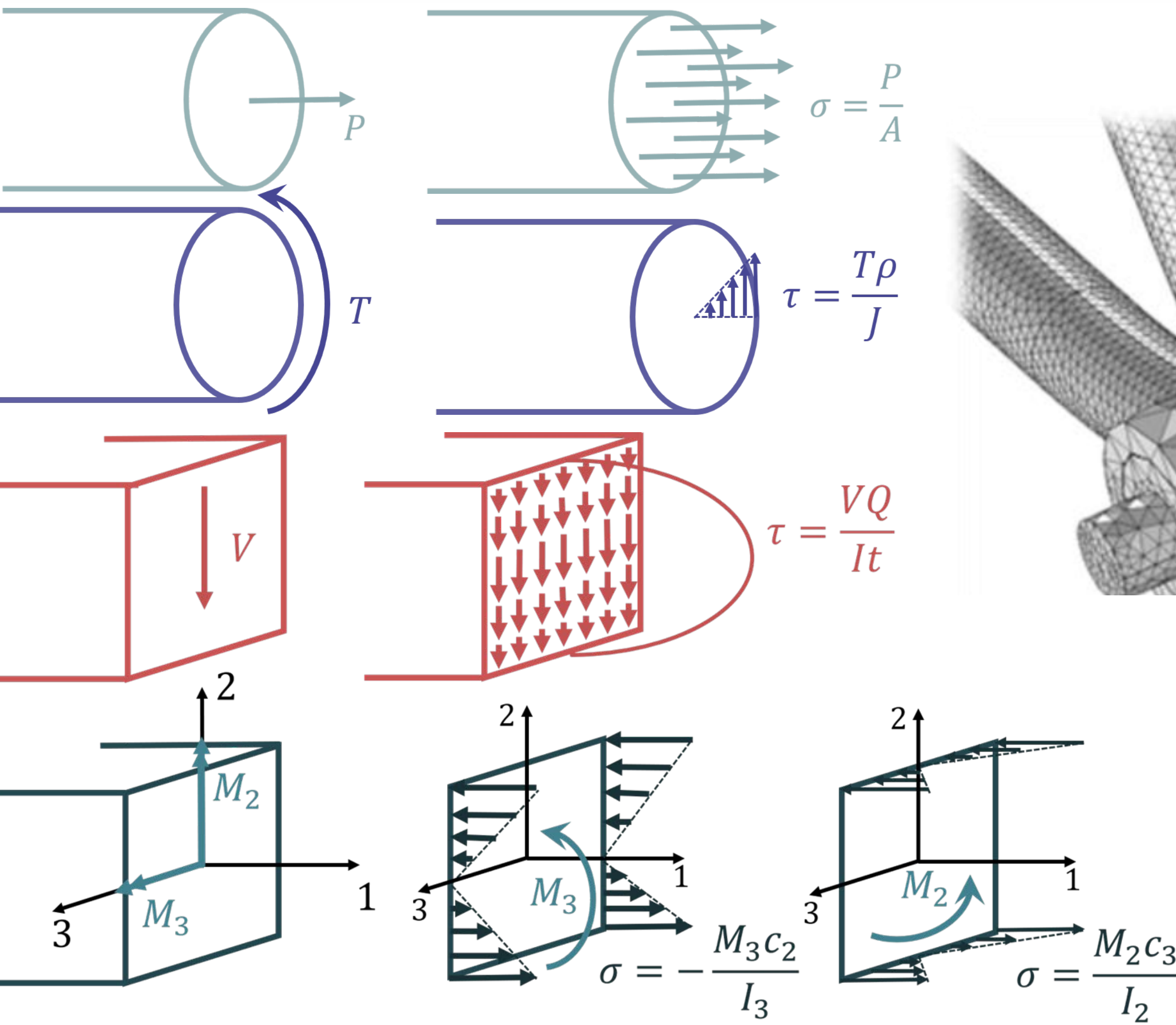
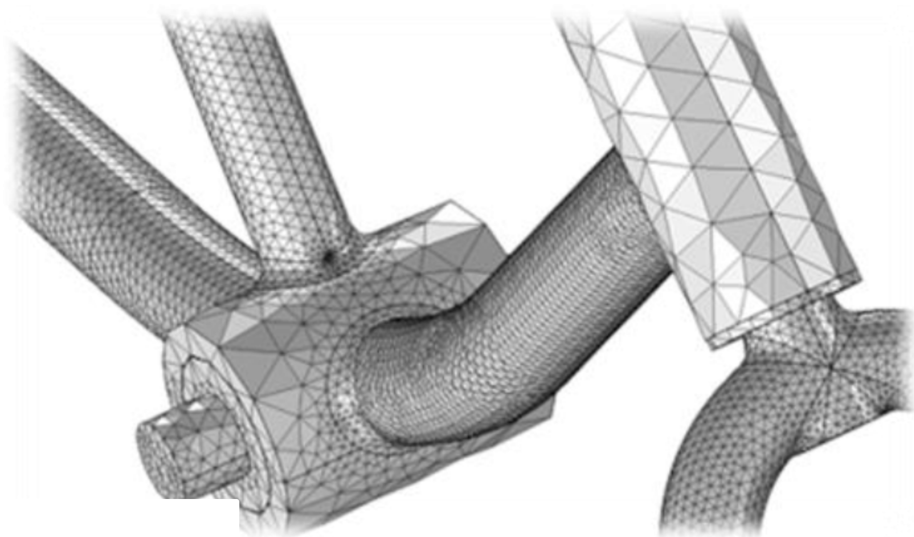
Product Development 1

Introduction to **Mechanical Engineering Design**

Spring 2023

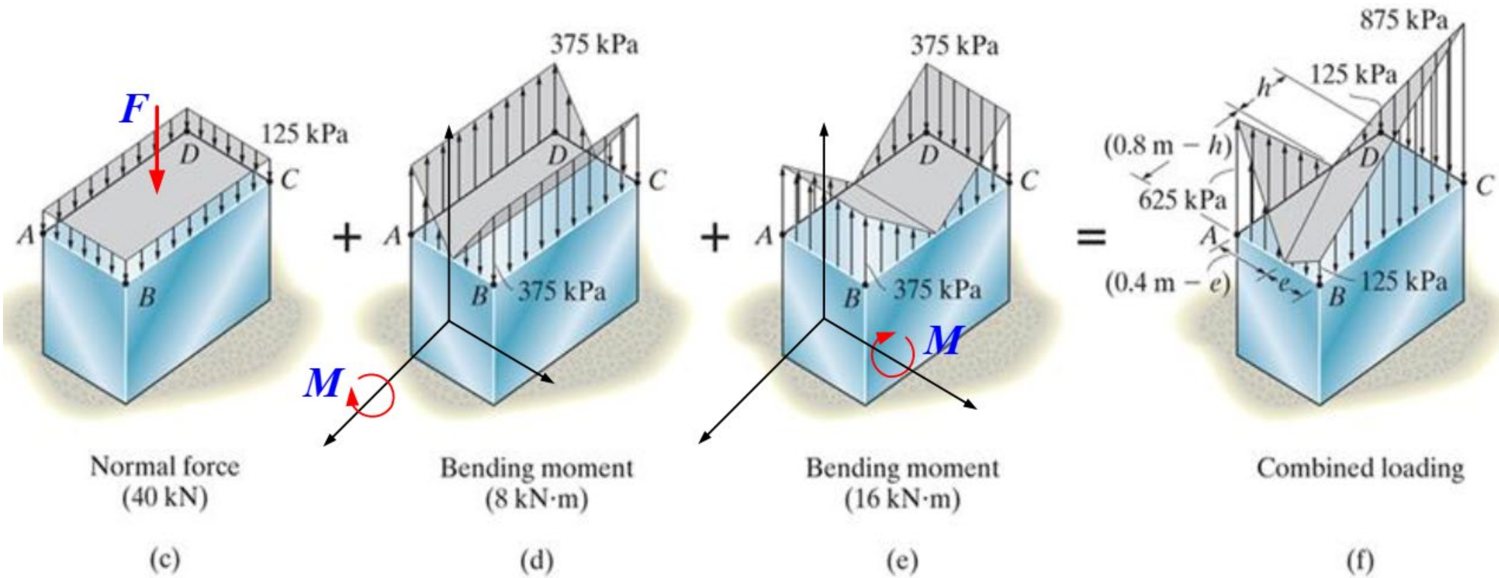
Combined loading





Introduction

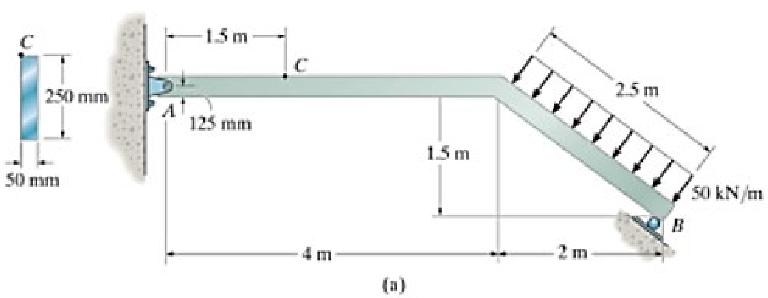
This presentation serves as a review of the stress analysis that has been developed in the previous lecture regarding *axial load*, *torsion*, *bending*, and *shear*. We will discuss the solution of problems where several of these internal loads occur simultaneously on a member's cross section.

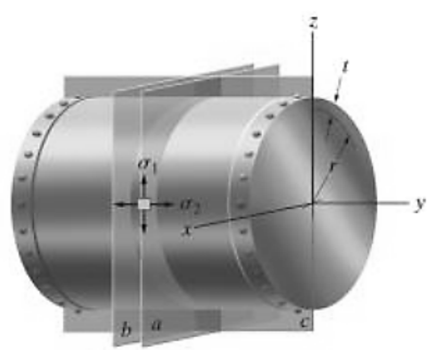


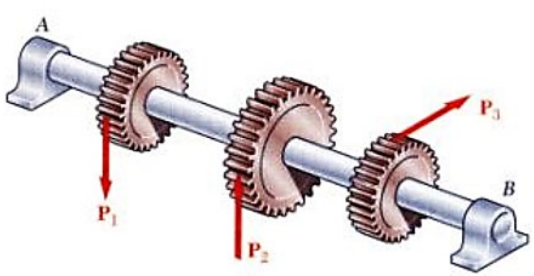
Introduction

- In many structures the members are required to resist more than one kind of loading (combined loading). These can often be analyzed by superimposing the stresses and strains cause by each load acting separately.
- Superposition of stresses and strains is permissible only under the following conditions:
  I. The stresses and the strains must be a linear function of the applied loads (Hooke's law must be obeyed and the displacements must be small).
  II. There must be no interaction between the various loads.

