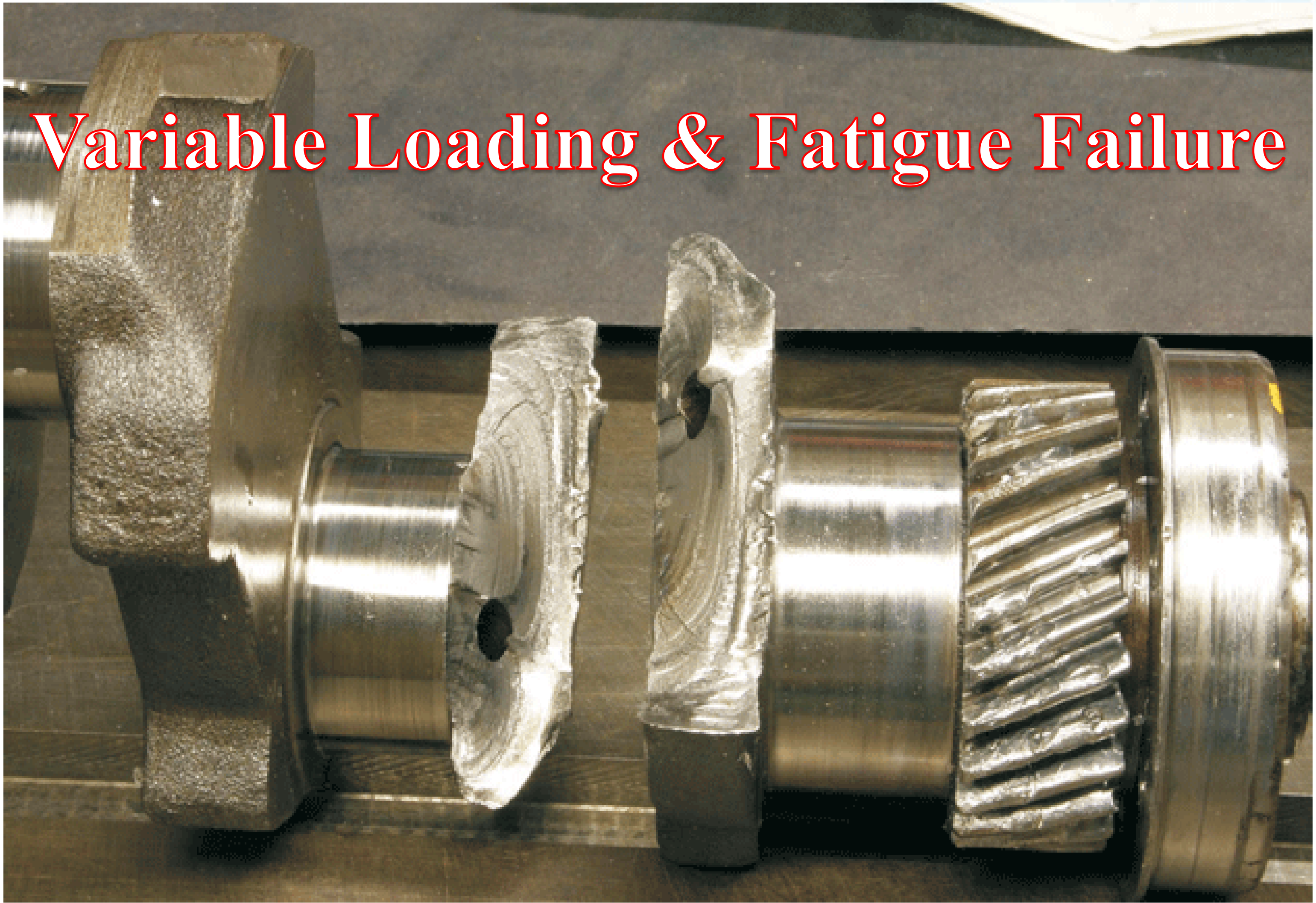
Product Development 1

Introduction to **Mechanical Engineering Design**

Spring 2023

Variable Loading & Fatigue Failure

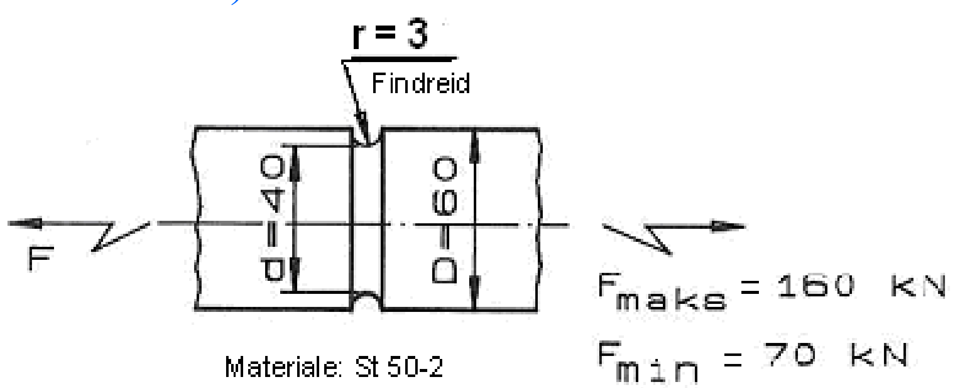


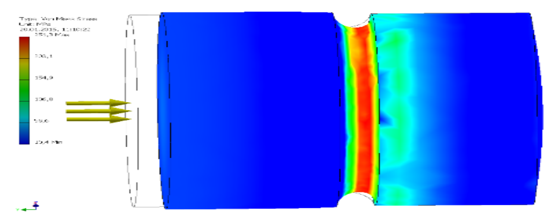
Sample problem 2
PPT-11

The figure shows part of a shaft subjected to a variable axial load (tensile) that varies between Fmin = 70 kN and Fmax = 160 kN.

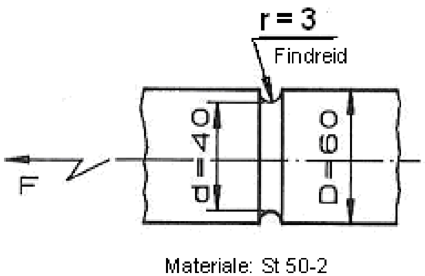
Check if the safety factor at the groove is satisfactory against fatigue failure.

