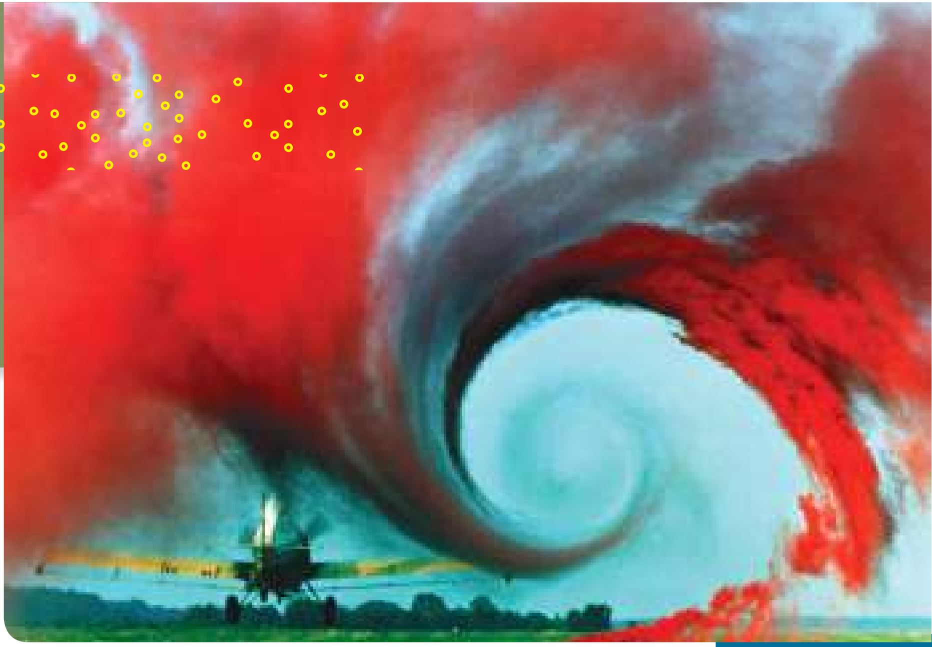
CHAPTER 1



Source: NASA/Langley Research Center (NASA-LaRC)

Physics, the Fundamental Science

Chapter Overview

The main objective of this chapter is to help you understand what physics is and where it fits in the broader scheme of the sciences. A secondary purpose is to acquaint you with the metric system of units and the advantages of the use of simple mathematics.

Chapter Outline

1. **What about energy?** What do concerns about global warming and climate change have to do with energy? How is physics involved in these discussions?
2. **The scientific enterprise.** What is the scientific method? How do scientific explanations differ from other types of explanation?
3. **The scope of physics.** What is physics? How is it related to the other sciences and to technology? What are the major subfields of physics?
4. **The role of measurement and mathematics in physics.** Why are measurements so important? Why is mathematics so extensively used in science? What are the advantages of the metric system of units?
5. **Physics and everyday phenomena.** How is physics related to everyday experience and common sense? What are the advantages of using physics to understand common experience?

Imagine that you are riding your bike on a country road on an Indian-summer afternoon. The sun has come out after a brief shower, and as the rain clouds move on, a rainbow appears in the east (fig. 1.1). A leaf flutters to the ground, and an acorn, shaken loose by a squirrel, misses your head by only a few inches. The sun is warm on your back, and you are at peace with the world around you.

No knowledge of physics is needed to savor the moment, but your curiosity may bring some questions to mind. Why does the rainbow appear in the east rather than in the west, where it may also be raining? What causes the colors to appear? Why does the acorn fall more rapidly than the leaf? Why is it easier to keep your bicycle upright while you are moving than when you are standing still?

Your curiosity about questions like these is similar to what motivates scientists. Learning to devise and apply theories or models that can be used to understand, explain, and predict such phenomena can be a rewarding intellectual game. Crafting an explanation and testing it with simple experiments or observations is fun. That enjoyment is often missed when the focus of a science course is on accumulating facts.

This book can enhance your ability to enjoy the phenomena that are part of everyday experience. Learning to produce your own explanations and to perform simple experimental tests can be gratifying. The questions posed here lie in the realm of physics, but the spirit of inquiry and explanation is found throughout science and in many other areas of human activity. The greatest rewards of scientific study are the fun and excitement that come from understanding something that has not been understood before. This is true whether we are talking about a physicist making a major scientific breakthrough or about a bike rider understanding how rainbows are formed. There are also benefits to understanding the physics concepts that underlie issues arising in political and policy debates. The next section introduces questions in the very important areas of energy use and climate change. These involve everyday phenomena of a more pressing nature than rainbows.

Figure 1.1 Rainbows appear to the east in late afternoon. How can this phenomenon be explained? (See everyday phenomenon box 17.1.) *Courtesy of Sally Cantrell Griffith*

Study Hint

If you have a clear idea of what you want to accomplish before you begin to read a chapter, your reading will be more effective. The questions in the chapter outline—as well as those in the subheadings of each section—can serve as a checklist for measuring your progress as you read. A clear picture of what questions are going to be addressed and where the answers will be found forms a mental road map to guide you through the chapter. Take a few minutes to study the outline and fix this road map in your mind. It will be time well spent.

1.1 What about Energy?

Suppose that you have just emerged from a heated argument with a friend about global warming and energy. Your friend has a different political bent than your own and you suspect that his or her opinions on global warming are simply a matter of political bias. However, since you may know very little about the details of energy issues, you are really not in a position to counter the arguments. Where do you go from there?

All of us find ourselves in this position from time to time. Energy issues lie at the heart of the political debate on global warming and climate change. Understanding the basics of these issues is important to politicians, policymakers, and ordinary citizens who discuss these issues and vote for or against ballot measures and candidates.

What is energy and how is it used? Which energy sources are renewable and which are not? What can you do to understand and coherently discuss energy issues?

How is energy involved in the global warming debate?

Much of our use of energy involves the burning of fossil fuels. The carbon that is released in this process was locked up millions of years ago in coal, oil, and natural gas. Therefore, this carbon has not been a part of ongoing processes that absorb and release carbon dioxide. From the perspective of geological time frames, this burning of fossil fuels is happening on a very short timescale. It is a geologic flash in the pan. (See fig. 1.2.)

What are the natural ongoing processes involving carbon? Trees and other green plants absorb carbon dioxide from the atmosphere—it is essential to their growth. When the plants die, they decay, releasing some carbon dioxide back to the atmosphere. Forest or brush fires release carbon dioxide to the atmosphere more quickly. A small portion of the carbon in plants may get buried and may ultimately, over a period of many millions of years, be converted to a fossil fuel. When we burn wood as a fuel, we release carbon dioxide, but this has no long-term effect on greenhouse gases, because the carbon dioxide released was absorbed from the atmosphere not too long ago. Wood burning does emit particles of ash and other pollutants that can have undesirable effects.

The reduction of forest cover to create cities, highways, and the like therefore also affects the balance of carbon dioxide in the atmosphere. But it is the burning of fossil fuels that has the greatest impact, and that is where the focus must be if we are to change the rate at which greenhouse gases are increasing. This, then, gets us into the familiar debates on how we produce energy, how we use energy, and what can be done to change these patterns.

But what is energy? Although the term is bandied about all the time and we all think we have some sense of what it means, it turns out that providing a satisfactory definition is not a trivial matter. Many of the misunderstandings involved in the global-warming debate result from poor understanding of what energy is. For example, is hydrogen a source of energy or merely a means of transporting energy, and what is the difference (see everyday phenomenon box 18.1)? Much of the political hoopla regarding the hydrogen economy failed to address this basic question.

In this book, we will define energy initially in chapter 6, titled "Energy and Oscillations." The prior chapters on mechanics provide the underpinnings for the introduction of the energy concept. In fact, it is difficult to understand how energy is defined without having some knowledge of mechanics. Following the introduction in chapter 6, energy ideas appear and are expanded in all of the chapters that follow. These ideas are central to all of physics.

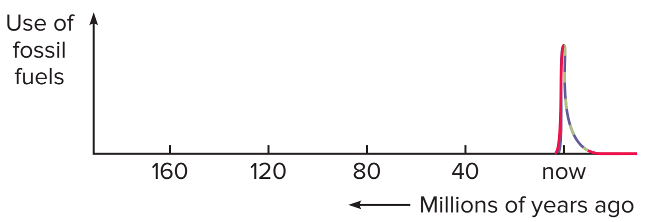


Figure 1.2 A schematic sketch of our use of fossil fuels on a geologic timescale. Coal, oil, and natural gas were produced from 40 million to 200 million years ago.

Physics and energy

Understanding the definition of energy is obviously a good starting point for discussions of energy policies. The meaning of energy and the nature of energy transformations are firmly within the realm of physics. How we convert one form of energy to another, how we can use energy efficiently, and what it means to conserve energy are all topics that will come up in this book and in the study of physics more generally.

Many other topics within the realm of physics also play important roles in addressing energy issues. For example, transportation is a major area of energy use in our society. Cars, trucks, airplanes, boats, and trains are all part of the mix. They all utilize energy in some manner, but their basic physics can be understood from ideas in mechanics that are discussed in the early chapters of this book before energy ideas are introduced.

In the short term, one of our best options for reducing our use of fossil fuels involves energy conservation. Changes can be made in this realm more quickly than in the development of alternative energy resources. The rising costs of gasoline, diesel fuel, and fuel oil for heating have already been shown to significantly affect our energy consumption. Strictly speaking, we do not really consume energy—we simply convert it to less usable forms (see chapter 6 and chapter 11). The study of the mechanics of transportation (chapters 2–4) and the thermodynamics of engines (chapters 9–11) play important roles in energy conservation.

Questions regarding choices on how to generate usable forms of energy all involve physics concepts. Is it better to use natural gas than nuclear power (fig. 1.3), for example? Nuclear power has been a particularly contentious issue for many years and has suffered somewhat from the whims of political fashion. What is nuclear energy, and should we be rushing into a new commitment to its use, or should we be afraid of going there? Natural gas releases less carbon dioxide per unit of energy generated than does coal or oil, and it is a relatively clean fuel. It is, however, an emitter of greenhouse gases, and its long-term supply is questionable.

Figure 1.3 Are nuclear power plants our salvation or relics of the past? *Steve Allen/Brand X Pictures/Alamy Stock Photo*

Nuclear power does not involve the burning of a carbon-based fuel, so it does not release carbon dioxide into the atmosphere. For this reason, it is now receiving renewed attention as a possible resource for reducing our "carbon footprint." Nuclear power does involve the mining of a limited resource, uranium, and has serious environmental issues associated with mining, possible accidents, and waste disposal. However, the utilization of any energy resource has environmental consequences, so the weighing of such issues must be an important aspect of our decision making.

We will not provide a definitive answer to the questions we have just raised. What we will do is discuss some of the basic physics underlying nuclear power, natural-gas power plants, and other resources used in electric power generation. Fossil-fuel power plants are discussed in chapter 11, and nuclear power is addressed in chapter 19. Many other means of generating energy will also be discussed, and some of the pros and cons of their use will be indicated in many sections of the book.

After studying these issues, will you win your argument with your friend? Perhaps not, but you will be in a much better position to debate the questions. Both of you may come to a better understanding of the real issues involved.

> Political debates on climate change and energy utilization are important features of current events. The two topics are intimately related, because the burning of fossil fuels for energy generation is the primary cause of release into the atmosphere of the greenhouse gas carbon dioxide. Physics is the science of energy and is therefore heavily involved in decisions on energy conversion and utilization. Thus, the study of physics provides a basis for understanding some of the fundamental issues in these debates.

1.2 The Scientific Enterprise

How do scientists go about explaining something like the temperature change of the Earth or the rainbow described in the introduction? How do scientific explanations differ from other types of explanations? Can we count on the scientific method to explain almost anything? It is important to understand what science can and cannot do.

Philosophers have devoted countless hours and pages to questions about the nature of knowledge, and of scientific knowledge in particular. Many issues are still being refined and debated. Science grew rapidly during the twentieth century and has had a tremendous impact on our lives. Innovations in medicine, communications, transportation, and computer technology all have resulted from advances in science. What is it about science that explains its impressive advances and steady expansion?

Science and rainbows

Let's consider a specific example of how a scientific explanation comes to be. Where would you turn for an explanation of how rainbows are formed? If you return from your bike ride with that question on your mind, you might look up *rainbow* in a textbook on physics or on the Internet and read the explanation found there. Are you behaving like a scientist?

The answer is both yes and no. Many scientists would do the same if they were unfamiliar with the explanation. When we do this, we appeal to the authority of the textbook author and to those who preceded the author in inventing the explanation. Appeal to authority is one way of gaining knowledge, but you are at the mercy of your source for the validity of your explanation. You are also hoping that someone has already raised the same question and done the work to create and test an explanation.

Suppose you go back three hundred years or more and try the same approach. One book might tell you that a rainbow is a painting of the angels. Another might speculate on the nature of light and its interactions with raindrops but be quite tentative in its conclusions. Either of these books might have seemed authoritative in its day. Where, then, do you turn? Which explanation will you accept?

If you are behaving like a scientist, you might begin by reading the ideas of other scientists about light and then test these ideas against your own observations of rainbows. You would carefully note the conditions when rainbows appear, the position of the sun relative to you and the rainbow, and the position of the rain shower. What is the order of the colors in the rainbow? Have you observed that order in other phenomena?

You would then invent an explanation, or **hypothesis,** using current ideas on light and your own guess about what happens as light passes through a raindrop. You could devise

Figure 1.4 French physicist and chemist Marie Curie (1867–1937) gives a lecture to an audience of men and women at the Conservatory of Arts and Crafts, Paris, 1925. Curie won Nobel prizes for both Physics (1903) and Chemistry (1911). *Jacques Boyer/Roger Viollet/Getty Images*

experiments with water drops or glass beads to test your hypothesis. (See chapter 17 for a modern view of how rainbows are formed.)

If your explanation is consistent with your observations and experiments, you could report it by giving a paper or talk to scientific colleagues. They may critique your explanation, suggest modifications, and perform their own experiments to confirm or refute your claims. If others confirm your results, your explanation will gain support and eventually become part of a broader **theory*** about phenomena involving light. The experiments you and others do may also lead to the discovery of new phenomena, which will call for refined explanations and theories.

What is critical to the process just described? First is the importance of careful observation. Another aspect is the idea of testability. An acceptable scientific explanation should suggest some means to test its predictions by observations or experiment. Saying that rainbows are the paintings of angels may be poetic, but it certainly is not testable by mere humans. It is not a scientific explanation.

Another important part of the process is a social one, the communication of your theory and experiments to colleagues (fig. 1.4). Submitting your ideas to the criticism (at times blunt) of your peers is crucial to the advancement of science. Communication is also important in assuring your own care in performing the experiments and interpreting the results. A scathing attack by someone who has found an important error or omission in your work is a strong incentive for being more careful in the future. One person working alone cannot hope to think of all the possible ramifications, alternative explanations, or potential mistakes in an argument or theory. The explosive growth of science has depended heavily on cooperation and communication.

What is the scientific method?

Is there something we can call the **scientific method** within this description, and if so, what is it? The process just described is a sketch of how the scientific method works. Although there are variations on the theme, this method is often described as shown in table 1.1.

The steps in table 1.1 are all involved in our description of how to develop an explanation of rainbows. Careful observation may lead to **empirical laws** for when and where rainbows appear. An empirical law is a generalization derived from experiments or observations. An example of an empirical law is the statement that we see rainbows with the sun at our backs as we look at the rainbow. This is an important clue for developing our hypothesis, which must be consistent with this rule. The hypothesis, in turn, suggests ways of producing rainbows artificially that could lead to experimental tests and, eventually, to a broader theory.

This description of the scientific method is not bad, although it ignores the critical process of communication. Few scientists are engaged in the full cycle that these steps suggest. Theoretical physicists, for example, may spend all of their time with step 3. Although they have some interest in experimental results, they may never do any experimental work themselves. Today, little science is done by simple observation, as step 1 may seem to imply. Most observations are designed to test a hypothesis or existing theory and often involve carefully controlled

Table 1.1

Steps in the Scientific Method

1. Careful observation of natural phenomena.
2. Formulation of rules or empirical laws based on generalizations from these observations and experiments.
3. Development of hypotheses to explain the observations and empirical laws, and the refinement of hypotheses into theories.
4. Testing of the hypotheses or theories by further experiment or observation.

*The concept of a theory, as used in science, is often misunderstood. It is much more than a simple hypothesis. A theory consists of a set of basic principles from which many predictions can be deduced. The basic principles involved in the theory are often widely accepted by scientists working in the field.

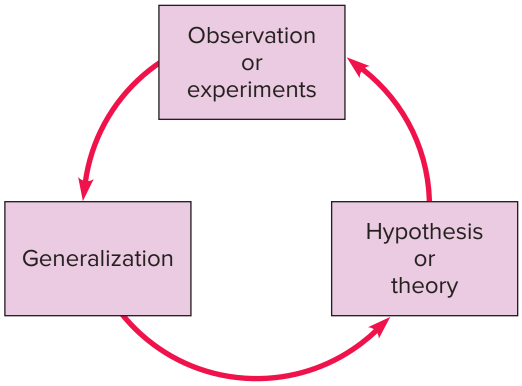


Figure 1.5 The scientific method cycles back to observations or experiments as we seek to test our hypotheses or theories. Communication with peers is involved in all stages of the process.

experiments. Although the scientific method is presented here as a stepwise process, in reality these steps often happen simultaneously with much cycling back and forth between steps (fig. 1.5).

The scientific method is a way of testing and refining ideas. Note that the method applies only when experimental tests or other consistent observations of phenomena are feasible. Testing is crucial for weeding out unproductive hypotheses; without tests, rival theories may compete endlessly for acceptance. Example box 1.1 provides a sample question and response illustrating these ideas.

How should science be presented?

Traditional science courses focus on presenting the results of the scientific process rather than the story of how scientists arrived at these results. This is why the general public often sees science as a collection of facts and established theories. To some extent, that charge could be made against this book, because it describes theories that have resulted from the work of others without giving the full picture of their development. Building on the work of others, without needing to repeat their mistakes and unproductive approaches, is a necessary condition for human and scientific progress.

This book attempts to engage you in the process of making your own observations and developing and testing your own explanations of everyday phenomena. By doing home experiments or observations, constructing explanations of the results, and debating your interpretations with your friends, you will appreciate the give-and-take that is the essence of science.

Whether or not we are aware of it, we all use the scientific method in our everyday activities. The case of the malfunctioning coffeemaker described in everyday phenomenon box 1.1 provides an example of scientific reasoning applied to ordinary troubleshooting.

Example Box 1.1

Sample Question: How Reliable Is Astrology?

Question: Astrologers claim that many events in our lives are determined by the positions of the planets relative to the stars. Is this a testable hypothesis?

Answer: Yes, it could be tested if astrologers were willing to make explicit predictions about future events that could be verified by independent observers. In fact, astrologers usually carefully avoid doing this, preferring to cast their predictions as vague statements subject to broad interpretation. This prevents clean tests. Astrology is not a science!

> The process of science begins with, and returns to, observations of or experiments on natural phenomena. Observations may suggest empirical laws, and these generalizations may be incorporated into a more comprehensive hypothesis. The hypothesis is then tested against more observations or by controlled experiments to form a theory. Working scientists are engaged in one or more of these activities, and we all use the scientific method on everyday problems.

Debatable Issue

We are often told that there is a strong consensus among climate scientists that global warming and climate change are being caused by human activity that is producing growing amounts of greenhouse gases, particularly carbon dioxide, in the atmosphere. Does a strong consensus among scientists imply that this idea is correct? Why or why not?

1.3 The Scope of Physics

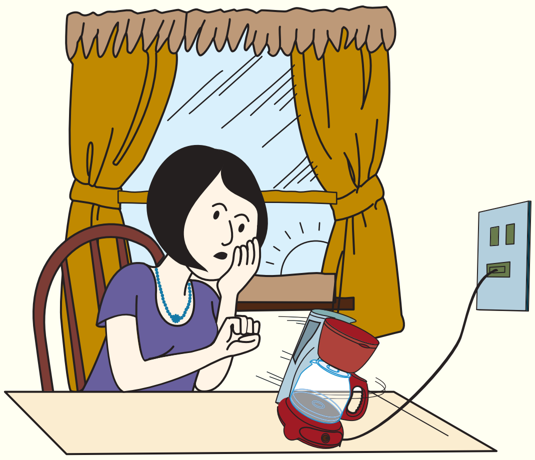
Where does physics fit within the sciences? Since this book is about physics, rather than biology, chemistry, geology, or some other science, it is reasonable to ask where we draw the lines between the disciplines. It is not possible, however, to make sharp distinctions among the disciplines or to provide a definition of physics that will satisfy everyone. The easiest way to give a sense of what physics is and does is by example—that is, by listing some of its subfields and exploring their content. First, let's consider a definition, however incomplete.

Everyday Phenomenon Box 1.1

The Case of the Malfunctioning Coffeemaker

The Situation. It is Monday morning and you are, as usual, only half-awake and feeling at odds with the world. You are looking forward to reviving yourself with a freshly brewed cup of coffee when you discover that your coffeemaker refuses to function. Which of these alternatives is most likely to work?

1. Pound on the appliance with the heel of your hand.
2. Search desperately for the instruction manual, which you probably threw away two years ago.
3. Call a friend who knows about these things.
4. Apply the scientific method to troubleshoot the problem.



Fixing a malfunctioning coffee pot—alternative 1.

The Analysis. All of these alternatives have some chance of success. The sometimes positive response of electrical or mechanical appliances to physical abuse is well documented. The second two alternatives are both forms of appeal to authority that could produce results. The fourth alternative, however, may be the most productive and quickest, barring success with alternative 1.

How would we apply the scientific method as outlined in table 1.1 to this problem? Step 1 involves calmly observing the symptoms of the malfunction. Suppose that the coffeemaker simply refuses to heat up. When the switch is turned on, no sounds of warming water are heard. You notice that no matter how many times you turn the switch on or off, no heat results. This is the kind of simple generalization called for in step 2.

We can now generate some hypotheses about the cause of the malfunction, as suggested in step 3. Here are some candidates:

a. The coffeemaker is not plugged in.
b. The external circuit breaker or fuse has tripped.
c. The power is off in the entire house or neighborhood.
d. An internal fuse in the coffeemaker has blown.
e. A wire has become loose or burned through inside the coffeemaker.
f. The internal thermostat of the coffeemaker is broken.

No detailed knowledge of electrical circuits is needed to check these possibilities, although the last three call for more sophistication (and are more trouble to check) than the first three. The first three possibilities are the easiest to check and should be tested first (step 4 in our method). A simple remedy such as plugging in the coffeemaker or flipping on a circuit breaker may put you back in business. If the power is off in the building, other appliances (lights, clocks, and so on) will not work, either, which provides an easy test. There may be little that you can do in this case, but at least you have identified the problem. Abusing the coffeemaker will not help.

The appliance may or may not have an internal fuse. If it is blown, a trip to the hardware store may be necessary. A problem like a loose wire or a burnt-out connection often becomes obvious by looking inside after you remove the bottom of the coffeemaker or the panel where the power cord comes in. (You must unplug the appliance before making such an inspection!) If one of these alternatives is the case, you have identified the problem, but the repair is likely to take more time or expertise. The same is true of the last alternative.

Regardless of what you find, this systematic (and calm) approach to the problem is likely to be more productive and satisfying than the other approaches. Troubleshooting, if done this way, is an example of applying the scientific method on a small scale to an ordinary problem. We are all scientists if we approach problems in this manner.

How is physics defined?

Physics can be defined as the *study of the basic nature of matter and the interactions that govern its behavior.* It is the most fundamental of the sciences. The principles and theories of physics can be used to explain the fundamental interactions involved in chemistry, biology, and other sciences at the atomic or molecular level. Modern chemistry, for example, uses the physical theory of *quantum mechanics* to explain how atoms combine to form molecules. Quantum mechanics was developed primarily by physicists in the early part of the twentieth century, but chemists and chemical knowledge also played important roles. Ideas about energy that arose initially in physics are now used extensively in chemistry, biology, and other sciences.

The general realm of science is often divided into the life sciences and the physical sciences. The life sciences include the various subfields of biology and the health-related disciplines that deal with living organisms. The physical sciences deal with the behavior of matter in both living and nonliving systems. In addition to physics, the physical sciences include chemistry, geology, astronomy, oceanography, and meteorology (the study of weather). Physics underlies all of them.

Physics is also generally regarded as the most quantitative of the sciences. It makes heavy use of mathematics and numerical measurements to develop and test its theories. This aspect of physics has often made it seem less accessible to students, even though the models and ideas of physics can be described more simply and cleanly than those of other sciences. As we will discuss in section 1.4, mathematics serves as a compact language, allowing briefer and more precise statements than would be possible without its use. However, the quantitative skills needed to understand this book are minimal.

What are the major subfields of physics?

The primary subfields of physics are listed and identified in table 1.2. Mechanics, which deals with the motion (or lack of motion) of objects under the influence of forces, was the first subfield to be explained with a comprehensive theory. Newton's theory of mechanics, which he developed in the last half of the seventeenth century, was the first full-fledged physical theory that made extensive use of mathematics. It became a prototype for subsequent theories in physics.

The first four subfields listed in table 1.2 were well developed by the beginning of the twentieth century, although all have continued to advance since then. These subfields—mechanics, thermodynamics, electricity and magnetism, and optics—are sometimes grouped as **classical physics.** The last four subfields—atomic physics, nuclear physics, particle physics, and condensed-matter physics—are often grouped under the heading of **modern physics,** even though all of the subfields are part of the modern practice of physics. The distinction is made because the last four subfields all emerged during the twentieth century and only existed in rudimentary forms before the turn of that century. In addition to the subfields listed in table 1.2, many physicists work in interdisciplinary fields such as biophysics, geophysics, or astrophysics.

The photographs in this section (fig 1.6, fig. 1.7, fig. 1.8, and fig. 1.9) illustrate characteristic activities or applications

Table 1.2

The Major Subfields of Physics

Mechanics. The study of forces and motion.

Thermodynamics. The study of temperature, heat, and energy.

Electricity and magnetism. The study of electric and magnetic forces and electric current.

Optics. The study of light.

Atomic physics. The study of the structure and behavior of atoms.

Nuclear physics. The study of the nucleus of the atom.

Particle physics. The study of subatomic particles (quarks, etc.).

Condensed-matter physics. The study of the properties of matter in the solid and liquid states.

Figure 1.6 A surgeon using a laser. *Larry Mulvehill/Corbis/SuperStock*

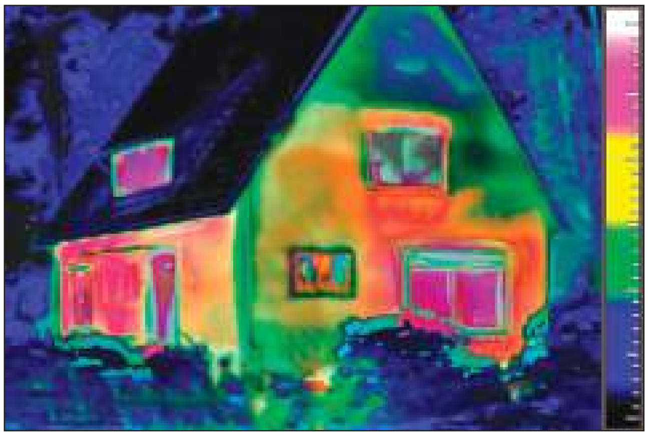


Figure 1.7 An infrared photograph showing patterns of heat loss from a house is an application of thermodynamics. *Dirk Püschel/Hemera/Getty Images*

Figure 1.8 A power plant at Nellis Air Force Base utilizes photovoltaic solar cells. *Stocktrek Images/Getty Images*

Figure 1.9 The Large Hadron Collider (LHC) is an accelerator used to study interactions of subatomic particles at very high energies. It is located at CERN, the European Particle-physics laboratory in Switzerland. *Fabrice Coffrini/AFP/Getty Images*

of the subfields. The invention of the laser has been an extremely important factor in the rapid advances now taking place in optics, as well as many advances in the medical field (fig. 1.6). The development of the infrared camera has provided a tool for the study of heat flow from buildings, which involves thermodynamics (fig. 1.7). The rapid growth in consumer electronics, as seen in the availability of laptop computers, smartphones, and many other "essential" personal paraphernalia, has been made possible by developments in condensed-matter physics. These developments, as well as the development of photovoltaic solar cells (fig 1.8), all involve applications of semiconductors. Particle physicists use particle accelerators to study the interactions of subatomic particles in high-energy collisions. The Large Hadron Collider (fig. 1.9) was used in the discovery of the Higgs Boson in 2012.

Science and technology depend on each other for progress. Physics plays an important role in the education and work of engineers, whether they specialize in electrical, mechanical, nuclear, or other engineering fields. In fact, people with physics degrees often work as engineers when they are employed in industry. The lines between physics and engineering, or research and development, often blur. Physicists are generally concerned with developing a fundamental understanding of phenomena, and engineers with applying that understanding to practical tasks or products, but these functions often overlap.

One final point: Physics is fun. Understanding how a bicycle works or how a rainbow is formed has an appeal that anyone can appreciate. The thrill of gaining insight into the workings of the universe can be experienced at any level. In this sense, we can all be physicists.

> Physics is the study of the basic characteristics of matter and its interactions. It is the most fundamental of the sciences; many other sciences build on ideas from physics. The major subfields of physics are mechanics, electricity and magnetism, optics, thermodynamics, atomic and nuclear physics, particle physics, and condensed-matter physics. Physics plays an important role in engineering and technology, but the real fun of physics comes from understanding how the universe works.

1.4 The Role of Measurement and Mathematics in Physics

If you go into your college library, find a volume of *Physical Review* or some other major physics journal, and open it at random, you are likely to find a page with many mathematical symbols and formulas. It would probably be incomprehensible to you. In fact, even many physicists who are not specialists in the particular subfield covered by the article might have difficulty making sense of that page, because they would not be familiar with the particular symbols and definitions.

Why do physicists make such extensive use of mathematics in their work? Is knowledge of mathematics essential to understanding the ideas being discussed? Mathematics is a compact language for representing the ideas of physics that makes it easier to precisely state and manipulate the relationships between the quantities that we measure in physics. Once you are familiar with the language, its mystery disappears and its usefulness becomes more obvious. Still, this book uses mathematics in a very limited manner, because most ideas of physics can be discussed without extensive use of mathematics.

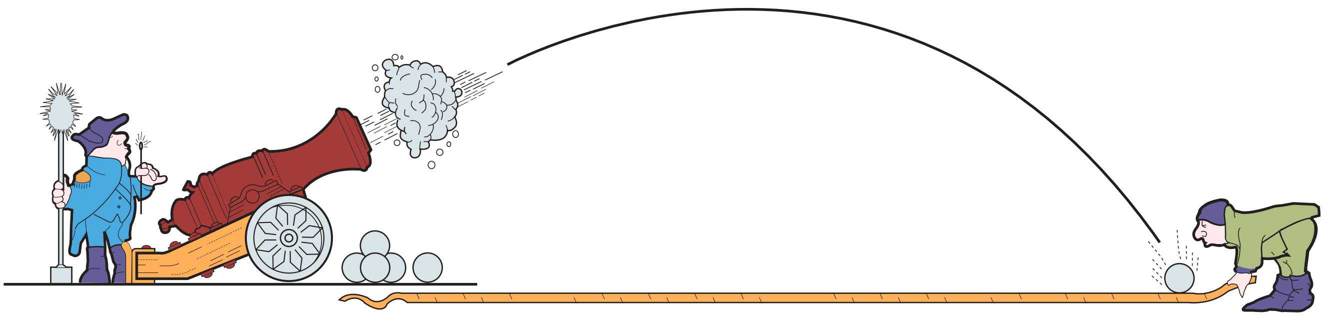


Figure 1.10 Cannonballs and a measuring tape: the proof lies in the measurement.

Why are measurements so important?

How do we test theories in physics? Without careful measurements, vague predictions and explanations may seem reasonable, and making choices between competing explanations may not be possible. A quantitative prediction, on the other hand, can be tested against reality, and an explanation or theory can be accepted or rejected based on the results of measurements. If, for example, one hypothesis predicts that a cannonball will land 100 meters from us and another predicts a distance of 200 meters under the same conditions, firing the cannon and measuring the actual distance provide persuasive evidence for one of these hypotheses (fig. 1.10). The rapid growth and successes of physics began when the idea of making precise measurements as a test was accepted.

Life is full of situations in which measurements and measurable quantities, as well as the ability to express relationships between them, are important. Imagine that you are working on a tree farm and want to impress your boss. You have just finished planting 300 seedlings on 2.4 acres, when your boss asks how many seedlings will be needed for a second location of 4 acres. You don't want to waste money on too many seedlings; nor do you want too few. Perhaps you can solve the problem in your head, but this might be hard to describe in words to your boss and might just confuse the issue.

How can mathematics help?

You can reduce the confusion by creating a word statement for this situation. You can say, "The quantity needed for 4 acres is related to the quantity for 2.4 acres as 4 is to 2.4." This still takes quite a few words, though, and such a **proportion** statement may not be familiar to the boss.

To make your statement briefer, you can use the symbols Q to represent the quantity of seedlings needed for 4 acres and Q_o to represent the quantity needed for the original 2.4 acres, then express them as a mathematical equation:

$$\frac{Q}{Q_o} = \frac{4}{2.4}$$

Using symbols is simply a compact way of saying the same thing you previously expressed in words. This compact statement also has the advantage of making it easier for you to manipulate the relationship. For example, if you multiply both sides of this equation by Q_o, it takes the form

$$Q = \left(\frac{4}{2.4}\right) Q_o$$

which, in words, says that the quantity needed for 4 acres is (4/2.4) times the quantity needed for 2.4 acres. If, instead, you want to find the number needed for 8 acres, you can easily see it is just $Q = (8/2.4)\ Q_o$. If you are comfortable with fractions, you can use this relationship to quickly find the proper quantity for any number of acres.

A common, everyday situation in which you might use this idea of proportion to your advantage is with recipes. Everyday phenomenon box 1.2 addresses the challenge of taking a frosting recipe for two cakes and scaling it up for five cakes.

There are a few important points to these examples. First, making measurements and using measurable quantities are both a routine and an important part of everyday experience. Second, using symbols to represent quantities in a mathematical statement is a shorter way of expressing an idea involving numbers than the same statement in words. Using mathematics also makes it easier to manipulate relationships to construct concise arguments. These are the reasons that physicists (and many other people) find mathematical statements useful.

Why are metric units used?

Units of measurement are an essential part of any measurement. We do not communicate clearly if we just state a number. If you just talked about adding $1^1/_3$ of milk, for example, your statement would be incomplete. You need to indicate whether you are talking about cups, pints, or milliliters.

The liter and milliliter are *metric* units of volume. Cups, pints, quarts, and gallons are holdovers from the older English system of units. Most countries have now adopted the **metric system,** which has several advantages over the English system still used in the United States. The main advantage of the metric system is its use of standard prefixes to represent multiples of 10, making unit conversion within the

Everyday Phenomenon Box 1.2

Scaling a Recipe

The Situation. You have an excellent recipe for vanilla frosting, which you have made to ice two cakes. However, your friends want you to take five cakes to your annual club meeting. How do you scale up the recipe, so that you have just enough icing for five cakes? How can your newfound understanding of proportions help you?

Putting finishing touches on a cake.

The Analysis. According to our understanding of proportions, the quantity of frosting needed for five cakes is to the quantity needed for two cakes as 5 is to 2. Another way we can look at this is by using symbols. If we let Q_2 be the quantity needed for two cakes and Q_5 be the quantity needed for five cakes, then we can write the relationship as

$$\frac{Q_5}{Q_2} = \frac{5}{2}$$

We can rewrite this equation to solve for Q_5 (which is the quantity we are looking for) by multiplying both sides by Q_2:

$$(Q_2)\left(\frac{Q_5}{Q_2}\right) = (Q_2)\left(\frac{5}{2}\right)$$

Notice that on the left-hand side the Q_2 in the numerator cancels the Q_2 in the denominator, so the equation becomes

$$Q_5 = (Q_2)\left(\frac{5}{2}\right)$$

In words, this translates to "the quantity needed for five cakes is equal to the quantity used for two cakes times the ratio (5/2)."

Now, let's look at the recipe. The recipe for two cakes is as follows:

- 16 cups confectioners' sugar
- 1 cup butter, softened
- 1 cup whole milk
- 1 tablespoon vanilla extract

Combine butter, milk, and vanilla. Beat on medium speed until smooth. Gradually add sugar while beating until smooth and fluffy.

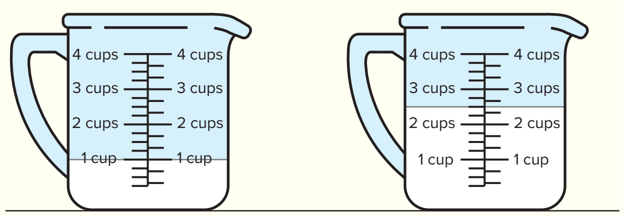
To expand the recipe for five cakes, we would multiply each quantity by (5/2). Let's look at the amount of milk:

$$(1\ \text{cup})\left(\frac{5}{2}\right) = \frac{5}{2}\ \text{cups} = 2\frac{1}{2}\ \text{cups} = 2.5\ \text{cups}$$

We can do the same for the other ingredients. Our revised frosting recipe for five cakes is thus

- 40 cups confectioners' sugar
- 2.5 cups butter, softened
- 2.5 cups whole milk
- 2.5 tablespoons vanilla extract

Once you know how to use proportions to scale recipes up (or down), you will be much more confident when determining the quantities needed. This skill is useful in many aspects of everyday living. Determining quantities such as the amount of paint needed for a project, or the number of pizzas to order, becomes less problematic. In fact, you just might find yourself the person your friends turn to when scaling quantities up or down.



Two measuring cups, one containing enough milk for frosting two cakes and one for frosting five cakes.

system quite easy. The fact that a kilometer (km) is 1000 meters and a centimeter (cm) is $^1/_{100}$ of a meter, and that the prefixes *kilo* and *centi* always mean 1000 and $^1/_{100}$, makes these conversions easy to remember (see table 1.3). To convert 30 centimeters to meters, all we have to do is move the decimal point two places to the left to get 0.30 meter. Moving the decimal point two places to the left is equivalent to dividing by 100.

Table 1.3

Commonly Used Metric Prefixes

Prefix	Meaning: in figures		in scientific notation	in words
tera	1 000 000 000 000		$= 10^{12}$	= 1 trillion
giga	1 000 000 000		$= 10^{9}$	= 1 billion
mega	1 000 000		$= 10^{6}$	= 1 million
kilo	1000		$= 10^{3}$	= 1 thousand
centi	$^{1}/_{100}$	= 0.01	$= 10^{-2}$	= 1 hundredth
milli	$^{1}/_{1000}$	= 0.001	$= 10^{-3}$	= 1 thousandth
micro	$^{1}/_{1\,000\,000}$	$= {^{1}/_{10^6}}$	$= 10^{-6}$	= 1 millionth
nano	$^{1}/_{1\,000\,000\,000}$	$= {^{1}/_{10^9}}$	$= 10^{-9}$	= 1 billionth
pico		$= {^{1}/_{10^{12}}}$	$= 10^{-12}$	= 1 trillionth

Table 1.3 is a list of the common prefixes used in the metric system. (See appendix B for a discussion of the **powers of 10,** or **scientific notation,** used for describing very large and very small numbers.) The basic unit of volume in the metric system is the liter (L), which is slightly larger than a quart (1 liter = 1.057 quarts). A milliliter (mL) is $^{1}/_{1000}$ of a liter, a convenient size for quantities in recipes. One milliliter is also equal to 1 cm^3, or 1 cubic centimeter, so there is a simple relationship between the length and volume measurements in the metric system. Such simple relationships are hard to find in the English system, where 1 cup is $^{1}/_{4}$ of a quart, and a quart is 67.2 cubic inches.

Example Box 1.2

Sample Exercise: Length Conversions

If you are told that there are 2.54 cm in 1 inch,

a. How many centimeters are there in 1 foot (12 inches)?
b. How many meters does 1 foot represent?

a. 1 inch = 2.54 cm
1 foot = 12 inches
1 foot = ? (in cm)

$$(1\ \cancel{\text{ft}})\left(\frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}}\right) = 30.5\ \text{cm}$$

1 foot = 30.5 cm

b. 1 foot = 30.5 cm
1 m = 100 cm
1 foot = ? (in m)

$$(1\ \cancel{\text{ft}})\left(\frac{30.5\ \cancel{\text{cm}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right) = 0.305\ \text{m}$$

1 foot = 0.305 m

Lines drawn through the units indicate cancellation.

The metric system predominates in this book. English units will be used occasionally because they are familiar to many and can help in learning new concepts. Most people in the United States still relate more readily to distances in miles than in kilometers, for example. That there are 5280 feet in a mile is a nuisance, however, compared to the tidy 1000 meters in 1 kilometer. Becoming familiar with the metric system is a worthy objective. Your ability to participate in international trade (for business or pleasure) will be enhanced if you are familiar with the system of units used in most of the world. Example boxes 1.2 and 1.3 provide unit conversion exercises involving metric units.

Example Box 1.3

Sample Exercise: Rate Conversions

If the rate of flow in an automatic watering system is 2 gallons/hour, how many milliliters per minute is this?

1 gallon = 3.786 liters
1 liter = 1000 mL
2 gal/h = ? (in mL/min)

$$\left(\frac{2\ \cancel{\text{gallons}}}{\cancel{\text{hour}}}\right)\left(\frac{3.786\ \cancel{\text{liter}}}{1\ \cancel{\text{gallon}}}\right)\left(\frac{1000\ \text{mL}}{1\ \cancel{\text{liter}}}\right)\left(\frac{1\ \cancel{\text{hour}}}{60\ \text{min}}\right)$$
$$= 126.2\ \text{mL/min}$$

2 gallons/hour = **126.2 mL/min**

Lines drawn through the units indicate cancellation.

Stating a result or prediction in numbers lends precision to otherwise vague claims. Measurement is an essential part of science and of everyday life. Using mathematical symbols and statements is an efficient way of stating the results of measurements and eases manipulating the relationships between quantities. Units of measurement are an essential part of any measurement, and the metric system of units used in most of the world has a number of advantages over the older English system.

1.5 Physics and Everyday Phenomena

Studying physics can and will lead us to ideas as earthshaking as the fundamental nature of matter and the structure of the universe. With ideas like these available, why spend time on more mundane matters like explaining how a bicycle stays upright or how a flashlight works? Why not just plunge into far-reaching discussions of the fundamental nature of reality?

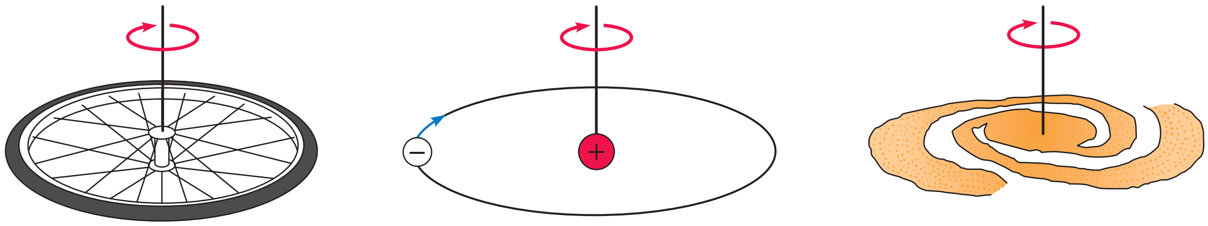


Figure 1.11 A bicycle wheel, a model of an atom, and a galaxy all involve the concept of angular momentum.

Why study everyday phenomena?

Our understanding of the fundamental nature of the universe is based on concepts such as mass, energy, and electric charge that are abstract and not directly accessible to our senses. It is possible to learn some of the words associated with these concepts and to read and discuss ideas involving them without ever acquiring a good understanding of their meaning. This is one risk of playing with the grand ideas without laying the proper foundation.

Using everyday experience to raise questions, introduce concepts, and practice devising physical explanations has the advantage of dealing with examples that are familiar and concrete. These examples also appeal to your natural curiosity about how things work, which, in turn, can motivate you to understand the underlying concepts. If you can clearly describe and explain common events, you gain confidence in dealing with more abstract concepts. With familiar examples, the concepts are set on firmer ground, and their meaning becomes more real.

For example, why a bicycle (or a top) stays upright while moving but falls over when at rest involves the concept of angular momentum, which is discussed in chapter 8. Angular momentum also plays a role in our understanding of atoms and the atomic nucleus—both in the realm of the very small—and the structure of galaxies at the opposite end of the scale (fig. 1.11). You are more likely to understand angular momentum, though, by discussing it first in the context of bicycle wheels or tops.

The principles explaining falling bodies, such as the acorn mentioned in the chapter introduction, involve the concepts of velocity, acceleration, force, and mass, which are discussed in chapter 2, chapter 3, and chapter 4. Like angular momentum, these concepts are also important to our understanding of atoms and the universe. Energy ideas, introduced initially in chapter 6, appear throughout the chapters that follow in the textbook. These ideas are crucial to the understanding of the universe, as well as to everyday concerns such as climate change and energy conservation.

Our "common sense" sometimes misleads us in our understanding of everyday phenomena. Adjusting common sense to incorporate well-established physical principles is one of the challenges we face in dealing with everyday experience. By performing simple experiments, either in laboratories and demonstrations associated with your course in physics or at home (as is often suggested in this book), you can take an active part in building your own scientific worldview.

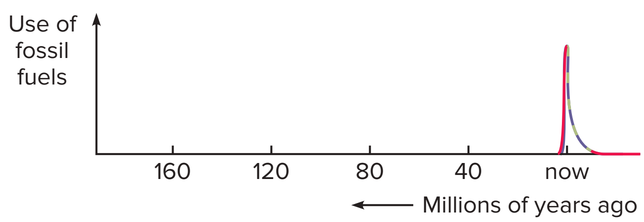
Although it may seem like an oxymoron, everyday experience is extraordinary. A bright rainbow is an incredible sight. Understanding how rainbows originate does not detract from the experience. It adds excitement to explain such a beautiful display with just a few elegant concepts. In fact, people who understand these ideas see more rainbows because they know where to look. This excitement, and the added appreciation of nature that is a part of it, is accessible to all of us.

> Studying everyday phenomena can make abstract ideas more accessible. These ideas are needed to understand the fundamental nature of matter and the universe, but they are best encountered first in familiar examples. Being able to explain common phenomena builds confidence in using the ideas and enhances our appreciation of what happens around us.

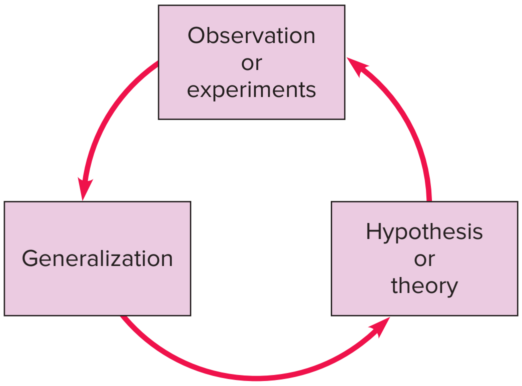
Summary

This first chapter introduces the connections between physics and everyday phenomena, including current issues involving energy. We also introduce the scientific enterprise and its methods, the scope of physics, and the use of mathematics and measurement in physics. The key points include the following:

1 **What about energy?** Most of our use of energy involves the burning of fossil fuels, which release carbon, and this affects many aspects of the Earth's climate, including global warming. The definition and science of energy are in the realm of physics, and therefore some understanding of this physics is crucial to meaningful participation in these debates.

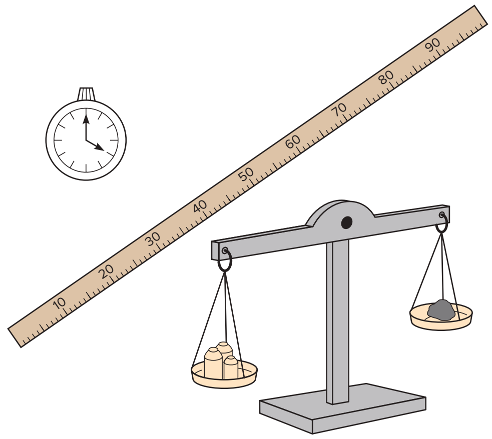


2 **The scientific enterprise.** Scientific explanations are developed by generalizing from observations of nature; forming hypotheses, or theories; and then testing these theories by further experiments or observations. This process is often called the *scientific method,* but actual practice may depart in various ways from this model.



3 **The scope of physics.** Physics is the most fundamental of the natural sciences, because physical theories often underlie explanations in the other sciences. Its major subfields include mechanics, thermodynamics, electricity and magnetism, optics, atomic physics, nuclear physics, condensed-matter physics, and particle physics.

4 **The role of measurement and mathematics in physics.** Much of the progress in physics can be attributed to its use of quantitative models, which yield precise predictions that can be tested by making physical measurements (with well-defined units). Mathematics is a compact language for describing and manipulating these results. The basic concepts of physics can often be described and understood with a minimum of mathematics.



5 **Physics and everyday phenomena.** Many of the basic concepts of physics become clearer if applied to everyday phenomena. Being able to understand and explain familiar phenomena makes the concepts more vivid. This adds to the enjoyment of studying physics.

If you want an extra drill in doing algebraic manipulations of equations, or doing unit conversions, try the practice problems in Connect.

Key Terms

Hypothesis, 4
Theory, 5
Scientific method, 5
Empirical law, 5
Classical physics, 8
Modern physics, 8
Mechanics, 8
Thermodynamics, 8
Electricity and magnetism, 8
Optics, 8
Atomic physics, 8
Nuclear physics, 8
Particle physics, 8
Condensed-matter physics, 8
Proportion, 10
Metric system, 10
Powers of 10, 12
Scientific notation, 12

Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available on Connect

Q1. Are fossil fuels produced within a few hundred years after dead plant remains are covered by dirt? Explain.

Q2. Do trees and other green plants have any impact on the amount of carbon dioxide in the atmosphere? Explain.

Q3. Since burning wood releases carbon dioxide to the atmosphere, should wood be considered a fossil fuel? Explain.

*Q4. If the amount of carbon dioxide in the atmosphere is increasing with time, should we expect an increase in the average global temperature? Explain.

Q5. Has the burning of fossil fuels been taking place on a significant scale over many thousands of years? Explain.

Q6. Does the use of nuclear power significantly increase the amount of carbon dioxide in the atmosphere? Explain.

*Q7. Which of these criteria best distinguish between explanations provided by science and those provided by religion: truth, testability, or appeal to authority? How do religious explanations differ from scientific explanations?

Q8. A person claiming to have paranormal powers states that she can predict which card will come up next in a shuffled deck of cards simply by exercising her mental powers. Is this a testable claim? Explain.

Q9. Historians sometimes develop theories to explain observed patterns in the history of different countries. Are these theories testable in the same sense as a theory in physics? Explain.

*Q10. Over the years, there have been several credible claims by experienced observers of sightings of unidentified flying objects (UFOs). Despite this, scientists have shied away from taking up a serious study of UFOs, although there are ongoing searches for signals from extraterrestrial intelligent beings. Can you think of reasons that scientists have not taken UFOs seriously? What problems can you see in trying to study UFOs?

Q11. Suppose that your car will not start and you form the hypothesis that the battery is dead. How would you test this hypothesis? Explain. (See everyday phenomenon box 1.1.)

Q12. Suppose that your phone has not rung in several days, but a friend tells you he has tried to call. Develop two hypotheses that could explain why the phone has not rung and state how you would test these hypotheses. (See everyday phenomenon box 1.1.)

*Q13. Suppose that a friend states the hypothesis that the color of socks that he wears on a given day, brown or black, will determine whether the stock market will go up or down. He can cite several instances in which this hypothesis has been apparently verified. How would you go about evaluating this hypothesis?

Q14. Which of the three science fields—biology, chemistry, or physics—would you say is the most fundamental? Explain by describing in what sense one of these fields may be more fundamental than the others.

Q15. Based on the brief descriptions provided in table 1.2, which subfield of physics would you say is involved in the explanation of rainbows? Which subfield is involved in describing how an acorn falls? Explain.

Q16. Based on the descriptions provided in table 1.2, which subfields of physics are involved in explaining why an ice cube melts? Which subfields are involved in explaining how an airplane flies? Explain.

Q17. Suppose you are told that speed is defined by the relationship $s = d/t$, where s represents speed, d represents distance, and t represents time. State this relationship in words, using no mathematical symbols.

Q18. Impulse is defined as the average force acting on an object multiplied by the time the force acts. If we let I represent impulse, F the average force, and t the time, is $I = F/t$ a correct way of expressing this definition? Explain.

Q19. The distance that an object travels when it starts from rest and undergoes constant acceleration is one-half the acceleration multiplied by the square of the time. Invent your own symbols and express this statement in symbolic form.

Q20. What are the primary advantages of the metric system of units over the older English system of units? Explain.

Q21. What are the advantages, if any, of continuing to use the English system of units in industry and commerce rather than converting to the metric system? Explain.

Q22. Which system of units, the metric system or English system, is used more widely throughout the world? Explain.

Q23. The width of a man's hand was used as a common unit of length several hundred years ago. What are the advantages and disadvantages of using such a unit? Explain.

Q24. A pirate map indicates that a treasure is buried 50 paces due east and 120 paces due north of a big rock. Will you know where to dig? Explain.

Q25. List the following volumes in descending order: gallon, quart, liter, milliliter. The conversion factors given in appendix E may be useful.

Q26. List the following lengths in descending order: kilometer, feet, mile, centimeter, inch. The conversion factors given in appendix E may be useful.

Exercises

E1. Suppose that a pancake recipe designed to feed five people calls for 310 grams of flour. How many grams of flour would you use if you wanted to reduce the recipe to only feed two people? (See everyday phenomenon box 1.2.)

E2. Suppose that a cupcake recipe designed to produce 16 cupcakes calls for 240 grams of flour. How many grams of flour would you use if you wanted to make 20 cupcakes? (See everyday phenomenon box 1.2.)

E3. It is estimated that eight medium pizzas are about right to serve a physics club meeting of 32 students. How many pizzas would be required if the group, due to a conflicting math club meeting, were only going to have 20 students in attendance? (See everyday phenomenon box 1.2.)

E4. A child uses her hand to measure the width of a tabletop. If her hand has a width of 8 cm at its widest point, and she finds the tabletop to be 16.5 hands wide, what is the width of the tabletop in cm? In meters?

E5. A small woman's foot is 7 inches long. If she steps off the length of a room by placing one foot directly in front of the other, and finds the room to be 15 foot-lengths long, what is the length of the room in inches? In feet?

E6. A paperback book is 220 mm in height. What is this height in centimeters? In meters?

E7. A crate has a mass of 8.30×10^6 mg (milligrams). What is this mass in grams? In kilograms? (Hint: See table 1.3.)

E8. A tank holds 5260 L (liters) of water. How many kiloliters is this? How many milliliters? (Hint: See table 1.3.)

E9. A mile is 5280 ft long. The sample exercise in example box 1.2 shows that 1 foot is approximately 0.305 m. How many meters are in a mile? How many kilometers (km) are in a mile?

E10. If a mile is 5280 ft long and a yard contains 3 ft, how many yards are in a mile?

E11. Area is found by multiplying the length of a surface times the width. If a floor measures 5.28 m^2, how many square centimeters does this represent? How many square centimeters are in 1 m^2?

E12. A common speed limit in Vancouver, British Columbia, is 70 km/h. If you are going 45 MPH, are you speeding? Show by converting 45 MPH to km/h using the conversion factors in appendix E.

E13. If gas costs \$1.27 a liter, how much does a gallon of gas cost? Show by converting gallons to liters using the conversion factors in appendix E.

E14. The volume of a cube is found by multiplying length times width times height. If an object has a volume of 1.44 m^3, what is the volume in cubic centimeters? Remember to multiply each side by the conversion factor.

E15. If the area of a square has increased by a factor of 16, by how much has each side increased?

E16. A cube has a certain volume. If the length of each side is tripled, by what factor will the volume increase?

Synthesis Problems

SP1. Astrologers claim that they can predict important events in your life by the configuration of the planets and the astrological sign under which you were born. Astrological predictions, called horoscopes, can be found in most daily newspapers. Find these predictions in a newspaper and address the questions:

a. Are the astrological predictions testable?
b. Choosing the prediction for your own sign, how would you go about testing its accuracy over the next month or so?
c. Why do newspapers print these readings? What is their appeal?

SP2. In the United States, a common quantity of hard liquor was historically a fifth, which represents a fifth of a U.S. gallon. However, because the United States wants to market its alcohol globally, and everyone else uses the metric system, it has retooled its packaging, so a common quantity is now 750 mL.

a. How many liters are in a fifth?
b. How many milliliters are in a fifth?
c. Which is larger, 750 mL or a fifth of a gallon?

SP3. A compact fluorescent light (CFL) bulb is very energy-efficient. A 22W CFL bulb has the same brightness as a 100W incandescent bulb. (Note that these calculations ignore the fact that the CFL bulb can last up to 10 times as long as the incandescent bulb. This would increase your savings even more.)

a. If you have this lightbulb on for 5 hours a day, for 350 days during a year, how many hours is it on?
b. A kilowatt is 1000 watts. The kilowatt-hour is a common unit for energy, obtained by multiplying the power in kilowatts by the time used in hours. How many kilowatt-hours (kWh) will you use when burning the 100W bulb for the year (don't forget to convert the 100W to kW first)?
c. How many kilowatt-hours (kWh) will you use when burning the 22W bulb for the year (be sure to convert the 22W to kW before multiplying by the time in hours)?
d. Assuming that the cost of electricity is 15¢ per kWh, what is the cost of using the 100W incandescent bulb for the year?
e. Assuming this same cost, what is the cost of using the 22W CFL bulb for the year?
f. How much do you save per year by using the 22W CFL bulb?
g. How much would you save every year if you replaced 20 of the 100W incandescent bulbs with the 22W CFL bulbs?

Home Experiments and Observations

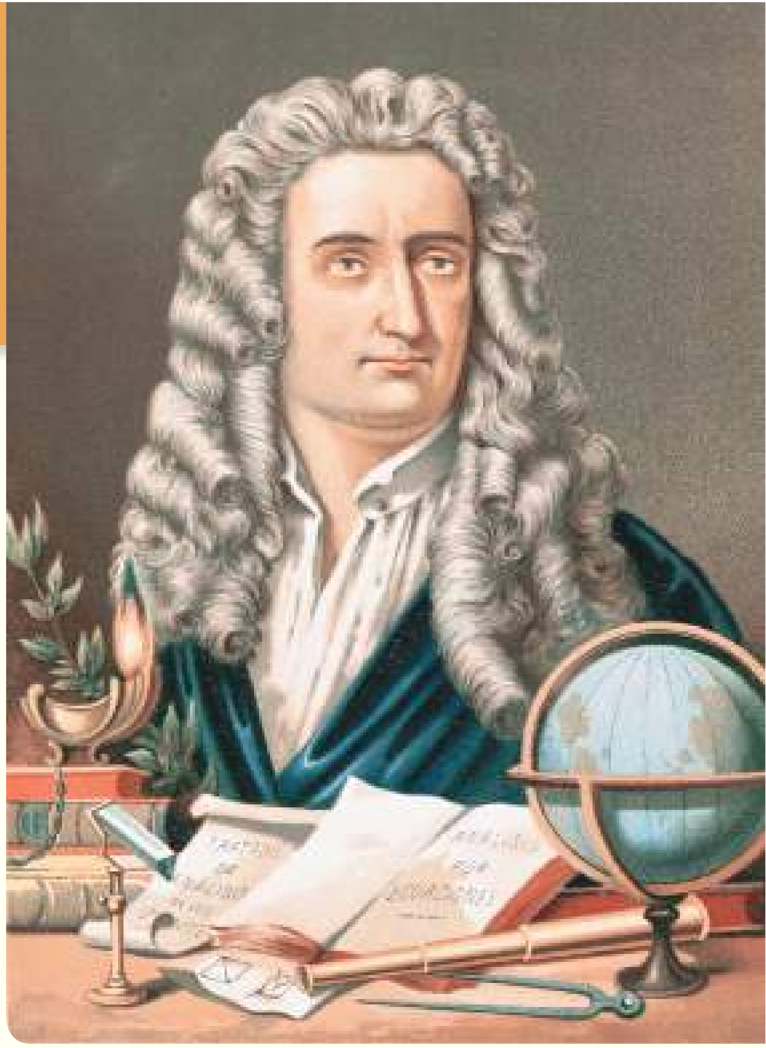
HE1. Look around your house, car, or dormitory room to see what measuring tools (rulers, measuring cups, speedometers, etc.) you have handy. Which of these tools, if any, provides both English and metric units? For those that do, determine the conversion factor needed to convert the English units to metric units.

