# Timeslides Calculations

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## 1 Introduction

There is a series of questions that still need to be answered when it comes to estimating the background during a coincident inspiral search for GW when using the timeslides method, in the specific case of searches associated with GRBs. Suppose one performs $S$ timeslides with a slide amount which is much larger than the largest possible coincidence window on a stretch of data of length $T$. One needs to answer these questions before concluding that by doing timeslides one will gain in sensitivity:

- What is the desired sensitivity of the search, in other words, what is the range of false alarm rates (FAR) (or probability, FAP) a detection result should be quoted with? This should be answered by using an astrophysical model of the source distribution and not using statistical properties of the GW data.

- What is the maximum number of timeslide that one can perform so that one can still gain in senisitivity – that is, up to what $S^{max}$ can one go so that the FAR (FAP) will still decrease and reach the minimum quoted by answering the above question?

- How independent will be the resulting coincidences from doing slides, knowing that the timeslides method does not produce new triggers but rather recycles the existing triggers present in the initial stretch of data and rearranges them to create new coincidences? That is, having a Poisson distributed collecton of single detector triggers, that will be creating coincidences, which triggers will be prone to repeat in coincidences more often than the others, if there is any preference towards such a behavior at all?

- How will a loud noise trigger and a glitch affect the statistics by propagating in coincidence and being the loudest event in coincidence?

- What is the nominal sensitivity of the search, in other words, what is the maximum false alarm rate (FAR) (or probability, FAP) and its error a detection result should be quoted with, with figures extracted from the statistical properties of the GW data? Is this sufficient, i.e. how does it compare to the one quoted by answering the first question?

# 2 What is the astrophysically motivated desired sensitivity of the search?

When are we going to claim we made a detection? The lower estimate of the FAR in order for one to claim a detection is obtained by appying an astrophysical model, i.e. prior, on the distribution of short GRBs in the Local Universe.

One way of answering this question is by quoting an occurrence rate for short GRBs in the Local Universe (at redshift zero). From [?] we learn that the range for the volume rates for short hard GRBs considered compact binary mergers, at redshift null, can be written as:

$$10 \;\leq\; \mathrm{R}_{\mathrm{GRB=merger}}(z=0) \;\leq\; 10^{4}\mathrm{Gpc}^{-3}\mathrm{yr}^{-1} \tag{1}$$

Assuming a maximum range of 40 Mpc for the LIGO I detectors, the volume rate can be expressed as a detector detection rate or true alarm rate:

$$7\times10^{-4} \;\leq\; \mathrm{R}_{\mathrm{GW,GRB}} \;\leq\; 7\times10^{-1}\mathrm{yr}^{-1} \tag{2}$$

or

$$2\times10^{-11} \;\leq\; \mathrm{R}_{\mathrm{GW,GRB}} \;\leq\; 2\times10^{-8}\mathrm{Hz} \tag{3}$$

According to (3), if we did a single search over a one year peried and found a loud candidate, the range of false alarm rates one should associate with it should be $2\times10^{-11}$ and $2\times10^{-8}$ Hz but if we did a search over a span of roughly 2000s one would want to quote a larger FAR:

$$3\times10^{-7} \;\leq\; \mathrm{FAR}_{\mathrm{GRB},2000s} \;\leq\; 3\times10^{-4}\mathrm{Hz} \tag{4}$$

An alternative way of computing the desired FAR is to look at how the GRBs are distributed in volume. Short hard GRBs are cosmological events but a volume distribution function has not been derived yet. The lack of statistical information concerning a putative volume distribution in the present scientific literature prompted a hands-down look at the available data. The table below contains the distances in Gpc and volumes in Gpc$^3$ to the Swift short GRBs that have a confidentially associated host galaxy, hence known redshift $z$.

The volume listed in the table is the comoving volume, the volume measure in which number densities of non-evolving objects (transients) locked into Hubble flow are constant with redshift. The comoving volume is approximated by the physical volume for distances of up to 4 Gpc:

Assuming a constant number density for short GRBs within a physical spherical volume with a radius of 4 Gpc and assuming an optimistic average

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D(Gpc) | 1.108 | 13.8 | 3.14 | 1.46 | 0.509 | 5.24 | 5.85 | 2.53 | 2.05 | 21.8 | 5.84 | 9.1 |
| V(Gpc$^3$) | 3.102 | 502.6 | 35.3 | 6.16 | 0.402 | 98.6 | 121.4 | 22 | 13.68 | 924 | 121.2 | 263.7 |

Table 1: Swift short GRBs with associated host galaxies: distances and astrophysical volumes

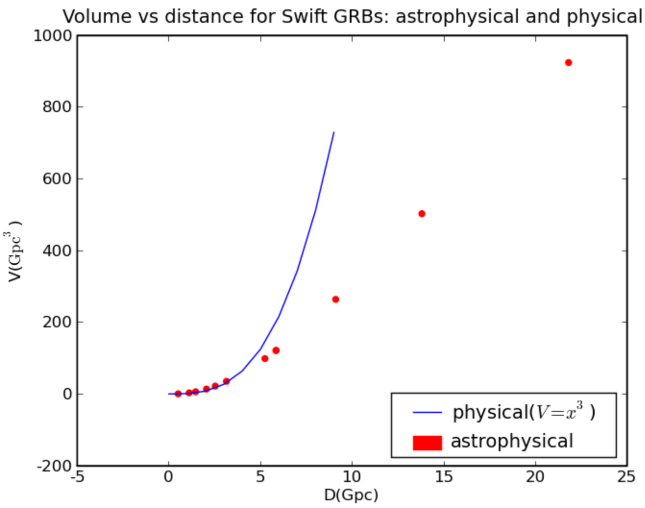


Figure 1: Comoving volume (Hubble volume) compared to physical volume for the Swift GRBs with known hosts and redshifts. The red dots represent the GRBs.