<u>Examples:</u> wide-flange beam supported by a cable (combined bending and axial load), cylindrical pressure vessel supported as a beam, and shaft in combined torsion and bending.







Method of analysis

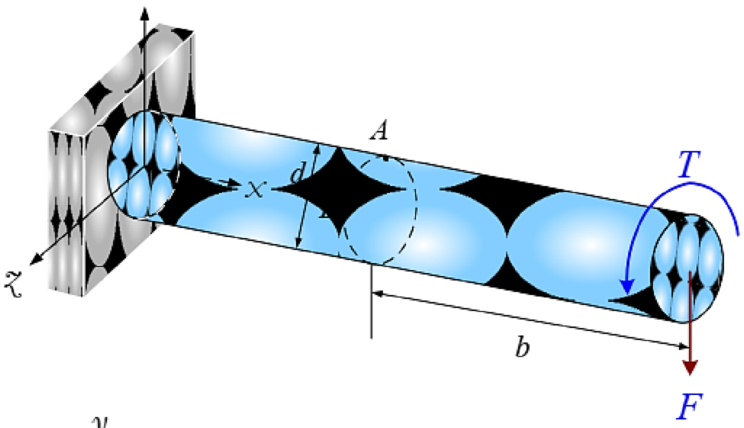
I. Select the point on the structure where the stresses and the strains are to be determined.
II. For each load on the structure, determine the stress resultant at the cross section containing the selected point.
III. Calculate the normal and shear stresses at the selected point due to each of the stress resultant.
IV. Combine the individual stresses to obtain the resultant stresses at the selected point.
V. Determine the principal stresses and maximum shear stresses at the selected point.
VI. Determine the strains at the point with the aid of Hooke's law for plane stress.
VII. Select additional points and repeat the process.

$$\sigma_{axial} = \frac{F}{A} \qquad \tau_{torque} = \frac{T\rho}{J} \text{ and } \tau_{torque(\max)} = \frac{Tc}{J}$$

$$\sigma_{bend.} = -\frac{My}{I} = -\frac{M}{Z} \qquad \tau_{shear\ Force} = \frac{V}{A} \qquad \tau_{bend.shear} = \frac{VQ}{Ib}$$

Illustration of the method

The bar shown is subjected to two types of loads: a torque **T** and a vertical load **F**. Let us select randomly two points. Point A (top of the bar) and point B (side of the bar - in the same cross section).



The resulting stresses acting across the section are the following:

- A twisting moment equal to the torque T.
- A bending moment M equal to the load F times the distance b.
- A shear force V equals to the load F.

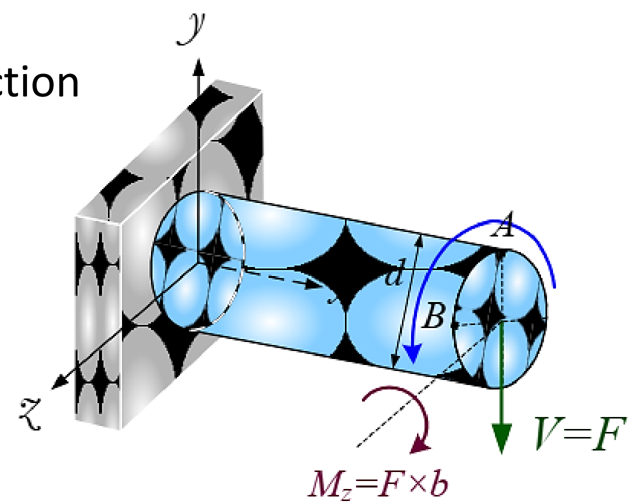
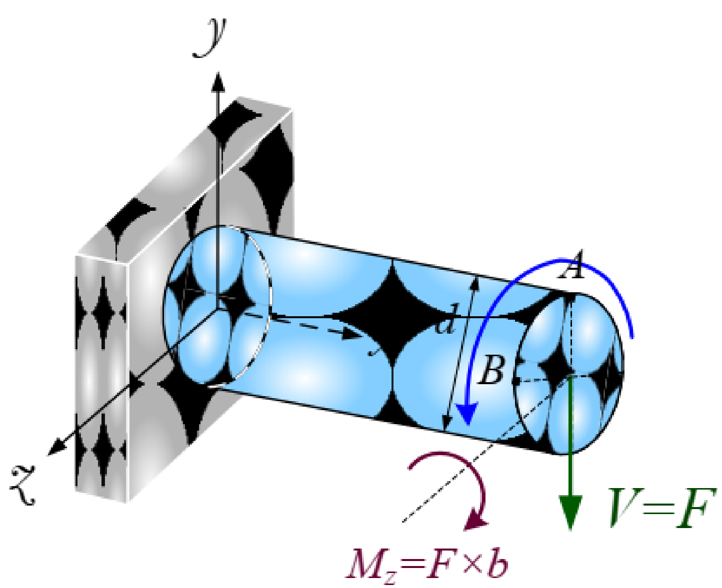


Illustration of the method:
Transvers and torsional loads

The twisting moment T produces a torsional shear stresses

$$\tau_{torsion} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

The stress $\boldsymbol{\tau_1}$ acts horizontally to the left at point *A* and vertically downwards at point *B*



The bending moment M produces a tensile stress at point *A* :

$$\sigma_{bending} = -\frac{Mc}{I} = -\frac{4M}{\pi c^3}$$

However, the bending moment produces no stress at point B, because B is located on the neutral axis.

Illustration of the method:
Transvers and torsional loads

The shear force V produces no shear stress at the top of the bar (point A), but at point B the shear stress is as follows:

$$\tau_{bend.shear} = \frac{VQ}{Ib} = \frac{4V}{3A}$$

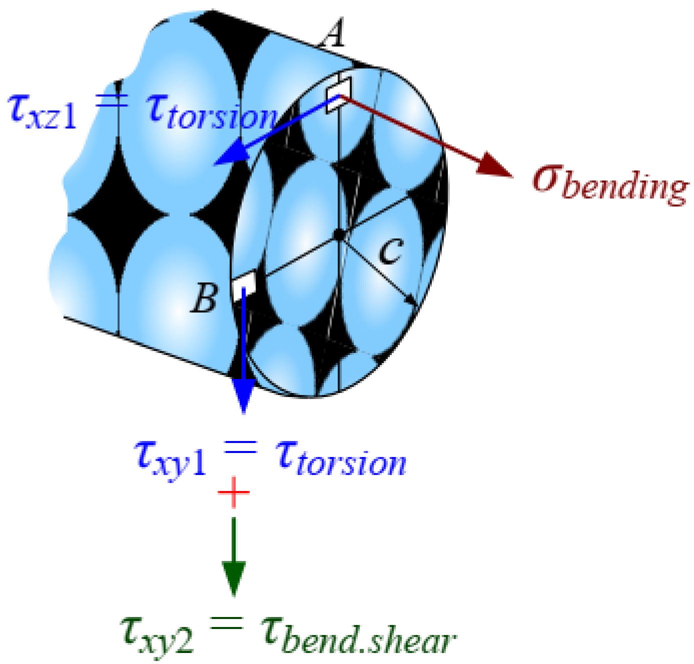
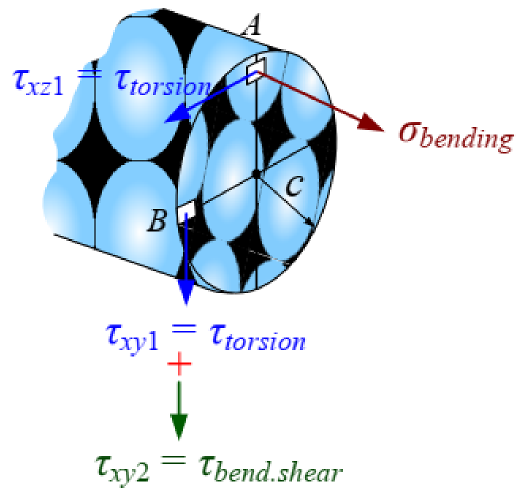
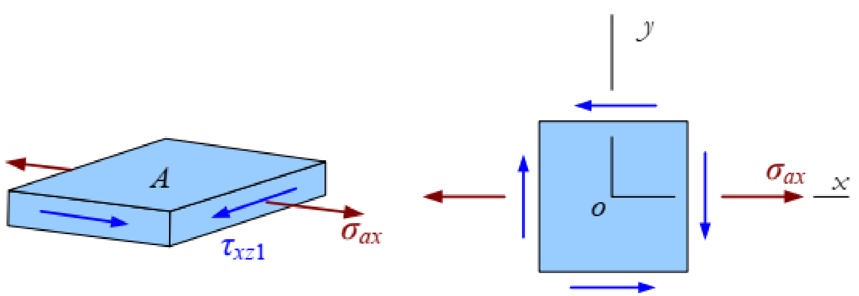


Illustration of the method:
Transvers and torsional loads

σ_A and τ_1 are acting in point A, while the τ_1 and τ_2 are acting in point *B*.
Note that the element is in plane stress with
$\sigma_x = \sigma_A$, $\sigma_y = 0$, and $\tau_{xz} = -\tau_1$
A stress element in point *B* is also in plane stress and the only stresses acting on this element are the shear stresses τ_1 and τ_2. Therefore
$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -(\tau_1 + \tau_2)$



At point *A*: $\sigma_x = \sigma_a$, $\sigma_y = 0$ and $\tau_{xz} = -\tau_1$



At point B: $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -(\tau_1 + \tau_2)$

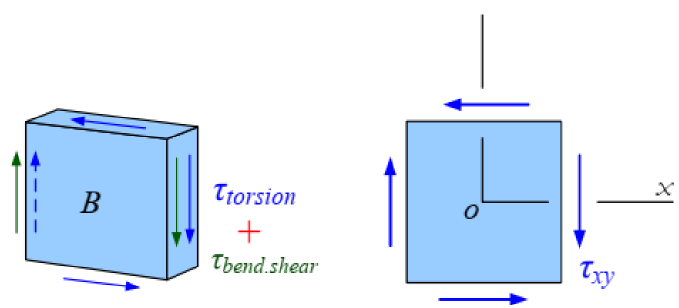
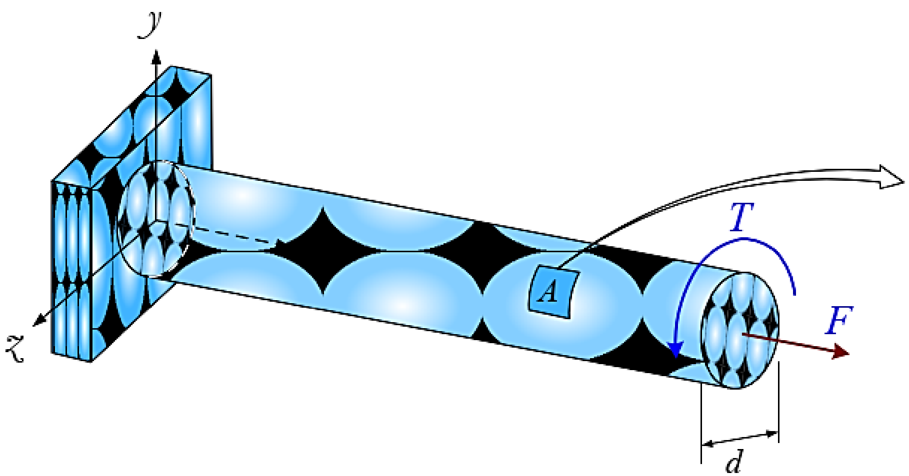
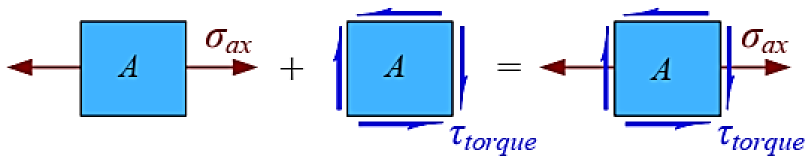