HE2. Find 200 pennies (or other small, identical objects—cardboard squares will work). Create a square with 3 pennies in each of 3 rows. Represent the area as width × length, or 3 × 3 pennies, or 9 pennies.

a. Double the number of pennies in each row and column. What is the area now? How much bigger is it than the original area? Is it twice as big? Is it four times as big?
b. Now double the number of pennies in each row and column once more. Can you correctly predict the area?
c. What is the pattern? If you started with a square with 2 pennies to the side, and you tripled the number of pennies in each row and column, how much greater would the area be? See if you can predict the answer before trying it.
d. Can you state the rule? If you multiply each side of an area by "n," by how much does the area increase, in terms of "n"?

UNIT 1

The Newtonian Revolution



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In 1687, Isaac Newton published his *Philosophiae Naturalis Principia Mathematica,* or *Mathematical Principles of Natural Philosophy.* This treatise, often called simply Newton's *Principia,* presented his theory of motion, which included his three laws of motion and his law of universal gravitation. Together these *laws* explain most of what was then known about the motion of ordinary objects near Earth's surface (terrestrial mechanics), as well as the motion of the planets around the sun (celestial mechanics). Along the way, Newton had to invent the mathematical techniques that we call calculus.

Newton's theory of mechanics described in the *Principia* was an incredible intellectual achievement that revolutionized both science and philosophy. The revolution did not begin with Newton, though. The true rebel was Italian scientist Galileo Galilei, who died just a few months after Newton was born in 1642. Galileo championed the sun-centered view of the solar system proposed a hundred years earlier by Nicolaus Copernicus and stood trial under the Inquisition for his pains. Galileo also challenged the conventional wisdom, based on Aristotle's teachings, about the motion of ordinary objects. In the process, he developed many of the principles of terrestrial mechanics that Newton later incorporated into his theory.

Although Newton's theory of motion does not accurately describe the motion of very fast objects (which are now described using Einstein's theory of relativity) and very small objects (for which quantum mechanics must be used), it is still used extensively in physics and engineering to explain motion and to analyze structures. Newton's theory has had enormous influence over the last three hundred years in realms of thought that extend well beyond the natural sciences and deserves to be understood by anyone claiming to be well educated.

Central to Newton's theory is his second law of motion. It states that the acceleration of an object is proportional to the net force acting on the object and inversely proportional to the mass of the object. Push an object and that object accelerates in the direction of the applied force. Contrary to intuition and to Aristotle's teachings, acceleration, not velocity, is proportional to the applied force. To understand this idea, we will thoroughly examine acceleration, which involves a *change* in the motion of an object.

Rather than plunging into Newton's theory, we begin this unit by studying Galileo's insights into motion and free fall. This provides the necessary foundation to tackle Newton's ideas. To see well, we need to stand on the shoulders of these giants.

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CHAPTER 2

Describing Motion

Chapter Overview

The main purpose of this chapter is to provide clear definitions and illustrations of the terms used in physics to describe motion, such as the motion of the car described in this chapter's opening example. Speed, velocity, and acceleration are crucial concepts for the analysis of motion in later chapters. Precise description is the first step toward understanding. Without it, we remain awash in vague ideas that are not defined well enough to test our explanations.

Each numbered topic in this chapter builds on the previous section, so it is important to obtain a clear understanding of each topic before going on. The distinctions between speed and velocity and velocity and acceleration are particularly important.

Chapter Outline

1 **Average and instantaneous speed.** How do we describe how fast an object is moving? How does instantaneous speed differ from average speed?

2 **Velocity.** How do we introduce direction into descriptions of motion? What is the distinction between speed and velocity?

3 **Acceleration.** How do we describe changes in motion? What is the relationship between velocity and acceleration?

4 **Graphing motion.** How can graphs be used to describe motion? How can the use of graphs help us gain a clearer understanding of speed, velocity, and acceleration?

5 **Uniform acceleration.** What happens when an object accelerates at a steady rate? How do the velocity and distance traveled vary with time when an object is uniformly accelerating?

Imagine that you are in your car, stopped at an intersection. After waiting for cross traffic, you pull away from the stop sign, accelerating eventually to a speed of 56 kilometers per hour (35 miles per hour). You maintain that speed until a dog runs in front of your car and you hit the brakes, reducing your speed rapidly to 10 km/h (fig. 2.1). Having missed the dog, you speed up again to 56 km/h. After another block, you come to another stop sign and reduce your speed gradually to zero.

We can all relate to this description. Measuring speed in miles per hour (MPH) may be more familiar than the use of kilometers per hour (km/h), but speedometers in cars now show both. The use of the term *acceleration* to describe an increase in speed is also common. In physics, however, these concepts take on more precise and specialized meanings that make them even more useful in describing exactly what is happening. These meanings are sometimes different from those in everyday use. The term *acceleration,* for example, is used by physicists to describe any situation in which velocity is changing, even when the speed is decreasing or only the direction of the motion is changing.

How would you define the term *speed* if you were explaining the idea to a younger brother or sister? Does *velocity* mean the same thing? What about *acceleration*—is the notion vague or does it have a precise meaning? Is it the same thing as velocity? Clear definitions are essential to developing clear explanations. The language used by physicists differs from our everyday language, even though the ideas are related and the same words are used. What are the exact meanings that physicists attach to these concepts, and how can they help us to understand motion?

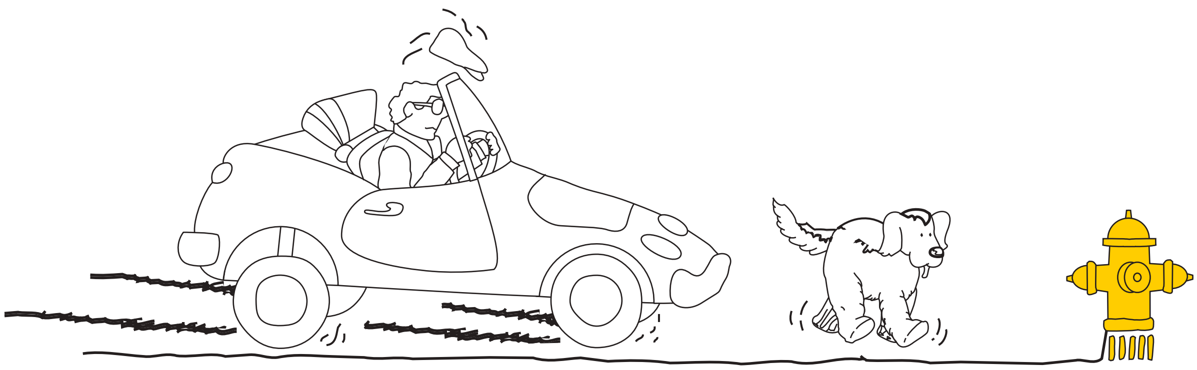


Figure 2.1 As the car brakes for the dog, there is a sudden change in speed.

2.1 Average and Instantaneous Speed

Since driving or riding in cars is a common activity in our daily lives, we are familiar with the concept of speed. Most of us have had experience in reading a speedometer (or perhaps failing to read it carefully enough to avoid the attention of law enforcement). If you describe how fast something is moving, as we did in our example in the introduction, you are talking about **speed.**

How is average speed defined?

What does it mean to say that we are traveling at a speed of 55 MPH? It means that we would cover a distance of 55 miles in a time of 1 hour if we traveled steadily at that speed. Carefully note the structure of this description: There is a number, 55, and some units or dimensions, miles per hour. Numbers and units are both essential parts of a description of speed.

The term *miles per hour* implies that miles are divided by hours in arriving at the speed. This is exactly how we would compute the **average speed** for a trip: Suppose, for example, that we travel a distance of 260 miles in a time of 5 hours, as shown on the road map of figure 2.2. The average speed is then 260 miles divided by 5 hours, which is equal to 52 MPH. This type of computation is familiar to most of us.

We can also express the definition of average speed in a word equation as

Average speed equals the distance traveled divided by the time of travel.

or

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of travel}}$$

We can represent the same definition with symbols by writing

$$s = \frac{d}{t}$$

where the letter s represents the speed, d represents distance, and t represents the time. As noted in chapter 1, letters or

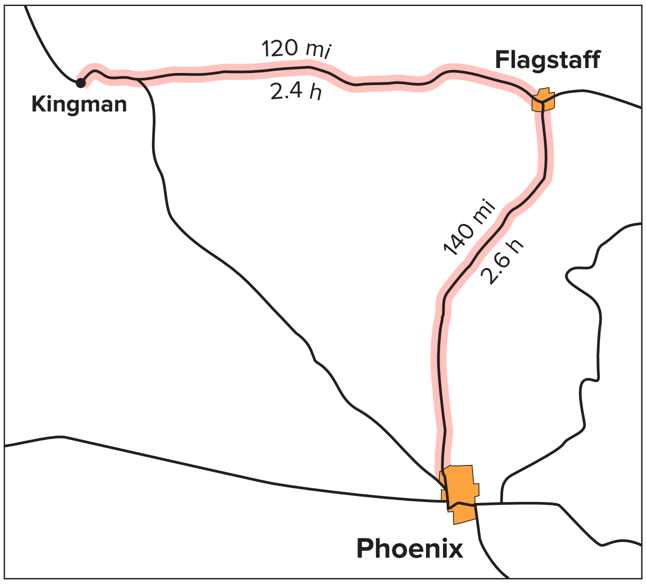


Figure 2.2 A road map showing a trip of 260 miles, with driving times for the two legs of the trip.

symbols are a compact way of saying what could be said with words (and a little more effort and space). Judge for yourself which is the more efficient way of expressing this definition of average speed. Most people find the symbolic expression easier to remember and use.

The average speed that we have just defined is the **rate** at which distance is covered over time. Rates always represent one quantity divided by another. Gallons per minute, pesos per dollar, and points per game are all examples of rates. If we are considering time rates, the quantity that we divide by is *time,* which is the case with average speed. Many other quantities that we will be considering involve time rates.

Example Box 2.1

Sample Exercise: Speed Conversions

Convert 90 kilometers per hour to (a) miles per hour and (b) meters per second.

a. 1 km = 0.6214 miles
90 km/h = ? (in MPH)

$$\left(\frac{90\ \cancel{\text{km}}}{\text{h}}\right)\left(\frac{0.6214\ \text{miles}}{\cancel{\text{km}}}\right) = 55.9\ \text{MPH}$$

90 km/h = **55.9 MPH**

b. 1 km = 1000 m

$$\left(\frac{90\ \cancel{\text{km}}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{\cancel{\text{km}}}\right) = 90{,}000\ \text{m/h}$$

$$\text{However, } (1\ \cancel{\text{h}})\left(\frac{60\ \cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(\frac{60\ \text{sec}}{\cancel{\text{min}}}\right) = 3600\ \text{s}$$

$$\left(\frac{90{,}000\ \text{m}}{\cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right) = 25.0\ \text{m/s}$$

90 km/h = **25.0 m/s**

Part b can also be done using the conversion factors for speed in appendix E:

1 km/h = 0.278 m/s

$$(90\ \cancel{\text{km/h}})\left(\frac{0.278\ \text{m/s}}{1\ \cancel{\text{km/h}}}\right) = 25.0\ \text{m/s}$$

90 km/h = **25.0 m/s**

Lines drawn through the units indicate cancellation.

What are the units of speed?

Units are an essential part of the description of speed. Suppose you say that you were doing 70—without stating the units. In the United States, that would probably be understood as 70 MPH, because that is the unit most frequently used. In Europe, on the other hand, people would probably assume you were talking about the considerably slower speed of 70 km/h. If you do not state the units, you will not communicate effectively.

It is easy to convert from one unit to another if the conversion factors are known. For example, if we want to convert kilometers per hour to miles per hour, we need to know the relationship between miles and kilometers. A kilometer is roughly $^6/_{10}$ of a mile (0.6214, to be more precise). As shown in example box 2.1, 90 km/h is equal to 55.9 MPH. The process involves multiplication or division by the appropriate conversion factor.

Units of speed will always be a distance divided by a time. In the metric system, the fundamental unit of speed is meters per second (m/s). Example box 2.1 also shows the conversion of kilometers per hour to meters per second, done as a two-step process. As you can see, 90 km/h can also be expressed as 25.0 m/s. This is a convenient size for discussing the speeds of ordinary objects. (As shown in example box 2.2, the convenient unit for measuring the growth of grass has a very different size.) Table 2.1 shows some familiar speeds expressed in miles per hour, kilometers per hour, and meters per second to give you a sense of their relationships.

Table 2.1

Familiar Speeds in Different Units

20 MPH	=	32 km/h	=	9 m/s
40 MPH	=	64 km/h	=	18 m/s
60 MPH	=	97 km/h	=	27 m/s
80 MPH	=	130 km/h	=	36 m/s
100 MPH	=	160 km/h	=	45 m/s

Example Box 2.2

Sample Question: Watching Grass Grow

Question: The units km/h and m/s have an appropriate size for moving cars and people. Many other processes move much more slowly, though. What units would have an appropriate size for measuring the average speed with which a blade of grass grows?

Answer: When grass is well fertilized and watered, it is not unusual for it to grow 3 to 6 centimeters in the course of a week. This can be seen by measuring the length of the clippings after mowing. If we measured the speed in m/s, we would obtain an extremely small number that would not provide a good intuitive sense of the rate of growth. The units of cm/week and mm/day would provide a better indication of this speed.

What is instantaneous speed?

If we travel a distance of 260 miles in 5 hours, as in our earlier example, is it likely that the entire trip takes place at a speed of 52 MPH? Of course not; the speed goes up and down as the road goes up and down, when we overtake slower vehicles, when rest breaks occur, or when the highway patrol looms on the horizon. If we want to know how fast we are going at a given instant in time, we read the speedometer, which displays the **instantaneous speed** (fig. 2.3).

How does instantaneous speed differ from average speed? The instantaneous speed tells us how fast we are going at a given instant but tells us little about how long it will take to travel several miles, unless the speed is held constant. The average speed, on the other hand, allows us to compute how long a trip might take but says little about the variation in speed during the trip. A more complete description of how the speed of a car varies during a portion of a trip could be provided by a graph such as that shown in figure 2.4. Each point on this graph represents the instantaneous speed at the time indicated on the horizontal axis.

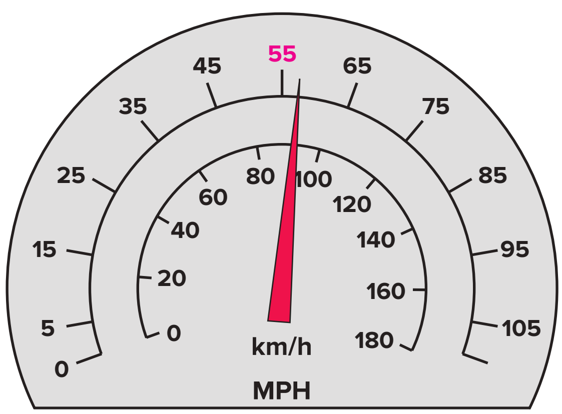


Figure 2.3 A speedometer with two scales for measuring instantaneous speed: MPH and km/h.

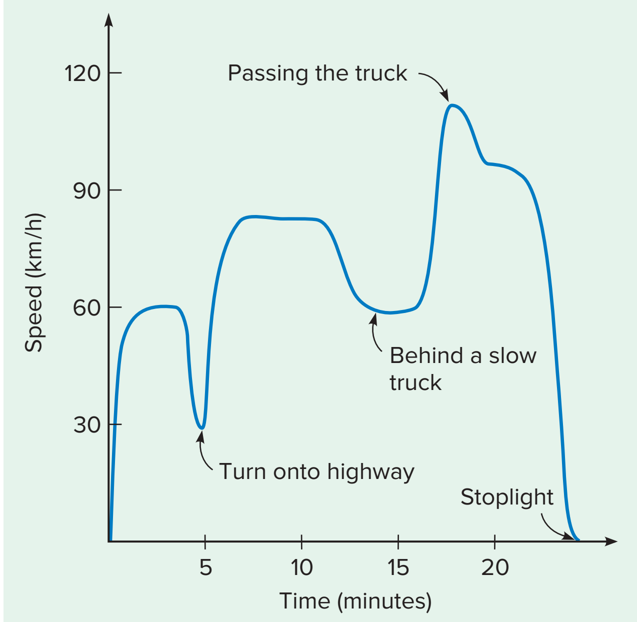


Figure 2.4 Variations in instantaneous speed for a portion of a trip on a local highway.

Even though we all have some intuitive sense of what instantaneous speed means from our experience in reading speedometers, computing this quantity presents some problems that we did not encounter in defining average speed. We could say that instantaneous speed is the rate that distance is being covered at a given instant in time, but how do we compute this rate? What time interval should we use? What is an instant in time?

Our solution to this problem is simply to choose a very short interval of time during which a very short distance is covered and the speed does not change drastically. If we know, for example, that in 1 second a distance of 20 meters was covered, dividing 20 meters by 1 second to obtain a speed of 20 m/s would give us a good estimate of the instantaneous speed, provided that the speed did not change much during that single second. If the speed was changing rapidly, we would have to choose an even shorter interval of time. In principle, we can choose time intervals as small as we wish, but in practice it can be hard to measure such small quantities.

If we put these ideas into a word definition of instantaneous speed, we could state it as

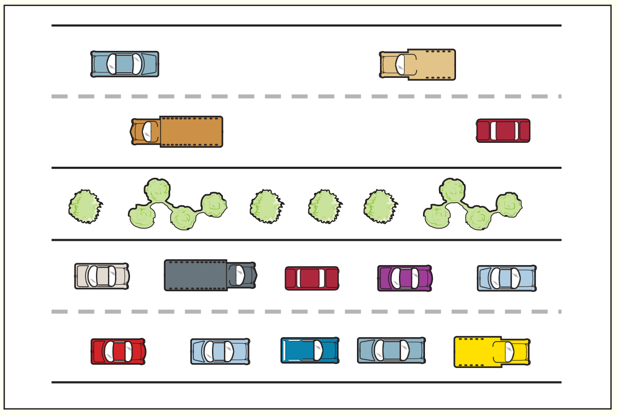
> Instantaneous speed is the rate at which distance is being covered at a given instant in time. It is found by computing the average speed for a very short time interval in which the speed does not change appreciably.

Instantaneous speed is closely related to the concept of average speed but involves very short time intervals. When discussing traffic flow, average speed is the critical issue, as shown in everyday phenomenon box 2.1.

Everyday Phenomenon Box 2.1

Transitions in Traffic Flow

The Situation. Jennifer commutes into the city on a freeway every day for work. As she approaches the city, the same patterns in traffic flow seem to show up in the same places each day. She will be moving with the flow of traffic at a speed of approximately 60 MPH when suddenly things will come to a screeching halt. The traffic will be stop-and-go briefly and then will settle into a wavelike mode, with speeds varying between 10 and 30 MPH. Unless there is an accident, this will continue for the rest of the way into the city.



The traffic in the upper lanes is flowing freely with adequate spacing to allow higher speeds. The higher-density traffic in the lower lanes moves much more slowly.

What causes these patterns? Why does the traffic stop when there is no apparent reason such as an accident? Why do ramp traffic lights seem to help the situation? Questions like these are the concern of the growing field of traffic engineering.

The Analysis. Although a full analysis of traffic flow is complex, there are some simple ideas that can explain many of the patterns that Jennifer observes. The density of vehicles, measured in vehicles per mile, is a key factor. Adding vehicles at entrance ramps increases this vehicle density.

When Jennifer and other commuters are traveling at 60 MPH, they need to keep a spacing of several car lengths between vehicles. Most drivers do this without thinking about it, although there are always some who follow too closely, or *tailgate.* Tailgating runs the risk of rear-end collisions when the traffic suddenly slows. Reaction time, and its implications for tailgating, is discussed in everyday phenomenon box 3.1.

When more vehicles are added at an entrance ramp, the density of vehicles increases, reducing the distance between vehicles. As the distance between vehicles decreases, drivers should reduce their speed to maintain a safe stopping distance. If this occurred uniformly, there would be a gradual decrease in the average speed of the traffic to accommodate the greater density. This is not what usually happens, however.

A significant proportion of drivers will attempt to maintain their speed at 50 to 60 MPH, even when densities have increased beyond the point where this is advisable. This creates an unstable situation. At some point, usually near an entrance ramp, the vehicle density becomes too large to sustain these speeds. At this point, there is a sudden drop in average speed and a large increase in the local density. As shown in the drawing, cars can be separated by less than a car length when they are stopped or moving very slowly.

Once the average speed of a few vehicles has slowed to less than 10 MPH, vehicles moving at 50 to 60 MPH begin to pile up behind this slower-moving jam. Because this does not happen smoothly, some vehicles must come to a complete stop, further slowing the flow. At the front end of the jam, on the other hand, the density is reduced due to the slower flow behind. Cars can then start moving at a speed consistent with the new density, perhaps around 30 MPH. If every vehicle moved with the appropriate speed, flow would be smooth and the increased density could be safely accommodated. More often, however, overanxious drivers exceed the appropriate speed, causing fluctuations in the average speed as vehicles begin to pile up again.

Notice that we are using *average speed* with two different meanings in this discussion. One is the average speed of an individual vehicle as its instantaneous speed increases and decreases. The other is the average speed of the overall traffic flow involving many vehicles. When the traffic is flowing freely, the average speed of different vehicles may differ. When the traffic is in a slowly moving jam, the average speeds of different vehicles are essentially the same, at least within a given lane.

Traffic lights at entrance ramps that permit vehicles to enter one-at-a-time at appropriate intervals can help to smoothly integrate the added vehicles to the existing flow. This reduces the sudden changes in speed caused by a rapid increase in density. Once the density increases beyond the certain level, however, a slowing of traffic is inevitable. The abrupt change from low-density, high-speed flow to higher-density, slow flow is analogous to a phase transition from a gas to a liquid. (Phase transitions are discussed in chapter 10.) Traffic engineers have used this analogy to better understand the process.

If we could automatically control and coordinate the speeds of all the vehicles on the highway, the highway could carry a much greater volume of traffic at a smooth rate of flow. Speeds could be adjusted to accommodate changes in density, and smaller vehicle separations could be maintained at higher speeds, because all the vehicles would be moving in a synchronized fashion. Better technology may someday achieve this dream.

We find an average speed by dividing the distance traveled by the time required to cover that distance. Average speed is therefore the average rate at which distance is being covered. Instantaneous speed is the rate that distance is being covered at a given instant in time and is found by considering very small time intervals or by reading a speedometer. Average speed is useful for estimating how long a trip will take, but instantaneous speed is of more interest to the highway patrol.

Debatable Issue

A radar gun used by a police officer measures your speed at a certain instant in time, whereas an officer in a plane measures the time it takes for you to travel the known distance between two stripes painted on the highway. What is the difference in nature between these two types of measurement and which is the fairer basis for issuing a speeding ticket?

2.2 Velocity

Do the words *speed* and *velocity* mean the same thing? They are often used interchangeably in everyday language, but physicists make an important distinction between the two terms. The distinction has to do with direction: Which way is the object moving? This distinction turns out to be essential to understanding Newton's theory of motion (introduced in chapter 4), so it is not just a matter of whim or jargon.

What is the difference between speed and velocity?

Imagine you are driving a car around a curve (as illustrated in fig. 2.5) and you maintain a constant speed of 60 km/h. Is your velocity also constant in this case? The answer is no, because **velocity** involves the direction of motion, as well as how fast the object is going. The direction of motion is changing as the car goes around the curve.

To simply state this distinction, speed as we have defined it tells us how fast an object is moving but says nothing about the direction of the motion. Velocity includes the idea of direction. To specify a velocity, we must give both its size or **magnitude** (how fast) and its direction (north, south, east, up, down, or somewhere in between). If you tell me that an object is moving 15 m/s, you have told me its *speed.* If you tell me that it is moving due west at 15 m/s, you have told me its *velocity.*

At point A on the diagram in figure 2.5, the car is traveling due north at 60 km/h. At point B, because the road curves, the car is traveling northwest at 60 km/h. Its velocity at point B is different from its velocity at point A (because the directions are different). The speeds at points A and B are the same.

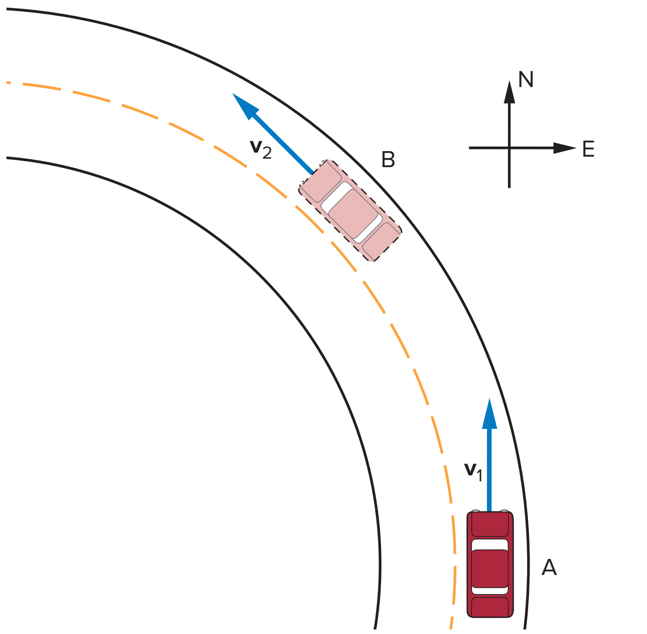


Figure 2.5 The direction of the velocity changes as the car moves around the curve, so that the velocity v_2 is not the same as the velocity v_1 even though the speed has not changed.

Direction is irrelevant in specifying the speed of the object. It has no effect on the reading on your speedometer.

Changes in velocity are produced by forces acting upon the car, as we will discuss further in chapter 4. The most important force involved in changing the velocity of a car is the frictional force exerted on the tires of the car by the road surface. A force is required to change either the size or the direction of the velocity. If no net force were acting on the car, it would continue to move at constant speed in a straight line. This happens sometimes when there is ice or oil on the road surface, which can reduce the frictional force to almost zero.

Study Hint

Science has always relied on pictures and charts to get points across. Throughout the book, a number of concepts will be introduced and illustrated. In the illustrations, the same color will be used for certain phenomena.

- Blue arrows are velocity vectors.
- Green arrows depict acceleration vectors.
- Red arrows depict force vectors.
- Purple arrows show momentum, a concept we will explore in chapter 7.

What is a vector?

Velocity is a quantity for which both the size and direction are important. We call such quantities **vectors.** To describe

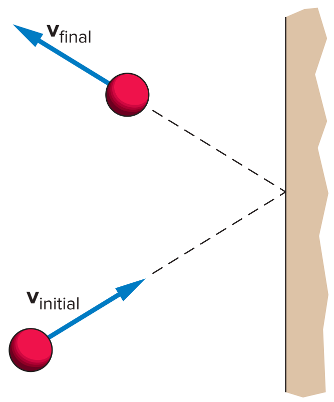


Figure 2.6 The direction of the velocity changes when a ball bounces from a wall. The wall exerts a force on the ball in order to produce this change.

these quantities fully, we need to state *both the size and the direction.* Velocity is a vector that describes how fast an object is moving and in what direction it is moving. Many of the quantities used in describing motion (and in physics more generally) are **vector quantities.** These include velocity, acceleration, force, and momentum.

Think about what happens when you throw a rubber ball against a wall, as shown in figure 2.6. The speed of the ball may be about the same after the collision with the wall as it was before the ball hit the wall. The velocity has clearly changed in the process, though, because the ball is moving in a different direction after the collision. Something has happened to the motion of the ball. A strong force had to be exerted on the ball by the wall to produce this change in velocity.

The velocity vectors in figures 2.5 and 2.6 are represented by arrows. This is a natural choice for depicting vectors, because the direction of the arrow clearly shows the direction of the vector, and the length can be drawn proportional to the size. In other words, the larger the velocity, the longer the arrow (fig. 2.7). In the text, we will represent vectors by printing their symbols in boldface and making them larger than other symbols: $\mathbf{v}$ is thus the symbol for velocity. A fuller description of vectors can be found in appendix C.

How do we define instantaneous velocity?

In considering automobile trips, *average* speed is the most useful quantity. We do not really care about the direction of motion in this case. *Instantaneous* speed is the quantity of interest to the highway patrol. **Instantaneous velocity,** however, is most useful in considering physical theories of motion. We can define instantaneous velocity by drawing on our earlier definition of instantaneous speed.

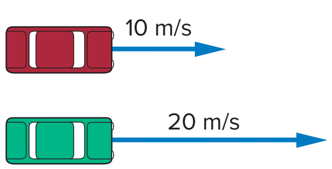


Figure 2.7 The length of the arrows show the relative size of the velocity vectors.

> Instantaneous velocity is a vector quantity having a size equal to the instantaneous speed at a given instant in time and having a direction corresponding to that of the object's motion at that instant.

Instantaneous velocity and instantaneous speed are closely related, but velocity includes direction as well as size. It is *changes* in instantaneous velocity that require the intervention of forces. These changes will be emphasized when we explore Newton's theory of mechanics in chapter 4. We can also define the concept of average velocity, but that is a much less useful quantity for our purposes than either instantaneous velocity or average speed.*

> To specify the velocity of an object, we need to state both how fast and in what direction the object is moving; velocity is a vector quantity. Instantaneous velocity has a magnitude equal to the instantaneous speed, and it points in the direction that the object is moving. Changes in instantaneous velocity are where the action is, so to speak, and we will consider these in more detail when we discuss acceleration in section 2.3.

2.3 Acceleration

Acceleration is a familiar idea. We use the term in speaking of the acceleration of a car away from a stop sign or the acceleration of a running back in football. We feel the effects of acceleration on our bodies when a car's velocity changes rapidly and even more strikingly when an elevator lurches downward, leaving our stomachs slightly behind (fig. 2.8). These are all accelerations. You can think of your stomach as an acceleration detector—a roller coaster gives it a real workout!

Understanding acceleration is crucial to our study of motion. **Acceleration** is the rate at which velocity *changes.* (Note that we said velocity, not speed.) It plays a central role in Newton's theory of motion. How do we go about finding

*Strictly speaking, velocity is the change in displacement divided by time, where displacement is a vector representing the change in position of an object. See appendix C and figure C.2 for a discussion of displacement vectors. In one-dimensional motion when an object does not change direction, the distance traveled is equal to the magnitude of the displacement.

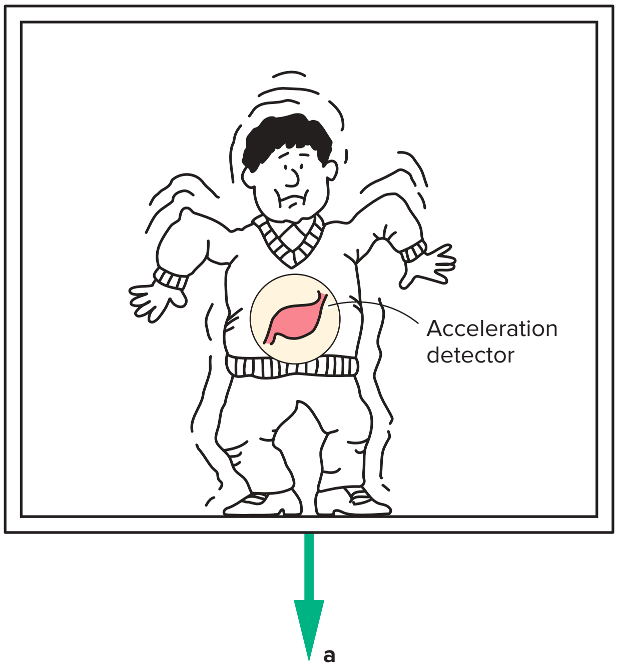


Figure 2.8 Your acceleration detector senses the downward acceleration of the elevator.

a value of an acceleration, though? As with speed, it is convenient to start with a definition of average acceleration and then extend it to the idea of instantaneous acceleration.

How is average acceleration defined?

How would we go about providing a quantitative description of an acceleration? Suppose that your car, pointing due east, starts from a full stop at a stop sign, and its velocity increases from zero to 20 m/s, as shown in figure 2.9. The change in velocity is found simply by subtracting the initial velocity from the final velocity (20 m/s − 0 m/s = 20 m/s). To find its *rate of change,* however, we also need to know the time needed to produce this change. If it took just 5 seconds for the velocity to change, the rate of change would be larger than if it took 30 seconds.

Suppose that a time of 5 seconds was required to produce this change in velocity. The rate of change in velocity could then be found by dividing the size of the change in velocity by the time required to produce that change. Thus, the size

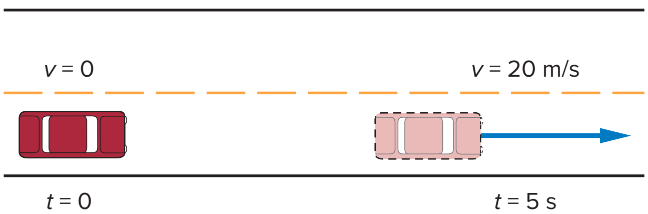


Figure 2.9 A car, starting from rest, accelerates to a velocity of 20 m/s due east in a time of 5 s.

of the **average acceleration,** *a,* is found by dividing the change in velocity of 20 m/s by the time of 5 seconds,

$$a = \frac{20 \text{ m/s}}{5 \text{ s}} = 4 \text{ m/s/s}$$

The unit m/s/s is usually written m/s^2 and is read as *meters per second squared.* It is easier to understand it, however, as *meters per second per second.* The car's velocity (measured in m/s) is changing at a rate of 4 m/s every second. Other units could be used for acceleration, but they will all have this same form: distance per unit of time per unit of time. In discussing the acceleration of a car on a drag strip, for example, the unit *miles per hour per second* is sometimes used.

The quantity we have just computed is the size of the average acceleration of the car. The average acceleration is found by dividing the total change in velocity for some time interval by that time interval, ignoring possible differences in the rate of change of velocity that might be occurring within the time interval. Its definition can be stated in words as follows:

> Average acceleration is the change in velocity divided by the time required to produce that change.

We can restate it in symbols as

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{elapsed time}}$$

or

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t}$$

Because change is so important in this definition, we have used the special symbol Δ (the Greek letter delta) to mean a change in a quantity. Thus, $\Delta \mathrm{v}$ is a compact way of writing *the change in velocity,* which otherwise could be expressed as $\mathrm{v}_f - \mathrm{v}_i$, where v_f is the final velocity and v_i is the initial velocity. Because the concept of change is critical, this delta (Δ) notation will appear often.

The idea of change is all-important. Acceleration is *not* velocity over time. It is the *change* in velocity divided by time. It is common for people to associate large accelerations with large velocities when, in fact, the opposite is often true. The acceleration of a car may be largest, for example, when it is just starting up and its velocity is near zero. The rate of change of velocity is greatest then. On the other hand, a car can be traveling at 100 MPH but still have a zero acceleration if its velocity is not changing.

What is instantaneous acceleration?

Instantaneous acceleration is similar to average acceleration, with an important exception. Just as with instantaneous speed or velocity, we are now concerned with the rate of change at a given instant in time. It is *instantaneous* acceleration that our stomachs respond to. It can be defined as follows:

> Instantaneous acceleration is the rate at which velocity is changing at a given instant in time. It is computed by finding the average acceleration for a very short time interval during which the acceleration does not change appreciably.

If the acceleration is changing with time, choosing a very short time interval guarantees that the acceleration computed for that time interval will not differ too much from the instantaneous acceleration at any time within the interval. This is the same idea used in finding an instantaneous speed or instantaneous velocity.

What is the direction of an acceleration?

Like velocity, acceleration is a vector quantity. Its direction is important. The direction of the acceleration vector is that of the *change* in velocity $\Delta\mathbf{v}$. If, for example, a car is moving in a straight line and its velocity is increasing, the change in velocity is in the same direction as the velocity itself, as shown in figure 2.10. The change in velocity $\Delta\mathbf{v}$ must be added to the initial velocity $\mathbf{v}_i$ to obtain the final velocity $\mathbf{v}_f$. All three vectors point forward. The process of adding vectors can be readily seen when we represent the vectors as arrows on a graph. (More information on vector addition can be found in appendix C.)

If the velocity is decreasing, however, the change in velocity $\Delta\mathbf{v}$ points in the opposite direction to the two velocity vectors, as shown in figure 2.11. Because the initial velocity $\mathbf{v}_i$ is larger than the final velocity $\mathbf{v}_f$, the change in velocity must point in the opposite direction to produce a shorter $\mathbf{v}_f$ arrow. The acceleration is also in the opposite direction to the velocity, because it is in the direction of the *change* in velocity.

In Newton's theory of motion (chapter 4), the force required to produce this acceleration would also be opposite in direction to the velocity. It must push backward on the car to slow it down. In general, if an object's velocity is decreasing, velocity and acceleration vectors point in opposite directions and therefore have opposite signs. If an object is speeding up, the signs of the velocity and acceleration are the same.

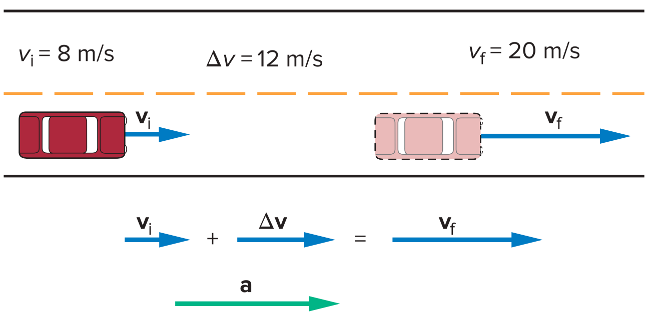


Figure 2.10 The acceleration vector is in the same direction as the velocity vectors when the velocity is increasing.

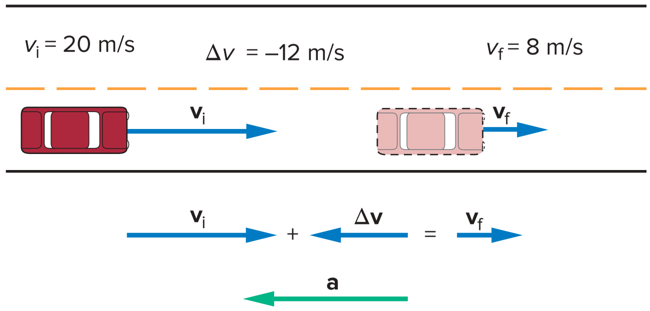


Figure 2.11 The velocity and acceleration vectors for decreasing velocity: $\Delta\mathbf{v}$ and $\mathbf{a}$ are now opposite in direction to the velocity. The acceleration $\mathbf{a}$ is proportional to $\Delta\mathbf{v}$.

The term *acceleration* describes the rate of *any* change in an object's velocity. The change could be an increase (as in our initial example), a decrease, or a change in direction. The term applies even to decreases in velocity (*decelerations*). To a physicist these are simply accelerations with a direction opposite that of the velocity. If a car is braking while traveling in a straight line, its velocity is decreasing and its acceleration is negative if the velocity is positive. This situation is illustrated in the sample exercise in example box 2.3. If, on the other hand, the car is moving in the negative direction and slowing down, the acceleration is in the positive direction. When an object is slowing down, the velocity and acceleration vectors point in opposite directions.

The minus sign is an important part of the result in the example in example box 2.3, because it indicates that the change in velocity is negative. The velocity is getting smaller. We can call it a deceleration if we like, but it is the same thing as a negative acceleration. One word, *acceleration,* covers all situations in which the velocity is changing.

Example Box 2.3

Sample Exercise: Negative Accelerations

The driver of a car steps on the brakes, and the velocity drops from 30 m/s due east to 10 m/s due east in a time of 4.0 seconds. What is the acceleration?

$v_i = 30$ m/s due east

$v_f = 10$ m/s due east

$t = 4.0$ s

$\mathbf{a} = ?$

$$a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$$

$$= \frac{10\text{ m/s} - 30\text{ m/s}}{4.0\text{ s}}$$

$$= \frac{-20\text{ m/s}}{4.0\text{ s}}$$

$$= -5\text{ m/s}^2$$

$\mathbf{a} = 5.0\text{ m/s}^2$ due west

Notice that when we are dealing just with the magnitude of a vector quantity, we do not use the boldface notation. The sign can indicate direction, however, in a problem involving straight-line motion.

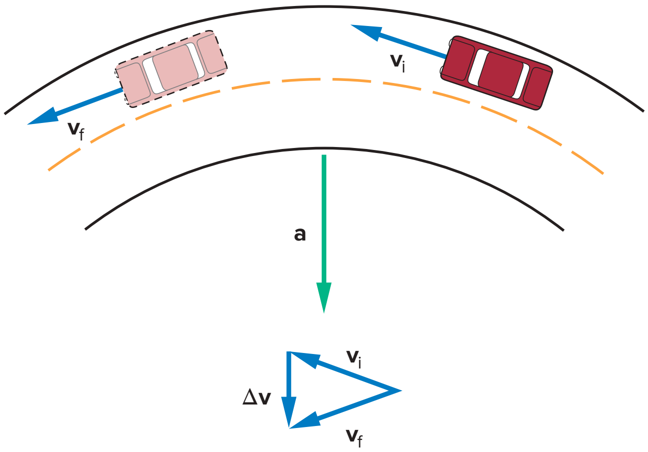


Figure 2.12 A change in the direction of the velocity vector also involves an acceleration, even though the speed may be constant.

Can a car be accelerating when its speed is constant?

What happens when a car goes around a curve at constant speed? Is it accelerating? The answer is yes, because the direction of its velocity is changing. If the direction of the velocity vector is changing, the velocity is changing. This means there must be an acceleration.

This situation is illustrated in figure 2.12. The arrows in this drawing show the direction of the velocity vector at different points in the motion. The change in velocity $\Delta \mathbf{v}$ is the vector that must be added to the initial velocity $\mathbf{v}_i$ to obtain the final velocity $\mathbf{v}_f$. The vector representing the change in velocity points toward the center of the curve, and therefore the acceleration vector also points in that direction. The size of the change is represented by the length of the arrow $\Delta \mathbf{v}$. From this, we can find the acceleration.

Acceleration is involved whenever there is a change in velocity, regardless of the nature of that change. Cases like figure 2.12 will be considered more fully in chapter 5, where circular motion is discussed.

> Acceleration is the rate of change of velocity and is found by dividing the change in the velocity by the time required to produce that change. Any change in velocity involves an acceleration, whether an increase or a decrease in speed, or a change in direction. Acceleration is a vector having a direction corresponding to the direction of the change in velocity, which is not necessarily the same direction as the instantaneous velocity itself. The concept of change is crucial. The graphical representations in section 2.4 will help you visualize changes in velocity as well as changes in other quantities.

2.4 Graphing Motion

It is often said that a picture is worth a thousand words, and the same can be said of graphs. Imagine trying to describe the motion depicted in figure 2.4 precisely in words and numbers. The graph provides a quick overview of what took place. A description in words would be much less efficient. In this section, we will show how graphs can also help us to understand velocity and acceleration.

What can a graph tell us?

How can we produce and use graphs to help us describe motion? Imagine you are watching a battery-powered toy car moving along a meter stick (fig. 2.13). If the car were moving slowly enough, you could record the car's position while also recording the elapsed time using a digital watch. At regular time intervals (say, every 5 seconds), you would note the value of the position of the front of the car on the meter stick and write down these values. The results would be something like those shown in table 2.2.

How do we graph these data? First, we create evenly spaced intervals on each of two perpendicular axes, one for distance traveled (or position) and the other for time. To show how distance varies with time, we usually put time on the horizontal axis and distance on the vertical axis. Such a graph is shown in figure 2.14, where each data point from table 2.2 is plotted and a line is drawn through the points. To make sure you understand this process, choose different points from table 2.2 and find where they are located on the graph. Where would the point go if the car were at 21 centimeters at 25 seconds?

The graph summarizes the information presented in the table in a visual format that makes it easier to grasp at a glance. The graph also contains information on the velocity and acceleration of the car, although that is less obvious.

Figure 2.13 A toy car moving along a meter stick. Its position can be recorded at different times.
Michelle Mauser/McGraw-Hill Education

Table 2.2

Position of the Toy Car along the Meter Stick at Different Times

Time	Position
0 s	0 cm
5 s	4.1 cm
10 s	7.9 cm
15 s	12.1 cm
20 s	16.0 cm
25 s	16.0 cm
30 s	16.0 cm
35 s	18.0 cm
40 s	20.1 cm
45 s	21.9 cm
50 s	24.0 cm
55 s	22.1 cm
60 s	20.0 cm

For example, what can we say about the average velocity of the car between 20 and 30 seconds? Is the car moving during this time? A glance at the graph shows us that the distance is not changing during that time interval, so the car is *not* moving. The velocity is zero during that time, which is represented by a horizontal line on our graph of distance versus time.

What about the velocity at other points in the motion? The car is moving more rapidly between 0 and 20 seconds than it is between 30 and 50 seconds. The distance curve is rising more rapidly between 0 and 20 seconds than between 30 and 50 seconds. Because more distance is covered in the same time, the car must be moving faster there. A steeper slope to the curve is associated with a larger speed.

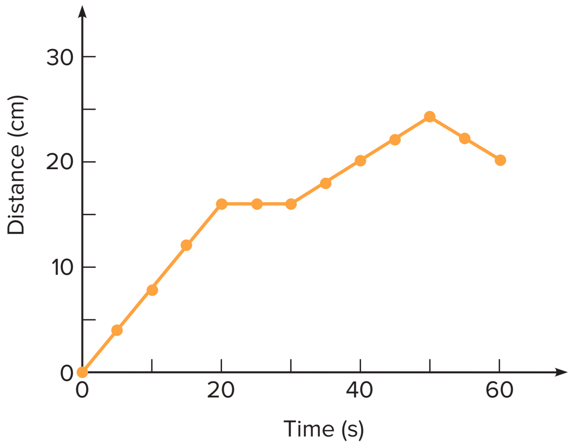


Figure 2.14 Distance plotted against time for the motion of the toy car. The data points are those listed in table 2.2.

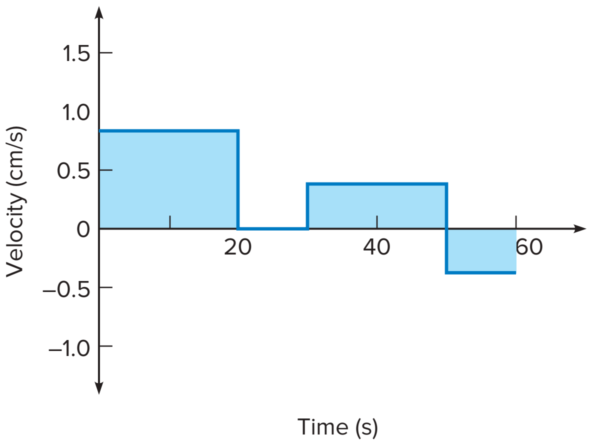


Figure 2.15 Instantaneous velocity plotted against time for the motion of the toy car. The velocity is greatest when distance traveled is changing most rapidly.

In fact, the **slope** of the distance-versus-time curve at any point on the graph is equal to the *instantaneous velocity* of the car.* The slope indicates how rapidly the distance is changing with time at any instant in time. The rate of change of distance with time is the instantaneous speed according to the definition given in section 2.1. Because the motion takes place along a straight line, we can then represent the direction of the velocity with plus or minus signs. There are only two possibilities, forward or backward. We then have the instantaneous velocity, which includes both the size (speed) and direction of the motion.

When the car travels backward, its distance from the starting point decreases. The curve goes down, as it does between 50 and 60 seconds. We refer to this downward-sloping portion of the curve as having a *negative slope* and say that the velocity is negative during this portion of the motion. A large upward slope represents a large instantaneous velocity, a zero slope (horizontal line) a zero velocity, and a downward slope a negative (backward) velocity. Looking at the slope of the graph tells us all we need to know about the velocity of the car.

Velocity and acceleration graphs

These ideas about velocity can be best summarized by plotting a graph of velocity against time for the car (fig. 2.15). The velocity is constant wherever the *slope* of the distance-versus-time graph of figure 2.14 is constant. Any straight-line segment of a graph has a constant slope, so the velocity

*Because the mathematical definition of slope is the change in the vertical coordinate Δd divided by the change in the horizontal coordinate Δt, the slope, $\Delta d/\Delta t$, is equal to the instantaneous velocity, provided that Δt is sufficiently small. It is possible to grasp the concept of slope, however, without appealing to the mathematical definition.

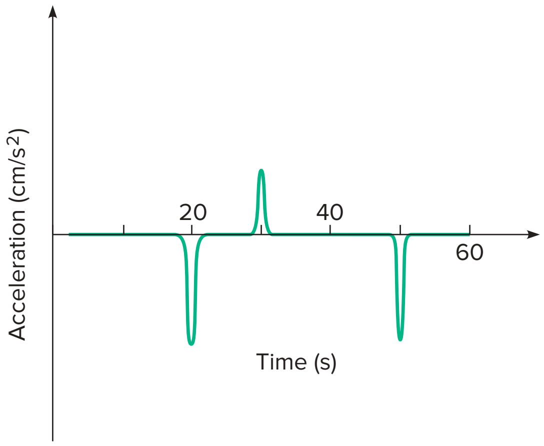


Figure 2.16 An approximate sketch of acceleration plotted against time for the toy-car data. The acceleration is nonzero only when the velocity is changing.

changes only where the slope of the distance graph in figure 2.14 changes. If you carefully compare the graph in figure 2.15 to the graph in figure 2.14, these ideas should become clear.

What can we say about the acceleration from these graphs? Because acceleration is the rate of change of velocity with time, the velocity graph (fig. 2.15) also provides information about the acceleration. In fact, the instantaneous acceleration is equal to the slope of the velocity-versus-time graph. A steep slope represents a rapid change in velocity and thus a large acceleration. A horizontal line has zero slope and represents zero acceleration. The acceleration turns out to be zero for most of the motion described by our data. The velocity changes at only a few points in the motion. The acceleration would be large at these points and zero everywhere else.

Because our data do not indicate how rapidly the changes in velocity actually occur, we do not have enough information to say just how large the acceleration is at those few points where it is not zero. We would need measurements of distance or velocity every tenth of a second or so to get a clear idea of how rapid these changes are. As we will see in chapter 4, we know that these changes in velocity cannot occur instantly. Some time is required. Thus, we can sketch an approximate graph of acceleration versus time, as shown in figure 2.16.

The spikes in figure 2.16 occur when the velocity is changing. At 20 seconds, there is a rapid decrease in the velocity represented by a downward spike or negative acceleration. At 30 seconds, the velocity increases rapidly from zero to a constant value, and this is represented by an upward spike or positive acceleration. At 50 seconds, there is another negative acceleration as the velocity changes from a positive value to a negative value. If you could put yourself inside the toy car, you would definitely feel these accelerations. (Everyday phenomenon box 2.2 provides another example of how a graph is useful for analyzing motion.)

Can we find the distance traveled from the velocity graph?

What other information can be gleaned from the velocity-versus-time graph of figure 2.15? Think for a moment about how you would find the distance traveled if you knew the velocity. For a constant velocity, you can get the distance simply by multiplying the velocity by the time, $d = vt$. In the first 20 seconds of the motion, for example, the velocity is 0.8 cm/s and the distance traveled is 0.8 cm/s times 20 seconds, which is 16 cm. This is just the reverse of what we used in determining the velocity in the first place. We found the velocity by dividing the distance traveled by the time.

How would this distance be represented on the velocity graph? If you recall formulas for computing areas, you may recognize that the distance d is the area of the shaded rectangle on figure 2.15. The area of a rectangle is found by multiplying the height times the width, just what we have done here. The velocity, 0.8 cm/s, is the height, and the time, 20 seconds, is the width of this rectangle on the graph.

It turns out that we can find the distance this way even when the areas involved on the graph are not rectangles, although the process is more difficult when the curves are more complicated. The general rule is that the distance traveled is equal to the area under the velocity-versus-time curve. When the velocity is negative (below the time axis on the graph), the object is traveling backward and its distance from the starting point is decreasing.

Even without computing the area precisely, it is possible to get a rough idea of the distance traveled by studying the velocity graph. A large area represents a large distance. Quick visual comparisons give a good picture of what is happening without the need for lengthy calculations. This is the beauty of a graph.

> A good graph can present a picture of motion that is rich in insight. Not only does distance traveled plotted against time tell us where the object is at any time, but also its slope indicates how fast it was moving. The graph of velocity plotted against time also contains information on acceleration and on the distance traveled. Producing and studying such graphs can give us a more general picture of the motion and the relationships between distance, velocity, and acceleration.

Everyday Phenomenon Box 2.2

The 100-m Dash

The Situation. A world-class sprinter can run 100 m in a little under 10 s. The race begins with the runners in a crouched position in the starting blocks, waiting for the sound of the starter's pistol. The race ends with the runners lunging across the finish line, where their times are recorded by stopwatches or automatic timers.

Runners in the starting blocks, waiting for the starter's pistol to fire.
Stockbyte/Getty Images

What happens between the start and finish of the race? How do the velocity and acceleration of the runners vary during the race? Can we make reasonable assumptions about what the velocity-versus-time graph looks like for a typical runner? Can we estimate the maximum velocity of a good sprinter? Most importantly for improving performance, what factors affect the success of a runner in the dash?

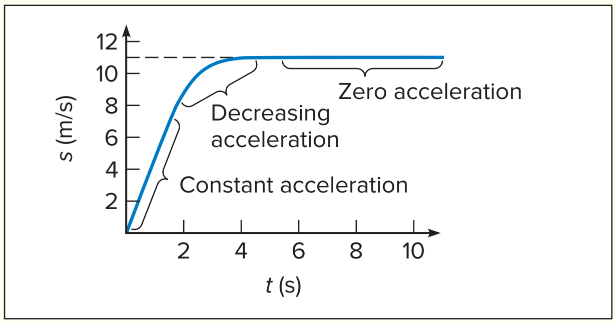
The Analysis. Let's assume that the runner covers the 100-m distance in a time of exactly 10 s. We can compute the average speed of the runner from the definition $s = d/t$:

$$s = \frac{100 \text{ m}}{10 \text{ s}} = 10 \text{ m/s}$$

Clearly, this is not the runner's instantaneous speed throughout the course of the race, because the runner's speed at the beginning of the race is zero and it takes some time to accelerate to the maximum speed.

The objective in the race is to reach a maximum speed as quickly as possible and to sustain that speed for the rest of the race. Success is determined by two things: how quickly the runner can accelerate to this maximum speed and the value of this maximum speed. A smaller runner often has better acceleration but a smaller maximum speed, while a larger runner sometimes takes longer to reach top speed but has a larger maximum speed.

The typical runner takes at least 10 to 20 m to reach top speed. If the average speed is 10 m/s, the runner's maximum speed must be somewhat larger than this value, because we know that the instantaneous speed will be less than 10 m/s while the runner is accelerating. These ideas are easiest to visualize by sketching a graph of speed plotted against time, as shown. Because the runner travels in a straight line, the magnitude of the instantaneous velocity is equal to the instantaneous speed. The runner reaches top speed at approximately 2 to 3 s into the race.



A graph of speed versus time for a hypothetical runner in the 100-m dash.

The average speed (or velocity) during the time that the runner is accelerating is approximately half of its maximum value if the runner's acceleration is more or less constant during the first 2 s. If we assume that the runner's average speed during this time is about 5.5 m/s (half of 11 m/s), then the speed through the remainder of the race would have to be about 11.1 m/s to give an average speed of 10 m/s for the entire race. This can be seen by computing the distance from these values:

$$\begin{aligned} d &= (5.5 \text{ m/s})(2 \text{ s}) + (11.1 \text{ m/s})(8 \text{ s}) \\ &= 11 \text{ m} + 89 \text{ m} = 100 \text{ m} \end{aligned}$$

What we have done here is make some reasonable guesses for these values that will make the average speed come out to 10 m/s. We then checked these guesses by computing the total distance. This suggests that the maximum speed of a good sprinter must be about 11 m/s (25 MPH). For the sake of comparison, a distance runner who can run a 4-minute mile has an average speed of about 15 MPH, or 6.7 m/s.

The runner's strategy should be to get a good jump out of the blocks, keeping the body low initially and leaning forward to minimize air resistance and maximize leg drive. To maintain top speed during the remainder of the race, the runner needs good endurance. A runner who fades near the end needs more conditioning drills. For a given runner with a fixed maximum speed, the average speed depends on how quickly the runner can reach top speed. This ability to accelerate rapidly depends on leg strength (which can be improved by working with weights and other training exercises) and natural quickness.

2.5 Uniform Acceleration

If you drop a rock, it falls toward the ground with a constant acceleration, as we will see in chapter 3. An unchanging, or **uniform, acceleration** is the simplest form of accelerated motion. It occurs whenever there is a constant force acting on an object, which is the case for a falling rock, as well as for many other situations.

How do we describe the resulting motion? The importance of this question was first recognized by Galileo, who studied the motion of balls rolling down inclined planes as well as objects in free fall. In his famous work *Dialogues Concerning Two New Sciences,* published in 1638 near the end of his life, Galileo developed the graphs and formulas that are introduced in this section and that have been studied by students of physics ever since. His work provided the foundation for much of Newton's thinking a few decades later.

How does velocity vary in uniform acceleration?

Suppose a car is moving along a straight road and accelerating at a constant rate. We have plotted the acceleration against time for this situation in figure 2.17. The graph is very simple, but it illustrates what is meant by uniform acceleration. A uniform acceleration is one that does not change as the motion proceeds. It has the same value at any time, which produces a horizontal-line graph.

The graph of velocity plotted against time for the same situation tells a more interesting story. From our discussion in section 2.4, we know that the slope of a velocity-versus-time graph is equal to the acceleration. For a uniform positive acceleration, the velocity graph should have a constant upward slope; the velocity increases at a steady rate. A constant slope produces a straight line, which slopes upward if the acceleration is positive, as shown in figure 2.18. In plotting this graph, we assume that the initial velocity is zero.

This graph can also be represented by a formula. The velocity at any time t is equal to the original velocity *plus* the velocity that has been gained because the car is accelerating. The change in velocity Δv is equal to the acceleration times

Figure 2.17 The acceleration graph for uniform acceleration is a horizontal line. The acceleration does not change with time.

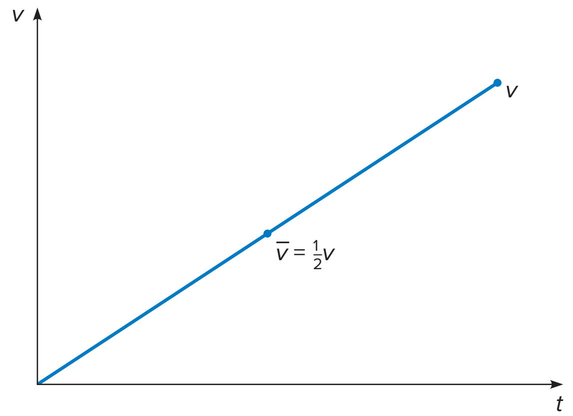


Figure 2.18 Velocity plotted against time for uniform acceleration, starting from rest. For this special case, the average velocity is equal to one-half the final velocity.

the time, $\Delta v = at$ because acceleration is defined as $\Delta v/t$. These ideas result in the relationship

$$v = v_0 + at$$

The first term on the right, v_0, is the original velocity (assumed to be zero in figure 2.18), and the second term, at, represents the change in velocity due to the acceleration. Adding these two terms together yields the velocity at any later time t.