LIGO/Virgo detector range of 40 Mpc and maximum angualr sensitivity within this volume, one can estimate the maximum FAP we should claim for a possible detection statement for a GRB search:

$$\mathrm{FAP}_{max} = \frac{dN_{\mathrm{observable}}(\mathrm{V}_{\mathrm{IFO}} = 64 \times 10^{-6}\mathrm{Gpc}^3)}{dN(\mathrm{V}_{\mathrm{uniform}} = 64\mathrm{Gpc}^3)} \approx 10^{-6} \tag{5}$$

# 3 Maximum number of slides, statistical independence of slides and maximum FAR/P

## 3.1 Time only coincidences

One assumes that the single-detector triggers are Poisson distributed with a constant occurrence rate that varies from one detector to another and from time to time within the same detector (due to excessively noisy or glitchy times, detector maintenance, bad weather, etc). The Poisson approximation is applied for a GRB search that spans a time of around 2000s.By inspecting the first inspiral files one could get the total number of single detector triggers $N_i^1$ and $N_i^2$ and the trigger rates $R_i^1$ and $R_i^2$ knowing that the total analysis time is $T$ and that we use the three chirp mass bins. Suppose we analyze data from two detectors, 1 and 2:

$$R_i^1 = \frac{N_i^1}{T}, R_i^2 = \frac{N_i^2}{T} \tag{6}$$

where $i$ represents the index for each chirp mass bin (with 0.86,3.48,7.40,17.50 boundaries) and $L, V$ the detector indeces.

The probabilities of having at least one trigger in a coincidence window $\delta$t given the single detector trigger rates ($R_i^1, 2$ will be:

$$\mathrm{p_i} = \mathrm{p}(R_i^1) = 1 - \mathrm{e}^{-R_i^1 \delta t} \tag{7}$$

$$\mathrm{q_i} = \mathrm{q}(R_i^2) = 1 - \mathrm{e}^{-R_i^2 \delta t} \tag{8}$$

this by choosing a fixed time coincidence window $\delta$t. According to [?], the average number of coincidences will be:

$$\mathrm{FAR_i} = \frac{n \mathrm{p_i q_i}}{T} = \frac{1}{\delta t}(\mathrm{p_i} \times \mathrm{q_i}) \tag{9}$$

where $n$ is the total number of coincidence windows $n = T/\delta t$. The variance will be, knowing that $S$ is the total number of timeslides:

$$\sigma_i = \sqrt{\mathrm{Var}_i} = \sqrt{\frac{n \mathrm{p_i q_i}}{T^2} \times \frac{1 + \mathrm{p_i q_i} - (\mathrm{p_i} + \mathrm{q_i}) + S(\mathrm{p_i} + \mathrm{q_i} - 2\mathrm{p_i q_i})}{S}} \tag{10}$$

## 3.2 Time and mass coincidences in the offsource

**Time-mass coincidence window** We have written the resulting FAR and its error for the case in which we apply a one dimensional time-only coincidence window. In the standard inspiral CBC analysis three dimensional coincidence elliptical windows are used to find coincidences. Let's consider a rectangular coincidence window for ease of calculation and use notations and numerical results from [?] and $\delta t$ from the above section:

$$\begin{aligned} x(\eta, M, d/c, \rho_0) &= \Delta_w t_c(d/c, \rho_0) \times \Delta_w \tau_0(\eta, M, \rho_0) \times \Delta_w \tau_3(\eta, M, \rho_0) \\ &\approx \frac{2 a_{\tau_0} a_{\tau_3}}{\rho_0^2} \delta t \\ &\approx \frac{4}{\rho_0^2} \delta t \text{ (s)} \end{aligned} \tag{11}$$

where $t_c$ is the coalescence time, $\tau_0$ and $\tau_3$ are two functions of total mass $M$ and $\eta$, fraction of reduced mass and total mass, that maximize the signal-to-noise ratio recovered from filtering with a certain template, $d/c$ is the light travel time between the two considered detectrors, $a_{\tau_0}$ and $a_{\tau_3}$ are two constants estimated in [**?**] and $\rho_0$ is the signal-to-noise ratio threshold.

Mass coincidences ($\tau_0$, $\tau_3$) introduce a dimensionless factor $\epsilon(\eta, M, \rho_0) \approx \epsilon(\rho_0) \approx 4/\rho_0^2$ of order 0.16 at threshold signal-to-noise ratio $\rho_0 = 5$. Grossly approximating the factor as constant across each chirp mass bin, a one dimensional time-only coincidence window $\delta t$ becomes a two-dimensional mass-time coincidence window denoted by $x_i = \epsilon \times \delta t$ in each chirp mass bin $i$ of order

$$x_i = \epsilon \times \delta t \sim 0.1 \times 25 \text{ ms} \approx 2 \times 10^{-3} \text{ s} \tag{12}$$

Any two events (triggers) from detectors 1 and 2 separated by

$$|\Delta_{12} t_c| \leq |\Delta_w t_c|, |\Delta_{12} \tau_0| \leq |\Delta_w \tau_0|, |\Delta_{12} \tau_3| \leq |\Delta_w \tau_3| \tag{13}$$

are considered coincident.

**False Alarm Rate and its error** Replacing equations (7) and (8) in (9) and making the appropriate approximations in the exponential expressions, knowing that the exponents are $\ll 1$:

$$p_i = p(R_i^1) = 1 - \mathrm{e}^{-R_i^1 x_i} \approx R_i^1 x_i \tag{14}$$

$$q_i = q(R_i^2) = 1 - \mathrm{e}^{-R_i^2 x_i} \approx R_i^2 x_i \tag{15}$$

we can get an expression for the FAR:

$$\begin{aligned} \mathrm{FAR_i} &= \frac{n p_i q_i}{T} \\ &= \frac{(1 - \mathrm{e}^{-R_i^1 \epsilon \delta t})(1 - \mathrm{e}^{-R_i^2 \epsilon \delta t})}{\epsilon \delta t} \\ &= \frac{(1 - \mathrm{e}^{-R_i^1 x_i})(1 - \mathrm{e}^{-R_i^2 x_i})}{x_i} \\ &= x_i R_i^1 R_i^2 \end{aligned} \tag{16}$$

