Product Development 1

Introduction to **Mechanical Engineering Design**

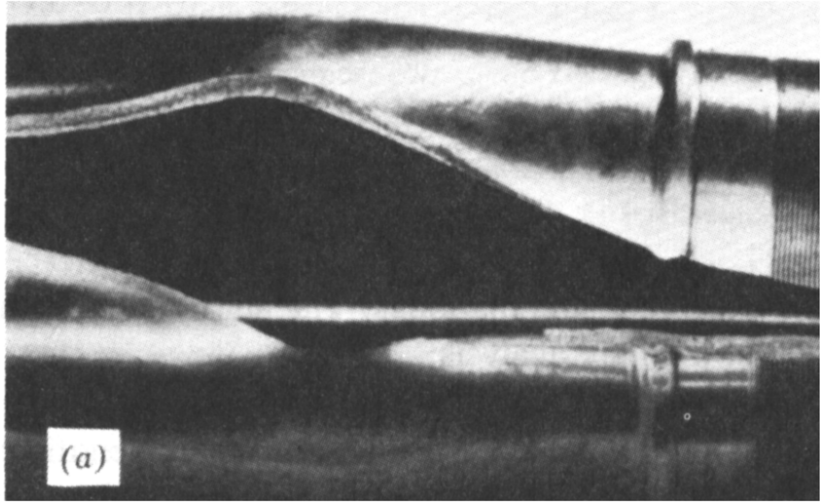
Spring 2023

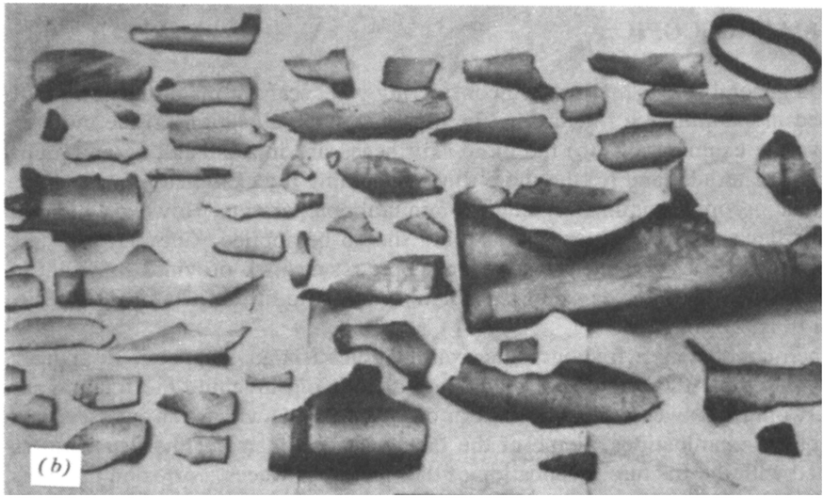
Failure Theories

Introduction

- **Ductile failure:**
  - one piece
  - large deformation

- **Brittle failure:**
  - many pieces
  - small deformation





Figures from V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 4.1(a) and (b), p. 66 John Wiley and Sons, Inc., 1987.

Mechanical Failure

- **Yielding- This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in ductile materials.**

Ductile material:

- Its resistance to sliding is smaller than its resistance to separation.
- Failure takes place by yielding
- These materials can have same yielding point value for both tension and compression

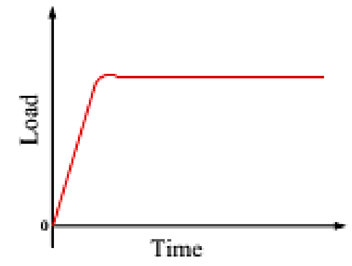
- **Fracture- in this case the component tears apart in two or more parts. This mostly occurs in brittle materials.**

Brittle material:

- Its resistance to separation is smaller than its resistance to sliding.
- Failure takes place by fracture
- These materials mostly have higher ultimate value in compression than tension

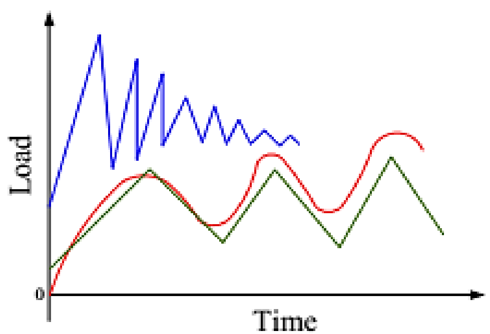
Load Types

i) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.



Static Loading

ii) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge. Load may change in direction, for example, load on piston rod of a double acting cylinder.



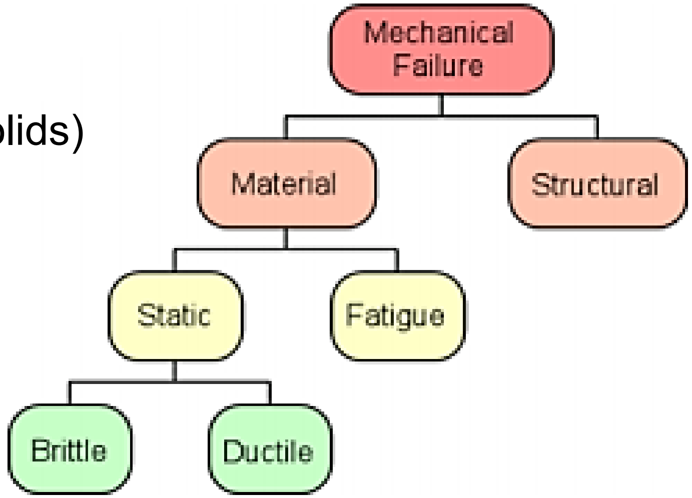
Dyamic Loading

Material Failure-
Static Loading

- Failure of a loaded structural assembly or component can be regarded as any behaviour that renders it unsuitable for its intended use.

- Examples of different failure modes are:
  - Yielding (what we use in Solid Mechanics course)
  - Fatigue (Machine Design)
  - Buckling (Solid Mechanics/Machine Design)
  - Fracture (Machine Design/Advanced Solids)
  - Excessive elastic deflection, etc. (Advanced Solids)

- Static failure
  - Ductile
  - Brittle
  - Stress concentration

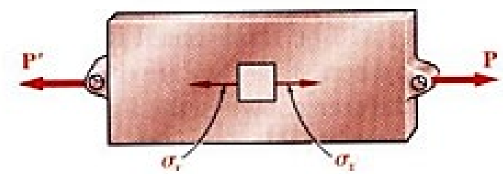


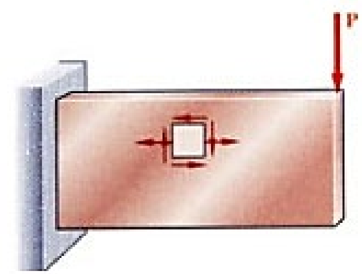
Material Failure-
Static Loading

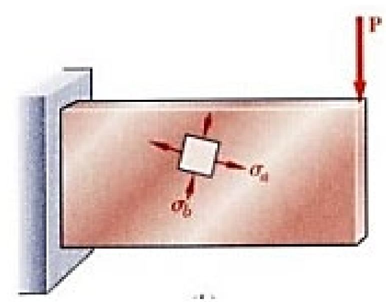
- When an engineer is faced with the problem of design using a specific material, it becomes important to place an upper limit on the state of stress that defines the material's failure.
- In this section we will discuss three theories that are often used in engineering practice to predict the failure of a material subjected to a multiaxial state of stress.
- No single theory of failure, however, can be applied to a specific material at all times, because a material may behave in either a ductile or brittle manner depending on the temperature, rate of loading, chemical environment, or the way the material is shaped or formed.
- When using a particular theory of failure, it is first necessary to calculate the normal and shear stress components at points where they are the largest in the member.
- In any case, once this state of stress is established, the principal stresses at these critical points are then determined, since each of the following theories is based on knowing the principal components.

Yield Criteria for Ductile Materials Under Plane Stress

- Failure of a machine component subjected to uniaxial stress is directly predicted from an equivalent tensile test.
- Failure of a machine component subjected to plane stress cannot be directly predicted from the uniaxial state of stress in a tensile test specimen.
- It is convenient to determine the principal stresses and to base the failure criteria on the corresponding biaxial stress state.
- Failure criteria are based on the mechanism of failure. Allows comparison of the failure conditions for a uniaxial stress test and biaxial component loading.



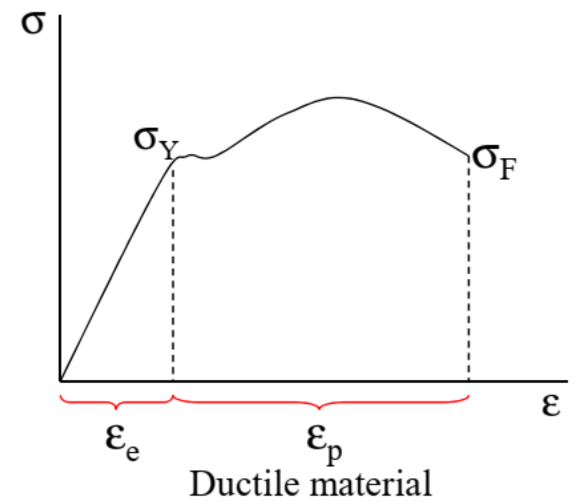




Failure from Static Loading

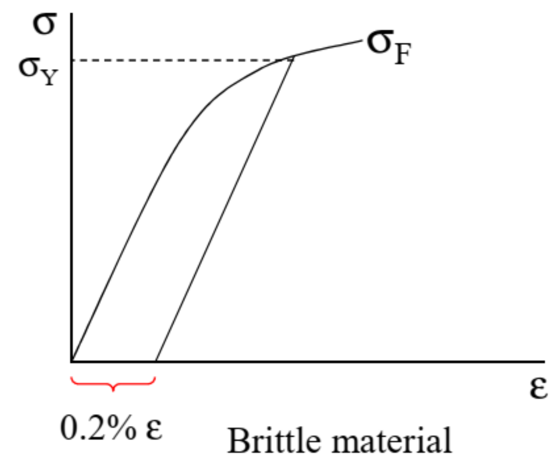
Ductile failures – Yield criteria

- ❑ Maximum Shear Stress Theory
- ❑ Distortion Energy Theory
- ❑ Coulomb-Mohr Theory