A numerical example applying these ideas to an accelerating car is found in part a of example box 2.4. The car could not keep on accelerating indefinitely at a constant rate, because the velocity would soon reach incredible values. Not only is this dangerous, but physical limits imposed by air resistance and other factors prevent this from happening.

Example Box 2.4

Sample Exercise: Uniform Acceleration

A car traveling due east with an initial velocity of 10 m/s accelerates for 6 seconds at a constant rate of 4 m/s^2.

a. What is its velocity at the end of this time?
b. How far does it travel during this time?

a. $v_0 = 10$ m/s, $a = 4$ m/s^2, $t = 6$ s, $v = ?$

$$\begin{aligned} v &= v_0 + at \\ &= 10 \text{ m/s} + (4 \text{ m/s}^2)(6 \text{ s}) \\ &= 10 \text{ m/s} + 24 \text{ m/s} \\ &= \mathbf{34\ m/s} \end{aligned}$$

v = 34 m/s due east

b.
$$\begin{aligned} d &= v_0t + \frac{1}{2}at^2 \\ &= (10 \text{ m/s})(6 \text{ s}) + \frac{1}{2}(4 \text{ m/s}^2)(6 \text{ s})^2 \\ &= 60 \text{ m} + (2 \text{ m/s}^2)(36 \text{ s}^2) \\ &= 60 \text{ m} + 72 \text{ m} = \mathbf{132\ m} \end{aligned}$$

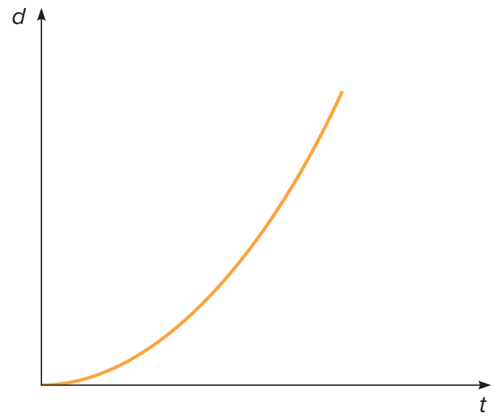


Figure 2.19 As the car accelerates uniformly, the distance covered grows more and more rapidly with time, because the velocity is increasing.

What happens if the acceleration is negative? Velocity would decrease rather than increase, and the slope of the velocity graph would slope downward rather than upward. Because the acceleration is then negative, the second term in the formula for v would subtract from the first term, causing the velocity to decrease from its initial value. The velocity then decreases at a steady rate.

How does distance traveled vary with time?

If the velocity is increasing at a steady rate, what effect does this have on the distance traveled? As the car moves faster and faster, the distance covered grows more and more rapidly. Galileo showed how to find the distance for this situation.

We find distance by multiplying velocity by time, but in this case we must use an average velocity, because the velocity is changing. By appealing to the graph in figure 2.18, we can see that the average velocity should be just half the final velocity, v. If the initial velocity is zero, the final velocity is *at,* so multiplying the average velocity by the time yields

$$d = \frac{1}{2}at^2$$

The time t enters twice, once in finding the average velocity and then again when we multiply the velocity by time to find the distance.*

The graph in figure 2.19 illustrates this relationship; the distance curve slopes upward at an ever-increasing rate as the velocity increases. This formula and graph are only valid if the object starts from rest, as shown in figure 2.18. Because distance traveled is equal to the area under the velocity-versus-time curve (as discussed in section 2.4), this expression for distance can also be thought of as the area under the triangle in figure 2.18. The area of a triangle is equal to one-half its base times its height, which produces the same result.

*Expressing this argument in symbolic form, it becomes the average velocity $\overline{v} = \frac{1}{2}v = \frac{1}{2}at$ $d = \overline{v}t = \left(\frac{1}{2}at\right)t = \frac{1}{2}at^2$.

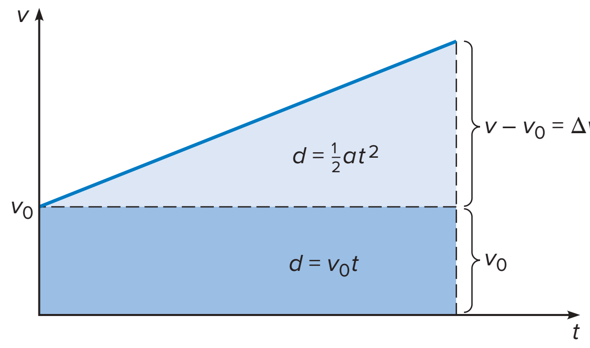


Figure 2.20 The velocity-versus-time graph redrawn for an initial velocity different from zero. The area under the curve is divided into two portions, a rectangle and a triangle.

If the car is already moving before it begins to accelerate, the velocity graph can be redrawn as in figure 2.20. The total area under the velocity curve can then be split into two pieces, a triangle and a rectangle, as shown. The total distance traveled is the sum of these two areas,

$$d = v_0t + \frac{1}{2}at^2$$

The first term in this formula represents the distance the object would travel if it moved with constant velocity v_0, and the second term is the additional distance traveled because the object is accelerating (the area of the triangle in figure 2.20). If the acceleration is negative, meaning that the object is slowing down, this second term will subtract from the first.

This more general expression for distance may seem complex, but the trick to understanding it is to break it down into its parts, as just suggested. We are merely adding two terms representing different contributions to the total distance. Each one can be computed in a straightforward manner, and it is not difficult to add them together. The two portions of the graph in figure 2.20 represent these two contributions.

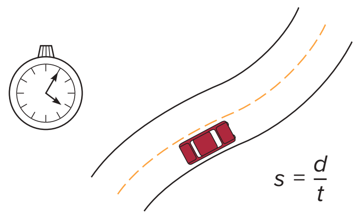
The sample exercise in example box 2.4 provides a numerical example of these ideas. The car in this example accelerates uniformly from an initial velocity of 10 m/s due east to a final velocity of 34 m/s due east and covers a distance of 132 meters while this acceleration is taking place. Had it not been accelerating, it would have gone only 60 meters in the same time. The additional 72 meters come because the car is accelerating.

Acceleration involves change, and uniform acceleration involves a steady rate of change. It therefore represents the simplest kind of accelerated motion that we can imagine. Uniform acceleration is essential to an understanding of free fall, discussed in chapter 3, as well as to many other phenomena. Such motion can be represented by either the graphs or the formulas introduced in this section. Looking at both and seeing how they are related will reinforce these ideas.

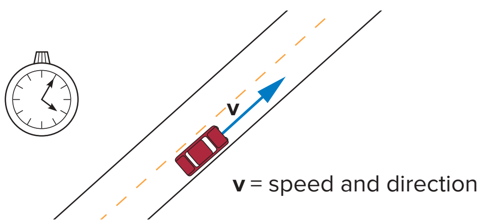
Summary

The main purpose of this chapter is to introduce concepts that are crucial to a precise description of motion. To understand acceleration, you must first grasp the concept of velocity, which in turn builds on the idea of speed. The distinctions between speed and velocity, and between velocity and acceleration, are particularly important.

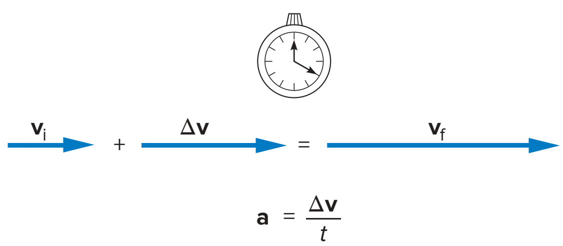
1 Average and instantaneous speed. Average speed is defined as the distance traveled divided by the time. It is the average rate at which distance is covered. Instantaneous speed is the rate at which distance is being covered at a given instant in time and requires that we use very short time intervals for computation.



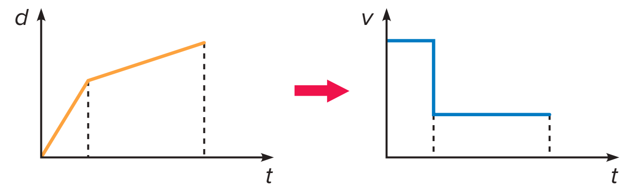
2 Velocity. The instantaneous velocity of an object is a vector quantity that includes both direction and size. The size of the velocity vector is equal to the instantaneous speed, and the direction is that of the object's motion.



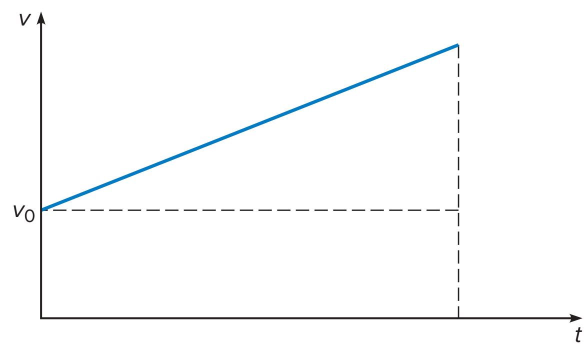
3 Acceleration. Acceleration is defined as the time rate of change of velocity and is found by dividing the *change* in velocity by the time. Acceleration is also a vector quantity. It can be computed as either an average or an instantaneous value. A change in the direction of the velocity can be as important as a change in magnitude. Both involve acceleration.



4 Graphing motion. Graphs of distance, speed, velocity, and acceleration plotted against time can illustrate relationships between these quantities. Instantaneous velocity is equal to the slope of the distance-time graph. Instantaneous acceleration is equal to the slope of the velocity-time graph. The distance traveled is equal to the area under the velocity-time graph.



5 Uniform acceleration. When an object accelerates at a constant rate, producing a constant-slope graph of velocity versus time, we say that it is uniformly accelerated. Graphs help us to understand the two formulas, describing how velocity and distance traveled vary with time for this important special case.



$$v = v_0 + at$$

$$d = v_0 t + \tfrac{1}{2}at^2$$

The concepts of velocity and acceleration discussed in this chapter are often difficult to understand, particularly because we use the same terms in everyday life but often with different meanings. There are mastery quizzes and other helpful resources in Connect that will help you clarify your understanding of these ideas. We encourage you to try them.

Key Terms

Speed, 19
Average speed, 19
Rate, 20
Instantaneous speed, 21
Velocity, 23
Magnitude, 23
Vector, 23
Vector quantity, 24
Instantaneous velocity, 24
Acceleration, 24
Average acceleration, 25
Instantaneous acceleration, 25
Slope, 28
Uniform acceleration, 31

Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available in Connect

Q1. Suppose that critters are discovered on Mars that measure distance in *boogles* and time in *bops.*
  a. What would the units of speed be in this system? Explain.
  b. What would the units of velocity be? Explain.
  c. What would the units of acceleration be? Explain.

Q2. Suppose we chose inches as our basic unit of distance and days as our basic unit of time.
  a. What would the units of velocity and acceleration be in this system? Explain.
  b. Would this be a good choice of units for measuring the acceleration of an automobile? Explain.

Q3. What units would have an appropriate size for measuring the rate at which fingernails grow? Explain.

Q4. A tortoise and a hare cover the same distance in a race. The hare goes very fast for brief intervals but stops frequently, whereas the tortoise plods along steadily and finishes the race ahead of the hare.
  a. Which of the two racers has the greater average speed over the duration of the race? Explain.
  b. Which of the two racers is likely to reach the greatest instantaneous speed during the race? Explain.

Q5. A driver states that she was doing 80 when stopped by the police. Is that a clear statement? Would this be interpreted differently in England than in the United States? Explain.

Q6. Does the speedometer on a car measure average speed or instantaneous speed? Explain.

Q7. Is the average speed over several minutes more likely to be close to the instantaneous speed at any time for a car traveling in freely flowing, low-density traffic or for one traveling in high-density traffic? Explain.

*Q8. The highway patrol sometimes uses radar guns to identify possible speeders and at other times uses associates in airplanes, who note the time taken for a car to pass between two marks some distance apart on the highway. What does each of these methods measure, average speed or instantaneous speed? Can you think of situations in which either one of these methods might unfairly penalize a driver? Explain.

Q9. Is the term *vehicle density* (as used in everyday phenomenon box 2.1) related to the weight of an individual vehicle, or does it refer to a property of several vehicles? Explain.

Q10. Under what traffic conditions is the average speed of several vehicles equal to the average speed of individual vehicles within the group? Explain. (See everyday phenomenon box 2.1.)

Q11. At the front end of a traffic jam, is the vehicle density higher or lower than at the back end of the traffic jam? Explain. (See everyday phenomenon box 2.1.)

Q12. A hockey puck is sliding on frictionless ice. It slams against a wall and bounces back toward the player with the same speed it had before hitting the wall. Does the velocity of the hockey puck change in this process? Explain.

Q13. A ball attached to a string is whirled in a horizontal circle such that it moves with constant speed.
  a. Does the velocity of the ball change in this process? Explain.
  b. Is the acceleration of the ball equal to zero? Explain.

*Q14. A ball tied to a string fastened at the other end to a rigid support forms a pendulum. If we pull the ball to one side and release it, the ball moves back and forth along an arc determined by the string length.
  a. Is the velocity constant in this process? Explain.
  b. Is the speed likely to be constant in this process? What happens to the speed when the ball reverses direction?

Q15. A dropped ball gains speed as it falls. Can the velocity of the ball be constant in this process? Explain.

Q16. A driver of a car steps on the brakes, causing the velocity of the car to decrease. According to the definition of acceleration provided in this chapter, does the car accelerate in this process? Explain.

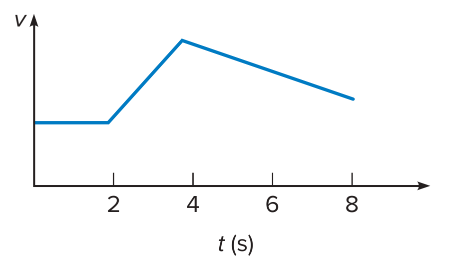
Q17. At a given instant in time, two cars are traveling at different velocities, one twice as large as the other. Based on this information, is it possible to say which of these two cars has the larger acceleration at this instant in time? Explain.

Q18. A car just starting up from a stop sign has zero velocity at the instant it starts. Must the acceleration of the car also be zero at this instant? Explain.

Q19. A car traveling with constant speed rounds a curve in the highway. Is the acceleration of the car equal to zero in this situation? Explain.

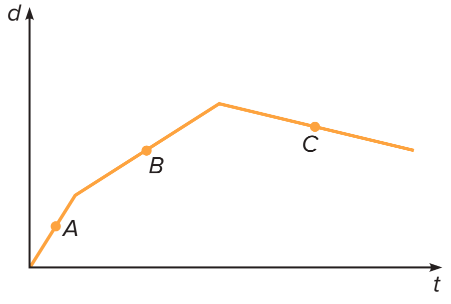
Q20. A racing sports car traveling with a constant velocity of 100 MPH due west startles a turtle by the side of the road and the turtle begins to move out of the way. Which of these two objects is likely to have the larger acceleration at that instant? Explain.

Q21. In the graph shown here, velocity is plotted as a function of time for an object traveling in a straight line.
  a. Is the velocity constant for any time interval shown? Explain.
  b. During which time interval shown does the object have the greatest acceleration? Explain.



Q21 Diagram

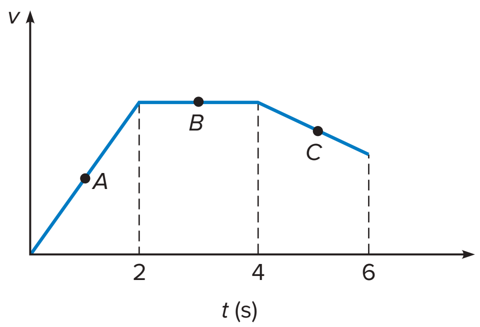
Q22. A car moves along a straight line, so that its position (distance from some starting point) varies with time as described by the graph shown here.
   a. Does the car ever go backward? Explain.
   b. Is the instantaneous velocity at point *A* greater or less than that at point *B?* Explain.



Q22 Diagram

Q23. For the car whose distance is plotted against time in question 22, is the velocity constant during any time interval shown in the graph? Explain.

Q24. A car moves along a straight section of road, so that its velocity varies with time as shown in the graph.
   a. Does the car ever go backward? Explain.
   b. At which of the labeled points on the graph, *A, B,* or *C,* is the magnitude of the acceleration the greatest? Explain.



Q24 Diagram

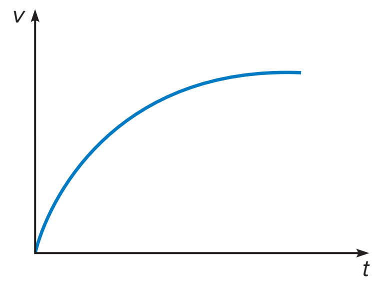
Q25. For the car whose velocity is plotted in question 24, in which of the equal time segments 0–2 seconds, 2–4 seconds, or 4–6 seconds is the distance traveled by the car the greatest? Explain.

Q26. Look again at the velocity-versus-time graph for the toy car shown in figure 2.15.
   a. Is the instantaneous speed greater at any time during this motion than the average speed for the entire trip? Explain.
   b. Is the car accelerated when the direction of the car is reversed at $t = 50$ s? Explain.

Q27. Suppose the acceleration of a car increases with time. Could we use the relationship $v = v_0 + at$ in this situation? Explain.

Q28. When a car accelerates uniformly from rest, which of these quantities increases with time: acceleration, velocity, and/or distance traveled? Explain.

Q29. The velocity-versus-time graph of an object curves as shown in the diagram. Is the acceleration of the object constant? Explain.



Q29 Diagram

Q30. For a uniformly accelerated car, is the average acceleration equal to the instantaneous acceleration? Explain.

Q31. A car traveling in the forward direction experiences a *negative* uniform acceleration for 10 seconds. Is the distance covered during the first 5 seconds equal to, greater than, or less than the distance covered during the second 5 seconds? Explain.

Q32. A car starts from rest, accelerates uniformly for 5 seconds, travels at constant velocity for 5 seconds, and finally decelerates uniformly for 5 seconds. Sketch graphs of velocity versus time and acceleration versus time for this situation.

Q33. Suppose that two runners run a 100-meter dash, but the first runner reaches maximum speed more quickly than the second runner. Both runners maintain constant speed once they have reached their maximum speed and cross the finish line at the same time. Which runner has the larger maximum speed? Explain. (See everyday phenomenon box 2.2.)

Q34. Sketch a graph showing velocity-versus-time curves for the two runners described in question 33. Sketch both curves on the same graph, so that the differences are apparent. (See everyday phenomenon box 2.2.)

*Q35. A physics instructor walks with increasing speed across the front of the room, then suddenly reverses direction and walks backward with constant speed. Sketch graphs of velocity and acceleration consistent with this description.

Q36. Draw a velocity-versus-time graph for the situation described in example box 2.4. Then, determine the distance traveled from the area under the curve.

Q37. Return to example box 2.4, but this time assume the acceleration is a negative 1 m/s^2; in other words, the car is slowing down. Using the same equations (and same initial velocity of +10 m/s), determine the final velocity after 6 seconds, and the distance traveled after 6 seconds. Then, graph the velocity-versus-time graph and determine the distance traveled from the area under the curve.

Exercises

E1. A traveler covers a distance of 413 miles in a time of 7 hours. What is the average speed for this trip?

E2. A walker covers a distance of 2.4 km in a time of 30 minutes. What is the average speed of the walker for this distance in km/h?

E3. Grass clippings are found to have an average length of 5.2 cm when a lawn is mowed 2 weeks (14 days) after the previous mowing. What is the average speed of growth of this grass in cm/day?

E4. A driver drives for 2.5 hours at an average speed of 68 MPH. What distance does she travel in this time?

E5. A woman walks a distance of 504 m, with an average speed of 1.4 m/s. What time was required to walk this distance?

E6. A person in a hurry averages 70 MPH on a trip covering a distance of 500 miles. What time was required to travel that distance?

E7. A hiker walks with an average speed of 1.3 m/s. What distance in kilometers has the hiker traveled after 1.5 hours at this pace?

E8. A car travels with an average speed of 28 m/s.
a. What is this speed in km/s?
b. What is this speed in km/h?

E9. A car travels with an average speed of 65 MPH. What is this speed in km/h? (See example box 2.1.)

E10. Starting from rest and moving in a straight line, a cheetah can achieve a velocity of 31 m/s (approximately 69 mph!) in 4 seconds. What is the average acceleration of the cheetah?

E11. The velocity of a car decreases from 28 m/s to 20 m/s in a time of 4 seconds. What is the average acceleration of the car in this process? (See example box 2.3.)

E12. Starting from rest, a car accelerates at a rate of 6.7 m/s^2 for a time of 4 seconds. What is its velocity at the end of this time? (See example box 2.4a.)

E13. A car traveling with an initial velocity of 27 m/s slows down at a constant rate of 5.4 m/s^2 for 3 seconds. What is its velocity at the end of this time?

E14. A car traveling with an initial velocity of 16 m/s accelerates at a constant rate of 2.8 m/s^2 for a time of 3 seconds. (See example box 2.4.)
a. What is its velocity at the end of this time?
b. What distance does the car travel during this process?

E15. A runner traveling with an initial velocity of 1.1 m/s accelerates at a constant rate of 0.8 m/s^2 for a time of 2 seconds.
a. What is his velocity at the end of this time?
b. What distance does the runner cover during this process?

E16. A car moving with an initial velocity of 32 m/s slows down at a constant rate of -4 m/s^2.
a. What is its velocity after 5 seconds of deceleration?
b. What distance does the car cover in this time?

E17. A runner moving with an initial velocity of 4.0 m/s slows down at a constant rate of -1.6 m/s^2 over a period of 2 seconds.
a. What is her velocity at the end of this time?
b. What distance does she travel during this process?

E18. If a world-class sprinter ran a distance of 100 meters starting at his top speed of 9 m/s and running with constant speed throughout, how long would it take him to cover the distance?

E19. In 1950, the world record holder for the 400-meter dash was Jamaican George Rhoden. If his average speed during his record dash was 8.73 m/s, how long did it take him to run the race?

E20. Starting from rest, a cheetah accelerates at a constant rate of 7.75 m/s^2 for a time of 4 seconds.
a. Compute the velocity of the cheetah at 1 s, 2 s, 3 s, and 4 s and plot these velocity values against time.
b. Compute the distance traveled by the cheetah for these same times and plot the distance values against time.

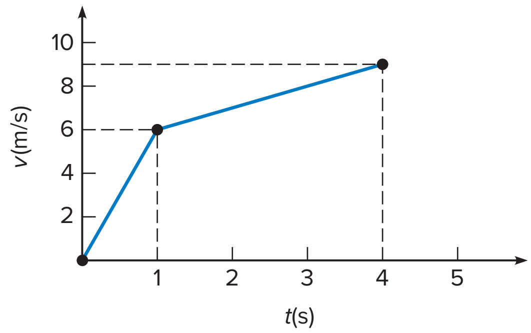
E21. A race car has a speed of 90 m/s. The driver hits the brakes, causing the car to slow down at a constant rate of 11 m/s^2 for a period of 8 seconds.
a. Compute the speed of the car at 2 s, 4 s, 6 s, and 8 s after the driver hits the brakes, and plot these values against time.
b. Compute the distance traveled by the car for the same times, and plot the distance values against time.

Synthesis Problems

SP1. A railroad engine moves forward along a straight section of track for a distance of 70 m due west at a constant speed of 5 m/s. It then reverses its direction and travels 32 m due east at a constant speed of 4 m/s. The time required for this deceleration and reversal is very short due to the small speeds involved.
a. What is the time required for the entire process?
b. Sketch a graph of average speed versus time for this process. Show the deceleration and reacceleration upon reversal as occurring over a very short time interval.
c. Using negative values of velocity to represent reversed motion, sketch a graph of velocity versus time for the engine (see fig. 2.15).
d. Sketch a graph of acceleration versus time for the engine (see fig. 2.16).

SP2. The velocity of a car increases with time, as shown in the graph.
a. What is the average acceleration between 0 seconds and 1 second?

b. What is the average acceleration between 1 second and 4 seconds?
c. What is the average acceleration between 0 seconds and 4 seconds?
d. Is the result in part c equal to the average of the two values in parts a and b? Compare and explain.



SP2 Diagram

SP3. A car traveling due west on a straight road accelerates at a constant rate for 10 seconds, increasing its velocity from 0 to 30 m/s. It then travels at a constant speed for 10 seconds and decelerates at a steady rate for the next 5 seconds to a velocity of 20 m/s. It travels at this velocity for 5 seconds and then decelerates rapidly to a stop in a time of 4 seconds.

a. Sketch a graph of the car's velocity versus time for the entire motion. Label the axes of your graph with the appropriate velocities and times.
b. Sketch a graph of acceleration versus time for the car.
c. Does the distance traveled by the car continually increase in the motion described? Explain.

SP4. A car traveling in a straight line with an initial velocity of 10 m/s accelerates at a rate of 3.0 m/s^2 to a velocity of 34 m/s.

a. How much time does it take for the car to reach the velocity of 34 m/s?
b. What is the distance covered by the car in this process?
c. Compute values of the distance traveled at 1-second intervals and carefully draw a graph of distance plotted against time for this motion.

SP5. Just as car A is starting up, it is passed by car B. Car B travels with a constant velocity of 7 m/s, while car A accelerates with a constant acceleration of 4.2 m/s^2, starting from rest.

a. Compute the distance traveled by each car for times of 1 s, 2 s, 3 s, and 4 s.
b. At what time, approximately, does car A overtake car B?
c. How might you go about finding this time exactly? Explain.

Home Experiments and Observations

HE1. How fast do you normally walk? Using a meter stick or a string of known length, lay out a straight course of 40 or 50 meters. Then, use a watch with a second hand or a stopwatch to determine

a. Your normal walking speed in m/s.
b. Your walking speed for a brisk walk.
c. Your jogging speed for this distance.
d. Your sprinting speed for this distance.

Record and compare the results for these different cases. Is your sprinting speed more than twice your speed for a brisk walk?

HE2. The speed with which hair and fingernails grow provides some interesting measurement challenges. Using a millimeter rule, estimate the speed of growth for one or more of these: fingernails, toenails, facial hair if you shave regularly, or hair near your face (such as sideburns) that will provide an easy reference point. Measure the average size of clippings or of growth at regular time intervals.

a. What is the average speed of growth? What units are most appropriate for describing this speed?
b. Does the speed appear to be constant with time? Does the speed appear to be the same for different nails (thumb versus fingers, fingernails versus toenails) or, in the case of hair, for different positions on your face?

Studio Photogram/Alamy Stock Photo

CHAPTER 3

Falling Objects and Projectile Motion

Chapter Overview

Our main purpose in this chapter is to explore how objects move under the influence of the gravitational acceleration near the Earth's surface. Uniform acceleration, introduced in chapter 2, plays a prominent role. We begin by considering carefully the acceleration of a dropped object, and then we extend these ideas to thrown objects or objects projected at an angle to the ground.

Chapter Outline

1. **Acceleration due to gravity.** How does a dropped object move under the influence of the Earth's gravitational pull? How is its acceleration measured, and in what sense is it constant?
2. **Tracking a falling object.** How do velocity and distance traveled vary with time for a falling object? How can we quickly estimate these values knowing the gravitational acceleration?
3. **Beyond free fall: Throwing a ball upward.** What changes when a ball is thrown upward rather than dropped? Why does the ball appear to hover near the top of its flight?
4. **Projectile motion.** What determines the motion of an object that is fired horizontally? How do the velocity and position of the object change with time in this case?
5. **Hitting a target.** What factors determine the trajectory of a rifle bullet or football that has been launched at some angle to the horizontal to hit a target?

Have you ever watched a leaf or a ball fall to the ground? At times during your first few years of life, you probably amused yourself by dropping an object repeatedly and watching it fall. As we grow older, that experience becomes so common that we usually do not stop to think about it or to ask why objects fall as they do, yet this question has intrigued scientists and philosophers for centuries.

To understand nature, we must first carefully observe it. If we control the conditions under which we make our observations, we are doing an experiment. The observations of falling objects that you performed as a young child were a simple form of experiment, and we would like to rekindle that interest in experimentation here. Progress in science has depended on carefully controlled experiments, and your own progress in understanding nature will depend on your active testing of ideas through experiments. You may be amazed at what you discover.

Look around for some small, compact objects. A short pencil, a rubber eraser, a paper clip, or a small ball will all do nicely. Holding two objects at arm's length, release them simultaneously and watch them fall to the floor (fig. 3.1). Be careful to release them from the same height above the floor without giving either one an upward or downward push.

How would you describe the motion of these falling objects? Is their motion accelerated? Do they reach the floor at the same time? Does the motion depend on the shape and composition of the object? To explore this last question, you might take a small piece of paper and drop it at the same time as an eraser or a ball. First, drop the paper unfolded. Then, try folding it or crumpling it into a ball. What difference does this make?

Figure 3.1 An experimenter dropping objects of different mass. Do they reach the ground at the same time?
Jill Braaten/McGraw-Hill Education

From these simple experiments, we can draw some general conclusions about the motion of falling objects. We can also try throwing or projecting objects at different angles to study the motion of a projectile. We will find that a constant downward gravitational acceleration is involved in all of these cases. This acceleration affects virtually everything we do when we move or play on the surface of the Earth.

3.1 Acceleration Due to Gravity

If you dropped a few objects as suggested in this chapter's introduction, you already know the answer to one of the questions posed there. Are the falling objects accelerated? Think for a moment about whether the velocity is changing. Before you release an object, its velocity is zero, but an instant after the object is released, the velocity has some value different from zero. There has been a change in velocity. If the velocity is changing, there is an acceleration.

Things happen so rapidly that it is difficult, just from watching the fall, to say much about the acceleration. It does appear to be large, because the velocity increases rapidly. Does the object reach a large velocity instantly, or does the acceleration occur more uniformly? To answer this question, we must slow down the motion somehow, so that our eyes and brains can keep up with what is happening.

How can we measure the gravitational acceleration?

There are several ways to view the action in slow motion. One was pioneered by Italian scientist Galileo Galilei (1564–1642), who was the first to accurately describe the *acceleration due to gravity.* Galileo's method was to roll or slide objects down a slightly inclined plane. This allows only a small portion of the gravitational acceleration to come into play, just that part in the direction of motion along the plane. Thus, a smaller acceleration results. Other methods (not available to Galileo) use time-lapse photography, ultrasonic motion detectors, or video recording to locate the position of the falling object at different times.

If you happen to have a grooved ruler and a small ball or marble handy, you can make an inclined plane. Lift one end of the ruler slightly by placing a pencil under one end, and let the ball or marble roll down the ruler under the influence of gravity (fig. 3.2). Can you see it gradually pick up speed as it rolls? Is it clearly moving faster at the bottom of the incline than it was halfway down?

Galileo was handicapped by a lack of accurate timing devices. He often had to use his own pulse as a timer. Despite this limitation, he was able to establish that the acceleration was uniform, or constant, with time and to estimate its value using inclined planes. We are more fortunate. We have devices that allow us to study the motion of a falling object more directly. One such device is a stroboscope, a rapidly blinking light whose flashes occur at regular intervals in time. Figure 3.3 is a photograph taken using a stroboscope to illuminate an object as it falls. The position of the object is pinpointed every time the light flashes.

Figure 3.2 A marble rolling down a ruler serving as an inclined plane. Does the velocity of the marble increase as it rolls down the incline?
Jill Braaten/McGraw-Hill Education

If you look closely at figure 3.3, you will notice that the distance covered in successive time intervals increases regularly. The time intervals between successive positions of the ball are all equal. (If the stroboscope light flashes every 1/20 of a second, you are seeing the position of the ball every 1/20 of a second.) Since the distance covered by the ball in equal time intervals is increasing, the velocity must be increasing. Figure 3.3 shows a ball whose velocity is steadily increasing in the downward direction. (For this example, it is convenient to choose downward as positive.)

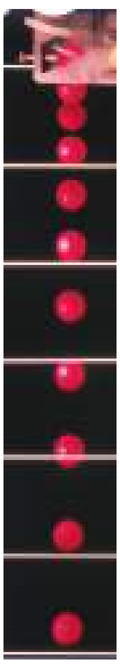


Figure 3.3 A falling ball is illuminated by a rapidly blinking stroboscope. The stroboscope blinks at regular time intervals.
Richard Megna/Fundamental Photographs, NYC

Computing values of the average velocity for each time interval will make this even clearer. The computation can be done if we know the time interval between flashes and can measure the position of the ball from the photograph, knowing the distance between the grid marks. Table 3.1 displays data obtained in this manner. It shows the position of a ball at intervals of 1/20 of a second (0.05 second).

To see that the velocity is indeed increasing, we compute the average velocity for each successive time interval. For example, between the second and third flashes, the ball traveled a distance of 3.6 centimeters, which is found by subtracting 1.2 centimeters from 4.8 centimeters. Dividing this distance by the time interval of 0.05 second yields the average size of the velocity:

$$v = \frac{3.6 \text{ cm}}{0.5 \text{ s}} = 72 \text{ cm/s}$$

You could verify the other values shown in the third column of table 3.1 by doing similar computations.

It is clear in table 3.1 that the velocity values steadily increase. To see that velocity is increasing at a constant rate, we can plot velocity against time (fig. 3.4). Notice that each velocity data point is plotted at the midpoint between the two times (or flashes) from which it was computed. This is because these values represent the average velocity for the short time intervals between flashes. For constant acceleration, the

Table 3.1

Distance and Average Velocity Values for a Falling Ball

Time	Distance	Average Velocity
0	0	
		24 cm/s
0.05 s	1.2 cm	
		72 cm/s
0.10 s	4.8 cm	
		124 cm/s
0.15 s	11.0 cm	
		174 cm/s
0.20 s	19.7 cm	
		218 cm/s
0.25 s	30.6 cm	
		268 cm/s
0.30 s	44.0 cm	
		320 cm/s
0.35 s	60.0 cm	
		368 cm/s
0.40 s	78.4 cm	
		416 cm/s
0.45 s	99.2 cm	
		464 cm/s
0.50 s	122.4 cm	

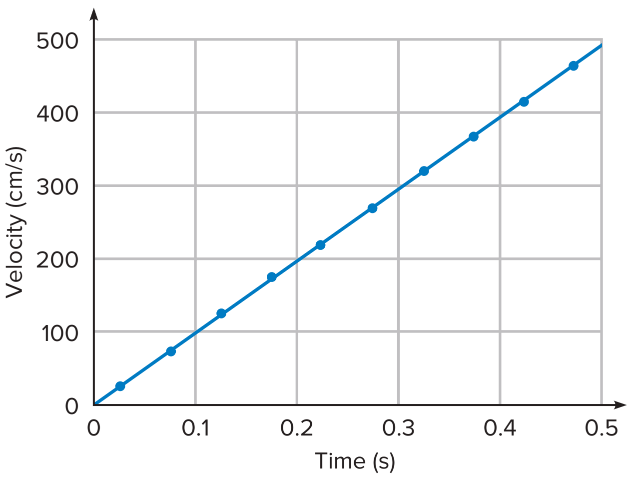


Figure 3.4 Velocity plotted against time for the falling ball. The velocity values are those shown in table 3.1.

average velocity for any time interval is equal to the instantaneous velocity at the midpoint of that interval.

Did you notice that the slope of the line is constant in figure 3.4? The velocity values all fall approximately on a constant-slope straight line. Because acceleration is the slope of the velocity-versus-time graph, the acceleration must also be constant. The velocity increases uniformly with time.

To find the value of the acceleration, we choose two velocity values that lie on the straight line and calculate how rapidly the velocity is changing. For example, the last velocity value, 464 cm/s, and the second value, 72 cm/s, are separated by a time interval corresponding to eight flashes, or 0.40 second. The increase in velocity Δv is found by subtracting 72 cm/s from 464 cm/s, obtaining 392 cm/s. To find the acceleration, we divide this change in velocity by the time interval ($a = \Delta v/t$),

$$a = \frac{392 \text{ cm/s}}{0.4 \text{ s}} = 980 \text{ cm/s}^2 = 9.8 \text{ m/s}^2$$

This result gives us the **acceleration due to gravity** for objects falling near the Earth's surface. Its value actually varies slightly from point to point on the Earth's surface because of differences in altitude and other effects. This acceleration is used so often that it is given its own symbol, g, where

$$g = 9.8 \text{ m/s}^2$$

Called the *gravitational acceleration* or *acceleration due to gravity,* it is valid only near the Earth's surface and thus is not a fundamental constant.

How did Galileo's ideas on falling objects differ from Aristotle's?

There is another sense in which the gravitational acceleration is constant, which takes us back to the experiments suggested in the chapter opener: When you drop objects of different sizes and weights, do they reach the floor at the same time? Except for an unfolded piece of paper, it is likely that all of the objects that you test, regardless of their weight, reach the floor at the same time when released simultaneously. This finding suggests that the gravitational acceleration does not depend on the weight of the object.

Galileo used similar experiments to prove this point. His experiments contradicted Aristotle's view that heavier objects fall more rapidly. How could Aristotle's idea have been accepted for so long when simple experiments can disprove it? Experimentation was not part of the intellectual outlook of Aristotle and his followers; they valued pure thought and logic more highly. Galileo and other scientists of his time broke new ground by using experiments as an aid to thinking. A new tradition was emerging.

On the other hand, Aristotle's view agrees with our intuition that heavy objects do fall more rapidly than some lighter objects. If, for example, we drop a brick together with a feather or an unfolded piece of paper (fig. 3.5), the brick will reach the floor first. The paper or feather will not fall in a straight line but instead will flutter to the floor much as a leaf falls from a tree. What is happening here?

You will probably recognize that the effects of **air resistance** impede the fall of the feather or paper much more than the fall of the brick, a steel ball, or a paper clip. When we crumple the piece of paper into a ball and drop it simultaneously with a brick or other heavy object, the two objects reach the floor at approximately the same time. We live at the bottom of a sea of air, and the effects of air resistance can be substantial for objects like leaves, feathers, or pieces of paper. These effects produce a slower and less regular flight for light objects that have a large surface area.

If we drop a feather and a brick simultaneously in a vacuum or in the very thin atmosphere of the moon, they do reach the ground at the same time. Moonlike conditions are not part of our everyday experience, however, so we are used to seeing feathers fall more slowly than rocks or bricks. Galileo's insight was that the gravitational acceleration is the same for all objects, regardless of their weight, provided that

Figure 3.5 When a brick and a feather are dropped at the same time, the brick reaches the floor first.
Jill Braaten/McGraw-Hill Education

the effects of air resistance are not significant. Aristotle did not separate the effect of air resistance from that of gravity in his observations.

> The gravitational acceleration for objects near the Earth's surface is uniform and has the value of 9.8 m/s^2. It can be measured by using stroboscopes or similar techniques to record the position of a falling object at regular, very small time intervals. This acceleration is constant in time. Contrary to Aristotle's belief, it also has the same value for objects of different weight.

3.2 Tracking a Falling Object

Imagine yourself dropping a ball from a sixth-story window, as in figure 3.6. How long does it take for the ball to reach the ground below? How fast is it traveling when it gets there? Things happen quickly, so the answers to these questions are not obvious.

If we assume that air-resistance effects are small for the object we are tracking, we know that it accelerates toward the ground at the constant rate of 9.8 m/s^2. Let's make some quick estimates of how these values change with time without doing detailed computations.

How does the velocity vary with time?

In making estimates of velocity and distance for a falling object, we often take advantage of the fact that the gravitational-acceleration value of 9.8 m/s^2 is almost 10 m/s^2 and round it up. (*Here we are choosing the downward direction as positive.*) This makes the numerical values easier to calculate without sacrificing much in accuracy. Multiplying by 10 is quicker than multiplying by 9.8.

How fast is our dropped ball moving after 1 second? An acceleration of 10 m/s^2 means that the velocity is increasing by 10 m/s each second. If its original velocity is zero, then after 1 second its velocity has increased to 10 m/s, in 2 seconds to 20 m/s, and in 3 seconds to 30 m/s. For each additional second, the ball gains 10 m/s in velocity.*

To help you appreciate these values, look back at table 2.1, which shows unit comparisons for familiar speeds. A velocity of 30 m/s is roughly 70 MPH, so after 3 seconds the ball is moving quickly. After just 1 second, it is moving with a downward velocity of 10 m/s, which is over 20 MPH. The ball gains velocity at a faster rate than is possible for a high-powered automobile on a level surface.

*In section 2.5, we noted that the velocity of an object moving with uniform acceleration is $v = v_0 + at$, where v_0 is the original velocity and the second term is the change in velocity, $\Delta v = at$. When a ball is dropped, $v_0 = 0$, so v is just at, the change in velocity.

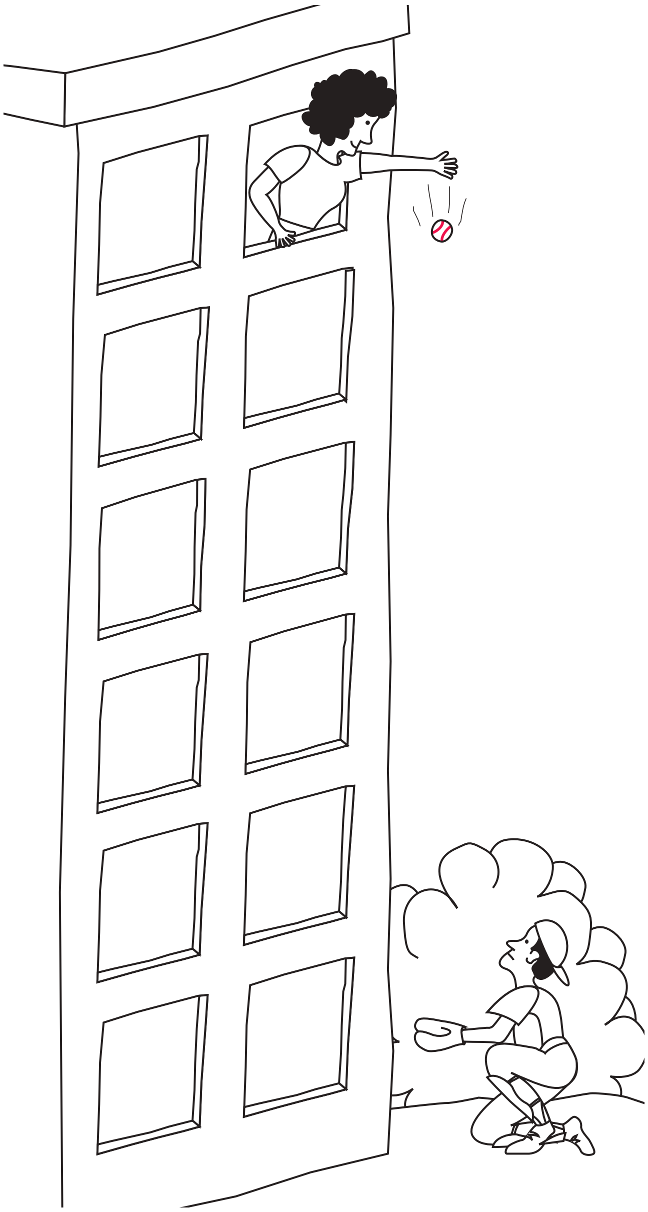


Figure 3.6 A ball is dropped from a sixth-story window. How long does it take to reach the ground?

How far does the ball fall in different times?

The high velocities are more meaningful if we examine how far the ball falls during these times. As the ball falls, it gains speed, so it travels farther in each successive time interval, as in the photograph in figure 3.3. Because of uniform acceleration, the distance increases at an ever-increasing rate.

During the first second of motion, the velocity of the ball increases from zero to 10 m/s. Its average velocity during that first second is 5 m/s, and it travels a distance of 5 meters in that second. This can also be found by using the relationship between distance, acceleration, and time in section 2.5.

If the starting velocity is zero, we found that $d = 1/2\,at^2$. After 1 second, the ball has fallen a distance

$$d = \frac{1}{2}(10\text{ m/s}^2)(1\text{ s})^2 = 5\text{ m}$$

Because the height of a typical story of a multistory building is less than 4 meters, the ball falls more than one story in just a second.

During the next second of motion, the velocity increases from 10 m/s to 20 m/s, yielding an average velocity of 15 m/s for that interval. The ball travels 15 meters in that second, which, when added to the 5 meters covered in the first second, yields a total of 20 meters. After 2 seconds, the distance fallen is four times as large as the 5 meters traveled after 1 second.* Because 20 meters is roughly five stories in height, the ball dropped from the sixth story will be near the ground after 2 seconds. Everyday phenomenon box 3.1 discusses how to approximate your reaction time using these ideas.

Figure 3.7 gives the velocity and distance fallen at half-second time intervals for a ball dropped from a six-story building. Notice that in just half a second, the ball falls 1.25 meters. An object dropped to the floor from an outstretched arm therefore hits the floor in roughly half a second. This makes it difficult to time with a stopwatch. (See example box 3.1.)

The change in velocity is proportional to the size of the time interval selected. In 1 second, the change in velocity is 10 m/s, so in half a second the change in velocity is 5 m/s. In each half-second, the ball gains approximately 5 m/s in velocity, as illustrated in figure 3.7. As the velocity gets larger, the arrows representing the velocity vectors grow. If we plotted these velocity values against time, we would get a simple, upward-sloping, straight-line graph, as in figure 3.4.

What does the graph of the distance values look like? The distance values increase in proportion to the square of the time, which means they increase more and more rapidly as time elapses. Instead of being a straight-line graph, the graph of the distance values curves upward, as in figure 3.8. The rate of change of distance with time is itself increasing with time.

Throwing a ball downward

Suppose that instead of just dropping the ball, we throw it straight down, giving it a starting velocity v_0 different from zero. How does this affect the results? Will the ball reach the ground more rapidly and with a larger velocity? You would probably guess correctly that the answer is yes.

*This is a result of the time being squared in the formula for distance. Putting 2 s in place of 1 s in the formula $d = \frac{1}{2}at^2$ multiplies the result by a factor of $4(2^2 = 4)$, yielding a distance of 20 m.

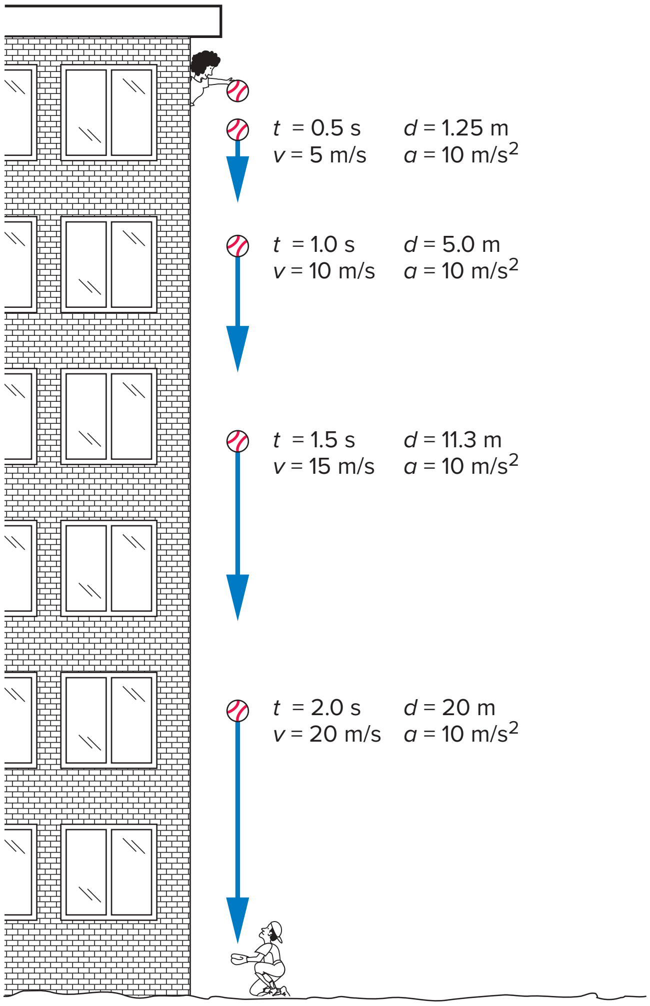


Figure 3.7 Approximate velocity and distance values for the dropped ball shown at half-second time intervals (using the approximate value $g = 10\text{ m/s}^2$).

Example Box 3.1

Sample Question: Using a Pulse Rate to Time a Falling Object

Question: Suppose that Galileo's resting pulse rate was 60 beats per minute. Would his pulse be a useful timer for getting position-versus-time data for an object dropped from the height of 2 to 3 meters?

Answer: A pulse rate of 60 beats per minute corresponds to one beat per second. In the time of 1 second, a dropped object falls a distance of approximately 5 m. (It falls 1.22 m in just half a second, as seen in table 3.1.) Thus, this pulse rate (or most pulse rates) would not be an adequate timer for an object dropped from a height of a few meters. It could be slightly more effective for an object dropped from a tower several stories in height.

Everyday Phenomenon Box 3.1

Reaction Time

The Situation. Have you ever wondered how fast your reaction time is? Many activities in our everyday lives are affected by how quickly we can react. The time it takes us to respond to a sudden slowdown in traffic when driving is one of the most important. Your score on video games is also a direct result of your reaction time to different stimuli.

How can we measure reaction time? A simple measurement can accomplish this without much difficulty. The Home Experiment HE2 at the end of this chapter describes a procedure in which a partner drops a meter stick placed between your open fingers. You catch it and measure how far it has fallen in the time that it takes for you to react to its motion and close your fingers. How does this permit you to calculate your reaction time, and what is the significance of the result?

Jill Braaten

The Analysis. Home Experiment HE2 uses the equation relating the distance traveled by an accelerated object to the time of flight, $d = 1/2at^2$. Is the meter stick accelerating? Of course, it is, because its velocity is changing from an initial velocity of zero at its release to an increasing velocity as it falls. An object is said to be in **free fall** if the only force on the object is the gravitational force. The only other force acting on the meter stick is that of air resistance, which is discussed in chapter 4. However, air resistance is essentially negligible in the early part of the flight of an object such as a falling meter stick.

Thus, the acceleration of the meter stick is that due to gravity, $a = g = 9.8\ \text{m/s}^2$, as discussed in this section. In the Home Experiment, we solve the distance equation for the time, so if we know the distance that the meter stick has fallen until your

(continued)

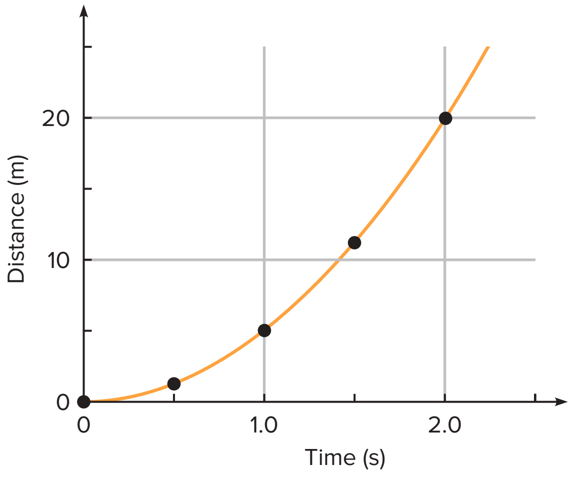


Figure 3.8 A plot of approximate distance versus time for the dropped ball.

In the case of the velocity values, the effect of the starting velocity is not difficult to see. The ball is still being accelerated by gravity, so that the change in velocity for each second of motion is still $\Delta v = 10$ m/s, or for a half-second, 5 m/s. If the initial downward velocity is 20 m/s, after half a second the velocity is 25 m/s, and after 1 second it is 30 m/s. We simply add the change in velocity to the initial velocity, as indicated by the formula $v = v_0 + at$.

In the case of distance, however, the values increase more rapidly. The full expression for distance traveled by a uniformly accelerated object (introduced in section 2.5) is

$$d = v_0t + \frac{1}{2}at^2$$

The first term is the distance the ball would travel if it continued to move with just its original velocity. This distance fingers close, we can calculate the time of flight. This time is your reaction time.

Some issues arise in determining an accurate reaction time. Those measuring the reaction time should not look at the person dropping the meter stick—they should keep their eyes on the stick itself. It is extremely difficult for your partner to avoid indicating in subtle, but recognizable, ways that he or she is about to release the meter stick. A few practice trials are in order to get a feel for the procedure. Doing the experiment several times and averaging the results will produce a much more accurate value than that of a single trial.

What processes are taking place when you react to a falling meter stick? Obviously, the sight of the meter stick dropping must be transmitted to the brain, and then the brain must signal the fingers to close. Details of these processes lie in the realm of biological physics and neurobiology. A "normal" reaction time for a young adult is between 0.2 and 0.25 second. Does your reaction time fall in this range?

How does your reaction time affect your stopping distance when driving a car? A general guideline when driving is to allow at least one car length (usually assumed to be 16 ft, or 4.9 m) between you and the car in front of you for every 10 MPH of your speed. This guideline takes into account not only your reaction time but also how long it takes to stop the car once you step on the brakes. How far will you travel while your brain is processing that you need to stop and then while a signal from the brain is reaching your foot?

The accompanying table gives the distance in meters that you would travel at different speeds before you even began to apply the brakes, assuming that you had a reaction time of 0.25 second. Thus, if you are traveling at 50 MPH, you will travel more than a car length before you touch the brakes. Even at 40 MPH, you will travel almost one car length. Are you surprised?

MPH	km/hr	Meters Covered in 0.25 sec
10	16	1.1
20	32	2.2
30	48	3.4
40	64	4.5
50	81	5.6
60	97	6.7
70	113	7.8

These ideas demonstrate the danger of tailgating. If you are tailgating a car at 50 MPH and you see its brake lights go on, you will travel 5.6 meters before you can apply your brakes. Because the other car is already slowing down, this is likely to result in a rear-end collision, assuming that your brakes are no better than the other car's.

Once you know how to compute your reaction time using this simple technique, you can explore how various conditions affect it. Does sleep deprivation affect your reaction time? If you consume a lot of caffeine, does your reaction time improve or get worse? Does your reaction time change during the course of the day? You can do simple scientific experiments to answer these questions. Perhaps you will discover the best conditions to improve your video game scores, or when it is safest to get behind the wheel of a car.

also increases with time. The second term is due to the acceleration and has the same values as shown in figures 3.7 and 3.8.

In the sample exercise in example box 3.2, we calculate velocity and distance traveled during the first 2 seconds of motion for a ball thrown downward. Notice that after 2 seconds the ball has traveled a distance of 60 meters, much farther than the 20 meters when the ball is simply dropped. After just 1 second, the ball has already traveled 25 meters, which means it would be near the ground if thrown from our sixth-story window.

Keep in mind, though, that we have ignored the effects of air resistance in arriving at these results. For a compact object falling just a few meters, the effects of air resistance are very small. These effects increase as the velocity increases, however, so that the farther the object falls, the greater the effects of air resistance. In chapter 4, we will discuss the role of air resistance in more depth in the context of sky diving.

When an object is dropped, its velocity increases by approximately 10 m/s every second due to the gravitational acceleration. The distance traveled increases at an ever-increasing rate, because the velocity is increasing. In just a few seconds, the object is moving very rapidly and has fallen a large distance. In section 3.3, we will explore the effects of gravitational acceleration on an object thrown upward.

Example Box 3.2

Sample Exercise: Throwing a Ball Downward

A ball is thrown downward with an initial velocity of +20 m/s.*
Using the approximate value 10 m/s² for the gravitational acceleration, find (a) the velocity and (b) the distance traveled at 1-s time intervals for the first 2 s of motion.

a. $v_0 = 20$ m/s $\quad v = v_0 + at$

$a = 10$ m/s² $\quad$ for $t = 1$ s

$v = ?$ $\quad v = 20 \text{ m/s} + (10 \text{ m/s}^2)(1 \text{ s})$

$= 20 \text{ m/s} + 10 \text{ m/s}$

$= \mathbf{30\ m/s}$

$t = 2$ s $\quad v = 20 \text{ m/s} + (10 \text{ m/s}^2)(2 \text{ s})$

$= 20 \text{ m/s} + 20 \text{ m/s} = \mathbf{40\ m/s}$

b. $d = ?$ $\quad d = v_0 t + \frac{1}{2}at^2$

$t = 1$ s $\quad d = (20 \text{ m/s})(1 \text{ s}) + \frac{1}{2}(10 \text{ m/s}^2)(1 \text{ s})^2$

$= 20 \text{ m} + 5 \text{ m} = \mathbf{25\ m}$

$t = 2$ s $\quad d = (20 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(10 \text{ m/s}^2)(2 \text{ s})^2$

$= 40 \text{ m} + 20 \text{ m} = \mathbf{60\ m}$

*choosing downward as positive

3.3 Beyond Free Fall: Throwing a Ball Upward

In section 3.2, we discussed what happens when a ball is dropped or thrown downward. In both of these cases, the ball gains velocity as it falls due to the gravitation acceleration. What if the ball is thrown upward instead, as in figure 3.9? How does gravitational acceleration affect the ball's motion? What goes up must come down—but when and how fast are interesting questions with everyday applications.

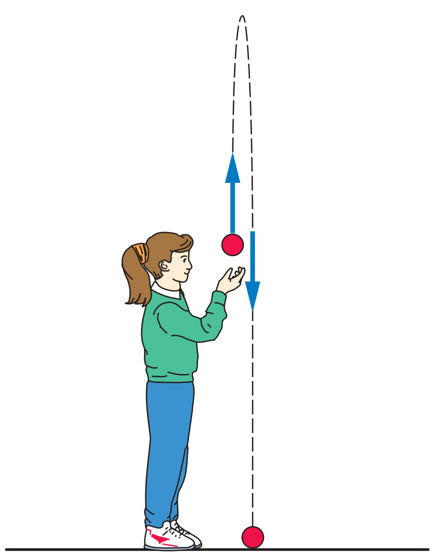


Figure 3.9 A ball thrown upward returns to the ground. What are the magnitude and direction of the velocity at different points in the flight?

The directions of the acceleration and velocity vectors merit our close attention. The gravitational acceleration is always directed downward toward the center of the Earth, because that is the direction of the gravitational force that produces this acceleration. This means that the acceleration is in the opposite direction to the original upward velocity.

How does the ball's velocity change?

Suppose we throw a ball straight up with an original velocity of 20 m/s. Many of us can throw a ball at this velocity: It is approximately 45 MPH. This is a lot slower than a 90-MPH fastball, but throwing a ball upward with good velocity is harder than throwing it horizontally.

Once the ball leaves our hand, the primary force acting on it is gravity, which produces a downward acceleration of 9.8 m/s² or approximately 10 m/s². (*If we now choose the upward direction as positive, this acceleration is negative, because it is downward.*) Every second, there is a change in velocity of 10 m/s. This change in velocity is directed downward, however, opposite the direction of the original velocity. It subtracts from the original velocity rather than adding to it.

Once you are aware of how important direction is in observing the ball thrown upward, finding the velocity at different times is not hard. After 1 second, the velocity of the ball has decreased by 10 m/s, so if it started at +20 m/s (*choosing the positive direction to be upward in this case*), it is now moving upward with a velocity of just +10 m/s. After 2 seconds, it loses another 10 m/s, so its velocity is then zero. It does not stop there, of course. In another second (3 seconds from the start), its velocity decreases by another 10 m/s, and it is then moving downward at −10 m/s. *The sign of the velocity indicates its direction.* All of these values can be found from the relationship $v = v_0 + at$, where $v_0 = +20$ m/s and $a = -10$ m/s².

Clearly, the ball has changed direction, as you might expect. Just as before, the velocity changes steadily at −10 m/s each second, due to the constant downward acceleration. After 4 seconds, the ball is moving downward with a velocity of −20 m/s and is back at its starting position. These results are illustrated in figure 3.10. The high point in the motion occurs at a time 2 seconds after the ball is thrown, where the velocity is zero. If the velocity is zero, the ball is moving neither upward nor downward, so this is the turnaround point.

An interesting question, a favorite on physics tests (and often missed by students), asks for the value of acceleration at the high point in the motion. If the velocity is zero at this point, what is the value of the acceleration? The quick, but incorrect, response given by many people is that the acceleration must also be zero at that point. The correct answer is that the acceleration is still −10 m/s². The gravitational acceleration is constant and does not change.