(Ans. n = 2.1)



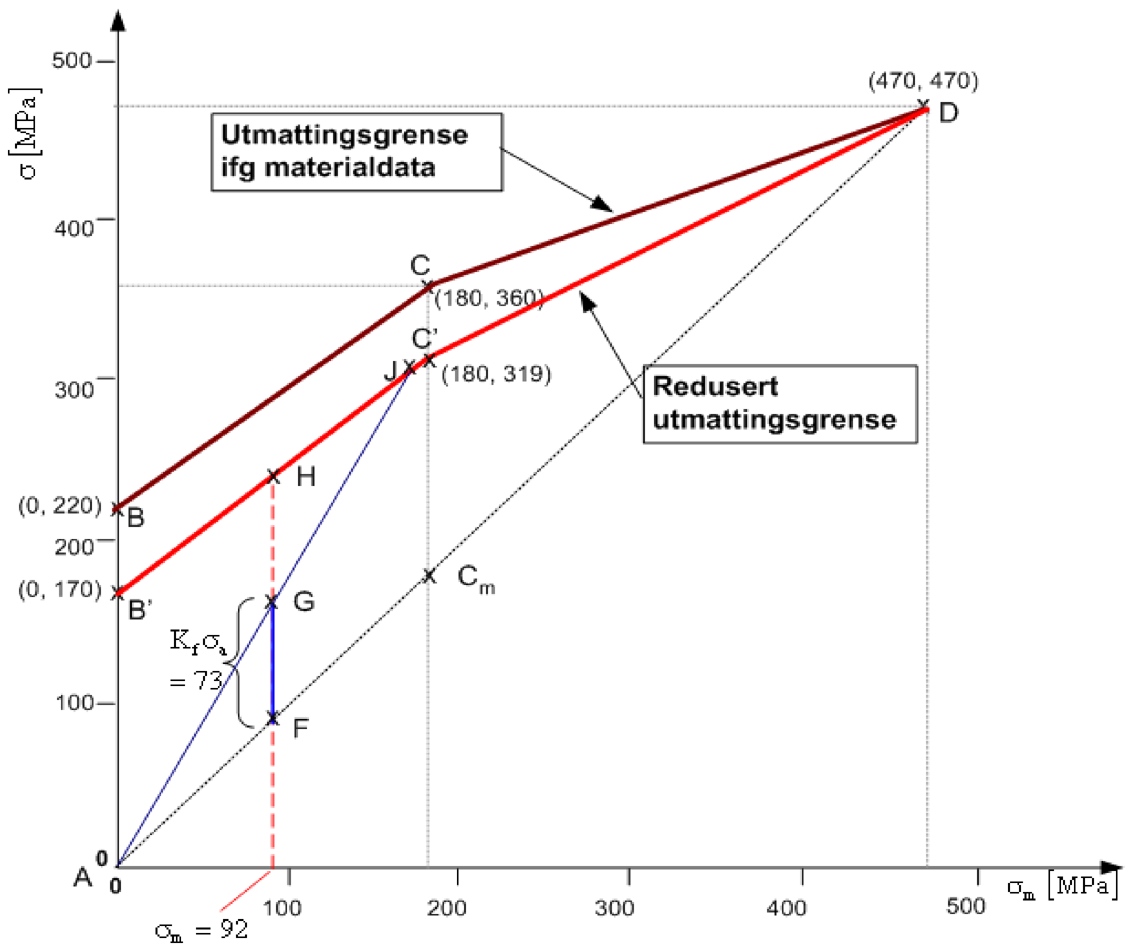


Solution

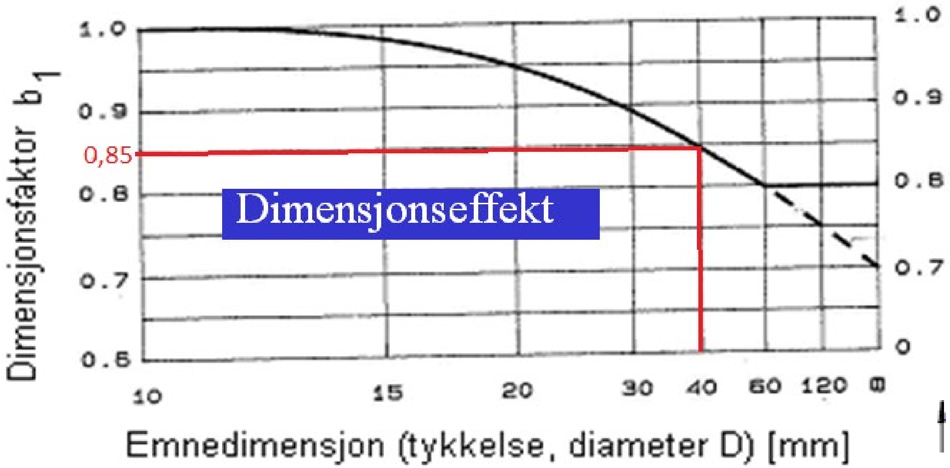


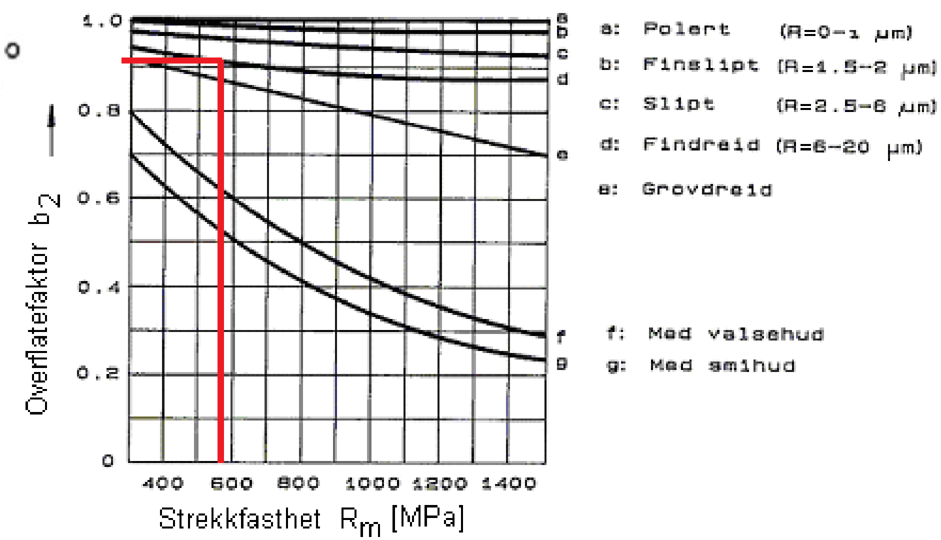
F_{maks} = 160 kN

F_{min} = 70 kN



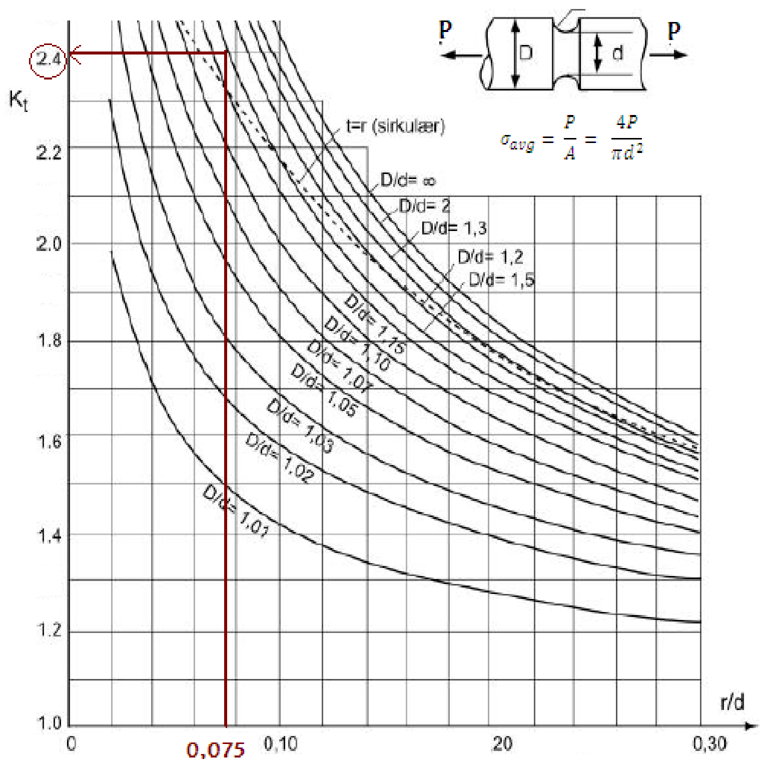
Solution



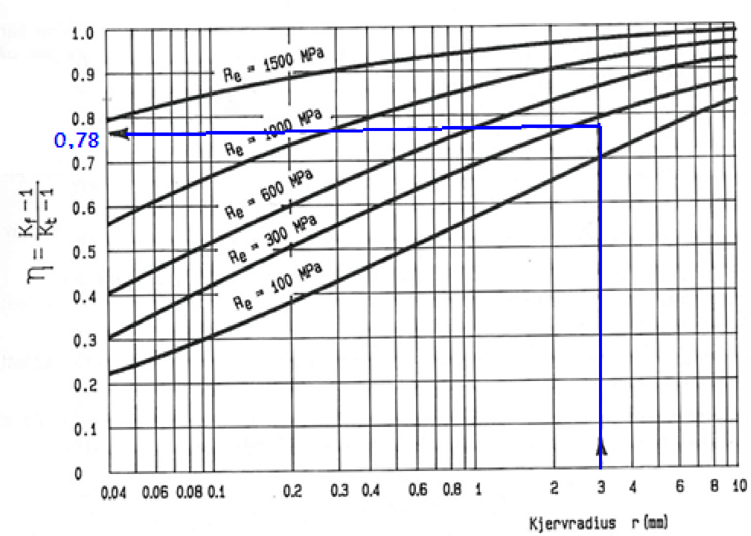


σ_a [MPa]	$b\sigma_a$ [MPa]	$\sigma_m + b\sigma_a$ [MPa]	comment