Brittle failures – Fracture criteria

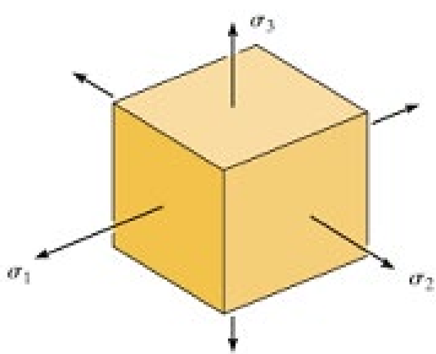
- ❑ Maximum Normal Stress Theory
- ❑ Modified Mohr Theory

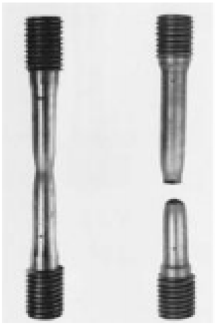


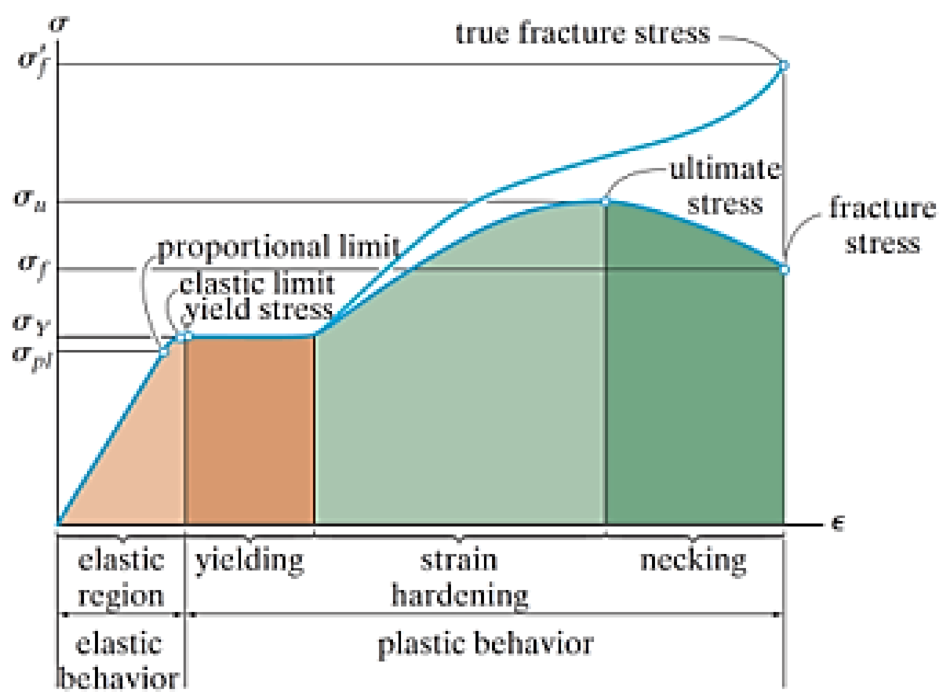
Yield Criteria for Ductile Materials

The "*theory*" behind most statics failure criteria (for ductile materials) is that whatever is responsible for failure in the simple tensile test also is responsible for failure under combined loading.

Given: Stress-strain data for simple uniaxial tension.
Find: When failure occurs for general state of stress.





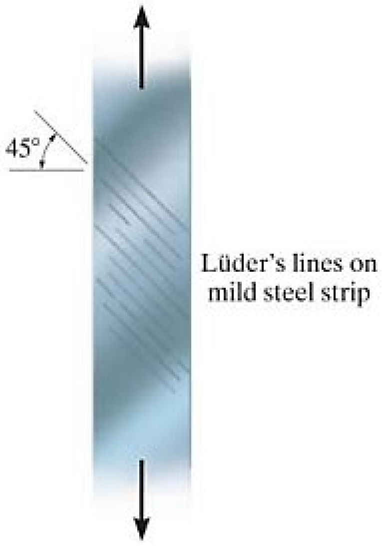


Yield Criteria for Ductile Materials

- The most common cause of yielding of a ductile material such as steel is slipping, which occurs along the contact planes of randomly ordered crystals that make up the material.

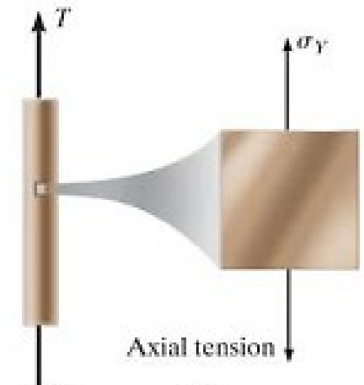
- This slipping is due to *shear stress*, and if we make a specimen into a highly polished thin strip and subject it to a simple tension test we can see how it causes the material to yield,

- Luder's lines clearly indicate the slip planes in the strip, which occur at approximately 45° with the axis of the strip.



Yield Criteria for Ductile Materials

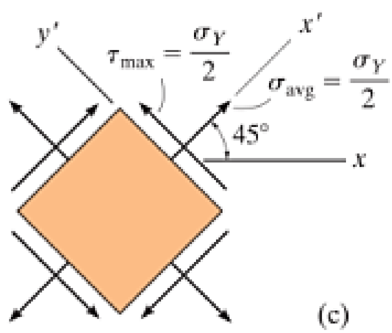
Consider now an element of the material taken from a tension specimen, which is subjected only to the yield stress σ_Y .



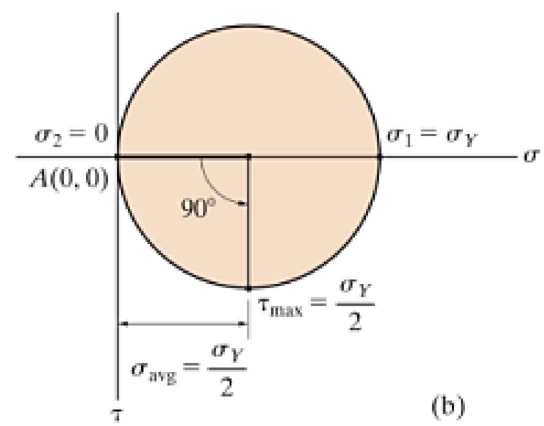
(a)

The maximum shear stress can be determined by drawing Mohr's circle for the element.

$$\tau_{\max} = \tau_{y(shear)} = \frac{\sigma_y}{2}$$



(c)



(b)

Allowable Stress-
Safety Factor

For design purpose an allowable stress is used in place of the ultimate / yield strength:

It takes into account the uncertainties namely:
- Uncertainty in loading
- Inhomogeneity of materials
- Various material behaviours (e.g. corrosion, creep)
- Residual stress due to manufacturing process
- Fluctuating loading (fatigue loading)
- Safety and reliability

Allowable stress is set considerably lower than the ultimate strength / yield strength

$$\text{Factor of safety } (F.S) = \frac{\text{Yielding / Ultimate Strength}}{\text{Allawable Stress}} = \frac{\sigma_{y\ or}\ \sigma_u}{\sigma_{allow}}$$

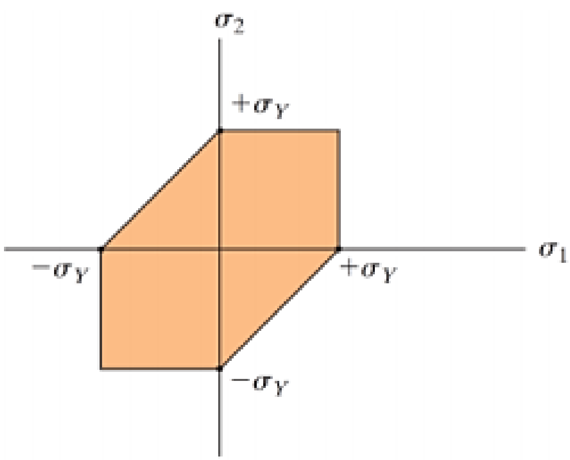
The above ratio must always be greater than unity!

Maximum Shear Stress Theory (Tresca)

Using this idea, that ductile materials fail by shear, Henri Tresca 1868 proposed the maximum-shear-stress theory. The maximum-shear-stress theory states that yielding of the material begins when the absolute maximum shear stress in the material reaches the shear stress that causes the same material to yield when it is subjected only to axial tension. To avoid failure, therefore, the maximum-shear-stress theory requires τ_{max} in the material to be less than or equal to $\sigma_Y/2$, where σ_Y is determined from a simple tension test.

$$\left.\begin{aligned} |\sigma_1| &= \sigma_Y \\ |\sigma_2| &= \sigma_Y \end{aligned}\right\} \quad \sigma_1, \sigma_2 \text{ have same signs}$$

$$|\sigma_1 - \sigma_2| = \sigma_Y\} \quad \sigma_1, \sigma_2 \text{ have opposite signs}$$

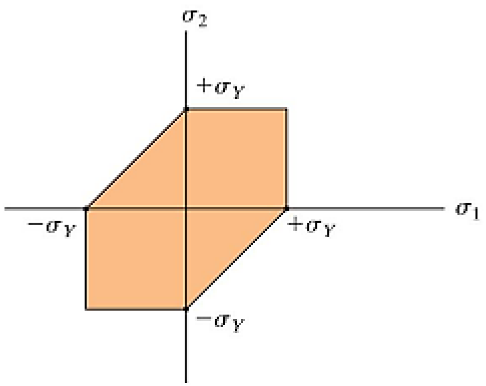


Maximum-shear-stress theory

Maximum Shear Stress Theory (Tresca)

According to this theory, the failure or yielding of mechanical component subjected to biaxial or tri-axial stress occurs when the maximum shear stress at any point in the component becomes equal to the shear stress at yield point in a simple tension test. Mathematically:

$$\tau_{\max} = \frac{\tau_{y(shear)}}{F.S.}$$



Maximum-shear-stress theory

Where

τ_{max}: Maximum shear stress in an axial stress system.