What if the acceleration were zero? That would imply the velocity was not changing. Because the instantaneous velocity is zero at the high point of the motion, it would remain zero. In other words, it would not move and would stay suspended at the top of the motion forever. When asked for a value of acceleration, we need to think about whether the velocity is

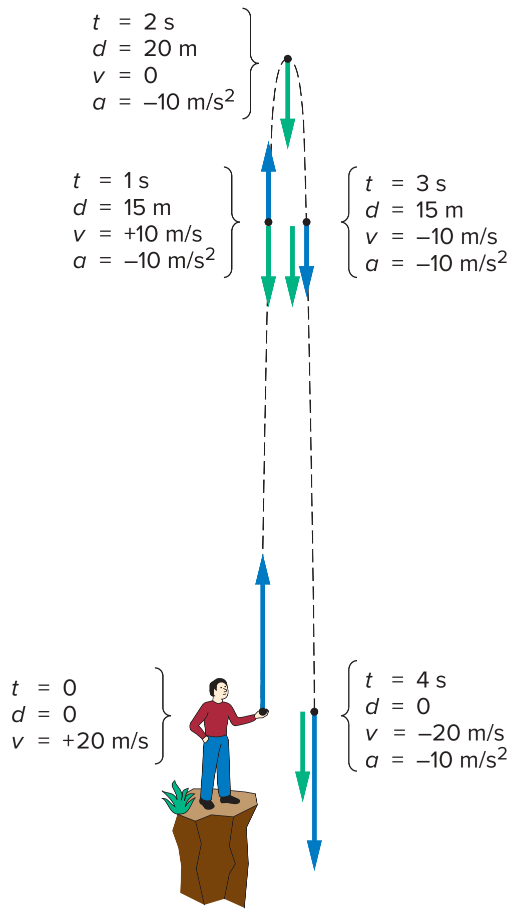


Figure 3.10 The changing velocity is indicated by the blue velocity vectors at different points in the flight of a ball thrown upward with a starting velocity of +20 m/s. The constant downward acceleration is shown as a green vector at each point.

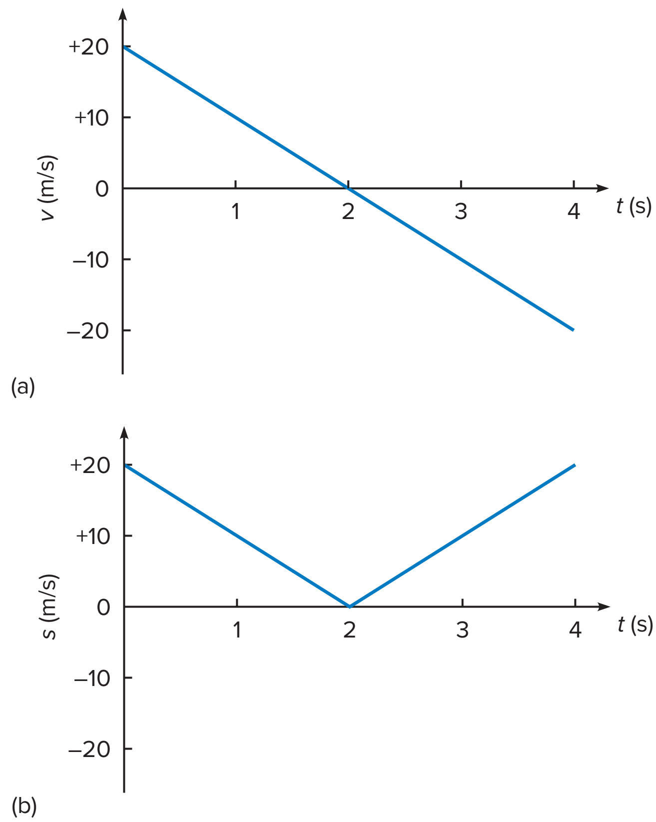


Figure 3.11 A plot of velocity versus time (a) and speed versus time (b) for a ball thrown upward with an initial velocity of +20 m/s. The negative values of velocity represent downward motion. The speed is always positive. The ball slows down on its way upward and speeds up on its way downward.

changing. In this instant, the ball's velocity is still changing, from a positive to a negative value, even though the instantaneous velocity is zero. Acceleration is the *rate of change* of velocity and is *unrelated* to the size of the velocity.

What would a graph of velocity plotted against time look like for the motion just described? If we make the upward direction of motion positive, the velocity starts with a value of +20 m/s and changes at a steady rate, decreasing by −10 m/s each second. This is a straight-line graph, sloping downward, as in figure 3.11. The positive values of velocity represent upward motion, where the size of the velocity is decreasing, and the negative values of velocity represent downward motion. If the ball did not hit the ground, but was thrown from the edge of a cliff, it would continue to gain negative velocity as it moved downward.

How high does the ball go?

The position or height of the ball at different times can be computed using the methods in section 3.2. These distance computations involve the formula for uniform acceleration developed in section 2.5. In the sample exercise in example box 3.3, we compute the height or distance traveled at

Example Box 3.3

Sample Exercise: Throwing a Ball Upward

A ball is thrown upward with an initial velocity of +20 m/s.*
Find its estimated height at 1-s intervals for the first 5 s of its flight, using the approximate value of $g = 10\ \text{m/s}^2$.

$d = ?$ $\qquad d = v_0 t + \frac{1}{2}at^2$

$t = 1\ \text{s}$ $\qquad = (20\ \text{m/s})(1\ \text{s}) + \frac{1}{2}(-10\ \text{m/s}^2)\,(1\ \text{s})^2$
$= 20\ \text{m} - 5\ \text{m} = \mathbf{15\ m}$

$t = 2\ \text{s}$ $\qquad d = (20\ \text{m/s})(2\ \text{s}) + \frac{1}{2}(-10\ \text{m/s}^2)(2\ \text{s})^2$
$= 40\ \text{m} - 20\ \text{m} = \mathbf{20\ m}$

$t = 3\ \text{s}$ $\qquad d = (20\ \text{m/s})(3\ \text{s}) + \frac{1}{2}(-10\ \text{m/s}^2)(3\ \text{s})^2$
$= 60\ \text{m} - 45\ \text{m} = \mathbf{15\ m}$

$t = 4\ \text{s}$ $\qquad d = (20\ \text{m/s})(4\ \text{s}) + \frac{1}{2}(-10\ \text{m/s}^2)(4\ \text{s})^2$
$= 80\ \text{m} - 80\ \text{m} = \mathbf{0\ m}$

$t = 5\ \text{s}$ $\qquad d = (20\ \text{m/s})(5\ \text{s}) + \frac{1}{2}(-10\ \text{m/s}^2)(5\ \text{s})^2$
$= 100\ \text{m} - 125\ \text{m} = \mathbf{-25\ m}$

*choosing upward as positive

1-second intervals for the ball thrown upward at +20 m/s, using $-10\ m/s^2$ for the gravitational acceleration.

What should you notice about these results? First, the high point of the motion is 20 meters above the starting point. The high point is reached when the velocity is zero, and we determined earlier that this occurs at a time of 2 seconds. This time depends on how fast the ball is thrown initially. The larger the original velocity, the greater the time to reach the high point. Knowing this time, we can use the distance formula to find the height.

You should also notice that after just 1 second, the ball has reached a height of 15 meters. It covers just 5 additional meters in the next second of motion, then falls back to 15 meters in the following second. The ball spends a full 2 seconds above the height of 15 meters, even though it reaches a height of only 20 meters. The ball is moving more slowly near the top of its flight than it is at lower points—this is why the ball appears to "hang" near the top of its flight.

Finally, the time the ball takes to fall back to its starting point from the high point is equal to the time it takes to reach the high point in the first place. It takes 2 seconds to reach the high point and another 2 seconds to return to the starting point. (For times greater than 4 seconds, the distance d becomes negative, indicating the ball is now below its starting point.) The total time of flight to return to the starting point is just twice the time needed to reach the high point. In this case, 4 seconds is the total time in the air. A larger starting velocity would produce a higher turnaround point and a greater "hang time" for the ball.

> A ball thrown upward is slowed by the constant downward gravitational acceleration until its velocity is reduced to zero at the high point. Remember that although the velocity is zero at the high point, the gravitational acceleration is not; it remains constant. The ball then falls from that high point, accelerating downward at the same constant rate as when it was rising. The ball travels more slowly near the top of its flight, so it appears to "hang" there. It spends more time in the top few meters than it does in the rest of the flight. We will find that these features are also present when a ball is projected at an angle to the horizontal, as discussed in section 3.5.

3.4 Projectile Motion

Suppose that instead of throwing a ball straight up or down, you throw it horizontally from some distance above the ground. What happens? Does the ball go straight out until it loses all of its horizontal velocity and then starts to fall, like the perplexed coyote in the *Roadrunner* cartoons (fig. 3.12)? What does the real path, or **trajectory,** look like?

Cartoons give us a misleading impression. In fact, two different things are happening at the same time: (1) The ball is accelerating downward under the influence of gravity, and (2) the ball is also moving sideways with an approximately constant horizontal velocity. Combining these two motions gives the overall trajectory.

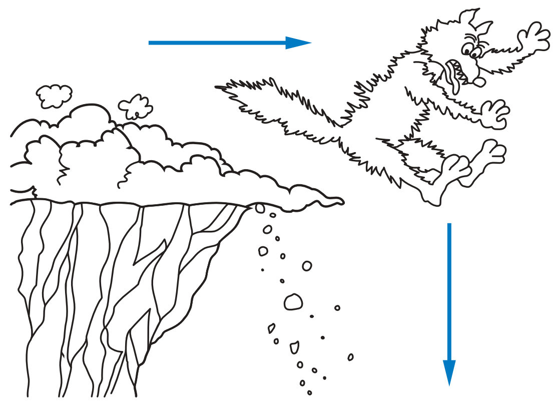


Figure 3.12 A cartoon coyote falling off a cliff. Is this a realistic picture of what happens?

What does the trajectory look like?

You can perform a simple experiment to help you visualize the trajectory that the projectile follows. Roll a marble or small ball along the top of a desk or table, and let it roll off the edge. What does the trajectory of the ball look like as it travels through the air to the floor? Is it like the coyote in figure 3.12? Roll the ball at different velocities and see how the trajectory changes. Try to sketch the trajectory after making these observations.

How do we go about analyzing this motion? The key lies in thinking about the horizontal and vertical components of the motion separately and then combining them to get the actual trajectory (fig. 3.13).

The acceleration of the horizontal motion is zero, provided that air resistance is small enough to be ignored. This implies that the ball moves with a constant horizontal velocity once it has rolled off the table or has left the hand. The ball travels equal horizontal distances in equal time

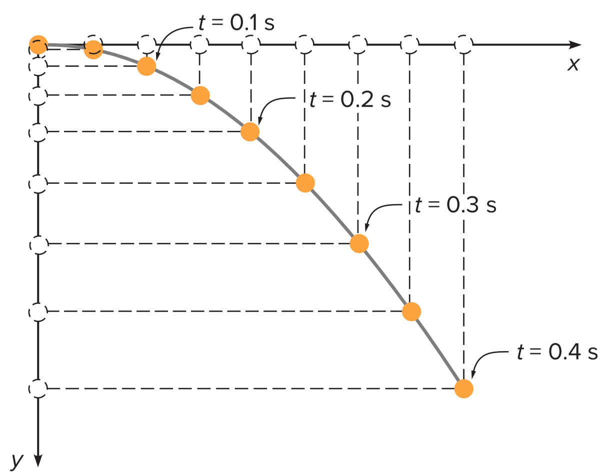


Figure 3.13 The horizontal and vertical motions combine to produce the trajectory of the projected ball. The vertical and horizontal positions are shown at regular time intervals.

intervals, as shown across the top of figure 3.13. In constructing this diagram, we assume an initial horizontal velocity of 2 m/s for the ball. Every tenth of a second, then, the ball travels a horizontal distance of 0.2 meter.

At the same time that the ball travels with constant horizontal velocity, it accelerates downward with the constant gravitational acceleration *g*. Its vertical velocity increases exactly like that of the falling ball photographed for figure 3.3. This motion is depicted along the left side of figure 3.13. In each successive time interval, the ball falls a greater distance than in the time interval before, because the vertical velocity increases with time.

Combining the horizontal and vertical motions, we get the trajectory shown curving downward in figure 3.13. For each time shown, we draw a horizontal dashed line locating the vertical position of the ball and a vertical dashed line for the horizontal position. The position of the ball at any time is the point where these lines intersect. The resulting trajectory (the solid curve) should look familiar if you have performed the simple experiments suggested in the first paragraph in this section.

If you understand how we obtained the trajectory of the ball, you are well on your way to understanding **projectile motion.** The total velocity of the ball at each position pictured is in the direction of the trajectory at that point, because this is the actual direction of the ball's motion. This total velocity is a vector sum of the horizontal and vertical components of the velocity (fig. 3.14). (See appendix C for a discussion of vector components.) The horizontal velocity remains constant, because there is no acceleration in that direction. The downward (vertical) velocity gets larger and larger.

Study Hint

If you are not familiar with vectors, you should take the time to read and do the exercises in appendix C. Appendix C describes what vectors are, how they are added using simple graphical procedures, and how vector components are defined. In this section, we use the ideas that a vector quantity such as velocity can have both horizontal and vertical components and that these components add up to give the total velocity. These concepts are critical to your understanding of projectile motion. Vector addition and vector components are also used in many other situations we will encounter in later chapters.

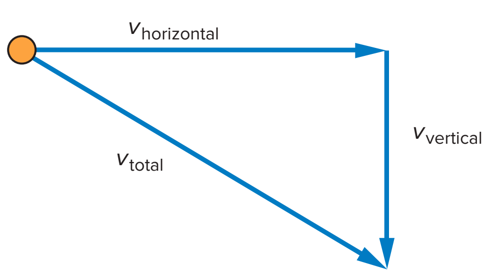


Figure 3.14 The total velocity at any point is found by adding the vertical component of the velocity to the horizontal component.

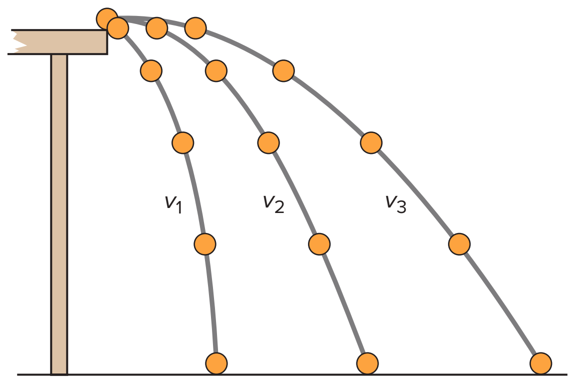


Figure 3.15 Trajectories for different initial velocities of a ball rolling off a table: v_3 is larger than v_2, which in turn is larger than v_1. The positions are shown at equal time intervals.

The actual shape of the trajectory followed by the ball depends on the original horizontal velocity given the ball by throwing it or rolling it from the tabletop. If this initial horizontal velocity is small, the ball does not travel very far horizontally. Its trajectory will then be like the smallest starting velocity v_1 in figure 3.15.

The three trajectories shown in figure 3.15 have three different starting velocities. As you would expect, the ball travels greater horizontal distances when projected with a larger initial horizontal velocity.

What determines the time of flight?

Which of the three balls in figure 3.15 would hit the floor first if all three left the tabletop at the same time? Does the time taken for the ball to hit the floor depend on its horizontal velocity? There is a natural tendency to think that the ball that travels farther takes a longer time to reach the floor.

In fact, the three balls should all reach the floor at the same time. The reason is that they are all accelerating downward at the same rate of 9.8 m/s^2. This downward acceleration is not affected by how fast the balls travel horizontally. The time taken to reach the floor for the three balls in figure 3.15 is determined strictly by how high above the floor the tabletop is. The vertical motion is *independent* of the horizontal velocity.

This fact often surprises people. It contradicts our intuitive sense of what is going on but can be confirmed by doing simple experiments using two similar balls (fig. 3.16). If you throw one ball horizontally at the same time that you simply drop the second ball from the same height, the two balls should reach the floor at roughly the same time. They may fail to hit at the same time, but this is likely because it is hard to throw the first ball completely horizontally and to release both balls at the same time. A special spring gun, often used in demonstrations, can do this more precisely.

If we know how far a ball falls, we can compute the time of flight. This can then be used to determine the horizontal distance the ball will travel, if we know the initial horizontal velocity. The sample exercise in example box 3.4 shows this type of analysis. Notice that the horizontal distance traveled is determined by two factors: the time of flight and the initial velocity.

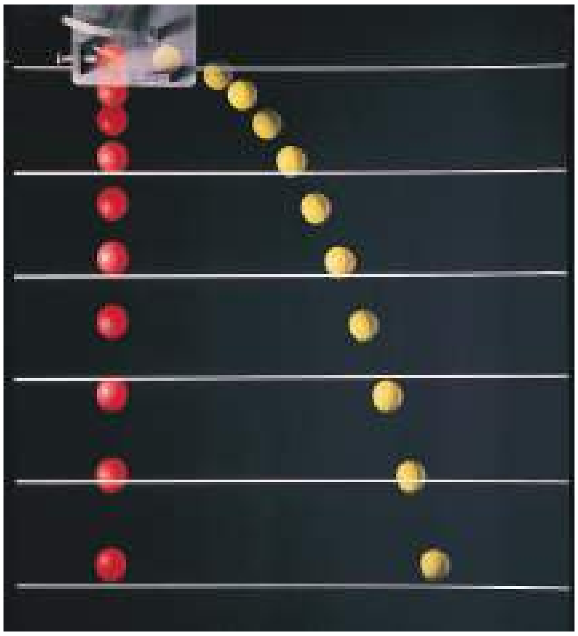


Figure 3.16 A ball is dropped at the same time that a second ball is projected horizontally from the same height. Which ball reaches the floor first?
Richard Megna/Fundamental Photographs, NYC

Example Box 3.4

Sample Exercise: Projectile Motion

A ball rolls off a tabletop with an initial velocity of 3 m/s. If the tabletop is 1.25 m above the floor,

a. Estimate the time it takes for the ball to hit the floor (using the approximate value of $g = 10\ \text{m/s}^2$)?
b. How far does the ball travel horizontally?

a. In figure 3.7, we saw that a ball will fall a distance of 1.25 m in approximately half a second. This can be found directly in this way:

$d_{\text{vertical}} = 1.25\ \text{m}$ $\qquad d_{\text{vertical}} = \frac{1}{2}at^2$

$a = g = 10\ \text{m/s}^2$ $\qquad$ Solving for t^2:

$t = ?$ $\qquad t^2 = \dfrac{d}{\frac{1}{2}a}$

$$= \frac{1.25\ \text{m}}{5\ \text{m/s}^2}$$
$$= 0.25\ \text{s}^2$$

Taking the square root to get t:

$$t = \mathbf{0.5\ s}$$

b. Knowing the time of flight t, we can now compute the horizontal distance traveled:

$v_0 = 3\ \text{m/s}$ $\qquad d_{\text{horizontal}} = v_0 t$

$t = 0.5\ \text{s}$ $\qquad = (3.0\ \text{m/s})(0.50\ \text{s})$

$d_{\text{horizontal}} = ?$ $\qquad = \mathbf{1.5\ m}$

Treating the vertical motion independently of the horizontal motion and then combining them to find the trajectory is the secret to understanding projectile motion. A horizontal glide combines with a vertical plunge to produce a graceful curve. The downward gravitational acceleration behaves the same as for any falling object, but there is no acceleration in the horizontal direction if air resistance can be ignored. The projectile moves with constant horizontal velocity while it is accelerating downward.

3.5 Hitting a Target

As long as humans have been hunters or warriors, they have wanted to predict where a projectile such as a cannonball will land after it is fired. Being able to hit a target such as a bird in a tree or a ship at sea has obvious implications for survival. Being able to hit a catcher's mitt with a baseball thrown from center field is also a highly valued skill.

Does the bullet fall when a rifle is fired?

Imagine you are firing a rifle at a small target some distance away, with the rifle and target at exactly the same distance above the ground (fig. 3.17). If the rifle is fired directly at the target in a horizontal direction, will the bullet hit the center of the target? If you think of the ball rolling off the table in section 3.4, you should conclude that the bullet will strike the target slightly below the center. Why? The bullet will be accelerated downward by Earth's gravitational pull and will fall slightly as it travels to the target.

Because the time of flight is small, the bullet does not fall very far, but it falls far enough to miss the center of the target. How do you compensate for the fall of the bullet? You aim a little high. You correct your aim either through trial and error or by adjusting your rifle sight, so that your aim is automatically a little above center. Rifle sights are often adjusted for some average distance to the target. For longer distances you must aim high, for shorter distances a little low.

If you aim a little high, the bullet no longer starts out in a completely horizontal direction. The bullet travels up slightly during the first part of its flight and then comes down to meet the target. This also happens when you fire a cannon or throw a ball at a distant target.

A frequently used demonstration to illustrate the independence of the vertical and horizontal motions of projectiles is referred to as "Shoot the Monkey" or "Monkey in a Tree." A projectile is aimed directly at a toy monkey (or other suitable target) hanging from the ceiling. An electronic trigger allows the target to drop at the same time the projectile is launched. The target falls straight down at a rate governed by the acceleration of gravity. The projectile starts to move toward

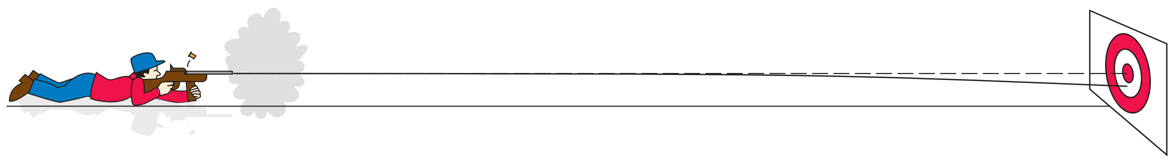


Figure 3.17 A target shooter fires at a distant target. The bullet falls as it travels to the target.

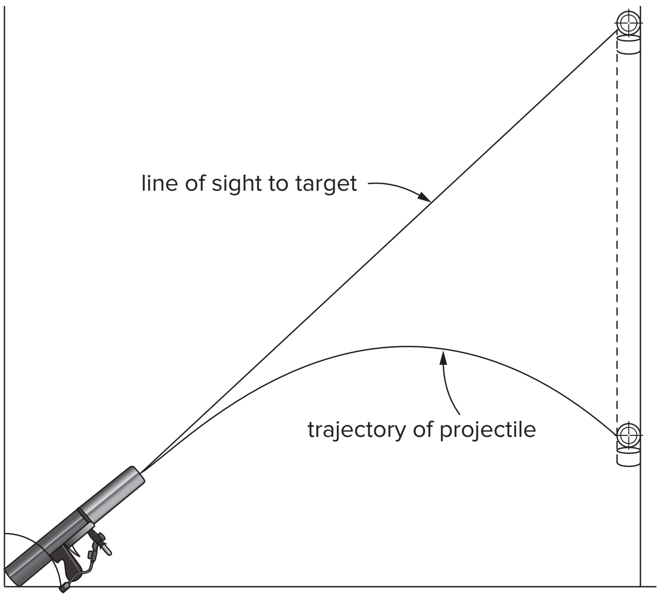


Figure 3.18 If the projectile is launched at the same time the target is dropped, will it hit the target?

the initial position of the target but also starts to fall at a rate governed by the acceleration of gravity.

Due to the fact that both the projectile and the target begin falling in the vertical direction at exactly same time and with the same downward acceleration, the projectile will always hit the target (fig. 3.18). It is crucial to recognize that the projectile hits below where it was aimed by an amount equal to the vertical distance the target drops, because the acceleration of gravity has the same effect on both the projectile and the target. It is also important that the target is released and the projectile is fired ***at exactly the same time.***

If the target were stationary, the projectile would have to be aimed above the target to compensate for the vertical drop due to the acceleration of gravity.

The flight of a football

Whenever you throw a ball, such as a football, at a somewhat distant target, the ball must be launched at an angle above the horizontal, so that the ball does not fall to the ground too soon. A good athlete does this automatically as a result of practice. The harder you throw, the less you need to direct the ball upward, because a larger initial velocity causes the ball to reach the target more quickly, giving it less time to fall.

Figure 3.19 shows the flight of a football thrown at an angle of 30° above the horizontal. The vertical position of the ball is plotted on the left side of the diagram, as in figure 3.13 for the horizontally projected ball. The horizontal position of the ball is shown across the bottom of the diagram. We have assumed that air resistance is small, so the ball travels with a constant horizontal velocity. Combining these two motions yields the overall trajectory.

As the football climbs, the vertical component of its velocity decreases because of the constant downward gravitational acceleration. At the high point, this vertical component of the velocity is zero, just as it is for a ball thrown straight upward. The velocity of the ball is completely horizontal at this high point. The ball then begins to fall, gaining downward velocity as it accelerates. Unlike the ball thrown straight upward, however, there is a constant horizontal component to the velocity throughout the flight. The horizontal component is not zero at the top. We need to add this horizontal motion to the up-and-down motion described in section 3.3.

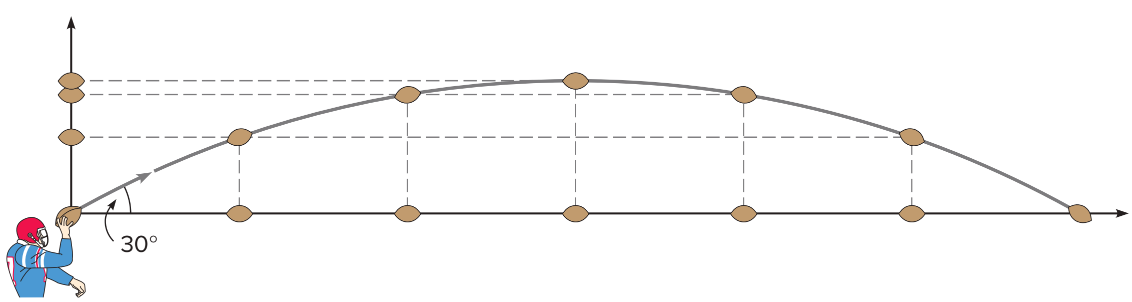


Figure 3.19 The flight of a football launched at an angle of 30° to the horizontal. The vertical and horizontal positions of the ball are shown at regular time intervals.

Everyday Phenomenon — Box 3.2

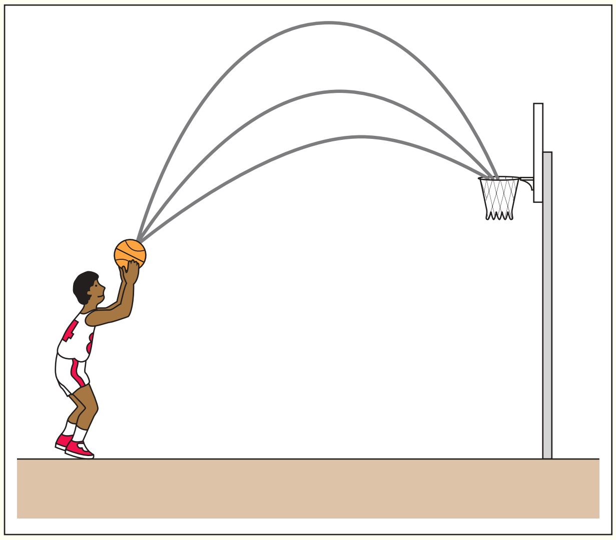
Shooting a Basketball

The Situation. Whenever you shoot a basketball, you unconsciously select a trajectory for the ball that you believe will have the greatest likelihood of getting the ball to pass through the basket. Your target is above the launch point (with the exception of dunk shots and sky hooks), but the ball must be on the way down for the basket to count.

What factors determine the best trajectory? When is a high, arching shot desirable, and when might a flatter trajectory be more effective? Will these factors be different for a free throw than for a shot taken when you are guarded by another player? How can our understanding of projectile motion help us answer these questions?

The Analysis. The diameter of the basketball and the diameter of the basket opening limit the angle at which the basketball can pass cleanly through the hoop. The second drawing shows the range of possible paths for a ball coming straight down and for one coming in at a 45° angle to the basket. The shaded area in each case shows how much the center of the ball can vary from the center line if the ball is to pass through the hoop. As you can see, a wider range of paths is available when the ball is coming straight down. The diameter of the basketball is a little more than half the diameter of the basket.

The second drawing illustrates the advantage of an arched shot. There is a larger margin of error in the path that the ball can take and still pass through the hoop cleanly. For the dimensions of a regulation basketball and basket, the angle must be at least 32° for a clean shot. As the angle gets larger, the range of possible paths increases. At smaller angles, appropriate spin on the basketball will sometimes cause the ball to rattle through, but the smaller the angle, the less the likelihood of that happening.



Different possible trajectories for a basketball free throw. Which has the greatest chance of success?

(continued)

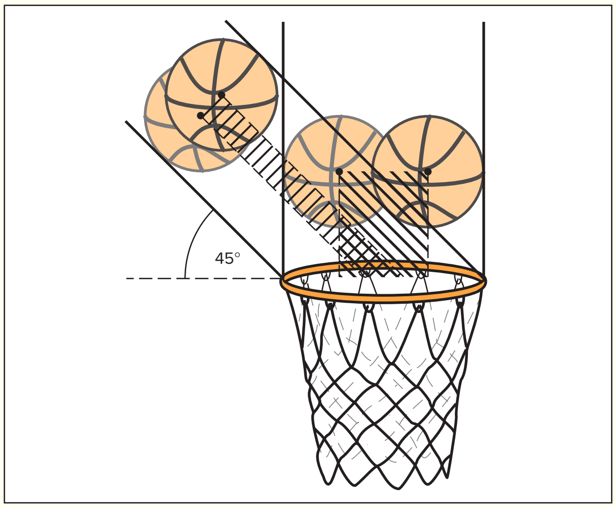
In throwing a ball, you can vary two quantities to help you hit your target. One is the initial velocity, which is determined by how hard you throw the ball. The other is the launch angle, which can be varied to fit the circumstances. A ball thrown with a large initial velocity does not have to be aimed as high and will reach the target more quickly. It may not clear the onrushing linemen, however, and it might be difficult to catch because of its large velocity.

There is no time like the present to test these ideas. Take a page of scrap paper and crumple it into a compact ball. Then, put your wastebasket on your chair or desk. Throwing underhand, experiment with different throwing speeds and launch angles to see which is most effective in making a basket. Try to get a sense of how the launch angle and throwing speed interact to produce a successful shot. A low, flat-trajectory shot should require a greater throwing speed than a higher, arching shot. The flatter shot must also be aimed more accurately, because the effective area of the opening in the basket is smaller when the ball approaches at a flat angle. The ball "sees" a smaller opening. (This effect is discussed in everyday phenomenon box 3.2.)

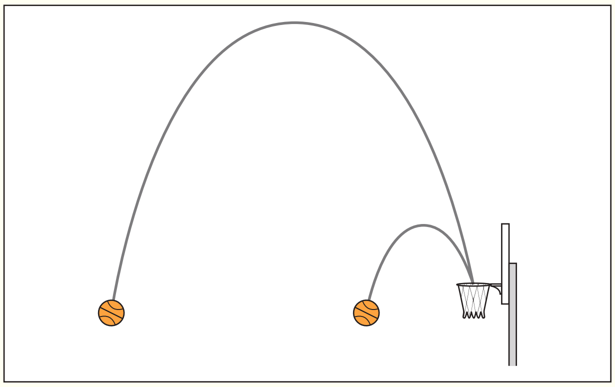
How can we achieve maximum distance?

In firing a rifle or cannon, the initial velocity of the projectile is usually set by the amount of gunpowder in the shell. The launch angle is then the only variable we can change in attempting to hit a target. Figure 3.20 shows three possible trajectories for a cannonball fired at different launch angles for the same initial speed. For different launch angles, we tilt the cannon barrel by different amounts from the position shown.

Note that the greatest distance is achieved using an intermediate angle—an angle of 45° if the effects of air resistance are negligible. The same considerations are involved in the shot put in track-and-field events. The launch angle is



Possible paths for a basketball coming straight down and for one coming in at a 45° angle. The ball coming straight down has a wider range of possible paths.



An arched shot launched from a large distance stays in the air longer than one launched at the same angle from much closer to the basket.

The disadvantage of the arched shot is less obvious. As you get farther away from the basket, launching conditions for an arched shot must be more precise for the ball to travel the horizontal distance to the basket. If an arched shot is launched from 30 ft, it must travel a much higher path than a shot launched at the same angle closer to the basket, as shown in the third drawing. Because the ball stays in the air for a longer time, small variations in either the release speed or the angle can cause large errors in the distance traveled. This distance depends on both the time of flight and the horizontal component of the velocity.

A highly arched shot is more effective when you are close to the basket. You can then take advantage of the greater range of paths available to the arched shot without suffering much from the uncertainty in the horizontal distance. Away from the basket, the desirable trajectories gradually become flatter, permitting more accurate control of the shot. An arched shot is sometimes necessary from anywhere on the court, however, to avoid having the shot blocked.

The spin of the basketball, the height of the release, and other factors all play a role in the success of a shot. A fuller analysis can be found in an article by Peter J. Brancazio in the *American Journal of Physics* (April 1981) entitled "Physics of Basketball." A good understanding of projectile motion might improve the game of even an experienced player.

very important and, for the greatest distance, will be near 45°. Air resistance and the fact that the shot hits the ground below the launch point are also factors, so the most effective angle is somewhat less than 45° in the shot put.

Thinking about what happens to the horizontal and vertical components of the initial velocity at different launch angles will show us why the angle for maximum distance is approximately 45°. (See fig. 3.21.) Velocity is a vector, and its horizontal and vertical components can be found by drawing the vector to scale and adding dashed lines to the horizontal and vertical directions (fig. 3.21). This process is described more fully in appendix C.

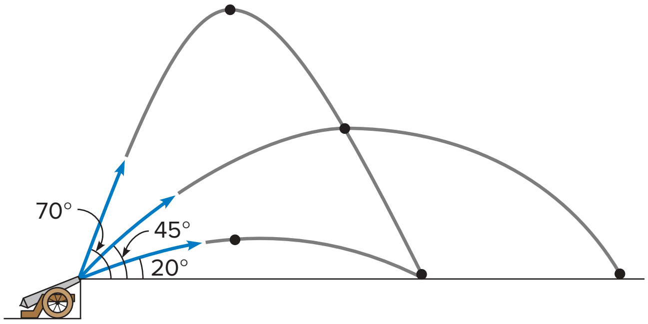


Figure 3.20 Cannonball paths for different launch angles but the same initial launch speed.

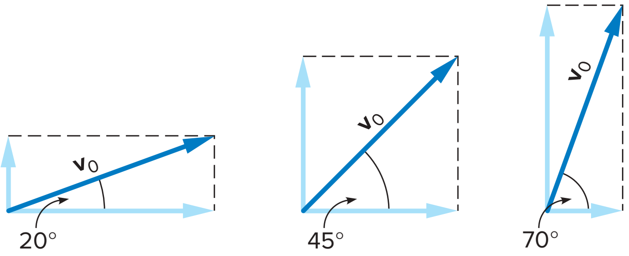


Figure 3.21 Vector diagrams showing the horizontal and vertical components of the initial velocity for the three cases illustrated in figure 3.20.

For the lowest launch angle of 20°, we see that the horizontal component of the velocity is much larger than the vertical. Because the initial upward velocity is small, the ball does not go very high. Its time of flight is short, and it hits the ground sooner than in the other two cases shown. The ball gets there quickly because of its large horizontal velocity and short travel time, but it does not travel very far before hitting the ground.

The high launch angle of 70° produces a vertical component much larger than the horizontal component. The ball thus travels much higher and stays in the air for a longer time than at 20°. It does not travel very far horizontally, however, because of its small horizontal velocity. The ball travels the same horizontal distance as for the 20° launch, but it takes longer getting there.* (If we shot it straight up, the horizontal distance covered would be zero, of course.)

The intermediate angle of 45° splits the initial velocity into equal-sized horizontal and vertical components. The ball therefore stays in the air longer than in the low-angle launch but also travels with a greater horizontal velocity than in the high-angle launch. In other words, with relatively large values for both the vertical and horizontal components of velocity, the vertical motion keeps the ball in the air long enough for the horizontal velocity to be effective. This produces the greatest distance of travel.

The time of flight and the horizontal distance traveled can be found if the launch angle and the size of the initial velocity are known. It is first necessary to find the horizontal and vertical components of the velocity to do these computations, however, and this makes the problem more complex than those discussed earlier. The ideas can be understood without doing the computations. The key is to think about the vertical and horizontal motions separately and then combine them.

*The angles 20° and 70° are complementary, because their sum is 90°. Any pair of complementary launch angles (30° and 60°, for example) yields the same horizontal range as each other.

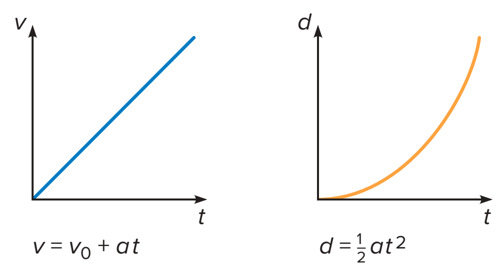
> For a projectile launched at an angle, the initial velocity can be broken down into vertical and horizontal components. The vertical component determines how high the object will go and how long it stays in the air, while the horizontal component determines how far it will go in that time. The launch angle and the initial speed interact to dictate where the object will land. Through the entire flight, the constant downward gravitational acceleration is at work, but it changes only the vertical component of the velocity. Producing or viewing such trajectories is a common part of our everyday experience.

Summary

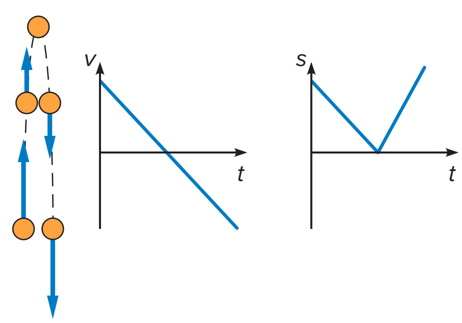
The primary aim in this chapter has been to introduce you to the gravitational acceleration for objects near the Earth's surface and to show how that acceleration affects the motion of objects dropped or launched in various ways.

1 Acceleration due to gravity. To find the acceleration due to gravity, we use measurements of the position of a dropped object at different times. The gravitational acceleration is 9.8 m/s^2. It does not vary with time as the object falls, and it has the same value for different objects regardless of their weight.

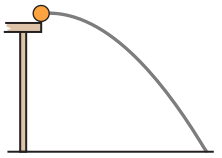
2 Tracking a falling object. The velocity of a falling object increases by approximately 10 m/s every second of its fall. Distance traveled increases in proportion to the square of the time, so that it increases at an ever-increasing rate. In just 1 second, a dropped ball is moving with a velocity of 10 m/s and has traveled 5 meters.



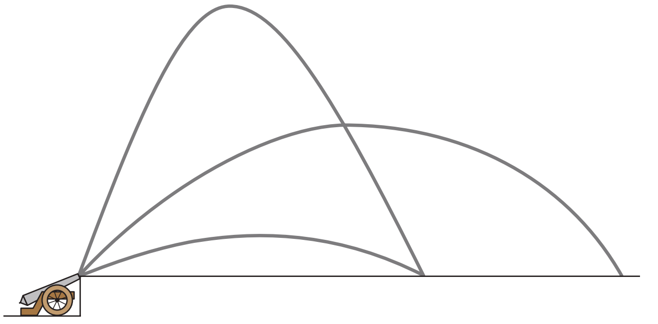
3 Beyond free fall: Throwing a ball upward. The speed of an object thrown upward first decreases due to the downward gravitational acceleration, passes through zero at the high point, and then increases as the object falls. The object spends more time near the top of its flight, because it is moving more slowly there. If we choose the upward direction as positive, the velocity values change from positive to negative as the ball changes direction.



4 **Projectile motion.** If an object is launched horizontally, it moves with a constant horizontal velocity at the same time it accelerates downward due to gravity. These two motions combine to produce the object's curved trajectory.



5 **Hitting a target.** Two factors, the launch speed and the launch angle, can be varied to determine the path of an object launched at an angle to the horizontal. Once again, the horizontal and vertical motions combine to produce the overall motion as the projectile moves toward a target.



One of the key points of this chapter is that you can't turn off gravity. Therefore, the acceleration in the downward vertical direction remains the same all the time! Try your hand at the mastery quizzes in Connect. They will give you some useful practice at clarifying these ideas.

Key Terms

Acceleration due to gravity, 41
Air resistance, 41
Free fall, 44
Trajectory, 48
Projectile motion, 49

Conceptual Questions

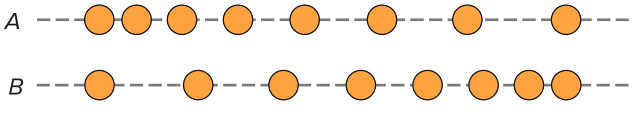
* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available in Connect

Q1. A small piece of paper is dropped and flutters to the floor. Is the piece of paper accelerating at any time during this motion? Explain.

Q2. The diagram shows the positions at intervals of 0.10 second of a ball moving from left to right (as in a photograph taken with a stroboscope that flashes every tenth of a second). Is the ball accelerated? Explain.

Q2 Diagram

Q3. The diagram shows the positions at intervals of 0.05 second of two balls moving from left to right. Is either or both of these balls accelerated? Explain.



Q3 Diagram

Q4. A lead ball and an aluminum ball, each 1 inch in diameter, are released simultaneously and allowed to fall to the ground. Due to its greater density, the lead ball has a substantially larger mass than the aluminum ball. Which of these balls, if either, has the greater acceleration due to gravity? Explain.

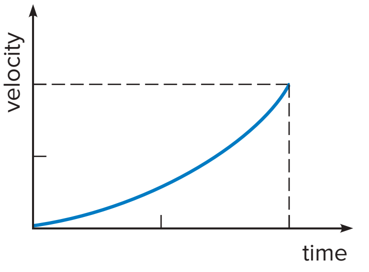
Q5. Two identical pieces of paper, one crumpled into a ball and the other left uncrumpled, are released simultaneously from the same height above the floor. Which one, if either, do you expect to reach the floor first? Explain.

Q6. Two identical pieces of paper, one crumpled into a ball and the other left uncrumpled, are released simultaneously from inside the top of a large evacuated tube. Which one, if either, do you expect will reach the bottom of the tube first? Explain.

*Q7. Aristotle stated that heavier objects fall faster than lighter objects. Was Aristotle wrong? In what sense could Aristotle's view be considered correct?

Q8. A rock is dropped from the top of a diving platform into the swimming pool below. Will the distance traveled by the rock in a 0.1-second interval near the top of its flight be the same as the distance covered in a 0.1-second interval just before it hits the water? Explain.

Q9. The graph shows the velocity plotted against time for a certain falling object. Is the acceleration of this object constant? Explain.



Q9 Diagram

Q10. If cars are traveling on a highway at constant speed, what is the advantage, if any, of remaining more than a car length behind the car in front of you? Explain. (See everyday phenomenon box 3.1.)

Q11. If an object is in free fall, what forces are acting on it? (See everyday phenomenon box 3.1.)

Q12. A ball is thrown downward with a large starting velocity.
a. Will this ball reach the ground sooner than one that is just dropped at the same time from the same height? Explain.
b. Will this ball accelerate more rapidly than one that is dropped with no initial velocity? Explain.

Q13. A ball thrown straight upward moves initially with a decreasing upward velocity. What are the directions of the velocity and acceleration vectors during this part of the motion? Does the acceleration decrease also? Explain.

Q14. A rock is thrown straight upward, reaching a height of 20 meters. On its way up, does the rock spend more time in the top 5 meters of its flight than in the first 5 meters of its flight? Explain.

Q15. A ball is thrown straight upward and then returns to the Earth. Choosing the positive direction to be upward, sketch a graph of the velocity of this ball against time. Where does the velocity change direction? Explain. Indicate this point on your graph.

Q16. A ball is thrown straight upward and then returns to the Earth. Sketch a graph of the speed (not velocity) of this ball against time. Where does the direction of the ball's motion change? Explain. Indicate this point on your graph. Compare your graph to the graph of velocity versus time from question 15.

Q17. A ball is thrown straight upward. At the very top of its flight, the velocity of the ball is zero. Is its acceleration at this point also zero? Explain.

Q18. A ball is thrown straight upward and then returns to the Earth. Does the acceleration change direction during this motion? Explain.

*Q19. A ball rolls up an inclined plane, slows to a stop, and then rolls back down. Do you expect the acceleration to be constant during this process? Is the velocity constant? Is the acceleration equal to zero at any point during this motion? Explain.

Q20. A ball rolls rapidly along a tabletop, off its edge, and then to the floor. At the instant the ball leaves the edge of the table, a second ball is dropped from the same height. Which ball, if either, reaches the floor first? Explain.

Q21. For the two balls in question 20, which, if either, has the larger total velocity when it hits the floor? Explain.

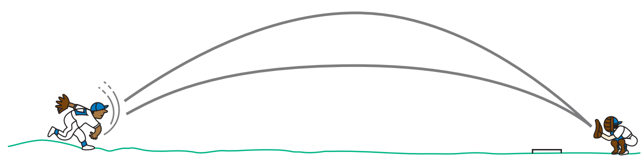
Q22. Is it possible for an object to have a horizontal component of velocity that is constant at the same time that the object is accelerating in the vertical direction? Explain by giving an example, if possible.

Q23. A ball rolls off a table with a large horizontal velocity. Does the direction of the velocity vector change as the ball moves through the air? Explain.

Q24. A ball rolls off a table with a horizontal velocity of 5 m/s. Is this velocity an important factor in determining the time it takes for the ball to hit the floor? Explain.

Q25. An expert marksman aims a high-speed rifle directly at the center of a nearby target. Assuming that the rifle sight has been accurately adjusted for more distant targets, will the bullet hit the near target above or below the center? Explain.

Q26. In the diagram, two different trajectories are shown for a ball thrown by a center fielder to home plate in a baseball game. Which of the two trajectories (if either), the higher one or the lower one, will result in a longer time for the ball to reach home plate? Explain.



Q26 Diagram

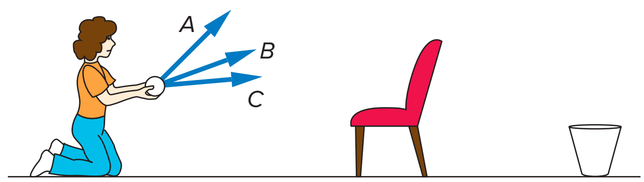
Q27. For either of the trajectories shown in the diagram for question 26, is the velocity of the ball equal to zero at the high point in the trajectory? Explain.

Q28. Assuming that the two trajectories in the diagram for question 26 represent throws by two different center fielders, which of the two is likely to have been thrown by the player with the stronger arm? Explain.

Q29. A cannonball fired at an angle of 70° to the horizontal stays in the air longer than one fired at 45° from the same cannon. Will the 70° shot travel a greater horizontal distance than the 45° shot? Explain.

Q30. Will a shot fired from a cannon at a 20° launch angle travel a longer horizontal distance than a 45° shot? Explain.

Q31. The diagram shows a wastebasket placed behind a chair. Three different directions are indicated for the velocity of a ball thrown by the kneeling woman. Which of the three directions—A, B, or C—is most likely to result in the ball landing in the basket? Explain.



Q31 Diagram

Q32. In the situation pictured in question 31, is the magnitude of the velocity important to the success of the shot? Explain.

Q33. In shooting a free throw in basketball, what is the primary advantage that a high, arching shot has over one with a flatter trajectory? Explain. (See everyday phenomenon box 3.2.)

Q34. In shooting a basketball from greater than free-throw range, what is the primary disadvantage of a high, arching shot? Explain. (See everyday phenomenon box 3.2.)

*Q35. A football quarterback must hit a moving target while eluding onrushing linemen. Discuss the advantages and disadvantages of a hard, low-trajectory throw over a higher-lofted throw.

Exercises

For the exercises in this chapter, use the approximate value of $g = 10 \text{ m/s}^2$ for the acceleration due to gravity.

E1. A steel ball is dropped from a diving platform (with an initial velocity of zero). Using the approximate value of $g = 10 \text{ m/s}^2$,
   a. What is the velocity of the ball 0.7 second after its release?
   b. What is its velocity 1.4 seconds after its release?

E2. A large rock is dropped from the top of a high cliff. Assuming that air resistance can be ignored and that the acceleration has the constant value of 10 m/s^2, how fast would the rock be traveling 6 seconds after it is dropped? What is this speed in MPH? (See conversion factors in appendix E.)

E3. For the ball in exercise E1,
   a. Through what distance does the ball fall in the first 0.7 second of its flight? (Assume $g = 10 \text{ m/s}^2$.)
   b. How far does it fall in the first 1.4 seconds of its flight?

E4. Suppose Galileo's pulse rate was 75 beats per minute.
   a. How many beats per second is this?
   b. What is the time in seconds between consecutive pulse beats?
   c. How far (in meters) does an object fall in this time when dropped from rest?
   d. What is this distance in feet (use the conversion factors in appendix E.)?

E5. A ball is thrown downward with an initial velocity of 14 m/s. Using the approximate value of $g = 10 \text{ m/s}^2$, what is the velocity of the ball 3.0 seconds after it is released? (See example box 3.2.)

E6. A ball is dropped from a high building. Using the approximate value of $g = 10 \text{ m/s}^2$, find the *change* in velocity between the first and sixth seconds of its flight.

E7. A ball is thrown upward, from the ground, with an initial velocity of 13 m/s. Using the approximate value of $g = 10 \text{ m/s}^2$, what are the magnitude and direction of the ball's velocity (see example box 3.3)
   a. 1 second after it is thrown?
   b. 2 seconds after it is thrown?

E8. How high above the ground is the ball in exercise 7
   a. 1 second after it is thrown?
   b. 2 seconds after it is thrown?

E9. At what time does the ball in exercise E7 reach the high point in its flight? (Use the approximate value of $g = 10 \text{ m/s}^2$, and remember that the velocity is equal to zero at the highest point.)

E10. Suppose that the gravitational acceleration on a certain planet is only 4.0 m/s^2. A space explorer standing on this planet throws a ball straight upward with an initial velocity of 17 m/s.
   a. What is the velocity of the ball 3 seconds after it is thrown?
   b. How much time elapses before the ball reaches the high point in its flight?

E11. A bullet is fired horizontally with an initial velocity of 800 m/s at a target located 200 m from the rifle.
   a. How much time is required for the bullet to reach the target?
   b. Using the approximate value of $g = 10 \text{ m/s}^2$, how far does the bullet fall in this time?

E12. A ball rolls off a shelf with a horizontal velocity of 3 m/s. At what horizontal distance from the shelf does the ball land if it takes 0.45 second to reach the floor?

E13. A ball rolls off a table with a horizontal velocity of 3 m/s. If it takes 0.45 second for the ball to reach the floor, how high above the floor is the tabletop? (Use $g = 10 \text{ m/s}^2$.)

E14. A ball rolls off a table with a horizontal velocity of 3 m/s to the right. If it takes 0.4 second for it to reach the floor,
   a. What is the downward component of the ball's velocity just before it hits the floor? (Use $g = 10 \text{ m/s}^2$.)
   b. What is the horizontal component of the ball's velocity just before it hits the floor?

E15. A ball rolls off a platform that is 3 meters above the ground. The ball's horizontal velocity as it leaves the platform is 5 m/s.
   a. How much time does it take for the ball to hit the ground? (See example box 3.4; use $g = 10 \text{ m/s}^2$.)
   b. How far from the base of the platform does the ball hit the ground?

E16. A projectile is fired at an angle such that the vertical component of its velocity and the horizontal component of its velocity are both equal to 50 m/s.
   a. Using the approximate value of $g = 10 \text{ m/s}^2$, how long does it take for the projectile to reach its high point?
   b. What horizontal distance does the projectile travel in this time?

Synthesis Problems

For the synthesis problems in Chapter 3, use the approximate value of $g = 10 \text{ m/s}^2$ for the acceleration due to gravity. Ignore air resistance.

SP1. A ball is thrown straight upward with an initial velocity of 18 m/s. Use $g = 10 \text{ m/s}^2$ for computations listed here.
   a. What is its velocity at the high point in its motion?
   b. How much time is required to reach the high point?
   c. How high above its starting point is the ball at its high point?
   d. How high above its starting point is the ball 3 seconds after it is released?
   e. Is the ball moving up or down 2 seconds after it is released?

SP2. Two balls are released simultaneously from the top of a tall building. Ball A is simply dropped with no initial velocity, and ball B is thrown downward with an initial velocity of 15 m/s.
   a. What are the velocities of the two balls 1.3 seconds after they are released?
   b. How far has each ball dropped in 1.3 seconds?
   c. Does the difference between the velocities of the two balls change at any time after their release? Explain.

SP3. Two balls are rolled off a tabletop that is 0.7 m above the floor. Ball A has a horizontal velocity of 4 m/s, and that of ball B is 6 m/s.
   a. Assuming $g = 10 \text{ m/s}^2$, how long does it take each ball to reach the floor after it rolls off the edge?
   b. How far does each ball travel horizontally before hitting the floor?

c. If the two balls started rolling at the same time at a point 1.5 m behind the edge of the table, will they reach the floor at the same time? Explain.

SP4. A cannon is fired over level ground at an angle of 35° to the horizontal. The initial velocity of the cannonball is 500 m/s, but because the cannon is fired at an angle, the vertical component of the velocity is 287 m/s and the horizontal component is 410 m/s.

a. How long is the cannonball in the air? (Use $g = 10\ \text{m/s}^2$ and the fact that the total time of flight is twice the time required to reach the high point.)
b. How far does the cannonball travel horizontally?
c. Repeat these calculations, assuming that the cannon was fired at a 55° angle to the horizontal, resulting in a vertical component of velocity of 410 m/s and a horizontal component of 287 m/s. How does the distance traveled compare to the earlier result? (See fig. 3.20.)

SP5. An excellent major league pitcher can throw a baseball at a speed of 100 MPH. The pitcher's mound is approximately 60 ft from home plate.

a. What is the speed in m/s?
b. What is the distance to home plate in meters?
c. How much time is required for the ball to reach home plate?
d. If the ball is launched horizontally, how far does the ball drop in this time, ignoring the effects of spin?
e. What is this distance in feet (this explains why the pitchers stand on a mound)?

SP6. An archeologist is running at 8 m/s with her hands outstretched above her head (1.85 m from feet to fingertips) while being chased by a tiger. She runs exactly horizontally off the edge of a chasm and attempts to grab on to the opposite side.

a. If the chasm is 5.0 meters wide, how long does she take to cover this distance?
b. During this time, what distance has she fallen vertically (use $g = 10\ \text{m/s}^2$)?
c. How far above or below the edge of the opposite side do her fingertips fall? (Use + to indicate distances above the edge, − to indicate distances below the edge.)

Home Experiments and Observations

HE1. Gather numerous small objects and drop them from equal heights, two at a time. Record which objects fall significantly more slowly than a compact, dense object such as a marble. Rank order these slower objects by their time of descent. What factors seem to be important in determining this time?

HE2. Working with a partner, you can get an estimate of your reaction time by catching a falling meter stick. (See everyday phenomenon box 3.1.) Have your partner hold the meter stick from a point near the top while you place the finger and thumb of your catching hand about an inch apart on both sides of the 50-cm mark. Without giving any cues, your partner then drops the meter stick; when you see it move, react to catch it by closing your finger and thumb. Record the distance that the meter stick moves between the times that your partner releases it and you catch it.

a. Repeat this process several times for each partner and compute the average distance the meter stick traveled for each partner. Tabulate your results.
b. Because the distance traveled in the time t that it takes for you to react is $d = 1/2\ gt^2$, the time of travel is $t = \sqrt{\frac{2d}{g}}$. Use a calculator to compute the reaction time t for each partner from the average distance d (expressed in meters). Use $g = 10\ \text{m/s}^2$. How does your average reaction time compare to your partner's?
c. A "normal" reaction time is between 0.2 and 0.25 second. Is your reaction time close to this? If not, explain why you think your reaction time is different.

HE3. Try dropping a ball from one hand at the same time you throw a second ball with your other hand. At first, try to throw the second ball horizontally, with no upward or downward component to its initial velocity. (It may take some practice.)

a. Do the balls reach the floor at the same time? (It helps to enlist a friend for making this judgment.)
b. If the second ball is thrown slightly upward from the horizontal, which ball reaches the ground first?
c. If the second ball is thrown slightly downward from the horizontal, which ball reaches the ground first?

HE4. Take a ball outside and throw it straight up in the air as hard as you can. By counting seconds, or by enlisting a friend with a watch, estimate the time that the ball remains in the air. From this information, can you find the initial velocity that you gave to the ball? (The time required for the ball to reach the high point is just half the total time of flight.)

HE5. Take a stopwatch to a football game and estimate the hang time of several punts. Also note how far (in yards) each punt travels horizontally. Do the highest punts have the longest hang times? Do they travel the greatest distances horizontally?

HE6. Using rubber bands and a plastic rule or other suitable support, design and build a marble launcher. By pulling the rubber band back by the same amount each time, you should be able to launch the marble with approximately the same speed each time. (*Warning:* Leave yourself ample room free of breakable objects!)

a. Produce a careful drawing of your launcher and note the design features you used. (Prizes may be available for the best design.)
b. Placing your launcher at a number of different angles to the horizontal, launch marbles over a level surface and measure the distance they travel from the point of launch. Which angle yields the greatest distance?
c. Fire the marbles at different angles from the edge of a desk or table. Which angle yields the greatest horizontal distance?

HE7. Try throwing a ball or wadded piece of paper into a wastebasket placed a few meters from your launch point.

a. Which is most effective, an overhanded or underhanded throw? (Five practice shots followed by 10 attempts for each might produce a fair test.)
b. Repeat this process with a barrier such as a chair placed near the wastebasket.

CHAPTER 4

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Newton's Laws: Explaining Motion

Chapter Overview

The primary purpose of this chapter is to explain Newton's three laws of motion and how they apply in familiar situations. We begin with a historical sketch of their development and then proceed to a careful discussion of each law. The concepts of force, mass, and weight play critical roles in this discussion. We conclude the chapter by applying Newton's theory to several familiar examples.

Chapter Outline

1. **A brief history.** Where do our ideas and theories about motion come from? What roles were played by Aristotle, Galileo, and Newton?

2. **Newton's first and second laws.** How do forces affect the motion of an object? What do Newton's first and second laws of motion tell us, and how are they related to each other?

3. **Mass and weight.** How can we define mass? What is the distinction between mass and weight?

4. **Newton's third law.** Where do forces come from? How does Newton's third law of motion help us to define force, and how is the third law applied?

5. **Applications of Newton's laws.** How can Newton's laws be applied in different situations, such as pushing a chair, skydiving, throwing a ball, and pulling two connected carts across the floor?

A large person gives you a shove, and you move in the direction of that push. A child pulls a toy wagon with a string, and the wagon lurches along. An athlete kicks a football or a soccer ball, and the ball is launched toward the goal. These are familiar examples involving forces in the form of pushes or pulls that cause changes in motion.

To pick a less complex example, imagine yourself pushing a chair across a wood or tile floor (fig. 4.1). Why does the chair move? Will it continue its motion if you stop pushing? What factors determine the velocity of the chair? If you push harder, will the chair's velocity increase? Up to this point, we have introduced ideas useful in describing motion, but we have not talked much about what causes changes in motion. Explaining motion is more challenging than describing it.

You already have some intuitive notions about what causes the chair to move. Certainly, the push you exert on the chair has something to do with it. But is the strength of that push more directly related to the velocity of the chair or to its acceleration? At this point, intuition often serves us poorly.

Over two thousand years ago, Greek philosopher Aristotle (384–322 B.C.) attempted to provide answers to some of these questions. Many of us would find that his explanations match our intuition for the case of the moving chair, but they are less satisfactory in the case of a thrown object where the push is not sustained. Aristotle's ideas were widely accepted until they were replaced by a theory introduced by Isaac Newton in the seventeenth century. Newton's theory of motion has proved to be a much more complete and satisfactory explanation of motion, and it permits quantitative predictions that were largely lacking in Aristotle's ideas.