220	170	170	Gives pt B'
180	139	319	Gives pt C'

Solution

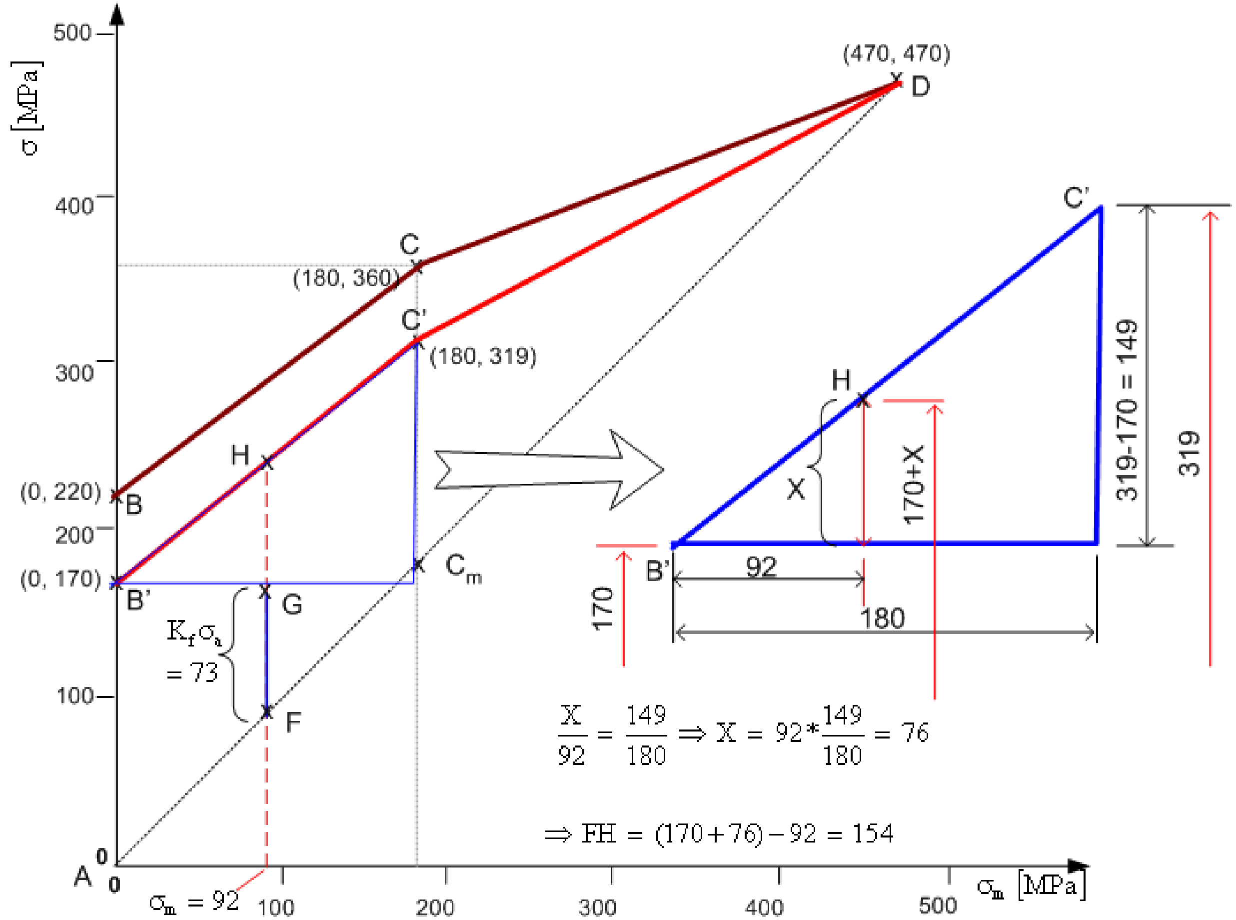


P
D
d
P
$\sigma_{avg} = \frac{P}{A} = \frac{4P}{\pi d^2}$
K_t
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
t=r (sirkulær)
D/d= ∞
D/d= 2
D/d= 1,3
D/d= 1,2
D/d= 1,5
D/d= 1,15
D/d= 1,10
D/d= 1,07
D/d= 1,05
D/d= 1,03
D/d= 1,02
D/d= 1,01
r/d
0
0,075
0,10
20
0,30