τ_{sy}: Shear stress at yield point from simple tension test.

F.S.: Factor of safety.

$$\text{Since, } \tau_{y(shear)} = 0.5\sigma_Y$$

$$\tau_{\max} = \frac{\sigma_Y}{2F.S.}$$

Maximum Shear Stress Theory (Tresca)

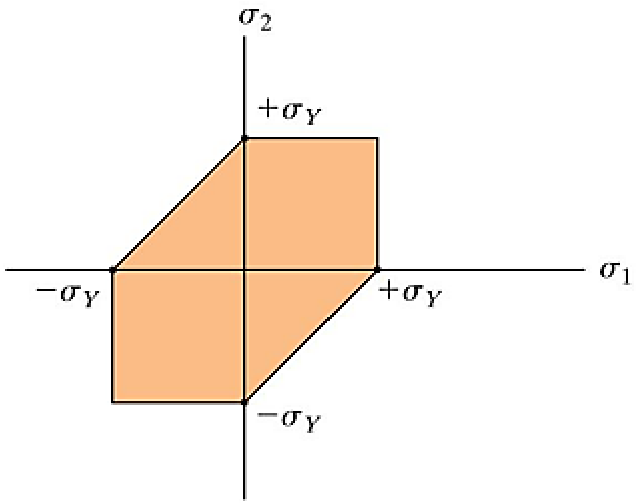
Material yields (fails) when:

1) $\tau_{max} \geq \dfrac{\sigma_Y}{2}$ or

2) $(\sigma_1 - \sigma_2) \geq \sigma_Y$

– Factor of Safety:

$$F.S. = \frac{\sigma_Y}{(\sigma_1 - \sigma_2)} = \frac{\sigma_Y}{2\tau_{max}}$$



Maximum-shear-stress theory

Maximum Shear Stress Theory (Tresca)

Case 1: $\sigma_1 \geq \sigma_2 \geq 0$, For this case, it reduces to a yield condition of,

$$\sigma_1 \geq \sigma_Y$$

Case 2: $\sigma_1 \geq 0 \geq \sigma_2$
For this case, it reduces to a yield condition of,

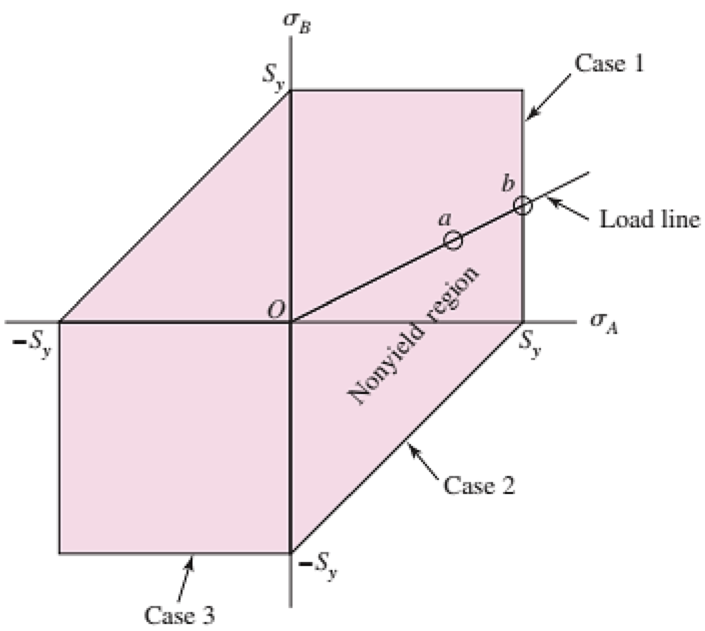
$$(\sigma_1 - \sigma_2) \geq \sigma_Y$$

Case 3: $0 \geq \sigma_1 \geq \sigma_2$
For this case, it reduces to a yield condition of,

$$\sigma_2 \geq \sigma_Y$$

or

$$\sigma_2 \leq -\sigma_Y$$



Maximum Energy of Distortion Theory (2D von-Mises)

The maximum distortion energy theory ,also known as the Von Mises theory, was proposed by M.T.Huber in 1904 and further developed by R. von Mises (1913). In this theory failure by yielding occurs when at any point in the body, the distortion energy per unit volume in a state of combined stress becomes equal to that associated with yielding in a simple tension test.

So for the general state of stress is given, yield is predicted if,

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right]}\geq\sigma_Y$$

The von-Mises effective stress (σe) also sometimes referred to as equivalent stress is defined as the uniaxial tensile stress that would create the same distortion energy as is created by the actual combination of applied stresses.

$$\sigma_e=\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right]}$$

And failure occuare when;

$$\sigma_e\geq\sigma_Y$$

Maximum Energy of Distortion Theory (2D von-Mises)

For two dimensional stress state ($\sigma_3 = 0$), the equations reduces to

$$\sigma_e = \sqrt{\sigma_1^{\ 2} - \sigma_1\sigma_2 + \sigma_2^{\ 2}}$$

In terms of applied stresses in coordinate directions

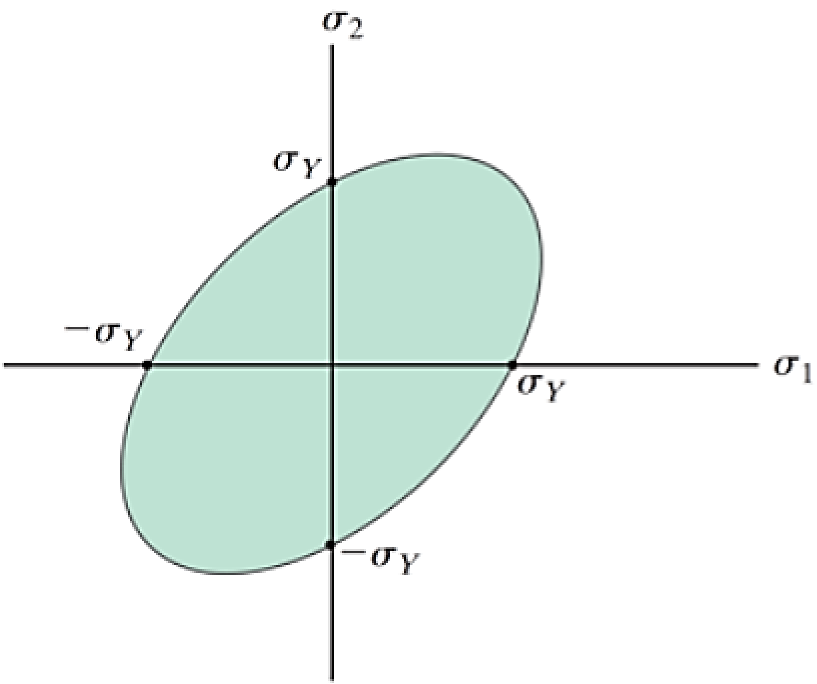
$$\sigma_e = \sqrt{\sigma_x^{\ 2} - \sigma_x\sigma_y + \sigma_y^{\ 2} + 3\tau^2_{\ xy}}$$

According to theory failure criteria is

$$\sigma_e \geq \sigma_Y$$

Safety factor

$$F.S. = \frac{\sigma_y}{\sigma_e}$$



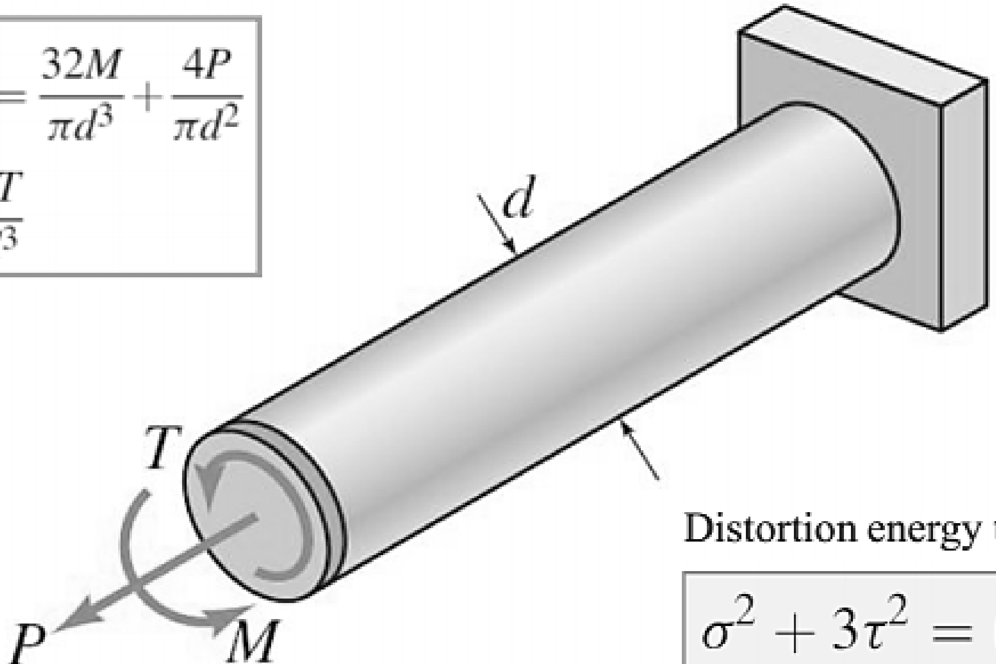
Maximum-distortion-energy theory

Maximum Energy of Distortion Theory (2D von-Mises)

Stress components:

$$\sigma = \frac{My}{I} + \frac{P}{A} = \frac{32M}{\pi d^3} + \frac{4P}{\pi d^2}$$