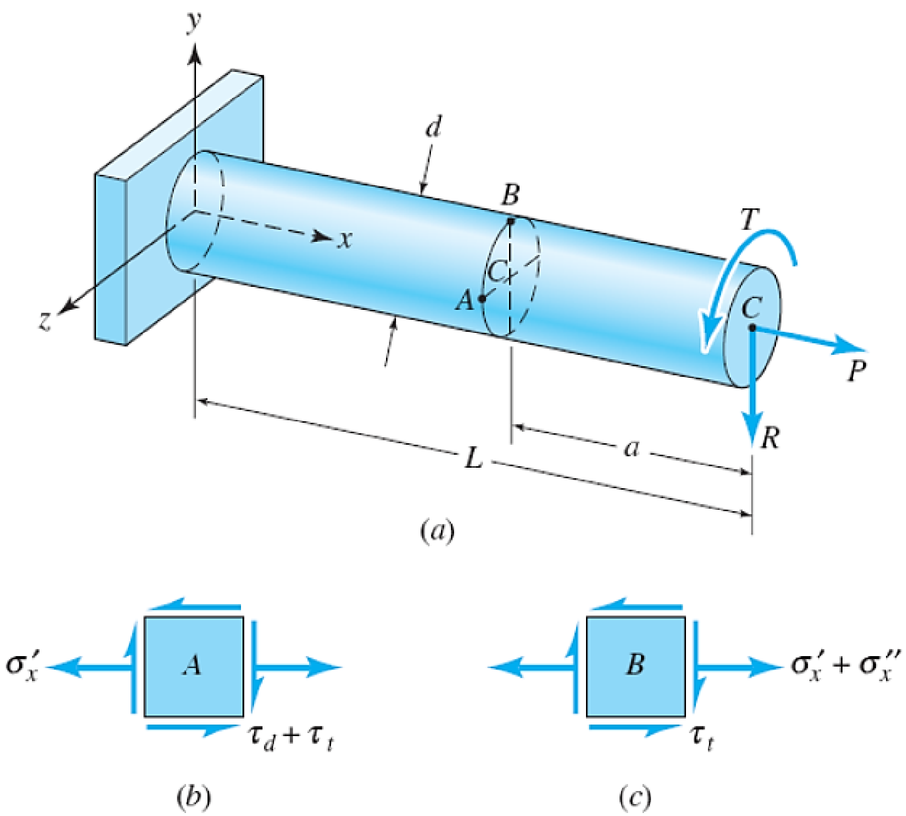
Illustration of the method:
Axial and torsional loads





$$\sigma_x = \frac{P}{A} = \frac{4P}{\pi d^2} \qquad \tau_{xy} = -\frac{Tc}{J} = -\frac{16T}{\pi d^3}$$

Axial, Transverse, and Torsional Loads



(a) a cantilever bar with three loads;

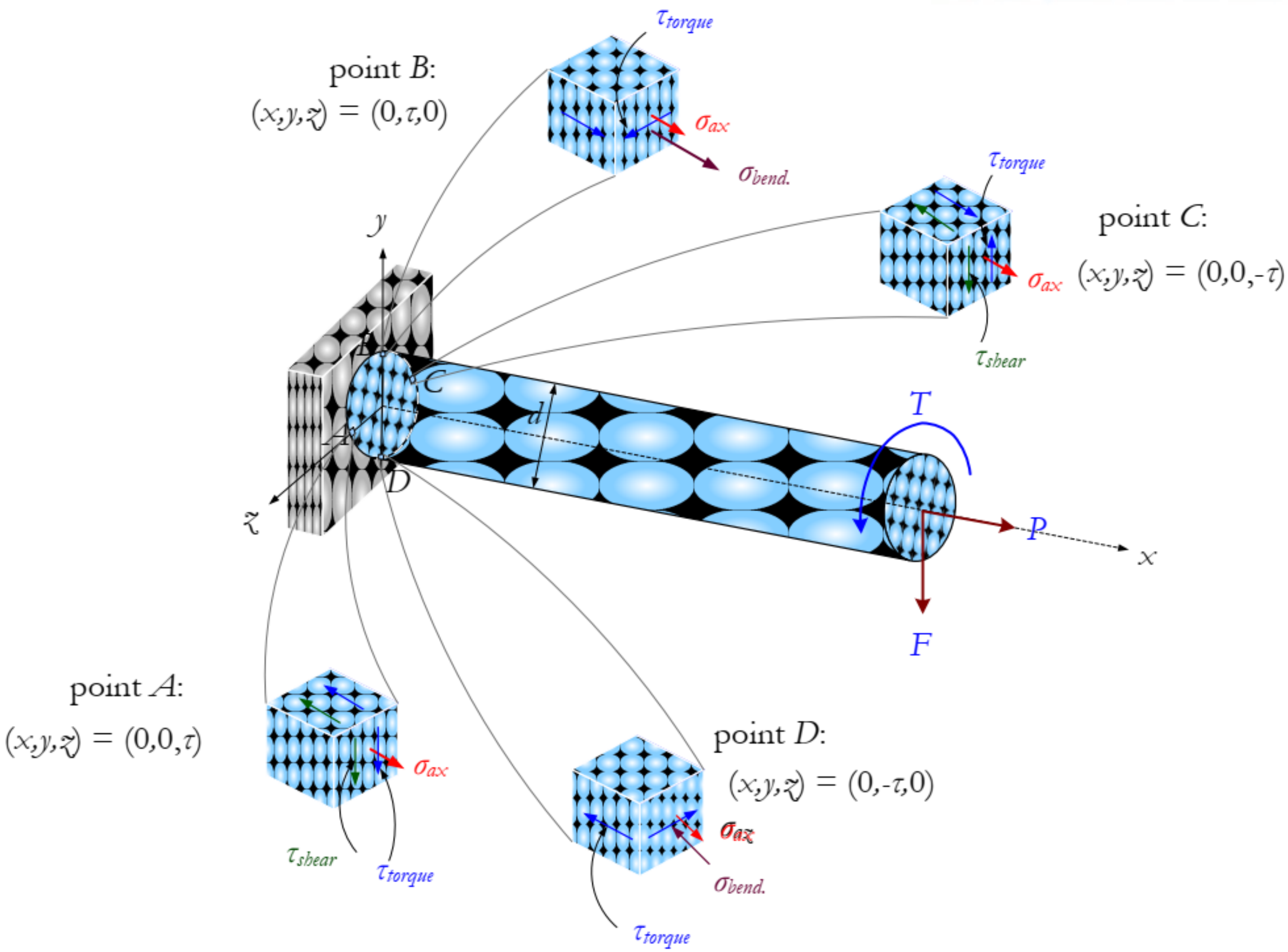
(b) combined stresses due to axial load, direct shear, and torsion at point A;

(c) combined stresses due to torsion, axial load, and bending on an element at point B.

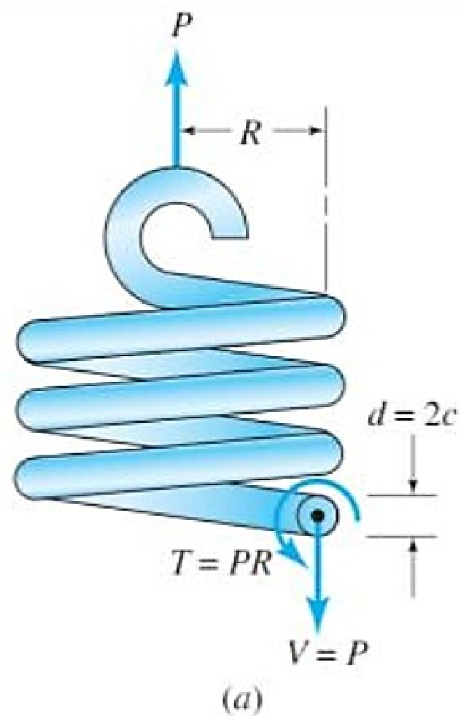
$$\sigma'_x = \frac{P}{A} \qquad \tau_t = \frac{Tc}{J}$$

$$\sigma''_x = \frac{Mc}{I} \qquad \tau_d = -\frac{VQ}{Ib}$$

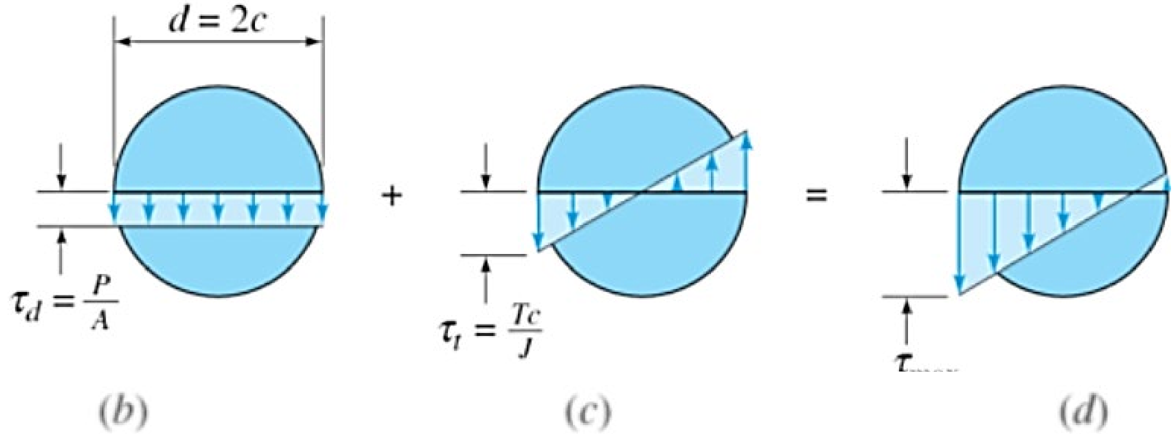
Stresses at which point?
General Case



Direct Shear and Torsional Loads: Helical Springs



(a) a helical tension spring under combined shear loading



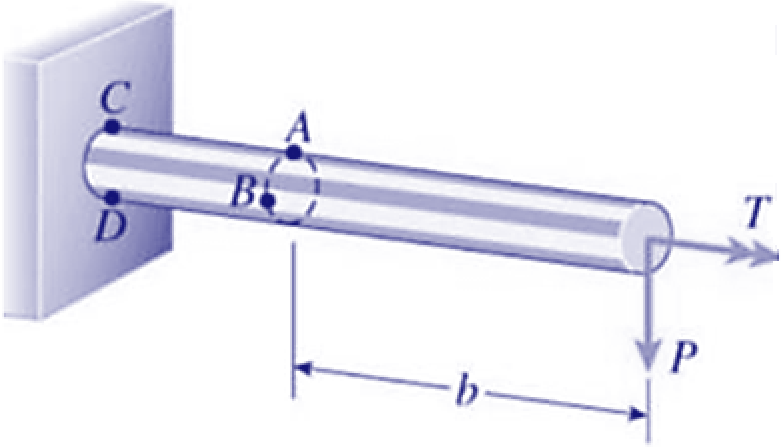
(b) direct shear due to load P; (c) torsional shear stress; (d) combined shear stress.