$\eta = \frac{K_f - 1}{K_t - 1}$
1.0
0.9
0.8
0,78
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_e = 1500 MPa
R_e = 1000 MPa
R_e = 600 MPa
R_e = 300 MPa
R_e = 100 MPa
0.04
0.06
0.08
0.1
0.2
0.3
0.4
0.6
0.8
1
2
3
4
6
8
10
Kjervradius r (mm)

Solution



$$\frac{X}{92} = \frac{149}{180} \Rightarrow X = 92 * \frac{149}{180} = 76$$

$$\Rightarrow FH = (170 + 76) - 92 = 154$$

Torsional Fatigue Strength

I. For ductile, polished, notch-free, and cylindrical materials: The existence of a torsional steady-stress component not more than the torsional yield strength has no effect on the torsional endurance limit!

II. For materials with stress concentration, notches, or surface imperfections: The torsional fatigue limit decreases monotonically with torsional steady stress (Goodman diag.):

$$S_{su} = 0.67 S_{ut}$$

$$S_{sy} = 0.577 S_{yt}$$

Fatigue – Combination of Loading Modes

Considering that the bending, torsional, and axial stresses have alternating and midrange components, the von Mises stresses for the two stress elements can be written as:

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

$$\sigma'_m = \left\{ \left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

Example 1

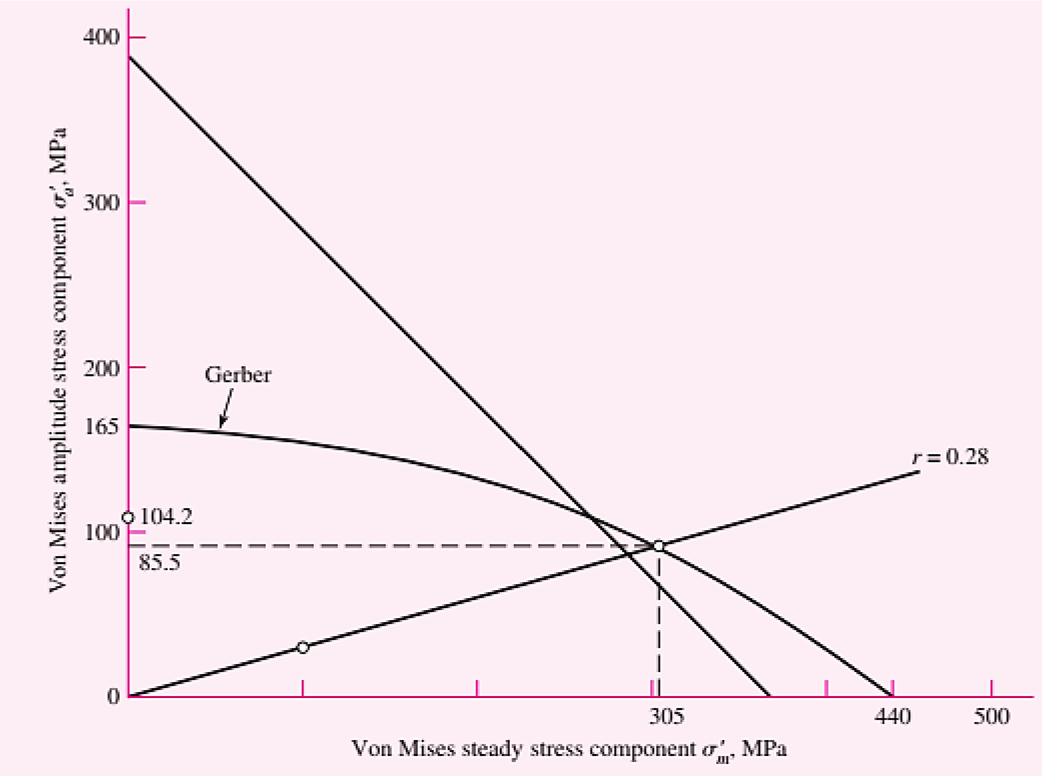
A rotating shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

(a) The shaft is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.

(b) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m

(Example 6-14, Shigley-9th edition)

Solution



400
300
200
165
100
0
104.2
85.5
Gerber
r = 0.28
305
440
500
Von Mises amplitude stress component σ'_a, MPa
Von Mises steady stress component σ'_m, MPa

Application of Soderberg's Equation - Combined Loading

We have seen that according to Soderberg's equation:

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(For reversed bending load)}$$

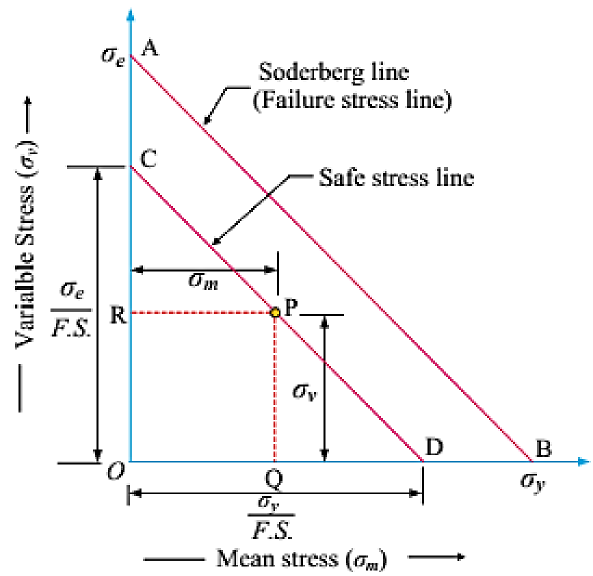
This equation may also be written as:

$$\frac{\sigma_y}{F.S.} = \boxed{\sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}}$$

Since the factor of safety based on yield strength is the ratio of the yield point stress to the working or design stress, therefore we may write this equation for working or design stress as:

$$\text{Working or Design stress} = \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v$$

And known as equivalent normal stress due to reversed bending.



