Product Development 1

Introduction to **Mechanical Engineering Design**

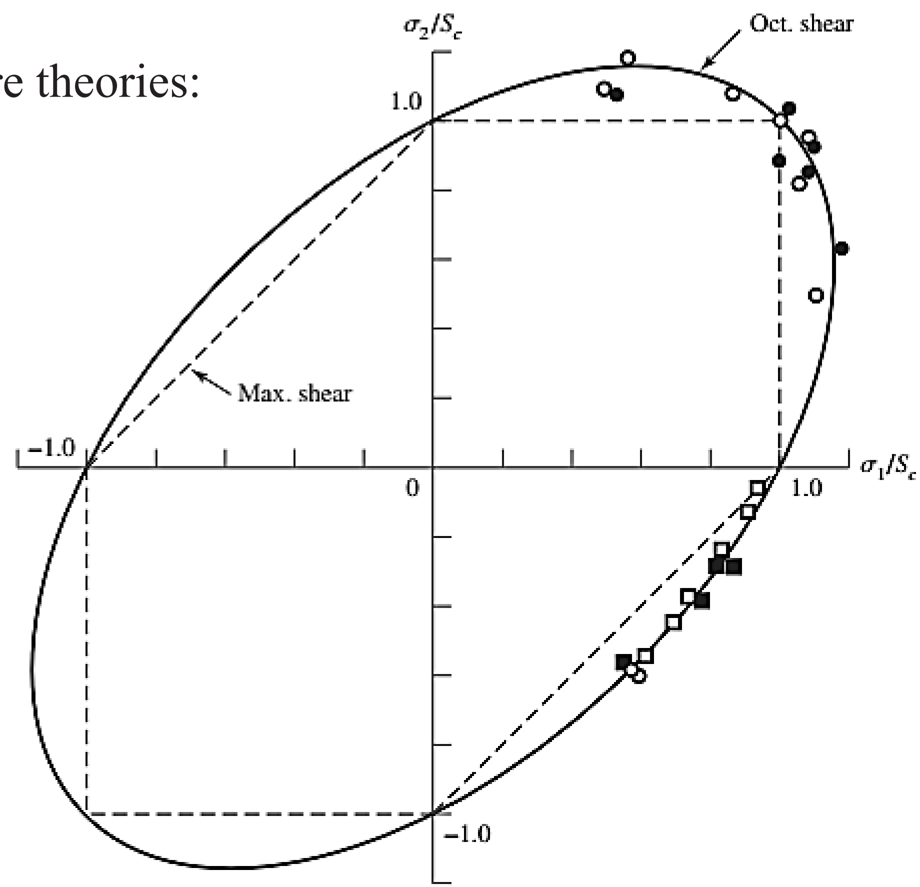
Spring 2023

Failure of Ductile Materials: A review!

Experimental data superposed on failure theories:

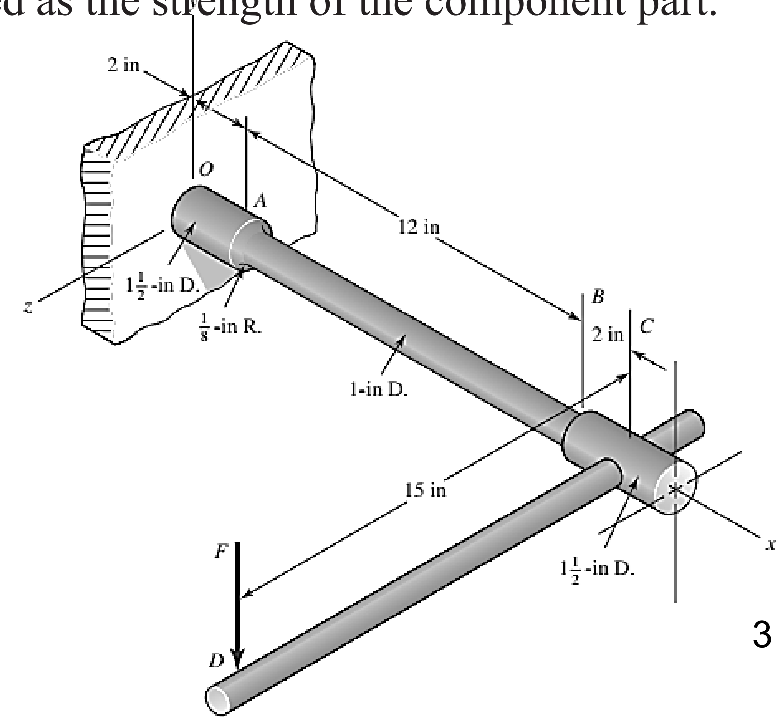
Yielding ($S_c = S_y$)

- ○ Ni-Cr-Mo steel
- • AISI 1023 steel
- □ 2024-T4 Al
- ■ 3S-H Al



Failure of Ductile Materials: A review!

A certain force *F* applied at *D* near the end of the 15-in lever shown in the figure, which is quite similar to a socket wrench, results in certain stresses in the cantilevered bar *OABC*. This bar (*OABC*) is of AISI 1035 steel, forged and heat-treated so that it has a minimum yield strength of 81 kpsi. We presume that this component would be of no value after yielding. Thus the force *F* required to initiate yielding can be regarded as the strength of the component part. Find this force.



Failure of Ductile Materials: A review!

$$\sigma_x = \frac{M}{I/c} = \frac{32M}{\pi d^3} = \frac{32(14F)}{\pi(1^3)} = 142.6F$$

$$\tau_{zx} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(15F)}{\pi(1^3)} = 76.4F$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{zx}^2\right)^{1/2} = [(142.6F)^2 + 3(76.4F)^2]^{1/2} = 194.5F$$

$$F = \frac{S_y}{194.5} = \frac{81\ 000}{194.5} = 416 \text{ lbf}$$

DE

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{\left(\frac{142.6F}{2}\right)^2 + (76.4F)^2} = 104.5F$$

$$F = \frac{81\ 000/2}{104.5} = 388 \text{ lbf}$$

MSS

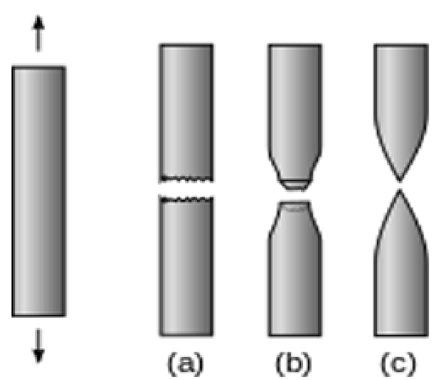
Failure of

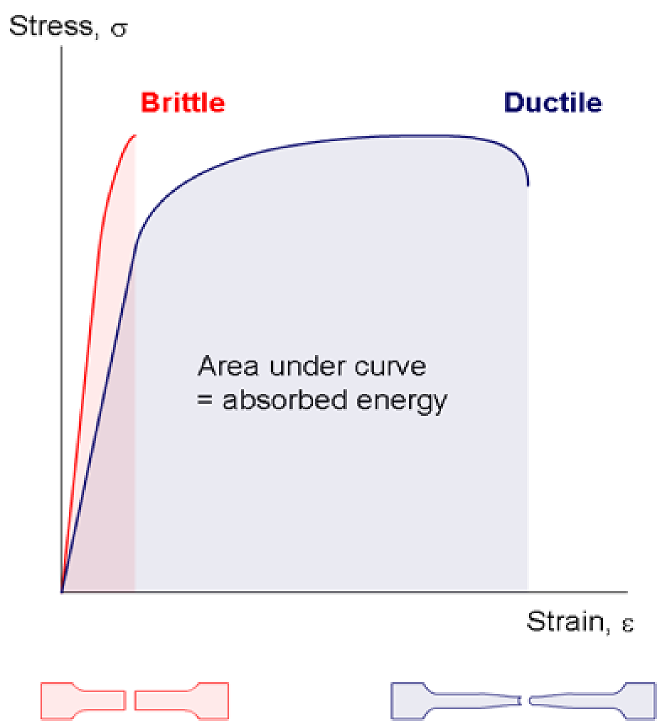
Brittle Materials

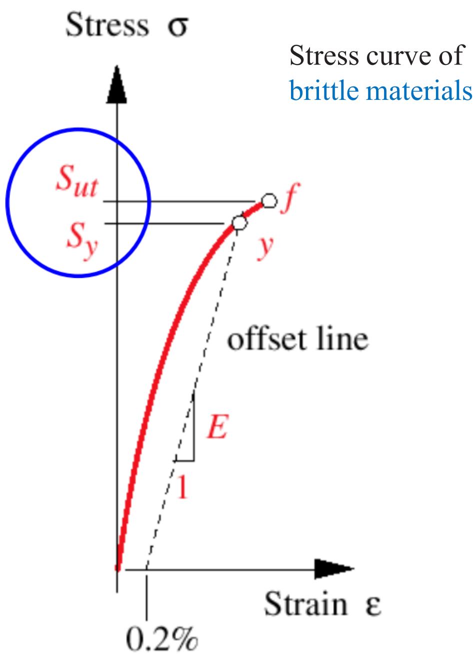
Reference: *Shigley's Mechanical Engineering Design, Ch. 5*

Introduction

Brittle materials are characterized by little deformation, poor capacity to resist impact and vibration of load, high compressive strength, and low tensile strength.