In the case of an inspiral GRB search one can estimate both the rates $R_i^{1,2}$ and the mass-time coincidence window $x_i$, hence get an estimate of the FAR, by making a series of assumptions: the coincidence window is constant across each of the three chirp mass bins and does depend only on the threshold SNR, $\rho_0$, (chosen to be constant for the whole search); the rates depend on the threshold SNR and may be approximated as constant across the search when using a constant threshold and apply proper data quality cuts. They can be regarded as simply the total number of single detector triggers divided by analysis time, as in equation (6). For GRB090809B we have the following numbers of first stage (of the pipeline) single detector triggers and rates, listed in Tables 2 and 3. Note that the detectors are 1=L and 2=V and the division in chirp masses is expressed by the indeces.

| $N_1^L$ | $N_2^L$ | $N_3^L$ | $N_1^V$ | $N_2^V$ | $N_3^V$ |
|---|---|---|---|---|---|
| 86310 | 9435 | 1423 | 267503 | 19673 | 1494 |

Table 2: GRB090809B: first inspiral trigger counts for L and V detectors for the three chirp mass bins

| $R_1^L$ | $R_2^L$ | $R_3^L$ | $R_1^V$ | $R_2^V$ | $R_3^V$ |
|---|---|---|---|---|---|
| 39.41 | 4.31 | 0.68 | 122.15 | 8.98 | 0.68 |

Table 3: GRB090809B: trigger rates in units of Hz

The variance can be easily derived from equation (10) assuming that $p_i \ll 1$ and $q_i \ll 1$.

$$\begin{aligned} \sigma_i = \sqrt{\mathrm{Var_i}} &\approx \sqrt{\frac{np_iq_i}{\mathrm{T}^2}(\frac{1}{S} + p_i + q_i)} \\ &= \sqrt{\frac{x_iR_i^1R_i^2}{\mathrm{T}}[\frac{1}{S} + (R_i^1 + R_i^2)x_i]} \end{aligned} \tag{17}$$

The number of timeslides at which the Poisson error terms $x_i(R_i^1 + R_i^2)$ equalize the error from doing timeslides, $1/S$, will be improperly called the maximum number of timeslides, $S^{\mathrm{max}}$. It is improperly called maximum since one may choose to do as many timeslides as one wants and there is no physical limit for $S$ simply imposed by the Poisson counting errors. It is just that by increasing the number of timeslides beyond $S^{\mathrm{max}}$ there is very limited gain in sensitivity. Hence, $S^{\mathrm{max}}$ is given by:

$$S_i^{\mathrm{max}} = \frac{1}{x_i(R_i^1 + R_i^2)} \tag{18}$$

In the case of our example GRB, we will *estimate* the magnitude of the coincidence window $x$ in each chirp mass bin by using the single detector rates from Table 3 and equation (16). The false alarm rates are the number of backgound coincidences divided by the analysis time $T$. This estimation is done for the zerolag and the timeslides (160 timeslides) methods of background analysis and are tabled in Table 4, together with the corresponding FAR's.

| $x_1$ | $x_2$ | $x_3$ | $\text{FAR}_1$ | $\text{FAR}_2$ | $\text{FAR}_3$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ |
|---|---|---|---|---|---|---|---|---|
| 0.00009 | 0.0046 | 0.0346 | 0.435 | 0.179 | 0.016 | 0.0026 | 0.0026 | 0.0007 |
| 0.00010 | 0.0041 | 0.0389 | 0.468 | 0.158 | 0.018 | 0.0025 | 0.0022 | 0.0007 |

Table 4: GRB090809B: coincidence windows $x_i$ (s), FAR's and their errors (Hz) obtained using data from the zerolag (S=0, first line) and timeslides (S=160, second line)

$S^{\max}$ is decisevely larger for the low chirp mass bin, at about ∼70 slides, whreas for the other two chirp mass bins it's not more than ∼20 slides.

**Equal trigger rates** Let's suppose both detectors have the same trigger rate $R_i^1 = R_i^2 = R$ within the same chirp mass bin $i$ and we consider working in only one of the chirp mass bins, say the low mass one. One can express the false alarm rate in the considered chirp mass bin as $\langle \text{FAR} \rangle = \text{FAR}(1 \pm f) = xR^2(1 \pm f)$ for a fixed (by the analysis) coincidence window $x$ and with $f$ given by the following:

$$f(R, S) = \frac{1}{\sqrt{T}} \sqrt{\frac{2}{R} + \frac{1}{xR^2 S}} \tag{19}$$

By *fixing* the number of timeslides at maximum $S = S^{\max} = 1/2xR$ we have:

$$f(R, S = S^{\max}) = \frac{2}{\sqrt{RT}} \tag{20}$$

Considering that $f(R)$ should be no more than 0.05 for a 95% exact result of the false alarm rate, one can extract a desired rate of triggers function of the total time of analysis:

$$R \approx \frac{1600}{T} \text{Hz} \tag{21}$$

$$\text{FAR} \approx \frac{2.6 \times 10^4 \times x}{T^2} \text{Hz} \tag{22}$$

In other words this would yield a 5% error of the FAR at maximum number of timeslides if one has roughly 1600 single detector triggers counted in the analysis time $T$.

Since we would ideally want a FAR of about $10^{-5}$ and knowing that for a 5% FAR error the single detector trigger rate at threshold SNR is given by (21), one gets the desired width of the coincidence window for such a FAR:

$$x \approx 4T^2 \times 10^{-11}\text{s} \tag{23}$$

For a standard GRB search of about 2000s the coincidence window should be of $\approx 2 \times 10^{-4}$ s and the maximum number of timeslides $S^{\max} \approx 10^6/3$. In conclusion one can get close to a FAR of $10^{-5}$ and a $\sigma$ of $5 \times 10^{-7}$ if the single detector trigger rates approach 1 Hz and 300,000 timeslides are performed with an analysis time of 2000s per slide.