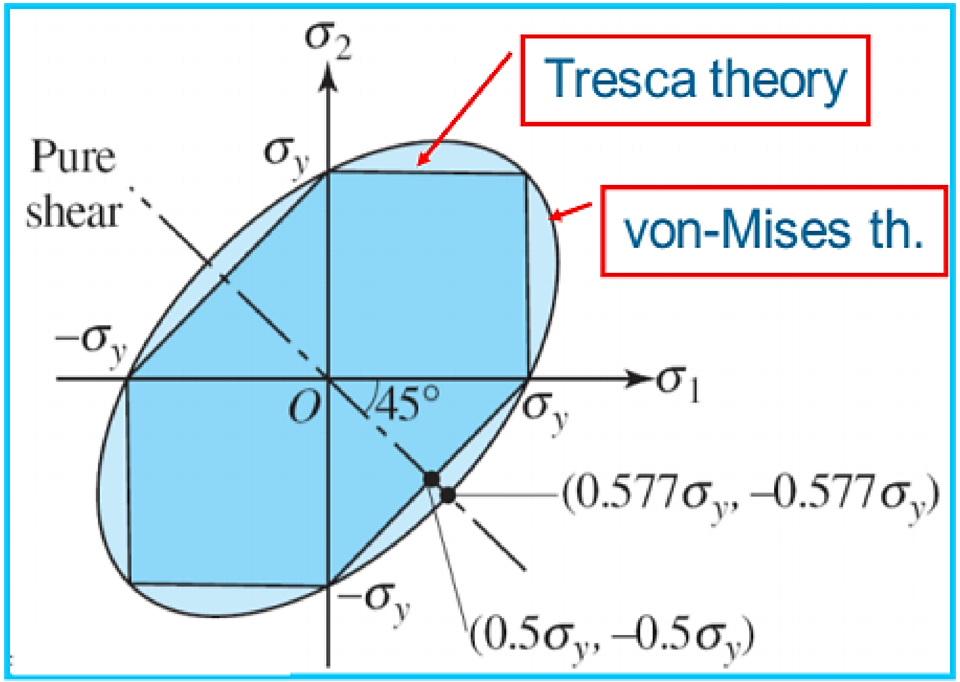
$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$



Distortion energy theory:

$$\boxed{\sigma^2 + 3\tau^2 = (\sigma_{\text{all}})^2}$$

Comparison of Failure Theories – Ductile Materials

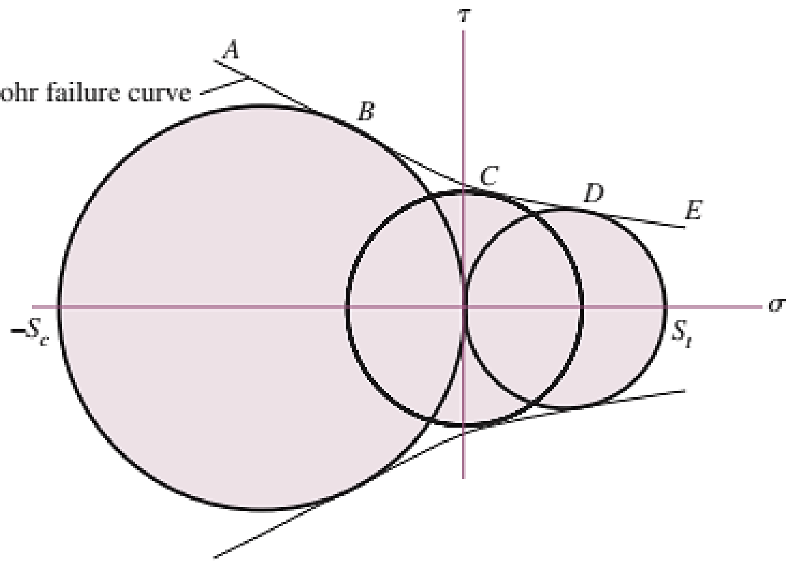


A comparison of the above two failure criteria is shown in the figure. Note that both theories give the same results when the principal stresses are equal.

Coulomb-Mohr Theory for Ductile Material

✓ The idea of Mohr is based on three "simple" tests: tension, compression, and shear, to yielding if the material can yield, or to rupture. It is easier to define shear yield strength as S_{sy} than it is to test for it.

✓ Three Mohr circles, one for the uniaxial compression test, one for the test in pure shear, and one for the uniaxial tension test, are used to define failure by the Mohr hypothesis. The strengths *Sc* and *St* are the compressive and tensile strengths, respectively; they can be used for yield or ultimate strength.



Coulomb-Mohr Theory for Ductile Material

$$\frac{B_2C_2 - B_1C_1}{OC_2 - OC_1} = \frac{B_3C_3 - B_1C_1}{OC_3 - OC_1}$$

$$\frac{B_2C_2 - B_1C_1}{C_1C_2} = \frac{B_3C_3 - B_1C_1}{C_1C_3}$$

$$\Rightarrow \quad \frac{\dfrac{\sigma_1 - \sigma_3}{2} - \dfrac{S_t}{2}}{\dfrac{S_t}{2} - \dfrac{\sigma_1 + \sigma_3}{2}} = \frac{\dfrac{S_c}{2} - \dfrac{S_t}{2}}{\dfrac{S_t}{2} + \dfrac{S_c}{2}} \quad \Rightarrow \quad \frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

Case 1: $\sigma_A \geq \sigma_B \geq 0$. For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$.

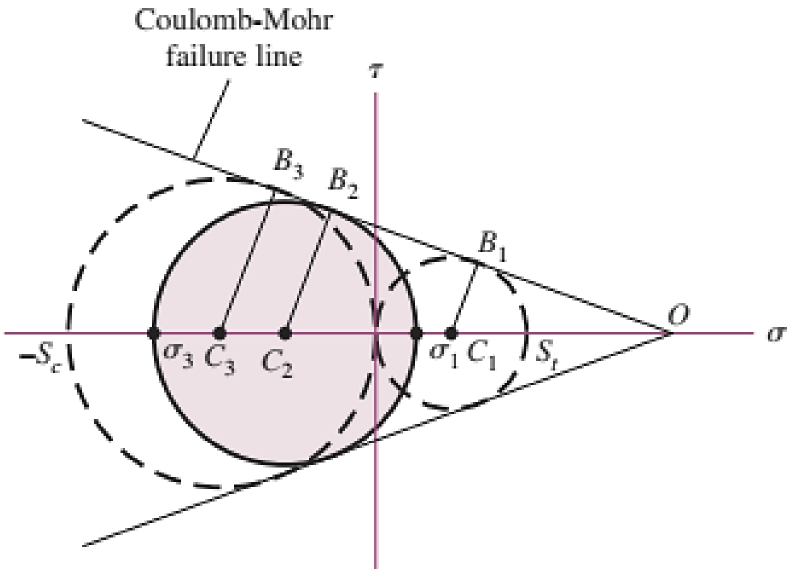
$$\sigma_A \geq S_t$$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$. Here, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$

$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1$$

Case 3: $0 \geq \sigma_A \geq \sigma_B$. For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$

$$\sigma_B \leq -S_c$$

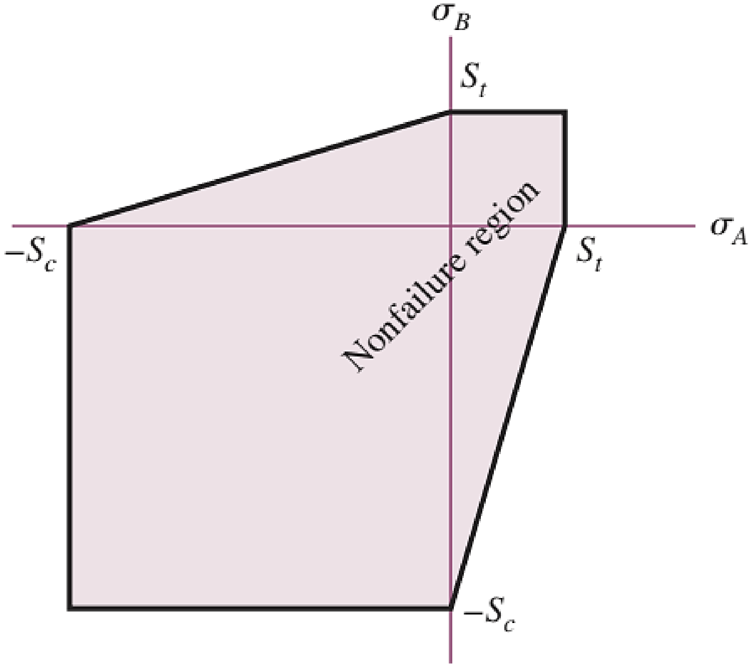


Coulomb-Mohr Theory for Ductile Material

For design equations, incorporating the factor of safety *n*, divide all strengths by *n*.
For example, the design equation can be written as:

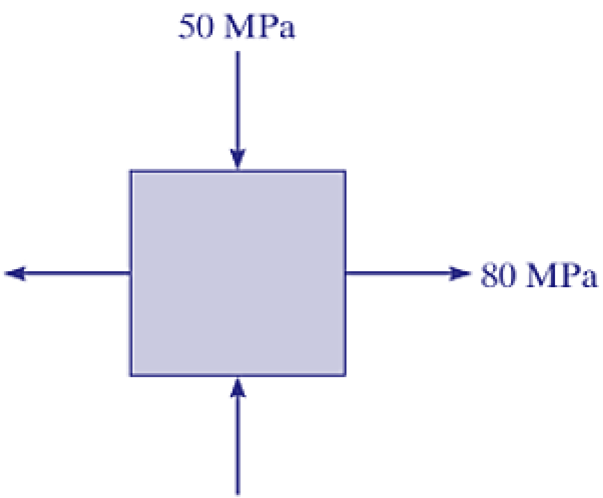
$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

$$S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}}$$



Example 1

The principal plane stresses acting on a differential element are shown. If the material is machine steel having a yield stress of σY = 700 MPa, determine the factor of safety with respect to yielding if the maximum-shear-stress theory is considered.