$$\tau_{max} = \frac{P}{A} + \frac{T_c}{J} = \frac{4P}{\pi d^2} + \frac{16PR}{\pi d^3}$$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

Illustration of the method

Of interest are the points where the stresses calculated from the flexure and shear formulas have maximum or minimum values, called critical points!

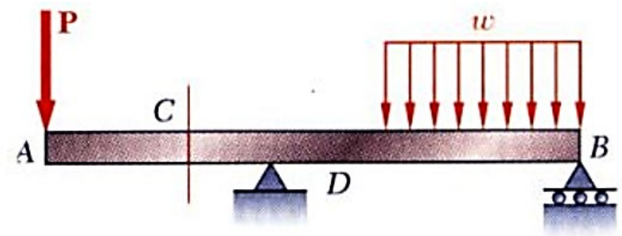


For instance, the normal stresses due to bending are largest at the cross section of maximum bending moment, which is at the support. Therefore, points C and D at the top and bottom of the beam at the fixed ends are critical points where the stresses should be calculated.

Selection of critical planes/points

If the objective of the analysis is to determine the largest stresses anywhere in the structure, then the critical points should be selected at cross sections where the stress resultants have their largest values. Furthermore, within those cross sections, the points should be selected where either the normal stresses or the shear stresses have their largest values.

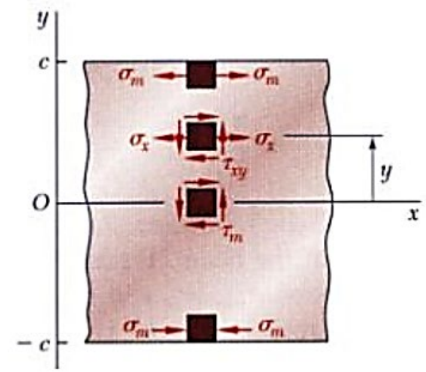
Principle Stresses in Beam



- Prismatic beam subjected to transverse loading

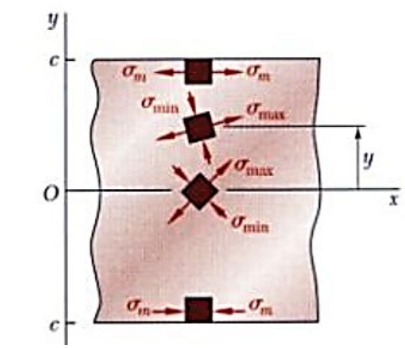
$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{Mc}{I}$$

$$\tau_{xy} = -\frac{VQ}{It} \qquad \tau_m = \frac{VQ}{It}$$



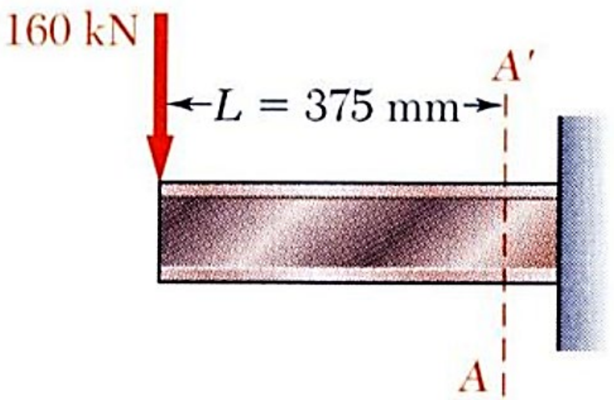
- Can the maximum normal stress within the cross-section be larger than

$$\sigma_m = \frac{Mc}{I}$$



Example 1

A 160-kN force is applied at the end of a W200x52 rolled-steel beam. Neglecting the effects of fillets and of stress concentrations, determine whether the normal stresses satisfy a design specification that they be equal to or less than 150 MPa at section AA'.



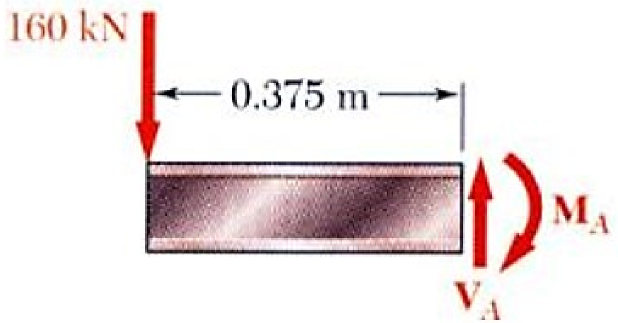
- Determine shear and bending moment in Section A-A'

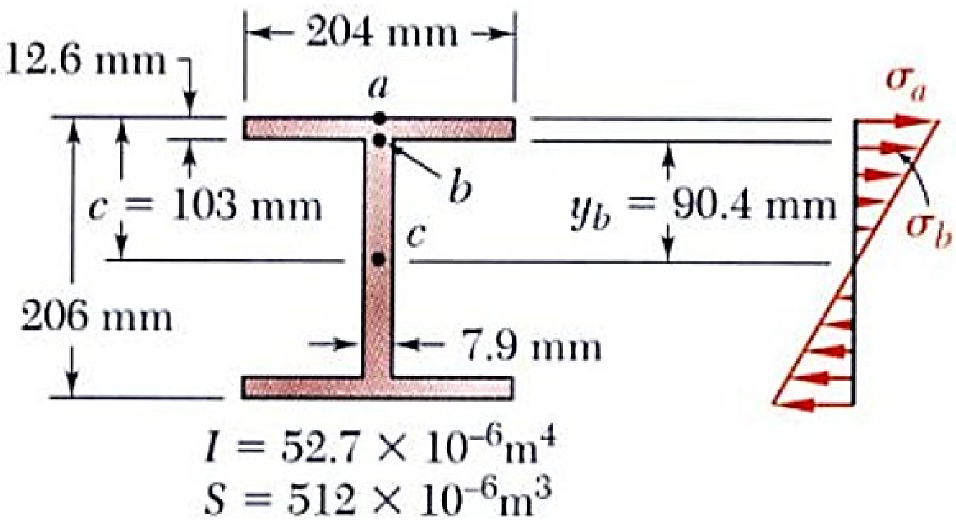
- Calculate the normal stress at top surface and at flange-web junction.

- Evaluate the shear stress at flange-web junction.

- Calculate the principal stress at flange-web junction.
(the next lectures on stress transformation)

Solution





- Determine shear and bending moment in Section A-A’

$$M_A = (160\,\text{kN})(0.375\,\text{m}) = 60\,\text{kN - m}$$
$$V_A = 160\,\text{kN}$$

- Calculate the normal stress at top surface and at flange-web junction.

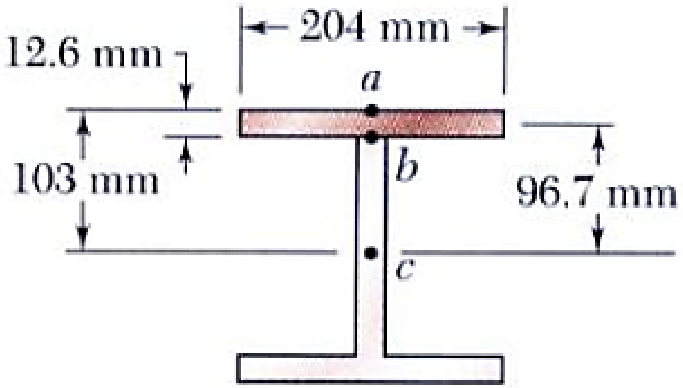
$$\sigma_a = \frac{M_A}{S} = \frac{60\,\text{kN}\cdot\text{m}}{512\times10^{-6}\,\text{m}^3}$$
$$= 117.2\,\text{MPa}$$

$$\sigma_b = \sigma_a \frac{y_b}{c} = (117.2\,\text{MPa})\frac{90.4\,\text{mm}}{103\,\text{mm}}$$
$$= 102.9\,\text{MPa}$$

Solution

- Evaluate shear stress at flange-web junction.

$$Q = (204 \times 12.6)96.7 = 248.6 \times 10^3\,\text{mm}^3$$
$$= 248.6 \times 10^{-6}\,\text{m}^3$$
$$\tau_b = \frac{V_A Q}{It} = \frac{(160\,\text{kN})(248.6 \times 10^{-6}\,\text{m}^3)}{(52.7 \times 10^{-6}\,\text{m}^4)(0.0079\,\text{m})}$$
$$= 95.5\,\text{MPa}$$



Example 2

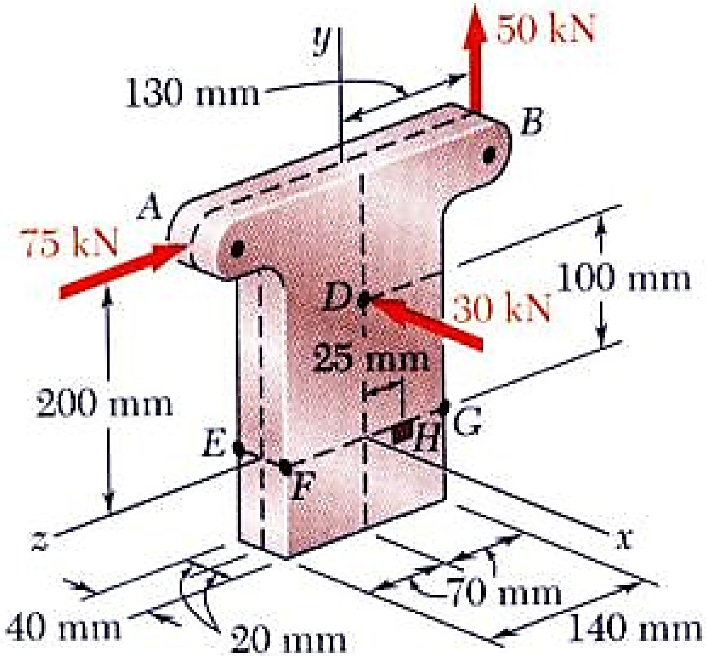
Three forces are applied to a short steel post as shown in the figure. Determine the principle stresses, principal planes and maximum shearing stress at point H.