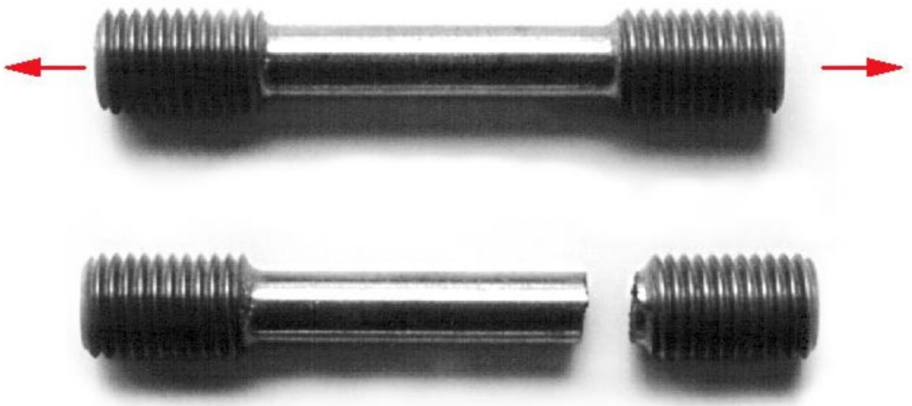
Introduction

Ductile	Brittle
Ductile materials can be drawn into wires by stretching	Brittle materials breaks, crack or snap easily
Ductile materials show deformation	Brittle materials do not show deformation
Ductility is affected by temperature	Brittleness is affected by pressure
Major examples for ductile materials are metals	Examples of brittle materials include ceramic & glass

Introduction

Accepted failure theories that apply to **brittle materials**:

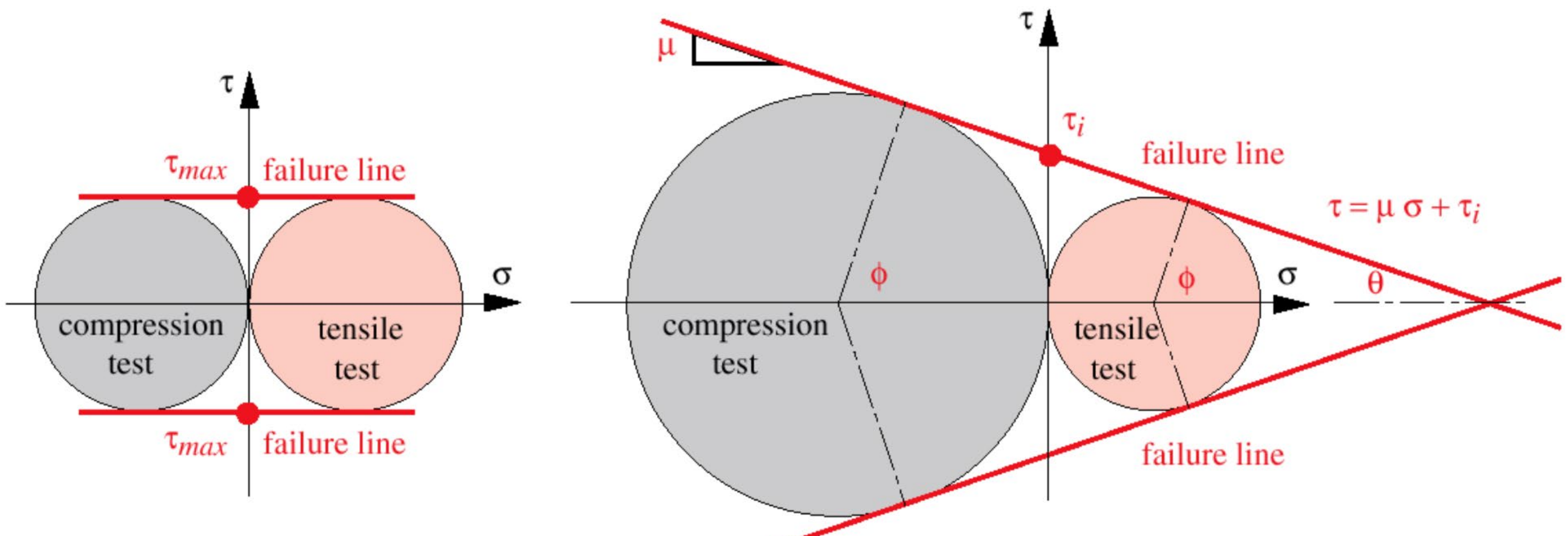
- Maximum normal stress theory (even material)
- Maximum normal stress theory (uneven material)
- Coulomb-Mohr theory
- Modified Mohr theory



Introduction

Brittle materials even and uneven material:

Mohr's circles for both compression and tensile tests showing the failure envelopes for (a) even and (b) uneven materials.



(a) An even material — $S_{uc} = -S_{ut}$ (b) An uneven material — $|S_{uc}| > |S_{ut}|$

Maximum-Normal Stress Theory for Brittle Materials

The maximum-normal-stress (MNS) theory states that *failure occurs whenever one of the three principal stresses equals or exceeds the strength*. Again we arrange the principal stresses for a general stress state in the ordered form $\sigma_1 \geq \sigma_2 \geq \sigma_3$. This theory then predicts that failure occurs whenever:

$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc}$$

where S_{ut} and S_{uc} are the ultimate tensile and compressive strengths, respectively, given as positive quantities.

For plane stress, with the principal stresses given by Eq.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum-Normal Stress Theory for Brittle Materials

with $\sigma_A \geq \sigma_B$,

$$\sigma_A \geq S_{ut} \qquad \text{or} \qquad \sigma_B \leq -S_{uc}$$

As before, the failure criteria equations can be converted to design equations. We can consider two sets of equations where $\sigma_A \geq \sigma_B$ as

$$\sigma_A = \frac{S_{ut}}{n} \qquad \text{or} \qquad \sigma_B = -\frac{S_{uc}}{n}$$

Modifications of the Mohr Theory for Brittle Materials

Two modifications of the Mohr theory for brittle materials:

1- The Brittle-Coulomb-Mohr (BCM) theory

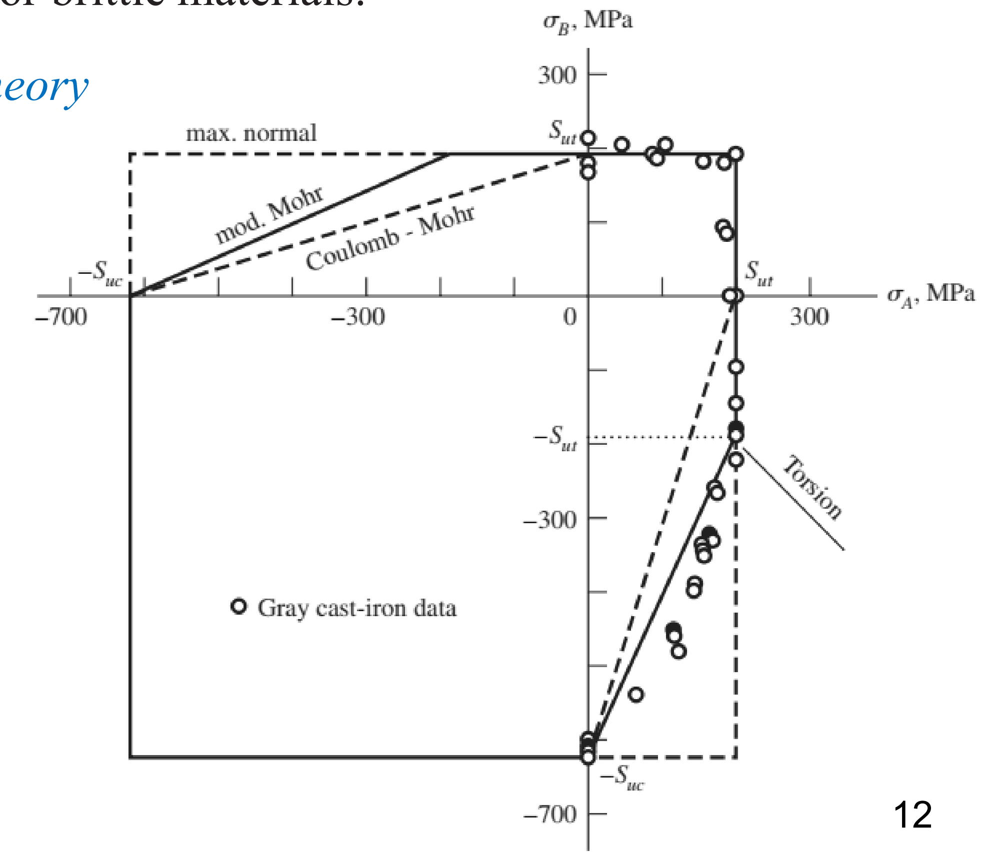
2- The modified Mohr (MM) theory

Brittle-Coulomb-Mohr

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$



Modifications of the Mohr Theory for Brittle Materials

Modified Mohr

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

Data are still outside this extended region. The straight line introduced by the modified Mohr theory,

for $\sigma_A \geq 0 \geq \sigma_B$ and $|\sigma_B/\sigma_A| > 1$,

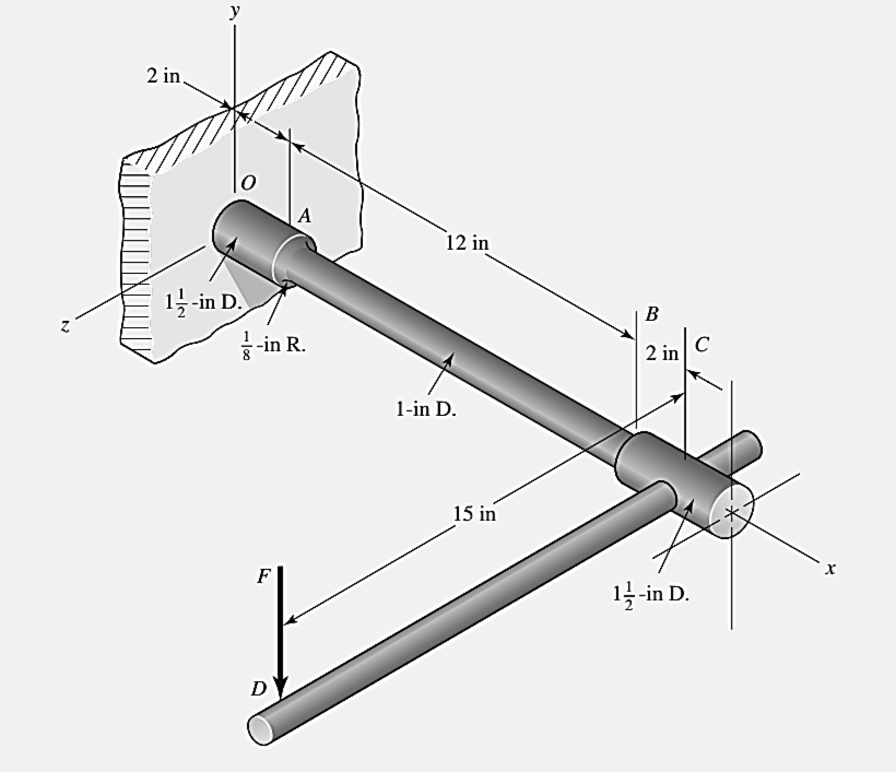
can be replaced by a parabolic relation which can more closely represent some of the data.

Example 1

Consider the wrench in this Fig, as made of cast iron, machined to dimension. The force F required to fracture this part can be regarded as the strength of the component part. If the material is ASTM grade 30 cast iron, find the force F with:

(a) Coulomb-Mohr failure model.

(b) Modified Mohr failure model.



Solution

Mechanical Properties of Three Non-Steel Metals

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi Tension†	Modulus of Elasticity, Mpsi Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

Solution (continued)

$$\sigma_x = K_t \frac{M}{I/c} = K_t \frac{32M}{\pi d^3} = (1)\frac{32(14F)}{\pi(1)^3} = 142.6F$$

$$\tau_{xy} = K_{ts} \frac{Tr}{J} = K_{ts} \frac{16T}{\pi d^3} = (1)\frac{16(15F)}{\pi(1)^3} = 76.4F$$

The nonzero principal stresses σ_A and σ_B are

$$\sigma_A, \sigma_B = \frac{142.6F + 0}{2} \pm \sqrt{\left(\frac{142.6F - 0}{2}\right)^2 + (76.4F)^2} = 175.8F, -33.2F$$

This puts us in the fourth-quadrant of the σ_A, σ_B plane.
(*a*) For BCM, with $n = 1$ for failure.

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{175.8F}{31(10^3)} - \frac{(-33.2F)}{109(10^3)} = 1$$

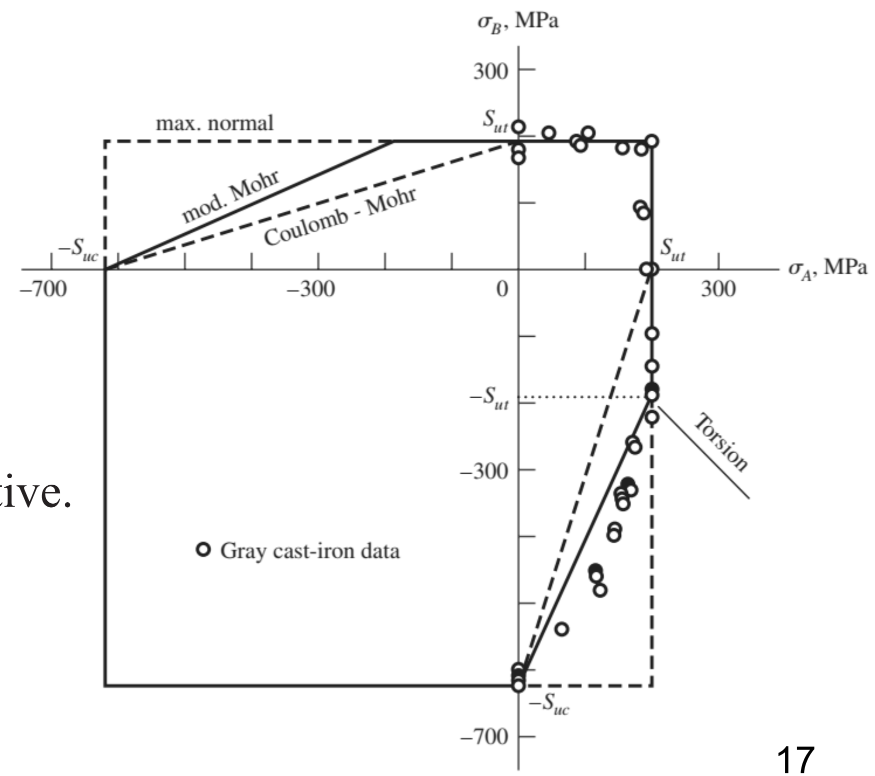
Solution (continued)

Solving for *F* yields: $F = 167\ lbf$

(*b*) For MM, the slope of the load line is $|\sigma_B/\sigma_A| = 33.2/175.8 = 0.189 < 1$.

