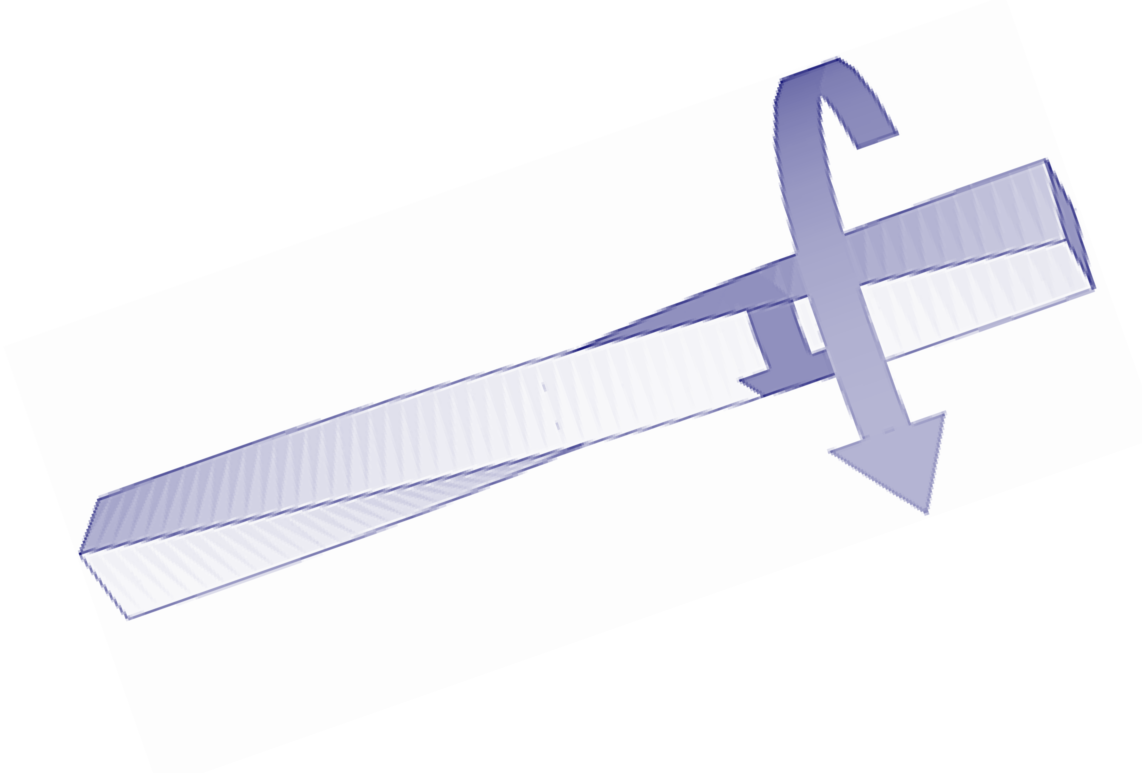
Product Development 1

Introduction to **Mechanical Engineering Design**

Spring 2023

Torsion

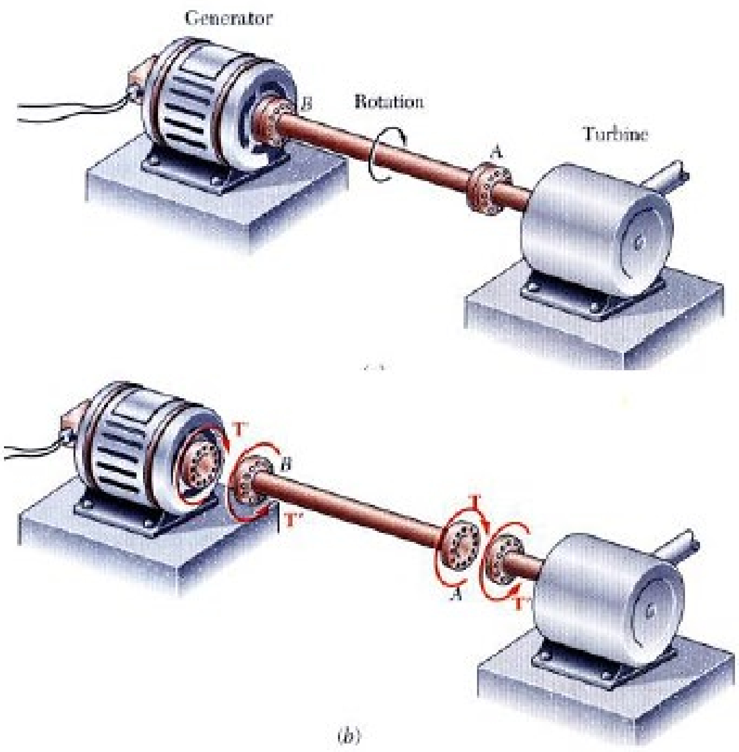


Specific Instructional Objectives

After completing this lesson one will be able to:

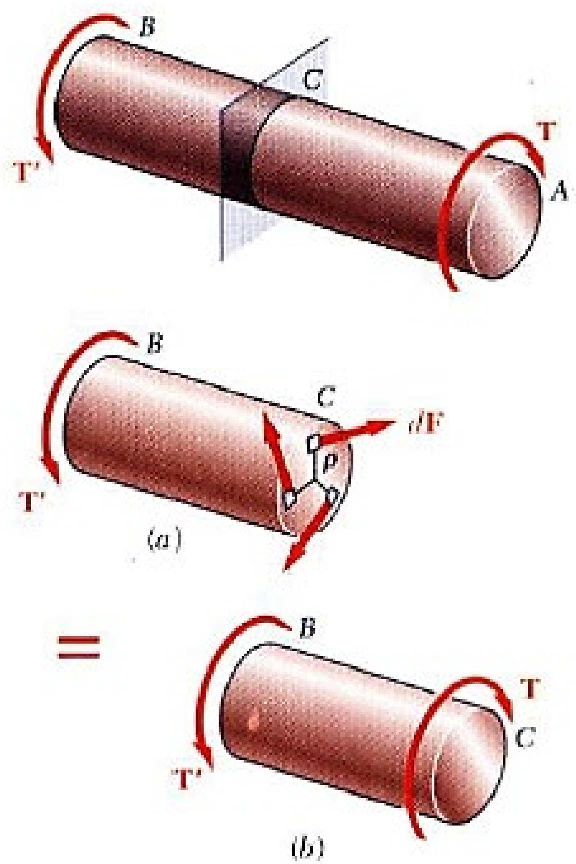
- ✓ Understand the concept of twisting moment and its effect on bars of circular cross section
- ✓ Understand the concept of shear stress du to torsion
- ✓ Evaluate stresses and deformation in Solid or Hollow circular bars due to torsion

Torsional Loads on Circular Shafts



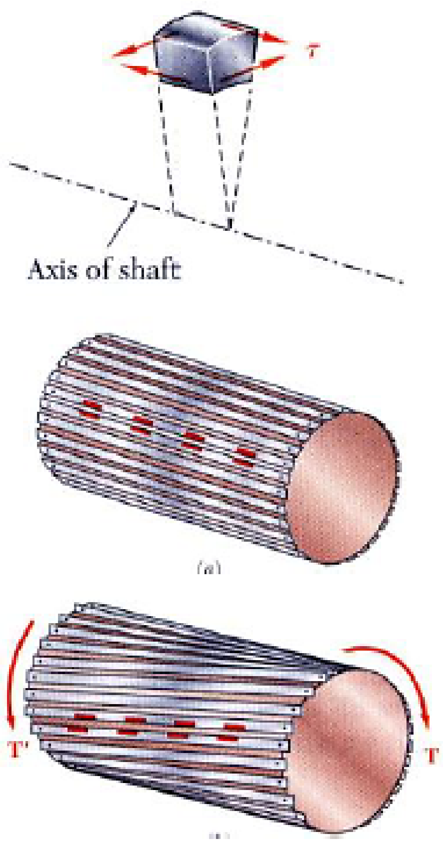
- ✓ Interested in stresses and strains of circular shafts subjected to twisting couples or *torques*
- ✓ Turbine exerts torque *T* on the shaft
- ✓ Shaft transmits the torque to the generator
- ✓ Generator creates an equal and opposite torque *T*'

Net Torque Due to Internal Stresses



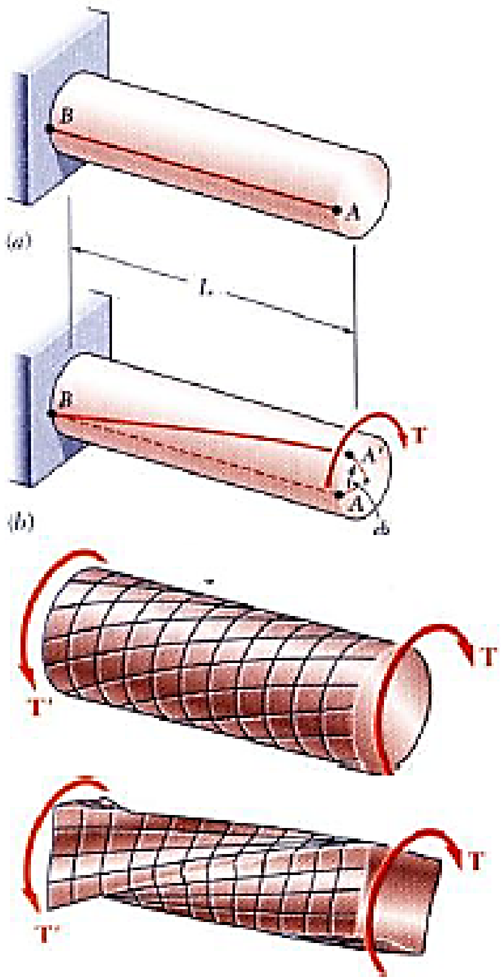
- ✓ Net of the internal shearing stresses is an internal torque, equal and opposite to the applied torque.
- ✓ Although the net torque due to the shearing stresses is known, the distribution of the stresses is not.
- ✓ Distribution of shearing stresses is statically indeterminate – must consider shaft deformations.
- ✓ Unlike the normal stress due to axial loads, the distribution of shearing stresses due to torsional loads can not be assumed uniform.