Application of Soderberg's Equation - Combined Loading

Equivalent normal stress due to reversed bending:

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

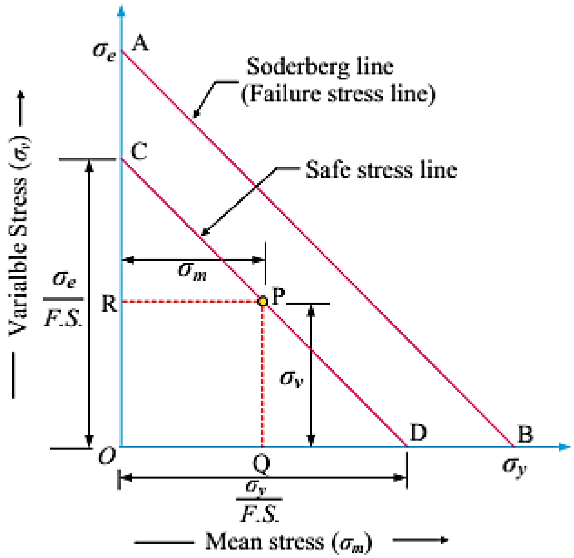
Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

and total equivalent normal stress, $\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{\sigma_y}{F.S.}$

In a similar manner and for reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$



The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

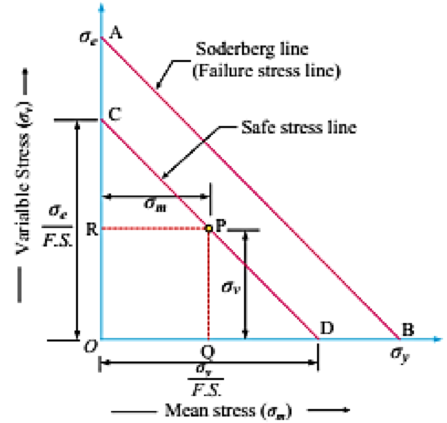
Application of Soderberg's Equation - Combined Loading

Let us now consider the use of Soderberg's equation to a ductile material under the following loading conditions.

1. Axial loading

In case of axial loading, we know that the mean or average stress:

$\sigma_m = W_m / A$



and variable stress, $\sigma_v = W_v / A$

where

W_m = Mean or average load,

W_v = Variable load, and

A = Cross-sectional area.

The equation $= \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v$ on the previous slide may now be written as follows:

$$\text{Working or Design stress} = \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v = \frac{W_m}{A} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{W_v}{A} = \frac{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}{A}$$

$$F.S. = \frac{\sigma_y \times A}{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}$$

Application of Soderberg's Equation - Combined Loading

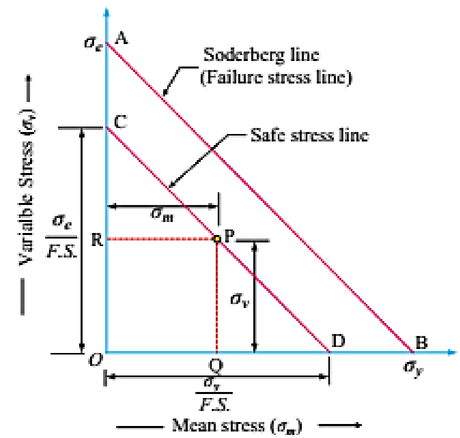
2.Simple bending

In case of simple bending, we know that the bending stress,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{Z}$$

I = section moment of inertia

$$Z = \frac{I}{c} = \text{section modulus}$$



Now, Mean or average bending stress is $\sigma_m = M_m / Z$

and variable bending stress, $\sigma_v = M_v / Z$

where

M_m = Mean bending moment,

M_v = Variable bending moment, and

Z = Section modulus.

The equation $= \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v$ may now he written as follows :

Working or bending Design stress $\sigma_b = \frac{M_m}{Z} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{M_v}{Z}$

Application of Soderberg's Equation - Combined Loading

Working or bending Design stress

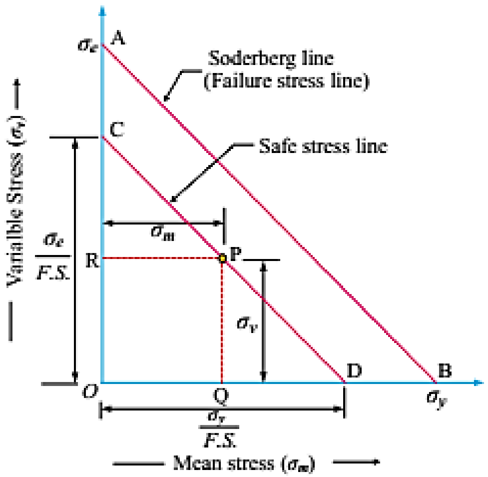
$$\sigma_b = \frac{M_m}{Z} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{M_v}{Z}$$

$$= \frac{M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v}{Z}$$

$$= \frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]$$

$$F.S. = \frac{\sigma_y}{\frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]}$$

For circular shaft $Z = \frac{\pi}{32} \times d^3$



Application of Soderberg's Equation - Combined Loading

3. Simple torsion of circular shafts

In case of simple torsion, we know that the torque,

$$T = \frac{\pi}{16} \times \tau \times d^3 \quad \text{or} \quad \tau = \frac{16\,T}{\pi\,d^3}$$

Mean or average shear stress (τ_m), $\tau_m = \dfrac{16\,T_m}{\pi\,d^3}$

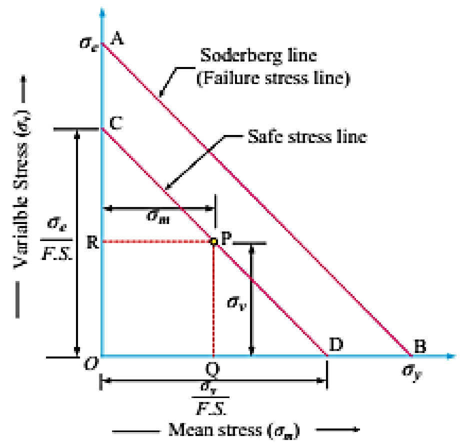
and variable shear stress(τ_v) $\tau_v = \dfrac{16\,T_v}{\pi\,d^3}$

where

T_m = Mean or average torque,

T_v = Variable torque, and

d = Diameter of the shaft.