Solution:

- In plane principal stresses: Since no shear acts on the element in its orientation

$$\sigma_1 = \sigma_x = 80MPa \qquad \sigma_2 = \sigma_y = -50MPa$$

- The max absolut shear stress in the material:

$$\tau_{\substack{abs\\max}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{80-(-50)}{2} = 65MPa$$

- The yeilding shear strength of material:

$$\tau_y = \frac{\sigma_Y}{2} = \frac{700}{2} = 350MPa$$

- Thus the factor of safety is:

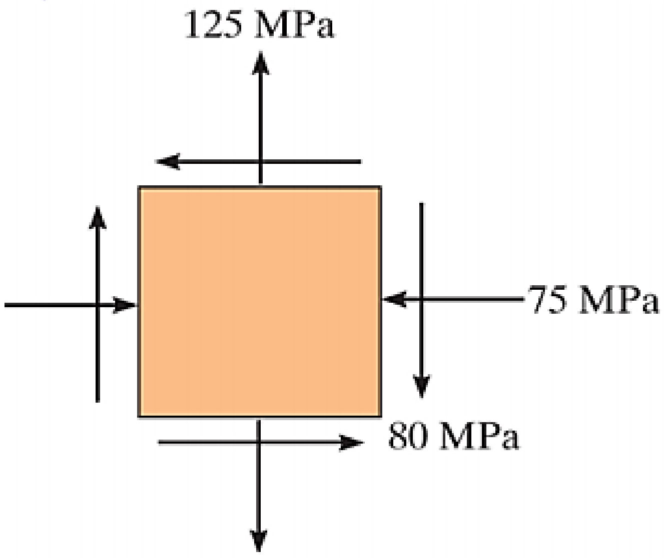
$$F.S = \frac{\tau_y}{\tau_{\substack{abs\\max}}} = \frac{350}{65} = 5.38 \qquad \text{ans}$$

Example 2

The components of plane stress at a critical point an A-36 structural steel shell with σY = 250 MPa) are shown. Determine if failure (yielding) has occurred on the basis of:

a) the maximum-shear-stress theory.

b) the maximum-distortion-energy theory.



Solution

- In plane Principal Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-75+125}{2} \pm \sqrt{\left(\frac{-75\text{-}125}{2}\right)^2 + (-80)^2} = 24.00 \pm 128.06$$

$$\sigma_1 = 153.06 \qquad \sigma_2 = -103.06\text{MPa}$$

- Maximum Shear Stress Theory: σ_1 and σ_2 has opposite sign

$$(\sigma_1 - \sigma_2) = 153.06 - (-103.06) = 256.12 > \sigma_Y$$

- Thus the material will yield according to the max shear stress theory.
- Maximum Distortion Energy Theory:

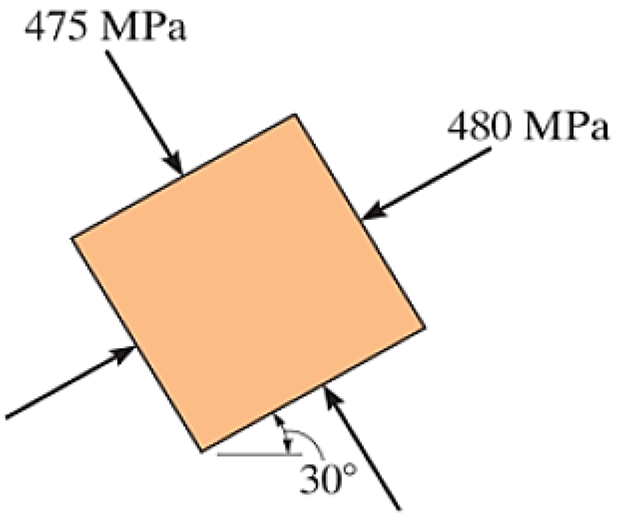
$$\sigma_1^{\ 2} - \sigma_1\sigma_2 + \sigma_2^{\ 2} \le \sigma_Y^{\ 2}$$

$$(153.06)^2 - 153.06 \times (-103.06) + (-103.06)^2 = 49825 \le 62500$$

- Thus the material will not yield according to the Maximum Distortion Energy Theory.

Example 3

The principal plane stresses acting on a differential element are shown in the figure. If the material is machine steel having a yield stress of σY = 700 MPa, determine the factor of safety with respect to yielding using the maximum-distortion-energy theory



Solution:

- In plane principal stresses: Since no shear acts on the element in its orientation

$$\sigma_1 = \sigma_y = -475 MPa \qquad \sigma_2 = \sigma_x = -480 MPa$$

- Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$(-474)^2 - -474 \times (-480) + (-480)^2 = \sigma_{allow}^2$$

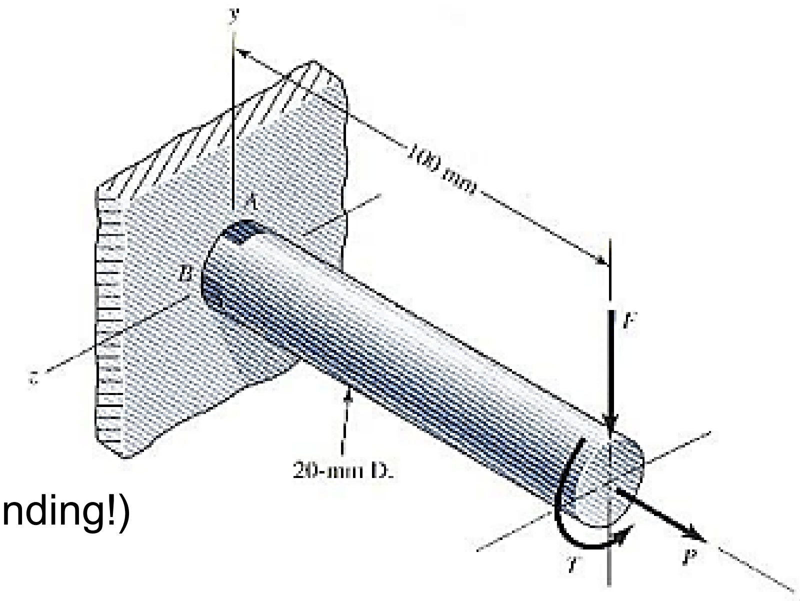
$$\sigma_{allow} = 477.52 MPa$$

- Thus the factor of safety is:

$$F.S = \frac{\sigma_y}{\sigma_{allaw}} = \frac{700}{477.52} = 1.47 \qquad \text{ans}$$

Example 4

- **Given:**
  - Bar is AISI 1020 hot-rolled steel, a DUCTILE material
  - *F* = 0.55 kN
  - *P* = 8.0 kN
  - *T* = 30 N.m



- **Find:**
  - Factor of safety F.S.

- **Two areas of interest:**
  - *A*
    - Top – where max normal stress is seen (bending!)
  - *B*
    - Side – where max shear stresses seen

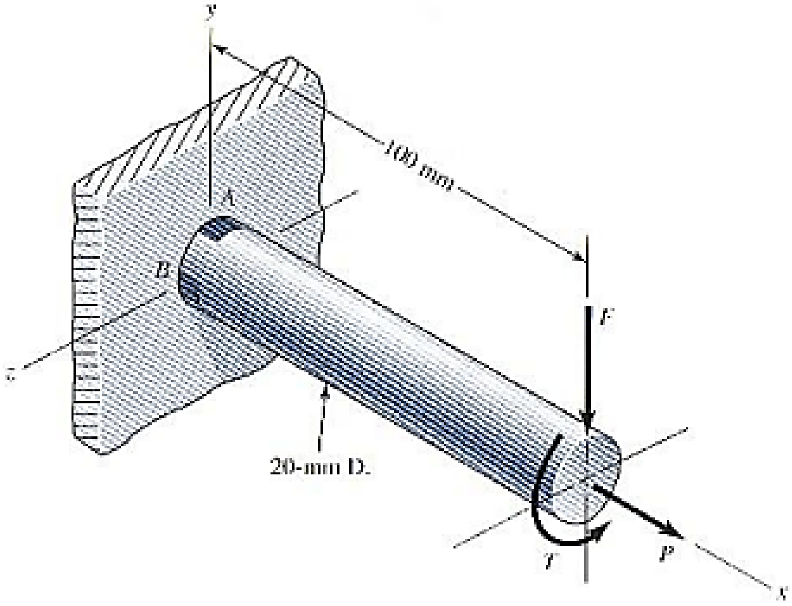
Solution

Element *A*

- Consider the types of loading we have
- Axial?
    - Yes – due to *P*

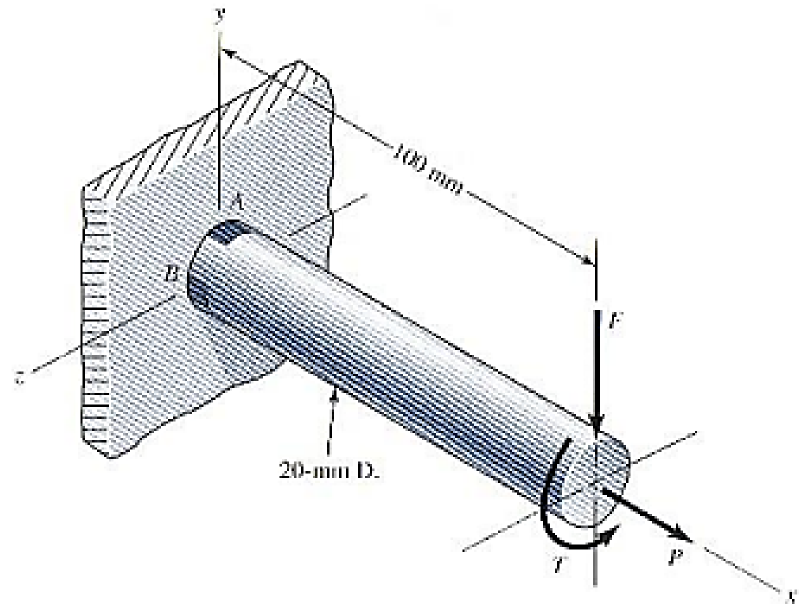
- Bending?
    - Recall that bending produces σ and τ, depending on the element of interest
    - Yes – due to *M* (σ at *A*, τ at *B*)

- Torsion?
    - Yes – due to *T*



Solution

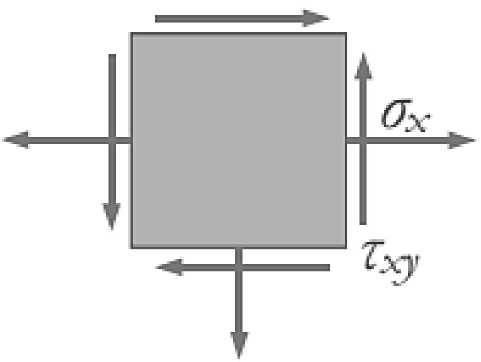
- Calculate stresses due to each load
- Axial: $$\sigma_x = \frac{P}{A} = \frac{P}{\left(\frac{\pi D^2}{4}\right)} = \frac{4P}{\pi D^2}$$
- Bending: $$\sigma_x = \frac{My}{I} = \frac{(FL)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{64}\right)} = \frac{32FL}{\pi D^3}$$
- Shear: $\tau_{xy} = 0$
- Torsion: $$\tau_{xz} = \frac{Tc}{J} = \frac{(T)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{32}\right)} = \frac{16T}{\pi D^3}$$



