advantage of this camera is that each pixel is independent from the others. Each pixel has is own scintillator, photodiode (PD), amplifier. This means that in principle, in this detector, two gamma rays may interact simultaneously on two different pixel, and we carry out them simultaneously, we have no pile up in terms of gamma ray detector, because in principle different pixels may absorb simultaneously different gamma rays. The only probability to be excluded is that two gamma rays interact in the same pixel, but it is a very rare case.

Disadvantage: it is a segmented scintillator, very complicated because we need to manufacture very small pixels. Moreover, the resolution, the spatial resolution, is given by the dimension of the pixel. The better we want the resolution, the smaller we need to manufacture the scintillator.
For instance, a clinical PET detector requires pixel of 3mm. If we want a better resolution PET detector we need to build pixel of 1mm or even 0.5mm (best case), but it is complicated, because we need to make the pixel small and increase the number of readout channel.

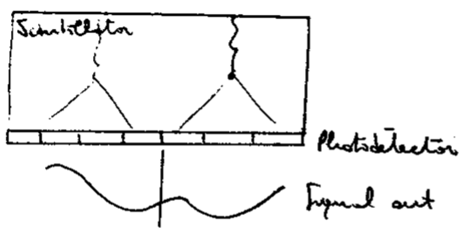
The other popular scintillation detector in nuclear medicine is the 'Anger camera'. It is based on a unique monolithic scintillator read out by an array of photodetector. The light is spread all over several photodetectors and from this light spread, using centroid reconstruction algorithm, or center of mass algorithm, we reconstruct the coordinate of interaction.
So we take the gamma ray, it interacts in the scintillator and we have a flash of light that is not confined into a single photodetector like in the pixelated camera, but it is spread over several photodetectors. Then, the photodetector that has collected more light will dominate in the center of mass formula.
So we get a PSF like in the image (it is not a pillbox), and it is not a Dirac delta because the light contribution to the various photodetector is a statistical process and we have also electronic noise of the photodetectors, so we cannot rely that we will have always the same coordinates from the center

of mass calculation → we have a distribution of probability, that is the PSF by definition, that can be for instance a Gaussian PSF.

Advantage: the lower number of photodetectors. The coverage is the same, but we cover the same area with a larger PD or photodetector, which means a lower density of electronic channels, a lower number of amplifiers. In fact, we are covering the same area with a smaller number of photodetectors (but the photodetectors are larger). Even if larger, the number of amplifiers will drop down with the square of the reduction. If we use 10 times larger PD, we don't use 10 times lower number of amplifiers, but the reduction is in both dimension → 100.
Moreover, we also reduce the complexity of the scintillator, that is now much cheaper to be manufactured (we need to create a single piece of scintillator, and not an array of small scintillators).

Disadvantage: a single gamma ray interaction involves, in principle, al the photodetectors. It is not like in pixelated camera where only the photodetector coupled with the scintillator is involved in the readout. Here we have a single gamma ray that provides a signal to all the photodetectors.
So if we imagine to have two simultaneous gamma rays interacting in the scintillator, we will have the superposition of the light created by the photodetector and it becomes a mess. The superposition of the light doesn't allow to reconstruct the center of mass.

72.



I have not a cloud of signal associated to each gamma ray

If I apply I blind center of mass reconstruction, the output is the coordinate x and not the two coordinates corresponding to the gamma rays.
A smart readout algorithm may reconstruct the position in this case, but if the situation is worst, the output can be more complicated due to the superposition and it is very difficult to reconstruct the two independent events → **Anger camera cannot resolve simultaneous gamma rays interactions (the pixelated system can)**.

ORGANIC SCINTILLATORS

The working principle is basically the excitation of the material, molecules or crystal. The scintillator can be divided in organic scintillators, composed by organic molecules (like plastic), and inorganic scintillators that are crystals. Although the physics of these material is different, the principle is still the same.

Working principle

In an ionizing detector the energetic electron created by either photoelectric or Compton effect was travelling around the material and provoking ionization. Here is always the energetic electron our main guest, but this electron, while travelling in the scintillator, instead of creating electrical charge, it excites energetic levels of the molecules.

Organic molecules can in fact be described with energy levels, so we have a ground state where the molecule is at rest, but the molecule may be excited, if we provide energy, to higher energetic levels. This is right what happens after the absorption of the gamma ray. After the absorption of the gamma ray, the energetic electron that was travelling around the material is going to excite the molecule, so providing quanta of energy.

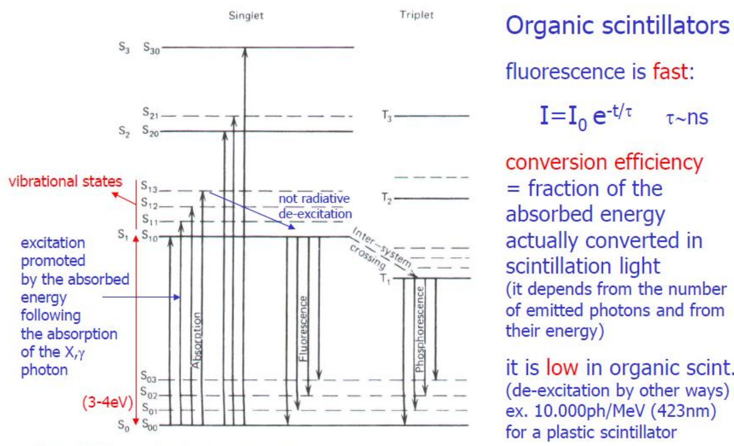
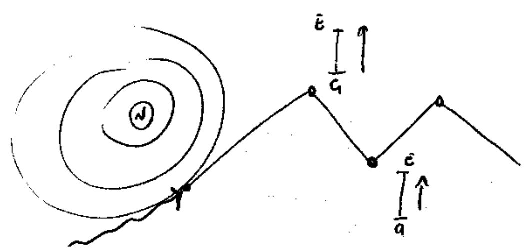


Figure 8.1 Energy levels of an organic molecule with π-electron structure. (From J. B. Birks, *The Theory and Practice of Scintillation Counting*. Copyright 1964 by Pergamon Press, Ltd. Used with permission.)

So we have the atom, the nucleus, the radiation kicks out an electron and the electron is going to hit everywhere in the material and if previously was causing ionization (separation of positive and negative charges), in the scintillator is producing excitation, and so a molecule is promoted from the ground state to the excited one.

73.



Now, the molecules don't stand excited forever, but they immediately come back to the ground state and release energy in the form of optical photons. This is the principle of scintillation. **The scintillation is due to the excitation of the molecules and then the immediate production of photons**.

Organic molecules

The prompt emission of radiation is called fluorescence. However it could be that, during the excitation, the molecule transits to a different status, in a process called intersystem crossing, where the molecule remains excited for long time. This is the phenomenon of phosphorescence. It is the delayed fluorescence because time to time the molecule gets locked into a status where the deexcitation is delayed.

The phosphorescence is a problem for scintillators, because scintillators are used to detect radiation, and we need to have a ready response to the radiation. So the phosphorescence emission on scintillator material causes the phenomenon called afterglow (a kind of permanent and diffused luminosity of the crystal), absolutely of no usable for detection.

Vibrational state

The energetic states are not just separated in energy by large gaps, but there are also substates due to vibrational quanta of energy (not of our interest).

Interesting parameters

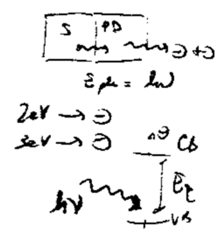
- Speed: how fast is the release of fluorescence. The release of the light in a scintillator is described by an exponential decay. We assume that the intensity of the photon flux is given by a sharp rise in the order of pico-seconds but it is the release that is not delta like. The fluorescence follows an exponential decay, the deexcitation follows an exponential decay with a time constant that is called **scintillator time constant**.
  In the organic scintillator the speed is in the order of few nano seconds (tau), that is very nice, because the response of the scintillator is almost a Dirac delta.
- Efficiency: we have a gamma ray, for instance coming form a PET (511keV). Which is the fraction of energy absorbed that is converted in scintillation light? We know the incoming energy and we can count and sum the optical photons; each individual photon has an energy (typically between 2 and 4 eV), so we count the photons and we have the total energy.
  So we can compute this output energy vs input energy. The ratio between the sum of the energy of the photons divided by the energy of the gamma rays is called **conversion efficiency**.

Hence we know that energetic efficiency is given by the sum of the energy of the exiting photons divided by the energy of the gamma ray, that can be for instance 35% (the scintillator is pretty inefficient in terms of energy conversion).
This is one approach to the problem, but there is also another more practical way to look at the efficiency, that is the following.

74. $E_{out} = \sum_i h\nu$

η_E (energetic efficiency) $= \dfrac{\sum_i h\nu}{E_\gamma} \simeq 32\%$ (I DON'T CARE ABOUT THIS)

Incoming gamma ray



At the end of the process, I have the scintillator and the photodetector. The photodetector, in case of 100% efficiency, takes 1 photon and produces 1 electron-hole pair, and we collect one of the two. So what matters in practice is the number of collected electrons, which is, intern, the number of emitted photons.
Since the ration 1/1 is quite independent from the energy of the photon, if I have a photon of 2eV or 3eV we get 1 electron (to create a couple in the photodetector we need to be at least beyond the energy gap with the energy of the photon). But once the energy of the optical photon is sufficient, it doesn't matter how large is this energy with respect to the energy gap. Once I overcame the energy threshold, I don't care if the photon is very energetic.
More important than the energy conversion, where the energy of the photon matters, what matters is the granularity, the number of optical photons delivered by the scintillator. The signal is photoelectrons will be the number of electrons created. If the electron has been created by one photon of 2eV or by one photon of 3eV I don't care, what matters is the number of photoelectrons.

If we look to a table of scintillator, the energy efficiency is never quoted, but it is quoted a parameter called gain or **conversion efficiency that is the number of emitted photons per unit of energy**.
It is not a ratio, a percentage, but it is the delivered number of photons per absorbed energy, I care just to the number, not to the energy. The higher the number, the better.
In organic scintillator this number is 10'000 phs/MeV, so the number of photons emitted per MeV of energy absorbed by the material.
Then if the photons are 2 or 3 eV I don't care, because what matters are the 10'000 photons that will be transformed in 10'000 electrons (even if in reality it is impossible, the photodector has a quantum efficiency).

INORGANIC SCINTILLATORS

They are crystals, no more molecules. When we have a crystal, we have a material with a regular geometry → we have not the energetic levels of the single molecules, but we have energy bands all over the crystal, like in Si (Si is a crystal, but not scintillating).

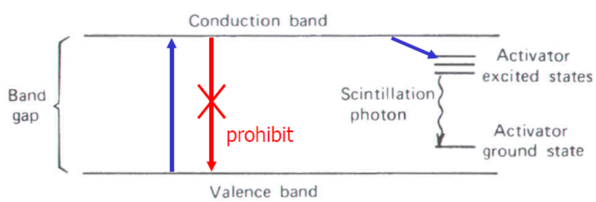


Figure 8.6 Energy band structure of an activated crystalline scintillator.

- scintillation allowed thanks to the presence of activators
- fast scintillators (~ 20-50ns) usually less efficient (~ 10-30×10^3 ph/MeV)
- efficient scintillators (~ 30-60×10^3 ph/MeV) usually slow (~ 0.3-1μs)
(recent scintillators, e.g. $LaBr_3$, have ~ 50×10^3 ph/MeV and ~ 20ns but are hygroscopic)

We have a similar process than before; the energetic and erratic electron travels everywhere and now it excites an electron form the valence band of the crystal to the conduction band (individual excitation). In principle we are expecting the fluoresce also in this case.
There are crystals which scintillates, but they are very few; sometimes the fluorescence transition is somehow inhibited by quantum mechanics → we have to help the scintillator to emit light and so we dope it with some atoms called activators.

Also the activators have two states, one ground state and one activated state. They are placed here and there in the crystal and we need simply to wait for an electron in the conduction band to fall inside one excited state of the activator, then the activator emits light and moves to the ground state and a scintillation photon is emitted.
The figure of merits of these scintillators are:

- Speed: the exponential law for the fluorescence emission still applies but the difference is that while plastic scintillators are very fast (nanoseconds), inorganic scintillators may range from few tens of ns to microseconds, so they can be very slow.
- Efficiency: usually better than plastic scintillators but usually they are divided into classes; there are fast scintillators (ns) with a lower efficiency or we may have very efficient scintillators but slower. The only exception is LaBr.

Properties of scintillators

Table 8.3 Properties of Common Inorganic Scintillators

	Specific Gravity	Wavelength of Max. Emission	Refractive Index	Decay Time (µs)	Abs. Light Yield in Photons/MeV	Relative Pulse Height Using Bialk. PM tube	References
Alkali Halides							
NaI(Tl)	3.67	415	1.85	0.23	38 000	1.00	
CsI(Tl)	4.51	540	1.80	0.68 (64%), 3.34 (36%)	65 000	0.49	78, 90, 91
CsI(Na)	4.51	420	1.84	0.46, 4.18	39 000	1.10	92
Li(Eu)	4.08	470	1.96	1.4	11 000	0.23	
Other Slow Inorganics							
BGO	7.13	480	2.15	0.30	8200	0.13	
$CdWO_4$	7.90	470	2.3	1.1 (40%), 14.5 (60%)	15 000	0.4	98–100
ZnS(Ag) (polycrystalline)	4.09	450	2.36	0.2		1.3[a]	
CaF_2 (Eu)	3.19	435	1.47	0.9	24 000	0.5	
Unactivated Fast Inorganics							
BaF_2 (fast component)	4.89	220		0.0006	1400	na	107–109
BaF_2 (slow component)	4.89	310	1.56	0.63	9500	0.2	107–109
CsI (fast component)	4.51	305		0.002 (35%), 0.02 (65%)	2000	0.05	113–115
CsI (slow component)	4.51	450	1.80	multiple, up to several µs	varies	varies	114, 115
CeF_3	6.16	310, 340	1.68	0.005, 0.027	4400	0.04 to 0.05	76, 116, 117
Cerium-Activated Fast Inorganics							
GSO	6.71	440	1.85	0.056 (90%), 0.4 (10%)	9000	0.2	119–121
YAP	5.37	370	1.95	0.027	18 000	0.45	78, 125
YAG	4.56	550	1.82	0.088 (72%), 0.302 (28%)	17 000	0.5	78, 127
LSO	7.4	420	1.82	0.047	25 000	0.75	130, 131
LuAP	8.4	365	1.94	0.017	17 000	0.3	134, 136, 138
Glass Scintillators							
Ce activated Li glass[b]	2.64	400	1.59	0.05 to 0.1	3500	0.09	77, 145
Tb activated glass[b]	3.03	550	1.5	~3000 to 5000	~50 000	na	145
For comparison, a typical organic (plastic) scintillator:							
NE102A	1.03	423	1.58	0.002	10 000	0.25	

This table shows several scintillators type with respect to different figures of merit. The red marked are the most important one to be considered when choosing a scintillator.

Specific gravity

It is the density (g/cm3). It is important because the first duty of a detector is to stop efficiently the radiation, and density is a cumulative parameter that tells us the stopping power of the scintillator. For instance, organic scintillators (bottom line) are very poor because the density is 1, so they are very bad (not used in medical imaging for this reason, although they are very fast).

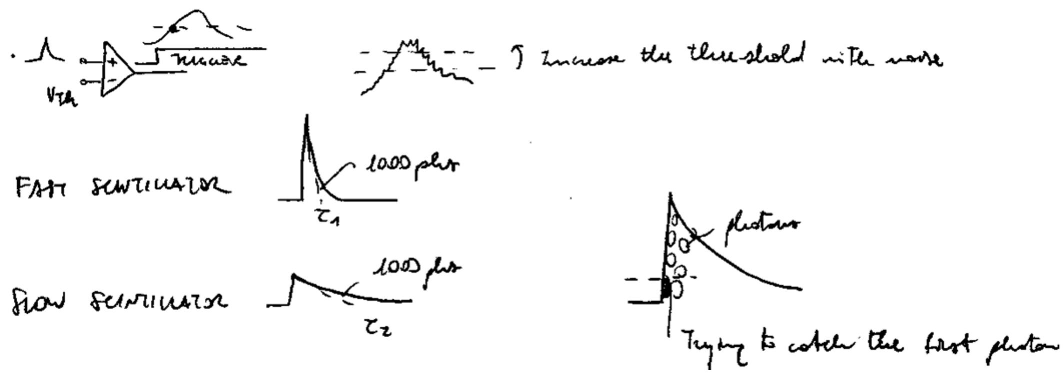
Decay time (us)

The faster, the better the response. We have various decay time, like ultrafast scintillators (LSO). They are not as fast as plastic scintillators however.
The speed of the scintillator is important because we have, in nuclear medicine, techniques such as PET, which rely on timing, coincidence (in PET we need to make the coincidence between two gamma rays back to back) inside a coincidence window. The easiest way to make timing is to take a signal, for instance out of the scintillator, we covert it with a photodetector and we apply a threshold to detect when the pulse overcome the threshold.
Of course, we don't have only a nice shaped-signal, but random noise on the signal. When we have it, in order not to be triggered by the noise above the threshold, we typically need to increase the threshold.

Let's now assume we have two scintillators that produce the same number of photons (the integral of the flash of light is 10'000 phs), and these are distributed in a very steep signal with a very short scintillation time constant in scintillator 1.
Now let's assume that the scintillator 2 has a much longer time constant (but with the same area to have a fair comparison).

It is evident that the better scintillator for timing is the 1. In fact, with a very steep rise of the signal, even in case of noise we can put a relatively safe threshold because we are sure that the signal crosses the threshold.
Conversely, with the slower scintillator is much trickier, because the rise of the pulse is less steep and so it is more tricky to place the same threshold, because it risks to be not sufficient to get the triggering from the signal. So the faster the better for discrimination.

75.



In reality, a scintillator waveform is composed by the superposition of single scintillations photons. 10'000 photons are real quanta covered by the shape.
Modern techniques of timing are trying to catch up with threshold the earliest photon in the family, of the waveform.

If I distribute the same number of photons on a longer time constant, of course the precision in the decision where to trigger is more lossy. If we have a larger amount of photons accumulated in short time, also the statistic probability to measure the first one among them has less jitter than having the same number of photons spread wherever, because already the jitter in the measurement of the first one will be more spread. The more the photons are squeezed in a short time, the lower is the jitter (oscillazioni) in the measurement.

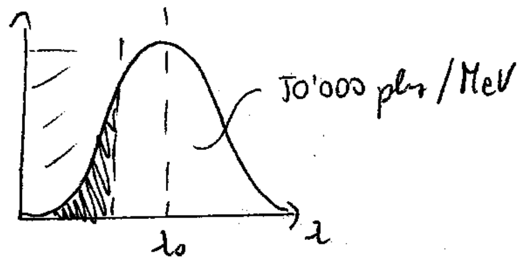
Deficiency – light yield (gain of the scintillator)
Number of photons emitted by the scintillator per unit of energy. It is quoted in phs/MeV.

Wavelength of maximum emission of photons
A scintillator is not like a laser. The characteristic of the emission of a laser is that usually lasers are almost monochromatic, they emit at a given wavelength.
Scintillators have a broader emission spectrum. What we quote is the lambda of the peak emission, and it doesn't tell us how much broad the emission is, but tells us more or less where is located the emission of the light (e.g. 200nm or 500nm).
This is important not as a property of the scintillator itself, but it is important for the choice of the photodetector. In fact, photodetectors are efficient only in specific wavelength ranges.
For instance, if we have the emission like in the image and we choose photodetector that it is efficient only up to a certain wavelength, we are measuring only a part of the emission spectrum of the scintillator. So the scintillator could be optimal because it emits 50'000phs/MeV but in reality with a

wrong selection of the photodetector we are using only partially the emission spectrum of the scintillator.

In conclusion, lambda tells us more or less where the emission of the light is.

76.



Example: choice of scintillators for PET and SPECT

Let's consider the three main figures of merit: stopping power (density), decay time and light yield. We have to choose what is mandatory and what can be relaxed.

PET

- **Stopping power**: in PET we have 511keV, that is a very large energy. If we recall the plot of u', we are in the region where u' (mass attenuation coefficient) has dropped a lot and we re dominated by Compton (not so nice, the full absorption of the gamma ray is at risk with Compton). To have photoelectric dominant we need to choose a high Z, which means high density. So 511keV in PET is a high energy and it is of paramount importance to choose a very efficient scintillator because we need to stop 511keV.
  For example, we can use BGO. Nowadays also LSO is very popular. We cannot use NaI (too small density).
- **Decay time**: I need a fast oscillator → we need to discard the tau in the order of us. In LS= we have a better tau than BGO; it has good density and good speed.
- **Light yield**: in PET it is not important because the energy of the photon is 511keV. This creates a big problem in stopping power, but in terms of signal generation is good, great. 511keV provides us an enormous quantity of photons. We are not forced to have a very efficient scintillator because the starting energy is already enormous. This is why BGO was chosen for PET despite of the 8200 phs/MeV. Even LSO is not so luminous.

In conclusion for PET, higher priority for density and time and we can relax the gain.

SPECT

- **Stopping power**: the energy in SPECT is 140keV, so the density is less critical, because the photon is much less energetic → we can relax a bit more the stopping power, and so the density, because the efficiency is already enough high. In SPECT we can consider crystals like CsI.
- **Decay time**: it is a single positron emission computer tomography, so we don't have to do any coincidence, it is a technique without timing → we can relax the decay time and the choice of the scintillator.

- **Light yield**: it is important, because now we have only 140keV of Tc99, and so we need to preserve the signal. So we need to choose a high gain scintillator, like NaI or CsI.

LSO is chosen for PET. It is based on lutetium, that is an element that has a radioactive isotope → inside the crystal we have self-radioactivity due to a radioactive component of lutetium. It seems not so nice to have a crystal to measure radiation that is radioactive itself. So the question is: is this radioactivity a problem in PET?
No, because since PET is based on coincidence, a valid event needs to pass the coincidence. Is a single radioactive event in one of the two crystals pass such a coincidence test? Very rarely.
So radioactivity of lutetium is not a problem in PET, while it is a problem in SPECT, because SPECT is based on single photon detection, and if it is from an internal source or not, they cannot be distinguished. → for SPECT we can't use a radioactive crystal.

Comparison between NaI and CsI

NaI(Tl)	CsI(Tl)
$G \sim 38\times10^3$ ph/MeV	$G \sim 65\times10^3$ ph/MeV (+71%)
$\lambda = 415$nm (well matched with PMT)	$\lambda = 540$nm (well matched with PD)
⇒ fraction of converted energy $= hc/\lambda\times G \sim 3\text{eV}\times G = 0.11$	⇒ fraction of converted energy $= hc/\lambda\times G \sim 2.3\text{eV}\times G = 0.15$ (+36%)

notes:
- limited efficiency of the energy conversion
- what matters more is the number of generated photons, more then E. efficiency
- the generated charge then depends on the quantum efficiency QE of the used photodetector
- CsI(Tl) better than NaI(Tl), in principle, but the final result depends on the QE of the used photodetector (which depends on λ)

G indicates the emission, the one of CsI is higher than NaI → better to select CsI for SPECT.
The lambda of NaI is instead well matched with photomultiplier tubes. Typically, photomultiplier tubes are responding well at short wavelength (the photodiode at long wavelength). Hence the choice of the best photodetector depends also on the emission spectrum of the scintillator. There are scintillators emitting at long wavelength; CsI emits at more than 500nm, so a Si photodiode is good, but if we take for instance NaI, it is more peaked at shorter wavelength, where the photomultiplier tube is competitive in terms of performances with the Si photodiode.

So **if the scintillator emits a short wavelength we may consider photomultiplier tube**, because Si efficiency drops down; **if the scintillator emits at higher wavelengths, we may consider photodiode**.

Which is the fraction of converted energy, the fraction of gamma rays converted in optical photons?
I take the gain G (number of photons normalized per MeV of gamma ray energy), I multiply for the energy of the single photon and I have the efficiency. The energy of the single photon is lower in CsI. At the end of the calculation we have the efficiencies above.
Regarding these results, we can say that scintillators are very inefficient detectors, because they convert only 10-20% of the energy of the gamma ray → **scintillation is a very inefficient mechanism**.

Moreover, it is more important to look at G than at the efficiency calculated. In fact, the efficiency tells us just that CsI is 36% more efficient than NaI in terms of energy efficiency. But the number G tells us that the efficiency of CsI in terms of number of photons is much larger (71%).
G is different from the calculated efficiency because I have a different energy of the photon in the two materials. But what matters is not the energy of the optical photon, but the number of photons, and it is much larger in CsI.

Time constant

CsI has two time constants, not just one. Scintillators may have two time constants, and we have to specify the weight for each contribution.

PHOTODETECTORS FOR SCINTILLATORS

THE PHOTOMULTIPLIER TUBE (1)

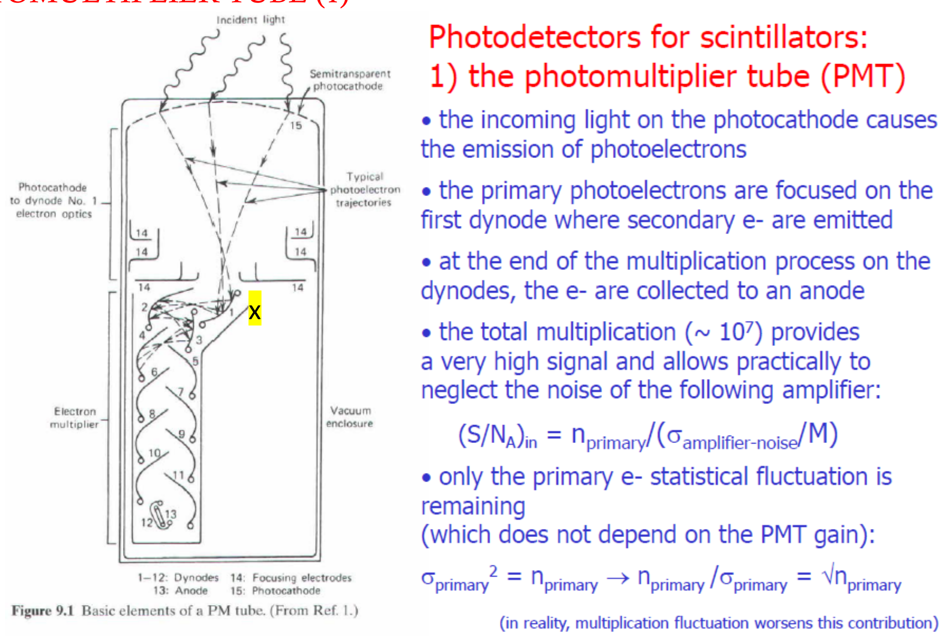


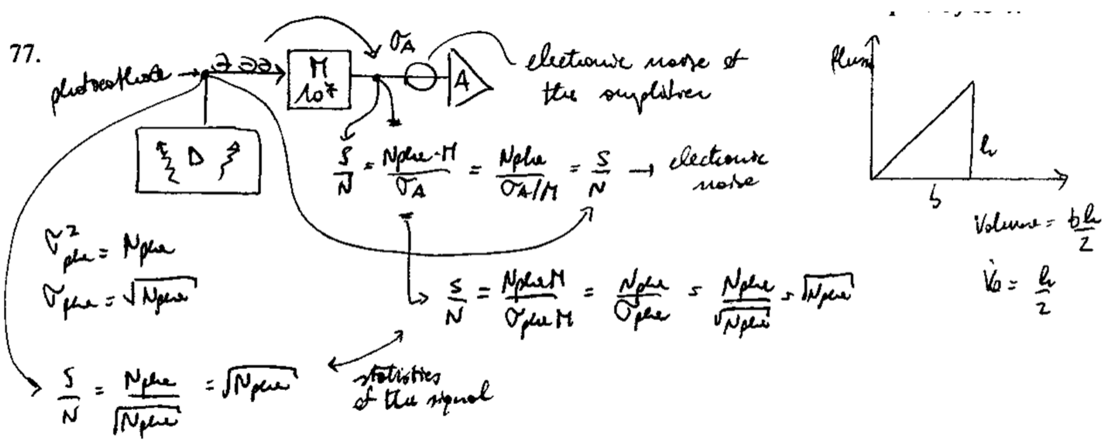
Figure 9.1 Basic elements of a PM tube. (From Ref. 1.)

The photomultiplier tube is a photodetector where the primary charge created by the light is multiplied. It has an internal gain like silicon photomultipliers. **The main reason to have a multiplication is to make negligible the noise contribution of the following amplifier**. There are also some cons, and the main one is that multiplication is not a deterministic property, but it owns its own statistical fluctuation, the multiplication fluctuates itself → worsening in the overall resolution that will affect in particular the component associated to the signal.

It is used to reduce the electronic noise and in practice is not a really user friendly device, because it has to be operated at high voltages.
It is a vacuum component, so everything is occurring in a vacuum enclosure (not ultravacuum however, but at least absence of air or other gasses), we have an entrance windows where we have a photocathode, that is responsible for the conversion of the single photon to a photoelectron through a quantum efficiency. So the conversions of photon to photoelectron is not 100% efficient but in

photomultiplier tube between 20 to 40%. Then we have the electron that is accelerated by a very large voltage difference and the trajectory of the electrons, wherever they are created, are focused by an electrodes (labelled as 14 in the image) so that electrons will crash into the same electrode called **dynode** (x). The dynode is responsible for the multiplication. So one electron is crashing into the dynode and 10^7 electrons are exiting from the anode. Primary electrons are the one created in the photocathode, secondary electrons are emitted by the dynodes.
Then we have several dynodes and the electric field in the multiplication chain is so that the secondary electrodes emitted by the first dynode are focused and crashing on a second dynode and so on.
Each time each electron is multiplied by a certain quantity and at the end of the chain we have a collecting electron (called **anode**) that are multiplied. The total multiplication can be up to 10^7 electrons. And 10^7 is called multiplication factor M.

This principle is appliable to all the application for single photons detection, but we are connecting scintillators to a photodetector, so we don't have one photon, but what said before is applied to all the scintillation photons that are emitted by the scintillator. So we have 1000 photons out of the scintillator and among 1000 only around 300 will be created on the photocathode an will be multiplied by 10^7.



We have the detector, which is providing a number of primary photoelectrons (we are at the level of the photocathode). Then we have the multiplication (10^7) and then we have the amplifier that is amplifying and filtering the signal. Each amplifier has its own electronic noise, due to the internal components of the amplifier. We can identify the electronic noise at the input with a cumulative quantity sigma-a (sigma-amplifier, that groups all the noise fluctuations of the amplifier).
Now the charge is multiplied by M and it is facing the noise of the amplifier with a beneficial multiplication number M. So if I compute the SNR after the multiplication and before the amplifier, I can obtain the result in the formula. The noise is given by the sigma-a, while the signal is the number of photonelectrons multiplied.
We can clearly see that a photodetector with an internal gain improves the SNR thanks to the M at the numerator.

The SNR can be expressed as number of photoelectrons divided by sigma-a divided by M. This manipulation holds an alternative and powerful way to see the same benefit → we are killing the noise of the amplifier from a factor M (alternative point of view, that is better). In this case the SNR can be

seen not as the SNR the entrance of the amplifier, but prior to the multiplication. The signal is N and the noise is the noise of the amplifier demultiplied by M.
NB: the multiplication is doing nothing directly on the noise sources.

Additional elements to noise

Also the generation of the signal is statistical. So, completely separated from the noise of the amplifier, I must consider the fluctuation of the signal, because the scintillator doesn't provides always the same number of photons per delivered energy and also the quantum efficiency of the photodetector is not deterministic. There is a new statistical spread that is **the spread of the number of photoelectrons**.

The spread of the number of photoelectrons in scintillation detectors is dominated by Poisson statistic. So the scintillation phenomenon and the corresponding conversion in PE is described as a Poisson mechanism.
The main property of the Poisson statistic is that the variance of the number of photoelectron is equal to the number of photoelectrons themselves.
If now I compute the SNR prior to the M and after the generation of photoelectrons (PE), the signal is the number of PE and the noise is the sigma, given by the sqrt(PE).
Hence the SNR just due to the intrinsic fluctuation of the signal goes with a square root of the signal. This explains also why we are interested in scintillators with a high light yield. The larger it, the better is the SNR in the measurement.

If now we calculate the SNR after the multiplication, it is given by:

- Signal: number of PE multiplied by M
- Noise: sigma of the noise in this point. From statistic, if we have, for instance, a variable Y given by Y = M*X, the sigma square of the variable is (78):

$$Y = MX$$
$$\sigma_Y^2 = M^2 \sigma_X^2 \qquad \text{if } M = \text{constant}$$

  So the sigma due to the intrinsic fluctuation after the multiplication is the sigma of PE multiplied by M, because the sigma-square is equal to M^2 and sigma is just multiplied by M.

In conclusion, multiplication doesn't change the statistic of the signal, doesn't have any impact on the spread of the signal, the same spread is maintained after multiplication. So photomultipliers are good because they reduce the impact of the amplifier noise but they have no effect on the statistics.
In reality this is not true because we have assumed M constant, but it cannot be, the multiplication is itself a statistical mechanism. We cannot rely on that one photoelectron produces always the same amount of multiplied electrons → this formula SNR = sqrt(Nphs) is not true (true only if M is constant) and so **there will be a penalty factor at the denominator of the formula**. Hence multiplication has a cost, and the cost is the worsening of the intrinsic statistical property of the signal. Multiplication is beneficial towards the amplifier but it has a negative effect with respect to the intrinsic statistic of the generation of photons in the scintillators.

Photocathode

Responsible for the generation of the PE. Materials involved as photodetectors can be described in energy bands; electrons normally stay in the valence band, then there is a forbidden energy gap and then eventually we have electrons in the conduction band, and they can move freely in the material.

Photodetection means creating electrons in the CB. The arriving photon must have sufficient energy to move an electron in the upper CB. This is the minimum goal in a Si photodetector. After has been promoted, thanks to drifting field, we can collect inside the device the electron with an electrode.

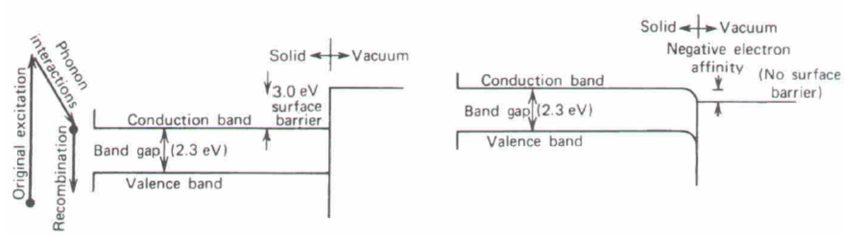
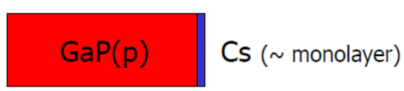


Figure 9.4 Band structure near the surface for conventional semiconductors (left) and NEA materials (right). (Adapted from Krall et al.[6])



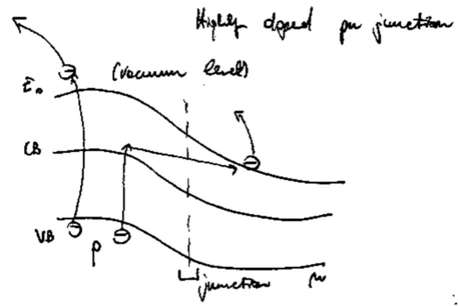
- GaP is doped *p* (ex. $10^9/cm^3$ Zn)
- Cs is electropositive (it releases e-)
- At the surface, the acceptors actract e- from Cs which is ionized

$\Rightarrow$ at the surf. bands bend and the vacuum level is lowered with respect to the bottom of the conduction band inside

In the photomultiplier we are not simply asking the photon to generate the photoelectron and then go somewhere in the cathode and return home, but we are asking to the photon to create the PE and the PE to physically leave the material. the electron must be free to travel in vacuum not belonging to the material → high requirement from the energetic point of view, not just to move to the CB, but to a higher energy level called vacuum level (or surface barrier). It is an energetic level where the electron is energetically free (even if physically it may still be inside the material). So photomultiplier tubes require that at the photocathode the photon has sufficient energy to go beyond the vacuum level → energy cost is higher.
Not only the energy cost is higher, but as long as the electron has not left, it is still physically inside the material and it may hit different atoms in the material (scattering around) and it may loose energy (**phonon interaction**, a phonon is a quantum of mechanical hit) and if it looses energy it moves down to the CB and eventually to the VB.
So it is a really costly and low probability process, and this is the main reason why photomultiplier tubes have a quantum efficiency of 40%, while Si photodiodes may have it up to 90%.
This because in a photodiode we simply ask the electron to be promoted in the CB, while in the photomultiplier tube we ask more, the photoelectron to be promoted in the vacuum level and it may very easily loose this energy.

There are some special photocathodes called NEA (Negative Electron Affinity materials) that are composed by a bulk material like gallium phosphide (red) that is layered (coupled) with a very thin layer of material that is an electropositive material (like an n-doped semiconductor). So it is like a pn junction, the GaP is the p and Cs the n.
The coupling of the materials is in a way that the energy bands in the junction of the materials are bending down so heavily (right scheme in the image) that the vacuum level in the Cs is bended so low that there is a chance that an electron in the CB of the GaP is able to jump into the vacuum. It is like pushing down the vacuum level so that we have a ‘negative electron affinity’, because the jump between the CB and the vacuum level is negative.

79.



It is not strictly speaking a pn junction, but let's suppose we create a very highly doped pn junction. We have CB and VB. Then there is also the vacuum level, that is the one that an electron has to reach to escape from the material. now, if VB and CB go down, also the vacuum level goes.
If the electron is very close to the junction, not in the bulk, there is a nonnegligible probability that the electron is promoted in the CB and then it could jump down in the vacuum level of the other side → it jumps out from the material from the most convenient vacuum level. So we need simply that the photon provides a sufficient energy to reach the CB and if so and we are close to the boundary, we can escape the vacuum level jumping down → **negative affinity, because we are exiting going down instead of going up**.

We can see that the photocathode has a high quantum efficiency because the energetic cost is low.

Quantum efficiency

Quantum efficiency vs wavelength of photon energy plot (not the isolines). We have various types of photocathodes; the quantum efficiency is not larger than 40% in classical PMT. Especially, we have a severe cut off; **after 500nm the quantum efficiency is very bad** → in the scintillator we need also to look at the wavelength, not only at the number of PE of the scintillator. In fact, if we have CsI that is beautiful, but the peak emission is at 550nm and we look at the graph, at 550nm the quantum efficiency has dropped to 10%. So we have chosen a beautiful scintillator but coupled with a wrong photodetector.

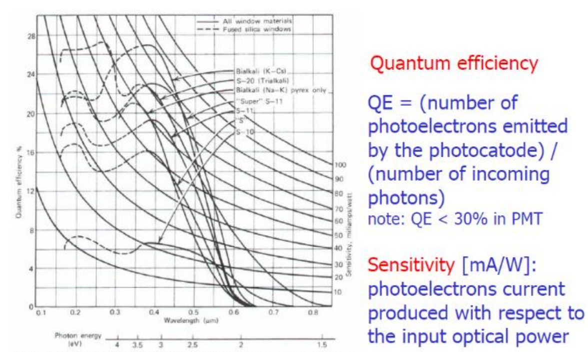


Figure 9.2 The spectral sensitivity of a number of photocathode materials of interest in PM tubes. The use of silica or quartz windows is necessary to extend the response into the ultraviolet region. (Courtesy of EMI GENCOM Inc., Plainview, NY.)

In the plot is also plotted the **sensitivity**, that is an equivalent figure of merit of the quantum efficiency. But if the quantum efficiency is the number of exiting PE divided by the number of entering photons, the sensitivity is quoted as mA of electron current divided by Watt of optical power. So it is a flux divided by flux. An identical quantum efficiency line (e.g. 20% horizontal) is not corresponding to an identical sensitivity line. This is due to the fact that the optical power doesn't depend only on the number of photons, but also on the energy of the single photon, so on the wavelength.
Hence if we have 100 photons but with different wavelengths, the optical power at the denominator of the sensitivity is different.

Multiplication factor at dynodes

The electron is reaching the dynode and crashing into it, producing secondary electrons.

The mechanism of production of secondary electrons is ionization (the same of ionizing detector). We have an energetic electron with a lot of kinetic energy that is crashing in the material and it is loosing its energy by ionizing several electron-ions or electron-hole pairs. The created secondary electrons belong to the material but since between the first dynode and the second dynode we apply an electric field, we can extract the secondary electrons from the first dynode. So they are extracted from the material, taken away and redirected to the second dynode.

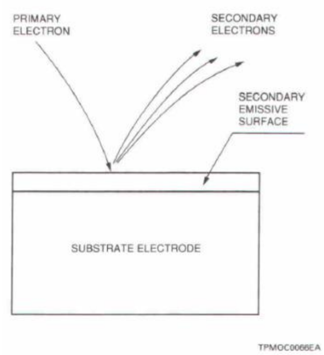


Figure 2-6: Secondary emission of dynode

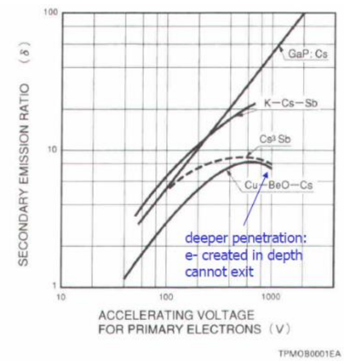


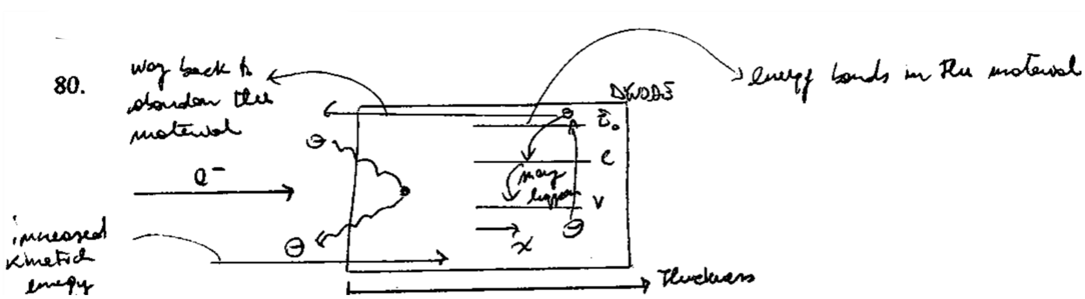
Figure 2-7: Secondary emission ratio

δ= number of secondary electrons emitted for each incoming primary electron

Delta is the number of secondary electrons emitted for each entering photoelectron.
Since the multiplication depends on the kinetic energy of the primary PE, we should be tempted to increase as much as we can the voltage difference between the photocathode and the dynode, because the larger the voltage difference we apply, the larger the kinetic energy and so the delta.
This is true (in the plot we have delta vs voltage difference), delta can reach a value of 10 → we increase the M factor. However, this increase reaches a maximum and it can eventually reduce.

Why the reduction of delta?

80.



Let's suppose we have the dynode and an entering electron. We know the thickness of the dynode.
The entering electron is ionizing and then I have some secondary electrons that are exiting the material. However, if we increase the kinetic energy of the electron, the electron enters deeper in the material (the more energetic the electron, the more will enter in the depth of the material).

If in the dynode we have our energy bands along the thickness of the material, a very energetic electron is of course able to promote an electron from the valence band to the vacuum level. However, from the energetic point of view the electron is free, but it still belongs to the geometrical boundaries of the material → the electron has to move back to exit the material and the deeper is created, the longer is the path it has to take to live the material.
Moreover, the longer the path, the more could be the dangers, and the dangers are phonons, mechanical hits, scattering in the material (we are not in vacuum yet). Hence during the trip the electron may loose energy and falling in the conduction band and may then fall down to the VB and it's over.

In conclusion, dynodes don't increase secondary emission when we increase the voltage because primary electrons are ionizing too into the depth and so we reduce the chances of exiting for secondary electrons. We can use NEA material also as dynodes because they are less demanding from the energetic point of view; in a NEA material as dynode, the exiting electron can stay in the CB, not in the vacuum level. And if we are lucky that they are in the CB, they have the chance to fall down at the boundary due to the negative affinity. Of course, also for the NEA materials there is a chance that from the conduction band the electron falls down in the valence band, and then it is over even for the NEA material.
However, for NEA materials the escaping path is facilitated.

So in the plot standard materials reach a maximum and then drop down, while NEA materials (straight lines) are very good, because delta can increase a lot with applied voltage.

Total multiplication factor

$$\boxed{\text{total gain} = \alpha\, \delta^N}$$

α= fraction of collected photoelectrons (~1)
δ= multiplication factor of single dynode
N= number of multiplication stages

$\delta = 5 \Rightarrow G = 5^{10} \sim 10^7$ with 10 stages
$\delta = 55$ (NEA) $\Rightarrow G = 55^4 \sim 10^7$ with 4 stages

multiplication statistics

δ fluctuates statistically event-by-event

variance related to 1 dynode (Poisson): $(\sigma/\delta)^2 = \delta/\delta^2 = 1/\delta$

variance of total G: $\boxed{(\sigma_G/\delta^N)^2 = 1/\delta + 1/\delta^2 + 1/\delta^3 + \ldots + 1/\delta^N \sim 1/(\delta - 1)}$

(note: variance dominated by the fluctuations of the first dynode for $\delta >> 1$)

If we have a N number of dynodes and each dynode has a multiplication factor of delta, the overall multiplication factor is given by the formula in the upper box. We have a alpha factor in front that is describing how efficient is the focusing action (we cannot guarantee that 100% of PE are really focused on the first dynode, we may loose few of them). So the alpha factor represents the effectiveness of the focusing action.

In conclusion, if we want to reach an overall gain of 10^7 and we use standards dynodes, we have delta = 5 and we need 10 stages. Differently for NEA (4 stages are enough). Is then up to us if it is more practical to have more stages of a low performance dynode or less stages of a high performance dynode.

Multiplication statistics

We cannot rely that all the time we have the same number of PE generated, because **delta fluctuates as described by a Poisson statistic**.
The Poisson statistic tells us that if we calculate the 'noise to signal squared' (sigma over delta), so the noise normalized to the signal (it is squared to have sigma squared, that in Poisson statistic is equal to the signal, delta), in the end we have 1/delta.
This is the problem; we cannot rely that delta is a fixed number, each time it varies. However **what matters is the variance of the total gain, we need to calculate the sigma squared of the total gain divided by the total gain itself (that is delta^N)**.

This calculation is not easy because there are some correlations between dynodes and in the final formula we have that the relative spread of the gain is 1/(delta-1).

This tells us that if we can approximate this final formula with one of the single dynode, it s like to think that the 'game is over' already at the first dynode in fact, if the total fluctuation at the end of the device is 1/(delta-1), which is practically 1/delta, it is like if all the spread has been played in the first dynode and then in the rest of the cascade there are just averaging phenomena, the rest of the chain is not worsening too much the statistic.

The conclusion is that, in practice, **we have to choose very good quality dynode**; the better the delta of the dynode, the smaller the overall statistical spread. If we are interest in the lowest possible spread of the multiplication, we have to invest money in a quality PMT with high quality dynodes.

PMTs configurations

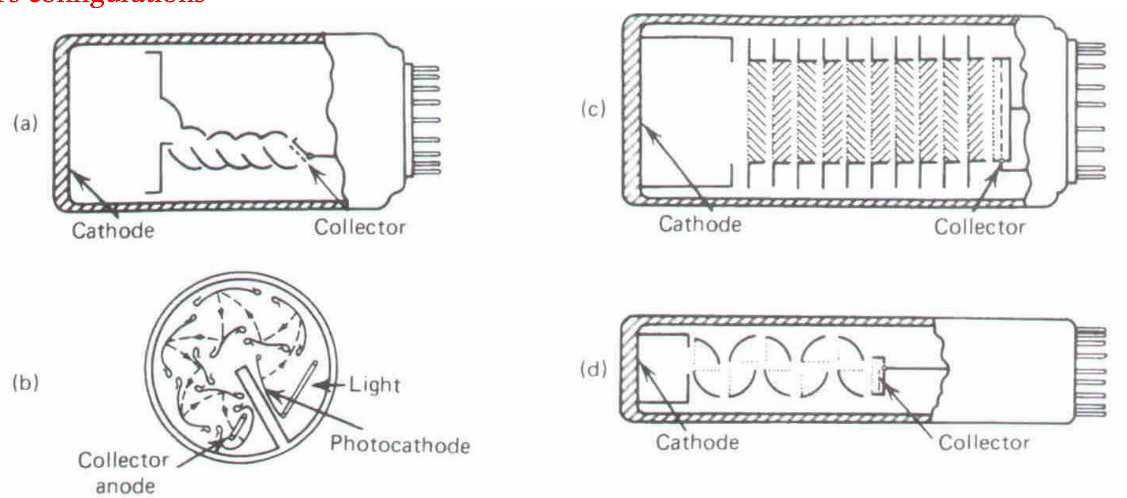
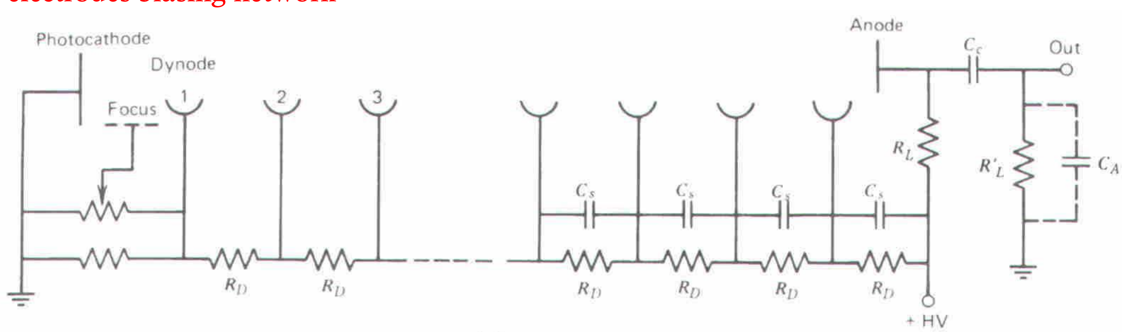


Figure 9.7 Configurations of some common types of PM tubes. (*a*) Focused linear structure. (*b*) Circular grid. (*c*) Venetian blind. (*d*) Box-and-grid. (Courtesy of EMI GENCOM Inc., Plainview, NY.)

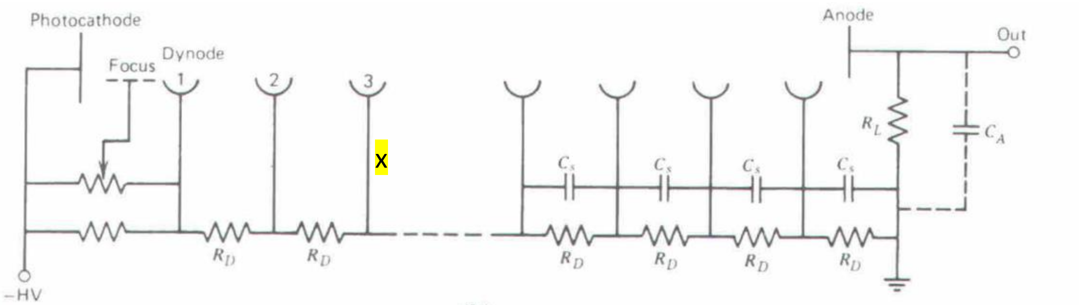
PMT electrodes biasing network



(a)

A PMT has a cathode, then dynodes which need to be bias at more positive voltage (in the chain voltages are increasing because electrons are moving towards positive voltages).
We don't apply independent voltages, but in practice the PMT is biased through a voltage divider. We apply the big voltage difference between the photocathode and the anode. In the scheme we ground the photocathode and we apply +2000V to the anode. We have create a big voltage difference.

Then with a chain of resistors we create all the intermediate voltages; all the dynodes are simply taken as intermediate voltages in this chain.



(b)

Figure 9.13 Typical wiring diagrams for the base of a PM tube. Scheme (*a*) utilizes positive high voltage and a grounded photocathode. Scheme (*b*) uses negative high voltage, and the photocathode must be isolated from ground. Values of the divider string resistors R_D are chosen using criteria given in the text. The equivalent anode load resistance is R_L in (*b*), and the parallel combination of R_L and R'_L in (*a*). Other identifications are given in the text.

The alternative solution is that we apply a negative voltage on the photocathode, and we ground the anode. The importance is that the voltage difference between anode and cathode is positive.

Advantages and drawbacks

The advantage of the first solution is that the photocathode is grounded, that is a relief because in this way we have the window of the photomultiplier tube which is touching the scintillator that is grounded. The disadvantage is that we have the amplifier connected to the anode, for instance the charge preamplifier.
Of course we cannot connect an amplifier to 2000V, so we need some decoupling, for instance the capacitor Cc, that is used to decouple the 2000V of the anode with the amplifier. → we need a AC coupling through a capacitor.

The advantage of the second solution is that the anode is grounded and can be connected to the virtual ground of the amplifier → the anode is to the virtual ground of the amplifier.
The disadvantage is that the photocathode is at -2000V (so the coupling with the scintillator is at -2000V).

Moreover, in the resistive divider there are also some capacitors placed between resistors, there is a chain of capacitors in parallel to the resistors, and not on all the dynodes, but just on the final ones.
We have these capacitors because if we bias the dynodes through the voltage divider, where does the electron released into the vacuum come from? They come from the chain of resistors.
So we can imagine to have electrons emitted from the dynodes and hance a corresponding current drawn form the lines x.
This current comes from the resistors; if this signal current is too large, and especially in the last dynodes (because the first dynode has to emit few electrons, but the final dynode has to emit 10^7 electrons), this current flows into the resistors Rd and create a voltage drop. This voltage drop changes then the voltages applied to the dynodes themselves → nonlinearity is introduced, because the change is signal-dependent (= nonlinearity).

We use the capacitors because the capacitors are an escape path for this signal current. In fact, a capacitor in AC with fast signals are considered short circuits → chain of shortcircuits, so the current from the dynodes doesn't take the path through the resistors, but it takes the path through the capacitors. At first approximation, the current for the dynode is supplied through the capacitors.

POSITON-SENSITIVE PMTs

The PMT is not able to identify the position of interaction of the photon, because the path of the PE is always the same and we simply get a total charge corresponding to the photon detection. It would be convenient to have the possibility to also identify the photon position of interaction → position sensitive PMTs.

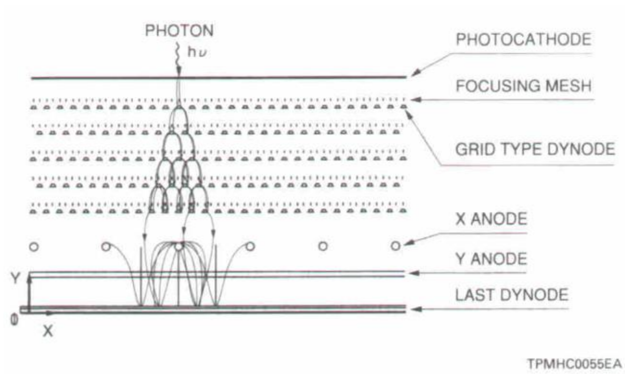
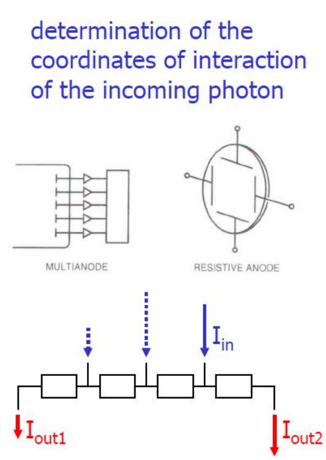


Figure 6-2: Electrode structure and electron trajectories in a position-sensitive photomultiplier tube

It is based on the same topology and components of the classical PMT, but the exit anodes are not just one electrode, but they are several and perpendicular and thanks to this subdivision we can have the total number of electrons at the output and moreover, depending on the charge collected by each anode, we can extract the position of interaction.

So, we have one photon interacting in the photocathode, we produce a single PE that is accelerated and we have that the dynodes are now segmented in a mesh of several dynodes. So we have a first layer of dynodes, then the production of secondary electrons that are spread and accelerated in the lower second floor of dynodes and so on. We have a cascade that enlarges and that is symmetric, because the device is symmetric. We have no more a tiny path of PE, but a cloud of PE.



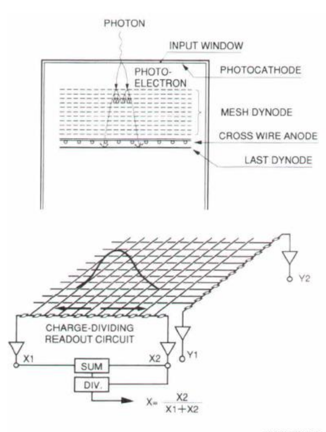


Figure 6-6: Center-of-gravity detection method for position-sensitive photomultiplier tubes using cross-wire anodes

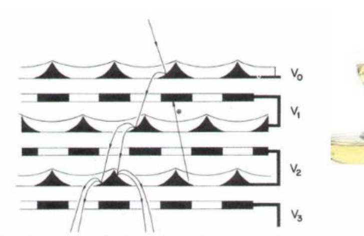
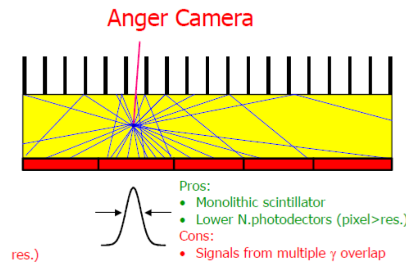


Figure 9.23 Cross section of a focused mesh electron multiplier. Each cusp-shaped dynode layer has an associated guard plate with its holes aligned over the points of the cusps. The guard plate helps focus the emitted secondary electrons onto the next dynode layer and also stops any secondary ions (the upward arrow shown) from causing ion feedback. (From Vallerga et al.[83])

We have a cloud of electrons and then finally the electrons are collected by X anodes and Y anodes. In this way we have a sampling of the cloud; having a sampling of the cloud means that the anode closest to the centroid of the loud will receive more charge, the anode further from the cloud will receive less charge and the anode completely outside the cloud will receive 0 charge. Hence **we can calculate the centroid of the cloud that is associated to the direction of the photon**.
In reality, if we couple this device with a scintillator, the interest in gamma ray detection is not the single photo position. If we imagine to have an Anger camera for instance, we have a cloud of optical photons, but the device works in the same way; instead of finding the position of the PE as the centroid of the cloud, we will have a cloud of photons that will create a cloud of PE, so finding the center of mass of the electrons is a way to find the center of mass of the optical photons.



If we consider the Anger camera structure and the scintillator light spread all over the photodetectors (red), we have two different possibilities:

- For each photodetector (red ones) we make a single PMT. So no position sensitivity. Each one phtodetector is individual one, we measure the charge and then we compute the center of mass
- We cover the entire surface with a position sensitive PMT. So it is a single device that holds intrinsically the position sensitivity, it is no more segmented. Now we shine the device with a cloud of photons, the device determines the center of mass of electrons and the center of mass of photoelectrons electrons will be automatically the point of interaction of the gamma ray.

Details

It is not true that the charge will go directly from the dynode layer before the anode to the anode; they are going first to a last bottom dynode that is common to everyone, not segmented as the others. There is a strong hit and multiplication in from this dynode and the charge exiting from this dynode is collected. This is done for two reasons:

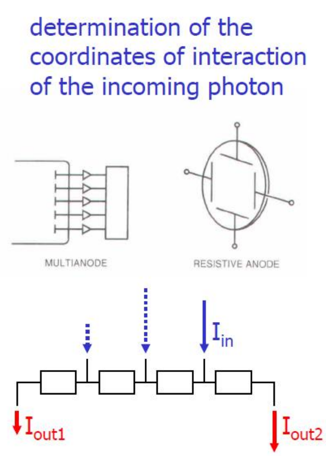
1. It is used to provide the charge with a larger width. In fact, if we would rely on the chances that the charges exiting form the normal last dynode would reach the anode, this occurs maybe yes, maybe no. it would be a disaster if all the charges of the las segmented dynode will go to only one single anode, because the spatial resolution will be simply the pitch between the electrodes. On the contrary, if we let the charge to spread over more than one anode, by the calculation of center of mass of a spread signal, we can find the position with a resolution that is better than the pitch between the anodes.
   If the current out of the dynodes will reach just a single anode, the resolution is the resolution of the single anode. So if two anodes are separated by 2mm, the resolution is 2mm. But if we can spread the charge over multiple anodes, there will be of course a dominant anode, but there will be neighours that will receive a 'piece of signal'. This is the best resolution we may have to have a 'subpixel resolution', because when we calculate the center of mass, we may find the position with a resolution that is better than the distance between the anodes.

2. For timing reasons; we have some techniques like PET in which we need timing. Where to collect the signal for the timing?
   We would like to have the charge spread over the anode, not all the charges going in one single electrode, because this is not beneficial for timing, I would like to have the highest possible

signal. This is what is done by the last dynode, because it will receive the total signal, and so it is the best candidate for timing.
So I let the anodes to be used for position and retrieval, and the last dynode for timing purposes, because it have the largest signal.

Readout

I have now multiple anodes and I have to collect and measure the currents and, thanks to an algorithm, find the center of mass of the charges.



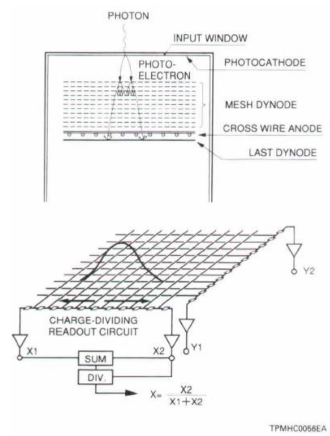


Figure 6-6: Center-of-gravity detection method for position-sensitive photomultiplier tubes using cross-wire anodes

The brute force approach is to connect an amplifier to each anode (multianode). It is however very expensive. There is a smarter solution that is using voltage divider, a resistive chain. Instead of reading out each individual anode, we connect all the anodes to a chain of equal value resistors and we read the currents at the left and right end (Iout1 and Iout2).

We can imagine that the ratio between the two currents Iout1 and Iout2 depends on where the cloud has been collected in the anodes. For instance, let's suppose that we have a single current on the anodes Iin; this current will see less impedance on the right and more impedance on the left. So the current will go preferably to the right, so if we measure Iout2/Iout1 it will be a large ratio.
It Iin would flow in the middle, we would have an equipartition of the current, so the two boundary currents will be the same.

So we use the ratio of the currents to measure the centroid of the cloud. More in detail, we measure the two currents, we make the sum and then we calculate the coordinate X by taking one current and divide by the sum. We also do it for the Y.

It is a 'wired' center of mass calculation, done on hardware. To calculate it in a more sophisticated way we need to extract the currents with the amplifiers.

Spatial resolution

The following is a plot of the spatial resolution, that is in the order of mm. We can change the resolution by changing the voltage difference between the photocathode and the first dynode. By changing the voltages we squeeze or enlarge the electron cloud, so we change the spatial resolution.

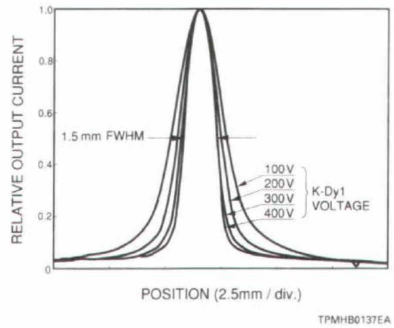
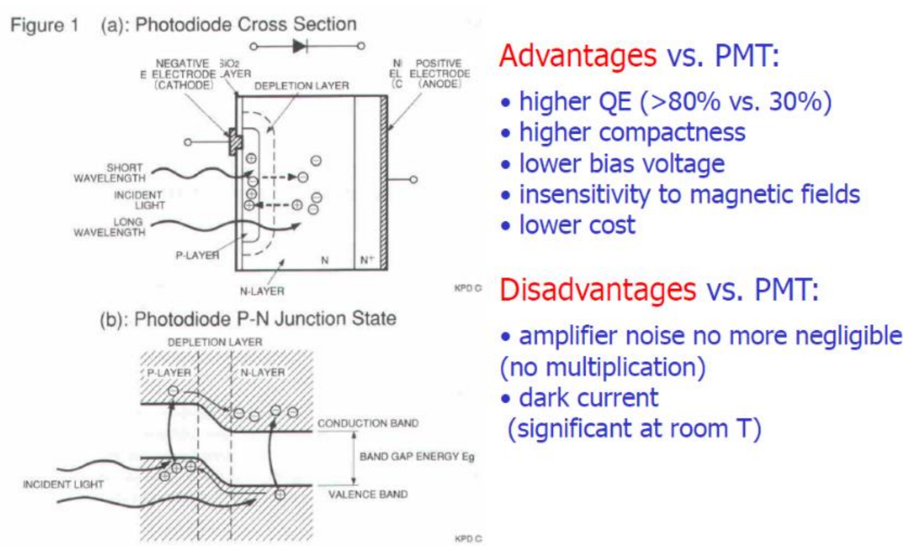


Figure 6-4: Spatial resolution between the photocathode and the anode at different photocathode-to-first dynode voltages

THE PHOTODIODE (2)



A Si PD is a pn junction. **The energy cost for the photon is the one to promote an electron from the VB to the CB**, no more to the vacuum level, because the electron must not be separated from the material, I will collect the electron by an electrode in the device. Electrons will be collected on the n electrode (anode) and holes in the p electrode (cathode).
The depth of interaction of optical photons allows us to say that short wavelength photons are absorbed in the layer closer to the surface of the device. Longer wavelength, moving toward infrared, are more penetrating in the device and so the promotion is happening in the n region.
NB: of course also for optical detection the wavelength matters, because shorter one are absorbed at the surface and longer into the depth, but there isn't the problem of xray absorption, where I need a very large depletion layer. In a normal Si photodiode the depletion region is reasonably small and all the photons will be absorbed. It is a completely different business the photodetection from the radiation detection.

Advantages vs PMT

- High quantum efficiency
- They are 2D devices, because they are made on a Si wafer, while a PMT is a 3D device.

- Insensitivity to magnetic field: nowadays there is a big trend of multimodality imaging and so can be placed in a high magnetic field. PMT cannot be used in magnetic resonance because the magnetic resonance deflects the path of the primary electrons. They deflect because the path is millimeters or centimeters long, so the deflection by the Lorentz force is enormous. In PD of course the same law applies, but what matters is that the travel distance is much smaller. Of course, we have the deflection due to the magnetic field, but the path is of microns. That's the reason why in MRI systems only Si readout of scintillators are accepted.

Disadvantage vs PMT

- Lack of gain: the amplifier noise must be accounted, cannot be considered negligible as in the PMT, so we need to design the amplifier carefully because its noise matters.
- The dark current, that is due to the thermal promotion of electron to the CB, is producing shot noise. In PMT the electrons must be promoted over the vacuum level (and this was a problem), but this implies that the dark current is negligible, because the probability that an electron is promoted outside the vacuum level and is able to escape from the cathode is negligible. So PMT has negligible dark current.

Quantum efficiency

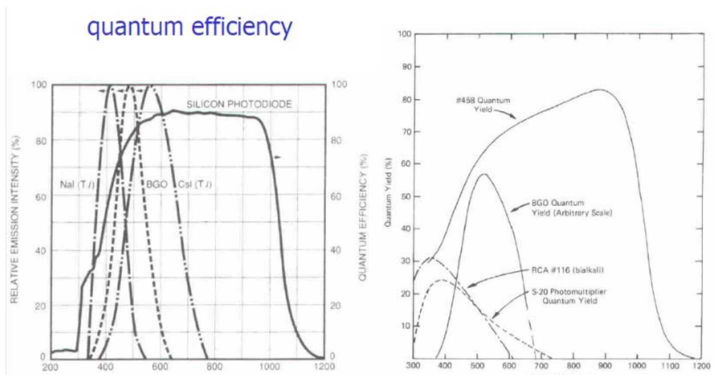


Figure 9.15 A comparison of the quantum efficiency of a silicon photodiode (labeled #458) with representative bialkali and S-20 photocathode quantum efficiencies. The emission spectrum from a BGO scintillator is shown for reference. (From Groom.[63])

The quantum efficiency of the PD is very good, especially in the high wavelength ranges where PMT are dropping down the QE dramatically. We have on the left the QE of a Si PD compared to some emission spectra of notable scintillators.

AVALANCHE PHOTODIODE (3)

It is a trial to combine the benefits of PMT and the PD (multiplication of PMT and quantum efficiency of PD). The **APD is very good because it has a very good quantum efficiency and multiplication**.

In APD, the classical pn diode is a reversed pn junction; image 1: p electrode on the left and is assumed to be the entering site for the light, and n electrode on the right. In addition we add a thin layer of p doping close to the n side. This p doping, together with the n doping, creates a very highly doped pn junction. If we have a highly doped pn junction we have a very high electrical field in the juntion.
On the bottom (2), we see the electric field across the device. In the internal region of the device we have the classical electric field responsible for the separation of charges (moderate electrical field just with the goal of separate charges), but on the right side we have the 'multiplying region' where we

have a peak of electric field. All the charges entering in this region will make **impact ionization**, so will be accelerated and after acceleration they will start to create ionization, electron-hole pairs.

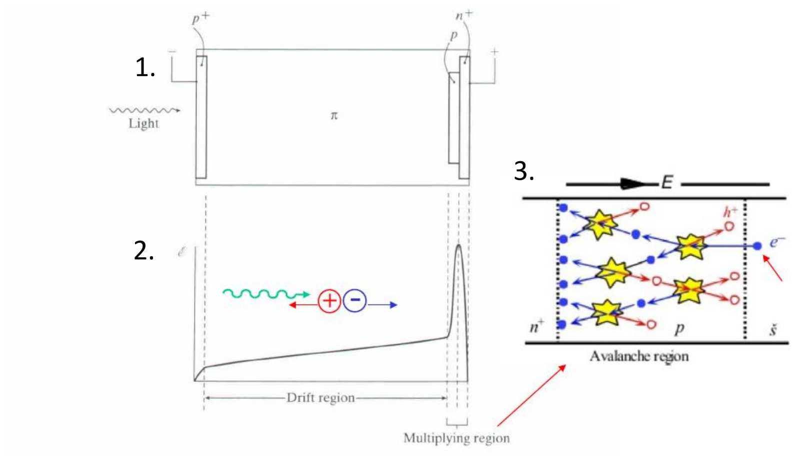


Figure 9.17 The reach-through configuration for an avalanche photodiode is sketched at the top of the figure. Below is a plot of the resulting electric field when a bias voltage is applied.

Mechanism

If I assume that the light is coming from the left, I will have the creation of electron-hole pairs on the left with respect to the multiplication region. Holes will go to the left and won't experience multiplication, electrons will go to the right and so we will deal with electrons in the multiplication.
It could be also the opposite, but it is not the case of drawing 2.
Drawing 3 represents what happens. Take care, the image is inverted with respect to the 2, photon is entering form the right.

If an electron is entering from the multiplication region, it is accelerated immediately, because the electric field is enormous, is making an impact ionization and creating an electron hole pair. So now I have two electrons, the starting one and the secondary one. They are accelerating and creating other electron-hole pairs, and so I have multiplication of electrons.
Also holes, which were not existing before are accelerated and also doing impact ionization. Hence it is an enormous multiplication.
At the end of the story, electron will exit from the left, holes from the right and so I will collect a number M of charges, either on the right or on the left.

At the end of the story, the n+ electrode that has been elected as the collecting one, where the amplifier is placed, will collect the enormous number of electrons at the exit of the avalanche region. On this electrode I get the electrons multiplied.

However, there is a problem, and **the problem is the dark current**; in the material I have spontaneous generation of dark currents. So time to time I have the creation of electron-hole pairs and the electrons also enter in the multiplication region. The shot noise of the dark current is already a problem because it is a source of noise, and in addition it is multiplied.
Hence multiplication doesn't properly elevate the signal against the noise, because the noise of the amplifier can be neglected, but the shot noise of the dark current experiences the same multiplication.
→ problem.

Output signal

The output signal from the multiplication will be given by the primary electrons created in the material multiplied by M. We can see that M labelled with 'e', so Me, means that the multiplication has been initiated by one electron. Conversely, if the avalanche is initiated by one hole, the multiplication factor will be different (and lower). This because holes ionize less than electrons. The fact that the initial hit is made by a hole makes the whole consecutive avalanche lower.

output signal $\quad N_{Sout} = N_S M_{(e-)} \quad$ N_S: primary e- generated
$M_{(e-)}$: multiplication coeff. for e-

variance of the output signal $\quad \sigma^2_{Sout} = N_S M_{(e-)}^2 F_{(e-)} \quad$ $F_{(e-)}$: excess noise factor (multiplication due to e-)

$$\boxed{S/N = N_{Sout}/\sigma_{Sout} = \sqrt{(N_s/F_{(e-)})}}$$

$F_{(e-)} \sim 2\text{-}3$

worsened with respect to:
$S/N = \sqrt{(N_s)}$

output noise spectral density due to dark current (I_D) $\quad = 2qI_D M_{(e-)}^2 F_{(e-)}$

this noise component also worsens by a factor $F_{(e-)}$ (with respect to $2qI_D M_{(e-)}^2$)

Variance of the output signal

The output variance in multiplication is equal to the input variance that is Ns (due to Poisson statistic) multiplied by the square of the multiplication.
Unfortunately, the formula Ns*M^2 is not rigorous because it doesn't take into account that the multiplication is a statistical phenomenon, and not a fixed parameter.
Hence we have a worsening factor in the formula, Fe, called excess noise factor, which takes into account the spread of the multiplication.

This quantity comes from the fact that Fe is equal to the sigma square we have (the one we really measure) with respect to the sigma square we expect, that is the number of photoelectrons time M^2 (the one we expect if the multiplication would be deterministic).
If we flip this formula, the sigma at the output is the noise we would expect times F.

81. $$F = \frac{\sigma^2}{N_{ple} M^2} \longrightarrow \sigma^2 = N_{ple} \cdot M^2 \cdot F$$

The F factor, that is a factor larger than 2, ideally should be one.
If we compute the SNR, we take the signal that is Ns*M; we take the sigma at the denominator just calculated and the ratio is the one in the formula.
The SNR is we would expect for a deterministic multiplication is sqrt(Ns), if we include the worsening due to the multiplication, we have sqrt(Ns/F), and since F>2, it is worse. This is the price to be paid for multiplication.
The same formula applies for the PMT (the worsening factor is called differently from F).

Contribution of the dark current

Moreover, not only the SNR has worsened, but the shot noise due to the dark current, where 2qId is the initial shot noise in the drift region (not the multiplying region), is worsened by the multiplication, because this noise current is also entering in the multiplication; it is worsened by the same factor F, because 2qId is the variance, multiplied then for M^2 and F.
This is a problem, because I'm multiplying the dark current, which is a problem in photodiodes.

There is a way to reduce the effect of the dark current. About the statistic of the signal, we cannot do much, no way to reduce the variance of the output signal. Unfortunately, F at first approximation is proportional to M → the more we increase M, the more we boost the multiplication to reduce the effect of the amplifier noise (we consider multiplication to drop the noise of the amplifier indeed), the more F increases.

Improvement of the shot noise contribution

The shot noise is a problem related to the fact that if we put the multiplication on the right side and the light enters from the left, signal and dark current follow the same story, there is no way to split them. We can split them if we consider a different geometry of the device (x), where the avalanche region has been placed on the left.
When the multiplying region is on the right, we add a p implantation close to the n implantation, whereas if we want to create it on the left, the polarity of the multiplication is n, we have to add an n silicon.

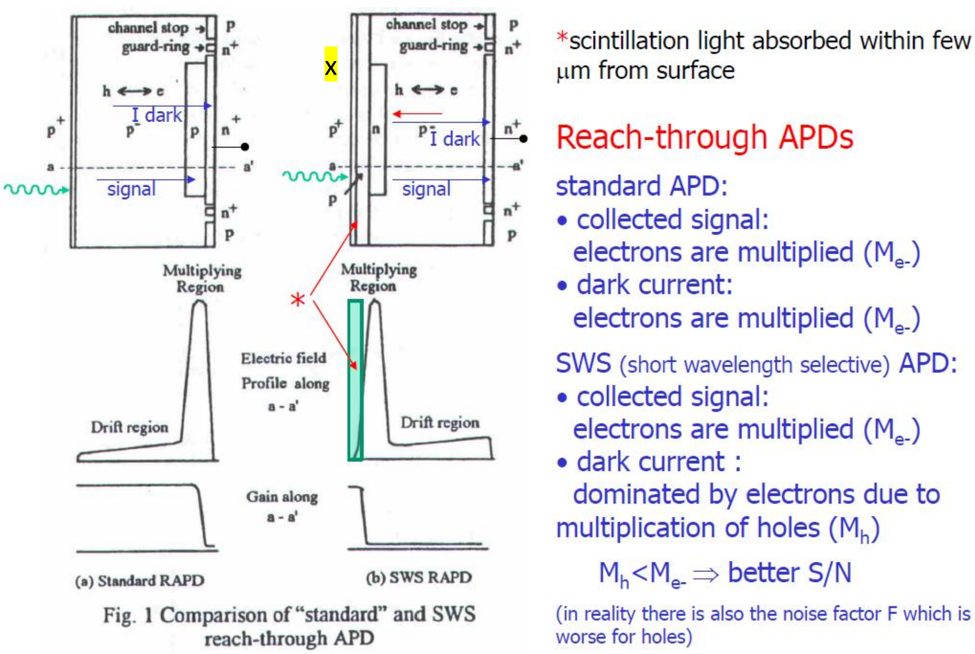


Fig. 1 Comparison of "standard" and SWS reach-through APD

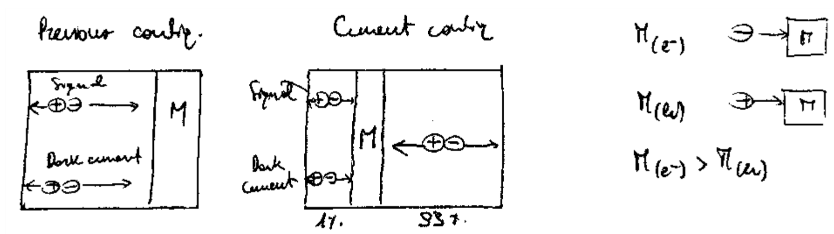
SWS – Short Wavelength Selective

Besides the previous details, we still leave a shallow region close to p where the radiation can be absorbed. We have to imagine that the device, called short wavelength selective, is designed for short wavelength light, so close to the UV, that are absorbed more to the region of the device close to the implantation, while long wavelength light is absorbed more internally.
If we consider a short wavelength and the n implantation is not placed really close to the contact, but we have the p+ contact, then a region where we absorb photon and the n the n implantation, we have

the multiplying region on the left, but not really close to the contact, we still have a region where we absorb photons.

If photons are of short wavelengths, the signal is generated only in the green region of the image. This region in green is on the left of the multiplying region, so are holes or electrons entering in the multiplication region? Electrons. So the electrons are still entering to the left of the multiplication region. So having moved the region is irrelevant for what concerns the signal, because it is entering in the region from the left anyway.
What is different is the dark current, because now most of the dark current is generated on the right of the multiplication region, not on the left as before. So now are holes and not electrons entering in the multiplication region.

82.



In the previous configuration, the multiplying region is totally on the left. Hence for the signal we have electrons that are multiplied. For what concerns the dark current it is the same, the electrons of the dark current are entering the multiplication region.

Now the device is different; we have a thin layer of generation of the signal, because we have short wavelengths, so electrons/holes pairs are generated at few microns from the surface; then we have the multiplication region and the question is: which is entering in the multiplication region? For the signal are still electrons entering in the multiplication region.
As for the dark current, I can consider it in the volume of the device, so generated on the right of the multiplication region. So now dark current electrons are collected without entering in the region, while holes are entering → **now the signal electrons are multiplied, for the dark current holes**.

We may have dark current in the small piece of volume on the left of the multiplication. In this case that small amount of dark current experiences the multiplication as before. But, overall, the largest contribution of the dark current is created in the rest of the volume, not in the thin layer.

We have done this configuration because the multiplication factor when it is an electron entering in the multiplication region is larger than the multiplication factor when it is a hole entering in the multiplication region. So if an electron is starting the multiplication, the number of secondary charges will be larger than in the case of holes, because holes have lower ionization properties than electrons, they are more lazy to be accelerated and impact in other charges.

Knowing this property (that is that avalanche is more trigger by electrons than holes), having split the story of electrons and holes, electrons experience the multiplication factor of the electrons, which is good, because for the signal I'm glad that I can use the large multiplication; as for the dark current, I have to compute its multiplication considering the multiplication factor for the holes, that is lower. The hope is that at the end of the story, the SNR will be better than in the previous case, because signal and dark current don't experience anymore the same multiplication factor (that previously was given only by electrons).

Computations of the SNR

Conventional Reach-through APD

$N_{Sout} = N_S M_{(e-)}$

$\sigma^2_{I\ dark\ out} = \sigma^2_{I\ dark(e-)} M_{(e-)}^2 F_{(e-)}$

$\boxed{S/N_{dark} = N_s / (\sigma_{I\ dark(e-)} \sqrt{F_{(e-)}})}$

$\sigma^2_{I\ dark}$: variance associated to the shot noise of the dark current ($2qI_D$)

Reach-through SWS APD

$N_{Sout} = N_S M_{(e-)}$

$\sigma^2_{I\ dark\ out} = \sigma^2_{I\ dark(h)} M_{(h)}^2 F_{(h)}$

$\boxed{S/N_{dark} = N_s \times M_{(e-)}/M_{(h)} / (\sigma_{I\ dark(h)} \sqrt{F_{(h)}})}$

$\boxed{\begin{array}{l} M_{(e-)} > M_{(h)} \\ F_{(h)} > F_{(e-)} \end{array}}$

it is higher than the previous thanks to $M(e-)/M(h)$

Conventional reach through APD

Let's consider the SNR for the two devices. As for the conventional one (the one on the left), it is given by the number of charges Ns per the multiplication factor Me.

The sigma square of the dark current is the one of the dark current originated in the body multiplied by M^2 and F. 'Sigma-square-Idark' is the shot noise (that is a noise spectral density) integrated over the bandwidth of the amplifier. So the noise spectral density doesn't give us directly the sigma of the noise, we have to integrate all over the bandwidth of the amplifier. The important thing is that, at the end of the calculation, whatever is the bandwidth of the amplifier, I call Sigma-square-Idark the fluctuation due to the dark current. This fluctuation is then multiplied by M^2 and F.

If we take the signal Nsout and we divide it by the sigma of the noise, again M cancels out because they are identical. The one obtained is the SNR between signal and dark current.

Reach through SWS APD

The signal is still given as in the conventional APD, but the formula for the dark current, its sigma square, is different. It is given by the sigma square of the original dark current (same as before), but this value must be multiplied by an Mh factor, that is the multiplication factor if a hole is entering in the multiplication region.

Then we have also a different F, Fh, because it Mh is different, also the statistic of the multiplication is different.

If we compute the SNR now, in the computation the M doesn't cancel out. And moreover, the Me/Mh ratio is larger than 1, because Me>Mh. → beneficial factor.

We simply have to take into account that the excess noise factor F is the one due to holes now at the denominator, and unfortunately, Fh>Fe.

In conclusion, due to the fact that Me>Mh we have a benefit, but in reality this advantage is a little bit reduced because Fh>Fe.

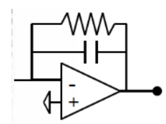
Last details

This configuration of the APD has been done to reduce the impact of the dark current. But the absorption layer for the signal is limited (the mechanism works only for short wavelengths, otherwise also for the signal the avalanche would be triggered by holes).
But if the signal is absorbed so close to the surface, why not just reducing the thickness of the device? I could make it very short, so to have a volume of dark current much smaller, and solve the problem of the dark current.
The reason why this is not done is because if a very shallow pn junction is made (with just the multiplication in between), the capacitance of the junction would be very large → if the depletion region is very small, the depletion capacitance is very large, while if I make a pn junction very wide, the capacitance between the two terminals is very small.

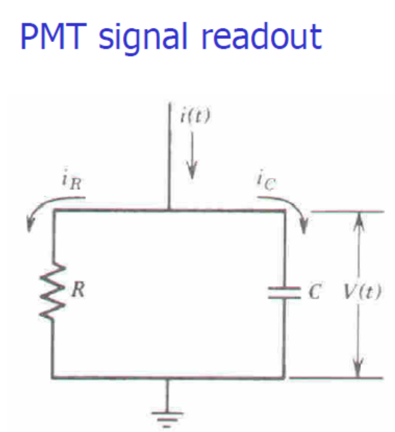
We will see later in the course that the capacitance of the readout electrode has a fundamental importance for the electronic noise. **The larger the capacitance of the electrode where we collect the charges, the much larger is the noise**.

PMT SIGNAL READOUT

i(t) is the current at output of the photodiode. The reasoning is done over the PMT, but the same applies for the avalanche photodiode or the photodiode.
Whatever is the readout, the current at the output electrode is sent to a parallel of a capacitor and a resistor. It could be that we connect directly the current to the parallel of a resistor and a capacitor or alternatively we can use the solution of a charge preamplifier, so we connect the exit node to a virtual ground and the current is integrated in the parallel of a capacitor and a resistor.



The current is read out into a parallel of a resistor and a capacitor. Which is the optimal tau of this network?



$$\boxed{V(t) = 1/(\lambda-\theta)\,\frac{\lambda Q}{C}\,(e^{-\theta t} - e^{-\lambda t})}$$

$$\theta = 1/RC$$

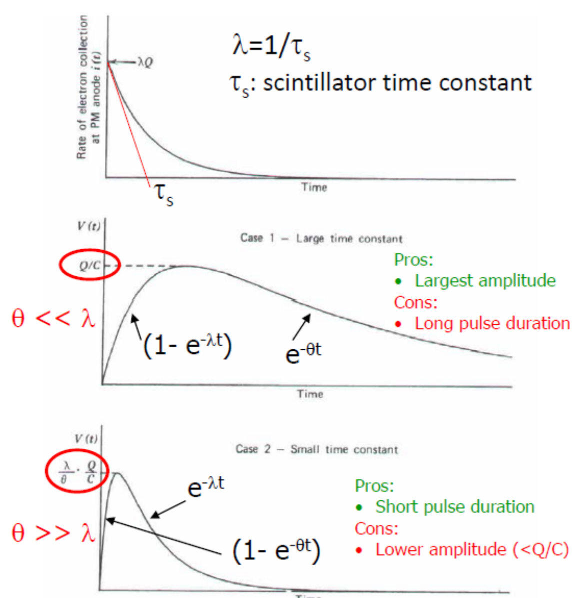


Figure 9.19 For the assumed exponential light pulse shown at the top, plots are given of the anode pulse $V(t)$ for the two extremes of large and small anode time constant. The duration of the pulse is shorter for Case 2, but the maximum amplitude is much smaller.

If we consider the resistor very large, in principle infinite, all the current will integrate over the C and we will have a step. On the contrary, is R is small, while we are integrating the current on the C, the charge is simultaneously discharged through the RC network.

We need to take the shape of the current and then we make a convolution with the RC network. Which is the shape of i(t), the current exiting the photodetector?
Since we are discussing scintillator photodetectors, the shape of i(t), if we assume an extremely fast photodetector, so the photodetector is not relevant in terms of signal generation (the collection properties of the signal and the travelling time of the signal in the photodetector are zero), who is dominating the time response of the signal? The scintillator.
Hence **the signal is assumed to be exponential, and the tau of the exponential is the scintillator's time constant** (tabulated). Tau-s is the scintillator response, lambda its reverse.
Q is the integral of the current pulse (total charge delivered by the photodetector). So the first plot is the shape of the input current.

Then we make a convolution of this signal with the time response of the RC network and V(t) is the output of the network, our measurement.
We can see that the output is the combination of two exponential factors; one is associated to the **tau of the scintillator**, one is characterized by theta, that is **1/tau-RC**.

Plots 2 and 3 are the two extreme cases of the output voltage.

- **Plot 2 (theta << lambda)**: the tau of the RC network is much larger than the one of the scintillator (like if the resistor was infinite), or on the contrary theta factor is much smaller than lambda factor. In this case we can simply conclude that the peak value of the signal is the charge divided by the capacitance. In fact, if the resistor is very large, the total charge will integrate over the capacitor C. The peak value is Q/C.
  There is a rise time and rise time of the function is dominated by the scintillator time constant. The rise time has the 'shape' of the scintillator response; so the faster the scintillator, the steeper the response, but the peak is anyway Q/C.
  The fall time of the pulse is dominate by the RC network; the exponential decay is dominated by theta, so by the time constant of the circuit.

  In this case the advantage is that this readout modality provides the largest possible amplitude, that is Q/C, we can't do better. The cons is that the pulse takes a lot of tie mot be extinguished, and this is not good because of pile up → tails of pulses may pile up one over the other.

- **Plot 3 (theta >> lambda)**: the alternative is making the RC network much faster than the scintillator time constant (small R). If R is small, the resistor is going to draw charge while the current is integrating over the capacitor. By looking at the graph, we can imagine that with this modality I cannot reach the same peak as before, because the charge will never be fully integrated on the capacitor → the peak is no more Q/C, but lower. The advantage of this solution is that the rise time is now dominated by the RC network, and the fall time by the scintillator; hence the faster the scintillator, the shorter the pulse we get. The choice of the scintillator has a direct impact in the duration of the pulse.