The equation $= \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v$ may now he written as follows :

Working or Design stress for shear = $\tau = \dfrac{16\,T_m}{\pi d^3} + \left(\dfrac{\tau_y}{\tau_e}\right) K_{fs} \times \dfrac{16\,T_v}{\pi d^3} = \dfrac{16}{\pi d^3}\left[T_m + \left(\dfrac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]$

$$F.S. = \frac{\tau_y}{\dfrac{16}{\pi d^3}\left[T_m + \left(\dfrac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]}$$

where K_{fs} = Fatigue stress concentration factor for torsional or shear loading.

Note : For shafts made of ductile material, $\sigma_y = 0.5\sigma_{ut}$, and $\tau_y = 0.5\sigma_e$ may be taken.

Application of Soderberg's Equation - Combined Loading

In case of combined bending and torsion of circular shafts, the maximum shear stress theory may be used. According to this theory, maximum shear stress:

$$\tau_{max} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left[\frac{32}{\pi d^3}\left\{M_m + \left(\frac{\sigma_y}{\sigma_e}\right)K_f \times M_v\right\}\right]^2 + 4\left[\frac{16}{\pi d^3}\left\{T_m + \left(\frac{\tau_y}{\tau_e}\right)K_{fs} \times T_v\right\}\right]^2}$$

$$= \frac{16}{\pi d^3}\sqrt{\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right)K_f \times M_v\right]^2 + \left[T_m + \left(\frac{\tau_y}{\tau_e}\right)K_{fs} \times T_v\right]^2}$$

The majority of rotating shafts carry a steady torque and the loads remain fixed in space in both direction and magnitude. Thus during each revolution every fibre on the surface of the shaft wider-goes a complete reversal of stress due to bending moment_ Therefore for the usual case when $M_m = 0$, $M_v = M$, $T_m = T$ and $T_v = 0$, the above equation may be written as

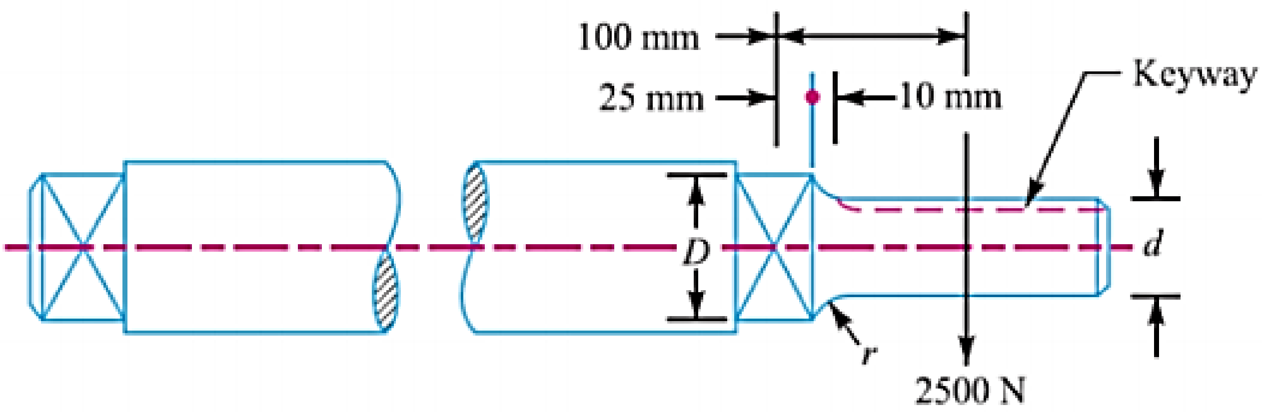
$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right)K_f \times M\right]^2 + T^2}$$

Note: The above relations apply to a solid shaft. For hollow shaft, the left hand side of the above equation must be multiplied by (1 - k_4). where k is the ratio of inner diameter to outer diameter.

Example 2

A centrifugal blower rotates at 600 r.p.m. A belt drive is used to connect the mower to a 5 kW and 1750 r.p.m. electric motor. The belt forces a torque of 250 N-m and a force of 2500 *N* on the shaft. The fig shows the location of bearings, the steps in the shaft and the plane in which the resultant belt force and torque act. The ratio of the journal diameter to the overhung shaft diameter is 1.2 and the radius of the, fillet is 1/10th of overhung shaft diameter. Find the shaft diameter. Journal diameter and radius of fillet to have a factor of safety 3. The blower shaft is to be machined from hot rolled steel having the following values of stresses:

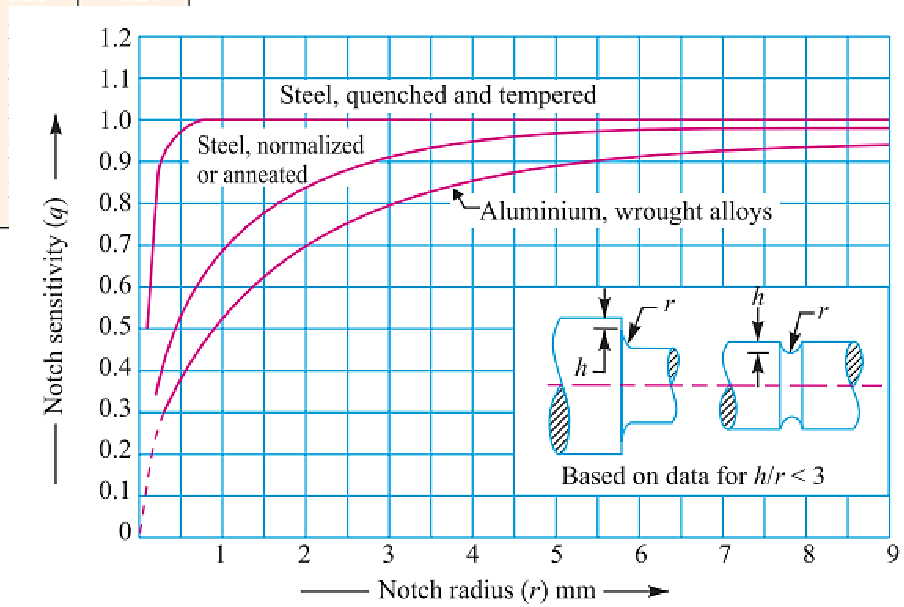
Endurance limit = 180 MPa; Yield point stress = 300 MPa; Ultimate tensile stress = 450 MPa.



Solution

Table 6.4. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in bending.