Axial Shear Components



- ✓ Torque applied to shaft produces shearing stresses on the faces perpendicular to the axis.
- ✓ Conditions of equilibrium require the existence of equal stresses on the faces of the two planes containing the axis of the shaft.
- ✓ The existence of the axial shear components is demonstrated by considering a shaft made up of axial slats.
- ✓ The slats slide with respect to each other when equal and opposite torques are applied to the ends of the shaft.

Shaft Deformations



- From observation, the angle of twist of the shaft is proportional to the applied torque and to the shaft length.

$$\phi \propto T$$

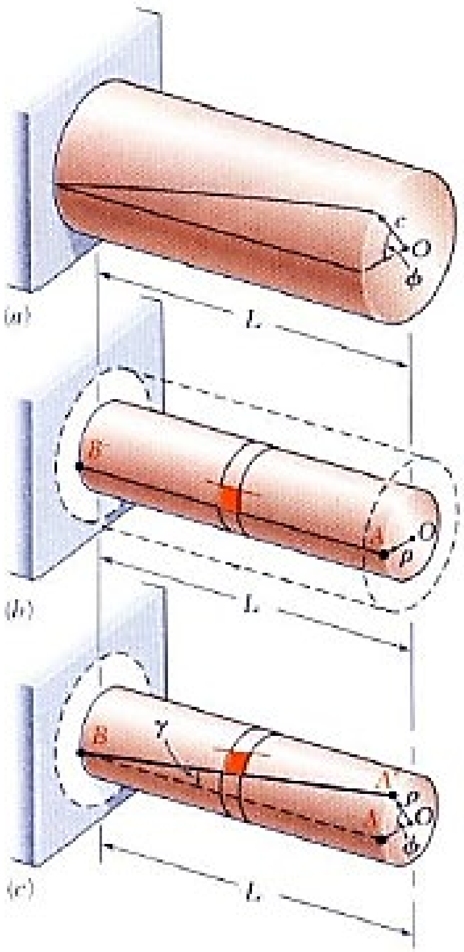
$$\phi \propto L$$

- When subjected to torsion, every cross-section of a circular shaft remains plane and undistorted.

- Cross-sections for hollow and solid circular shafts remain plain and undistorted because a circular shaft is axisymmetric.

- Cross-sections of noncircular (non-axisymmetric) shafts are distorted when subjected to torsion.

Shearing Strain



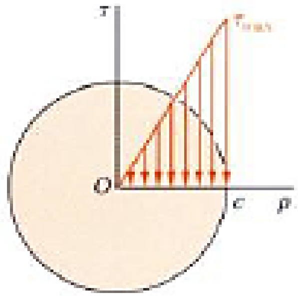
- Consider an interior section of the shaft. As a torsional load is applied, an element on the interior cylinder deforms into a rhombus.
- Since the ends of the element remain planar, the shear strain is equal to angle of twist.
- It follows that

$$L\gamma = \rho\phi \quad \text{or} \quad \gamma = \frac{\rho\phi}{L}$$

- Shear strain is proportional to twist and radius

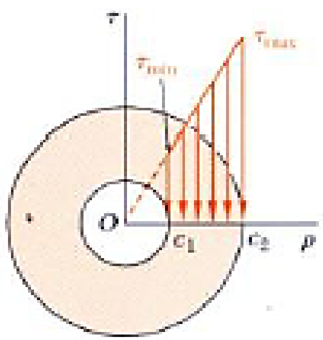
$$\gamma_{\max} = \frac{c\phi}{L} \quad \text{and} \quad \gamma = \frac{\rho}{c}\gamma_{\max}$$

Stresses in Elastic Range



Solid Shaft

$$J = \frac{1}{2}\pi c^4$$



Tublar Shaft

$$J = \frac{1}{2}\pi\left(c_2^4 - c_1^4\right)$$

- Multiplying the previous equation by the shear modulus,

$$G_\gamma = \frac{\rho}{c} G\gamma_{max}$$

- From Hooke's Law, $\tau = G_\gamma$, so

$$\tau = \frac{\rho}{c}\tau_{max}$$

- The shearing stress varies linearly with the radial position in the section.
- Recall that the sum of the moments from the internal stress distribution is equal to the torque on the shaft at the section,

$$T = \int \rho r \, dA = \frac{\tau_{max}}{c}\int \rho^2 \, dA = \frac{\tau_{max}}{c} J$$

The results are known as the elastic torsion formulas,

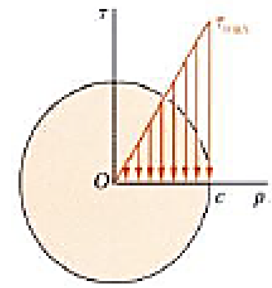
$$\tau_{max} = \frac{T_c}{J} \quad and \quad \tau = \frac{T\rho}{J}$$

Stresses in Elastic Range

Combining the two equations, the maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

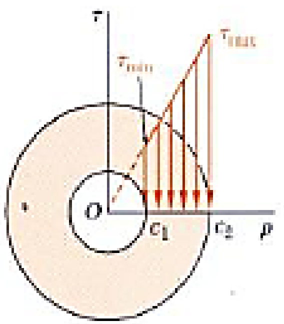
$$\frac{\tau_{\max}}{c} = \frac{T}{J} = \frac{G\phi}{L}$$

τmax = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,
c = Radius of the shaft,
T = Torque or twisting moment,
J = Second moment of area of the section about its polar axis or polar moment of inertia,
G = Modulus of rigidity for the shaft material,
L = Length of the shaft, and
φ = Angle of twist in radians on a length *L*.



Solid Shaf

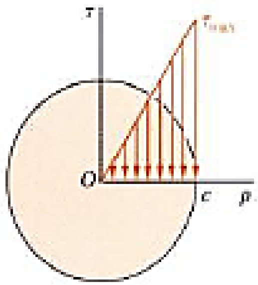
$$J = \frac{1}{2}\pi c^4$$



Tublar Shaf

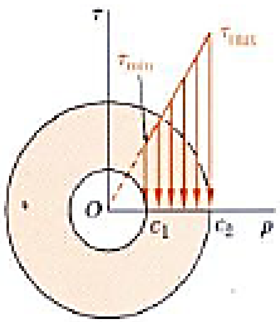
$$J = \frac{1}{2}\pi\left(c_2^4 - c_1^4\right)$$

Stresses in Elastic Range



Solid Shaf

$$J = \tfrac{1}{2}\pi c^4$$



Tublar Shaf

$$J = \tfrac{1}{2}\pi\left(c_2^4 - c_1^4\right)$$

✓From equation $\frac{\tau_{max}}{c} = \frac{T}{J} = \frac{G\phi}{L}$ we know that

$$\frac{\tau_{max}}{c} = \frac{T}{J} \text{ or } T = \tau_{max}\frac{J}{c}$$

✓For a solid shaft of diameter (*d*), *the polar moment of inertia,*

$$J = I_{xx} + I_{yy} = \frac{\pi}{64}\times d^4 + \frac{\pi}{64}\times d^4 = \frac{\pi}{32}\times d^4$$

$$\text{since } T = \tau_{max}\frac{J}{c}$$

$$T = \tau_{max}\frac{\pi}{32}\times d^4 \times \frac{2}{d} = \frac{\pi}{16}\times \tau_{max} d^3$$

✓In case of a hollow shaft with external diameter (do) and internal diameter (di), the polar moment of inertia,

$$J = \frac{\pi}{32}\left[\left(d_0^4\right) - \left(d_i^4\right)\right] \text{ and } c = \frac{d_o}{2}$$

$$\tau = \frac{T}{J}r = \frac{T}{(\pi(d_0^4 - d_i^4)/32)}\frac{d_o}{2} = \frac{16T}{\pi(d_0^4 - d_i^4)}\times d_o$$

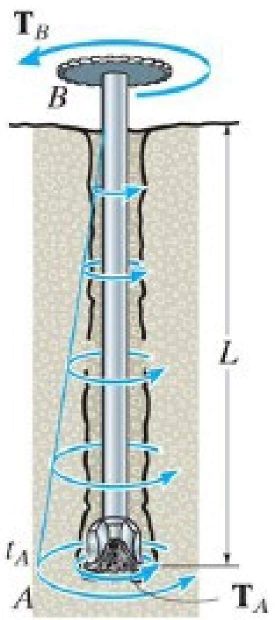
$$T = \frac{\pi}{16}\times\tau\left[\frac{\left(d_0^4\right)-\left(d_i^4\right)}{d_o}\right]$$

Angle of Twist

Occasionally the design of a shaft depends on restricting the amount of rotation or twist that may occur when the shaft is subjected to a torque. Furthermore, being able to compute the angle of twist for a shaft is important when analyzing the reactions of bearings in shaft design.