$$\frac{\sigma_A}{S_{ut}} = \frac{175.8F}{31(10^3)} = 1$$

$$F = 176\ \text{lbf}$$

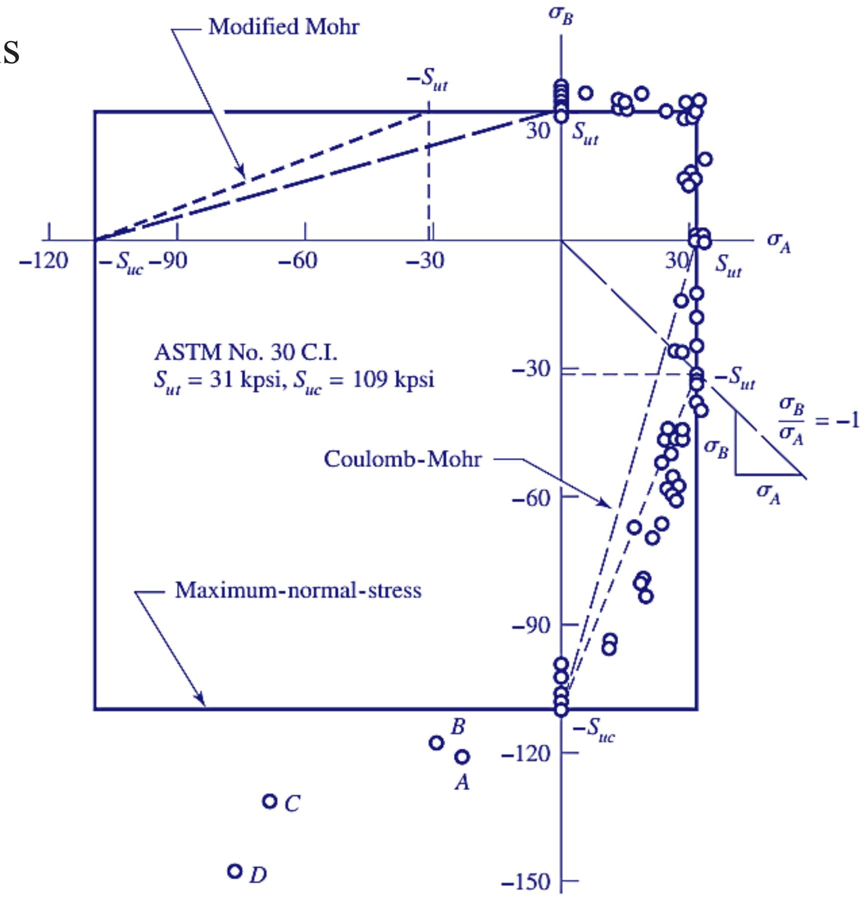


As one would expect from inspection of this Fig, Coulomb-Mohr is more conservative.

Failure of Brittle Materials: Review!

- Failure or strength of brittle materials is related to those materials whose true strain at fracture is 0.05 or less.

- ductile materials may develop a brittle fracture or crack if used below the transition temperature.



Failure of Brittle Materials: Review!

- In the first quadrant the data appear on both sides and along the failure curves of maximum-normal-stress, Coulomb-Mohr, and modified Mohr. All failure curves are the same, and data fit well.

- In the fourth quadrant the modified Mohr theory represents the data best, whereas the maximum-normal-stress theory does not.

- In the third quadrant the points *A*, *B*, *C*, and *D* are too few to make any suggestion concerning a fracture locus

Failure Criteria Selection

Ductile

- MSS is conservative, often used for design where higher reliability is desired
- DE is typical, often used for analysis where agreement with experimental data is desired
- If tensile and compressive strengths differ, use Ductile Coulomb-Mohr

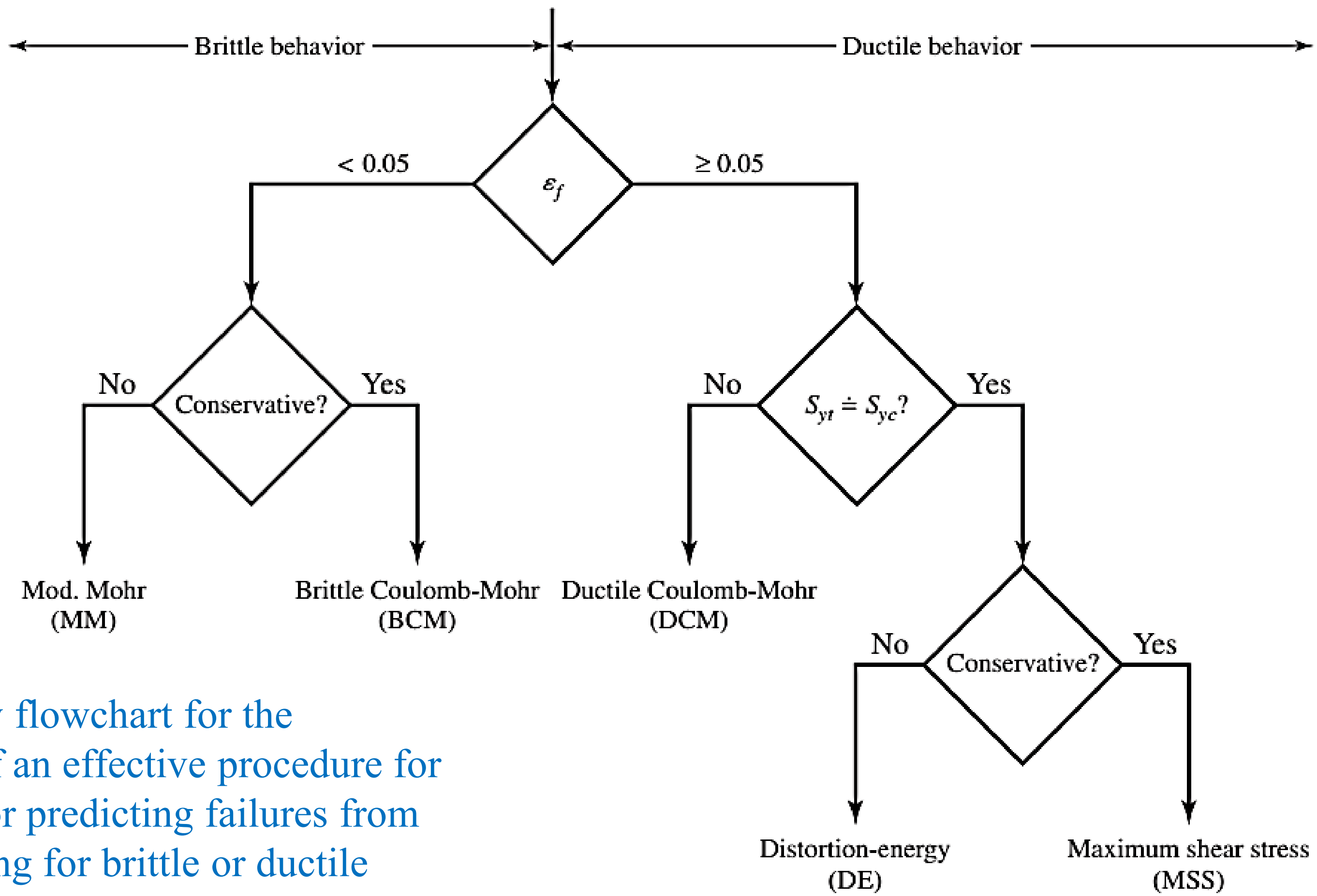
For ductile behavior the preferred criterion is the distortion-energy theory. In the rare case when $S_{yt} = S_{yc}$, the ductile Coulomb-Mohr method is employed

Brittle

- Mohr theory is best, but difficult to use
- Brittle Coulomb-Mohr is very conservative in 4th quadrant
- Modified Mohr is still slightly conservative in 4th quadrant, but closer to typical

For brittle behavior, the original Mohr hypothesis, constructed with tensile, compression, and torsion tests, with a curved failure locus is the best hypothesis we have.

Failure Criteria Selection



A summary flowchart for the selection of an effective procedure for analyzing or predicting failures from static loading for brittle or ductile behavior.

Fracture Mechanics: An Intro!

The idea that cracks exist in parts even before service begins, and that cracks can grow during service, has led to the descriptive phrase "damage-tolerant design." The focus of this philosophy is on crack growth until it becomes critical, and the part is removed from service. The analysis tool is **linear elastic fracture mechanics (LEFM).**

Linear Elastic Fracture Mechanics (LEFM) is the basic theory of fracture, originally developed by Griffith (1921 to1924) and completed in its essential form by Irwin (1957, 1958) and Rice (1968 a,b). LEFM is a highly simplified, yet sophisticated, theory that deals with sharp cracks in elastic bodies.