Conversely, by considering a variable number of slides $S$, an equal single detector trigger rate of R=0.32 Hz would yield a FAR=$10^{-5}$ and and a $\sigma \approx 10^{-6}$ for 5000 timeslides, as seen in Figure 3.2.

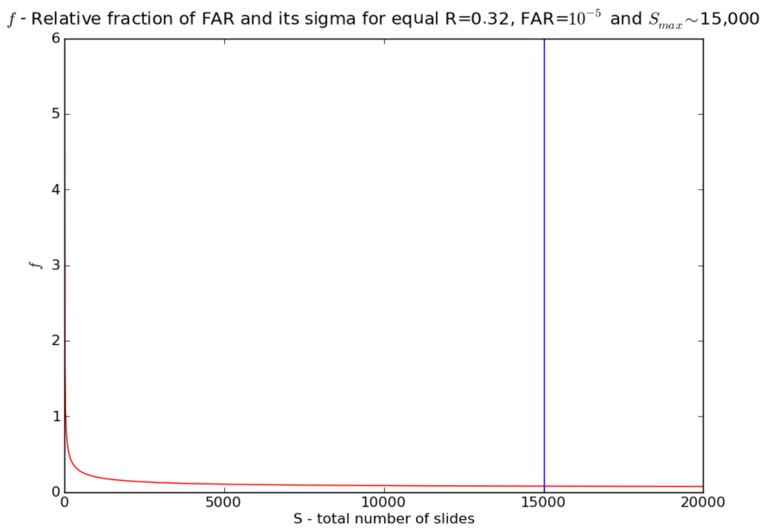


Figure 2: Fraction $f$ versus number of slides $S$ with $f$(5,000 slides)=0.1, $f$(10,000 slides)=0.08 and $f$(15,000 slides)$\approx$ $f$(20,000 slides)=0.07

An analysis time of T=2000s and a trigger rate R=0.32Hz would mean accepting on average about N=650 triggers per detector. This can be easily achieved by applying a higher SNR threshold

**Unequal trigger rates** Consider now that the single detector trigger rates are very different: suppose we take e.g. $R^1 \ll R^2$ hence one can write $f$ as:

$$f(R^1 \ll R^2, S) = \sqrt{\frac{1}{TR^1}} \sqrt{1 + \frac{1}{xSR^2}} \tag{24}$$

$$f(R^1 \ll R^2, S = S^{\max}) \approx \sqrt{\frac{2}{TR^1}} \tag{25}$$

Employing the same kind of reasoning as in the above paragraph, we would want to *fix* the number of timeslides at $S = S^{\max}$ and work with a FAR $= 10^{-5}$ with a fractional error of $f = 0.05$. From equation (25) one can write down the rate and the false alarm rate:

$$R^1 \approx \frac{800}{T} \text{ Hz} \tag{26}$$

$$\text{FAR} \approx \frac{800 R^2 x}{T} \sim 10^{-5} \tag{27}$$

For an analysis time of 2000s and a trigger rate $R^2$ of order 100 Hz one may be able to reaqch a FAR of $10^{-5}$ if and only if the coincidence window is of order $10^{-4}$ s or smaller.

For a practical example, seen in Figure 3 $R^1 = 0.32$ Hz and $R^2 = 100$ Hz. In such a case the FAR$\approx 10^{-2}$ Hz and $S^{\max} \approx 100$ as seen from Figure 3.

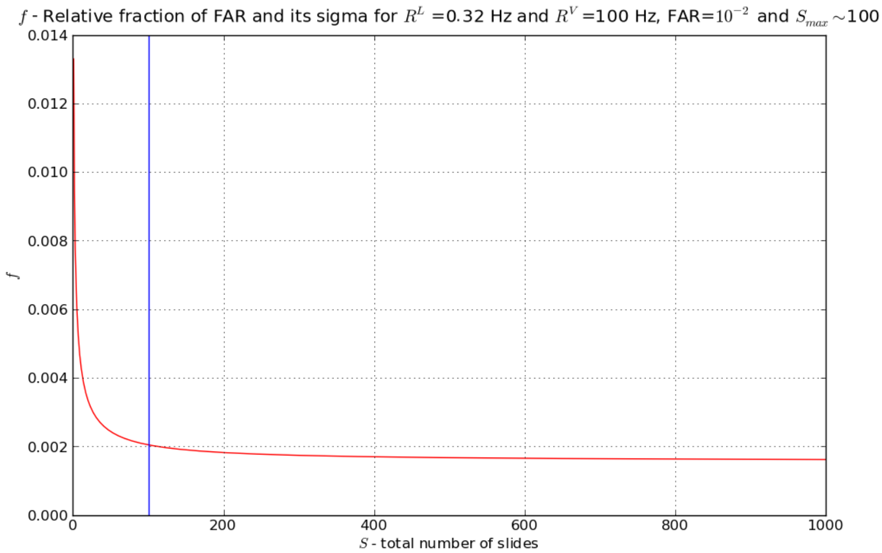


Figure 3: Fraction $f$ versus number of slides $S$ with f(100 slides) $\approx 0.002$ and f(1000 slides) $\approx 0.0016$

**Repeating triggers and trigger occurrence rates** The same single detector triggers may participate in numerous coincidences when doing timeslides, since the timeslides method of "extending" the background doesn't produce new triggers but rather reuses the same triggers within the analysis time $T$ to create new coincidences at every slide step. Whichever triggers are more prone to repeat in coincidences is not yet known and remains to be further investigated into, but we can create plots that help us understand if the number of times a trigger repeats is a function of any parameter such as chirp mass or SNR, etc. Figure 4 shows such a plot for the example GRB, GRB090809B: number of repeating triggers versus chirp mass.