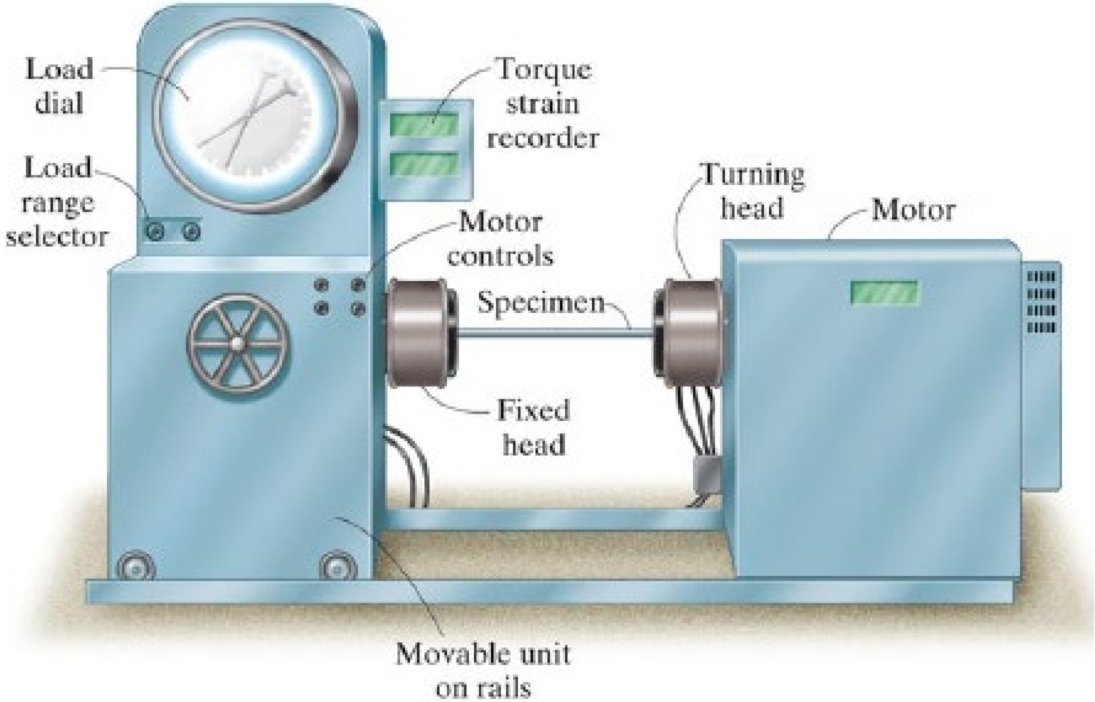
Oil wells are commonly drilled to depths exceeding a thousand meters. As a result, the total angle of twist of a string of drill pipe can be substantial and must be computed.

Angle of Twist:
An example



When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A.

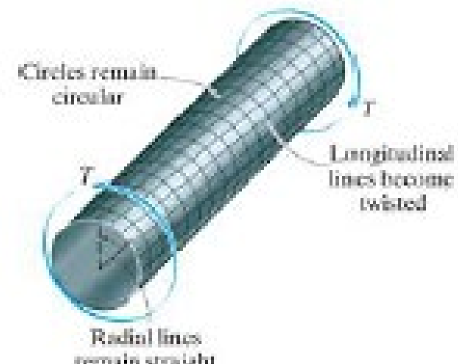
Angle of Twist: Torsion test



We can use the equation to determine the shear modulus of elasticity G of the material. To do so, a specimen of known length and diameter is placed in a torsion testing machine. The applied torque T and angle of twist φ are then measured between a gauge length L and calculate G using $\boldsymbol{G = TL/J\varphi}$.

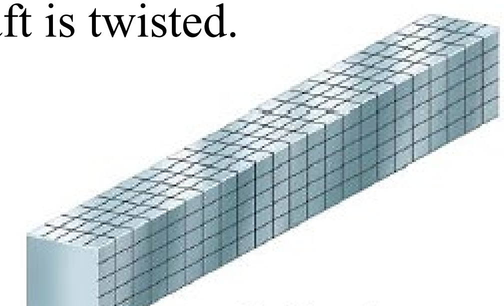
Solid Noncircular Shafts

When a torque is applied to a shaft having a circular cross section, one that is axisymmetric, the shear strains vary linearly from zero at its center to a maximum at its outer surface. Furthermore, due to the uniformity of the shear strain at all points on the same radius, the cross section does not deform, but rather remains plane after the shaft has twisted.

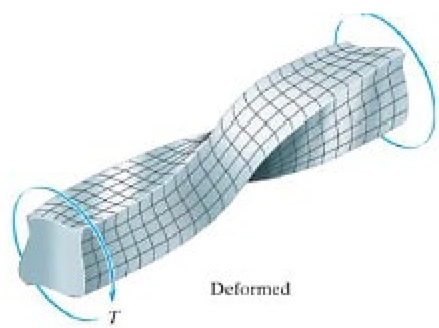


After deformation
(b)

Shafts that have a noncircular cross section, however, are not axisymmetric, and because the shear stress over their cross section is distributed in a very complex manner, their cross sections will bulge or warp when the shaft is twisted.



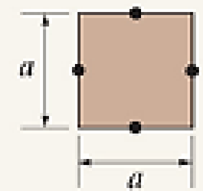
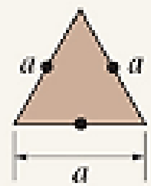
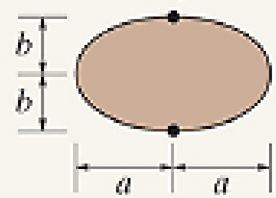
Undeformed



Deformed

Solid Noncircular Shafts

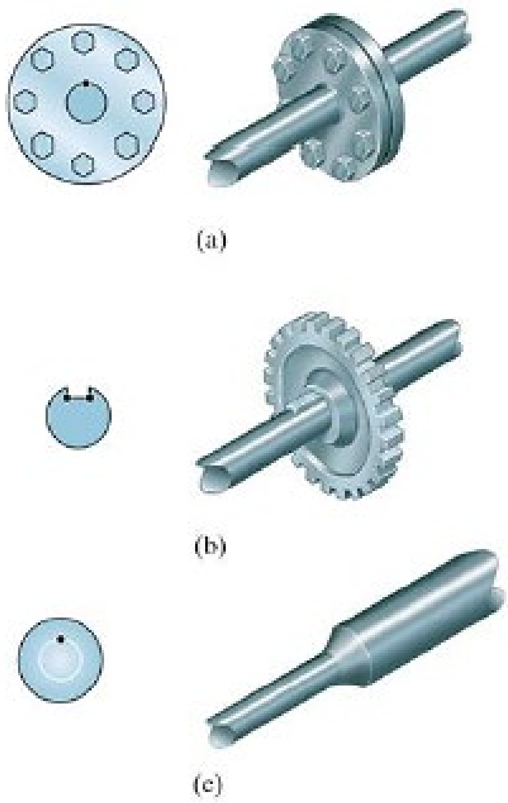
Using a mathematical analysis based on the theory of elasticity, however, it is possible to determine the shear-stress distribution within a shaft of some uniform cross sections. The results of the analysis for square cross sections, along with other results from the theory of elasticity, for shafts having triangular and elliptical cross sections, are reported in the table here.

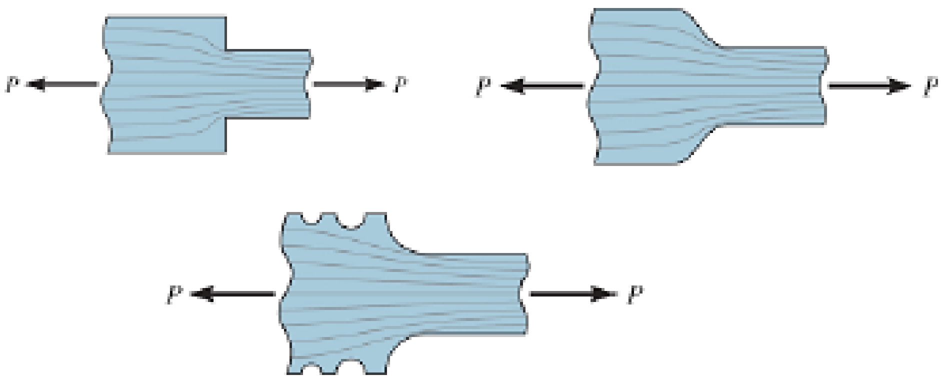
Shape of cross section	τ_{max}	ϕ
Square 	$\dfrac{4.81\,T}{a^3}$	$\dfrac{7.10\,T}{a^4G}$
Equilateral triangle 	$\dfrac{20\,T}{a^3}$	$\dfrac{46\,TL}{a^4G}$
Ellipse 	$\dfrac{2\,T}{\pi ab^2}$	$\dfrac{(a^2+b^2)TL}{\pi a^3b^3G}$

Stress Concentration

The torsion formula, $\tau_{max} = Tc/J$, can be applied to regions of a shaft having a circular cross section that is constant or tapers slightly. When sudden changes arise in the cross section, both the shear-stress and shear-strain distributions in the shaft become complex.

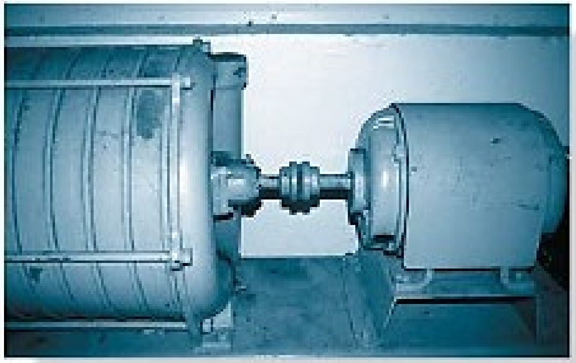
In order to eliminate the necessity for the engineer to perform a complex stress analysis at a shaft discontinuity, the maximum shear stress can be determined for a specified geometry using a torsional stress-concentration factor, ***K***.