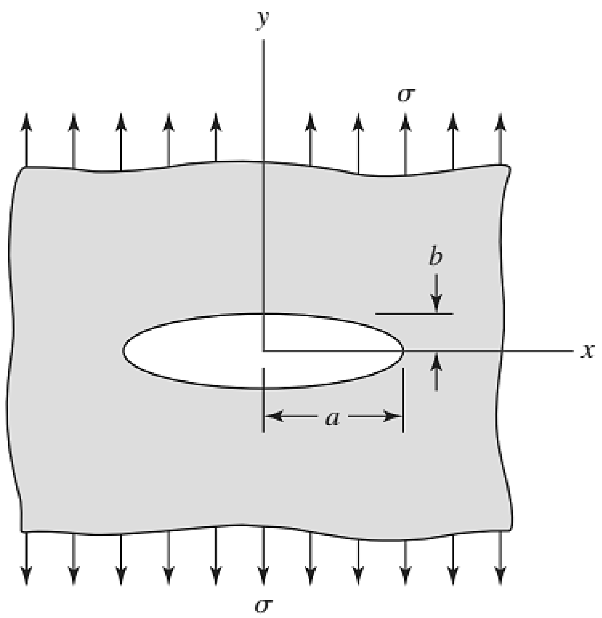
The theory of **Linear Elastic Fracture Mechanics (LEFM)** has been developed using a stress intensity factor (K) determined by the stress analysis, and expressed as a function of stress and crack size.

Fracture Mechanics: An Intro!

- ***Quasi-Static Fracture***

For the infinite plate loaded by an applied uniaxial stress σ in this figure , the maximum stress occurs at ($\pm a$, 0) and is given by:

$$(\sigma_y)_{\max} = \left(1 + 2\frac{a}{b}\right)\sigma$$



Note that when $a = b$, the ellipse becomes a circle and the above Eq. gives a stress concentration factor of 3.

Fracture Mechanics: An Intro!

The crack growth occurs when the energy release rate from applied loading is greater than the rate of energy for crack growth.

Crack growth

- Stable
- Unstable

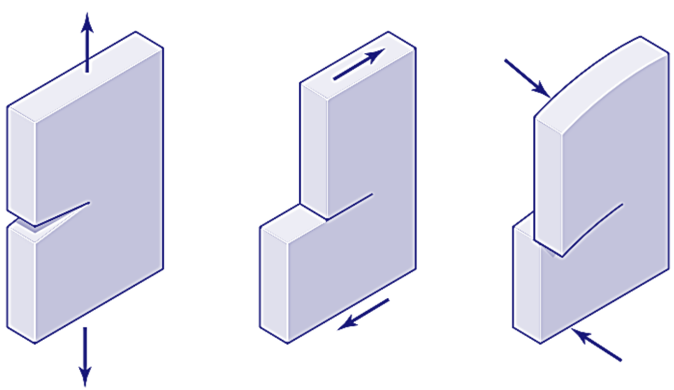
Unstable crack growth occurs when the *rate* of change of the energy release rate relative to the crack length is equal to or greater than the *rate* of change of the crack growth rate of energy.

Crack Modes and the Stress Intensity Factor

An object can be loaded in any direction relative to a crack. The sketch at the right shows a force vector at such a random orientation. It is mostly perpendicular to the crack, but also contains components that produce in-plane and out-of-plane shear.

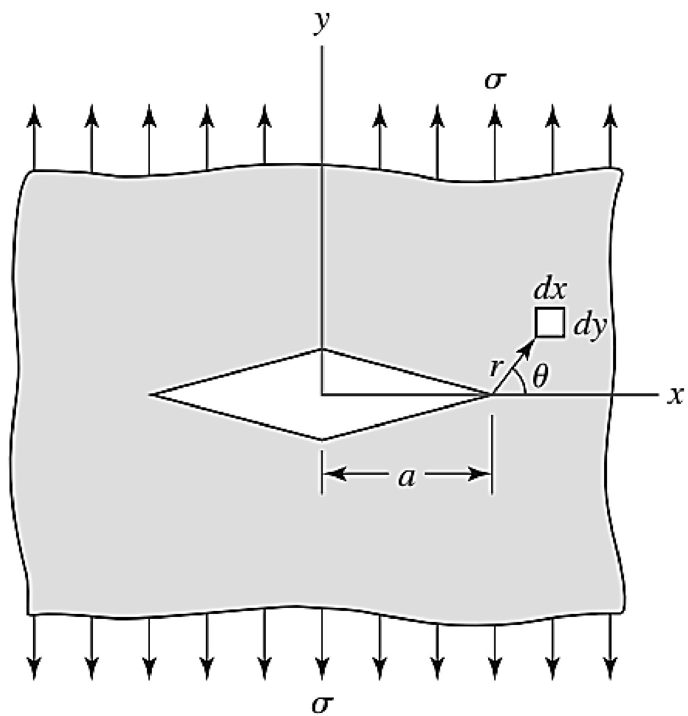
When this occurs, the logical thing to do is partition the force into its fundamental components. This process leads to the three loading modes shown below.

- Mode I, the *opening crack propagation mode* is the most common in practice
- Mode II is the *sliding mode,* is due to in-plane shear
- Mode III is the *tearing mode,* which arises from out-of-plane shear



(*a*) Mode I (*b*) Mode II (*c*) Mode III

Mode I: Opening crack propagation mode



By using complex stress functions, it has been shown that the stress field on a *dx dy* element in the vicinity of the crack tip is given by:

$$\sigma_x = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \sigma\sqrt{\frac{a}{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$$

Mode I: Opening crack propagation mode

The stress σ_y near the tip, with θ = 0, is

$$\sigma_y|_{\theta=0} = \sigma\sqrt{\frac{a}{2r}}$$ Eq. (a)

As with the elliptical crack, we see that $\sigma_y|_{\theta=0} \rightarrow \infty$ as $r \rightarrow 0$, and again the concept of an infinite stress concentration at the crack tip is inappropriate.

The quantity $\sigma_y|_{\theta=0}\sqrt{2r} = \sigma\sqrt{a}$, , however, does remain constant as $r \rightarrow 0$. It is common practice to define a factor K called the *stress intensity factor* given by

$$K = \sigma\sqrt{\pi a}$$ Eq. (b)

Mode I: Opening crack propagation mode

where the units are: $\text{MPa}\sqrt{\text{m}}$ or $\text{kpsi}\sqrt{\text{in}}$.

Since we are dealing with a mode I crack, Eq. (*b*) is written

$$K_I = \sigma\sqrt{\pi a}$$

Thus:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

Mode I: Opening crack propagation mode

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$$

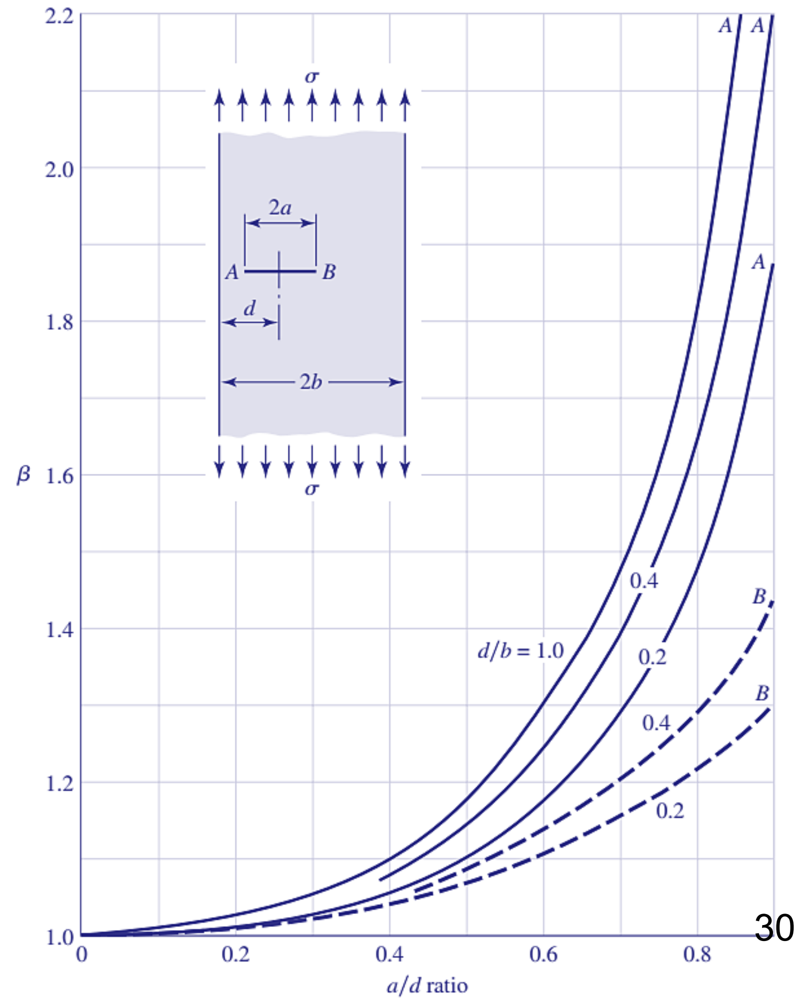
The stress intensity factor is a function of geometry, size and shape of the crack, and the type of loading. For various load and geometric configurations

