Question:	1	2	3	Total
Points:	3	3	14	20
Score:				

Submission instructions: Please submit your solutions electronically as a **zip file in StudIP** (.zip only). The zip file must contain **one pdf** that has your solutions to the Exam-type and Theory exercises, and then **one jupyter notebook** (add/import additional files if you like) containing the code for the programming exercise (3). The notebook must be executable in the gwdg jupyter-hub. Submissions that are not executable by us will not be graded! Answers to the questions of the programming exercise should also be given in the jupyter notebook in markdown cells. Use complete and correct sentences unless otherwise noted. Please be succinct. Use your own words. Write down concise reasoning, not just the result.
We expect you to do the exercises on your own or with maximally two additional partners as a group. If you are solving the exercise sheet as a group, please also submit as a group and not multiple, individual submissions. Include your name(s) and immatriculation number(s) in the name of the zip file (i.e. `exercise1_nameA_1234_nameB_6789.zip`).
Defense: If you are willing to defend your solution, please remark, "Willing to defend" on top of the pdf that you submit. You will be considered for defense as long as you score at least 60% of the total points on this exercise sheet.

1. **Exam-Type:** Consider the following situation: You're coming home at noon on a Monday and you notice that your flatmate is home and in her room. You are trying to figure out why your flatmate is not at work. There are two possible reasons that you can think of. Either your flatmate is sick with a cold or she has taken a day of vacation. You also are aware that both explanations depends on the season. For example, in summer it is very likely that she took vacation and rather unlikely that she got sick.

[1] (a) From the situation explained above and the variables cold (C), season (S), vacation (V) and home (H), write down the graphical model.

[1] (b) Compute $P(H = 0, V = 1, C = 0, S = \text{spring})$. The conditional probabilities are given by the following tables:

S	$P(C = 1\|S)$
spring	0.25
summer	0.05
autumn	0.80
winter	0.90

S	$P(V = 1\|S)$
spring	0.30
summer	0.95
autumn	0.20
winter	0.60

C	V	$P(H = 1\|C, V)$
1	1	0.95
1	0	0.90
0	1	0.80
0	0	0.05

The prior probability of each of the four seasons is 0.25.

[1] (c) With the same probability values from (b), compute: If your flatmate does not have a cold and she has not taken vacation, how probable is winter?

[3] 2. **Theory:** In this exercise we are going to prove closure of Gaussians under linear transformation. Let x represent an n-variate random variable with Gaussian density $\mathcal{N}(\mu_x, \Sigma_x)$ and let its linear transformation be $y = Ax$ where $A \in R^{n\times n}$. Prove that y is Gaussian distributed with density $\mathcal{N}(\mu_y, \Sigma_y)$, where $\mu_y = A\mu_x$ and $\Sigma_y = A\Sigma_x A^\top$.

Important: In the lecture, we derived the mean and covariance of the linear transformation $y = Ax$, *knowing* that y is Gaussian. Here, you must prove that y is indeed Gaussian with the given mean and covariance.

[14] 3. **Programming:** For the practical part of this exercise, we will implement a simple normalizing flow model, step-by-step. Please refer to the corresponding jupyter-notebook which contains the tasks in detail.