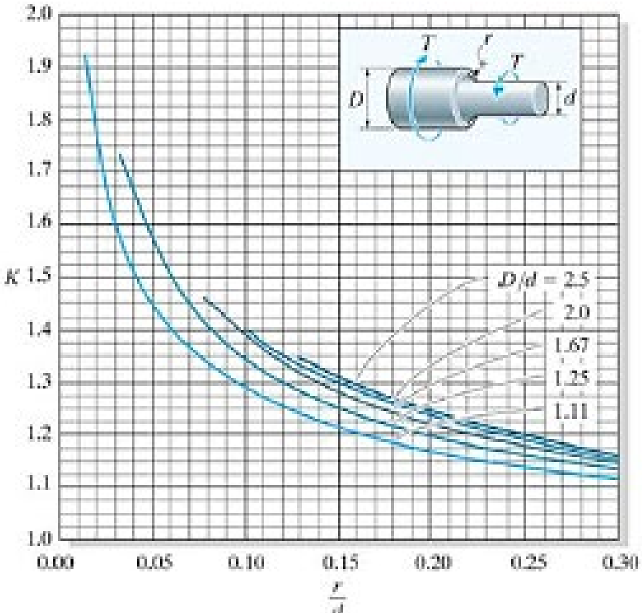




Stress Concentration

Stress-concentration factor *K* is usually taken from a graph. To use this graph, one first finds the geometric ratio D/d to define the appropriate curve, and then once the abscissa r/d is calculated, the value of *K* is found along the ordinate. The maximum shear stress is then determined from the equation.

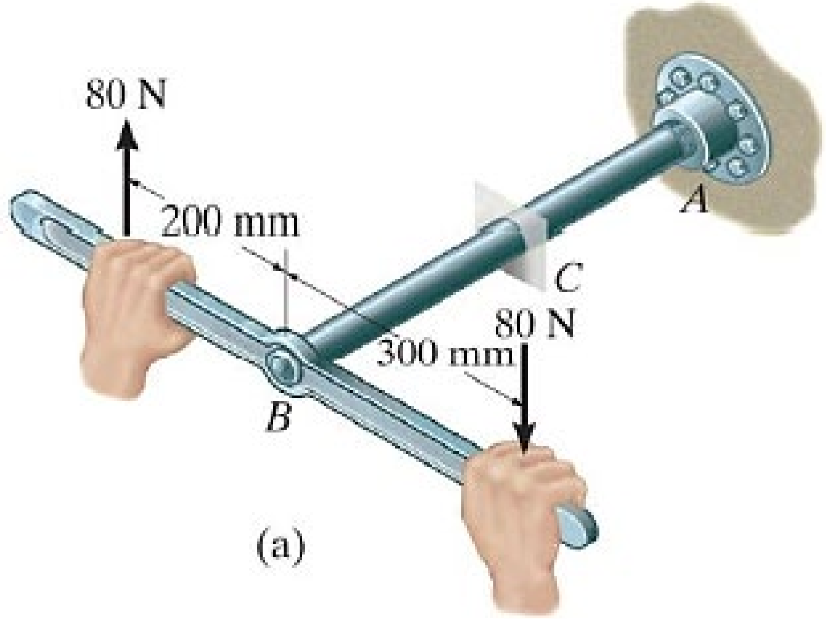




It can be noted from the graph that an increase in fillet radius *r* causes a decrease in *K*. Hence the maximum shear stress in the shaft can be reduced by increasing the fillet radius. Also, if the diameter of the larger shaft is reduced, the *D/d* ratio will be lower and so the value of *K* and therefore τ_{max} will be lower.

Example 1

The pipe shown in the figure has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at A using a torque wrench at B, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



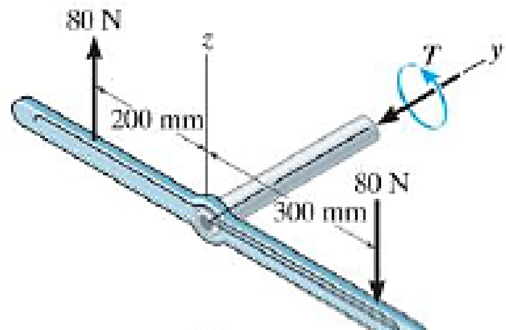
(a)

Solution

Internal Torque. A section is taken at an intermediate location C along the pipe's axis. Fig. 5-13b. The only unknown at the section is the internal torque T. Force equilibrium and moment equilibrium about the x and z axes are satisfied. We require:

$$\sum M_y = 0 ; \quad 80\text{ N }(0.3\text{ m}) + 80\text{ N }(0.2\text{ m}) - T = 0$$

$$T = 40\text{ N. m}$$



(b)

Section property. The polar moment of inertia for the pipe's cross sectional area is

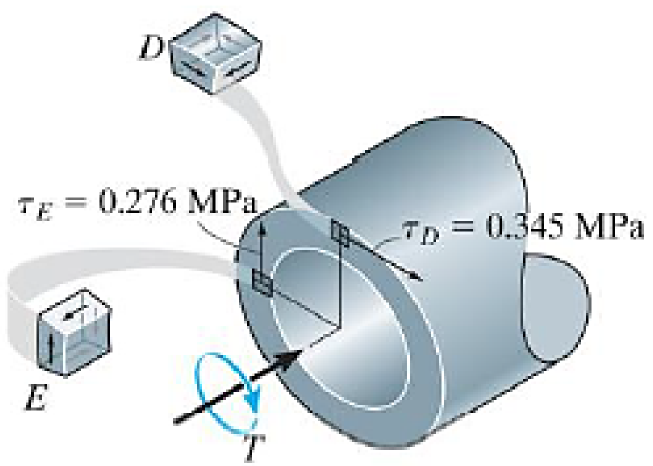
$$J = \frac{\pi}{2}[(0.05\,m)^4 - (0.04\,m)^4] = 5.80(10^{-6})m^4$$

Shear Stress. For any point lying on the outside surface of the pipe. $\rho = C_O = 0.05\,m$, we have

$$\tau_o = \frac{TC_O}{J} = \frac{40\,N.m\,(0.05\,m)}{5.80\,(10^{-6})m^4} = 0.345\,MPa$$

And for any point located on the inside surface, $\rho = C_i = 0.04\,m$, so that

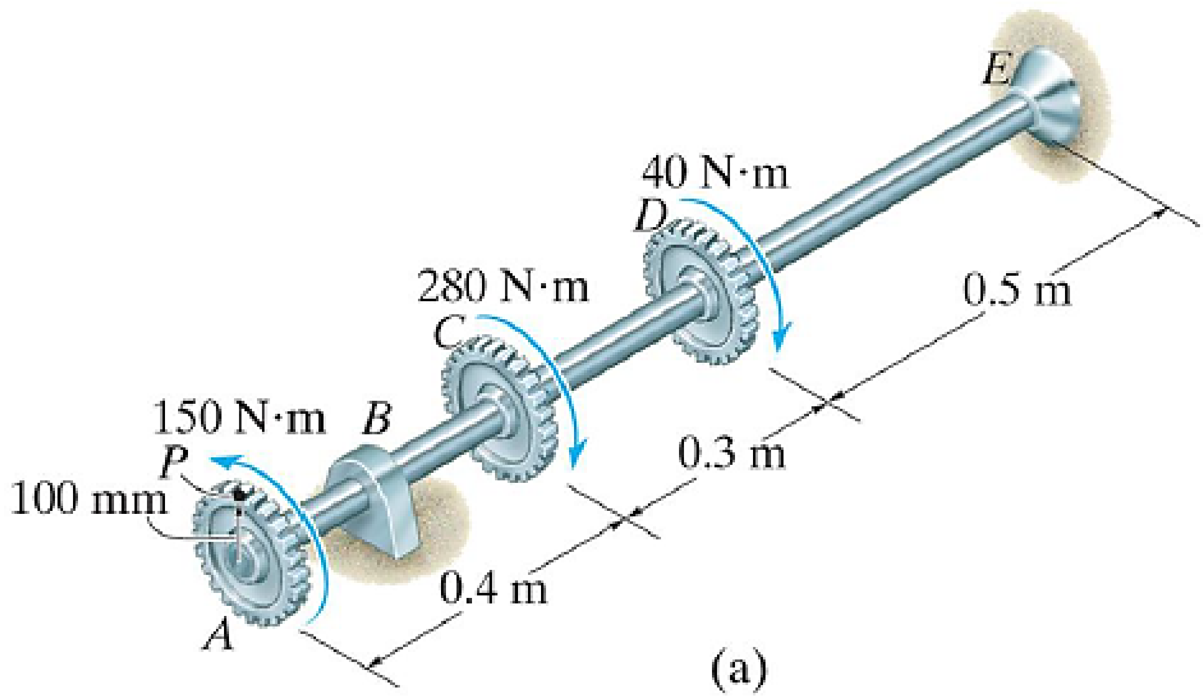
$$\tau_i = \frac{TC_i}{J} = \frac{40\,N.m\,(0.04\,m)}{5.80\,(10^{-6})m^4} = 0.276\,MPa$$



(c)

Example 2

The gears attached to the fixed-end steel shaft are subjected to the torques shown in figure below. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth P on gear A. The shaft turns freely within the bearing at B.