The occurrence rate $z$ of a trigger in coincidences can be defined as the number of times a same trigger shows up in different coincidences with triggers from the opposite detector. The avergae occurrence rate per detector per chirp mass bin $i$ can be written as:

$$z_i^1 = x_i R_i^2 \tag{28}$$

$$z_i^2 = x_i R_i^1 \tag{29}$$

It is useful to look at the trigger occurrence rates $z$ collected from GRB090809B analysis to have an idea of the magnitude order and moreover to look at a comparison between occurrences in timeslides background and in zerolag background. This data is listed in Table 5. The trigger occurrence rates listed in Table 5 have been obtained by employing a computer program that simply counts the triggers and their occurrences and does not use the equations (28) - (29).

| $z_1^L$ | $z_2^L$ | $z_3^L$ | $z_1^V$ | $z_2^V$ | $z_3^V$ |
|---|---|---|---|---|---|
| 0.019 | 0.047 | 0.037 | 0.012 | 0.025 | 0.035 |
| 0.012 | 0.037 | 0.026 | 0.004 | 0.018 | 0.026 |

Table 5: GRB090809B: trigger occurrence rates in coincidences for individual L and V triggers for zerolag and timeslides data, in units of Hz

The Table 5 has two implications:

- The first implication is that individual trigger occurrence rates are much higher in the medium and high chirp mass bins. So if a glitch or any unwanted loud trigger has a medium or high chirp mass it will be part of more coincidences than a similar one that has a low chirp mass.

- The second implication is that by looking at both the zerolag and the timeslides numbers we see almost to no change , hence equations (28) - (29) stand fine since they are independent of the number of trials.

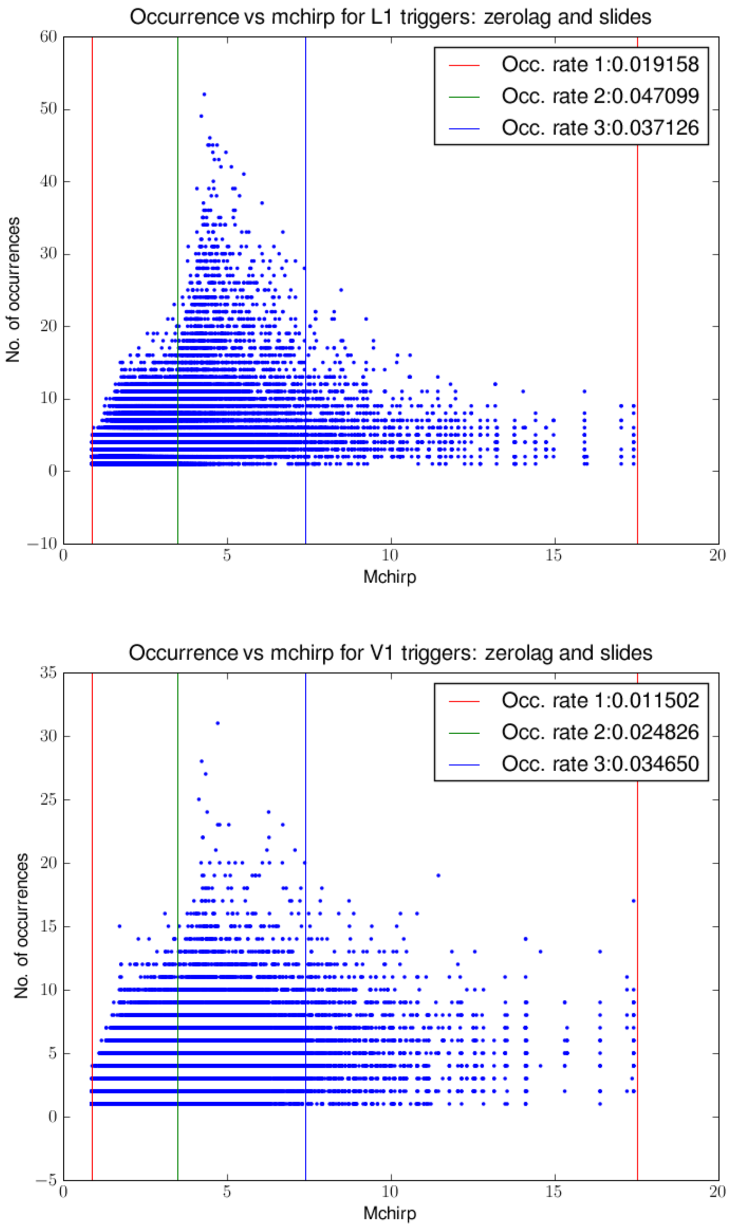


Figure 4: GRB090809B: number of occurrences of L and V triggers (N, counts) versus chirp mass

An alternative way of *estimating* the coincidence window $x$ in the case of GRB090809B is to use equations (28) - (29) and Tables 3 and 5: by averaging over the two occurrence rates per chirp mass bin we get similar results to the ones in Table 4 - low chirp mass bin coincidence window $x_1 \approx 2 \times 10^{-4}$, medium chirp mass bin coincidence window $x_2 \approx 5 \times 10^{-3}$ and high chirp mass bin coincidence window $x_3 \approx 5 \times 10^{-2}$.

The probability that a trigger will form $k$ coincidences when doing an analysis comprising $S$ timeslides may be approximated with a Poisson process probability:

$$p_i^{1,2}(k, S) = \frac{\sum_l N_l^{1,2}(k, S)}{N_0} \approx \frac{\left(z_i^{1,2} S\right)^k}{k!} \mathrm{e}^{-z_i^{1,2} S} = \frac{\left(x_i R_i^{2,1} S\right)^k}{k!} \mathrm{e}^{-x_i R_i^{2,1} S} \tag{30}$$

where $\sum_l N_l^{1,2}(k, S)$ represents the summing numbers of different triggers that each shows up in exactly $k$ coincidences, and $N_0$ is the total number of coincidences.

Figures 5 - 6 are histograms of the number of trigger occurrences fitted with Poisson distributions given by (30) for GRB090809B. The fit is not perfect but is close enough.