$\frac{D}{d}$	*Theoretical stress concentration factor (K_t)*									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.01	1.85	1.61	1.42	1.36	1.32	1.24	1.20	1.17	1.15	1.14
1.02	1.97	1.72	1.50	1.44	1.40	1.32	1.27	1.23	1.21	1.20
1.05	2.20	1.88	1.60	1.53	1.48	1.40	1.34	1.30	1.27	1.25
1.10	2.36	1.99	1.66	1.58	1.53	1.44	1.38	1.33	1.28	1.27
1.20	2.52	2.10	1.72	1.62	1.56	1.46	1.39	1.34		
1.50	2.75	2.20	1.78	1.68	1.60	1.50	1.42	1.36		
2.00	2.86	2.32	1.87	1.74	1.64	1.53	1.43	1.37		
3.00	3.00	2.45	1.95	1.80	1.69	1.56	1.46	1.38		
6.00	3.04	2.58	2.04	1.87	1.76	1.60	1.49	1.41		



Solution (Continued)

Let D = Journal diameter,

d = Shaft diameter, and r = Fillet radius.

∴ Ratio of journal diameter to shaft diameter,

$$D/d = 1.2 \qquad \text{...(Given)}$$

and radius of the fillet, $r = 1/10 \times$ Shaft diameter $(d) = 0.1\ d$

$$\therefore \quad r/d = 0.1 \qquad \text{...(Given)}$$

From Table 6.3, for $D/d = 1.2$ and $r/d = 0.1$, the theoretical stress concentration factor,

$$K_t = 1.62$$

The two points at which failure may occur are at the end of the keyway and at the shoulder fillet. The critical section will be the one with larger product of $K_f \times M$. Since the notch sensitivity factor q is dependent upon the unknown dimensions of the notch and since the curves for notch sensitivity factor (Fig. 6.14) are not applicable to keyways, therefore the product $K_t \times M$ shall be the basis of comparison for the two sections.

∴ Bending moment at the end of the keyway,

$$K_t \times M = 1.6 \times 2500\ [100 - (25 + 10)] = 260 \times 10^3 \text{ N-mm}$$

...($\because$ K_t for key ways = 1.6)

and bending moment at the shoulder fillet,

$$K_t \times M = 1.62 \times 2500\ (100 - 25) = 303\ 750 \text{ N-mm}$$

Since $K_t \times M$ at the shoulder fillet is large, therefore considering the shoulder fillet as the critical section. We know that

$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right)K_f \times M\right]^2 + T^2}$$

Solution (Continued)

$$\frac{0.5 \times 300}{3} = \frac{16}{\pi d^3} \sqrt{\left[\left(\frac{300}{180} \times 303\,750\right)^2 + (250 \times 10^3)^2\right]}$$

... (Substituting, $\tau_y = 0.5\ \sigma_y$)

$$50 = \frac{16}{\pi d^3} \times 565 \times 10^3 = \frac{2877 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 2877 \times 10^3/50 = 57\ 540$ or $d = 38.6$ say 40 mm **Ans.**

Note: Since r is known (because $r/d = 0.1$ or $r = 0.1d = 4$ mm), therefore from Fig. 6.14, the notch sensitivity factor (q) may be obtained. For $r = 4$ mm, we have $q = 0.93$.

$\therefore$ Fatigue stress concentration factor,

$$K_f = 1 + q\,(K_t - 1) = 1 + 0.93\,(1.62 - 1) = 1.58$$

Using this value of K_f instead of K_t, a new value of d may be calculated. We see that magnitudes of K_f and K_t are very close, therefore recalculation will not give any improvement in the results already obtained.

Cumulative Fatigue Damage

A machine part, at a critical location, is subjected to:

- A fully reversed stress σ_1 for n_1 cycles, σ_2 for n_2 cycles, ..., or
- A wiggly time line of stress exhibiting many and different peaks and valleys.

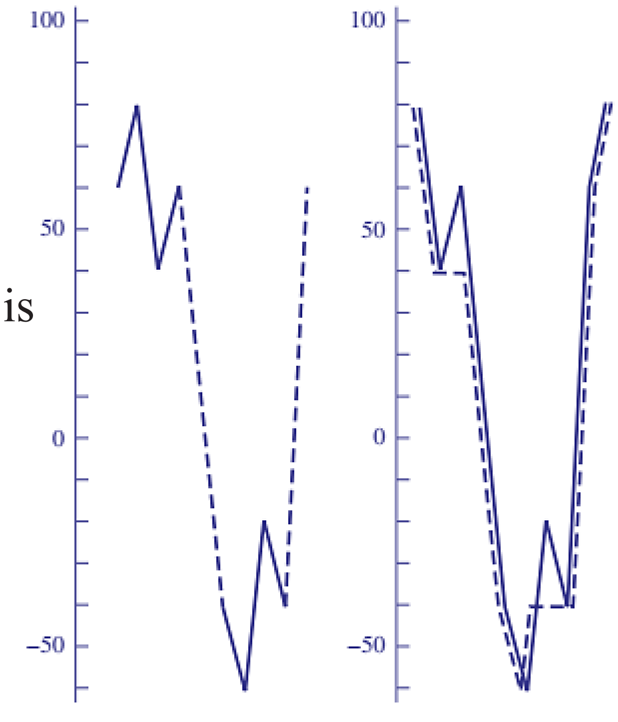
Therefore, Miner's rule:

$$\sum \frac{n_i}{N_i} = c$$

where n_i is the number of cycles at stress level σ_i and N_i is the number of cycles to failure at stress level σ_i.

Using the deterministic formulation as a linear damage rule we write:

$$D = \sum \frac{n_i}{N_i}$$

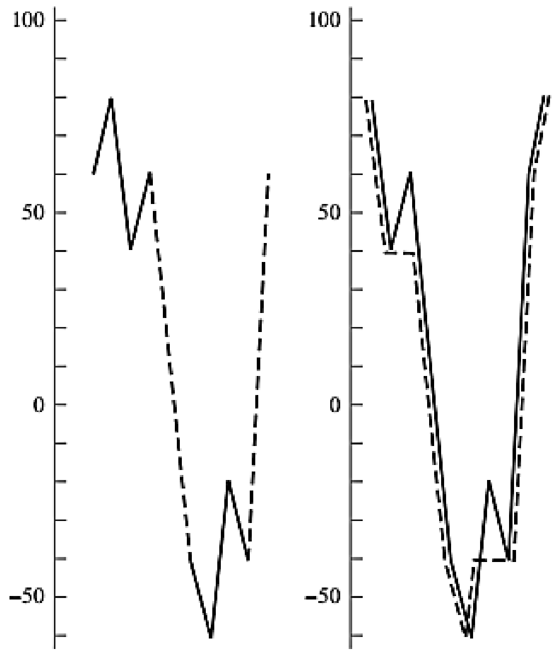


(a) (b)