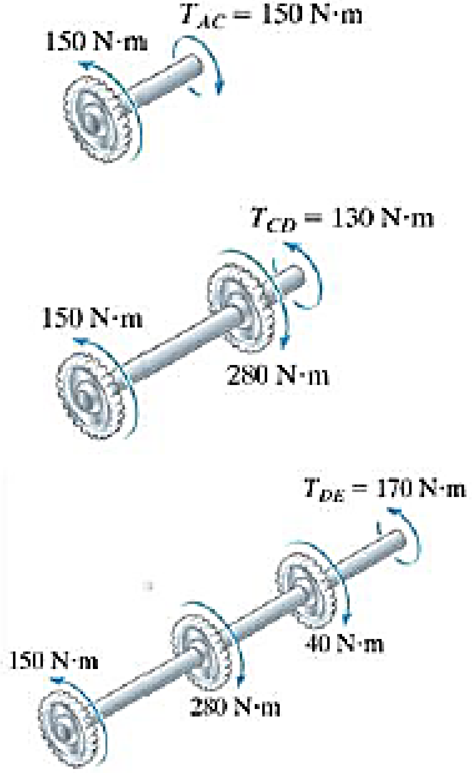
(a)

Solution

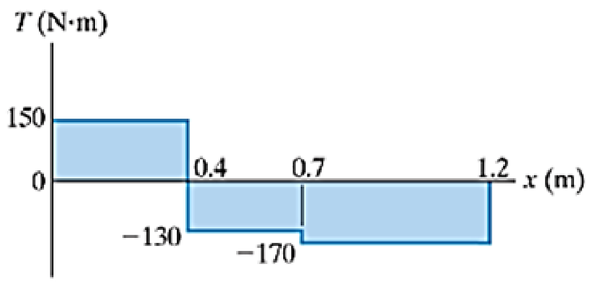
Internal Torque. By inspection is segments AC, CD, and DE are different yet constant throughout each segment. Free body diagrams of appropriate segments of the shaft along with the calculated internal torques are shown in Fig 5-20b. Using the right hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

$$T_{AC} = +150\ N.m \qquad T_{CD} = -130\ N.m \qquad T_{DE} = -170\ N.m$$

These results are also shown on the torque diagram, Fig 5-20c.



(b)



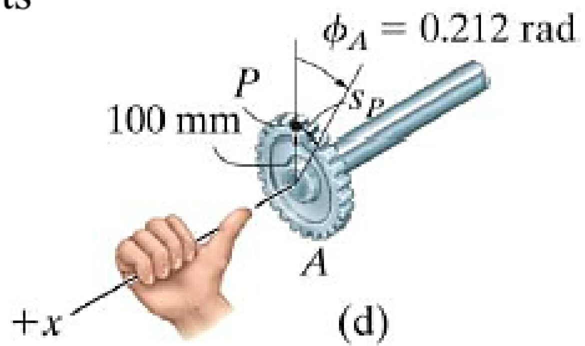
(c)

Solution

Angle of Twist. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.007\ m)^4 = 3.77(10^{-9})m^4$$

Applying Eq. 5-16 to each segment and adding the results algebraically, we have



(d)

$$\emptyset_A = \sum \frac{TL}{JG} = \frac{(+150\ N.m)(0.4\ m)}{3.77(10^{-9})m^4[80(10^9)]N/m^2} + \frac{(-130N.m)(0.3\ m)}{3.77(10^{-9})m4[80\ (10^9)]N/m^2} + \frac{(-170N.m)(0.5\ m)}{3.77(10^{-9})m4[80\ (10^9)]N/m^2} = \text{-0.212 rad}$$

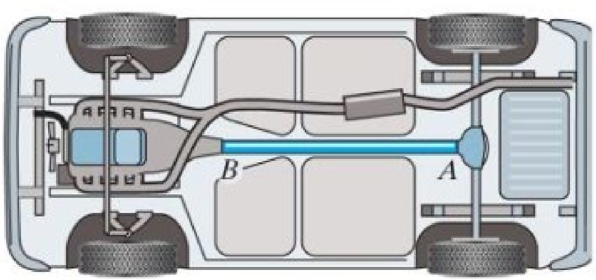
Since the answer is negative, by the right-hand rule the thumb is directed toward the end E of the shaft, and therefore gear A ill rotate as shown in Fig. 5-20d.

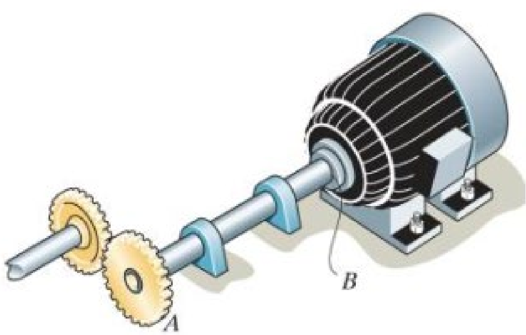
The displacement of tooth P on gear A is:

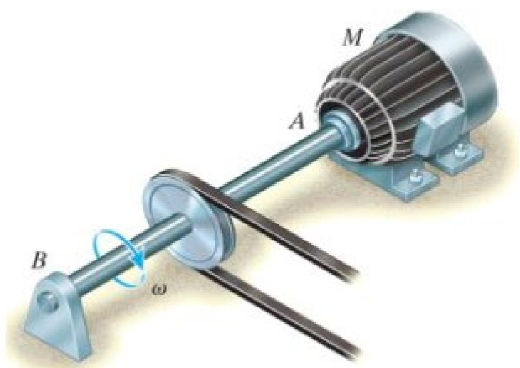
$$S_P = \emptyset_A r = (0.212\ rad)(100mm) = 21.2mm$$

Power transmission

Shafts and *tubes* having circular cross sections are often used to transmit power developed by a machine. When used for this purpose, they are subjected to torques that depend on the power generated by the machine and the angular speed of the shaft.







Power transmission

The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the equations below are used. The power transmitted by the shaft (in watts) is given by:

$$P = \frac{2\pi N . T}{60} = T.\omega \qquad \left(\because \omega = \frac{2\pi N}{60}\right)$$

T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Power is expressed in watts when torque is measured in newton-meters (N.m) and angular speed is in radians/sec.

Example. *A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.*

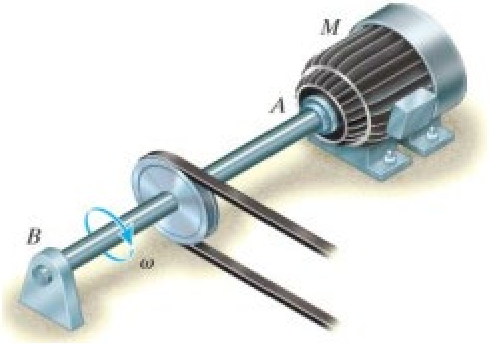
$$100 \times 10^3 = \frac{2\pi N . T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\, T_{mean}$$

$$T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm}$$



Design of Transmission Shafts

- ✓ Principal transmission shaft performance specifications are: power & speed.
- ✓ Designer must select shaft material and cross-section to meet performance specifications without exceeding allowable shearing stress.

- Determine torque applied to shaft at specified power and speed,

$$P = T\omega = 2\pi f T$$

$$T = \frac{P}{\omega} = \frac{P}{2\pi f}$$

- Find shaft cross-section which will not exceed the maximum allowable shearing stress,

$$\tau_{\max} = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{T}{\tau_{\max}} \quad \text{(solid shafts)}$$

$$\frac{J}{c_2} = \frac{\pi}{2c_2}\left(c_2^4 - c_1^4\right) = \frac{T}{\tau_{\max}} \quad \text{(hollow shafts)}$$

Example 3

A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 meters. Take C=80 GPa.

Solution: We know that the power transmitted by the shaft (P):

$$97.5 \times 10^3 = \frac{2\,\pi\,N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\,T$$

$$T = 97.5 \times 10^3/18.852 = 5172 \text{ N-m} = 5172 \times 10^3 \text{ N-mm}$$

Now let us find the diameter of the shaft based on the strength and stiffness.

Considering strength of the shaft

We know that the torque transmitted (T):

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

$$d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3 \quad \text{or} \quad d = 76 \text{ mm}$$

$$\frac{T}{J} = \frac{\tau}{r}$$

Considering stiffness of the shaft

Polar moment of inertia of the shaft:

$$J = \frac{\pi}{32} \times d^4 = 0.0982\,d^4$$

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{5172 \times 10^3}{0.0982\,d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \quad \text{or} \quad \frac{52.7 \times 10^6}{d^4} = 0.464$$

$$d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6 \quad \text{or} \quad d = 103 \text{ mm}$$

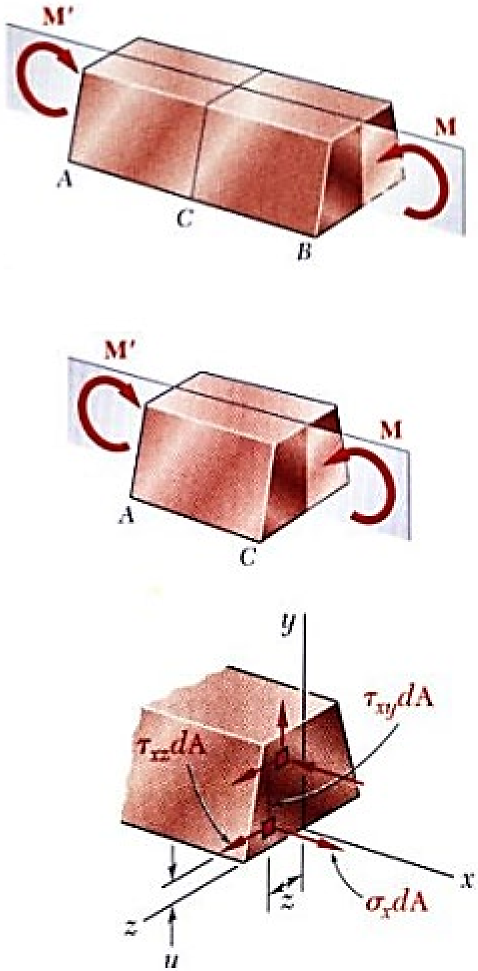
Taking larger of the two values, we shall provide $d = 103$ say 105 mm.