**Finding a conservative upper limit for $S^{\max}$** Equation (30) approximates the probability of finding a trigger $k$ times in $k$ different coincidences when doing $S$ timeslides. Let's consider the case of the low chirp mass bin for which we take the coincidence window constant across the bin and fixed at $x_1 = x = 10^{-4}$ s and plot the probabilities (30) for different $k$'s for either detector 1 or 2 as function of rate of either detector 2 or 1 multiplied by the number of timeslides:

$$p_k^{1,2} = p_k^{1,2}(R^{2,1} \times S), \quad k = \overline{1, N} \tag{31}$$

An example of such plot is shown in Figure 7 for $k$=1,2,3,5 and 10.

In an ideal case the number of times $k$ a trigger forms coincidences should be as low as possible, prefrerably one, *i.e.* every trigger forms one single coincidence and does not get recycled in more coincidences with the incease of number of timeslides. By looking at Figure 7 we can set a conservative limit for $R \times S$ by imposing:

$$p_{k=1}^{1,2}(R^{2,1} \times S) = p_{k=2}^{1,2}(R^{2,1} \times S) \tag{32}$$

$$\min\left(\frac{2}{xR^1}, \frac{2}{xR^2}\right) \;\leq\; S_{\mathrm{lim}}^{\max} \;\leq\; \max\left(\frac{2}{xR^1}, \frac{2}{xR^2}\right) \tag{33}$$

Ideally, we would want to choose an $S$ to have $p_{k=1}^{1,2}(R^{2,1} \times S) = \max$ hence $dp_{k=1}^{1,2}(R^{2,1} \times S)/d(R^{2,1} \times S) = 0$ and that yields a reccommended

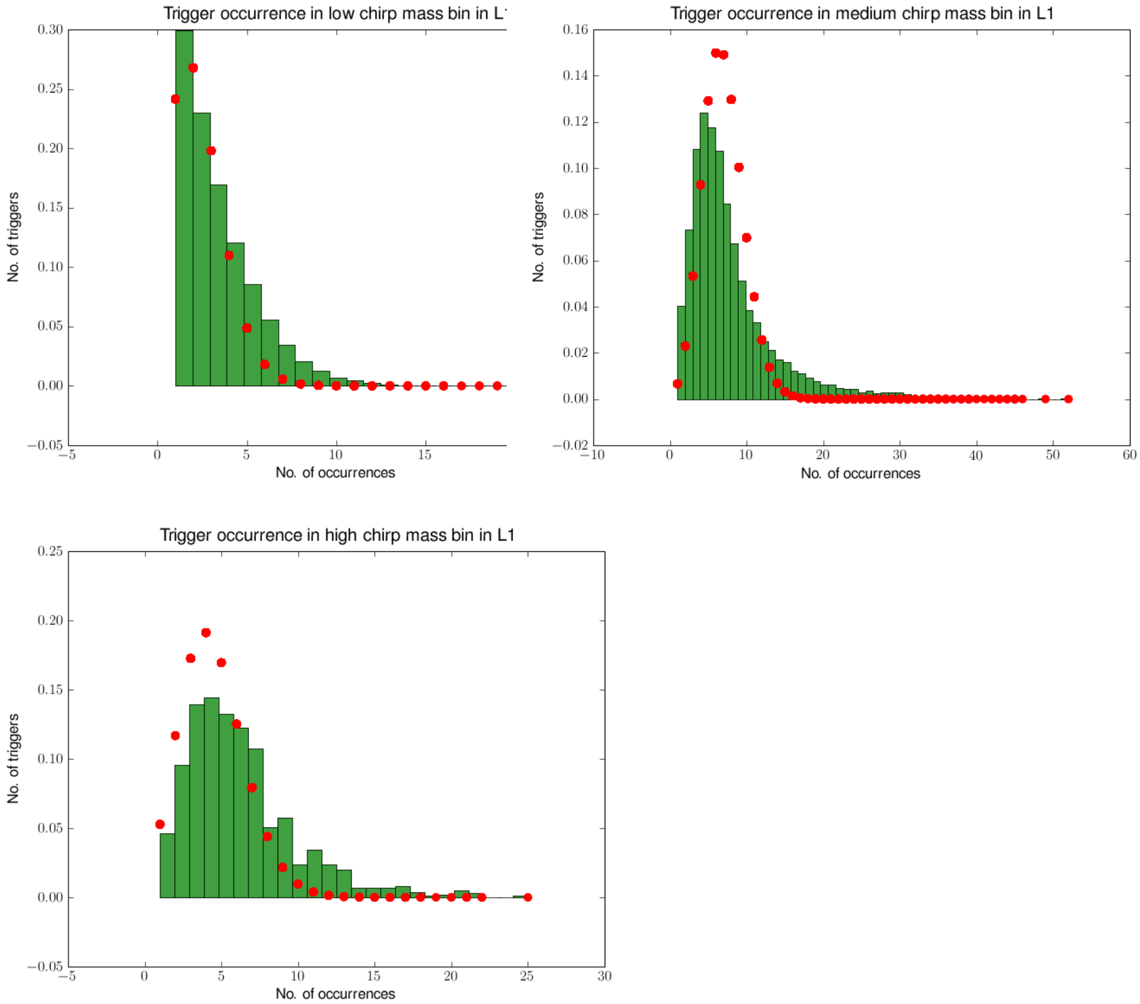


Figure 5: Histogram of number of trigger occurrences in coincidences for GRB090809B, detector 1 (L): low, medium and high chirp mass bins

number of timeslides $S^{\max}_{\text{recc}}$ given the two single detector trigger rates $R^{1,2}$, the coincidence window $x$ and the fact that we want to keep $k = 1$:

$$\min(\frac{1}{xR^1}, \frac{1}{xR^2}) \;\leq\; S^{\max}_{\text{recc}} \;\leq\; \max(\frac{1}{xR^1}, \frac{1}{xR^2}) \tag{34}$$

**Threshold SNR** The single detector rates depend intrinsically on the signal-to-noise ratio threshold $\rho_0$, fixed during a certain search. According to [] one can write the functional variation of the rate with threshold SNR:

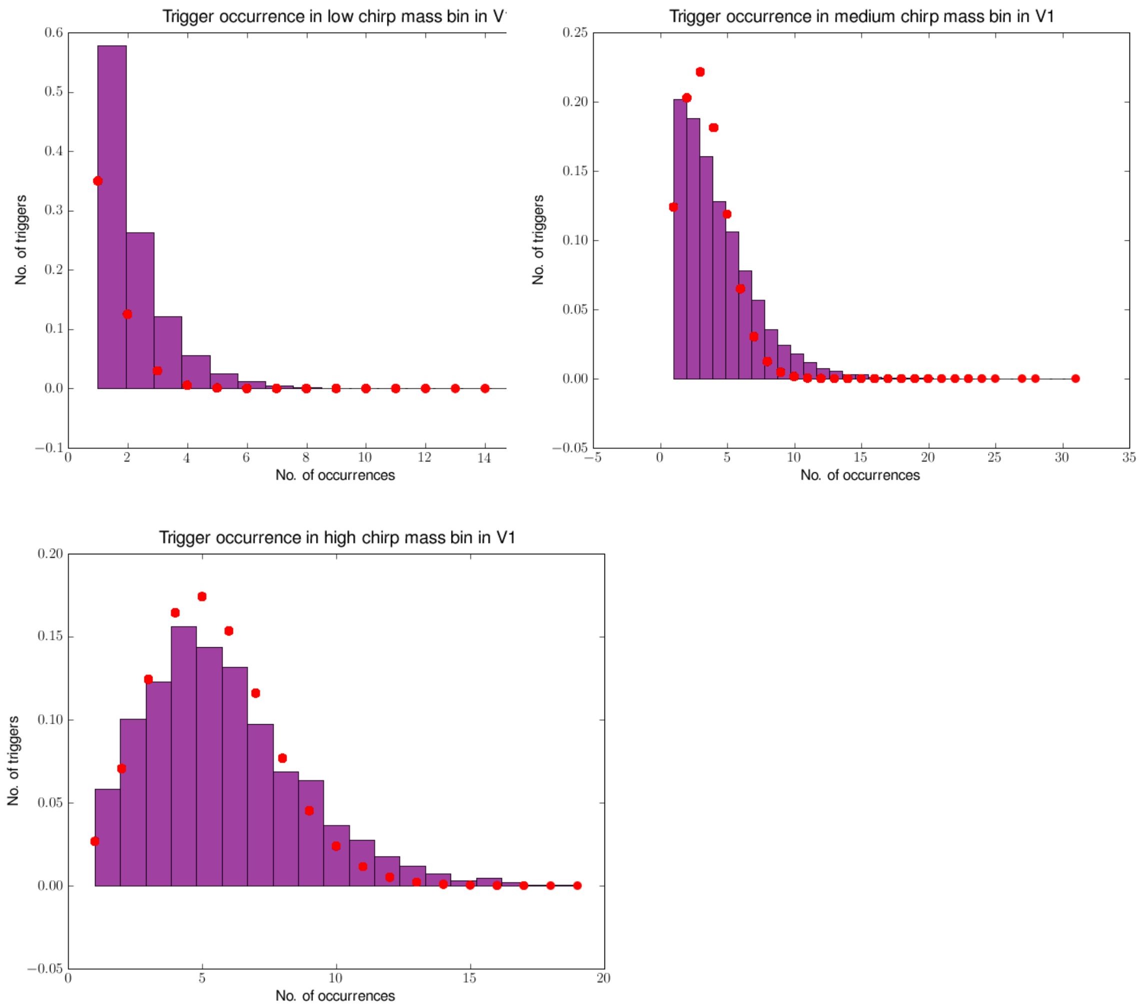


Figure 6: Histogram of number of trigger occurrences in coincidences for GRB090809B, detector 2 (V): low, medium and high chirp mass bins

$$R(\rho_0) = C\mathrm{e}^{-\frac{\rho_0^2}{2}} \tag{35}$$

where $C$ is a constant that folds in characteristics of the template bank used during the search.

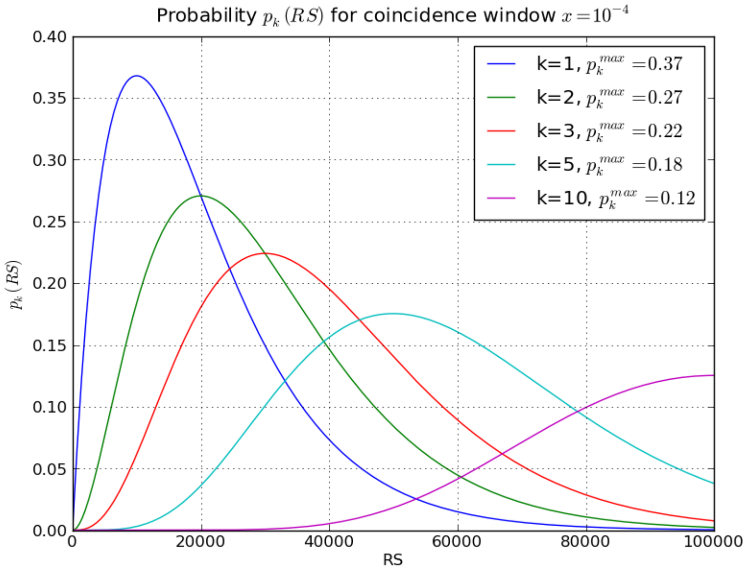


Figure 7: Probability of finding a trigger in $k$ different coincidences function of the $RS$ (Hz) product for different $k$'s

# 4 Conclusions

Reiterating the items listed in the introductory section, we find answers to the questions posed to what is the best way to estimate background by doing timeslides in the CBC coincident GRB search:

- What is the desired sensitivity of the search, in other words, what is the maximum false alarm rate (FAR) (or probability, FAP) a detection result should be quoted with?