Newton's three laws of motion form the foundation of his theory. What are these laws, and how are they used in explaining motion? How do Newton's ideas differ from those of Aristotle, and why do Aristotle's ideas often seem to fit our commonsense notions of what is happening? A good understanding of Newton's laws will permit you to analyze and explain almost any simple motion. This understanding will provide you with insights useful in driving a car, moving heavy objects, and many other everyday activities.

Figure 4.1 Moving a chair. Will the chair continue to move when the person stops pushing?
Keith Eng 2008

4.1 A Brief History

Did some genius, sitting under an apple tree, concoct a full-blown theory of motion in a sudden, blinding flash of inspiration? Not quite. The story of how theories are developed and gain acceptance involves many players over long periods of time.

Let's highlight the roles of a few key people whose insights produced major advances. A glimpse of this history can help you appreciate the physical concepts we will discuss by showing when and how the theories emerged. It is important, for example, to know whether a theory was just proposed yesterday or has been tried and tested over a long time. Not all theories carry equal weight in their acceptance and use by scientists. Aristotle, Galileo, and Newton were major players in shaping our views of the causes of motion.

Aristotle's view of the cause of motion

Questions about the causes of motion and changes in motion perplexed philosophers and other observers of nature for centuries. For over a thousand years, Aristotle's views prevailed. Aristotle was a careful and astute philosopher of nature. Aristotle investigated an incredible range of subjects, and he (or perhaps his students) produced extensive writings on topics such as logic, metaphysics, politics, literary criticism, rhetoric, psychology, biology, and physics.

In his discussions of motion, Aristotle conceived of force much as we have talked about it to this point: as a push or pull acting on an object. He believed that a force had to act for an object to move and that the velocity of the object was proportional to the strength of the force. A heavy object would fall more quickly toward the Earth than a lighter object, because there was a larger force pulling the object to the Earth. The strength of this force could be appreciated simply by holding the object in your hand.

Aristotle was also aware of the resistance that a medium offers the motion of an object. A rock falls more rapidly through air than through water. Water provides greater resistance to motion than air, as you surely know from trying to walk through waist-deep water at the beach. Aristotle thus saw the velocity of the object as being proportional to the force acting on it and inversely related to the resistance, but he never defined the concept of resistance quantitatively. He did not distinguish acceleration from velocity, and he spoke of velocity by stating the time required to cover a fixed distance.

Aristotle was an observer of nature rather than an experimenter. He did not make quantitative predictions that he checked by experiment. Even without such tests, however, some problems with his basic ideas of motion troubled Aristotle himself, as well as later thinkers. For example, in the case of a thrown ball or rock, the force that initially propels the object no longer acts once the ball leaves the hand. What keeps the ball moving?

Because the ball does keep moving for some time after leaving the hand that throws it, a force was necessary, according to Aristotle's theory. He suggested that the force that maintains the motion once the ball leaves the hand is provided by air rushing around to fill the vacuum in the spot where the ball has just been (fig. 4.2). This flow of air then pushes the ball from behind. Does this seem reasonable?

Following the decline of the Roman Empire, only fragments of Aristotle's writings were known to European thinkers for several centuries. His complete works, which had been preserved by Arab scholars, did not resurface in Europe until the twelfth century. Along with the work of other Greek thinkers, Aristotle's works were translated into Latin during the twelfth and thirteenth centuries.

How did Galileo challenge Aristotle's views?

By the time Italian scientist Galileo Galilei (1564–1642) came on the scene, Aristotle's ideas were well established at European universities, including the universities of Pisa and Padua, where Galileo studied and taught. In fact, education at the universities was organized around the disciplines defined by Aristotle, and much of Aristotle's natural philosophy had been incorporated into the teaching of the Roman Catholic Church. Italian theologian Thomas Aquinas had carefully interwoven Aristotle's thinking with the theology of the church.

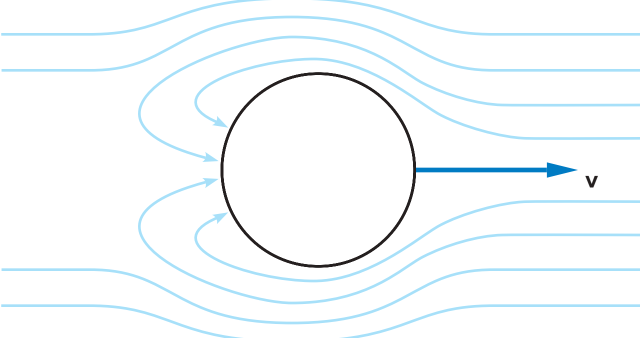


Figure 4.2 Aristotle pictured air rushing around a thrown object to continue pushing the object forward. Does this picture seem reasonable?

To challenge Aristotle was equivalent to challenging the authority of the church and could carry heavy consequences. Galileo was not alone in questioning Aristotle's ideas on motion; others had noted that dropped objects of similar form but radically different weights fall at virtually the same rate, contrary to Aristotle's theory. Although Galileo may never have dropped objects from the Leaning Tower of Pisa, he did perform careful experiments with dropped objects and actively publicized his results.

Galileo's primary problems with the church came from advocating the ideas of Copernicus. Copernicus had proposed a sun-centered (*heliocentric*) model of the solar system (discussed in chapter 5), which opposed the prevailing Earth-centered models of Aristotle and others. Galileo was an activist on several fronts in challenging Aristotle and the traditional thinking. This placed him in conflict with many of his university colleagues and with members of the church hierarchy. He was eventually tried by the Inquisition and found guilty of heresy. He was placed under house arrest and forced to retract some of his teachings.

In addition to his work on falling objects, Galileo developed new ideas on motion that contradicted Aristotle's theory. Galileo argued that the natural tendency of a moving object is to continue moving: No force is required to maintain this motion. (Think about the pushed chair again. Does this statement make sense in that situation?) Building on the work of others, Galileo also developed a mathematical description of motion that included acceleration. The relationship $d = \frac{1}{2}at^2$ for the distance covered by a uniformly accelerating object was carefully demonstrated by Galileo in experiments involving balls rolling down an inclined plane. He published many of these ideas near the end of his life in his famous *Dialogues Concerning Two New Sciences.*

What did Newton accomplish?

Isaac Newton (1642–1727; fig. 4.3) was born in England within a year of Galileo's death in Italy. Building on the work of Galileo, he proposed a theory of the causes of motion that could explain the motion of any object—the motion of ordinary objects such as a ball or chair as well as the motion of heavenly bodies such as the moon and the planets. In the Greek tradition, celestial motions were thought of as in an entirely different realm from Earthbound motions, thus requiring different explanations. Newton abolished this distinction by explaining both terrestrial and celestial mechanics with one theory.

The central ideas in Newton's theory are his three laws of motion (discussed in sections 4.2 and 4.4) and his law of

Figure 4.3 A portrait of Isaac Newton.
Pixtal/age Fotostock

universal gravitation (discussed in chapter 5). Newton's theory provided successful explanations of aspects of motion already known, as well as offered a framework for many new studies in physics and astronomy. Some of these studies led to predictions of phenomena not previously observed. For example, calculations applying Newton's theory to irregularities in the orbits of the known planets led to the prediction of the existence of Neptune, which was quickly confirmed by observation. Confirmed predictions are one of the marks of a successful theory. Newton's theory served as the basic theory of mechanics for over two hundred years and is still used extensively in physics and engineering.

Newton developed the basic ideas of his theory around 1665, when he was still a young man. To avoid the plague, he had returned to his family's farm in the countryside, where he had time to engage in serious thought with little interruption. The story has it that seeing an apple fall led to his insight that the moon also falls toward the Earth and that the force of gravity is involved in both cases. (See chapter 5.) Flashes of insight or inspiration were surely a part of the process.

Although Newton developed much of his theory and its details in 1665, he did not formally publish his ideas until 1687. One reason for this delay was his need to develop some of the mathematical techniques required to calculate the effects of the proposed gravitational force on objects such as planets. (He is generally credited with being the co-inventor of what we now call *calculus.*) The English title of Newton's 1687 treatise is *The Mathematical Principles of Natural Philosophy* (*Philosophiae Naturalis Principia Mathematica* in Latin), which is often referred to as Newton's *Principia.* His development and use of calculus in physics are considered a major milestone.

Scientific theories like Newton's do not just emerge in an intellectual vacuum. They are products of both their time and the state of knowledge and current worldviews. They usually replace earlier and often cruder theories. The accepted theory of motion in Newton's day was still that of Aristotle, although it had come under attack by Galileo and others. Its shortcomings were generally recognized. Newton provided the capstone for a revolution in thought that was already well under way.

Although Aristotle's ideas on motion are now considered unsatisfactory and are worthless for making quantitative predictions, they do have an intuitive appeal much like that of our own untrained thinking about motion. For this reason, we often speak of the need to replace Aristotelian ideas about motion with Newtonian concepts as we learn mechanics. Even though our own naive ideas about motion are not usually as fully developed as those of Aristotle, you may find that some of your commonsense notions will require modification.

Newton's theory, in turn, has been partially superseded by more sophisticated theories that provide more accurate descriptions of motion. These include Einstein's theory of relativity and the theory of quantum mechanics, both of which arose early in the twentieth century. Although the predictions of these theories differ substantially from Newton's theory in the realm of the very fast (in the case of relativity) and the very small (quantum mechanics), they differ insignificantly for the motion of ordinary objects traveling at speeds much less than that of light. Newton's theory was a tremendous step forward and is still used extensively to analyze the motion of ordinary objects.

> Aristotle's ideas on motion, although not capable of making quantitative predictions, provided explanations that were widely accepted for many centuries and that fit well with commonsense thinking. Galileo challenged Aristotle's ideas on free fall, as well as his general assumption that a force was required to keep an object in motion. Building on Galileo's work, Newton developed a more comprehensive theory of motion that replaced Aristotle's ideas. Newton's theory is still widely used to explain the motion of ordinary objects.

4.2 Newton's First and Second Laws

If we push a chair across the floor, what causes the chair to move or to stop moving? Newton's first two laws of motion address these questions and, in the process, provide part of a definition of **force.** The first law tells us what happens in the absence of a force, and the second describes the effects of applying a force to an object.

We discuss the first and second laws of motion together because the first law is closely related to the more general second law. Newton felt the need to state the first law separately, however, to counter strongly held Aristotelian ideas about motion. In doing so, Newton was following the lead of Galileo, who had stated a principle similar to Newton's first law several years earlier.

Newton's first law of motion

In language not too different from his own, **Newton's first law of motion** can be stated as follows:

> An object remains at rest, or in uniform motion in a straight line, unless it is compelled to change by an externally imposed force.

In other words, unless there is a force acting on the object, its *velocity* will not change. If it is initially at rest, it will remain at rest; if it is moving, it will continue to do so with constant velocity (fig. 4.4). In essence, Newton was reinforcing one of Galileo's key ideas of motion in stating the first law.

Notice that in paraphrasing Newton's first law, we have used the term *velocity* rather than the term *speed*. Constant velocity implies that neither the direction nor the magnitude of the velocity changes. When the object is at rest, its velocity is zero, and that value remains constant in the absence of an external force. If there is no force acting on the object, the acceleration of the object is zero. The velocity does not change.

Although this law seems simple enough, it directly contradicts Aristotle's ideas (and perhaps your own intuition as well). Aristotle believed that a force is required to keep an object moving. His views make intuitive sense if we are talking about moving a heavy object, such as the chair mentioned in our introduction. If you stop pushing, the chair stops moving. This view encounters problems, however, if we consider the motion of a thrown ball, or even a chair moving on a slippery surface. These objects continue to move after the initial

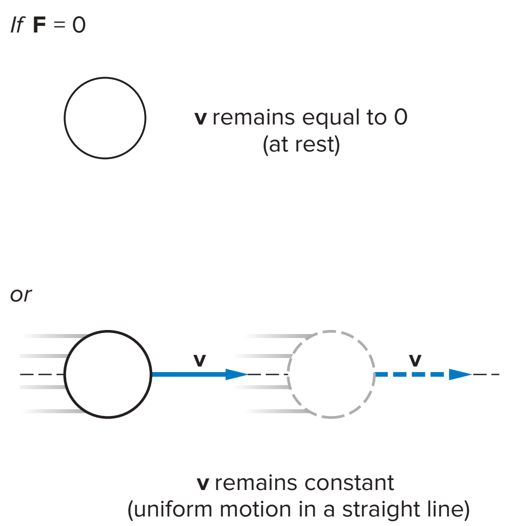


Figure 4.4 Newton's first law: In the absence of a force, an object remains at rest or moves with constant velocity.

push. Newton (and Galileo) made the strong statement that no force is needed to keep an object moving.

How can Aristotle's ideas be so different from those of Newton and Galileo and yet seem so reasonable in some situations? The key to answering that question involves the existence of resistive, or **frictional, forces.** The chair does not move far after you stop pushing because the frictional forces of the floor acting on the chair cause the velocity to quickly decrease to zero. A thrown ball would eventually stop moving, even if it did not fall to the ground, because the force of air resistance is pushing against it. It is quite difficult to find a situation in which there are *no* forces acting upon an object. Aristotle recognized the presence of air resistance and similar effects but did not treat them as forces in his theory.

How is force related to acceleration?

Newton's second law of motion is a more complete statement about the effect of an imposed force on the motion of an object. Stated in terms of acceleration, it says

> The acceleration of an object is directly proportional to the magnitude of the imposed force and inversely proportional to the mass of the object. The acceleration is in the same direction as that of the imposed force.

This statement is most easily grasped in symbolic form. By choosing appropriate units for force, we can state the proportionality of Newton's second law as the equation

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$

where $\mathbf{a}$ is the acceleration, $\mathbf{F}_{\text{net}}$ is the total, or **net, force** acting on the object, and m is the mass of the object. Because the acceleration is directly proportional to the imposed force, if we double the force acting on the object, we double the acceleration of the object. The same force acting on an object with a larger mass, however, will produce a smaller acceleration (fig. 4.5).

Note that the *acceleration* is directly related to the imposed force, not the velocity. Aristotle did not make a clear distinction between acceleration and velocity. Many of us also fail to make the distinction when we think informally about motion. In Newton's theory, this distinction is critical.

Newton's second law is the central quantitative idea of his theory of motion. According to this law, the acceleration of an object is determined by two quantities: the net force acting on the object and the mass of the object. In fact, the concepts of force and mass are, in part, defined by the second law. The net force acting on the object is the cause of its acceleration. Newton's third law, discussed in section 4.4, completes the definition of force by making clear that forces result from interaction of the object with other objects.

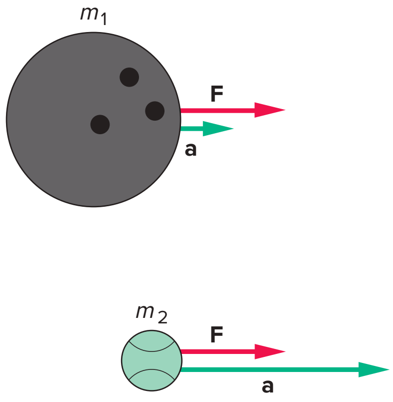


Figure 4.5 The smaller-mass object experiences a larger acceleration than the larger-mass object when identical forces are applied to the two objects.

The **mass** of an object is a quantity that tells us how much resistance an object has to a change in its motion, as indicated by the second law. We call this resistance to a change in motion **inertia,** following Galileo. (See everyday phenomenon box 4.1.) We can define mass as follows:

> Mass is a measure of an object's inertia, the property that causes it to resist a change in its motion.

The standard metric unit for mass is the kilogram (kg). We will say more about the determination of mass and its relationship to the weight of an object in section 4.3.

Units of force can also be derived from Newton's second law. If we solve for $\mathbf{F}_{net}$ by multiplying both sides of the second-law equation by the mass, it can be expressed as

$$\mathbf{F}_{net} = m\mathbf{a}$$

The appropriate unit for force must therefore be the product of a unit of mass and a unit of acceleration, or in the metric system, kilograms times meters per second squared. This frequently used unit is called the **newton** (N). Accordingly,

$$1 \text{ newton} = 1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

How do forces add?

Our version of the second law implies that the imposed force is the total, or *net, force* acting on the object. Force is a vector quantity whose direction is clearly important. If there is more than one force acting on an object, as there often is, we must then add these forces as vectors, taking into account their directions.

This process is illustrated in figure 4.6 and the sample exercise in example box 4.1. A block is being pulled across a table by a force of 10 N applied through a string attached to the block. A frictional force of 2 N acts on the block, a result of contact with the table. What is the total force acting on the block?

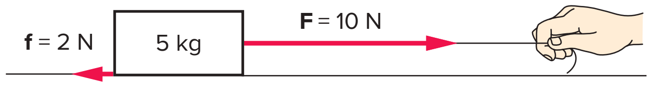


Figure 4.6 A block being pulled across a table. Two horizontal forces are involved.

Example Box 4.1

Sample Exercise: Finding the Net Force

A block with a mass of 5 kg is being pulled across a tabletop by a force of 10 N applied by a string tied to the front end of the block (fig. 4.6). The table exerts a 2-N frictional force on the block. What is the acceleration of the block?

$\mathbf{F}_{string} = 10$ N (to the right)
$\mathbf{f}_{table} = 2$ N (to the left)
$m = 5$ kg
$\mathbf{a} = ?$

$F_{net} = F_{string} - f_{table}$
$= 10 \text{ N} - 2 \text{ N} = 8 \text{ N}$
$\mathbf{F}_{net} = 8$ N (to the right)

$$\mathbf{a} = \frac{F_{net}}{m} = \frac{8 \text{ N}}{5 \text{ kg}} = \mathbf{1.6\ m/s^2}$$

($\mathbf{a} = 1.6$ m/s^2 to the right)

Is the net force the numerical sum of the two forces, 10 N plus 2 N, or 12 N? Looking at the diagram in figure 4.6 should convince you this cannot be true. The two forces oppose each other. Because the forces are in opposite directions, the net force is found by subtracting the frictional force from the force applied by the string, resulting in a net force of 8 N. We cannot ignore the directions of the forces involved.

That forces are vectors whose directions must be taken into account when finding the net force is an important aspect of the second law. For forces restricted to one dimension, as in example box 4.1, finding the net force is not difficult. In problems involving forces in two or three dimensions, addition is more complex but can be accomplished using techniques described in appendix C. In this chapter, we will consider only one-dimensional cases.

A final point about Newton's first and second laws bears repeating: The first law is contained within the second law, but it was very important for Newton to state the first law as a separate law so as to counter long-standing beliefs about motion. The relationship between the two laws can be demonstrated by asking what happens, according to the second law, when the net force acting on an object is zero. In this case, the acceleration $\mathbf{a} = \frac{\mathbf{F}_{net}}{m}$ must also be zero. If the acceleration is zero, the velocity must be constant. The first

Everyday Phenomenon Box 4.1

The Tablecloth Trick

The Situation. When he was a child, Ricky Mendez saw a magician do the tablecloth trick. A full dinner place setting, including a filled wineglass, sat on a tablecloth covering a small table. The magician, with appropriate fanfare, pulled the tablecloth from the table without disturbing the dinnerware. Ricky ended up in the doghouse, however, when he tried this at home, with disastrous results.

More recently, Ricky saw his physics instructor do a similar trick with a simpler place setting. The students were told that the demonstration had something to do with inertia. Why does the trick work, and how is inertia involved? Why did the trick not work when Ricky tried it at home as a child?

The Analysis. The magician's trick, which is frequently used as a physics demonstration, is indeed an illustration of the effects of inertia. Because the nature of frictional forces also plays a role, the choice of a smooth material for the tablecloth is important. (Butcher paper is sometimes substituted in physics demonstrations.) Some practice is usually essential to the successful execution of the trick.

The performer, be it a magician, an instructor, or a student, must pull the cloth or paper very quickly, giving it a large initial acceleration. Pulling slightly downward across the edge of the table helps to assure that there is no upward component to the acceleration and that the acceleration is reasonably uniform across the width of the tablecloth. As the tablecloth accelerates, it exerts a frictional force on the tableware. If we pulled slowly, this frictional force would pull the dishes and glasses along with the tablecloth.

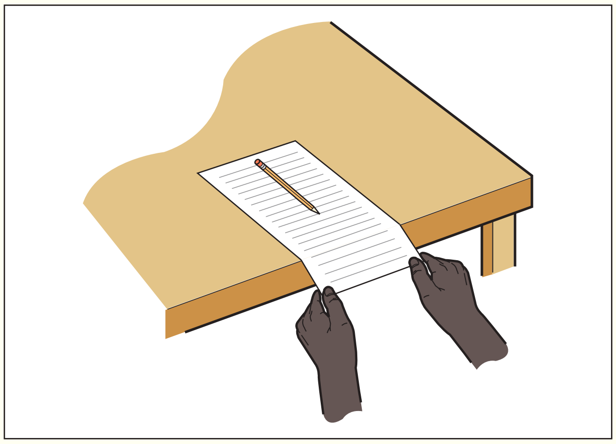
Inertia is the tendency of an object (related to its mass) to resist a change in its motion. When an object is at rest, it remains at rest unless a force is applied. There *is* a force acting on the plates and glasses, however—the frictional force exerted by the tablecloth. If the tablecloth is pulled quickly enough, the frictional force is in effect for only a very short time, so the acceleration of the objects is very brief. The objects will accelerate slightly, but not nearly as much as the tablecloth.

Two aspects of the frictional force are important to our understanding of what happens. One is that the force of static friction (in effect when the surfaces are not sliding relative to each other) has a maximum value that is determined by the nature of the contacting surfaces and by the force pushing the surfaces together. The second is that once the objects start to slide, kinetic, or sliding, friction comes into play. The force of kinetic friction is usually smaller than that of static friction.

When the tablecloth is given a large lateral acceleration, the force needed to also accelerate the tableware ($F_{net} = ma$) exceeds the maximum force of static friction between the dish or glass and the tablecloth. The tablecloth then begins to slide underneath the dish, reducing the size of the frictional force. If the surfaces are smooth, the frictional force is never large enough to produce an acceleration of the dish or glass that is anywhere near the size of the acceleration of the tablecloth. In the fraction of a second that this force acts, it does not have a chance to increase the velocity very much or to move the object very far. (See synthesis problem SP3.) Once the tablecloth is no longer in contact with the object, the frictional force exerted by the table quickly decelerates the object.

You can test these ideas yourself with a pencil, cup, or similar object (preferably nonbreakable) and a sheet of smooth tablet paper. Place the paper on a smooth desk or table surface with the end of the paper extending over the edge. Grasping the paper with both hands near the corners, as shown in the drawing, pull it downward across the edge of the desk or table. Notice that a slow pull brings the object along with the paper, but a very rapid pull leaves the object essentially in place. (The objects will usually move slightly in the direction of the pull.)

Before you graduate to tablecloths and full dinner place settings, a few cautions are in order. Objects that can tip, like filled wineglasses, are more difficult to work with. The bottom may start to move while the top portion (with its greater inertia) remains in place, causing the glass to tip and spill the wine or water. Also, the larger the tablecloth, the more difficult it is to pull it clear of the table—your hands must move very rapidly through a large distance in the pull. Practice is essential, which is the case for most of the tricks that magicians (and physics instructors) perform.



Grasp the paper near the corners and pull slightly downward across the edge of the table. A quick pull will leave the pencil near its initial position.

law tells us that if the net force is zero, the object moves with constant velocity (or remains at rest). Newton's first law addresses the special case of the second law in which the net force acting on an object is zero.

> The central principle in Newton's theory of motion is his second law of motion. This law states that the acceleration of an object is proportional to the net force applied to the object and inversely proportional to the mass of the object. The mass of an object is its inertia, or resistance to change in motion. Newton's first law is a restatement of Galileo's principle of inertia, which defines what happens when the net force acting on the object is zero. To find the net force acting on the object, we take into account the directions of the individual forces and add them as vectors.

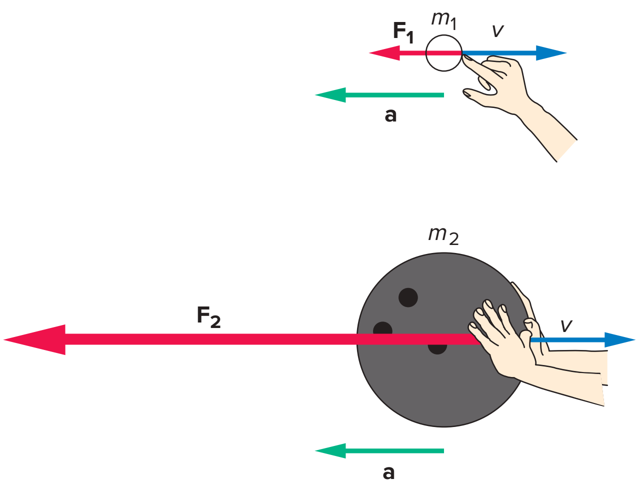


Figure 4.7 Stopping a bowling ball and a Ping-Pong ball. A much larger force is required to produce the same *rate* of change in velocity for the larger mass.

4.3 Mass and Weight

What exactly is weight? Is your *weight* the same as your *mass,* or is there a difference in the meaning of these two terms? Clearly, mass plays an important role in Newton's second law. *Weight* is a familiar term often used interchangeably with *mass* in everyday language. Here again, physicists are very specific with the words they use. There is a distinction between mass and weight that is important to Newton's theory.

How can masses be compared?

From the role that mass plays in Newton's second law, we can devise experimental methods of comparing masses. Mass is defined as the property of matter that determines how much an object resists a change in its motion. The greater the mass, the greater the *inertia,* or resistance to change, and the smaller the acceleration provided by a given force. Imagine, for example, trying to stop (decelerate) a bowling ball and a Ping-Pong ball that are moving initially with equal velocities (fig. 4.7). A much greater force is required to stop the bowling ball than to stop the Ping-Pong ball because of the difference in mass. According to the second law, the force required is proportional to the mass.

In effect, we are using Newton's second law to define mass. If we used the same force to accelerate different masses, the different accelerations could be used to compare the masses involved. If we choose one mass as a standard, any other mass can be measured against the standard mass by comparing the accelerations produced by equal forces. We could, in principle, determine the mass of any object this way.

How do we define weight?

In practice, the method just described is not convenient for comparing masses because of the difficulty of measuring acceleration. The more common method of comparing masses is to "weigh" the objects on a balance or scale (fig. 4.8). What we actually do in weighing is to compare the gravitational force acting on the mass we wish to measure with that acting on some standard mass. The gravitational force acting on an object is the **weight** of the object. As a force, weight has different units (newtons) than mass (kilograms).

How is weight related to mass? From our discussion of gravitational acceleration in chapter 3, we know that objects of different mass experience the same gravitational acceleration near the Earth's surface ($g = 9.8\ \text{m/s}^2$). This acceleration is caused by the gravitational force exerted by the Earth on the object, which is the weight of the object. According to Newton's second law, the force (the weight) is equal to the mass times the acceleration, or

$$\mathbf{W} = m\mathbf{g}$$

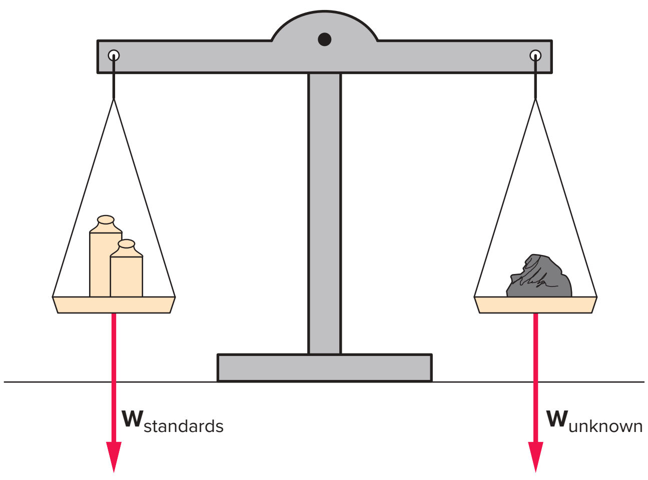


Figure 4.8 Comparing an unknown mass to standard masses on a balance.

Example Box 4.2

Sample Exercise: Computing Weights

Suppose that a woman has a mass of 50 kg. What is her weight in

a. newtons?
b. pounds?

a. $m = 50$ kg $\quad W = mg$

$W = ?$ $\quad = (50 \text{ kg})(9.8 \text{ m/s}^2)$

$\quad = \mathbf{490\ N}$

b. $W = ?$ in pounds

1 lb = 4.45 N $\quad W = \dfrac{490 \text{ N}}{4.45 \text{ N/lb}}$

$\quad = \mathbf{110\ lb}$

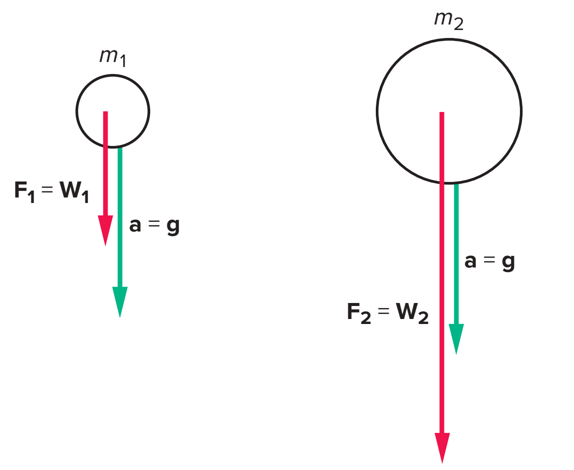


Figure 4.9 Different gravitational forces (weights) act on falling objects of different masses, but because acceleration is inversely proportional to mass, the objects have the same acceleration.

The symbol **W** represents the weight. It is a vector whose direction is straight down toward the center of the Earth.

If we know the mass of an object, we can then compute its weight. An example is provided in example box 4.2, in which a woman with a mass of 50 kg has a weight of 490 N. Since we are more used to expressing weights in the English system, we also convert her weight in newtons to pounds (lb), which yields a weight of 110 lb. The pound is most commonly used as a unit of *force,* not mass, in the English system. A mass of 1 kg weighs approximately 2.2 lb near the Earth's surface.

Although weight is proportional to mass, it also depends on the gravitational acceleration *g*. Because *g* varies slightly from place to place on the surface of the Earth—and has a much smaller value on the moon or the smaller planets—the weight of an object clearly depends on where that object is. On the other hand, the mass of an object is a property of the object related to the quantity of matter making up that object and does not depend on the location of the object.

The gravitational acceleration on the moon is approximately one-sixth that on the Earth's surface. If we transported the woman whose weight we have just determined to the moon, her weight would decrease to about 18 lb (or 82 N), one-sixth her weight on Earth. The woman's mass would still be 50 kg, provided that the trip did not take too much out of her. The mass of an object changes only if we add or subtract matter from it.

Why is the gravitational acceleration independent of mass?

The distinction between weight and mass can provide insight into why the gravitational acceleration is independent of mass. Let's turn to the case of a falling object and consider its motion using Newton's second law. Reversing the argument that we used in defining weight, we use the gravitational force (the weight) to determine the acceleration. By Newton's second law, the acceleration can be found by dividing the force ($\mathbf{W} = m\mathbf{g}$) by the mass:

$$\mathbf{a} = \frac{m\mathbf{g}}{m} = \mathbf{g}$$

Mass cancels out of the equation when we compute the acceleration for a falling object. The gravitational force is proportional to the mass, but by Newton's second law, the acceleration is inversely proportional to the mass: These two effects cancel each other. This holds true only for falling objects. In most other cases, the net force does not depend directly on the mass.

Force and acceleration are *not* the same, although they are closely related by Newton's second law. A heavy object experiences a larger gravitational force (its weight) than a lighter object, but the two objects will have the same gravitational acceleration (fig. 4.9). Because the gravitational force is proportional to mass, we find the same acceleration for different masses. The gravitational force will be discussed further in chapter 5 when we take up Newton's law of gravitation, a critical piece of his overall theory of motion.

Weight and mass are not the same. Weight is the gravitational force acting on an object, and mass is an inherent property related to the amount of matter in the object. Near the Earth's surface, weight is equal to the mass multiplied by the gravitational acceleration ($\mathbf{W} = m\mathbf{g}$), but the weight would change if we took the object to another planet where **g** had a different value. The reason that all objects experience the same gravitational acceleration near the Earth's surface is that the gravitational force is proportional to the mass of the object, but acceleration is equal to the force divided by the mass.

4.4 Newton's Third Law

Where do forces come from? If you push on a chair to move it across the floor, does the chair also push back on you? If so, how does that push affect your own motion? Questions like these are important to what we mean by *force*. Newton's third law provides some answers.

Newton's third law of motion is an important part of his definition of force. It is an essential tool for analyzing the motion or lack of motion of real objects, but it is often misunderstood. For this reason, it is good to take a careful look at the statement and use of the third law.

How does the third law help us to define force?

If you push with your hand against a large chair or any other large object, such as the wall of your room, you will feel the object push back against your hand. A force is acting on your hand that you can sense as it compresses your hand. Your hand is interacting with the chair or wall, and that object pushes back against your hand as you push against the object.

Newton's third law contains the idea that forces are caused by such interactions of two objects, each exerting a force on the other. It can be stated thus:

> If object A exerts a force on object B, object B exerts a force on object A that is equal in magnitude but opposite in direction to the force exerted on B.

The third law is sometimes referred to as the **action/reaction principle**—for every action there is an equal but opposite reaction. Note that the two forces always act on two *different* objects, never on the same object. Newton's definition of force includes the idea of an *interaction* between objects. The forces represent that interaction.

If you exert a force $\mathbf{F}_1$ on the chair with your hand, the chair pushes back on your hand with a force $\mathbf{F}_2$ that is equal in size but opposite in direction (fig. 4.10). Using this notation, Newton's third law can be stated in symbolic form as

$$\mathbf{F}_2 = -\mathbf{F}_1$$

The minus sign indicates that the two forces have opposite directions. The force $\mathbf{F}_2$ acts on your hand and partly determines your own motion, but it has nothing to do with the motion of the chair. Of this pair of forces, the only one that affects the motion of the chair is the one acting on the chair, $\mathbf{F}_1$.

Our definition of force is now complete. Newton's second law tells us how the motion of an object is affected by a force, and his third law tells where forces come from. They come from interactions with other objects. With a suitable definition of mass, which also depends on the second law, we know how to measure the size of forces by determining the acceleration that they produce ($\mathbf{F} = m\mathbf{a}$). Both the second and third laws are necessary to define what we mean by *force*.

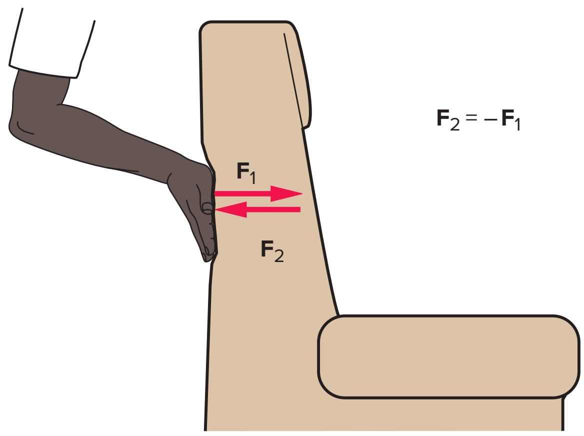


Figure 4.10 The chair pushes back on the hand with a force $\mathbf{F}_2$ that is equal in size but opposite in direction to the force $\mathbf{F}_1$ exerted by the hand on the chair.

How can we use the third law to identify forces?

How do we identify the forces that act on an object to analyze how that object will move? First, we identify other objects that interact with the object of interest. Consider a book lying on a table (fig. 4.11). What objects are interacting with the book? Because it is in direct contact with the table, the book must be interacting with the table, but it also interacts with the Earth through the gravitational attraction.

The downward pull of gravity that the Earth exerts on the book is the book's weight **W.** The object interacting with the book to produce this force is the Earth itself. The book and the Earth are attracted to each other (through gravity) with equal and opposite forces that form a third-law pair. The Earth pulls down on the book with the force **W,** and the

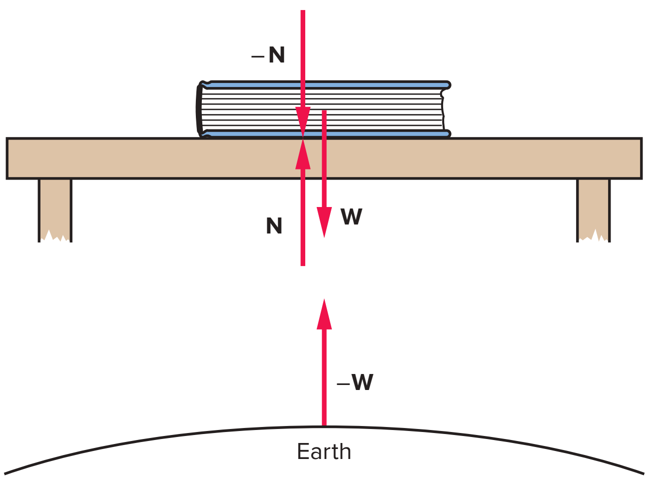


Figure 4.11 Two forces, **N** and **W,** act on a book resting on a table. The third-law reaction forces **–N** and **–W** to act on different objects: the table and the Earth.

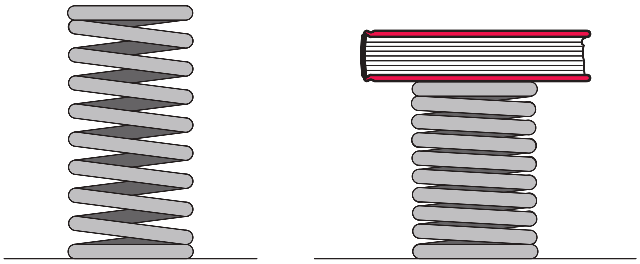


Figure 4.12 An uncompressed spring and the same spring supporting a book. The compressed spring exerts an upward force on the book.

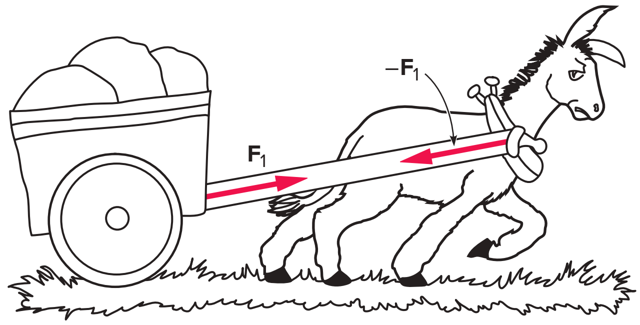


Figure 4.13 A mule and a cart. Does Newton's third law prevent the mule from moving the cart? There are additional forces not shown here (see the text).

book pulls upward on the Earth with the force $-\mathbf{W}$. Because of the Earth's enormous mass, the effect of this upward force on the Earth is extremely small.

The second force acting on the book is an upward force exerted on the book by the table. This force is often called the **normal force,** where the word *normal* means "perpendicular" rather than "ordinary" or "usual." The normal force $\mathbf{N}$ is always perpendicular to the surfaces of contact. The book, in turn, exerts an equal but oppositely directed downward force $-\mathbf{N}$ on the table. These two forces, $\mathbf{N}$ and $-\mathbf{N}$, constitute another third-law pair. They result from the mutual compression of the book and table as they come into contact with each other. You could think of the table as a large and very stiff spring that compresses ever so slightly when the book is placed on it (fig. 4.12).

The two forces acting on the book, the force of gravity and the force exerted by the table, also happen to be equal in size and opposite to each other, but this is *not* due to the third law. How do we know that they must be equal? Because the book's velocity is not changing, its acceleration must be zero. According to Newton's *second* law, the net force $\mathbf{F}_{net}$ acting on the book must then be zero, because $\mathbf{F}_{net} = m\mathbf{a}$ and the acceleration $\mathbf{a}$ is zero. The only way that the net force can be zero is for the two contributing forces, $\mathbf{W}$ and $\mathbf{N}$, to cancel each other. They must be equal in magnitude and opposite in direction for their sum to be zero.

Even though equal in size and opposite in direction, these two forces do not constitute a third-law action/reaction pair. They both act on the *same* object, the book, and the third law always deals with interactions between *different* objects. Thus, $\mathbf{W}$ and $\mathbf{N}$ are equal in size and opposite in direction in this case as a consequence of the second law rather than the third law. If they did not cancel each other, the book would accelerate away from the tabletop. (Both the second and third laws are critical to the analysis of the elevator example in everyday phenomenon box 4.2.)

Can a mule accelerate a cart?

Consider the story of a stubborn mule, which, having had a brief exposure to physics, argued to his handler that there was no point in pulling on the cart to which he was connected. According to Newton's third law, the mule argued, the harder he pulls on the cart, the harder the cart pulls back on him (fig. 4.13). The net result is, therefore, nothing. Is he right, or is there a fallacy in his argument?

The fallacy is simple but perhaps not obvious. The motion of the cart is affected by only one of the two forces that the mule is talking about—namely, the force that acts *on the cart.* The other force in this third-law pair acts on the mule and must be considered in conjunction with other forces that act on the mule to determine how he will move. The cart will accelerate if the force exerted by the mule *on the cart* is larger than the frictional forces acting *on the cart.* Try placing yourself in the role of the handler and explain the fallacy to the mule.

What force causes a car to accelerate?

As with the mule, the **reaction force** to a push or pull exerted by an object is often extremely important in describing the motion of the object itself. Consider the acceleration of a car. The engine cannot push the car because it is part of the car. The engine drives either the rear or front axle of the car, which causes the tires to rotate. The tires in turn push against the road surface through the force of friction $\mathbf{f}$ between the tires and the road (fig. 4.14).

According to Newton's third law, the road must then push against the tires with an equal but oppositely directed force $-\mathbf{f}$. This external force causes the car to accelerate.

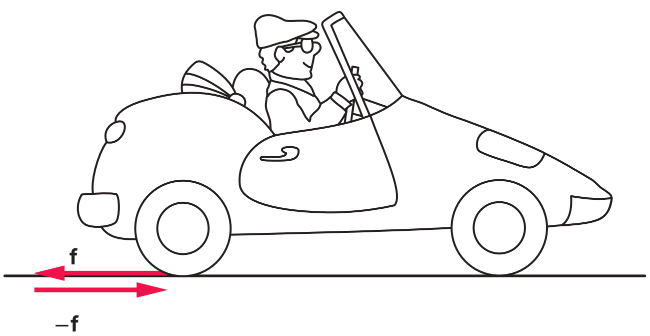


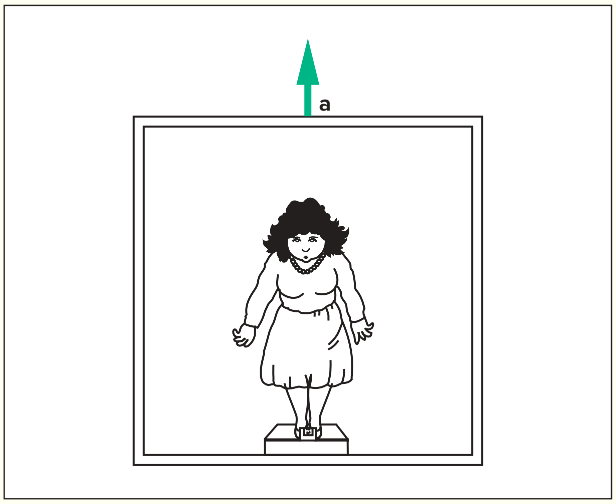
Figure 4.14 The car pushes against the road; in turn, the road pushes against the car.

Everyday Phenomenon Box 4.2

Riding an Elevator

The Situation. We have all had the experience of riding an elevator and feeling sensations of heaviness or lightness as the elevator accelerates up or down. The feeling of lightness as the elevator accelerates downward is generally more striking, particularly if the acceleration is not smooth.

Do we really weigh more or less than usual in these situations? If you took a bathroom scale into the elevator, would it read your true weight when the elevator is accelerating? How can we apply Newton's laws of motion to explore these questions?

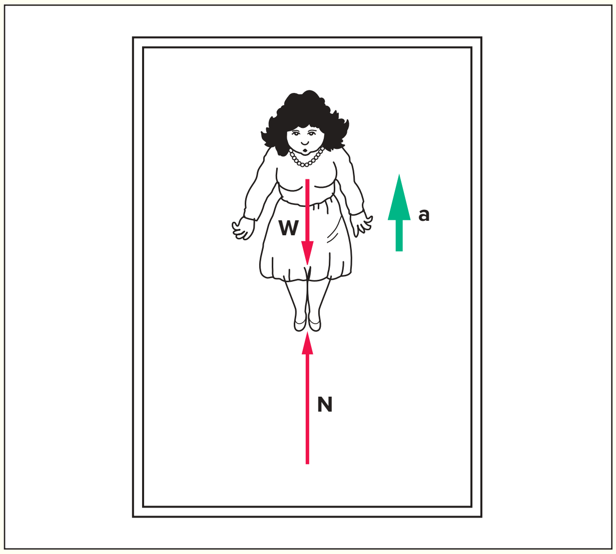


A woman standing on a bathroom scale inside an accelerating elevator. Will she read her true weight on the scale?

The Analysis. The first step in analyzing any situation using Newton's laws is to isolate the body of interest and carefully identify the forces that act on just that body. Different choices are possible for which objects to isolate, but some choices will be more productive than others. In this case, it makes sense to isolate the person standing on the scale, because her weight is the focus of our questions. The second drawing shows a **free-body diagram** of the woman indicating just those forces that act on her.

In this case, just two other objects interact with the woman, resulting in two forces. The Earth pulls downward on the woman through the force of gravity **W.** The scale pushes upward on her feet with a force **N,** the normal force. The vector sum of these two forces determines her acceleration. If the elevator is accelerating upward with an acceleration **a,** the woman must also be accelerating upward at that rate. The net force must also be upward, which implies that the normal force **N** is larger than the gravitational force **W.** Using signs to indicate direction, and letting the positive direction be upward, Newton's second law requires that

$$\mathbf{F}_{net} = \mathbf{N} - \mathbf{W} = m\mathbf{a}$$



A free-body diagram of the woman in the elevator when accelerating upward. Why is the normal force **N** larger than the weight **W?**

What about the scale reading? According to Newton's third law, the woman exerts a downward force on the scale equal in size to the normal force **N,** but opposite in direction. Because this is the force pushing down on the scale, the scale should read the value N, the magnitude of the normal force. The woman's true weight has not changed, but her apparent weight as measured by the scale has increased by an amount equal to $m\mathbf{a}$. (Rearranging the second-law equation yields $\mathbf{N} = \mathbf{W} + m\mathbf{a}$.)

What happens when the elevator is accelerating downward? In that case, the net force acting upon the woman must be downward, and the normal force must be less than her weight. The scale reading N will then be less than the woman's true weight by the amount ma, perhaps producing a smile rather than a scowl.

If the elevator cable breaks, we have a particularly interesting special case. Both the woman and the elevator will accelerate downward with the gravitational acceleration **g.** Because the woman's weight is all that is required to give her that acceleration, the normal force acting on her feet must then be zero. The scale reading will likewise be zero, and the woman is apparently weightless!

The sensation of our own weight is produced in part by the pressure on our feet and forces in our leg muscles needed to maintain our posture. The woman will feel weightless in this situation even though her true weight (the gravitational force acting on her) has not changed. In fact, she will be able to float around in the elevator just as astronauts do in the orbiting space station. (The space station is also falling toward the Earth as it moves laterally in its orbit.) This happy scenario will come to a crashing halt for the woman, however, when the elevator reaches the bottom of the shaft.

Obviously, friction is desirable in this case. Without friction, the tires would spin, and the car would go nowhere. The case of the mule is similar. The frictional force exerted by the ground on his hooves causes him to accelerate forward. This frictional force is the reaction to his pushing against the ground.

Think about this next time you find yourself walking. What external force causes you to accelerate as you start out? What is your role, and that of friction, in producing this force? How would you walk on an icy or slippery surface?

To figure out what forces are acting on any object, we need first to identify the other objects with which it is interacting. Some of these will be obvious. Any object in direct contact with the object of interest will presumably contribute a force. Interactions producing other forces, such as air resistance or gravity, may be less obvious but still recognizable with a little thought. The third law is the principle we use to identify any of these forces.

> Newton's third law of motion completes his definition of force. The third law notes that forces arise from interactions between different objects. If object A exerts a force on object B, object B exerts an equal-size but oppositely directed force on A. In order to apply the second law of motion, we use the third law to identify the external forces that act on an object.

4.5 Applications of Newton's Laws

We have now discussed Newton's laws of motion and the definitions of force and mass within these laws. To appreciate their usefulness, however, we must be able to apply them to some familiar examples, such as pushing a chair or throwing a ball. How do Newton's laws help us make sense of these motions? Do they provide a satisfactory picture of what is going on?

What forces are involved in moving a chair?

We have returned from time to time to the example of a chair being pushed but have not yet analyzed how and why it moves. As indicated in section 4.4, the first step in any analysis is to identify the forces that act on the chair. As shown in figure 4.15, four forces act on the chair from four separate interactions:*

1. The force of gravity (the weight) **W** due to interaction with the Earth.
2. The upward (normal) force **N** exerted by the floor due to compression of the floor.

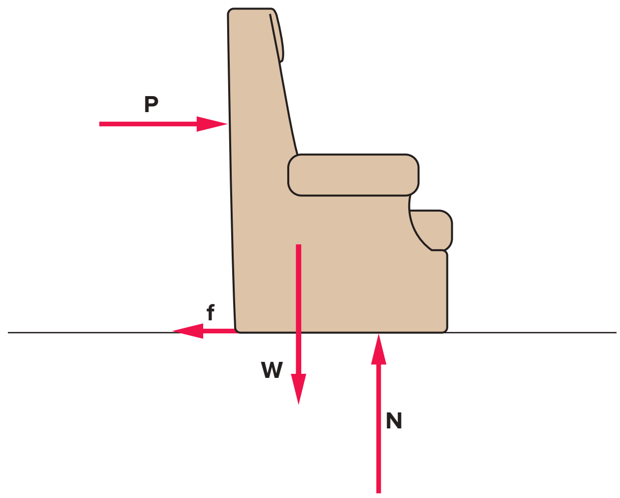


Figure 4.15 Four forces act on a chair being pushed across the floor: the weight **W,** the normal force **N,** the force **P** exerted by the person pushing, and the frictional force **f.**

3. The force exerted by the hand of the person pushing, **P.**
4. The frictional force **f** exerted by the floor.

Two of these forces, the normal force **N** and the frictional force **f,** are actually due to interactions with a single object, the floor. Because they are due to different effects and are perpendicular to one another, they are usually treated separately.

The effects of the two vertical forces acting on the chair, the weight **W** and the normal force **N,** cancel each other. Like the book on the table in section 4.4, this results because there is no acceleration of the chair in the vertical direction. According to Newton's second law, the sum of the vertical forces must then be zero, which implies that the weight **W** and the normal force **N** are equal in size but opposite in direction. (But they do not constitute a third-law pair.) They play no direct role in the horizontal motion of the chair.

The other two forces, the push of the hand **P** and the frictional force **f,** do not necessarily cancel. These two forces together determine the horizontal acceleration of the chair. The push **P** must be larger than the frictional force **f** for the chair to accelerate. In the most likely scenario for moving the chair, you first give a push with your hand that is larger than the frictional force. This produces a total force, with magnitude $P - f$, in the forward direction, causing the chair to accelerate.

Once you have accelerated the chair to a reasonable velocity, you reduce the strength of your push **P,** so that it is equal in size to the frictional force. The net horizontal force becomes equal to zero, and the horizontal acceleration is also zero by Newton's second law. If you sustain the push at this level, the chair moves across the floor with constant velocity.

Finally, you remove your hand and its push **P,** and the chair quickly decelerates to zero velocity under the influence of the frictional force **f.** If you happen to have a chair and a smooth floor handy, try to produce this motion. See if you can feel differences in the force you are exerting with your

*A figure such as figure 4.15, showing all the forces acting on an object, is often called a free-body diagram. See also everyday phenomenon box 4.2.

hand at various points in the motion. The force should be largest at the beginning of the motion.

The size of the force needed to keep the chair moving with constant velocity is determined by the strength of the frictional force, which, in turn, is influenced by the weight of the chair and the condition of the floor surface. If you fail to recognize the importance of the frictional force, you may be led, like Aristotle, to think that a force is always needed to keep an object moving. Frictional forces are almost always present, but they are not as obvious as the forces applied directly.

Does a skydiver continue to accelerate?

In chapter 3, we considered the fact that an object falls with constant acceleration **g** if air resistance is not a significant factor. What about objects such as skydivers who fall for large distances? Do they continue to accelerate at this rate, gaining larger and larger downward velocities? Any person with experience in skydiving knows that this does not happen. Why not?

If air resistance were not a factor, a falling object would experience only the gravitational force (its weight) and would indeed continue to accelerate. In skydiving, air resistance is an important factor, and its effects get larger as the velocity of the skydiver (or any object) increases. The skydiver has an initial acceleration of **g,** but as her velocity increases, the force of air resistance becomes significant. Her acceleration decreases (fig. 4.16).

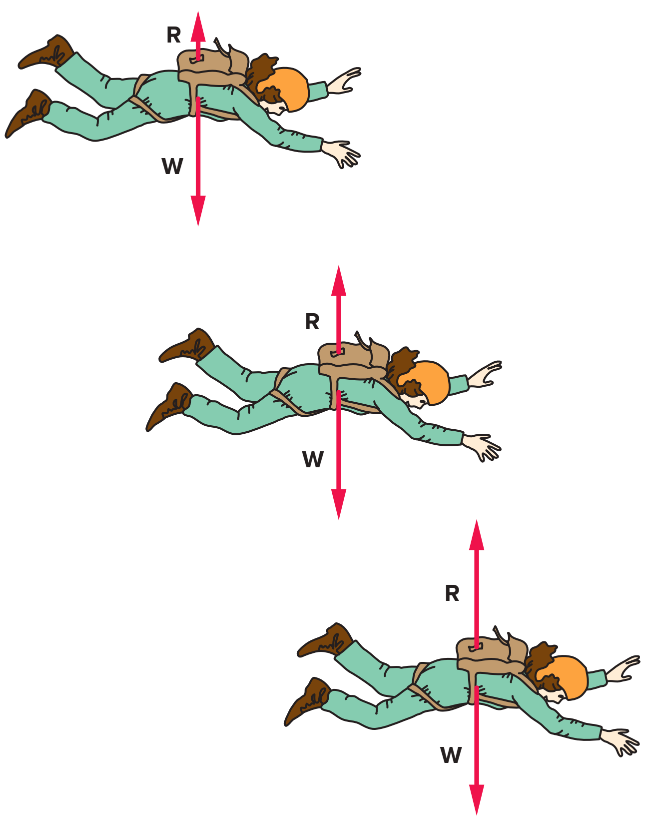


Figure 4.16 The force of air resistance **R** acting on a skydiver increases as the velocity increases.

Example Box 4.3

Sample Exercise: Air Resistance

As an 8-kg rock is falling from a high cliff, it experiences air resistance, which increases as the velocity of the rock increases. At a particular instant in time, assume the rock experiences an *upward* force due to air resistance of 30 N.

a. What is the gravitational force on the rock? What is the direction of this force?
b. What is the net force on the rock? What is the direction of this force?
c. What are the magnitude and direction of the acceleration on the rock at this instant in time?

a. $m = 8$ kg $\quad W = mg = (8\text{ kg})(9.8\text{ m/s}^2)$
$g = 9.8\text{ m/s}^2$ $\quad$ **W = 78.4 N, downward**

b. There are two forces on the rock; the upward force of air resistance and the downward gravitational force. The net force is the difference between the two. Since the gravitational force is greater the net force will be downward. We will assume upward is positive.

$\mathbf{F_{air}} = 30$ N, upward
W = 78.4 N, downward

$$F_{net} = F_{air} - W$$
$$= 30\text{ N} - 78.4\text{ N}$$
$$= -48.4\text{ N}$$
$$\mathbf{F_{net} = 48.4\text{ N, downward}}$$

c. $m = 8$ kg
$F_{net} = -48.4$ N

$$F_{net} = ma$$
$$a = F_{net}/m$$
$$a = -48.4\text{ N/8 kg}$$
$$a = -6.05\text{ m/s}^2$$
$$\mathbf{a = 6.05\text{ m/s}^2\text{, downward}}$$

For small velocities, the air-resistive force **R** is small, and the weight is the dominant force. As the velocity increases, the air-resistive force gets larger, causing the total magnitude of the downward force, $W - R$, to decrease. Because the net force is responsible for the acceleration, the acceleration will also decrease. A sample calculation is shown in example box 4.3. Ultimately, as the velocity continues to increase, the air-resistive force reaches a value equal in size to the gravitational force. The net force is then zero, and the skydiver stops accelerating. We say that she has reached **terminal velocity,** and from there on, she moves downward with constant velocity. This terminal velocity is usually between 100 and 120 MPH.

Frictional or resistive forces play a critical role in analyzing the motion. Aristotle did not have the opportunity to try skydiving (nor have many of us), so this example was not a part of his experience. He did observe the terminal velocity, however, of very light objects, such as feathers or leaves. The weight of such objects is small and the surface area is large relative to the weight, so the air-resistive force **R** becomes equal in size to the weight much sooner than for a heavier object.

Try tearing a small corner from a piece of paper and watching it fall. Does it appear to reach a constant (terminal) velocity? It will flutter as it falls, but it does not seem to accelerate much for most of its downward motion. You can see why Aristotle concluded that heavier objects fall faster than lighter objects. Dropping heavier objects through water can also show the terminal velocity. Water exerts a larger resistive force at lower velocities than air.

What happens when a ball is thrown?

Aristotle had trouble explaining the motion of a thrown object, such as a ball, once it had left the thrower's hand. Let's reconsider this example from a Newtonian perspective. Do we need a force to keep the ball moving? Not according to Newton's first law. Three forces, however, are involved in the flight of the ball: the initial push by the thrower, the downward pull of gravity, and (once again) air resistance (fig. 4.17).

To highlight Newton's approach, it is best to break down the motion into two different spans of time. The first is the process of throwing, when the hand is in contact with the ball. During this interval, the force **P** exerted by the hand dominates the motion. The combined effects of the other forces (gravity and air resistance) must be smaller than the force **P** if the ball is to accelerate. Thus, **P** accelerates the ball to a velocity that we often refer to as the *initial* velocity. The magnitude and direction of the initial velocity are determined by the strength and direction of the force **P** and the length of time it acts on the ball. Because this force usually varies with time, a full analysis of the process of throwing gets quite complex.

Once the ball leaves the hand, however, we are in the second time period, where **P** is no longer a consideration. During this interval, the gravitational force **W** and the air-resistive force **R** produce changes in the ball's velocity. From this point on, the problem becomes one of projectile motion (see section 3.4). The gravitational force accelerates the ball downward, and the air-resistive force acts in a direction opposite to the velocity, gradually reducing the ball's velocity.

Contrary to Aristotle's view, no forces are needed to keep the ball moving once it has been thrown. In fact, if an object were thrown in deep space, where air resistance is nonexistent and gravitational forces are very weak, it would keep moving with constant velocity, as stated in Newton's first law.

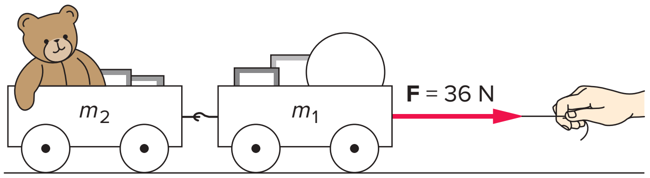


Figure 4.18 Two connected carts being accelerated by a force **F** applied by a string.

Because the air-resistive force or the push exerted by a person throwing a ball varies with time, we have avoided working out numerical examples for these situations. Just identifying the forces involved and their causes due to third-law interactions with other objects provides a useful description of what is happening.

How do we analyze the motion of connected objects?