  In this case the pro is that the pulse is much shorter than before and so we can avoid pile up effect, while the cons is that the amplitude is lower.

In the dimensioning of the RC network we hence have to decide if we privilege speed, a narrow pulse or amplitude (integrate as much as we can the charge).

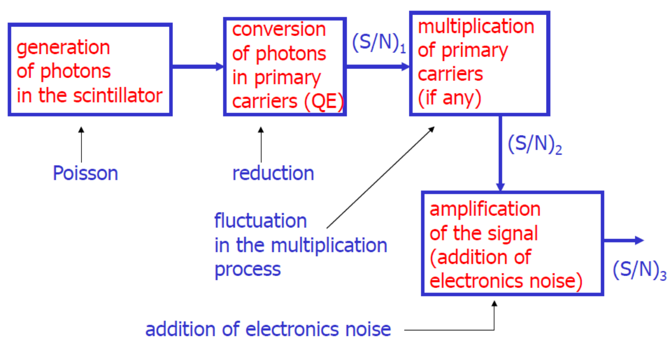
COMPARISON BETWEEN DIFFERENT SCINTILLATION PHOTODETECTORS

PMT
- gain
- reliable/mature fabrication and several available topologies
- low quantum efficiency 30%
- costs, size
- high bias voltage, sensitivity to magnetic fields

PD
- high quantum efficiency 80%
- small size, low bias voltage, low cost
- fabrication of arrays of uniform units
- lack of gain (amplifier noise plays a role)

APD
- gain
- high quantum efficiency
- small size
- gain fluctuation, high bias voltage
- difficulty to fabricate arrays of uniform units

Comparison: the fluctuation of the produced signal



Let's consider just the SNR. The two main sources of noise are the intrinsic fluctuation of the signal itself, because starting from the scintillator, the scintillator produces a different number of photons for the same energy, and then I have the electronic noise.
Multiplication reduces the impact of the electronic noise, but on the other size it worsens the statistic of the signal. How to combine this in a single formula?

The SNR has changed along different steps. At the beginning, the scintillator is producing photons, that are delivered with a Poisson statistic (statistic that describes very well a scintillator). Hence the variance of the emitted photons is equal to the average number of photons.
Then I have the quantum efficiency of the photodetector; in fact, not all the scintillation photons are converted in photoelectrons, but a lower number depending on the quantum efficiency. The statistical model that describes the SNR after the conversion, if we take a Poisson statistic followed by a quantum efficiency (whose exit is 1 or 0, converted or not converted photon), is still Poisson statistic. Hence a Poisson statistic at the scintillator followed by a quantum efficiency (that is also a statistical phenomenon), then we have still a Poisson statistic.
The statistic is hence still Poisson, but with a worse SNR, because the number of photoelectrons is smaller than the number of photons.

The SNR is given by the square root of the number of the carriers. The first number of carriers is Ns, the number of scintillation photons, while in the second case is the number of photoelectrons, that is smaller due to quantum efficiency.

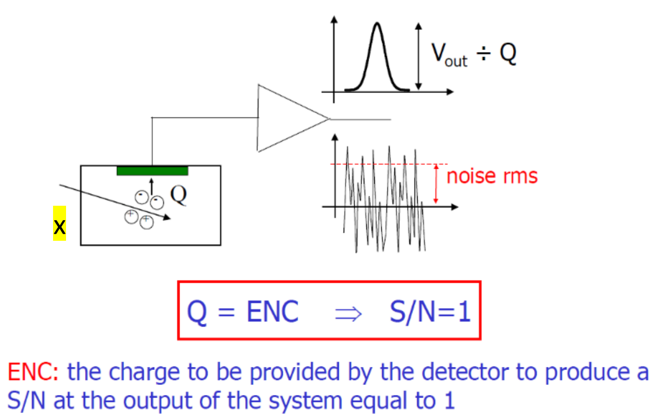
83. N_s = n° of scintillation photons produced by a scintillator → $SNR = \sqrt{N_s}$
After the conversion, the n° of photoelectrons is: $N_{phe} = \underbrace{N_s QE}_{< N_s}$ → $SNR = \sqrt{N_{phe}}$

Scintillator produces a number of scintillation photons Ns, and SNR is SNR = sqrt(Ns).
After the conversion I will have the number of photoelectrons, Nphs, given by Nphs = Ns*QE, where QE is the quantum efficiency and Nphs<Ns.
Since we still have a Poisson, SNR = sqrt(Nphs), that is worse because Nphs<Ns.

We are just at the level of the photocathode of the PMT or in the region prior to the multiplication (SNR1) → the next step is the multiplication. Also the multiplication changes the SNR (SNR2), because of the worsening due to multiplication, and in the end we have also the amplifier noise that changes the SNR. We need to find the SNR after the signal has passed the amplifier (SNR3).

Before doing so, we need to quote the noise of the amplifier with a definition which is consistent with all the other statistical definitions, which are defined by the sigma-square, the variance of the fluctuation of the signal.

Amplifier electronic noise: the Equivalent Noise Charge (ENC)
The amplifier noise, which is usually defined as noise spectral densities, equivalent noise generators, is not homogeneous with the theory of the previous steps, because it was based on sigma-squares of the signals, fluctuations of the signals.
I need to explain the noise of the system in an equivalent fluctuation of the signal, and this can be done with the definition of the noise as the ENC. It is an equivalent fluctuation of the signal due to the noise of the amplifier.

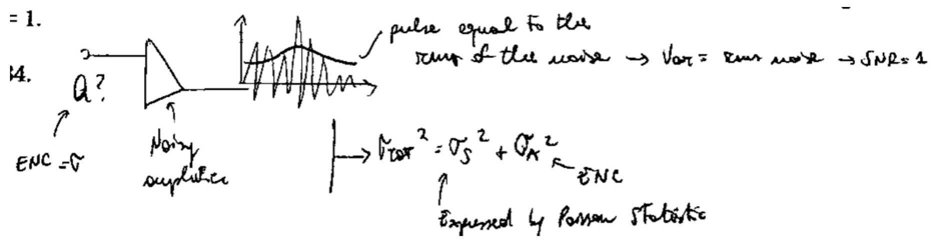


We have a detector (x), the detector is providing a charge Q that passes through the electronics and produces a pulse. The amplitude of the pulse is corresponding to the charge delivered by the detector, so the larger the charge the larger the voltage.

Now, the amplifier is noisy. This means that if we don't provide charge to it, at the output of the amplifier we still see a statistical fluctuation that is due to the various noise generators inside the amplifier. So we don't have any variation but we see the noise.

To quote such noise at the output, we quote it as the rms. But, if we have the calculation for a charge Q that is converted in a voltage Vout and now we have the quote in voltage of the noise, we can define an equivalent charge at the input that provides a pulse matching the noise value, so matching the SNR = 1.

84.



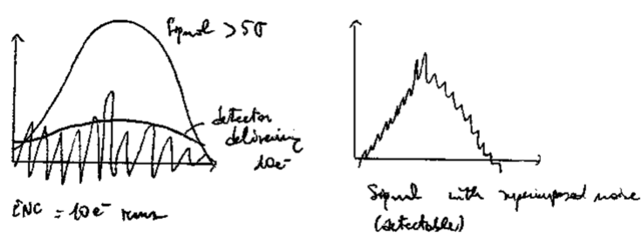
We have the amplifier, the amplifier is noisy and now we simply want to find the charge Q that, delivered to the system, provided a pulse with the amplitude equal to the rms of the noise? This means a SNR = 1 at the output.
This charge Q assumes an important role and it is called equivalent noise charge. <u>It is the charge, expressed in Coulomb or number of electrons, that provides at the output a voltage equal to the noise</u>.
Hence the ENC has now the statistical meaning of a sigma, because it is a charge, a signal, matching the sigma of the noise at the output. So it is the statistical variable I was looking for, because in all my SNR considerations the noise is always expressed as sigma.
In the end, the total sigma of the fluctuation will be the total sigma due to the signal plus the total sigma due to the amplifier. In fact, **electronic noise and fluctuation of the signal are statistically independent processes**.

Indeed, the Poisson fluctuation of the scintillator has nothing related to the thermal noise of the input transistors of the amplifier; scintillator fluctuates by its own and amplifier is noisy by its own → we can sum the statistical fluctuations as a sum of variances.

NB: ENC has a relevant physical meaning. It tells us in one single number the noise of the amplifier expressed either in Coulomb [C] or in number of electrons [ne-] (the number of electrons is the noise in [C] divided by the charge of the electron).
This is important because if we know that my electronics, my noisy electronics, has an equivalent noise charge of 10 e- rms, it practically means that, if I have a detector delivering 10 e-, the pulse will be basically not distinguishable from the noise.

Hence we know that this electronics can see pulses only if > 5*σ of the noise. So ENC tells us how much noisy is the amplifier we have.

85.



Conclusions

$(S/N)_1 = \sqrt{(N_s)}$

N_s : primary carriers generated after photodetection (note: Poisson statistics is applied where the signal is smaller)

$(S/N)_2 = \sqrt{(N_s/F)}$

F : worsening factor of the statistics of carriers due to the multiplication (e.g. noise factor in APD)

$$(S/N)_3 = \frac{N_s M}{\sqrt{(N_s M^2 F + ENC^2)}} = \frac{N_s}{\sqrt{(N_s F + ENC^2/M^2)}}$$

SNR3 is after amplification. As for the signal in this formula, I'm considering the general case of a multiplying photodetector, so Ns*M. Then, if we consider a photodiode, we put M = 1 in the formula. At the denominator we have the sigma of the noise; the sigma of the noise is the square root of the sum of all the variances of the processes involved.
The two variances summed are one associated to the fluctuation of the signal, that obeys to the Poisson statistic (Ns is the signal in electrons at the output of the photodetector) and is the variance of the signal after the process of multiplication, the other is the ENC. ENC is squared because ENC is a sigma, and if I'm looking for I variance I have to square it.

Marked in red we can see pros and cons of multiplication. Pros are that the **multiplication is at the denominator of the electronic noise**. However, the con of the multiplication is the excess noise factor F in the other term. In the classical photodiode F = 1, but in an avalanche photodiode or in a PMT it is larger than 1.

So in conclusion, should I multiply or not?

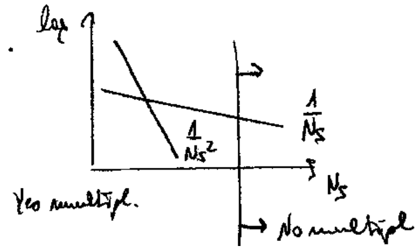
$$(S/N)_3 = \frac{N_s}{\sqrt{(N_s F + ENC^2/M^2)}} = \frac{1}{\sqrt{(F \times 1/N_s + ENC^2/M^2 \times 1/N_s^2)}}$$

1) photodetectors with multiplication improves the component related to electronics noise but worsens the 'statistics' component
2) 'electronics' component dominates for low signals with respect to 'statistics' component as $1/N_s^2$ increases faster than $1/N_s$ when N_S decreases ($\Rightarrow$ for low signals PMT/APD better than PD)
3) comparison of APD with PMT:
a) APD may have a better 'statistics' component because of the higher number of primary carried generated N_s because of the better QE
b) it is necessary to compare the multiplication statistics for them (usually better for PMT)
c) PMT has M much larger than APD. Moreover ENC^2 includes the component due to the dark current , which is multiplied by M^2 and therefore is not reduced by the multiplication. This component is usually larger in APDs

If I multiply, I reduce one contribution but increase the other. To draw conclusion, I normalize the formula by Ns.

I have that the contribution due to the statistic of the signal goes as 1/Ns, whereas the one due to the electronic noise goes as 1/Ns^2. This makes a difference.
I have also the factor in green that is the behaviour of the two contribution with respect to the amount of the signal. The term with 1/Ns^2 increases as Ns becomes small.

86.



These are the plots of the two contributions in a log scale.
We can see that the contribution proportional to 1/Ns^2, that is the one associated to the electronic noise, is getting dominant for small Ns, and is becoming negligible for large Ns.
Hence the conclusion is that multiplying or not is not an absolute decision, but **if we have a very weak signal** (e.g. very weak scintillator, the signal made out of few electrons) we are in the region where the electronic noise component is going to be dominant (small Ns) and in this case is **better to use** a PMT, so **multiplication**, because since the electronic noise is dominant, we have to reduce it as much as possible. Moreover, in this case we don't care about the other contribution to noise and the excess noise factor.

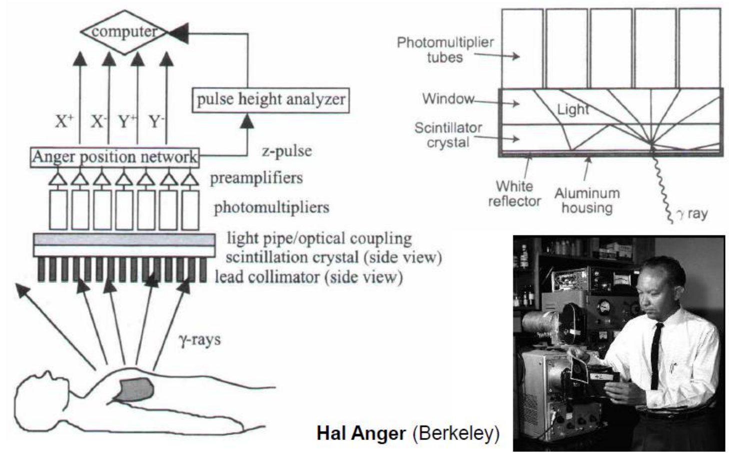
On the contrary, if we have an application with an enormous signal, the electronic noise itself is negligible, while the intrinsic contribution of the signal is going to be dominant and having a large excess noise factor is not good, because it is worsening the dominant component. In this situation **with large signal better not to use a photodetector with multiplication**, because the excess noise factor F can worsen the statistic.

As usual, the reality is in between these two situations.

If we then draw the conclusion that we need a multiplying photodetector, now we have a choice: PMT or avalanche photodiode.
To choose, we have to consider that APD has a better quantum efficiency, so 1/Ns^2 is better in the APD, because in APD for the same scintillator we have a larger number of photons.
On the contrary, PMT has a much larger M than APD. Moreover, usually the fluctuation (the statistic) of the PMT gain is lower than the statistic of the APD gain.

GAMMA CAMERAS FOR SPECT AND PET

ANGER CAMERA



Hal Anger (Berkeley)

This is the architecture of an Anger camera. It is used for scintigraphy or SPECT, but nowadays there is a trend of using more and more the Anger camera topology also or PET.
The concept is to have a unique scintillator, a homogeneous monolithic scintillator. The collimator is an external component with respect to the camera and is selecting only those gamma rays that are passing through the collimator's slabs. If one gamma ray survives the collimator, it interacts in the monolithic scintillator and produces a flash of photons that are spread over several photodetectors, that in the example of the image are PMT.

The principle of the AnC (Anger camera) is that there is an algorithm which, from the light measured on different photodetectors, through some centroids' algorithms, reconstruct the position of the gamma ray.
The advantage is that we can use pretty large PMT or photodiode and thanks to the center of mass reconstruction we can have a spatial resolution that is much better than the dimension of the pixels we are using. This is advantageous because we can use larger channel and so a smaller number of photodetectors to be read out.

Rule of thumb: the gain factor in terms of resolution vs dimension of the photodetector (the resolution we get is better than the pitch between photodetectors) is of 1/10, so the resolution can be 1/10 of the pitch between pixels. If we have pixels of 1cm we can target a spatial resolution at the mm level.
If we would like to have a 1mm resolution with a pixelated camera instead, we can only use mm pixels, because the resolution in that case is equal to the pitch of the pixels.
So **in AnC we can use a photodetector 10 times larger than the resolution we are targeting**.

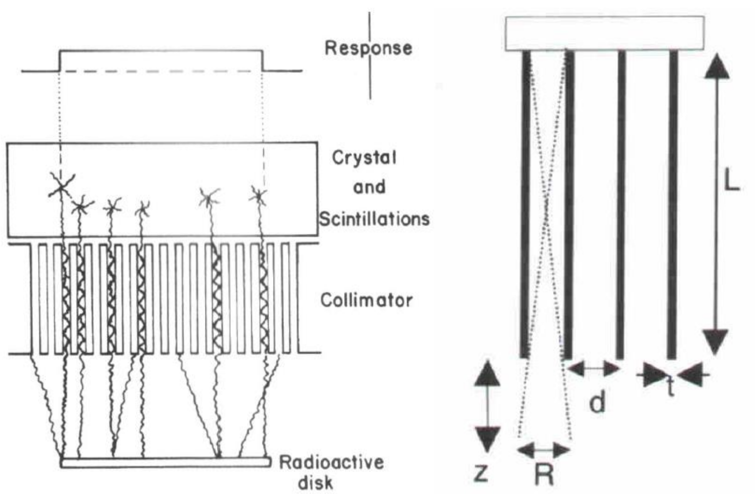
If a factor of 10 is the gain factor in terms of pixel dimension, in terms of number of channels the gain factor is 10^2, 100, because if we increase by 10 the pitch of the detector linearly, if we have to sample a surface there is a gain factor that is the square of the linear increase → potentially in an AnC we can use 100 less amplifiers.

The AnC is based on the principle that the light is spread over several photodetectors. If unfortunately the light would be collected by a single photodetector, this would be dramatic, because in this case the spatial resolution would be the pitch of the photodetector.

So in order to be sure that the light is spread all over the photodetector, in an AnC, very often, between the scintillator and the photodetector layer, there is a window of glass or quartz which is devoted to artificially spread the light further.
So if we are afraid that the light spread out of the scintillator is not sufficient, we use an interposed transparent material with an index of refraction lower than the one of the scintillator, the light is not focusing on one single point.

Moreover, to have the **highest simplicity as possible**, not only we have to have a lower number of photodetectors, but in some cases we can create some Anger positioning network, so the reconstruction of the center of mass, via hardware.
Hence in principle we can have only 4 outputs, two for X and two for Y. Of course, simplicity is not always compatible with precision.

THE COLLIMATOR



The collimator has to select only those gamma rays passing through the slabs of the collimator.
The gamma ray passes through the holes of the collimator, then the position of interaction in the scintillator is reconstructed and so once we have the position of reconstruction and we know the position of the collimator, we can backtrace the direction of arrival of gamma ray.

Of course, the geometry of the slabs of the collimator is not always parallel, we may have diverging slabs. However, the goal is always to have a mechanical mean to allow the backtrace into the patient volume what is sensed by the sensor.

Parallel holes collimator

There is a dependency of the resolution on the dimensions. A collimator with a non-zero hole don't let just the fully parallel gamma ray to pass through, but also the gamma rays that are a bit tilted with respect to the collimator walls.

In the image on the left, the two gamma rays, one coming from the right and one coming from the left are both passing through the collimator. **The tilting allowed depends on the collimator's walls**.

R is the resolution, the resolving capability. Two points separated by R are all providing a signal passing through the hole of the collimator. So the discrimination capability of the hole of the collimator is given by R.

Of course, this discrimination capability R is primarily associated to the **hole dimension** (the smaller the hole, the higher the resolution), then there is a dependency with the **slab length** (the longer the slab, less tilted are gamma rays passing through the same hole, and then there is also a dependency with z, that is not depending on the collimator itself, but it is the **distance between the collimator and the patient**.

Hence the resolution is not an absolute number, but it varies depending on where you place the collimator with respect to the organ to be imaged. The higher the distance, the worse the resolution. That's the reason why **the best scintigraphy is achieved when the gamma camera is placed close to the organ**.

Spatial resolution

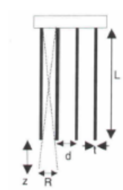
Rg is the geometrical resolution of the collimator.

R_G : geometric resolution

R_i : intrinsic resolution of the detector

$$\boxed{R_{tot}^2 = R_G^2 + R_i^2}$$

$$\boxed{R_G = \frac{d\,(L + z)}{L}}$$



Sensitivity (efficiency)

$$\boxed{S = k\left(\frac{d^2}{L\,(d + t)}\right)}$$

k : costant dependent on the detector geometry

note: for $d \rightarrow 0$
$R_G \rightarrow 0$ but also $S \rightarrow 0$
$\Rightarrow$ resolution $\longleftrightarrow$ sensitivity

Of course, the geometrical resolution Rg is only one aspect of the collimator, it second aspect is its efficiency (or sensitivity).
Indeed, the question is: how many gamma rays do I have to sacrifice in order to have a given resolution?
The answer to the question is the efficiency, that is the percentage of gamma rays that are passing through the collimator. Unfortunately, **a collimator designed for high resolution doesn't provide high sensitivity**.

Intrinsic resolution of the detector

Another trade-off in the choice of the collimator is that the **imaging resolution is not determined only by the collimator, but also by the resolution of the AnC** in reconstructing the position of interaction. So the collimator provides one contribution to the resolution, but then the camera has to provide the precision in reconstructing the point of interaction.

The resolution in reconstructing the point of interaction in the scintillator is called **intrinsic resolution of the detector**.

In conclusion, we have 2 'resolutions':

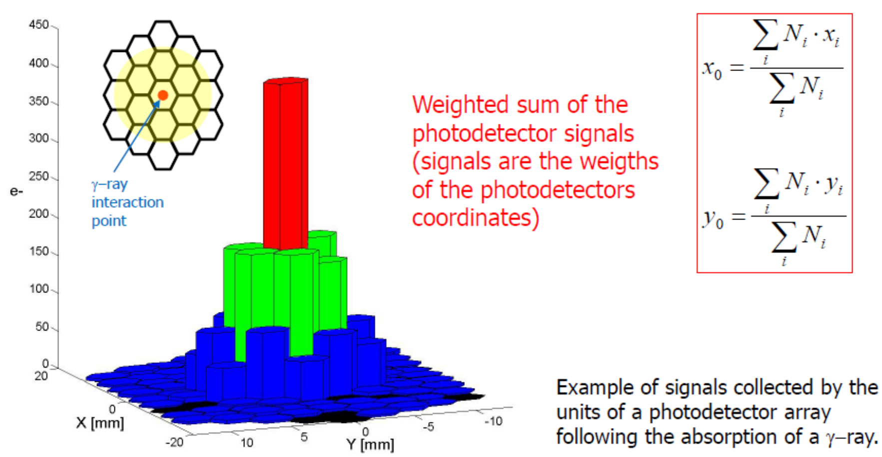
- A mechanical one due to the collimator geometry
- A physical one in the scintillator in reconstructing the position of interaction.

The two phenomena are statistically independent and so the total resolution of the imaging detector is the sum of the two contributions.

Also here we have to take some decisions. Does it make sense to use a collimator with 1mm resolution if the AnC has 10mm intrinsic resolution? No. hence in choosing the collimator we have to look also at the other parameter.
The same applies when designing an Anger camera. If the collimator has a resolution of 10mm, no sense to design a camera with an intrinsic resolution much lower.

TECHNIQUES FOR THE RECONSTRUCTION OF THE POSITION OF INTERACTION

THE CENTROID METHOD (CENTER OF MASS)



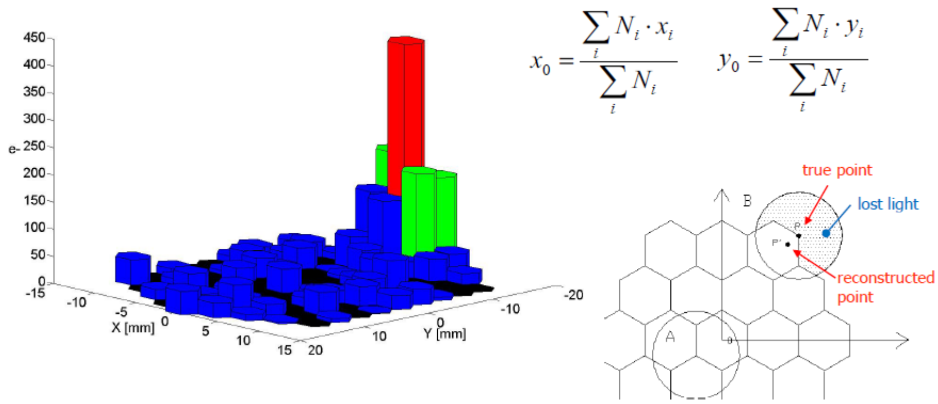
We want to reconstruct the center of gravity (or the position of interaction) starting from the signals of the photodetectors.
Let's suppose we have a scintillator read out by an array of photodetectors (PD, PMT or APD). The hexagonal shape is an adopted geometry because the best way to make a two dimensional sampling of a cloud is to use an hexagonal form, because it is the most homogeneous in the two dimensions.

We could also use squares, but if we have to make a 2D sampling, hexagons provide a more uniform sampling.

On the z axis we have the light expressed in terms of number of photoelectrons collected by the various photodetector, and it corresponds to a gamma ray interaction in the red point.

The simplest reconstruction algorithm is the centroid algorithm, shown on the right.
We make a weighted sum of the coordinates of the photodetector, where the weights are the signals collected by the photoelectrons. For instance, the coordinate of interaction X is given by the weighted sum of the coordinates xi of each photodetector where each coordinate is weighted more or less depending if the photodetector got more or less electrons. Then the sum is normalized by the total signal, that is the total number of electrons.

Since it is a weighted sum, the photodetector dominates more the sum the more electrons they get. But at the same time a big limitation of the method. In fact, the gamma camera is not an infinite plane, at a given point there is a boundary.



Missing contributions in the weighted sum due to border effects and damaged units may result in biased and distorded result of the recontruction

What we see in the plot is the reconstruction of a gamma ray falling close to the boundary of the Anger camera. The boundary means that we have photodetector close to the edge, but then there is still (in the gray region) a region of cloud of light that is sampled by nobody.

Hence this light has lost the opportunity to be represented in the formula, because in the formula only the existing photodetectors are present. The number of photoelectrons that are falling outside has no representation, no weight in the formula.
So the result of the formula is pushed more towards the photodetector that has received the light.

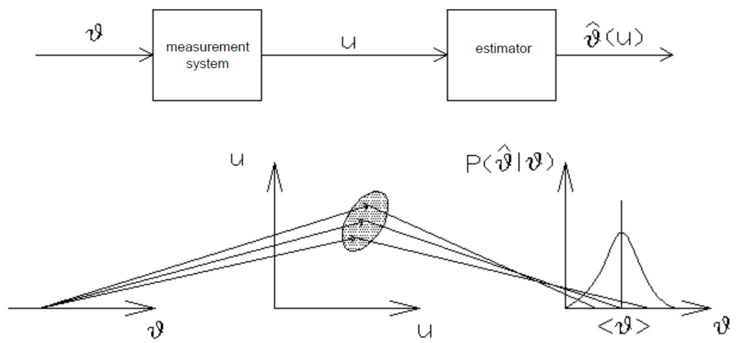
This means that if we have, for instance, a true point p, so we assume the gamma ray arriving in point p, in the reconstruction, the reconstructed coordinate won't be corresponding to p, but to p', that is a more squeezed point, a biased point. This because the weighted sum has weighted more the internal photodetectors and not the external ones (that however are not existing). → boundaries are very badly reconstructed.

The second problem is that time to time, **it may happen that a photodetector is broken**. Hence the PMT may be broken and not taking any signal. Substitution may be done but it is costly.

The broken photodetector is however now missing in the formula, the light that it takes is not represented in the formula.
If so, the result will be more favorable with respect to the other working photodetectors. This means that the result of the formula will be biased toward the non-broken photodetectors → gamma ray position will be reconstructed as shifted.

If in a pixelated camera having a broken photodetector leads to the complete missing of the information, in the AnC this is in principle recoverable. In fact, even if we miss the signal provided by one pixel, there are the other pixels that collect the light → the information is not completely lost.
It is indeed dramatic if we use the formula of the centroid with the weighted sum, because we are missing an important contribution in the sum and so the result of the weighted sum will be more pushed toward the other photodetectors that are contributing to the sum → distortion of the image.

THE MAXIMAL LIKELIHOOD



Construction of an 'estimator': the reconstructed interaction point (X,Y,Z) is the one which maximizes the probability for the estimation to correspond to the measurements (best estimate)

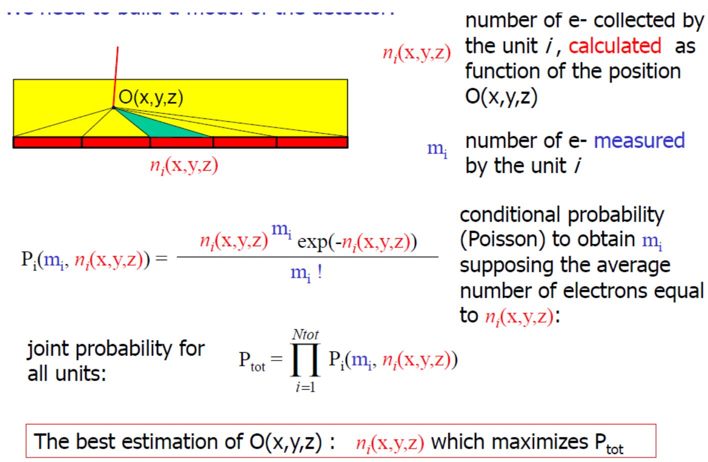
More sophisticated technique. It is a statistical reconstruction technique based on maximal likelihood; it is used to reconstruct the position of interaction starting from the measurement on the photodetector.

The maximum likelihood (ML) is based on building an estimator. We have our measuring system that starts from the coordinates of the source of interaction, represented by theta (set of x, y and z interaction of the gamma ray in the crystal), and then the system provides us u, that is a set of photodetector signals.
Building an estimator means building a function which takes as input the measurement and provides as an output an estimation of the coordinates. It is a function built out of probability, a function of theta.
Determining the position of interaction means, once we have such estimator, finding the set of theta, so x, y, z, that maximize this function.
So the coordinates are not found as an analytical output like in the center of mass, but we create a function which expresses the probability that the measurement corresponds to a given set of coordinates and, since this function is constructed above probability, the set of coordinates will be the one that maximize the probability that we indeed get the measurement.

Modeling of the detector

First of all, we need to create a model of the photodetector. This is one difference with the centroid method, because the centroid is just an analytical formula, is ignoring the structure of the detector.

The difficulty in this technique is to build a reliable model of the detector. A model of the detector is shown in the image below.



We have a physical model of the detector; the detector is the monolithic scintillator (yellow) and the photodetectors are the red ones.

We need to have a look up table where the output is the number of photoelectrons ni collected by each photodetector 'i' as a function of the position of interaction O. O is unknown, its coordinates are unknown. But supposing that in the model we have a position x, y, z, we are able to calculate or simulate or measure the amount of electrons ni, that depends on x, y and z.

The model can be build using very simple assumption. The simplest way is to use the formula of the solid angle. For instance, we can calculate ni with a solid angle coverage in green.

Finally, we may also use measurements; we take a source of gamma ray, we scan experimentally point by point the crystal and we record the measurement.

So, let's suppose now that the model has been done; now I have the task to reconstruct the position of interaction given a set of measurement mi → we have a vector of numbers corresponding to the photodetector response.

NB: ni are function of x, y, z, measurements mi are numbers. So ni is the variable, mi a number.

The next step is starting to build a conditional probability; it is the probability to have measured mi supposing that the signal in the pixels is ni in average. So I focus on a single pixel (e.g. pixel 3), pixel n° 3 has measured 140 electrons (mi), so I build a model, a function, that is the probability that for a given average signal ni, which is the probability to have measured mi, that is 140?

This probability can be described by the Poisson probability.

In principle, I could stop here, because I already have built an estimator for a single pixel. Now we need to find the maximum of this function. The maximum is when ni = mi.

If I have measured 140 electrons, the highest probability is when also the average value is 140.

Now if in my model I associate to one pixel the value (e.g. 140 in the example), I can backtrace that the position of interaction is at a given coordinate, because there are only few coordinates O that can provide 140 electrons.
However, I don't stop here because it is a measurement, and the measurement is affected by statistical error, and it is a little bit brutal to assume that the answer to the problem is exactly equal to the measurement, because the measurement may be wrong. → stopping to just one or two pixel is too brutal.

Joint probability

What we can do more is to build a joint probability. It is the product of the conditional probabilities above, so it is a product extended to all pixels. Not just the probability of one pixel, but the probability of one pixel joint with the probability of all the other pixels of the camera.
Maximizing this function is much stronger, because the coordinate we will find (x,y,z) will be the one which will not just maximize the probability of just one single pixel, but will be the one that overall maximizes the probability of all pixels.

The function is the following: $P_{tot} = \prod_{i=1}^{Ntot} P_i(m_i, n_i(x,y,z))$

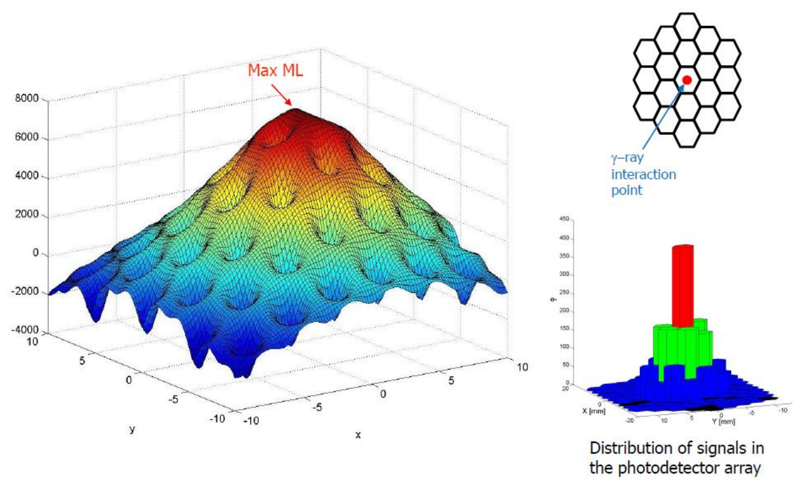
The method is simply the maximization of this function that we have constructed and that is a function of x, y and z. We will find the coordinates x, y and z, which will be the one that provides the solution that maximizes the probability that the measurement we have measured with the photodetector are the good one.

Of course, the result we find may not maximize the individual probability, but maximizes the product of the probabilities.

Example

In this example I have a gamma ray interaction at the center of the camera, made out of hexagonal pixels. In the RGB image on the right we have the distribution of the mi in the different pixels.

Example of the function of maximum likelyhood corresponding to a measured event

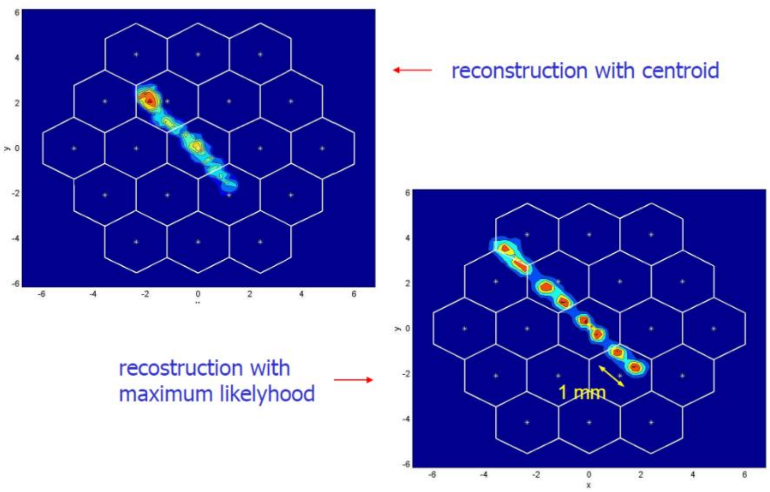


I build a model, the condition al probability for each individual hexagon and finally I build a function as a product (joint probability). The function we get is the one in the central image.

It has an absolute maximum. So I take as coordinates the maximum of the function.

What are the holes in the function? What are those local minima? *(Centers of neighbour pixels)*
These holes are unlikely positions, positions which don't correspond to the irradiation. Where are geometrically placed these holes?

In the following image there is the comparison between the two techniques. The one with likelihood underlines the presence of a set of irradiation points, separated by 1mm. With centroids I cannot distinguish the points and the image is compressed toward the center. With maximum likelihood the points are well resolved and especially we can reconstruct points at the boundaries which are not compressed.

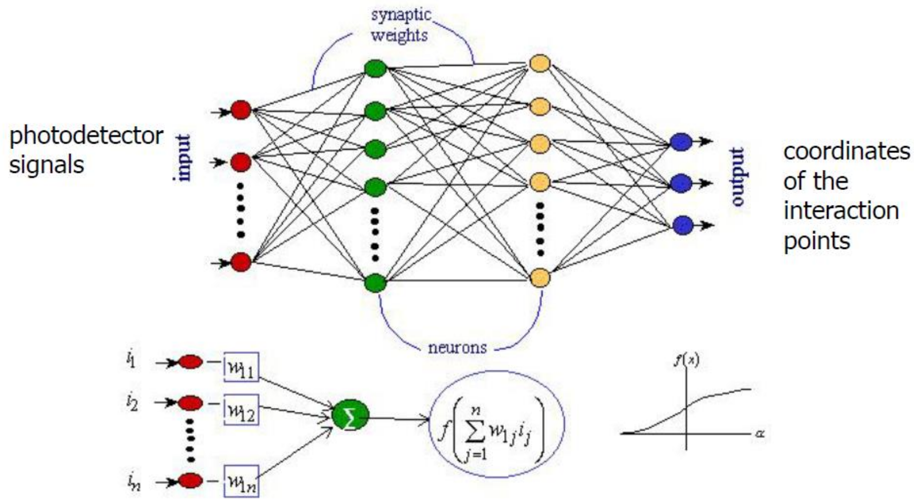


Hence this very last method solves the problem of boundaries and broken units because each detector is playing a role in the formula and the formula is not just an 'algebric domination of the sum' like in the centroid method, but it sis just a probability. So each detector is simply playing a role we are looking to the signal in this photodetector, the average signal that is closes to the measurement. So we can even exclude detectors in the formula. Simply, we are not adding information in the formula, but the estimator can be conceptually build by reducing the subset of photodetector where we expect the signal that they measure is consistent with the irradiation corresponding to a given point of irradiation.

Hence there is no reason that in this method that we have a bias if we have a missing detector. We can even exclude the detector in the center. We simply take a joint probability using the light taken by the other photodetectors.
This difference tells us that is not mandatory to use all the photodetectors in the formula, because the formula itself is much complicated and the more photodetectors we add in the formula, the more complicated is the maximization. → we can consider in the formula only those detectors which have a signal that is relevant. The other detectors at the boundaries may only provide noise.

Usually, what is done is finding the detector with the largest signal and we include in the formula only the cluster of neighbours.

THE NEURAL NETWORK (NN)



NN are a model, but differently form maximum likelihood, that is a conscious model because it is a model where we introduce the physical knowledge of the system, a NN is a mathematical model where at the beginning we have connections between neurons that are not totally knowledgeable.

A neural network is a net where we have some inputs, and in our case the inputs are the photodetectors signals, so the mi. The output of the network are the three coordinates of the interaction. In between we have weighting and sum, which means that each photodetector signals are weighted and then finally they are summed. Then the result of the sum is passing through a nonlinear function, typically a sigmoid.

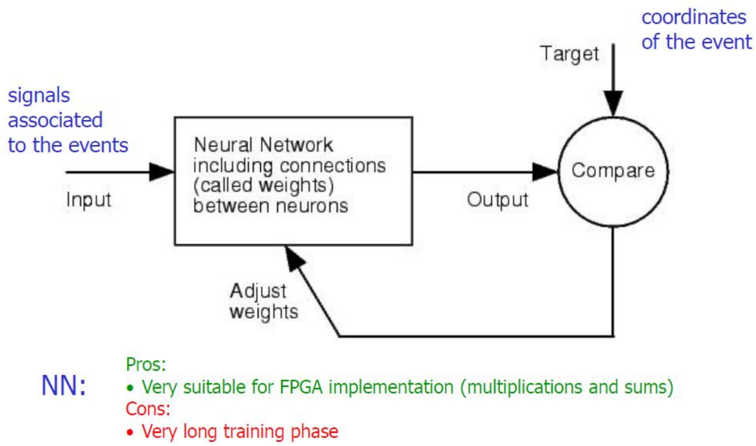
So in the first layer we take the measurement, we multiply by some coefficients, we sum the results, we pass the results through a nonlinear function and we are now in the second layer (green nodes). Each of the green nodes are a result of multiplication and sum of the set of input signals, and we have several nodes because we have several way in which we can weight the signal.
Then we repeat the procedure for all the layers, with new weights and nonlinear functions.

At the end we have the output coordinates. Of course, at the beginning the network is completely unconscious, the result are not calibrated → we need to do a training of the network.
The training of the network is a very extensive and time consuming procedure where we need to shape the network, which means providing reasonable values for the weights.

Training phase

We take a set of data, measurements on the detector, corresponding to known irradiation positions. Again, the data can be built by simulator, so we take the simulator and we make the scan of an hypothetical source inside the simulator and we record the measurement, or we can do it experimentally, so we scan with a focused beam the detector and we record the measurement.

So we provide in input to the NN the signal associated to events, the first output of the network will be completely wrong and so we will compare this crazy output with the true coordinates and we adjust the weights. The procedure is repeated until the output matches the true coordinates, with several irradiations.

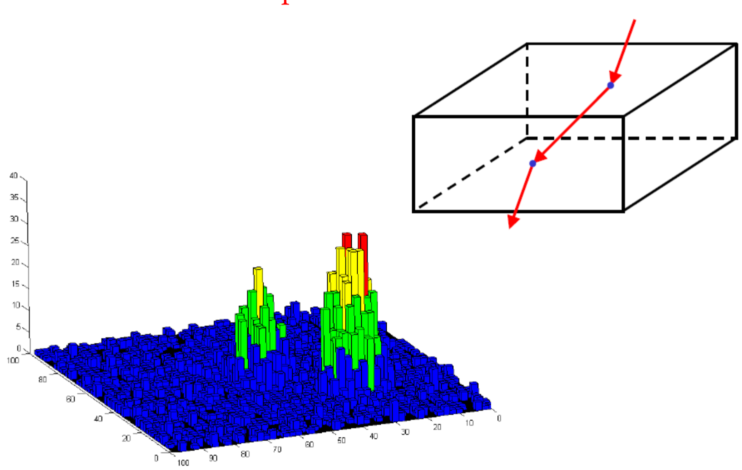


Advantages and disadvantages

Maximal likelihood is pretty, but for real time application is very heavy, because if we want to use a FPGA, in that case the FPGA needs to do a very nasty maximization of the function in the case of the anger camera, it is quite though to reconstruct events real time.
NN have a huge training phase (hours) but once we have the network ready, the real reconstruction is just a set of product and sum, and product and sum are things that modern FPGA does in few clock cycles.

So machine learning techniques are very popular because they are very compatible with FPGA and graphic cards.

Application of NN: reconstruction of Compton events

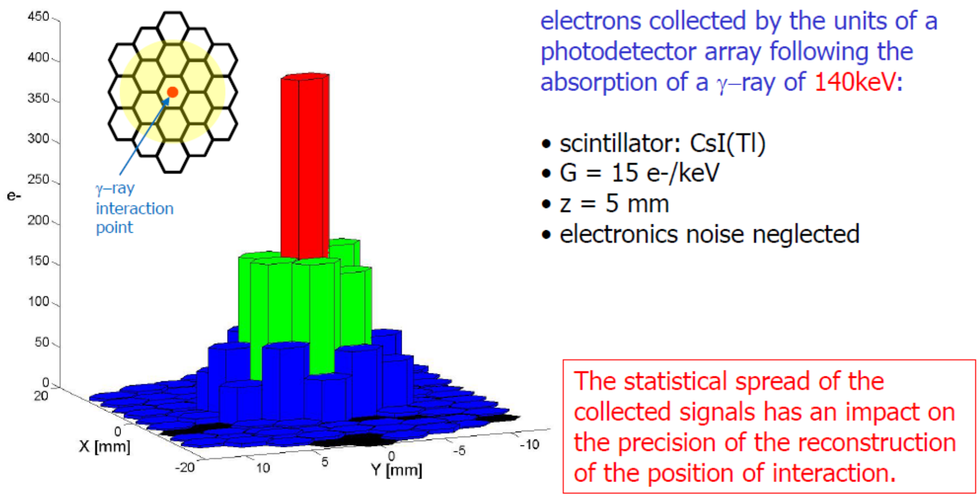


These are more complicated interactions. It is nasty to be reconstructed, because if I have for instance two interactions in the crystal, like in the image, in the photodetector we have two simultaneous flashes of light that can be either well separated like in the image or superposed.
This are impossible to be reconstructed with maximum likelihood, because our model should be so sophisticated that should foreseen all the possible combinations of interactions, and this is impossible.

In principle, in NN nothing is impossible, because we train the network to let the network now that also multiple interactions are possible.

In principle, we could reconstruct also very sophisticated interactions and not just very simplified models of interactions.

IMPACT OF THE PHOTODETECTOR/AMPLIFIER ELECTRONICS NOISE ON THE SPATIAL RESOLUTION OF THE ANGER CAMERA



For instance, in maximum likelihood, instead of using all the data, I use only data that are significant, which means above the electronics noise.
Electronics noise is important because the statistical fluctuations due to the electronics noise sums up to the intrinsic fluctuation of the signal.

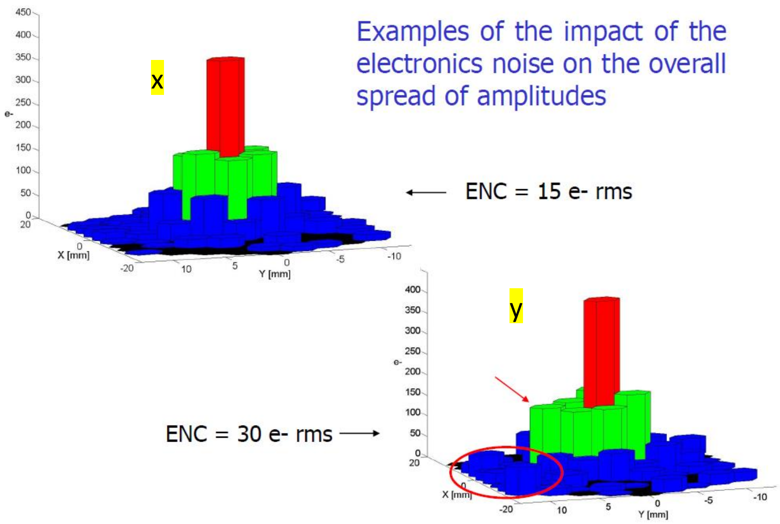
In the image we have again the gamma ray interaction and the result of the measurement (RGB plot). In the simplest case we neglect the electronic noise (black). But even without the electronics noise, despite the choice of a totally symmetrical point in the simulation, the irradiation is not symmetric at all; if we take a single event, not an average, we see that the contributions in green are not the same, despite the hexagons are symmetric and the interaction occurs in the center of the array.
This because irradiation is a statistical process, we cannot rely that the light taken by a photodetector is always the same, it depends; there are random reflections on the top of the scintillator. Also the isotropic emission in the entire solid angle is not uniform → statistical process, and from a statistical process we must expect to have a statistical result of the measurement.

But this means that if we now apply each one of the reconstruction algorithm, we will have an error.

Reminder: the PSF of the AnC is a Gaussian, and it is a gaussian because it is the error; each time we repeat the measurement we get a different configuration of signals and so each time we have a different reconstructed coordinate. The fluctuation of this reconstructed coordinate reconstruction covers a gaussian. It has an average point, that is the center, and a spread.

Hence the AnC has not an infinite precision and doesn't reconstruct the position with an infinite precision because each time we repeat the measurement we find different distribution of photodetectors.

Moreover, in addition to this, we have also the electronics noise.
The electronics noise of the amplifier can be related with a physical meaning, the ENC. The noise is represented in terms of electrons.
So now in the 3D plot we should add the electronics noise. If we do so, we obtain something like this.



X is the same distribution of photoelectrons as before but in the simulation it has been switched on the electronics noise; we see we have 15 electrons rms. It means that each photodetector, each hexagon, has, in addition to the previously mentioned fluctuation of the signal, +-15 electrons of noise → each column of image x is fluctuating with this sigma, and also we have the a ground signal (blue ones), because photodetectors that were supposed to have no signal have instead some background.

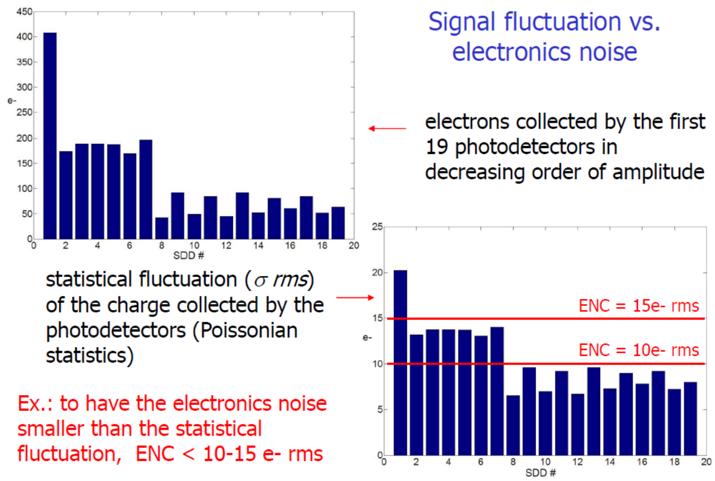
In image y we can see that the situation is even worse if we increase the noise to 30 electrons rms.
30 e- rms are even able to let the green detectors to change color. Previously they were blue, but now they have accumulated so much noise that their contribution in the center of mass reconstruction becomes green → this will impact a lot in the position reconstruction.

In conclusion, to estimate the spatial resolution we get in our camera, we need to calculate the statistical fluctuation of the signal itself, and, moreover, we need to design carefully the electronics (amplifier, filters, ecc.) so that they will not dominate in the spread.

Signal fluctuation vs electronics noise

I take the values of signal without noise and I put in them in the first plot in descending order of amplitude. We can see that the detector in the center collects 400 electrons, the detectors in the border 50 e-.
Now I wonder: how will each photodetector fluctuate statistically? Which is the intrinsic statistical fluctuation of each column?

I take as usual Poisson as a good statistical model and what we have in the second plot is the sigma of the fluctuation calculated according to Poisson → this means that sigma square of the first detector is 400, so the sqrt of 400 is 20.
So in the second plot we have the relative fluctuations of each photodetector.



Coming to the amplifier, how much good must it be?
As a rule of thumb, ENC<10 or 15 e- rms.
In this case if ENC is 15 e- we are practically adding a contribution in the spread which is the same as the intrinsic one of the photodetector. It would be no-sense to create electronics with 1 e- rms, because 1 e- rms would be anyway dominated by the intrinsic fluctuation of the signal.
On the other hand, it is neither a good idea to use electronics of 100e- rms, because otherwise we would be dominated by the electronics noise fluctuation.

When we design such systems, **the better the scintillator and the photodetector, the larger is the number of electrons collected and lower is the relative fluctuations**.

ENERGY RESOLUTION

$$E_g = G \times \sum_i (N_i + noise_i)$$

N_i : electrons collected by the unit *i* of the matrix

$noise_i$: electrons associated to the electronics noise of the unit *i*

G : gain factor keV/e-

⇒ the sum of the electronics noise of all the units of the array (larger than a given threshold) contributes to the statistical fluctuation of the computed energy

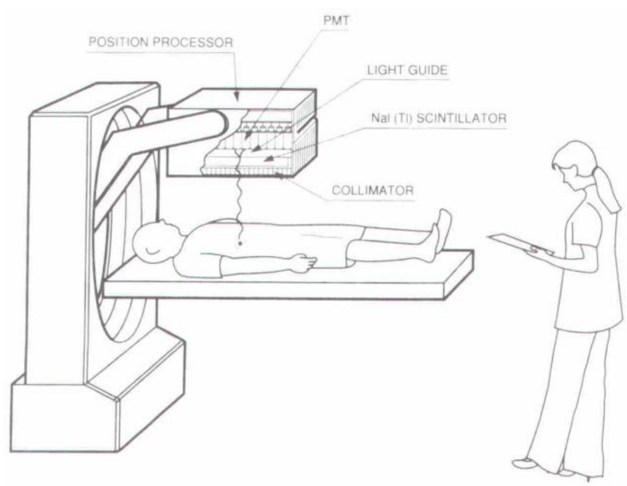
The same concept applies for energy resolution.
Energy means reconstructing not only the position of the interaction of the gamma ray but also the energy. The energy allows us to reject Compton scattering in the patient, and so it is very important.

For example, if in SPECT we have 140keV, if we measure with the detector 100keV we know that this photon has suffered by Compton interaction in the patient.

Fortunately, energy reconstruction is very simple in detectors. The simplest way is just to make the sum of the signals; we take the contributions and we make the sum.
Usually the energy is given by the sum of the photodetectors signal multiplied by a calibration coefficient, because the signal is expressed in electrons, the signal is given in keV.
The calibration coefficient is found by irradiating the detector with a known radionuclide, like 99Tc, and if the sum is always 1400 electrons, we found the conversion coefficient.

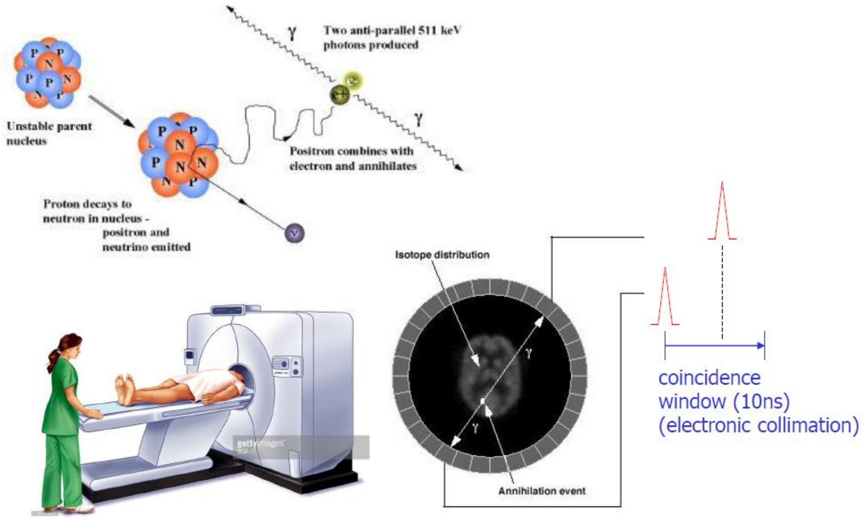
However, also here, if we add not only the photodetector signals but also the electronics noise, the reconstruction of the energy is worsened. And the worse the energy resolution, the worse the capability to distinguish Compton interactions in the patient. So the electronics noise play also a role in energy resolution.

ANGER CAMERA FOR SPECT



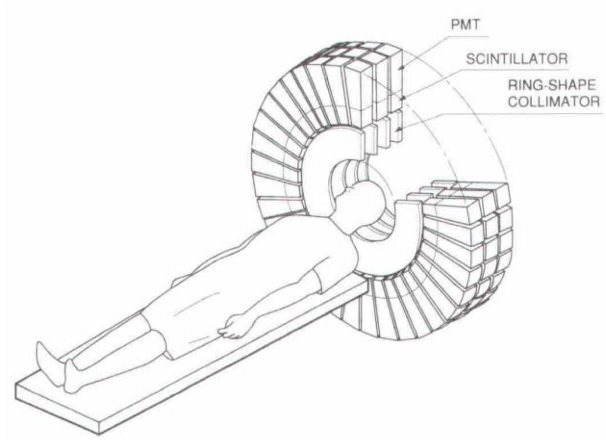
Anger camera is the most used detector for scintigraphy and SPECT. Scintigraphy is a static recording of the gamma rays, while SPECT is a computer tomography, the camera is moved around the patient. The typical SPECT scanner is made out of 3 cameras covering the entire circle and we are rotating them simultaneously.

POSITRON EMISSION COMPUTED TOMOGRAPHY (PET)



The radioemitter is emitting positrons that are travelling a short track called positron range and when positron annihilates with electrons it produces a couple of gamma rays that are travelling back to back, they are detected by a detector element and the technique is based on the fact that two elements trigger almost instantaneously (or with a time difference if ns) and then we draw a line connecting the two detectors.
We need a coincidence window to exclude other interactions that doesn't belong to the coupling of a pair of gamma rays.

GAMMA CAMERA FOR PET



It is based on radial detectors, so detectors placed across a gantry. They may be composed by one single pixel or by the detectors in the image, called **block detectors**.

The PET system is in principle collimator-free, because the collimation is overcame by the geometrical connection of the detectors. However, there are still some circular slabs called collimators. These collimators are included in the design to simplify the association of the various photodetectors.
We have the gantry with all the detectors, but if we look the camera from the longitudinal direction, we have a quite relevant number of detectors.

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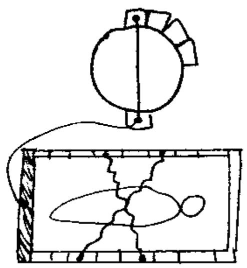
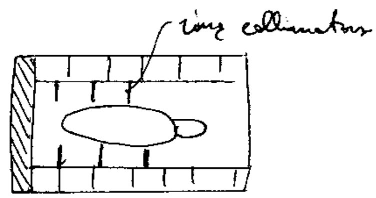




Gamma rays may be belonging to a single plane but in principle they can be emitted also intercepting different detectors' ring in the system.
So when we have a pretty complex scanner, and we are waiting for all possible combinations of back to back gamma rays, we could expect everything. The electronics should be ready to connect all possible combinations of all detectors.
This may be quite nasty. Nowadays, thanks to the enormous power of computing components like graphic cards we can do something like in the image, but in more simplified systems we introduce some rings collimators because we are overwhelmed by the capability to track all possible coincidence, and so thanks to these rings we limit the interactions that we want to reconstruct only to the one parallel to the ring.
So such collimators force the system to reconstruct only interactions happening in planes defined by the collimators.

Without collimator, the efficiency is in principle 4*pi, if the detectors are all along the patient. In conventional PET it is smaller, because the detector is not all along the patient, the scanner is limited in size.

THE BLOCK DETECTOR FOR PET

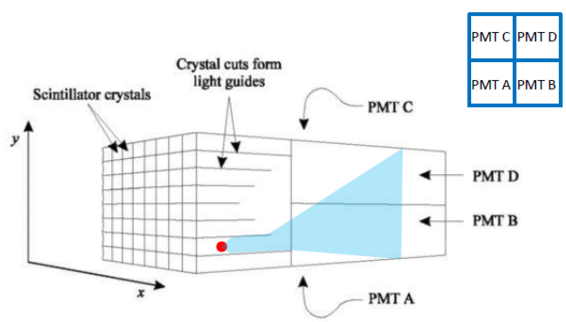
Most used detector in PET.

In a block detector, a 2D array of crystals is attached to 4 PMTs.
The array is cut from a single crystal and the cuts filled with light-reflecting material. When a photon is incident on one of the crystals, the resultant light is shared by all 4 PMTs. Information on the position of the detecting crystal may be obtained from the PMT outputs by calculating the following ratios and comparing them to pre-set values:

$$R_x = \frac{A+B}{A+B+C+D}$$

$$R_y = \frac{A+C}{A+B+C+D}$$

where A, B, C and D are the fractional amounts of light detected by each PMT

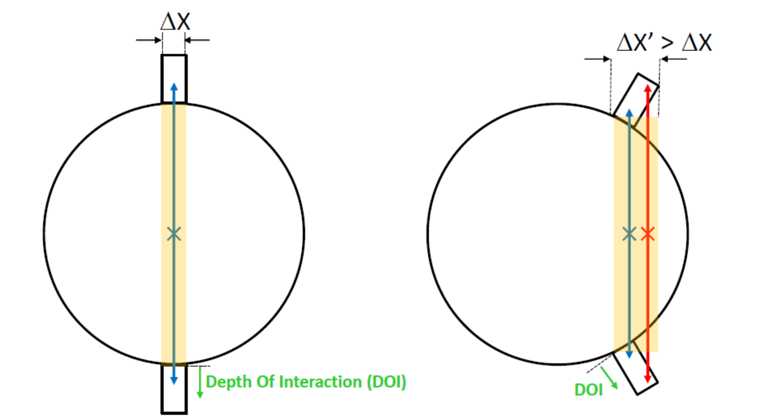


The block detector is a hybrid between a fully pixelated detector and an Anger camera.
It is selected a pixelated detector because the scintillator starts as a pixelated crystal, we have pixels.
So the goal is to reconstruct the coordinated of the pixels.

Since it is a pixelated detector, we would expect to have a single combination pixel-scintillator-photodetector.
In reality, we adopt a similar concept than the Anger camara. The scintillator is not pixelated up to the end, but we make some cuts at the beginning of the crystal and the remaining part is monolithic, not segmented. The light at the extremities of the scintillator is able to spread away (light blue).
Then the photodetector is composed by an array of 4 PMTs, and we measure the light collected by each PMT and we apply a kind of centroid formula.

For instance, the interaction corresponding to the red dot will shine more photodetectors B and partially D and less photodetector A and C. Then by using the formula on the left we reconstruct the coordinates Rx and Ry of the pixel.
NB: we reconstruct the coordinate of a pixel, not like in the AnC whatever coordinate in the space of variables.

THE PARALLAX ERROR



Here, resolution is limited by pixel size (ΔX) only, independently on the information about the DOI

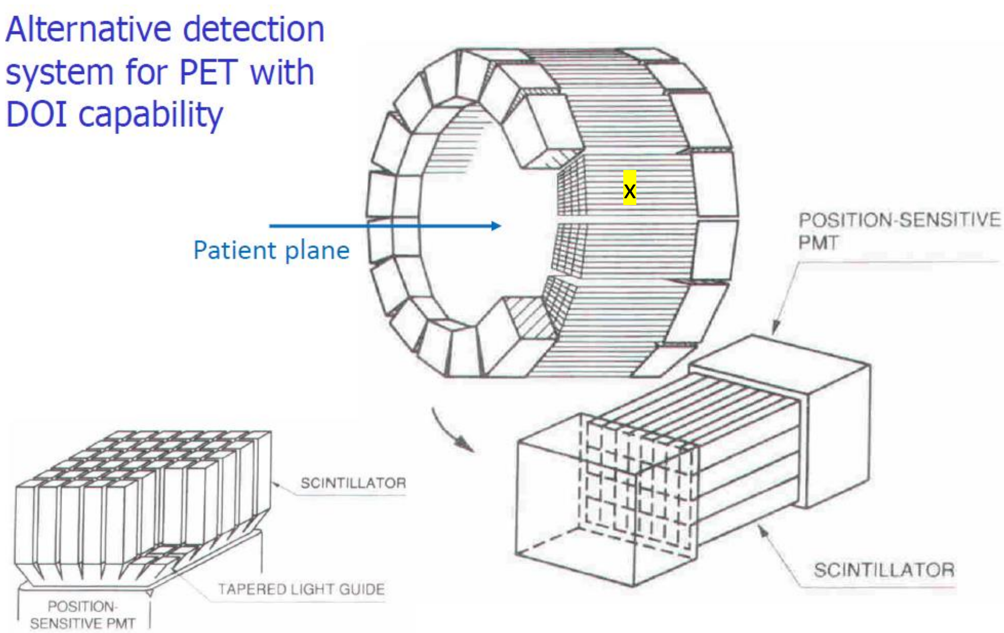
Here, resolution is affected also by parallax error, and it could be recovered to ΔX by the knowledge provided by the DOI

When we take detectors with a radial configuration, whatever they are fully pixelated or as a block detector, practically we are insensitive to the third coordinate, that is the depth of interaction.
This is not a problem for gamma ray interactions that are in parallel to the direction of the detector.

Let's assume that we have a PET scanner and the individual pixel resolution is delta x (2-3 mm in PET), placed as in the image (it could be the dimension of the pixel). The yellow area in the image is the resolution, so all couples of gamma rays belonging to the yellow area will produce the same coordinate of interaction, and this is the spatial resolution related to the pixel.
However, for the tilted detector elements like in the right, the resolution is not delta x, but even worse, because gamma rays are detected by the same detector element when the uncertainty area in yellow is not as large as delta x, but it is as large as delta x'. This is called parallax error, due to the fact that the detector elements parallel to the gamma rays are 'lucky', whereas detector elements tilted with respect to the direction of gamma rays are 'unlucky', in the sense that the two gamma rays red and blue are associated to the same coordinate → no resolution inside the pixel.
Of course, the red rays can be detected by another pixel, but we must consider that we need sufficient material to be efficient il capturing the ray. The higher the thickness of detector, the higher the parallax problem.

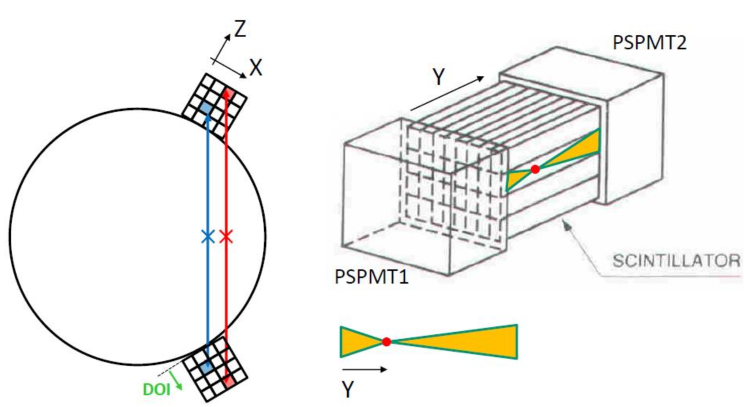
To solve this problem, we need the information of the interaction point also in the third coordinate, that is marked in green and called depth of interaction (DOI), or z-coordinate.
This coordinate is normally not available, we don't now where the gamma ray has interacted in the crystal, but if we would know also this coordinate, we could distinguish the blue and red cases.

Alternative detection system for PET with DOI capability



Now there is a ring of pixelated crystals and the pixelization is not radial but longitudinal. The arrays, the scintillator bars x are parallel to the patient. In the image of the gamma camera for PET the bars are radial, so orthogonal to the patient direction, while here the arrays are parallel to the patient.

So if now, with a position sensitive photodetector (PMT) which provides the coordinates of each single element, I have now the capability to distinguish each element of the array, thanks to a position sensitive photodetector.



Pixelization in Z direction (in addition to X) allows to implement DOI reconstruction

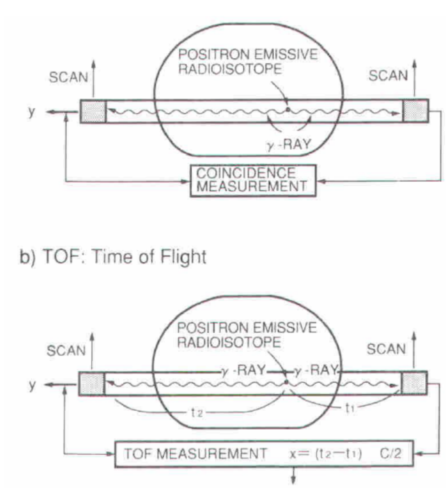
Reconstruction on Y coordinate is obtained by the ratio of the signals collected by the PMTs which is dependent on the position of interaction

Now if I take the cross section again, the detector is pixelated along the z and x direction, so the PMT is able to detect the pixel in blue and the pixel in red and so thanks to this information (DOI) I'm able to distinguish the two tracks.
What is missing is the y direction, so the direction parallel to the patient, that was available before.
To reconstruct the interaction along the y direction I exploit two photodetectors. I don't use just one detector on one side, but I use detector on the other side (PSPMT1 and PSPMT2).
In this way, when I have a gamma ray interaction in the red dot, I will have an amount of light taken by photodetector 1, and an amount taken by photodetector 2. Since the light is depending on the distance, due to the loss of light in the scintillator, the signal on photodetector 1 will be for instance larger than on photodetector 2.
I use a weight to reconstruct the difference and I can reconstruct the interaction also in this coordinate y.

So on two coordinates (x and z) I use the pixelization, in the other coordinate I use the different light sharing → fully 3D reconstruction of the gamma ray interaction.

PET-TOF (Time of flight)

It represents the most advanced topology for PET scanners.
The concept is illustrated in the figures. On the top we have the basic principle of PET detection; so we have the isotope that annihilates with electron, there is an emission of 2 back to back gamma rays and they are detected by two detectors.
The two gamma rays reach the detectors, and we have a coincidence unit that checks if the time difference between the two gamma rays is between a reasonable interval and, if this is the case, we draw a line, the line of response, that connects the two detectors, and the information we get is just that the source of the signal belongs to the line.
But in reality, if we would know precisely the time difference between the two hits (we don't know the absolute time between gamma generation and gamma detection, but we know the time difference t2-t1), if the source of the gamma rays is placed more on the right we know that t2 > t1. And so by using simple kinematics formulas we can determine the coordinate of the origin of the gamma rays inside the line of response.
If the origin was in the middle, the difference is 0 and the coordinate x is 0. If the point is close to one of the two detectors we have the largest possible time difference.



a bit of PET history:

- PET: 1950
- PET-TOF: 1960
- PET-TOF first prototype: 1980
- PET-TOF clinical scanner: 2005

Q: why 25 years to get a first useful instrument?

In the formula there is not speed of light, but speed of light divided by 2 simply because we are considering a reference system of coordinates at the center of the system. So if we consider it at the center, when we have a source completely to the right, the time difference multiplied by the speed of light is the total length of the line. So we would get the total distance between the two detectors, but the system of reference is in the middle.

The concept is so simply that the obvious question in: why does it take so long to get a first useful real instrument PET TOF? Because of electronics.
PET as a general concept (figure a) has been proposed in 1950. PET TOF (figure b) was proposed in 1960. Then it took 20 years to build the first PET TOF prototype.
We had to wait so long because the timing resolution to implement this concept was not sufficient for a long time.

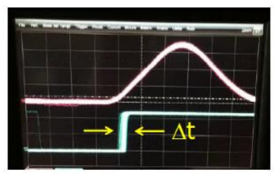
Numerical considerations

$$\boxed{\Delta x = \Delta t \times c/2}$$

$$c = 30\text{cm/ns}$$

$$\text{if } \Delta t \sim 3\text{ns}$$

$$\Rightarrow \quad \Delta x \sim 45\text{cm}$$



..the uncertainty in the position of annihilation is of the same order of the diameter of the body of the patient….!

The big point is: which is the timing precision for t2 and t1? Which is the precision in measuring such a time difference?
Given the delta t, the time jitter in the measurement, the corresponding uncertainty in the spatial coordinates goes with the same formula.
So if we have a delta t time coincidence in the measurement between time t2 and t1 (we measure the difference, not the absolute value), delta x is given by delta t time c/2.

Now we have for instance a signal out of the amplifier (image in black) and we put a threshold with a comparator to check when the waveform passes through the threshold and this event represents the time stamp, shown in the bottom trace. But unfortunately, due to many factors (some related to the scintillator and photodetector, but many associated to the electronic chain) we have a time jitter, delta t is the uncertainty, the jitter of our signal.

How much is typically a time jitter?
If we consider a time jitter of 3 ns, which is not so bad, we get a delta x of 45 cm, that is unacceptable → means that the resolving capability is 45cm, that is basically the diameter of the patient.
This is the reason why it took so long to develop a working PET TOF.

Nowadays, electronics is much better.
Hence the time jitter has been dropped down up to 300ps, that is a great time capability for a scintillator detector, then resulting in a delta x of 4,5cm. 4,5 cm still looks far from the PET resolution, that is few mm.

We have still a gap of one order of magnitude with respect to classical PET.

$$\boxed{\Delta x = \Delta t \times c/2}$$

c = 30cm/ns

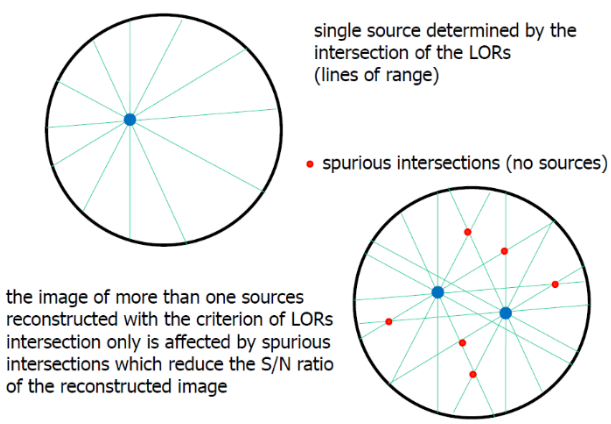
if $\Delta t \sim$ 300ps (now state of the art)

$\Rightarrow \quad \Delta x \sim$ 4.5cm

- recent improvements in electronics timing resolution have pushed an 'old' idea of PET TOF to reality....**but**
- resolution still not adequate for the reconstruction of the annihilation point with a resolution required for PET (~mm)

In reality, PET TOF started to be used because producers find a smart way to exploit these 4,5 cm. the smart way was to give up the spatial resolution, but to improve the SNR in the image recording.

Improvement of the SNR



We have to make a step back in what is the conventional PET reconstruction.
Let's take the simplified object in the top of the image that is a single atom, let's consider a single point-like source.
A single point-like source would emit several lines of response in the traditional PET; we have several couples of gamma rays.
The reconstruction of the position requires the determination of the geometrical point of the intersection between the lines of response. Of course, we could have noise and for example a parallax error (if we have this error, we won't have a line but a rectangle).

If then we assume to have two sources and apply the same algorithm to determine the point of crossing of the lines of response, what we get is that we have a very high density of crossing in correspondence of the two blue sources, which is good, but, unfortunately, we have also some spurious crossing, marked in red.
Since the algorithm to determine the crossing is unique, we have also some unfortunate crossing. These are of course not corresponding to any point of the image → it is noise in the final PET image.

We have this problem because in the line of response we are attributing an equal probability of existence of the source. So we are looking to any possible crossing because the lines of response, since we don't know where the source is, we have to attribute in the line of response the uniform probability. The source can be everywhere.

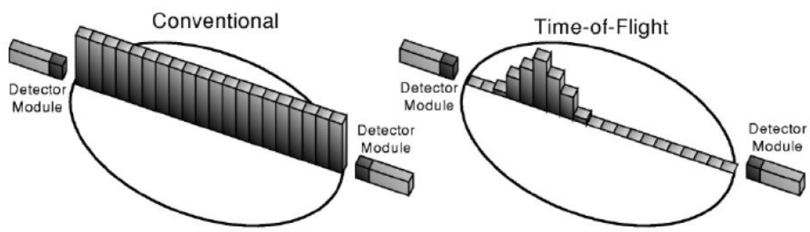
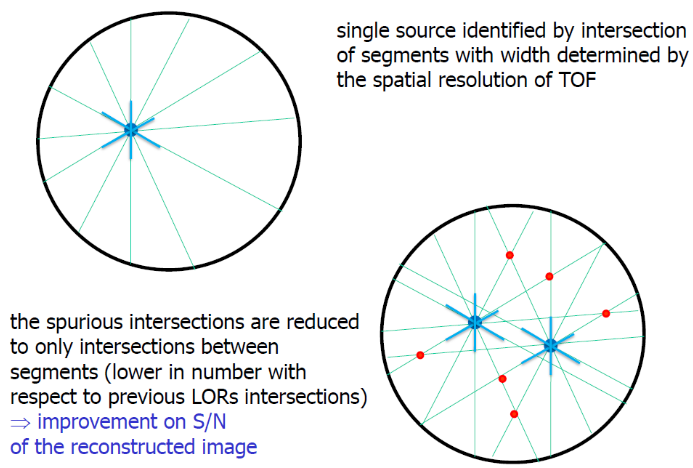


Fig. 2. TOF reconstruction. With conventional reconstruction (shown on the left), all pixels along the chord are incremented by the same amount. With TOF reconstruction (shown on the right), each pixel on the chord is incremented by the probability (as determined by the TOF measurement) that the source is located at that pixel.

If we would use TOF, we have available a resolution along the line of response 4,5 cm wide, which is not sufficient to find the precise position of the radiotracer, but at least we can limit the probability of existence of the source to a segment (of course not a sharp segment, but a gaussian).

Now, if we rely on this segment, we can repeat the procedure. Hence now I'm not looking to intersection of the lines of response, but to the intersections of those segments reconstructed by the TOF.



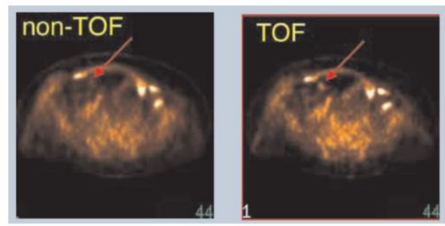
The rule is looking at the coincidences of segments, and not coincidences of the entire line. For the single source it makes no difference, while for multiple sources if we look to the crossing of only segments of 4,5cm width, we can distinguish very well the true sources, and the red spots are not existing in reality. The only crossings are the one marked with bold segments → we are eliminating the spurious crossing, and improving the SNR of the image.

So we are not gaining in spatial resolution, we are gaining in SNR of the image.

Of course, if the two sources are close each other, we may have also spurious crossing of segments, but at least in the field of view we are eliminating a lot of spurious spots.

It can be demonstrated that the SNR (or the variance) improves by a factor that is given by the diameter of the scanner (so the field of view) divided by delta x, our spatial resolution.

> • **Variance Reduction Given by $D/\Delta x = 2D/c\Delta t$**
> **(D: diameter of the object)**
> • **500 ps Timing Resolution $\Rightarrow$ 8 cm localization**
> **$\Rightarrow$ 5x Reduction in Variance!**



If spatial resolution is as large as the patient, we have no gain, but if now the diameter of the patient is 40cm and we have 500ps (more conservative value), we have 8cm localization variability → 40/8 = 5x reduction of the variance, that is enormous. So, either we have a better image, or we have the same image but with 5 time less radiotracer in the patient.

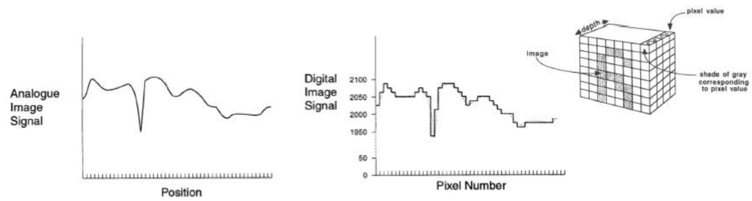
In conclusion, the timing resolution is the thing that really makes the difference with respect to the old approach.
Moreover, the future goals are to reach a delta t of 10ps so to have a 1,5mm spatial resolution. With 1,5mm it would be possible to reconstruct the origin of the gamma rays in the line with just a couple of gamma rays → we can avoid the reconstruction based on crossing of the lines.

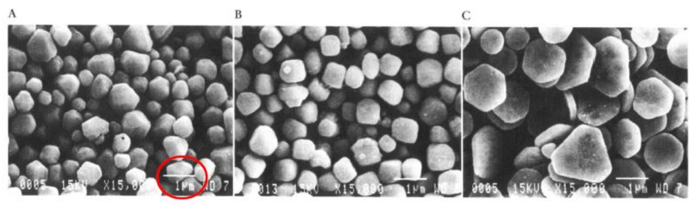
To reach 10ps, we need to have less time in performing all the steps. But 10ps is already the travelling time of the light in a bar of a PET detector.

DIGITAL RADIOGRAPHY

In a radiographic system we have an xray tube that is shining an object and then radiation can be either absorbed in the object (hard tissues) and the surviving radiation is recorded into a slab.

ANALOG VS DIGITAL RADIOGRAPHY





Photographic emulsion for traditional analog radiography

The photographic emulsion was considered the analog recording of an image. We record for each position of the space an analog value of the intensity of the xrays.
The slab for this analog radiography is composed by grains, and when such grains are shined by xray radiations, they change properties, and this will reflect in a different gray level in the image recorded.

On the contrary, we can also make a quantization of the level of intensity we want to record and so basically the image is discretized in pixels, and also the intensity is discretized in energy levels, that are then converted in digital code.
If the discretization of the image is very fine, we could not detect the discretization in the digital image. In reality, the quality of the image depends on the discretization capability.

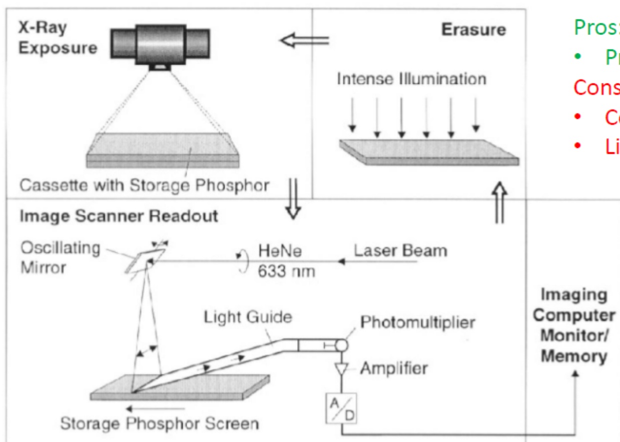
RADIOGRAPHIC SPECIFICATIONS

Clinical Task →	Chest radiology	Mammography	Fluoroscopy
Detector size	35 cm × 43 cm	18 cm × 24 cm	25 cm × 25 cm
Pixel size	200 μm × 200 μm	50 μm × 50 μm	250 μm × 250 μm
Number of pixels	1750 × 2150	3600 × 4800	1000 × 1000
Readout time	~ 1 s	~ 1 s	1/30 s
X-ray spectrum	120 kVp	30 kVp	70 kVp
Mean exposure	300 μR	12 mR	1 μR
Exposure range	30 - 3000 μR	0.6 – 240 mR	0.1 - 10 μR
Radiation (quantum) noise	6 μR	60 μR	0.1 μR

The typical radiographic panel is in the order of 40 x 40 cm^2 as an order of magnitude. For mammography it is smaller (18x24).
The pixel size in a classical radiography is 200um, for mammography 50um, we need a better resolution because we need to look at microclassifications.
The number of pixels is in the order of 2000x2000 in classical radiography and a bit higher for mammography.
Another parameter to be known is the recording time, that is in 1s both in radiography and mammography. This means that in 1s the statistical content of my image must be sufficient.

The trend in radiography is, more than improving resolution, is to reduce dose to the patient.

PHOTOSTIMULABLE PHOSPHOR DIGITAL RADIOGRAPHY SYSTEM



Pros:
- Practical, simple cassette

Cons:
- Complex readout system
- Limited spatial resolution

The optical signal is not derived from the light that is emitted in prompt response to the incident radiation, but from subsequent emission when electrons and holes are released from traps in the material. By stimulating the crystal with red light (by a laser raster scan), electrons are released from the traps and raised to the conduction band, then triggering the emission of shorter-wavelength (blue) light (photostimulated luminescence).

It is the most adopted digital recording of radiography. It is a passive recording of the image, in the sense that we have a phosphorus screen that is photostimulated and then it is recorded.
The other topology are instead active electronics panels.

A photostimulable phosphor is a cassette of the right dimension made out of a material where the energy taken by xrays is producing trapping of charges. It means that if we have a crystal and we have the energy gap, electrons, by absorption of energy, may fall into such traps.

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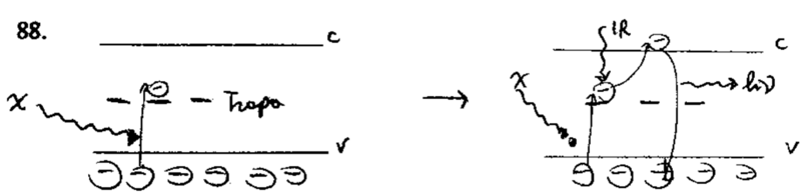


These are the energetic levels; usually electrons stay in the valence band, they don't participate to conduction, but inside the main material we may have some traps.

And since traps are due to atoms not belonging to the main material, they have their own energy level, they don't need to respect the subdivision into VB and CB.
So, when we have an absorption of xrays, it may happen that electrons are promoted not to the CB, but to the traps, and they remain in the traps as long as we don't do something.
So during the recording with a cassette this is the process that is going to happen, and the trapping of electrons is proportional to the intensity of the absorbed xrays, and this is the recording.

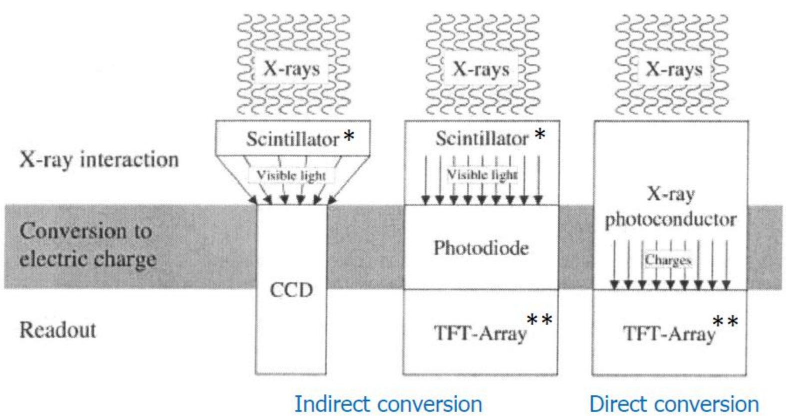
Hence in the recording the information is not accessible, because it is inside the density of trapped electrons. Then we take the cassette, we go to a system with a laser scanning, the system provides infrared irradiation that is able to let the trapped electrons to make the remaining part of the jump in the CB → electrons escape from the traps to reach CB.
Then, if the electrons reach the CB, they will easily go back to the valence band (thing that they were not able to do before because they were trapped) and this will correspond to the emission of a UV (shorter wavelength, because the energy is larger → no interference between the exciting light (infrared) and the emitting light).

Finally, the amount of photon we record exiting is proportional to the trapping. The amount of light we record is proportional to the stored trapping.

Pros and cons

It is a very practical solution because we don't have electrons moving when recording the image but we don't get a prompt signal out of the sensor, we need to have an offline readout system.

DETECTORS FOR DIGITAL RADIOGRAPHY



* or phosphor ** Thin-film transistor

This is the detectors we are interest about, that are active detectors in the sense that they are providing electrical signals while they are recording xrays.

A first classification is that these detectors can be either:

- Direct detectors: we have direct conversion of xrays into charges. However, the material responsible for such direct conversion is not a depleted pn junction, but it is a particular material non-depleted called **photoconductor**. A photoconductor is a material which changes its conducibility with radiation.

- Indirect detectors: scintillator-based photodetectors. We have a scintillator, we may read out the scintillator by a Si photodetector, either CCD or a CMOS imager, or we may have a scintillator read out by a photodiode.

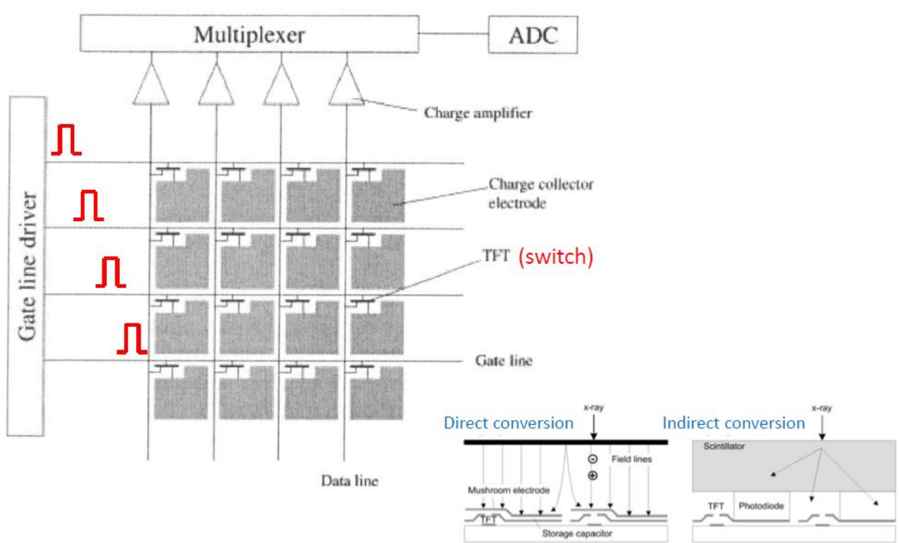
Why don't we use all the time Si photodetectors or CMOS imagers?
Because of costs, the cost is a dominating factor in this technology. The issue of cost is in the radiographic panel, that can be very huge. A panel for radiography cannot cost more than 40.000 euros.

Because of this, if we really want to use Si devices like CCD or CMOS imagers, we need to focus the light out of the scintillator into a smaller field of view Si sensor (first scheme in the image). The only exception is mammography; mammography has a reduced field of view so we can afford to have a cassette where we have a 1 to 1 ratio between the area of the scintillator and the area of the CCD, otherwise we need to use some focusing systems that are, anyway, still expensive.

Another popular technology, besides Si one, is thin film transistors (TFT). It is a chip technology based on not crystalized materials, but amorphous materials. This is the reason why a very popular semiconductor for this technology is Selenium amorphous, which is an amorphous material.
Amorphous means that we can build and stack several layers of material without the need to respect crystallin alignment (with silicon, everything has to respect the silicon crystal orientation).

The TFT are arrays and once we have this thin film technology, it can be used both for indirect detection with photodiodes and for direct detection with photoconductors.

TFT ARRAY: ARCHITECTURE



We are talking about a pixelated sensor, and a pixelated sensor is an array of pixels that are organized in rows and columns and in each pixel unit we have a charge collecting electrodes. So we have for each pixel an electrode, that can be represented as a capacitor, which is collecting the charge.
The collected signal may be the result of a direct interaction or of an indirect one.

In the bottom of the image we can see that in case of direct conversion detectors with photodetectors we have a semiconductor an we have direct creation of electron-hole pairs directly in the material by xrays. Then such charge is collected by one electrode.