$$K_I = \beta\sigma\sqrt{\pi a}$$

where β is the ***stress intensity modification factor***. Tables for β are available in the literature for basic configurations

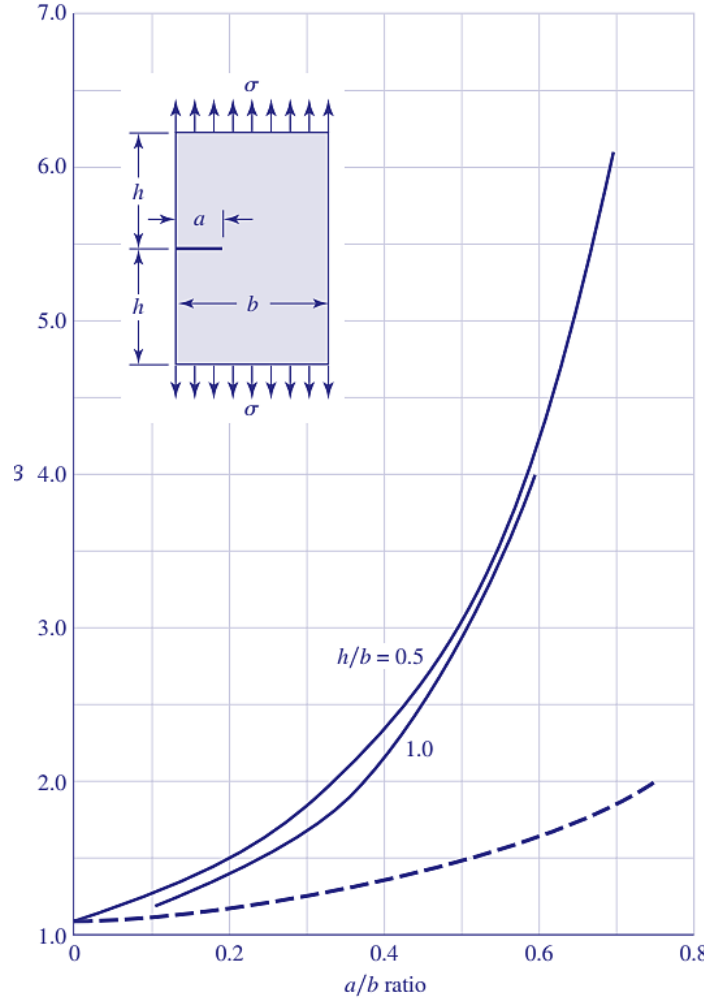
Mode I: Examples of β for crack propagation

- Off-center crack in a plate in longitudinal tension; solid curves are for the crack tip at *A*; dashed curves are for the tip at *B*.



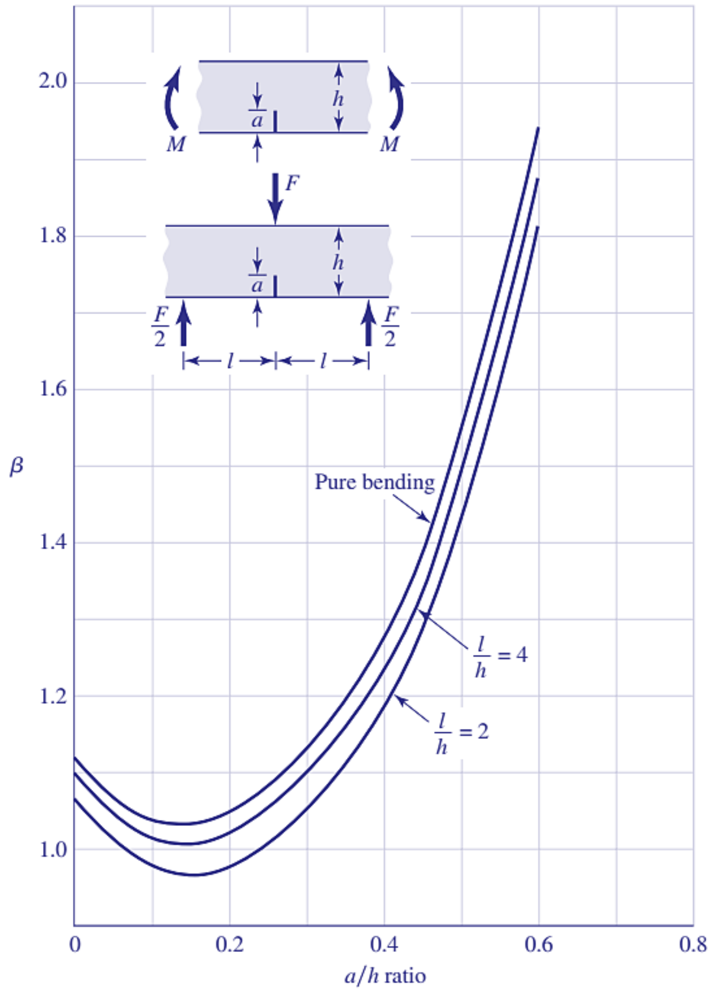
Mode I: Examples of β for crack propagation

- Plate loaded in longitudinal tension with a crack at the edge; for the solid curve there are no constraints to bending; the dashed curve was obtained with bending constraints added.