Verification of Newton's laws of motion came initially from simpler examples that could be easily set up in the laboratory. One example not difficult to picture and set up in a physics laboratory (or even at home if suitable toys are available) is two connected carts accelerated by the pull of a string (fig. 4.18). To keep things simple, we will assume that the carts have excellent wheel bearings, so that they roll with very little friction. We will also assume that a scale is available to determine the masses of the carts and their contents.

To measure the magnitude of the force applied by the string, we would have to insert a small spring balance somewhere between the hand and the carts. The trickiest part of the entire experiment is applying a steady force with this arrangement while the carts are accelerating.

If we know the masses of the carts and their contents, and the magnitude of the force applied by the string, we should be able to predict the value of the acceleration of the system from Newton's second law. (See example box 4.4.) For the masses given in the example, and an applied force of 36 N, we find an acceleration of 2.0 m/s^2 for the two carts. The acceleration could be verified experimentally by measuring the time required for the carts to travel a fixed distance and using the equations developed for constant acceleration in chapter 2 to calculate an experimentally determined value.

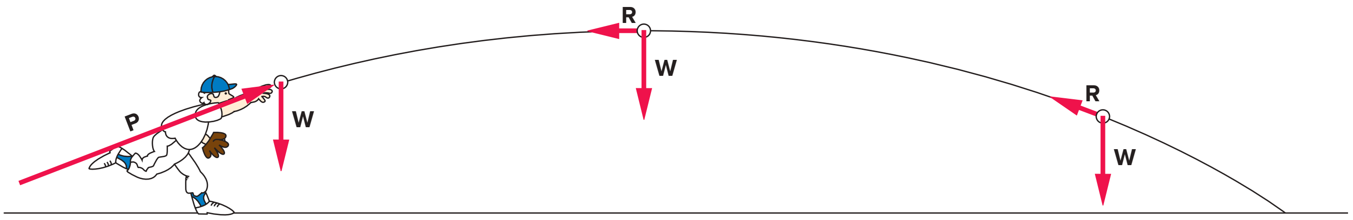


Figure 4.17 Three forces act on a thrown ball: the initial push **P**, the weight **W**, and air resistance **R**.

Example Box 4.4

Sample Exercise: Connected Objects

Two connected carts are pulled across the floor under the influence of a force of 36 N applied by a string (fig. 4.18). The forward cart and its contents have a mass of 10 kg, and the second cart and contents have a mass of 8 kg. Assuming that frictional forces are negligible,

a. What is the acceleration of the two carts?
b. What is the net force acting on each cart?

a. Defining the system as both carts, as discussed in the text,

$m_1 = 10$ kg $\qquad F_{net} = ma$

$m_2 = 8$ kg

$F = 36$ N $\qquad$ or: $a = \dfrac{F_{net}}{m} = \dfrac{36\text{ N}}{10\text{ kg} + 8\text{ kg}}$

$\mathbf{a} = ?$ $\qquad = \dfrac{36\text{ N}}{18\text{ kg}} = \mathbf{2.0\ m/s^2}$

$\mathbf{a} = 2.0\ \text{m/s}^2$ in the forward direction

b. Treating each cart separately:

$F_{net} = ?$ (for each cart)

first cart

$$F_{net} = ma = (10\text{ kg})(2\text{ m/s}^2) = \mathbf{20\ N}$$

$\mathbf{F_{net} = 20\ N}$ in the forward direction

second cart

$$F_{net} = ma = (8\text{ kg})(2\text{ m/s}^2) = \mathbf{16\ N}$$

$\mathbf{F_{net} = 16\ N}$ in the forward direction

In example box 4.4, we first treat the two carts as a single system to find the acceleration. Suppose, however, that we want to know the magnitude of the force exerted by the hooks connecting the two carts. In this case, it makes sense to treat the motion of the individual carts separately. Once we know the acceleration, we again apply Newton's second law to find the net force acting on each cart. This computation is done in the second part of example box 4.4 and is illustrated in figure 4.19.

For the second cart, a force of 16 N is required to produce the acceleration of 2 m/s^2. By Newton's third law, there should then be a force of 16 N pulling back on the first cart.

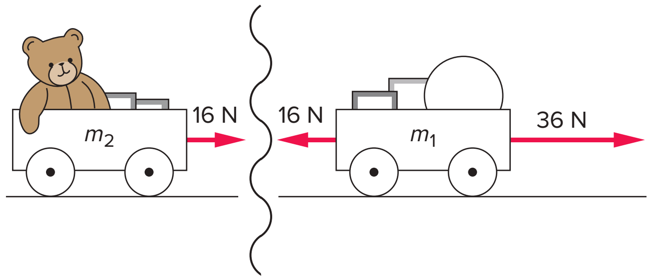


Figure 4.19 The interaction between the two carts illustrates Newton's third law.

Combined with the forward force of 36 N applied by the string, this results in a net force of 20 N acting on the first cart (36 N − 16 N). This is exactly the value required to give the first cart an acceleration of 2 m/s^2.

From this example, we see that Newton's laws provide a completely consistent picture of the forces and accelerations of the different parts of the connected-cart system. This is a necessary condition for us to accept the laws as valid. Obviously, another condition is that any predictions be confirmed by experimental measurements. This has been done many times by experiments similar to the one we have dealt with here.

We could try many variations on this experiment in the laboratory to see if the results agree with predictions derived from Newton's laws. Even with careful experimental technique using accurate stopwatches and balances, however, our results are unlikely to agree exactly with our predictions. It is impossible to eliminate the effects of friction completely, and none of our measurements can be made with infinite precision. The art of the experimentalist is to reduce these inaccuracies to a minimum, as well as to predict how they affect our results.

> Newton's laws of motion provide both qualitative and quantitative explanations of any familiar motion. First, we identify the forces acting on the object by examining interactions with other objects. The relative sizes of these forces, when added together, give the acceleration of the object. The acceleration may change as the forces change with time, as in the case of a skydiver. Newton's laws have been verified many times by experimental tests of their quantitative predictions. They are a much more consistent theory of the causes of motion than the older Aristotelian view.

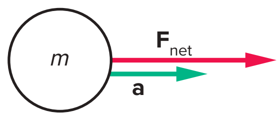
Summary

In 1685, Newton published his *Principia,* in which he introduced three laws of motion as the foundation of his theory of mechanics. These laws continue to serve as an extremely useful model for explaining the causes of motion and for predicting how objects will move in many familiar situations.

1 **A brief history.** Newton's theory was constructed on groundwork laid by Galileo and replaced a much earlier and less quantitative model developed by Aristotle to explain motion. Newton's theory had much greater predictive power than

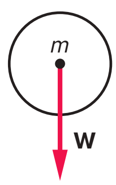
Aristotle's ideas. Although we now recognize its limitations, Newton's theory is still used extensively to explain the motion of ordinary objects.

2 **Newton's first and second laws.** Newton's second law states that the acceleration of an object is proportional to the net external force acting on that object and inversely proportional to the mass of the object. The first law, the principle of inertia, describes what happens when the net force is zero. The acceleration must be zero, when the net force is zero, and the object then moves with constant velocity.



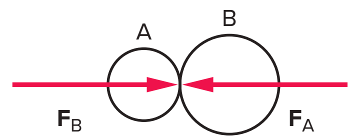
$$\mathbf{a} = \frac{\mathbf{F}_{net}}{m}$$

3 **Mass and weight.** Newton's second law defines the inertial mass of an object as the property that causes the object to resist a change in its motion. The weight of an object is the gravitational force acting on the object and is equal to the mass multiplied by the gravitational acceleration **g.** The weight of an object may vary as **g** varies, but mass is an inherent property of the object related to its quantity of matter.



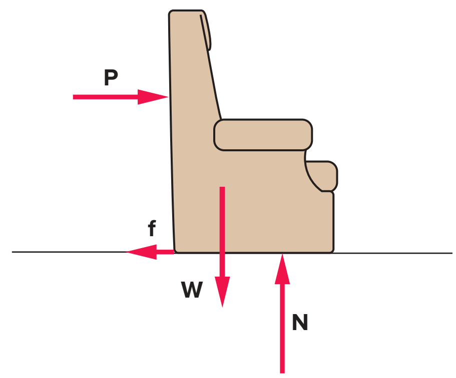
$$\mathbf{W} = m\mathbf{g}$$

4 **Newton's third law.** Newton's third law completes the definition of force by showing that forces result from interactions between objects. If object A exerts a force on object B, then object B exerts an equal-size but oppositely directed force on object A.



$$\mathbf{F}_A = -\mathbf{F}_B$$

5 **Applications of Newton's laws.** In analyzing the motion of an object using Newton's laws, the first step is to identify the forces that act on the object due to interactions with other objects. The strength and direction of the net force then determine how the object's motion will change.



The study hints found on Connect are a concise summary of the chapter. You might find them useful as a review before an exam.

Key Terms

Force, 62
Newton's first law of motion, 63
Frictional forces, 63
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Net force, 63
Mass, 64
Inertia, 64
Newton, 64
Weight, 66
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Action/reaction principle, 68
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Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available on Connect

Q1. Did Galileo's work on motion precede that of Aristotle or Newton? Explain.

Q2. Why did Aristotle believe that heavier objects fell faster than lighter objects? Explain.

Q3. Aristotle believed that a force was necessary to keep an object moving. Where, in his view, did this force come from in the case of a ball moving through the air? Explain.

*Q4. How did Aristotle explain the continued motion of a thrown object? Does this explanation seem reasonable to you? Explain.

Q5. Did Galileo develop a more complete theory of motion than that of Newton? Explain.

Q6. Two equal forces act on two different objects, one of which has a mass 10 times as large as the other. Will the more massive object have a larger acceleration, an equal acceleration, or a smaller acceleration than the less massive object? Explain.

Q7. A 3-kg block is observed to accelerate at a rate twice that of a 6-kg block. Is the net force acting on the 3-kg block therefore twice as large as that acting on the 6-kg block? Explain.

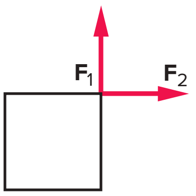
Q8. Two equal-magnitude horizontal forces act on a box as shown in the diagram. Is the object accelerated horizontally? Explain.

Q8 Diagram

Q9. Is it possible that the object pictured in question 8 is moving, given the fact that the two forces acting on it are equal in size but opposite in direction? Explain.

Q10. Suppose that a bullet is fired from a rifle in outer space, where there are no appreciable forces due to gravity or air resistance acting on the bullet. Will the bullet slow down as it travels away from the rifle? Explain.

Q11. Two equal forces act on an object in the directions pictured in the diagram. If these are the only forces involved, will the object be accelerated? Explain, using a diagram.



Q11 Diagram

Q12. An object moving horizontally across a table is observed to slow down. Is there a nonzero net force acting on the object? Explain.

Q13. A car goes around a curve, traveling at constant speed.
a. Is the acceleration of the car zero in this process? Explain.
b. Is there a nonzero net force acting on the car? Explain.

*Q14. Is Newton's first law of motion explained by the second law? Explain. Why did Newton state the first law as a separate law of motion?

Q15. Is the mass of an object the same thing as its weight? Explain.

Q16. The gravitational force acting on a lead ball is much larger than that acting on a wooden ball of the same size. When both are dropped, does the lead ball accelerate at the same rate as the wooden ball? Explain, using Newton's second law of motion.

Q17. The acceleration due to gravity on the moon is approximately one-sixth the gravitational acceleration near the Earth's surface. If a rock is transported from the Earth to the moon, will either its mass or its weight change in the process? Explain.

Q18. Is mass a force? Explain.

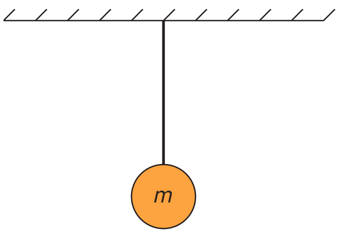
Q19. Two identical cans, one filled with lead shot and the other with feathers, are dropped from the same height by a student standing on a chair.
a. Which can (if either) experiences the greater force due to the gravitational attraction of the Earth? Explain.
b. Which can (if either) experiences the greater acceleration due to gravity? Explain.

Q20. A boy sits at rest on the floor. What two vertical forces act upon the boy? Do these two forces constitute an action/reaction pair as defined by Newton's third law of motion? Explain.

Q21. The engine of a car is part of the car and cannot push directly on the car in order to accelerate it. What external force acting on the car is responsible for the acceleration of the car on a level road surface? Explain.

Q22. It is difficult to stop a car on an icy road surface. Is it also difficult to accelerate a car on the same icy road? Explain.

Q23. A ball hangs from a string attached to the ceiling, as shown in the diagram.
a. What forces act on the ball? How many are there?
b. What is the net force acting on the ball? Explain.
c. For each force identified in part a, what is the reaction force described by Newton's third law of motion?



Q23 Diagram

Q24. Would the tablecloth trick (see everyday phenomenon box 4.1) work if a bath towel were used instead of a tablecloth? Explain.

*Q25. When a magician performs the tablecloth trick (see everyday phenomenon box 4.1), the objects on the table do not move very far. Is there a horizontal force acting on these objects while the tablecloth is being pulled off the table? Why do the objects not move very far? Explain.

Q26. A sprinter accelerates at the beginning of a 100-meter race and then tries to maintain maximum speed throughout the rest of the race.
a. What external force is responsible for accelerating the runner at the beginning of the race? Explain carefully how this force is produced.
b. Once the runner reaches her maximum velocity, is it necessary to continue pushing against the track in order to maintain that velocity? Explain.

Q27. A mule is attempting to move a cart loaded with rock. Since the cart pulls back on the mule with a force equal in size to the force that the mule exerts on the cart (according to Newton's third law), is it possible for the mule to accelerate the cart? Explain.

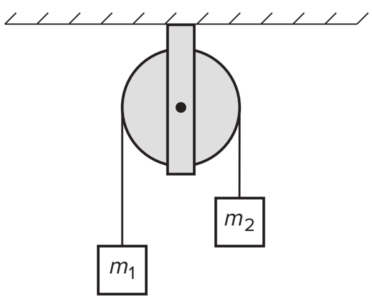
Q28. The upward normal force exerted by the floor on a chair is equal in size but opposite in direction to the weight of the chair. Is this equality an illustration of Newton's third law of motion? Explain.

Q29. A toy battery-powered tractor pushes a book across a table. Draw separate diagrams of the book and the tractor, identifying all of the forces that act upon each object. What is the reaction force described by Newton's third law of motion for each of the forces that you have drawn?

Q30. If you get into an elevator on the top floor of a large building and the elevator begins to accelerate downward, will the normal force pushing up on your feet be greater than, equal to, or less than the force of gravity pulling downward on you? Explain. (See everyday phenomenon box 4.2.)

Q31. If the elevator cable breaks and you find yourself in a condition of apparent weightlessness as the elevator falls, is the gravitational force acting upon you equal to zero? Explain. (See everyday phenomenon box 4.2.)

Q32. Two masses, m_1 and m_2, connected by a string, are placed on a fixed, frictionless pulley, as shown in the diagram. If m_2 is larger than m_1, will the two masses accelerate? Explain.

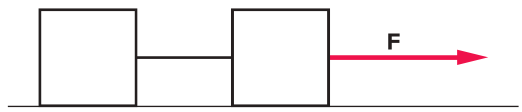


Q32 Diagram

Q33. Two blocks with the same mass are connected by a string and are pulled across a frictionless surface by a constant force, **F,** exerted by a string (see diagram).

a. Will the two blocks move with constant velocity? Explain.

b. Will the tension in the connecting string be greater than, less than, or equal to the force **F?** Explain.



Q33 Diagram

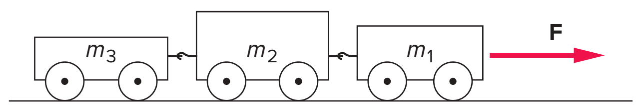
*Q34. Suppose that a skydiver wears a specially lubricated suit that reduces air resistance to a small, constant force that does not increase as the diver's velocity increases. Will the skydiver ever reach a terminal velocity before opening her parachute? Explain.

Q35. Does a skydiver experience constant acceleration throughout her descent from the plane to the ground?

Q36. When a skydiver reaches the point where the upward force of air resistance is equal to her weight, does her velocity continue to increase?

Q37. Does the force of gravity on a skydiver change as he falls toward Earth?

Q38. You have three carts connected by strings, as shown in the diagram. Assume their masses m_1, m_2, and m_3 are different from one another. If a force, **F,** is applied to m_1, how do their accelerations compare? Are they the same? Are they different? Explain.



Q38 Diagram

Q39. You have three carts connected by strings, as shown in the diagram for question 38. Assume their masses m_1, m_2, and m_3 are different from one another. If a force, **F,** is applied to m_1, how does the net force on each cart compare? Are the net forces the same? Are they different? Explain.

Exercises

For the exercises in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8\ \text{m/s}^2$ for the acceleration due to gravity.

E1. A single force of 42 N acts upon a 6-kg block. What is the magnitude of the acceleration of the block?

E2. A heavy ball with a mass of 4.5 kg is observed to accelerate at a rate of 7.0 m/s^2. What is the size of the net force acting upon this ball?

E3. A net force of 32 N acting upon a wooden block produces an acceleration of 4.0 m/s^2 for the block. What is the mass of the block?

E4. A 4.0-kg block being pulled across a table by a horizontal force of 70 N also experiences a frictional force of 15 N. What is the acceleration of the block?

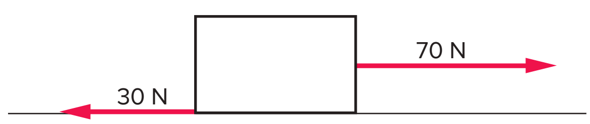
E5. A pulled tablecloth exerts a frictional force of 3.6 N on a plate with a mass of 0.4 kg. What is the acceleration of the plate?

E6. A 5-kg block being pushed across a table by a force **P** has an acceleration of 6.0 m/s^2.

a. What is the net force acting upon the block?

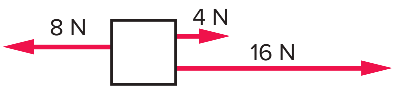
b. If the magnitude of **P** is 38 N, what is the magnitude of the frictional force acting upon the block?

E7. Two forces, one of 70 N and the other of 30 N, act in opposite directions upon a box, as shown in the diagram. What is the mass of the box if its acceleration is 5.0 m/s^2?



E7 Diagram

E8. A 5-kg block is acted upon by three horizontal forces, as shown in the diagram.
   a. What is the net horizontal force acting upon the block?
   b. What is the horizontal acceleration of the block?



E8 Diagram

E9. A 9-kg sled sliding freely on an icy surface experiences a 3-N frictional force exerted by the ice and an air-resistive force of 0.6 N.
   a. What is the net force acting upon the sled?
   b. What is the acceleration of the sled?

E10. What is the weight of a 45-kg mass?

E11. What is the mass of a 735-N weight?

E12. Patricia has a weight of 125 lb.
   a. What is her weight in newtons? (1 lb = 4.45 N)
   b. What is her mass in kilograms?

E13. One of the authors of this text has a weight of 660 N.
   a. What is her mass in kilograms?
   b. What is her weight in pounds? (1 lb = 4.45 N)

E14. Who has the larger mass, a woman weighing 160 lb or one weighing 690 N?

E15. At a given instant in time, a 6-kg rock that has been dropped from a high cliff experiences an upward force of air resistance of 12 N.
   a. What is the gravitational force on the rock?
   b. What is the net force on the rock? (Note the directions of the two forces!)
   c. What are the magnitude and direction of the acceleration of the rock?

E16. At a given instant in time, a 12-kg rock is observed to be falling with an acceleration of 6.25 m/s^2.
   a. What are the magnitude and direction of the net force on the rock?
   b. What are the magnitude and direction of the gravitational force on the rock?
   c. What is the magnitude of the force of air resistance acting on the rock at this instant?

E17. A 0.8-kg book rests on a table. A downward force of 12 N is exerted on the top of the book by a hand pushing down on the book.
   a. What is the net force on the book? Is it accelerating?
   b. What is the magnitude of the gravitational force acting upon the book?
   c. What is the magnitude of the upward (normal) force exerted by the table on the book? (Is the book accelerated?)

E18. An upward force of 32.6 N is applied via a string to lift a ball with a mass of 2.8 kg.
   a. What is the gravitational force acting upon the ball?
   b. What is the net force acting upon the ball?
   c. What is the acceleration of the ball?

E19. A 75-kg woman in an elevator is accelerating upward at a rate of 0.6 m/s^2.
   a. What is the net force acting upon the woman?
   b. What is the gravitational force acting upon the woman?
   c. What is the normal force pushing upward on the woman's feet? (Note: A picture of the forces on her will help.)

Synthesis Problems

For the synthesis problems in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8$ m/s^2 for the acceleration due to gravity.

SP1. A constant horizontal force of 28 N is exerted by a string attached to an 8-kg block being pulled across a tabletop. The block also experiences a frictional force of 6 N due to contact with the table.
   a. What is the horizontal acceleration of the block?
   b. If the block starts from rest, what will its velocity be after 3 seconds?
   c. How far will it travel in these 3 seconds?

SP2. A rope exerts a constant horizontal force of 350 N to pull a 40-kg crate across the floor. The velocity of the crate is observed to increase from 1 m/s to 9 m/s in a time of 2 seconds under the influence of this force and the frictional force exerted by the floor on the crate.
   a. What is the acceleration of the crate?
   b. What is the net force acting upon the crate?
   c. What is the magnitude of the frictional force acting upon the crate?
   d. What force would have to be applied to the crate by the rope in order for the crate to move with constant velocity? Explain.

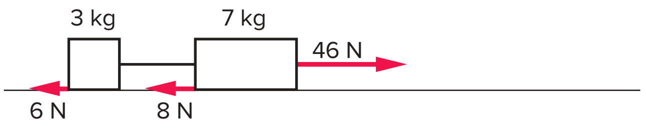
SP3. A dish with a mass of 0.3 kg has a force of kinetic friction of 0.18 N exerted on it by a moving tablecloth for a time of 0.2 s.
   a. What is the acceleration of the dish?
   b. What velocity does it reach in this time, starting from rest?
   c. How far (in cm) does the dish move in this time?

SP4. A 60-kg crate is lowered from a loading dock to the floor using a rope passing over a fixed support. The rope exerts a constant upward force on the crate of 500 N.
   a. Will the crate accelerate? Explain.
   b. What are the magnitude and direction of the acceleration of the crate?
   c. How long will it take for the crate to reach the floor if the height of the loading dock is 1.4 m above the floor?
   d. How fast is the crate traveling when it hits the floor?

SP5. Two blocks tied together by a horizontal string are being pulled across the table by a horizontal force of 46 N, as shown in the diagram. The 3-kg block has a 6-N frictional force exerted on it by the table, and the 7-kg block has an 8-N frictional force acting on it.
   a. What is the net force acting upon the entire two-block system?
   b. What is the acceleration of this system?

c. What force is exerted on the 3-kg block by the connecting string? (Consider only the forces acting upon this block. Its acceleration is the same as that of the entire system.)

d. Find the net force acting upon the 7-kg block, and calculate its acceleration. How does this value compare to that found in part b?



SP5 Diagram

SP6. An 85-kg man is in an elevator that is accelerating downward at the rate of 1.3 m/s^2.

a. What is the true weight of the man in newtons?

b. What is the net force acting upon the man required to produce the acceleration?

c. What is the force exerted on the man's feet by the floor of the elevator?

d. What is the apparent weight of the man in newtons? (This is the weight that would be read on the scale dial if the man were standing on a bathroom scale in the accelerating elevator.)

e. How would your answers to parts b through d change if the elevator were accelerating upward with an acceleration of 1.3 m/s^2?

SP7. A skydiver has a weight of 850 N. Suppose that the air-resistive force acting on the diver increases in direct proportion to his velocity such that for every 10 m/s that the diver's velocity increases, the force of air resistance increases by 100 N.

a. What is the net force acting upon the skydiver when his velocity is 30 m/s?

b. What is the acceleration of the diver at this velocity?

c. What is the terminal velocity of the skydiver?

d. What would happen to the velocity of the skydiver if, for some reason (perhaps a brief down draft), his velocity exceeded the terminal velocity? Explain.

Home Experiments and Observations

HE1. Collect a variety of small objects such as coins, pencils, keys, and bottle caps. Ice cubes, if they are available, also make excellent test objects. Try sliding these objects across a smooth surface such as a tabletop or floor, being as consistent as possible in the initial velocity that you give to them.

a. Do the objects slide the same distance after they leave your hand? What differences are apparent, and how are they related to the nature of the surface and size of the objects? Which objects come closest to demonstrating Newton's first law of motion?

b. What factors seem to be important in reducing the frictional force between the objects and the surface on which they are sliding? If you see some general principle at work, test this idea by finding other objects that would support your hypothesis.

HE2. Place a sheet of paper under a medium-sized book lying on a smooth tabletop or desktop.

a. Try to accelerate the book smoothly by exerting a constant pull on the sheet of paper. What happens if you try to accelerate the book too rapidly? Can you pull the paper cleanly from underneath the book without moving the book? Explain your observations in terms of Newton's laws of motion.

b. Repeat these observations with a few books in a stack. How does increasing the mass of the books affect the results?

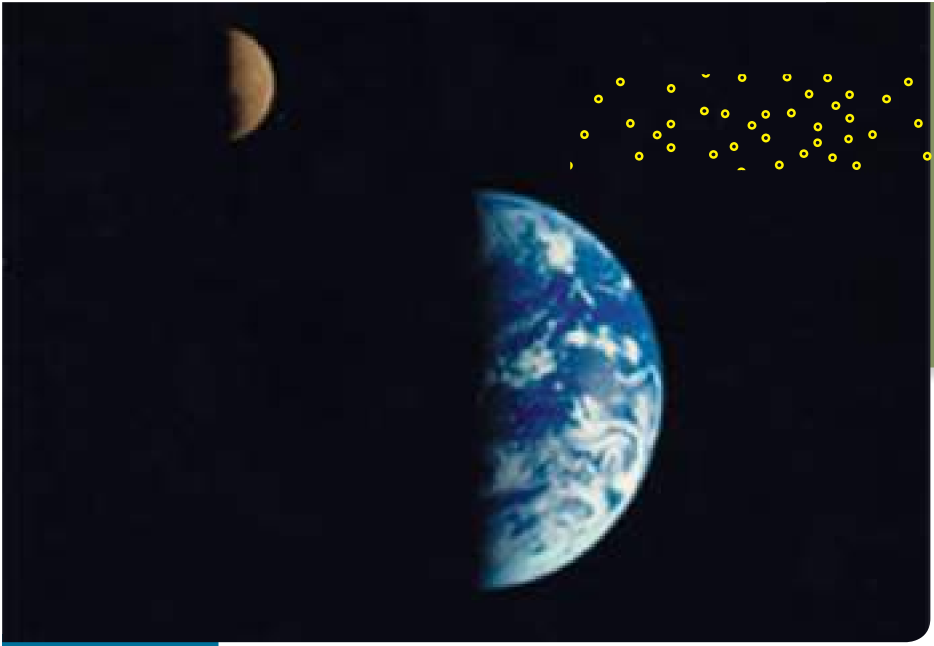
c. Try other objects. Which objects move the least when the paper is pulled rapidly?

HE3. Falling objects whose surface area is large relative to their weight will reach terminal velocity more readily than a ball or a rock. Test several objects, such as a balloon, small pieces of paper, plant parts (leaves, flowers, or seeds), or anything else you think might work. Do these objects reach a terminal velocity? How far does each object fall before reaching constant velocity? How does the rate of fall differ for different objects when dropped at the same time? Which of the objects tested produces the clearest demonstration of terminal velocity, showing first a brief acceleration followed by a constant velocity?

HE4. Using elevators in your dormitory or other campus buildings, observe the effects of the elevator's acceleration. Most elevators accelerate briefly as they start, and again as they stop (deceleration). Express elevators in high-rise buildings are best for observing the effects of acceleration.

a. If you have a bathroom scale, see how much your apparent weight differs from your true weight when the elevator is stopping or starting. Can you estimate the rate of acceleration from this information? (See everyday phenomenon box 4.2 and synthesis problem 6.)

b. Try holding your arm away from your body and maintaining it in this position as the elevator accelerates. How difficult is this to do for different conditions during the motion of the elevator? Explain your observations.



Source: NASA/JPL

CHAPTER 5

Circular Motion, the Planets, and Gravity

Chapter Overview

Using the example of a ball on a string, we first examine the acceleration involved in changing the direction of the velocity in circular motion (centripetal acceleration). Next, we consider the forces involved in producing a centripetal acceleration in different cases, including that of a car rounding a curve. Kepler's laws of planetary motion are then examined, and Newton's law of universal gravitation is introduced to explain the motion of the planets. We also show how this gravitational force relates to the weight of an object and the gravitational acceleration near the Earth's surface.

Chapter Outline

1. **Centripetal acceleration.** How can we describe the acceleration involved in changing the *direction* of an object's velocity? How does this acceleration depend on the object's speed?
2. **Centripetal forces.** What types of forces are involved in producing centripetal accelerations in different situations? What forces are involved for a car rounding a curve?
3. **Planetary motion.** How do the planets move around the sun? How has our understanding of planetary motion changed historically? What are Kepler's laws of planetary motion?
4. **Newton's law of universal gravitation.** What is the fundamental nature of the gravitational force, according to Newton? How does this force help to explain planetary motion?
5. **The moon and other satellites.** How does the moon orbit the Earth? How do the orbits of artificial satellites differ from those of the moon and from those of one another?

"The car failed to negotiate the curve." How many times have you seen a phrase like that in an accident report in the newspapers? Either the road surface was slippery or the driver was driving too fast for the sharpness of the curve. In either case, poor judgment and probably a poor sense of the physics of the situation were at work (fig. 5.1).

When a car goes around a curve, the direction of its velocity changes. A change in velocity means acceleration, and according to Newton's second law, an acceleration requires a force. The situation has much in common with a ball being twirled in a circle at the end of a string, as well as other examples of circular motion.

What forces keep a car moving around a curve? How does the force required depend on the speed of the car and the sharpness of the curve? What other factors are involved? Finally, what does the car rounding a curve have in common with the ball on a string and the motions of the planets around the sun?

The motions of the planets around the sun and the moon around the Earth played important roles in the development of Newton's theory of mechanics. Newton's law of universal gravitation was a crucial part of that theory. The gravitational force explains the behavior of objects falling near the Earth's surface, but it also explains why the planets move in curved paths about the sun. Circular motion is a very important special case of motion in two dimensions, both in the history of physics and in our everyday experience.

Figure 5.1 The white SUV apparently failed to negotiate the curve and slid into the oncoming lane. How do Newton's laws handle this situation?
eyecrave/Getty Images

5.1 Centripetal Acceleration

Suppose that we attach a ball to a string and twirl the ball in a horizontal circle (fig. 5.2). With a little practice, it is not hard to keep the ball moving with a constant speed, but the direction of its velocity changes continually. A change in velocity implies an acceleration, but what is the nature of this acceleration?

The key to this situation involves taking a careful look at what happens to the velocity vector as the ball moves in a circle. How does this vector change as the path of the ball changes direction?

Can we evaluate the size of this change and how it is related to the speed of the ball or the radius of the curve? To define the concept of *centripetal acceleration,* we need to answer these questions.

Figure 5.2 A ball being twirled in a horizontal circle. Is the ball accelerated?
Mark Dierker/McGraw-Hill Education

What is a centripetal acceleration?

What do we have to do to get the ball on the string to change its direction? If you try twirling a ball as pictured in figure 5.2, you will feel a tension in the string. In other words, you have to apply a force by pulling on the string to cause the change in direction of the ball's velocity.

What would happen if this force were not present? According to Newton's first law of motion, an object will continue moving in a straight line with constant speed if there is no net force acting on the object. If the string breaks, or if we let go of the string, this is exactly what will happen. The ball will fly off in the direction in which it was traveling when the string broke (fig. 5.3). Without the pull of the string, the ball will move in a straight line. It will also fall, of course, as it is pulled down by the gravitational force.

According to Newton's second law of motion, if there is a net force, there must be an acceleration ($\mathbf{F}_{net} = m\mathbf{a}$). Acceleration can be associated with the change in the magnitude of the velocity or the *direction* of the velocity, or both. In the case of the ball on the string, the string pulls the ball toward the center

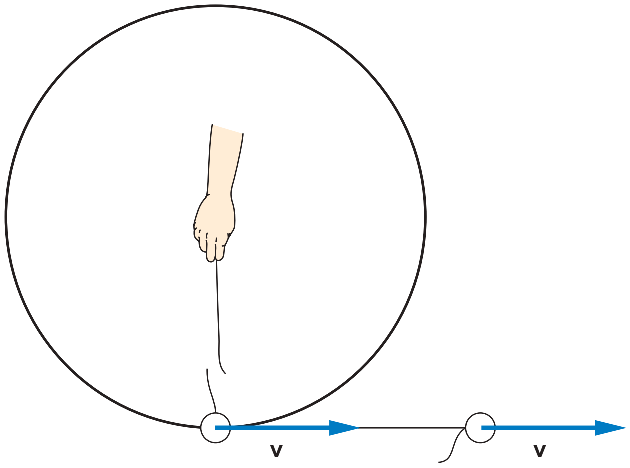


Figure 5.3 If the string breaks, the ball flies off in a straight-line path in the direction in which it was traveling at the instant the string broke.

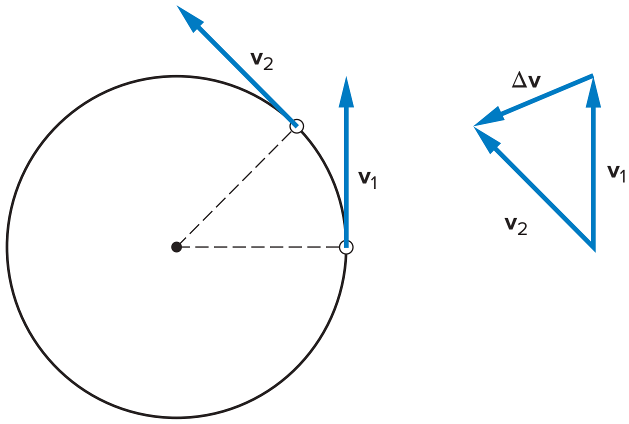


Figure 5.4 The velocity vectors for two positions of a ball moving in a horizontal circle. The change in velocity, $\Delta\mathbf{v}$, adds to $\mathbf{v}_1$ to yield $\mathbf{v}_2$.

of the circle, causing the direction of the velocity vector to change continually. The direction of the force, and of the acceleration it produces, is *toward the center* of the circle. We call this acceleration the **centripetal acceleration:**

> Centripetal acceleration is the rate of change in velocity of an object that is associated with the change in *direction* of the velocity. Centripetal acceleration is always perpendicular to the velocity vector itself and toward the center of the curve.

To find the size of the centripetal acceleration, we need to determine how fast the velocity is changing. You might guess that this depends on how rapidly you are twirling the ball, but it also depends on the radius of the curve—the size of the circle.

How do we find the change in velocity Δv?

Figure 5.4 shows the ball and string as seen from above. The ball is moving in a horizontal circle. Velocity vectors are drawn on the circle at two positions separated by a short time interval. The velocity $\mathbf{v}_2$ occurs a short time after the velocity $\mathbf{v}_1$, as the ball moves counterclockwise around the circle. These two vectors are drawn with the same length, indicating that the speed of the ball is unchanged.

The change in velocity, $\Delta\mathbf{v}$, is the difference between the initial velocity and the final velocity for a given time interval. In other words, the change in velocity is a vector that is added to the initial velocity to produce the final velocity. Adding $\Delta\mathbf{v}$ to $\mathbf{v}_1$ produces $\mathbf{v}_2$. This vector addition is shown in the vector triangle to the right of the circle in figure 5.4. (See appendix C for a discussion of vector addition by graphical methods.)

Note that the vector $\Delta\mathbf{v}$ has a direction different from that of either of the velocity vectors. If we choose a short enough time interval between the two positions, the direction of the *change* in velocity points toward the center of the circle, the direction of the instantaneous acceleration of the ball. (Acceleration always has the same direction as the change in velocity.) The ball is being accelerated toward the center of the circle, the direction of the tension in the string. This is consistent with Newton's second law from chapter 4. Recall that the acceleration is in the direction of the net force on an object.

What is the size of the centripetal acceleration?

How large is this centripetal acceleration, and how does it depend on the speed of the ball and the radius of the curve? The triangle illustrating the vector addition in figure 5.4 can be used to explore these questions. There are three effects to consider:

1. As the speed of the ball increases, the velocity vectors become longer, which makes $\Delta\mathbf{v}$ longer. The triangle in figure 5.4 becomes larger.
2. The greater the speed of the ball, the more rapidly the direction of the velocity vector changes, because the ball reaches the second position in figure 5.4 more quickly.
3. As the radius of the curve decreases, the rate of change in velocity increases, because the direction of the ball changes more rapidly. A tight curve (small radius) produces a large change, but a gentle curve (large radius) produces a small change.

The first two effects indicate that the rate of change in velocity will increase with an increase in the speed of the ball. Combining these two effects suggests that the centripetal acceleration should be proportional to the square of the speed. We need to multiply by the speed twice. The third effect suggests that the rate of change of velocity is inversely proportional to the radius of the curve. The larger the radius, the smaller the rate of change. Taken together, these effects produce the expression

$$a_c = \frac{v^2}{r}$$

for the size of the centripetal acceleration, $\mathbf{a}_c$. It is proportional to the square of the speed and inversely proportional to the radius, *r,* of the curve. The direction of the centripetal acceleration vector $\mathbf{a}_c$ is always toward the center of the curve, the direction of the change in velocity $\Delta\mathbf{v}$.

The ball moving in a circle is accelerated, even though its speed remains constant. To change the direction of the velocity vector is to change the velocity, and an acceleration is involved. People often resist this idea: We use the term *acceleration* in everyday language to describe increases in speed without taking into account changes in direction.

What force produces the centripetal acceleration?

Because an object moving in a circle is accelerated, a force must be acting to produce that acceleration, according to Newton's second law. For the ball on the string, the tension in the string pulling on the ball provides the centripetal acceleration. A closer look shows that this tension has both horizontal and vertical components, because the string is not completely within the horizontal plane. As shown in figure 5.5, the horizontal component of the tension pulls the ball toward the center of the horizontal circle and produces the centripetal acceleration.

The total tension in the string is determined by both the horizontal and the vertical components of the tension. The vertical component is equal to the weight of the ball, because the net force in the vertical direction should be zero. The ball stays in the horizontal plane of the circle and is not accelerated in the vertical direction. In example box 5.1, the weight of the ball is approximately 0.50 N ($\mathbf{W} = m\mathbf{g}$), so that becomes the value of the vertical component of the tension.

The ball in example box 5.1 has a slow speed. Even at this low speed, the horizontal component of the tension is larger than the vertical component. As the ball twirls at a faster rate, the centripetal acceleration increases even more rapidly, because it is proportional to the square of the speed of the ball. The horizontal component of the tension then becomes much larger than the vertical component, which remains equal to the weight of the ball (fig. 5.6). These effects can be readily observed with your own ball and string. Give it a try. You will feel the tension increase with increasing speed.

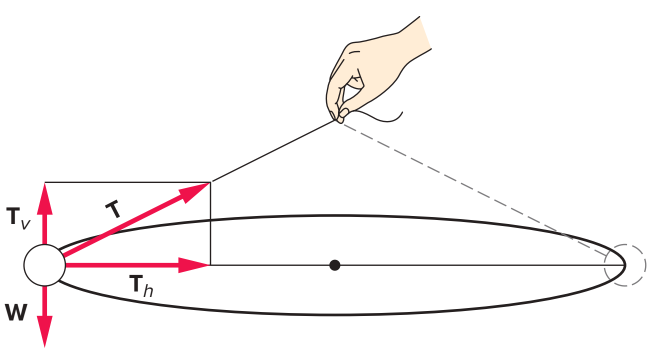


Figure 5.5 The horizontal component of the tension, $\mathbf{T}_h$, is the force that produces the centripetal acceleration. The vertical component of the tension, $\mathbf{T}_v$, is equal to the weight of the ball.

Example Box 5.1

Sample Exercise: Circular Motion of a Ball on a String

A ball has a mass of 50 g (0.050 kg) and is revolving at the end of a string in a circle with a radius of 40 cm (0.40 m). The ball moves with a speed of 2.5 m/s, or one revolution per second. (See fig. 5.5.)

a. What is the centripetal acceleration?
b. What is the horizontal component of the tension needed to produce this acceleration?

a. $v = 2.5$ m/s, $r = 0.40$ m, $a_c = ?$

$$a_c = \frac{v^2}{r} = \frac{(2.5\text{ m/s})^2}{(0.4\text{ m})} = \mathbf{15.6\ m/s^2}$$

b. $m = 0.05$ kg, $T_h = ?$

$$F_{\text{net}} = T_h = ma = (0.05\text{ kg})(15.6\text{ m/s}^2) = \mathbf{0.78\ N}$$

The horizontal component of the tension must equal 0.78 N in magnitude. The vertical component of the tension must equal the weight of the ball (0.50 N), as discussed in the text.

Centripetal acceleration involves the rate of change in the direction of the velocity vector. Its size is equal to the square of the speed of the object divided by the radius of the curve ($a_c = v^2/r$). Its direction is toward the center of the curve. Just as with any acceleration, there must be a force acting on the object to produce the centripetal acceleration. For a ball on a string, that force is the horizontal component of the tension in the string.

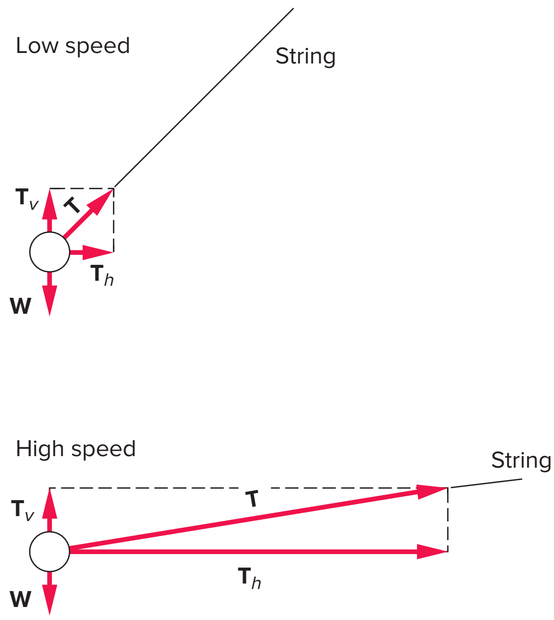


Figure 5.6 At higher speeds, the string comes closer to lying in the horizontal plane because a large, horizontal component of the tension is needed to provide the required centripetal force.

5.2 Centripetal Forces

For a ball twirled at the end of a string, the string pulls inward on the ball, providing the force that causes the centripetal acceleration. For a car rounding a curve, however, there is no string attached. Different forces must be at work to provide the centripetal acceleration. A person riding on a Ferris wheel also experiences circular motion. What forces produce centripetal acceleration in these situations?

The *net* force that produces a centripetal acceleration is often referred to as the **centripetal force.** This term is sometimes a source of confusion, because it implies that a special force is somehow involved. In fact, centripetal forces are *any* force, or combination of forces, that acts on an object in certain situations to *produce* the centripetal acceleration. Almost any force can play this role: pulls from strings, pushes from contact with other objects, friction, gravity, and so on. We need to analyze each situation separately to identify the forces and determine their effects.

What force helps a car negotiate a flat curve?

What forces are involved in producing the centripetal acceleration for a car rounding a curve? It depends on whether or not the curve is banked. The easiest situation in which to analyze when a curve is not banked is a flat road surface.

For a flat road surface, friction alone produces the necessary centripetal acceleration. The tendency of the car to move in a straight line causes the tires to pull against the pavement as the car turns. By Newton's third law, the pavement then pulls in the opposite direction on the tires (fig. 5.7).

Figure 5.7 The centripetal acceleration of a car rounding a level curve is produced by frictional forces exerted on the tires by the road surface.
Takeshi Takahara/Science Source

The frictional force acting on the tires points toward the center of the curve. If this force were not present, the car could not turn.

The size of a frictional force depends on whether or not there is motion along the surfaces of contact producing the friction. If there is no motion in the direction of the force, we call it the **static force of friction.** If the object is sliding, as it might on a wet or icy surface, the **kinetic force of friction** is involved. Usually, the kinetic force of friction is smaller than the maximum possible static force of friction, so whether or not the car is skidding becomes an important factor.

Unless the car has already begun to skid, the *static* force of friction produces the centripetal acceleration for the car rounding the curve. The part of the tire in contact with the road is momentarily at rest on the road; it does not slide along the road. If the tires do not move in the direction of the frictional force, the static force is in effect.

How large is the required frictional force? It depends on the speed of the car and the radius of the curve. From Newton's second law, we know that the magnitude of the required force is $F_{net} = ma_c$, where the centripetal acceleration a_c is equal to v^2/r. Putting these two ideas together, we see that the frictional force f must be equal to mv^2/r, because it is the only force operating to produce the centripetal acceleration. The speed of the car is a critical factor in determining how large a force is needed, which is why we often slow down when approaching a curve.

If the mass times the centripetal acceleration is greater than the maximum possible frictional force, we are in trouble. Because the square of the speed is involved in this relationship, doubling the speed would require a frictional force four times as large as that for the lower speed. Also, because we cannot control the frictional force, a sharper curve with a smaller radius r requires a lower speed. Both the speed and the radius must be considered in making driving judgments.

What happens if the required centripetal force is larger than the maximum possible frictional force? The frictional force cannot produce the necessary centripetal acceleration, and the car begins to skid. Once it is skidding, kinetic friction rather than static friction comes into play. Because the force of kinetic friction is generally smaller than that of static friction, the frictional force decreases and the skid gets worse. The car, like the ball on the broken string, follows its natural tendency to move in a straight line.

The maximum possible value of the frictional force is dictated by the road and tire conditions. Any factor that reduces the force of static friction will cause problems. Wet or icy road surfaces are the usual culprits. In the case of ice, the force of friction may diminish almost to zero, and an extremely slow speed will be necessary to negotiate a curve. There is nothing like driving on an icy road to give you an appreciation of the value of friction. Newton's first law is illustrated vividly. (See also everyday phenomenon box 5.1.)

What happens if the curve is banked?

If the road surface is properly banked, we are no longer totally dependent on friction to produce the centripetal acceleration. For the banked curve, the normal force between the car's tires and the road surface can also be helpful (fig. 5.8). As discussed in chapter 4, the **normal force, N,** is always perpendicular to the surfaces involved, so it points in the direction shown in the diagram. The total normal force acting on the car (indicated in the diagram) is the sum of those for each of the four tires.

Because the car is not accelerated vertically, the net force in the vertical direction must be zero. The vertical component of the normal force $\mathbf{N}_v$ must be equal in magnitude to the weight of the car to yield a net vertical force of zero. This fact determines how large the normal force will be. Only the horizontal component of the normal force $\mathbf{N}_h$ is in the appropriate direction to produce the centripetal acceleration.

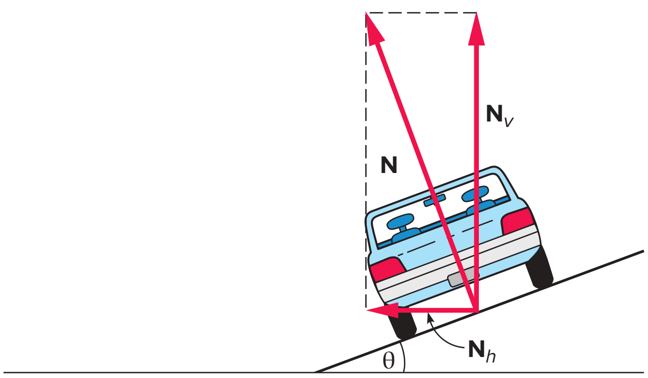


Figure 5.8 The horizontal component of the normal force $\mathbf{N}_h$ exerted by the road on the car can help to produce the centripetal acceleration when the curve is banked.

The angle of the banking and the weight of the car determine the size of the normal force. They also determine the size of its horizontal component. At the appropriate speed, this horizontal component pushing on the tires of the car is all that is needed to provide the centripetal acceleration. The higher the speed, the steeper the required banking angle, because a steeper angle produces a larger horizontal component for the normal force. Fortunately, because both the normal force and the required centripetal force are proportional to the mass of the car, the same banking angle will work for vehicles of different mass.

A banked curve is designed for a particular speed. Because friction is also usually present, the curve can be negotiated at a range of speeds above and below the intended speed. Friction and the normal force combine to produce the required centripetal acceleration.

If the road is icy and there is no friction, the curve can still be negotiated at the intended speed. Speeds higher than that speed will cause the car to fly off the road, just as on a flat road surface. Speeds too low, on the other hand, will cause the car to slide down the icy banked incline toward the center of the curve.

What forces are involved in riding a Ferris wheel?

Riding a Ferris wheel is another example of circular motion that many of us have experienced. On a Ferris wheel, the circular motion is vertical, unlike the horizontal circles of our previous examples.

Figure 5.9 shows the forces exerted on the rider at the bottom of the circle as the Ferris wheel turns. At this point in the ride, the normal force acts upward and the weight downward. Because the centripetal acceleration of the rider is directed upward, toward the center of the circle, the net force acting on the rider must also be upward. In other words, the normal force of the seat pushing on the rider must be larger than the weight of the rider.

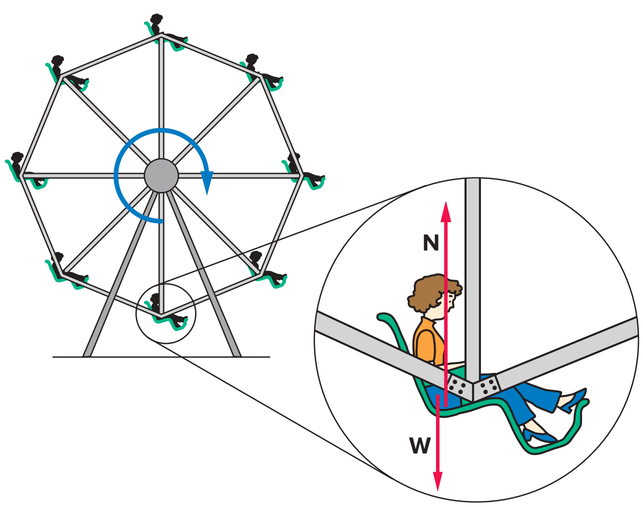


Figure 5.9 At the bottom of the cycle, the weight of the rider and the normal force exerted by the seat combine to produce the centripetal acceleration for a rider on a Ferris wheel.

Everyday Phenomenon Box 5.1

Seat Belts, Air Bags, and Accident Dynamics

The Situation. In automobile accidents, serious or fatal injuries are often the result of riders' being thrown from the vehicle. Since the 1960s, federal regulations have required that cars be equipped with seat belts. Since 1998, front-seat air bags have also been required in an effort to reduce the carnage. Still, we often read of people being thrown from their vehicle in accident reports.

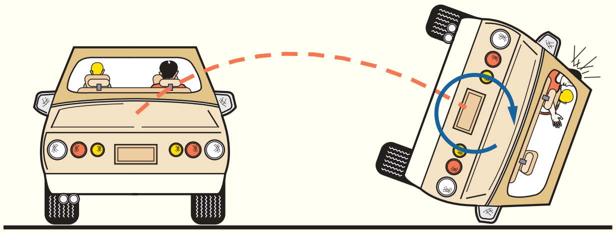
How do air bags and seat belts help? If your car is equipped with air bags, as most now are, is it still necessary to wear your seat belt? In what situations are air bags most effective, and when are seat belts essential?

The Analysis. Except in high-speed collisions where the passenger compartment of the vehicle is crushed, most injuries and fatalities are caused by motion of the rider within, and outside of, the vehicle. The vehicle stops or turns suddenly due to the collision and the rider continues to move in a straight line, following Newton's first law of motion.

In a head-on collision, the car stops while the rider continues to move forward unless constrained. In the absence of either seat belts or air bags, front-seat riders hit the windshield or the steering column, resulting in serious head or chest injuries. Seat belts can prevent this when used properly, but air bags are also designed to protect against these injuries. As the rider begins to move forward relative to the vehicle, the air bag inflates rapidly, providing a cushion between the rider and other objects in the car. The rider then decelerates more gradually, involving a smaller force and less trauma. (This idea is best understood in terms of the concept of impulse, discussed in chapter 7.) Air bag usage has resulted in a significant reduction in serious head and chest injuries in head-on collisions with other vehicles or with fixed objects.

Head-on collisions are not the most frequent type of serious accident, however. Rollover accidents involving single vehicles are common, and vehicles can also collide in intersections, providing impacts

In a head-on collision, the air bag inflates rapidly to prevent the rider from moving forward and colliding with the windshield or steering column.
fStop Images GmbH/Alamy Stock Photo



As the vehicle rolls, a rear-seat passenger is thrown against the side of the vehicle (viewed from the back). A properly adjusted seat belt and shoulder harness can prevent this.

to the side of the car. In the latter case, the struck vehicle will often go into a spin. In both of these cases, the vehicle undergoes rotational motion while the rider moves forward in a straight line.

These are the accidents in which the rider is likely to be thrown from the vehicle.

Will air bags help in these situations? Air bags are most effective in head-on collisions and do not provide much protection against sideways motion of the rider. (Some newer vehicles do come equipped with air bags in the front-seat doors, which can protect against sideways movement, but air bags are not usually provided for the rear seats.) In a rollover accident, the vehicle goes into a spin about an axis through its long dimension. The doors will sometimes open or the windows will shatter during the first roll, providing openings for the rider to fly through as he or she continues to move forward while the vehicle turns. In some cases, the rider is thrown from the vehicle and the vehicle then rolls over the victim.

Seat belts can make a big difference. Because the vehicle is turning rapidly in a rollover accident, a centripetal force acting on the rider is necessary to hold the rider against the seat and keep him or her from moving forward in a straight line. In the absence of such a force, the rider is thrown outward against the sides of the vehicle. Attempts by riders to brace themselves are usually totally inadequate to provide the required centripetal force. The seat belt and shoulder harness, on the other hand, can provide the force necessary to hold the rider in place.

Statistics on accident fatalities are compelling. In rollover accidents, riders who are wearing their seat belts generally survive, while those who are not using their belts and shoulder harnesses are frequently killed or seriously injured. Often, those killed are thrown from the vehicle, but even when riders remain inside the vehicle, trauma from being thrown around inside the vehicle can be fatal. Statistics indicate that a high percentage of the deaths in rollover accidents involve riders ejected from the vehicle.

Newton's first law of motion is vividly illustrated in automobile accidents. An object keeps moving in a straight line with constant speed unless acted upon by an external force. Air bags and seat belts can provide that force, but seat belts provide better protection for all passengers in rollover accidents.

According to Newton's second law, the net force must be equal to the mass times the centripetal acceleration. In this case, the centripetal force is the difference of two forces, the upward normal force and the downward weight of the rider, so

$$F_{net} = N - W = ma_c$$

Because the normal force is larger than her weight, she feels heavy in this position ($N = W + ma_c$). The situation is similar to that in an upward accelerating elevator (see everyday phenomenon box 4.2).

As the rider moves up or down along the sides of the circle, a horizontal component of the normal force is needed to provide the centripetal acceleration. This horizontal component may be provided by the frictional force exerted by the seat on the rider, by the seat back pushing on the rider on the left side of the cycle, or by a seat belt or handbar on the right side of the cycle. The latter case is more exciting.

At the top of the cycle, the weight of the rider is the only force (other than a possible seat-belt force) in the appropriate direction to produce the centripetal acceleration. Again, from Newton's second law, the net force must equal the mass of the rider times the centripetal acceleration, which is now directed downward. This yields the relationship

$$F_{net} = W - N = ma_c$$

As the speed gets larger and the centripetal acceleration, $a_c = v^2/r$, increases, the normal force must get smaller to increase the total force. Usually, the top speed of the Ferris wheel is adjusted, so that the normal force is small when the rider is at the top of the cycle. Because the force exerted by the seat on the rider is small, the rider feels light, part of the thrill of the ride.

If there is one nearby, take a break and go ride a Ferris wheel. There is nothing like direct experience to bring home the ideas we have just described. As you ride, try to sense the direction and magnitude of the normal force. The light feeling at the top and the sense of plunging outward in the downward portion of the cycle are what the price of the ride is all about.

> A centripetal force is any force or combination of forces that produces the centripetal acceleration for an object moving around a curve. In the case of a car negotiating a curve on a flat road surface, the centripetal force is provided by friction. If the road surface is banked, the normal force of the road pushing on the tires of the car also helps. In the case of a Ferris wheel, the weight of the rider and the normal force exerted by the seat on the rider combine to provide the centripetal force. We use Newton's laws of motion to identify the forces and analyze each situation.

5.3 Planetary Motion

Have you ever watched Venus or Mars in the night sky and wondered how and why their positions change from night to night? From the standpoint of the history of science, the motions of the planets are the most important examples of centripetal acceleration. These objects are a part of our everyday experience, yet many of us are surprisingly unaware of how they move. How do the sun, the stars, and the planets move? How can we make sense of the motions?

Early models of the heavens

Observing the heavens was probably a more popular pastime when there were fewer roofs over our heads. If you have ever spent a night in a sleeping bag under the stars, you probably experienced a sense of wonder and amazement at all of those bright objects out there. If you spent night after night observing the stars, you might have noticed, as the ancients did, that some of the brightest objects move relative to the other stars.

These wanderers are the planets. The so-called fixed stars always maintain the same position relative to one another as they move across the sky (fig. 5.10). The Big Dipper never seems to change its shape, but the planets roam about with respect to the fixed stars in a regular but curious fashion. Their motions excited the curiosity of ancient observers of the heavens. They were carefully tracked and often incorporated into religious and cultural beliefs.

Suppose you are an early philosopher-scientist trying to make sense of these motions. What kind of model might you develop? Some features seem simple and regular. The sun, for example, moves across the sky each day, from east to west, as if it were at the end of an enormously long and invisible rope tethered at the center of the Earth. The stars

Figure 5.10 A time-lapse photograph showing the apparent motion of stars in the northern sky. Polaris (the "North Star") lies near the center of the pattern and does not appear to move very much. The entire pattern appears to rotate during the night around a point near Polaris. *Photo by Vincent Ting/Moment/Getty Images*

follow a similar pattern. Their apparent motion as seen from Earth could be explained by picturing them as lying on a giant sphere that revolved around the Earth. This Earth-centered, or *geocentric,* view of the universe seemed natural and reasonable.

The moon also moves across the sky in an apparently circular orbit around the Earth. Unlike the stars, the moon does not reappear in the same position each night. Instead, it goes through a series of regular changes in position and phase in a cycle of approximately 30 days. How many of us can provide a clear explanation of the phases of the moon? The motion of the moon will be considered more fully in section 5.5.

Early models of the motions of the heavenly bodies developed by Greek philosophers involved a series of concentric spheres centered on the Earth. Plato and others of his time viewed spheres and circles as ideal shapes that would reflect the beauty of the heavens. The sun, the moon, and the five planets known then each had its own sphere. The fixed stars were on the outermost sphere. These spheres were thought to revolve around the Earth in ways that explained the positions of the heavenly bodies.

Unfortunately, the planets do not behave as though they were on a continuously revolving sphere. The planets sometimes appear to move backward relative to their normal direction of motion against the background of the fixed stars. We call this **retrograde motion.** It takes a few months for Mars to trace one of these retrograde patterns (fig. 5.11).

To explain the apparent retrograde motion of these planets, Ptolemy (Claudius Ptolemaeus), working in the second century A.D., devised a more sophisticated model than the one used by earlier Greek philosophers. Ptolemy's model used circular orbits rather than spheres but was still geocentric. He invented the idea of **epicycles,** circles that rolled along the larger basic orbit of the planet around the Earth (fig. 5.12). The epicycles accounted for the retrograde motion and could be used to explain other irregularities in planetary orbits.