- Determine internal forces in Section EFG.

- Calculate principal stresses and maximum shearing stress. Determine principal planes.
(the next lectures on stress transformation)

- Evaluate shearing stress at H.

- Evaluate normal stress at H.



Solution

- Determine internal forces in Section *EFG*.

$$V_x = -30\,\text{kN} \quad P = 50\,\text{kN} \quad V_z = -75\,\text{kN}$$

$$M_x = (50\,\text{kN})(0.130\,\text{m}) - (75\,\text{kN})(0.200\,\text{m})$$

$$= -8.5\,\text{kN}\cdot\text{m}$$

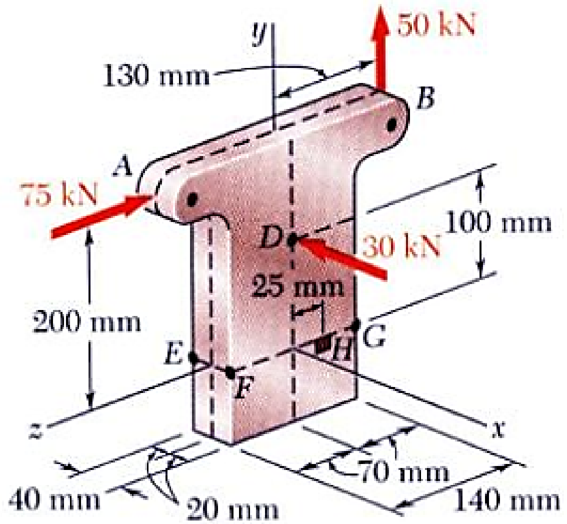
$$M_y = 0 \quad M_z = (30\,\text{kN})(0.100\,\text{m}) = 3\,\text{kN}\cdot\text{m}$$

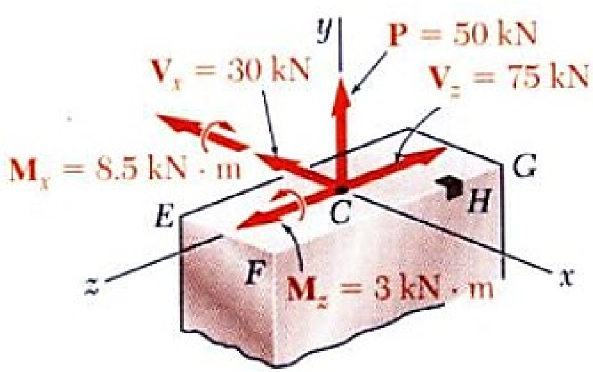
Note: Section properties,

$$A = (0.040\,\text{m})(0.140\,\text{m}) = 5.6\times10^{-3}\,\text{m}^2$$

$$I_x = \tfrac{1}{12}(0.040\,\text{m})(0.140\,\text{m})^3 = 9.15\times10^{-6}\,\text{m}^4$$

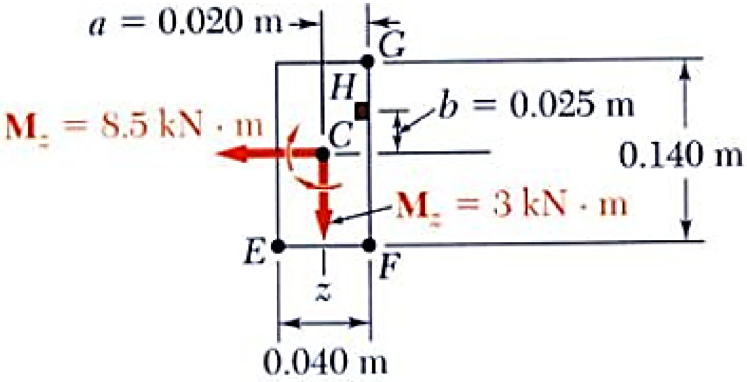
$$I_z = \tfrac{1}{12}(0.140\,\text{m})(0.040\,\text{m})^3 = 0.747\times10^{-6}\,\text{m}^4$$





Solution

- Evaluate shear stress at flange-web

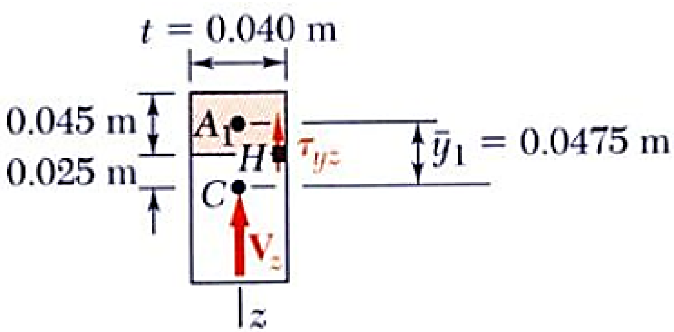


$$\sigma_y = +\frac{P}{A} + \frac{|M_z|a}{I_z} - \frac{|M_x|b}{I_x}$$

$$= \frac{50\,\text{kN}}{5.6\times10^{-3}\,\text{m}^2} + \frac{(3\,\text{kN}\cdot\text{m})(0.020\,\text{m})}{0.747\times10^{-6}\,\text{m}^4} - \frac{(8.5\,\text{kN}\cdot\text{m})(0.025\,\text{m})}{9.15\times10^{-6}\,\text{m}^4}$$

$$= (8.93 + 80.3 - 23.2)\,\text{MPa} = 66.0\,\text{MPa}$$

- Evaluate shearing stress at H.



$$Q = A_1\bar{y}_1 = [(0.040\,\text{m})(0.045\,\text{m})](0.0475\,\text{m})$$

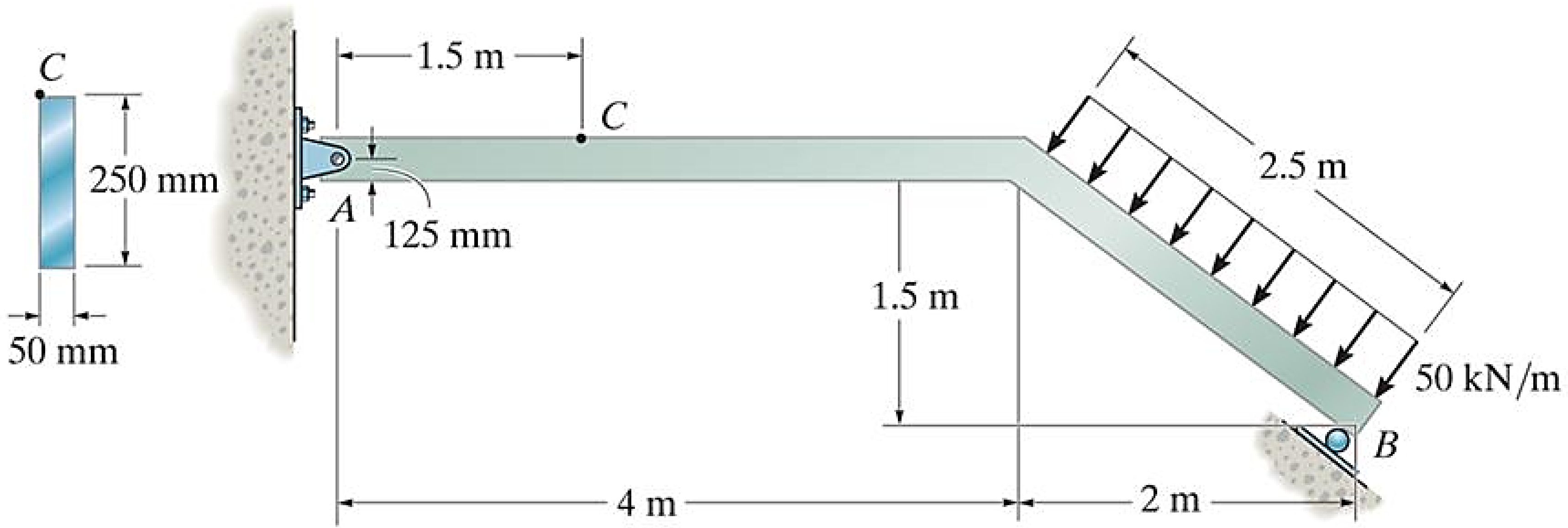
$$= 85.5\times10^{-6}\,\text{m}^3$$

$$\tau_{yz} = \frac{V_z Q}{I_x t} = \frac{(75\,\text{kN})\left(85.5\times10^{-6}\,\text{m}^3\right)}{\left(9.15\times10^{-6}\,\text{m}^4\right)(0.040\,\text{m})}$$

$$= 17.52\,\text{MPa}$$

Example 3

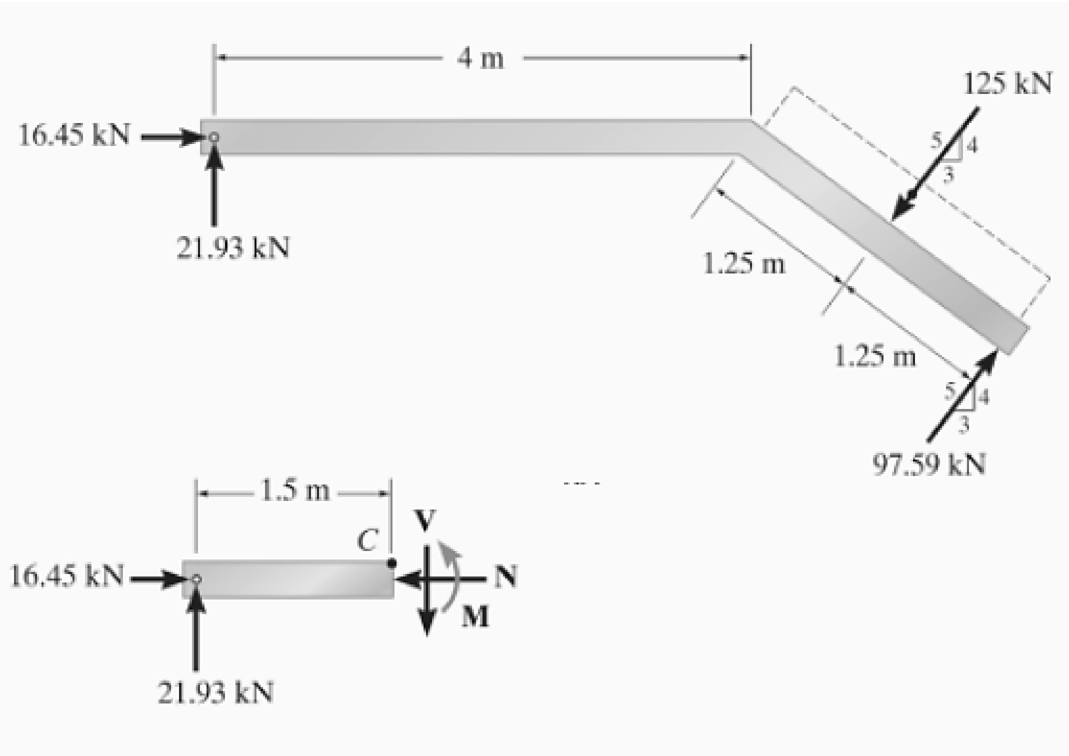
The member shown in the figure has a rectangular cross section. Determine the state of stress that the loading produces at point C