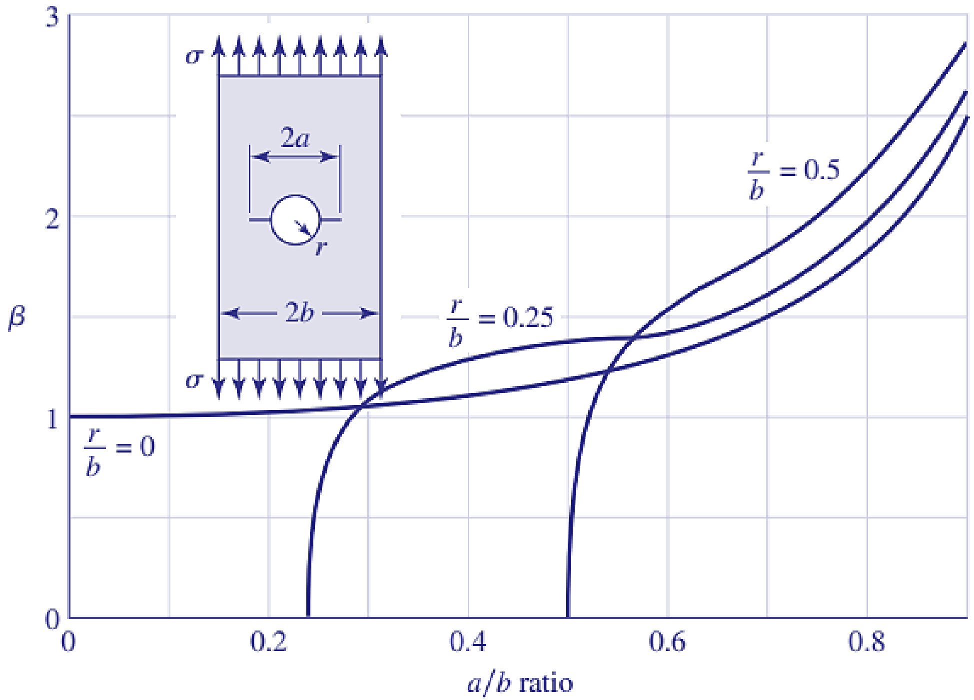


Mode I: Examples of β for crack propagation

- Beams of rectangular cross section having an edge crack

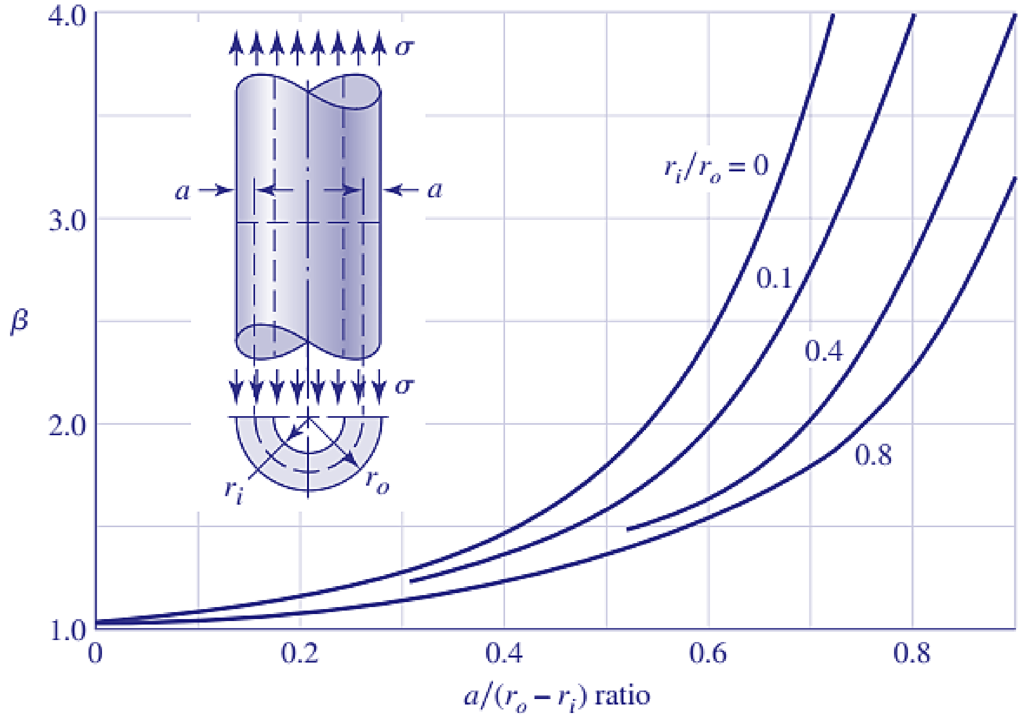


Mode I: Examples of β for crack propagation



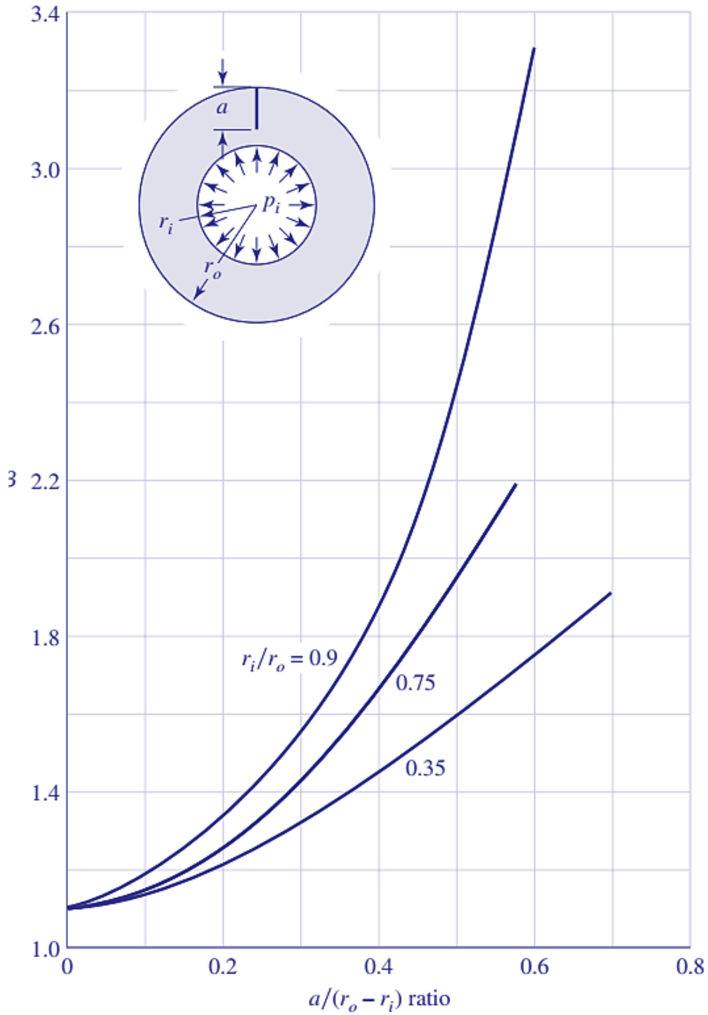
- Plate in tension containing a circular hole with two cracks

Mode I: Examples of β for crack propagation



- A cylinder loading in axial tension having a radial crack of depth *a* extending completely around the circumference of the cylinder.

Mode I: Examples of β for crack propagation



Cylinder subjected to internal pressure *p*, having a radial crack in the longitudinal direction of depth *a*.

Use Eq:

$$d = \left(\frac{64 P_{cr} l^2}{\pi^3 C E}\right)^{1/4}$$

for the tangential stress at $r = r_0$.

Fracture Toughness

When the magnitude of the mode I stress intensity factor reaches a critical value, K_{Ic}, crack propagation initiates.

The *critical stress intensity factor* $\boldsymbol{K_{Ic}}$ is a material property also called the *fracture toughness* of the material, that depends on:

- Material,
- Crack mode,
- Processing of the material,
- Temperature, loading rate,
- The state of stress at the crack site (such as plane stress versus plane strain).

The fracture toughness for plane strain is normally lower than that for plane stress. For this reason, the term K_{Ic} is typically defined as the *mode I, plane strain fracture toughness*.

Fracture Toughness

The range of fracture toughness K_{Ic}

- **Engineering metals,**

$$20 \leq K_{Ic} \leq 200 \text{ MPa}\sqrt{\text{m}};$$

- **Engineering polymers and ceramics,**

$$1 \leq K_{Ic} \leq 5 \text{ MPa}\sqrt{\text{m}};$$

- For a **4340 steel** where the yield strength due to heat treatment ranges from 800 to 1600 MPa, K_{Ic} *decreases* from 190 to 40 MPa $\cdot \sqrt{\text{m}}$;

Fracture Toughness

The strength-to-stress ratio K_{Ic}/K_I can be used as a factor of safety as

$$n = \frac{K_{Ic}}{K_I}$$

This table gives some approximate typical room-temperature values of K_{Ic} for several materials:

Material	K_{Ic}, MPa$\sqrt{m}$	S_y, MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490
Titanium		
Ti-6AL-4V	115	910
Ti-6AL-4V	55	1035
Steel		
4340	99	860
4340	60	1515
52100	14	2070

Example 2

A steel ship deck plate is 30 mm thick and 12 m wide. It is loaded with a nominal uniaxial tensile stress of 50 MPa. It is operated below its ductile-to-brittle transition temperature with K_{Ic} equal to 28.3 MPa. If a 65-mm-long central transverse crack is present, estimate the tensile stress at which catastrophic failure will occur. Compare this stress with the yield strength of 240 MPa for this steel.