In the right we can see instead the indirect conversion version of the detector, where we have the scintillator, the xrays are converted in optical light in the scintillator, and then we have photodiode, and the photodiode converts the light into electrical charge.
At the end we have an electrical charge in both processes, but the two processes are different, but **the readout is the same**, at the end we have in both cases some charge accumulated on one electrode.

Then the charge accumulated is proportional to the exposure to xrays. These detectors are integrating, in the sense that when the xray tube is activated, we have xrays for a certain period of time (typically 1s), we collect in the sensor the charge created by xrays; but this charge is the superposition of xrays, the accumulation of xrays recorded for 1s, we don't have the capability to distinguish interactions ray by ray → it is a cumulative information, it is the total charge accumulated by the absorption of xrays in the recording time.

Differently, in PET we have the scintillator, a single gamma ray that creates a pulse and we record the pulse. This modality is called pulsing mode, because we are recording single events. Here instead we are integrating information, we are loosing any knowledge about individual rays. This thing makes the digital radiography system very easy, but on the other side we are loosing information; for instance, we are loosing information about the energy of the xrays. **Digital radiography is not able, at least in the traditional way, to record the image depending on the specific energy of the xray tube**.

Organization of the array

Of course, we cannot apply one amplifier for each pixel, we need to scan the array. And the array is scanned row by row, in the sense that the pixels are connected column-wise to a single amplifier. All the pixels in a column are connected to the same amplifier. It is not a permanent connection because there are switches, and only the pixels connected to the same row are connected to the set of amplifiers each time.
When we apply a pulse to a row gate line driver, we switch on the transistors of one row and only these are connected to the amplifiers. → **the number of readout lines is equal to the number of columns, not to the number of pixels**.

Then eventually we can also multiplex the signal of the amplifiers, and sample them one by one using a single ADC converter.

CCD-BASED READOUT SYSTEMS

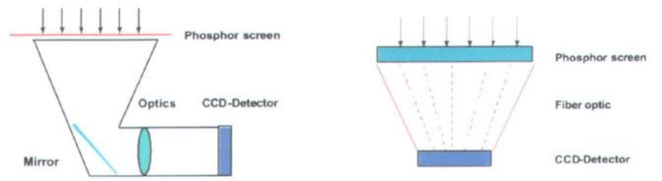


Figure 7. Collection of light from an X-ray absorbing phosphor or CsI(Tl) scintillator with optical lenses and mirrors (left) or with the use of fibre optic (right). (Figures supplied by Siemens).

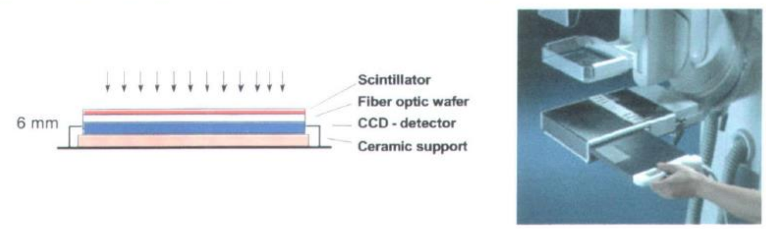


Figure 8. A CsI(Tl)-CCD detector with a 4.9 cm x 8.5 cm field of view. The principle of operation is shown (left) as well as the use of the detector in a cassette that can replace a film cassette in a mammography unit (right). (Figures supplied by Siemens).

We can either focus the image in the CCD area using focusing optic or optical fibers or we can use a 1 to 1 coupling of the CCD with a scintillator but with a smaller active area.

CCD: charge couple device

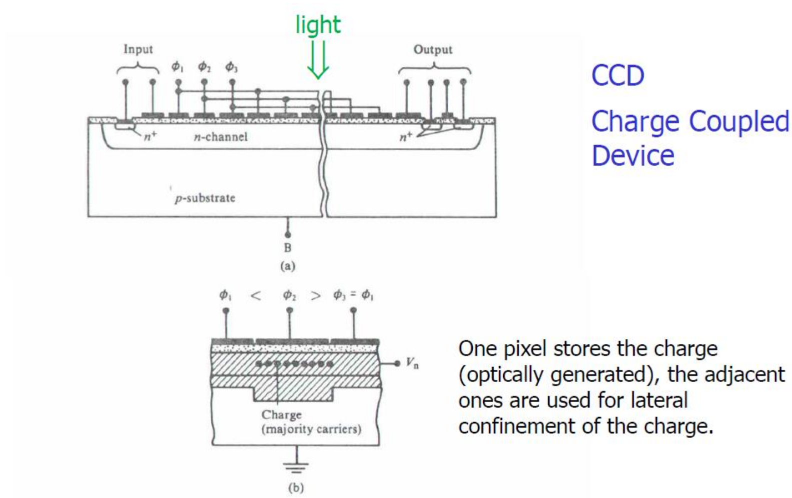
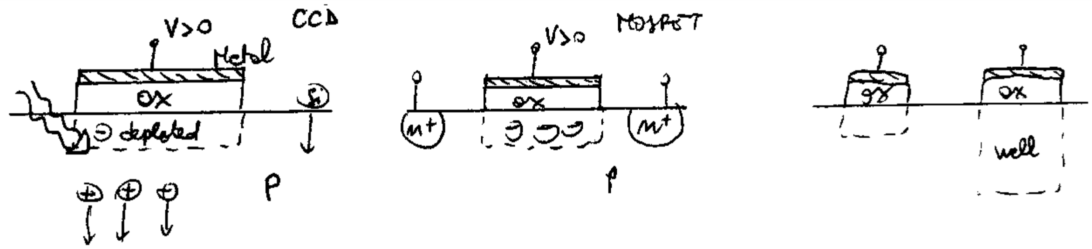


Fig. 3.3 Bulk-channel charge-coupled device (BCCD). The structure in (a) is a three-phase structure, showing the n-channel on a p-substrate; and (b) shows the charge stored in the n-channel as majority carriers. Reprinted after Ref. [4] with permission.

The CCD is an optical sensor. On the top we have the structure on one dimension.
It is a silicon sensor based on an array of MOS capacitors similar to the one used in MOSFET transistor; the difference is that MOSFET transistors are not just MOS capacitors but also include electrodes.
These arrays of MOS capacitors are aligned on the surface, the light is collected by the same side of the device, it is creating charges that are accumulated at the interface between oxide and silicon (bottom figure). The charge created by the light is accumulated in this interface and this is the **photodetection phase**, where the light is converted in charge. Then there is the **readout phase**.

Basics of a MOS junction

89.



In a MOS junction we have silicon, the oxide and a metal. When we apply a voltage larger than 0 between the metal and the silicon, and the Si is of the p type, the holes go away from the region close to the oxide (we are depleting the region below the interface with the oxide, because holes don't like to stay close to positive potential) and we create a region of depleted Si.
Then, in MOSFET theory, when the voltage V is particularly high, we create accumulation of electrons in the interface (the so-called inversion), we create a channel and, by adding source and drain, we have a connection between source and drain through this inversion channel.

In the CCD we have to forget these last things. We have no source and drain and no inversion layer. The CCD is just a MOS capacitor (metal, oxide and silicon), and we simply create the depletion region, we are not interested in inversion.

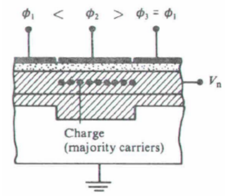
Then the optical light coming from the top of the device will create electrons that will be attracted by the positive voltage of the metal and will be accumulated and stored at the interface.
Hence <u>we have created an MOS structure with depleted silicon ready to accept charges created by optical light (otherwise the depletion region is empty), and then the charges are accumulated in the interface and, as long as we don't do anything, they don't move</u>.

If now I replicate this structure multiple times, depending on the difference voltages I apply to the gates, the amount of depletion region will be different.
For instance, if I apply a relatively low voltage, I will have less depletion and less attraction capability for electrons; if I apply a larger voltage, I will have a deeper **well** (it is the name of the depletion region) and the capability for attractions of electrons will be larger in the second well → electrons will prefer to go to the bigger well, because electrons like positive voltages, and so they go where the positive voltage is larger.

With these reasonings I have introduced the concept of **spatial confinement of the charges**. It means that, depending on the voltage between neighbours, the charges prefer to stay in one structure and it is indirectly confined with respect to the previous one. Depending on the relative voltage, one volume is confined with respect to the other, because electrons prefer to go into the volume where I have applied the larger voltage.

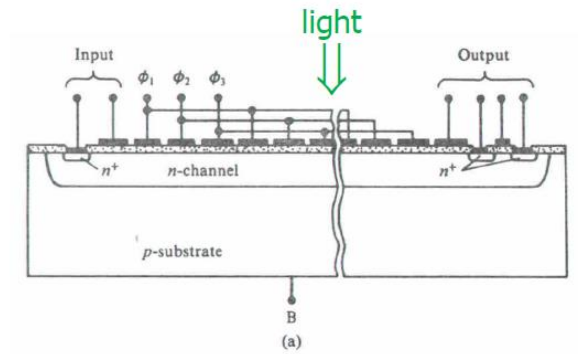
In this drawing there are 3 units of CCD, 3 MOS structure. We see that the voltage on the middle one, phi2, is larger than phi1 on the left and phi3 on the right. If I have charge around, I suppose that any charge create in the depletion region by optical radiation will stay below phi2, and it is confined with respect to the neighbour phi1 and phi3.

In the drawing it is also reported a higher depletion region.
So, this introduces the concept that, among several of the metallic contacts, of the MOS structure, one at a time will be devoted to accumulation, and the two neighbours will be devoted to confinement, and this will be repeated every 3 electrodes. We have to imagine to have triplets of electrodes, where the middle one will be used for accumulation, and the two neighbours for confinement.



One pixel stores the charge (optically generated), the adjacent ones are used for lateral confinement of the charge.

We can understand why the voltages are given the same every three electrodes.



Phi1 is not given to one single electrode, but also to the other electrodes every 3. We don't apply individual voltages, but triplets of voltages.
This because one electrode every three will be used as accumulation and the other for confinement.

Charge storage in the pixel

We can see that we have a single MOS. If I apply a voltage I create a potential well where electrons will fall down.
If we look at the bands (last images), we can see a potential well, so a region where the bands are bent and electrons can be confined in this well.

The amount of electron themselves conditionate the depth of the well. More electrons we have in the well, higher becomes the top level of the potential of the well, which means that more electrons are filled in the well, the more capability of the well to collect additional electrons reduced.

This means that a well has a maximum capability to accumulate charge, and this is a parameter we find in CCD specifications. It is the charge handling capability.

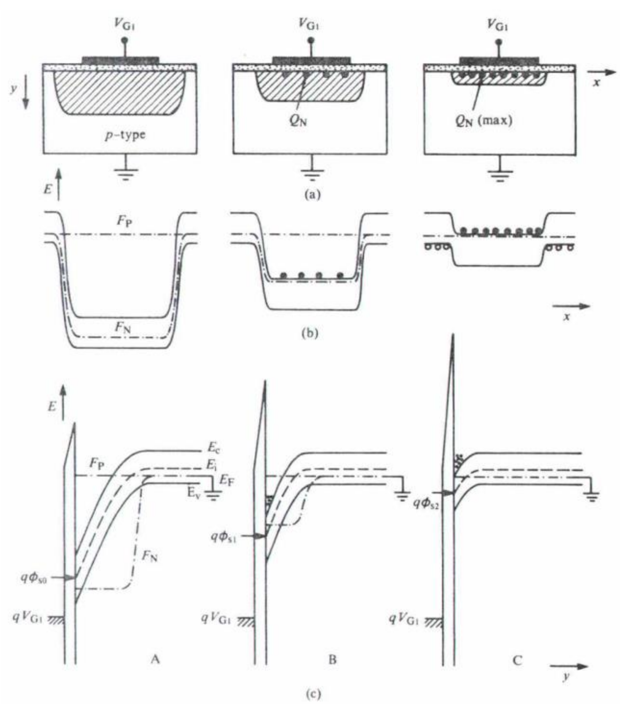


Fig. 3.6 MOS-C of Fig. 3.5 for points A, B, and C. (a) shows the space-charge regions, (b) shows the potential diagrams along x for $y = 0$, and (c) shows the potential diagrams along y into the semiconductor. The inversion charge is indicated by the solid circles.

It means that each well is able to receive 1 milions of electrons, and no one more → hence we cannot illuminate the CCD with whatever light we like, we have to be careful about the maximum intensity of the light we apply, because the more light we provide, the more we fill the single well of the device.

Charge transfer (3 phases) – LINEAR CCD

How we read the charge?

So far, the charge can remain forever in the pixels, so haw can we read out it?

The charge is read out by a peculiar method of the CCD called PERISTALTIC TRANSPORT OF THE CHARGE.

First of all, on the top of the figure we can see the accumulation situation. So we have irradiated the CCD and we will have more or less charge accumulated in a MOS structure.

Once again, we can note that the electrodes which are able to host the charge are 1 every 3.

The other has been suitably biased to create confinement, otherwise we would have a uniform lake of electrons.

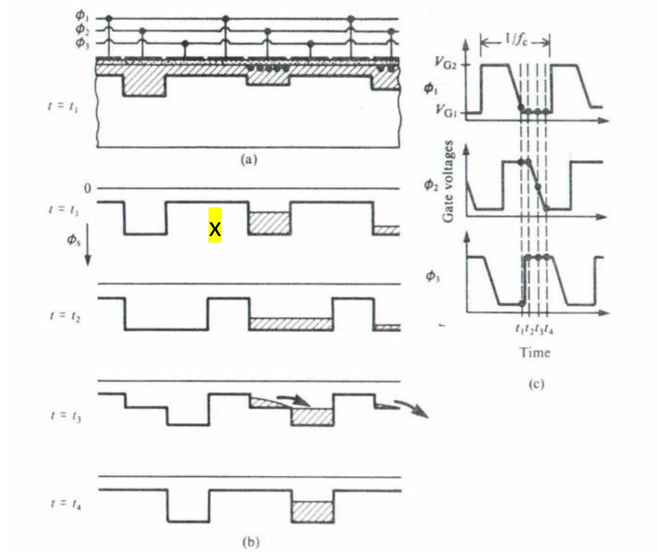


Fig. 3.11 (a) Device cross section, and (b) surface potential diagrams for times t_1, t_2, t_3, and t_4 shown on the clock-voltage waveform diagram in (c).

NB: from the device point of view, I can say that I have a pixel for each electrode, but from the imaging point of view, the pixel of the image is as large as 3 electrodes. There is a factor of 3 between the pixel of the image and of the CCD cell.

How to transfer the charge?

We have the situation in which phi2 is much larger than phi1 and phi3, which are for confinement. X is the equivalent hydraulic model.

Now, what we do is to rise the voltage phi3 up to the same potential of phi2. If we do so, we create a deep well on the right (phi3 is the electrode on the right). → water (aka charge) is spread between the two electrodes.

Then we lower the voltage of phi2; in this way we push up its potential minimum, and so the water will be finally transferred to the electrode of phi3.

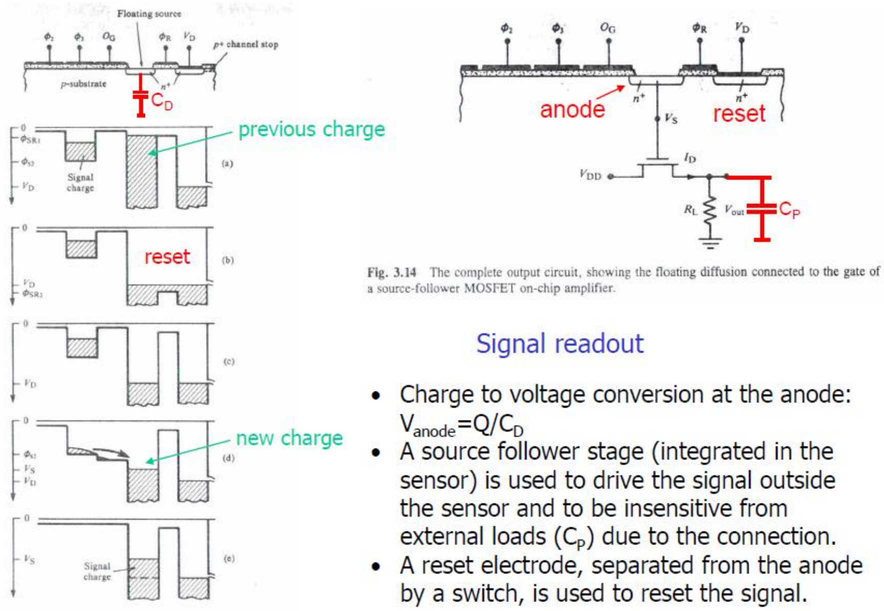
Phi1 is not operational, remains always to the same voltage.

The result of this voltage variations is that we have transferred the charge of one segment. Then we iterate this procedure. We will lower phi1 and so on → peristaltic transport: we switch on and off the potential on the pixels and in the end the charge will transfer.

The speed of the transfer will be the clock period. How many steps will be made? With 10 clock cycles we move the charge in 10 pixels for example.

Now we have several electrodes, and clock after clock the charge will arrive to an output node that is made as following.

Output node readout

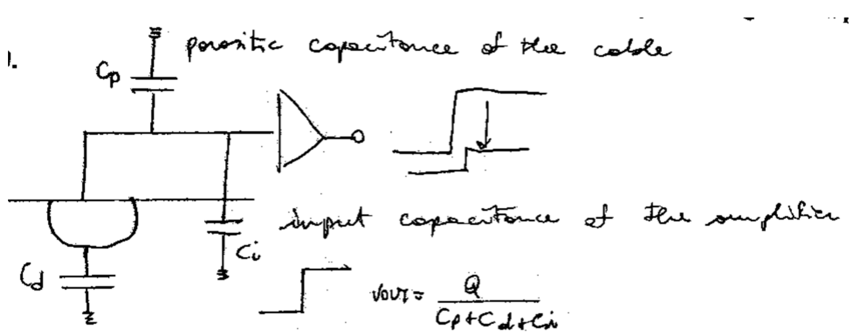


We have a floating electrode and hence a floating capacitor. So let's imagine we have a floating capacitor Cd.
Once we make the peristaltic transport, the charge will go into the capacitor Cd and create a voltage step. How much is the amplitude of this voltage step?
It is equal to the charge divided by the value of the parasitic capacitor Cd. When we drop a charge into a capacitor, we have a Heaviside step that is in fact the charge divided by the capacitor.

So we have an electrode with a voltage step and this in principle can be enough, with an amplifier we can record the step.
In conclusion, the great advantage of a CCD is that it is a pixelated imager but the charge is readout not pixel by pixel like in a CMOS sensor, but in a single electrode.

In reality, to read the charge on the capacitance Cd we would need to connect with a wire the Cd to an amplifier, but the capacitance of the wire and the one of the amplifier (input capacitance) would be in parallel to Cd. Hence in the conversion I would have not Q/Cd but Q/Ctot in parallel.

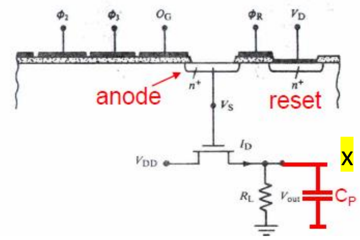
90.



All these capacitances are in parallel with the capacitance of the charge to voltage conversion → the voltage step is as in the formula.

Hence we are going to get a very small output signal at the output of the CCD, so small that we can't measure it anymore with the amplifier → this measurement system is not good.

To solve this problem, we put into the CCD a transistor in source follower configuration. The output is x.



The voltage is still the same as before, but now, since the transistor is very close to the readout anode in the silicon, the parasitic capacitance of the connection is disconnected. The capacitance Cp is no more in parallel with Cd.

So we are recording Q/Cd without the effect of the external capacitance, because now the follower is driving the voltage, but the follower is now at low impedance.

Of course, connected to the anode there is the gate capacitance of the follower, that is indeed in parallel to Cd. But the follower is so small that the capacitance of the metal connections is very small → **the conversion is not corrupted, and by using the follower we have preserved the conversion of the charge into the anode capacitance, and then, once we have the voltage step, the follower provides the voltage step at low impedance**. Hence I don't care about Cp.

Once the charge has reached the anode of the follower, we need to reset the charge, and this is done by one electrode, that is the reset electrode. It is an electrode that is separated form the anode with another gate (phiR) and so when we switch on the gate of the reset, the anode voltage is set equal to the reset voltage.

So anode, gate (phiR) and reset creates like an nMOSFET. When the voltage phiR is low, we have a total disconnection between reset and anode, and this occurs when the anode is accumulating charge. But when the measurement is over, we switch on the gate, it is like to have a nMOSFET on and so like to have a drain shorted to source, so we impose to the anode the voltage on the reset.

Then we switch off and it is ready to collect charge again.

This configuration of CCD is called 3 phases CCD because one electrode accumulates the charge and we need two other electrodes for confinement.

There is also a two phases CCD

TWO PHASES CCD

We try to remove at least one of the two electrodes for confinement creating an internal confinement.

Now we can see that in this type of CCD, inside one electrode, the oxide layer is different.

We have one single metal, but the single metal is superposed to two different internal oxide thickness.

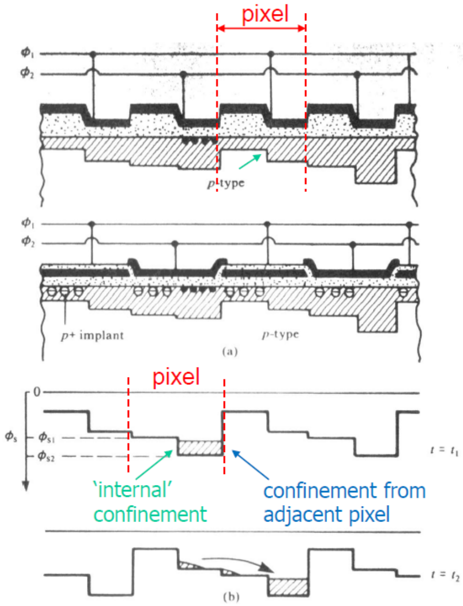
In particular, where we have the shorter oxide thickness, we have a deeper well, and where we have the thicker oxide we have the lower depth.

This means that inside each electrode, we have two internal different depths.

Each pixel now has internally two different bottoms. If the bottom is uniform, the water goes everywhere, but if we have two levels, the water will go only on the half of it.

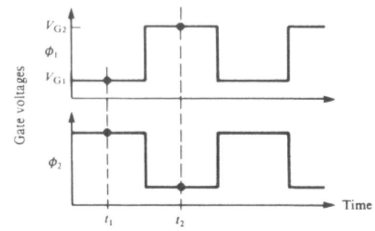
Now, inside one phase, I have an internal confinement on the left. So the water is confined on the right half of the pixel, because the left part is used to do the confinement.

In this way we need only two phases, because for confinement we need only the phase on the right, on the left the charge is internally confined inside the electrode.



Charge transfer (2 phases)

A step in the potential well inside the pixel (by an additional implantation or by a different oxide thickness) creates an 'internal' confinement inside the pixel itself (marked in the figure)



TYPES OF 2 DIMENSONAL CCD

Frame-transfer CCD

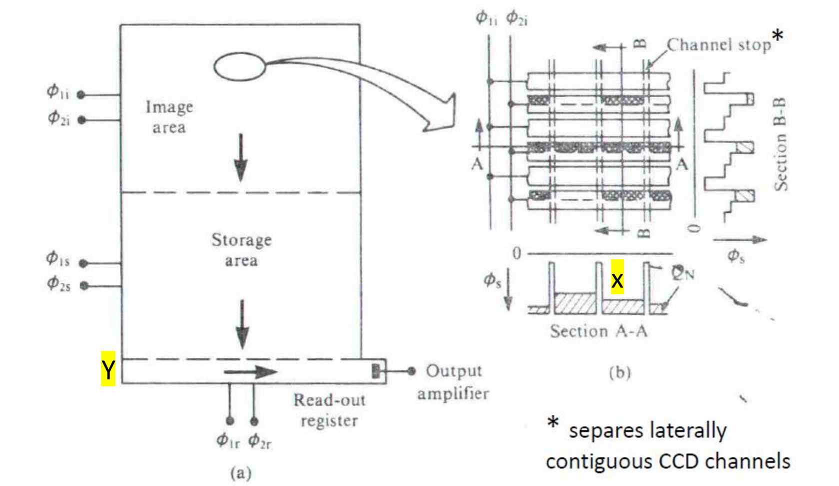


Fig. 4.12 (a) Schematic of a frame-transfer CCD imager, and (b) an enlarged section of the image area, showing the gate structure and surface potential profiles. A two-phase CCD is used.

In frame-transfer CCD we have an imaging area where the charge is created and stored, and then we have a storage area where the charge is transferred and digested out of the device while in the imaging area we are recording a new image.
So on the top we have only area exposed to light illumination, while the storage area is dark, covered with a dark layer.
So light is entering in the imaging area. Once we have recorded sufficiently the light, we transfer by peristaltic transportation the charge into the storage area and we are recording the image.
While we are recording a new image, the storage area is read out row by row by a linear CCD in the bottom.

In the image area we have a superposition of parallel linear CCDs. It is a 2-dimensional CCD because it is the parallel of several unidimensional devices.

How can the linear devices not share charge among them? How the charge remains confined in each column and not spread to the other neighbour column?
We use some electrodes called channel stop (x). They are barriers; the charge in fact is confined in each column and it cannot be spread in the adjacent columns because we have grown some potential barriers.

So the charge can be moved only vertically, it cannot move horizontally.

Then, on the bottom we have a linear CCD (y), so we have to imagine that we take one row in the storage area, we transfer it to the linear CCD and then we read it out horizontally, and then we flip down another row and read it out horizontally and so on.
Of course, the clock of the horizontal CCD must be much faster than the one of the vertical transfer, because for one clock cycle of the vertical transfer we read out an entire row → it is important the synchronization of the clocks.

Interline transfer CCD

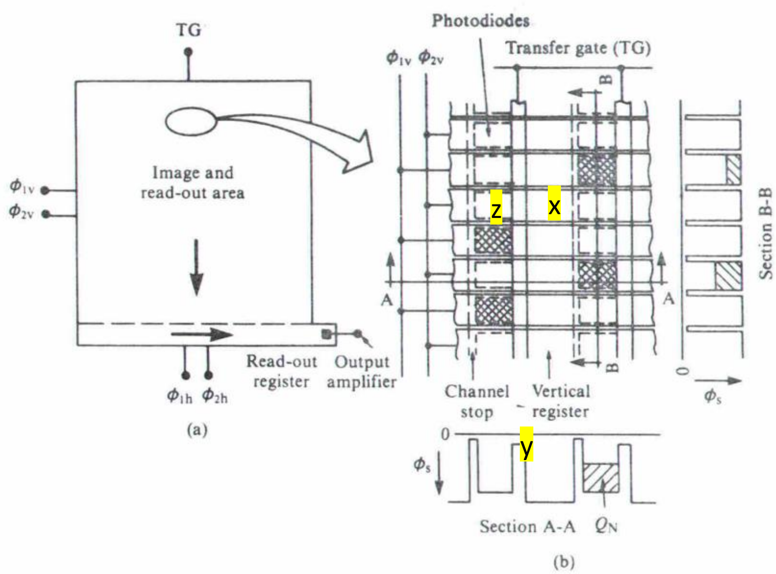


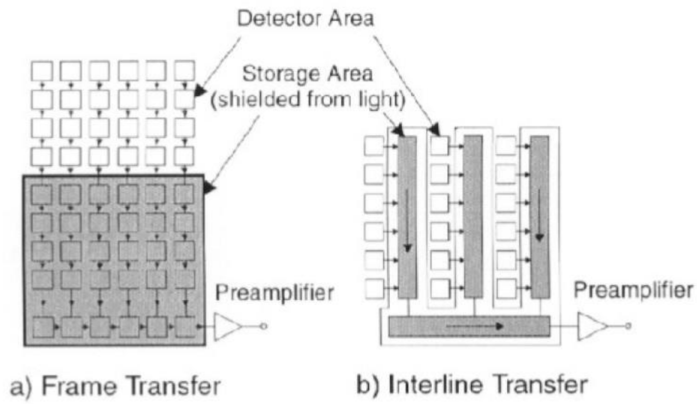
Fig. 4.13 (a) Schematic of an interline-transfer CCD imager, and (b) an enlarged section of the image area, showing the gate structure and surface potential profiles. A two-phase CCD is used.

In this case, imaging area and transfer area are the same. How is it possible that the charge is accumulated and transferred at the same time?
In this case we don't have CCDs in parallel, but we have an alternation of standard photodiodes and linear CCDs (x).
The photodiodes are exposed to light, the CCDs are not exposed to light, they are darkened, and just used as transportation path. So photodiodes collect the light and while the light is collected, the transfer gate y is a high barrier, there is no possibility that charge in the photodiode falls in the CCD channels.

Then we remove the barrier, the charge of the photodiodes is transferred in the CCD channels and then we rise again the barrier y.

Let's assume the charge is accumulated in photodiodes z. The barrier is high, so no charge can move in the CCDs. Then we lower the barrier, the charge flows laterally to the CCDs and we rise up again the barrier.
Now, while the photodiodes can record new fresh charges, the CCD is slowly transferring the previous charges.

Summary of the CCDs family



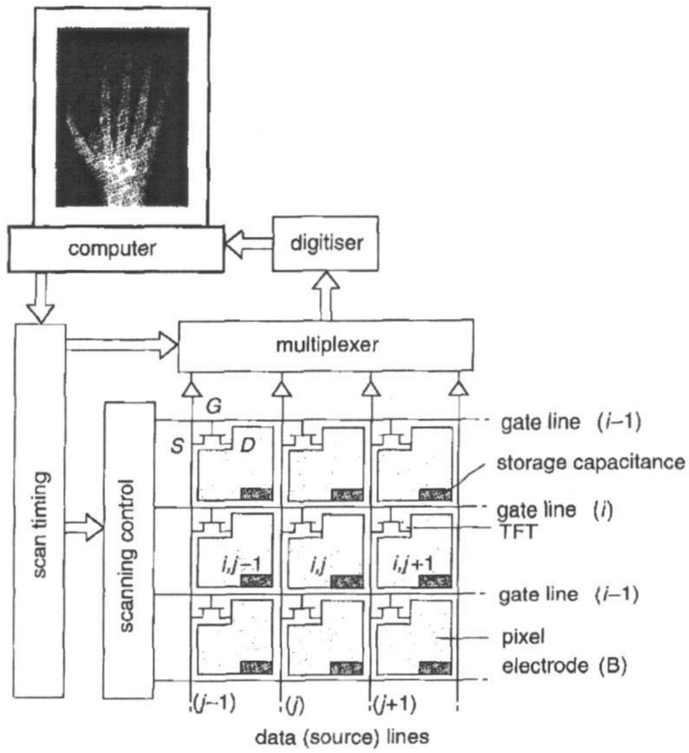
- 100% efficient detector area (although occupying 50% device foot-print area)
- Longer transfer time from detector to storage area

- 50% efficient detector area (although inside the full device foot-print area)
- Quick transfer from detector to storage area

We have to choose the one we prefer according to what we need.
In the frame transfer, one part is exposed to light with an efficiency 100% and the other part is shielded from light and we transfer the light in the darkened one. The advantage is that we have at least one part fully efficient for the scintillator, so we could for instance focus the scintillator in the first region that is 100% efficient; the disadvantage is that the footprint of the device is only 50%, because only 50% can be illuminated. And, moreover, the transfer of charge from the top part to the bottom one needs to work with the clock of the CCD (peristaltic clock).

The interline transfer has the advantage that the footprint of the device is fully available. We could make 1 to 1 matching between scintillator and the footprint of the entire device, but the disadvantage is that only 50% of the internal surface is efficient to light, the bars in dark are the transfer channels which are used only for transferring the charge → not 100% optical coupling as in the other.
Finally, the other advantage of the interline transfer is that the transfer of the charge from the detector area to the transportation area is 1 clock cycle, hence very quick, while in the other one we have to wait the CCD.

READOUT SCHEME OF PIXELS WITH TFT



Type of sensors based on TFT. They are **based on a matrix of collectors of charges**; the common point of the different derivations is that we have a *storage capacitor* where we store the charge created by direct (photoconductor) or indirect detection (scintillator + diode).

We have, looking from the top, an array of pixels connected column-wise to a common amplifier. A single pixel is activated only when the corresponding row is activated. Only the pixels of the row are connected to the amplifiers, one row at a time, and row by row we scan all the pixels.
To do so, we need switches. The transistors activated by the gate line are switches that connect the storage capacitor to the input node of the preamplifier, otherwise the pixel is floating.
Normally **during image recording the pixels are floating**, waiting for the charges to be integrated, and **then when it is time to readout the signal they are connected to the amplifiers and read**. Eventually, also the amplifiers are scanned with a multiplexer and a unique digitizer to make the ADC conversion of the amount of charge stored on the capacitor.

DIRECT CONVERSION X-RAY IMAGER

Direct conversion = we expect the xray to be absorbed in the material mainly by means of photoelectric effect. In fact, we have several competing absorption mechanisms (Compton and photoelectric), but in radiography the energy involved are of 100-140keV, so in this range photoelectric effect is still dominant and we may have also Compton effect in the high range of energy. By means of photoelectric effect, we create charges in the material that are collected by two electrodes (holes on the bottom and electrons on the top) thanks to the applied electric field, and then on the bottom we have a storage capacitor.

So we have the sensor material x, to the material we apply a positive voltage on the top electrode A, and then on the bottom electrode we connect a capacitor which is connected to a ground electrode. So at the beginning, when at t = 0 no charge is preexisting on the capacitor, we expect the y electrode at 0 → ground on both the plates of the storage capacitor.

Hence if we have a plate at 0V and the other (A) at 100V, of course the charges created by ionization will see electrons moving towards the positive voltage and holes towards the lower voltage. We don't care about electrons collected on the top, while holes will be collected and stored on the top plate of the capacitor → storage capacitor Cij.

The storage capacitor is the capacitor Cij, that is storing the positive charges that are drifting toward electrode y after the interaction of the xray in the material.

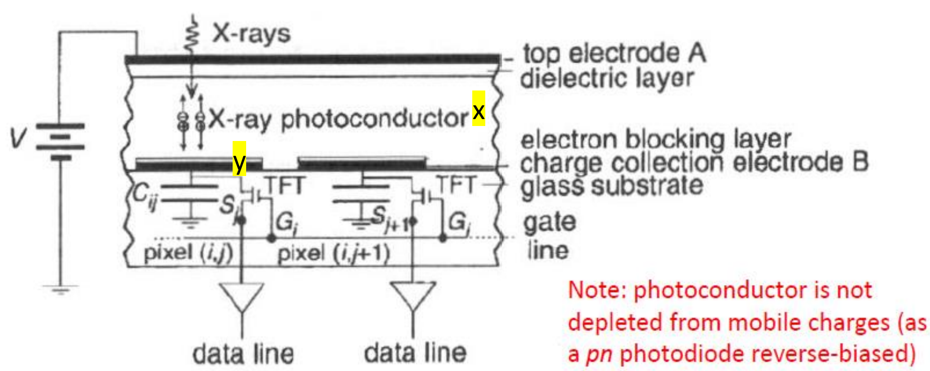


Fig. 4 *Simplified schematic diagram of cross-sectional structure of two pixels of a-Se self-scanned X-ray image detector*

NB: this is an integrator detector, in the sense that we are integrating charge and we are accumulating the charge created by the flux of xrays, we are loosing the information regarding the single photon.

We are not detecting the single photon, but the integral of thousands of photons and we are integrating the energy pf photons, but the energy of the single photon is lost.
So most of the imagers for digital radiography are of integration mode. Nowadays there are also new imager based on pulse mode, able to detect photon by photon → we can get a nice information about the energy of the single photon, because our body is sensitive to the different photons' energies.
In fact, we may have tissues for example more sensitive to 100keV and less sensitive to 10keV. Hence it is important, in order to achieve contrast in tissue imaging, to have information about the energy, but this is possible if photons by photons can be detected.

Detectors in pulse mode are not so widespread because the electronics must be very fast to distinguish photons by photons in very intense xray irradiation.

When we do a radiography, we have plenty of photons that are arriving on the detector, and it is very difficult to have the capability to distinguish photon by photon; this is the reason why the most popular detectors are integrating, losing the information about the single photon.

Coming back to the image, this is a traditional ionizing detector, the charge is stored on each capacitor and now we can understand that the detectors y are pixelated. So the imaging capability is given by the pitch between these electrodes. The electrode on the top is not pixelated, just used to provide the other side of the biasing.

Then we have a transistor, a MOSFET, which connects, when we need to read out the array, the capacitor (where the charge has been stored) to the data line row by row.

The Photoconductor

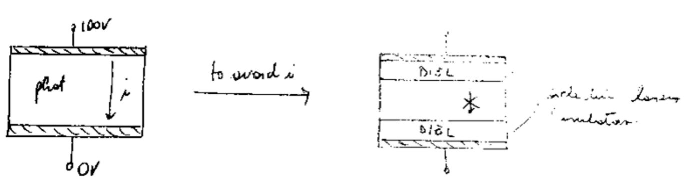
Material where the conducibility changes with irradiation. It is a material able to absorb optical light or xrays and it changes its conducibility due to this absorption of light. It varies because the light creates charge in the material by ionization.
Usually, photoconductors are read out by circuits that sense the change of conducibility. Hence the photoconductor changes the conductance, and if we put this conductance in a system where it can be measure, we can measure the signal.

However, in this case the photoconductor is used as ionization material, it is a material where you create e-/holes pairs and we don't measure the change of conductance, but the charge reaching one electrode.
Independently on the name, this is a traditional ionization material, it is like a pin diode (even in Si pin diode we create ionization and we collect charges).

The main difference between ptc (photoconductors) and pin diode is that pin diode is a reversed bias junction which makes the volume free of mobile charges, so no mobile charges exist in the material, and so we have only to collect the charge created by the radiation. On the contrary, in ptc mobile charges exist; it is not a reverse bias junction, but a piece of material where we place two electrodes and create a drift field, but we still have charges in the material.
This is important because I have to be careful about the presence of mobile charges in the material, because the mobile charges may go to recombine the charges created by the photons.
We will see this concept later, but we can say that we cannot exceed a given thickness of the material, otherwise the charges will never reach the electrode, because they will never survive if they have to travel a lot to reach the electrode.

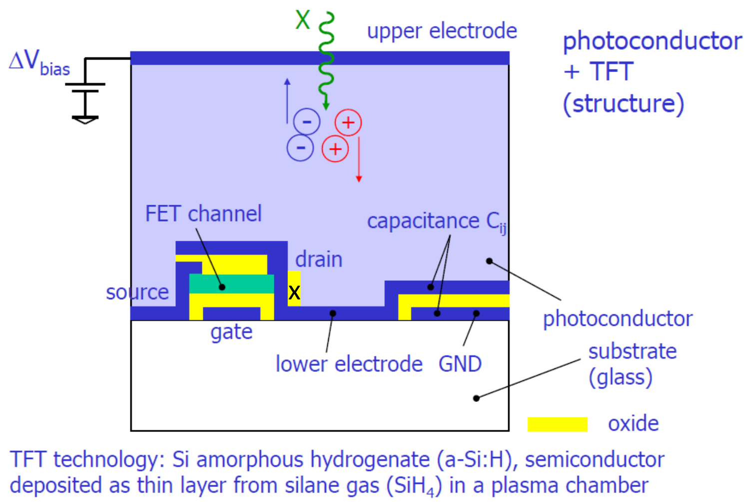
Moreover, in the image we can see that there is a dielectric on the top electrode and another one on the bottom electrode (called electron blocking layer) → the ptc is not let to be conductive on both sides, but there is a thin layer of dielectric in between. This because if we take two pieces of metal and we put them in contact with a conductive material, we have a huge current flowing in the material.
And in principle is not good to have a permanent current flowing in an electrode that is supposed to collect only the signal current.
Hence the dielectric layers are an attempt to avoid the injection of current through the contact; in fact having a piece of conductive material between two layers of insulator prevents the permanent injection of charge in the electrodes.

91.



So we have our photoconductor material, we place an electrode on the top and one on the bottom and if the two electrodes are at different voltages, we have current flowing through the material. In order to avoid this, we insert in the material two dielectric layers; now the charge cannot be injected externally because we have the insulator. We still apply the voltage difference but charge is no injected in the material.

Photoconductor + TFT



The only missing layers in the drawing are the dielectric ones.
This drawing introduce the TFT technology. The TFT technology is a very simple technology to build the electrodes and the device we need in this system. In fact, the components I need in the system are: active layer of ptc, a plate for the capacitor to collect the charge, the bottom plate of the capacitor to ground and a MOSFET that works as a switch.
In principle, I could have had all these devices in a Si wafer, but we don't use Si because it is a crystalline structure, so we cannot take Si an build over it whatever material we like, because we have to respect the crystalline coupling of the layers of Si.

This is an **amorphous technology** (amorphous = there is not the need to respect some crystalline structure). So we start from a substrate and then, by spattering, we deposit layers of materials that we need to build the electrodes, but these layers don't have to couple particular crystalline orientations.

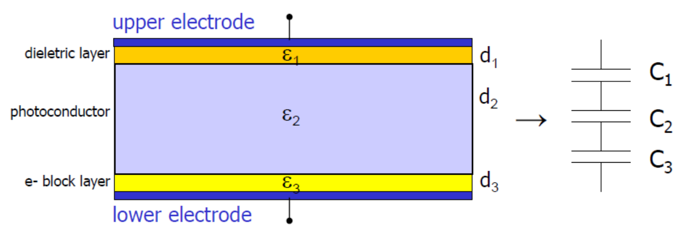
We can see that in the layering in the bottom we have built a capacitor (where the bottom plate is to ground and in between there is a layer of oxide). Then we have a FET (transistor) with drain, source, a channel in between (so it is a depletion mode MOSFET, the channel is already existing) and the channel is controlled by a gate.
The drain of the transistor has to be connected to the top capacitor, while the gate to the gate line that controls the gates of all the MOSFETs.

NB: what is in the schematic just represented as an electrical connection between drain and top plate of the capacitor Cij is in reality a very extended layer of metal. In principle, the drain could stop in x, but it is extended all over the top because this metal is also the collecting electrode for the charge. To have 100% collection efficiency, in principle the top blue electrode should be extended all over the distance of the pixel.

Then, once we have created the layers, we can grow on top of them the photoconductor we like. It is like if we build a scintillator over a ptc. It is impossible to build a scintillator over a ptc, because they don't match each other, while in this technology yes, because the materials are amorphous.
The pixel pitch is 139 um, the electrode (mushroom electrode) is a bit smaller. If it would be as large as the pixel size, it would short and touch the contiguous pitch → we need a small gap between pixels.

Pixel capacitance

If I have a 'sandwich' of material composed by the main ptc (dielectric), two layers of dielectric (1 and 3), if I consider the bipole with and without the dielectric layers, does it change from the electrical point of view?. No, because when we take 3 materials of different dielectric constants and we put them together, it is equivalent to place 3 capacitors in series.
The series of 3 capacitor provides a total capacitance Cp (pixel capacitance), that is the resulting capacitance between the two electrodes (it is NOT the collecting capacitance).



$$1/C_P = 1/C_1+1/C_2 +1/C_3$$

$$C_X = e_X S/d_X$$

$$\Rightarrow C_1, C_3 >> C_2 \qquad \Rightarrow \boxed{C_P \sim C_2}$$

Moreover, a capacitor is proportional to the dielectric constant and reversely proportional to the thickness of the capacitor → in the series, the smaller d the larger the C. But in the series of the 3 capacitors the smaller is the winner.
Hence in the formula the Cp will be equal to the ptc capacitance.

In conclusion, even if we have placed the two layers of dielectric for the previous mentioned reasons, it is like if they are not present, because ptc capacitance dominates the bipole capacitance.

Having clarified this, how is the applied voltage distributed across the series of the two capacitors (pixel and storage ones)?

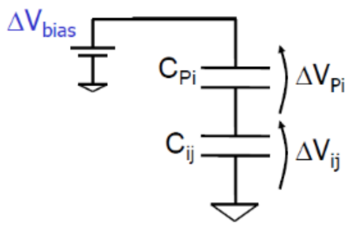
Biasing

We have two capacitances in series, the pixel capacitance and the storage capacitance, and the pixel capacitance can be assumed to be equal to the ptc capacitance.
I'm interested in knowing how the bias voltage drops across the two capacitors. Where I'm more satisfied the bias voltage to be dropped? Across the ptc or the storage capacitor? Across the ptc capacitor, because it is where I want to rise the electric field to separate charges.
I'm not interested in having a DC voltage on the storage capacitor, it should just receive the signal.

If we consider a very simple partition, the voltage across the pixel is given by the bias voltage times the storage capacitor divided by the Cp + Cij. It is sufficient to choose Cij>>Cp to neglect in the formula the Cp, hence drawing the conclusion that the voltage across the pixel is indeed all the biasing one.

When we instead have a signal generated by charges, it is like to have a current generator between the upper and lower electrode. Hence in our modelling is like to have charge placed as a generator across the pixel. This charge now integrates all over the parallel of the two capacitors.
Once again, if Cij >>Cp, the voltage on Cij is indeed the charge divided by Cij.

Biasing

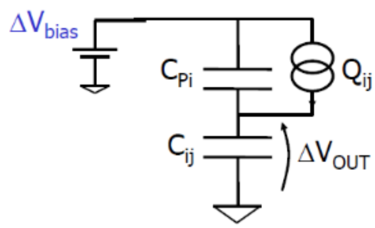


$$\Delta V_{Pi} = \Delta V_{bias}\, C_{ij}/(C_{ij}+C_{Pi})$$

$$C_{ij} >> C_{Pi} \qquad (C_{ij} \sim 1\text{pF})$$

$$\Rightarrow \boxed{\Delta V_{Pi} \sim \Delta V_{bias}} \quad (\sim\text{kV})$$

Signal integration
(signal produced by X-ray absorption in the exposure time)

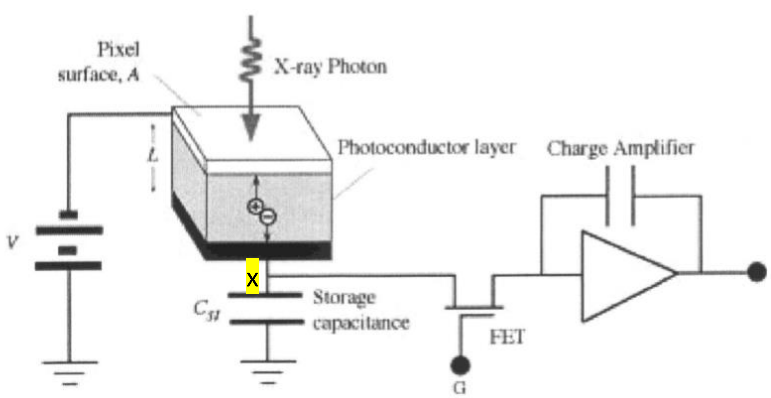


$$\Delta V_{OUT} = Q_{ij}/(C_{ij}+C_{Pi})$$

$$C_{ij} >> C_{Pi} \qquad (C_{ij} << C_3)$$

$$\Rightarrow \boxed{\Delta V_{OUT} \sim Q_{ij}/C_{ij}}$$

In reality, we don't read out the voltage on the capacitor through a voltage amplifier. Hence the computation of the voltage drop across the storage capacitor is a bit useless. In reality we connect the node between Cp and Cij with a charge preamplifier and so if we do so, the voltage goes to the virtual ground of the charge preamplifier and the stored charge goes through the feedback capacitor of the amplifier.



when a pixel is connected by switching-on the FET to the common output line connected to the preamplifier:

$$V_{OUTpre} = Q_{ij}/C_F$$

Let’s assume the switch G is open. We are integrating charge on the storage capacitor, voltage x is rising but then, when we close the switch, we force the node x to the virtual ground of the amplifier and the charge moves into the feedback capacitance.
In this way the output voltage we measure is the charge Q divided by the feedback capacitor.

In conclusion, to read out the charge we have two alternatives. <u>We either read the voltage directly on the storage capacitor and the value is Qij/Cij, or we read out the charge preamplifier and the voltage we read is Qij/Cf</u>.

Solutions to protect the pixel form over-voltages due to Idark

1) inversion of the bias voltage (TFT turns on when over-voltage occurs)
2) electric field lowering due to trapped electrons
3) protection devices

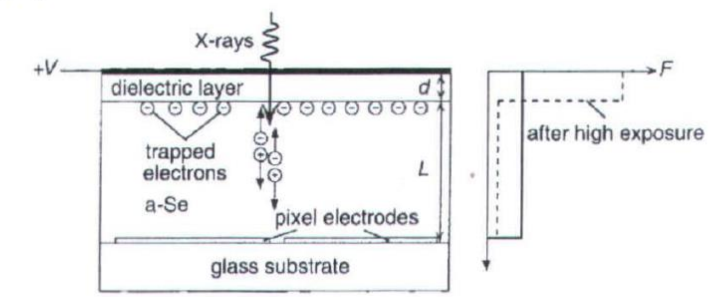
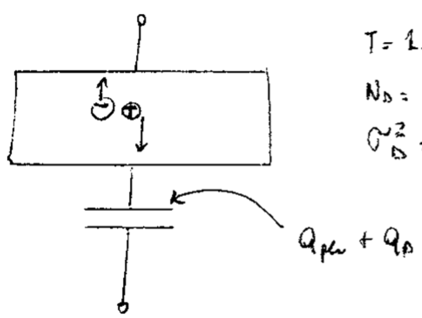


Fig. 5 *Trapped electrons at interface between dielectric layer and photoconductor collapse field in photoconductor so that no further charge can accumulate on pixel electrodes; thus TFT is prevented from breakdown under high exposure*

There is a problem: the ptc has also **thermal generated charge**. The material, similarly to a Si material, has thermally promoted electron/hole pairs. Hence we have the problem of the dark current.
Not only xrays generate e-/h pairs, but we have also pairs generated spontaneously thermally by the material.

From the point of view of the SNR this is a problem.

92.



$T = 1s$

$N_D = 1000$ holes

$\sigma_D^2 = N_D \implies \sigma \sim 100$

It is a problem because if we have our sensor, the storage capacitor and, due to thermal generation, we accumulate on the capacitor a number of thermally generated charges. If a radiography recording lasts 1s, the number of dark electrons or holes collected by the capacitor are 1000.

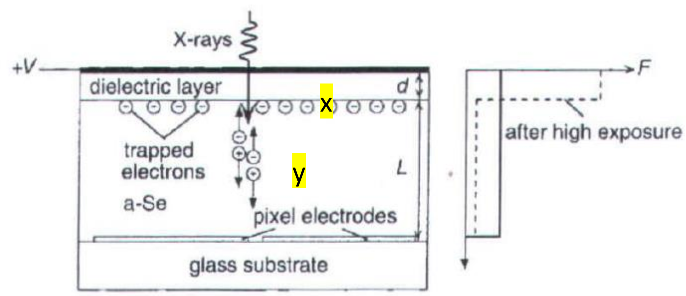
If temperature doesn't change, this is a value that we can subtract from the image. The image is given by the charge created by the photon plus the charge created by the dark charge. Hence we take in principle an image in dark conditions, without xrays, and then, when we do the image recording we subtract the one recorded with the dark image.

Unfortunately, we can subtract the average value but we cannot subtract the sigma of the charge and, considering the Poisson statistic, the sigma square of the charges is equal to Nd, which means that the sigma, in the example, is 100 → we have noise in the image due to the fact that, even if we subtract the average value of the dark charge, we cannot subtract its statistical fluctuation.

There is another problem.
If we don't read out and discharge the capacitor for long time, we could destroy the sensor. In fact, if we are doing nothing, the positive charges due to dark current, even if the xray tube is off and we are not recording images, go in the capacitor and rise the potential voltage over the capacitor. The voltage rises and we may destroy the switch connected to the node. If the gate line of the switch is 0 and the drain is rising, at a given point we broke the oxide between gate and drain of the MOSFET.

If there are dark holes going in the bottom electrode, on the other side there are dark electrons; if these electrons are trapped in the dielectric layer we applied to avoid injection, they practically reduce the electric field across the device.

The solid line is the electric field F across the device at the beginning, responsible for charge separation. If we have accumulation of negative charges in the barrier (x), the new F is the dashed one in the right plot.



So we have a peak of electric field inside the dielectric, and we don't care about this, and, correspondingly, we have a very low electric field inside the material which automatically reduces the further separation of charges.

We move from the situation of the continuous line to the one of the dashed for the following reason.
Let's consider the application of a positive voltage like having put positive charges on the electrode.
When I have +V on the upper electrode, it means, in practice, to have put a number of positive charges on this electrode. Now, if we have a layer of negative charges just below the dielectric, accordingly to the Gauss theorem, these negative charges are going to compensate the positive ones.
It is like to have a layer of positive charges on the top of the dielectric due to the application of V and, few microns away, there is a layer of negative charges.
Hence by the Gauss theorem, due to compensation, the potential in the region y is lowered.

Previously I had the electric field because I have only positive charges on the electrode; then, due to the superposition of positive and negative charges the electric field inside the ptc is going to 0.
A field going to zero prevents further charges thermally generated to separate.

Instead, across the dielectric on the top the electric field is very high because we have a huge accumulation of opposite layers of charges.
This is the solution of our problem: if electrons are not swept away, they are going to nullify the electric field in the material (solution 2)).

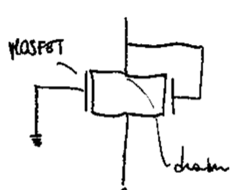
If this solution is not satisfactory, we have also other two solutions. One is the inversion: we don't apply a positive voltage, but a negative voltage on the top electrode.

2. Inversion
By inverting the bias voltage, we collect holes on the top and electrons on the pixel electrode. We have transformed a readout of positive charges in a readout of negative ones.
This solution is no more dangerous because now, in the transistor, the voltage is no more increasing due to thermal generation, but decreasing. In fact, the pixelated electrode is charged by thermally generated electrons, and so its voltage lowers.
This lowering of the voltage is no more breaking the device because the drain voltage is now going negative.
But if in a MOSFET the drain voltage gets more negative than the gate voltage (Vgd>0), what happens is that the transistor turns on. In the previous case the transistor was not on, it was more and more reverse bias.
Turning on the transistor is not nice, because it means that we have pixels randomly connected to the readout line (but at least they are not destroyed) → **transistors spontaneously turns on and the panel cannot be used as an imager**.

3. Protection devices
Further solution; protection devices means devices that prevents the node x, in right polarity, to rise too much.
So the node x can increase too much and destroy the transistor but either we apply a Zener diode so that the voltage is clamped and the node is no more able to rise.

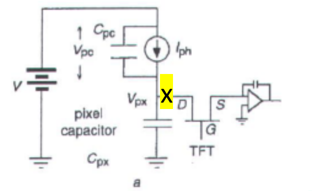
As a more clever solution, once we spatter the gate of the MOSFET, we add another gate. When we have a channel, we can have oxide and one gate or, if we have a 3D device we can apply another parallel gate in between. This top gate TG is shortened with the drain.

93.

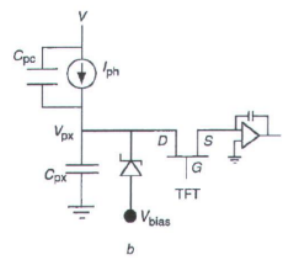


We create another twin MOSFET in parallel that shares the same channel and its gate is shortened with the drain.

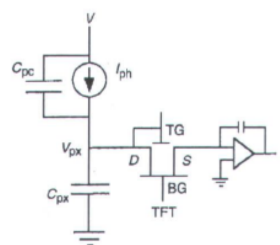
Normally this secondary transistor is off, but when the drain voltage rises, before that the main transistor is destroyed, since the drain voltage is also the gate voltage of the secondary transistor, this transistor turns on and works as a discharging safety device.



conventional readout



Zener protection



protection with a Top gate
in a 'transdiode' configuration

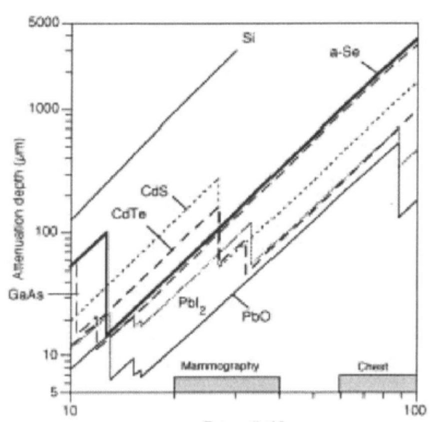
Absorption efficiency

Coming back to the ptc, how much thick should we do the ptc? Which are the boundaries in selecting the thickness?

The first requirement is that the material as to be thick enough to absorb efficiently xrays. Given a material of thickness L, is capability to absorb xrays is given by the formula below.

$$\eta_{assorb.} = 1 - \exp(-\mu L)$$

L : sensor thickness
$\delta = 1/\mu$: absorption length
($L=\delta$ absorbs 63% of the radiation)
is it good to have at least $L=2\delta$
ex. a-Se: 20keV 100μm, 60keV 2000μm



L not too small for efficiency but
L not too large (thick) because:

- it increases the probability of trapping/recombination of the charge
- difficult to grow on extended areas with large thickness without defects
- the bias voltage to apply to reach a given electric field F increases with L

So we need to have L large enough. There is an alternative quantity for u that is delta = 1/u, called **absorption length**, and it provides the absorption properties of a material. For instance, if L = delta, we have -1 in the exponential and we have absorbed 63% of the radiation → delta provides us a rule of thumb to select the thickness of the panel. In fact, choosing delta = 1 as thickness allows us to absorb 63% of the radiation.

In the plot, we have the different delta for different materials and the range of energy for different radiographic systems. In chest radiography we have delta of about 1mm. The table tells us that if we choose a-Se, the material has to be at least 2mm thick to have almost 90% of absorption.

So we know that the panel should be sufficiently thick. But why not growing it much more than what necessary?
There are technology issues; the thicker is the photoconductor, the more difficult is to have a ptc uniform in a panel.

Moreover, the more important reason is that we are dealing with a ptc full of charges. The thicker it, the less the chances that our charges have to travel across it and reach the electrode, because they are going to be recombined. If we have holes travelling for several mm, soon or later they will be recombined by electrons → we don't accumulate any charge on the storage capacitor. So the panel must be not too thin and not too thick.
Is there a way to quantify the survival probability that the charges have when travelling the ptc?

This survival probability is called lifetime t, that is the time in which a travelling charge can survive in a semiconductor of a given type and a given doping. t is strictly related to doping, if highly doped, the survival time is much smaller.
By the kinematic point of view, remembering that in semiconductor the speed is given by the mobility times the electric field, if we take the velocity and multiply it by the carrier lifetime, the result is a length.
The meaning of this length is the carrier mean path. It is the path that in average a carrier running at a given speed is covering before 'dying'. It is the average survival distance. It can be easily calculated because we have the electric field (we know the voltage applied), we have the mobility (tabulated) and we have the carrier lifetime.

Now, the criterium for choosing the length of the panel is that L (the thickness) much be much lower than the carrier mean path → we don't have only an upper bound to L, but also lower bound. If the panel is too thick, the thickness will be larger than the carrier mean path

Figure of merits of various materials

In the table (not to be known) we have some values associated to materials. We have some figure of merits:

- **Density**: the more dense the material, the better the absorption.
- **Delta**: absorption length, provided for various energies, and of course the smaller delta the better, because the smaller the thickness we have to choose for the panel.
- **Energy gap**: it is fundamental for the thermal promotion of the dark current. The larger the energy gap, the lower is the expected dark current → better to choose semiconductors with large energy gap.

- **W**: epsilon, amount of charge generated per unit of energy → the smaller the better, because more charge I have. There is a proportionality between epsilon and the energy gap. If we have a large energy gap, we have a lower thermal promotion but we also have a lower generation of charges by ionization.

Table 1: Densities, attenuation depths ($\delta = 1/\alpha$) at photon energy of 20 and 60 keV and energy bandgaps (E_g) of selected potential X-ray photoconductor materials

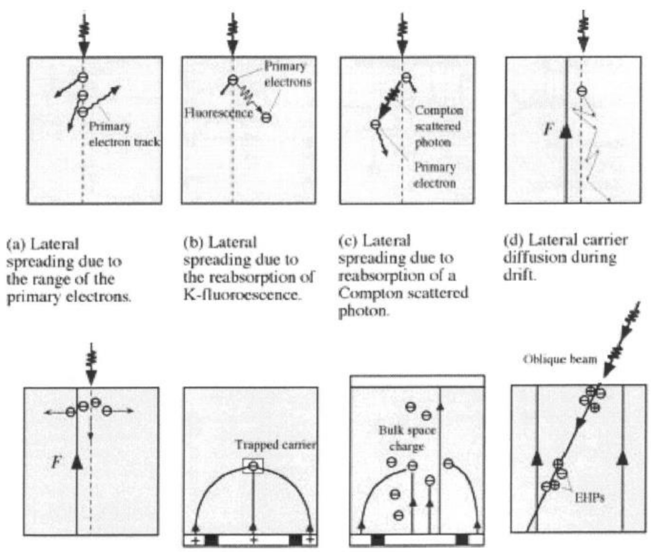
Photoconductor	TlBr	PbO	PbI_2	HgI_2	Ge	GaAs	a-Se	GaSe	ZnTe	CdS	CdSe	CdTe
Density ($g\,cm^{-3}$)	7.5	9.8	6.1	6.3	5.32	5.31	4.3	4.6	6.34	4.82	5.81	6.06
δ (μm) at 20 keV	18	11.8	28	32	44	44	48	49	58	127	56	77
δ (μm) at 60 keV	317	218	259	252	929	926	976	1026	300	439	385	250
E_g (eV)	2.7	1.9	2.3	2.1	0.7	1.42	2.3	2.0	2.26	2.3	1.8	1.5
$W_\pm$ (eV)	6.5	8–20	5	4–7	1.5	6.3	45^*, $20^\dagger$	6.3	7	7.2	5	4.65

*at $F = 10\,V/\mu m$
†at $F = 30\,V/\mu m$
X-ray mass attenuation coefficients data from http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html

Figures of merit:
- low absorption length δ of the material at the energy E of interest
- high sensitivity of the photoconductor: $Q=qE/\varepsilon$ $\quad \varepsilon \sim 2.8E_{gap}$ (ε=W in Table1)
- low carriers recombination in the bulk during drift and low trapping (large carrier mean path : $\mu\tau F >> L$
  μ = mobility, τ = carrier life time, F = electric field, L = thickness)
- low dark current (not-injecting contacts, low thermal generation)
  (note: thermal generation $\div 1/E_{gap}$ in conflict with sensitivity)
- polycrystalline photoconductor material of easy grow on large areas

Reasons for limited spatial resolution in pixelated detector

When we have xrays, why the charge should not fall in one pixel but spread also in other pixels? First of all, we have scattering electrons, that go around to create charges around a sphere of about 10-20 um of radius → we don't have a delta like charge created (a).



(a) Lateral spreading due to the range of the primary electrons. (b) Lateral spreading due to the reabsorption of K-fluorescence. (c) Lateral spreading due to reabsorption of a Compton scattered photon. (d) Lateral carrier diffusion during drift. (e) Lateral spreading due to space charge effects (Coulombic repulsion) (f) Lateral spreading due to bulk trapped carriers (the average filed is relatively unaffected) (g) Lateral spreading due to bulk space charge modifying the field. (h) Lateral spreading due to oblique incidence.

Causes of loss of spatial resolution

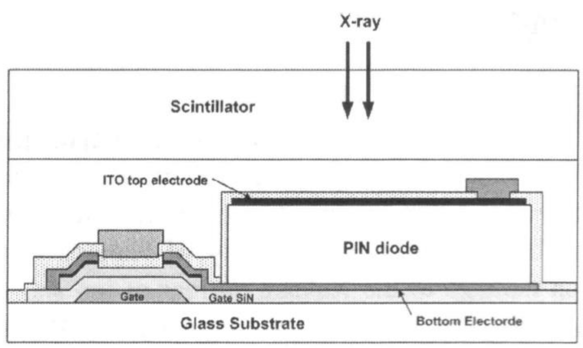
Moreover, upon a photoelectric effect we may have a fluorescence photon (b). If the fluorescence photon is absorbed elsewhere, we cannot assume that the total charge is created in one point.

As for Compton (c), we have absorption in one point and then the Compton is absorbed elsewhere → charge spread not in a delta-like point.

Then diffusion (d); if we have a cloud of charges, the charges diffuse. They may also spread due to Columbian repulsion (e).
Then we have trapping (f); if a charge is trapped, it induces charges on the electrodes nearby, a signal is induced not only in one electrode, but also in the other pixels.

Then with a lot of charge we can modify locally the electric field (g) and in the end, if the xray is not parallel to the panel but tilted, we cannot guarantee that the absorption point is always at the same coordinate, we have an uncertainty along the track of the xray (h).

TFT TECHNOLOGY WITH SCINTILLATOR

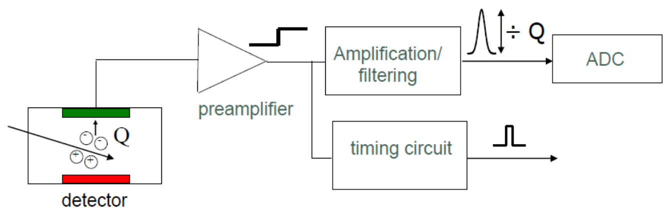


PIN diode in amorphous Si (same material used in solar cells), slightly reverse-biased or not biased

example of pixel dimension = 139μm with 57% of sensitive area

We have a scintillator, a photodiode integrated in the TFT technology and then we have the bottom electrode, the transistor and the same components. It is the same structure than the previous one, but here I have an indirect interaction (scintillator + pin diode).
Also in this case everything is amorphous. Of course, the top electrode (called ITO) is transparent so that scintillator passes through and is not reflected (like in classical metals).

ELECTRONICS FOR THE READOUT AND FILTERING OF SIGNALS FROM RADIATION DETECTORS



Measurement of the charge and/or of the time of occurrence
⇒ Measurement of the energy released by the radiation
⇒ Measurement of the arrival time of the event
⇒ Measurement of the position of interaction

Digital radiography is an exception because the panel is simply an integration of the charge delivered by the signal.
Now we are interested in recording the pulse delivered by a single photon rather than integrating the charge delivered by several photons. The model of the detector is a current generator.

Usually, the first stage of the readout electronics is a charge preamplifier, that integrates the charge into a capacitor and provides a step and then we have different stages based on the information I need to record.

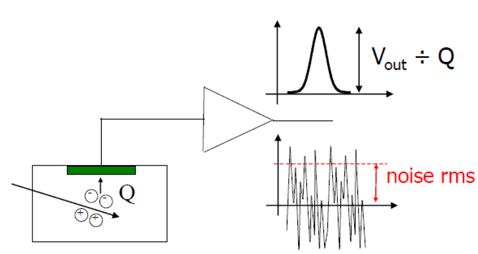
The main reasons to measure the charge are:

- Measurement of the energy released by the radiation.
- Measurement of the arrival time of the event.
- Measurement of the position of interaction: more a matter of reconstruction than electronics themselves. It is a manipulation of the amplitude provided by the electronics.

In the block diagram we can already see that usually the measurement is done separately. Most of the system have a chain measuring the time stamp and another one measuring the amplitude of the pulse.

THE ELECTRONIC NOISE: ENC

It will be the guiding parameter both for amplitude and timing measurement. It is the key parameter that tells us how much good we are measuring timing and amplitude.



$\boxed{Q = ENC \quad \Rightarrow \quad S/N=1}$

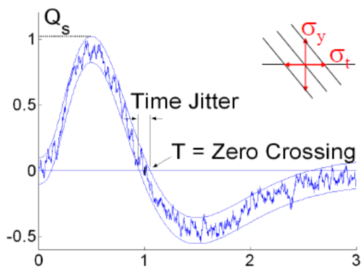
ENC: the charge to be release in the detector to produce a S/N at the output of the system equal to 1

The cumulative quantity that specifies very well the electronic noise in detector readout is the ENC, which is the charge delivered by the detector that provides at the output of the electronics an output equal to the rms noise of the amplifier and so the larger is the rms noise of the amplifier, larger is the ENC that the detector need to supply to at least match the noise. If ENC is high, we cannot detect weak signals.

Importance of electronic noise

$$(S/N) = \frac{N_s}{\sqrt{(N_s F + ENC^2/M^2)}}$$

Amplitude measurement



Time measurement

$$\sigma_t = \frac{\sigma_y}{y'_{riv}\big|_{t=T}} \longleftarrow ENC$$

Amplitude measurement

As for amplitude measurement, we must recall the SNR formula we found for scintillators. The noise is given by the quadratic sum of the intrinsic spread of the signal charge, eventually worsened by the noise excess factor F due to the multiplication, plus a statistical independent factor that is the ENC, eventually reduced by the multiplication factor.

So either we design well the electronics so that we don't need multiplication → ENC is intrinsically so low we don't need to use a PMT or an APD. Otherwise, we don't care much about the amplifier noise and we use a multiplier photodetector, which however has a drawback that is F.

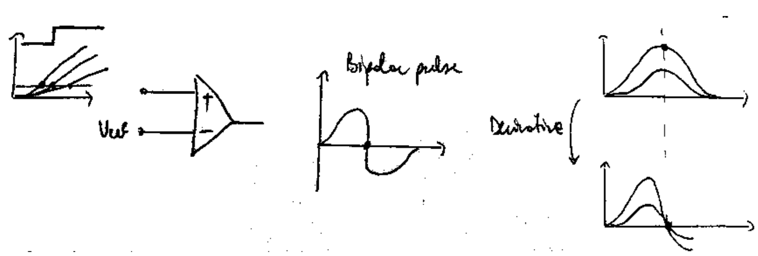
Time measurement

For time measurements, it is a bit more tricky to see why the electronic noise is important. One possibility to make timing measurement is to use a classical comparator. We have a reference voltage, a signal, and we check with an open loop amplifier when the signal has overcome the reference voltage. The comparator will provide a digital signal, a time stamp, when the voltage has overcome the reference threshold. This is the easiest way to perform timing (with an open loop amplifier).

The intrinsic weakness of this method is that, if we can rely that the amplitude of the pulse is always the same, we can expect that the crossing point is always the same. But if on the contrary the voltage is going to have different amplitudes, the crossing point of the voltage with respect to the threshold will change, it is signal dependent. This annoying phenomenon is called amplitude walk.

It is the shift of the time stamp of the trigger position depending on the amplitude of the signal. There are more sophisticated techniques to limit this problem and one technique requires a bipolar pulse.

94.



We need to shape the signal to get a bipolar pulse and we assume as time stamp the zero crossing of the bipolar pulse, when the pulse crossing the zero. This complicated choice is better because it can be easily verified that if we have a bipolar pulse and we change the amplitude because the detector is

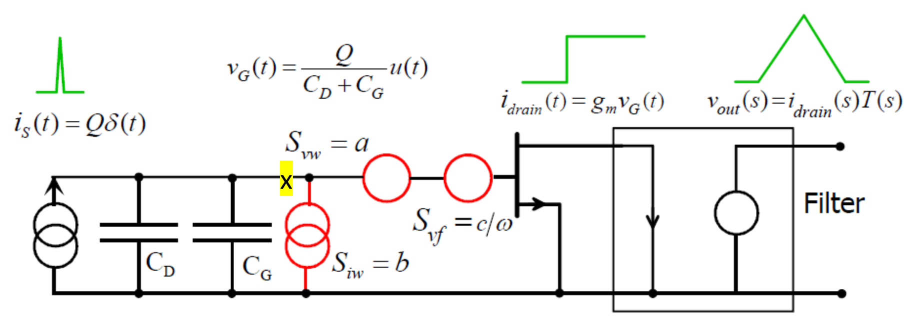
changing charges, the peak of the bipolar pulse changes, but the zero crossing point remains the same for all amplitudes → independent on the amplitude of the signal.
In fact, if we take a unipolar pulse and we make a derivative (passing it through a CR derivator), we get a bipolar pulse

Having said that bipolar pulse is nice because the zero crossing point is independent by the detector amplitude, we must also consider the electronics noise. This noise is not a deterministic change of the pulse, and we hence have a noisy waveform.

If we zoom in the region of zero crossing point, it is not true that the zero point is always the same, because the local amplitude fluctuation determines a jitter in the zero crossing point.
So we have to imagine that any statistical fluctuation on the vertical axis, that is the electronic noise, means an horizontal uncertainty in the zero crossing point. Hence the wider the vertical band of voltage uncertainty, wider the horizontal band of uncertainty when the noisy waveform crosses zero.

The weighting factor between vertical and horizontal jittering is the derivative; the steeper the zero crossing slope, the better is the insensitivity of the zero crossing time with respect to the vertical fluctuation.
If we would take an infinite slope, I could not care about fluctuation → ideally the slope should be infinite.
In conclusion, <u>electronics noise (vertical fluctuation) is not corrupting only the amplitude, but also the time peak off measurement</u>.

READOUT SCHEME WITH A SIMPLE VOLTAGE AMPLIFIER (with charge preamplifier the conclusions are the same)
In the image we see the main scheme of an electronics readout.



$$\frac{S}{N} = \frac{v_{so\,peak}}{\sqrt{\langle v_{no}^2 \rangle}} = \frac{Q \cdot Max\left[v_{so\,u\delta}(t)\right]}{\sqrt{\langle v_{no}^2 \rangle}}$$

$v_{so\,u\delta}(t)$ Output amplitude in correspondence of Q=1

$$\Rightarrow \quad ENC = \frac{\sqrt{\langle v_{no}^2 \rangle}}{Max\left[v_{so\,u\delta}(t)\right]}$$

u(t): Heaviside

In the scheme we have a **detector represented by a current generator with a delta like pulse of area Q**. In fact, we assume the detector is releasing so quickly the charge that we have a delta like generator with charge Q. The only exceptions are scintillators, that have an exponential delivery due to scintillator decay time.

The detector is represented by its capacitance (Cd). Then I have the readout; to simplify the calculation, the readout is made out of a common source transistor (a JFET) and then the current of the transistor is going into a filter. So the readout is divided in: detector, which is generator + capacitor, transistor (represented by transconductance of the transistor and its gate capacitance in parallel to the detector capacitance, and then the current of the transistor goes into a filter which produces a voltage output which, most of the time, has a peak shape, it is a pulse (and I want to measure the peak of the pulse).

Practically speaking, a delta like current on the detector will provide a Heaviside step on the gate of the transistor and the Heaviside step is given by the charge integrated over the two capacitors multiplied by u(t), Heaviside function. Then the step on the gate will further provide a drain current step, given by the step of the gate multiplied by the transconductance of the transistor.
And then, I assume that the current is integrated into a virtual ground of an amplifier and the amplifier will provide a triangular pulse where I'm interested in measuring the peak (filter output).

The filter output is described by its Laplace transfer function.

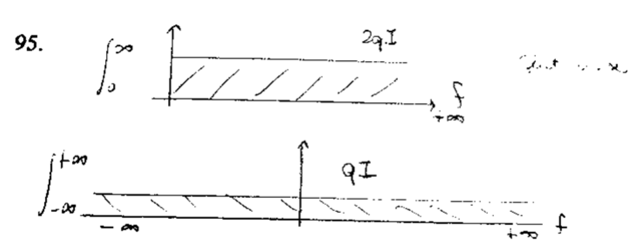
It can be demonstrated that, if I use a charge preamplifier (so I don't use an open loop transistor but a charge preamplifier and the charge is integrated on the feedback capacitance), the results are the same.

Now, the green color identifies good things, the signal, and red colors the bad things, aka noise sources.
The noise sources are represented by two equivalent generators. One is a current generator, and one is a voltage generator, that is then split in two generators where the first one is characterized by white noise (constant spectral density), while the second one is a 1/f noise, so the noise spectral density goes with 1/omega.
In case of good design of the system, these two generators are dominated only by the input devices in node x, that are detector and input transistor → all the noise of the second stage of the filter can be neglected.

Main noise sources

The noise source has been described as mathematical spectral noise densities, which means that the noise densities go from – to + infinite of the frequencies. In physical noise spectral densities we move from 0 to + infinite → now noise sources are divided by a factor of 2 to get the same integral in the two cases.

95.



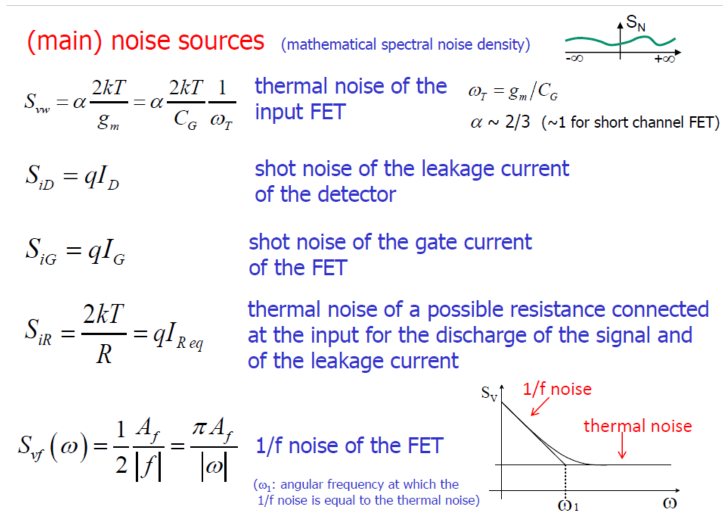
For example, shot noise is 2*qI. But this formula is valid from physical noise spectral density, that goes from 0 to + infinite.

In the notation we are using, we are using an alternative notation where the noise is described form – infinite to + infinite. So the noise spectral density of the shot noise must be qI. In fact, a noise integrated up to fmax must be the same if integrated between [-fmax, +fmax] in the new notation.
So the difference is that in a) integrals are made from 0 to +inf, in b) in [-inf, +inf].

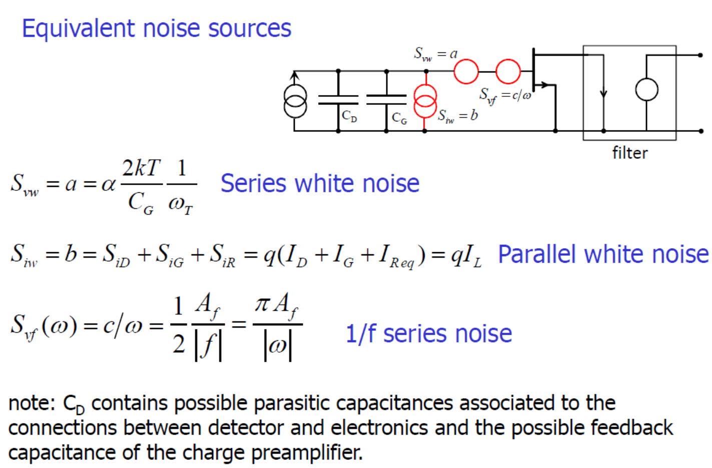
Current noise generator

We have:

- Shot noise due to the leakage current of the detector. It is a current noise (current generator).
- Shot noise due to the gate current of the transistor (JFET and BJT have gate and base current, MOSFET has no shot noise on the gate) → added to the current generator.
- Thermal noise we have because in any front end we have a discharge resistor for the capacitor, and the resistor is noisy because of the thermal noise. The resistor is not shown in the image of the previous time, but its contribution is in the current noise generator.



All these 3 noise generators are in parallel, and the current generator in the circuit is the sum of these three contributions. The noise b is constant with frequency because it is a white noise (shot noise and thermal noise are white).



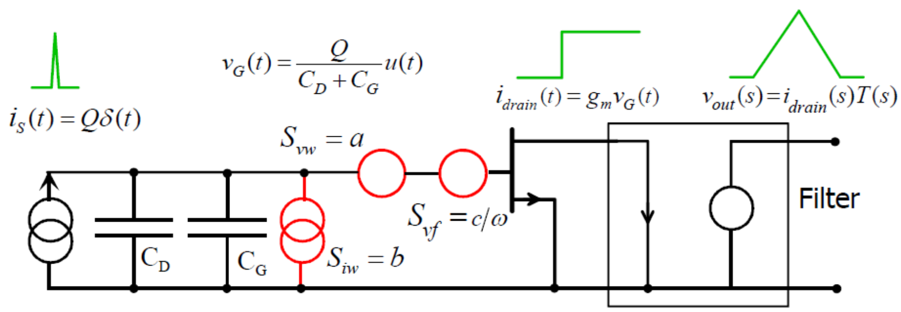
Voltage noise generator

The voltage generator called a is the thermal noise of the input transistor, and thermal noise is given by an alpha factor that depends on the channel length of the transistor, times 2kT/gm.
If we consider that the cut off frequency of a transistor is given by its transconductance divided by the gate capacitance, we can then substitute and get an alternative expression for the noise.

In conclusion, the other generator is the 1/f noise.
A transistor, especially a JFET, has a thermal noise component and a 1/f noise component; thermal noise is inside Svw, while 1/f noise is in the contribution Svf.

Overall, the equivalent noise sources of our electronics are given by the parallel white noise current generator, the voltage white noise called 'series white noise' and the 1/f noise.
Moreover, the detector capacitance Cd may also include parasitic capacitances due to the physical connection between the detector and the electronics → to be added in parallel to the Cd.
Eventually, if we have a discharging resistor, we have also the thermal noise of this resistor. Theoretically it should be a voltage noise generator, but it is represented as a current noise generator so that from an aesthetical point of view all the contributions are summed easily. Two sources of the noise are shot noise, the third is the thermal noise.

ENC definition



$$\frac{S}{N} = \frac{v_{so\,peak}}{\sqrt{\langle v_{no}^2 \rangle}} = \frac{Q \cdot Max\left[v_{so\,u\delta}(t)\right]}{\sqrt{\langle v_{no}^2 \rangle}}$$

$v_{so\,u\delta}(t)$ Output amplitude in correspondence of Q=1

$$\Rightarrow \quad ENC = \frac{\sqrt{\langle v_{no}^2 \rangle}}{Max\left[v_{so\,u\delta}(t)\right]}$$

u(t): Heaviside

It is the charge Q given by the detector that provides at the output a signal that has the same value as the rms value of the noise. So we compute the SNR at the output of the filter.
The signal will be the peak of the triangle and, since the peak of the triangle is proportional to the charge, we represent it as charge time the peak of the triangle when we have a unitary charge.
At the denominator we have the rms of the noise; so we consider the noise spectral density previously calculated, we integrate it over the bandwidth of the filter and we get the rms noise at the denominator.

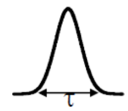
Once we consider the SNR given by the formula above, the ENC is the Q in the formula corresponding to a SNR = 1. So we put 1 in the formula, we flip it and get ENC, that is the rms of the noise divided by the peak of the pulse corresponding to unitary charge.

Final ENC as a function of the noise sources and of the filter

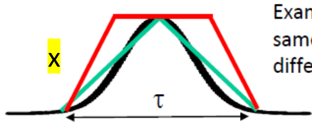
Given some noise sources wit coefficients a, b, c, and given a filter shape (triangle, gaussian, trapezoid, …, the importance is that it is the shape of a peaked function), the ENC is given by the final formula below.

$$ENC^2 = \left(C_D + C_G\right)^2 a \frac{1}{\tau} A_1 + \left(C_D + C_G\right)^2 c A_2 + b\, \tau A_3$$

τ : shaping time (characteristic time related to the **width** of the pulse at the output of the filter)



A_1, A_2, A_3 : coefficients that depend only on the **shape** of the pulse at the output of the filter and not from its width
(note: the coefficients depends on the definition of τ, see later)



Example: pulses of same width but different shapes

Example: pulses of same shape but different widths

The formula is expressed in square because, according to the calculations, the square of the ENC is represented by the sum of 3 terms, and each term is related to one type of noise.
This is also the reason why the noise generators have been split in 3 terms. We have in the formula a term including a, the thermal noise, a term including c, the 1/f noise and a term including b, the parallel noise (the one related to shot noise sources).
In addition to this, the first term, proportional to the thermal noise, is also proportional to the square of the sum of all capacitances at the input → better to keep the capacitances of the detector and the transistor as small as possible.

The second term is related to the 1/f noise and includes also the capacitances term and the third term simply include the shot noise term.
However, we see also terms in the formula that are related to the filter characteristic.

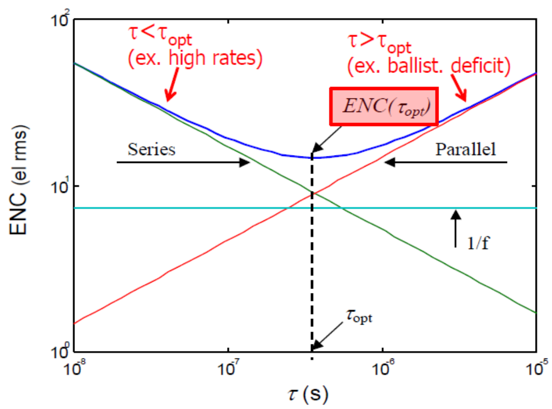
In particular, the filter characteristics are split in filter duration and filter shape. The filter duration is described by tau, the **shaping time**, that is a characteristic time related to the width, how large is the filter response.
The second set of parameters (3, A1, A2 and A3) are related only to the shape of the filter, not the duration.

Having established such a distinction, now we can realize that we may have pulses of different shapes, so different sets of A1, A2, A3 but same duration tau (case x) or we may have pulses of the same shape but of different durations (case y).
This differentiation is done because usually in readout electronics, once the filter has been chosen, we establish its coefficient, we cannot change them. On the contrary, we still may be flexible to play with the duration of the filter.
So we buy a filter, we plug the coefficients and don't touch them anymore, but we can make the filter larger or shorter → reason why the description of the filter is differentiated in shape vs duration.

Optimization of the noise

Given a filter, so a set of parameters A1, A2, A3, the first contribution drops by increasing the filter width. The last contribution increases a lot by increasing the width tau, while the middle contribution is totally independent from the filter width → there must be a noise minimization, we want the lowest possible noise.

Choice of the optimum shaping time for minimal noise



$$ENC^2 = (C_D + C_G)^2 a \frac{1}{\tau} A_1 + (C_D + C_G)^2 c A_2 + b \tau A_3$$

In this graph we have a logarithmic scale for ENC and shaping time tau.
The series noise decreases by increasing tau, the parallel noise increases by increasing tau and 1/f cannot however be optimized with filter shape.

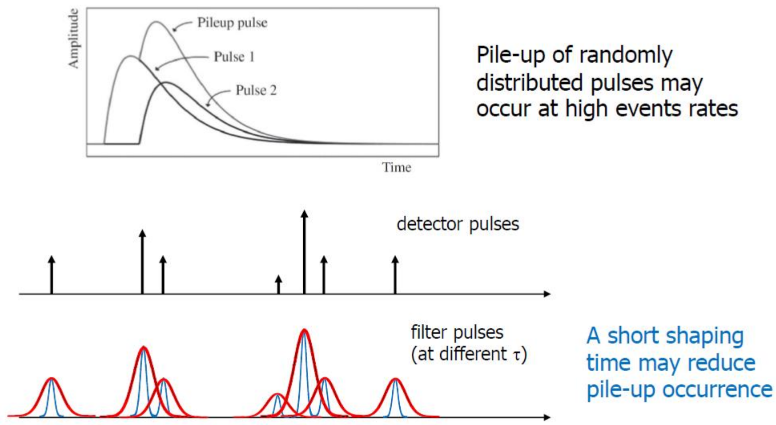
Unless we have other constrains, what we have to do is to choose a tau called optimum shaping time, where we minimize the noise. Noise source and filter coefficient are given, we play with the duration. However, there are some situations in which this cannot be freely done, and we may be forced by other constrains to work in suboptimal conditions → we may be forced to use a pulse duration lower than the optimal one.
This is not good, because the noise is suboptimal. But if we are forced to work in the range with small tau, we can see that not all the noise components are dominant, mainly the series noise it is.
Once we have this constrain, at least we can try to reduce the thermal noise a as much as we can, without paying any care to the shot noise, because it is important for higher tau.

The same consideration applies if for any reason we are forced to work in the longer shaping time range, with bigger tau. Also here we are suboptimal.
There are two examples of the extreme cases.

High rates

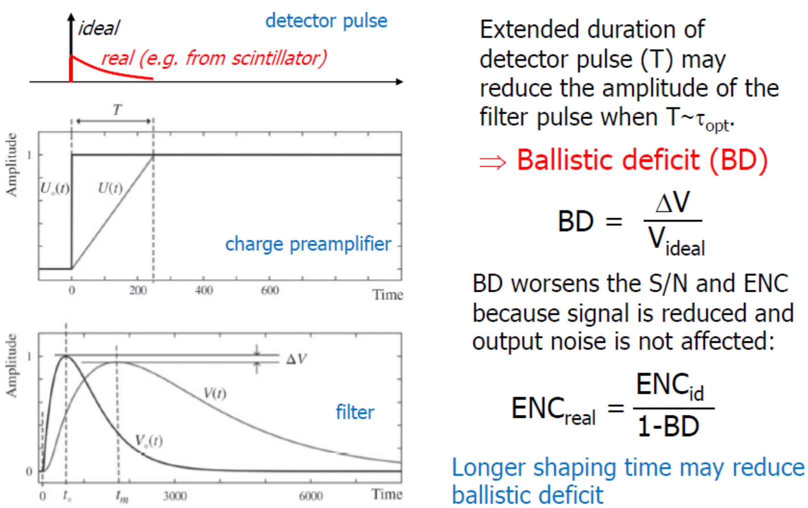
When we are forced to choose a duration of the pulse shorter than the optimal. It is a typical case when we have two frequent pulses → the rate of occurrence of the pulses is too high.
So if we have two pulses and the pulses are lasting too much because we want to target the optimum shaping time, but at the optimum shaping time we have a nonzero probability of pile up and so the resulting pile up pulse must be discarded, the measurement is completely wrong.

