Discrete Mathematics Assignment 3

Gormery K. Wanjiru

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Problem 1

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 10$ if x_1, x_2, x_3, x_4 are nonnegative integers?

Solution: This problem is solved using the stars and bars theorem. The formula to find the number of nonnegative integer solutions is given by:

$$\binom{n+k-1}{k-1}$$

where $n = 10$ and $k = 4$.

$$\binom{10+4-1}{4-1} = \binom{13}{3} = 286$$

Problem 2

Three officers—a president, a treasurer, and a secretary—are to be chosen from among four people: Ann, Bob, Cyd, and Dan. Bob is not qualified to be treasurer and Cyd is not qualified to be secretary. How many ways can the officers be chosen?

Solution:

$$\text{Choices for president} = 4$$

$$\text{Choices for treasurer (excluding Bob)} = 3$$

$$\text{Choices for secretary (excluding Cyd)} = 2$$

$$\text{Total ways} = 4 \times 3 \times 2 = 24$$

Problem 3

Find the minimum number of students needed to guarantee that 10 of them were born on the same day of the week.

Solution: By the pigeonhole principle:

$$\text{Minimum students} = 7 \times 9 + 1 = 64$$

Problem 4

Given sets $A, B \subseteq U$, $A \cap B = \varnothing$, $n(A) = 12$, $n(B) = 10$, what is the probability that the selection contains 4 elements from A and 3 from B?

Solution:

$$\text{Favorable outcomes} = \binom{12}{4} \times \binom{10}{3}$$

$$\text{Total outcomes} = \binom{22}{7}$$

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{\binom{12}{4} \times \binom{10}{3}}{\binom{22}{7}} \approx 0.3483$$

Problem 5

How many five-person teams contain at most one man from a group of five men and seven women?

Solution:

$$\text{Teams with no men} = \binom{7}{5}$$

$$\text{Teams with one man} = \binom{5}{1} \times \binom{7}{4}$$

$$\text{Total teams} = \binom{7}{5} + \binom{5}{1} \times \binom{7}{4} = 196$$

Problem 6

How many pairs of two distinct integers from the set $\{1, 2, 3, ..., 100, 101\}$ have a sum that is even?

Solution:

$$\text{Even pairs (from even numbers)} = \binom{50}{2}$$

$$\text{Odd pairs (from odd numbers)} = \binom{51}{2}$$

$$\text{Total even pairs} = \binom{50}{2} + \binom{51}{2} = 2500$$

Problem 7

Eight people are attending the movies together, but two of them do not want to sit next to each other. How many ways can they be seated?

Solution:

$$\text{Total arrangements} = 8!$$

$$\text{Unacceptable arrangements (two specific people next to each other). I count them as a unit} =$$

$$\text{Acceptable arrangements} = 8! - 2 \times 7! = 30240$$

Its 2 times the Unacceptable arrangements because the two can switch positions. Albeit sit together in two different ways