Sample problem!

A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 meters not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.

Bending

Symmetric Member in Pure Bending



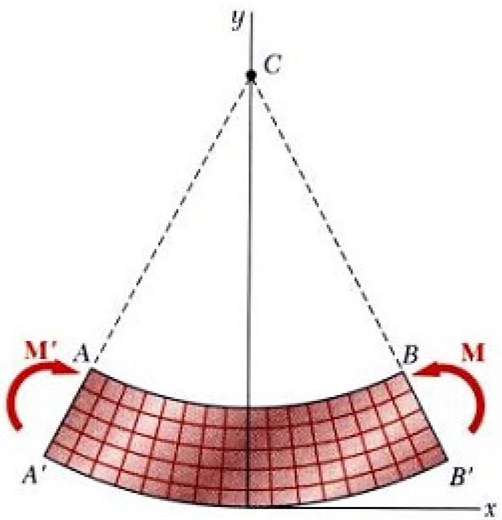
- ✓ Internal forces in any cross section are equivalent to a couple. The moment of the couple is the section **bending moment.**
- ✓ From statics, a couple M consists of two equal and opposite forces.
- ✓ The sum of the components of the forces in any direction is zero.
- ✓ The moment is the same about any axis perpendicular to the plane of the couple and zero about any axis contained in the plane.
- ✓ These requirements may be applied to the sums of the components and moments of the statically indeterminate elementary internal forces

$$F_x = \int \sigma_x \, dA = 0$$

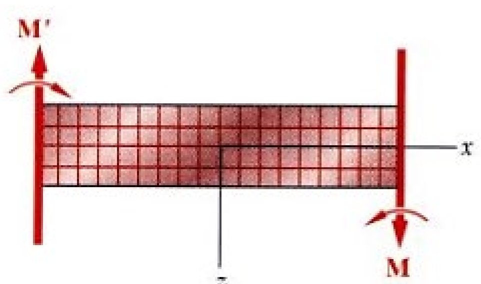
$$M_y = \int z\sigma_x \, dA = 0$$

$$M_z = \int -y\sigma_x \, dA = M$$

Bending Deformations



(a) Longitudinal, vertical section (plane of symmetry)



(b) Longitudinal, horizontal section

- ✓ beam with a plane of symmetry in pure bending:
member remains symmetric,
bends uniformly to form a circular arc, and
cross-sectional plane passes through arc center and remains planar.
- ✓ length of top decreases and length of bottom increases.
- ✓ a **neutral surface** must exist that is parallel to the upper and lower surfaces and for which the length does not change.
- ✓ stresses and strains are negative (compressive) above the neutral plane and positive (tension) below it.

Stress due to Bending

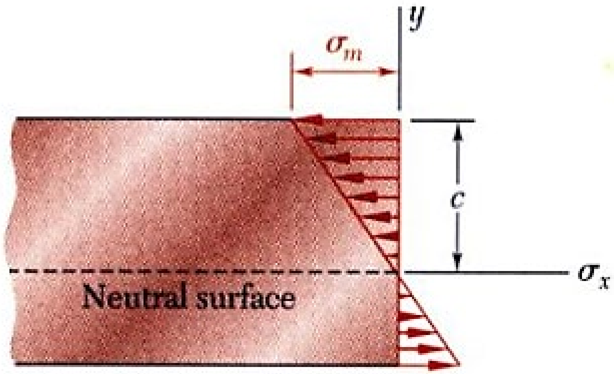
✓ For static equilibrium:

$$M = \frac{\sigma_{\max}}{c}\int y^2\, dA = \frac{\sigma_m I}{c}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M}{Z}$$

Substituting $\sigma_x = -\frac{y}{c}\sigma_{\max}$

$$\sigma_x = -\frac{My}{I}$$



✓ The maximum normal stress due to bending:

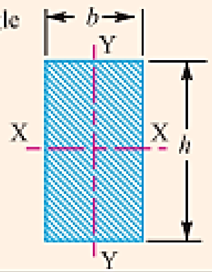
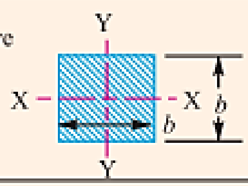
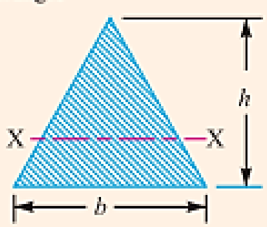
$$\sigma_m = \frac{Mc}{I} = \frac{M}{Z}$$

$I =$ section moment of inertia

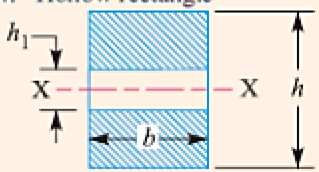
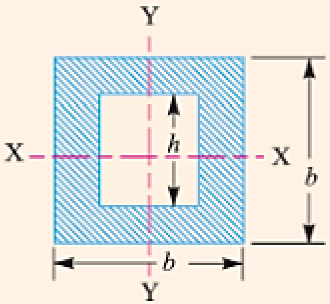
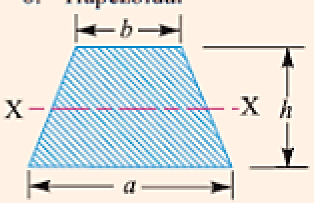
$Z = \frac{I}{c} =$ section modulus

✓ The ratio I/y is known as **section modulus** and is denoted by Z.A beam section with a larger section modulus will have a lower maximum stress.

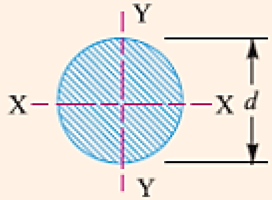
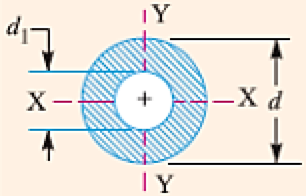
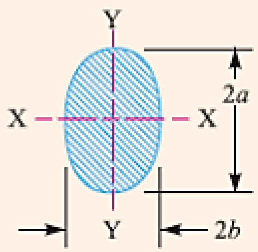
Beam Section Properties

Section	*Area* *(A)*	*Moment of inertia* *(I)*	*Distance from the neutral axis to the extreme fibre (y)	*Section modulus* $\left[Z = \frac{I}{y}\right]$	*Radius of gyration* $\left[k = \sqrt{\frac{I}{A}}\right]$
1. Rectangle 	bh	$I_{xx} = \frac{b.h^3}{12}$ $I_{yy} = \frac{h.b^3}{12}$	$\frac{h}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{b.h^2}{6}$ $Z_{yy} = \frac{h.b^2}{6}$	$k_{xx} = 0.289\ h$ $k_{yy} = 0.289\ b$
2. Square 	b^2	$I_{xx} = I_{yy} = \frac{b^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^3}{6}$	$k_{xx} = k_{yy} = 0.289\ b$
3. Triangle 	$\frac{bh}{2}$	$I_{xx} = \frac{b.h^3}{36}$	$\frac{h}{3}$	$Z_{xx} = \frac{bh^2}{12}$	$k_{xx} = 0.2358\ h$

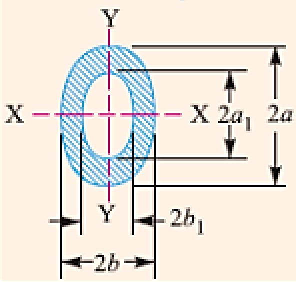
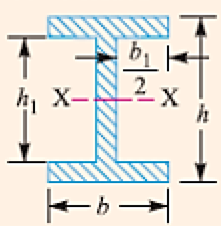
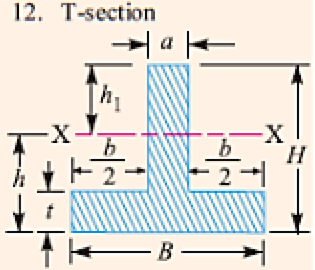
Beam Section Properties

Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
4. Hollow rectangle 	$b\,(h - h_1)$	$I_{xx} = \frac{b}{12}(h^3 - h_1^3)$	$\frac{h}{2}$	$Z_{xx} = \frac{b}{6}\left(\frac{h^3 - h_1^3}{h}\right)$	$k_{xx} = 0.289\sqrt{\frac{h^3 - h_1^3}{h - h_1}}$
5. Hollow square 	$b^2 - h^2$	$I_{xx} = I_{yy} = \frac{b^4 - h^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^4 - h^4}{6b}$	$0.289\sqrt{b^2 + h^2}$
6. Trapezoidal 	$\frac{a + b}{2} \times h$	$I_{xx} = \frac{h^2\,(a^2 + 4ab + b^2)}{36\,(a + b)}$	$\frac{a + 2b}{3\,(a + b)} \times h$	$Z_{xx} = \frac{a^2 + 4ab + b^2}{12\,(a + 2b)}$	$\frac{0.236}{a+b}\sqrt{h\,(a^2 + 4ab + b^2)}$