To solve this problem, we have to make the pulses narrower.



Let's suppose we have the set of detector pulses that are the currents given by the detector (we assume to be a Dirac delta); in red there is the processing with optimum pulse duration, and we can see many pile ups. If we use shorter shaping time, we see that we can skip most of the pile up occurrences, and we can identify correctly the pulses.
Of course, we cannot avoid completely the pile up, if two pules occur simultaneously we can distinguish them.

Tau greater that tau optimal



We are forced to use pulses with longer duration than the optimal one.
NB: we must not have pile up in this case, otherwise it is no-sense to use a longer shaping time than the optimal one.

The reason to use it is related to the fact that is not true that the detector pulse is a Dirac delta, that was our initial assumption. If we have a scintillator, we have a pulse far from a delta, we have an exponential decay pulse.
This is important now because, if we have delta and we integrate over the capacitances Cd and Cg, we have a perfect Heaviside step.

However, if we don’t have a delta but we have for instance an exponential decay, the preamplifier has a rising shape, not a step → the charge preamplifier response is far from Heaviside.
This is true not only if the detector is **not delivering a perfect delta pulse**, but it is also true if the amplifier itself has a **finite bandwidth**. In fact, an amplifier delivering a Heaviside step has infinite bandwidth.

The consequences for the final filter are that the filter in case of perfect delta has the shape Vs(t), in case of a not perfect delta is V(t). The characteristic of V(t) is that the amplitude has been reduced → the longer the exponential decay, the smaller the value of the peak.

The problem is that this peak is smaller and this reduction of the peak is called ballistic deficit. For the same amount of charge we get a lower peak. It is defined as the ratio between the drop we have with respect to the ideal pulse and the ideal pulse amplitude. It is the percentage of loss in amplitude.
The problem is that we don’t have only reduced the amplitude, but we have also equivalently increased the noise. In reality, the noise generators are the same, the coefficients a, b, c are the same → the ballistic deficit is not affecting the noise, only the signal.
But the ENC definition was based on the SNR ratio, not on the noise itself. It is a definition where the ENC is the charge delivered by the detector that matches the noise. But is the signal has been reduced, to match the same noise I need more charge to be delivered.

Although the noise sources are the same, the **ENC**, so our capability to distinguish the charged delivered by the detector from noise, **increases**.

With a ballistic deficit, the real ENC is given by the old ideal one divided by 1 – ballistic deficit in percentage.

BD worsens the S/N and ENC because signal is reduced and output noise is not affected:

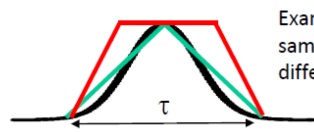
$$ENC_{real} = \frac{ENC_{id}}{1\text{-}BD}$$

Longer shaping time may reduce ballistic deficit

A quick solution to the problem is to increase the pulse width. In fact, if I make the pulse much longer, the effect of the exponential decay is smaller, and so the exponential may be approximated as a delta like, and so I restore the original amplitude.
→ **A longer shaping time can for instance used with a slow scintillator** if we want to recover all the charges.

Moreover, we can also say that not all the shapes are prone to this ballistic deficit; for instance, the triangle is worse than the trapezoid if the shaping time is the same. The BD with the trapezoid is much smaller than the one with the triangle.

Filter coefficients vs definition of the shaping time
Depending on the filter, we may have different tau and different shaping factors.
Is it possible to compare filters? Yes, if we describe filters with the same shaping time tau. For instance, in the image we have compared 3 different filters but with the same duration at the base of the filter.



Example: pulses of same width but different shapes

One very popular definition of the filter is indeed its duration at the base, because it defines the time occupation of the filter.
If we suppose that the time definition is different, how can we compare them?

By means of a transformation formula x.

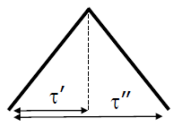
$$ENC^2 = (C_D + C_G)^2 \, a \frac{1}{\tau} A_1 + (C_D + C_G)^2 \, 2\pi a_f A_2 + b \tau A_3$$

Given a filter, if we change the definition of τ,
also the factors A_1, A_2, A_3 changes according to the formula:

x $\boxed{\tau'' = k\tau'}$

y $\boxed{\begin{aligned} A_1(\tau'') &= k A_1(\tau') \\ A_2(\tau'') &= A_2(\tau') \\ A_3(\tau'') &= \frac{1}{k} A_3(\tau') \end{aligned}}$

Example: triangular filter



$A_1(\tau'')=2 \cdot A_1(\tau')$
$A_2(\tau'')=A_2(\tau')$
$A_3(\tau'')=0.5 \cdot A_3(\tau')$

Useful transformation to compare different filters performances at an equivalent shaping time.

If we have the coefficients A1, A2, A3 defined according to a definition of tau' and now we want to see how the coefficients change with tau'' (because the coefficients are depending on the definition of tau), how do coefficients change? According to table y.

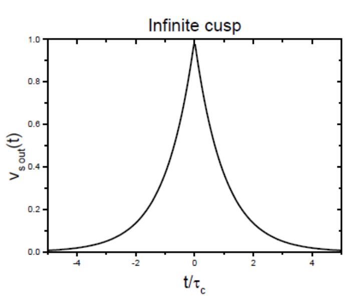
For instance, if we have the coefficients of a triangular filter defined as tau' peaking time (time to reach the peak) and now we want to calculate the filter coefficients for a new definition tau'' that is not the peak but the duration, the coefficient k = 2 for geometrical considerations.
With these rules we can compare filters.

OPTIMIZATION OF THE NOISE – THEORY OF THE OPTIMUM FILTER

If we take the problem of noise minimization in general, we should describe the theory of optimum filter. Once we have our processing system and our noise generators, which are fixed, the only thing we can do is to make the best filtering process.

We can indeed choose an optimum filter. For example, in case of only series and parallel white noises, so in case we have only series noise and parallel noise and we can neglect the 1/f noise, the shape is the infinite cusp below. It is a filter given by the formula below.

Example: optimum filter in presence of white noise generators only (1/f assumed negligible)



$$v_{so}(t) = \exp\left(-\frac{|t|}{\tau_C}\right)$$

$$\tau_c = (C_D + C_G)\sqrt{\frac{a}{b}}$$

The optimum shaping time is the tau-c, depending on the coefficient of the thermal noise a and shot noise b.

Typical practical non ideal filters

Table 1
Behaviour of A_1, A_2 and A_3 for different $h(f)$ functions

	Shaping	h (t) Function		A_2	$\sqrt{A_1 A_3}$	$\frac{A_2}{\sqrt{A_1 A_3}}$	A_1	A_3	$\sqrt{\frac{A_1}{A_3}}$
1	indefinite cusp	$e^{t/\tau}$, $e^{-t/\tau}$		0.64 $\left(\frac{2}{\pi}\right)$	1	0.64	1	1	1
2	truncated cusp	$k = \tau'/\tau$	k=1	0.77	1.04	0.74	2.16	0.51	2.06
			k=2	0.70	1.01	0.69	1.31	0.78	1.30
			k=3	0.67	1	0.67	1.31	0.91	1.10
3	triangular			0.88 $\left(\frac{4}{\pi}\ln 2\right)$	1.15 $\left(\frac{2}{\sqrt{3}}\right)$	0.76	2	0.67 $\left(\frac{2}{3}\right)$	1.73
4	trapezoidal			1.38	1.83	0.76	2	1.67	1.09
5	piecewise parabolic			1.15	1.43	0.80	2.67	0.77	1.86
6	sinusoidal lobe			1.22	1.57	0.78	2.47	1	1.57
7	RC-CR			1.18	1.85	0.64	1.85	1.85	1
8	semigaussian (n = 4)			1.04	1.35	0.77	0.51	3.58	0.38
9	gaussian	$e^{-1/2(t/\tau)^2}$		1	1.26	0.79	0.89	1.77	0.71
10	clipped approximate integrator			0.85	1.34	0.63	2.54	0.71	1.89
11	bipolar triangular			2	2.31	0.87	4	1.33	1.73

Comparison between typical practical (not ideal) filters

$$ENC^2 = A_1 (C_D + C_G)^2 S_{Vw} \frac{1}{\tau} + A_2 (C_D + C_G)^2 2\pi S_f f + A_3 S_{Iw} \tau$$

In reality, it is impractical to implement an optimal filter, because in many applications you can't spend a lot of resources to have each detector processing with an optimum filter.

In the table we have a variety of practical filters with the coefficients A1, A2 and A3 calculated. The best filter is the indefinite cusp seen before, and the three coefficients are the smallest in the table. Then we have a truncated cusp (whose time don't start at infinite), then the triangle (not a bad approximation of a cusp, and it can be realized with an integrator.

In conclusion, once we have choose the type of filter we can implement, we get from this table a triplet of numbers, we put them in the formula and we see the noise we get. It won't be as good as the noise with the cuspid.

SUMMARY FOR ENC MINIMIZATION

$$ENC^2 = \boxed{(C_D + C_G)^2 a}\boxed{\frac{1}{\tau}}\boxed{A_1} + \boxed{(C_D + C_G)^2 c}\boxed{A_2} + \boxed{b}\boxed{\tau}\boxed{A_3}$$

minimize noise sources
(vs. detector parameters)

chose the best filter
(vs. other requirements)

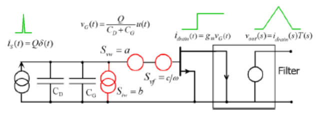
chose the best shaping time
(vs. det./appl. parameters,
e.g. count rate, ballistic def., ...)

To make the measurement as sensitive as possible, we can work on 3 domains:

- Yellow refers to the minimization of noise sources, independently on the purpose of the filter. These terms are thermal noise of the transistor, capacitances, 1/f noise, the dark current.

- Red refers to the choice of the filter that provides us the triplet of values which minimizes the noise, once put in the formula.
- Once we have choose the filter, blue refers to the optimization of the shaping time, so trying to choose the shaping time that minimizes the noise, unless we have other constrains. If we have other constrains we may have a bit of recursive process, in the sense that all the different factors must be tuned iteratively until a satisfactory condition is met.

ENC CALCULATION (not to be known)



time domain → Laplace

$$i_D(t) = Q\delta(t) \quad \rightarrow \quad I_D(s) = Q$$

$$i_{DRAIN}(t) = \frac{Q}{C_D + C_G} g_m u(t) \quad \rightarrow \quad I_{DRAIN}(s) = \frac{1}{s}\frac{Q}{C_D + C_G} g_m$$

$$v_{so}(t) = \frac{Q}{C_D + C_G} g_m \cdot \mathrm{L}^{-1}\left[\frac{1}{s}T(s)\right] \quad \leftarrow \quad V_{so}(s) = \frac{Q}{C_D + C_G} g_m \frac{1}{s} T(s)$$

T(s): transfer function of the filter

L^{-1}: inverse operator of the Laplace transform

u(t): Heaviside

$$S_{no}(\omega) = \left(a + \frac{2\pi a_f}{|\omega|} + \frac{b}{\omega^2 (C_D + C_G)^2}\right) g_m^2 |T(j\omega)|^2$$

Power noise spectrum at the filter output

$$\langle v_{no}^2 \rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{no}(\omega)\, d\omega$$

noise at the filter output (rms²)

$$\Downarrow$$

$$ENC^2 = \frac{\langle v_{no}^2 \rangle}{Max^2(v_{so\,u\delta})} = \frac{\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(a + \frac{2\pi a_f}{|\omega|} + \frac{b}{\omega^2 (C_D + C_G)^2}\right) g_m^2 |T(j\omega)|^2\, d\omega}{\frac{g_m^2}{(C_D + C_G)^2} Max^2\left(L^{-1}\left[\frac{1}{s}T(s)\right]\right)}$$

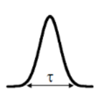
Considerations:

1) ENC does not depend on the filter gain
(which changes signal and noise in the same way)

$\Rightarrow$ let us assume $Max\{L^{-1}[T(s)/s]\} = 1$

2) we represent the frequency response of the filter as function of the dimensionless variable $x = \omega\tau$

where τ is a characteristic time associated to the time width of the pulse



$$ENC^2 = (C_D + C_G)^2 a \frac{1}{\tau}\frac{1}{2\pi}\int_{-\infty}^{+\infty} |T(x)|^2\, dx +$$
$$+ (C_D + C_G)^2\, 2\pi a_f \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{|x|}|T(x)|^2\, dx +$$
$$+ b\,\tau \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{x^2}|T(x)|^2\, dx$$

We define the shaping factors A_1, A_2, A_3 :

$$A_1 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |T(x)|^2\, dx$$

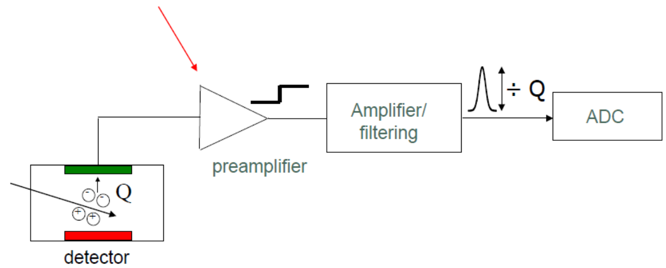
$$A_2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{|x|}|T(x)|^2\, dx$$

$$A_3 = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{x^2}|T(x)|^2\, dx$$

Note:
These factors depend only on the "shape" of the filter and not on the width of its response, as the integrals are calculated as function of the dimensionless variable x

The same derivation for A1, A2 and A3 can be done either in the time domain or in the frequency domain.

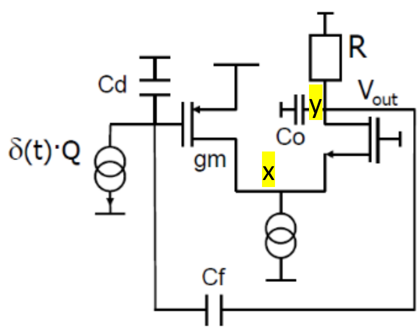
THE CHARGE PREAMPLIFIER



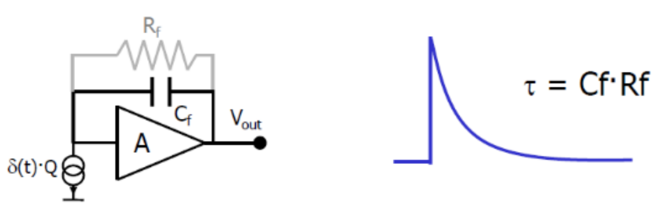
purpose: convert the generated charge into a voltage step, minimizing the added noise

The role of the charge preamplifier is the production of a Heaviside step.

THE CASCODE CONFIGURATION



- V_{out}: voltage step of amplitude Q/Cf
- $A = -gm\cdot R$, $tau_{pole} = R\cdot Co$
- GBWP ~ $A/tau_{pole} = gm/Co$
- $Gloop = A\cdot Cf/(Cd+Cf)$
- tau_{cl} of the closed loop dominant pole ~ $tau_{pole}/Gloop = Co/gm\cdot(Cd+Cf)/Cf$
- need for a feedback component to discharge the signal charge and the dark current (typically a resistor Rf)



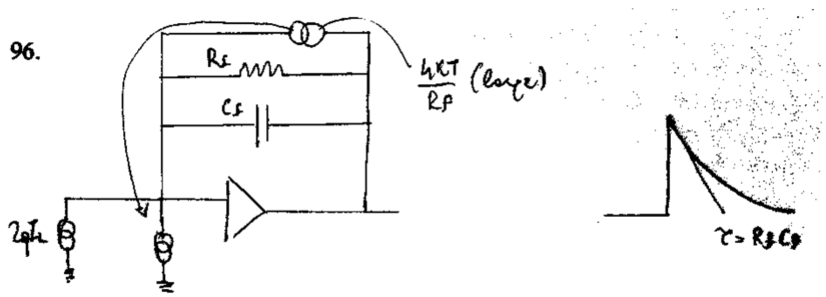
It is a classical configuration. The cpa is an integrator, composed by a main voltage amplifier with amplification A and a capacitor Cf in feedback. The delta like charge is integrated in Cf producing a step. The step cannot last forever and we need some restoring component, that is the resistor Rf that determines a decay time that restore the preamplifier output to its baseline. There are also other possibilities, like the pulse reset, a switch in parallel to the capacitor. Normally it is an open circuit, but time to time it is closed to eliminate the charge on the capacitor.

The advantage with continuous reset and the resistor is that we don't have to do anything, while in pulse reset we need a circuitry that is sensing when the voltage of the pile up of steps is going close to the saturation voltage of the amplifier.

We don't use all the time the passive solution RC because of the noise of the resistor. In parallel to the resistor there is a noise source. It is a noise generator given by 4kT/Rf.

This thermal noise generator, since the output node of the amplifier is a low impedance node (like ground), it is like to be at the input of the amplifier. So it is in parallel to the other current noise generators, for instance due to the shot noise of the dark current of the detector.
Hence, basically, we have to be careful about the resistor Rf, because we need to choose a value of Rf so large that its noise is negligible with respect to the main noise of the detector. The idea is to prevent this contribution to be dominant with respect to the unavoidable one, that is the one of the detector. So the resistor should be large.

96.



This is important for two main issued related to Rf:

- The Rf value determines the discharge time of the amplifier (RfCf), and the larger Rf the larger the response (risk of pile up)
- We are looking for CMOS implementation of such circuits, and the larger the resistor Rf (10^8 Ohm), more difficult is to implement in CMOS technology.

These considerations on noise are not present in the pulse reset, we have no noise of the switch when it is open.

Let's now analyze the core amplifier, the A block between input and output node. It is a common source (in the image there is a pMOS implementation due to noise reasons, because the 1/f noise is lower for pMOS than nMOS). Then if we consider to have an input step voltage the current is given by the input step voltage times the transconductance gm of the mosfet. Then we have a folded cascode configuration → the current is not going directly to the load R but it is passing through a **cascode** and then is converted into voltage at the output by the resistor R.
So the voltage gain intended as Vout/Vin is -gm*R.

The advantages of a cascode configuration are that we avoid the Miller effect and some other listed before. We can have also a larger output impedance R (that can be a resistor or active loads).
The limit in the bandwidth is due to the dominant pole at the output node itself, Co*R. The open loop tau is given by Co*R.
The capacitance Co may be a parasitic capacitance, not necessarily a capacitance placed on purpose. However, whatever it is, the pole that it generates is dominating the bandwidth.

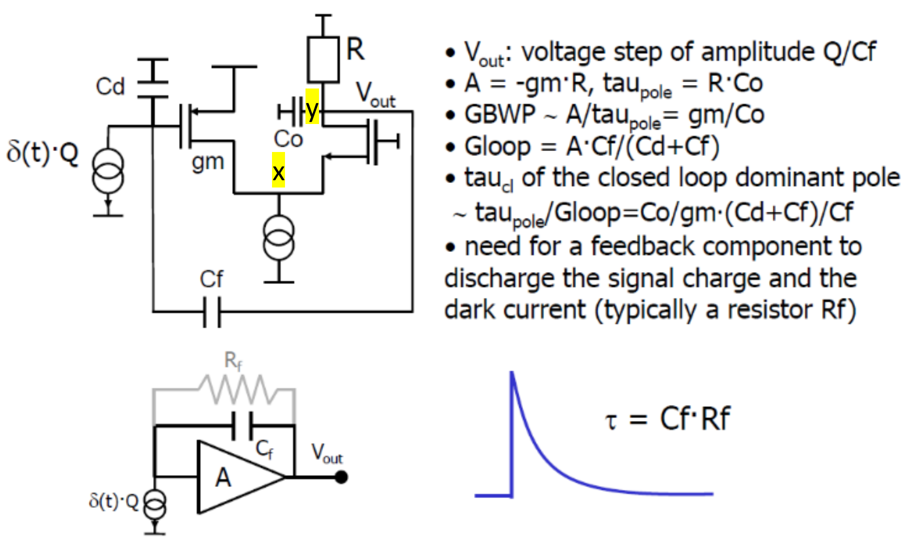
NB: there are also secondary poles, like a pole in node x, where we have also capacitive loads and impedance, where the impedance is the 1/gm of the cascode, so very small → smaller time constant and pole at much higher frequencies.
The main two poles of the charge preamplifier are the dominating one at the output node and the one at the input of the cascode. But if we keep these two far away we don't have problem of stability.

Moreover, node y is a high impedance node: if we want to exit the preamplifier with a low impedance we have to add a buffer implemented as a source follower → this may add an additional pole.

The GBWP of this amplifier is the product between A and 1/tau and given by gm/Co. It depends on the transconductance of the transistor and on the capacitance Co → Co to be kept as small as possible. As for the transconductance, I would be tempted to have it as large as possible (big geometrical dimensions of the transistor, W and L). This can be a problem because the gate capacitance matters for the noise, and if we increase gm we increase the speed of the circuit but it is not a good choice in terms of noise.

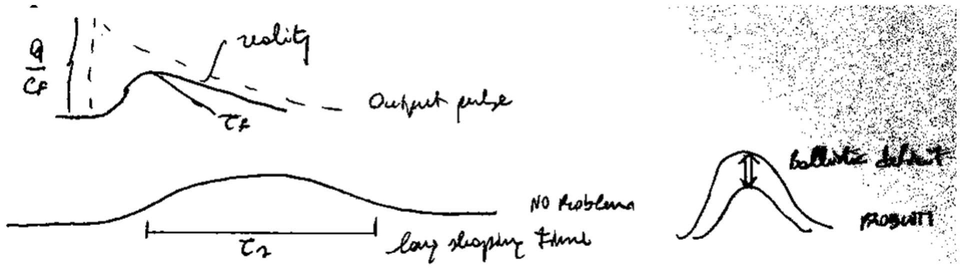
The loop gain is A*Cf/(Cd+Cf) (we cut the loop at the output, we apply a test generator, we go through the partition Cf and Cd and then we have A). A is a negative term (-gm*R). We know that the LG must be usually high, at least larger than 100 to have a good feedback and that all the charge goes into the feedback capacitor due to the virtual ground. In fact, the charge Q, thanks to the virtual ground at the input, goes in Cf. The LG have to guarantee this.
To increase LG, we can increase A but there is a penalty in the LG; Cd is the detector capacitance and usually is given, we cannot choose it, but we can choose the feedback capacitance Cf. The smaller Cf, the steeper, the larger the step in output of the preamplifier for a given charge. Having a larger step is important because the larger the signal, the lower the noise of the following stages brought back at the input. In fact, we have also noise sources at the input of the filter, not only the charge preamplifier. To consider them, we calculate them and we bring them at the input of the preamplifier by means of the factor Cf, so to have the noise in charge.
So smaller is the capacitance Cf, the smaller the second stage noise.

However, I cannot consider the smallest Cf as possible, because in LG it is also at the numerator, not only at the denominator → it reduces the LG. hence the choice of Cf is a compromise between the gain of the amplifier and second noise stage suppression and loop gain, because if the LG is too small, the feedback of the circuit is open.



As for the CL speed, which is the rise of the pulse (related to the CL dominant pole), once we have the OL tau, the CL pole is the frequency of the OL pole multiplied by the LG. Hence the tau of the closed loop of the amplifier is tauOL/LG-gain. This tau is the speed of the response of the preamplifier.

97.



The output pulse should be a Heaviside step given by Q/Cf, but in reality it is given by the continuous line. The tau of the rise is the one of the CL, and it is the speed of the preamplifier.
The importance of this tauCL depends on the situation. If the shaping time of the following filter is very very long, so much larger than the rise of the preamplifier, this is not a problem. But if the shaping time of the filter is as short as the rise time of the amplifier we run into the **Ballistic deficit**. This means a reduction of the amplitude of the filter.

NB: previously, the ballistic deficit was due to the scintillation decay time, because if the input current of the amplifier is an exponential pulse, when we integrate and exponential pulse over the charge amplifier we still get an exponential pulse in output → the non perfect steepness of the preamplifier pulse is due to the exponential delivery of the photons by the scintillator, and this is one problem.

However, even if we would have a perfect delta like charge by the scintillator, we still have the non-perfect Heaviside step of the preamplifier because of the bandwidth of the preamplifier itself, and this can cause ballistic deficit → due to scintillator and CL bandwidth of the amplifier..

To boost the bandwidth of the amplifier, we try to increase the gm and reduce the Co.

1/f NOISE: CHOICE OF THE INPUT FET, PMOS VS NMOS

It is the only responsible for the noise if we design the circuit well. I'm discarding BJT, that are not popular for detector readout, due to the shot noise of the base current.
If we consider FET, we have two choices: JFET or MOSFET. Nowadays, JFET are not used in integrated circuits, but only as discrete component circuits, so the MOSFET are used.

However, MOSFETs have a worse 1/f noise than JFETs. But since we are motivated to work in CMOS technology, we have to use them.
We need now to investigate the physical causes of the 1/f noise if we deal with MOSFETs.

1/f noise models

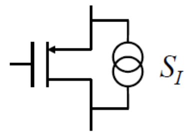
We have two ways to model it, according to two different physical point of view:

- Hooge model: 1/f caused by mobility fluctuations of the carriers inside the channel. If previously we have assumed that the channel in the MOSFET is uniform once created, and so we have a uniform flow of electrons because the electrons find a uniform conducibility, this model doubts about this assumption → the conduction of electrons is through a medium with different conducibility regions. So electrons move with variable velocity. This creates fluctuations in current.

- McWorther model: the channel is now considered uniform, but time to time we have traps where electrons are trapped and then released after a while. This means that the output current is constant +- some drops due to electrons trapping. The Fourier transform of such a behaviour is the 1/f noise spectrum.

These two models are valid depending on the specific transistor. If the electron flow is close to the interface between Si and oxide, there trapping is dominant, because the coupling between oxide and Si is not a perfect match, time to time we have some lack of crystal structure.

1/f noise models (strong inversion)



$$S_{I(\Delta\mu)} = \left(\frac{q\sqrt{2\mu}\,\alpha_H}{\sqrt{C_{ox}'}}\right)\left(\frac{1}{\sqrt{W L^3}}\right) I^{3/2}\frac{1}{f} \quad [\mathrm{A}^2/\mathrm{Hz}]$$

Hooge model
(carrier mobility fluctuations)

$$S_{I(\Delta N)} = \left(\frac{q^2 \mu\, kT\, N_T}{\gamma C_{ox}'}\right)\left(\frac{1}{L^2}\right) I \frac{1}{f} \quad [\mathrm{A}^2/\mathrm{Hz}]$$

McWorther model
(channel conductivity modulation caused by trapping/detrapping)

Having said so, what matters for MOSFETs is that, once we have assumed that the transistor is responding to one of the two models, that have in common the 1/f dependency, what is different is the dependency with the other parameters. There are some physical parameters that are in the first parenthesis.
Then there are some dependencies on the dimensions of the transistor W and L and on the current flowing in the transistor. This is important, because once we have chosen the technology, we cannot do much about the factors in the first parenthesis. For instance, I cannot do much about trap density (in case of trapping), it is given but we can act on W, L and the current.

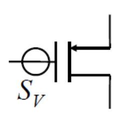
Computations

Since the noise of the transistor can be seen as input noise voltage generator, I need to transfer the noise from the current to the input voltage.
I take the transconductance formula, I take the current, divide by it and we have the voltage noise spectral density. We divide by the square because we are speaking about noise.

NB: the McWorther formula can be simplified in a form (x) with a parameter Kf. This is an ultrasimplified generic formula for

$$g_m^2 = 2\mu C_{ox}'\frac{W}{L} I$$

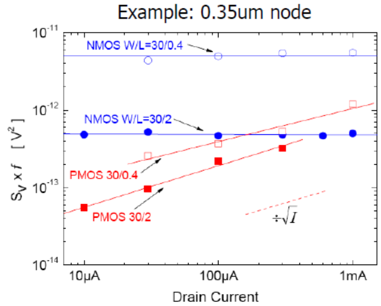
$$S_{V(\Delta\mu)} = \left(\frac{q\,\alpha_H}{\sqrt{2\mu}\sqrt{C_{ox}'^3}}\right)\left(\frac{1}{\sqrt{W^3 L}}\right)\sqrt{I}\frac{1}{f} \quad [\mathrm{V}^2/\mathrm{Hz}]$$



$$S_{V(\Delta N)} = \left(\frac{q^2\, kT\, N_T}{2\gamma C_{ox}'^2}\right)\left(\frac{1}{W L}\right)\frac{1}{f} \quad [\mathrm{V}^2/\mathrm{Hz}]$$

$$S_{V(\Delta N)} = \left(\frac{K_F}{C_{ox}' W L}\right)\frac{1}{f} = \left(\frac{K_F}{C_G}\right)\frac{1}{f} \quad [\mathrm{V}^2/\mathrm{Hz}] \quad \mathrm{x}$$

Example: 0.35um node



- PMOS follows $\Delta\mu$
- NMOS follows ΔN

But McWorther model seems more suitable for both NMOS and PMOS for more scaled technologies:

$S_V = K_f/C_G \cdot 1/f$

1/f noise in MOSFET transistors. It tells us that the **1/f noise is reversely proportional to the dimension of the transistor W and L**. It is a strong message because it means that if for any reason we are tempted to increase the capacitance Cg we lower the 1/f, but we increase the capacitance in the formula of the noise.

Moreover, there is a difference between the two models, in particular on the dependency of the current the transistor.
We can see that the McW model, considered as input voltage, has lost the dependency on the current. So whatever current we bias the transistor, the 1/f noise is the same.
In Hooge model, instead, the noise goes with the square root of the current → better try to reduce as much as we can the current to reduce the noise.

In the plot there are the plot of a technology of 0.35 um node. We can see that in this technology node, pMOSFETs follow the square root of the current, so follows more the Hooge model, while nMOSFETs have noise independent on the current.

This tells us that if we use this technology node, in pMOS we can play with current, in nMOS not. If the technologies are even smaller, McW model is suitable for both polarities.

Alpha and Nt

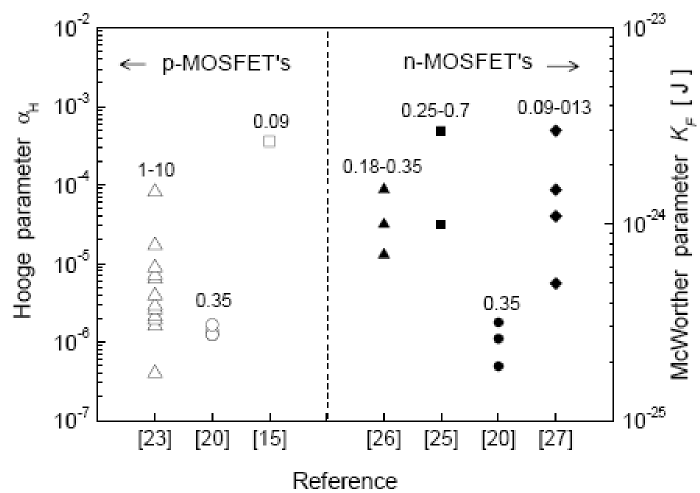


Fig. 7 Experimental Hooge parameter for different p-channel MOSFET's (white symbols) and K_F parameter (eq.36) for n-channel MOSFET's (black symbols) taken from different references. The numbers indicate the technology node (minimum gate length).

This plot shows us the technology parameters; they are parameters like alpha or Nt that depend, in the formula of the noise, on the technology. In this plot are shown alpha and Kf (the cumulative factor) for different technology nodes.
NB: the two plots are separated, we must distinguish pMOS and nMOS.
We can see that we have a huge variety of 1/f values of the parameters depending on the technology.

Now, we will try to optimize the noise acting on W, L and current, but the impact of the technology is enormous → with the wrong technology we may have troubles in terms of 1/f.

Moreover, **scaling is not necessarily good for 1/f noise**.

Pmos vs nmos comparison

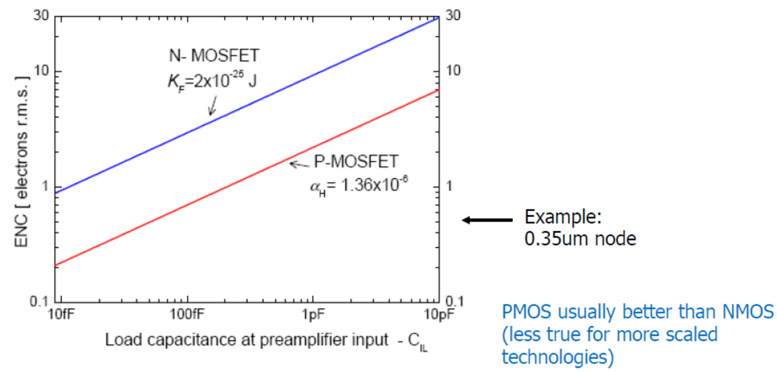


Fig. 10 Comparison between the minimums ENC_{1f} achievable at different C_{IL} with N and P-MOSFET's. The considered noise parameter values K_f and α_H are in between the lowest ones experimentally measured (see Fig. 7).

Pmos are better than nmos in terms of 1/f noise. This is a usual trend for not ultrascaled technologies. The gap then eventually reduces in ultrascaled technologies.

OPTIMIZATION AND DESIGN OF THE INPUT MOSFET

We consider first 1/f only

$$S_{V(\Delta\mu)} = \left(\frac{q\,\alpha_H}{\sqrt{2\mu}\sqrt{C_{ox}^{'3}}}\right)\left(\frac{1}{\sqrt{W^3 L}}\right)\sqrt{I}\,\frac{1}{f} \quad [\mathrm{V^2/Hz}]$$

$$S_{V(\Delta N)} = \left(\frac{q^2\,k\,T\,N_T}{2\,\gamma\,C_{ox}^{'2}}\right)\left(\frac{1}{W\,L}\right)\frac{1}{f} \quad [\mathrm{V^2/Hz}]$$

$$\boxed{ENC^2_{1/f} = \frac{2\pi}{q^2} A_2\, A_f\,(C_{IL}+C_G)^2}$$

x

$$S_V = \frac{A_f}{f} \qquad C_{IL} = C_D + C_f + C_T + C_S \qquad C_G = C_{ox}^{'} WL$$

C_{IL}: total capacitance
C_D: detector
C_f: feedback
C_T: test cap.
C_S: parasitic capacitance

$$\boxed{\begin{aligned} ENC^2_{1/f(\Delta\mu)} &= 2\pi\, A_2\left(\frac{\alpha_H\, L}{q\sqrt{2\mu}}\right)\left[\frac{(C_{IL}+C_G)^2}{\sqrt{C_G{}^3}}\right]\sqrt{I} \\ ENC^2_{1/f(\Delta N)} &= 2\pi\, A_2\left(\frac{k\,T\,N_T}{2\,\gamma\,C_{ox}^{'}}\right)\left[\frac{(C_{IL}+C_G)^2}{C_G}\right] \end{aligned}}$$

Difference between models:
- dependence or not on I
- different dependence on C_G

$I \geq I_{min} = R_{min} I_S$ (strong inversion)

So we have the two models. Now I take the noise formula of ENC (x), and in particular only the 1/f contribution. We have the sum of the two capacitances, the gate capacitance of the transistor Cg and the detector capacitance Cil, that is the sum of several contributions plugged at the input node, not only the detector capacitance.
The test capacitance is used to test the amplifier, to see if it works (not important). The important is the Cs, parasitic capacitance; in fact, if we have some distance between the detector and the preamplifier, the cabling capacitance has to be added in the sum. → in conclusion Cil is not just the detector capacitance, but the capacitance of whatever we have connected to the input.

Af is what matters now, that is the factor between the two set of parentheses in the Sv formula. If we plug in Af in the ENC^2 formula we get the formulas in the red box in the bottom of the image.

Moreover, since we have in the formula the Cg (gate capacitance) that is a worsening factor, but also in the models Sv we have the Cg (that is W*L *Cox), we can conclude that in the formula everything can be represented as Cg.
Now we can understand why increasing Cg or not is tricky; in fact, it is not to choose Cg as large as I can, because the Cg is both at numerator and denominator → we need to find a trade off.
Moreover, at least in the Hooge model, we have the square root of the current.

We will focus now in Hooge model, because it is the one that applies better for pMOS.

Current
In the formula I can say that if I want to reduce the noise, I should reduce the current as much as I can. In reality, I cannot be so aggressive because these two models are valid only in strong inversion → there is a minimum current Imin where these assumptions are verified (MOSFET can work in strong inversion, weak inversion and subthreshold). So to draw conclusions in weak inversion we need to refer to other models.

$$ENC^2_{1/f(\Delta\mu)} = 2\pi\, A_2 \left(\frac{\alpha_H\, L}{q\sqrt{2\mu}}\right)\left[\overset{x}{\frac{(C_{IL}+C_G)^2}{\sqrt{C_G{}^3}}}\right]\sqrt{I}$$

We want to minimize the term x to select the best Cg, which means best choice of the W and L of the transistor.
We can notice that in the formula we have a lonely L.
Guidelines:

1. Smallest current 'I' as possible.
2. Lowest possible L
3. Optimize the fraction x

Optimization of the fraction

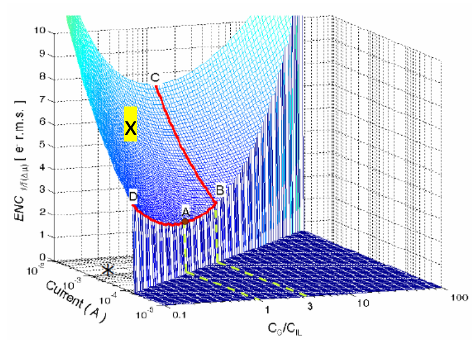


Fig. 2 $\Delta\mu$-type MOSFET's : $ENC_{1/f}$ given by eq. 8 as function of the bias current and the ratio C_G/C_{IL} between the gate capacitance and the load capacitance at the amplifier input. The $ENC_{1/f}$ has been set to zero outside of the field of validity of eq. 8 given by eq. 10. The absolute minimum at C_G/C_{IL}=1 (point A) and the locus of relative minima given by the line D-A-B-C are shown.

If we are free to choose I:
⇒ absolute minimum ($I=I_{min}$)
for $C_G=C_{IL}$

If we are not free to choose I
(I fixed, e.g. for Gloop and speed):
⇒ minimum for I fixed
for $C_G=3C_{IL}$

* considered domain:
strong inversion only
$I \geq I_{\min} = R_{\min} I_S$

This is the ENC vs current vs Cg/Cil plot.

Not all the domain of the current is available, we cannot choose a current lower than a certain value, and this is due to the strong inversion regime.

Then we can minimize the noise. The absolute minimum of the noise is the point A, which belongs to the minimum current, so to have chosen 'I' the minimum to stay in strong inversion.
So once the current is belonging to the minimum, we are in the parabola B-A-D. The point A is the one that corresponds to a 1 in the Cg/Cil ratio.

In conclusion, to minimize the previous formula we have to:

1. Choose the minimum current, staying in strong inversion.
2. Lower L as possible
3. Select Cg = Cil (condition called capacitive matching, given the detector we match the amplifier input capacitance), which means, in practice, since Cox is given by the technology and L has already been minimized independently, to choose the optimal W → W can be choose optimally using this relationship.

NB: for W there is not an absolute good value, but it depends on the detector capacitance. If we have a detector capacitance of 10pF we have a given W, differently if we have 1pF, but we have the relationship to do the correct choice.

Moreover, this equality stands also for the McW model, not only the Hooge one. Simply there is no minimization of the current because there is no current in the model (we can use the current we like), but as for the Cg the result is the same, that is that the ratio is minimal when Cg/Cil = 1.
So the rule is valid even if we have for instance a nMOS fulfilling the McW model.

However, this rule Cg = Cil is based on the fact that we can operate the transistor with a very low minimum current, but this is not always possible to be in the D-A-B line, because the current in the transistor has an impact also in the transconductance, and the transconductance has an impact in the dynamic (speed of the amplifier) and gain properties of the amplifier.
→ There are some cases where, if we choose the lowest possible current, the gm is so bad that the speed of the preamplifier is dramatically bad.

Current not minimal

Hence, this means that it could be that we are not fully free to choose the smallest possible current. If so, we have to consider the part x of the plot, the one for a current higher than the minimum, not the minimal one. The noise can still be minimized, but it won't be the minimum noise we can achieve, but at least, depending on the value of the current, we can search local minima. The local minima in the plot are in the red line B-C.

The B-C line corresponds to a new factor Cg/Cil = 3. So if we are forced to use a current larger than the minimum, the best noise is achieved with a Cg = 3*Cil.

Inclusion of all the noise contributions

We have concentrated on 1/f noise up to now, but we have also the white series noise proportional to 1/tau and reversely proportional to the gm, and the parallel noise, proportional to the shaping time.

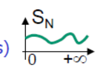
These two noise contributions are introduced because I want to check if my previous choices for W and L are also compatible with the other noise contributions, so to check if W and L can be selected independently on the other noise contributions.

The answer regarding the parallel noise is that it doesn't matter in choosing <W and L, because it depends only on the shot noise of the detector current. Of course in principle we have also the shot noise of the gate current, and larger the transistor the larger its contribution, but at first approx., if we are dominated by the dark current of the detector the transistor can be any and the parallel noise will be insensitive.

We consider now all noise contributions
(PMOSFET to be optimized now for both series and 1/f noise)

$$\boxed{ENC_{tot}^2 = ENC_{ws}^2 + ENC_{1/f(\Delta\mu)}^2 + ENC_{wp}^2}$$

(physical noise spectra densities)



$$\rightarrow ENC_{ws}^2 = \frac{A_1}{q^2} S_V (C_{IL} + C_G)^2 \frac{1}{\tau} \qquad S_V = \beta \frac{4kT}{g_m} = \frac{\beta\, 4kT}{\sqrt{2\mu C'_{ox} \frac{W}{L}}\sqrt{I}}$$

$$\rightarrow ENC_{1/f(\Delta\mu)}^2 = 2\pi A_2 \left(\frac{\alpha_H L}{q\sqrt{2\mu}}\right)\left[\frac{(C_{IL}+C_G)^2}{\sqrt{C_G{}^3}}\right]\sqrt{I}$$

$$\rightarrow ENC_{wp}^2 = \frac{A_3}{q^2} S_I \tau \qquad S_I = 2q\left(I_{\det} + I_G + I_F\right)$$

This is not the case for the series noise. In fact, in the series noise there is a dependency with the thermal noise of the transistor, which depends on the current and W/L. This tells us that we are not completely free to make the choices of 'I', W and L on the 1/F noise without any consequences over the series noise.

From the formula below of the total ENC we can see the dependencies with the dimensions of the transistor and the current.

$$\begin{cases} ENC_{tot(\Delta\mu)}^2 = k_{ws} \dfrac{(C_{IL}+C_G)^2}{\sqrt{C_G}} \dfrac{1}{\sqrt{I}} \dfrac{1}{\tau} + k_{\Delta\mu} \dfrac{(C_{IL}+C_G)^2}{\sqrt{C_G^3}} \sqrt{I} + k_{wp}\tau \\ I \geq R_{\min} I_S \end{cases}$$

$$k_{ws} = A_1 \left(\frac{\beta\, 4kT\, L}{q^2 \sqrt{2\mu}}\right) \; ; k_{\Delta\mu} = 2\pi A_2 \left(\frac{\alpha_H L}{q\sqrt{2\mu}}\right) ; k_{wp} = \frac{A_3}{q^2} S_I$$

- if we are free to choose an optimum $I=I_{opt}$: $C_{Gopt}=C_{IL}$
- if I is fixed: $1/3C_{IL}<C_{Gopt}<3C_{IL}$ (depending on the constraint on I)

- Example for $I<I_{opt}$: minimum power consumption (e.g. many electronics channels)
- Example for $I>I_{opt}$: large g_m to increase Gloop and speed of preamplifier

Marked with red arrows, we can see in the formula that the 1/f contribution is reduced by decreasing the current, but unfortunately, the series noise has the current at the denominator → it is not true that the lowest current the better, but there must be an optimal current which is the best compromise for 1/f and series noise.

Once we have found this optimal current, is it possible to establish a condition regarding the capacitances as did before? The answer is yes, because there are terms depending on Cg on both the noises.

The conclusion for the capacitances is the same as before. If we have optimized the current, the best choice for the transistor capacitance (or dimension, because when talking about Cg we are making decisions about W) is still Cg = Cil.

If, for any reason, I'm not able to operate the transistor at the optimal current, the range of optimal capacitance is 1/3Cil < Cg < 3Cil.
This can be seen on the plot below (the same as before).

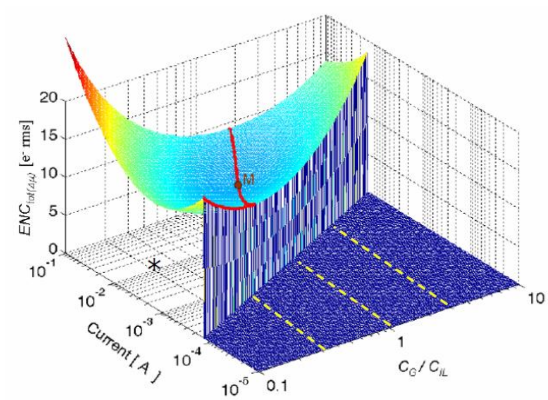


Fig. 4 Example of plot of $ENC_{tot(\Delta\mu)}$ given by eq. 23 as function of the bias current and the ratio C_G/C_{IL} between the gate capacitance and the load capacitance at the amplifier input. The *ENC* has been set to zero outside of the field of validity of eq. 23. The absolute minimum (point M) is at C_G/C_{IL}=1 and $I=I_{opt(tot)}$=14 mA (eq. 25). The red line on the surface is the locus of the relative minima as the bias current is varied, as given by eq. 28.

* considered domain:
strong inversion only
$I \geq I_{\min} = R_{\min} I_S$

We see that there is an absolute minimum now, M, that is not belonging to the minimum current, because there is no more a minimum current, but there is an optimal current, that is the one corresponding to M. if for any reason we are either above or below the optimal current, we have to look at the range of capacitances, and the range of capacitances is not at a fixed number as before (1 or 3), but, depending on the branch we are, we range between the 3 and 1/3.

Examples why we cannot choose the optimal current
The reason why we cannot choose a too small current, so we need to operate at a larger current than the optimal is due to the loop gain and speed. In fact, it could be that the optimum current derived by the formula is not sufficiently high to provide a good electrical characteristic in terms of speed and loop gain.

On the contrary, the main reason why we could be forced to use a lower current than the optimal, is power consumption. For instance, if the optimization tells us that the optimal current is 10mA and this current needs to be multiplied by 1000 in a PET apparatus, the power consumption would be not tolerable → in many applications we are forced to use the transistor at lower current than the optimal because of power saving (and we have to accept a worst noise).

THE SHAPING AMPLIFIER (FILTER)

We have a variety of possible filters synthesis (see table previous chapter) and our wish is to choose the best filter to implement to have a triplet of values related to the filter shape A1, A2 and A3 that overall minimizes the noise.
The best choice would be the infinite cusp, if we have only white noises. Then we have other suboptimal filters in terms of electronic noise, but they provide a realistic possibility to be implemented.

In particular, a good approx. of the infinite cusp is the Gaussian shaper; if we compare the performances in terms of noise given filtering shaping factors for a gaussian, they are not as good as the indefinite cusp but not so bad.
However, gaussian is practically impossible to be implemented with practical RC networks in analog circuitry. There is a way to synthetize an almost gaussian filter (semi-gaussian) which represents the best approximation of the gaussian one.

THE SEMIGAUSSIAN FILTER WITH CONJUGATE COMPLEX POLES

Purpose: approximate a "Gaussian" filter (which is a 'good' approximation of the optimum filter but which cannot be implemented in practice) with a "Semi-Gaussian" filter which can be implemented with R-C networks.

$$f(t) = a_0 \, e^{-\frac{1}{2}t^2/\sigma^2} \qquad F(\omega) = a_0 \sqrt{(2\pi)}\, \sigma \, e^{-\frac{1}{2}\sigma^2 \omega^2}$$

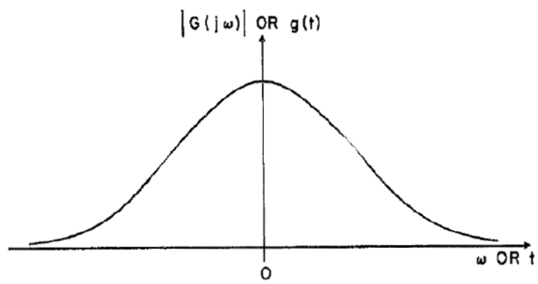


Fig. 1—Gaussian response curve.

We want to implement the semigaussian filter, that represents the best approx. of the gaussian filter.

If we consider a gaussian filter, the characteristics of a gaussian response in the time domain is that the corresponding Fourier transform is also gaussian (bell-shaped).
We are looking for a transfer function H(s) to be the best approx. we can of a gaussian transfer function → H(s) = H0/Q(s).
H0 is a fixed constant, the denominator is polynomial → we have only poles, and these poles have to approx. at best the gaussian response, and on the other side they should be implemented with networks that we can design in practice.

We can see that if we make the product of H(jw)*H(-jw), the result is in the formula below.

The square modulus of H can be put equal to the modulus of the gaussian transfer function we want to synthetize.

We look for a transfer function H(s) of the Gaussian filter expressed as:

$$H(s) = H_0/Q(s)$$

where Q(s) is a polynomial in which the zeros are therefore the poles of the filter.

We observe that: $H(j\omega)\cdot H(-j\omega) = |H(\omega)|^2 = [F(\omega)]^2$ $\quad F(\omega) = a_0\sqrt{(2\pi)}\,\sigma\, e^{-\frac{1}{2}\sigma^2\omega^2}$

$$\Rightarrow \quad Q(j\omega)\cdot Q(-j\omega) = \frac{1}{2\pi}\left(\frac{H_0}{a_0\sigma}\right)^2 e^{\sigma^2\omega^2} \qquad \text{x}$$

$$Q(s)\cdot Q(-s) = \frac{1}{2\pi}\left(\frac{H_0}{a_0\sigma}\right)^2 e^{-\sigma^2 s^2} \qquad s = j\omega$$

by normalization: $Q(p)\cdot Q(-p) = e^{-p^2}$ y $\qquad$ with $p = \sigma s$

Considering that Q(s) is simply H(s) flipped divided by a constant factor H0, we come to the x equation. It is simply the gaussian flipped.
Then, since s is a dimension variable, I can even make a change of variable using the dimension level variable p equal to sigma*s.
Sigma is the time width of the gaussian, it has a real physical meaning, it is the duration of the gaussian not expressed in terms of base duration of the gaussian, but in terms of the sigma.

Now I have to look for a Q(p) so that y is satisfied.

Since exp(-p^2) is impossible to be synthetized with a RC network, what is possible to be implemented are real poles and circuits of the second orders in the sense of circuit with c.c. poles (e.g. Sallen-Key cells) → we can use a Taylor expansion.

the exponential can be approximated with a Taylor series:

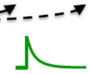
$$Q(p)\cdot Q(-p) = 1 - p^2 + \frac{p^4}{2!} - \frac{p^6}{3!} + \ldots + (-1)^n\frac{p^{2n}}{n!}$$

we factorize the term on the right of the equation to the same form of the term on the left to determine Q(p)

Example: n=1

$$Q(p)\cdot Q(-p) = 1 - p^2 = (1+p)\,(1-p)$$

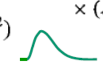
$$\Rightarrow \quad Q(p) = 1 + p$$



Example: n=2

$$Q(p)\cdot Q(-p) = 1 - p^2 + \frac{p^4}{2!} = \tfrac{1}{2}[\sqrt{2} + \sqrt{(2+2\sqrt{2})}\,p + p^2] \times$$
$$\times\,(\sqrt{2} - \sqrt{2+2\sqrt{2})}\,p + p^2).$$

$$\Rightarrow \quad Q(p) = \frac{1}{\sqrt{2}}(\sqrt{2} + \sqrt{(2+2\sqrt{2})}\,p + p^2)$$



The product Q(p)*Q(-p) is represented as the Taylor expansion of the exponential function.

Now I need to choose for a good polynomial Q in order to stop at a desired number of terms in the Taylor series.

Case n = 1

The first easiest thing that we can do is to stop at the first order, so at 1 – p^2. Now the polynomial Q(p) that multiplied by Q(-p) produces 1 – p^2 is 1 + p. → our gaussian filter has been represented as H0/(1+p). It is a single pole filter.
The time response of a single pole transfer function is the one in green, an exponential decay.
However, this is a bit brutal approximation of a gaussian.

Case n = 2

I add another term of the Taylor expansion. Again, we can get the final Q(p), that is a polynomial of the second order with two c.c. poles. The response in this case doesn't start steep, it reaches a maximum and then it has a decay.
Also this one is not a very nice representation of the gaussian, but it is at least better than case n = 1. What is missing is the symmetry in the response, the response is not symmetric.

In principle, we could continue as deep as we like with the approximation, adding more and more poles.

General case

The t.f. H(s) can be represented as below.

In general, the transfer function of the Semigaussian filter is obtained as:

$$H(s) = \frac{A_0 \prod_{i=1}^{k} \{A_i^2 + W_i^2\}}{(\sigma s + A_0) \prod_{i=1}^{k} \{(\sigma s + A_i)^2 + W_i^2\}} \qquad \text{n. odd poles}$$

1 real pole

couples of complex conjugate poles

$$H(s) = \frac{\prod_{i=1}^{k} \{A_i^2 + W_i^2\}}{\prod_{i=1}^{k} \{(\sigma s + A_i)^2 + W_i^2\}} \qquad \text{n. even poles}$$

$s = j\omega$

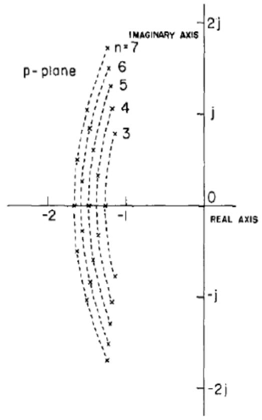
σ of the Gaussian

couples of complex conjugate poles

At the denominator I have the products of polynomial of the second order. So we have a H(s) with no zeros and the denominator, depending on where we stop in the Taylor series, if we are look at an even number of poles, it is in the form of sigma*s plus a coefficient Ai and another coefficient Wi.
If the poles are odd, we have still the product of polynomial of the second order, but we need to add also a real pole, otherwise we cannot implement an odd number of poles.

The coefficients of these poles can be calculated. We have a constellation of poles for the various order. If we stop at n = 1, we have only a real pole (not shown in the plot). If we have a 3rd order, we need to implement a real pole and a couple of c.c. poles.

for n≥3 the zeros of Q(p) (poles of the filter) are determined numerically:



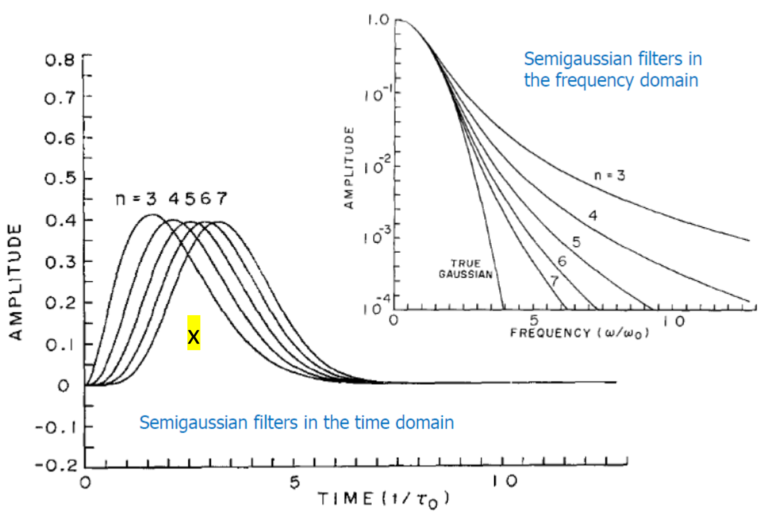
Pole locations of the Gaussian filters.

	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
A_0	1.2633573		1.4766878		1.6610245
A_1	1.1490948	1.3553576	1.4166647	1.5601279	1.6229725
W_1	0.7864188	0.3277948	0.5978596	0.2686793	0.5007975
A_2		1.1810803	1.2036832	1.4613750	1.4949993
W_2		1.0603749	1.2994843	0.8329565	1.0454546
A_3				1.2207388	1.2344141
W_3				1.5145343	1.7113028

Note: A_i and W_i are independent from the pulse width (shaping time) but depend just on the filter order.

Then, in n = 4 we have no real poles, only two couples of c.c. poles and so on.

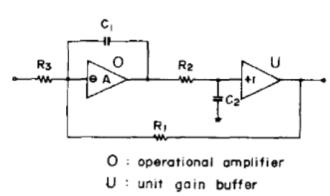
Time and frequency implementation of the filter



for the definition of t_0 see in the following

X is the time implementation: by increasing the order of the filter the approx. with a gaussian look much better; a filter of the 7th order looks very similar to a gaussian, so I'm expecting that the triplet of factor will be almost identical to the one of the Gaussian.
In the frequency domain y we see the corresponding Fourier transform of the filters, and we can see that the more we increase the order of the filter, better we approximate the gaussian Fourier transform.

Having a good approximation is important because the general formula provides the opportunity to implement the networks simply with a cascade of networks of the second order. I need to synthetize a chain of t.f. with c.c. poles.
For instance, I can make a use of the following network.



It is based on an amplifier with a feedback capacitor and then a second loop with another feedback capacitor and the loop is closed with a R1, we compute the t.f. and we get a t.f. of the 2nd order.
Then we get two c.c. poles, we plug them where we like in the gauss plane (the one with the constellation) and then coefficients Ai and Wi are related to the components of the circuit.

$$A_i = \frac{\sigma_0 \tau_0}{2 C_2 R_2},$$

$$W_i = \left(\frac{\sigma_0 \tau_0}{2 C_2 R_2}\right) \sqrt{\left(\frac{4 C_2 R_2}{C_1 R_1} - 1\right)}$$

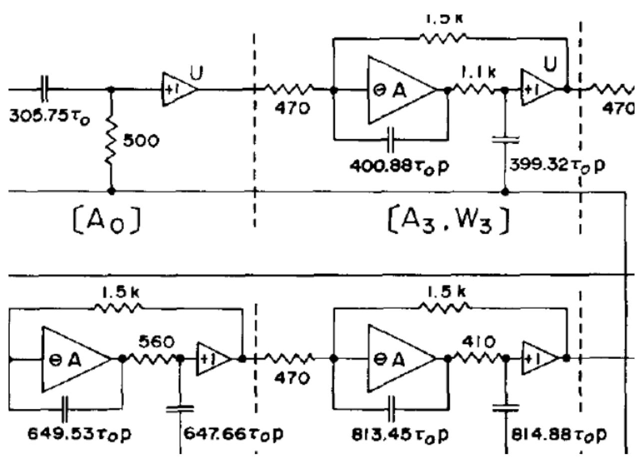
In the formula we have sigma0 and tau0, that are:

$$\sigma_0 = (e/\sqrt{2\pi})$$
$$\tau_0 = 0.9221\sigma$$

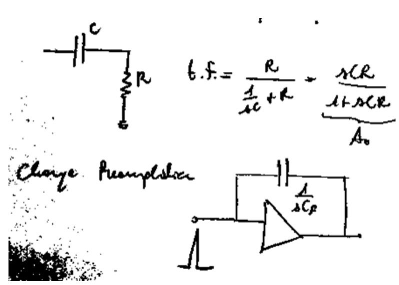
So if we want to synthetize a gaussian of a given sigma and we want the poles to have certain values, we need to take the value of R and C in the circuit so that, given the sigma of the gaussian, fulfill A and W.

NB: **A and W are shaping time-independent**, because of how they are derived. They are simply associated to the order of the filter we want to implement. The width of the filter is inside tau0 in the formula.
This means that if we have a filter, for instance of the 3rd order, and we want to keep it of the 3rd order but simply to enlarge the sigma, we change tau0 in the formula, but we also change R and C in a way that W remains the same, because W represents the order of the filter, not the duration.
→ We have a very practical way to change the shaping time without changing the shape of the pulse, we simply change the components. In fact, we don't change the shape as long as W and A remains the same.

NB: to increase the filter order, we need to add a lot of cells, but this is not a problem, we create a cascade of several cells of the same type. In the image there is an example of the seven order (with a real pole that is simply a derivator, a capacitor and a resistor).



98.

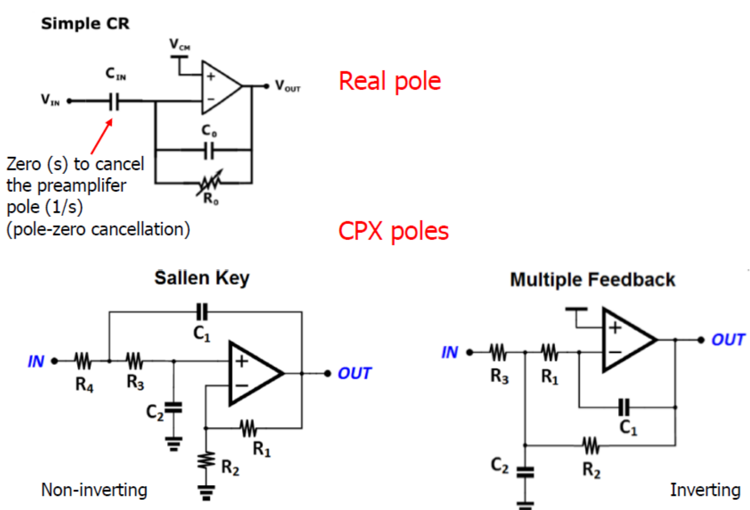


This is the real pole I put into the constellation, the one corresponding to the coefficient A0.
However, I have also a zero in the origin; how to cope with this if there are no zeros in the origin?
What H(s) has to synthetize is the entire t.f., from the detector to the end. But this filter is not the only circuits I have, I have already the charge preamplifier → this is a filter, but before I have the charge preamplifier, and the t.f. of the charge preamplifier is 1/s (in Laplace domain), the transfer between the delta like charge and the output is 1/sCf.
The zero in the origin has the role to cancel the pole in the origin of the charge preamplifier, otherwise I won't have all the H(s) made by poles only.
So I don't have to forget the charge preamplifier, because it is adding a not necessary pole in the origin.
→ we must cancel this unwanted pole in the origin, and this can be done only with a zero in the origin (**pole-zero cancellation**).

Coming back to the example, we have a derivator because the derivator provides a zero in the origin and then the pole where we like it (A0, this network has to implement the pole in position A0).
Then we have a first cell that implements a couple of c.c. poles corresponding to A3 and W3. Then I need another cell connected to the previous one to get A2 and W3 and then one last again.
So we take 3 network of the same type, we put a cascade and we make the Taylor approximation we like. Practically speaking, exceeding the seventh order is not necessary.

NB: the coefficients of the polynomials are constant, given the order of the polynomial. They are independent on the shaping time. The shaping time is then changed with sigma.
To make the filter larger or shorter, we change the values of R and C to change tau0, and keeping constant the coefficients.

Examples of elementary cell for poles implementation
There are different cells, like the multiple feedback cell and the Sallen-Key cell, to implement c.c. poles.
For instance, SK cell is not inverting, the polarity of the signal is the same, while the multiple feedback is inverting. This detail has relevance if we have a cascade of several cells, because maybe we are



interested in the fact that the single cell is inverting or not. If we use an even number of cells the polarity is the same even in cascade, but if odd, we will have finally an inversion of the signal.
Another aspect is the LG. These circuits implement a CL t.f. only under the hypothesis that the LG is large enough, which means that the amplifier needs to have a very high gain.
Especially when we synthetize the poles at very high frequency, to really get the poles at a certain freq, we need the LG to be high enough at high freq. This is not a problem for poles in the low frequency region.

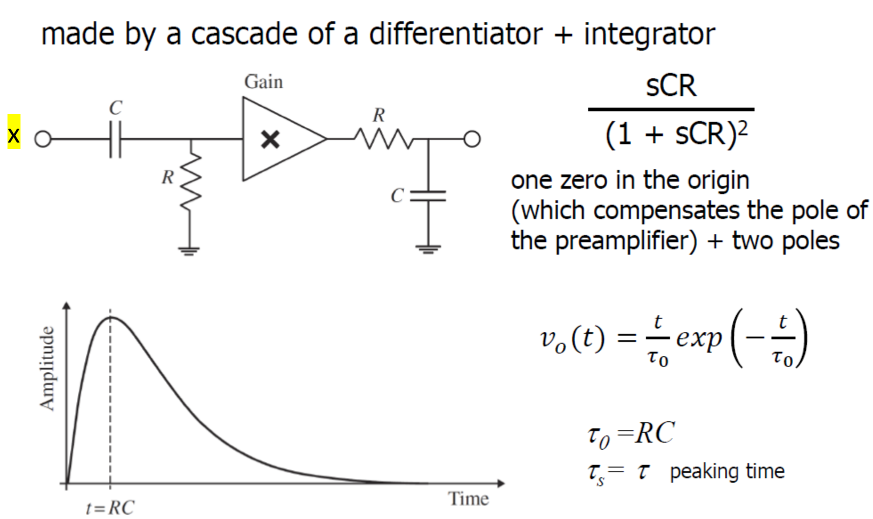
When we need to implement a true semigaussian filter of the odd order, we need to implement a real pole that can be done with a passive derivator or by using a real integrator stage (upper left).

This concludes the approx. of the semigaussian filter with c.c. poles and the usage of the constellation of poles.

There is an alternative way to implement semigaussian shapers, that is not rigorous mathematically like the Taylor expansion, but more a synthesis of gaussian filter with something that looks like a gaussian filter. These techniques are very simple to be implemented and they are based on cascades of simple derivators and integrators.

THE CR-RC FILTER

It is a filter composed by a cascade of a derivative network followed by an integrator. It can be made by 4 resistors in principle plus a buffer in between, because we can assume that the two time constants are independent if they are separated by a buffer, so that the two networks don't see each other.
Then we compute the t.f. as the product between t.f.. We have a zero in the origin and two identical poles if the R C values are the same.



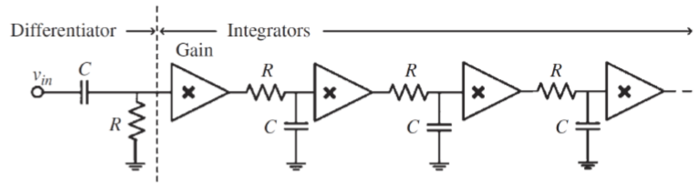
The zero is not required in the final t.f., but it is needed to cancel the pole in the origin of the charge preamplifier. In fact, the input x of the network is not directly connected to the detector, but to a circuit, the charge preamplifier, which has a pole to convert the delta-like charge in a Heaviside step, and the Heaviside step has a function 1/s → we need to cancel the 1/s.

Then in the time domain we have an exponential time response, that is different from the gaussian shape, because very asymmetric, but it is already something.

The time to reach the peak (a figure of merit of the filter) is equal to the RC time constant of both derivator and integrator.

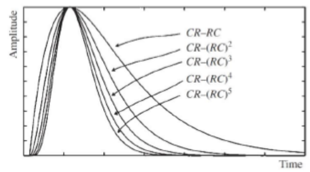
THE CR-(RC)^n WITH REAL POLES

made by a cascade of a differentiator + *n* integrators



$$H(s) = \left(\frac{s\tau_o}{1+s\tau_o}\right)\left(\frac{A_{sh}}{1+s\tau_o}\right)^n$$

one zero in the origin (which compensates the pole of the preamplifier) + *(n+1)* poles



$$v_o(t) = \frac{A}{n!}\left(\frac{t}{\tau_s}\right)^n e^{\frac{-nt}{\tau_s}}$$

$\tau_0 = RC$

$\tau_s = n\,\tau_0$ peaking time

We add more integrator separated by buffer to separate the tau (that can be considered additive and not interacting), with a single derivator. With n integrators we add n poles with the result that the number of poles is n+1. The +1 comes from the derivator.

The peaking time of the pulse now is given not only by the time constant, but by the time constant multiplied by n.
If we take filters and we want to compare them on the same plot by the same peaking time, we cannot use the same R and C for all the filters, because the higher the order, the higher the tau → we need to scale down the value of the components.

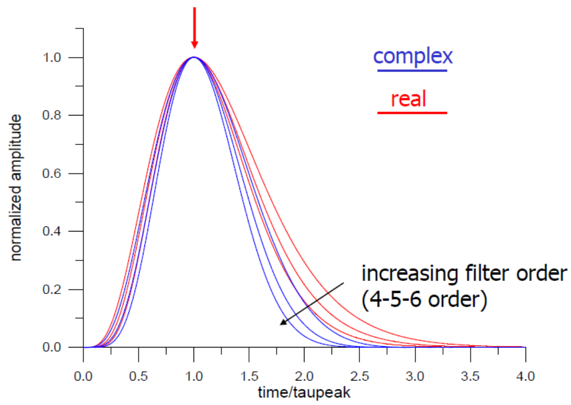
Moreover, **real poles filters are modular, based on a copy and paste replica of the same pole**. This cannot be done for c.c. poles filters, because we need the constellation of poles, and each pole is different.

The effect of increasing the order of the filter is to make the response more symmetric → the higher the order, the more similar to a gaussian the shape. This is the reason why this implementation is called semigaussian but it has no mathematical foundation.
The noise is very good indeed, similar to the one of a semigaussian shape.

SHAPING TIME DEFINITION AND COMPARISON BETWEEN FILTERS

We have generally introduced the shaping time as a parameter proportional to the time duration of the filter response. But this definition is arbitrary, because the width of a pulse can be quoted in different ways.
Now we have 3 main definitions of shaping time. The definition is important to evaluate the noise performances.

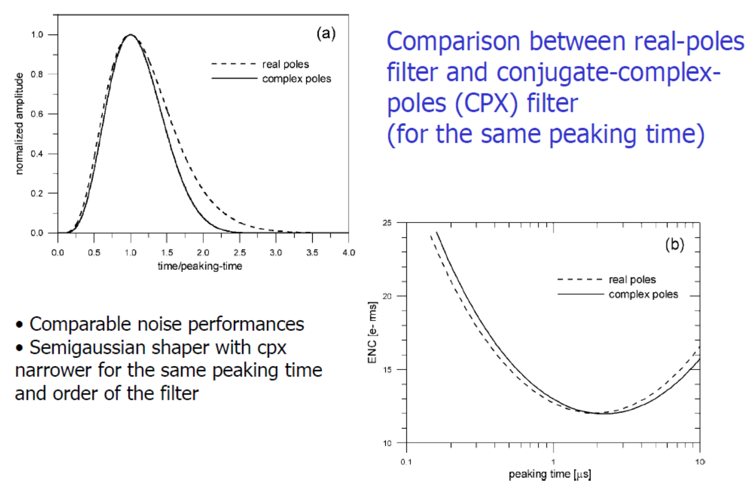
1. Shaping time as the peaking time
   The shaping time is the time necessary to reach the peak. It is a reasonable definition. However, this definition doesn't tell us everything about the filters in the comparison. In fact, if we look at the image, we have filters of different families (c.c. poles or real poles), and all the filters are



represented by increasing the filter order (3 filters orders per type). From the comparison, it is evident that the duration of the pulse is different, even if the peaking time is the same. Increasing the order of the filter, for the same peaking time the duration is squeezed, that is good, because there is less risk of pile up.
Moreover, for the same order, if we consider real or c.c. poles filter, the c.c. poles filter is narrower, its time occupation is narrower. This is why the c.c. poles approximate better the gaussian shape.

Electronic noise

Instead, on the plot on the right we see the electronic noise for a given detector and a change in peaking time. The two filters (real and c.c.) are not identical, but what is not much different is the minimum, which is the optimal peaking time to obtain the lowest noise if we have no other constrains. So is it better to use a fitler with real or c.c. poles? From the point of view of noise there is no difference, given the same order. In the terms of width, the c.c. is better, because it is narrower for a given peaking time.

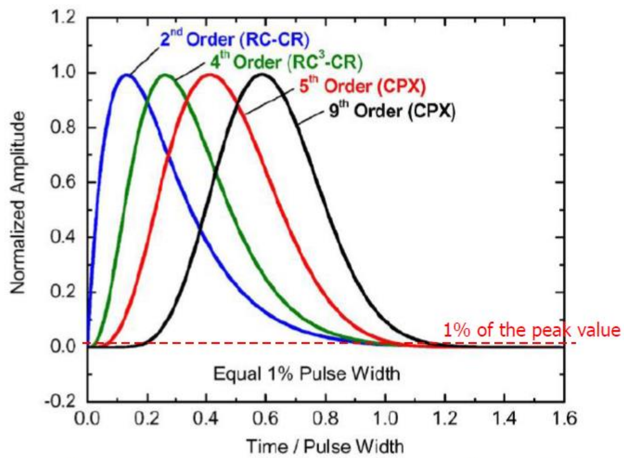


So the big advantage of rea poles filter is that they are very easy to be implemented, the noise is good, but the duration is larger.

2. Shaping time as pulse width at 1% of the peak value

The definition is the shaping time as the time occupation of the filter response. However, we have a problem, and the problem is that semigaussian response never ends, they have an exponential decay. So we need to agree on a common definition of when a semigaussian

response ends. Realistically speaking, we can consider a pulse to end when it reaches the 1% of its peak value, when it is below it.
1% is not 'official' but it is a reasonable practical value because it is in the order of the electronic noise. Let's imagine we have a noisy waveform; when a pulse is below the 1% of its peak value it is no more distinguishable from its noise.

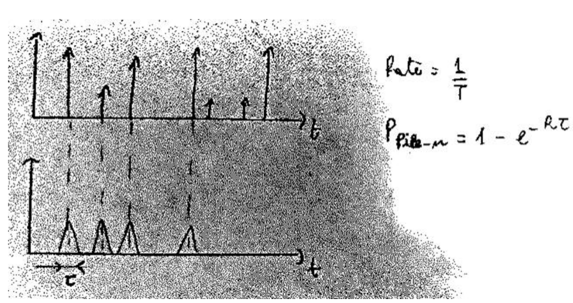


Note: larger is the order, lower is the ENC and the ballistic deficit immunity

Once we agree on a given percentage to define the pulse duration, it is worth to compare pulses not for the same peaking time, not for the same order, but for the same width. The pulse in blue has the same 1% duration as the pulse in black, although they are very different.
This definition of shaping time is important when we study the counting rate capability of our electronics.

Let's assume we have a detector providing delta like pulses in a totally random way. How can we understand if we have pile up?
We know the rate of arrival of the pulses, that is 1/T, where T is the average distance between pulses. To set the electronics, we set it in a way so that pulses are not overlapped. We choose the width of the duration at the base of the filter in a way that the pulses are not overlapped. This is where the definition of the shaping time as pulse duration is important, because given the pulse duration we can establish the probability of pile up of pulses.
In Poisson statistics, a rule of thumb is that the probability of pile up is given by 1-exp(-rate*tau), where tau is the duration of the pulse.

Given a rate (R) and given a single pulse duration tau, the probability that pulses overlap is given by below. This means that the larger is the average rate and the larger the time occupation tau, the more the probability increases → **it is important for a given rate to evaluate the pulse duration**



Which filter to choose?

In blue we have a filter of low order (RC-CR), in green higher order and in black a filter which is a Taylor approx. of the 9^{th} order (ultra-performing filter).

The good reasons to use the black filter are 2:

A. If we consider the triplet A1, A2 and A3, the best combination of the three numbers that gives the best **electronic noise** is given by filters of higher order. For the same noise, having a c.c. or real filter leads more or less to the same noise.

B. **Ballistic deficit**: it occurs when the time response of the preamplifier is not a perfect Heaviside step but a slow rise in the same order of the filter shaping time. It is a relative matter between the preamplifier rise time and filter response → depends on the preamplifier but also on the filter; in fact, with a longer shaping time we reduce the ballistic deficit.

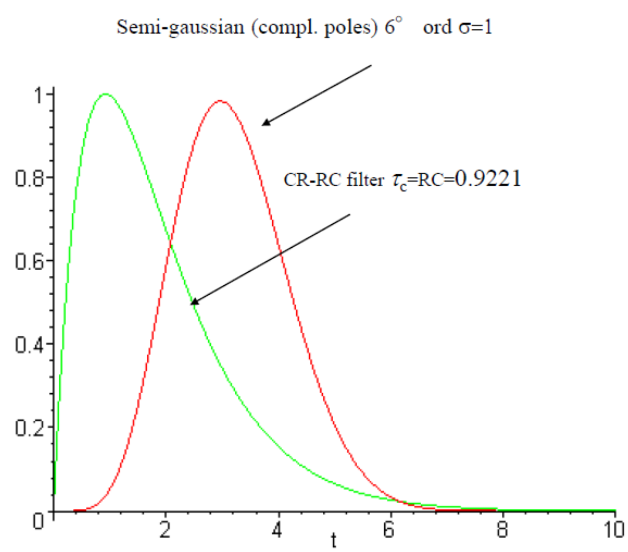
   Given a rise time, it is more prone to ballistic deficit the filter in blue or in black? These two filters have the same duration, but the filter in blue reaches the peak immediately, the one in black reaches the peak afterwards. The filter in blue is more prone to ballistic deficit, because it has a steep rise right in the region where the preamplifier has a rise. On the contrary, for the filter in black, the preamplifier is like a Heaviside step → filters of the higher order, for the same duration, are less prone to ballistic deficit.

3. Shaping time of a filter defined as the time constant of the filter CR-RC which provides the same peak value and the same area of the Gaussian approximated by the given filter.

   The most used one, even if meaningless.

   The shaping time is defined as **the time constant of an equivalent CR-RC filter of the first order which provides the same peak value and area of the Gaussian approximated by my filter**.

   We have our filter, for instance the one in red; this filter approximates a gaussian of a given width sigma (e.g. 1us). The shaping time is not 1us, but it is the peaking time of the filter in green, that is the CR-RC filter which has the same peak amplitude and area of our filter (red). Once we have done this, the tau of the filter in green is the shaping time.



X is the time response of the filter of the first order; we calculate the area y of this filter of the first order. Then we compute the area z of the gaussian we are approximating (which is also the area of our filter), that will be expressed in terms of sigma. Then we equalize the two areas and we have a correspondence between the tau of the CR-RC filter and the sigma of the gaussian. For example, if our filter is approximating a gaussian of 1us, the RC is 0.92us, and it is the shaping time.

x $$f(t) = \left(\frac{t}{CR}\right) e^{-t/CR}$$

y $$S_{CR} = e \int_0^{\infty} \left(\frac{t}{CR}\right) e^{-t/CR}\, dt = e \cdot CR$$

area of the pulse CR-RC with amplitude normalized to 1

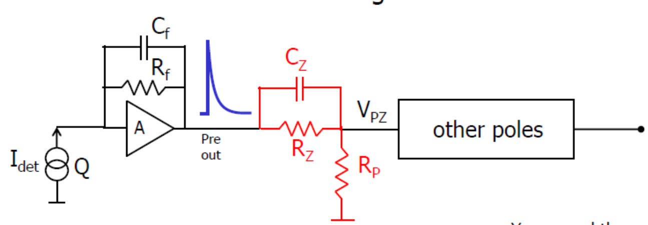
z $$S_G = \int_{-\infty}^{+\infty} e^{-\frac{1}{2}t^2/\sigma^2}\, dt = \sqrt{(2\pi)}\, \sigma$$

area of the Gaussian with amplitude normalized to 1

$$\boxed{\sigma = \frac{e}{\sqrt{(2\pi)}} \cdot CR = 1.0844\,\tau_0} \qquad \boxed{\tau_0 = 0.9221\sigma}$$

POLE-ZERO COMPENSATION

Purpose: compensate (cancel) the pole of the preamplifier with a zero in a the derivative stage



$$V_{PZ} = Q\, \frac{R_f}{\cancel{1 + sC_fR_f}}\; \frac{\cancel{1 + sC_ZR_Z}}{1 + sC_Z\underbrace{R_Z//R_P}_{\sim R_P}}\; R_P/(R_Z+R_P)$$

(choosing $R_Z >> R_P$)

You cancel the pole of the preamplifier (e.g. constrained by noise requirements) to obtain another pole (e.g. constrained to be the first pole of the filter)

The first thing we have to do with a shaping amplifier is to introduce a zero in the origin to compensate the pole in the origin of the Heaviside step of the charge preamplifier. However, things are more complicated than just a zero in the origin and a pole in the origin, because the preamplifier may not have a pole in the origin.

If the preamplifier, in addition to the feedback capacitor, has a discharging resistor, the pole is not in the origin, it given by 1+(sCfRf). Generally speaking, with a passive discharge of the preamplifier, the pole is not in the origin (it is not 1/sCf).

We need to make a more general cancellation of the pole of the preamplifier when not in the origin. The first option is the pole zero network in red in the image.

So we have the preamplifier and the decay time in blue given by the time constant CfRf, then we have a CR network followed by a resistor. Then Vpz is a voltage, and then we let the shaper to add additional poles.
The t.f. of the red block is given by the formula in red. We have a zero that is given by CzRz and a pole given by Cz and the parallel between Rz and Rp → we cancel the pole of the preamplifier with the zero of the red network. Of course, the pole is cancelled if CfRf = CzRz. Typically, Rz is a potentiometer and we change the resistor value as long as the zero is perfectly cancelling the pole of the preamplifier.

The result of the cancellation is that we have a new pole, which is at the bottom. We can use this pole as first real pole of the shaper. So we have an additional pole that forces us to choose a shaper of the odd order (in case of c.c. poles).

So I have the real pole of the shaper that has to stay in a fixed position in the constellation. But the pole depends on the Rz//Rp, and if I change with a potentiometer Rz to compensate the pole of the preamplifier, I could change the pole of the shaper (not stable pole). We solve this problem by choosing Rp always smaller than Rz, so that Rp dominates the parallel and, whatever the value of Rz changed by the potentiometer, the parallel is fixed by Rp and the pole is stable.

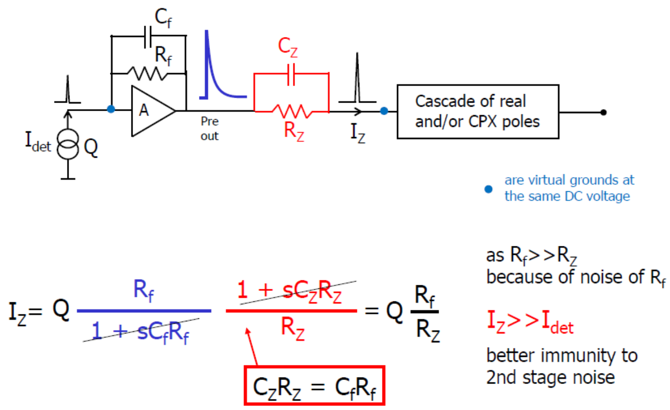
The second observation is that, apparently, I have cancelled the pole of the preamplifier and as a result I got another pole. Why not keeping the pole of the preamplifier as the real pole of the filter?
The answer is that, in most of the cases, the value of the pole of the preamplifier is not fitting the value of the pole of the shaper we need. So the pole of the shaper, which is lying in the constellation, is choose by freely choosing Cz and Rp. The main problem with Cf and Rf is that Rf in the preamplifier has to be large due to noise reasons.
Rf has a corresponding noise generator 4kT/Rf, and the larger Rf the smaller the noise. This generator is at the input and it is providing parallel noise. This generator dominates the noise if Rf is not sufficiently large → constrain on Rf, which in 99% of the case has to have a value not compatible with the real pole we need in the constellation.

So we need to decouple the two things; the pole of the preamplifier is good for noise and we cancel it, then we introduce the one for the constellation. This problem of noise doesn't apply for Rp, the thermal noise of Rp is not critical; a thermal noise generation is position of Rp is less critical then in the position of Rf, simply because the position of Rp is more ahead in the chain.

Example of POLE ZERO COMPENSATION
The network in red is now plugged between the output of the preamplifier and the virtual ground of the filter. This means that the node in blue is a virtual ground; it is a strong difference than the previous case. In the previous schematic, the node where we had Vpz was free to move, it is not a virtual ground. Hence, we can only compute the current Iz, that if for instance goes in the feedback of an integrator generates a real pole.

If we compute again the t.f. of the combination of preamplifier in blue and pole-zero network in red, we still have the zero (and we use it with a potentiometer to compensate for the preamplifier), but there is no pole. Hence the output current (the t.f. is between current at the input and current at the output) is also a delta current (as it was in input).

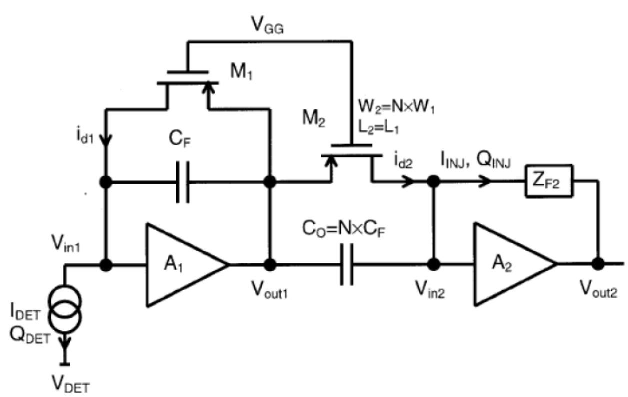
Moreover, the additional bonus for this pole-zero cancellation is that Iz is amplified with respect to the current Iq, by the factor Rf/Rz, and since Rf must be larger than Rz due to noise reasons, with this pole zero cancellation we get also an amplification of the signal, which is good, because **if we amplify the signal, we can neglect the second stage noise** → noise dominated by the charge preamplifier stage.



Implementation of the previous solution with CMOS (integrated circuits)

With IC we have the problem that very large resistors like Rf and Rz cannot be implemented, and we need to find an alternative.

An alternative is in the implementation below.



The virtual ground is the one of Vin2, and it must be at the same voltage as Vin1.

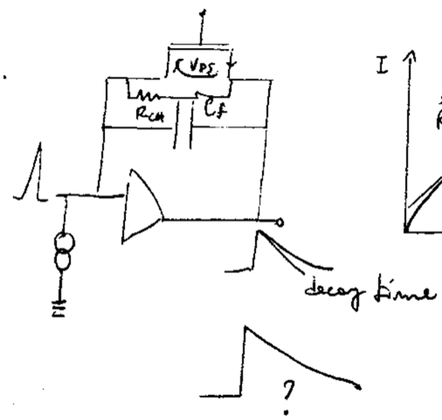
We have the capacitor Cf and the capacitor Cz (called Co here), that is a n-time replica of Cz. If Rf must be much larger than Rz and we have to respect that CfRf = CzRz, this means that Cz must be much larger than Cf.

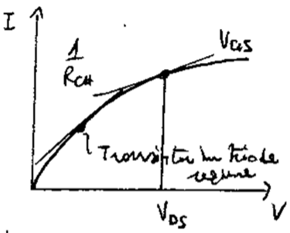
This means that the capacitor Co is larger by n time and **the resistor is obtained by means of two MOSFETs in triode regime**. A MOSFET in the triode regime can be indeed used as a large resistor, as large as we like by changing the gat voltage.

NB: we would like to have Rf>>Rz, and to reach this we can make the two transistor of the same channel length L and make the W of the second larger to increase the equivalent resistance.

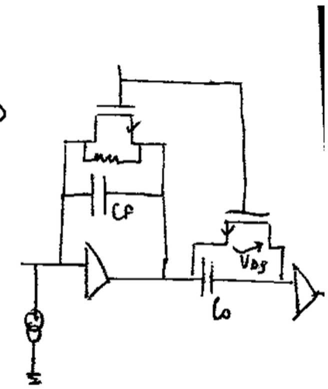
We are in the triode regime of the transistor, and in this regime the transistor is equivalent to a resistance R = 1/2kVos with k = 1/2CoxW/L. So if I make a W larger, I get a smaller resistance.

100.





$R_{CH} = \frac{1}{2KV_{DS}} \Rightarrow$



In conclusion, in the pole zero network I have Co that is n time the Cf, and I have a transistor that is n time larger than the corresponding transistor in the feedback network.
The two resistors are biased with the same Vgs, so they are identical except for the W → in this way the two resistors are one n time the other.

But implementing a resistor with a MOSFET in triode is not like to have a constant value resistor, because if we change the Vds across the transistor, we change the resistance. But in the charge preamplifier, when we have a charge, we have a pulse at the output of the amplifier, and this pulse changes the Vds of the transistor that implements Rf. So the transistor is going to change its resistance during the application of the signal → this also means that the decay time is not constant, but it depends on the value of the output voltage. For instance, at the beginning when we have the step we have a large Vds and the resistance is large. So we start with a large resistance and then it is not constant → with a change of the output voltage of the preamplifier we cannot trust the resistance to be constant.

However, this is not a problem in our pole zero cancellation because also the other mosfet representing Rz, since it is sharing the same Vgs, shares the same Vd, the same Vs (Vin1 = Vin2) and so the Vds voltages of the two transistors are the same. We have two devices with identical extreme voltages and the same common voltage.
Since we have the golden rule that CfRf = CzRz, the two resistors Rf and Rz change with the same behaviour and so the equality is still conserved.

So this solution is working because the two MOSFETs share the same behaviour.

Summary of the network

- based on the principle of the last mentioned pole-zero compensation
- V_{DS} equal for M_1 and M_2 also under signal application

(if virtual grounds of A_1 and A_2 have equal DC)
⇒ automatic compensation also if the resistance associated to M_1 and M_2 is not linear!