Solution

with $d = b$, $2a$ = 65 mm and $2b$ = 12 m, so that d/b = 1 and a/d =65/12(10^3) = 0.00542.
Since a/d is so small, β = 1, so that

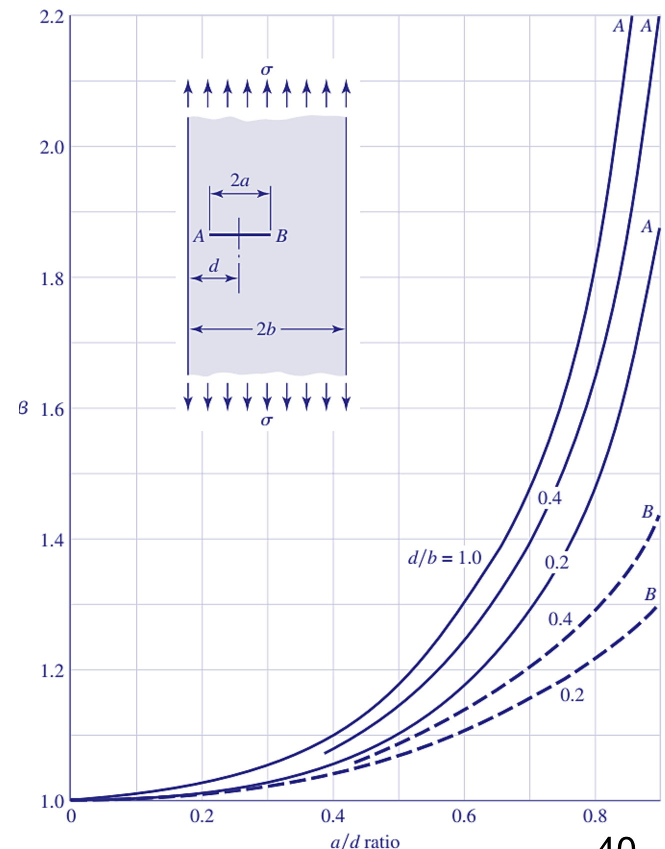
$$K_I = \sigma\sqrt{\pi a} = 50\sqrt{\pi(32.5 \times 10^{-3})} = 16.0 \text{ MPa}\sqrt{\text{m}}$$

$$n = \frac{K_{Ic}}{K_I} = \frac{28.3}{16.0} = 1.77$$

The stress at which catastrophic failure occurs is

$$\sigma_c = \frac{K_{Ic}}{K_I}\sigma = \frac{28.3}{16.0}(50) = 88.4 \text{ MPa}$$

The yield strength is 240 MPa, and catastrophic failure occurs at 88.4/240 = 0.37, or at 37 percent of yield. The factor of safety in this circumstance is
K_{Ic}/K_I = 28.3/16 = 1.77 and *not* 240/50 = 4.8.



Example 3

A plate of width 1.4 m and length 2.8 m is required to support a tensile force in the 2.8-m direction of 4.0 MN. Inspection procedures will detect only through-thickness edge cracks larger than 2.7 mm. The two Ti-6AL-4V alloys in Table below are being considered for this application, for which the safety factor must be 1.3 and minimum weight is important. Which alloy should be used?

Material	K_{Ic}, MPa$\sqrt{m}$	S_y, MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490
Titanium		
Ti-6AL-4V	115	910
Ti-6AL-4V	55	1035
Steel		
4340	99	860
4340	60	1515
52100	14	2070

Solution

(a) We elect first to estimate the thickness required to resist yielding. Since $\sigma = P/wt$, we have $t = P/w\sigma$. For the weaker alloy, we have, from the Table, $S_y = 910$ MPa.

Thus,

$$\sigma_{\text{all}} = \frac{S_y}{n} = \frac{910}{1.3} = 700 \text{ MPa}$$

$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10)^3}{1.4(700)} = 4.08 \text{ mm or greater}$$

For the stronger alloy, we have,

$$\sigma_{\text{all}} = \frac{1035}{1.3} = 796 \text{ MPa}$$

Solution (continued)

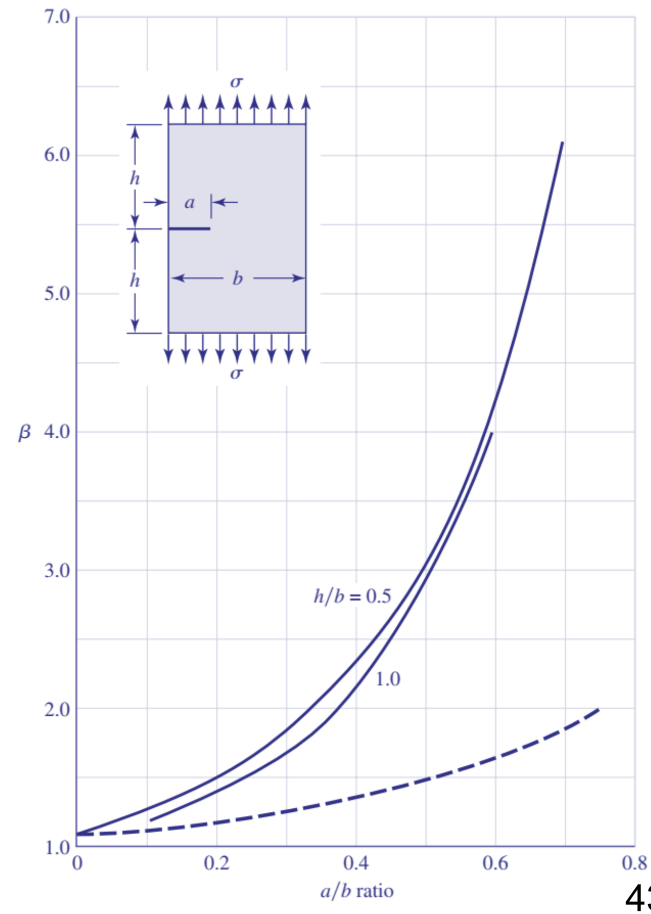
and so the thickness is

$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10)^3}{1.4(796)} = 3.59 \text{ mm or greater}$$

(b) Now let us find the thickness required to prevent crack growth. Using this Fig, we have:

$$\frac{h}{b} = \frac{2.8/2}{1.4} = 1 \qquad \frac{a}{b} = \frac{2.7}{1.4(10^3)} = 0.001\,93$$

Corresponding to these ratios we find from β =. 1.1, and $K_I = 1.1\sigma\sqrt{\pi a}$.



Solution (continued)

$$n = \frac{K_{Ic}}{K_I} = \frac{115\sqrt{10^3}}{1.1\sigma\sqrt{\pi a}}, \qquad \sigma = \frac{K_{Ic}}{1.1n\sqrt{\pi a}}$$

K_{Ic} = 115 MPa √m for the weaker of the two alloys. Solving for σ with n = 1 gives the fracture stress

$$\sigma = \frac{115}{1.1\sqrt{\pi(2.7\times10^{-3})}} = 1135 \text{ MPa}$$

which is greater than the yield strength of 910 MPa, and so yield strength is the basis for the geometry decision. For the stronger alloy S_y = 1035 MPa, with n = 1 the fracture stress is

$$\sigma = \frac{K_{Ic}}{nK_I} = \frac{55}{1(1.1)\sqrt{\pi(2.7\times10^{-3})}} = 542.9 \text{ MPa}$$

Solution (continued)

which is less than the yield strength of 1035 MPa. The thickness t is

$$t = \frac{P}{w\sigma_{\text{all}}} = \frac{4.0(10^3)}{1.4(542.9/1.3)} = 6.84 \text{ mm or greater}$$

This example shows that the fracture toughness K_{Ic} limits the geometry when the stronger alloy is used, and so a thickness of 6.84 mm or larger is required. When the weaker alloy is used the geometry is limited by the yield strength, giving a thickness of only 4.08 mm or greater. Thus the weaker alloy leads to a thinner and lighter weight choice since the failure modes differ.

Sample problem

A brittle material has the properties Sut = 210 MPa and Suc = 630 MPa.
Using the brittle Coulomb-Mohr and modified-Mohr theories, determine the factor of safety for the following states of plane stress:

(a) $\sigma_x = 170$ MPa, $\sigma_y = 100$ MPa
(b) $\sigma_x = 100$ MPa, $\sigma_y = -100$ MPa
(c) $\sigma_x = 140$ MPa, $\tau_{xy} = -70$ MPa
(d) $\sigma_x = -100$ MPa, $\sigma_y = 70$ MPa, $\tau_{xy} = -100$ MPa
(e) $\sigma_x = -140$ MPa, $\sigma_y = -140$ MPa, $\tau_{xy} = -100$ MPa

End of today’s lecture!