**Transfer functions and frequency responses (Lab 04)**

The following VI displays the magnitude and impulse response of a general transfer function

$$H(z) = \frac{\sum_{i=0}^{\infty} c_i z^{-i}}{\sum_{i=0}^{\infty} d_i z^{-i}} \quad , \tag{1}$$

where $c_i$ is the forward coefficients and $d_i$ is the reverse coefficients. In this VI, an impulse is generated (1024 long) and sent to an IIR filter[1] in the middle of the figure. The reverse coefficients and the forward coefficients can be configured in the front panel. The output of the filter has two paths. The upper one is thourgh an FFT and then the magnitude is plotted, meaning that this is the spectrum of the impulse response of the filter. The bottom one is directly shown in the time domain as impulse response. To check the correlations of the coefficients in Eq. (1) and the coefficients defined in the IIR Filter VI, please go to the detailed help of this VI[2].

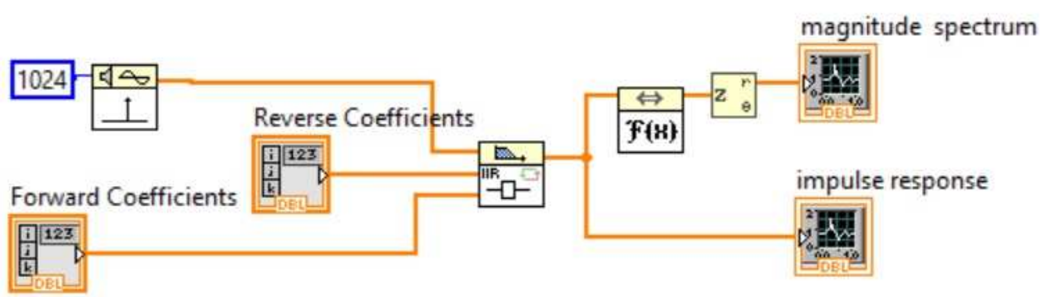


Figure. 1: Transfer function realized by an IIR filter block

Here we have four groups of transfer functions, 1) to 4), with different configurations.

1) $H_1(z) = 1 - b_1 z^{-1}$. By giving $b_1$ one of the following values, i.e., 1, -1, 0.5, and -0.5, at a time, we can generate four transfer functions for this case in total.

2) $H_2(z) = \frac{1}{1-a_1 z^{-1}}$. By giving $a_1$ one of the following values, i.e., 0.9, -0.9, 0.5, and -0.5, at a time, we can generate four transfer functions for this case in total.

[1]Note that a FIR filter can be considered as a special case of IIR filter if $d_0 = 1$ and $d_i = 0$, $\forall i > 0$ in Eq. (1).

[2]Note that it is very important to check how coefficients are defined in a filter function (in this case, the IIR Filter VI). They may be defined in different ways in distinct software platforms. The filters frequency response will become completely different if a wrong coefficient is typed in.

3)

$$H_3(z) = \frac{(z - ke^{j\varphi})(z - ke^{-j\varphi})}{z^2},$$

k = 0.5 or 1, $\phi = \pi/4$ *or* $\pi/2$ (again, four transfer functions for this case in total).

4)

$$H_4(z) = \frac{(z - e^{j\pi/2})(z - e^{-j\pi/2})}{(z - 0.99e^{j\pi/2})(z - 0.99e^{-j\pi/2})}.$$

**Questions:**

All transfer functions from 1) to 4) with its specified parameters are to be examined.

a) Find the difference equations for all transfer functions.

b) Run the Vi and save the screen shots with different parameters for $H_1(z), H_2(z), H_3(z)$, and $H_4(z)$. For example, for $H_1(z)$ when $b_1 = 1$, we have the result[3] as shown in Fig. 2. Compare the impulse responses with the difference equations and explain if they are IIR or FIR type.

c) Derive the expressions for $\mid H(\omega T) \mid$. Insert $0, \pi/2, \pi$, for $\omega T$ into $\mid H(\omega T) \mid$ and compare these results with the magnitude responses you find by running the program.

d) Find the pole-zero locations. Check the relationship between the pole-zero diagrams and magnitude responses.

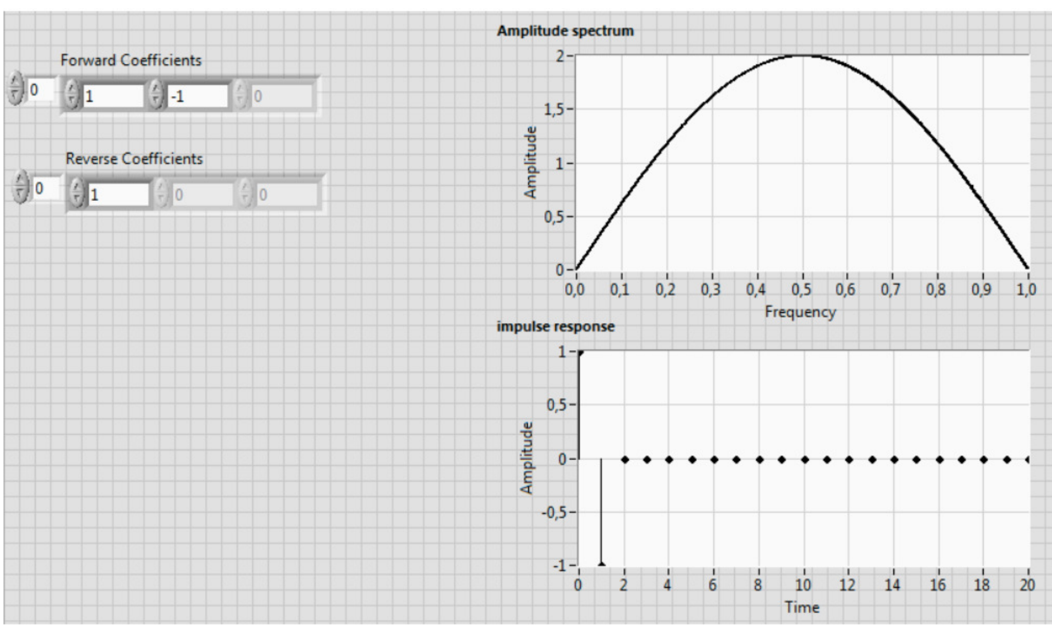


Figure. 2: Example for H1(z) when $b_1$=1

[3]Note that the spectrum has been normalized, i.e., 1 means sampling frequency.