(a)

Solution

Internal Loadings: The support reactions on the member have been determined in statics and are shown on the figure. If the left segment AC of the member is considered, the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. Solving:



$N = 16.45\ kN \qquad V = 21.93\ kN \qquad M = 32.89\ kN.m$

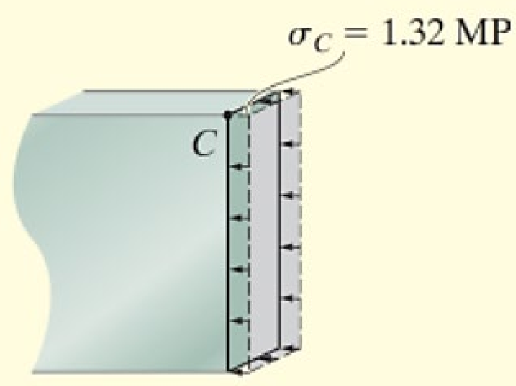
Solution

Stress Components:

Normal Force:

The uniform normal-stress distribution acting over the cross section is produced by the normal force. At point C:

$$\sigma_c = \frac{P}{A} = \frac{16.45\,(10^3)N}{(0.050m)(0.250m)} = 1.32\ MPa$$

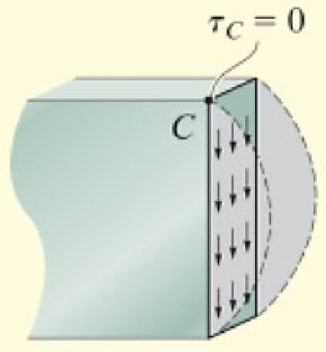


Normal Force

Shear Force:

Here the area since point C is located at the top of the member. Thus and for C, the shear stress:

$$\tau_c = 0$$

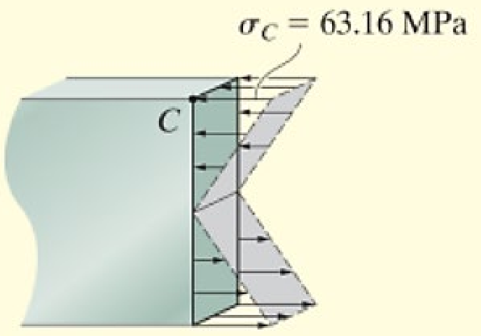


Shear Force

Solution

Bending Moment:
Point C is located at distance c from
the neutral axis, so the normal stress at C, is:

$$\sigma_c = \frac{M_C}{I} = \frac{(32.89\,(10^3)N.m)(0.125\,m)}{[\frac{1}{12}(0.050m)(0.250m)^3]} = 63.16\,MPa$$



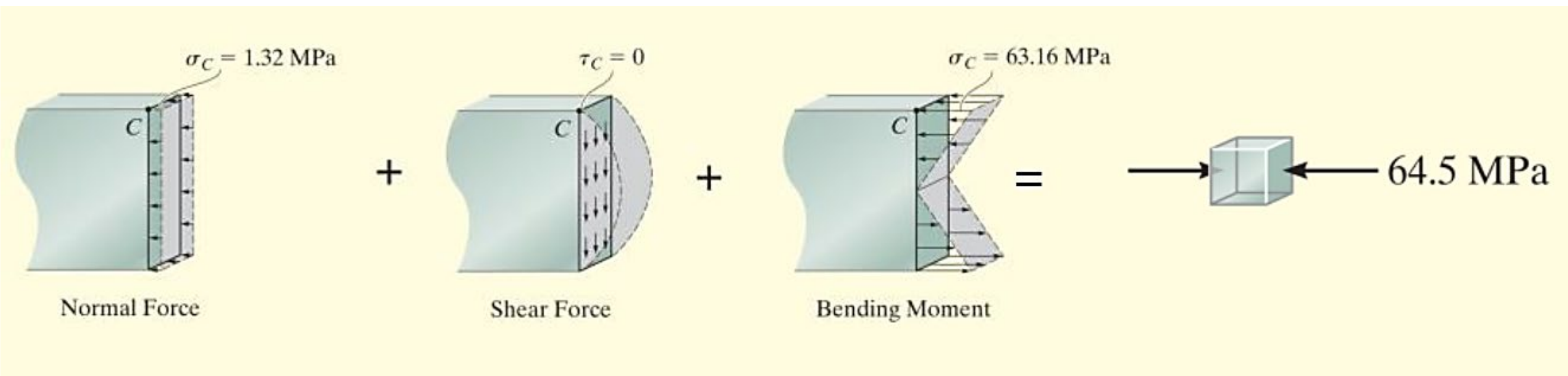
Bending Moment

<u>Superposition:</u>
The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of:

$$\sigma_c = 1.32\,MPa + 63.16\,MPa = 64.5\,MPa$$

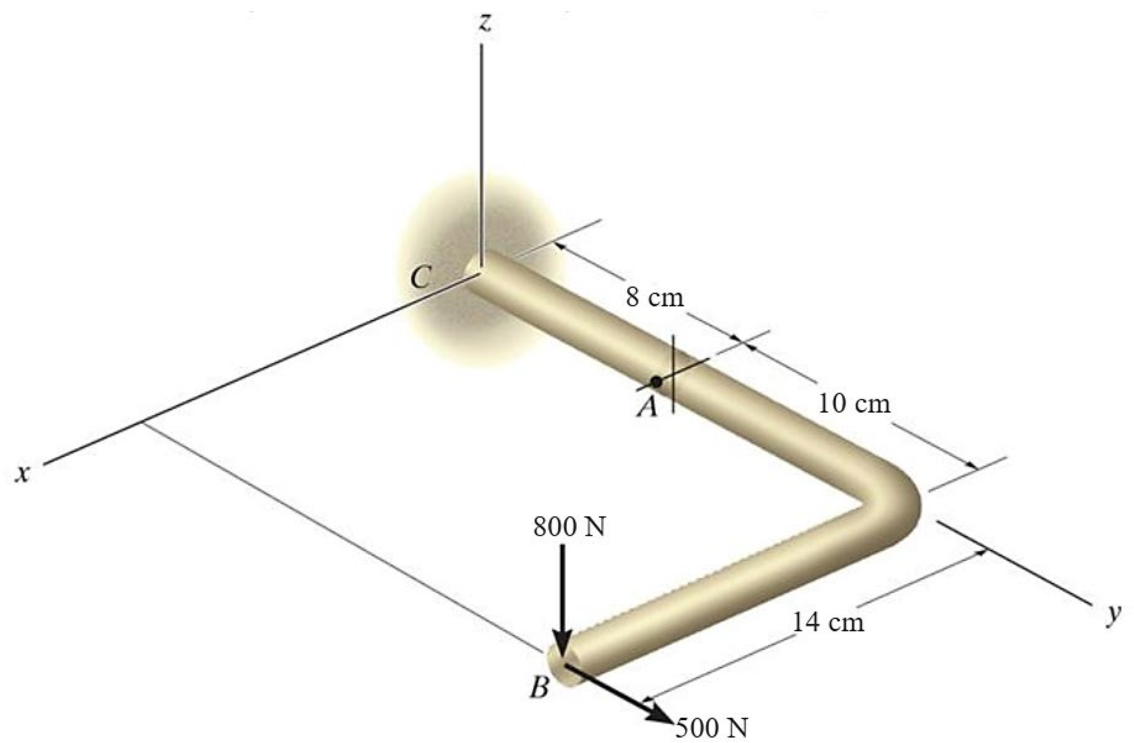
Solution

This result, acting on an element at *C*, is shown in the figure.



Example 4

The solid rod shown in the figure has a radius 0.75 cm. If it is subjected to the loading shown, determine the state of stress at point A.



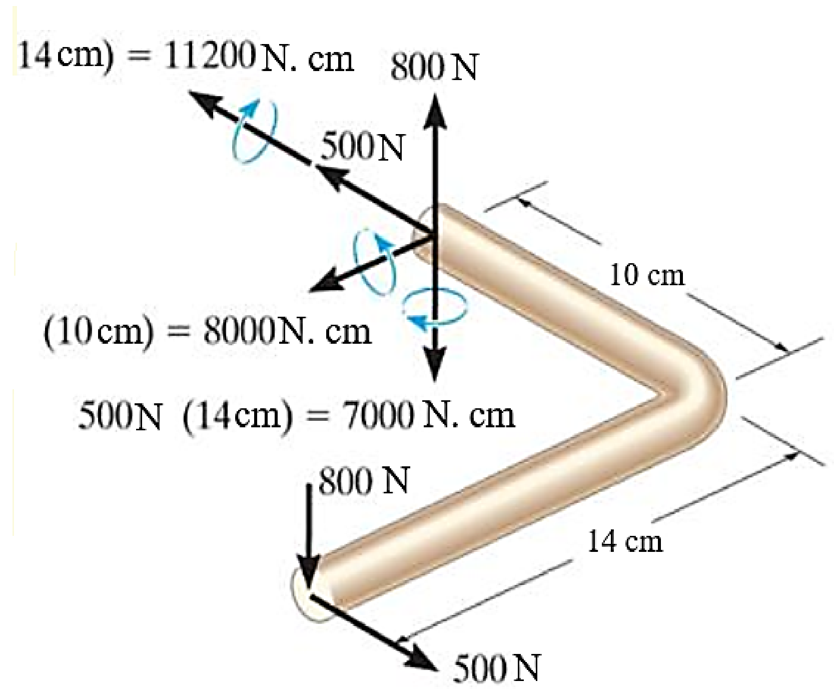
(a)

Solution

<u>Internal Loadings:</u>

The rod is sectioned through point A. Using the free-body diagram of segment AB, the resultant internal loadings can be determined from the six equations of equilibrium. (Verify these results!)