  *ANSWER*: By looking at the local rates for short hard GRBs, one would want a FAR that is at most of the same order as the rate so would accept a detection at a FAR of say at most $10^{-4}$.

- What is the maximum number of timeslide that one can perform so that one can still gain in senisitivity – that is, up to what $S^{max}$ can one go so that the FAR (FAP) will still decrease and reach the minimum quoted by answering the above question?

  *ANSWER*: The maximum number of timeslides depends on two issues: first, if we do too many timeslides we don't gain anymore in sensitivity ([**?**]) and second, if we do too many timeslides we risk an oversaturation of triggers in coincidences , that is, the probability that a trigger will form multiple coincidences with a multiplicity $k > 1$ will be larger than the same probability for $k = 1$. Whereas equation (18) may shed light on the number of timeslides up to which the search can still gain in sensitivity, in practice it is found thet usually that $S^{\max}$ is too small to be used. A few times $S^{\max}$ will be used in practice and an interval for $S$ is given by equation (34). That shows us that the lower the single detector trigger rates the more timeslides one can do. In the standard S6 GRB case we have about 100 to 200 slides in the low chirp mass bin given by equation (34).

- How independent will be the resulting coincidences from doing slides, knowing that the timeslides method does not produce new triggers but rather recycles the existing triggers present in the initial stretch of data and rearranges them to create new coincidences? That is, having a Poisson distributed collecton of single detector triggers, that will be creating coincidences, which triggers will be prone to repeat in coincidences more often than the others, if there is any preference towards such a behavior?

  *ANSWER*: Single detector triggers follow an almost Poisson distribution in the timeslides coincidences; there has been no priviliged occurrence observed, as function of SNR. Medium and high chirp mass bin triggers show up more often relative to low mass triggers due to a more loose mass-time coincidence window constraint.

- How will a loud trigger/glitch affect the statistics by propagating in coincidence and being the loudest event in coincidence?

  *ANSWER*: A loud single detector trigger deemed as background trigger will follow the same statistics as the other triggers. Glitches are a different story.

- What is the nominal sensitivity of the search, in other words, what is the maximum false alarm rate (FAR) (or probability, FAP) and its error a detection result should be quoted with, with figures extracted from the statistical properties of the GW data? Is this sufficient, *i.e.* how does it compare to the one quoted by answering the first question?

  *ANSWER*: The answer to this question is not exactly an easy one. Say we would tighten the coincidence window to $x = 10^{-5}$, decrease the single detector trigger rates to $R \approx 10$ Hz and increase the analysis time to $T = 4000$ s. We may use equation (34) to have an idea on how many timeslides to do, that would yield about 10,000 slides - and in this case we are sure that we minimize the repetitivity of triggers in coincidences (the probability in equation (30) $p_k(R, S)$ is maximum for $k = 1$). All of this would give us a FAR $\approx 10^{-3}$ Hz and a $\sigma \approx 10^{-5}$ Hz (from equations (16)-(17)-(19)), which is a good starting point.

## 4.1 APPENDIX: Study on four S6A GRBs

By repeating the calculation steps described above, one can look at other GRBs and in this case the tables below show how the parameters change when looking at four different S6A GRBs.

| GRB | IFOs | $R_1^L$ | $R_2^L$ | $R_3^L$ | $R_1^V$ | $R_2^V$ | $R_3^V$ |
|---|---|---|---|---|---|---|---|
| 090709B | L1V1 | 57.59 | 9.06 | 1.16 | 125.11 | 8.86 | 0.65 |
| 090727 | L1V1 | 51.38 | 7.26 | 1.03 | 112.75 | 9.43 | 0.82 |
| 090809B | L1V1 | 39.41 | 4.31 | 0.68 | 122.15 | 8.98 | 0.68 |
| 090814B | L1V1 | 60.35 | 9.31 | 1.1 | 127.91 | 10.77 | 0.86 |
| GRB | IFOs | $x_1$ | $x_2$ | $x_3$ | $S_1^{max}$ | $S_2^{max}$ | $S_3^{max}$ |
| 090709B | L1V1 | 0.00019 | 0.0091 | 0.1061 | 30 | 7 | 5 |
| 090727 | L1V1 | 0.00013 | 0.0047 | 0.0474 | 50 | 13 | 12 |
| 090809B | L1V1 | 0.00009 | 0.0046 | 0.0346 | 70 | 17 | 21 |
| 090814B | L1V1 | 0.00014 | 0.0058 | 0.063 | 40 | 9 | 8 |

Table 6: Four S6A GRBs and their numbers

| GRB | IFOs | $z_1^L$ | $z_2^L$ | $z_3^L$ | $z_1^V$ | $z_2^V$ | $z_3^V$ |
|---|---|---|---|---|---|---|---|
| 090709B | L1V1 | 0.024 | 0.081 | 0.069 | 0.011 | 0.082 | 0.123 |
| 090727 | L1V1 | 0.015 | 0.044 | 0.039 | 0.007 | 0.034 | 0.049 |
| 090809B | L1V1 | 0.011 | 0.041 | 0.024 | 0.004 | 0.020 | 0.024 |
| 090814B | L1V1 | 0.018 | 0.062 | 0.054 | 0.008 | 0.054 | 0.069 |

Table 7: Expected occurrence rates in coincidences of individual L and V triggers for the four S6A GRBs (Hz)

By averaging the values in the tables (even though the statistical sample of four GRB is not yet enough, but it is a good indicator still) one can get a table with the take-home values for the parameters we would be interested in:

$$< x_1 >= 0.00014s, < x_2 >= 0.0061s, < x_3 >= 0.063s \quad (36)$$

$$< S_1^{max} >\sim 50, < S_2^{max} >\sim 12, < S_3^{max} >\sim 12 \quad (37)$$

$$< R_1^L >\sim 52Hz, < R_2^L >\sim 7.5Hz, < R_2^L >\sim 1Hz \tag{38}$$

$$< R_1^V >\sim 122Hz, < R_2^V >\sim 9.5Hz, < R_2^V >\sim 0.75Hz \tag{39}$$