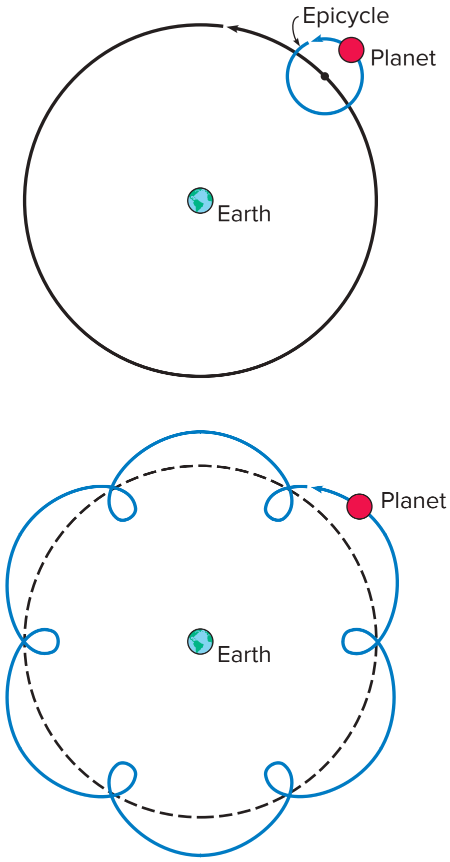


Figure 5.12 Ptolemy's epicycles were circles rolling along the circular orbits of the planets. This model explained the retrograde motion observed for the outer planets.

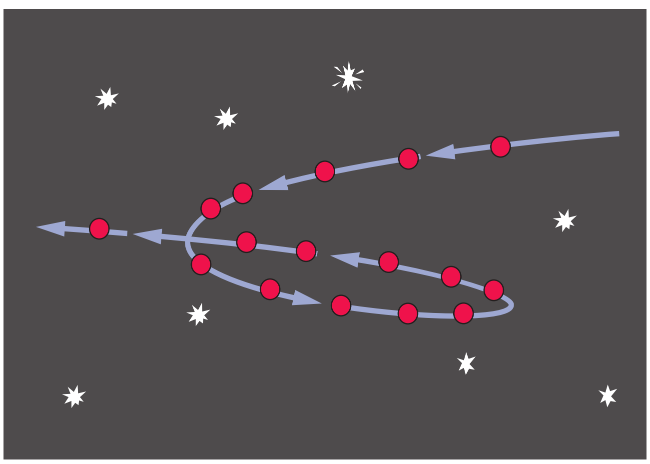


Figure 5.11 An example of the retrograde motion of Mars relative to the background of fixed stars. These changes take place over a period of several months.

Ptolemy's model accurately predicted where to find the planets at any given time of any year. As more accurate observations became available, refinements were needed, however, to improve the predictions. In some cases, this meant adding epicycles to epicycles, but the basic scheme of circles was retained. Ptolemy's system became part of the accepted knowledge during the Middle Ages and was incorporated, along with many of Aristotle's works, into the teachings of the Roman Catholic Church and the emerging European universities.

How did the Copernican model differ from Ptolemy's conception?

Ptolemy's model is not the one you were introduced to in elementary school. It has been superseded. During the sixteenth century, a Polish astronomer, Nicolaus Copernicus (1473–1543), put forth a sun-centered, or **heliocentric,** view, later championed by Galileo. Copernicus was not the first to suggest such a model, but earlier heliocentric versions had not taken hold. Copernicus spent many years working out the details of his model, but he did not publish it until within a year of his death.

Galileo was an early advocate of the Copernican model and promoted it more vigorously than Copernicus himself. In 1610, hearing of the invention of the telescope, Galileo built his own improved version and turned it to the heavens. He discovered that the moon has mountains, that Jupiter has moons, and that Venus goes through phases like our moon. He showed that the phases of Venus could be explained better by the Copernican model than by a geocentric model. Galileo became famous throughout Europe for his discoveries and ended up in trouble with church authorities, a problem not to be taken lightly in his day. People had been burned at the stake for similar offenses.

Copernicus placed the sun at the center of the circular orbits of the planets and demoted the Earth to the status of just another planet. Also, the Copernican model requires that the Earth *rotate* on an axis through its center—thus explaining the daily motions of the sun and the other heavenly bodies (including the fixed stars). This idea was revolutionary at the time. Why are we not blown away by the enormous winds that rotation might produce? Perhaps the air near the Earth's surface is dragged along with the Earth.

The advantage of the Copernican view is that it does not require complicated epicycles to explain retrograde motion, although epicycles were still used to make other adjustments to planetary orbits. Retrograde motion comes about because the Earth is orbiting the sun along with the other planets. The position of Mars appears to change as *both* Mars and Earth move in the same direction against the background of the fixed stars (fig. 5.13). As the more rapidly moving Earth passes Mars, Mars slips behind and briefly appears to move backward.

Accepting the Copernican model meant giving up the Earth-centered view of the universe to endorse what seemed to some an absurd proposition: that the Earth rotates, with a frequency of one cycle per day. Because an approximation of the radius of Earth was known (6400 km), rotation implied that we must be moving at roughly 1680 km/h (or just over 1000 MPH) if we are standing near the equator on the Earth! We certainly do not feel that motion.

Because Copernicus assumed the planets' orbits to be circular, the accuracy of his model for predicting was no better than Ptolemy's model. In fact, it required some adjustments (for which Copernicus used epicycles) just to make it agree with already known astronomical data. Settling the controversy generated by the competition between the two models called for more accurate observations, a project undertaken by a Danish astronomer, Tycho Brahe (1546–1601).

Tycho was the last great naked-eye astronomer. He developed a large quadrant (fig. 5.14), which he used to make very accurate sightings of the positions of the planets and stars. It was capable of measuring these positions to an accuracy of $^1/_{60}$ of a degree, considerably better than the accuracy of previously available data. Tycho spent several years painstakingly collecting data on the precise positions of the planets and other bodies—all without the benefit of a telescope.

Kepler's laws of planetary motion

Analyzing the data collected by Tycho fell to his assistant, Johannes Kepler (1571–1630), after Tycho's death. It was an enormous task requiring the transformation of the data to coordinates around the sun and then numerical trial and error to find regular planetary orbits. It was already known that these orbits are not perfect circles. Kepler was able to show that the orbits of the planets around the sun are ellipses, with the sun at one focus.

An **ellipse** can be drawn by attaching a string between two fixed foci and then moving a pencil around the perimeter of the path allowed by the string (fig. 5.15). A circle is a special case of an ellipse in which the two foci coincide. The orbits of most of the planets are very close to being circles, but Tycho's data were so precise that they

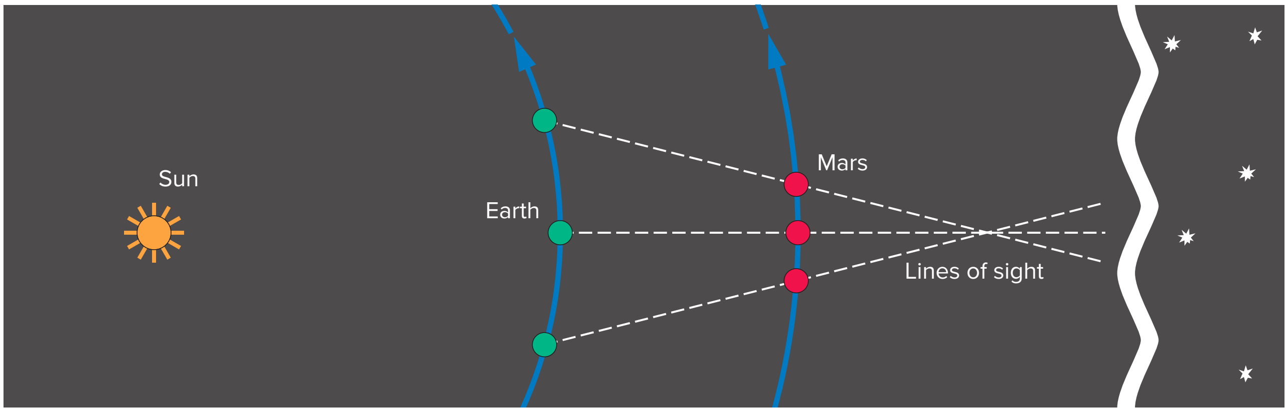


Figure 5.13 As the Earth passes the more slowly moving Mars, Mars appears to move backward as seen against the background of the much more distant fixed stars. (Not drawn to scale.)

Figure 5.14 Tycho Brahe's large quadrant permitted accurate measurement of the positions of the planets and other heavenly bodies. *Photo Josse/Leemage/Getty Images*

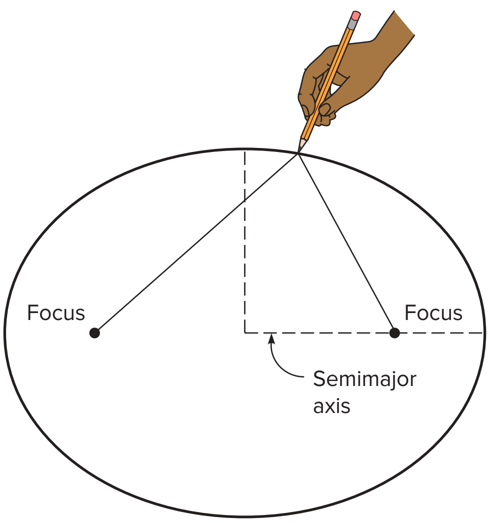


Figure 5.15 An ellipse can be drawn by fixing a string at two points (foci) and moving a pencil around the path permitted by the string.

showed a difference between a perfect circle, on one hand, and an ellipse with two closely spaced foci. Kepler's first law of planetary motion states that the orbits of the planets are ellipses.

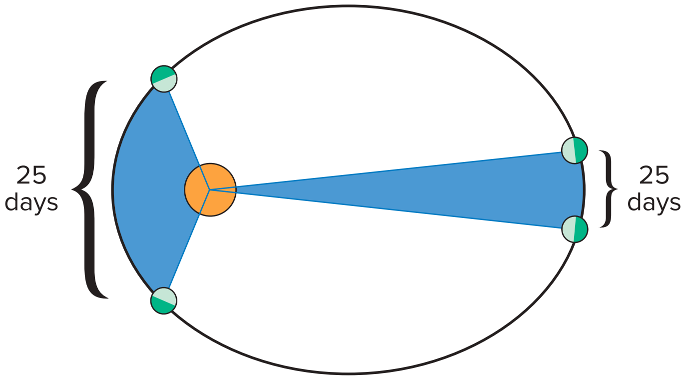


Figure 5.16 Because planets move faster when nearer to the sun, the radius line for each planet sweeps out equal areas in equal times (Kepler's second law). In other words, the two blue sections each cover the same span of time and have the same area. (Not drawn to scale.)

Kepler's other two laws of planetary motion came after even more laborious numerical trial and error with Tycho's data. Kepler's second law describes how the planets move faster when they are nearer to the sun, so that an imaginary line drawn from the sun to the planet moves through equal areas in equal times regardless of where it is in its orbit (fig. 5.16).* The first two laws were published in 1609.

The third law (published in 1619) states a relationship between the average radius of the orbit and the time taken for one complete cycle around the sun (the **period** of the orbit). Kepler found his third law after trying many other possible relationships between the periods, T, and the average radii of the planetary orbits, r, all without the use of a computer! To a high degree of accuracy, he found that the ratio of the square of the period to the cube of the radius (T^2/r^3) was the same for all of the known planets (see example box 5.2). The behavior of the planets is surprisingly regular. Kepler published his findings in papers that also contained elaborate speculations on numerical mysticism and musical harmonies associated with the planets. Some of these ideas must have seemed strange to Galileo and others who admired Kepler's work.

Kepler's laws added to the accuracy with which we can predict the positions of the planets as they appear to wander among the fixed stars. Like the Copernican model, Kepler's model was heliocentric (sun-centered), so it supported Galileo's efforts to overthrow the geocentric (Earth-centered) model of Ptolemy. More important, however, Kepler's laws described a new set of precisely stated relationships that called for explanation. The stage was set for Isaac Newton to incorporate these relationships into a grand theory that

*The second law turns out to be a consequence of conservation of angular momentum, which is discussed in chapter 8.

Example Box 5.2

Sample Exercise: Using Kepler's Third Law

How long does it take Mars to complete one orbit around the sun? The distance of Mars from the sun is approximately 1.5 astronomical units (AU). (An AU is the average distance from the Earth to the sun; thus, the radius of the Earth's orbit, r_{Earth}, is just 1 AU.)

$r_{\text{Earth}} = 1$ AU = the distance from Earth to the sun

$r_{\text{Mars}} = 1.5$ AU = the distance from Mars to the sun

$T_{\text{Earth}} = 1$ Earth year (yr)

$T_{\text{Mars}} = ?$ (in Earth years)

Kepler's third law for this case can be stated as

$$\frac{r^3_{\text{Mars}}}{T^2_{\text{Mars}}} = \frac{r^3_{\text{Earth}}}{T^2_{\text{Earth}}}$$

Cross multiplying, we find $\left(r^3_{\text{Earth}}\right)\left(T^2_{\text{Mars}}\right) = \left(r^3_{\text{Mars}}\right)\left(T^2_{\text{Earth}}\right)$.

Therefore,

$$T^2_{\text{Mars}} = \frac{\left(r^3_{\text{Mars}}\right)\left(T^2_{\text{Earth}}\right)}{r^3_{\text{Earth}}}.$$

Inserting the values for the planets listed above,

$$T^2_{\text{Mars}} = \frac{\left((1.5\ \text{AU})^3\right)\left((1\ \text{yr})^2\right)}{(1\ \text{AU})^3},$$

$T^2_{\text{Mars}} = 3.4\ \text{yr}^2$ (notice that the AU units cancel)

$T_{\text{Mars}} = \sqrt{3.4\ \text{yr}^2} \approx$ **1.8 Earth years**

explains both celestial mechanics (the motion of the heavenly bodies) and the more mundane motion of everyday objects near the Earth's surface.

Kepler's Laws of Planetary Motion

1. The planets all move in elliptical orbits about the sun, with the sun located at one focus of the ellipse.
2. An imaginary line drawn from the sun to any planet moves through equal areas in equal intervals of time.
3. If T is the amount of time taken for the planet to complete one full orbit around the sun (*period*) and if r is the average radius of the distance of the orbit around the sun for each planet, then the ratio of the square of the period to the cube of the radius (T^2/r^3) is the same for all of the known planets.

Many of the early models for describing the motion of the planets were geocentric (Earth-centered). Ptolemy's model included epicycles to explain the apparent retrograde motion of the planets. Copernicus introduced a heliocentric (sun-centered) model, which explained retrograde motion more simply. This model was championed by Galileo. Galileo was one of the first scientists to use a telescope systematically, and he made significant discoveries supporting the heliocentric view. Kepler refined the heliocentric model by showing that planetary orbits are ellipses with some surprising regularities.

5.4 Newton's Law of Universal Gravitation

Planetary motion and centripetal acceleration lead us to the next question. If the planets are moving in curved paths around the sun, what force must be present to produce the centripetal acceleration? You are probably aware that gravity is involved, but that involvement was not at all obvious when Newton began his work. How did Newton put it all together?

What was Newton's breakthrough?

Newton realized that there is a similarity between the motion of a projectile launched near the Earth's surface and the orbit of the moon. To illustrate this point, Newton produced a famous drawing similar to that shown in figure 5.17.

Figure 5.17 In a diagram similar to this, Newton imagined a projectile fired from an incredibly high mountain. If fired with a large enough horizontal velocity, the projectile falls toward the Earth but never gets there.

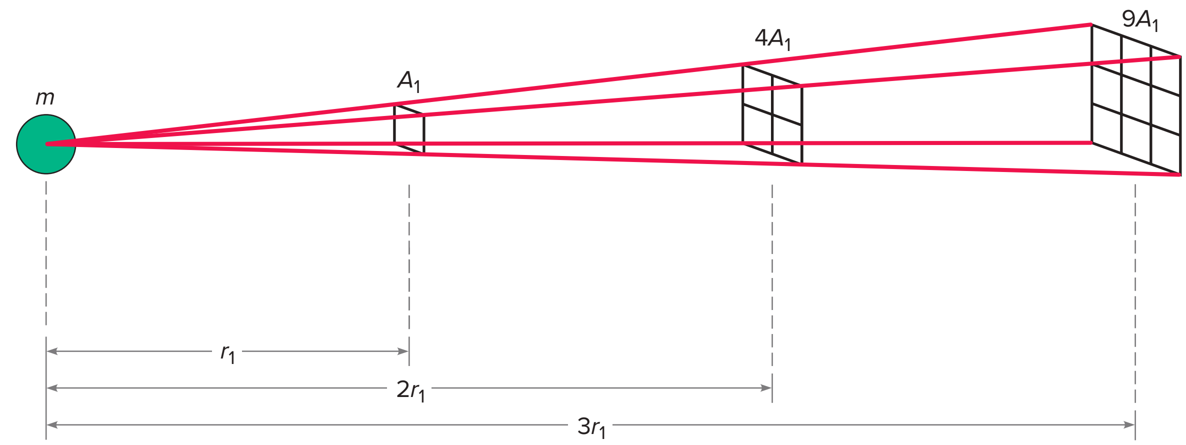


Figure 5.18 If lines are drawn radiating outward from a point mass, the areas intersected by these lines increase in proportion to r^2. Does this suggest that the force exerted by the mass on a second mass might become weaker in proportion to $1/r^2$?

The idea is simple but earthshaking. Imagine, as Newton did, a projectile being launched horizontally from an incredibly high mountain. The larger the launch velocity, the farther away from the base of the mountain the projectile will land. At very large launch velocities, the curvature of the Earth becomes a significant factor. In fact, if the launch velocity is large enough, the projectile will never reach the Earth's surface. It keeps falling, but the curvature of the Earth falls away, too. The projectile goes into a circular orbit around the Earth.

Newton's insight was that the moon, under the influence of gravity, is actually falling, just as a projectile does. The moon, of course, is at a distance from the Earth much greater than the height of any mountain. The same force that accounts for the acceleration of objects near the Earth's surface, as described by Galileo, explains the orbit of the moon.

Newton's explanation of Kepler's laws

From Galileo's work, Newton knew that near the Earth's surface the gravitational force is proportional to the mass of the object, $\mathbf{F} = m\mathbf{g}$. Mass, then, should be involved in any more general expression for the gravitational force.

Does the gravitational force vary with distance, though, and, if so, how? The idea that a force could influence two masses separated by a large distance was hard to accept in Newton's day (and, in some ways, is hard to accept even now). If such a force exists, we would expect that this force "acting at a distance" would decrease in strength as the distance increased. Using geometrical reasoning (fig. 5.18), other scientists had speculated that the force might be inversely proportional to the square of the distance r between the masses, but they could not prove it.

At this point, Kepler's laws of planetary motion and the concept of centripetal acceleration came into play. Newton was able to prove mathematically that Kepler's first and third laws of planetary motion could be derived from the assumption that the gravitational force between the planets and the sun falls off with the inverse square of the distance. The proof involved setting the assumed $1/r^2$ force equal to the required centripetal force in Newton's second law of motion. All of Kepler's laws are consistent with this assumption.

The proof that Kepler's laws could be explained by a gravitational force proportional to the masses of two interacting objects, and inversely proportional to the square of the distance between the objects, led to **Newton's law of universal gravitation.** This law and Newton's three laws of motion are the fundamental postulates of his theory of mechanics. The law of gravitation can be stated as follows:

The gravitational force between two objects is proportional to the mass of each object and inversely proportional to the square of the distance between the centers of the masses:

$$F = \frac{Gm_1m_2}{r^2}$$

where G is a constant. The direction of the force is attractive and lies along the line joining the centers of the two masses (fig. 5.19).

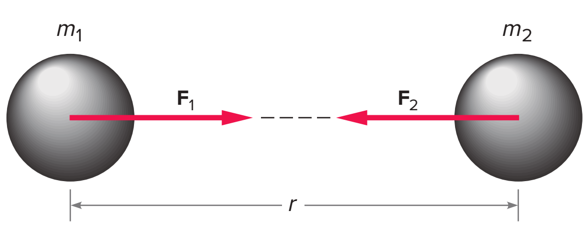


Figure 5.19 The gravitational force is attractive and acts along the line joining the center of the two masses. It obeys Newton's third law of motion ($\mathbf{F}_2 = -\mathbf{F}_1$).

For this statement to be completely valid, the masses in question must be either point masses or perfect spheres.

In Newton's law of gravitation, G is the **universal gravitational constant.** It has the same value for any two objects. Newton did not actually know the value of this constant, because he did not know the masses of the Earth, the sun, and the other planets. Its value was determined more than a hundred years later in an experiment done by Henry Cavendish (1731–1810) in England. Cavendish measured the very weak gravitational force between two massive lead balls for different distances of separation. In metric units, the value of G is

$$G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$$

The power-of-10 notation (see appendix B) is useful here, because G is a very small number. The power −11 means that the decimal point is located 11 places to the left of where it is shown. If we did not use power-of-10 notation, the number would appear as

$$G = 0.000\ 000\ 000\ 066\ 7\ \text{N·m}^2/\text{kg}^2$$

Because of the small size of this constant, the gravitational force between two ordinary-sized objects, such as people, is extremely small and not usually noticeable. Cavendish's experiment required ingenuity to measure such a weak force.

How is weight related to the law of gravitation?

Suppose that one of the masses is a planet or other very large object. The force of gravity then can be quite large because one of the masses is very large. Consider the force exerted on a person standing on the surface of the Earth. As figure 5.20 illustrates, the distance between the centers of the two objects, the person and the Earth, is essentially the radius of the Earth, r_e.

From Newton's law of gravitation, the force on the person must be $F = Gmm_e/r_e^2$, where m is the mass of the person and m_e is the mass of Earth. Because this gravitational force is the weight of the person, we can also express the force as $F = W = mg$. For these two expressions for F to be the same, g, the gravitational acceleration, must be related to the universal gravitational constant G by $g = Gm_e/r_e^2$.

The gravitational acceleration near the Earth's surface g is therefore *not* a universal constant. It will be different on different planets and even slightly different at different points on the Earth because of variations in the radius of the Earth and other factors. The constant G *is* a universal constant of nature that can be used to find the gravitational acceleration for any planet if we know the radius and mass of the planet.

If we know the gravitational acceleration near the Earth's surface, it is easier to use the expression $\mathbf{F} = m\mathbf{g}$ to compute a weight than to use the law of universal gravitation. This computation is done both ways in the sample exercise in example box 5.3. Either way, we get the same result. (When dealing with weight, we will use $g = 9.8\ \text{m/s}^2$, instead of the approximation of $10\ \text{m/s}^2$, to obtain more accurate values.) The weight of the 50-kg person is approximately 490 N. The mass of Earth, 5.98×10^{24} kg, is a very large number that was first determined by Cavendish when he measured the universal constant G. In a sense, Cavendish weighed the Earth by making that measurement.

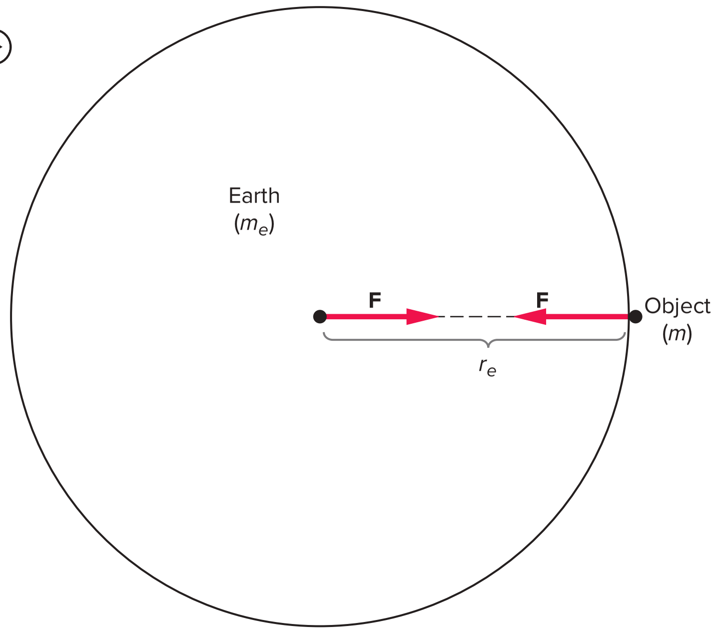


Figure 5.20 For the Earth and an object near the Earth's surface, the distance between the centers of the two objects is equal to the radius of the Earth.

Example Box 5.3

Sample Exercise: Gravity, Your Weight, and the Weight of the Earth

The mass of the Earth is 5.98×10^{24} kg, and its average radius is 6370 km. Find the gravitational force (the weight) of a 50-kg person standing on Earth's surface

a. By using the gravitational acceleration.
b. By using Newton's law of gravitation.

a. $m = 50$ kg $F = W = mg$
$g = 9.8\ \text{m/s}^2$ $= (50\ \text{kg})(9.8\ \text{m/s}^2)$
$F = ?$ $= \mathbf{490\ N}$

b. $m_e = 5.98 \times 10^{24}$ kg
$r_e = 6.37 \times 10^6$ m
$F = W = Gmm_e/r_e^2$

$$= \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)\ (50\ \text{kg})\ (5.98 \times 10^{24}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2}$$

$= \mathbf{490\ N}$

Most scientific calculators will handle the scientific notation directly. The powers add for multiplication and subtract in division.

If we wanted to know the gravitational force exerted on a 50-kg person in a space capsule several hundred kilometers above the Earth, we would have to use the more general expression in Newton's law of gravitation. Likewise, if we wanted to know the weight of this person when standing on the moon, we would need to use the mass and radius of the moon in place of those of the Earth in our calculation. The weight of a 50-kg person on the moon is only about one-sixth the value of 490 N that we computed for the same person standing on Earth. The expression $F = mg$ is valid *only* near the surface of the Earth.

The weaker gravitational force and acceleration of the moon are explained by the moon's smaller mass. Because our muscles are adapted to conditions on Earth, we would find that our smaller weight on the moon makes some amazing leaps and bounds possible. The smaller gravitational force on objects near the moon's surface also explains why the moon has essentially no atmosphere. Gas molecules escape the gravitational pull of the moon much more easily than that of Earth.

> Newton recognized that the moon is falling toward the Earth much like projectiles moving near the Earth's surface. He proposed that the gravitational force that explains projectile motion is also involved in the motions of the planets around the sun and of the moon around the Earth. Newton's law of universal gravitation states that the gravitational force between two masses is proportional to the product of the masses and inversely proportional to the square of the distance between them. Using this law and his laws of motion, Newton was able to explain Kepler's laws of planetary motion, as well as the motion of ordinary objects near the Earth's surface.

5.5 The Moon and Other Satellites

The moon has fascinated people as long as humanity has existed and wondered about nature. In the twentieth century, we actually visited the moon for the first time and brought back samples from its surface. That visit has not dulled the romance that the moon holds for us, but it may have reduced its mystery.

How are the *phases of the moon* associated with changes in its position? Are Kepler's laws of planetary motion valid for the moon? How are the orbits of other satellites of Earth similar to the moon's?

How do we explain the phases of the moon?

The moon was the only Earth satellite available to Newton and his predecessors to study. The moon played a pivotal role in Newton's thinking and in the development of his law of gravitation. Observations of the moon and its phases, however, go back much further than Newton's day. The moon figures in many early religions and rituals. Its course must have been carefully followed even in prehistoric times.

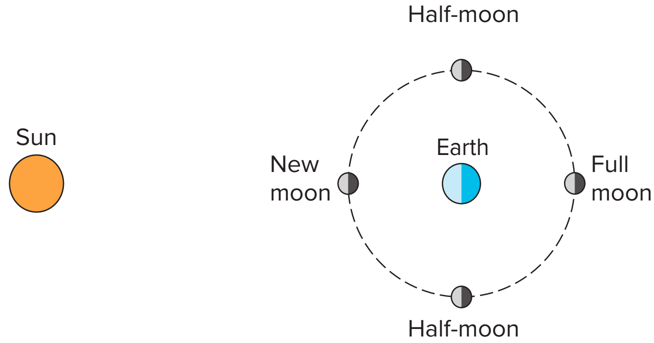


Figure 5.21 The phases of the moon depend on the positions of the sun, the moon, and the Earth. (Not drawn to scale.)

How do we explain the **phases of the moon?** Is the time that the moon rises in the evening related to whether it will be a full moon or not? Moonlight is reflected sunlight. So to understand the moon's phases, we have to take into account the positions of the sun, the moon, and the observer (fig. 5.21). When the moon is full, it is on the opposite side of the Earth from the sun, and we see the side that is fully illuminated by the sun. The full moon rises in the east about the same time that the sun sets in the west. These events are determined by the Earth's rotation.

Because Earth and the moon are both small compared to the distances between Earth, the moon, and the sun, they do not usually get in the way of light coming from the sun. When they do, however, there is an **eclipse.** During a *lunar eclipse,* Earth casts a full or partial shadow on the moon. From figure 5.21, we can see that a lunar eclipse can only occur during a full moon. A *solar eclipse* happens when the moon is in the right position to cast a shadow on the Earth. During what phase of the moon will this occur?

At other times during the moon's 27.3-day revolution around the Earth, we do not see all of the illuminated side of the moon; we see a crescent, a half-moon, or some shape in between (fig. 5.22). The new moon occurs when the moon is on the same side of Earth as the sun and is more or less invisible. When we are a few days on either side of the new moon, we see the familiar crescent.

Figure 5.22 Photographs of different phases of the moon. When during the day will each rise and set? *somchaisom/iStock/360/Getty Images*

When the moon is between full moon and new moon, it can often be seen during daylight. In particular, when it is near half-moon, it rises around noon and sets around midnight (or vice versa, depending on where it is in its cycle). Under just the right conditions near sunset or sunrise, we can sometimes see the dark portion of the crescent moon illuminated by earthshine.

The next time you see the moon, think about where it is in the sky, when it will rise and set, and how this is related to its phase. Better yet, try explaining this to a friend. You, too, can be the wizard who predicts the motions of the heavens.

Does the moon obey Kepler's laws?

The moon's orbit around Earth is more complicated than those of the planets, because two bodies, Earth and the sun, exert strong forces on the moon, rather than just one (fig. 5.23). Earth is much closer to the moon than the sun is, but the sun has a much larger mass than Earth, so the sun's effect is still appreciable. First, let's consider just the effects of Earth on the moon's motion.

The physics of the situation is the same as that for the orbits of the planets around the sun. The gravitational attraction between the moon and Earth provides the centripetal acceleration to keep the moon moving in its roughly circular orbit. By Newton's law of gravitation, the gravitational force acting on the moon is proportional to $1/r^2$, where r is the distance between the center of the moon and the center of Earth. The tides can be explained by this dependence of the gravitational force on distance. (See everyday phenomenon box 5.2.)

Like the planets, the moon's orbit is an ellipse, but with the Earth at one focus of the ellipse rather than the sun. The sun also exerts a force on the moon that distorts the ellipse, causing the moon's orbit around Earth to oscillate about a true elliptical path as the moon and Earth orbit together around the sun. Calculating these oscillations was a problem that kept mathematical physicists busy for many years.

Kepler's first and second laws of planetary motion are approximately true for the moon, provided that we substitute the Earth for the sun in the statement of these laws. Kepler's third law shows some differences between the moon and the planets. When Newton derived the expression for the ratio in Kepler's third law, he arrived at the expression

$$\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_s}$$

where m_s is the mass of the sun. For the moon, we would replace the mass of the sun with the mass of Earth. We get a different ratio for the moon's orbit around Earth than for the orbits of the planets around the sun.

Orbits of artificial satellites

Any satellite orbiting the Earth must have the same value for the ratio T^2/r^3 as the moon. Kepler's third law holds for any satellite of Earth, then, as long as we keep in mind that the ratio will not have the same value as it does for the orbits of the planets. The value of this ratio for Earth satellites is calculated either from the Earth's mass or from the values of the period and average distance of the moon's orbit.

Any artificial satellite of Earth must have the same value for this ratio. If its distance from the center of the Earth r is smaller than the moon's distance, its orbital period T must also be smaller to keep the ratio T^2/r^3 the same. Using this ratio, we can calculate the appropriate distance from Earth for any satellite if we know its orbital period. For example, a satellite with a **synchronous orbit** has a period of 24 hours, which keeps it above the same point on the Earth as Earth rotates. From the third-law ratio, we find a distance r of 42,000 km for such a satellite (measured from the center of the Earth). Because Earth's radius is 6370 km, this is roughly seven times the radius of the Earth. This is quite a ways up, but not nearly as high as the moon (which is approximately 384,400 km away from the Earth).

Most artificial satellites are even closer to the Earth. The original Russian satellite, *Sputnik,* for example, had a period of about 90 minutes, or 1.5 h. Using the third-law ratio, this yields an average distance from the center of the Earth of 6640 km. Subtracting Earth's radius, 6370 km, indicates that this distance is only 270 km above the Earth's surface. The shorter the period, the closer the satellite is to the Earth. The orbital period cannot be much shorter than *Sputnik*'s before atmospheric drag becomes too large for motion to be sustained. Obviously, the orbit cannot have a radius smaller than Earth's radius.

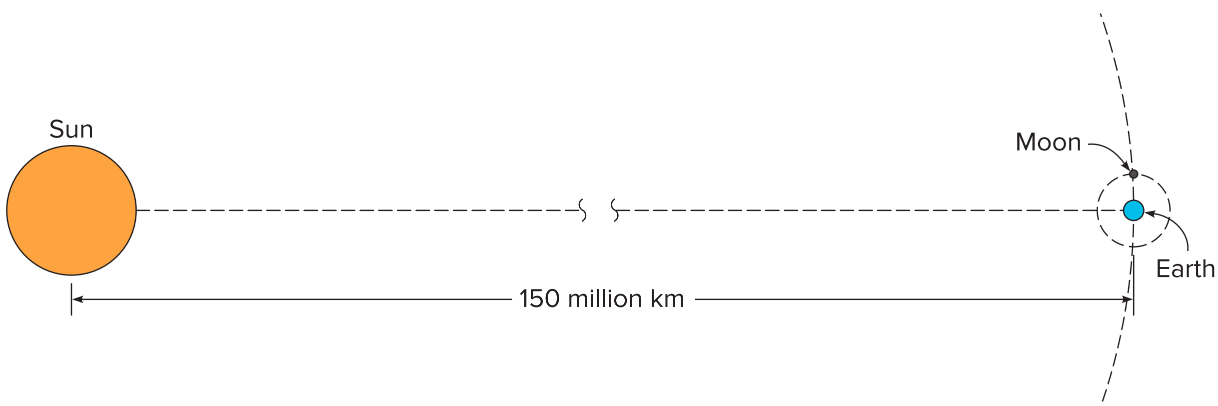


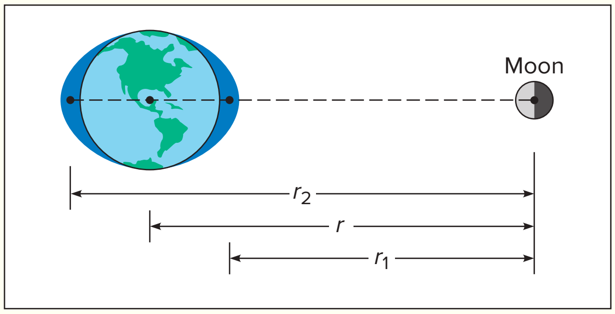
Figure 5.23 The moon is influenced by gravitational attraction to both the Earth and the sun. (Distances and sizes not drawn to scale.)

Everyday Phenomenon Box 5.2

Explaining the Tides

The Situation. Anyone who has lived near the ocean is familiar with the regular variation of the tides. Roughly twice a day the tides come in and go out again. The actual cycle of two high tides and two low tides is closer to 25 hours. Sometimes high tide is higher and low tide is lower than at other times—these times correspond to the full moon or the new moon.

The times when high tides and low tides happen shift from day to day because of their 25-hour cycle, but the pattern repeats monthly. How do we explain this behavior?



Because it depends on distance, the gravitational force per unit mass exerted by the moon on different parts of the Earth (and water in the oceans) gets weaker as we move from the side nearer the moon to the far side. (Not drawn to scale; the bulges here are greatly exaggerated.)

The Analysis. The monthly cycle and the correlations of the highest tides with the phase of the moon suggest a lunar influence. Both the moon and the sun exert gravitational forces on the Earth. The sun exerts the stronger force because of its much larger mass, but the moon is much closer, and variations in its distance from Earth may be significant. The gravitational force depends on $1/r^2$, so its strength will vary as the distance r varies, as indicated in the drawing shown below.

Because water is a fluid (except when frozen), the water that makes up the oceans moves over the more rigid crust of Earth. The primary force acting on the water is the gravitational attraction of Earth that holds the water to the Earth's surface. The gravitational force exerted by the moon on the water is also significant, however, and its strength per unit mass is greatest on the side of Earth closest to the moon and weakest on the opposite side of Earth because of the difference in distance.

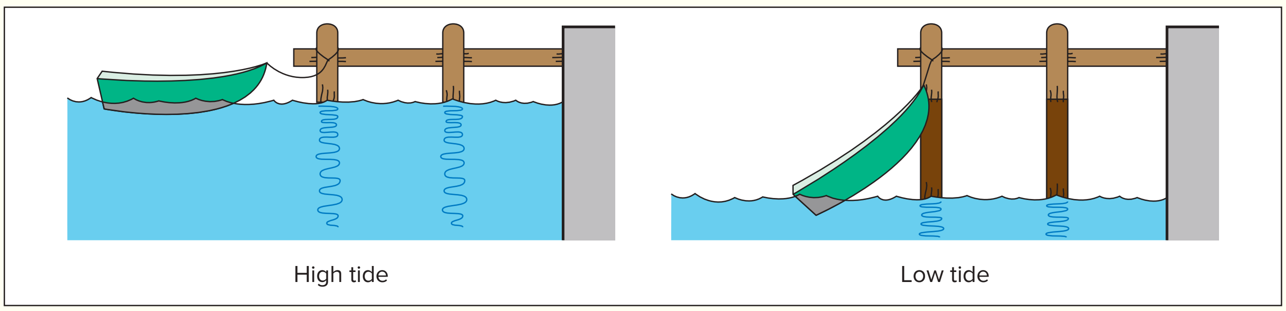
This difference in strength of the moon's pull produces a bulge in the water surface on both sides of the Earth. The bulge on the side nearest the moon results from the water's being pulled toward the moon by a stronger force per unit mass than the force per unit mass exerted on the rest of the Earth. This produces a high tide. The water will rise nearer to the top of the dock.

On the opposite side of the Earth, it is the Earth that is being pulled by the moon with a stronger force per unit mass than the water. Because the Earth is pulled away (slightly) from the water, this also produces a high tide. The forces exerted by the moon are small compared to the force that the water and the Earth exert on each other but are still large enough to produce the tides.

When the sun and the moon both line up with the Earth during the new moon or full moon, the sun also contributes to this difference in forces and produces bulges on either side of the Earth, adding to those produced by the moon. The highest tides occur during a full moon or a new moon because of this combination of the moon and sun.

Why is the cycle 25 hours rather than 24 hours? The high-tide bulges occur on either side of Earth along the line joining the moon and the Earth. The Earth rotates underneath these bulges with a period of 24 hours, but in this time, the moon also moves, because it orbits Earth with a period of 27.3 days. In 1 day, therefore, the moon has moved through roughly $^1/_{27}$ of its orbital cycle, causing the time when the moon again lines up with a given point on Earth to be a little longer than 1 day. This additional time is approximately $^1/_{27}$ of 24 hours, or a little less than an hour.

This model was conceived by Newton and accounts neatly for the major features of the tides. The variation of the gravitational force with distance is the key to the explanation.



High tide and low tide produce different water levels at the dock.

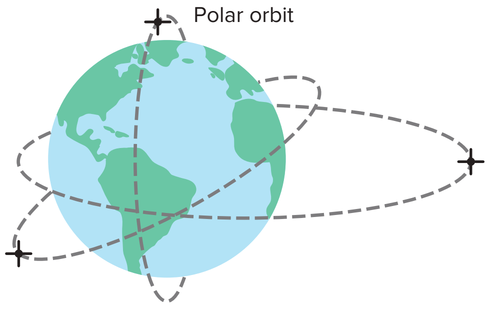


Figure 5.24 The orbits of different artificial satellites can have different orientations and elliptical shapes.

The orbits of different satellites are planned to meet different objectives. Some are close to circular, while others are much more elongated ellipses (fig. 5.24). The plane of the orbit can pass through the poles of the Earth (polar orbit) or take any orientation between the poles and the equator. It all depends on the mission of the satellite.

Artificial satellites have become a routine feature of today's world that did not exist before 1958, when *Sputnik* was launched. Their uses are many, including communications, surveillance, weather observations, and various military applications. The basic physics of their behavior is accounted for by Newton's theory. If Newton could return, he might be amazed at the developments, but for him the analysis would be routine.

The motion of the moon around Earth is governed by the same principles as that of the planets around the sun. The gravitational force provides the centripetal acceleration that keeps the moon in an approximately elliptical orbit. The moon is illuminated primarily by the sun, and the phases of the moon can be explained by the moon's position with regard to the sun and Earth. The full moon occurs when the sun and moon are on opposite sides of the Earth. Other satellites of Earth are governed by these principles, but Kepler's third-law ratio has a different value for satellites of Earth (including the moon) than for the planets. The moon is no longer alone; it has been joined by many, much smaller objects buzzing around the Earth in lower orbits.

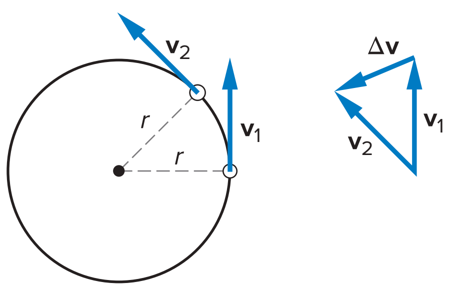
Debatable Issue

Some people believe that the moon landing in 1969 was just an elaborate hoax. Is this a reasonable belief? What evidence or arguments could you use to counter this claim?

Summary

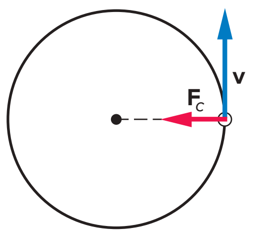
Objects moving in circular paths are accelerated, because the direction of the velocity vector continually changes. The forces involved in producing this centripetal acceleration were examined for the motion of a ball on a string, cars rounding curves, a rider on a Ferris wheel, and finally the planets moving around the sun. The force providing the centripetal acceleration for planetary motion is described by Newton's law of gravitation.

1 **Centripetal acceleration.** Centripetal acceleration is the acceleration involved in changing the direction of the velocity vector. It is proportional to the square of the speed of the object and inversely proportional to the radius of the curve.



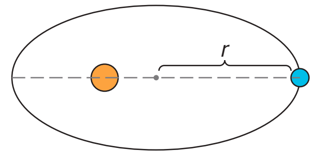
$$a_c = \frac{v^2}{r}$$

2 **Centripetal forces.** A centripetal force is any force or combination of forces that acts on a body to produce the centripetal acceleration, including friction, normal forces, tension in a string, or gravity. The net force is related to the centripetal acceleration by Newton's second law.



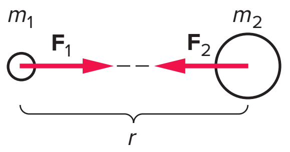
$$\mathbf{F}_c = m\mathbf{a}_c$$

3 **Planetary motion.** Kepler's three laws of planetary motion describe the orbits of the planets around the sun. The orbits are ellipses that sweep out equal areas in equal times (the first and second laws). The third law states a relationship between the period of the orbit and the distance of the planet from the sun.



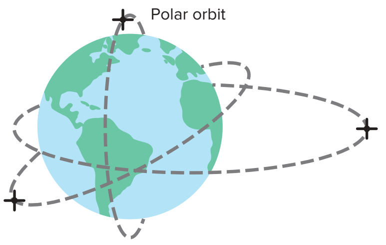
$$\frac{T^2}{r^3} = \text{constant}$$

4 **Newton's law of universal gravitation.** Newton's law of universal gravitation states that the gravitational force between two masses is proportional to each of the masses and inversely proportional to the square of the distance between the masses. Using this law with his laws of motion, Newton could derive Kepler's laws of planetary motion.



$$F = G\frac{m_1 m_2}{r^2}$$

5 **The moon and other satellites.** The moon's orbit around Earth can also be described by Kepler's laws, provided we substitute the mass of Earth for that of the sun in the expression for the period. Artificial satellites have the same ratio T^2/r^3 as that for the moon.



If you find it challenging to use the equations for Newton's law of universal gravitation and Kepler's third law, try the practice problems in Connect. Complete solutions (not just the answers) are provided, which should help you improve your ability to "plug and chug"!

Key Terms

Centripetal acceleration, 82
Centripetal force, 84
Static force of friction, 84
Kinetic force of friction, 84
Normal force, 85
Retrograde motion, 88
Epicycle, 88
Heliocentric, 88
Ellipse, 89
Period, 90
Newton's law of universal gravitation, 92
Universal gravitational constant, 93
Phases of the moon, 94
Eclipse, 94
Synchronous orbit, 95

Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available on Connect

Q1. Suppose that the speed of a ball moving in a horizontal circle is increasing at a steady rate. Is this increase in speed produced by the centripetal acceleration? Explain.

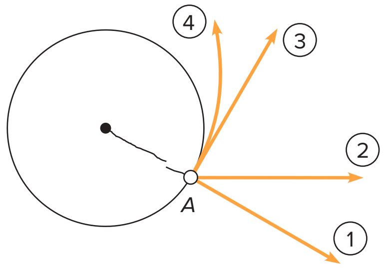
Q2. A car travels around a curve with constant speed.
a. Does the velocity of the car change in this process? Explain.
b. Is the car accelerated? Explain.

Q3. Two cars travel around the same curve, one at twice the speed of the other. After traveling the same distance, which car, if either, has experienced the larger change in velocity? Explain.

Q4. A car travels the same distance at constant speed around two curves, one with twice the radius of curvature of the other. For which of these curves is the change in velocity of the car greater? Explain.

*Q5. The centripetal acceleration depends on the square of the speed rather than just being proportional to the speed. Why does the speed enter twice? Explain.

Q6. A ball on the end of a string is whirled with constant speed in a counterclockwise, horizontal circle. At point A in the circle, the string breaks. Which of the curves sketched in the diagram most accurately represents the path that the ball will take after the string breaks (as seen from above)? Explain.



Q6 Diagram

Q7. Before the string breaks in question 6, is there a net force acting upon the ball? If so, what is its direction? Explain.

Q8. For a ball being twirled in a horizontal circle at the end of a string, does the vertical component of the force exerted by the string produce the centripetal acceleration of the ball? Explain.

Q9. A car travels around a flat (nonbanked) curve with constant speed.
a. Sketch a diagram showing all of the forces acting upon the car.
b. What is the direction of the net force acting upon the car? Explain.

Q10. Is there a maximum speed at which the car in question 9 will be able to negotiate the curve? If so, what factors determine this maximum speed? Explain.

Q11. If a curve is banked, is it possible for a car to negotiate the curve even when the frictional force is zero due to very slick ice? Explain.

*Q12. If a ball is whirled in a vertical circle with constant speed, at what point in the circle, if any, is the tension in the string the greatest? Explain. (Hint: Compare this situation to the Ferris wheel described in section 5.2.)

Q13. Sketch the forces acting upon a rider on a Ferris wheel when the rider is at the top of the cycle, labeling each force clearly. Which force is largest at this point, and what is the direction of the net force? Explain.

Q14. Which safety measure, seat belts or air bags, offers the most protection in head-on collisions? Explain. (See everyday phenomenon box 5.1.)

Q15. In a head-on collision between two vehicles, is there a force that propels a driver forward toward the windshield? Explain. (See everyday phenomenon box 5.1.)

Q16. If a car is equipped with air bags, should it be necessary to also wear seat belts? Explain. (See everyday phenomenon box 5.1.)

*Q17. In what way did the heliocentric view of the solar system proposed by Copernicus provide a simpler explanation of planetary motion than the geocentric view of Ptolemy? Explain.

Q18. Did Ptolemy's view of the solar system require motion of the Earth, rotational or otherwise? Explain.

*Q19. Heliocentric models of the solar system (Copernican or Keplerian) require that the Earth rotate on its axis, producing surface speeds of roughly 1000 MPH. If this is the case, why do we not feel this tremendous speed? Explain.

Q20. How did Kepler's view of the solar system differ from that of Copernicus? Explain.

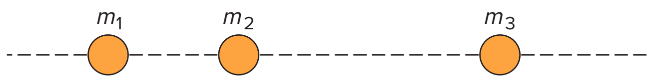
Q21. Consider the method of drawing an ellipse pictured in figure 5.15. How would we modify this process to make the ellipse into a circle, which is a special case of an ellipse? Explain.

Q22. Does a planet moving in an elliptical orbit about the sun move fastest when it is farthest from the sun or when it is nearest to the sun? Explain by referring to one of Kepler's laws.

Q23. Does the sun exert a larger force on the Earth than that exerted on the sun by the Earth? Explain.

Q24. Is there a net force acting on the planet Earth? Explain.

Q25. Three equal masses are located as shown in the diagram. What is the direction of the net force acting upon m_2? Explain.



Q25 Diagram

Q26. Two masses are separated by a distance *r*. If this distance is doubled, is the force of interaction between the two masses doubled, halved, or changed by some other amount? Explain.

Q27. A painter depicts a portion of the night sky as shown in the diagram below, showing the stars and a crescent moon. Is this view possible? Explain.

Q27 Diagram

Q28. At what times during the day or night would you expect the new moon to rise and set? Explain.

Q29. At what times of the day or night does the half-moon rise or set? Explain.

Q30. Are we normally able to see the new moon? Explain.

Q31. During what phase of the moon can a solar eclipse occur? Explain.

*Q32. A synchronous satellite is one that does not move relative to the surface of the Earth; it is always above the same location. Why does such a satellite not just fall straight down to the Earth? Explain.

Q33. Is Kepler's third law valid for artificial satellites orbiting about the Earth? Explain.

Q34. Since the Earth rotates on its axis once every 24 hours, why don't high tides occur exactly twice every 24 hours? Explain.

Q35. Why is there a high tide rather than a low tide when the moon is on the opposite side of the Earth from the ocean and the gravitational pull of the moon on the water is the weakest? Explain. (See everyday phenomenon box 5.2.)

Q36. Would tides exist if the gravitational force did not depend on the distance between objects? Explain. (See everyday phenomenon box 5.2.)

Exercises

For the exercises in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8\ \text{m/s}^2$ for the acceleration due to gravity.

E1. A ball is traveling at a constant speed of 4 m/s in a circle with a radius of 0.8 m. What is the centripetal acceleration of the ball?

E2. A car rounds a curve with a radius of 40 m at a speed of 18 m/s. What is the centripetal acceleration of the car?

E3. A ball traveling in a circle with a constant speed of 6 m/s has a centripetal acceleration of 20 m/s². What is the radius of the circle?

E4. How much larger is the required centripetal acceleration for a car rounding a curve at 60 MPH than for one rounding the same curve at 20 MPH?

E5. A 0.35-kg ball moving in a circle at the end of a string has a centripetal acceleration of 5 m/s^2. What is the magnitude of the centripetal force exerted by the string on the ball to produce this acceleration?

E6. A car with a mass of 1500 kg is moving around a curve with a radius of 45 m at a constant speed of 18 m/s (about 40 MPH).
   a. What is the centripetal acceleration of the car?
   b. What is the magnitude of the force required to produce this centripetal acceleration?

E7. A car with a mass of 1300 kg travels around a banked curve with a constant speed of 20 m/s (about 45 MPH). The radius of curvature of the curve is 35 m.
   a. What is the centripetal acceleration of the car?
   b. What is the magnitude of the horizontal component of the normal force that would be required to produce this centripetal acceleration in the absence of any friction?

E8. A Ferris wheel at a carnival has a radius of 8 m and turns so that the speed of the riders is 4.5 m/s.
   a. What is the magnitude of the centripetal acceleration of the riders?
   b. What is the magnitude of the net force required to produce this centripetal acceleration for a rider with a mass of 75 kg?

E9. What is the ratio of the Earth's period of rotation about its own axis to the Earth's orbital period about the sun?

E10. Dylan has a weight of 800 N (about 180 lb) when he is standing on the surface of the Earth. What would his weight (the gravitational force due to the Earth) be if he tripled his distance from the center of the Earth by flying in a spacecraft?

E11. Two masses are attracted by a gravitational force of 9.6 N. What will the force of attraction be if the distance between the two masses is quadrupled?

E12. Two 700-kg (1543-lb) masses are separated by a distance of 0.45 m. Using Newton's law of gravitation, find the magnitude of the gravitational force exerted by one mass on the other.

E13. Two masses are attracted by a gravitational force of 0.28 N. What will the force of attraction be if the distance between these two masses is halved?

E14. The acceleration of gravity at the surface of the moon is approximately one-sixth that at the surface of the Earth (9.8 m/s^2). What is the weight of an astronaut standing on the moon whose weight on Earth is 270 lb?

E15. The acceleration of gravity on the surface of Jupiter is approximately 25 m/s^2. What is the weight on Jupiter of a woman whose weight on Earth is 150 lb?

E16. The time separating high tides is 12 hours and 25 minutes. If high tide occurs at 1:10 P.M. on one afternoon,
   a. At what time will high tide occur the next afternoon?
   b. When would you expect low tides to occur the next day?

Synthesis Problems

For the synthesis problems in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8$ m/s^2 for the acceleration due to gravity.

SP1. A 0.25-kg ball is twirled at the end of a string in a horizontal circle with a radius of 0.45 m. The ball travels with a constant speed of 3.0 m/s.
   a. What is the centripetal acceleration of the ball?
   b. What is the magnitude of the horizontal component of the tension in the string required to produce this centripetal acceleration?
   c. What is the magnitude of the vertical component of the tension required to support the weight of the ball?
   d. Draw to scale a vector diagram showing these two components of the tension and estimate the magnitude of the total tension from your diagram. (See appendix C.)

SP2. A Ferris wheel with a radius of 15 m makes one complete rotation every 12 seconds.
   a. Using the fact that the distance traveled by a rider in one rotation is $2\pi r$, the circumference of the wheel, find the speed with which the riders are moving.
   b. What is the magnitude of their centripetal acceleration?
   c. For a rider with a mass of 50 kg, what is the magnitude of the centripetal force required to keep that rider moving in a circle? Is the weight of the rider large enough to provide this centripetal force at the top of the cycle?
   d. What is the magnitude of the normal force exerted by the seat on the rider at the top of the cycle?
   e. What will happen if the Ferris wheel is going so fast that the weight of the rider is not sufficient to provide the centripetal force at the top of the cycle?

SP3. A car with a mass of 1100 kg is traveling around a curve with a radius of 50 m at a constant speed of 25 m/s (56 MPH). The curve is banked at an angle of 12°.
   a. What is the magnitude of the centripetal acceleration of the car?
   b. What is the magnitude of the centripetal force required to produce this acceleration?
   c. What is the magnitude of the vertical component of the normal force acting upon the car to counter the weight of the car?
   d. Draw a diagram of the car (as in fig. 5.8) on the banked curve. Draw to scale the vertical component of the normal force. Using this diagram, find the magnitude of the total normal force, which is perpendicular to the surface of the road.
   e. Using your diagram, estimate the magnitude of the horizontal component of the normal force. Is this component sufficient to provide the centripetal force?

SP4. Assume that a passenger in a rollover accident must turn through a radius of 2.8 m to remain in the seat of the vehicle. Assume also that the vehicle makes a complete turn in 0.9 second.
   a. Using the fact that the circumference of a circle is $2\pi r$, what is the speed of the passenger?

b. What is the centripetal acceleration? How does it compare to the acceleration due to gravity?
c. If the passenger has a mass of 75 kg, what is the centripetal force required to produce this acceleration? How does it compare to the passenger's weight?

SP5. The sun's mass is 1.99×10^{30} kg, the Earth's mass is 5.98×10^{24} kg, and the moon's mass is 7.36×10^{22} kg. The average distance between the moon and the Earth is 3.82×10^{8} m, and the average distance between the Earth and the sun is 1.50×10^{11} m.
a. Using Newton's law of gravitation, find the average force exerted on the Earth by the sun.
b. Find the average force exerted on the Earth by the moon.
c. What is the ratio of the force exerted on the Earth by the sun to that exerted by the moon? Will the moon have much of an impact on the Earth's orbit about the sun?
d. Using the distance between the Earth and the sun as the average distance between the moon and the sun, find the average force exerted on the moon by the sun. Will the sun have much impact on the orbit of the moon about the Earth?

SP6. The period of the moon's orbit about the Earth is 27.3 days, but the average time between full moons is approximately 29.3 days. The difference is due to the motion of the Earth about the sun.
a. Through what fraction of its total orbital period does the Earth move in one period of the moon's orbit?
b. Draw a sketch of the sun, the Earth, and the moon with the moon in the full moon condition. Then, sketch the position the moon will be in 27.3 days later, when the Earth is in its new position. If the moon is in the same position relative to Earth as it was 27.3 days earlier, is this a full moon?
c. How much farther would the moon have to go to reach the full moon condition? Show that this represents approximately an extra 2 days.

Home Experiments and Observations

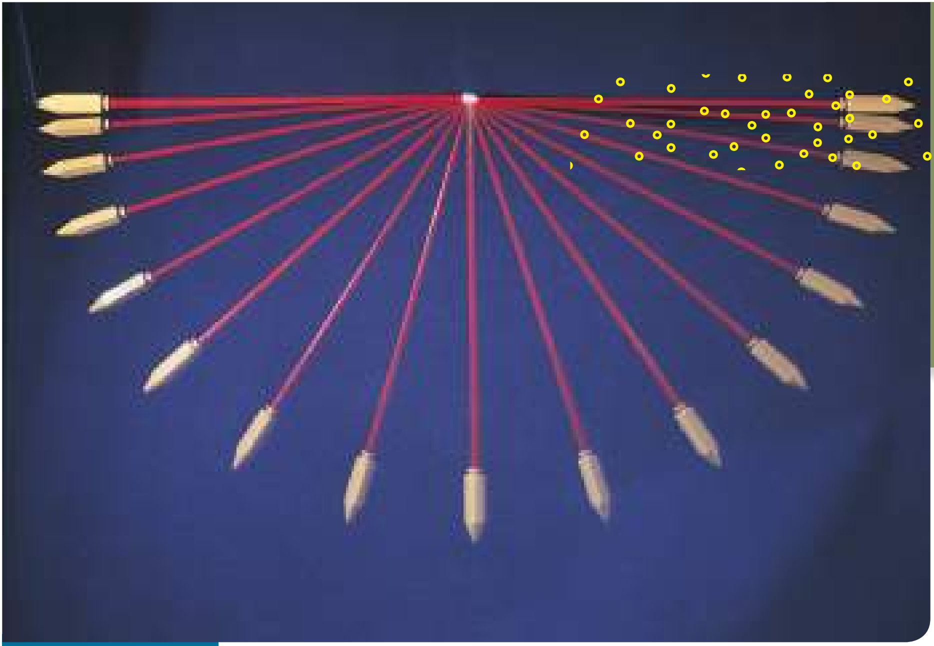
HE1. Tape a string half a meter or so in length securely to a small rubber ball. Practice whirling the ball in both horizontal and vertical circles and make these observations:
a. For horizontal motion of the ball, how does the angle that the string makes with the horizontal vary with the speed of the ball?
b. If you let go of the string at a certain point in the circle, what path does the ball follow after release?
c. Can you feel differences in tension in the string for different speeds of the ball? How does the tension vary with speed?
d. For a vertical circle, how does the tension in the string vary for different points in the circle? Is it greater at the bottom than at the top when the ball moves with constant speed?

HE2. Tie a small paper cup to a string, attaching it at two points near the rim, as shown in the diagram. Place a marble or other small object in the cup.
a. Whirl the cup in a horizontal circle. Does the marble stay in the cup? (Be careful! A flying marble can be dangerous.)
b. Whirl the cup in a vertical circle. Does the marble stay in the cup? What keeps the marble in the cup at the top of the circle?
c. Try slowing the cup down. Does the marble stay in the cup?
d. If you are brave, try replacing the marble with water. Under what conditions does the water stay in the cup?