- amplification of the current signal and $I_{leakage}$ at the input of A_2 equal to N (N^2 for the noise)

⇒ noise from the II stage (from A_2 on) negligible

- further stages of the same type can be used to increase the amplification of the system
- injection of a delta-like signal in A_2 (first stage of the shaper)

useful for the realization of a shaper with current input, which is compatible with any type of poles (real/cpx)

- in the layout of the circuit, pay attention to implement precise replicas of R_F and C_F, including parasitics (otherwise P-Z cancellation is not perfect)

Another advantage is that in the following stage we can implement even or odd filters, we are not constrained by the real pole.
Moreover, **the fact that Co is n time Cf and Rz is n time Rf must be checked carefully at the layout level**. If this is not true, the pole zero compensation is gone.
This can happen if for instance we decide that we have Cf and we make Co simply by creating a capacitor of n time larger value. This is not good because a capacitor value, in a real layout, doesn't depend just on the ideal 'parallel plates' design, but the layout may have some parasitic capacitances and so Cf may have a parasitic component Cp that, if we design Co = nCf, may not scale.

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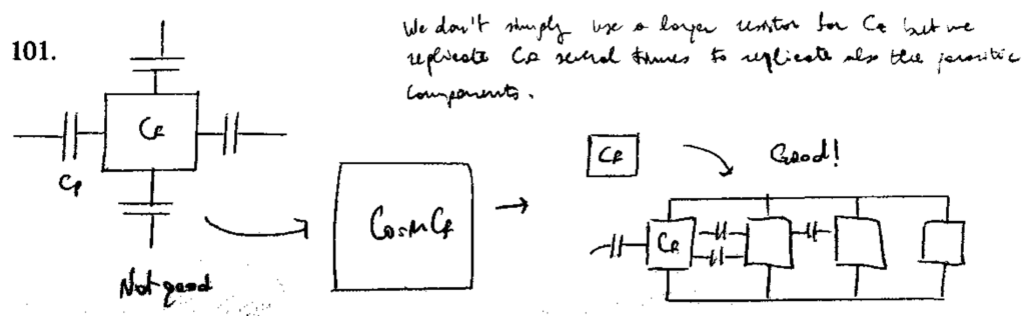
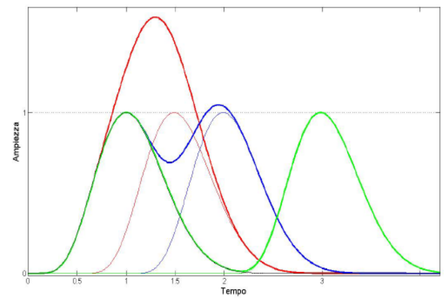


What we have to do is that we draw the layout of Cf, and we make copy and paste of the same layout n times and we connect them together. In this way we replicate exactly also the parasitic components, because the layouts are identical and when we multiply we multiply also the parasitic components.
The same applies for the transistors. The transistors should not be just a transistor with a W larger, we should take the smaller layout and replicate it n times, otherwise we won't have the pole zero cancellation.

PILE-UP OCCURRENCE BETWEEN NEIGHBOUR SIGNALS

Once a shaping time has been chosen, we may have pile up.
Having pile up means that, for instance, if the two pulses in green are well separated, the peaks (supposed to be identical) are not corrupted. But if we suppose to have two waveforms like the first

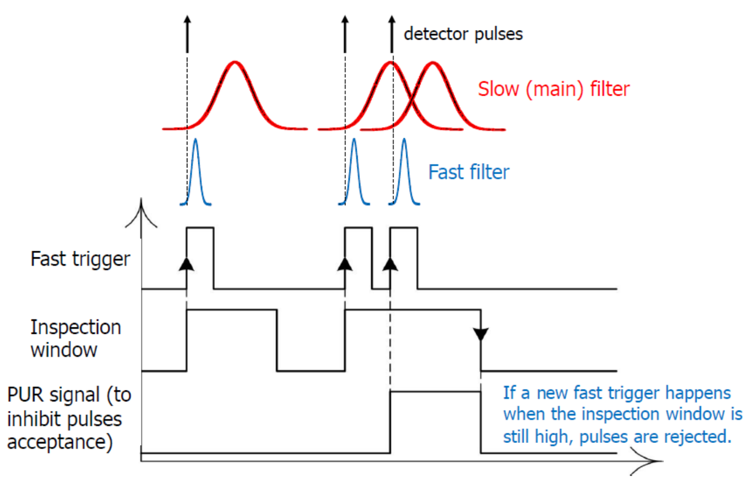
one in green and the one in blue that are close each other, the tail of the green pulse is superposed to the rising of the blue one. With the red the situation is even worse.
We may reduce pile up by squeezing the pulse (reducing the shaping time), but this is provided at the expense of the electronic noise.



• Nearby events produce superposed signals that can corrupt the peak amplitude of the corresponding pulses.
• The corrupted signals have to be identified and rejected by a suitable circuit (Pile-up Rejector or PUR).
• The rejection of the corrupted signals decreases the counting rate at the output of the system for a given counting rate at the input

In any case, once we have found a trade off between noise and pile up occurrence, pile up will occur anyway. It could occur 10% of the cases instead of 20%, but in any case we will have it → are necessary techniques to reject pile up. In fact, it is better to reject piled up images than having them in our detection.

Technique to reject pile up: the PILE-UP REJECTOR



It is an example of a popular one. We have the deltas representing the delta pulses; the red signals represent the slow filter (chosen for the best noise). Unfortunately, two pulses are overlapped and we want to discard at least one of them (usually the second one is corrupted and the first one can be saved) and we can perform, in our electronic, a parallel channel with a fast shaper. A fast shaper means a shaper with much shorter shaping time. Then we put a trigger on this shaper, so a discriminator to get fast digital signals, and then, when a fast digital signal arrives, we open an inspection window as wide as theory tells us. The wider, the more conservative.
The scope of the window is to see if a following faster trigger falls within the inspection window.

For instance, in the first inspection window no other pulse arrives, so everything is fine. But in the second case, the first pulse opens an inspection window and, before the inspection window is terminated, a second fast pulse arrives.
This is unacceptable, and so on the arrival of the second pulse we start a pile up rejection (PUR) signal that simply inhibits the data acquisition, avoiding the recording of the second pulse.

We need a fast filter because the fast filter is not used for careful measurements, but to generate the fast trigger; the faster is the trigger, the more peaks I detect, because the shorter is the shaping time of the fast filter, less probability of pile up in the fast filter itself I get.

However, I cannot take the peaks of these fast filters as measurement of the pulse because they are noisy. The fast filters are good to detect pulses, but in terms of noise vs shaping time they are suboptimal, they don't have the right shaping time to measure carefully the amplitude.

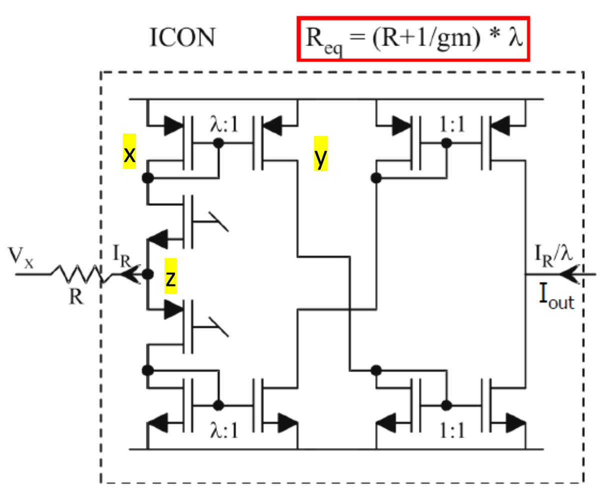
So we need the shaper in red to make a careful measurement of the amplitude, and the shaper in blue to detect pile ups.

RESISTANCE MULTIPLIER BY CURRENT DE-MULTIPLICATION

It is a **new technique devoted to the implementation of large time constants in IC circuits**. In IC it is difficult to implement filters when resistors are large. We cannot use the switched capacitor technique like in the pacemaker because we have indeed the same problem, but the domains of application are different, in the sense that in the pacemaker we need bandwidth of hundreds of Hz, very low bandwidth, while here we need filters with bandwidth up to MHz. Switched capacitor would hardly work transposed to such frequencies, so it cannot be used.

We cannot neither use the MOSFET in triode because it is unreliable, their value is not trustable, we cannot use them to create a very precise pole in the constellation.

One simple technique is to use a resistor together with a current mirror used as a demultiplier of the current of a resistor. We have a network like below.



(R.L.Chase, et.al, NUCLEAR INSTRUMENTS AND METHODS 409, 1998)

Principle:
the current flowing in R is demultiplied by a factor λ

An 'equivalent' resistance
$R_{eq} = V_X/I_{out} = R \cdot \lambda$
is obtained

Signal enters into N or P mirror depending on the direction

Second stage may operate also a demultiplication λ so reaching an overall amplification of R by λ^2!!

It is a stage similar to the one in the rectifier of the cochlear implant.
We have to branches, top and bottom, that are selected according to the polarity. There is a cascode taking the current and a mirror changing the sign of the current. This is identical, the difference is that the current is given by a resistor, we apply to the resistor a voltage Vx and we have a current Ir given, at first approx., by Vx/R, due to the low impedance of the transistor.
This current can entering either in the lower branch or upper branch depending on the sign of Vx.

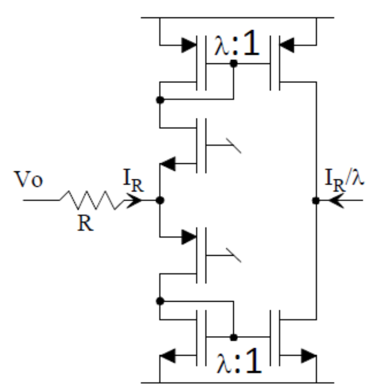
If it is flowing in the upper branch, now differently than in the cochlear implant, we have a demultiplication mirror. The mirror has a mirroring factor lambda:1, which means that the transistor x is lambda time larger than the transistor y.
So we demultiply the current, the current passes through the other mirror in the bottom right and then it is provided at the exit of the stage.

In conclusion, the value of the current at the output of the stage is Ir divided by lambda. If we consider this network as a 'black box' (called ICON) and try to find the equivalent t.f. of it, the corresponding t.f. is an Ohm law, it is current given by voltage divided by an equivalent resistance. The value of the equivalent resistance is the resistance R (because Ir = Vx/R) multiplied by lambda, the mirroring factor. This big bipole is hence a multiplier of Ohm law.

This is the large resistor I need, obtained by a small physical resistor R and the lambda factor, that is the mirroring factor (and the mirror is a very small circuit on Si). We have obtained a very large R with very low area occupation.

Details
Unfortunately, the resistor is not only R, but we must consider the 1/gm of the node z. if we consider it, the current Ir is not Vx/R, but Vx/(R+1/gm) → also the total resistance varies. This is not good because it means that the equivalent resistance depends on a transistor parameter that is, for instance, signal dependent. In fact, if Ir changes, also 1/gm changes.

Moreover, the second observation is: why do we need a second stage? Couldn't we simply exit at the output of the first mirror? The difference is in the direction of the current. In the image below we have in input an entering current and in output an exiting current → the t.f. will be a negative resistor.
If we don't want to have a negative resistor, we add an additional mirror and everything is restored, the output current has the same direction of the input current (ICON configuration).



So the secondary mirror is used to restore the sign, but we can make a better use of it. If in this mirror with a mirroring factor 1:1 we add another mirroring factor lambda:1, we demultiply the current by lambda^2. This means that we have lambda^2 in the formula of the equivalent resistance, so we could amplify even more the resistance.

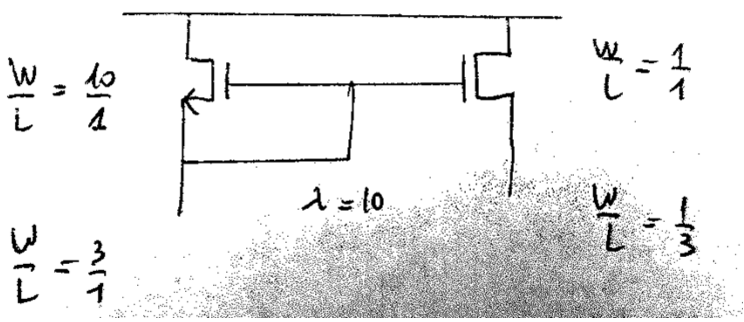
A last detail – implementation of lambda
The nice feature of the solution is that lambda is obtained at the cost of very small space on Si because transistor occupy less space than resistors. But is this true?

Let's suppose I need to implement a lambda = 10. I could make the transistor 10/1, which means W/L = 10u/1u, and the transistor on the right is W/L = 1/1, so very small space.

We can even do better, because what matters is the mirroring factor, so we can also do something like: W/L = 3/1 and W/L = 1/3 in the other. In fact, the mirroring factor is the ration between the W/L of the first transistor (left) divided by the other W/L of the other transistor.

In this way the first transistor in the first option is 10um^2, and the second 1um^2, so the overall area is 11um^2. In the second solution we have 3um^2 for both, and the overall area decreases, even if with the same lambda in the icon.

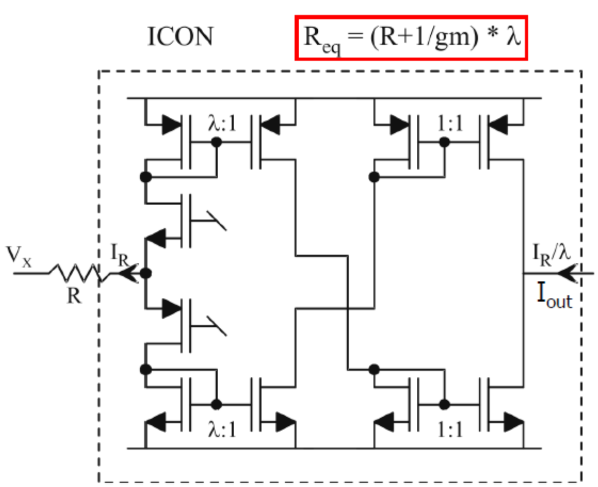
102.



So the ICON is a technique to multiply the value of a resistor in CMOS technology with the goal of reaching a larger resistance starting from a small physical resistor.

Reducing the effect of the 1/gm

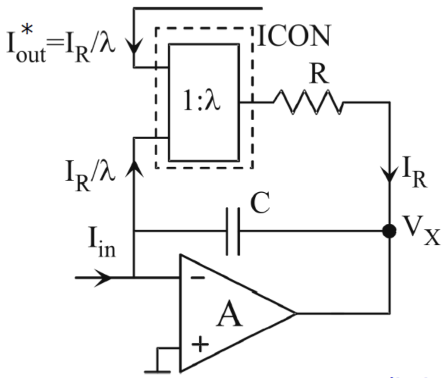
To reduce the problem of having a resistor depending on the 1/gm, we can establish a bias current in the branch, and this is done by providing suitable voltages to the gates (that in the drawing are grounded but in reality they are not grounded).
If we want the 1/gm not being dependent on the signal, we need to bias the branch with a current that must be dominating with respect to the signal current. In this way the 1/gm is established by the bias and not by the amount of signal.



However, the larger the current in the branch, the higher the noise.

IMPLEMENTATION OF A REAL POLE WITH THE ICON CELL

A real pole can be implemented with a R and a C in feedback of an amplifier; however, the problem is that in CMOS technology we cannot implement a physically large value of the resistor.
Hence we use the same topology (amplifier + feedback) but with an ICON.



$$\boxed{\tau = C\cdot(R+1/gm)\cdot\lambda}$$

$$\frac{V_X}{I_{IN}} = \frac{(R+1/gm)\cdot\lambda}{1+s\cdot C\cdot(R+1/gm)\cdot\lambda}$$

$$\approx \frac{R\cdot\lambda}{1+s\cdot C\cdot R\cdot\lambda}$$

* The secondary output I_{out} is obtained by duplicating the output MOSFETs of the ICON

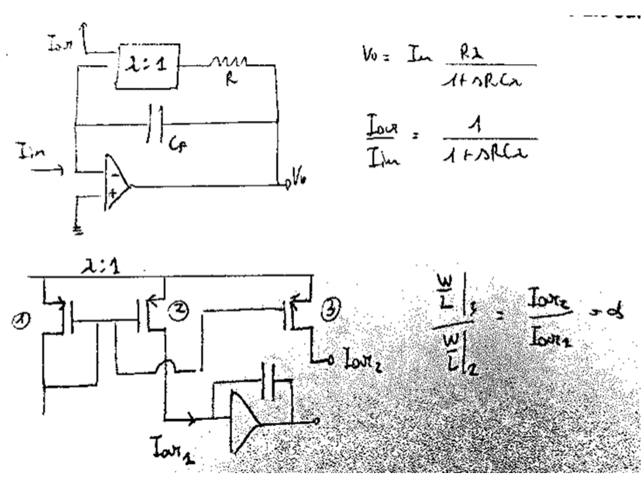
limit: dependence from gm

We have the capacitor in feedback, then the resistor is not directly closed to the virtual ground, but it is connected to the input of the ICON. The current Ir/lambda is connected to the capacitor C.
Hence the integrator, instead of realizing a pole R*C, is realizing a pole C*R*lambda.

The goal of the resistor R in feedback is to generate a current Ir that is fed into the capacitor and discharges the capacitor. Instead of feeding the capacitor with a current Ir, we feed it with a current lambda time smaller, and so the result is equivalent to a resistor lambda-time larger.

In terms of t.f., if we look the t.f. output voltage vs input current, we obtain a t.f. with a constant term at the numerator and a pole at the denominator.
If we are able to neglect the 1/gm, for instance because we use a R larger than 1/gm, then we have a nice pole in which 1/gm can be neglected.

This is the result if we want the output as a voltage. But if we want for instance to build a cascade, a chain of real poles, or c.c. poles, it is better to use a current output, so that it can be easily fed in the consecutive cells.
With this configuration we have a big opportunity, and the opportunity is to duplicate the current output with just two additional transistors. We have not only the output current Ir/lambda going in the capacitor, but we create a replica and then this current Iout* is the output.



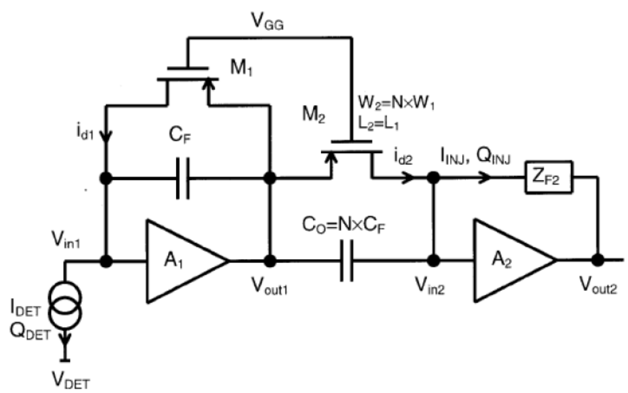
103.

We have R and the lambda of the ICON. if we take Vo and divide by R*lambda, we get the other t.f.. This tells us that the overall block implements a current mode filter; the input is in current, the output is in current and we have added a pole.
The potential of this current mode filter is that the Iout can be fed into a next identical block. We do a sort of copy and paste of the same structure.

If we look inside the ICON, the secondary output is obtained by taking a replica of the second transistor of the mirror. The secondary output costs two transistors, one per branch, more.

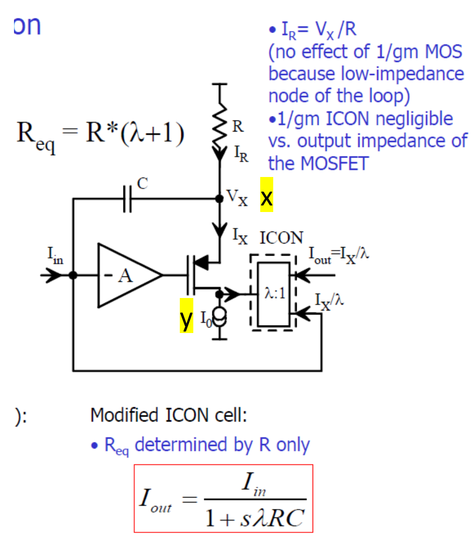
We can do even more. We are not forced to have transistor 3 equal to transistor 2, because the value of 2 is important because it is determining with transistor 1 the lambda factor, but the transistor 3 is in an open loop path, it is simply exiting from the structure, so the W/L of this transistor can be whatever. So the W/L of transistor 3 divided by the one of the transistor 2 gives to us a ratio Iout2/Iout1 → we have an amplification factor. The secondary factor has the same t.f. of Iout1/Iin, but with an alpha factor at the numerator, that is the ratio between the two currents.
So not only we can put in series a cascade of several of this blocks in current mode, but we can even amplify stage after stage the current by the alpha factor, simply at the cost of two more transistors.

If we come back to the pole-zero network with the two MOSFETs in triode regime, this solution was based on having a virtual ground at Vin2. The cell Zf2 is the one we have just seen, ICON + R.



IMPROVED ICON SOLUTION

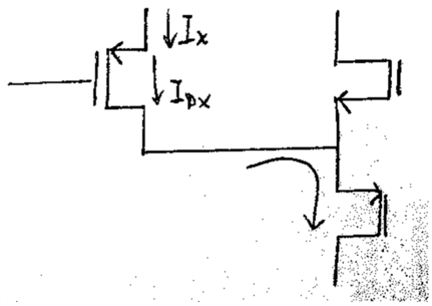
We want to solve the problem of 1/gm.
The solution is based on the main loop with amplifier, but the capacitor, connected at the virtual ground, is not directly connected at the output, but there is in between a pMOS in source follower. In terms of loop, for C it is equivalent as before; instead of having the amplifier directly connected to C, we have an amplifier, a follower (gain 1) and then the connection.



However, now we have the resistor R, and this is the resistor where we make the transformation V/R = Ir.
R is not in series with the 1/gm.

The generator I0 is essential to bias the network where we have R and the pMOS, otherwise the bias of the transistor should be a DC current going into the ICON.

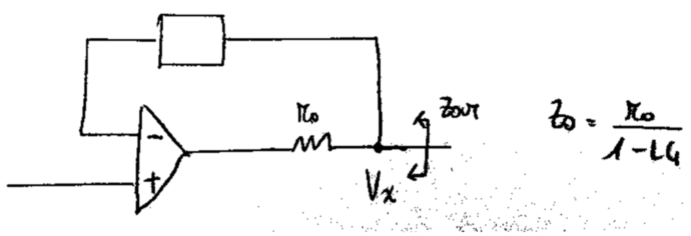
105.



In principle I could have connected the node out of the drain to the ICON, but since the pMOS need to be DC bias, the current must be provided externally, otherwise we impose to have a DC current at the input of the ICON.
But if we use a current generator I0, this generator provides the DC current, and we have no DC current in the ICON (NB: the current is imposed through the loop, not directly by the generator I0).

Now the resistor R is not at the entrance of an ICON, the R is simply connected to ground.
So Ir is simply Vx/R, and there is not the 1/gm of the pMOS, because node x is the node of a negative feedback loop, and the impedance at the output of a feedback loop is equal to the impedance in OL divided by the LG.

104.



If we have a feedback amplifier, for instance with an output impedance ro and a feedback, the Zout at the output node of a feedback amplifier is Zout = Zol/(1-LG). One of the big benefit of the LG is to kill the output impedance.
This means that if we have an infinite LG, the node at the output is a pure voltage node. That node, that in our case is Vx, is supplied at zero impedance.

In our specific case, the output impedance in node x (that in the drawing is ro) is the 1/gm of the pMOS (it is R || 1/gm, so basically 1/gm). But the node x is put to 0 impedance by means of the LG. So I can truly say that Ir is Vx/R, no 1/gm around.

Moreover, where is Ir going?
It is flowing in the pMOS and exiting the pMOS at the drain and it is entering in the ICON cell, and then the loop is closed to the capacitor. However, the current Ix is fed into the ICON not through the resistor, but through the drain of the pMOS. If we are at node y and we compute the Norton

equivalent, the Norton equivalent is the high impedance of the transistor, we don't see any more the resistor R. so the current Ix is entering in the ICON at high impedance, while before it was entering with R. Here we have a high impedance (in principle infinite) against 1/gm; it is like if we are feeding the current Ix into the ICON with an ideal current generator, so I don't care about the 1/gm.
→ The current Ix will be perfectly fed to the ICON and then demultiplied.

The conclusion is that Iout is equal to: $\boxed{I_{out} = \frac{I_{in}}{1 + s\lambda RC}}$

This result is rigorous, we are not neglecting anything (like the 1/gm) → no more the problem of 1/gm.
Moreover, in this loop, with respect to the previous one, we have an inversion, and so we have the version of the ICON cell with a negative resistance in order not to have a positive loop.

Precise calculation

This formula x includes the fact that the loop is not ideal, which means that the amplification A is not infinite. If A is not infinite, we have a t.f. with a zero and a more complicated pole.
But all the terms alfa, beta and gamma can be neglected if A→inf and the formula can be simplified.

x $$I_{out} = I_{in} \frac{1 + sCR(1+\alpha)}{1 + \beta + sCR\left[(\lambda+1)(1+\alpha) + \gamma\right]}$$

precise calculation

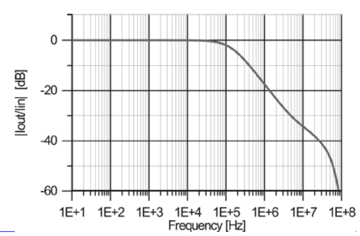
$$\alpha = 1/A \qquad \beta = \frac{\lambda R}{AR_o} + \frac{\lambda}{Ag_{mPMOS}R_o}$$

$$\gamma = \frac{\lambda}{Ag_{mPMOS}R_o} + \frac{\lambda}{Ag_{mPMOS}R}$$

$$A \to \infty$$

$$\boxed{I_{out} = I_{in} \frac{1 + sCR}{1 + sCR(\lambda + 1)}}$$

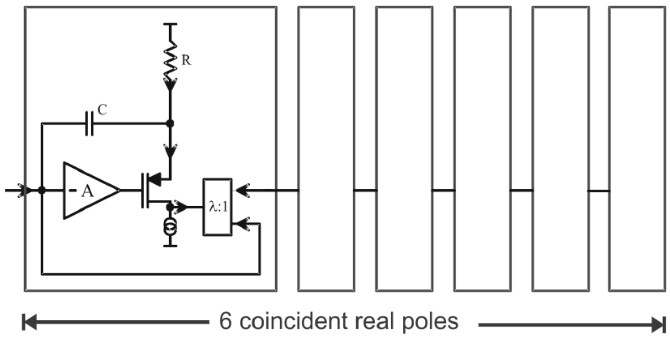
the zero is λ times faster than the pole
(be aware for small λ!)



There is a peculiarity. This particular implementation of the pole doesn't include only a pole, but also a zero C*R.
This additional zero is not a problem because the zero is lambda time faster than the pole, at a higher frequency. Since in this filter we are interested into poles, having a zero at high frequencies is not a problem.
The only situation where we have to remember the presence of the zero is when lambda is small. If so, we have a couple of pole and zero that are almost at the same frequency (but having a lambda small is not a realistic situation, otherwise we won't have used the ICON).

Moreover, this t.f. doesn't take into account the bandwidth of the amplifier (which has a pole) and the pole of the mirror. In fact, the bandwidth of the mirror is up to the gate capacitance times 1/gm of the mirror. But all these poles are at high frequencies → can be ignored.

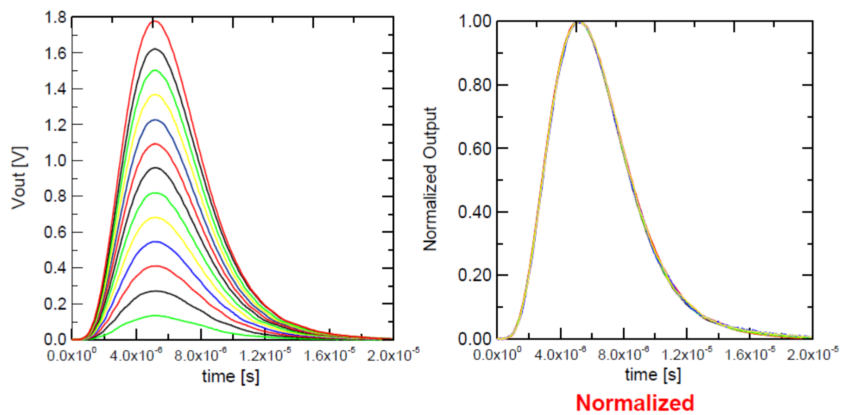
IMPLEMENTATION OF SEMIGAUSSIAN FILTER WITH REAL POLES



Note: very easy 'copy-and-paste' of the cell layout, in the design of an integrated filter.

We see how to make a semigaussian shaping amplifier of the 6^{th} order with real poles. We take one ICON cell and the output of the cell is fed into a completely identical cell, and so on. We can use also the cell with the Vx in output, simply we don't have the benefit of the 1/gm suppression.

FILTER SHAPE STABILITY VS INPUT SIGNAL AMPLITUDE



This is a measurement of the semigaussian shaper of the 6^{th} order with different input signals. If we take the plots on the left and we normalize with respect to the peak amplitude (we take the waveform and divide by the peak amplitude), we obtain the normalized waveforms on the right, that are all perfectly superposed.

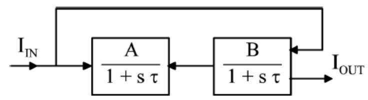
The fact that the waveforms are perfectly superposed means that the shape of the filter is perfectly signal independent. If the poles of the filter would have depended on the 1/gm, by changing the amplitudes the 1/gm would have changed and the filter response should be different. **The fact that we see perfectly superposed waveforms means that the 1/gm dependency has completely gone**.

IMPLEMENTATION OF C.C. POLES BY A CASCADE OF TWO REAL POLES

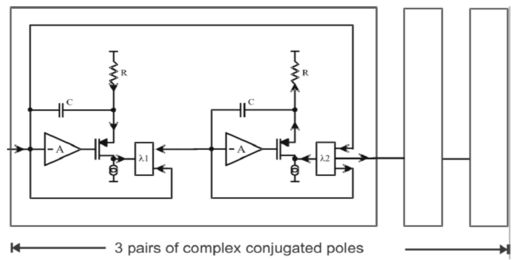
The improved ICON solution can be used also to implement shaping amplifiers with c.c. poles. If we look at the schematic on top right, we see that the schematic is composed by a cascade of two real poles of the type we have already implemented. If we realize this cascade and we close furthermore the output to the input, we create a second loop and we can calculate easily that the overall network

Iout/Iin implements a polynomial of the second order, which was the building block to realize c.c. poles in the constellation of the semigaussian shaping amplifier.

Implementation of conjugate complex poles by a cascade of two real poles



$$I_{out} = I_{in} \frac{AB}{(1+s\tau)^2 + AB}$$



Hence we can implement a second order polynomial with ICON cells, not only with Sallen key cells. So we take two ICONs with real poles and then we close the output to the input. The output of the second cell is a third replica. In fact, in the first ICON we have a local output and a general output, and in the second cell we have the same, but in addition we have a third replica to of the output fed back to the initial input (two transistors more). The tau in the formula is the tau of each single cell, so C*R*lambda.

The coefficients A and B comes from the secondary output, that may have different W/L. A and B are the alpha factors. A is the alpha factor of the first cell, B the alpha factor of the second cell.

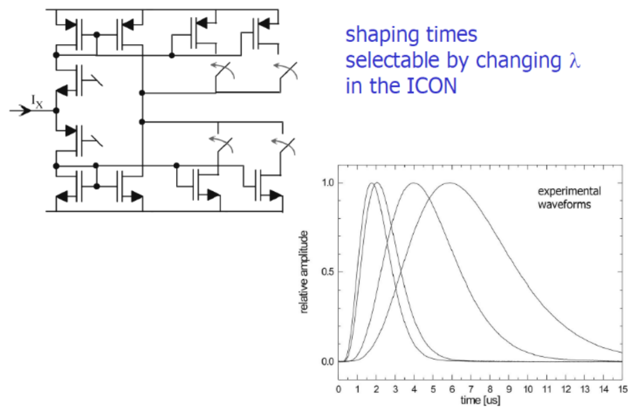
In this case, the implementation in the bottom is again a filter of the 6th order implemented with 3 pairs of c.c. poles, each one given by the formula in the red box. The difference with real poles where we make copy and paste of the same cell is that here we can't make the copy and paste, the 3 circuits are different because the positions in the Gauss plane is different, the values of the parameters in the circuits must be reshaped.

SELECTABLE SHAPING TIME

When we use this kind of filters, we may have benefits in the possibility of changing the shaping time. We need to change the shaping time for instance when we need to minimize the plot, when we need to fit the minimum of the ENC plot by tuning carefully the shaping time of the shaper.
However, CMOS technology is a bit rigid; if we use discrete components, we may use potentiometers to change the shaping time, but in CMOS the flexibility is limited.

To change the shaping time, at least to discrete value (among a set of possible values), is possible because the ICON cell provide the possibility of reprogramming the chip.
The tool to change the shaping time is the lambda factor, because it is the multiplier of the resistance, and if we change it, we change the filter poles.
But the lambda factor is the mirroring factor. So we can change the mirroring factor by putting more or less transistors in parallel in the secondary branch of the mirror. The W/L of the first transistor is

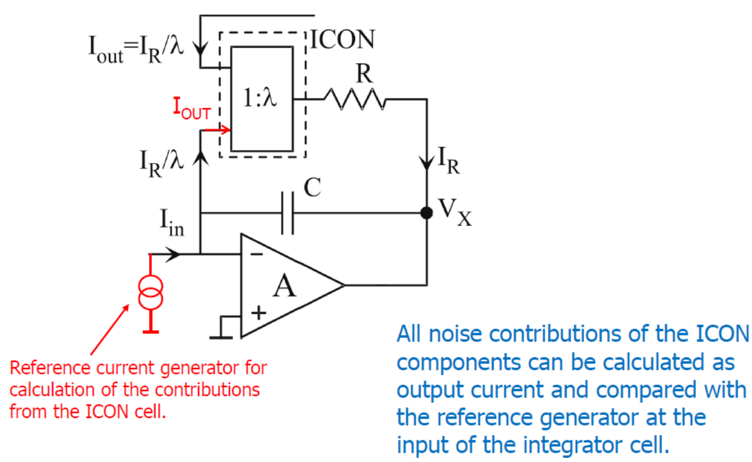
fixed, we change the one of the second (remember that the mirroring factor is given by the ratio between the two W/L).



The more secondary transistors we put in parallel, since the mirroring factor is lambda:1, the lambda is decreasing, so the more transistor we close, the shorter the shaping time, because the lambda gets smaller.
If we have the transistor in the left 10:1 and the transistor on the right is 1:1 and then we plug more transistors together, we have for instance 10:1 against 3:1, so we are reducing lambda.

This means that, depending on the shaping time, we need to choose the largest lambda corresponding to the longer shaping time we need, and then simply the more transistor we include, the shorter the shaping time.

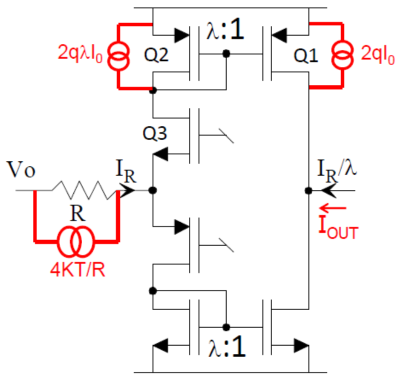
ELECTRONICS NOISE CONTRIBUTIONS OF THE ICON



Suppose that we have the real pole filter in the image; which are the noise contributions?
First of all, as in all noise analysis, I need to elect a reference point, I need to establish a reference parameter, and then all the noise contributions will be referred to this parameter.
In the real pole cell, a good equivalent generator to which refer all the noise is the red current generator at the input. It is already at the position of the signal, the signal is in the position of this equivalent noise generator, so once I have established the equivalent noise generator, is very easy to calculate the SNR.

Once the position of the noise to which I compare all the generators has been established, which is then the reference quantity I have to compute for the ICON noise? The output current.
This because the output current of the ICON cell is already in the right position to be comparable with the input current noise generator.

Now I take an ICON cell (in this case the simplified one) and I take all the transistors and transfer their noise generators to the output current Iout.



Q1: its noise is already at the output: $2qI_0$

Q2: its current is λ larger but its noise is demultiplied by λ^2 at the output: $2qI_0/\lambda$

Q3: its noise is cancelled, being a cascode

R: its noise generator is demultiplied by λ^2 at the output: $4KT/R \cdot 1/\lambda^2 = 4KT/(\lambda R) \cdot 1/\lambda = 4KT/(R_{eq}) \cdot 1/\lambda$

Notes: 1) all thermal noise generators have been approximated as shot noise, as MOSFETs are in weak inversion; 2) same calculation for the bottom stage.

The resistor noise is lower by a factor λ vs. the noise of the equivalent resistor $R_{eq}=\lambda R$!! ($\Rightarrow$ 'cold resistor')

Before doing so, we must say that all the transistors work in weak inversion, not in strong inversion, and it is a characteristic of low-power blocks. The electrical characteristic of transistors in weak inversion is more similar to BJT than to FET.
This analogy between MOSFET in weak inversion and BJT is underlined to introduce a further evidence that the thermal noise of the channel of the MOSFET that we used to consider as 4kT*(1/gm) is, in reality, better approximated by the shot noise → the noise generator in the channel of the transistor will be represented better with a shot noise formula than the classical formula.

Once this has been set, let's analyze transistor by transistor.

- Q1: transistor directly at the output of the ICON, and the noise is 2qI0. The contribution of this transistor at the output node is already the 2qI0 itself, is already at the output. So if I want to reduce this noise, I have to reduce the current I0. So <u>the transistors are biased in weak inversion because I want to reduce their noise</u>.
- Q2: the noise of Q2 Is 2q*lambda*I0. This doesn't necessarily means that the noise of Q2 is dominating with respect to Q1, because we have to transfer the generator at the output. The transfer factor between this generator and the output is 1/lambda^2. So if I take the generator that is lambda time larger but I divide by lambda^2 it is smaller by a factor lambda. The good news is that Q2 is negligible with respect to Q1.
- Q3: it is a cascode, and cascode is practically a zero-noise configuration, because the cascode recirculates its noise.

Then we have also to remember that we have the transistors in the lower branch that add their noises. In conclusion, the two main contributions of noise of the ICON cells are Q1 and the corresponding transistor on the bottom. To minimize this transistor noise we bias the transistor at a very low current.

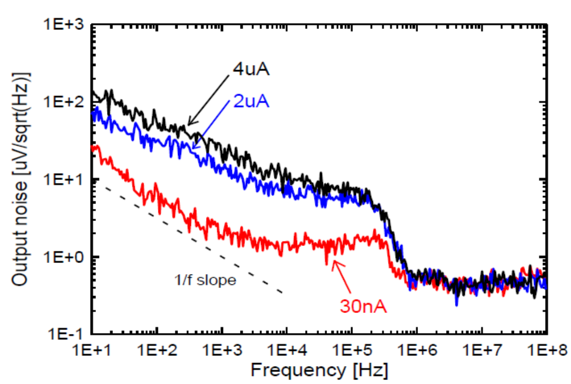
The consequence of the biasing of the transistor at low current is that also the input branch (the one on the left) is biased to low current. We are also reducing the current on the left branch, and the consequence is that the 1/gm matters.

In fact, the quickest way to neglect 1/gm was to bias the left branch to such a high current that the 1/gm is negligible, but we cannot do it because if we increase the current we increase also the current at the output and so we increase the noise. This is why the improved ICON configuration is important, because it allows to neglect the 1/gm without increasing the current.

As for the resistor, we must consider its thermal noise, that is 4kT/R. If we transfer this noise at the output, we divide by lambda^2.
Then I make an algebraic manipulation: one lambda is left to multiply R, and the other is brought outside. In the final formula for the resistor, the noise is 4kT/Req * 1/lambda → <u>the ICON solution has also provided a noise reduction of the resistor Req, it makes the resistor less noisy</u>.
This technique of the ICON can also be used to reduce the thermal noise of a resistor → cold resistor technique.

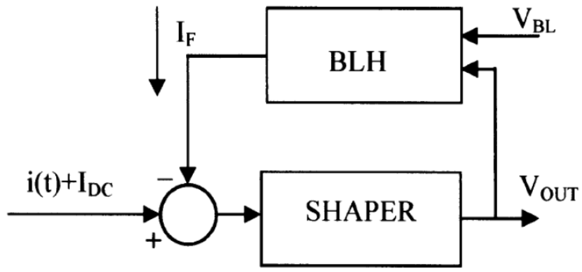
In conclusion, the noise of the ICON is dominated by the output MOSFETs. This noise can be reduced by reducing the current. This has the consequence of reducing the current in the input branch and therefore increasing the 1/gm.

The one below is the output plot of a shaping amplifier, and we see the noise depending on the biasing of the ICON cell. By reducing the current in the ICON from 4uA to 30nA we reduce a lot the noise of the ICON.



Measured noise at the output of a filter vs. different values of the current used in the ICON. Note that 1/f noise of the MOSFETs is also present, in addition to the white noise.

THE BASELINE HOLDER



It stabilizes the DC level of the baseline at the output of the shaper to V_{BL}

Auxiliary circuit operated in parallel with the shaping amplifier. The main goal of the circuit is to stabilize the DC level of the baseline of the shaping amplifier to a desired value Vbl, that may be typically ground or another value, for instance to match the dynamic range of the ADC.

In fact, the first thing that may go wrong is that the shaper, since it is a quite complex circuit with many stages that are DC connected, if the DC value is out of control, it is better to introduce an overall setting of the output baseline, otherwise among all the channels, each channel exhibits a random value of DC voltage.
The second reason for the baseline holder is that the shaper is fed by a current i(t) but in addition to i(t) that is the signal, we may have a steady DC level of current which internally comes from the detector. The detector provides a dark current that flows across the charge preamplifier, then the pole-zero network and then the shaping amplifier → DC path for the current from the detector.
If the detector changes this current, the DC current changes over time and if the shaping amplifier is locked to this current, while we change the temperature of the detector we have a shift in the shaping amplifier, and it is not nice to have a DC voltage floating.

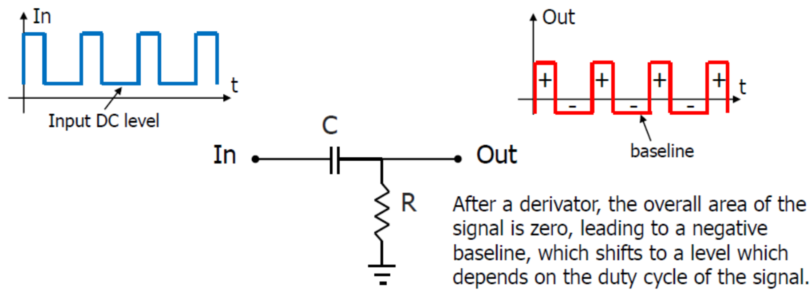
Another good reason is that there is also the possibility that the shaping amplifier has some AC coupling stages (with a capacitor block after block). In this case, when we have a train of pulses through an AC coupling stage, we have an undesirable effect, that is a baseline shift because of area compensation. This is not desirable, because the relative amplitude is kept but its absolute value is changed.

Goals of the BLH

To solve these problems we introduce the BLC block, a circuit that senses the DC output of the amplifier, compares it with a desired baseline and provides in output a current. If that is subtracted to the shaper to compensate the DC current coming from the previous stages or that can be fed to the shaping amplifier. It changes as long as the difference between Vout and Vbl is null.
Of course, the loop must be active only at low frequencies, where we want to compensate the DC; if operational at the frequencies of the gaussian pulse it would compensate also that, not acceptable → the system will have a low pass t.f. with a low frequency pole.

At the end, the circuit is equivalent to an AC coupling. If we want to break the DC level in a point of a circuit, in fact, we place a capacitor and make an AC coupling.

If we don’t like the DC value of the Vin, we use a derivator like in the center of the image, that at low frequencies has the capacitor open and the output controlled by R. In the image below R there is ground, but we are also free to put Vbl to fix the DC level of the output. At HF, where we have the gaussian pulse, the C is a shortcircuit. So apparently AC coupling is the quickest way to solve our problem.



$$V_{out}(s)= V_{in}(s)\,\frac{sCR}{1 + sCR}$$

$$\int V_{out}(t)dt = \lim_{s\to 0} \cancel{s}\cdot \frac{V_{out}(s)}{\cancel{s}} \to 0$$

Because of the zero in the origin from the derivator.
⇒ The signal area is zero

However, all AC couplings have a basic problem. If we suppose to feed in it a waveform like the blue one and we suppose that the DC level is not zero; after the derivator the signal is the red one. The signal response has been conserved thanks to the capacitor (same amplitude), but the problem is that after the AC coupling the area must be zero → we have introduced a negative baseline, shifting the signal. This baseline value depends on the frequencies of the pulses. If the pulses are very frequent, due to the matching of the area, the negative value will go down. If on the contrary we have one pulse every hour, the baseline will be very tiny because a tiny area integrated for one hour provides sufficient negative area to match the positive one.

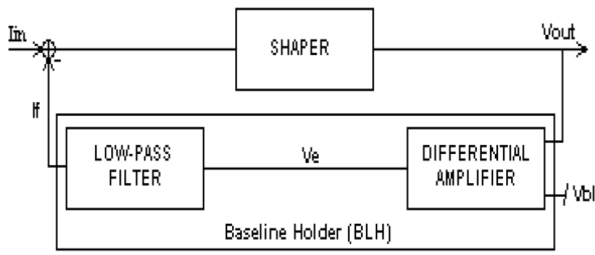
If the next circuitry is sensitive to the peak of the pulse and not to the amplitude (like an ADC), despite the pulses have the same relative amplitude, the top value can be different, and we have a problem. If the baseline shifts down, the peak will reach a lower value.
This is the reason why the very popular AC coupling has to be held carefully, because if we have 1 pulse every century is not a problem but if every microsecond it is a problem. Moreover, if we have random pulses, the baseline shifts every time, doesn’t remain constant like if the signal was a square wave.
The derivator imposes a total area equal to zero because it has a zero in the origin → **the area of the pulses after the derivator must be zero although the starting signal has no zero area**.

BASIC VERSION OF THE BLH

It is composed by a differential amplifier that senses the difference between output voltage and desired baseline level. Then it creates by a simple transistor (common source transistor) a current If that is sent to a subtracting node at the input of the shaper. So the input of the shaper ahs a differential node where the input of the shaper is the difference between the input signal and the feedback current.

In principle I should simply have the differential amplifier and the transistor for the voltage to current conversion, but we need to include a LP filter, because the loop must be broken in the frequency range of the signal, otherwise also If would be a perfect replica of the gaussian pulse, and would perfectly subtract the gaussian pulse → the circuit must be effective at LF, but the loop must be open at HF.

We are talking about low frequencies and not DC because I don't want to restrict the action of the circuit at 0Hz so that the circuit is effective also when we have slow shift of the baseline → not only a problem of DC, but also of DC changing every minute.

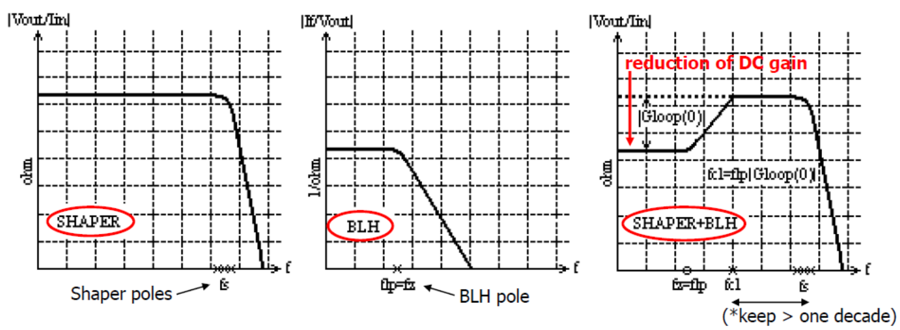


• at low frequency, the feedback provides a current If which is subtracted to Iin. With ~ zero error at the input of the diff. amplifier, the output voltage is determined by Vbl: Vout ≅ Vbl for any value of Iin

• at high frequency (in the signal bandwidth), the low-pass filter opens the loop and the output voltage is only determined by the signal Iin: $Vout=Iin(s)\cdot FDT(s)_{filter}$

⇒ the filter response with BLH can be approximated as an AC coupling

THE T.F. OF THE SHAPER + BLH



• the speed of the BLH to restore the baseline is related to f_{lp}
• f_{lp} difficult to place at very low frequency with an integrated circuit
• the precision of the baseline level, for a given offset at the output of the shaper, is determined by Gloop(0)
• pole f_{c1} (= f_z|Gloop(0)|) not to be placed at too high frequency (*), to not interfere with the poles of the shaper (f_s) and therefore to determine instability. This requirement is against a large Gloop(0).

We have the Bode plot of the shaper, that has current input – voltage output, so it is represented in terms of resistance (voltage divided by current). The shaper has a constant LF gain (that we don't like, because if it has a DC gain it can change the baseline) and in correspondence of the shaper poles we have the cut off frequency fs. If the shaper is on real poles, all the poles are at the same frequency fs, if the shaper is on c.c. poles, in the Bode plot we get something like in the image (eventually with an overshoot, and a cluster of poles close to fs).

As for the BLH, it has the opposite dimensions, because it is a current divided to an input voltage, so it has the dimension of 1/resistance. The action of the BLH is to have a constant gain at low frequencies and then the BLH has a cut off frequency at the frequency flp that is the pole of the BLH. We have to set flp so low that the BLH is not effective in the frequencies of the signal but also not too low, otherwise it wouldn't track slow variations of the baseline.

If we take a shaper response and a BLH and we put them together, we get the Bode plot on the right. It is identical to the one of the shaper in the HF range, so in the region of fs, and this is what we like, to have the shaper response unchanged at HF, but at LF the BLH drops the DC gain of the shaper at LF. So we have a reduction of the shaper gain at the DC level and the amount of this reduction is the LG. So we have to compute the LG of the BLH and the larger it, the lower the DC gain of the resulting circuit.

However, the CL t.f. has a zero, and this zero is the pole of the BLH. So the pole in the OL t.f. of the BLH becomes a zero. Then we have a rise and another pole when the rise reaches the shaper t.f..
Which is the frequency of the CL pole?
By the property of the Bode plot and the slope of 20dB/dec has a frequency that is the one of the zero multiplied by the LG. we need to be careful that this pole is sufficiently far from the shaper's poles.
Otherwise, if not, the pole is going to conflict with the intrinsic poles of the shaper. (the pole perturbates the shaper's response) → we need to keep at least 1 decade between the CL pole and the shaper's one.

This is critical because, given the position of the pole of the BLH Bode's that is fixed by constrains, if this that becomes a zero in the CL is fixed, and we increase the LG, the pole will shift on the right.

So we have the LF zero, than we have for instance 20dB/dec, the pole and then fs. Lp is fixed by technology; if we want to increase the LG, the new bode plot is the bold line → the pole has shifted to the right and we are going to corrupt the t.f. of the shaper.

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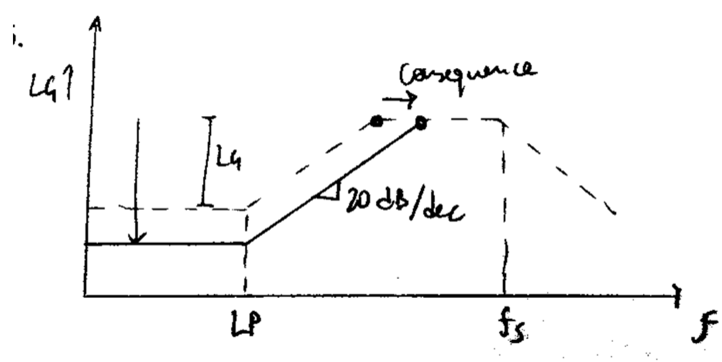
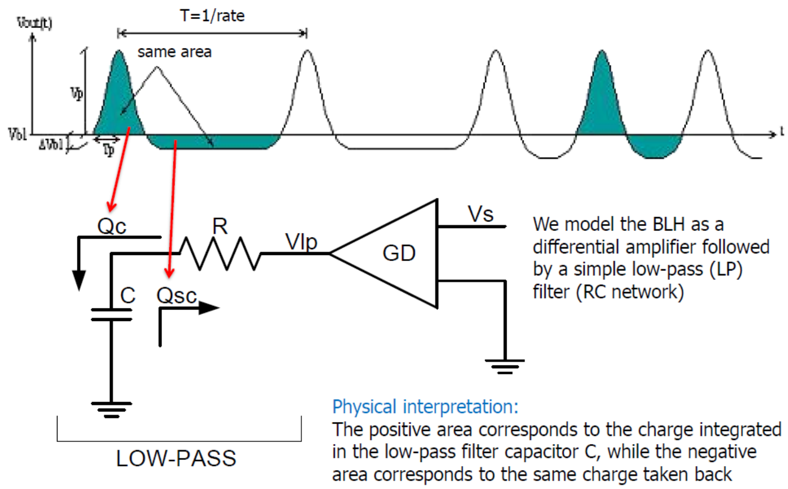


This situation imposes some conditions.

SHIFT OF THE BASELINE

We have a pulse which is supposed to be a triangle for simplicity. If we feed this pulse into the circuit below, we will have a positive area of the pulse and a negative one due to the baseline shift. Moreover, this baseline would shift to lower values if we decrease the distance between pulses.

There is a physical meaning of the ‘area matching’. To understand this, the baseline holder can be represented in the simplest way that is a differential amplifier followed by a low pass filter. The LP filter is represented in its simplest form by means of an integrator (supposed approximated linearly). The LP filter is a reasonable approximation of the real integrator (the one with an opamp) at the beginning.



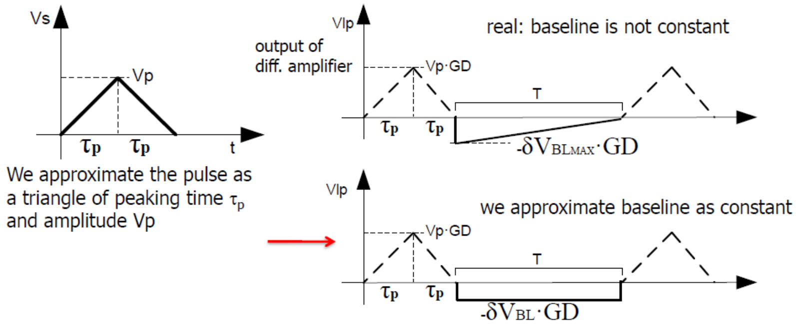
Having a positive area means that the signal Vs is larger than zero, Vlp is positive, if Vlp is positive a current flows in R and is integrated in C. So the green positive area means that the capacitor is integrating charge.
Then we have a negative baseline because we MUST have it to remove form the capacitor the charge that was integrated in the moments before. If we have a negative baseline, the current is directed in the other direction in the resistor (Vlp<0) and we are discharging the capacitor by the same amount of charge with which it was charged.
So the matching between negative and positive area means that the semigaussian pulse is integrated in the capacitor, and the baseline is a physiological result to remove such a current.

If this is true, the computation of the baseline is easy, simply obtained by imposing that the charge integrated in the capacitor is equal to the deintegrated one.

Computations

Let’s suppose for simplicity that the semigaussian pulse is a triangle of amplitude Vp and duaration 2*tau-p. Let’s suppose that the triangle is amplified by the amplifier GD of the previous image. So in the plot on the left I have the same triangle amplified by GD.
The baseline is negative, it has a peak -delta*Vbl an usually is not constant, it has a slow decay. We approximate the baseline by a rectangle (second plot) of amplitude delta*Vbl*GD, since if the triangle was amplified by GD, also the rectangle is. This rectangle will last T, that is the average distance between two consecutive pulses.

If we then take Vlp, what we need to calculate is the area. How much is the charge Qc integrated? It is equal to the current corresponding to the triangle divided by R and integrated for 2*tau-p, that is the duration of the pulse. The charge integrated in the capacitor is the area of the triangle (then divided by R since we integrate a current).



$$Q_C = \int_0^{2\tau p} \frac{V_{LP}}{R} dt = \frac{1}{R} V_P G_D \tau_p$$ (LP considered an ideal integrator)

$$Q_{SC} = \int_0^{T} \frac{|V_{LP}|}{R} dt = \frac{1}{R} \delta V_{BL} G_D T$$

shift of the baseline increases with the rate of occurence of the pulses

$$Q_C = Q_{SC} \quad \Rightarrow \quad \delta V_{BL} = \frac{V_P \tau_p}{T} = V_P \tau_p rate$$

The same calculation applies now for the rectangle. I need to integrate the rectangle; it is the integral from 0 to T.

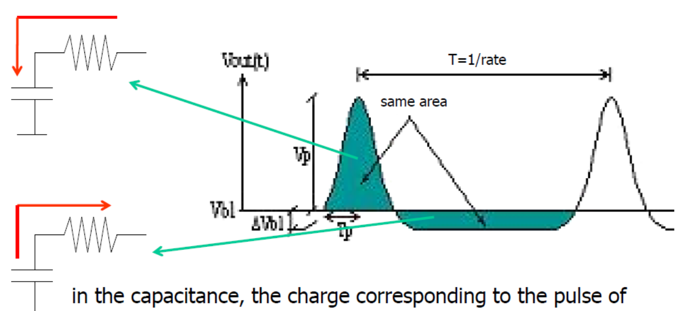
If we equalize, the R cancels out, and if we flip the formula, we can get the absolute value of the amplitude shift Vbl (reversely proportional to T). We can see that the longer the distance between pulses, the smaller the baseline, and this explains why in most of the cases we neglect baseline shift in AC couplings. In fact, in AC couplings, the distance between pulses so large that the baseline shift is always negligible. But if this is not the case, this is a problem.

Moreover, if T is not constant, it changes randomly with Poisson statistic (gamma rays arrive when they want) we have a random statistical fluctuation of the baseline.

T on average is the reverse of occurrence of the pulses → if rate changes, baseline shift changes.

SOLUTION OF THE PROBLEM

key point: integration of signal and baseline areas in the LP stage



in the capacitance, the charge corresponding to the pulse of amplitude Vp is integrated during the interval 2Tp and this charge is given back during the interval T=1/rate

idea: try to reduce the charge integrated (referred to Vout) in order to reduce also the charge given back (which creates the baseline shift)

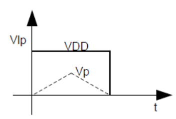
So far, the BLH looks like a more complicated way to implement an AC coupling. To solve the problem we must reason on the fact that the charge area produces charges integrated on the capacitor and the baseline area the opposite.

We can try to minimize the positive area integrated into the capacitor; if we minimize the positive area, referred to the output (node of the shaper), the capacitor will need to give back less charge, so also the baseline will be reduced.

This is done by means of two tricks:

1. We saturate the differential amplifier only when we have the pulse. So the pulse saturates the amplifier, not the baseline, the baseline is so small that is not able to saturate the amplifier in the other way → amplifier saturated only upon the positive pulse.
2. We will introduce a non linear buffer to get a slew rate (that is a current limitation, that usually is very bad, but we do this on purpose).

1[st] action: saturation of the differential stage



$$Q_C = \int_0^{2\tau p} \frac{V_{DD}}{R} dt = \frac{1}{R} V_{DD} 2\tau_p$$

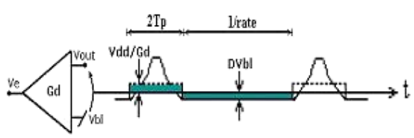
diff. amplifier is saturating to VDD for the signal

$$Q_C = Q_{SC} \qquad \frac{1}{R} V_{DD} 2\tau_p = \frac{1}{R} \delta V_{BL} G_D T$$

- $Q_C = Q_{SC}$ as before
- diff. amplifier is NOT saturating for the baseline ⇒ Q_{SC} expression is the same as before

$$\Rightarrow \quad \delta V_{BL} = \frac{V_{DD} 2\tau_p}{G_D} rate = V_p \tau_p rate \boxed{\frac{2V_{DD}}{V_p G_D}}$$

<1 for large G_D



note: the behavior of the circuit can be represented as an equivalent linear system where at the input of the differential stage there is a rectangle of amplitude V_{DD}/G_D instead of the shaper pulse

When we have the triangle and Vlp is the voltage at the output of the differential amplifier, this voltage at the output previously it was the triangle multiplied by GD, now it is saturated to Vdd (that is the power supply of the amplifier).
So in correspondence of the triangle we have a rectangle at the output of the amplifier, and the rectangle is Vdd high and 2 tau-p (peaking time) wide.

Now, I repeat the integration on the capacitor. What is integrated in the capacitor is now the rectangle of Vdd, a fixed voltage. The result is that the area integrated is Vdd*2*tau-p.
As for the baseline, the baseline is not saturating the amplifier, so for the baseline the calculation is the same seen before, I need to integrate its rectangle for the time T. But the baseline is stille depending on GD (that for the peak has disappeared now). So the term for the negative area is the same of before because we are not saturating.

So the charge Qc is matching the charge Qsc, the R cancels out and I calculate the baseline shift.

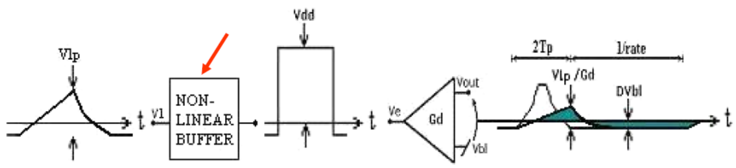
The final formula is that the baseline shift is equal to the shift calculated as before (in the previous conditions Vp*tau-p*rate) times a beneficial factor (red). This factor is lower than 1, because whatever is Vdd, Vp can be any voltage, but the differential gain GD can be enormous, like 100.
The magic is in having the GD in the formula, that was not appearing previously, because it was amplifying the triangle as well as the rectangle, so in the formula was cancelled.

Here GD is appearing only for the baseline, because for the triangle I have a saturation. This is the trick. I have saturated the triangle, and so I have increased the charge in the capacitor, but the point is not the amount of charge integrated in the capacitor. The point is that in the formula of the charge we take back there is baseline times GD. So the larger GD, the smaller the baseline. The trick is to have a large GD for the baseline so that the baseline is small, and not GD in the other polarity.

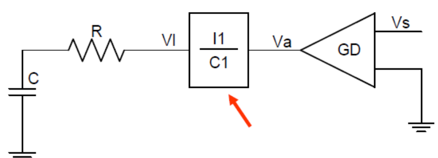
An intuitive way to understand the area matching now, if we look before the comparator (at the entrance), having this formula is like having not integrated a triangle, but a rectangle Vdd divided by GD. This formula is equivalent to have a matching of the negative area that always is the same (baseline times T), but the positive area is now the saturated rectangle divided by GD, no more the triangle. In fact if we take the rectangle and bring it back to the input we have to divide by GD.

In conclusion, <u>I have reduced the positive area from the full area of the triangle to a tiny rectangle Vdd/GD</u>. The higher the gain GD, the smaller the rectangle.

2nd action: introduction of a nonlinear buffer



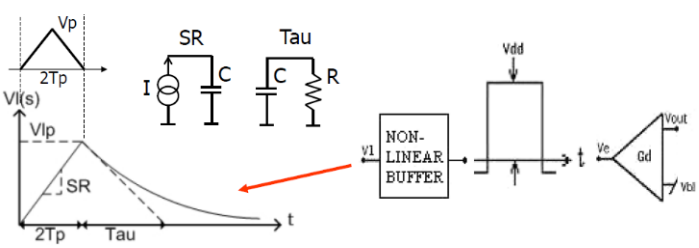
• a non-linear buffer is inserted between the differential stage and the low-pass filter in order to introduce a <u>slew-rate</u> which, therefore, further reduces the fraction of the area of the output pulse which is integrated



Introduced **to push furthermore having a smaller positive area**. I introduce voluntary a slew rate. In an amplifier we have a slew rate when we have a constant current that is integrated on a capacitor. This integration of a current over a capacitor produces a slope which is given by the current divided by the capacitor. Usually it is a nasty effect in amplifier, we want to avoid it to avoid limitation in the response.

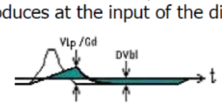
Now we introduce on purpose the slew rate in addition to the saturation. The slope is I1/C1.

Supposing we have the slew rate lasting 2*tau-p, we integrate the current on the capacitor and we have the ramp that reaches a value Vlp maximum. Then, when we have a slew rate and we remove the current, so the integration of current is finished, the capacitor needs to be discharged, we don't go immediately to 0.



The non-linear buffer produces a ramp on Vl(t) as long as the output of the amplifier is saturated; then, once reached the peak value Vlp, Vl(t) returns to zero with an exponential decay having time constant Tau (the first phase corresponds to the charge integrated in a capacitance in a slew-rate regime while the second phase regards the RC discharge of such capacitance – see later for implementation).

As seen before, such non-linear circuit can be represented as an equivalent linear circuit in which each event produces at the input of the differential amplifier the response of the figure above, scaled by a factor $1/G_D$:

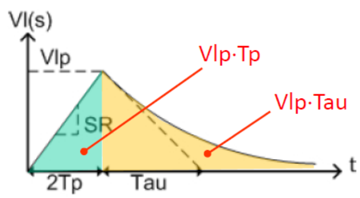


NB: the capacitor no2 is the one of the slew rate stage, not the one of the LP filter, that is placed after the capacitor of the slew rate stage → we have two capacitors

So we have a ramp because we integrate charge on the slew rate capacitor, but then the capacitor will go down with an impedance R connected to the capacitor → we have an exponential decay.
This overall rise + exponential is the one that is now integrated in the LP filter.

If previously we have a matching of area with a rectangle Vdd/GD, now it is the same, but it is like to have at the input the waveform rise + exponential divided by GD (gain of the differential amplifier), no more the rectangle. And I integrate this area.
So the softer the SR, the less area I have to integrate. More severe the SR, less area will be integrated in the LP filter and less area will be associated to the baseline.

Calculations



After the usual calculation $Q_c=Q_{SC}$:

$$\Delta(Vbl) \cong \frac{Vlp}{Gd}(Tp+\tau)\cdot rate \cong \frac{(2Vdd)(Tp)(rate)}{Gd}\cdot\frac{(Vlp)(Tp+\tau)}{(2Vdd)(Tp)}$$

$$\Delta(Vbl) \cong \frac{\boxed{(2Vdd)(Tp)}(rate)}{\boxed{Gd}}\cdot\boxed{K} \qquad K=\frac{(Vlp)(Tp+\tau)}{(2Vdd)(Tp)}$$

saturation non-linearity

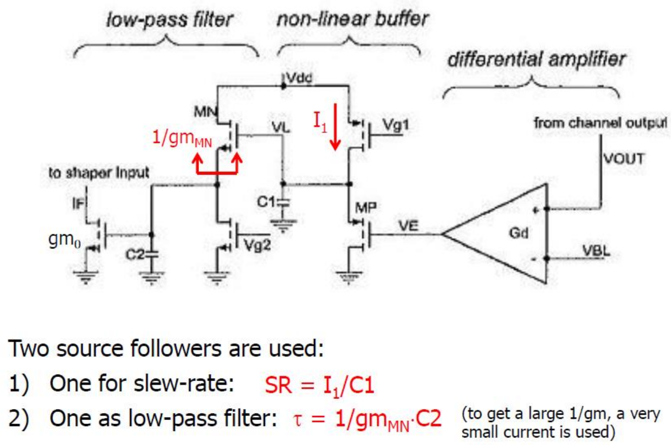
K expresses the ratio between the area at the output of the non-linear buffer and the area of the rectangle at the output of the differential stage and therefore the additional advantage in having introduced the SR effect

Now, to integrate Qc, I need to integrate a waveform composed by the part in green that is Vlp (peak reached by the SR) multiplied by 2*tau-p, and the area in yellow that is Vlp*tau (property of the exponential).
Then I divide by GD and multiply by the rate and I get the baseline shift.

Then we have some manipulation, we multiply and divide by 2*Vdd and tau-p because I want to represent the baseline shift as depending to the red term that was the term at the end of saturation. In this way I can identify the benefit coming from saturation and the one coming from slew rate. On the other side, K is a factor lower than 1 and represents the incremental benefit due to slew rate only.
This benefit of the slew rate is seen in Vlp, that is the top value of the curve and that can be reduced by making more severe the SR; the lower the slope of the SR, the lower Vlp, the lower the K factor.

We can balance then the two effect in conclusion. We can saturate only to reduce the baseline shift or we add SR.

CIRCUITAL IMPLEMENTATION



From the right, we have **the differential amplifier that is a very simple OTA**. The differential amplifier doesn't need special requirements, we have simply to set the gain GD so that the amplifier gets saturated to Vdd when we apply the pulse at the input. There are only minor requirements on the offset because it impacts on the difference between the desired baseline and the shaper baseline.

Then we have the nonlinear buffer (SR stage) that is a simple source follower based on a pMOS. Vg1 is the current generator active load at the source of the pMOS. The SR is generated when the pMOS MP is off and the constant current generator I1 integrates the current over the capacitor C1.

Then, we have the SR when we have the triangle of the signal. If we assume Vout is a triangle, if we apply the triangle to the positive input of Gd, the output is saturated to Vdd. If we take a pMOS and we push the gate to Vdd, the pMOS is off, so we have just the constant current of the transistor of Vg1 (Vg1 is a constant biasing reference), so I1 integrates over C1 so the SR we have in the formula is I1/C1. Given this SR that we can control with current and capacitor, we have simply to calculate where we 'go' after 2*tau-p → we have to calculate the amplitude reached at the top of the previous green zone.

The **SR can be slowed down by decreasing I1 and increasing C1**. When we have then the negative baseline, the amplifier is not saturated, the (+) is negative, the output is negative and the pMOS is on. So **in correspondence of the baseline we have truly a buffer with gain 1**. So the SR occurs when the transistor is off.

The LP filter is simply another follower. It is now an nMOS follower with an active load at the source, and the capacitor is C2. The time constant of this filter is the 1/gm of the MN transistor (that represents the resistance) times C2. In this application we use the 1/gm to implement an extremely large impedance, and this is possible by biasing the transistor with an extremely low current, so that the 1/gm becomes huge.

Finally, we need simply a voltage to current converter, and it is a transistor with transconductance 1/gm0.

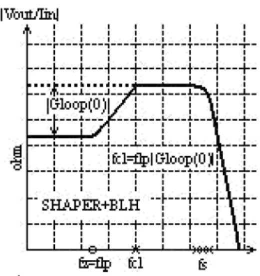
DESIGN EXAMPLE

The t.f. of just the baseline holder, from input to output current is given below.

Design example:

$$\frac{If(s)}{Vout(s)} = \frac{Gdgm_0}{\left(1+\cancel{\frac{sC_1 n_p Vth}{Id_p}}\right)\left(1+\frac{sC_2 n_n Vth}{Id_n}\right)}$$

neglected vs. low-pass pole *



$$|Gloop(0)| > 100 \qquad fcl < \frac{fs}{100} \qquad fcl = flp\,|Gloop(0)|$$

$$flp < \frac{fs}{(100\,|Gloop(0)|)} \quad fs \leftarrow 100Khz \qquad \frac{C_2 n_n Vth}{Id_n} > \frac{1}{2\pi 10Hz}$$

$$C_2 = 10pF \qquad Id_n < 2\pi \cdot 10Hz \cdot 10pF \cdot 25mV \cong 10pA$$

* gm=I/nVth (expression for weak inversion regime)

First of all we are analyzing the t.f. in linear operation, so we don't have SR, no saturations etc. In fact, when we are studying the Bode plot we are considering it with a linear behaviour.
In the linear behaviour we have the amplifier GD, the follower MP with a pole given by C1*(1/gm), then we have a second buffer with a second pole that is going to be the dominant one (MN) and then the transistor IF.

At the numerator I have the buffers gain, and at the denominator the poles of the two source followers (we neglect the pole of the SR stage, MP). In fact, the SR stage and the capacitor C1 are intended to operate in SR mode when needed, so when the transistor is off for large signal, but during normal linear operations I'm not interested in having two poles, I'm interested in having only one. This means that the capacitor C1 and the 1/gm of the MP are in a way that they will be at a higher frequency than the dominating pole, which is simply the pole of C2. If so, we don't care about this second pole → **second pole of the first buffer is neglected**.

As for the pole of the second buffer, it is given by the transconductance times C2, but the transistor is working in ultra weak inversion, because the current is very small. So the formula for a transconductance of a transistor in weak inversion is like the formula for a transconductance of a bipolar transistor, gm = I/nVth. It is the current divided by the thermal voltage multiplied by a weak inversion coefficient n.
This is the pole of the BLH that we can set by setting the current Idn, that is the tail current passing through Vg2.

As for the LG, we would like to have it **as larger as possible to have a good suppression of the DC gain**, but not that much (because of the problem of the pole interacting with the shaper pole), so let's assume 100.
Fcl is the frequency that must be far away from the fs pole, so two decades lower than the shaper constellation of poles.

We put everything in the formula; the fs is in the order of 100 kHz, the LF pole is given by the formula and so we can get all the parameters.
We impose the capacitance C2 by putting the largest capacitance we can place in CMOS circuits that is in the order of 10pF, and this means an Idn of 10pA.
So to provide a pole with the 1/gm at LF we need 10pA.

Is it possible to bias a transistor at 10pA?
It is difficult but possible.

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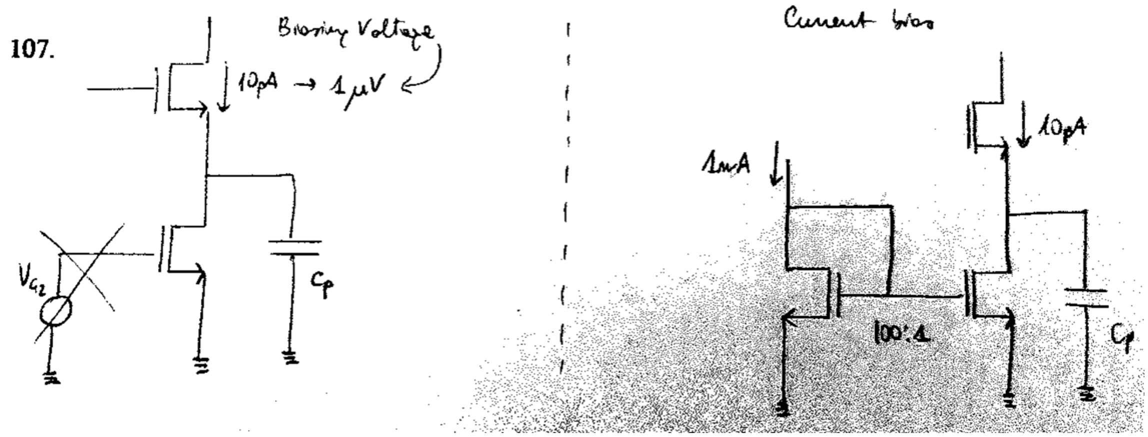


We are talking about biasing a follower with 10pA, that is really weak inversion. It is impossible to bias the transistor with voltage on the gate at 10pA, because just few volts would move the current from 10pA to 1A. So to bias the transistor a 10pA we use current mirrors.
With a current mirror, if we impose 1nA on one branch and we have e demultiplication factor of 100:1, from 1nA we gen 10pA. Of course they won't be exactly 10pA, but maybe 20pA, but it is not a problem because we are not dealing with the precise poles of a shaping amplifier, we are simply setting a LP filter.

The one below is a plot that is an example of measured t.f.

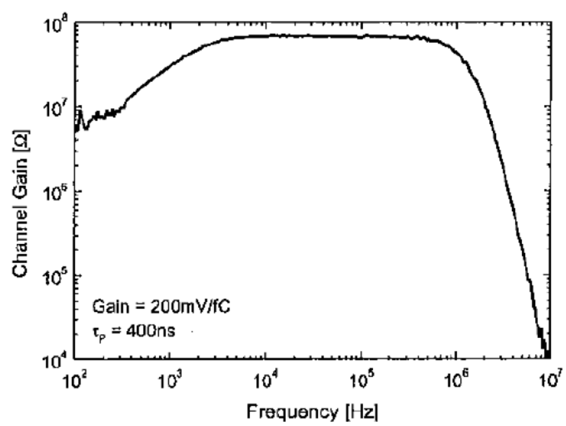


Figure 3: Measured channel small signal transfer function.

This is another measurement.

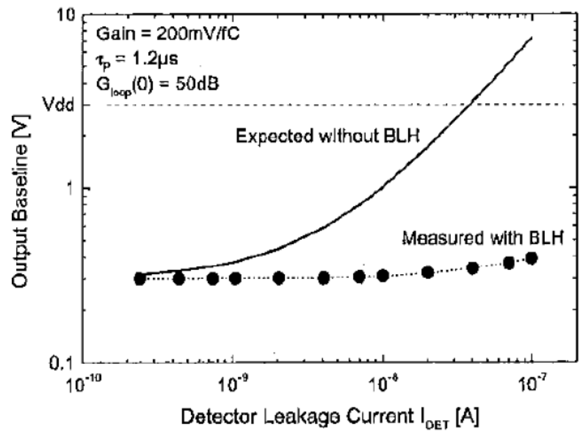


Figure 4: Output baseline dependence on detector leakage current I_{DET}.

The BLH was needed because for instance the baseline must be stable if the detector dark current changes due to temperature. In the plot we have the shaping amplifier baseline as a function of detector leakage current. If we don't use the BLH we have a dramatic change of the DC voltage of the amplifier (solid line), but if we use it, the output baseline remains stable even if the detector changes leakage current by two order of magnitudes (two decades in the log scale).

Even more important, we have to remember the sequence of pulses in AC coupling, and the fact that the AC coupling produces a shift downward of the rectangle. The image below represents an experiment where we have the baseline, that is 0, and we switch on a sequence of 500 semigaussian pulses. If we look the baseline after only 5 of them, we don't see changes; if we look after 500 pulses, if we don't use the baseline holder but standard AC coupling, the baseline has shifted downwards.

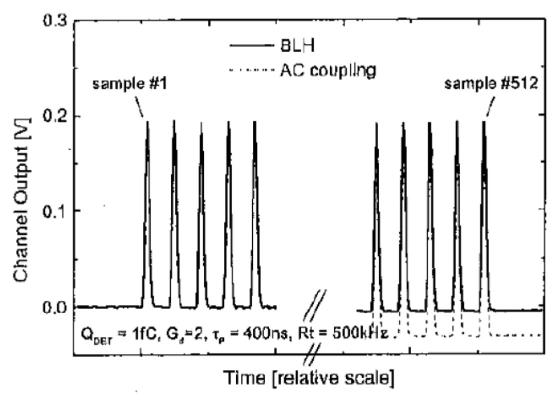
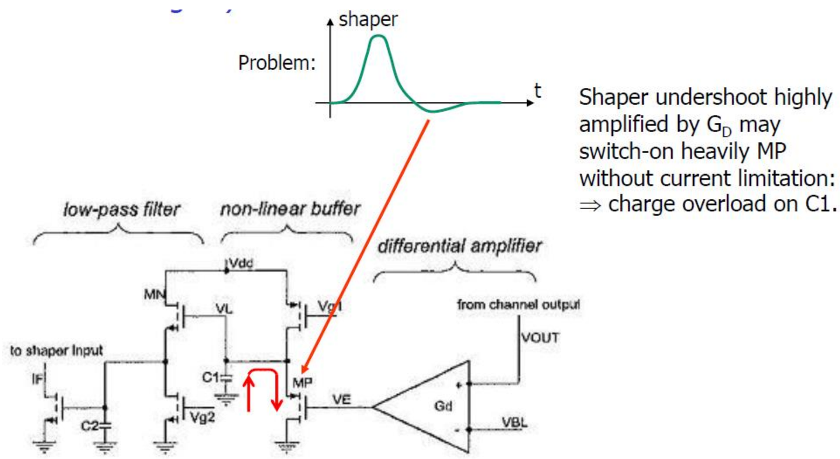


Figure 7: Experimental comparison for Q_{DET} = 1fC and Rt = 500kHz between ASIC channel output and case with BLH disabled and ac-coupling with same time constant of the approximated high-pass filter.

On the contrary, if we activate the BLH, the pulses are practically 'sitting' on an almost unchanged baseline (that has also a little bit dropped down).

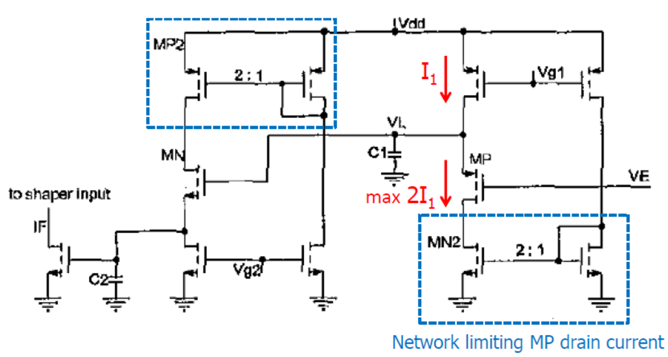
LIMITATION OF CURRENT IN NEGATIVE SWINGS

It may happen that we have designed the shaping amplifier very carefully but, due to tolerances of components, the constellation of poles is not perfect, especially the HF poles are not perfect.
Easily, in the shaper response there can be a small undershoot (of few tens of mV). This undershoot in terms of ENC has no relevance at all, we can forget it, but it could create a catastrophic effect in the BLH.



In fact, only the positive part of the shaper let the SR stage work in SR and the pMOS is off. But if we have a very tiny negative undershoot, this is a negative signal and, since the amplifier has been made purposely with a very large gain, we have a huge negative value at the output of the amplifier, it can saturate to the lower voltage. This means that the pMOS transistor may heavily switch on for the time of the undershoot and draws a lot of charges from the capacitor C1. This is unexpected, because the charged should be removed slowly by the baseline from C1 when we have the 'tiny rectangle'. But here we have an undershoot that draws a lot of charge.
This charge must be then restored in some way and this will produce some nasty waveforms in the shaping amplifier. (*Which waveforms?*)

Solution: limiting the maximum current in the followers

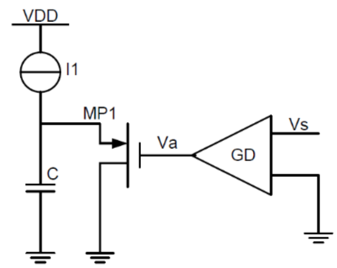


Question: in DC biasing conditions (current on C1 = 0), how may the current in MP matches the current on the upper generator (I_1)?

The problem is solved by limiting the current in the pMOS. So we have a pMOS follower, but the drain of the MOS is not placed to ground, but to a current generator. The current generator on the drain is introduced so that we are sure that, whatever happens to the source follower, it will be blocked

to a maximum current. So when the negative undershoot occurs, the pMOS switches on, but it won't have more current than the one established by the transistor MN2. This current is get by taking the current generator I1 at the top, we duplicate it and eventually multiply it with a mirror so to grant at the maximum the transistor has 2*I0, but not more than 2*I0 for the duration of the undershoot.
So for the duration of the undershoot the charge taken from C1 will be limited to this.

Let's suppose we are in DC, so no current flows in the capacitor C1. How is it possible to have I1 in the top and 2I1 at the bottom? Apparently we have a conflict in the Kirchhoff law; *Is it violated in node x?*



Is it possible to put the SR stage and LP filter stage in one single follower? In principle it is possible, we may have a single follower that works as SR when the pMOS is off an LP when the pMOS is on (figure).

NUMERICAL COMPARISON BETWEEN VARIOUS CONFIGURATIONS

I take some number and, given the general formula with the K factor depending on various benefits, we analyze different situations.
The semigaussian shaper is 1V high (Vp). All the other data are listed below. A rate of 100k counts/s means a T of 10us (the reverse of the rate).

Vp=1V	Gd=10	Vdd=1.7V
τ_p=4us	I1=10nA	
rate = 100k counts/s	C1=200fF	

$$\boxed{\delta V_{BL} = V_P \tau_p rate \cdot K}$$

	K	δV_{BL}
Basic	1	400mV
saturation of Gd	$\frac{2V_{DD}}{V_P G_D}$ = 0.34	136mV
satur.+SR	$\frac{I_1}{C_1}\frac{2\tau_s}{V_P G_D}$ = 0.04	16mV
single stage: sat.+SR+LP	$\frac{2V_{th}}{V_P G_D}$ = 0.005	2mV

Then we take the formula of the BLH that is given by Vp*tau-p*rate with K = 1 in the case of AC coupling (basic). With the data provided, the baseline shift is of 400mV, that is almost half of the amplitude of the pulse → catastrophic

If we activate only the saturation, we have the corresponding formula for K and we reduce the shift to 136mV (better).

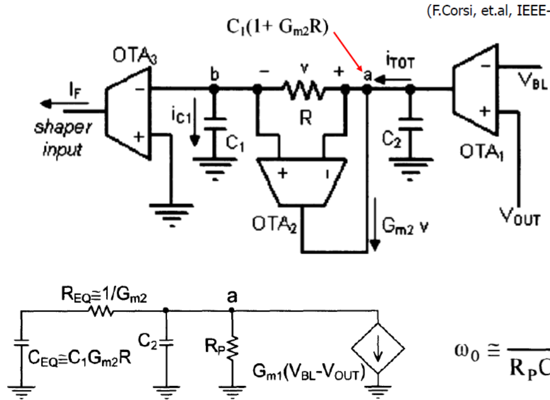
With saturation + SR, we have 16mV (enormous benefit). This is the typical residual shift of the baseline, that is small compared with the amplitude.

With the combined stage we can drop down to 2mV.

Example of implementation of the LF pole (not asked at the exam)

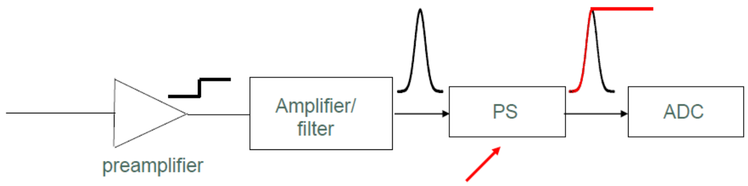
There is another way to implement the BLH; this solution is not based on 'starving' source followers (with 10pA) but it is based on a current that goes through a resistor, an OTA amplifies the voltage across the resistor and the OTA subtract current from the input node. With this loop it can be demonstrated that the resistor R is amplified a lot, because we reduce the current flowing in it.

(F.Corsi, et.al, IEEE-NSS, 2008)



$$\omega_0 \cong \frac{1}{R_P C_1 G_{m2} R}$$

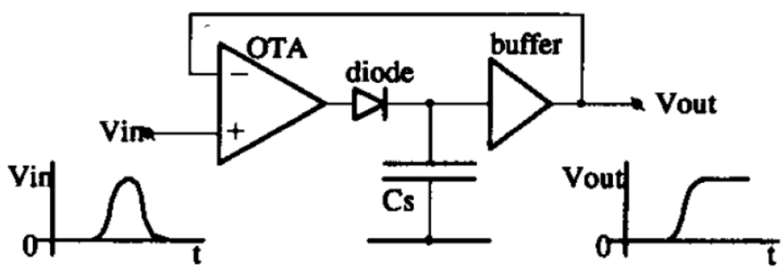
THE PEAK STRETCHER



The goal is to stretch the signal peak at the filter output for the time necessary for the ADC conversion. This circuit follows the shaping amplifier in order to take the peak of the pulse. In principle we can take the shape of the shaper and extract the peak with a fast ADC, but the implementing the peak stretcher is simpler because it tracks the pulse when it is rising and when it reaches the peak, the peak stretcher remains with a fixed value at the peak. Thus with the ADC we can take the peak value and convert it with one sampling, and moreover we don't need a fast ADC, but a one fast as the frequency of the pulses.
In reality this is not true because of the random occurrence of the pulses.

So the peak stretcher may be used also as 'derandomizer'. It could work as analog memory, we sample the analog value of the pulse and then wait for the ADC to be ready for the conversion.

THE TRADITIONAL PEAK STRETCHER



- Rise of the pulse: diode is on, loop is closed, Vout tracks Vin
- Fall of the pulse: $V-_{OTA}$ kept high by Cs, $V+_{OTA}$ decreasing, diode gets off, Vout value (Vin peak) is kept frozen by the capacitor Cs.
- Reset of Cs (not shown here) restore Vout to zero level.

It consists in an Opamp at which input we provide the pulse (with a positive polarity in the example) where we want the maximum. Then eventually we may have a buffer (not essential but recommended) and the n the loop is closed to the negative input of the OTA, so when the diode is on (shortcircuit) we have a negative loop. We can see that if the loop is closed, thanks to the virtual shortcircuit (epsilon = 0 between terminals) we have between the two input of the OTA, the output is a replica of the input.

Let's assume that at the beginning the capacitor is discharged, so we have zero at the right and of the diode. If the capacitor content is 0, also the negative input of the OTA is 0, because we are in open loop and we have a buffer.