The normal force (500N) and shear force (800N) must act through the centroid of the cross section and the bending moment component (8000 N.cm. and 7000 N.cm.) are applied about centroidal (principal) axes. In order to better "visualize" the stress distributions due to each of these loadings, we will *consider the equal but opposite resultants acting on AC.*

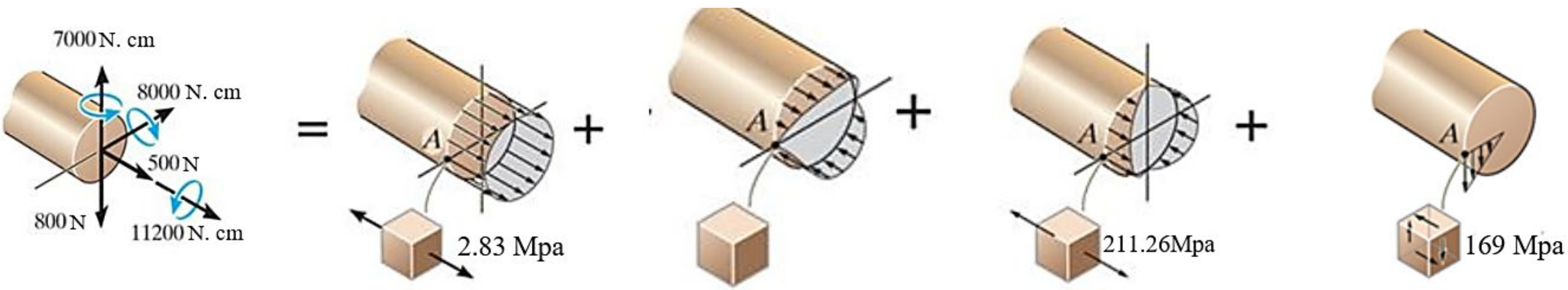


Solution

Stress Components:

Normal Force: The normal-stress distribution is shown in figure below. For point A, we have:

$$\sigma_A = \frac{P}{A} = \frac{500\ N}{\pi(0.75\ cm)^2} = 2.83\ MPa$$



Combined loading

Normal Force
500 N

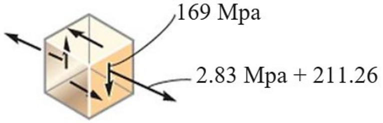
Bending moment
8000 N. cm.

Bending moment
7000 N. cm.

Torsional moment
11200 N. cm.

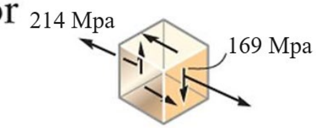
Solution

Bending Moments. For the 8000 N.cm. component, point A lies on the neutral axis, so the normal stress is:



$$\sigma_A = 0$$

or



For the 7000 N.cm. moment, c = 0.75 cm, so the normal stress at point A is

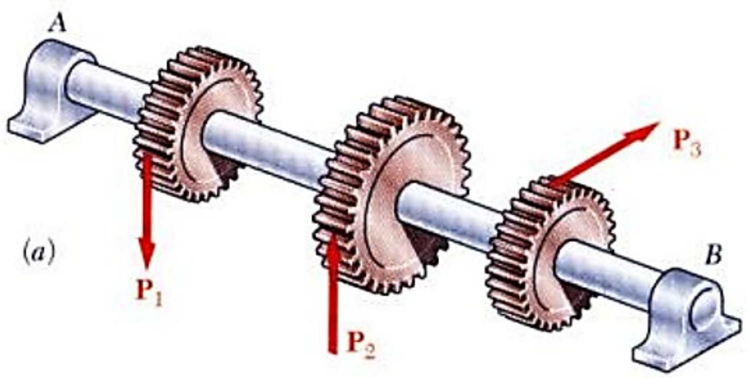
$$\sigma_A = \frac{Mc}{I} = \frac{7000\ N.cm.(0.75\ cm)}{[\frac{1}{4}\pi(0.75cm)^4]} = 21126\,\frac{\mathrm{N}}{\mathrm{cm}^2} = 211.26\ \mathrm{MPa}$$

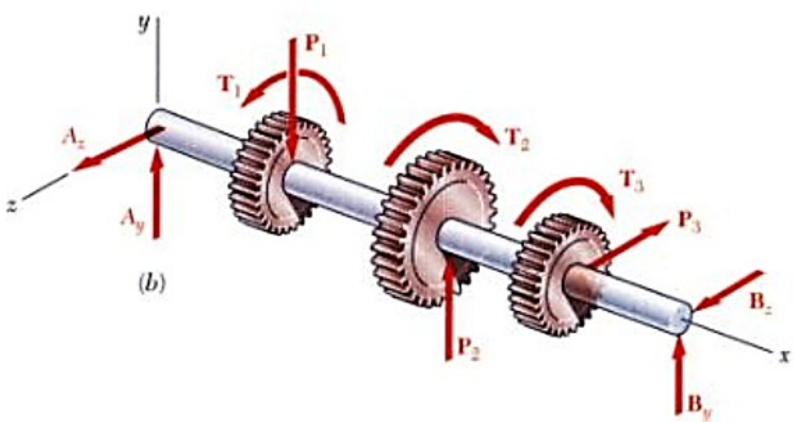
Torsional Moment. At point A, $\rho_A = C = 0.75\ cm, Fig. 8-6h.$ Thus the shear stress is

$$\tau_A = \frac{Tc}{J} = \frac{11200\ N.cm.(0.75\ cm)}{[\frac{1}{2}\pi(0.75cm)^4]} = 16901\,\frac{\mathrm{N}}{\mathrm{cm}^2} = 169\ \mathrm{MPa}$$

Superposition. When the above results are superimposed, it is seen that an element of material at A is subjected to both normal and shear stress components.

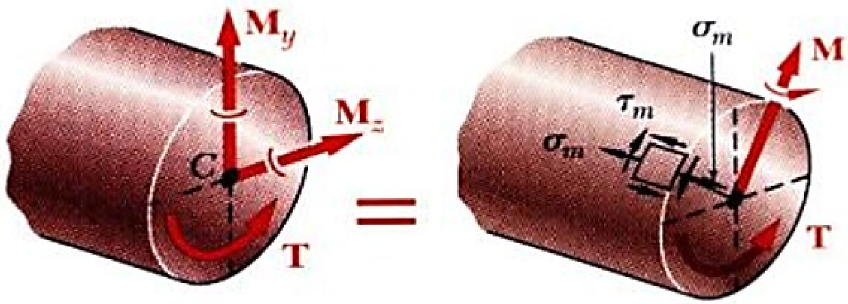
Design of a Transmission Shaft

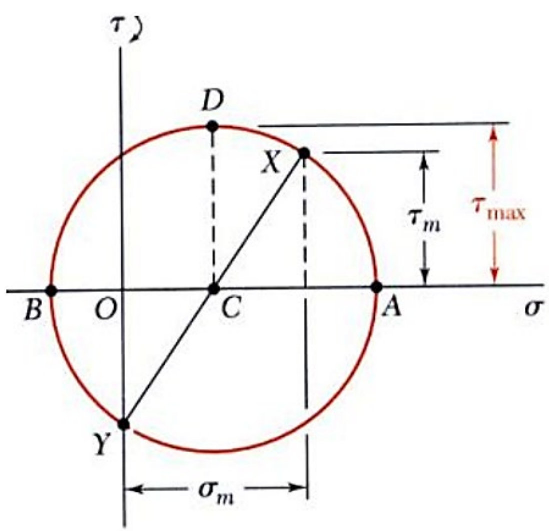




- If power is transferred to and from the shaft by gears or sprocket wheels, the shaft is subjected to transverse loading as well as shear loading.
- Normal stresses due to transverse loads may be large and should be included in determination of maximum shearing stress.
- Shearing stresses due to transverse loads are usually small and contribution to maximum shear stress may be neglected.

Design of a Transmission Shaft





- At any section:

$$\sigma_m = \frac{Mc}{I} \quad \text{where} \quad M^2 = M_y^2 + M_z^2$$

$$\tau_m = \frac{Tc}{J}$$

- Maximum shearing stress:

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + (\tau_m)^2} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

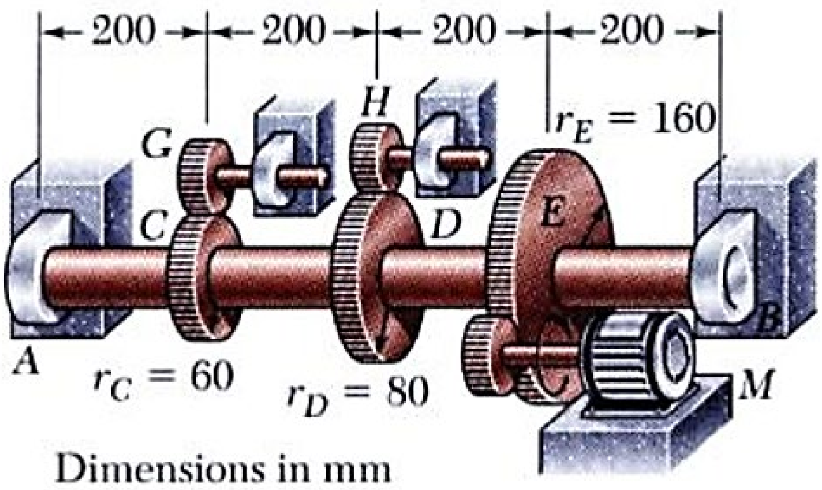
for a circular or annular cross - section, $2I = J$

$$\tau_{\max} = \frac{c}{J}\sqrt{M^2 + T^2}$$

- Shaft section requirement:

$$\left(\frac{J}{c}\right)_{\min} = \frac{\left(\sqrt{M^2 + T^2}\right)_{\max}}{\tau_{all}}$$

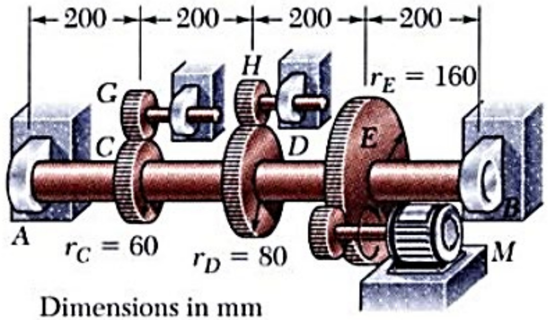
Example 5

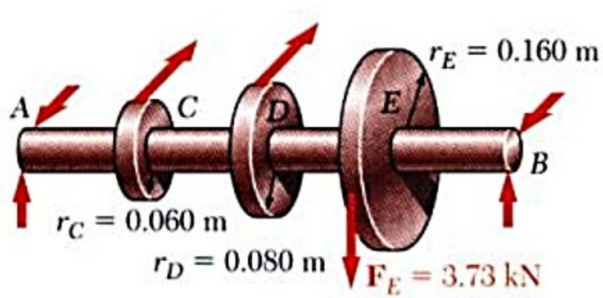


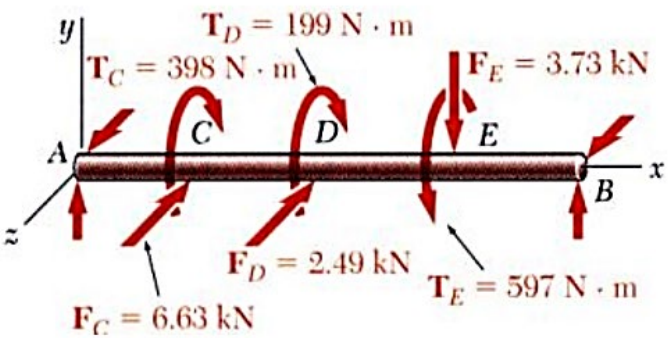
Solid shaft rotates at 480 rpm and transmits 30 kW from the motor to gears G and H; 20 kW is taken off at gear G and 10 kW at gear H. Knowing that s_{all} = 50 MPa, determine the smallest permissible diameter for the shaft.