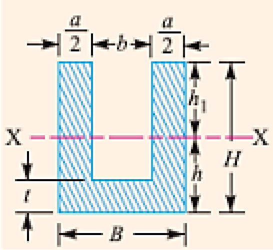
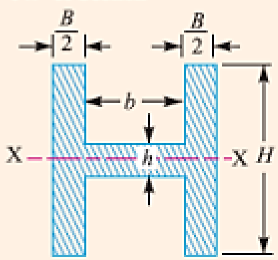
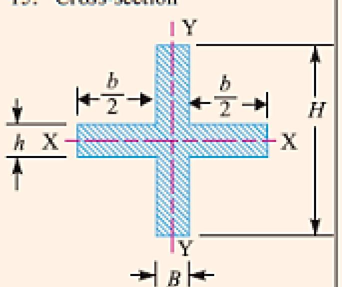
Beam Section Properties

Section	(A)	(I)	(y)	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
7. Circle 	$\frac{\pi}{4} \times d^2$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi d^3}{32}$	$k_{xx} = k_{yy} = \frac{d}{2}$
8. Hollow circle 	$\frac{\pi}{4}(d^2 - d_1^2)$	$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_1^4)$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi}{32}\left(\frac{d^4 - d_1^4}{d}\right)$	$k_{xx} = k_{yy} = \frac{\sqrt{d^2 + d_1^2}}{4}$
9. Elliptical 	πab	$I_{xx} = \frac{\pi}{4} \times a^3 b$ $I_{yy} = \frac{\pi}{4} \times ab^3$	a b	$Z_{xx} = \frac{\pi}{4} \times a^2 b$ $Z_{yy} = \frac{\pi}{4} \times ab^2$	$k_{xx} = 0.5a$ $k_{yy} = 0.5b$

Beam Section Properties

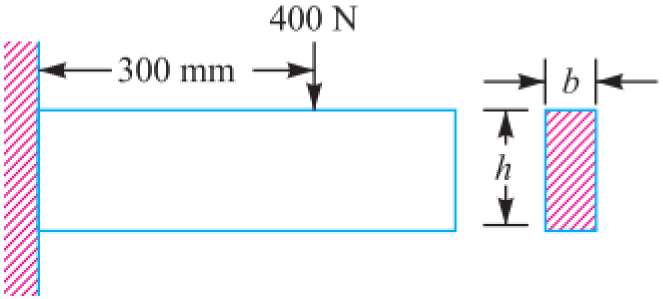
Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
10. Hollow elliptical 	$\pi(ab - a_1 b_1)$	$I_{xx} = \frac{\pi}{4}(ba^3 - b_1 a_1^3)$ $I_{yy} = \frac{\pi}{4}(ab^3 - a_1 b_1^3)$	a b	$Z_{xx} = \frac{\pi}{4a}(ba^3 - b_1 a_1^3)$ $Z_{yy} = \frac{\pi}{4b}(ab^3 - a_1 b_1^3)$	$k_{xx} = \frac{1}{2}\sqrt{\frac{ba^3 - b_1 a_1^3}{ab - a_1 b_1}}$ $k_{yy} = \frac{1}{2}\sqrt{\frac{ab^3 - a_1 b_1^3}{ab - a_1 b_1}}$
11. I-section 	$bh - b_1 h_1$	$I_{xx} = \frac{bh^3 - b_1 h_1^3}{12}$	$\frac{h}{2}$	$Z_{xx} = \frac{bh^3 - b_1 h_1^3}{6h}$	$k_{xx} = 0.289\sqrt{\frac{bh^3 - b_1 h_1^3}{bh - b_1 h_1}}$
12. T-section 	$Bt + (H - t)\,a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h = H - h_1$ $= \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H - t)a}}$

Beam Section Properties

Section	(A)	(I)	(y)	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
13. Channel Section 	$Bt + (H - t)\,a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h = H - h_1$ $= \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2\,I_{xx}(aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H - t)a}}$
14. H-Section 	$BH + bh$	$I_{xx} = \frac{BH^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$
15. Cross-section 	$BH + bh$	$I_{xx} = \frac{Bh^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$

Example 4

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam,if depth is twice that of width.



∴ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3}\ \text{mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3\ \text{N-mm}$$

We know that bending stress (σ_b),

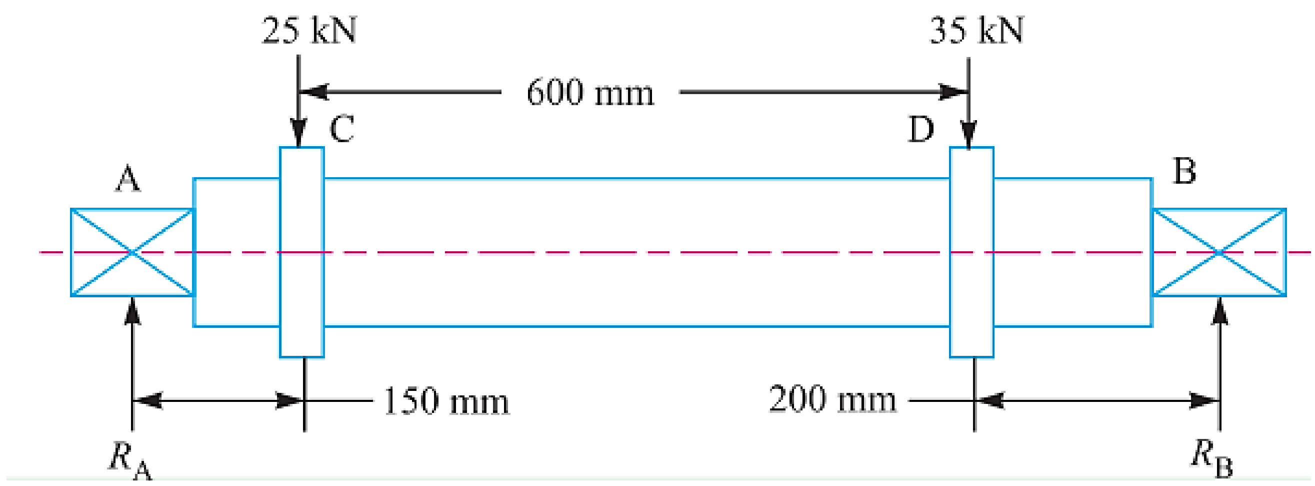
$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

$$\therefore \quad b^3 = 180 \times 10^3/40 = 4.5 \times 10^3 \quad \text{or} \quad b = 16.5\ \text{mm}$$

$$h = 2b = 2 \times 16.5 = 33\ \text{mm}$$

Example 5

A pump lever rocking shaft is shown in the figure. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.



Solution

Taking moments about A, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\ 000$$

$\therefore$ $$R_B = 30\ 000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

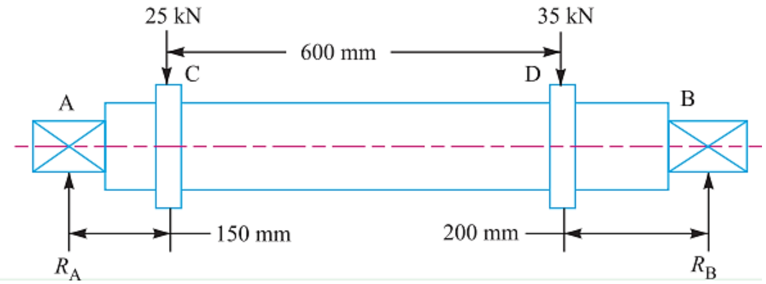
and $$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

$\therefore$ Bending moment at C

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $$D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6 \text{ N-mm}$$

We see that the maximum bending moment is at D, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.



Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3$$

$$= 0.0982\, d^3$$

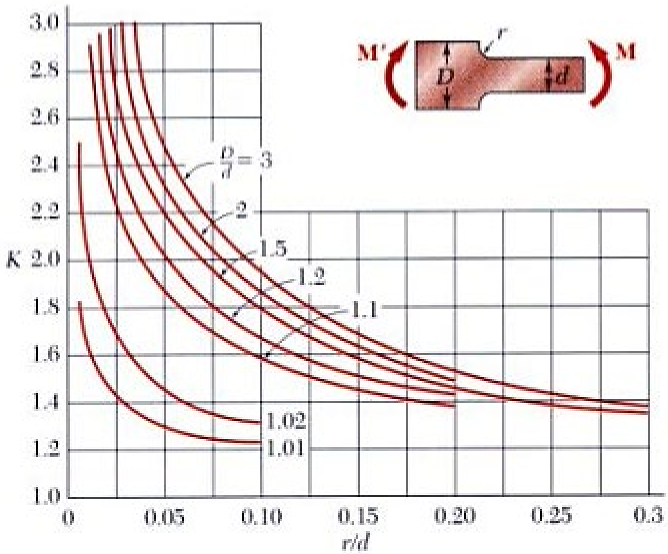
We know that bending stress (σ_b),

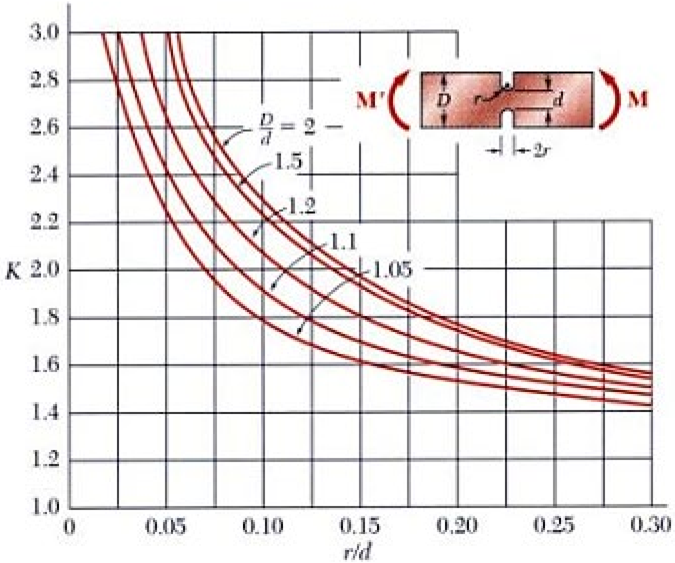
$$100 = \frac{M}{Z}$$

$$= \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$$d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3 \text{ or } d = 86.3 \text{ say } 90 \text{ mm}$$