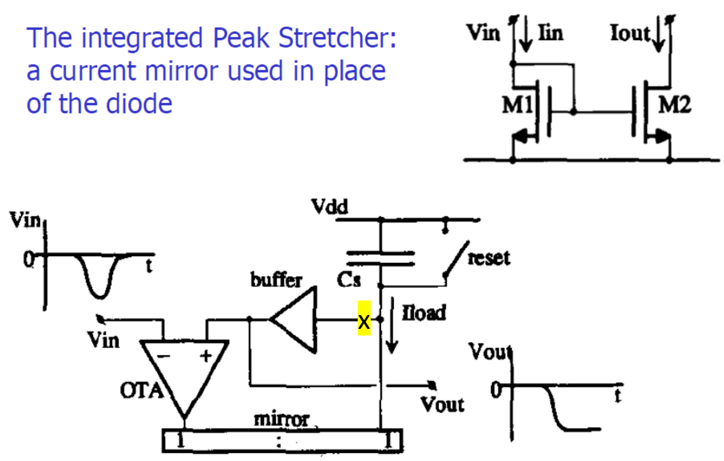
Then we have the signal, that starts to rise. If it starts to rise and the negative input is 0, the OTA will be biased with a positive input voltage between + and - → the output of the OTA will be positive. If so, the diode will turn on, because it has a positive voltage on the left and a zero on the right.
So now the loop is closed (negative feedback) and so we have a virtual shortcircuit between the two input pins. So the overall circuit acts like a buffer with a diode in the middle → the output tracks the input and nothing changes as soon as the input rises.

When the input reaches the top value and then starts to fall down, we have to remember that the negative input is somehow tracking what happened to the positive input. When the positive input decreases, for a short time the negative input is still held at the previous value. When we reach the maximum, the voltage stored on the capacitor is the maximum, so the capacitor has stored the maximum. So the negative input for a short while is at the maximum, but the positive input is decreasing → input voltage of the OTA is changing sign.
The virtual shortcircuit is virtual in the sense that is an effect of the loop taken in the right direction, it is not a wire connecting the + and the -, we have always to track the loop with the right direction. So the negative pin follows the positive one as long as the loop is closed, it is not permanent.
So since so, for a while the negative input is higher than the positive one that is dropping down, but this is sufficient to change the polarity at the input of the OTA.

But if so, the diode turns off, the loop is ISTANTANEOUSLY open and if the opening is instantaneous, across the capacitor we will have the remaining voltage, that is the peak voltage of the pulse, that was the last voltage recorded on the capacitor.
This is the reason why the output of the circuit remains to the peak value despite the input of the circuit is dropping down. We have frozen the peak value of the pulse and kept it available for the conversion of the ADC.

Once the peak is converted, the circuit must be rearmed to get a new pulse → there is a switch in parallel to the capacitor that activates when necessary to reset the capacitor to 0, so that the peak stretcher is ready to get a new pulse. If we don't reset it, it will remain always in the same situation.

THE INTERATED PEAK STRETCHER: A CURRENT MIRROR IN PLACE OF THE DIODE



The circuit above is designed for negative pulses (opposite polarity than in the previous example).
For positive pulses the schematic and the mirror have just to be flipped.

It is the implementation of the peak stretcher in CMOS technology. Diodes are not so common in CMOS, because it is difficult to have access to single electrodes of them, so better to substitute them with an element that provides the main feature of a diode, and the main feature of the diode is the rectification of the current in one single direction and not in the other.

A good substitute is the current mirror. It is a rectifying element because if we try to push the current at the input of it, it provides the current on the other side. But if we try to draw the current in the opposite direction (Iin in the opposite direction), the diode switches off, because the pMOS on the left cannot sustain a current going upwards (see image) → current mirror is a rectifying element.

Another detail when using current mirror as substitute of the diode is that it is inverting the direction of the current output. It is important to be considered in the loop. In fact to keep the loop negative with a current mirror we need to change the polarity at the input of the opamp, otherwise we have a positive loop.

On the bottom we have a CMOS peak stretcher with current mirror instead of a diode. The circuit is drawn for a negative polarity pulse.
So we have the input, an OTA (NB: the loop is closed to the positive input), the current mirror on the bottom, the capacitor Cs that is storing the analog value, the buffer and then the loop is closed.
If we assume that the capacitor is discharged and we apply a negative input to the OTA, considering the positive input is grounded, the current is exiting the OTA; it is the correct current for the current mirror, because if we have an exiting current from the OTA, we have an entering current into the mirror with the right polarity, so the mirror is on and taking the current from the OTA. The mirroring factor is 1.

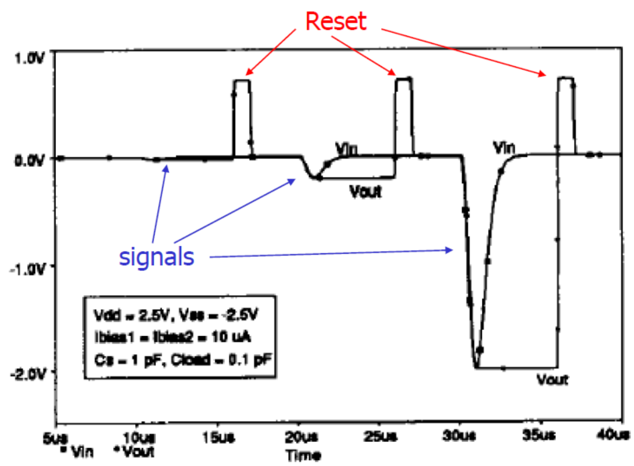
So we have an exiting current Iload and the current is exiting form node x, so the voltage of the node x is dropping down. So the other plate of the capacitor, connected to Vdd is fixed, the other one is becoming more negative due to this removal of current (positive charges). The decreasing voltage is buffered, given to the positive input, the loop is closed, we have a virtual shortcirctuit and the input is tracked.

When the pulse starts to rise again, the polarity of the OTA is inverted, it tries to get entering current in it. But an entering current in the OTA is not compatible with a current going upwards in the mirror, so it stops to work, we have no current Iload and so the output value is fixed to the peak value.
Now the loop is open because the current mirror is off.

We can appreciate the reset switch, typically a MOSFET, in parallel to Cs. If the Cs would be connected to ground and we close the reset, the output will go to ground. Since the Cs is connected to Vdd, the reset closes the capacitor to Vdd. It is an alternative way to do the reset, but in principle we could have connecte the capacitor to ground and reset Cs to ground.
However, this is an alternative choice because there is no other ground in the circuit.

Reset of the Cs to Vdd
In the plot we have the operation of the circuit in correspondence of 3 amplitude values. The first one cannot be seen, in the second one we see a negative gaussian and the third one is a very large negative gaussian. Vout is tracking the gaussian, then is keeping the output voltage when the loop is open and then, when we have the reset, the output is going to Vdd.

When we have the reset, the output is not going to 0, but to Vdd, and it remains connected to Vdd as long as the reset is closed.
When the reset is released, the input in the meantime has returned to 0, while the output is at Vdd. We are in a condition of the loop where the input is 0, output is Vdd so the loop is closed or open? The current in the OTA is exiting, so the loop is closed. If the loop is closed, the immediate attempt of the circuit will be to move the output to 0, to re-establish as soon as possible the virtual shortcircuit → the result is a very quick return to 0 of the output to restore everything for the next pulse.

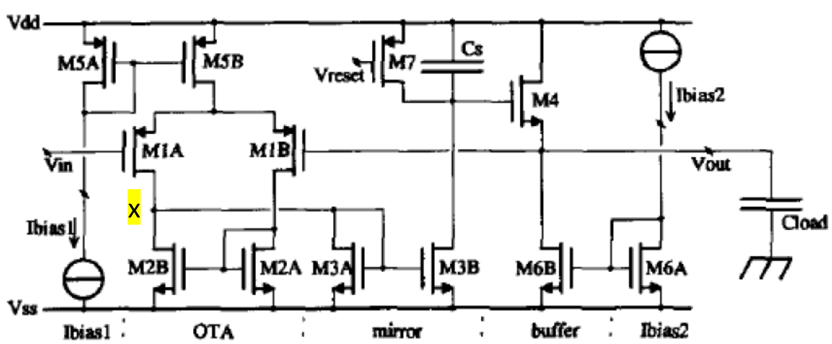


Note: reset of hold capacitor Cs is made on VDD and then the circuit closes the loop bringing the output to zero.

NB: we have to be sure that the gaussian in input has returned to 0, because the circuit output will track the input whatever it is. If I reset and the gaussian has not ended and returned to 0, the output will go to the value of the gaussian still in input → we need a sufficient delay when we start reset because we have to be sure that the gaussian will return to 0.

Independently on the amplitude of the gaussian, **the reset is always the same**.

IMPLEMENTATION IN CMOS TECHNOLOGY



Figures of merit:

1) $G_{loop} = GM_{OTA} \cdot 1/SC_S$ (to be high in the bandwidth of the signal)
2) "Droop rate" = $dV_{out}/dt = (I_{offM7} - I_{offM3B})/C_S$
(creates an error on the value sampled by the ADC)

It is the translation in transistors of the previous blocks (also bias generators can be done with transistors).

Ota

The most 'primitive' configuration is a differential pair with active loads, that are 4 transistors (5 if we include the tail generator). So the OTA is the pMOS M1A and M1B, the current mirror M2B and M2A and the output is at node x.

The current exits and enters in the mirror (if we have the right polarity, otherwise it cannot exit). M5A and M5B are biasing networks for the OTA. Then we have the mirror (M3A and M3B), the capacitor Cs close to Vdd and then we have the reset switch that is just a transistor (M7).

Buffer

The buffer is done with a source follower (M4), and M6B is the active load of the source follower (the equivalent resistance). It is a very simple solution, after the follower the loop is closed at the input.

From the loop point of view, the follower is not a problem, because it has a gain 1 as requested. The problem is that we have a DC drop between capacitor Cs and the output, because we have the Vgs voltage of the follower, which has to be higher than the threshold voltage.

Theoretically, the output should have the same voltage of the capacitor.

Hence here is not true that the output has the same voltage of the capacitor, it has the voltage of the capacitor minus the Vgs of the transistor. This is not a problem, because what matters is that the output voltage is equal to the input voltage thanks to the virtual shortcircuit. The **virtual shortcircuit matters**, not that the voltage over Cs is the same on output.

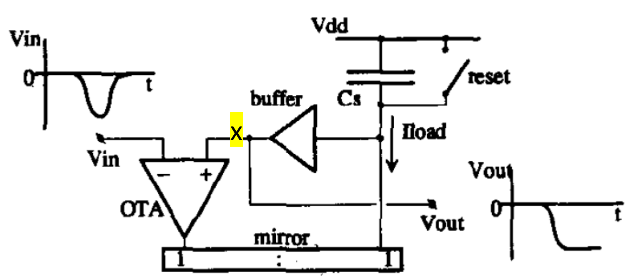
This means that when the loop is open, the output voltage will be equal to the voltage memorized on Cs minus Vgs, but it is just a rigid translation. Vgs is in fact fixed because it is a biasing voltage. What matters is simply that the output voltage is equal to the peak voltage at the input.

Ibias2 is then a biasing generator to provide biasing for the buffer.

FIGURES OF MERIT OF THE CIRCUIT

The parameters we have to pay attention are:

- **Good feedback loop gain**, the LG must be high enough. Let's now compute the LG. I may cut at the output of the buffer, x. When we compute the LG the input is put to 0. If we put a test voltage generator at node x, the output current is due to the transconductance of the OTA.



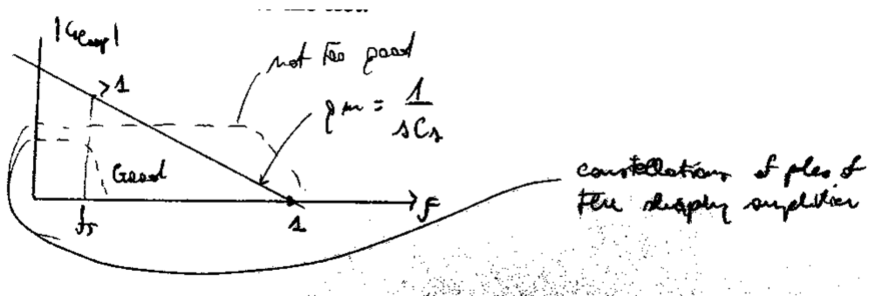
The circuit above is designed for negative pulses (opposite polarity than in the previous example).
For positive pulses the schematic and the mirror have just to be flipped.

So in the formula we have the transconductance of the OTA.

1) $G_{loop} = GM_{OTA} \cdot 1/SC_S$ (to be high in the bandwidth of the signal)

Then the current is mirrored with a factor 1. The voltage on the bottom plate of Cs is obtained thanks to the impedance (1/s*Cs) of the capacitor that converts Iload in the Vout. Then we have just a buffer and we return to the test.

108.



The Bode plot of this LG is not a constant one, but this is not a problem, what matters in the effectiveness of the LG is when the LG is larger than 1, so even the region of 20dB/dec matters, it's enough it is bigger than 1.

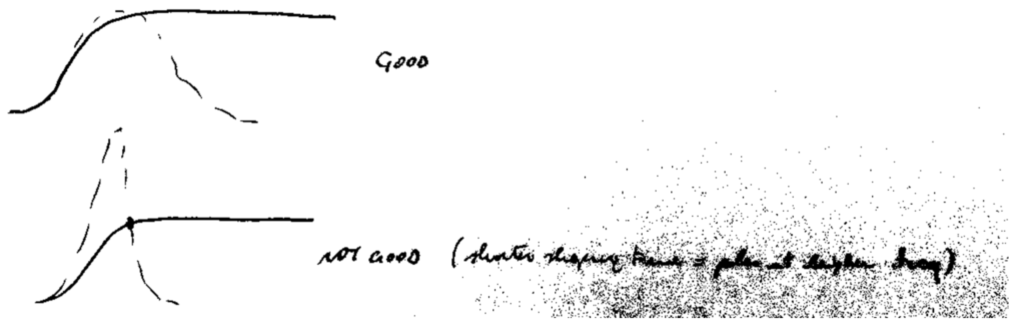
Since it is not constant, we have to check the LG in relation to the bandwidth (polse) of the shaping amplifier. We have to check where is the constellation of poles of the shaping amplifier. If the LG is still safely larger than 1 in correspondence of the poles of the shaping amplifier, we are in the 'safe side' → it is dropping but in correspondence of the frequency of the shaper fs the loop gain is robust.
If on the contrary the shaper has very fast poles (high frequency poles), and the LG of the stretcher is not high enough, it means that the peak stretcher is too slow.

So the goodness of the LG is relative to the situation of the shaping amplifier. We can increase the LG by making the Cs small, for instance, but this may provide problem.
So **the set of the LG depends on the speed of the signal**.

If we reason in the time domain, we have the shaping amplifier that is the dashed line. The LG must be sufficiently high that we can track perfectly the shaping amplifier (solid line) in the rising edge. In the bottom case the LG is so good that we can track perfectly the pulse.
However, if we consider another constellation of poles and a shorter shaping time, a shorter shaping time means that the constellation of poles is at larger frequencies (poles at higher frequency → shorter shaping time).

If we take the same peak stretcher, the LG is now not good, because the PS is 'always late', it cannot reach the peak because the LG is not high enough.
The problem is that in this way the PS tracks a wrong value once the pulse has extinguished.

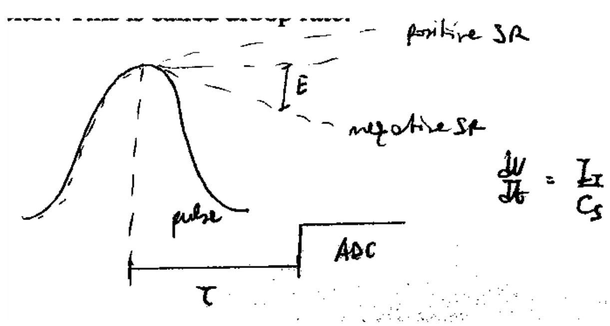
109.



- **Droop rate**: if the LG is a parameter when the loop is closed, this figure of merit occurs when the peak stretcher is in the 'hold phase', when the capacitor is fixed to a value. In principle, its value is fixed, but the capacitor is not 'alone', but connected to the output of the mirror and the reset. But the mirror is off, and also the switch; however, although a MOSFET is off, it doesn't mean that it doesn't carry a very small leakage current.
  Even a transistor in the off state has some current due to the reverse junction of its implantation. Although we keep the junction reverse bias, there is always the current of a diode in reverse bias, so with a reverse current. We usually neglect it, but here it is important because it discharges the capacitor. The Cs is discharged through M7 and M3B, the difference between their currents provides the discharge current.
  <u>If the current in M7 off is equal to the one in M3B off, no current will discharge Cs</u>, but this never happens.

  So we have a kind of slope (slew rate) given by the ration between the current and the capacitor. This is called droop rate.

110.



So we have a pulse, the peak stretcher, we think that the voltage is held forever but in reality we may have a negative slew rate dV/dt or a positive one. This is the droop rate, the change of voltage across the capacitor due to the leakage current of the transistors connected to the capacitor.

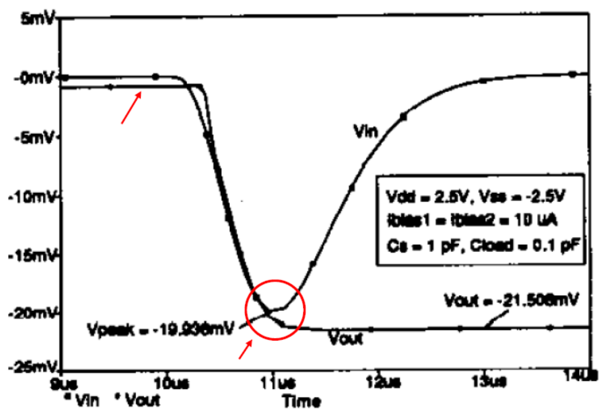
This introduces an error between the true value and the one sampled. This can be a problem or not. In fact, if the sampling by the ADC happens always at fixed delays after the peak has been detected, the error is not a problem because it is a constant error that can be cancelled from the measurements.

On the contrary, if we have a random start of conversion by the ADC, the error is a problem, because it can vary.

So the droop rate is a problem or not depending if the starting of conversion by the ADC is synchronous with the peak or not.

NB: the capacitance Cs is at the denominator of both the LG and droop rate, but with opposite effects, in the sense that if Cs is chosen small to increase the LG we are simultaneously increasing the droop rate → we have a trade off in the value of Cs in order to grant a sufficiently high LG and a satisfactory droop rate. If we don't find a compromise, we have to act on the other parameters, that are the Gm in the LG. or the current in the droop rate.

What happens if the loop is not perfect?



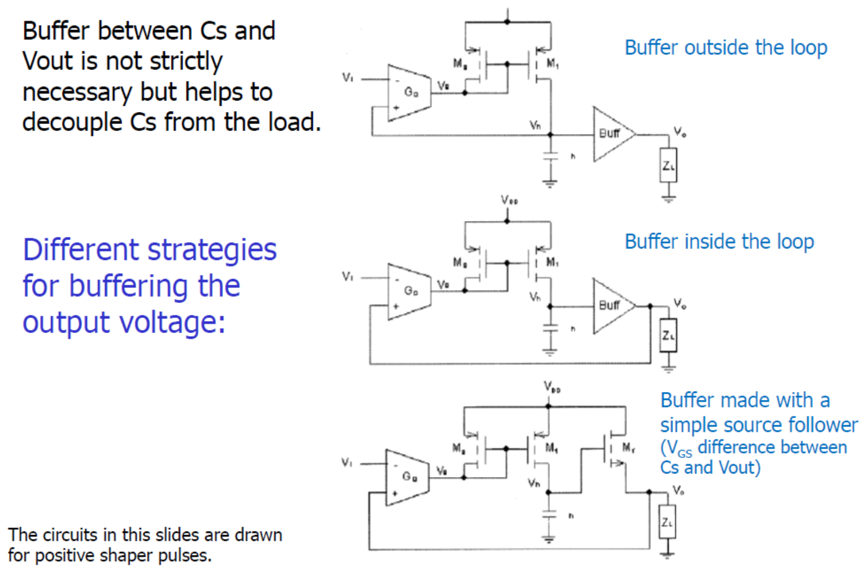
DC offset and error on V_{out} limited by OTA bandwidth and by the speed of the current mirror to switch off

We have a negative input pulse and we can see how the PS tracks it. There is an error, while the pulse is rising, the PS is for a short while continuing on the negative direction. This because the loop was not sufficiently fast to stop. The loop is limited, it cannot stop the mirror fast enough and the mirror is overcharging the capacitor. The same applies for the baseline → the baseline is a bit offset with respect to the semigaussian baseline, because the same problem also occurs in the reset, when we open the loop we have in fact an extra charge.

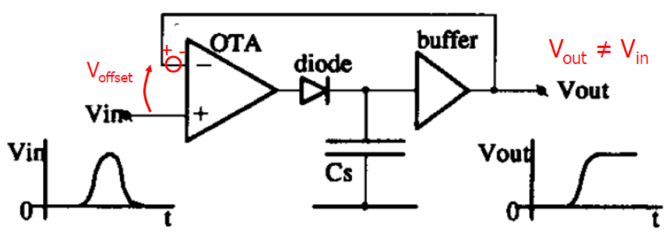
USE OF THE BUFFER

It is not strictly required in terms of operation of the loop, in principle we can skip it.
We use it because we have a external load; if we don't use the buffer, the Cload in output would be in parallel to Cs, so it dominates Cs. The buffer is hence essential.
In the image there are 3 options we can adopt for buffering:

1. We close the loop directly on an external buffer. The disadvantage is that the buffer is outside the loop.
2. The buffer is included in the loop (preferrable solution), because the loop is closed at the output of the buffer, and the solution uses a professional buffer (opamp).
3. Buffer inside the loop but implemented with a follower, not with a professional buffer.



LIMIT OF THE TRADITIONAL CONFIGURATION: OFFSET OF THE OTA



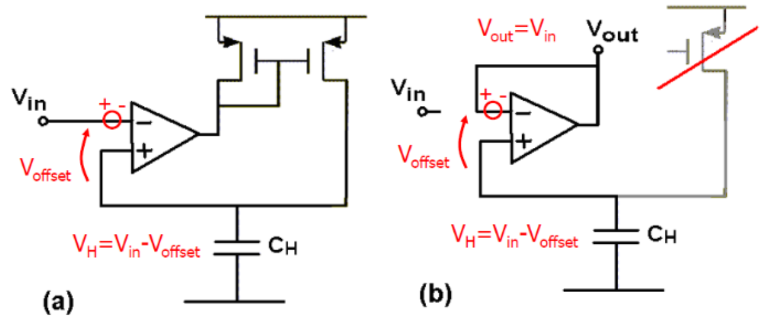
Operational amplifier may have an offset due to the mismatch between the input transistors. Because of this, with a zero voltage at the input we have a certain output.
Usually, the offset is represented by an equivalent voltage generator placed at one of the two inputs, so that the OTA can be represented as ideal. In case we have an OTA with offset at the input and we close the loop, the loop is of course effective but there is a voltage difference between output and input. There is a solution to solve the problem, that is the two phases peak stretcher.

Two phases peak stretcher

The polarity of the pulse is again flipped in the considerations now, the polarity is positive, and we have to flip the schematic, we make the pMOS nMOS and vice versa, and the capacitor Cs is on the bottom.

So we have the OTA and what is flipped is that the current of the OTA is entering in a p-mirror and the capacitor on the bottom. To simplify further the schematic, there is no buffer.
The offset is represented at the input of the OTA. If we are in the tracking phase, and we have an offset, the voltage stored on the capacitor is the input voltage, like the peak of the gaussian, minus the offset.
If we would now take as a result the voltage on the Cs, this voltage would be affected by the offset.

However, we don't simply open the circuit and take the voltage on the Cs to get the output, what we do more is that the OTA, that previously was floating when the loop was open, is closed as a buffer (b).

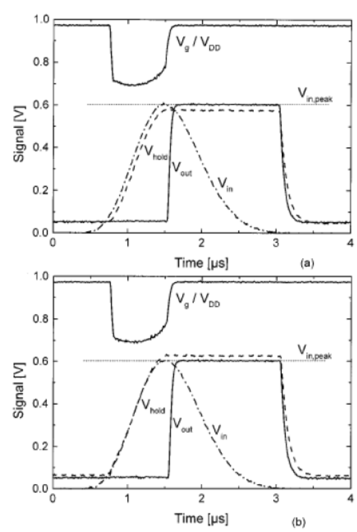


a) normal operation (sampling phase)

b) amplifier operated as buffer (hold phase)

If we do so, the content of the capacitor is buffered. So we elect as output not the content of the capacitor, but the output of the buffered OTA. Using as a buffer the OTA itself and not an extra buffer as before, we include in the t.f. the offset. In fact, now the output is equal to the voltage on Cs plus the offset. So the voltage on the capacitor was affected by a subtraction of the offset in the tracking phase, but in output the offset is summed and so they cancel out in the reading phase. So Vout = Vin.
It is an algebraic recover of the offset without using extra resources.

The one below is an example of the effectiveness of the solution.



example of response in case of offsets with opposite sign

(V_g: gate voltage of the mirror MOSFETs)

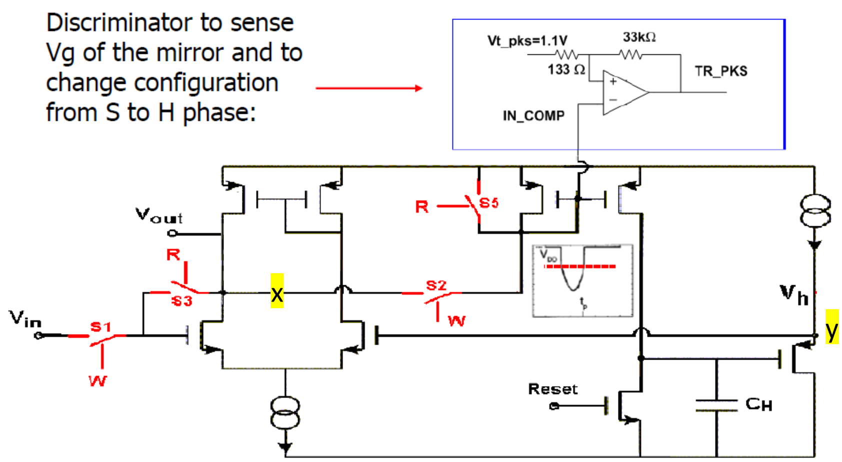
In the upper example we have the gaussian, and in dashed line we have the voltage over the capacitor. We can see that in solid line we have Vout, that is the voltage in output of the buffered OTA; in the tracking phase is nothing, when the OTA is buffered it is equal to the peak of the gaussian → we have perfectly recovered the offset.

In the example on the bottom, we have another case where the peak stretcher is affected randomly by a positive offset. In fact, the offset is a random statistical quantity; if we take for instance 100 chips, 50 will have negative offset and 50 positive one → it is a statistical thing the offset.

In this case, the capacitor has a positive offset, not negative, but the result is still the same, the output if the buffered OTA is still equal to the peak of the gaussian.

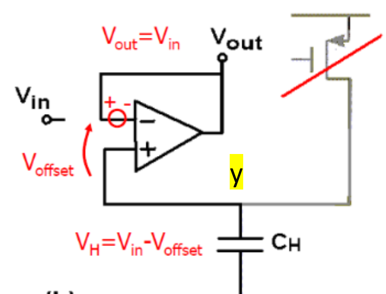
In practice, we have to implement this solution with something more.

Complete circuit



W: write (sampling) R: read (hold)

We have the OTA, the positive pMOS mirror, the capacitor Ch, the source follower and we close the loop. The connection I have to make to transform the OTA in buffer is the switch S3. So the output of the buffer (x) that was going in the mirror needs to be disconnected from the mirror and the output has to be connected to the negative input of the OTA.
This is done with the switch S3; this switch is closing the circuit as a buffer of the voltage stored on the capacitor. The node y that is buffered in the image is now node y in the other image.
The positive input is node y, that remains connected to the capacitor (with a follower in the middle).



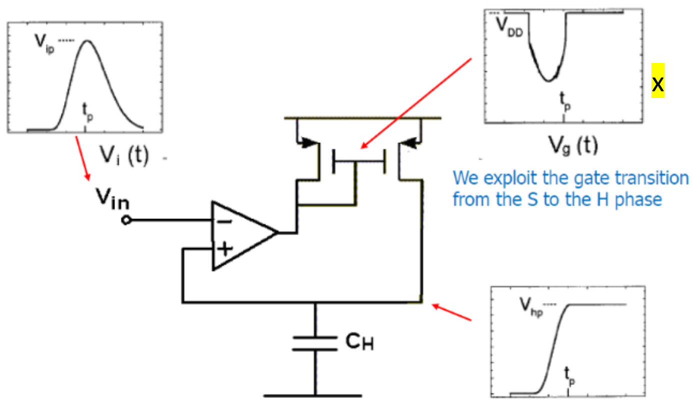
(b)

Previously the output pin of the peak stretcher was at node y, now is where I have Vout.

So I need a switch S3 to close the buffer; a switch S2 to disconnect the OTA from the mirror and a switch S1 because the buffered OTA has to be disconnected from the filter, otherwise we would have a conflict at the input of the OTA. Finally, it is not mandatory but there is a switch also to the mirror, S5, that is a bit redundant because the mirror is itself open, but in order to speed up the opening of the mirror we short the gate of the pMOS to Vdd.

We can see that the previous version of the circuit was spontaneously running from tracking phase to hold phase, now I have added 4 more switches. The switches driven by W (write) must be close when the circuit is in tracking (or sampling) mode. The switches driven by R (read) must be closed in the hold phase, when we are reading the capacitor.
The big question is: who drives the switches?

It is not a spontaneous modification we need to drive switches with digital signals. Once again, the circuit is doing the job by itself. I don't need anything more.
I need to find in the circuit an analog signal with a heavy transient so that I can use the signal to drive the switches. This signal is the gate of the mirror.
During the transition of the circuit is the mirror that is changing, it was on and then gets of. This means that if we take the gate of the mirror and we monitor its voltage, the voltage is making a negative swing. This because when we are tracking the rise of the gaussian, the mirror is on, so the gate is low (x) (p-mirror).

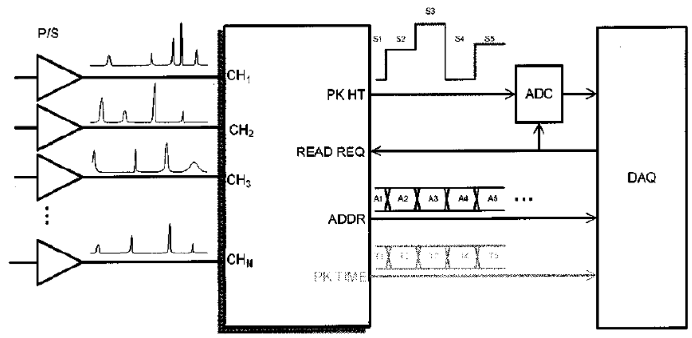


In the hold phase the mirror gets off, which means that the gate voltage rises heavily to have Vgs = 0. So if we want to sense the transition of the circuit from tracking mode to hold mode, we sense the voltage on the gate with a comparator (Schmidt trigger, blue box previous image) and when the node is rising and overcomes a threshold it means that the mirror is getting off and the trigger provides a voltage step, and we use this digital signal to drive R and W.

→ By sensing the gate of the mirror we sense that the circuit is changing.

So the mirror itself is setting the change of phases of the circuit. To choose the threshold value, we know that the top value is Vdd, while the bottom value reached by the gate when the mirror is on is the threshold voltage of the MOSFET. A typical value of a threshold value in MOSFET is 0.7V, so we have room because the threshold must be placed between Vdd and Vdd - 0.7V. So we can choose for instance half of the threshold voltage.

USE OF THE PEAK STRETCHER FOR THE DERANDOMIZATION OF EVENTS



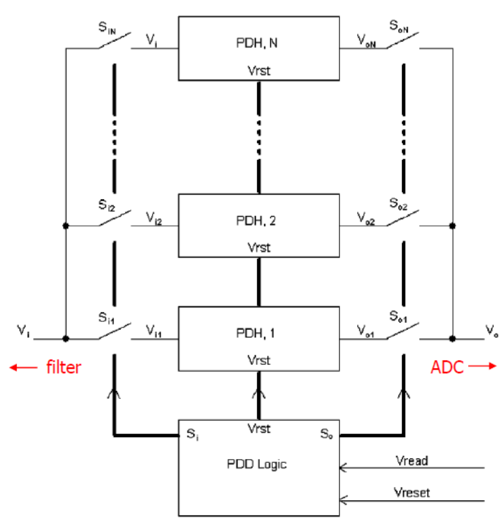
The problem: events arrive randomly (Poisson distributed). Constant-rate ADC should work at much higher frequency (> x10 of the average events rate) if events would not be derandomized (i.e. 'buffered' and processed a constant rate).

Pulses on my detector are arriving in a completely random way, distributed with Poisson statistic. If we have an ADC, the ADC is operated at its best if used at a constant conversion rate. So we have random pulses and we would like to feed the ADC with a constant rate sequence of analog values. The input of the ADC are the peaks of the pulses but distributed regularly in time. This is what wished.

Otherwise, the ADC must be much faster than the average arrival time of the event. Must be much faster because if we have e.g. 100 pulses per sec and the ADC can do right 100 conversion per sec, we can have situation of crowded pulses where in half a second we have plenty of pulses. The ADC is not able to track this fast concentration of pulses. So typically we need an ADC ten time faster than the average arrival rate of the pulses. In fact, a faster ADC is able to cope with the signals also when they arrive one close to the other. But this is a waste of resources.

The peaks stretcher ca be a suitable block to derandomize pulses, so to 'store' pulses amplitude waiting for the ADC to convert them.

USAGE OF THE PEAK STRETCHER



Solution:

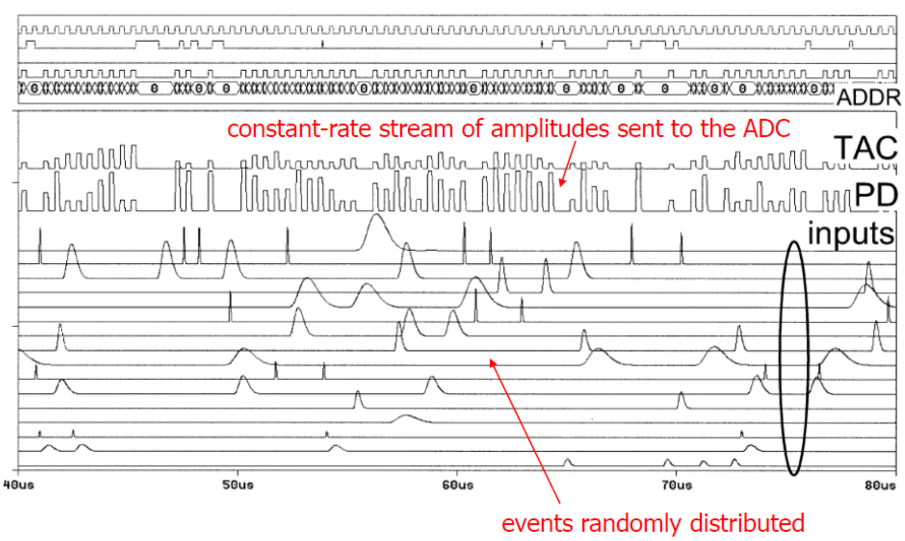
- A battery of peak stretchers is available for each analog channel;
- The pulse is stored in the first available (empty) peak stretcher;
- The ADC is sequentially connected to the peak stretchers which need to be readout.
- Events arrival occurrence and ADC conversion rate are decoupled:
  $\Rightarrow$ events rate ~ ADC rate
  (more efficient use of the ADC)

We don't have one peak stretcher per channel, but we allocate a battery of 3-4 peak stretcher each one potentially connected to a shaping amplifier. It is not a waste of resources because the peak stretcher is a compact a low-cost device. So we allocate few of them for each shaping amplifier.
When a pulse arrives, it is stored in the first PS. If a second arrives is stored in the second PS. So we have a battery of PS to store pulses of they are close. In the meantime, the ADC is reading at a constant rate the first PS, then moving to PS2 and so on. So the battery of PS is uploaded with random arrival pulses, but the ADC does its job regularly.

Of course, the rate of conversion of the ADC MUST be equal to the rate of arrival of events, but not extremely bigger.
We don't need more than 3-4 PS, in fact with some Poisson statistic calculation this is the number of PS we need.

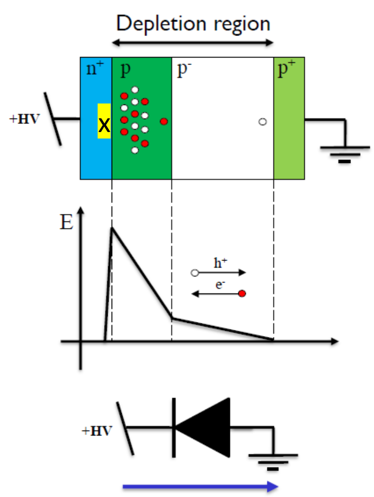
NB: we have a unique line driving the PS to the ADC, and we close to this line the first available PA ready to be read.

The one below is the final plot. We can see that we have a variety of pulses occurring randomly in time. However, we have a common line called PD that is an analog bus where all the PS which are filled by an event are discharged. We connect to this line with a constant rate, and we read the amplitudes of the gaussian.
Then the ADC is for instance a pipe-line ADC that is converting at a constant frequency these pulses.



SILICON PHOTOMULTIPLIERS (SiPMs) AND READOUT ELECTRONICS

IMPACT IONIZATION IN A PN DIODE



- High electric field (>5x10^5 V/cm) in the depletion region
- Charge carrier can be accelerated to create secondary charge pairs through impact ionization.

NB: for a given applied voltage, electrons have higher probability to start an avalanche than holes.

Two different working regimes, depending on the applied voltage to the reverse biased diode:

- APD (Avalanche photodiode): the output current is proportional to the input signal
- SPAD (Single Photon APD) or GM-APD (Geiger Mode APD): the output current is independent from the input signal

A PIN diode is a diode in which, in addition to the pn junction, we add another implantation that is an intrinsic Si implantation in order to increase the electric field (already seen for the APD). So in the depletion region we have a relatively soft electrical field that is responsible only for the separation and drift of positive and negative charges and then, close to the heavily doped junction x we have a steep increase of the electric field because we create a narrow volume where charges are accelerated and able to ionize and to create other couples of e/h pairs by impact ionization.

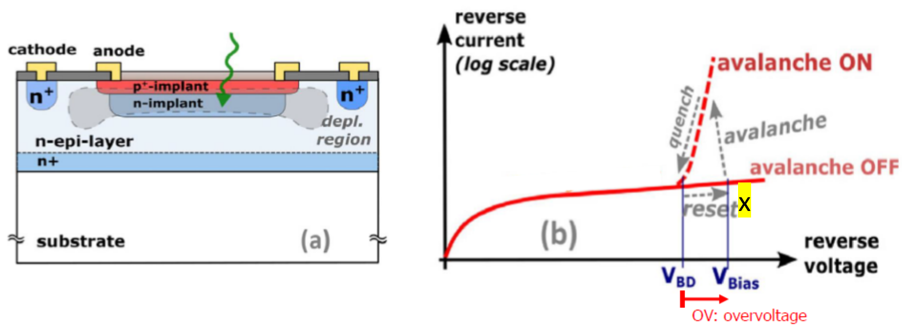
Considering this avalanche mechanism, there are two different ways to exploit it. One is the one introduced for the APD which is a proportional regime.
Proportional regime means that, despite one single charge is able to create many more charges, the amount of charges is proportional to the input signal, and the proportionality factor is the M multiplication factor seen for APD. So for every e-, I have for instance 1400 charges at the endo of the multiplication (+- an amount that is the statistical fluctuation of the multiplication).
So given an input signal of n photoelectrons, the output signal will be M*n.

There is another regime, called Geiger mode where the avalanche is triggered and then it is diverging. Diverging means that the avalanche mechanism is theoretically without an end. This introduces the concept of single photo APD (SPAD) where we basically have an overall avalanche just because of a single photoelectron, but we lose completely proportionality. The output is independent from the number of photons that have created the avalanche.

SPAD OPERATION

On the left we have a cross section of a SPAD element. It is realized in planar technology; we recognize a p implant that represent the p side of the diode, and n epi-layer that is then in contact with the n counterside.

In between, close to the p side, we have an n implant that creates the high electrical field region. So in gray we have the depletion region, and in it the region ready to create the avalanche.



The SPAD is biased above the breakdown voltage (V_{BD}). In such conditions, the electric field is so high that a single carrier generated into the depletion layer (by an incoming photon or by thermal generation) can trigger a self-sustaining avalanche process and a rapid increase of the current to a macroscopic level.
The current theoretically would continue to flow until the avalanche is quenched by lowering the bias voltage to or below the breakdown voltage, by a so called "quenching circuit". The bias voltage (V_{bias}) must then be restored in order to be able to detect another photon (reset phase).

From the point of view of the I vs V characteristic of the diode, we are dealing with a reverse bias characteristic of the diode. We apply a reverse bias on the diode and if we plot the current, the current in log scale increases slowly with the bias voltage but it is basically, at least before a given voltage (Vbreakdown) the classical reverse bias current.

However, above a given voltage called Vbd, avalanche can occur. Before Vdb the diode is reverse bias, but the field is not high enough to trigger an avalanche. Above this voltage even just one charge may trigger an avalanche. Of course, it is not convenient to bias the diode just right at the Vbd, it is 'too weak'. So we bias the diode at the voltage Vbias that is safely above the Vbd.

This difference between Vbias and Vbd is called OV, overvoltage. It is the extra voltage we use above breakdown.
If we are safely inside the breakdown region, in principle it happens nothing if the diode is 'alone in the universe', but if the diode receives even just a single optical photon, an avalanche is triggered and an enormous current is started inside the diode.
This is why usually, correspondingly to a bias voltage, we don't have one characteristic, but two characteristics. One is when the avalanche has not be triggered, the other when the avalanche has been started.
The responsible for the transition of the diode between avalanche off and avalanche on is the signal.

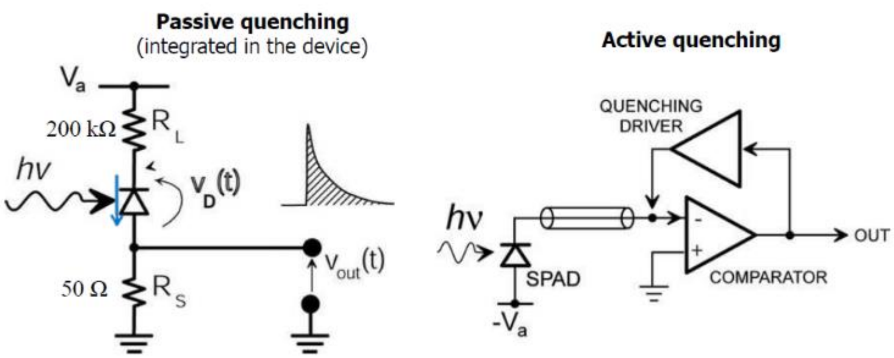
In this way we have a device that, depending on the current that is flowing is a '*digital device*'; the fact that we have a current on or off can be associated with the information that a photon has been detected or not.

This transition between avalanche off and avalanche on can be triggered not only by the optical photon (our goal), but also by thermal generated electrons (that is a problem). If while we are waiting for an optical photon to be detected an electron is thermally promoted in the high field region of the device, this electron is able to trigger an avalanche exactly in the same way as a photon. This is the **drawback**, that **thermally generated electrons that can trigger an avalanche** → cause of noise.

Once the avalanche is started, so we are in the dashed line, we need then to stop it, otherwise the diode cannot be rearmed for a new detection. The mechanism to stop the avalanche is the quenching.

We can quench the avalanche by reducing the voltage across the diode → we need to reduce it from the bias voltage just below Vbd. If we do so, the current will be reduced and the avalanche has been stopped.
Once we are just a little below Vbd, we need to go back to Vbias, otherwise we cannot make the device ready for the next photon. This turning of phases is shown by the grey arrows.

QUENCHING THE AVALANCHE



- The large signal current leads to a voltage drop at the bias resistor so that HV lowers
- Low counting rate ~ 100 kcps
- Worse photon timing (>100 ps)
- Less active area required

- A circuit detects a hit and lowers the voltage
- High counting rate ~ 1 Mcps
- Good photon timing (<100 ps)
- More active area required

Passive quenching is usually adopted for particle/gamma-ray detection

Passive quenching

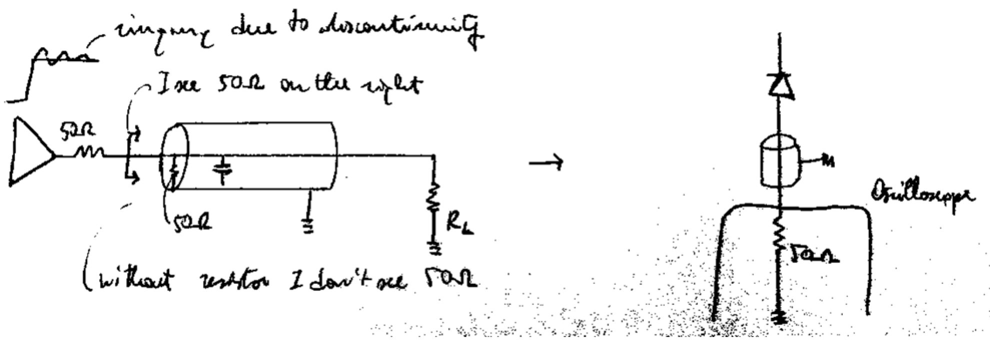
The easiest way to do quenching is to let the diode to quench by its own. This is called passive quenching.

We have a diode, a resistor Rs of 50 Ohms where we can read with a voltage the avalanche happening, and then a resistor Rl that is the quench resistor. Va is the reverse bias voltage. So at the beginning we apply to Va the bias voltage, the other node is put to ground through Rs.
Let's now suppose we are not in avalanche condition (no photons and no thermal electrons); the diode is reverse biased, but there is no current flowing in it. If we have no current in the diode, we have no drops on the resistors. By the Kirchhoff law, the bias voltage across the diode is indeed all Va.
We are right in point x of the previous image, where the entire Vbias is applied across the diode.

Now, a photon arrives. We have a huge increase of current and this huge amount of current will create a drop across Rs. So we will have a positive step of the voltage across Rs that is detected on Vout.

There is a problem. In electronics, where for instance we have an amplifier and a step on the amplifier output and we want to send the output to a load to read the signal, we may use a coaxial cable, a cylindrical conductor where the internal part is the signal conductor, and the external part is the shield, grounded.

111.



If we look at this cabling just as a discrete impedance model, at the best we may consider just a capacitance between the central wire of the cylinder and the shield. In reality, propagation of signal across conductors have a more sophisticated modelling in terms of propagations of electromagnetic waves. In fact, we are in reality propagating EM waves, and in this case the shield matters.
From the point of view of EM propagation, the cable shows an equivalent impedance that is 50 Ohm.

This means that if the 50Ohm impedance is not matched from both sides, so we don't put a 50 Ohm impedance in series, we have a discontinuity in EM propagation. This in the end means that the step in output of the amplifier is not totally corrupted, we have some *ringing* over the step.
This ringing comes from the fact that the EM wave instead of passing through two homogeneous media, has a part of the wave that is reflected back. This waveform coming back is superposing to the one coming forward → ringing.

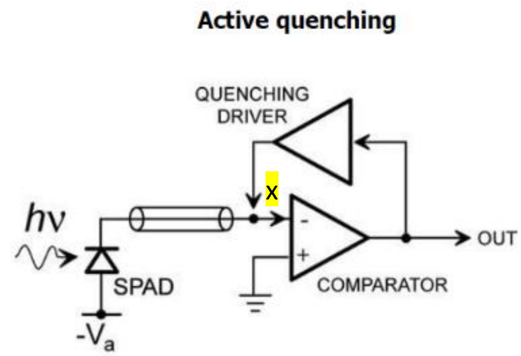
In conclusion, **the best way to match the impedance of a shielded cable is to let the cable to see 50 Ohm on the left and on the right**. This is the reason why it is convenient to read the SPAD with an oscilloscope of 50 Ohm impedance.

Coming back to the scheme, we are at the point where the avalanche is on. So we have the rise of the pulse on the 50 Ohm (otherwise we would have seen a ringing) but since we have a current, we start to have also a big drop on the resistor Rl → voltage across the diode is Vbias – Vrl -Vrs (negligible Vrs). So we have an automatic reduction of the reverse bias. This quench phase is created by the current itself. If then the avalanche stops, we have no more current, and the reset is automatic.
So **in passive quenching the quench and the reset are spontaneous**.

This solution is the most adopted in medical imaging and gamma ray detection because it is very easy to be implemented, but there are some limitations.
It is a spontaneous mechanism and since so, we have to wait the characteristic times of the mechanisms → how fast the quenching occurs depends on the device.

Active quenching
We have no resistors, we bias the SPAD between Vbias and a standard voltage on the (-) terminal of a comparator and when we detect by a comparator that the avalanche occurs, we have an active circuit which shuts down the voltage applied on the x side of the diode. So we have a circuit that normally

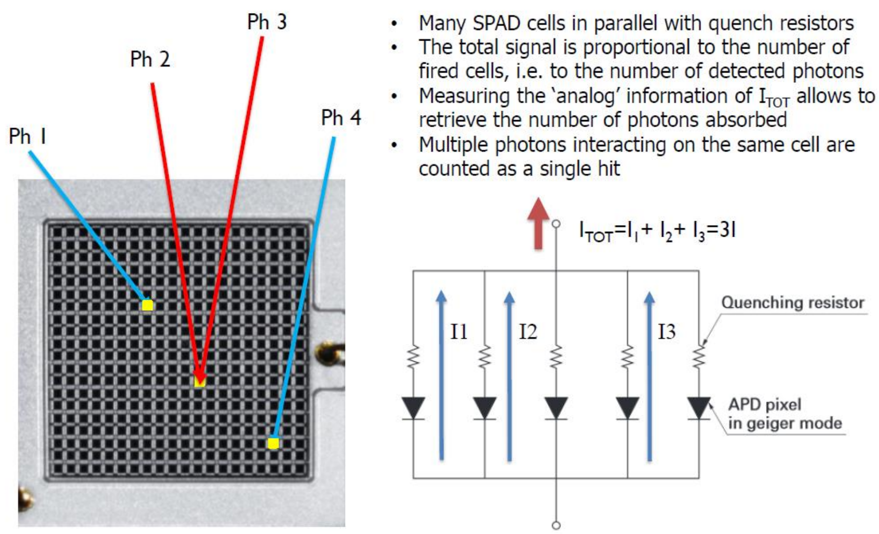
applies a voltage on node x that puts the diode in avalanche mode; when the avalanche occurs, the voltage on x is changed actively by an electronic circuit to stop the avalanche.



When the avalanche is stopped, the previous breakdown voltage is established again.

It is a solution that performs better because the quenching can be very fast and so we can accept photon with a higher rate, but of course we need a specific circuit for each cell.

THE SiPM PRINCIPLE



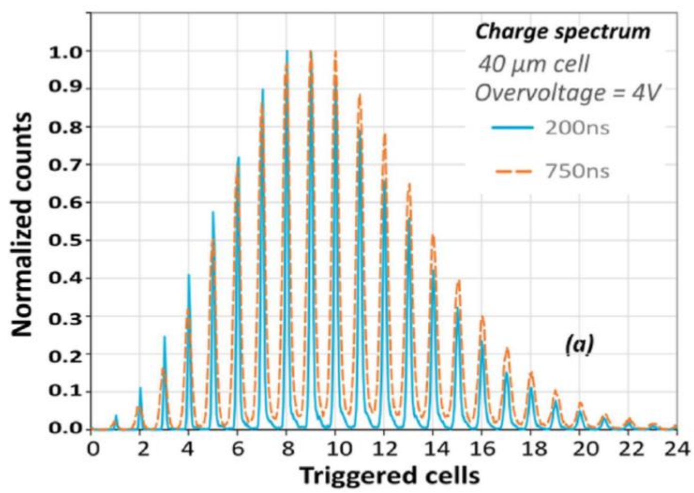
The sipm is a device where we put in parallel many of such SPAD cells. The one on the left is a sipm layout seen from the top, that consists of several cells, SPADs of the one seen so far. Each square in the image is a single SPAD; electrically, all these SPADs and their quenching resistors are put in parallel, which means that on one end of the parallel we apply the Vbias, on the other end we readout the current, for instance with the 50 Ohm resistor.

When nothing happens, we have no current everywhere, but if photons are detected, they start an avalanche and the total amount of current we read at the terminal of the device is the sum of the avalanche currents one per each fired cell.
If only one cell is fired by one photon, we will read out a single 'quantum' of current; if two or more cells are fired, we will count a number of quanta of currents that are simply the single current multiplied by the number of fired cells.

In this regard, we have a kind of analog-digital sensor, in the sense that the current is analog, but the amount of total current is digital in the sense that is the sum of off or on diodes.
The analog current we read out is a counter of how many photons have arrived (of course the device must be calibrated, to kwon the current produces by one photon).

It is a very popular device because it is not a proportional avalanche photodiode, in which the current is proportional to the number of photons in a proportional way. In this different way we count how many cells has been fired. This mechanism shows some limitations.
For instance, if the device is composed by 400 cells and we detect only photon 1 and photon 4, we have two hits. But if the number of photons starts to be so large that we have a non-negligible probability that two photons are hitting by chance the same cell, the amount of current I of this cell will still be the same.
So once we start the avalanche the current is still the same if the avalanche is triggered by one or more photons, we cannot distinguish → there is the risk to count less photons.
This means that higher is the optical signal, higher is **the probability of saturation of the device**. Saturation in the sense that the cell is occupied and extra photons won't be counted.
This is not a problem of APD, because they are in the proportional regime.

EXAMPLE OF SINGLE PHOTON DETECTION CAPABILITY



This plot shows the histogram of the signal at the output of the device. On the vertical axis we have the quantized current and we see that the current values are not all the possible values, but all the possible values of current are quantized.
This is true if the electronic noise allows us to distinguish one current step from the other. In fact, we don't have Dirac deltas of current, we have gaussians, and these gaussians come from the electronic noise of the amplifier → the electronic noise of the amplifier must be low enough to let us count current levels.
However, the help arrives from the multiplication: each current is multiplied, so I can neglect the electronic noise.

SiPMs PRODUCTION

There are different formats, from small to larger dimension; the difference is the number of cells.
A group of sipm cells composes a pixel and inside the pixel I have no imaging capability. Then I put together several pixels → we use arrays of sipm.
The array can be used to read a single scintillator, and so we have an imaging detector, or we can use an array to read out a monolithic scintillator, and in this case we have an Anger camera.

In fact, if an array of simp is coupled with an array of scintillator, we have a pixelated camera, for instance PET. Conversely, if we take a monolithic scintillator and we couple to an array of sipm we have an Anger camera. **Each pixel is the single SiPM, then internally to each pixel we have the array of SPADs**. The spatial resolution is given by the size of the pixel of SiPM.

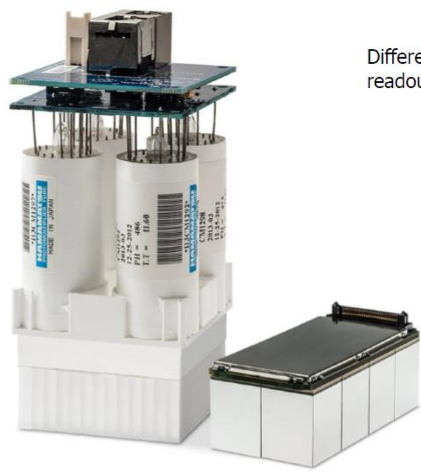
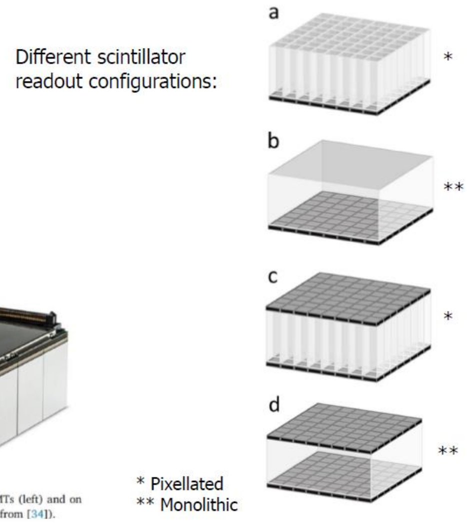


Fig. 2. Picture showing the comparison of PET detectors based on PMTs (left) and on SiPMs (right) in their implementation by Siemens Healthineers (picture from [34]).



The sipm are getting popular because allow to build more compact devices.
Moreover, we can even read out scintillator from both sides with sipm. The advantage of this (configurations c and d) is that if we use a double readout with sipm, we can get not only the information in the x and y coordinate, but also in the depth of interaction.
In fact, in principle to solve the parallax problem in PET we can keep the pixelated camera normally oriented, and we retrieve the depth of interaction information by looking to the ratio of the light collected by the two planes of sipm. If we use only one plane, the light we measure will eb the same irrespective of the depth of interaction. But if we make the double read out configuration, we can get the position in the depth → double layers of sipm allow to introduce the depth in scintillator easily.
It is impossible to make double readout with PMTs, because of dimension limitations.

Moreover, a thin layer of Si device is transparent to gamma ray, it doesn't absorb gamma ray.

SPECT-MRI COMPATIBILITY

Another advantage of sipm as a replacement of PMT is the insensitivity to magnetic field. Nowadays, the trend in medical imaging is multimodality, which means recording a SPECT or a PET image simultaneously with an MRI image, in order to get a functional image superposed to a morphological one.
The problem is that our SPECT or PET detector is inside a 3 Tesla magnetic field → we cannot put a PMT in a 3T magnetic field, because the tracks of the PMT from the photocathode to the dynodes are 'delicate' → electron may go everywhere. So the PMT has to stay outside the MRI.

With sipm this problem is much sensitive, not because trajectories of electrons in silicon are not affected by the magnetic field, because they are also deviated, but if in PMT electrons have to travel like 3 cm, in sipm they have to travel 40um → even if they are deviated, is not a problem.

PHOTODETECTION EFFICIENCY

The **photodetection efficiency (PDE)** of a SiPM is the product of 3 factors:

1. the geometrical efficiency (Fill Factor - FF)
2. the quantum efficiency (QE)
3. the turn-on probability (P_T)

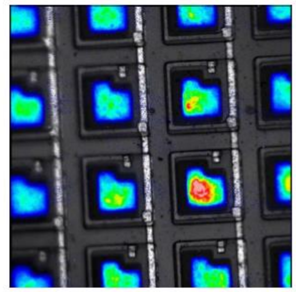
$$PDE = FF \cdot QE \cdot P_T$$

Geometrical efficiency (FF)
The ratio between the active area and the total device area.
Each SPAD cell is surrounded by a dead region determined by the guard ring and the structure preventing optical cross-talk.

Quantum efficiency (QE)
the probability for an impinging photon to create a primary electron–hole pair in the active volume.

Turn-on probability
Avalanche breakdown triggering probability.

FF: depends on the size of the cell.
Example:
50 μm cells, 45% fill-factor



It is the first figure of merit of a sipm. It is the efficiency for one photon to produce a photoelectron and an avalanche.
The difference between PDE and quantum efficiency is that QE is part of PDE, but not the only contribution. The other two contributions are the Fill-Factor (F) and the avalanche probability (Pt). of course the main contribution to PDE is the silicon QE, but not all, because there is also the geometrical efficiency.

Geometrical efficiency (FF)

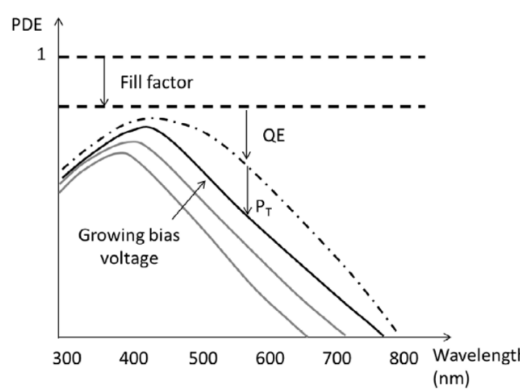
It is the ratio between the active area of the SPAD and the surrounding area. In fact, there are some boundaries around the SPAD that are not usable, but to separate the SPADs. So the FF is the active area of the cell (colored part, because avalanche produces also optical light) where we have the SPAD and the passive network. Unfortunately, the FF can be 50% more or less.

So while an APD or a pin diode is 100% efficient in terms of geometry, the sipm it is not.

Turn on probability (Pt)

It is the probability that a photon triggers an avalanche. It is very high, but not 100%, it could be that one photon is absorbed but it doesn’t generate an avalanche.

PLOT OF PDE vs WAVELENGTH

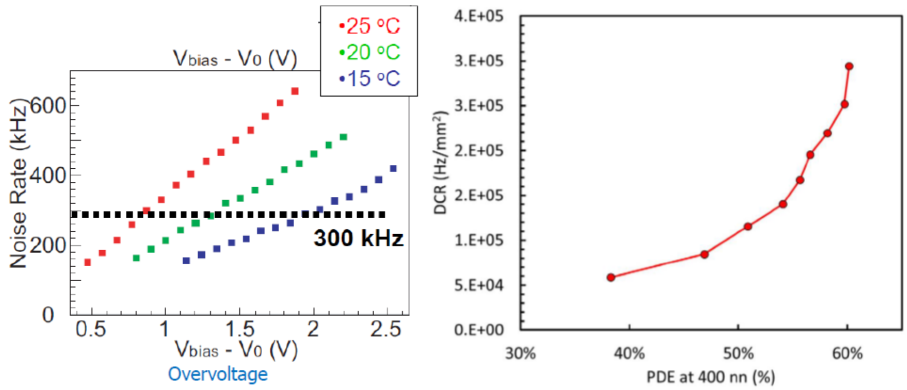


We can see the impact of all factors. The FF prevents the PDE to be 100% simply because we don’t have a 100% coverage of the area will SPAD cells.
Of course, the FF is, at first approx., wavelength independent, because it is a geometrical parameter.

As for QE, it is wavelength dependent; we have a peak around 400-500nm but then it drops down for UV and also for infrared (or near infrared).

Turn on avalanche probability maybe changed, boost, by growing the bias voltage. In fact, the larger the bias voltage, the higher the probability that an electron turns on the avalanche.
This means that growing the bias we increase the PDE, and this is something that is done. However, when we increase the OV, also the dark count increases.

DARK COUNT RATE



- DCR is the current generated in the silicon by thermal generation or tunneling
- DCR is measured in kHz/mm^2 and it increases with OV and decreases with T
- As both DCR and PDE increase with the overvoltage OV, a plot like the one on the right is useful to evaluate the effects on the S/N

It's the rate of thermally generated dark current. It's our main enemy.
It is a problem because the dark current mixes with the signal. As we can see in the left plot, the DCR, which is measured in kHz/mm^2 (Hz because it is an average rate of thermal spiking), increases with the OV, but it decreases with temperature. In fact, the lower the temperature, the less the probability to have thermally promoted electrons.

The dilemma now is: if I increase the OV, I increase the PDE (efficiency), but at the same time I increase the DCR. To understand better the situation, we can refer to the right plot.

Behind this plot there is the increase in bias voltage, each point corresponds to a different bias voltage; on the x axis we have PDE, on y axis the DCR. It is a plot showing the dilemma with respect to an increase of both the figure of merits. For instance, PDE can be increased from 40% to 60% which is very good, but the price is a much larger DCR → the optimal point must be chosen as an optimization of signal to noise.

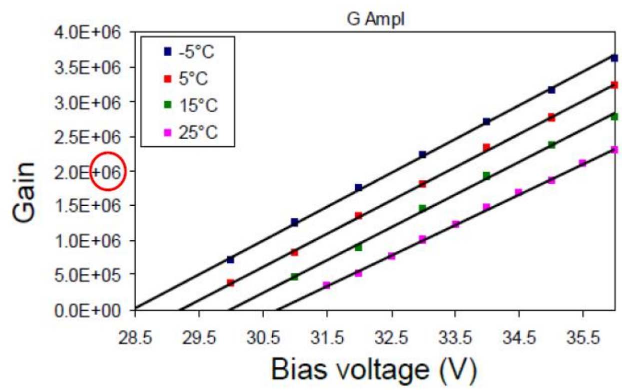
SPAD GAIN

It's another important figure of merit. The SPAD gain is the number of electrons for each starting photoelectron. This gain/multiplication factor is proportional also to the bias voltage, and it's enormous, 10^6 order of magnitude → **for 1 photon we have 10^6 photoelectrons**.

It is important to optimize this number because by doing so we can neglect the noise of the amplifier. We take the noise of the amplifier, we divide by this number and we have the ENC at the input of the chain.

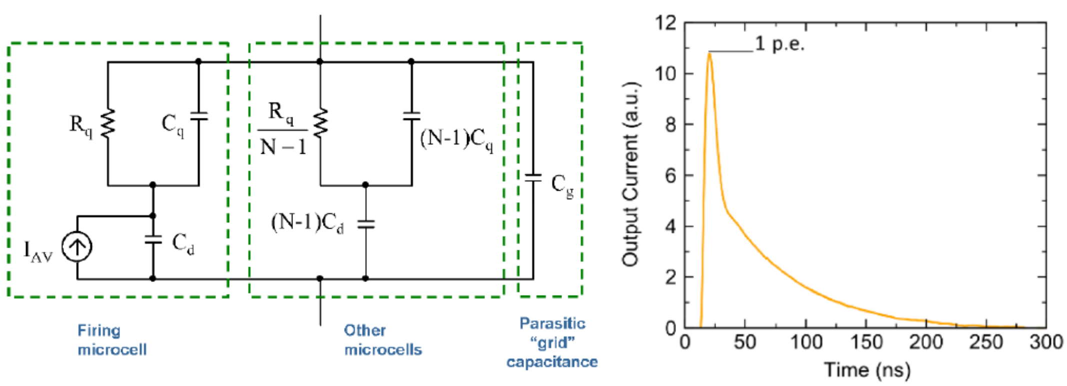
Why does the gain increase by decreasing the temperature?

Because in Si when we decrease the temperature we increase mobility; electrons can travel more free, reducing the probability of scattering. But if electrons are able to move faster, they ionize more → gain increases. It is a property of silicon.



- The multiplication factor M (or gain G) is proportional to the applied voltage.
- The gain increases reducing temperature because electron/hole mobility is higher at lower temperatures. Impact ionization is more effective.
- M is beneficial to reduce the impact of the electronics noise (as in PMT and APD)

SiPM SIGNAL SHAPE



The real output current is complex, because many SPADs are connected in parallel.
The model should include:

- the equivalent model of the fired cell
- the equivalent circuit of the other microcells
- the total parasitic capacitance associated to the large routing interconnections among all the micro-cells and the connection of the device to the front-end
- the front-end electronics input impedance

The output signal shows a fast rise time and a slow decay (with one or two components).

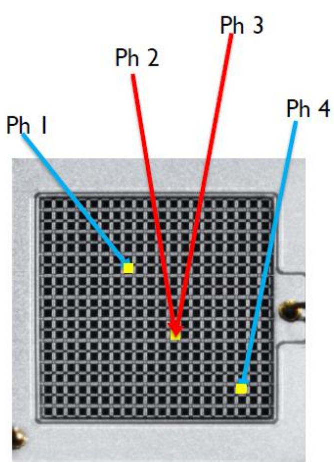
The electrical modellings of the sipm are quite complex, but there are some simplified, like the one below.
First of all, the model is divided in the cell that is firing and 'the rest'. So we have a sipm plenty of cells, one is firing and the others are silent.
So the model is composed by a current generator, that is the cell just firing, the capacitance of the junction of the cell Cd, the quenching resistor Rq and the capacitance across the quenching resistor. This is the microscopic structure of a single cell.

Then, the ‘rest of the world’ are all the other cells in parallel. This ‘rest of the world’ is represented by a detector capacitance which is Cd*(N-1), a quenching resistor that is in parallel for all cells, so it is divided by the number of cells and the capacitance of the quenching overall resistor that is multiplied by the number of cells. We are grouping all the cells in concentrated parameters.

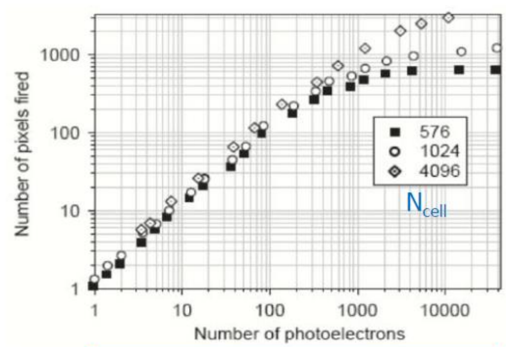
On top of this we have also the capacitance of the grid Cg. In fact, all the interconnections of several SPADs have a capacitance associated. All these Cg of all the sipm, together with the input impedance of the readout circuit, creates a RC time constant.

If we read a sipm, the Cg together with the input impedance of the amplifier creates a time constant that is appearing in the signal in output of the amplifier.
So we don’t have only the decay time constant, but also a rise constant related to this.

DYNAMIC RANGE



The output signal is proportional to the number of fired cells as long as the number of photons in a pulse (N_{ph}) times the PDE is significant smaller than the number of cells (N_{cell}).
Hp: photons are emitted all at the same time, so each single cell has no time to detect another photon.
Scintillation decay time << recovery time



This equation describes (approximately) the saturation of SIPM fired cells:

$$N_{fired} = N_{cell}\left(1 - e^{-\frac{N_{ph}\cdot PDE}{N_{cell}}}\right)$$

Sipm may saturate, cannot accept all the photons on the same cell. The saturation of the sipm is described by the formula in the red box, where the number of the fired cells is given by the number of available cells time the parenthesis. Nph*PDE is the number of photons per PDE, that is the number of photoelectrons. But if the number of photoelectrons is comparable with the total number of cells Ncell, we start to have more photons absorbed in the same cell.
At the beginning, the capability of collecting photons is linear, the formula can be linearized, but then we have a saturation. The maximum number of fired cells is the number of available cells.

NB: we have to operate far from the saturation regime, because if we operate the detector in this nonlinear regime the imaging processing may be affected, the algorithms for centroid identification will be corrupted by this nonlinearity.

NOISE IN SiPM

Noise in SiPMs is given by:

- **Primary source: dark count**

pulses triggered by non-photo-generated carriers (thermal / tunneling generation in the bulk or in the surface depleted region around the junction)

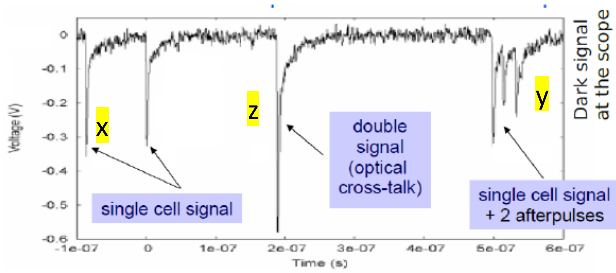
- **Secondary sources:**

After pulse

carriers can be trapped during an avalanche and then released triggering another avalanche

Cross talk

Photo-generation during the avalanche discharge. Some of the photons can be absorbed in the adjacent cell possibly triggering new discharges



The main noise sources are:

- Dark count. Cells can be fired bot only by photons, but also by themselves by thermal promotion of an electron from the valence band to the conduction band. This leads to the typical spots x. Single cell dark count can be recognized because they have the same amplitude, they correspond to single electrons. By the way, this can be also used to calibrate the detector; if we measure the dark count and we measure that for the dark count we have e.g. 47mV in the amplifier, we know that 47 mV corresponds to a single photoelectron. In this way, whatever optical signal we have, once we have the calibration, we can recognize the correspondence of the signal to the number of electrons.
- After pulse. These are carriers created during the avalanche which are trapped, and then released afterwards. Theoretically, in the avalanche all the charges should be released, but unfortunately, few charges are trapped and released later. So we have the normal signal plus afterpulses (y). Afterpulses are electrons released afterwards that trigger an avalanche → fake signal.
- Cross talk. It occurs during ionization; the ionization is expected to produce charges only, but in reality, since avalanche involves ionization of charges, some ionizations can promote electrons in the conduction band and these electrons, when relaxed, can emit photons. So in the avalanche region we may have also optical emission due to ionization → the avalanche region also emits photons.

  This is a problem if photons are able to travel enough distance to go to a neighbour cell, pixel. So the avalanche is occurring in one pixel, but if an optical photon created by the avalanche is travelling away, it can trigger an avalanche in a neighbour cell → not just one cell fired but multiple cells fired, but not by the original optical photons. This is catastrophic (z).

ENERGY RESOLUTION IN SCINTILLATION DETECTION

If we couple a sipm to a scintillator, we perform the measurement of photons generated by the scintillator, that are somehow counted by the sipm.

This is similar to what happens in the PMT and APD. The difference is that in PMT we have a current signal that is the sum of the current generated by the photoelectrons multiplied, while here it is different, because we count really the photons.

The one below is a formula that is the formula of the energy resolution.

$$\frac{\Delta E_{\gamma}}{E_{\gamma}} = 2.355\frac{\sigma_{N_{pe,out}}}{N_{pe,out}} = 2.355\sqrt{\underbrace{\upsilon(E_{int})}_{\text{Intrinsic resolution}} + \underbrace{\frac{ENF}{N_{pe}}}_{\text{Poisson resolution}} + \underbrace{\left(\frac{ENC_{TOT}}{N_{pe}M}\right)^2}_{\text{Electronic noise resolution}}}$$

$$N_{pe} = N_{ph}\eta = E_{\gamma}Y\eta$$

Parameters:

E_{γ}: gamma-ray energy	[keV]
Y: scintillator yield	[ph/keV]
η: photo-electron conversion efficiency	[e^-/ph]
$\upsilon(E_{int})$: intrinsic resolution of the scintillator	[ ]
N_{pe} : number of photoelectrons	[e^-]
M: multiplication gain	[ ]
ENF: excess noise factor of the multiplication process	[ ]
ENC_{TOT}: electronic noise (detectors and electronics)	[e^-]

If we recall the formula of the SNR of a scintillation detector, now we are simply flipping this formula, making a noise to energy ratio.

$$(S/N)_3 = \frac{N_s M}{\sqrt{(N_s M^2 F + ENC^2)}} = \frac{N_s}{\sqrt{(N_s F + ENC^2/M^2)}}$$

At the numerator in the blue formula we have the signal in terms of photoelectrons or converted in energy, while at the denominator we have the signal spread (Poisson spread) multiplied by the excess noise factor plus the electronic noise divided by M^2.

In the black formula it is shown the energy resolution, so the spread is already converted in eV by a conversion factor, normalized over the energy itself. 2.355 it is a term of conversion between sigma and FWHM. Then we have the ratio between the sigma in photoelectrons and the signal in photoelectrons. Npe is obtained because once we have the energy of the gamma ray we can obtain the number of photoelectrons we have in the detector. The conversion factor Y is the same at the numerator and denominator, so it cancels out.

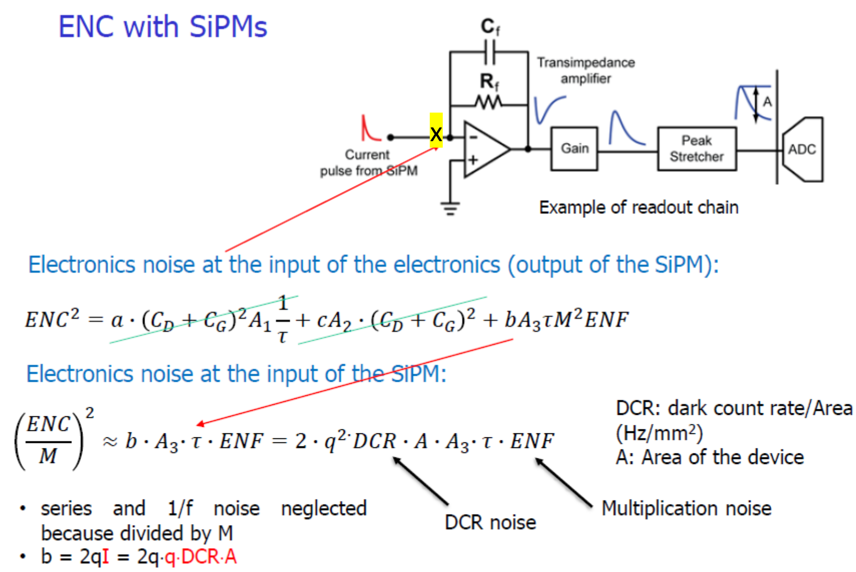
Sigma/Npe is given by the two contributions at the denominator of the blue formula, but simply flipped. The numerator is ENF (that is the same of F) divided by the sqrt(Npe); ENC is also divided by M and Npe.

In addition to this, we have another contribution, the intrinsic resolution, that is due to the scintillator itself. Independently on the photodetector and the amplifier, so on the other two contributions, the scintillator has a limited resolution itself that is physically associated to inhomogeneities in the crystal.

Inhomogeneities means that we cannot assume that the scintillation is uniform all over the crystal. The scintillation sites, the spots where scintillation light is emitted, are not uniform, because the crystal itself is not. So the scintillator is not a perfect emitter, is an inhomogeneous emitter and this inhomogeneity is not due to the detector or the amplifier.
It represents a sort of 'lower bound' because it cannot be eliminated.

To improve the performances, we should try to have the lowest possible ENF. Also sipm have an ENF because avalanche is not deterministic. Then, Npe depends on quantum efficiency, on PDE. The PDE provides in fact the highest probability of photoelectrons emission. Also multiplication M is important. In conclusion we have the electronics noise ENC.

ENC WITH SiPMs



From the sipm we can read the current, then we have for instance a transimpedance amplifier or a charge preamplifier with a discharge resistor, then we may have a gain (shaping amplifier), then the peak stretcher and ADC. This is a typical possible electronic chain.

In the formula of the ENC we have the series noise due to the thermal noise of the input transistor; the 1/f noise due to the 1/f noise of the input transistor; we have the parallel noise which is due to the factor b that is the shot noise associated to the dark current (thermally promoted electrons).
However, this dark current is subject to multiplication, unfortunately. So the shot noise b is multiplied by M^2, because the shot noise in the sipm is also multiplied by the gain of the sipm. Then we have the ENF because multiplication is not deterministic.

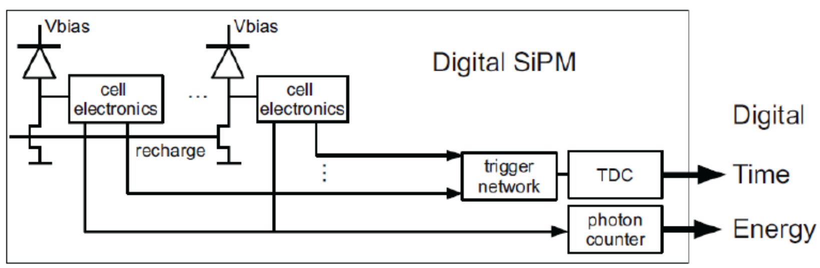
These three contributions are not all important, because if we take ne ENC at node x and we divide by M, as in the previous formula, what matters is ENC/M in terms of noise.
But if we take the first term of the ENC^2 formula ad we divide by M, this term is going to be negligible. Similarly for the second term. This is why front-end design is much relaxed → we can relax the noise property of the amplifier and also the capacitance. In the detector capacitance Cd is in fact included the parasitic capacitance of the cables, but if we divide by M if is negligible → **we can put the front end amplifier even meters away from the detector**.

This is not true for the parallel noise; M^2 cancels out and so the parallel noise in the formula is b*A*tau*ENF and we have to work on these quantities, they are not reduced by multiplication, but worsened by it, because ENF is introduced by multiplication.

b is represented by the formula of shot noise. In this formula we have the leakage current but in sipm the dark current is quoted in DCR, dark count rate per unit of area. It is the average frequency of dark counts in Hx normalized per square mm. To transform it in current, the current 'I' is the DCR multiplied by the area of the device to have the absolute number of spikes, and then we multiply by the charge q, because we need a current. This is the main noise we have to care about in sipm readout.

The shape of the filter is also important to collect the signal, that is the number at the denominator of the formula. But the question is: are we sure that we are collecting all our photoelectrons in the signal, that we are shaping the signal in a way that we maximize the collection of the signal?

DIGITAL SiMPs



- fabricated in CMOS process technologies
- each pixel contains one SPAD and its relative front-end electronics, which provides hit/no-hit information, as a digital signal
- the energy information is obtained by summing up all digital outputs
- 'hot' SPADs can be switched-off at cell level, so reducing significantly the overall dark-count noise (but also the PDE)

There is a special type of sipm called digital sipm. In terms of SPAD it is the same, it has individual SPADs, but in each single cell there is a custom electronics. This custom electronics in the cell provides a trigger, it is a kind of *local comparator*.
So in analog sipm we directly measure at the output terminals the analog current. In digital sipm all the firing is done on board of each single cell, because we have a discriminator. If on board of each single cell we have a discriminator, we measure the number of fired cells by counting the triggers.
In the periphery of the device we have a photon counter that is a counter that is increased for each trigger that has been fired on board of each sipm.
So if for instance 8 triggers are fired, we have 8 photons. Moreover, the single triggers may provide the timing information, because the first trigger among all that is fired provides perfectly the earliest possible time stamp. So **we don't have to pass the cumulative signal in a common discriminator to get the time stamp, because we have a local discriminator per cell**. This is very nice because we get the timing information at the earliest possible time.

Dark current diversity

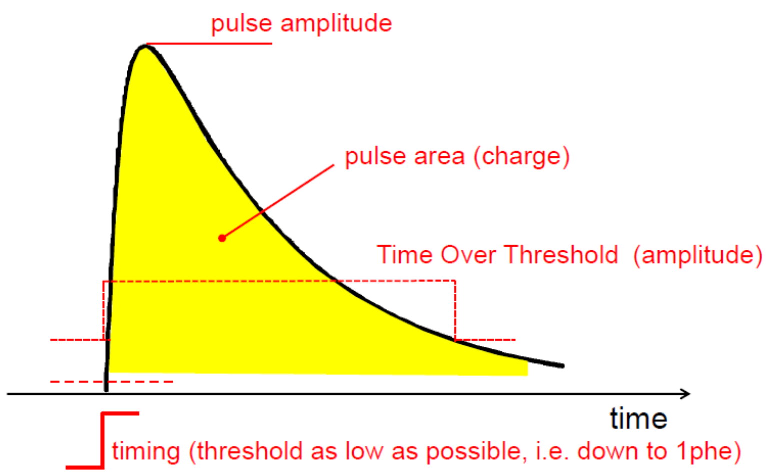
There is another advantage of digital sipm, that is the most important one.

When we are talking about DCR, our main enemy, in reality the sipm is not uniform, but the dark current is dominated by few other bad guys. So we don't have a uniform dark current everywhere, but we have few (5-10-100) hot spots. We have hot cells that are firing all the time.
If we would know which are such cells and we suppress them, we would highly suppress the dark current of the entire device.
If I suppress the hot spots, have little reduction of QE but negligible, but we have reduced the 99% of the main contributors to the DCR.

This is not possible in the analog sipm because everything is put in parallel, but possible in digital sipm because if we know, by investigating the local triggers, the triggers that are spiking most of the time, we can suppress the hot spots, we shortcitcuit the hot cells.

Disadvantages of digital SiPMs
However, we don't have only digital sipm because we need some 'intelligence' on the cell, and this takes area, space, so we reduce the PDE; moreover, we need to use CMOS technology, because we are not just building a sensor, but a sensor plus the transistors for the trigger. CMOS technology unfortunately is more noisy, the DRC is higher in CMOS technology than in custom photodetector technology.
So it is true that we can suppress hot cells, but the technology is intrinsically noisier → we need to find a trade-off.

MEASUREMENTS ON A SiPM SIGNAL



Let's suppose we have a sipm signal collected by a transimpedance amplifier. To take a measurement of the energy of the gamma ray that has produced the signal we can do two things:

- Measure the peak of the pulse. To do so we need a very quick peak stretcher, the bandwidth of the peak stretcher has to be as large as the bandwidth of the signal. If the peak stretcher is too slow, it cannot track the pulse amplitude.
- Measure the pulse area. If I measure the area of the pulse by a simple integrator (amplifier + capacitor in feedback), we have a beautiful measurement of the area in yellow, and we don't have bandwidth requirements.
- Measure the timing. If we have a very quick pulse, we take the pulse, we put at the input of a comparator, we put a threshold on the comparator as low as possible and once the threshold is triggered we have a timing signal. This is fundamental for PET, where we need timing.

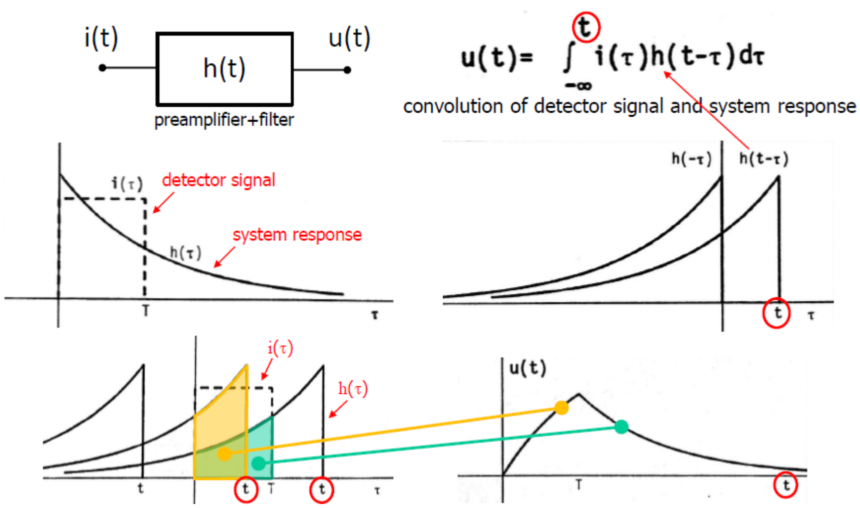
Threshold

In principle, the threshold should be as low as possible. This means **as low as the amplitude corresponding to a single photoelectron**, because the single photoelectron is the smallest possible amplitude.

NB: it is important that the electronic noise is lower than the amplitude corresponding to a single photoelectron. The electronic noise is typically small because its the contributions to the ENC is negligible.

If we have an electronics that is measuring timing we can do also a measurement called 'time over threshold'. We measure the time interval between the first firing triggered by the first photoelectron (step increase) and when the waveform goes again below the threshold. This quantity is proportional to the signal → the larger the energy of the signal, the larger the time over threshold.
This time over threshold is very often used in readout circuits to measure the timing and the energy. The advantage is that we don't need a separate electronics for energy and timing measurements, but we use a single electronic chain.
The disadvantage is that the relationship between this time over threshold and the event energy is not linear.

RECALL OF CONVOLUTION OF A DETECTOR SIGNAL i(t) WITH THE SYSTEM RESPONSE h(t)



Filter properties for noise reduction are embedded in the factor A3 of the formula. When a signal is delivered as an exponential delivery of photons, which is the best shape to be used in the filter?
h(t) is our signal processor, composed by preamplifier + filter. u(t) is the response of our processor for an incoming pulse i(t). u(t) is the convolution of the input signal and the processor transfer function h(t). The convolution operator is the integral from -inf up to time t of the signal i time the flipped and delayed h.
In practice, it means that we have our detector signal, that is for example a rectangle lasting t, but it could be a delta of Dirac.
The system response is instead an exponential decay, like the one provided by a transimpedance amplifier. So we take h(t), we flip it (h(-tau)) and we delay it by t. The convolution operator is the integral of the product then.

For instance, looking at the bottom couple of plots, if my signal is the i and then we have the time t where we are evaluating the convolution operation, the area I need to integrate is the area in yellow. The integral starts from -inf, but the signal starts from 0.
The value in yellow is then the point u(t) in the response.

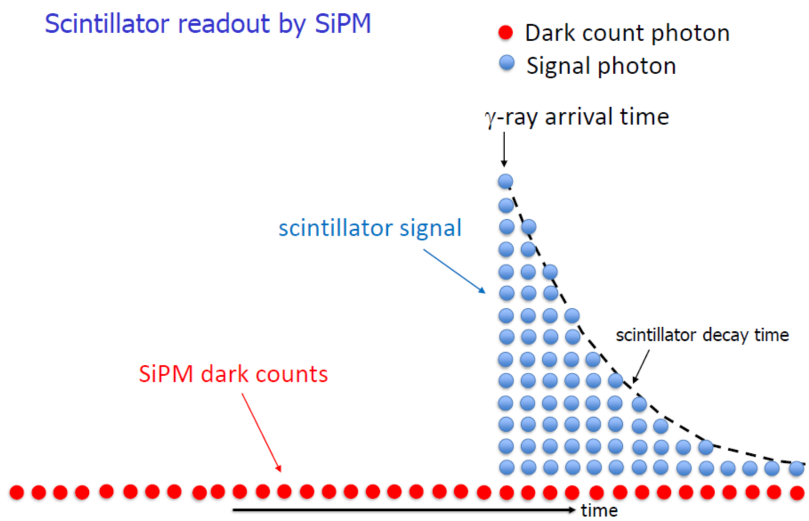
Then, if I'm evaluating the convolution at a different time t, I need to shift the exponential. But if I shift the exponential, the area superposed between exponential and detector signal is the green one. This can then be done iteratively to draw the system response.
The amplitude I take of the response is then the peak, neither the point in yellow nor the one in green.