Solution

- Look at a stress element
- Sum up stresses due to all the loads
- $$\sigma_x = \frac{4P}{\pi D^2} + \frac{32FL}{\pi D^3} = \frac{4PD + 32FL}{\pi D^3}$$

$$\tau_{xz} = \frac{16T}{\pi D^3}$$



$\sigma_x = 95.5$ MPa

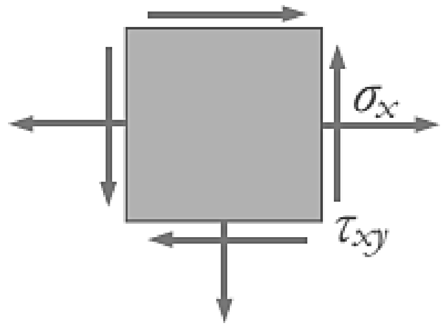
$\tau_{xz} = 19.1$ MPa

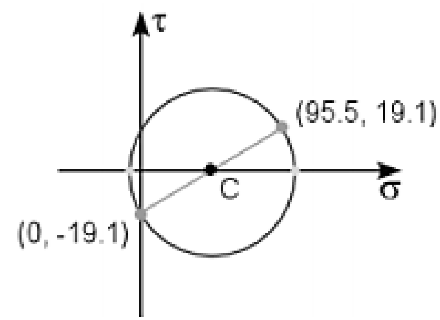
Solution

- Draw Mohr's Circle with the stresses that we calculated
  - ✓ $\sigma_x = 95.5$ MPa
  - ✓ $\tau_{xz} = 19.1$ Mpa

  - x at (σ_x, τ_{xz})
    - (95.5, 19.1)
  - y at (σ_y, τ_{zx})
    - $(\sigma_y, -\tau_{xz})$
    - (0, -19.1)
  - Find C
    - $$\left(\frac{\sigma_x+\sigma_y}{2},0\right)=\left(\frac{95.5-0}{2},0\right)=(47.8,0)$$
  - Find radius

$$R=\sqrt{(\sigma_x-C_x)^2+\tau_{xz}^{\ 2}}=\sqrt{(95.5-47.8)^2+19.1^2}=51.4$$





Solution

Out of Plane Maximum Shear for
Biaxial State of Stress:

Case 1

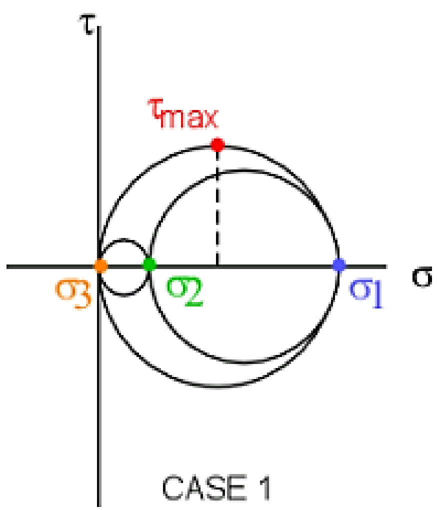
- $\sigma_{1,2} > 0$
- $\sigma_3 = 0$
- $\tau_{max} = \dfrac{\sigma_1}{2}$

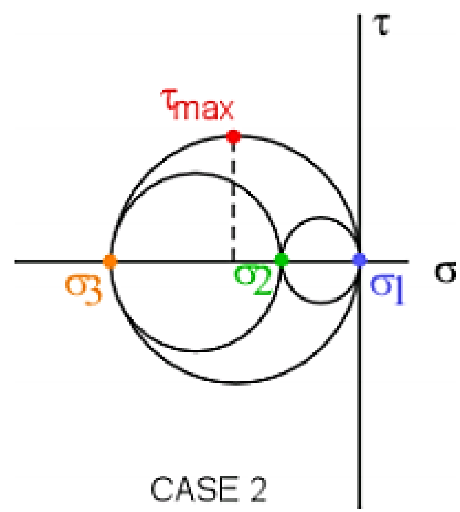
Case 2

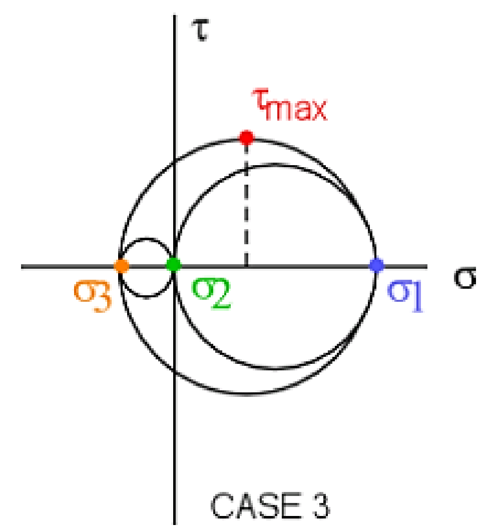
- $\sigma_{2,3} < 0$
- $\sigma_1 = 0$
- $\tau_{max} = \dfrac{|\sigma_3|}{2}$

Case 3

- $\sigma_1 > 0,\ s_3 < 0$
- $\sigma_2 = 0$
- $\tau_{max} = \dfrac{|\sigma_1 - \sigma_3|}{2}$



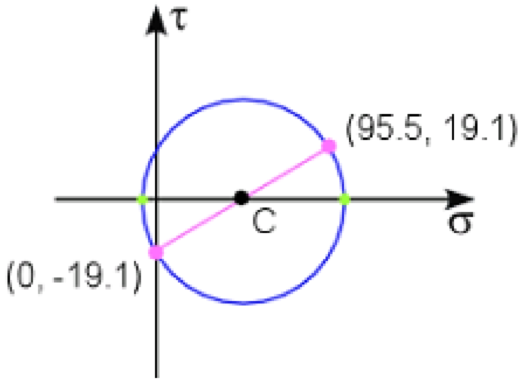


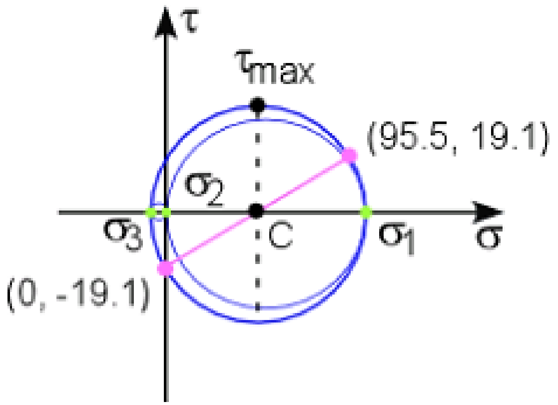


Solution

- Find principal stresses
  - σ1 = C + R
    - 99.2 MPa
  - σ2 = C - R
    - -3.63 MPa
  - Think about 3-D Mohr's Circle!
  - This is Case #3…
  - We want σ1 > σ2 > σ3
  - Assign σ3 = 0 and σ2 = -3.63 Mpa

- No failure theory was given, so use *MDE*





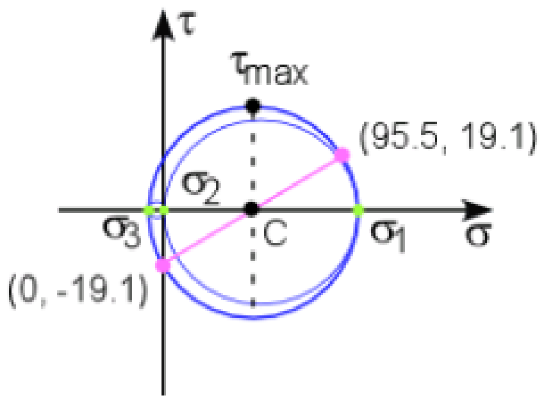
Solution

- Find the von Mises stress (σ_e)

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]}$$

$$\sigma_e = \sqrt{\frac{1}{2}\left[(99.2 - 0)^2 + (0 + 3.63)^2 + (99.2 + 3.63)^2\right]}$$

$$\sigma_e = 101 MPa$$

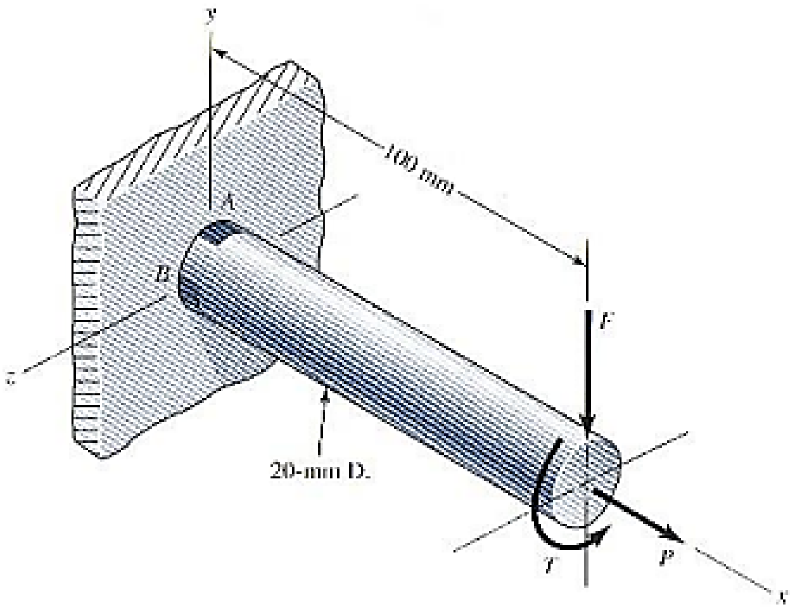


- σ_y for our material = 331 MPa
- Calculate the factor of safety
  - $F.S. = \frac{\sigma_y}{\sigma_e} = \frac{331}{101} = 3.28$ For yield