Stress Concentrations



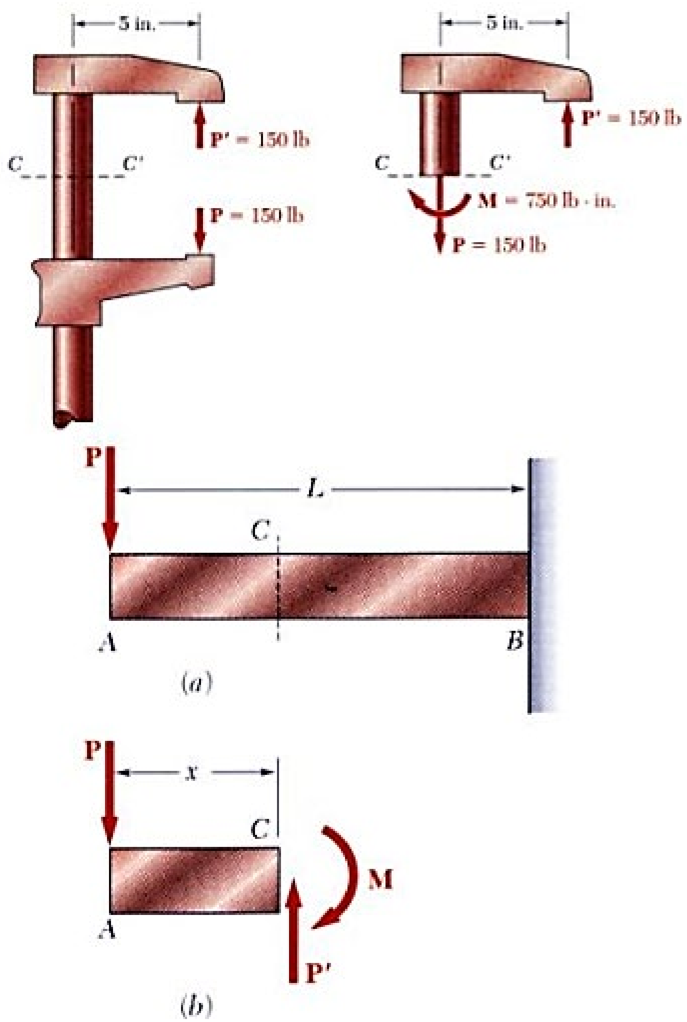


Stress concentrations (K) may occur:

- in the vicinity of points where the loads are applied.
- in the vicinity of sudden changes in cross section.

$$\sigma_m = K \frac{Mc}{I}$$

Other Loading Types

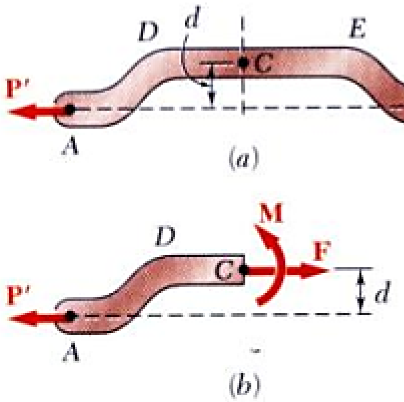


✓ Eccentric Loading: Axial loading which does not pass through section centroid produces internal forces equivalent to an axial force and a couple.

✓ Transverse Loading: Concentrated or distributed transverse load produces internal forces equivalent to a shear force and a couple.

✓ Principle of Superposition: The normal stress due to pure bending may be combined with the normal stress due to axial loading and shear stress due to shear loading to find the complete state of stress….

Eccentric Axial Loading in a Plane of Symmetry



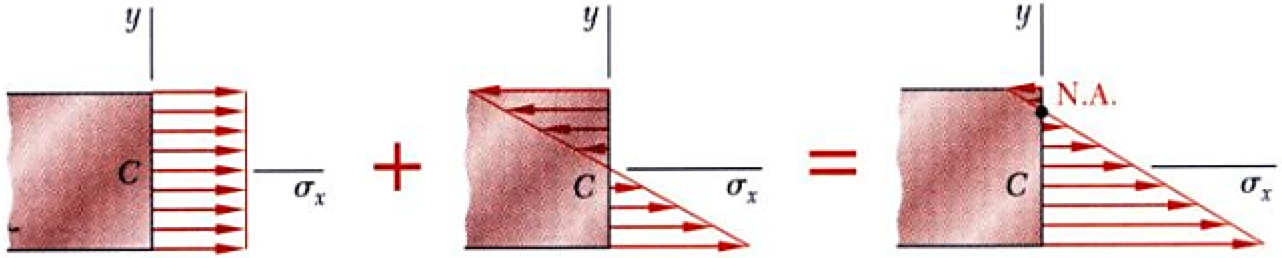
- Stress due to eccentric loading found by superposing the uniform stress due to a centric load and linear stress distribution due a pure bending moment

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$
$$= \frac{P}{A} - \frac{My}{I}$$

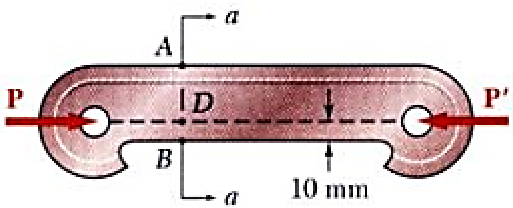
- Eccentric loading

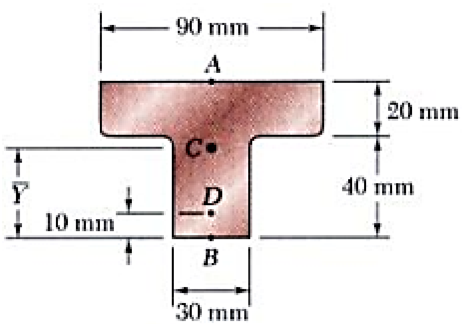
$$F = P$$
$$M = Pd$$

- Validity requires stresses below proportional limit, deformations have negligible effect on geometry, and stresses not evaluated near points of load application.



Example 6





Section a–a

Given:

$$A = 3\times10^{-3}\,\text{m}^2$$

$$\bar{Y} = 0.038\text{m}$$

$$I = 868\times10^{-9}\,\text{m}^4$$

The largest allowable stresses for the cast iron link are 30 MPa in tension and 120 MPa in compression. Determine the largest force *P* which can be applied to the link :

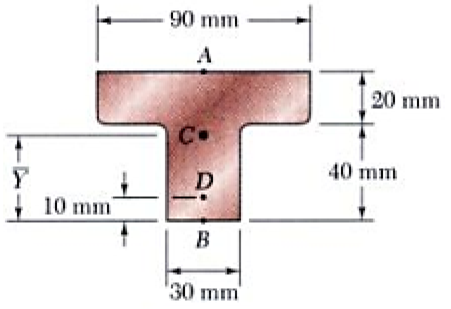
✓ Determine an equivalent centric load and bending moment.

✓ Evaluate the critical loads for the allowable tensile and compressive stresses.

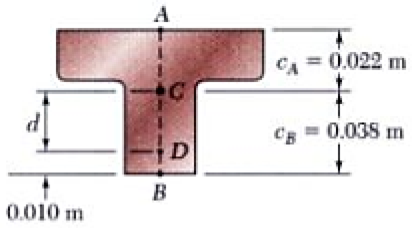
✓ The largest allowable load is the smallest of the two critical loads.

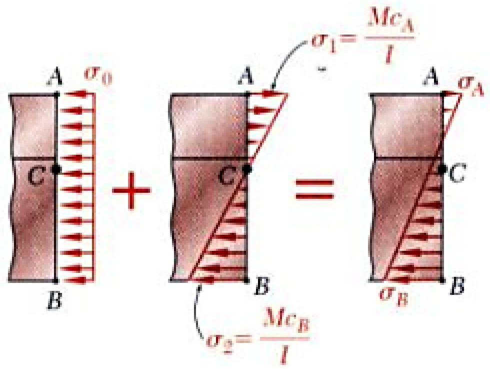
✓ Superpose the stress due to a centric load and the stress due to bending.

Solution



Section a–a





- ✓ Determine an equivalent centric and bending loads.

$$d = 0.038 - 0.010 = 0.028\text{m}$$

$$P = \text{centric load}$$

$$M = Pd = 0.028P = \text{bending moment}$$

- ✓ Superpose stresses due to centric and bending loads

$$\sigma_A = -\frac{P}{A} + \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} + \frac{(0.028P)(0.022)}{868\times10^{-9}} = +377P$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} - \frac{(0.028P)(0.022)}{868\times10^{-9}} = -1559P$$

- ✓ Evaluate critical loads for allowable stresses.

$$\sigma_A = +377P = 30\text{MPa} \qquad P = 79.6\text{kN}$$

$$\sigma_B = -1559P = -120\text{MPa} \qquad P = 79.6\text{kN}$$

- ✓ The largest allowable load

Sample Problem!

The shaft is supported by a smooth sleeve bearing at A at B and is subjected to the concentrated forces shown in the figure. If the shaft has a diameter of 50 mm. Determine the maximum bending stress in the shaft.