- Determine the gear torques and corresponding tangential forces.
- Find reactions at A and B.
- Identify critical shaft section from torque and bending moment diagrams.
- Calculate minimum allowable shaft diameter

Solution







Determine the gear torques and corresponding tangential forces.

$$T_E = \frac{P}{2\pi f} = \frac{30\,\text{kW}}{2\pi(80\,\text{Hz})} = 597\,\text{N}\cdot\text{m}$$

$$F_E = \frac{T_E}{r_E} = \frac{597\,\text{N}\cdot\text{m}}{0.16\,\text{m}} = 3.73\,\text{kN}$$

$$T_C = \frac{20\,\text{kW}}{2\pi(80\,\text{Hz})} = 398\,\text{N}\cdot\text{m} \qquad F_C = 6.63\,\text{kN}$$

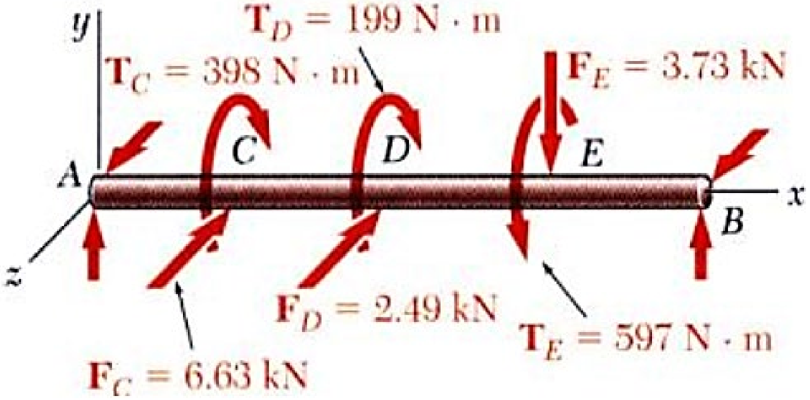
$$T_D = \frac{10\,\text{kW}}{2\pi(80\,\text{Hz})} = 199\,\text{N}\cdot\text{m} \qquad F_D = 2.49\,\text{kN}$$

Find reactions at *A* and *B*

$$A_y = 0.932\,\text{kN} \qquad A_z = 6.22\,\text{kN}$$

$$B_y = 2.80\,\text{kN} \qquad B_z = 2.90\,\text{kN}$$

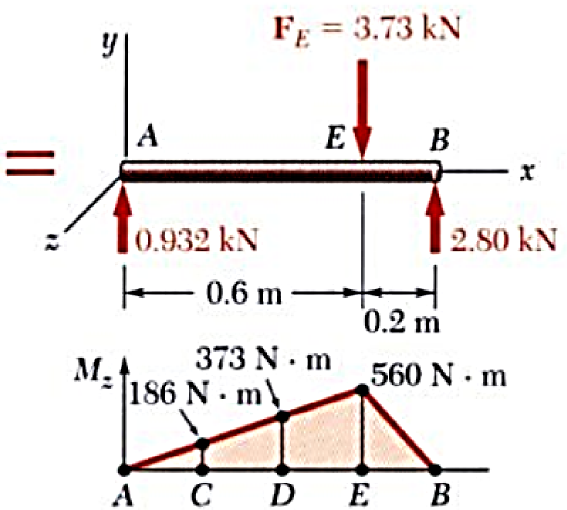
Solution

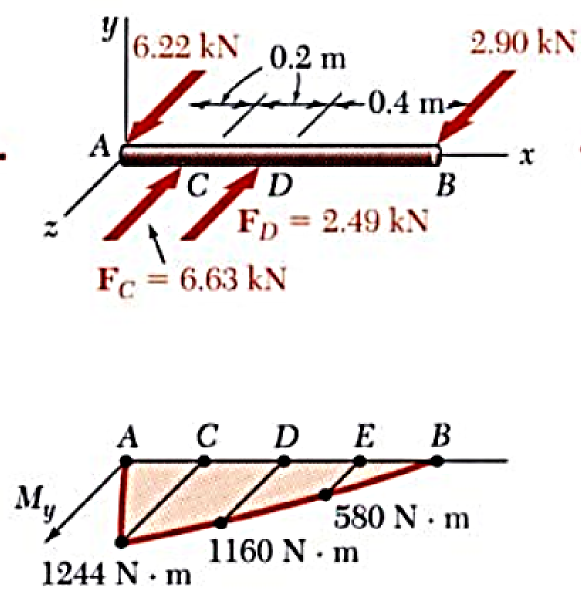


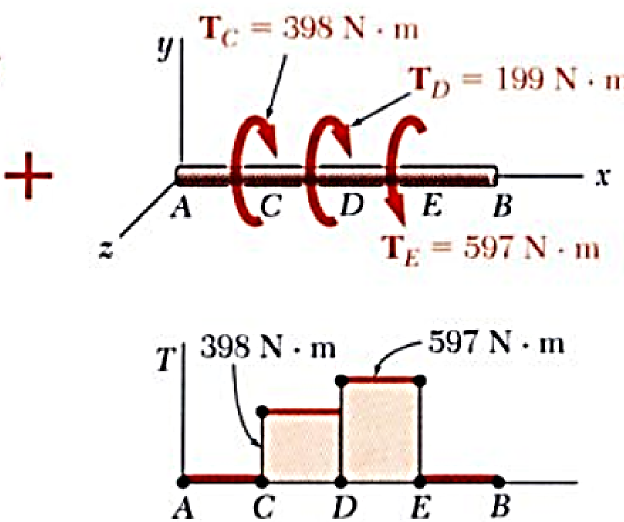
- Identify critical shaft section from torque and bending moment diagrams.

$$\left(\sqrt{M^2 + T^2}\right)_{max} = \sqrt{\left(1160^2 + 373^2\right) + 597^2}$$

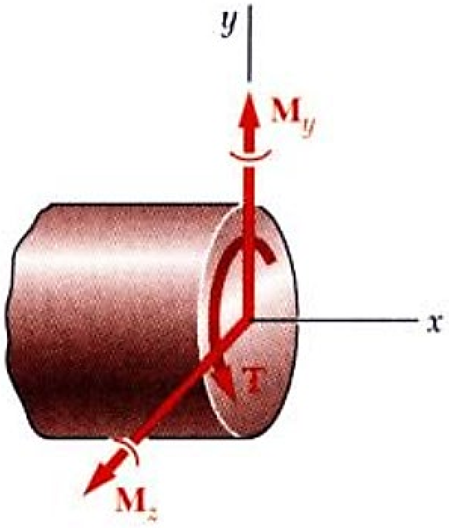
$$= 1357\,\text{N}\cdot\text{m}$$







Solution



- Calculate minimum allowable shaft diameter

$$\frac{J}{c} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{1357\ \mathrm{N \cdot m}}{50\,\mathrm{MPa}} = 27.14 \times 10^{-6}\,\mathrm{m}^3$$

For a solid circular shaft,

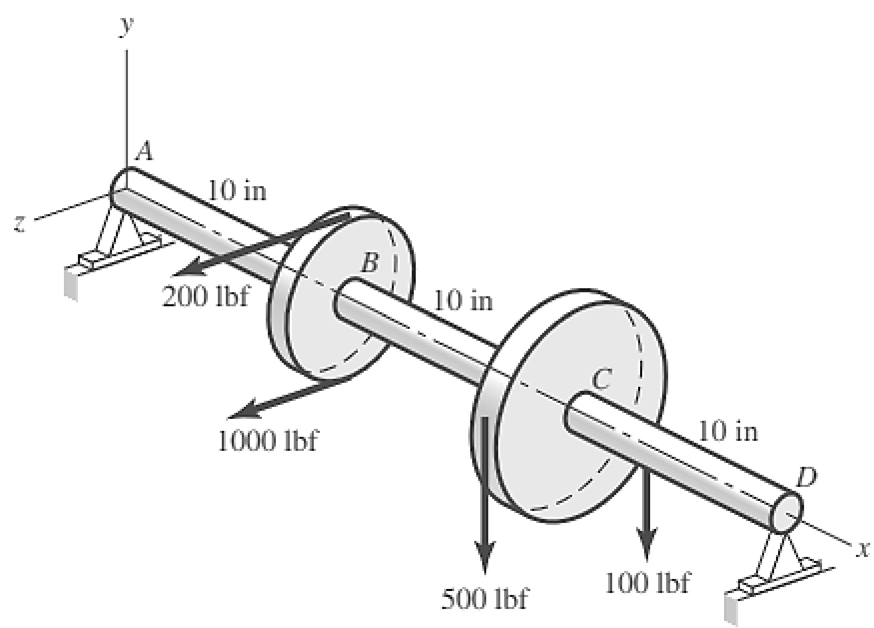
$$\frac{J}{c} = \frac{\pi}{2} c^3 = 27.14 \times 10^{-6}\,\mathrm{m}^3$$

$$c = 0.02585\,\mathrm{m} = 25.85\,\mathrm{m}$$

$$\boxed{d = 2c = 51.7\ \mathrm{mm}}$$

Sample Problem!

The 1.5-in-diameter solid steel shaft shown in the figure below is simply supported at the ends. Two pulleys are keyed to the shaft where pulley B is of diameter 4.0 in and pulley C is of diameter 8.0 in. Considering bending and torsional stresses only, determine the locations and magnitudes of the greatest tensile, compressive, and shear stresses in the shaft.



End of today's lecture!