Solution

Element *B*

- Consider the types of loading we have
- Axial?
  - Yes – due to *P*
- Bending?
  - Recall that bending produces σ and τ, depending on the element of interest
  - Yes – due to *M* (σ at A, τ at B)
- Torsion?
  - Yes – due to T



Solution

- Calculate stresses due to each load
- Axial:

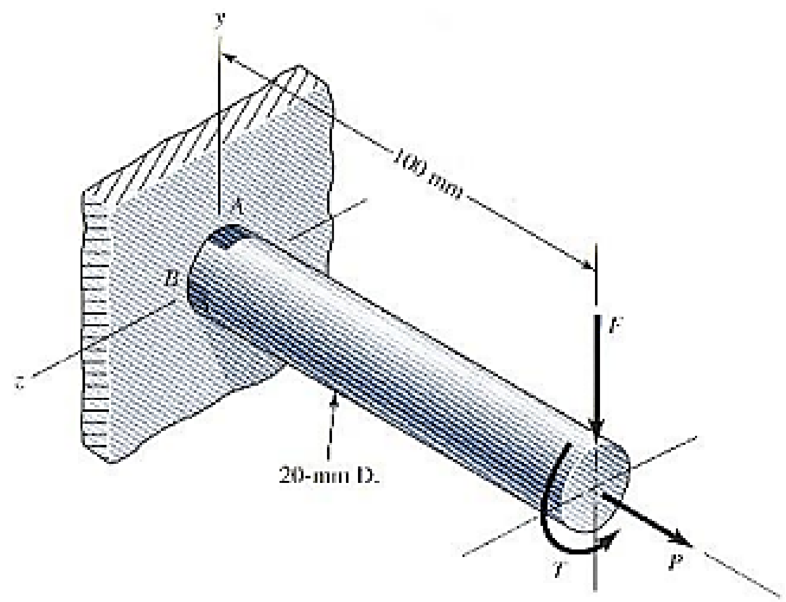
$$\sigma_x = \frac{P}{A} = \frac{P}{\left(\frac{\pi D^2}{4}\right)} = \frac{4P}{\pi D^2}$$

- Bending:
  – Use equation for round solid cross-section

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{4V}{3A} = \frac{(4F)}{3\left(\frac{\pi D^2}{4}\right)} = \frac{16F}{3\pi D^2}$$

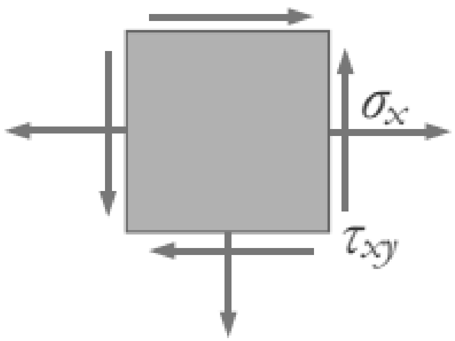
- Shear: $\tau_{xy} = 0$
- Torsion:

$$\tau_{xy} = \frac{Tc}{J} = \frac{(T)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{32}\right)} = \frac{16T}{\pi D^3}$$



Solution

- Look at a stress element
- Sum up stresses due to all the loads



$$\sigma_x = \frac{4P}{\pi D^2}$$

$$\tau_{xy} = \frac{16F}{3\pi D^2} + \frac{16T}{\pi D^3} = 19.1 + .002$$

σx = 25.5 MPa

τxz = 19.1 Mpa

- Note small contribution of shear stress due to bending!

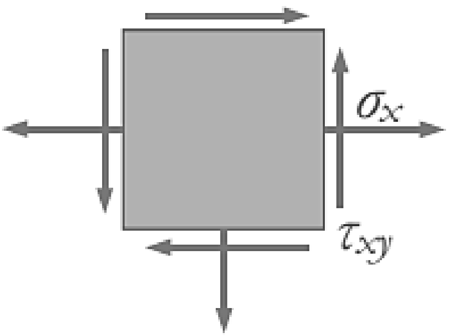
Solution

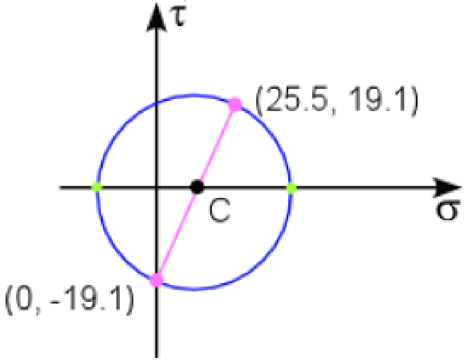
- Draw Mohr's Circle with the stresses that we calculated

  $\sigma_x = 25.5$ MPa

  $\tau_{xz} = 19.1$ MPa

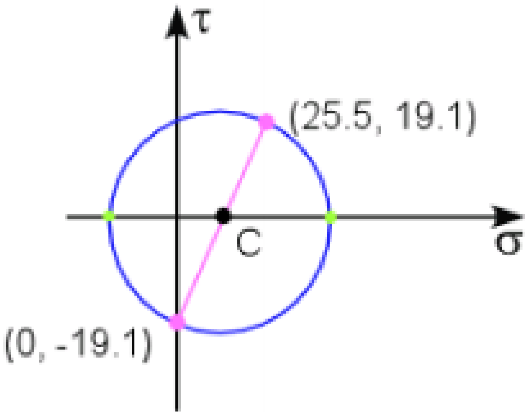
  - x at (σ_x , τ_{xz})
    - (25.5, 19.1)
  - y at (σ_y , τ_{xz})
    - $(\sigma_y , - \tau_{xz})$
    - (0, -19.1)
  - Find C
    - $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{25.5 - 0}{2}, 0\right) = (12.8, 0)$
  - Find radius
    - $R = \sqrt{(\sigma_x - C_x)^2 + \tau_{xz}^{\ 2}} = \sqrt{(25.5 - 12.8)^2 + 19.1^2} = 22.96$





Solution

- Find principal stresses
  $\sigma 1 = C + R$
  35.8 MPa
  $\sigma 2 = C - R$
  -10.2 MPa
  - Think about 3-D Mohr's Circle!
  - This is Case #3…
  - We want $\sigma 1 > \sigma 2 > \sigma 3$
  - Assign $\sigma 3 = 0$ and $\sigma 2 = -10.2$ Mpa



- No failure theory was given, so again use *MDE*

Solution

- Find the von Mises stress (σ_e)

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]}$$

$$\sigma_e = \sqrt{\frac{1}{2}\left[(35.8 - 0)^2 + (0 + 10.2)^2 + (35.8 + 10.2)^2\right]}$$

$$\sigma_e = 41.8MPa$$

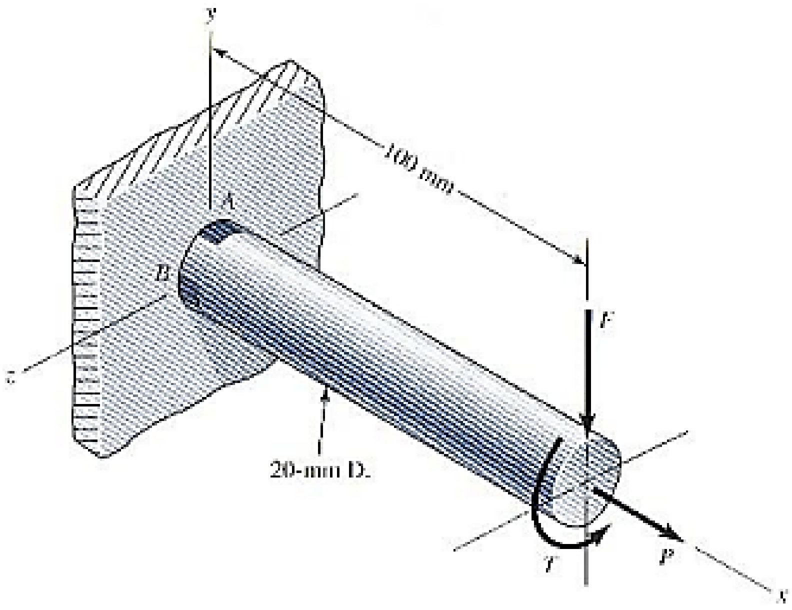
- σ_y for our material = 331 MPa
- Calculate the factor of safety

–

$$F.S = \frac{\sigma_y}{\sigma_e} = \frac{331}{41.8} = 7.91 \qquad \text{For yield}$$

Solution

- We found the factors of safety relative to each element, *A* and *B*
  - F.S @ *A* = 3.28
  - F.S @ *B* = 7.91



- A is the limiting factor of safety
  - F.S = 3.3

Example 5

A hot-rolled steel has a yield strength of $S_{yt} = S_{yc} = 100$ kpsi and a true strain at fracture of $\varepsilon_f = 0.55$. Estimate the factor of safety for the following principal stress states:

(*a*) 70, 70, 0 kpsi.
(*b*) 30, 70, 0 kpsi.
(*c*) 0, 70, -30 kpsi.
(*d*) 0, -30, -70 kpsi.
(*e*) 30, 30, 30 kpsi.

NOTE. Since $\varepsilon_f > 0.05$ and S_{yc} and S_{yt} are equal, the material is ductile and the distortion energy (DE) theory applies. The maximum-shear-stress (MSS) theory will also be applied and compared to the DE results. Note that cases a to d are plane stress states.

Solution

(a) The ordered principal stresses are $\sigma_A = \sigma_1 = 70$, $\sigma_B = \sigma_2 = 70$, $\sigma_3 = 0$ kpsi.

DE From Eq. (5–13),

$$\sigma' = [70^2 - 70(70) + 70^2]^{1/2} = 70 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{70} = 1.43$$

MSS Case 1, using Eq. (5–4) with a factor of safety,

$$n = \frac{S_y}{\sigma_A} = \frac{100}{70} = 1.43$$

(b) The ordered principal stresses are $\sigma_A = \sigma_1 = 70$, $\sigma_B = \sigma_2 = 30$, $\sigma_3 = 0$ kpsi.