With this example we can understand how much important is the shape of the system response. In fact, the shape of the system response flipped matters in the integral of convolution, and so the peak of u(t), the amplitude of our measurement, is the value that we put in the formula.

$$\frac{\Delta E_\gamma}{E_\gamma} = 2.355\frac{\sigma_{N_{pe,out}}}{N_{pe,out}} = 2.355\sqrt{\upsilon(E_{int}) + \frac{ENF}{N_{pe}} + \left(\frac{ENC_{TOT}}{N_{pe}M}\right)^2}$$

In the formula, the amplitude is expressed in Npe, but the number of photoelectrons depends on how much large is the signal that we have measured at our amplifier.
Maybe the scintillator provides 1000 photons, but it is up to us to make the best usage of these 1000 photons, and the best use is to choose a processor that provides the highest amplitude.

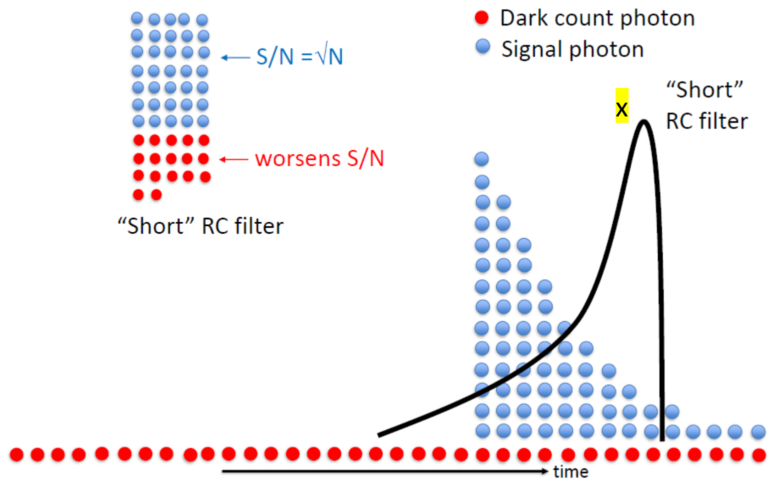
EXAMPLES ON THE SHAPE OF THE SIGNAL WE CHOOSE



Let's suppose we have a sipm and we have a scintillator. The scintillator provides the sequence of optical photons that are represented by the blue spots. Each blue spot represents the delivery of a single photon.
If we consider the scintillator decay time, this time is the envelope of all these emissions of single photons. So when we have the scintillator decay time, it doesn't mean that photons cannot eb delivered immediately. Of course some photons are delivered immediately, but then there are photons that are delivered afterwards, till the tail of the exponential envelope. This is the photon delivery.

Then, superposed to this, in red, there is the dark count. Dark count consists of single photons, it is a single fired cell at a time. Even if this dark photons are Poisson distributed, we assume them as a constant background.

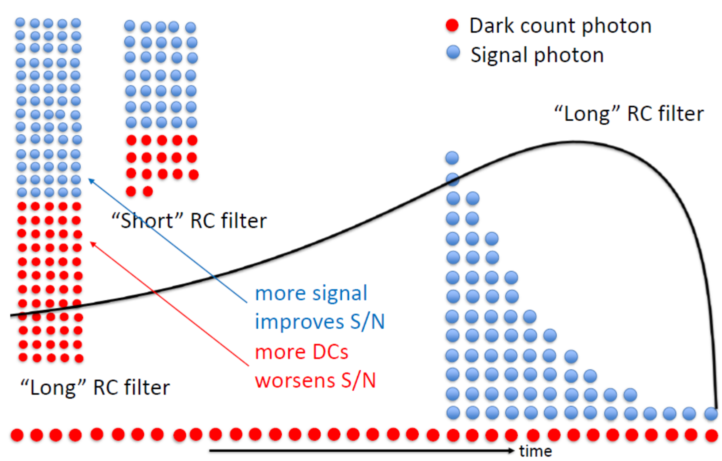
The choose of the filter matters because when for instance we have an RC filter, we take it, we flip it and we scroll all over the signal. The integral of the signal is the photons counting with the RC filter.



When we have the peak of the signal RC we are counting more balls (photons), in the tail of the RC signal we are counting less balls. So the shape of the filter matters because in the integral the filter is counting the signal at a given time but it is not counting effectively the signal in another time.

The counting is on the left, and are the photons counted by the filter in position x; the intrinsic noise of the signal is the square root of the number of counted photons. The less balls we have counted, the more spread is the signal.
In addition, we have counted also a number of red balls. The SNR is the signal in blue summed to the electronic noise. This situation should be optimized by moving the filter and match the peaks.

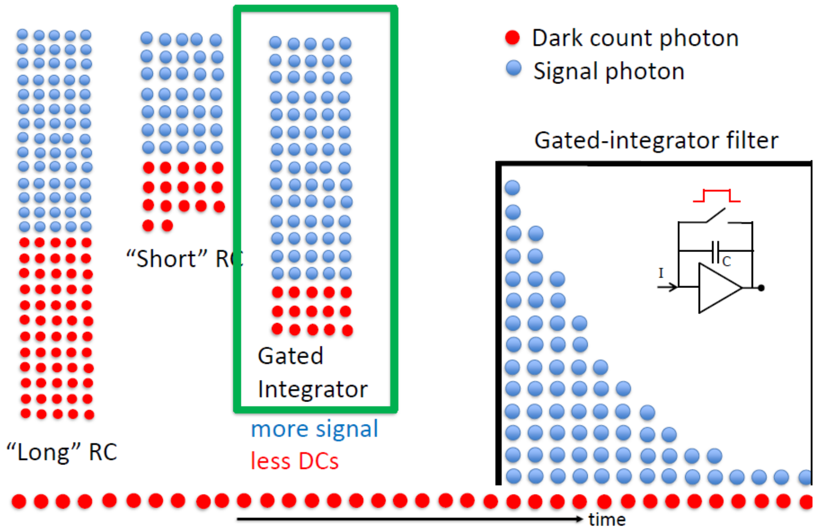
Otherwise, we can change the filter and take a longer filter, with a longer time constant, so a longer integrator.



This is beautiful for the signal, because with a longer tail in the convolution we count all the blue balls, photons → we have a larger amount of signal and the intrinsic relative fluctuation is lower. However, we have also counted many red balls. So we have increased the integral of the dark count rates.

In conclusion, the shape of the processor matters, not only because of the filter coefficients (A1, A2, A3) for the noise, but also for the way we collect the signal with respect to the way we collect the dark count rates.

One solution for the best filter: the gated integrator



The gated integrator is a simple integrator whose switch, and so the integration, is open when the signal arrives and the integration is closed when reasonably the signal has extinguished. Is up to us to select this interval.

This processor is the best because we have practically counted all the blue balls except for the ones after the end of integration, and we have counted only the red balls inside the integration window.

ELECTRONICS READOUT FOR SiPMs

We have 4 approaches to read out the signal of a sipm:

- Charge preamplifier
- Transimpedance amplifier
- Voltage readout
- Current readout

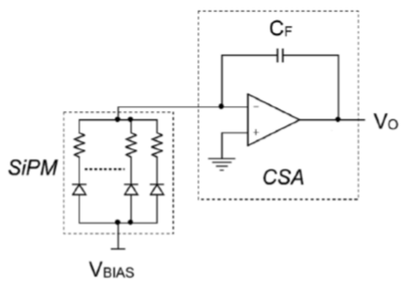
CHARGE PREAMPLIFIER

The reasoning done before on the 'blue and red balls' is how we process the signal, so preamplifier plus filter. Now it is just preamplifier, how to collect the original signal form the sipm.

The charge preamplifier was considered the best circuit for detectors because it integrates the charge from the sensor, it is very stable, the conversion is on a very reliable external component and so on. However, for sipm readout it is not convenient.

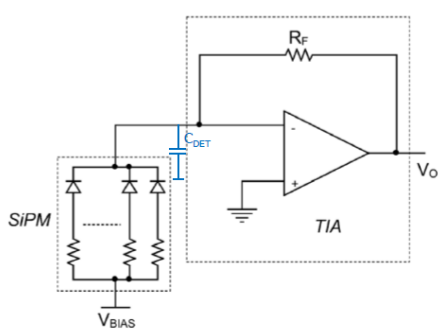
We can still use it, by connecting the virtual ground of the charge preamplifier to the sipm, the sipm delivers the current pulse, we integrate the current pulse over Cf and all is done.

The **problem is the amount of charge**, it is too much for the charge preamplifier.
For example, if we consider a pixel for PET (511 keV). In a scintillator like LYSO and considering a sipm with 40% PDE (40% efficiency), 1 gamma ray at 511keV produces 2000 e-. However, these are not 2000 electrons integrated in the feedback of the charge preamplifier, but we have 2000 SPAD cells fired and each cell produces charge of 10^6. One cell is fired by 1 photoelectron among the 2000, but then the cell produces a charge of 10^6. So we have to take 2000 electrons multiplied by 10^6 and we get 320pC, that is an enormous charge.
If we consider that an amplifier has a maximum range of 1V before saturation, if we want the amplifier not to saturate with 320pC over Cf, we need a Cf of 320pF to store them, otherwise we saturate.
However, in CMOS technology we cannot have more than 10pF → impossible in CMOS technology, only in a discrete component amplifier, but we are interested in CMOS.



The classic Charge Sensitive Amplifier (CSA), which is the standard front-end configuration for radiation detectors, often does not represent an optimal solution for a SiPM. Due to the large amount of charge at the output of the SiPM, a large value of the integration capacitance C_F is required, to avoid saturation of the CSA.
Example: in PET, 511keV produces about 2000e- in LYSO+SiPM. Considering $M=10^6$ of SiPM, the charge at the input of the CSA is about 320pC. If the maximum output voltage swing of the amplifier is 1V, the feedback capacitance needed would be 320 pF, which is not feasible in CMOS technology.

TRANSIMPEDANCE AMPLIFIER



The TIA converts the current pulse of the SiPM into a voltage and, if its bandwidth is large enough, it is able to preserve the fast rise time of the SiPM signal, thus enabling the achievement of good timing performance. A fast discriminator can be used after the TIA to extract the time information. Energy measurement by a following integrator may require again a voltage-to-current conversion by a resistor. Slow pole by R_F and C_{DET} in the loop may cause instability and may require a compensation cap. C_F in parallel to R_F.

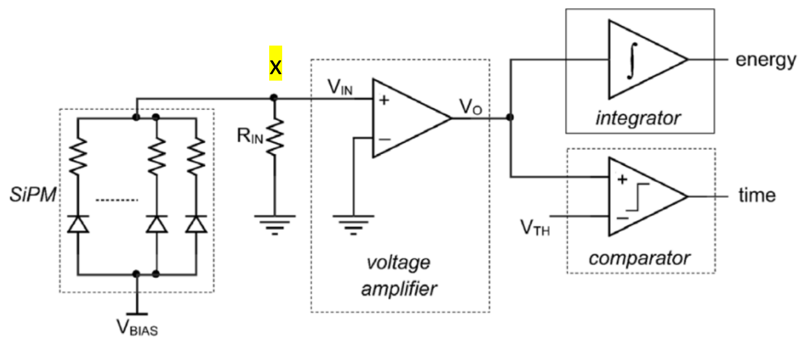
It is an amplifier with a resistor in feedback. Its output voltage is the input current multiplied by the resistance, and there are no problems of resistance. In this case, in order not to saturate the amplifier,

we don't need a large component like in the charge preamplifier, but a small one. Moreover, the transimpedance amplifier allows to have the same shape for the input and output pulse.
This was not the case of the charge preamplifier, which makes an integration → HF component of the signal is lost.

The reason why the transimpedance amplifier is a nice front end is that it converts the current signal of the sipm in voltage and then we can do whatever we want with the voltage, like integrate it and obtaining the energy.

The drawback is stability. Its low stability is due to the capacitance of the sipm, shown in blue. If we have a transimpedance amplifier, we cut the loop at the output and we calculate the loop gain, we have a pole due to the capacitance Cdet, and this may slow down the loop gain and eventually create instability (there is also another pole of the main amplifier in the loop).

VOLTAGE READOUT

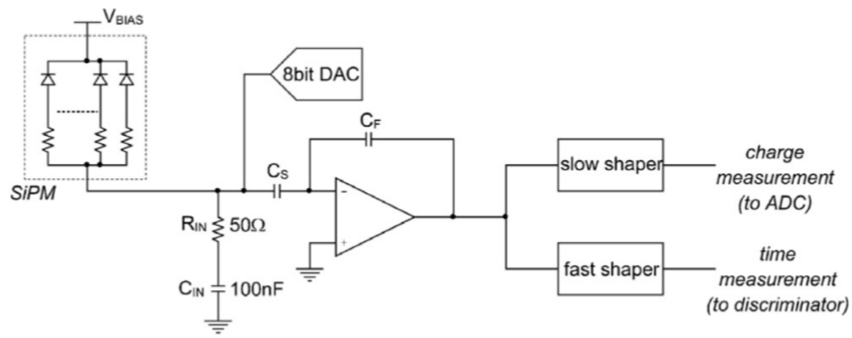


A very simple conversion of the signal current into voltage by R_{IN} is obtained, then a voltage amplifier is adopted. Bandwidth requirements for the voltage amplifier may be critical, to preserve signal speed. A voltage-to-current conversion by a resistor may be also required here to integrate the current in a integrator for amplitude measurement. Large value of R_{IN} may cause too large voltage variations across SiPM, changing the gain and introducing non linearity.

If the sipm provides a huge signal, so huge that it creates problem like in the charge preamplifier, why not converting the signal over a resistor?
It is possible, at the cost of one resistor; usually the resistor is then followed by a voltage amplifier (or a buffer if the amplitude is enough) and then with the output of the amplifier we can do whatever we like. For instance, if we send the signal to a comparator, we get the time peak off (the amplifier must be fast enough).

The drawback of this configuration is that while in the previous configuration the node (-) of the simp is at virtual ground and so the voltage across the sipm is fixed, in this readout node x is moving. But if this node is moving, the overall voltage across the sipm is changing, and the risk is that the sipm changes the multiplication gain, which depends on the overvoltage.
As long as the signal on the resistor is in the order of mV it is not a problem (50V of biasing of the sipm), but if the signal starts to be 1V we have a problem, we are changing multiplication.
Since this effect is signal dependent, we are introducing **non-linearities**.

Example of voltage readout



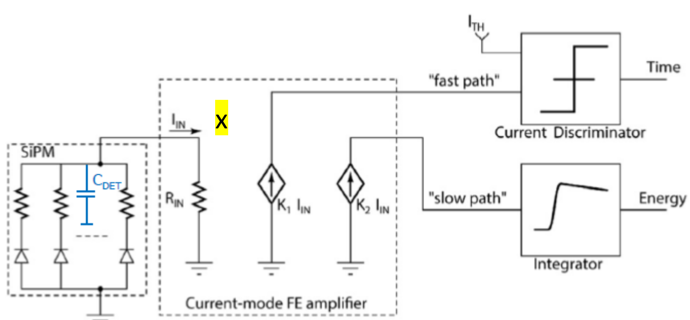
A DAC is used to adjust the bias of the SiPM. The capacitor C_{IN} is introduced to make a AC coupling of R_{IN} to ground and let the DAC set the DC voltage to the SiPM output electrode. $R_{IN}=50\Omega$ allows matching with cable impedance. The inverting voltage amplifier is made through capacitors (gain=C_S/C_F). The output voltage is processed by a slow shaper for amplitude measurement and a fast shaper for timing measurement.

We have the sipm, the conversion resistor Rin. Rin is 50 Ohm and in principle should be grounded; in reality, we have a decoupling capacitor Cin. If we consider HF, so the signal, the Cin is a shortcircuit, so the 50Ohm is grounded for the signal. In DC, the capacitor is open, the Rin is floating and so this allows the DAC to set and program the DC voltage of the sipm. So the sipm has a main bias voltage, but if we wanto to adjust the other end of the sipm with a certain voltage, we can use a DAC. The purpose is for instance to set a desired gain.
The DAC sets the gain, and the DAC is not interfering with the signal conversion.

Then we have the voltage amplifier, that in this case is made through capacitors because capacitors are open circuits in DC, so the virtual ground in (-) is not interfering with the voltage set by the DAC, because in DC Cs is in open circuit. In the signal regime it is instead an inverting configuration.

Then the output can go to a slow shaper for charge measurement or a fast shaper to have a steep pulse for threshold discrimination.

CURRENT READOUT



A current buffer with very low input impedance is coupled to the detector and exploits the advantage of a small R_{IN} value which preserves signal speed (R_{IN} and C_{DET} create a time constant at the input). The output signal of the buffer is a high impedance replica of the current pulse generated by the detector that can be easily reproduced with different scaling factors (K_1 and K_2) in different "fast" and "slow" signal paths, optimized for charge or time measurements. Typically, large bandwidths are easier to be achieved with current mode amplifiers.

The sipm is readout with a current buffer. The current buffer is a circuit that draws current at low impedance and gives back current at high impedance. It connects the sensor with a very low impedance (in principle 0Ohm), so it absorb any current delivered to node x, and it provides ideally current as a current generator at high impedance → no suffering from the impedance of the load after.

I have 2 possible options:

- Slow path, that is an integrator. So I take the current, that is the exact replica of the input current in terms of shape, and we integrate it to integrate the charge of the signal. So in this path we are performing the measurement of charge.
- Fast path: in this one the current is compared to a threshold → we get a digital trigger signal.

This current readout modality is popular because the output of the current buffer can be directly fed to an integrator, which is the best readout filter for sipm (see considerations on 'blue and red balls'). So if I have directly a current output, it is very easy to send the current into an integrator, while with a voltage readout integrating a voltage is more tricky: we have first to convert the voltage into current with a resistor and then we can use the integrator.
With the current readout we have current already provided.

But already the charge preamplifier is an integrator, and the problem of it was the need of a very large capacitor to store the charge coming from sipm. In principle, we should have the same problem also here, but we don't have.
Thanks to the current buffer, we can adopt an attenuation of the signal. The generator on the left is not exactly Iin, is K2*Iin, where **K2 is not a gain, it is an attenuation factor →** it is not simply a 1:1 current buffer, but an attenuation buffer. So if we attenuate the signal we can integrate it into an integrator with a smaller capacitance.

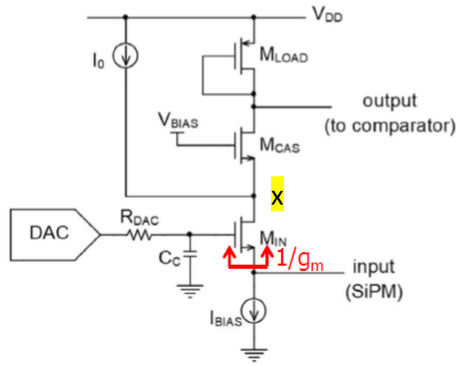
The price to be paid for this attenuation is that in electronics, any time we attenuate a signal, we have to cope with the noise of the following stages. It is the 'reverse story' of the early amplification, that is used to neglect the noise of the following stages. Here we are attenuating the signal and the noise sources of the input → any noise source of the integrator matters.
Of course, thanks to the multiplication of the sipm we can neglect the electronic noise of the amplifier, and it is still true now, but we don't have to abuse of this feature. If we introduce a massive demultiplication, we have to verify if the noise of the second stage is still negligible.

In parallel to the slow path we have a fsst one, that is a current discriminator: we take the current and we see if it overcomes a threshold. Also in this path we have an attenuation factor K1, different from K2. The reason of K1 != K2 is that we can use a different and less attenuated factor K1 because in the fast path we need just a discrimination, not an accurate measurement → we don't care if we are saturating the signal, our goal is simply to overcome a threshold. Hence usually K1>K2, because K2 needs to keep the signal in the dynamic range of the integrator, while K1 not.

Example of current readout – 1

The simplest current buffer we may use is the cascode, because the cascode Min is connected to the input of the sipm and the input impedance is the 1/gm of Min. The current is then provided at high impedance by the drain of Min to the load. In the case of the image, the load is a resistor implemented through a transdiode.

In this example we are making a current to voltage conversion, we are not integrating the current or using a threshold on it. Then the voltage is sent e.g. to a voltage comparator.
The equivalent impedance of the transdiode is 1/gm.



It is based on a open-loop input common-gate stage and a load resistor (transdiode) to form a voltage signal that is compared to a threshold by means of a fast comparator for the generation of the trigger signal. The input impedance R_{IN} is the 1/gm of the common-gate. Large current, and therefore power, on the input branch is required to reduce the 1/gm impedance, if needed.

Apparently, we have a conflict, because we want to keep 1/gm of Min small to have a small input impedance. A brute force approach to keep 1/gm of a transistor small is to use a large bias current, because the 1/gm is proportional to the current directly. However, if the same Ibias would go through Mload (all over the branch), while we are reducing the 1/gm of Min, we are reducing also the 1/gm of the Mload.

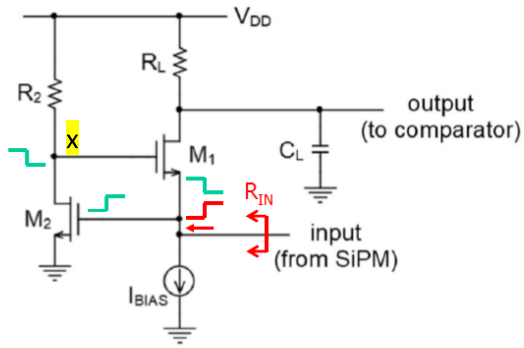
To solve this conflict, in node x we extract part of the Ibias. Through I0 part of the Ibias is taken out, and a part flows in Mload → Mload has a lower current than Min.
So we have the huge Ibias through Min that grants a small 1/gm and a little current on Mload and so a larger 1/gm for it.

Finally, the cascode voltage is not fixed, but it can be tuned on the gate of Min with a DAC. So in terms of impedance it is a cascode because the gate is fixed, but the DAC allows to tune the gate of the cascode and so the source of the cascode → <u>with the DAC we can tune the voltage on the sipm</u>.
From the point of view of the DAC, the cascode is a follower, transferring the voltage to the sipm.
The Rdac-Cc is a passive LP filter to filter the DAC noise.

Example of current readout – 2
We can try to have a lower impedance than 1/gm. This is done by the regulated cascode circuit.
We have still M1 that is the cascode, Rl is the load (acts like Mload as a conversion element) but, in addition to M1, M1 is not fixed to a DAC, but there is a loop formed by M2 and R2.

This is a negative feedback loop. A loop of this kind, when stimulated by a perturbation, it opposes to the perturbation. If we assume the perturbation is the sipm current in red (we are forcing a test current at the input of the circuit). If we stimulate the input node with a current, we get a positive step in the nose (we are pushing current in the node so it rises voltage). But this step in red is transimitted to the gate of M2. If the gate of M2 rises, the drain goes down, because M2 – R2 is a common source amplifier.

But if the drain of M2 goes down, also the gate of M1 goes down, and so the source of M2, that follows the gate → the loop counterbalances the initial perturbation in red.



It is based on a regulated common-gate configuration. The closed-loop impedance R_{IN} is equal to the open-loop input resistance $1/gm_1$, decreased by a factor equal to the loop gain $gm_2 \cdot R_2$. This configuration provides a better compromise between input impedance, and therefore speed, and power consumption. Stability is of concern in this kind of circuit, as it includes multiple poles.

Moreover, **if the LG would be infinite, the compensation of the perturbation would be perfect, in the sense that the green response of the loop would match perfectly the red stimulation** → net result would be zero.

What is the Rin impedance of a node where, when we inject a current, the voltage remains 0?
The Rin = 0.

So the regulated cascode is popular because the input impedance is not 1/gm, but 0 → perfect current buffer.
However, the LG is not infinite, but if I cut the loop at the gate of M2 and do a full turn, it is the product of 1/gm of M2 and R2, because M1 is a follower with gain 1. And the input impedance of a negative feedback closed loop is the OL impedance (that is 1/gm of M1, the cascode) divided by LG. So we take (1/gm)/(R2*gm2).

Problems
It is a negative feedback circuit, a closed loop circuit → the intrinsic problem of a closed loop circuit is stability. So with a CL circuit we have to check that in the LG we don't have more than 2 poles when we cross the 0 dB gain. If we have just 2 poles the phase margin is still larger than 0, but with more than 2 poles we have problems. In fact the perfect stable circuit is with one single pole.

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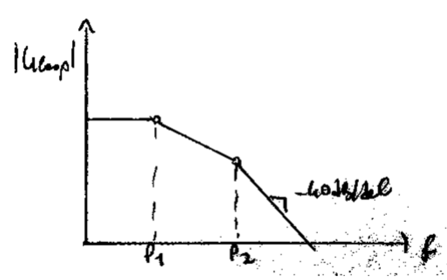
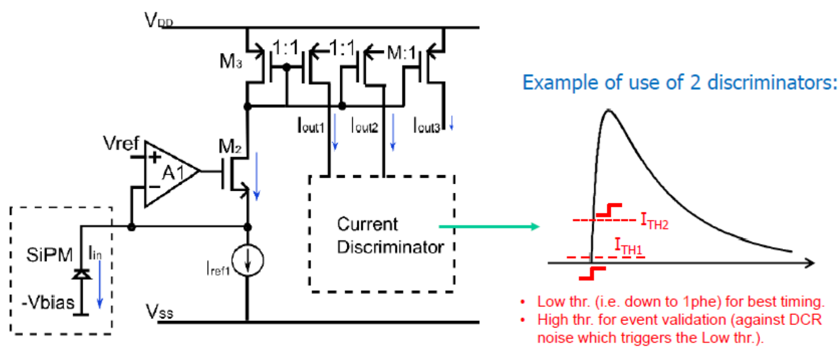


In our loop, we have 2 poles: one is in node x (it is a high impedance node because we have the resistor R2, the gate of M1 and the drain of M2) and one at the input.

The pole at the input is created by the capacitance of sipm together with the 1/gm of M1.
It can be verified that with these two poles in the loop the frequency response of the loop is with 2 c.c. poles, which is not a problem but it is neither ideal.

Example of current readout – 3



It is still based on a regulated common-gate configuration. The closed-loop impedance R_{IN} is equal to the open-loop input resistance $1/gm_1$, decreased by a factor equal to the gain of the amplifier A1. Vref can be used to bias the SiPM. The current at the output of the input branch (M_3) can go, without demultiplication (I_{out1}, I_{out2}), to current discriminators, while a third output (I_{out3}), can be used for amplitude measurement, with a demultiplication M introduced not to saturate the integrator stage.

It is still a regulated cascode, but the loop is made in a different way. Here we don't have a common source (M2 before) but an operational amplifier. The advantages are the same but the loop gain is much larger. Again, we must be careful about stability, because the opamp A1 may have multiple poles.

Current readout

This current regulated cascode configuration we can see how the current readout is made. It is made by a current mirror.
The current flowing at the input of the cascode is mirrored with the multiple outputs by means of a current mirror. So the two Iout of the original current readout are simply current mirrors, and the attenuation factors K1 and K2 are attenuations factor in the mirror.

What is indicated in the image as Iout3, that is simply the input demultiplied by M (K2 = 1/M), is sent to an integrator (not shown in the image).

The channel for the 'fast path' of the current discriminator doesn't require a particular attenuation for K1, in fact K1 = 1, the mirroring factor is 1:1 → we implement different attenuation with a single input.

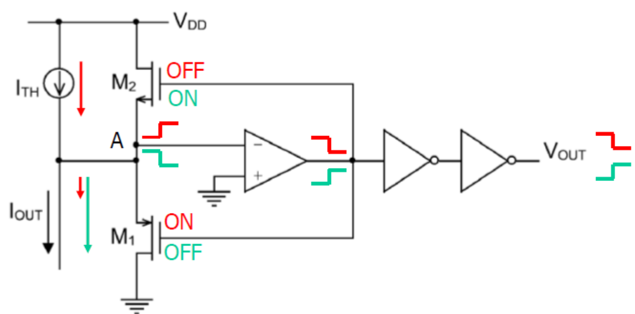
Moreover, differently from the original configuration where we have only one single current discriminator, here we have 2.
One is using an ultralow threshold Ith1, and the second a threshold Ith2. Why not using just a single discriminator with the lowest possible threshold (i.e. down to a single photoelectron)?
A single discriminator with the lowest possible threshold is good for timing, because being triggered by the first single photon is the very first time in which we get an information → with a very low threshold we are triggered by a single photon.

The problem is that if we lower the threshold too much, we may be triggered by dark counts. Dark counts are in fact as high as single photoelectrons. So if we put a threshold low as a single pe, we are triggered by a pe, but we are also triggered by dark counts.

Solution

To overcome this problem, we can rise the threshold, but it is a pity because we loose the opportunity to be triggered by the first photon. Hence we use the information of a second threshold; the threshold Ith2 can be triggered only by the true signal, not by the dark counts.
So if we have a logic (an AND logic) and we get the trigger by the lower threshold and the trigger from the upper threshold, we can say that we have a valid event → dark count is discarded.
The first threshold is kept to keep the good timing performances.

Example of a current discriminator



I_{OUT} is the output current of the front-end (see previous stages). In DC, with I_{OUT}=0 or small, the PMOS M_1 is ON and carries the difference between the threshold I_{TH} and I_{OUT}. The voltage on the node A is high, the output voltage of the inverting amplifier is low and sets further the PMOS to be ON. As soon as I_{OUT} overcomes the threshold I_{TH}, due to the arrival of a valid event, M_1 turns off and the NMOS M_2 turns on, the voltage at the node A goes down, thus the output of the amplifier makes a fast positive transition, which further sets OFF M_1 and ON M_2.

Iout is the current exiting from the mirror of the previous stage that we have to compare, and Ith is the threshold. When the current is lower than the threshold, so the current red Iout is smaller than the red Ith, we will have a negative output of the discriminator, while if the Iout > Ith, the output rises up.

In node A we have two cascode, n cascode on the top and p on the bottom. Then we have an amplifier that switches on one or the other cascode. Let's start from the condition Iout < Ith.

Iout < Ith

A net current enters in node A. If so, the voltage of node A rises up; if so, the output of the amplifier will be a negative step, so the p-mos gets on and takes the current, while the n-mos is off → it is a positive feedback, because the transistor M1 takes the current to reach an equilibrium point.
The conclusion is that the output of the amplifier remains low and after the two inverters is still low (the inverter are used to have a rail to rail output).

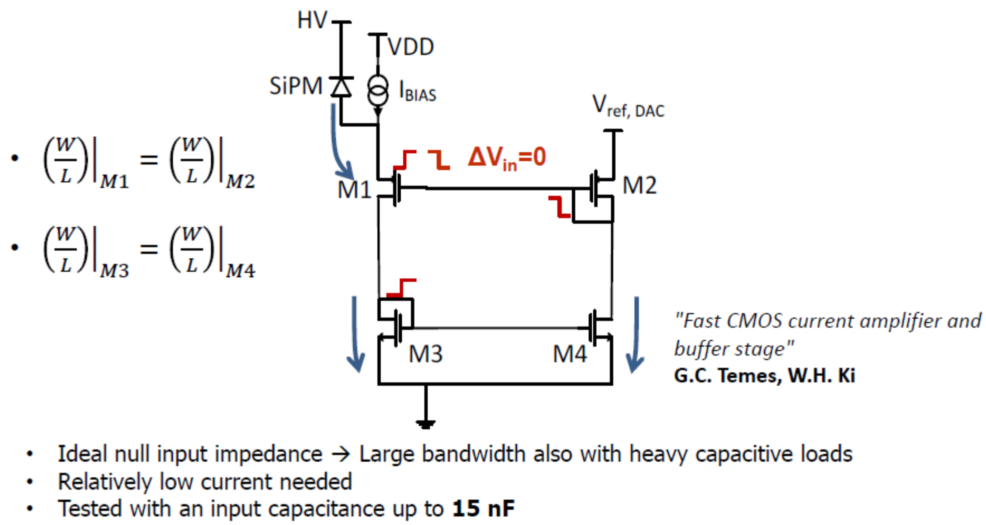
Iout > Ith

This occurs when the signal overcomes a threshold. Now the net current at node A is exiting the node, we have an excess of green current. So A goes down, the voltage at the output of the amplifier goes up, nmos is on, pmos is off and the current flows in M2 and everything is stable.

NB: this discriminator has two stability points, and reaches on or the other depending if the Iout overcomes or not the threshold. The circuit stays in one of the two conditions depending on the current.

Example of current readout – 4

It is based on positive feedback loop; usually, a positive feedback can still lead to stability of the response if the loop gain is lower than 1.

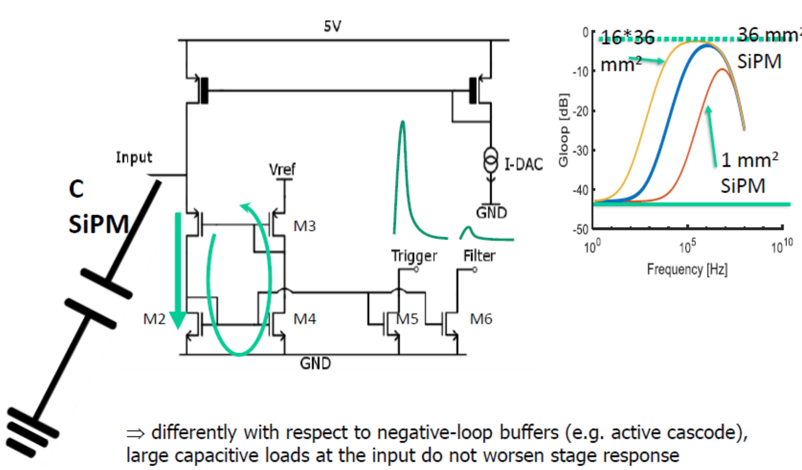


The loop is formed by M1, M2, M3, M4. This loop can still be a current buffer with almost zero input impedance.

Let's suppose we have the blue current from the sipm that will rise up the source of M1. This current also flows in M3. It is mirrored in M4 and then the current in M4 is sent to the transdiode M2 and pushes down the voltage at the gate of M2. Since the gate of M2 is the one of M1, the gate of M1 is going down and the source of M1 is going down.

It is a circuit that, when we have a perturbation at the input, the loop compensates this perturbation → this demonstrate that the input impedance is 0.

NB: regulated cascode may get unstable due to having 3 or more poles. This may get problematic if the capacitor of the sipm is huge.
This doesn't happened with this configuration, because the larger the Csipm, the loop gain is still stable due to the positive feedback. There is no crossing of the 0db because the loop gain is always lower than 0db → no problems of stability.



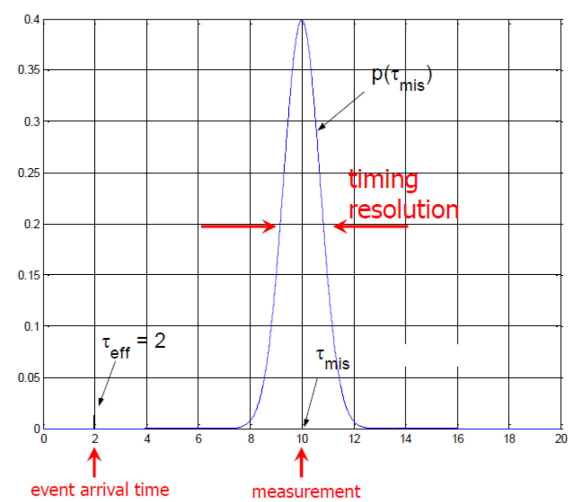
TIMING TECHNIQUES

<u>The goal is to measure the time occurrence of an event, or the time difference between events</u>.

A timing system has to provide a measurement τ_{meas} of this time which has to differ as less as possible from the time τ_{eff} in which the event actually occurs. Generally, τ_{meas} is a statistical variable characterized by an average value $\overline{\tau_{meas}}$ and by a variance σ_τ^2.
$\overline{\tau_{meas}}$ is always different from τ_{eff} because no measuring instrument reacts instantaneously to the event to be detected and therefore the output signal is delayed with respect to τ_{eff}. This type of error is systematic and therefore can be corrected.

As shown in the image it is almost physically impossible to have the measurement of the time coincident with the arrival time of the event due to phenomena of development of the signal in the detector, like the charge collection. Charge collection separates the arrival time of the photons with respect to the time when we can perform the measurement.

In PET, the delay between the gamma ray interaction and the detector timing signal is not critical as long as the delay is the same for all the detectors.



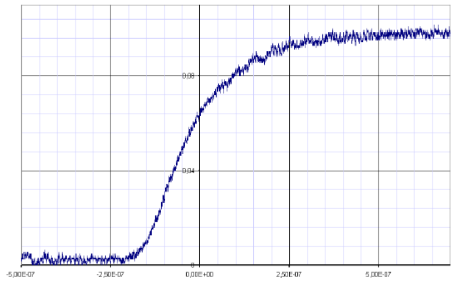
What is critical is the timing resolution, shown with the gaussian distribution of the measurement along the average value. Like in an amplitude measurement the fluctuations of the amplitude are critical, also here in timing measurement the spread is critical, because it represents the timing resolution.

TIME PEAK OFF TECHNIQUES

We have three techniques:

- Leading edge triggering: we check with a comparator when the signal crosses a threshold.
- Zero-crossing timing
- Constant fraction timing

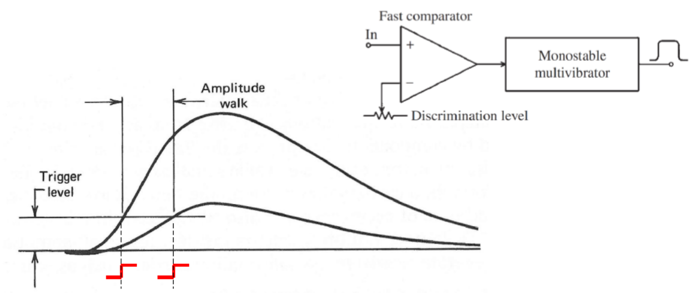
We have to consider the starting pulse for all these techniques. Usually, <u>the starting pulse is the preamplifier edge</u> (that is a steep pulse).
We may start to have problems when the waveform is no more as steep as we think. For instance, in the figure we have a charge preamplifier where the response is not a perfect Heaviside step, but it is a pulse with a non-zero rise time.
The causes for this rise time not to be zero are multiple; one may be the bandwidth of the preamplifier (if limited, the rise time will be limited by the opposite of the bandwidth), one may be the scintillator decay time (if we integrate a signal of a scintillator and the delivery of the photons is not fast, we have a rise time).



- Pulse at the output of the preamplifier, common to the 3 techniques considered here.
- The effect of the rise time of the preamplifier in the time pick-off technique has to be taken into account.

LEADING EDGE TRIGGERING

Goal: to detect the time in which the pulse crosses a given threshold



limitation: *amplitude walk*, variation of the *timing* signal with the signal amplitude

It is the simplest timing technique; it is a voltage comparison. We have our pulse (that can be a shaper's pulse or a preamplifier pulse) that we fed to the input pin of a comparator and we compare it with a threshold voltage that may be tuned by using a DAC or a resistor. Then the output of the comparator rises up when the pulse overcome the threshold.
If we want to narrow the pulse (not to have it lasting forever but for a short time) we can use a **monostable multivibrator**, that rises up but then autonomously goes down.

The one shown is a voltage comparison, but we can have also current discrimination. The problems and limitations are the same.

Problems

The main problem is called amplitude walk → the triggering time depends on the amplitude of the pulse. If the pulse is high, the triggering time is shorter. This is a problem because it introduces an uncertainty in the triggering time, and it is not a statistical spread, but a more deterministic problem.
In fact, there are ways to overcome this problem; since the amplitude walk is proportional to the amplitude, one way to overcome the problem is to correct it with the information of the amplitude.

There is a possibility to quantify this problem. A formula can quantify the problem, and it explains the options we have to try to improve the situation.

In the formula we assume to have a pulse with a rise that is represented by a linearly increasing pulse (approximation). We are not caring about what happens after the trigger.
Moreover, we suppose the pulse to be in average as large as y_{avg}, that is y-average.

We apply a threshold (y_{th}) and we measure when the pulse crosses the threshold.

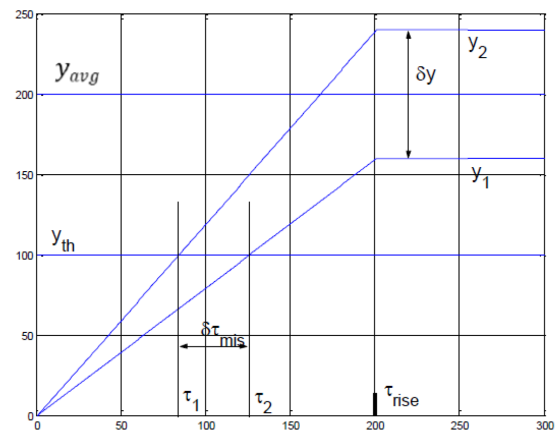
Supposing a linear rise of the signal, if its amplitude changes around an average value $\bar{y}$, the variation of τ_{meas} as a function of variations of the amplitude δy is given by:

$$\delta\tau_{meas} = -\frac{y_{th}\tau_{rise}}{y^2{}_{avg}}\delta y$$

where y_{th} is the level at which the threshold is placed

Time walk reduces with:

- Lower threshold (vs. noise)
- Shorter rise time (vs. noise)
- Larger signal (vs. S/N)



The problem is that the amplitude is changing within a delta-y, and is up to us to specify the amplitude spread. But, given the spread delta-y, the time peak-off measurement, the crossing of the threshold, has a spread between t1 and t2.

The formula simply gives to us the spread of the time peak off with respect to the spread of the amplitude (so the *amplitude walk*).
This formula tells us that the spread is proportional to the threshold, so lower the threshold, lower the spread; it is proportional to the rise time, so steeper the pulse, lower the spread; it is reversely proportional to the average amplitude, the larger the amplitude the steeper the pulse (relative slope steeper).

Sticking to the formula, the first thing we may be tempted to do is to lower the threshold as much as possible. For instance, in sipm as low as the single photoelectron.

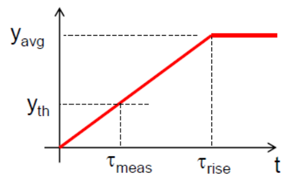
However, the limit in lowering the threshold is the electronic noise → when the threshold is 'embedded' in the noise, the discriminator is triggered most of the time by the noise. So given the noise, we have to keep the threshold safely above the noise. A rule of thumb, if the noise has a sigma, the threshold should be kept 3 or 4 times sigma.
So we cannot reduce the threshold as much as we can.

We may have a shorter rise time; with an ideal Heaviside step there is not a problem, because it crosses the threshold always at the same time, but if we use a charge preamplifier with a steep rise, we have also a high electronic noise, because the filter has not been applied in the chain → charge preamplifier is noisier but steeper, while the shaping amplifier is less noisy but smoother (so we have more amplitude walk).

Moreover, the larger the signal the steeper the pulse → if we choose the best scintillator we have the largest signal for a given energy.

The lower is the scintillation time, the more the photons are compressed in a steeper waveform. This is the reason why it is better a fast scintillator for timing, because we compress the photons almost in a delta like signal, that is perfect for timing.

Demonstration of the formula

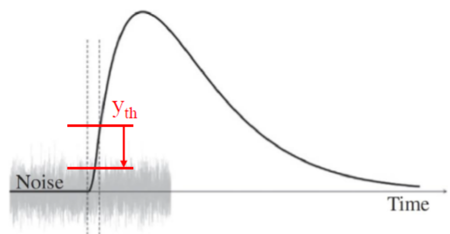


$$y = \frac{y_{avg}}{\tau_{rise}} t$$

$$y_{th} = \frac{y_{avg}}{\tau_{rise}} \tau_{meas}$$

$$\tau_{meas} = \frac{y_{th}}{y_{avg}} \tau_{rise}$$

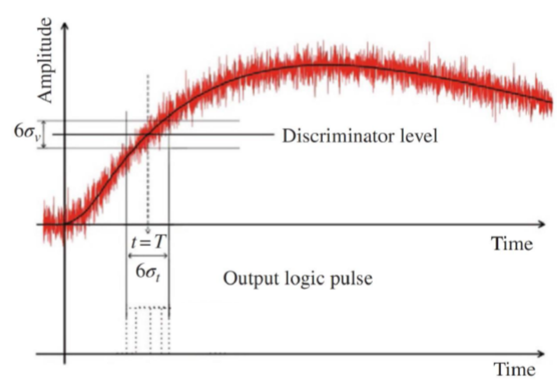
$$\boxed{\delta\tau_{meas} = -\frac{y_{th}\tau_{rise}}{y^2{}_{avg}} \delta y}$$



Note: lowering too much the threshold may set triggers by the noise.

We suppose to have a linear rise of the pulse up to y_{avg}, so we have the formula of the straight line. The measurement time tau-meas is when the line reaches the threshold, then we flip the formula, and we differentiate it (we compute the derivative of the formula with respect to an amplitude spread, differential).

TIME JITTER IN LEADING EDGE TRIGGERING



$$\sigma_T = \left.\frac{\sigma_v}{dv/dt}\right|_{t=T}$$

- Time jitter is the time uncertainty that is caused by noise and also by statistical fluctuations of the detector signals.
- It depends on the amplitude fluctuation and on the derivative at the point of time pick-off.

Besides the amplitude walk, that is somehow a deterministic problem, what about the *timing resolution*, so the spread of the firing of the trigger?

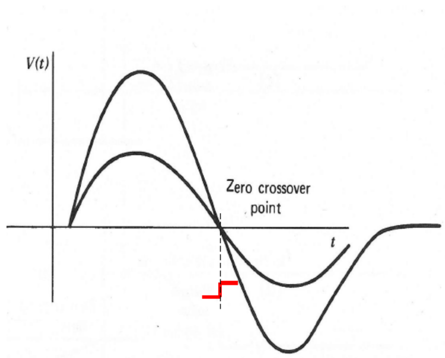
So, we have a waveform and we are doing leading edge triggering, but the waveform is noisy. We are supposing that the amplitude is always the same (no problem of amplitude walk). Since it is noisy, the crossing of the discrimination level will happen always at a different time → our problem is the time jitter, that is the horizontal spread of the timing crossing due to the vertical spread of the noisy waveform.

However, there is a very trivial way to estimate the time jitter, given the electronic noise, and it is given by a formula.
We take the waveform and linearize it at least in the region of the discriminator, and we can easily state that the time jitter (horizontal) is given by the vertical spread divided by the derivative. So the formula tells us that **the time jitter**, which is not recoverable, is our uncertainty in the timing precision, **is proportional to the electronic noise and inversely proportional to the derivative** → I may be tempted to make a very steep derivative to lower the time jitter for a given vertical jitter.
However, the steeper the pulse, the higher will be the noise. This is pretty intuitive; if we have an amplifier and we want it to be fast to have a steep derivative, it means that the bandwidth of the amplifier must be very large, so the amplifier collects any possible electronic noise around.

Typically, a steeper derivative allows the entrance of more noise.

ZERO-CROSSING TIMING

A method to eliminate the dependence of τ_{meas} from the signal amplitudes is based on the use of a bipolar pulse and on the detection of the time of zero crossing of this pulse.



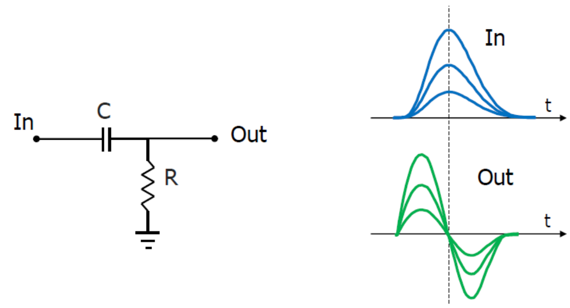
The measured time does not change, even if the two signals have different amplitudes

There is a relevant delay between event arrival and time pick-off, but this may be not an issue in coincidence timing where only time differences matters.

This technique eliminates the amplitude walk. It is based on generating with a custom filter (to be designed) a bipolar pulse. Then we elect the trigger instant not when the pulses cross a threshold, but when the pulse, coming back to zero, crosses the zero. So we check when the descending slope of the bipolar pulse crosses 0.

The problem of amplitude walk is solved because if we have pulses of different amplitudes, while the positive slope would have different crossing time, the zero crossing time is always the same.

HOW TO CREATE A BIPOLAR PULSE



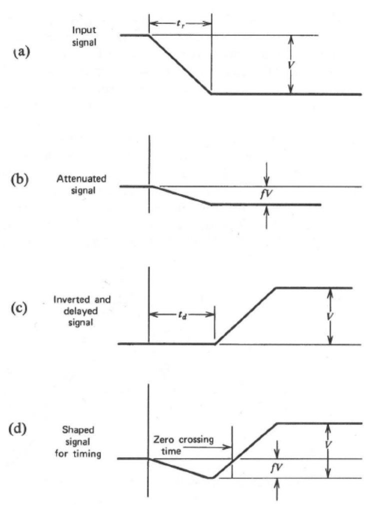
- To obtain the bipolar pulse, different types of shaping may be employed, among them a semigaussian signal suitably derived. These types may also offer the advantage to improve the S/N ratio at the output of the filter.
- Note: best shaping time for amplitude measurements is not necessarily best shaping time for timing $\Rightarrow$ separate channels for amplitude and timing

It can be created from any unipolar pulse, like the one of the semigaussian shaper, by passing it into a CR filter, because, by definition, the CR filter makes the derivative of the pulse (that is shown in green).
But since this simple stage makes the derivative, the zero derivative point of the gaussian is independent on the amplitude → peaking time is independent on amplitude → derived zero crossing time will be independent from amplitude as well.

So to create a bipolar pulse we create a semigaussian one and pass it in a CR filter.

Can we use the semigaussian already designed for the amplitude chain also in this case?
No, because of the **shaping time**. In fact, the semigaussian optimized for amplitude measurements may have its own shaping time, the one that minimizes ENC, but this time is not necessarily optimal for timing measurement → we need to design two shapers for amplitude and timing measurements.

CONSTANT FRACTION TIMING



It is similar to the previous method, except for the shaping of the bipolar pulse, $y_{bip}(t)$.

The input signal $y_{riv}(t)$ is taken, attenuated by a suitable factor f, and it is again added to an inverted and replica of $y_{riv}(t)$ delayed by a time τ_d.

Usually the best attenuation factor is ranging from 0,1 and 0,2, while τ_d has to be larger than the rise time τ_r of the pulse.

It is obtained:

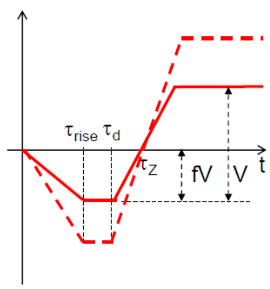
$$y_{bip}(t) = f y_{riv}(t) - y_{riv}(t - \tau_d)$$

The zero-crossing time results independent from the amplitude of the signal.

It pursues the zero crossing.

Let's suppose we have the preamplifier signal at the beginning (a) with a rise time r. It is negative, but it can be also positive.
Now, we take this signal and attenuate it by a factor f. Then we take the original signal, we flip it (making it positive) and we delay it by t_d. So we have taken an attenuated replica of the signal (b) and flipped and delayed the original signal (c). We make the sum of (a) and (c) and we get (d), that is an almost bipolar signal, and <u>the zero-crossing time of this signal is independent on the amplitude</u>.
Signal (d) given by the last formula (attenuated original signal minus the original signal flipped and delayed).

Zero crossing



$$y_{bip}(t) = f y_{riv}(t) - y_{riv}(t - \tau_d)$$

$$y_{bip}(\tau_Z) = 0$$

$$0 = -fV + \frac{V}{\tau_r}(\tau_Z - \tau_d)$$

$$\boxed{\tau_Z = f\tau_r + \tau_d}$$

independent from the amplitude V of the signal

Comparison with bipolar shaping:

- It is simpler (excepted for the delay line).
- It does not introduce shaping, therefore it may not reduce the noise.

In the image we can see two zero-crossing signals and the zero-crossing time t_z is independent from the amplitude. Beside the plot there is the demonstration of this independency.
We want to check when the bipolar signal crosses the zero, so we impose the bipolar signal equal to zero. Then we calculate the amplitudes of the two building blocks at the zero-crossing time t_z. Of course the attenuated signal at tz is equal to -fV, while the other is described by the equation.
Then in the formula the V cancels out, so the crossing time is only proportional to constant factors, but it doesn't depend on amplitude.

Comparison between bipolar shaping and constant fraction

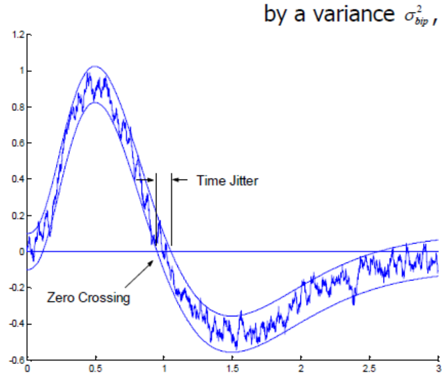
Constant fraction is usually simpler because it is just a quick manipulation of the preamplifier signal, we don't have to build complex semigaussian pulses. The only complicated stuff is the *delay line*, typically done with a coaxial cable → with a 3m of coaxial cable we create a 3ns delay, but creating delay in IC is more difficult.
On the other side, this waveform of the constant fraction timing is quite sharp, it has not introduced any shaping.

Instead, by introducing the bipolar by semigaussian shaping we have improved the SNR, we have introduced some filtering.

<u>In conclusion, constant fraction is very important with detectors where noise (ENC) is not crucial, like PMTs, while bipolar is more used when noise matters</u>.

TIME RESOLUTION IN THE ZERO-CROSSING TECHNIQUE

We consider a bipolar signal as a deterministic signal to which a statistical fluctuation due to the noise, characterized by a variance σ^2_{bip}, is added.

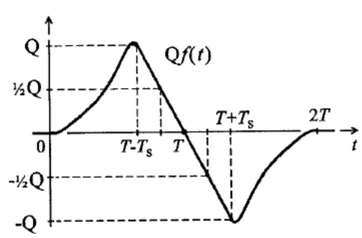


$$y_{bip}(t) = y_{\det}(t) + n(t)$$

The presence of $n(t)$ generates also statistical fluctuations on the measured variable τ_{meas}. This fluctuation has zero average level and can be described by the variance σ^2_τ.

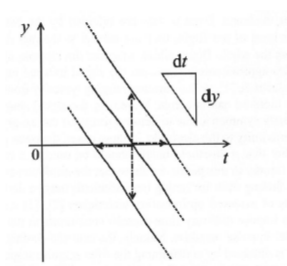
Also in bipolar pulse we have a noisy waveform as in the image. We have solved the amplitude walk issue, but not the electronics noise issue, the waveform is not deterministic but embedded in an envelope of noise (deterministic signal + statistical noise).
So unfortunately, even in the case of bipolar pulse, the zero amplitude is crossed at different time → we have a time jitter, that is similar to the one we have in crossing a threshold in leading edge triggering.

We would like to analyze more in detail this issue. Which are the dominating parameters? What can we do to try to reduce the jitter as much as possible?
The **time jitter obeys to the similar formula seen for the leading edge triggering**. So we have a bipolar pulse that is crossing zero at a zero-crossing time T and so the time jitter, given by geometry, is given by the vertical spread due to electronic noise divided by the derivative.



The noise can be held as a statistical fluctuation of the signal which can translate high or low in a random way the waveform $y_{\det}(t)$.
As first approximation, around the zero crossing point, it exists a linear relationship between amplitude and timing fluctuations (jiiter):

$$\sigma_\tau = \frac{\sigma_{bip}}{y'_{riv}\big|_{t=T}}$$



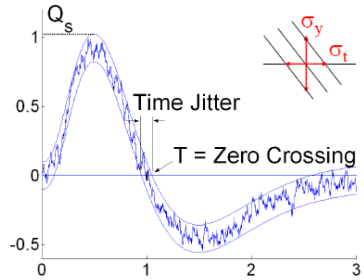
- The shaping realized with the Constant Fraction method leaves almost unchanged the value of σ_{bip} and is generally used for detectors in which the electronics noise is negligible, typically in those with internal gain $>>1$, like PMTs.
- With a suitable bipolar filter and the zero-crossing, it is possible to optimize the noise σ_{bip}.
- The derivative in T covers a role in both cases
- Note: the statistical fluctuation of Q does not count

In the image the pulse has been exaggerated to be linear, but it is not. What matters anyway is the local derivative of the pulse. So the problem is the vertical spread and the derivative.

While in amplitude measurements the intrinsic spread of the charge matters -the Poisson statistic of the signal itself matters-, in bipolar pulse the signal statistical spread doesn't matter in zero crossing. In fact, even if we have a charge fluctuation, there will be no fluctuation in the zero crossing time → in amplitude measurement we have the effect of charge statistic spread plus electronic noise, while in timing, if we use bipolar pulses, we have only the effect of electronic noise, because any charge spread would result in a different amplitude of the pulse but always with the same zero crossing point.
So we have to take care just about the electronic noise.

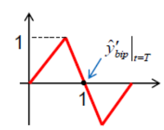
CALCULATION OF THE JITTER

Calculation of the time jitter:



x

$$\sigma_\tau = \frac{\sigma_{bip}}{y'_{bip}\big|_{t=T}} = \frac{\sigma_{bip}}{y_{MAX}} \cdot \frac{1}{y'_{bip}\big|_{t=T}\frac{T}{y_{MAX}}} T = \sigma^{\%}_{bip} \frac{1}{\hat{y}'_{bip}\big|_{t=T}} \cdot T$$



$$\boxed{\sigma_\tau = \frac{ENC}{Q_s} \frac{1}{\hat{y}'_{bip}\big|_{t=T}} T}$$

$\frac{ENC}{Q_s}$ = S/N

$\hat{y}'_{bip}\big|_{t=T}$ = derivative normalized in time and amplitude

Let's consider the derivative. A derivative of a signal is steep when the signal is large but also when the chosen zero crossing time is short. So the step forward we can do is not just to use in the formula the derivative as it is, but we would like to have a formula where we split the effects of signal amplitude and zero crossing time, so that we can act on one parameter and not on the other.
This is done with an algebraic manipulation of the formula in the blue box, where the denominator has been multiplied and divided by the signal amplitude (y-max), and also the numerator is multiplied and divided by T, the zero-crossing time.

However, now that we have introduced y-max and T, the sigma of the noise divided by the amplitude is the noise-to-signal ratio (x). The noise normalized to the signal is the NSR. If our signal in the oscilloscope is in mV, also the noise is in mV → this ratio is in mV over mV, so it can be represented also as ratio of corresponding charges, so ENC divided by the charge of the signal. The conversion factor is the same for both noise and signal.
So the ratio can be seen as ENC/Qs, where Qs is the signal → if we keep the **ENC small** is good, but we also **need to try to keep the charge delivered to the electrode as large as possible** (in this case we have to go back to the choices of the scintillators and photodetectors, because the conversion gain matters. In fact, the larger is the signal, better is the timing).

The other parameter in the formula is T, the zero crossing time. We have highlighted that reducing the zero crossing time improves the time jitter (because if we reduce T we have a steeper derivative).
So we have up to know highlighted the zero crossing time and the NSR.

What is remaining in the formula is the derivative y'-bip multiplied by T and divided by y-max. ŷ'-bip is a derivative normalized both in time and amplitude.
If I take the derivative and I explicitly extract the amplitude of the signal and the zero-crossing time, what remains is an absolute number ŷ'-bip, it is not with a unit of measure.

This absolute number is a derivative normalized in time and amplitude. It is a derivative in which both zero crossing and amplitude are unitary → the physical meaning is the intrinsic steepness of our processor.
The processed signal can be steep because we have reduced the zero crossing or raised the amplitude; but since we have extracted these two parameters in the formula, what remains is the intrinsic steepness of the waveform we have chosen.

So this formula in the red box has a more 'operative' meaning, we know on what act to reduce the sigma.

What to be done with the formula
We have to reduce the ENC and increase as much as possible the signal (e.g. by a more luminous scintillator, by using silicon as a photodetector because it has a higher quantum efficiency than PMT). Then we should choose the lowest possible T (so the shortest possible zero-crossing) and choose the largest possible normalized derivative.

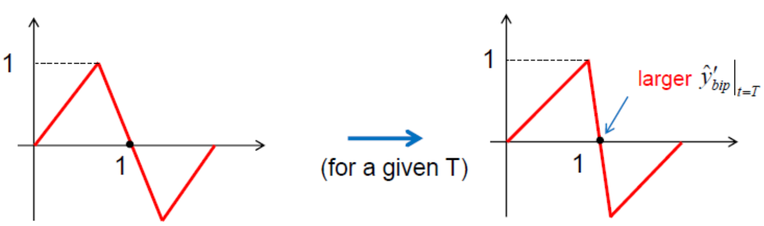
$$\boxed{\sigma_\tau = \frac{ENC}{Q_s}\frac{1}{\hat{y}'_{bip}\big|_{t=T}}T}$$

time jitter reduces with:

- small ENC (but look to dependency from T*)
- large Qs **
- small T (but be careful to preamp rise time)
- large $\hat{y}'_{bip}\big|_{t=T}$

*question: considering ENC dependence on T, do we overall improve or not σ_τ by reducing T?
**question: how the choice of the scintillator and photodetector have an impact here?

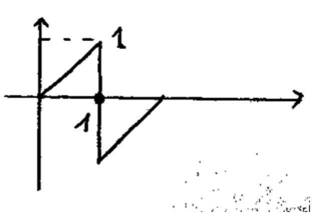
increasing $\hat{y}'_{bip}\big|_{t=T}$ to minimize time jitter?



but increasing $\hat{y}'_{bip}\big|_{t=T}$ has an effect also on ENC through the filtering factors

In the plots below we can see what increasing the derivative means. The two processors have the same amplitude and zero-crossing, but the one on the right has the largest intrinsic steepness.
The ideal processor we may wish is the following:

113.



It is a processor with a vertical derivative, so we would have 0 time jitter (infinite derivative in the formula).

Minimization of the shaping time T

Is minimization of T as easy as it seems?

No, because unfortunately, the parameters are correlated.

For instance, when we go with the zero crossing time close to zero, this is not independent on ENC, because the zero crossing time is associated to the shaping time of the filter. In fact, very obvious definition of shaping time in a bipolar filter is the zero crossing time.

So if we reduce the zero crossing time, and hence the shaping time, to zero, what happens to the ENC?

The ENC is increasing. So in the red box formula T is reducing sigma, while ENC is increasing it. Who is dominating? To be known for the examination (look at the ENC formula to find the answer, but it is the time).

Another limitation to the reduction of T is the preamplifier rise time. If the preamplifier was a perfect Heaviside step, the formula works well when T is minimized, but if the preamplifier ahs a limited rise time, we cannot squeeze T to zero as we like.

Increase of the normalized derivative

As for the normalized derivative, I'm tempted to make it as steepest as possible. But this has a side effect; also the shape of the signal applies in the ENC formula, in the A1, A2 and A3 coefficients, that depend on the shape by definition, not on the shaping time. So if we change the shape, we should expect the coefficients to change.

And it is not obvious that if we make the derivative steeper the coefficients change in our favor, it could be also the opposite.

Series noise of the ENC

$$ENC^2{}_{series} = aC_T{}^2 1/2\pi \int_{-\infty}^{+\infty} |T(j\omega)|^2 d\omega =$$

$$= aC_T{}^2 1/2\pi \int_{-\infty}^{+\infty} |H(j\omega)|^2 \omega^2 d\omega =$$

$H(j\omega)$ is the overall output response (preamp + filter $T(j\omega)$)

$$= [Parseval\ theorem] = aC_T{}^2 \int_{-\infty}^{+\infty} [h'(t)]^2 dt$$

$$h(t) = L^{-1}[H(s)] = L^{-1}[T(s)/s]$$

$h(t)$ is the output response in the time domain, $h'(t)$ its derivative

using $t' = t/\tau$ as dimensionless variable, i.e. the time normalized to the shaping time τ :

$dt = dt' \cdot \tau$ $\qquad$ $h'(t) = h'(t') \cdot 1/\tau$

I'm interested in the series noise of the ENC because if we are looking to reduce the shaping time to improve the jitter, the noise that matters is not the parallel noise, but the series noise dominates.

In the manipulation by the Parseval Theorem we can conclude that the A1 coefficient, that is the dominant one for timing, (NB: we are not in the condition of amplitude measurements where I have to look for the minimum of the combination of the three contributions, but we are at low shaping time, so we are looking for the series noise and A1 coefficient) is proportional to the integral of the derivative of the pulse response h(t), squared, and integrated over time.

$$ENC^2{}_{series} = aC_T{}^2\frac{1}{\tau}\int_{-\infty}^{+\infty}[h'(t')]^2dt'$$

introducing the shaping factor A_1 as done in a previous lesson:

$$A_1 = 1/2\pi\int_{-\infty}^{+\infty}|T(j\omega)|^2d\omega = \int_{-\infty}^{+\infty}[h'(t')]^2dt'$$

⇒ increasing the derivative of the normalized response may worsen the series noise, through A_1, which may be the dominant noise contribution at short shaping times (as required for timing)

(Note: A_3 can be equivalently calculated as:

$$A_3 = \int_{-\infty}^{+\infty}[h(t')]^2dt'$$

and it increases with $h(t')$ total area)

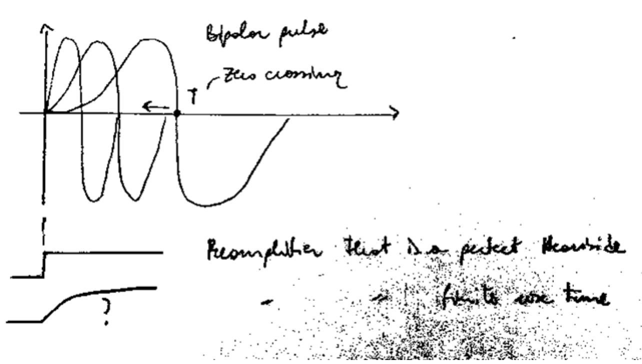
If we are thinking to make the derivative the steepest possible because of the previous sigma formula, we have to check the A1 coefficient because if, due to the vertical derivative, the A1 is increasing too much, we will have troubles in ENC.

In conclusion, we have a relationship between zero crossing and ENC. Moreover, we have a correlation between steepness of the derivative and ENC.

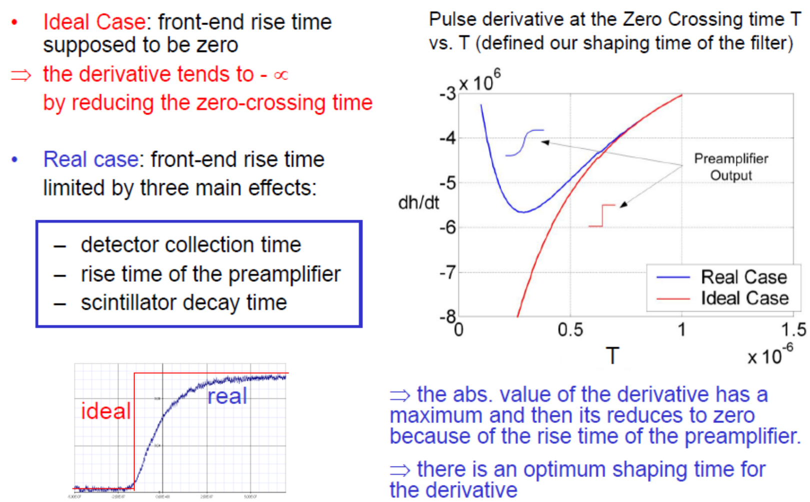
REDUCING THE ZERO CROSSING TIME VS PREAMPLIFIER RISE

If we are tempted to reduce the zero-crossing time to zero as the formula suggests, we have another problem depending on the shape of the preamplifier signal.
Remember, we have a charge preamplifier, so the bipolar pulse is the result of the complete shaping including the initial charge preamplifier. So we have the charge preamplifier Heaviside step, that is compensated with a zero in the origin.
The amplifier, in the ideal case is a Heaviside step. So if the preamplifier is a Heaviside step, it is obvious that the derivative that we see in the plot on the left (it is the absolute derivative, not the normalized one) as the red curve has a negative value and it goes to -inf if we reduce the zero-crossing time.

114.



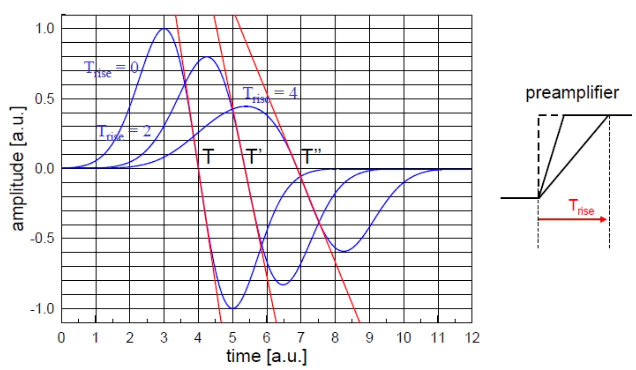
I'm taking a bipolar pulse with a zero crossing T and I'm reducing T. In fact, the formula says that if we reduce T, we improve the derivative.



But this is true if the preamplifier is a perfect Heaviside step, so that we have a perfect pole-zero cancellation and a perfect bipolar.

But if the preamplifier has a finite rise time, due to the finite bandwidth or due to the scintillator's scintillation finite time, and on the other side the filter is squeezed to very short zero crossing we have a conflict. If the amplifier takes time to reach the maximum I cannot expect that the signal remains the same simply with a steeper derivative, it is no sense.

EXAMPLE OF WORSENING OF THE DERIVATIVE BECAUSE OF THE RISE TIME



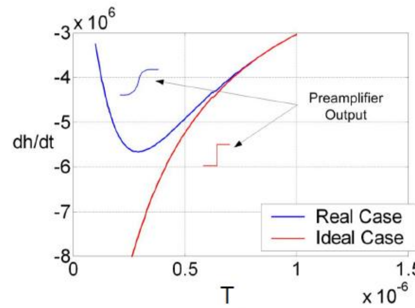
in this graph the nominal zero-crossing time (T) is kept constant, and the rise time of the preamplifier is increased ⇒ 1) the real zero-crossing time is increased (T→T',T'') and 2) the derivative is decreased (therefore the timing jitter is worsened)

In blue we see a bipolar pulse of a given normalized amplitude 1 for a given zero crossing time T. Now, the first waveform Trise = 0 corresponds to a 0 rise time of the preamplifier (perfect preamplifier). The following waveforms are plot for a preamplifier with increasing rise time, for instance due to an increasing scintillation decay time. If we take the same filter but we process with it a preamplifier response that becomes slower and slower, we can see two bad effects.

The first one is that the **amplitude goes down**, and if so we have the Ballistic deficit (apparent reduction of the amplitude not because the detector is supplying less charge, but because we have a lack in the processing of the signal → we are loosing signal). The second effect is that the **nominal derivative decreases**, and this is a problem.

The problem is that we have kept constant the shape and worsened the preamplifier in this plot. In (114) I keep constant the rise time of the preamplifier and reduce the zero crossing → same problem from a different perspective than before.
In this perspective now the bipolar shape is constant and we are worsening the preamplifier, in the perspective before the real preamplifier has a constant rise time and we are squeezing the bipolar pulse.

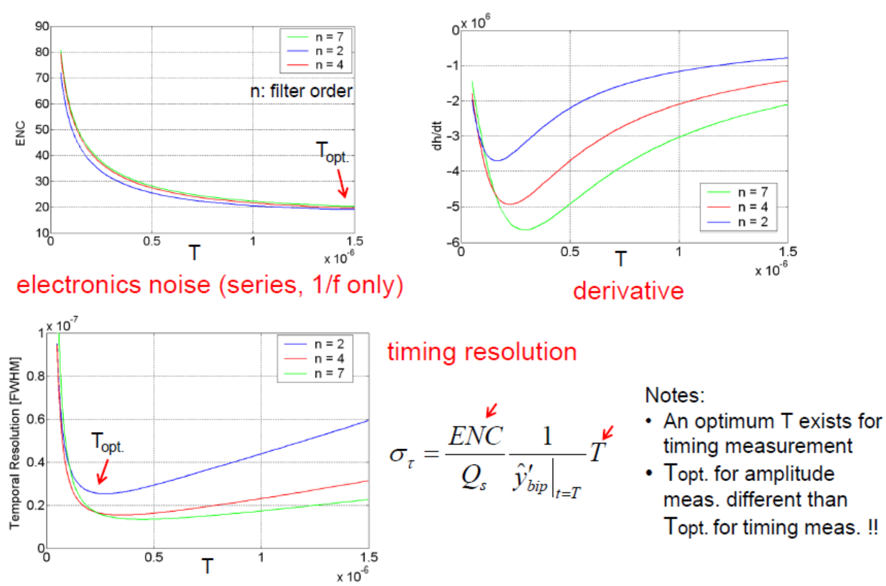
If we squeeze the bipolar pulse to lower zero-crossing time, we cannot expect the derivative to go to -inf, it will become more negative, reach a minimum and then it goes to 0. So it is not true that if we reduce the zero crossing time the derivative becomes steeper, because if we do so, the derivative becomes flat, because we have no more steepness of the charge preamplifier → no sense to make the bipolar steeper if the preamplifier is not steep in the real case.



The important conclusion is that, mainly due to the finite preamplifier rise time, **it is impossible to bring T close to 0, because we will find an optimum derivative, and if we go shorter the derivative will flatten** (blue plot).

EXAMPLE OF EVALUATION OF THE TIMING PERFORMANCES

We put all the ingredients. We have the formula, the plot of the noise and of the absolute derivative (not the normalized one) and I try to reduce the zero-crossing time.



As for the electronic noise, the shape of the electronic noise is due to the series noise and the 1/f noise, and in the plot there is not any parallel noise, that is discarded because we are focusing on the shortest

zero crossing times, so the contribution of the parallel noise is not important because it goes to 0 reducing the time.
In the plot of the noise we can see that **decreasing the zero crossing time the electronics noise goes to infinite**. So decreasing T the time jitter goes to zero, but the electronic noise goes to infinite. So apparently we have 0*inf. However, the winner is the zero-crossing time.
The dependency of the series noise on the shaping time is 1/sqrt(T), so the proportionality is:

115. $$ENC^2 \div \frac{1}{T}$$

$$ENC \div \frac{1}{\sqrt{T}}$$

$$\sigma_T \div \frac{1}{\sqrt{T}} \cdot T$$

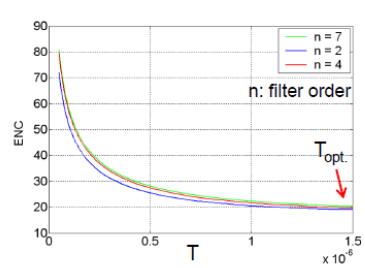
So the time jitter is proportional to a quantity that is going as T, and one as 1/sqrt(T), so T wins → <u>reducing the zero crossing is beneficial even if we increase the noise</u>.

Now that we have understood that the product of ENC vs zero-crossing is in our favour if we reduce zero-crossing, we should expect that reducing zero crossing brings jitter to zero, but there is unfortunately the preamplifier rise time limitation.
This is shown by the plot on the right, which is the plot of the derivative vs zero-crossing time. Unfortunately, moving zero-crossing time to zero doesn't bring the derivative to -inf, but it reaches a minimum and then goes to zero.

If we combine the effect of the electronic noise and derivative together, we get the plot on the bottom, that is the plot of the time jitter.
We can see that reducing T improves the time jitter, but up to a given minimum. If we reduce too much T, the time jitter explodes, and not because of the ENC, but because of the derivative that flattens.

In conclusion, a general rule that reducing the pulse width improves the time jitter is true, but up to a given limit given by the preamplifier rise time.

The second conclusion is that if we think to use the same semigaussian shaper for amplitude measurement and for timing measurements simply after a derivation, and we think to use the same shaping time, this time it is not necessarily optimized for both of them (amplitude and timing).
This can be seen from the plot aside, that suggests that for amplitude measurement only, where only ENC matters, we should use longer shaping time.



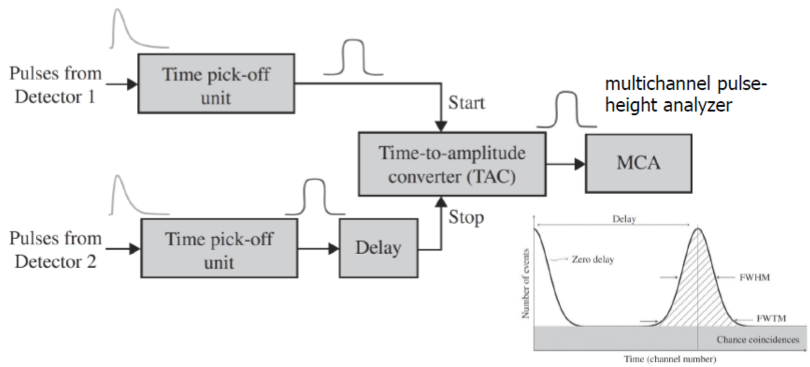
electronics noise (series, 1/f only)

But on the timing resolution plot a short shaping time is recommended for timing → in most of the cases for amplitude measurements and timing measurements we have to adopt two separated processing chains: one optimized for amplitude, and one optimized for timing.

TIME INTERVAL MEASURING TECHNIQUES

There are two popular techniques to measure time intervals. Let's suppose we have a time stamp; how do we combine with another detector, for instance, to make a coincidence (like in PET, where we need to measure time intervals)? Which is the technique I can use to obtain this time difference and compare it with a reference?

TIME-TO-AMPLITUDE CONVERTERS (TACs

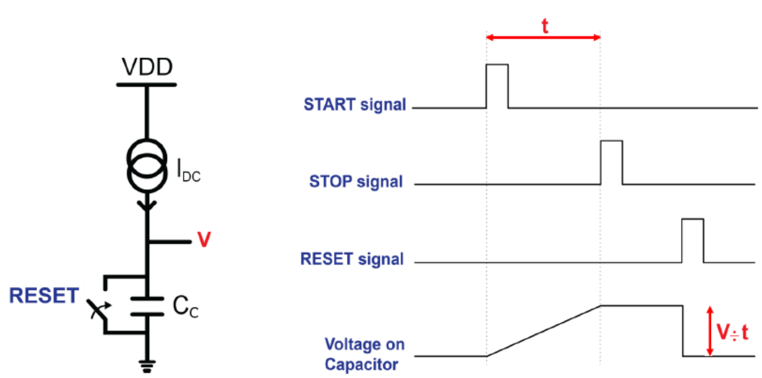


A TAC measures the time interval between pulses to its start and stop inputs and generates an analog output whose amplitude is proportional to the time interval. The stop pulse is normally delayed to ensure that the stop pulse arrives after the start pulse.

It converts the time difference into a voltage amplitude, which is easy to be measured. E.g. a time difference of 3ns into a pulse of 200mV, and if this pulse is proportional to the time difference, the electronic chain is very easy.

To do the measurement, we have the two detectors; supposing that one detector arrives first, we call its signal *start*, and the one of the other that arrives later *stop*. The time to amplitude converter is a circuit that produces an amplitude proportional to the difference between start and stop.

Implementation

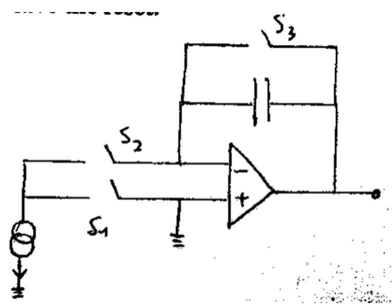


We have a constant current generator and a capacitor. We start to integrate the current in the capacitor, so we are producing a ramp and the slope of the ramp is the slew rate, current divided by capacitance (I/C). we simply start the ramp with a start signal and stop it with the stop signal (we have a switch somewhere that is closed when the charge has to be integrated on the capacitor).

The resulting voltage on the capacitor is proportional to the time difference between start and stop. Then the voltage on the capacitor can be read with a voltage buffer to an ADC.
NB: we need to calibrate the ADC.
Once the conversion is done, we reset the capacitor to zero charge with the reset switch.

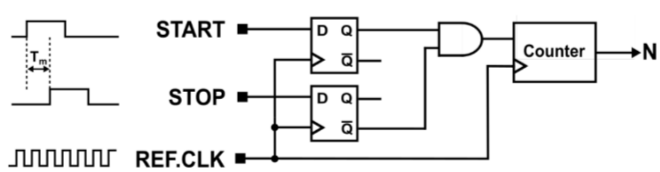
The real circuit is an integrator, then we have a generator and two switches, one going to the virtual ground and the other to the ground. Then we have the reset.
Normally, before we start the conversion, S1 is closed, so the generator is active but its charge is lost in the ground and no charge is integrated. When start arrives, S1 is opened and S2 closed. In this way, the generator is charging the capacitor.
When stop arrives, S1 is closed again and the generator is wasted into ground and S2 is opened. Then on top of all we have the reset.

116.



If we would have just start and stop, however, zero delay between start and stop would correspond to 0V. It is not very practical to measure separations, delays across 0, because it could happen that the stop arrives before the start. In order to avoid this, if we know which is start and which is stop, **in the stop channel we introduce on purpose a delay**.
This means that when the separation between start and stop is 0, in reality we have a constant difference of 5ns, which means a non-zero baseline in the amplitude. To have a variable 'start and stop' in PET, we add some intelligence to use a detector as start or stop.

TIME-TO-DIGITAL CONVERTER (TDC)



A time-to-digital converter (TDC) can directly convert the time intervals into a digital output. The simplest TDC is based on counting clock pulses from a stable reference oscillator during the time interval. The count of clock pulses starts with the start pulse, and at the arrival of the stop pulse, the counter stops, yielding a number proportional to the time interval between the pulses.
The time resolution is limited here to 1 clock cycle. Different techniques (e.g. interpolation, vernier, ...) are used to improve timing resolution.

The difference between start and stop is counted as number of clock cycles. We have a counter that is counting clock cycles and we simply let the counter start when the *start* arrives and we stop it when the *stop* arrives.
The number of ck cycles are the time measurements of the time interval.

The start is fed into a flip-flop, and so the stop. When we have just the start, we have 1 and 1 at the input of the AND, so the counter counts. When the stop arrivers, Q of the stop goes to 1, so the output of the AND is zero and the counter stops to count.

Accuracy

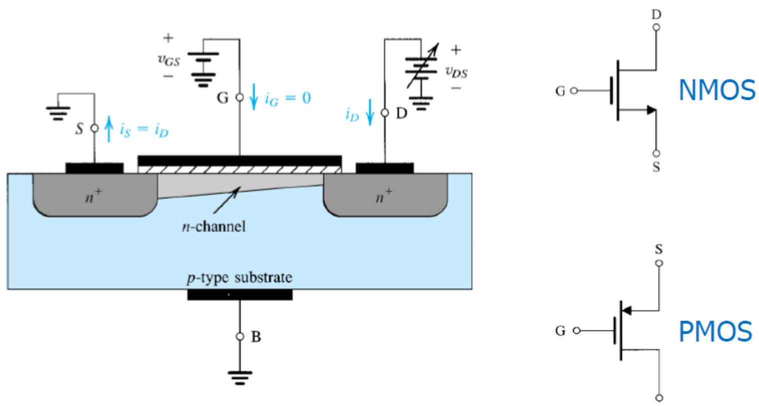
The accuracy is the ck period. If we want to increase accuracy we need to shorten the ck period and we increase the number of bits of the counter to cover the same full scale range.
However, if we need accuracies of sub-nanoseconds, like for PET-TOF (200-300ps of accuracy), we would have to use a clock above GHz, so there are TDC which can reach such accuracies not by boosting the ck frequency but using techniques that allow us to quote the time difference also within a ck cycle. We can differentiate start and stop not only as number of ck cycles, but also inside a ck cycle (interpolation TDC).

x x

COMPLEMENTS

TRANSISTORS AND CIRCUITS

MOSFET



N-channel MOSFET

It is a field-effect transistor based on conduction channel (in the figure is full of electrons, n channel). The channel is created thanks to the voltage applied to the gate, isolated from the channel by a layer of oxide. The channel is created by inversion when we apply a voltage above the threshold and connects the source and the drain. If we apply a Vdiff between drain and source, we create a current and the amount of current depends on the tension on the gate. The voltage on the drain also changes the concentration of electron in the channel (narrowing of the channel) and this create a different current when we increase the voltage (triode regime).

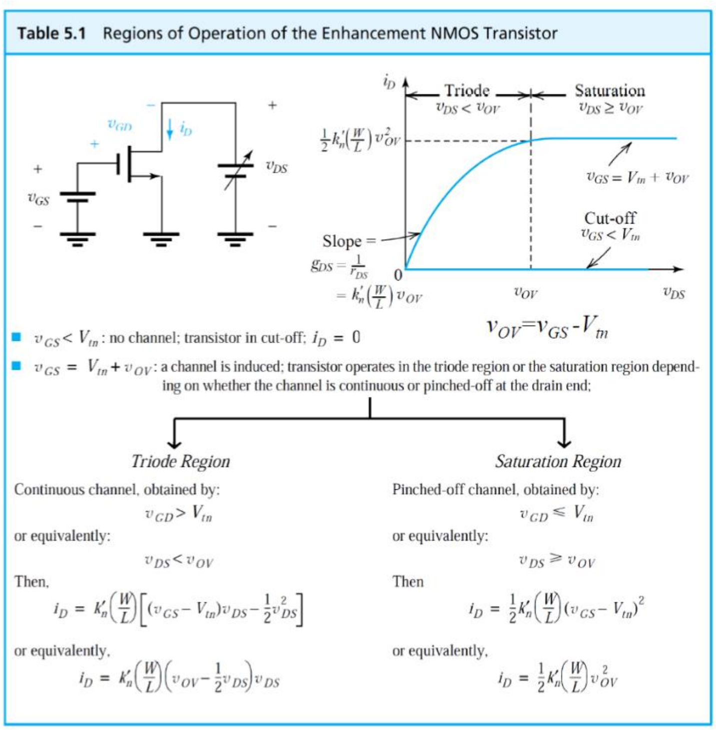


Table 5.1 Regions of Operation of the Enhancement NMOS Transistor

$v_{OV}=v_{GS}-V_{tn}$

- $v_{GS} < V_{tn}$: no channel; transistor in cut-off; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$: a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched-off at the drain end;

Triode Region

Continuous channel, obtained by:

$$v_{GD} > V_{tn}$$

or equivalently:

$$v_{DS} < v_{OV}$$

Then,

$$i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

or equivalently,

$$i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$$

Saturation Region

Pinched-off channel, obtained by:

$$v_{GD} \leq V_{tn}$$

or equivalently:

$$v_{DS} \geq v_{OV}$$

Then

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$

or equivalently,

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$$

In the first region is in the triode regime and the transistor is working as a controlled resistor, then when Vds>Vov (Vov = Vgs-Vth), the characteristic is constant and the current has a square dependency with the Vov (overvoltage).

Output resistance (impedance) r0

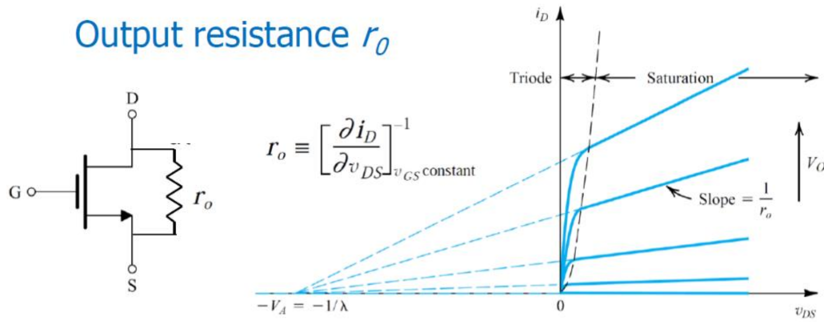
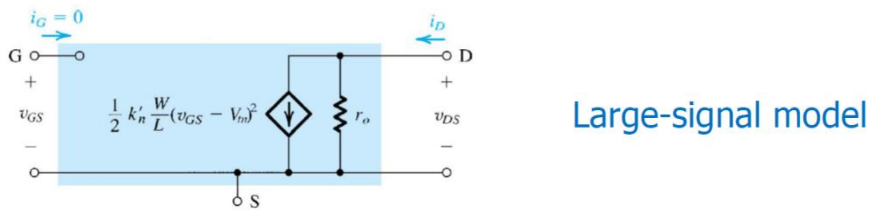


Figure 5.17 Effect of v_{DS} on i_D in the saturation region. The MOSFET parameter V_A depends on the process technology and, for a given process, is proportional to the channel length L.



Ideally, when we are in saturation and we change the Vds, the current is constant, but in reality this is not true, the current increase in a linear way and the slope of this behaviour in a plot voltage vs current is a conductance. The output resistance is the capability to modulate the drain current not because of the gate control but because of a Vds variation. This is the reason why this is represented simply by a resistor between drain and source.
On the bottom we have the large signal model of the transistor in saturation. The relevant infos are that the mosfet has no gate current due to the oxide. The current flowing in between drain and source is determined by a quadratic relation. W/L is the ratio between width and length of the transistor.