Example 3

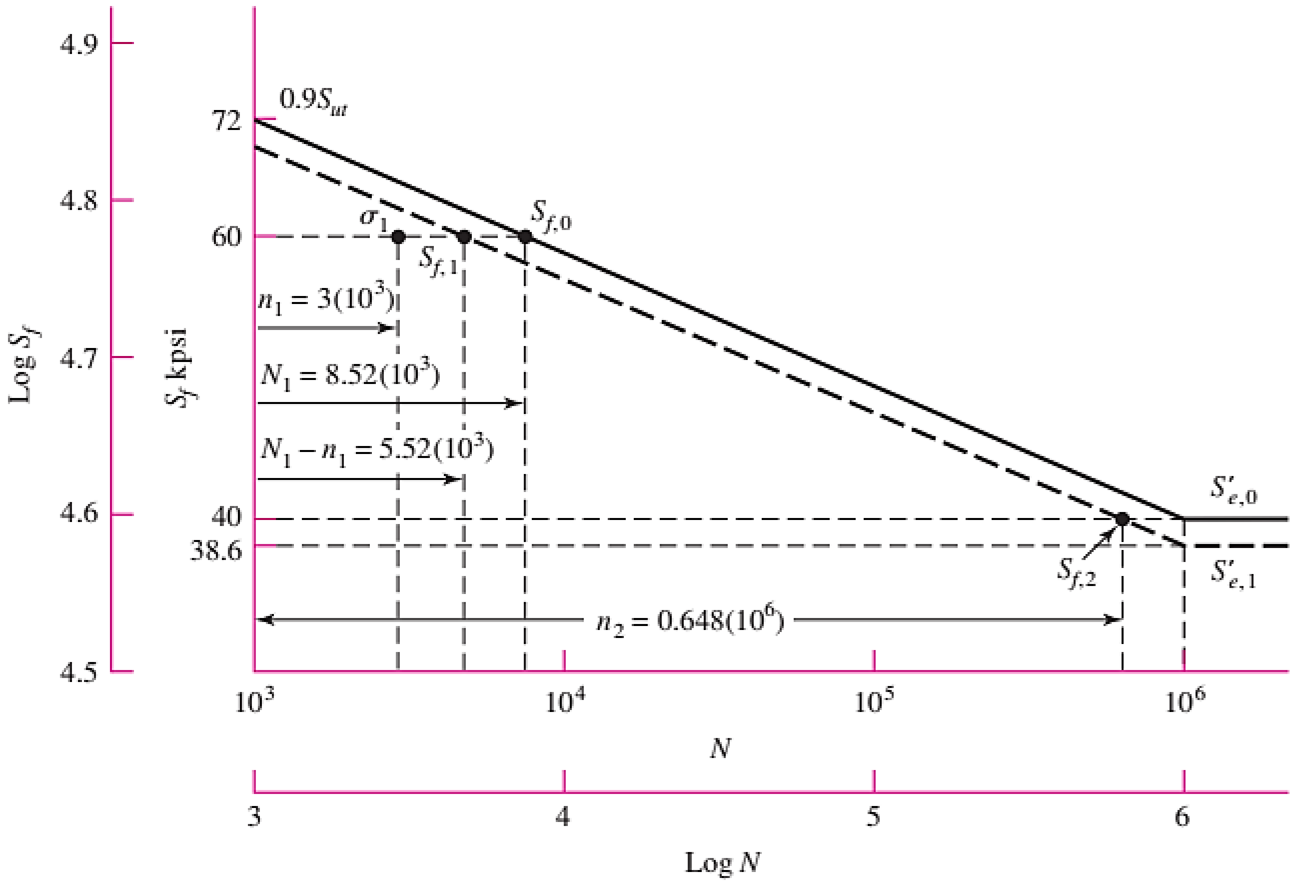
Given a part with Sut = 151 kpsi and at the critical location of the part, Se = 67.5 kpsi.
For the loading shown in the figure, estimate the number of repetitions of the stress-time block that can be made before failure.

(Example 6-15, Shigley-9th edition)



Miner Rule- More Discussion

(pp. 324, Shigley-9th edition)



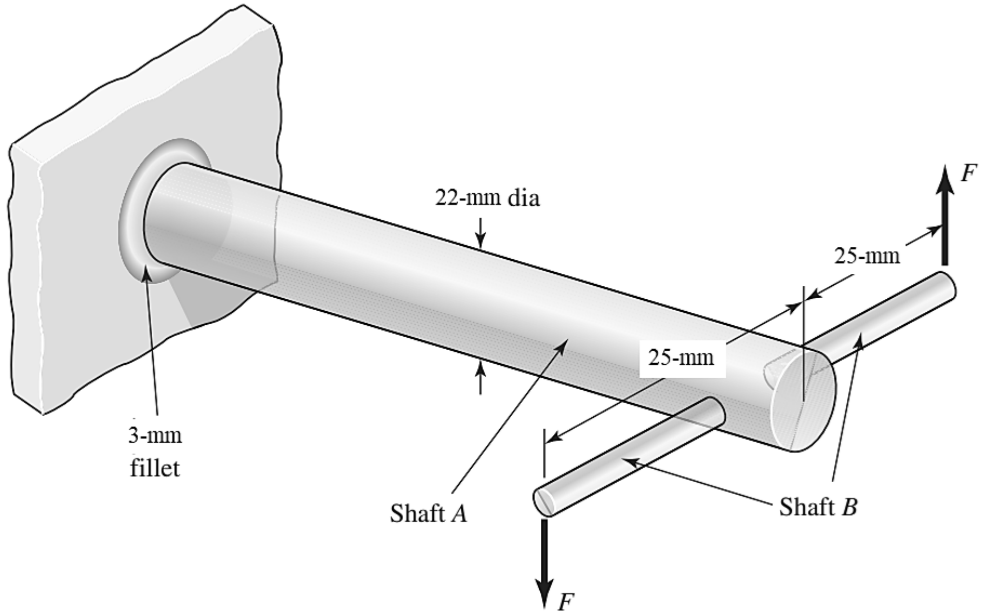
Sample Problem 1

In the figure shown, shaft A, made of AISI 1020 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces F via shaft B. A theoretical stress-concentration factor Kts of 1.6 is induced in the shaft by the 3-mm weld fillet. The length of shaft A from the fixed support to the connection at shaft B is 0.6 m. The load F cycles from 667 to 2224 N.

(a) For shaft A, find the factor of safety for infinite life using the Goodman fatigue failure criterion.

(b) Repeat part (a) using the Gerber fatigue failure criterion.

Ans. (a) $n_f = 2$, (b) $n_f = 2.5$



End of today's lecture!