DE
$$\sigma' = [70^2 - 70(30) + 30^2]^{1/2} = 60.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{60.8} = 1.64$$

MSS Case 1, using Eq. (5–4),

$$n = \frac{S_y}{\sigma_A} = \frac{100}{70} = 1.43$$

Solution

(c) The ordered principal stresses are $\sigma_A = \sigma_1 = 70$, $\sigma_2 = 0$, $\sigma_B = \sigma_3 = -30$ kpsi.

DE $$\sigma' = [70^2 - 70(-30) + (-30)^2]^{1/2} = 88.9 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{88.9} = 1.13$$

MSS Case 2, using Eq. (5–5),

$$n = \frac{S_y}{\sigma_A - \sigma_B} = \frac{100}{70 - (-30)} = 1.00$$

(d) The ordered principal stresses are $\sigma_1 = 0$, $\sigma_A = \sigma_2 = -30$, $\sigma_B = \sigma_3 = -70$ kpsi.

DE $$\sigma' = [(-70)^2 - (-70)(-30) + (-30)^2]^{1/2} = 60.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{60.8} = 1.64$$

MSS Case 3, using Eq. (5–6),

$$n = -\frac{S_y}{\sigma_B} = -\frac{100}{-70} = 1.43$$

Solution

(e) The ordered principal stresses are $\sigma_1 = 30$, $\sigma_2 = 30$, $\sigma_3 = 30$ kpsi

DE From Eq. (5–12),

$$\sigma' = \left[\frac{(30-30)^2 + (30-30)^2 + (30-30)^2}{2}\right]^{1/2} = 0 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{0} \to \infty$$

MSS From Eq. (5–3),

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{100}{30-30} \to \infty$$

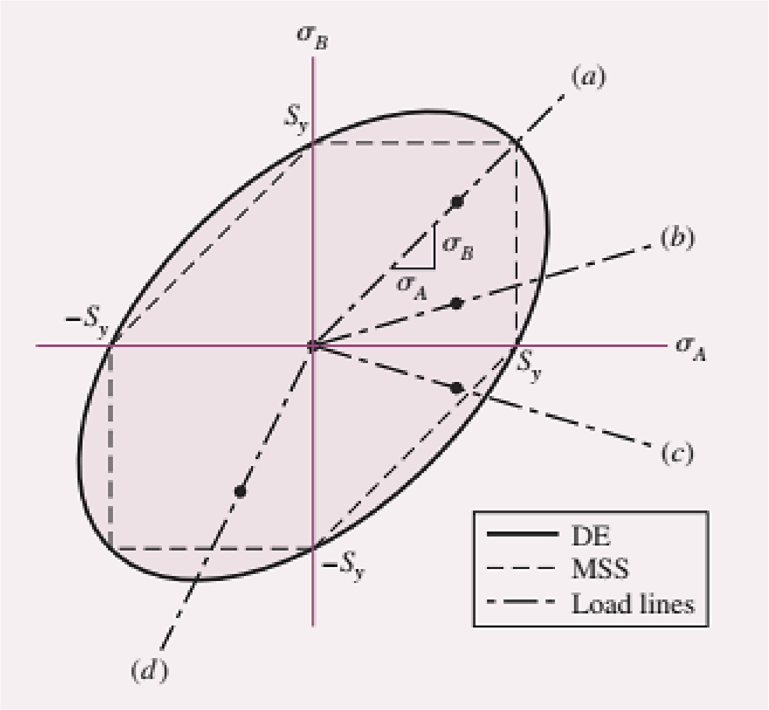
A tabular summary of the factors of safety is included for comparisons.

	(a)	(b)	(c)	(d)	(e)
DE	1.43	1.64	1.13	1.64	∞
MSS	1.43	1.43	1.00	1.43	∞

Solution

Since the MSS theory is on or within the boundary of the DE theory, it will always predict a factor of safety equal to or less than the DE theory, as can be seen in the table.

For each case, except case (*e*), the coordinates and load lines in the σ_A, σ_B plane are shown in Figure below. Case (*e*) is not plane stress. Note that the load line for case (*a*) is the only plane stress case given in which the two theories agree, thus giving the same factor of safety.



Example 6

A 25-mm-diameter shaft is statically torqued to 230 N · m. It is made of cast 195-T6 aluminum, with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. It is machined to final diameter. Estimate the factor of safety of the shaft.

The maximum shear stress is given by

$$\tau = \frac{16T}{\pi d^3} = \frac{16(230)}{\pi \left[25\left(10^{-3}\right)\right]^3} = 75\left(10^6\right)\text{N/m}^2 = 75\,\text{MPa}$$

The two nonzero principal stresses are 75 and −75 MPa, making the ordered principal stresses $\sigma_1 = 75$, $\sigma_2 = 0$, and $\sigma_3 = -75$ MPa. From Eq. (5–26), for yield,

$$n = \frac{1}{\sigma_1/S_{yt} - \sigma_3/S_{yc}} = \frac{1}{75/160 - (-75)/170} = 1.10$$

Alternatively, from Eq. (5–27),

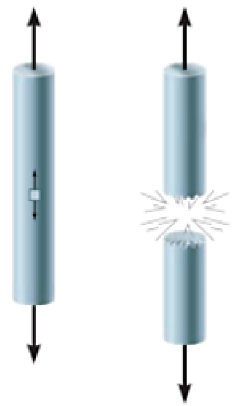
$$S_{sy} = \frac{S_{yt}S_{yc}}{S_{yt} + S_{yc}} = \frac{160(170)}{160 + 170} = 82.4\,\text{MPa}$$

and $\tau_{\max} = 75$ MPa. Thus,

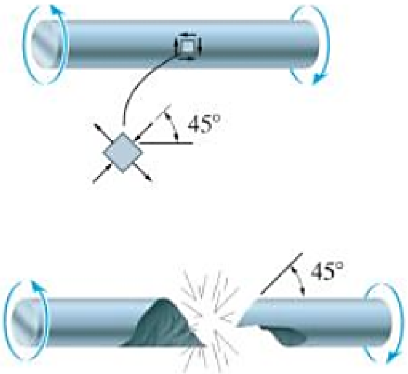
$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{82.4}{75} = 1.10$$

Fracture Criteria for Brittle Materials

Brittle materials, such as grey cast iron, tend to fail suddenly by fracture with no apparent yielding. In a tension test, the fracture occurs when the normal stress reaches the Ultimate Stress σu.



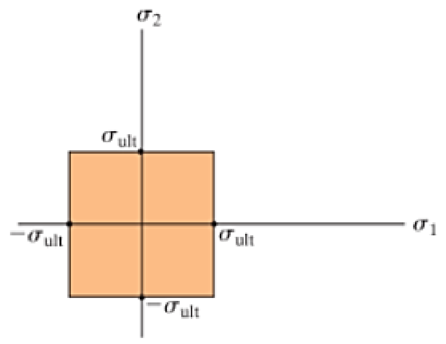
Failure of a brittle material in tension



Failure of a brittle material in torsion

Maximum Normal Stress Theory (Rankine)

$$\boxed{|\sigma_1| = \sigma_u \qquad \text{or} \qquad |\sigma_2| = \sigma_u}$$



Sample problem 1

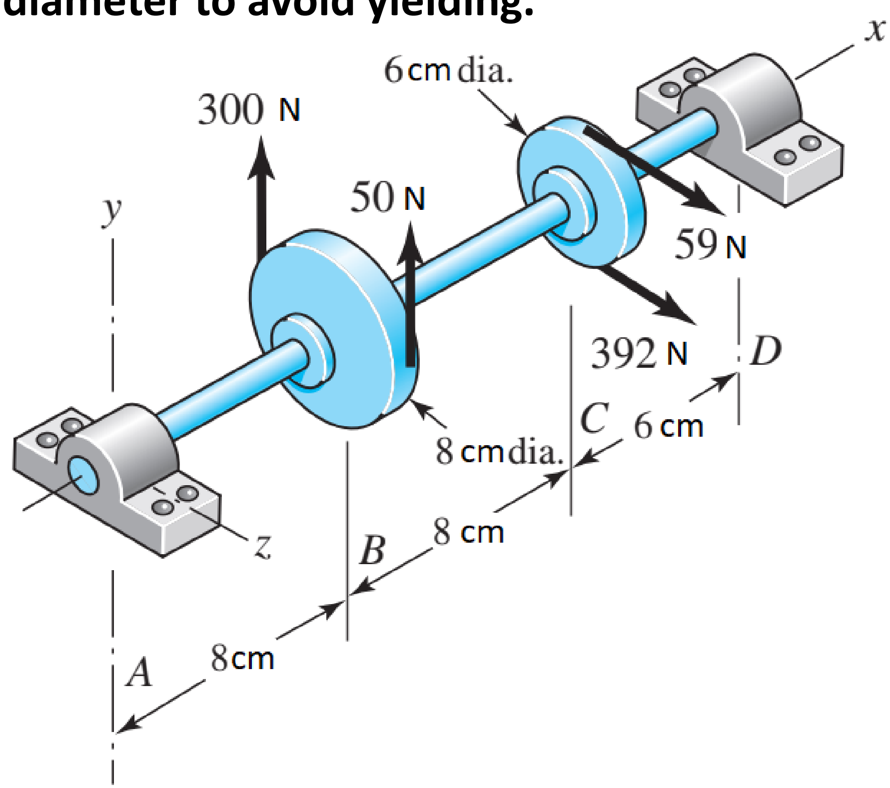
An AISI 1040 cold-drawn steel tube has an outside diameter of 50 mm and an inside diameter of 42 mm. The tube is 150 mm long and is capped at both ends. **Determine the maximum allowable internal pressure for a static factor of safety of 2 for the tube walls:**

(*a*) based on the maximum-shear-stress theory.

(*b*) based on the distortion-energy theory.

Sample problem 2

The figure shows a shaft mounted in bearings at *A* and *D* and having pulleys at *B* and *C*. The forces shown acting on the pulley surfaces represent the belt tensions. The shaft is to be made of AISI 1035 CD steel. **Using a conservative failure theory with a design factor of 2, determine the minimum shaft diameter to avoid yielding.**



End of today's lecture!