Small signal model

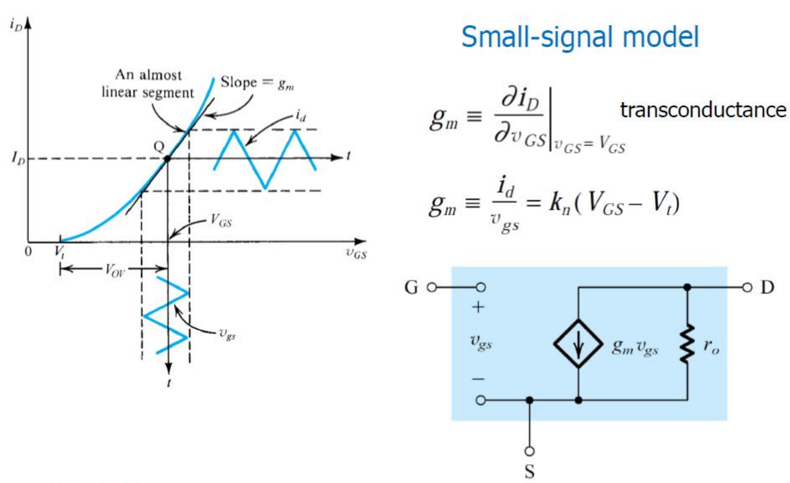
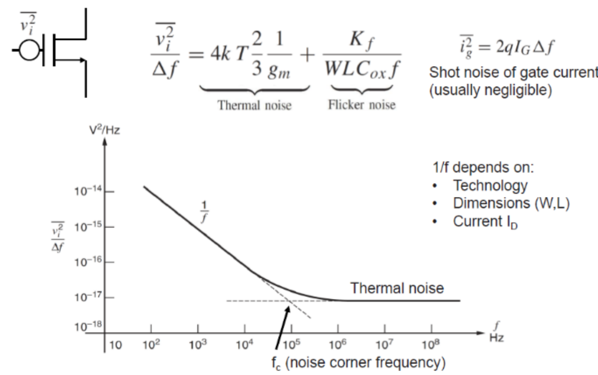


Figure 5.37 Small-signal models for the MOSFET: **(a)** neglecting the dependence of i_D on v_{DS} in saturation (the channel-length modulation effect); and **(b)** including the effect of channel-length modulation, modeled by output resistance $r_o = |V_A|/I_D$.

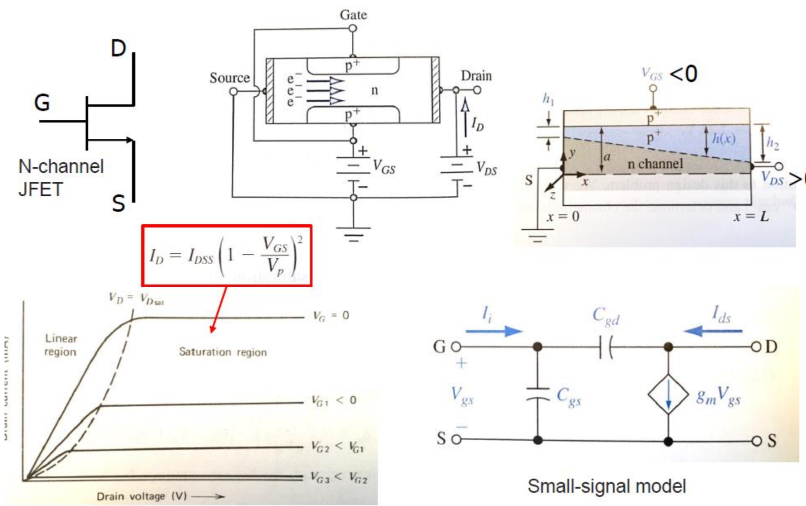
Approximation of the current vs Vgs in a given point of the quadratic characteristic. The transistor is biased in a given working point, we consider the derivative and not the quadratic relationship. The transconductance depends again on the Vov.

MOSFET noise sources

We have an equivalent voltage generator at the input. We have a thermal noise component with the transconductance term and the other one is the Flicker noise, that depends on the technology through the factor K, Cox. Moreover, we may have a shot noise if we have a gate current.



JFET



Transistor still belonging to the family of field effect transistor. The symbol is similar to the one of the MOSFET, but with one bar. It is a field effect transistor in the sense that it is still based on conduction channel between source and drain (**there is a pre-implanted channel**), but here the channel is created by doping, not by inversion like in the mosfet, so it is preexisting. The current is modulated by the gate, that can be on both sides or on one single side. Differently from the mosfet, where the gate is a metal over an oxide, here it is a silicon layer implanted in the device with an opposite type than the channel type (if channel is n, it is p).

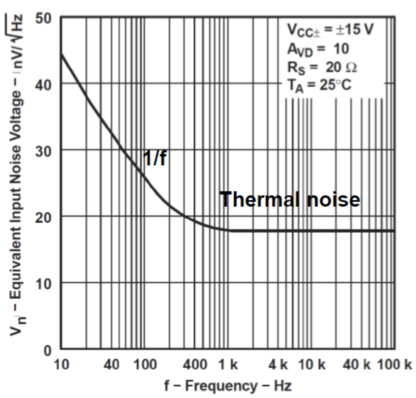
This because **the pn junction** (the gate channel junction) **is biased in reversed bias**. It consists in a kind of diode but the gate is biased more negative than the channel so that we are depleting (svuotare) more and more the channel. When we have a pn junction and we make the p voltage more negative than the n (voltage at the source for instance), we are extending the depletion region between p and n and so we are depleting the channel. Thus, if we are depleting the channel, we are reducing the current into the channel, because electrons have been removed from the channel. Overall, the JFET is a depletion transistor, so a transistor based on depleting the channel, thanks to a pn junction reversed biased.

In the characteristic drain-current vs Vds we still see a linear region or a saturation region; we see for instance a curve with a gate voltage equal to zero (which means that gate and channel are in equilibrium, they have no voltage difference, so we have a given current), but as soon as we start to

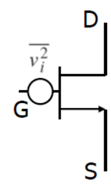
provide a negative voltage to the gate, the current drops down. More negative is the gate, more depleted is the channel in this pn junction and so less current we have in between source and drain. So differently from the mosfet where we increase the current by increasing the inversion, we have no inversion at all in the JFET, we are simpliy depleting the transistor.
When the voltage applied is sufficient to fully deplete the channel, the transistor is off (we have no more channel). The voltage we apply to deplete the channel is called **pinch-off voltage**. The formula for current is still quadratic, but it is in depletion, so when Vgs = Vpinch-off the current is zero because we have completely depleted the channel.

The small signal model is similar to the one of the mosfet. We have a transconductance and we have also the Cgd and Cgs, but they exist also in the mosfet.

JFET noise sources



Note: usually lower 1/f noise than MOSFETs



$$\frac{\overline{v_i^2}}{\Delta f} = \underbrace{4kT\frac{2}{3}\frac{1}{g_m}}_{\text{Thermal noise}} + \underbrace{\frac{A_f}{f}}_{\text{Flicker noise}}$$

$$\overline{i_g^2} = 2qI_G\Delta f$$

Shot noise of gate current
(small but not negligible)

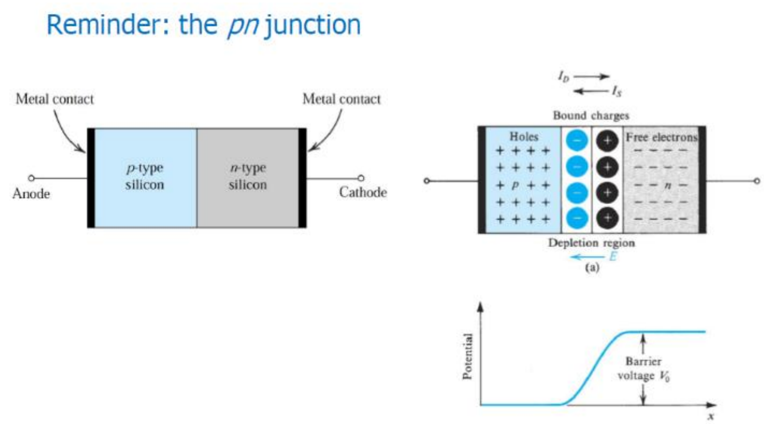
We can summarize the noise generator at the input and we have again a thermal noise on the channel (since it is conductive) and a Flicker noise. The formula is different since the parameter are all condensed in a single factor Af. In the JFET Af depends on dimension, technology, it is the noise at 1Hz.

The plot of the noise vs freq is similar to the one of the mosfet.
The JFET is interesting since it has in general lower Flicker noise with respect to the MOSFET (orders of magnitude smaller). If we consider that for biosignal they are below 1kHz, the 1/f noise is the one present → JFET is the preferrable input transistor in amplifiers.
Also in JFET we can have shot noise due to the gate current. With respect to the mosfet, in the mosfet we have the oxide and so theoretically 0 gate current, in the JFET we have a reversed bias pn junction, so like in all diodes in reverse regime the current is small but not negligible, so the **shot noise must not be neglected**.

BIPOLAR JUNCTION TRANSISTOR (BJT)

Completely different working principle. It is a transistor in the sense that we have current between two electrodes and the current is modulated by a 3rd electrode.

We must remember the pn junction. It is the contact between p type silicon full of holes and n type full of electrons. When they are in contact, holes move from the left to the right, and while doing so, they leave some uncovered dopants (remain as a layer of negative fixed charge). Similarly for the electrons, that leaves a layer of uncovered fixed positive charges. This creates an electromagnetic field that prevents further diffusion of the charges. Once the process happens and reaches equilibrium, there is the creation of a potential barrier that prevents further transfer of charges.



If we apply a positive voltage between n and p silicon we increase the voltage barrier and so there is no reason for charge to jump over the barrier (REVERSE BIAS REGION OF A DIODE). Conversely, if we apply a positive voltage between p and n silicon, we are lowering the barrier by a certain amount (the voltage we are applying) that makes easier the transfer of holes to the right and electrons to the left, there is a restart of current and this is an exponential rise of the current (FORWARD BIAS REGION OF THE DIODE). There is an exponential behaviour shown it the formula because for a given lowering of the barrier, the statistical probability to jump over the barrier increases exponentially. So the formula of the current has an exponential dependance on the ratio between the applied voltage and a quantity called thermal voltage given by the Boltzmann constant time the temperature divided by the charge of electrons.

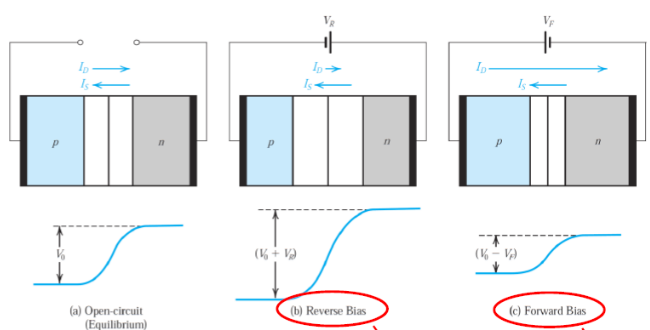
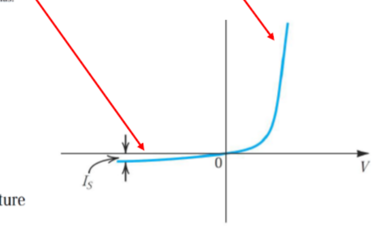


Figure 3.11 The *pn* junction in: **(a)** equilibrium; **(b)** reverse bias; **(c)** forward bias.

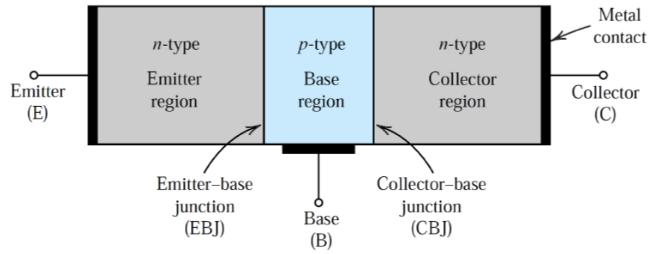
$$I = I_S(e^{V/V_T} - 1)$$

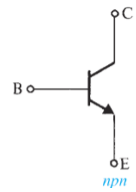


$$V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV at room temperature}$$

At room temperature, the thermal voltage Vt is of 25mV.

The BJT is a combination of 2 pn junctions. The p type silicon is only one, in the center (base), put in contact with the emitter region (left) and collector region (right). So we have two pn junctions. In the drawing of the npn BJT we identify that electrodes are called base, emitter and collector. Again the current flows between collector and emitter and it is controlled by the base electrode.





How the junctions are biased

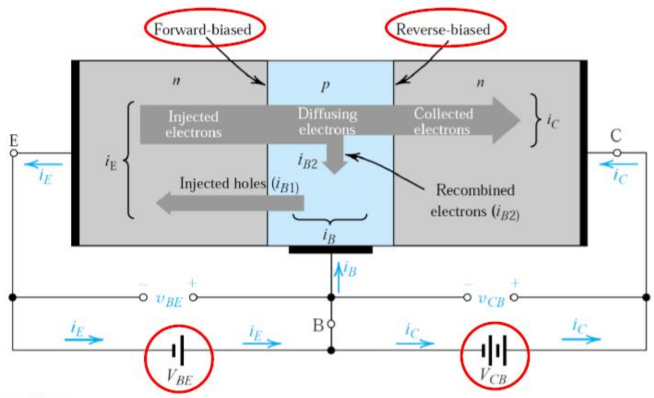
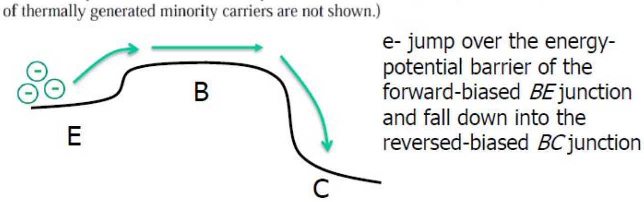


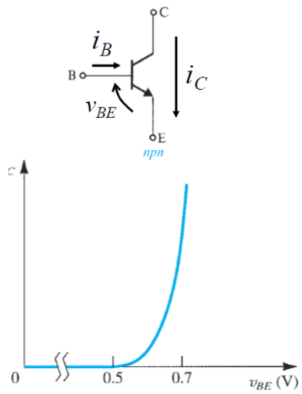
Figure 6.3 Current flow in an *npn* transistor biased to operate in the active mode. (Reverse current components due to drift of thermally generated minority carriers are not shown.)



The structure described is totally symmetric, but the biasing not. The junction on the left, the base to emitter junction, is biased positively (**forward biased**), while the one on the right, base-collector is **reversed bias**. If I look at the energy potentials barrier, on the left we have lowered the barrier between emitter and base (thanks to the forward bias), so electrons from the emitter are able to jump into the base (forward bias diode), however, thanks to the revers bias on the right, the barrier is increased a lot, and this prevents electrons from the collector to jump over the barrier, but the electrons jumping into the base are travelling through the base and then jumping down to the collector. A reversed bias junction avoids electrons to jump over the barrier (so from C to B), not electrons already over the barrier to jump down.

So basically we take electrons from the emitter (called so because it emits electrons), they move through the base and are collected by the collector. Since there is no current in the other direction thanks to the reverse bias, we have a net current of electrons from left to right and no viceversa. This means that if electrons move from emitter to collector, current is in the opposite direction. The arrows in the schematic is correct, because it is true that electrons move from the bottom to the top, but current is in the other direction (top-bottom) .

IV characteristic



$$\boxed{i_C = I_S e^{v_{BE}/V_T}}$$

$$i_B = \frac{i_C}{\beta}$$

$$i_E = i_C + i_B \qquad i_C = \alpha i_E \qquad \alpha = \frac{\beta}{\beta + 1}$$

$\beta = 100$, then $\alpha \simeq 0.99$.

Figure 6.15 The i_C–v_{BE} characteristic for an *npn* transistor.

The good news is that for what concerns the I-V (V: controlling voltage, base-emitter voltage, the one that lowers the barrier) curve, the BJT has the same characteristic formula of the diode. The current has an exponential dependency on the base-emitter voltage divided by the thermal voltage. In this formula we don't see at first approx. any dependance on the collector-base voltage, so apparently the junction that is reversed bias, the flow of current doesn't depend on how much the other connection is reversed biased. And this has a very intuitive explanation: the current is dominated by the jump over the barrier, but once the electrons are on the edge, why should they depend on how deep the fall is? The way to jump down a mountain doesn't depend on how deep the jump is. Hence the current doesn't depend on Vcb.

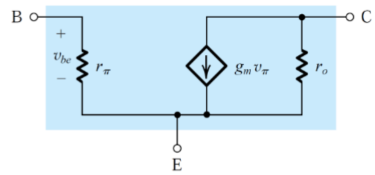
A peculiar feature of the BJT is that differently from the mosfet we have a base current. This current is given by the collector current divided by a figure of merit called beta. Indeed, when electrons travel through the base, we are like in an enemy territory, a region full of holes → some electrons recombine with holes. So if holes in the p electrode are lost by recombination, for the Kirchhoff law, these holes have to be reformed by the base electrode. So one reason for the base current is the current of holes taken from the network and going to recombine the electrons.
Moreover, another contribution is due to holes that are entering into the base and are injected into the emitter. In reality we have also holes that move in the other direction due to the forward bias; in reality transistors are made in a way so that holes are minimal, so that the injection of holes is reduced a lot.

These recombined holes may be reduced if we make the base very narrow. Thus we increase the probability that injected electron survive up to the collector without recombination in the base. If this is true, so we can reduce the effect, we have a very small base current, hence the beta parameter is very large → in ideal BJT we are not interested in the base current, doesn't play a relevant role. Hence beta are usually large (100 or 1000). However the current on the base is not negligible, because associated to it we have a shut noise.

By using the Kirchhoff law, we can have an equivalent equation where the collector current is proportional to an alpha factor times the emitter current and the alpha factor is closer to 1 if the larger the beta. I want alpha close to 1 because this means that most of the emitter current is collected at the collector, most of the electrons injected are collected in the emitter are collected at the collector.

If 100% of electrons are collected, it means that alpha is 1 and beta infinite.

we can build a small signal model, still with the voltage control generator and the transconductance, an output impedance but the transconductance is given by the current divided by the thermal voltage. The big difference between mosfet and BJT is that in mosfet and JFET we have no impedance on the third electrode, in the BJT we have it that is beta/gm. The larger beta, the more this impedance gets close to infinity.



$g_m = I_C/V_T$
$r_\pi = \beta/g_m$

Pnp BJT

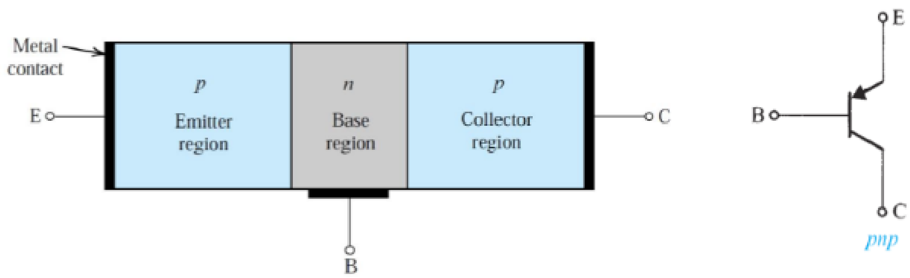


Table 6.2 Summary of the BJT Current–Voltage Relationships in the Active Mode

$$i_C = I_S e^{v_{BE}/V_T}$$
$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T}$$
$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$$

Note: For the *pnp* transistor, replace v_{BE} with v_{EB}.

$i_C = \alpha i_E$	$i_B = (1-\alpha) i_E = \dfrac{i_E}{\beta+1}$
$i_C = \beta i_B$	$i_E = (\beta+1) i_B$
$\beta = \dfrac{\alpha}{1-\alpha}$	$\alpha = \dfrac{\beta}{\beta+1}$

V_T = thermal voltage $= \dfrac{kT}{q} \simeq 25$ mV at room temperature

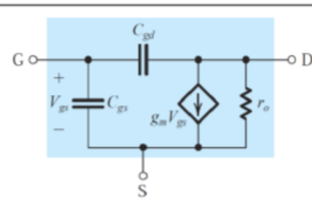
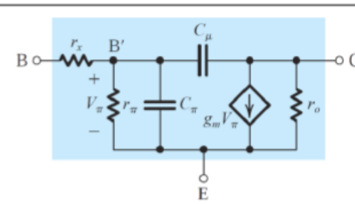
The base is n silicon and emitter and collector are p silicon. The current flows from the top to the bottom. The emitter is emitting holes in this case, it is in the top, and the holes follow the arrow, are injected into the base and they are collected by the collector.

COMPARATIVE SUMMARY

In the case of mosfet to switch on Vgs>Vthermal; in the BJT, the Vbe must be larger than the typical on voltage of a diode, that is around 0,6-0,7V. Hence the on voltage of a BJT is the classical 0,7V like in a diode.
The most relevant difference is the nonlinearity. In mosfet we have a square dependance of the drain current with gate voltage Vg (less different behaviour with respect to the linear one), while in the other case the behaviour is exponential (highly nonlinear). In BJT hence the linearity region is much smaller

than in the mosfet, even if the small signal circuits are similar. In mosfet you have to make a linearization of a square function, in BJT of an exponential one → region of validity of the linear model is much small.

Moreover, we have a resistance in the emitter (rπ) and one in series in the base called spreading resistance that is a resistance between the contact and the base region.
The spreading resistance is a resistance between the contact of the silicon and the region where the transportation of electrons take place. Its dimensions cannot be negligible.
In fact, in the true model of a BJT we have the emitter, the base and then the collector. We have a current flowing between emitter and collector and it passes through the base. If this is the real drawing of the BJT, the base contact is not so close, but far away, so we have a resistance between the electrode of the base and the region where 'the game is performed'. Of course then we have a noise source generator associated to rx.
1.

Transconductance g_m	$g_m = I_D/(V_{OV}/2)$ $g_m = (\mu_n C_{ox})\left(\frac{W}{L}\right)V_{OV}$ $g_m = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)I_D}$	$g_m = I_C/V_T$
Output Resistance r_o	$r_o = V_A/I_D = \frac{V_A' L}{I_D}$	$r_o = V_A/I_C$
Intrinsic Gain $A_0 \equiv g_m r_o$	$A_0 = V_A/(V_{OV}/2)$ $A_0 = \frac{2V_A' L}{V_{OV}}$ $A_0 = \frac{V_A'\sqrt{2\mu_n C_{ox} WL}}{\sqrt{I_D}}$	$A_0 = V_A/V_T$
Input Resistance with Source (Emitter) Grounded	∞	$r_\pi = \beta/g_m$
High-Frequency Model		

Noise sources for BJT

We have no thermal noise, only on the spreading resistance, but most of the noise in the collector is shot noise, which is quite understandable, because the bipolar transistor is based on junction, not on a conductor. So it is quite obvious to have a shot noise on the junction of the collector and on the base.

$$\overline{v_b^2} = 4kTr_b\,\Delta f$$

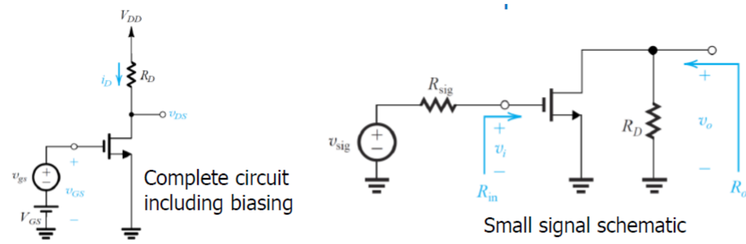
$$\overline{i_c^2} = 2qI_C\,\Delta f$$

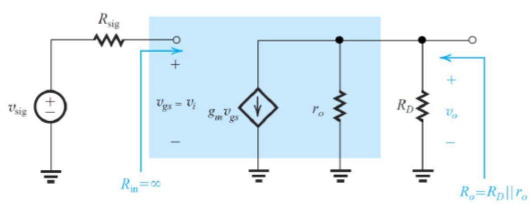
$$\overline{i_b^2} = \underbrace{2qI_B\,\Delta f}_{\text{Shot noise}} + \underbrace{K_1\frac{I_B^a}{f}\,\Delta f}_{\text{Flicker noise}} + \underbrace{K_2\frac{I_B^c}{1+\left(\frac{f}{f_c}\right)^2}}_{\text{Burst noise}}\Delta f$$

Then we have Flicker noise on the base. The only important thing is that if we build an amplifier base on BJT than on mosfet we have a relevant shot noise on the base, since the current on the base is not negligible. This shot noise is a killing factor for the noise in BJTs. For instance, if we suppose to have an ECG and there we have the electrodes, we have the noise due to current flowing into the base of the transistor.

MAIN SINGLE STAGE MOSFET AMPLIFIER CONFIGURATION

COMMON SOURCE AMPLIFIER





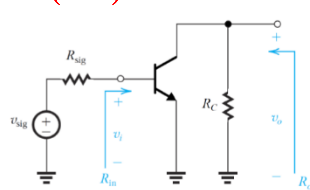
$$v_o = -(g_m v_{gs})(R_D \| r_o)$$

$$A_{vo} = -g_m(R_D \| r_o)$$

$$\boxed{A_{vo} \simeq (-g_m R_D)}$$

It is a mosfet with a source to zero and a small signal vgs superposed to Vgs (Vgs is the biasing voltage). On the drain we have a load resistor Rd. From the point of view of the gain we use the small signal model. We have the source of signal, the input impedance that is infinite in the case of mosfet, the controlled generator gm*vgs and the current given by the small signal generator is converted into voltage vds (output voltage). The output voltage is the current gm*vgs multiplied for the overall impedance in the output node, that is Rd || r0 (r0: transistor output impedance). We have a minus sign because if the current flows in the indicated direction, the voltage is negative.
Then we can extract the amplifier gain, where Rd>>r0, so r0 is discarded.

COMMON EMITTER AMPLIFIER (BJT)



$$G_v \equiv \frac{v_o}{v_{sig}} = -\frac{r_\pi}{r_\pi + R_{sig}}\, g_m(R_C \| r_o) \sim -g_m \cdot R_C \quad (\text{if } R_{sig} << r_\pi)$$

Note: the transfer functions of transistor stages with BJT are 'similar' to the corresponding ones with MOSFET (but pay attention to the finite *Base* impedance)

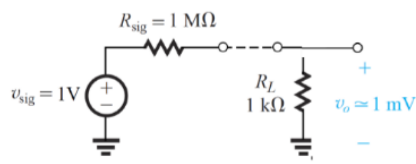
Similar to the one with MOSFET. The main changes is that it is now a common emitter amplifier, no more a common source amplifier, because the grounded electrode (that previously was the source) is the emitter. The gain is similar.

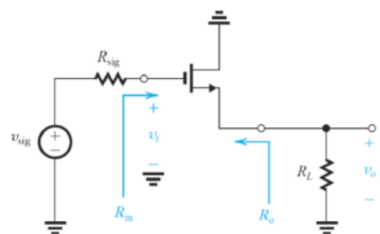
In reality, due to the input impedance, that is no more infinite but it is beta/gm, we have in reality a partition between the source impedance and the BJT impedance. All the rest is the same.
If we can neglect this impedance, the final formula is -gm*Rc which is equal to the one of the MOSFET.

So the new transistor, but if we use the BJT in the same configuration as the MOSFET, at first approximation formulas are the same, but we must pay attention to the input impedance. For instance if the source impedance is very large, the ratio is no more the unity, so we must consider the partition with the input impedance.

COMMON DRAIN AMPLIFIER (Source follower)

The need for Voltage Buffers





$$G_v = A_v = \frac{R_L}{R_L + 1/g_m} \sim 1$$

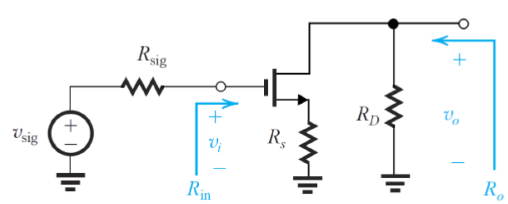
(if $R_L >> 1/g_m$)

$$R_o = 1/g_m$$

$$R_{in} = \infty$$

In this configuration the drain is grounded and we have an impedance on the source and the output of the circuit is on the source. At the end, the gain between input and source is as in the formula. The main goal of this circuit is not amplifying, but in this circuit the input impedance is infinite and the output one is 1/gm. These are the nice features of a voltage buffer. It has a gain equal to 1, a high input impedance and a small output one.
The voltage buffers are needed when you have a source with a relatively large source impedance and we have a load (upper circuit). If we connect the load directly to the source and the load has a much smaller impedance than the source, we have a very penalizing voltage division. If we put a buffer, the buffer has a large input impedance so it basically transfers the voltage to the output.

COMMON SOURCE AMPLIFIER WITH A SOURCE RESISTANCE



$$A_{vo} = \frac{v_o}{v_i} = -\frac{R_D}{1/g_m + R_s} \sim -R_D/R_S$$

(if $R_S >> 1/g_m$)

(gain depends only from resistors and not from g_m)

It is basically the same version of the common source but we have a resistance on the source and the gain is given by -Rd/Rs at first approx. The advantage is that while in the common source equation the gain depends on the transconductance gm that is a transistor-dependent parameter, in this configuration if we assume that 1/gm is negligible with respect to the source resistance, the amplification factor is a ratio between two resistances, which usually are quite stable components → amplification factor doesn't depend on the transistor itself. Hence it is a more reliable gain.

SUMMARY OF FORMULAS

Table 5.4 Characteristics of MOSFET Amplifiers

	Characteristics[a, b]				
Amplifier type	R_{in}	A_{vo}	R_o	A_v	G_v
Common source (Fig. 5.45)	∞	$-g_m R_D$	R_D	$-g_m(R_D \parallel R_L)$	$-g_m(R_D \parallel R_L)$
Common source with R_s (Fig. 5.47)	∞	$-\dfrac{g_m R_D}{1+g_m R_s}$	R_D	$\dfrac{-g_m(R_D \parallel R_L)}{1+g_m R_s}$ $-\dfrac{R_D \parallel R_L}{1/g_m + R_s}$	$-\dfrac{g_m(R_D \parallel R_L)}{1+g_m R_s}$ $-\dfrac{R_D \parallel R_L}{1/g_m + R_s}$
Common gate (Fig. 5.48)	$\dfrac{1}{g_m}$	$g_m R_D$	R_D	$g_m(R_D \parallel R_L)$	$\dfrac{R_D \parallel R_L}{R_{sig} + 1/g_m}$
Source follower (Fig. 5.50)	∞	1	$\dfrac{1}{g_m}$	$\dfrac{R_L}{R_L + 1/g_m}$	$\dfrac{R_L}{R_L + 1/g_m}$

[a] For the interpretation of R_{in}, A_{vo}, and R_o, refer to Fig. 5.44(b).

[b] The MOSFET output resistance r_o has been neglected, as is permitted in the discrete-circuit amplifiers studied in this chapter. For IC amplifiers, r_o must always be taken into account.

HOW TO INCREASE THE GAIN OF A COMMON SOURCE AMPLIFIER EXPLOITING MOSFET OUTPUT IMPEDANCE

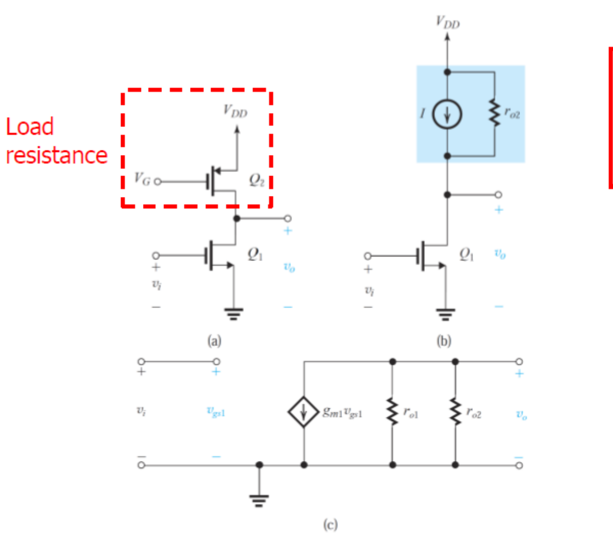


Figure 7.3 **(a)** The CS amplifier with the current-source load implemented with a *p*-channel MOSFET Q_2; **(b)** the circuit with Q_2 replaced with its large-signal model; and **(c)** small-signal equivalent circuit of the amplifier.

Substitute R_D (which cannot be increased indefinitely because of biasing issues) with the r_0 of a MOSFET

$$A_v = -\frac{1}{2} g_m r_o$$

Note: More generally, r_0 can be used to substitute R in MOSFET stages to increase the gain (e.g. also in the Source Follower to substitute the Source resistance R_S)

The gain of the common source is the transconductance times the parallel between the output resistance of the transistor and the load resistance connected to the drain. Very often, the output resistance of the transistor is so large that is dominating in the parallel → the resulting resistance for the gain is just the load resistance Rd. How to increase the gain? By increasing the resistance of the drain. However, I can do it only up to a maximum value, because the resistance Rd plays a role also in biasing of the branch. It means that if the current in the transistor is fixed but the Vgs and this current multiplied by the resistance creates a voltage drop, and the top side of the resistance is fixed at Vdd, the larger the value of the resistance the larger the voltage drop across the resistance for a given current. So the output node lowers its value, since the upper node of the resistance is fixed at Vdd. Si if the node goes down, it puts the transistor into triode regime. Indeed, if Vds becomes too small, the transistor enters in the triode region (where the current increases linearly) and so the current is reduced a lot (and it is no more in the saturation region).

The important observation is that there is a maximum gain we can reach with the formula gm*Rd because there is a maximal Rd value so not to leave the saturation region.
The goal is to have a large resistor but at the same time a resistor not involved in the biasing of the branch of the drain.

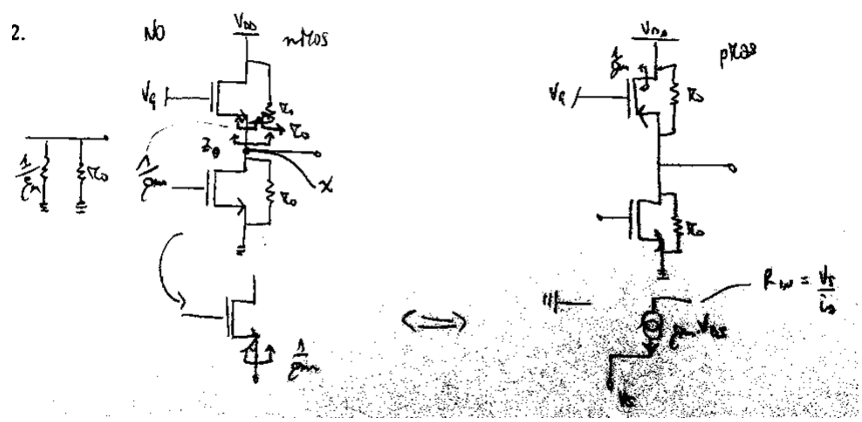
It is possible by substituting the Rd resistance with a transistor. We use a pMOSFET connected drain to drain. So this transistor that we will be bias in saturation region (it is a pMOSFET bias at a convenient voltage). Its equivalent model is a current generator, because a transistor bias in saturation region provides a constant current and we simply need this current to match the current on the bottom (by the Kirchhoff law the top current must be equal to the bottom current).
From the point of view of small signal analysis this transistor includes also a r0, its own output impedance.

If we suppose it is a good substitution of the Rd resistance and we go back to the calculation of the amplification, in the common source we have the parallel between Rd and r0 and the winner is r0. If we now consider the r0 at the top, we have the parallel between two r0, so the two resistances are almost the same.
Hence the new formula is that the amplification is given by r0 parallel to r0. But the parallel is r0/2. So the new formula is that the amplification is given by: (-1/2)*gm*r0.

This is a great gain, the largest gain we may have in an amplifier, because it is the multiplication of the transconductance times the output impedance of the transistor, which is a very large resistance, is much larger than whatever Rd we can put. And we have no problem of biasing because it is a resistance which doesn't depend on the biasing of the branch. → We substitute the resistor with an ACTIVE LOAD.

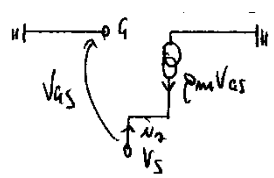
We can substitute whatever resistor by doing this.

2.



We have used a pMOSFET and not a nMOSFET for the following reason.
Is the impedance Z0 in that node just r0? No, because it is the source of a mosfet, and we have also the 1/gm. The impedance from the source of a MOSFET is 1/gm. It can be computed by an equivalent model, where we apply a voltage and measure a current.

3. Test of the output impedance



$$Z_{in} = \frac{V_s}{i_s} \; ; \quad i_s = -g_m V_{GS}$$

$$V_s = -V_{gs} \rightarrow Z_{in} = \frac{-V_{gs}}{-g_m V_{gs}} = \frac{1}{g_m}$$

We apply a voltage and we measure the current flowing. By definition, Zin = Vs/is. But the current is given by -gm*Vgs. So the general property of a transistor is that the impedance we look from the source is 1/gm, very small.

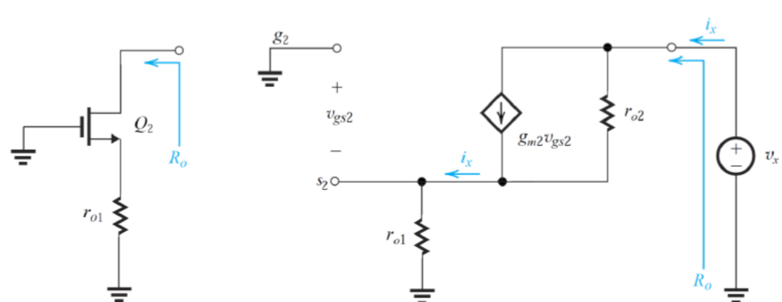
Coming back to the configuration with the two transistors, I have on the top 1/gm, and on the right r0. So I have a parallel between the two. The impedance connected to the node x is 1/gm || r0, but in the parallel 1/gm is the smalles. So if I put an nMOS, I have r0 in the bottom and 1/gm → instead of substituting the load with a large impedance, I would substitute it with a catastrophically small impedance.
Conversely, if I use a pMOS, the 1/gm is on the top, not on the bottom, so at node x the only connected impedance is r0. So I have the parallel between the two r0.

So the goal of the pMOS was to have for free a very large equivalent resistor → beautiful gain.

HOW TO INCREASE THE OUTPUT IMPEDANCE OF A MOSFET

The output impedance of a mosfet is already pretty large, r0, both on the top and on the bottom of the previous configuration. But can we do better? Yes.

Add another MOSFET at the Source of the main MOSFET



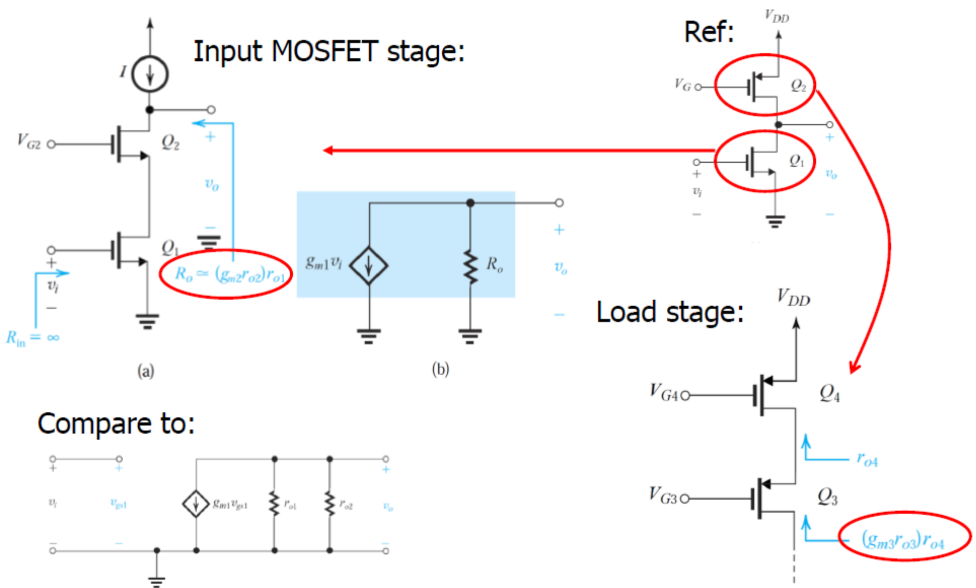
$$R_o \equiv \frac{v_x}{i_x} \qquad R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} \qquad \boxed{R_o \simeq (g_{m2} r_{o2}) r_{o1}}$$

Let's suppose to have a mosfet. If I put the source to ground, the output impedance is r0. Let's suppose we put a large resistor on the source, for example another transistor. What is the output resistance? We take the small signal model, we apply the rule: we take a generator Vx, we apply Vx an measure the current Ix absorbed by the network and by the definition the output impedance is the ratio between Vx and Ix.
If we do the calculation, we come to the result in the middle. Usually the last factor is greater than the previous two, so the conclusion is the red box, that is the output impedance of the network. gm*r0 is the factor multiplying the output resistance of the transistor, and it is usually larger than 1.

In conclusion, we have r0 multiplied by a nice gain. This is how to boost the output impedance of a transistor.

Increase further the gain of a Common Source amplifier

The latter considerations are useful because I want to improve the gain of my common source. It was not so bad, because it was gm*r0, but we can do better if we substitute to r0 the new amplified output

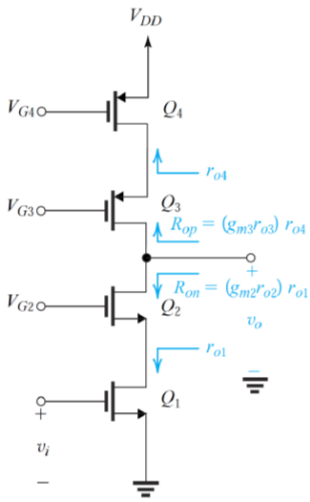


impedance.

To do so, I take the common source, I substitute its transistor Q1 with the same transistor but then I put another transistor in between. The new transistor (upper left) doesn't change the operation of the previous one, because when we have a current flowing, the current will flow into the top, will enter in the new transistor because I have the 1/gm. So the transistor couple on the left is a current buffer, takes a current and provide a current on the output.
Hence the transfer of the function is not changed, but the output impedance is no more r0, but is the new formula. If we look at the impedance from the top, it is r01*gm*r02. Of course having a large impedance for Q1 and a low one for Q2 (upper right) is not sufficient, because the lowest wins in the parallel of the two. So one has been boosted and the other remains the same → wins in the parallel.

Hence I have to do the same trick also in the top. I substitute a simple active load with a totem of two active load. The only different is that we have two pMOS, but the formula is the same.

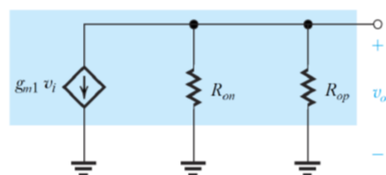
Now this is the complete circuit.



$$A_v = \frac{v_o}{v_i} = -g_{m1}[R_{on} \| R_{op}]$$

$$A_v = -g_{m1}\{[(g_{m2}r_{o2})r_{o1}] \| [(g_{m3}r_{o3})r_{o4}]\}$$

$$\boxed{A_v = -\frac{1}{2}(g_m r_o)^2 = -\frac{1}{2}A_0^2}$$

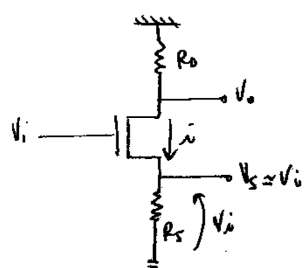


Vi is the input, Vo is the output and I have a totem on the bottom with total impedance Ron, then Rop; the amplification formula is the one in the box. I also suppose that the r0 and gm are the same.

In the new amplification formula we have (gm*r0)^2. gm*r0 was the gain with two transistors only; now thanks to the totem I have the square of the gain. So if gm*r0 is 10 for instance, the maximum amplification we have is 50 → we have increased the gain of one order of magnitude.

This trick can be applied everywhere, not only to the common source. In the common source with source resistance, for instance, the Rd can be substituted with a totem of pMOS.

4.



$G = \frac{V_o}{V_i}$

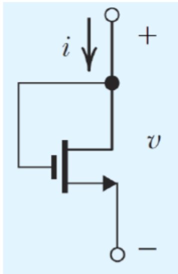
$i = \frac{V_i}{R_S}$

$V_o = -iR_D = -\frac{V_i}{R_S}R_D$

$G = \frac{V_o}{V_i} = -\frac{R_D}{R_S}$

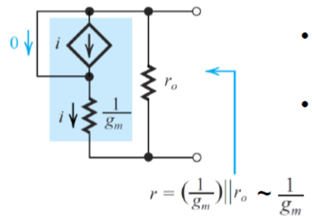
First of all, in the small signal analysis we put to down all the power supply (grounded). I want to compute the gain that is G = Vo/Vi.
The quick way to compute it is that from Vi to the source (Vs) it is a source follower, then as a good follower the voltage on the source is almost equal to the voltage of the input. So if Vs = V1, the voltage across Rs in Vi and the current 'I' flowing is Vi/Rs. This current flows also in the top and so Vo = -I*Rd.
To be more precise, we substitute with the model.

THE TRANSDIODE



$$i_D = \frac{1}{2}k'_n\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$

(MOSFET always in saturation)



- It is used as a 'diode': $V_{ON} = V_{GS} \sim V_{TH}$
- Its impedance is $1/g_m$

It is a bipole formed by a transistor where we short the gain with the drain. Hence the mosfet is always in saturation, because Vgd is always = 0 so smaller than the threshold.
The impedance of this bipole can be calculated with the small signal model and it is 1/gm || r0, so 1/gm. The conclusion is that the transdiode is very similar to a diode, because in terms of biasing Vgs is almost equal to the threshold voltage, even with a small current, when the transdiode is on.
As for the impedance, the impedance of the bipole is equal to the one of the source.

THE CMOS INVERTER

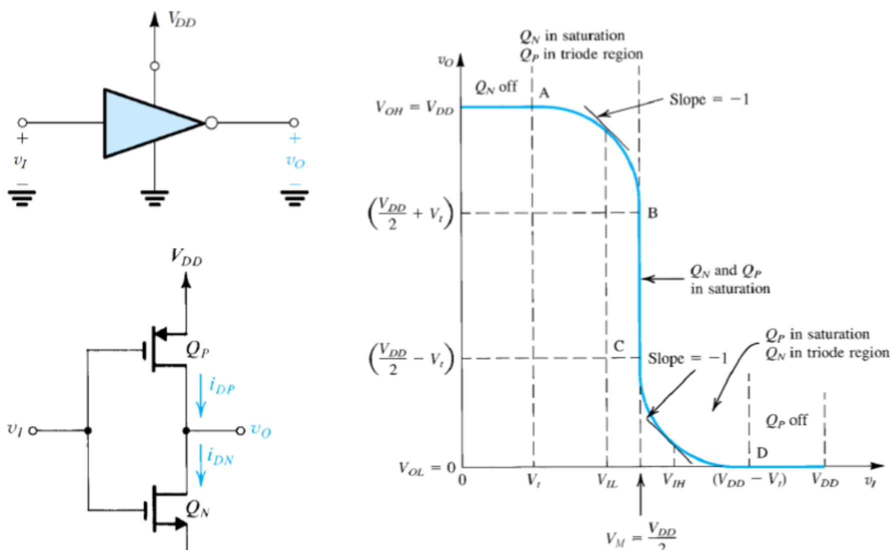
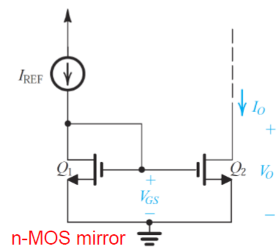


Figure 13.20 The voltage-transfer characteristic of the CMOS inverter when Q_N and Q_P are matched.

The inverter is made by a pMOS and a nMOS with common gate and common drain. When the input is low the nMOS is off the pMOS is on (in triode regime) and the output is high, when we increase the input viceversa, the nMOS enters in the triode regime and the output is low.

<u>The take-home message is that when we want to increase the gain, we use the r0 of the transistors.</u>

CURRENT MIRRORS

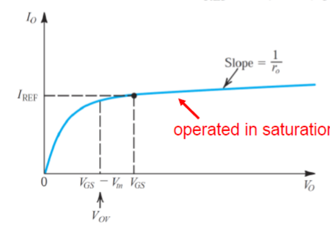


- Q_1 and Q_2 have the same V_{GS}, so if Q_2 is in saturation (Q1 for sure) and if the transistors have the same W/L*:

  $\Rightarrow I_0 = I_{REF}$

- If W/L are not equal: $\frac{I_O}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1}$ mirroring factor

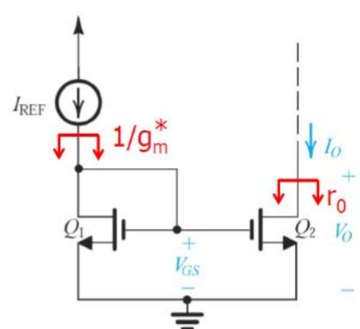
$$^{*}i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$



This is a stage widely employed in CMOS circuits. In the figure we see how the n-mirror is designed. It is based on a couple of nMOS transistors (there is also the p version). We have Q1 and Q2 that share the same Vgs voltage, and Q1 has the Vg shorted with the drain. We supposed to have a reference current flowing in Q1 and we are interested in knowing the current on the right branch I0. The result is that I0 = Iref, and this is the reason why the structure is called current mirror.

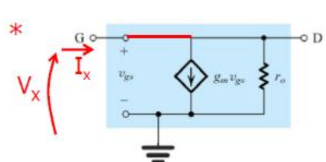
I0 = Iref because the two transistors have the same Vgs. If they work in saturation region, so with a constant current, we have the equation that applies for that regime. Since the Vgs is the same, Vov (overvoltage) is the same and also the current.
But are we sure they are in saturation? As for Q1 yes, because Vgd is 0 and so below the threshold by definition. As for Q2 we don't know, because we have no information on Vd. But because of the regular operation of the transistors as a current mirror, we assume that to whatever the node of the drain is connected, it is in saturation. Of course, if it is too low, we fall in triode regime, but we assume that Q2 is in saturation, while Q1 certainly is.

The two current are the same if we suppose that W/L is the same. If they are not equal, the difference in the current is not due to the overvoltage Vgs-Vth but due to W/L.
So we can have current mirrors with the same current or with different currents if the ratio is different.



Notes:

- Input impedance is low ($1/g_m$), which makes possible driving current into the input of the mirror
- Output impedance is high (r_0) which makes the mirror an ideal current generator



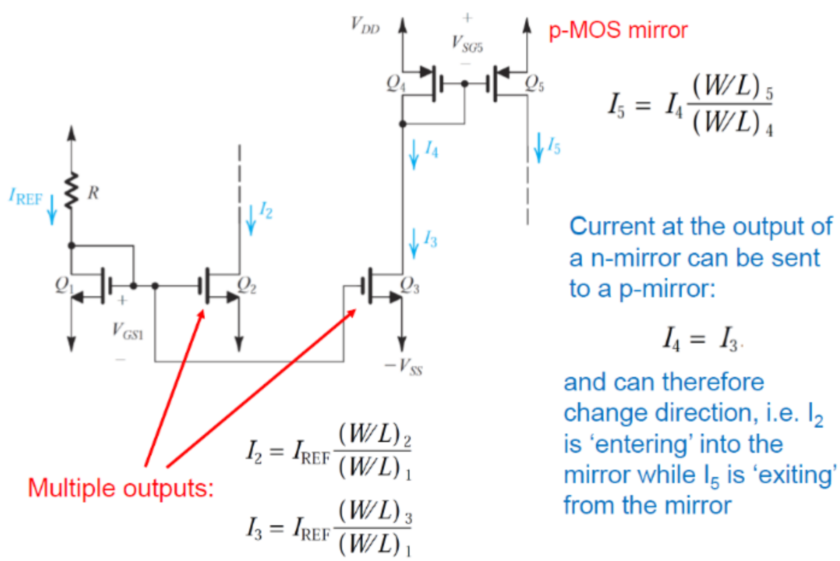
$V_{GS}=V_x$ $\quad I_x=g_mV_{GS}=g_mV_x$

$R_x=V_x/I_x=1/g_m$ (r_0 neglected)

The Q1 has a shortcircuit because it is a sink for the current, receives Iref because the impedance seen at the input of the current mirror is 1/gm → low impedance, and this is provided by the shortcircuit. The fact that the input impedance is 1/gm is due to the fact that the left branch is a transdiode.
We have the small signal model for the transistor, we apply a testing voltage and we measure the current absorbed. Since vgs is identical to Vx and Ix is equal to gm*vgs.

We draw the conclusion that the current mirror presents a low input impedance, and this is the reason why we can force the current Iref with a generator in 1/gm (branch connecting drain and gate) and then Q1 imposes the current to Q2.

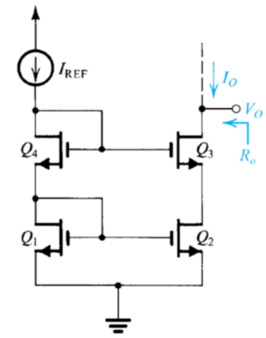
Let's now suppose we have the following current mirror (Q1 and Q2), eventually with a different mirroring factor.



Let's suppose now we want another current I3 to be a replica of Iref. We need to insert an additional transistor connected with the gate with the other. Because of this, what said before is still valid, the Vgs shared is the same. So we can replicate this concept many times.
It is hence a very common way to deliver currents to several points of the circuit.

Second observation: we see a pMOS mirror. It is based on pMOS transistor, with source connected to Vdd, so the common voltage is Vsg (source-gate) and we have that the drain voltage is shorted to the gate in the left transistor → operates in saturation. If we connected this mirror, in particular its input branch, with the output branch of another nMOS mirror, we have that I3 = I4. In particular, the current I5 is in relation to the current Iref.
If all the transistors have the same W/L (all the mirroring factor = 1), we have that Iref = I5. But is it really necessary? Yes, because the big difference is that I2 is a current entering into the mirror, form a hypothetical load into Q2, while I5 it is exiting. It is true that n and p mirror may have the same current, **but we use n mirror if we need a current entering in the mirror, and a p mirror if we need a current exiting it**.

Current mirror with improved performances



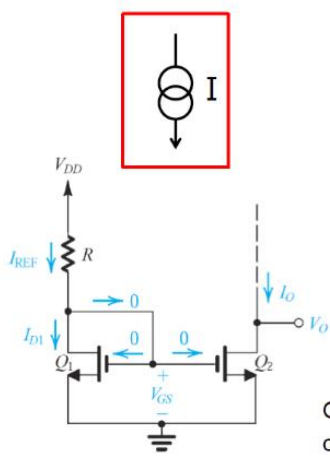
- I_{REF} flows in Q_4 and Q_1 while I_0 flows in Q_2 and Q_3
- The mirroring factor is determined by Q_1 and Q_2
- The output impedance is increased:

$$\boxed{R_o \simeq g_{m3} r_{o3} r_{o2}}$$

We see the same principles, but we have also a cascade of 2 identical cells. The mirror in term of mirroring is the same, since Iref enters in the 1/gm of Q4, it is exiting from Q4 and entering in Q1. So Iref sees two 1/gm in series, so it imposes the current in the right branch as in the single mirror. Then Iref is mirrored by Q2 and then the current of Q2 enters in Q3 and then exists from the drain. So basically the mirror is still on, and the mirroring factor depends on Q1 and Q2, because are the two transistors that share the same current.

The advantage is the output impedance. In the classical mirror it is r0, the one of just Q2. In this case when we have the cascade of two transistors, the output impedance is r0*gm*r0. This mirror is improved not because of the mirroring factor, but because the output impedance is better.

THE BASIC MOSFET CURRENT SOURCE



$$I_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2$$

$$I_{D1} = I_{REF} = \frac{V_{DD} - V_{GS}}{R}$$

$$I_O = I_{D2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 (V_{GS} - V_{tn})^2$$

Output impedance (ideally ∞ for an ideal current generator) given by r_0 of Q_2.

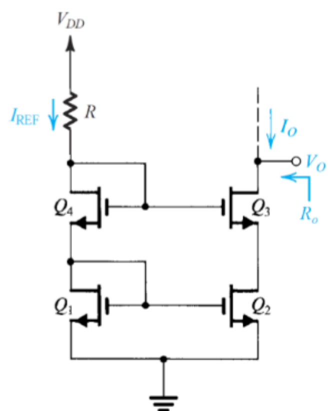
With a current mirror we can generate a current source. In the red box we have the current generator, that in practice is implemented with a current mirror.
The characteristic of a current generator is the output impedance, that is infinite in the ideal case, and also that they provide the current. So no voltage is able to change the current in the idea current generator.

The implementation is a current mirror, the left node is connected to a power supply by a resistor, so the network generates a current Iref, and it is mirrored to the right and supplied at high impedance (r0, the impedance of the output node of Q2).

As for the calculation, we have the equation of the current for the transistor on the left. But at the same time Id1 = Iref and Iref can be calculated by the ohm law as (Vdd-Vgs)/R. If we substitute in the transistor equation, we have an equation whose variable is Vgs.
It is an equation of the second order, and one solution must be discarded (if Vgs<0), and with Vgs we compute Id1. If we solve it, we see the current depends on the resistor.
Then the output current Id2 is given by the last equation.

So the current generator can be easily designed, and R can be a potentiometer to change the amount of current at the output. The output impedance isn't however infinite, but it is r0.

We can do better to have infinite impedance, thanks to the modified mirror.



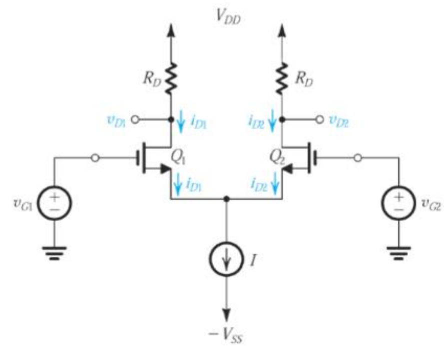
$$\boxed{R_o \simeq g_{m3} r_{o3} r_{o2}}$$

We take the network in the image, we take R connected to Vdd, we repeat the calculation as before, we compute a given current depending on R on Q3 and the good news is that the output impedance is no more only r0 but r0*gm*r0. So we have a higher value of output resistance, which makes this structure much closer to the ideal current generator.

In a CMOS amplifier, we may need a lot of current generators. But once we have created a starting structure, we can replicate the current in several points of the circuit by means of the replica of the gate contact, so to have different replicas of Iref with different mirroring factors.

We can use also BJT to implement current mirrors, and the architecture is the same, they share a common base emitter voltage, not the Vgs.

THE DIFFERENTIAL STAGE

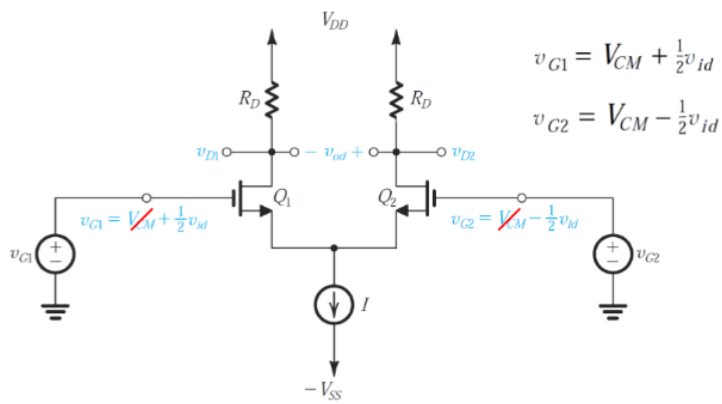


- It is an amplifier for a differential voltage given at the input (V_{G1}-V_{G2})
- It is commonly used as input stage of an operational amplifier

Usually, it is the input stage of any OpAmp. It is composed by two identical transistors, they cannot be different in terms of W/L, they share the same current generator 'I' and on both drain we have a load resistor Rd.

We have a voltage generator Vg1 and Vg2 on the two inputs **and the goal of this stage is to amplify not just a single voltage but the voltage difference (Vg1-Vg2) and provide a voltage difference between the drains**. The output is the voltage difference between the drains.

This structure is absolutely symmetrical, so the current 'I' flows half on the right and half on the left. As well as there is no reason, if Vg1 and Vg2, the drain voltages to be different → this structure is sensible to differences. It is a stage used to sense the differences, and not the absolute values.



- We split the input voltages in a common voltage V_{CM} and in a differential component V_{id}, which can be considered applied as $1/2V_{id}$ and $-1/2V_{id}$ to the inputs.
- The common mode V_{CM} is not producing effects at the output (V_{od}=0) because of the symmetry of the stage.

Vod is the voltage of interest. As for the input, it may be convenient that any voltage Vg1 in relation to any voltage Vg2 can be considered composed by a common component (common mode, Vcm) and a differential component (Vid), half attributed to one input and half (with a negative sign) to the other. So Vg1 = Vcm + 0.5Vid and Vg2 = Vcm - 0.5Vid.

The common mode Vcm is the average between the two voltages.

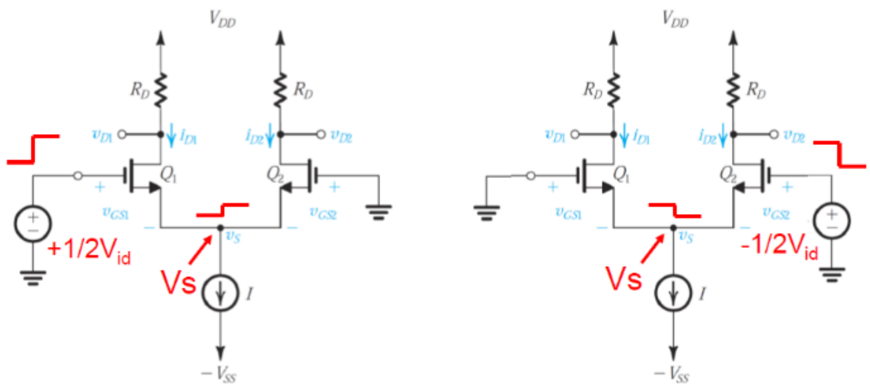
6. $V_{CM} = \frac{V_{G1} + V_{G2}}{2}$ and $V_{id} = V_{G1} - V_{G2}$ $\mapsto$ $V_{G1} = V_{CM} + \frac{1}{2}V_{id}$

$V_{G2} = V_{CM} - \frac{1}{2}V_{id}$

This alternative way to express the voltages Vg1 and Vg2 are useful because if we now apply a superposition of effect and we apply only Vcm, the circuit is symmetric and driven by symmetric voltage → Vod = 0. Hence the common mode can be neglected since it is not producing any voltage difference at the output (in an ideal stage). From now on we will consider just the application of the differential voltage, because the differential voltage is asymmetric.

We now consider the two voltages one at a time, using the superposition of effect. At first, we put the right voltage to ground, and we apply half of the differential voltage on the left.

If we consider the circuit on the left, Q1 is in source follower configuration, because the source is connected to an impedance, that is the 1/gm of Q2. So what is the effect of the input voltage on the source? For sure if Vgs1 will be a positive step, also Vs1 will be a positive step (I don't care about the amplitude, just the fact that the sign remains constant).

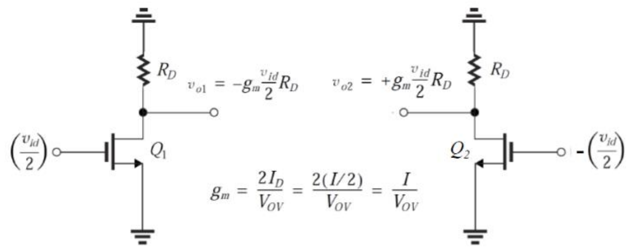


The overall effect on Vs is zero: Virtual Ground

As for the right circuit, the reasoning is the same (identical but reciprocal in terms of sign). The overall effect of Vs will be a perfect cancellation of any voltage of Vs. so when we apply the differential voltage the node where we have Vs is not moving → it is a **virtual ground**, the node is not physically grounded, but it is not moving if we apply signals.

The good reason to achieve this is that now I can study the circuit in two half circuits. As the common node is grounded, a virtual ground, I can move on in my computation considering the two half circuits and then superpose the effect.
NB: the goal is to reach Vod.

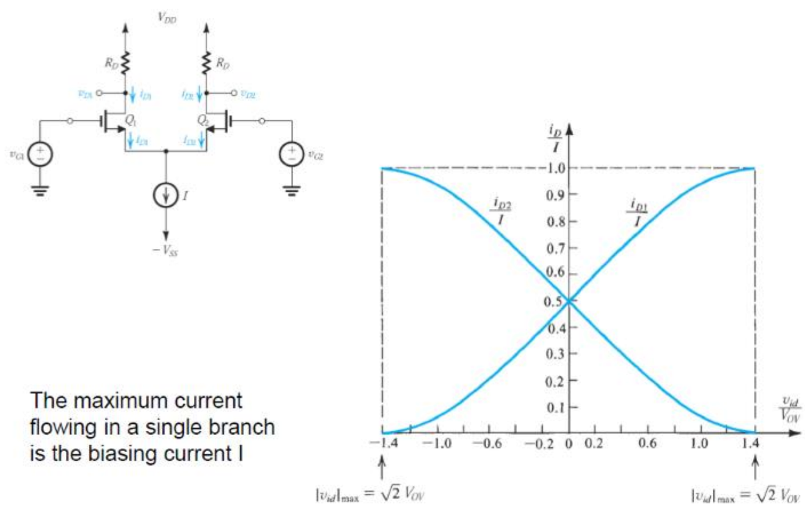
On the left branch I apply Vid/2, we have a common source configuration, and the same for the right branch. The overall gain is equal to the differences between the voltage difference divided by the input voltage: gm*Rd.



$$\frac{v_{o1}}{v_{id}} = -\frac{1}{2} g_m R_D \qquad \frac{v_{o2}}{v_{id}} = \frac{1}{2} g_m R_D \qquad \boxed{A_d \equiv \frac{v_{od}}{v_{id}} = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D}$$

It is identical to the gain of the common source. In conclusion, the gain of a differential stage is gm*Rd. In the middle we have the formula on how to calculate the transconductance gm.

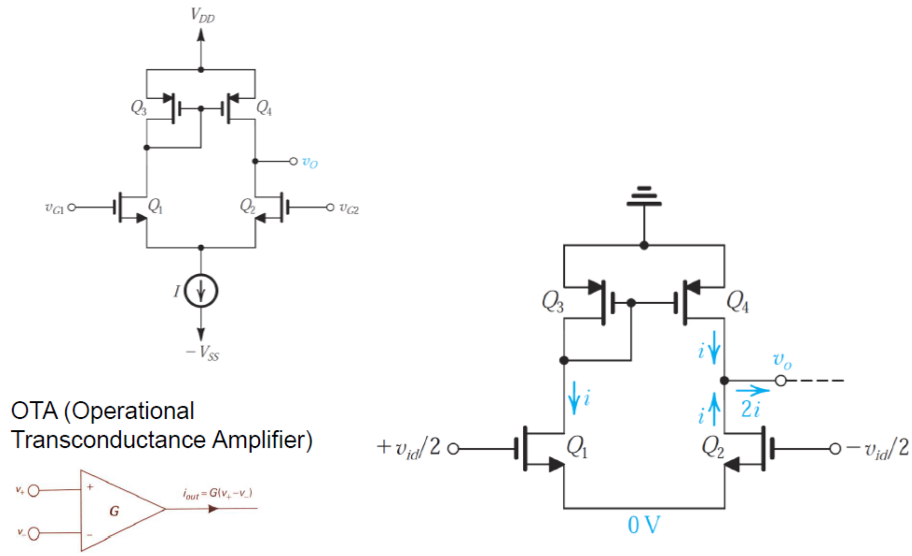
This analysis is possible because we have use transconductance, a small signal parameter for the transistor → signal much smaller than the biasing of the transistor.



However, for large signal, this is not true. When we have large signals, for instance Vg1-Vg2, we have more current on the left and less current on the right, so overall, if we apply a larger voltage on the left, the important conclusion is that Id1 and Id2 cannot increase indefinitely, because in the extreme case the current on the left increases so much that Id1 = I, and the branch on the right is 0, it remains with no current.
This explains the plot on the right, that is the ratio between each current and the differential voltage on the input. If we increase the voltage difference, Id1 increases and Id2 decreases, but the sum is equal to the constant current I. At the maximum voltage, the ratio id/I = 1.

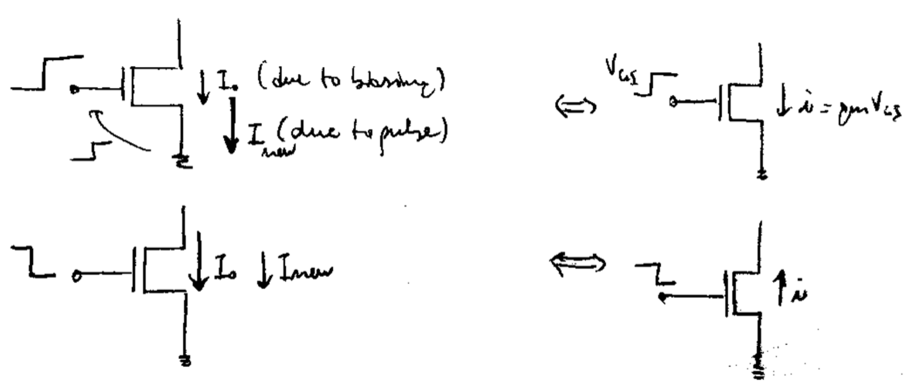
When we are analyzing the amplifier for small signal, we are in the middle, and the response of the amplifier is linear.

Is it possible to change the circuit to have a unipolar output, so referred to ground?



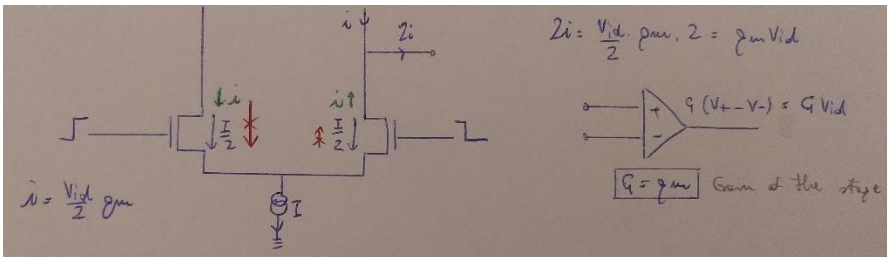
Yes, by a modification of the structure and by putting, instead of the resistors, current mirror. If we repeat the same analysis as before (applying a positive differential voltage on left and corresponding negative on right), we can imagine that due to the fact that the two input are the same but opposite, in small signal analysis we can say we have a current 'I' in the left and opposite current 'I' in the right, so the sum of the two is zero, because they are signal current and must compensate, because the overall current flowing in the current generator below must be constant. If now the current on the left is entering in the mirror, it is mirrored on the right. If I made the final balance in the output node, the two signal currents sum into an exiting current 2i.
I can conclude that in a stage like this with an active load, the output node draws the doble of the current generated in the input transistors. This intermediate block assumes the role of an amplifier and it is called OTA (operational transconductance amplifier).
It is an amplifier with differential input voltage and unipolar current output. We are talking about current, we don't know yet where the current is going, we don't care at the moment.

There are two ways to evaluate a currents in the amplifier. In the amplifier we have a current I0 due to the biasing. When I apply a pulse I increase the current, because I apply a pulse on the Vgs. This can be equivalently seen as an I0 and on top of that I add a signal current i. I don't consider the overall current I as the sum of the two, but I consider the increment of current i. This is the small signal analysis. In the same way, when we have a decrease of the current, the overall current decrease, because I'm reducing the Vgs voltage. But this is identical to say that due to the input signal I have a small current going upward, because in reality from the starting current I have a new current which is equal to the



old one minus a current going upwards. So when we analyze amplifiers, we never study the absolute new value, but on the differential new small signal value.

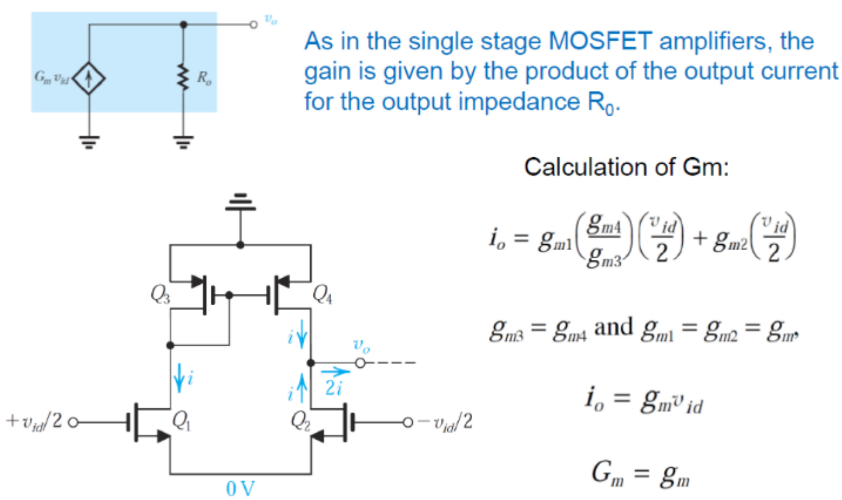
In the case of our amplifier, when I have a positive signal on the left and a corresponding negative on the right, while at the beginning I have I/2, then I should say that have a larger current on the left and a smaller on the right, and the sum is still I. But I don't say so, I prefer to say that I had an increment i on the left and an opposite increment i on the right but in the opposite direction, that corresponds to the reduction in the red arrow.

8. 

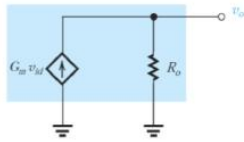
So from the absolute modification of the current we must focus just on the relative variation of the signal. Of course the sum variation of the two is zero.
This explains why in the image before I have the two current i:

- The one on the left is gm*vid/2
- The one on the right is -vid/2*gm

So the overall current at the output is gm*vid. Hence the overall gain is gm. Hence in my amplifier the gain is equal to gm. In the OTA the output current is: Io = gm*Vid.

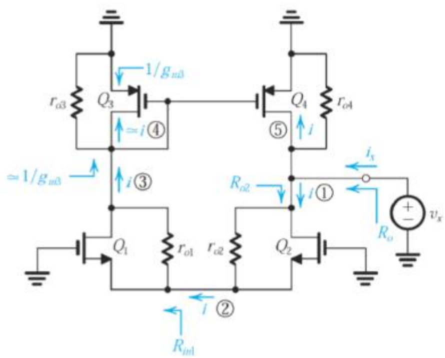


So from the point of view of the output current, we have like an equivalent generator. But what is the output unipolar voltage?
It is given but the current 2i that is going to the equivalent load r0 placed in the output node (if we have a load we will have Rl, external resistor). If the node is floating, I have to compute the output impedance r0 of my network, so that I can calculate gm*Vid*r0. I apply a voltage generator and calculate the current; the ratio will be the output impedance. It will be the parallel of the output impedance of the transistor Q4 (r04) in parallel to the impedance r02 (impedance seen from the bottom). If they are equal to a value r0, **the unipolar gain of my amplifier is gm*r0/2**. gm*r0 was the gain of the common source amplifier.



Calculation of R_0:

$$R_o \equiv \frac{v_x}{i_x}$$



$$R_o \equiv \frac{v_x}{i_x} = r_{o2} \parallel r_{o4}$$

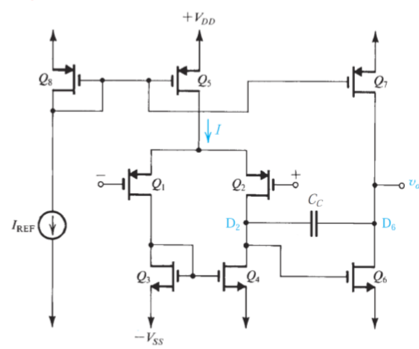
$$r_{o2} = r_{o4} = r_o$$

Calculation of the voltage gain:

$$A_d \equiv \frac{v_o}{v_{id}} = G_m R_o = g_m(r_{o2} \parallel r_{o4})$$

$$\boxed{A_d = \frac{1}{2} g_m r_o = \frac{A_0}{2}}$$

Two stages CMOS amplifier configuration



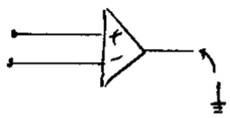
$$A_1 = -g_{m1}(r_{o2} \parallel r_{o4}) \qquad A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

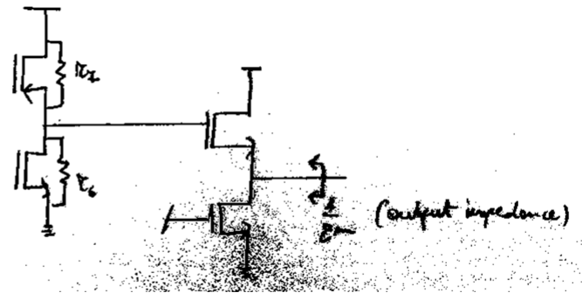
If we want a low output impedance (as in the ideal OPA) we have to add further a common source stage.

We have the differential pair in the middle (it is a pMOS one). The amplification up to node D2 is A1. There is a – sign because it depends on where we take the input, if it is on the same branch of the output we have a minus. Then we may have a second gate stage that is a common source with an active load (Q6 and Q7). The overall amplification is the multiplication of A1 and A2.
Q5 and Q8 is the current generator.

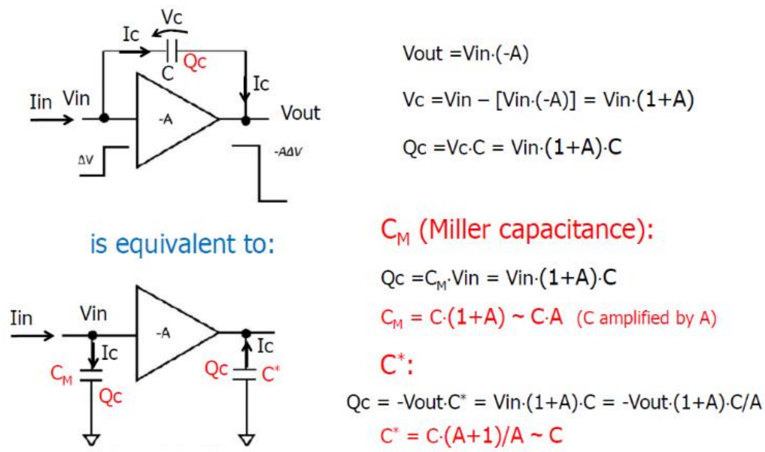
In the basic courses we have studied the opamp. The general opamp has a differential input and a unipolar output. Today we have built something similar, we have built a differential amplifier with a differential input, a pretty large gain given by the multiplication of the two gains of the two stages, but the remaining property not seen so far is that the output impedance is usually low. In the amplifier just seen, the output impedance is not small at all, because it is r06 and r07. This is not a problem, we can add a further stage which is a source follower. In this way the output impedance is the 1/gm of the source follower stage. It is not 0, but very close. This additional stage doesn't change the gain because it is 1.

9.





THE MILLER EFFECT



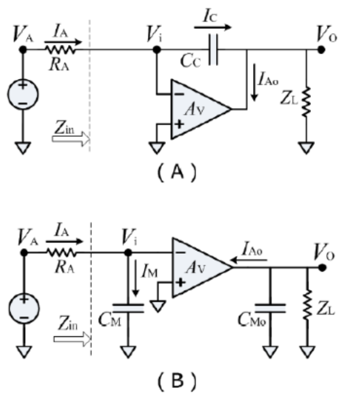
It is an effect occurring in a voltage amplifier when a capacitor is placed between the output and the input of the amplificator. Let's suppose to have a voltage amplifier and we have a capacitor. The miller effect implies that the capacitor in mid position is equivalent to have one capacitor at the input node which is equal to the original capacitor C but amplified by the voltage amplification (A or -A). if the amplification is -A is like to have at the input a capacitor C that is A times larger, and this is the problem. Less problematic is the equivalent capacitor C* that is equal to the original one.

The Mille effect is a problem because the input Miller capacitor Cm, together with the impedance of the source, creates a low pass filter, a pole.
So the output voltage is the amplification of Vi, but Vi is not Va, because we have a low pass filter. In this pole we have, the Miller capacitor is the physical capacitor Cc multiplied by the amplification gain A: Cm = Cc*A.

Demonstration of the Miller capacitance (upper image)
The voltage across the capacitor is Vin*(1+A). The charge Qc accumulated on the capacitor is Vc*C. Now we go to the equivalent circuit. The Miller capacitance is no more across the circuit; Cm must be hence charged by Ic. So Qc in the new schematic with Cm is Qc = Cm*Ic. Cm is the unknown.
Since the two Qc are the same, we can get that the Miller capacitance is equal to the physical capacitance multiplied by the gain of the amplifier.

By the same token, we can also compute C*, considering the same charge Qc on the plate. Practically, C* ~ C. Hence we can forget about it. The problem is the Cm, because the more we try to increase the gain of the amplifier, the more we increase it.

The problem so is that the more we want to amplify, the more we increase the Cm.

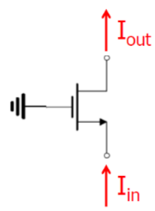


$$C_M = C_C(1 + A_v)$$

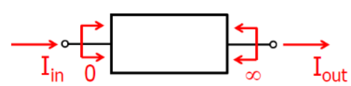
$$V_o = -A_v V_i = -A_v \frac{V_A}{1 + j\omega C_M R_A}$$

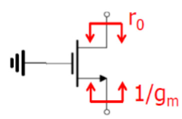
(low frequency pole from the Cc 'amplified' by the Miller effect)

THE CASCODE STAGE (also called common gate)



Use as current buffer:

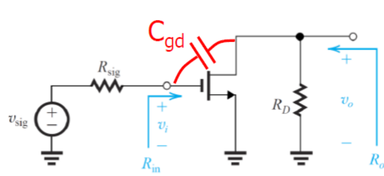


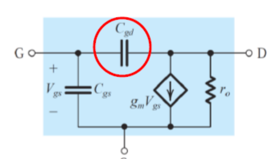


The gate is set to ground, it works as a current buffer. It takes an Iin from the source and provides an Iout with Iin = Iout. It is interesting since it acts as a current buffer.
The current buffer draws a current at 0 impedance (perfect ground) and provides in output the current at infinite impedance. The common gate is a good current buffer because the output current is equal to the output one, the input impedance is not 0 but 1/gm, which is the best approximation of 0 we may have in CMOS, in and the output impedance is not infinite but r0, that is the best approximation of infinite.

Application of the cascode with common source amplifier

Standard Common Source:



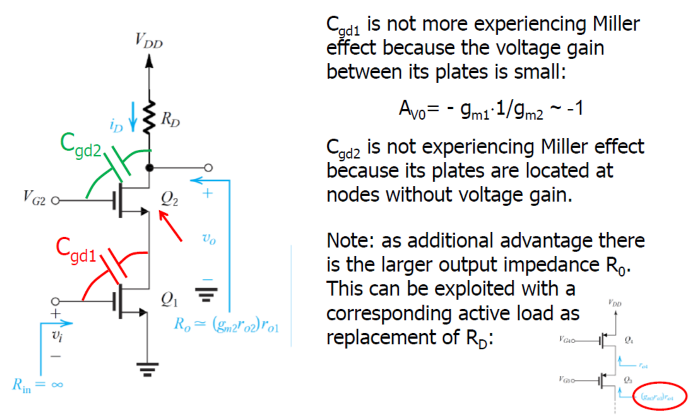


C_{gd} capacitance experiences Miller effect:

$$C_M = C_{gd} \cdot (1+|A_{v0}|) \qquad A_{vo} \simeq (-g_m R_D)$$

We highlight a parasitic effect, that is the gate-drain capacitance. Cgd is in a very dangerous position, since it connects the input and the output of the amplifier. So it experiences the Miller effect.
Cgd is equivalent to a capacitor at the input equal to the Cgs*A, where A = -gm*Rd. So the more I amplify with this amplifier, the more I increase the Miller effect.

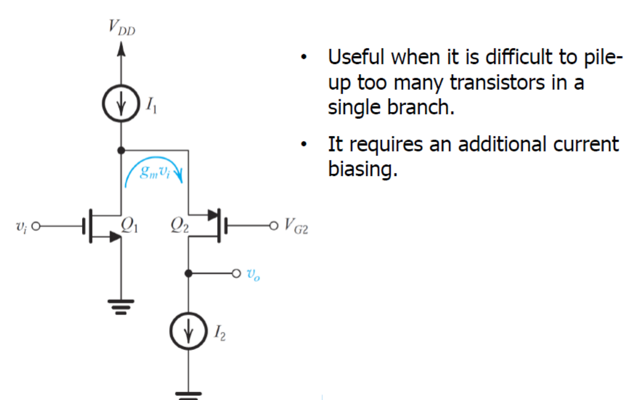
To solve this, we put between the common source and the load Rd a cascode. Now we haven't modified the overall gain, since the single gain on Q1 is still gm*Rd. As for the Miller effect, Cgd is no more connected between the input and the overall output. It is connected between the input and the node between the two transistors. It is still experiencing the miller effect, but the amplification I have to put in the formula for Cm is not between input and output, but between input and middle node. This amplification is Av0: the transconductance of the transistor times the impedance in the node on the right of Cgd1. This impedance is the 1/gm of the Q2. Hence the overall amplification is -1. So **Cgd1 is not sensing a big Miller effect**.



At the output I have the **Cgd2**. It **is not experiencing the Miller effect**, because it is not between the input and output of an amplifier, since Vg2 is fixed (= 0, grounded).
So none of the two capacitors plays a big role. This is why the cascode improves the frequency response of the standard common source amplifier.

Moreover, this transistor Q2 in cascode configuration increases also the output impedance. As for the amplification, having the small Rd and a high impedance is not advantageous, so we put also an active load on top instead of Rd (the active load is the one in the bottom left of the image).

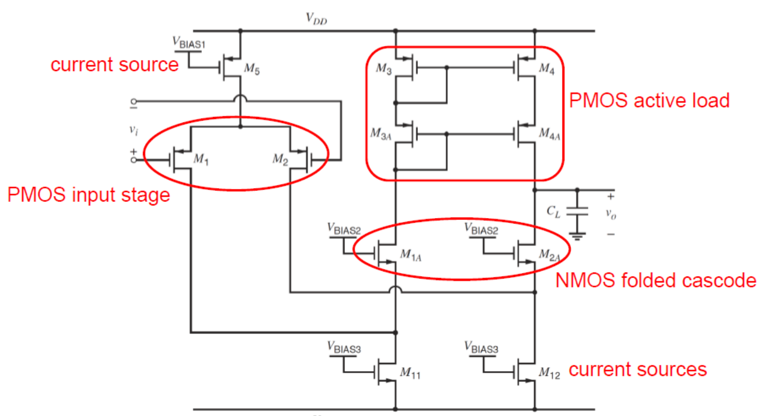
The folded cascode



In some amplifier, we cannot build a totem of many transistors, because of biasing problems, we have to guarantee that each transistor is above threshold. The folded cascode allows to implement the cascode solution by using instead of an nMOS cascode a pMOS cascode. They are identical. The signal current gm*vi is simply folded in the current buffer, but in this solution we have two transistors between Vdd and ground.
In the previous configuration the biasing current is the same for both the transistors, here we have the need of an additional transistor which has to supply the current to the first transistor and to the second.

EXAMPLE: CMOS OPAMP

Example: differential amplifier with PMOS inputs, folded cascode and active load

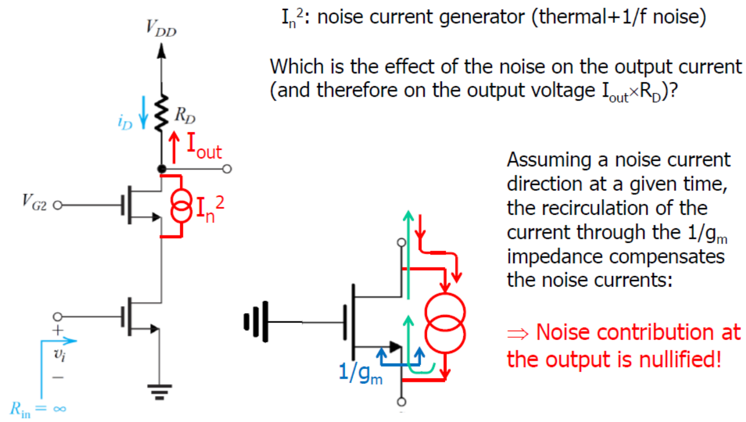


We see the input stage of the differential pair, made out of pMOS. We see the current source is made with a transistor that is probably the second stage of a current mirror; we see that we have a cascode but instead to put nMOS we have the folded cascode, with pMOS at input and nMOS as folded cascode. The right branch is M2 and M2a as a folded cascode and M1 and M1a as folded cascode.

What is missing is the mirror, we must have one. Since we have folded, we expect a p-mirror, because the path of the signal is going down and then up, and also we have not only a single mirror, but an improved mirror. If we look at the output impedance, we have r0 and r0, and looking from the node to the bottom M2a we have r0 and r0 (of M12), so we have also the nice output impedance.

If we have a pMOS stage without cascode, we could have used a nMOS current mirror. But if we go folding, we have to track the folding, so we need a pMOS active load. We have to follow the path of the signal current.

Noise in cascode stage



The cascode stage has another advantage: **it is noiseless**. Of course it has its noise generator 'In' that includes thermal and 1/f noise. But when we compute the effect of this generator on the output current, which is finally converted into voltage by the resistor Rd, the noise 'In' can be neglected.

We consider the generator and for a short instant let's consider it has a sign (in general they have not a sign, but instantaneously the noise generators have sign). The current is taken from the top, and the current delivered by the generator in the node below cannot go down to the bottom transistor, because I have a large impedance r0. So the only way is that it goes in the 1/gm impedance of the cascode itself (it becomes green). So the green current takes 1/gm, goes up and cancels up with the red one.

So the noise current is recirculating in the transistor itself but we have no net contribution going in the Rd. The same demonstration can be done by flipping the direction.

Of course in the bottom transistor we still have noise.