HE2 Diagram

HE3. Observe the position and phase of the moon on several days in succession and at regularly chosen times during the day and evening. (It is probably best to choose a point near the first quarter of the moon's cycle, when the moon is visible in the afternoon and evening.)
a. Sketch the shape of the moon on each successive day. Does this shape change for different times in the same day?
b. Can you devise a method for accurately noting changes in the position of the moon at a set time—say, 10 P.M.—on successive days? A fixed sighting point, a meter stick, and a protractor may be useful. Describe your technique.
c. By how much does the position of the moon change from one day to the next at your regular chosen time?

HE4. Consult your instructor or other sources to find out what planets are observable in the evening during the current month. Venus, Jupiter, and Mars are usually the best candidates.
a. Locate the planet visually and observe it with binoculars if possible. How does the planet differ in appearance from nearby stars?
b. Sketch the position of the planet relative to nearby stars for several nights. How does this position change?



Richard Megna/Fundamental Photographs, NYC

CHAPTER 6

Energy and Oscillations

Chapter Overview

We usually approach energy by first considering how it is added to a system. This involves the concept of work, which has a specialized meaning in physics. If a force does work on a system, the energy of the system increases. Work is a means of transferring energy.

We begin by defining work and showing how to find it in simple cases. In different circumstances, work done on a system increases either the kinetic energy or the potential energy of the system. Finally, we tie these ideas together by introducing the principle of conservation of energy and applying it to practical situations, including oscillations.

Chapter Outline

1. **Simple machines, work, and power.** What is a simple machine? How does the idea of work help us to understand the operation of simple machines? How do physicists define work, and how is work related to power?
2. **Kinetic energy.** What is kinetic energy? When and how does work change the kinetic energy of an object?
3. **Potential energy.** What is potential energy? When and how does work change the potential energy of an object?
4. **Conservation of energy.** What is the total energy of a system, and when is it conserved? How can we use these ideas to explain the motion of a pendulum and other phenomena?
5. **Springs and simple harmonic motion.** How is the motion of a mass on a spring like a pendulum? What is simple harmonic motion?

Have you ever watched a ball on the end of a string swing back and forth? A pendant on the end of a chain, a swing in the park, and the pendulum on a grandfather clock (fig. 6.1) all display the same hypnotic motion. Galileo (it is said) amused himself during boring sermons in church by watching the chandeliers sway slowly back and forth at the end of their chains.

What intrigued Galileo was the way a pendulum always seems to return to the same position at the end of each swing. It may fall a little short of the earlier position in successive swings, but the motion goes on for a long time before coming to a complete stop. On the other hand, the velocity is continually changing, from zero at the end points of the swing to a maximum at the low point in the path. How can the pendulum go through such changes in velocity and yet always return to its starting point?

Evidently, something is being saved, or *conserved.* The quantity that remains constant (and is conserved) turns out to be what we now call *energy.* Energy did not play a role in Newton's theory of mechanics. It was not until the nineteenth century that energy and energy transformations were elevated to the central position that they now hold in our understanding of the physical world.

The motion of a pendulum and other types of oscillation can be understood using the principle of conservation of mechanical energy. The potential energy that the pendulum has at its end points is converted to kinetic energy at the low point—and then back to potential energy. What is energy, though, and how does it get into the system in the first place?

Figure 6.1 A weight swinging at the end of a bar. Why does it return to approximately the same point after each swing?
Jonnie Miles/Getty Images

Why does energy now play a central role in physics and all of science?

Energy is the basic currency of the physical world. To spend energy wisely, we must understand it. That understanding begins with the concept of *work.*

6.1 Simple Machines, Work, and Power

If you make a pendulum by fastening a ball to the end of a string (fig. 6.2), what do you do to start it swinging? In other words, how do you get energy into the system? Usually, you would start by pulling the ball away from the center position directly below the point from which the string is suspended. To do so, you must apply a force to the ball with your hand and move the ball some distance.

To a physicist, applying a force to move an object some distance involves doing work, even though the actual exertion may be slight. Doing work on a system increases the energy of the system, and this energy can then be used in the motion of the pendulum. How do we define work, though, and how can simple machines demonstrate the usefulness of the idea?

Figure 6.2 The force applied does work to move the ball from its original position directly below the point of suspension.
James Ballard/McGraw-Hill Education

What are simple machines?

An early application of work was the analysis of the devices, such as levers, pulley systems, and inclined planes, that we call *simple machines.* A **simple machine** is any mechanical device that multiplies the effect of an applied force. A lever is one example of a simple machine. By applying a small force at one end of a lever, a larger force can be exerted on the rock at the opposite end (fig. 6.3).

What price do you pay for this multiplying effect of the applied force? To move the rock a small distance, the other end of the lever must move through a larger distance.

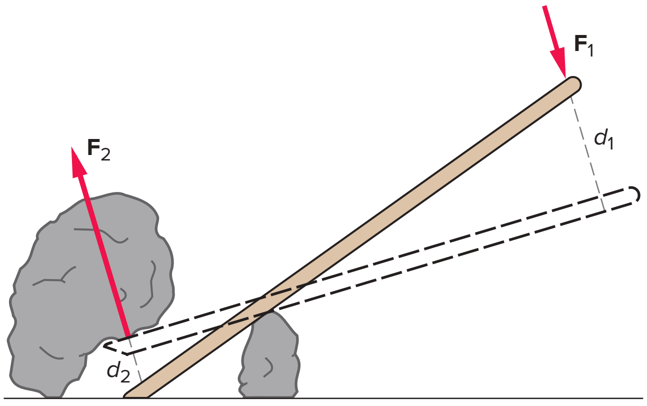


Figure 6.3 A lever is used to lift a rock. A small force $\mathbf{F}_1$ generates a larger force $\mathbf{F}_2$ to lift the rock, but $\mathbf{F}_1$ acts through a larger distance d_1 than does $\mathbf{F}_2$.

Generally, with simple machines, we get by with a small force if we are willing to apply that force over a large distance. The output force at the other end may be large, but it acts only over a short distance.

The pulley system shown in figure 6.4 is another simple machine that achieves a similar result. In this system, the tension in the rope pulls up on either side of the pulley supporting the weight. If the system is in equilibrium, the tension in the rope is only half the weight being lifted, because there are, in effect, two ropes pulling up on the pulley. But to lift the pulley and its load a certain height, the person must move the rope *twice* the distance the load moves. (Both rope segments on either side of the pulley must decrease in length by an amount equal to the increase in the height of the load.)

The net result of using the pulley system illustrated in figure 6.4 is that you can lift a weight a certain height by applying a force equal to only *half* the weight being lifted. However, you must pull the rope *twice* the distance the weight is lifted. This way, the product of the force and the distance moved will be the same for the input force applied by the person to the rope as for the output force exerted on the load. The quantity *force times distance* is thus conserved (if frictional losses are small). We call this product *work,* and the result for an ideal simple machine is

work output = work input

The ratio of the output force to the input force is called the **mechanical advantage** of the simple machine. For our pulley system, the mechanical advantage is 2. The output force that lifts the load is twice the input force exerted by the person pulling on the rope.

How is work defined?

Our discussion of simple machines shows that the quantity force times distance has a special significance. Suppose

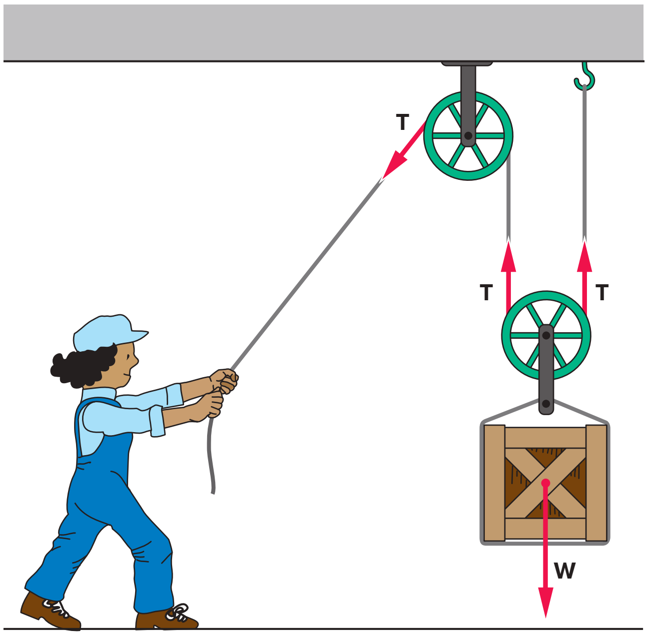


Figure 6.4 A simple pulley system is used to lift a weight. The tension in the rope pulls up on either side of the lower pulley, so the tension is only half the size of the supported weight.

you apply a constant horizontal force to a heavy crate to move it across a concrete floor, as illustrated in figure 6.5. You would agree that you have done work to move the crate and that the farther you move it, the more work you will do.

The amount of work that you do also depends on how hard you have to push to keep the crate moving. These are the basic ideas that we use in defining work: Work depends on both the *strength* of the applied force and the *distance* that the crate is moved. If the force and the distance moved are in the same direction, then **work** is the applied force multiplied by the distance that the crate moves under the influence of this force, or

work = force × distance

$$W = Fd$$

where W is the work and d is the distance moved. The units of work will be units of force multiplied by units of distance, or newton-meters (N·m) in the metric system. We call this unit a *joule* (J). The joule is the basic metric unit of energy (1 J = 1 N·m).

The first part of the sample exercise in example box 6.1 shows how we find the work done in a simple case. A horizontal force of 50 N is used to pull a crate a distance of 4 m, resulting in 200 J of work done on the crate by the applied force. In doing this work, we transfer 200 J of energy to the crate and its surroundings from the person applying the force. The person loses energy; the crate and its surroundings gain energy.

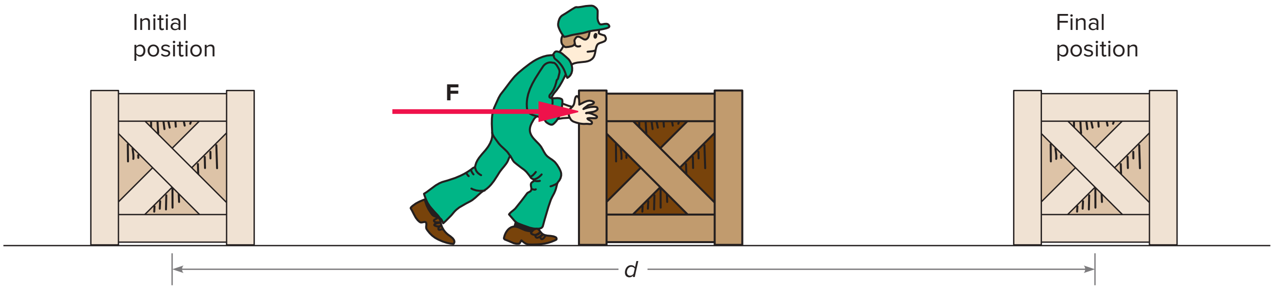


Figure 6.5 A crate is moved a distance *d* across a concrete floor under the influence of a constant horizontal force **F.**

Example Box 6.1

Sample Exercise: How Much Work?

A crate is pulled a distance of 4 m across the floor under the influence of a 50-N force applied by a rope to the crate. What is the work done on the crate by the 50-N force if

a. The rope is horizontal, parallel to the floor?
b. The rope pulls at an angle to the floor, so that the horizontal component of the 50-N force is 30 N (fig. 6.6)?

a. $F = 50\text{ N}$ $\quad W = Fd$

$d = 4\text{ m}$ $\quad = (50\text{ N})(4\text{ m})$

$W = ?$ $\quad = \mathbf{200\ J}$

b. $F_h = 30\text{ N}$ $\quad W = F_h d$

$d = 4\text{ m}$ $\quad = (30\text{ N})(4\text{ m})$

$W = ?$ $\quad = \mathbf{120\ J}$

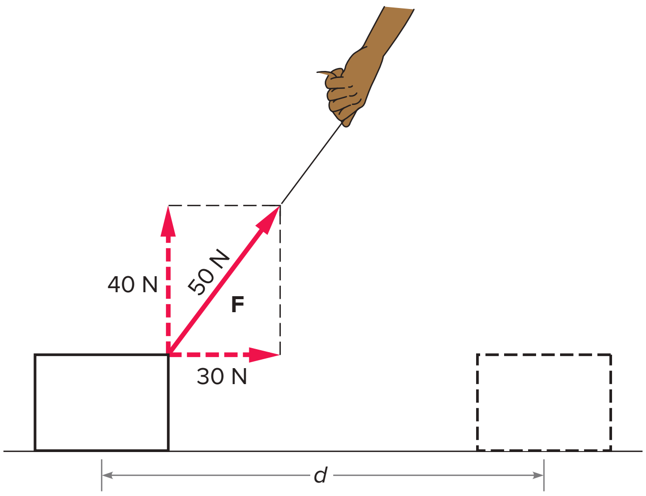


Figure 6.6 A rope is used to pull a box across the floor. Only the portion of the force that is parallel to the floor is used in computing the work.

Does any force do work?

In our initial example, the force acting on the crate was in the same direction as the motion produced. What about other forces acting on the crate—do they do work? The normal force of the floor pushes upward on the crate, for example, but the normal force has no direct effect in producing the motion, because it is perpendicular to the direction of the motion. Forces *perpendicular* to the motion, such as the normal force or the gravitational force acting on the crate, *do no work* when the crate moves horizontally.

What if the force acting on an object is neither perpendicular nor parallel to the direction of the object's motion? In this case, we do not use the total force in computing work. Instead, we use only that portion or component of the force in the direction of the motion. This idea is illustrated in figure 6.6 and in the second part of example box 6.1.

In figure 6.6, the rope used to pull the crate is at an angle to the floor, so that part of the applied force is directed upward rather than parallel to the floor. The box does not move in the direction of the force. Picture the force as having two components, one parallel to the floor and the other perpendicular to the floor. Only the component of the force in the direction of motion is used in computing the work. The component perpendicular to the motion does no work.

By taking direction into account, we can complete the definition of work:

> The work done by a given force is the product of the component of the force along the line of motion of the object multiplied by the distance that the object moves under the influence of the force.

How is power related to work?

When a car accelerates, energy is transferred from the fuel in the engine to the motion of the car. Work is done to move the car, but often we are more concerned with *how fast* this work is accomplished. The rate at which this work can be done depends on the **power** of the engine. The shorter the time, the greater the power. Power can be defined as follows:

> Power is the rate of doing work. It is found by dividing the amount of work done by the time required.
>
> $$\text{power} = \frac{\text{work}}{\text{time}}$$
>
> $$P = \frac{W}{t}$$

In the first part of the example in example box 6.1, we computed a work value of 200 J for moving a crate 4 m across the floor using a force of 50 N. If the crate is in motion for 10 seconds, the power is found by dividing 200 J by 10 seconds, yielding a power of 20 J/s. A joule per second (J/s) is called a *watt* (W), the metric unit of power. We use watts commonly in discussing electric power, but watts are also used more generally for any situation involving the rate of transfer of energy.

Another unit of power still used to describe the power of automobile engines is horsepower (hp). One horsepower is equal to 746 watts, or 0.746 kilowatt (kW). The day may come when we routinely compare the power of different engines in kilowatts rather than in horsepower, but we are not there yet. The relationship of horsepower to the typical horse is dubious, but comparing the iron horse to the flesh-and-blood kind still has a certain appeal.

> Work is the applied force times the distance moved, provided that the force acts along the line of motion of the object. In simple machines, work output can be no greater than work input, even though the output force is larger than the input force. Power is the rate of doing work: The faster the work is done, the greater the power. Doing work on an object increases the energy of the object or system, as in our initial example of pulling the pendulum bob away from its equilibrium position.

6.2 Kinetic Energy

Suppose that the force applied to move a crate is the only force acting on the crate in the direction of motion. What happens to the crate then? According to Newton's second law, the crate will accelerate, and its velocity will increase. Doing work on an object increases its energy. We call the energy associated with the motion of the object **kinetic energy.**

Because work involves the transfer of energy, the amount of kinetic energy gained by the crate should be equal to the amount of work done. How can we define kinetic energy so that this is indeed the case? Work serves as the starting point.

How do we define kinetic energy?

Imagine you are pushing a crate across the floor (fig. 6.5). If you place the crate on rollers with good bearings, the frictional forces may be small enough to be ignored. The force you apply will then accelerate the crate. If you knew the mass of the crate, you could find its acceleration from Newton's second law of motion.

As the crate gains speed, you will have to move faster to keep applying a constant force. As we have seen in chapter 2, an object moving with constant acceleration travels a distance proportional to the square of the final speed. The work done is therefore also proportional to the square of the speed.

Because the work done should equal the increase in kinetic energy, the kinetic energy must increase with the square of the speed. If the crate begins from rest, the exact relationship turns out to be

$$\text{Work done} = \text{change in kinetic energy} = \tfrac{1}{2}mv^2$$

We often use the abbreviation *KE* to represent kinetic energy.

> Kinetic energy is the energy of an object associated with its motion and is equal to one-half the mass of the object times the square of its speed.
>
> $$KE = \frac{1}{2}mv^2$$

Figure 6.7 illustrates the process. If the crate is initially at rest, its kinetic energy is equal to zero. After being accelerated over a distance *d,* it has a final kinetic energy of $\frac{1}{2}mv^2$, which is equal to the work done on the crate. The work done is actually equal to the *change* in kinetic energy. If the crate was already moving when you began pushing, its increase in kinetic energy would equal the work done.

In example box 6.2, we highlight these ideas by calculating the energy gained by the crate in two different ways.

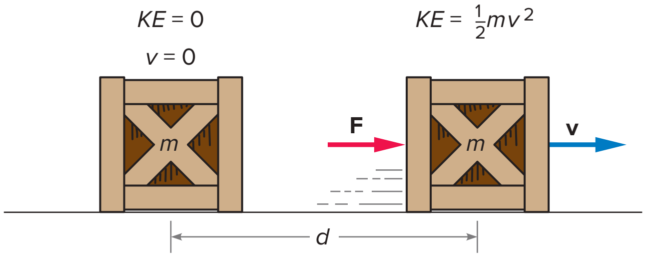


Figure 6.7 The work done on an object by the net force acting on the object results in an increase in the object's kinetic energy.

Example Box 6.2

Sample Exercise: Work and Kinetic Energy

Starting from rest on a frictionless floor, you move a 120-kg crate by applying a net force of 96 N for a time of 4 s. This results in a final speed of 3.2 m/s after the crate moves a distance of 6.4 m (see synthesis problem 2). Find the following:

a. The work done on the crate.
b. The final kinetic energy of the crate.

a. $F = 96\text{ N}$ $\quad W = Fd$
$d = 6.4\text{ m}$ $\quad = (96\text{ N})(6.4\text{ m})$
$W = ?$ $\quad = \mathbf{614\ J}$

b. $m = 120\text{ kg}$ $\quad KE = \frac{1}{2}mv^2$
$v = 3.2\text{ m/s}$ $\quad = \frac{1}{2}(120\text{ kg})\,(3.2\text{ m/s})^2$
$KE = ?$ $\quad = \mathbf{614\ J}$

In the first method, we use the definition of work. In the second, we use the definition of kinetic energy. We find that 614 J of work done on the crate results in an increase in kinetic energy of 614 J. It is no accident that these values are equal. Our definition of kinetic energy guarantees this to be true.

What is negative work?

If work done on an object increases its kinetic energy, can work also decrease the energy of an object? Forces can decelerate objects, as well as accelerate them. Suppose, for example, that we apply the brakes to a rapidly moving car, and the car skids to a stop. Does the frictional force exerted by the road surface on the tires of the car do work?

When the car skids to a stop, it loses kinetic energy. A decrease in kinetic energy can be thought of as a negative change in kinetic energy. If the change in kinetic energy is negative, the work done on the car should also be negative.

Note that the frictional force exerted on the car acts in the *opposite* direction to the motion of the car, shown in figure 6.8. When this is so, we say that the work done on the car by the force is **negative work,** removing energy from the system (the car) rather than increasing its energy. For a frictional force of magnitude f, the work done is $W = -fd$, if the car moves a distance d while decelerating. Because frictional forces always oppose the direction of motion, work done by friction is always negative.

Stopping distance for a moving car

The kinetic energy of the car is proportional not to the speed but rather to the *square* of the speed. If we double the speed, the kinetic energy *quadruples*. Four times as much work must be done to reach the doubled speed as was done to reach the original speed. Likewise, if we stop the car, four times as much energy must be removed.

A practical application is the stopping distances of cars traveling at different speeds. The amount of negative work required to stop the car is equal to the kinetic energy of the car before the brakes are applied. This amount of energy must be removed from the system. Because kinetic energy is proportional to the square of the speed, the work required (and the stopping distance) increases rapidly with the speed of the car. For example, the kinetic energy is four times as large for a car traveling at 60 MPH as for one traveling at 30 MPH. Doubling the speed requires four times as much negative work to remove the kinetic energy. The stopping distance at 60 MPH will be four times that required at 30 MPH, because the work done is proportional to the distance (assuming the frictional force is constant).

In fact, the frictional force varies with the speed of the car. If you look at the stopping distances in driver-training manuals, you will see that they do indeed increase rapidly with speed, although not exactly in proportion to the square of the speed. The more kinetic energy present initially, the more negative work is required to reduce this energy to zero, and the greater the stopping distance.

> Kinetic energy is the energy associated with an object's motion, and it is equal to one-half the mass of the object times the square of its speed. The kinetic energy gained or lost by an object is equal to the work done by the net force accelerating or decelerating the object. Friction always acts as a force opposite to the direction of motion.

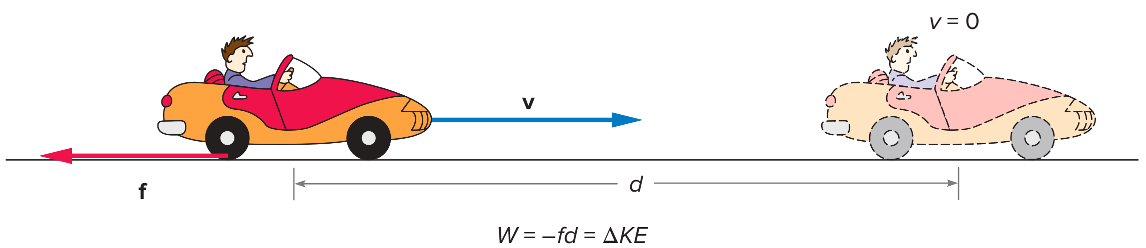


Figure 6.8 Frictional forces exerted on the car's tires by the road surface do negative work in stopping the car, resulting in a decrease in kinetic energy.

6.3 Potential Energy

Suppose we lift a crate to a higher position on a loading dock, as in figure 6.9. Work is done in this process, but no kinetic energy is gained if the crate ends up just sitting on the dock. Has the energy of the crate increased? What happens to the work done by the lifting force?

Drawing back a bowstring or compressing a spring is similar. Work is done, but no kinetic energy is gained. Instead, the **potential energy** of the system increases. How does potential energy differ from kinetic energy?

Gravitational potential energy

To lift the crate in figure 6.9, we need to apply a force that pulls or pushes upward on the crate. The applied force will not be the only force acting on the crate. The gravitational attraction of the Earth (the weight of the crate) pulls down on the crate. If we lift the crate with a force exactly equal to the force of gravity but opposite in direction, the net force acting on the crate will be zero, and the crate will not accelerate. We actually accelerate the crate a little bit at the start of the motion and decelerate it at the end of the motion, moving it with constant velocity during most of the motion.

The work done by the lifting force increases the **gravitational potential energy** of the crate. The lifting force and the gravitational force are equal in magnitude and opposite in direction, so the net force is zero and there is no acceleration. The lifting force does work by moving the object against the gravitational pull. If we let go of the rope, the crate will accelerate downward, gaining kinetic energy.

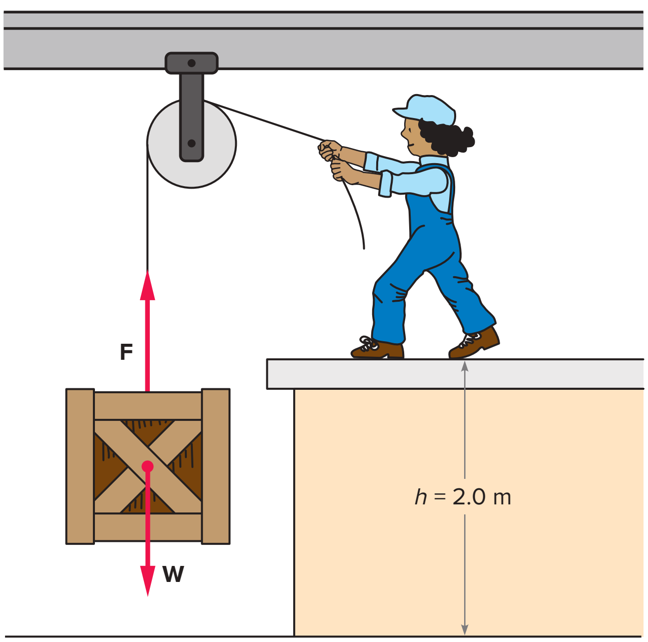


Figure 6.9 A rope and pulley are used to lift a crate to a higher position on the loading dock, resulting in an increase in potential energy.

How much gravitational potential energy is gained? The work done by the lifting force is equal to the size of the force times the distance moved. The applied force is equal to the weight of the crate mg. If the crate is moved a height h, the work done is mg times h or mgh. The gravitational potential energy is equal to the work done,

$$PE = mgh$$

where we use the abbreviation PE to represent potential energy.

The height h is the distance the crate moves above some reference level or position. In example box 6.3, we have chosen the original position of the crate on the ground to be our reference level. We usually choose the lowest point in the probable motion of the object as the reference level to avoid negative values of potential energy. The *changes* in potential energy are what is important, however, so the choice of reference level does not affect the physics of the situation.

The essence of potential energy

The term *potential energy* implies storing energy to use later for other purposes. Certainly, this feature is present in the situation just described. The crate could be left indefinitely higher up on the loading dock. If we pushed it off the dock, though, it would rapidly gain kinetic energy as it fell. The kinetic energy, in turn, could be used to compress objects lying underneath, to drive pilings into the ground, or for other useful mayhem (fig. 6.10). Kinetic energy also has this feature, however, so storing energy is not what distinguishes potential energy.

Potential energy involves *changing the position* of the object that is being acted on by a specific force. In the case of gravitational potential energy, that force is the

Example Box 6.3

Sample Exercise: Potential Energy

A crate with a mass of 100 kg is lifted onto a loading dock 2 m above ground level. How much potential energy has been gained?

$m = 100$ kg
$h = 2$ m
$g = 9.8$ m/s^2

$$\begin{aligned} PE &= mgh \\ &= (100\ \text{kg})(9.8\ \text{m/s}^2)(2\ \text{m}) \\ &= (980\ \text{N})(2\ \text{m}) \\ &= \mathbf{1960\ J} \end{aligned}$$

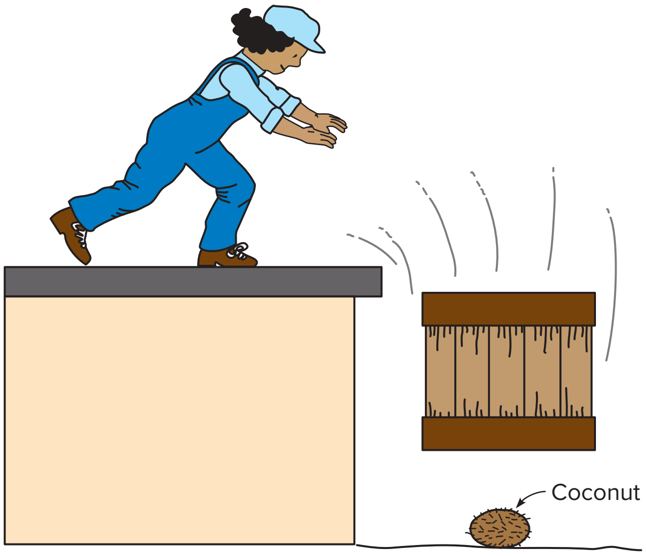


Figure 6.10 The potential energy of the raised crate can be converted to kinetic energy and used for other purposes.

gravitational attraction of the Earth. The farther we move the object away from the Earth, the greater the gravitational potential energy. Other kinds of potential energy involve different forces.

What is elastic potential energy?

What happens if we pull on a bowstring or stretch a spring? In these examples, work is done by an applied force against an opposing **elastic force,** a force that results from stretching or compressing an object. Imagine a spring attached to a post, as in figure 6.11, with a wooden block or similar object attached to the other end of the spring. If we pull the block from the original position where the spring was unstretched, the system gains *elastic potential energy.* If we let go, the block would fly back.

Because a force must be applied over some distance to move the block, work is done in pulling against the force exerted by the spring. Most springs exert a force proportional to the distance the spring is stretched. The more the spring is stretched, the greater the force. This can be stated in an equation by defining the **spring constant** k that describes the stiffness of the spring. A stiff spring has a large spring constant. The force exerted by the spring is given by the spring constant multiplied by the distance stretched, or

$$F = -kx$$

where x is the distance the spring is stretched, measured from its original, unstretched position. This is often called Hooke's Law, named after Robert Hooke (1635–1703). The minus sign indicates that the force exerted by the spring pulls back on the object as the object moves away from its equilibrium position. Thus, if the mass is moved to the right, the spring pulls back to the left. If the spring is compressed, it pushes back to the right.

How do we find the increase in potential energy of such a system? As before, we need to find the work done by the force involved in changing the position of the object. We want the block to move without acceleration so the net force acting on the block is zero. The applied force must be adjusted so it is always equal in magnitude but opposite in direction to the force exerted by the spring. This means the applied force must increase as the distance x increases (fig. 6.12).

The increase in **elastic potential energy** is equal to the work done by the average force needed to stretch the spring. Figure 6.12 suggests that the average force is one-half the magnitude of the final force kx. The work done is the average force $\frac{1}{2}kx$ times the distance x, so

$$PE = \frac{1}{2}kx^2$$

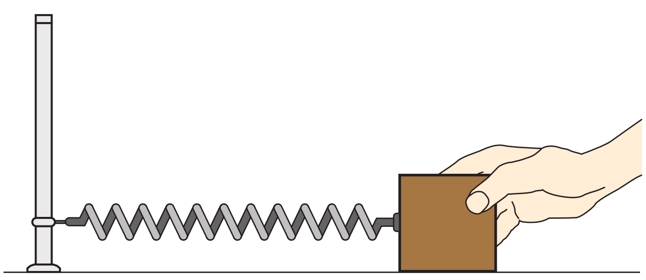


Figure 6.11 A wooden block is attached to a spring tied to a fixed support at the opposite end. Stretching the spring increases the elastic potential energy of the system.

Figure 6.12 The applied force used to stretch the spring varies with the distance stretched, going from an initial value of zero to a final value of kx. Work done is equal to the shaded area under the force-versus-distance curve.

The potential energy of the stretched-spring system is one-half the spring constant times the square of the distance stretched. The same expression is valid when the spring is compressed. The distance x is then the distance the spring is compressed from its original, relaxed position.

The potential energy stored in the spring can be converted to other forms and put to various uses. If we let go of the block when the spring is either stretched or compressed, the block will gain kinetic energy. Cocking a bow and arrow, squeezing a rubber ball, and stretching a rubber band are all familiar examples in which we generate elastic potential energy similar to the spring.

What are conservative forces?

Potential energy can result from work done against a variety of forces besides gravity and springs. Work done against frictional forces, however, does not result in an increase in the potential energy of the system. Instead, heat is generated, which either transfers energy out of the system or increases the internal energy of the system at the atomic level. As discussed in chapter 11, this internal energy cannot be completely recovered to do work on another object or system.

Forces such as gravity or elastic forces that lead to potential energy relationships are referred to as **conservative forces.** When work is done against conservative forces, the energy gained by the system is completely recoverable for use in other forms.

> Potential energy associated with some conservative force (such as gravity or the spring force) is an object's energy by virtue of its position rather than the object's motion. We find the potential energy by computing the work done to move the object against the conservative force. The system is then poised to release that energy, converting it to kinetic energy or work done on some other system.

6.4 Conservation of Energy

The concepts of work, kinetic energy, and potential energy are now available to us. How can they help explain what is happening in systems like a pendulum?

Conservation of energy is the key. The total energy, the sum of the kinetic and potential energies, is a quantity that remains constant (is conserved) in many situations. We can describe the motion of a pendulum by tracking the energy transformations. What can this tell us about the system?

Energy changes in the swing of a pendulum

Imagine a pendulum consisting of a ball initially hanging motionless at the end of a string attached to a rigid support. You pull the ball to the side and release it to start it swinging. What happens to the energy of the system?

In the first step, work is done on the ball by your hand. The net effect of this work is to increase the potential energy of the ball, because the height of the ball above the ground increases as the ball is pulled to the side. The work done transfers energy from the person doing the pulling to the system consisting of the pendulum and the Earth. It becomes gravitational potential energy, $PE = mgh$, where h is the height of the ball above its initial position (fig. 6.13).

When you release the ball, this potential energy begins to change to kinetic energy as the ball begins its swing. At the bottom of the swing (the initial position of the ball when it was just hanging), the potential energy is zero, and the kinetic energy reaches its maximum value. The ball does not stop at the low point; its motion continues to a point opposite the release point. During this part of the swing, the kinetic energy decreases, and the potential energy increases until it reaches the point where the kinetic energy is zero and the potential energy is equal to its initial value before release. The ball then swings back, repeating the transformation of potential energy to kinetic energy and back to potential energy (fig. 6.13).

What does it mean to say that energy is conserved?

As the pendulum swings, there is a continuing change of potential energy to kinetic energy and back again. The total mechanical energy of the system (the sum of the potential and kinetic energies) remains constant, because there is no work being done on the system to increase or decrease its energy. The swing of the pendulum demonstrates the principle of **conservation of energy:**

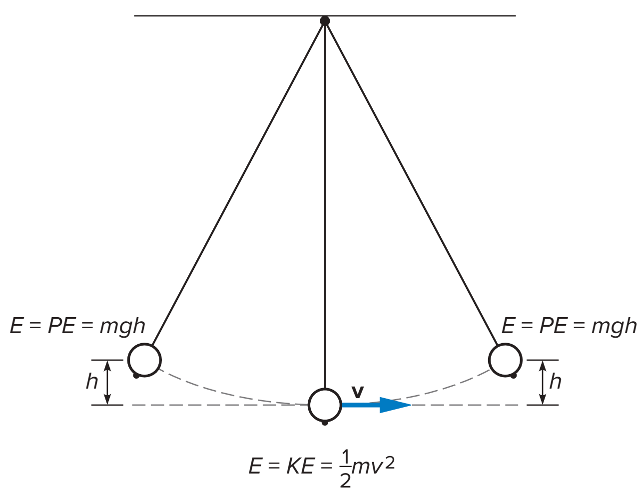


Figure 6.13 Potential energy is converted to kinetic energy and then back to potential energy as the pendulum swings back and forth.

When only conservative forces are involved, the total mechanical energy of the system (the sum of its kinetic energy and potential energy) remains constant.

Work is pivotal. If no energy is added or removed by forces doing work, the total energy should not change. In symbols, this statement takes this form:

$$\text{If } W = 0, \qquad E = PE + KE = \text{constant}$$

where E is the symbol commonly used to represent the total energy. A broader picture of the meaning of this very important principle is provided in everyday phenomenon box 6.1.

We applied conservation of energy in describing the motion of the pendulum. Some points deserve close attention: For example, why do we *not* consider the work done by gravity on the pendulum? The answer is that the gravitational force becomes part of the system by including the gravitational potential energy of the ball in our description. Gravity is a conservative force already accounted for by potential energy.

What other forces act on the ball? The tension of the string acts in a direction perpendicular to the motion of the ball (fig. 6.14). This force does no work, because it has no component in the direction of the motion. The only other force that need concern us is air resistance. This force does negative work on the ball, slowly decreasing the total mechanical energy of the system. The total energy of the system is not completely constant in this situation. It would be constant only if air resistance were negligible. The air-resistive effects are often small, however, and can be ignored.

Why do we use the concept of energy?

What are the advantages of using the principle of conservation of energy? Imagine trying to describe the motion of the pendulum by direct application of Newton's laws of motion. You would have to deal with forces that vary continually in direction and magnitude as the pendulum moves. A full description using Newton's laws is quite complex.

Using energy considerations, however, we can make predictions about the behavior of a system much more easily than by applying Newton's laws. To the extent that we can ignore frictional effects, for example, we can predict that the ball will reach the same height at either end of its swing. The kinetic energy is zero at the end points of the swing where the ball momentarily stops, and at these points, the total energy equals potential energy. If no energy has been lost, the potential energy has the same value it had at the point of release, which implies that the same height is reached ($PE = mgh$).

A demonstration sometimes performed in physics lecture rooms using a bowling ball as the pendulum bob illustrates this idea dramatically. The bowling ball is suspended from a support near the ceiling, so that when pulled to one side, the ball is near the chin of the physics instructor. The instructor pulls the ball to this position, releases it to allow it to swing across the room, and stands without flinching as the ball returns and stops just a few inches from his or her chin (fig. 6.15). Not flinching requires some faith in the principle of conservation of energy! The success of this demonstration depends on the ball's not being given any initial velocity when released. But what happens if it is pushed?

We can also use the principle of conservation of energy to predict what the speed will be at any point in the swing. The speed is zero at the end points and has its maximum

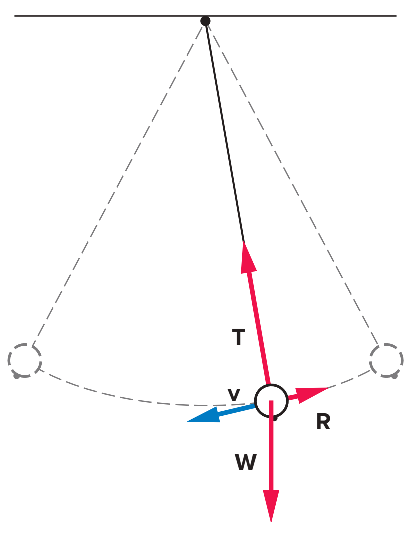


Figure 6.14 Of the three forces acting on the ball, only the force of air resistance does work on the system to change its total energy. The tension does no work, and the work done by gravity is already included in the potential energy.

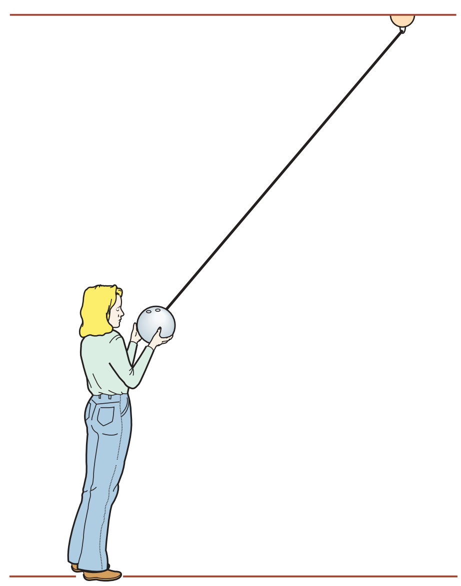


Figure 6.15 A bowling ball at the end of a cable suspended from the ceiling is released and allowed to swing across the room and back, stopping just in time.

Everyday Phenomenon Box 6.1

Conservation of Energy

The Situation. Mark Shoemaker had just come out of a physics lecture on the conservation of energy, and he was confused. His instructor had noted that the principle of conservation of energy in its most general form implies that energy can be neither created nor destroyed—it is always conserved.

On the other hand, Mark had been following news items regarding the need to conserve energy. If energy is always conserved, what is the problem that news commentators and environmentalists are harping on? Don't they understand physics?

The Analysis. When people talk about conserving energy in the context of everyday energy use, the term *conservation of energy* has a different meaning than that involved in the physics principle. Understanding these distinctions is critical to understanding the issues involving energy and the environment that have been hot topics in recent years.

The principle of conservation of energy stated in this section is limited to mechanical energy, the sum of the potential and kinetic energies of a mechanical system. In chapters 10 and 11, we will see that heat is also a form of energy and must be included in a more general statement of the conservation principle. Later, in chapter 20, we will find that even mass must be considered a form of energy. In this most general context, energy is always conserved.

However, not all forms of energy have equal value in our daily lives. To take an example, oil is an important energy resource. The energy stored in oil is a form of potential energy involving the electric forces that bind atoms together in molecules. When we use oil, we are converting this potential energy to other forms of energy, depending on the application.

To release this potential energy, we usually burn the oil. Burning generates heat, which is a form of energy. At high temperatures, this heat can be very useful for running heat engines of various kinds (discussed in chapter 11). The common gasoline engine in our cars is a heat engine, as are the diesel engines that power trucks and trains and the jet turbines that power airplanes. These engines transform some energy in the form of heat to kinetic energy for cars, trains, boats, and airplanes.

But what happens, ultimately, to the kinetic energy associated with the moving vehicles? Eventually, it is transformed to lower-temperature heat due to frictional effects in the engine and the tires and air resistance. The energy has not disappeared—it warms up the surroundings. However, it is now in a form that is much less useful than the original potential energy stored in the fuel. Heat at temperatures near those of the surroundings is sometimes called low-grade heat—its uses are limited to heating our homes or similar applications.

So what are we conserving when we talk about energy conservation in our daily lives? We are conserving high-value forms of energy by using them more wisely and, as much as possible, preventing them from being converted to less useful forms of energy. We are also limiting the environmental effects associated with burning oil and other fuels such as natural gas or coal. From the standpoint of physics, though, the energy itself is conserved in all situations.

If you have a choice of commuting to work or school by walking, riding a bicycle, or driving a fuel-efficient car as opposed to driving a large car or sports utility vehicle (SUV), your choice should be clear. By walking or riding a bicycle, as pictured in the photograph, you convert some energy obtained from the food you eat to low-grade heat, but much less high-value energy is converted than if you are driving an SUV or car. And, of course, your impact on the environment is much smaller when you are walking or riding a bicycle.

Later chapters in this book deal with many aspects of energy use. The laws of thermodynamics, discussed in chapters 10 and 11, are particularly important to understanding energy issues. Solar energy, geothermal power, and other methods of generating electricity are discussed in chapter 11. Chapter 13 addresses household electrical energy uses, while chapter 14 deals with electrical power generation, and chapter 19 discusses nuclear power.

The physics and economics of wise energy use are critical issues. To be involved in these debates, you should understand what energy conservation is all about. Energy can be neither created nor destroyed, but the ways in which it is transformed from one form to another are extremely important to the use of energy resources and to the environment.

Is energy conserved for all these commuters?
PhotoAlto

value at the low point of the swing. If we place our reference level for measuring potential energy at this low point, the potential energy will be zero there, because the height is zero. All of the initial potential energy has been converted to kinetic energy. Knowing the kinetic energy at the low point allows us to compute the speed, as shown in example box 6.4.

We could find the speed at any other point in the swing by setting the total energy at any point equal to the initial energy. Different values of the height h above the low point yield different values of the potential energy. The remaining energy must be kinetic energy. The system has only so much energy, either potential or kinetic energy, or some of both, but it cannot exceed the initial value.

How is energy analysis like accounting?

Both a roller coaster and a sled on a hill illustrate the principle of conservation of energy. Conservation of energy can be used to make predictions about the speed of the sled or roller coaster that would be hard to make by direct application of Newton's laws. An energy accounting provides a better overview. The pole-vaulting example in everyday phenomenon box 6.2 can also be analyzed in this way.

Consider the sled on the hill in figure 6.16. A parent pulls the sled to the top of the hill, doing work on the sled and rider that increases their potential energy. At the top of the hill, the parent may do more work by giving the sled a push, providing it with some initial kinetic energy. The total work done by the parent is the energy input to the system and equals the sum of the potential and kinetic energies shown in table 6.1.

Example Box 6.4

Sample Exercise: The Swing of a Pendulum

A pendulum bob with a mass of 0.50 kg is released from a position in which the bob is 12 cm above the low point in its swing. What is the speed of the bob as it passes through the low point in its swing?

$m = 0.5$ kg	The initial energy is
$h = 12$ cm	$E = PE = mgh$
$v = ?$	$= (0.5\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m})$
(at the low point)	$= 0.588$ J

At the low point, the potential energy is zero, so

$$E = KE = 0.588\text{ J}$$

$$\frac{1}{2}mv^2 = 0.588\text{ J}$$

Dividing both sides by $\frac{1}{2}m$,

$$v^2 = \frac{KE}{\frac{1}{2}m} = \frac{(0.588\text{ J})}{\frac{1}{2}(0.5\text{ kg})} = 2.35\text{ m}^2/\text{s}^2$$

Taking the square root of both sides,

$v = \mathbf{1.53\ m/s}$

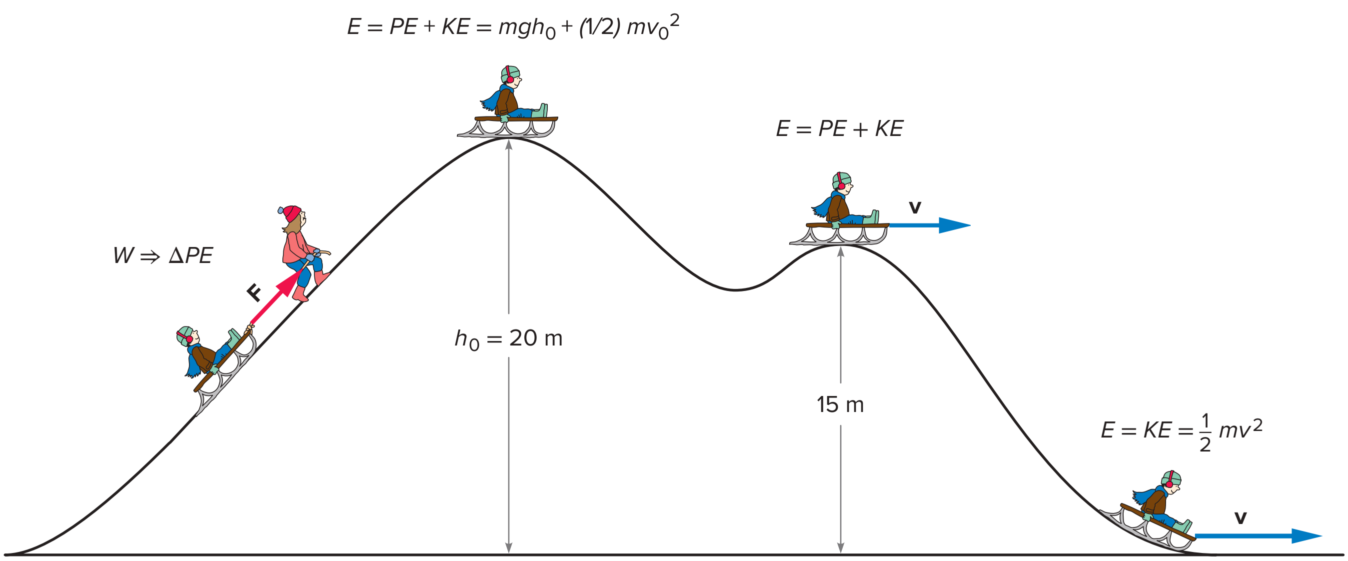


Figure 6.16 Work done in pulling a sled up a hill produces an increase in potential energy of the sled and rider. This initial energy is then converted to kinetic energy as they slide down the hill.

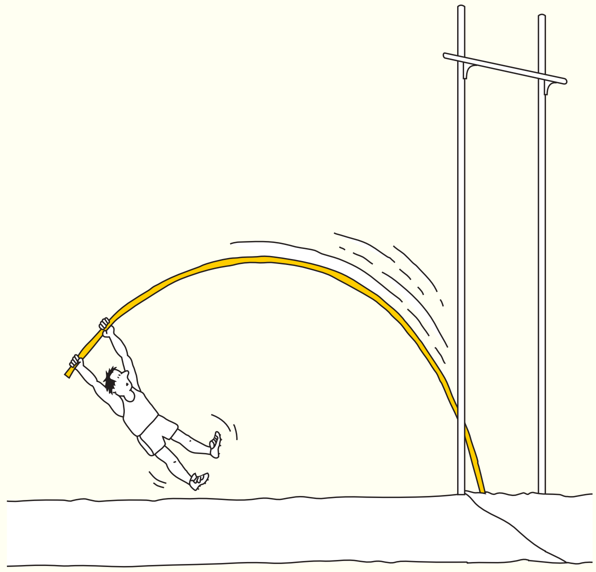
Everyday Phenomenon Box 6.2

Energy and the Pole Vault

The Situation. Ben Lopez goes out for track. He specializes in the pole vault and helps out sometimes with the sprint relays, where his speed can be used to good advantage. His coach, aware that Ben is also taking an introductory physics course, suggests that Ben try to understand the physics of the pole vault. What factors determine the height reached? How can he optimize these factors?

The coach knows that energy considerations are important in the pole vault. What types of energy transformation are involved? Could understanding these effects help Ben's performance?

A pole-vaulter on the way up. What energy transformations are taking place?
Patrik Giardino/Getty Images



The flexibility of the pole and the point at which the vaulter grasps the pole are important to the success of the vault.

The Analysis. It was not difficult for Ben to describe the energy transformations that take place in the pole vault: The vaulter begins by running down a path to the vaulting standard and pit. During this phase, he is accelerating and increasing his kinetic energy at the expense of chemical energy stored in his muscles. When he reaches the standard, he plants the end of the pole in a notch in the ground. At this point, some of his kinetic energy is stored in the elastic potential energy of the bent pole, which acts as a spring. The rest is converted to gravitational potential energy as he begins to rise over the standard.

Near the top of the vault, the elastic potential energy in the bent pole converts to gravitational potential energy as the pole straightens out. The vaulter does some additional work with his arm and upper-body muscles to provide an extra boost. At the very top of his flight, his kinetic energy should be zero, with only a minimal horizontal velocity left to carry him over the standard. Too large a kinetic energy at this point would indicate that he had not optimized his jump by converting as much energy as possible into gravitational potential energy.

What can Ben learn from his analysis? First is the importance of speed. The more kinetic energy he generates during his approach, the more energy is available for conversion to gravitational potential energy (mgh), which will largely determine the height of his vault. Successful pole-vaulters are usually good sprinters.

The characteristics of the pole and Ben's grip on it are also important factors. If the pole is too stiff, or if he has gripped it too close to the bottom, he will experience a jarring impact in which little useful potential energy is stored in the pole, and some of his initial kinetic energy will be lost in the collision. If the pole is too limber, or if Ben's grip is too far from the bottom, it will not spring back soon enough to provide useful energy at the top of his vault.

Finally, upper-body strength is important in clearing the standard. Good upper-body conditioning should improve Ben's pole vaulting. Timing and technique are also critical and can be improved only through practice. As far as his coach is concerned, that may be the most important message.

Table 6.1

Energy Balance Sheet for the Sled

A parent pulls a sled and rider with a combined weight of 50 kg to the top of a hill 20 m high and then gives the sled a push, providing an initial velocity of 4 m/s. Frictional forces acting on the sled do 2000 J of negative work as the sled moves down the hill.

Energy input	
Potential energy gained by work done in pulling sled up the hill:	
$PE = mgh = (50\ \text{kg})(9.8\ \text{m/s}^2)(20\ \text{m})$	9800 J
Kinetic energy gained by work done in pushing the sled at the top:	
$KE = \frac{1}{2}mv^2 = \frac{1}{2}(50\ \text{kg})\ (4\ \text{m/s})^2$	400 J
Total initial energy:	10,200 J
Energy expenditures	
Work done against friction as the sled slides down the hill:	
$W = -fd$	−2000 J
Energy balance:	8200 J

Where did this initial energy come from? It came from the body of the parent doing the pulling and pushing. Muscle groups were activated, releasing chemical potential energy stored in the body. That energy came from food, which in turn involved solar energy stored by plants. A parent who does not eat a good breakfast, or attempts too many trips up the hill, may not have enough energy to get to the top.

If the sled and rider slide down the hill with negligible friction and air resistance, energy is conserved, and the total energy at any point during the motion should equal the initial energy. It is more realistic to assume that there is some friction as the sled slides down the hill (fig. 6.17). Although it is difficult to predict the amount of work done against friction precisely, we can make an estimate if we know the total distance traveled and make some assumptions about the size of the average frictional force. In the energy accounting done in table 6.1, we assume that 2000 J of work has been done against friction by the time the sled reaches the bottom of the hill.

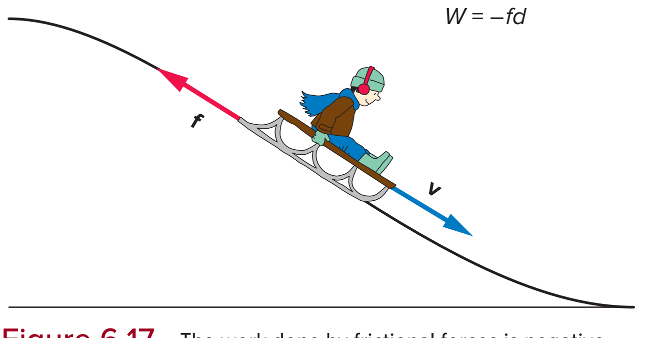


Figure 6.17 The work done by frictional forces is negative, and it removes mechanical energy from the system.

The work done against friction removes energy from the system and shows up as an expenditure on the account sheet. The energy balance at the bottom of the hill is 8200 J rather than 10,200 J. This will lead to a smaller, more realistic value for the speed of the sled and rider at the bottom of the hill than if we ignored friction. Although precise calculations are not always possible, energy accounting sets limits on what is likely and helps us understand the behavior of systems such as the sled on the hill.

> Energy is the currency of the physical world; an understanding of energy accounting is relevant to both science and economics. Doing work on a system puts energy in the bank. Total energy is then conserved, provided that only conservative forces are at work. Many aspects of the motion of the system can be predicted from a careful energy accounting.

Debatable Issue

Does the bicycle rider pictured in the photograph in everyday phenomena box 6.1 use any energy in his commute? If so, where does this energy come from, and how can we claim that he is conserving energy by commuting by bike?

6.5 Springs and Simple Harmonic Motion

If conservation of energy explains the motion of a pendulum, what about other systems that oscillate? Many systems involve springs or elastic bands that move back and forth, with potential energy being converted to kinetic energy and then back to potential energy repeatedly. What do such systems have in common? What makes them tick?

A mass on the end of a spring is one of the simplest oscillating systems. This system, and the simple pendulum described in section 6.4, are examples of *simple harmonic motion.*

Oscillation of a mass attached to a spring

If we attach a block to the end of a spring, as in figure 6.18, what happens when we pull it to one side of its equilibrium position? The equilibrium position is where the spring is neither stretched nor compressed. Doing work to pull the mass against the opposing force of the spring increases the potential energy of the spring-mass system. The potential energy in this case is elastic potential energy, $\frac{1}{2}kx^2$, rather than the gravitational potential energy associated with the pendulum. Increasing the potential energy of the mass on the spring is similar to cocking a bow and arrow or slingshot.

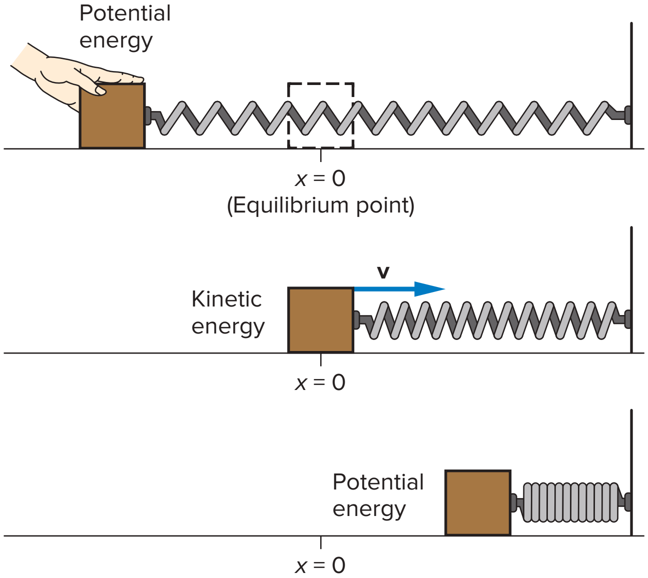


Figure 6.18 Energy added by doing work to stretch the spring is then transformed back and forth between the potential energy of the spring and the kinetic energy of the mass.

Once the mass is released, potential energy is converted to kinetic energy. As for the pendulum, the motion of the mass carries it beyond the equilibrium position, and the spring is compressed, gaining potential energy again. When the kinetic energy is completely reconverted to potential energy, the mass stops and reverses, and the whole process repeats (fig. 6.18). The energy of the system changes continually from potential energy to kinetic energy and back again. If frictional effects can be ignored, the total energy of the system remains constant while the mass oscillates back and forth.

Using a video camera or other tracking techniques, it is possible to measure and plot the position of a pendulum bob or mass on a spring as it varies with time. If we plot the position of the mass against time, the resulting curve takes the form shown in figure 6.19. The mathematical functions that describe such curves are called "harmonic" functions, and the motion is called **simple harmonic motion,*** a term probably borrowed from musical descriptions of sounds produced by vibrating strings, reeds, and air columns. (See chapter 15.)

The line at zero on the graph in figure 6.19 is the equilibrium position for the mass on a spring. Points above this line represent positions on one side of the equilibrium point, and those below the line represent positions on the other side. The motion starts at the point of release, where the distance of the mass from equilibrium is a maximum. As the mass moves toward the equilibrium position ($x = 0$ on the graph), it gains speed, indicated by the increasing slope of the curve. (See section 2.4.) The object's position changes most rapidly when it is near the equilibrium point, where the kinetic energy and speed are the greatest.

*If you have studied trigonometry, you may know that the curve plotted in figure 6.19 is a cosine function. Sines and cosines are collectively referred to as harmonic functions.

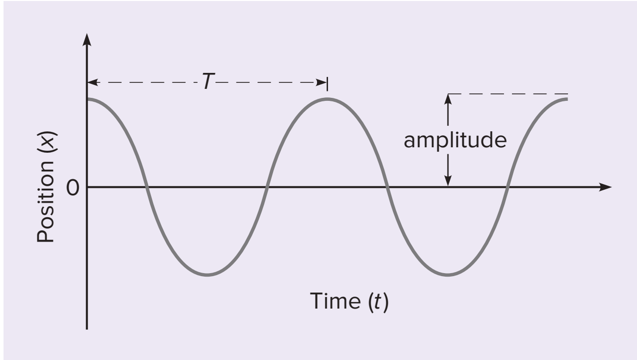


Figure 6.19 The horizontal position x of the mass on the spring is plotted against time as the mass moves back and forth. The resulting curve is a harmonic function.

As the mass passes through the equilibrium position, it starts to move away from equilibrium in the direction opposite to its initial position. The force exerted by the spring is now in the direction opposite to the velocity and is decelerating the mass. When the mass reaches the point farthest from its release point, the speed and kinetic energy are again zero, and the potential energy has returned to its maximum value. (See example box 6.5.) The slope of the curve is zero at this point, indicating that the mass is momentarily stopped (its velocity is zero). The mass continually gains or loses speed as it moves back and forth.

Example Box 6.5

Sample Exercise: Motion of a Mass on a Spring

A 500-g (0.50-kg) mass is undergoing simple harmonic motion at the end of a spring with a spring constant of 800 N/m. The motion takes place on a frictionless horizontal surface as pictured in figure 6.18. The speed of the mass is 12 m/s when it passes through the equilibrium point.

a. What is the kinetic energy at the equilibrium point?
b. How far does the mass travel from the equilibrium point before it turns around?

a. $m = 0.50\text{ kg}$ $\quad KE = \frac{1}{2}mv^2$

$v = 12\text{ m/s}$ $\quad KE = \frac{1}{2}(0.50\text{ kg})(12\text{ m/s})^2$

$KE = ?$ $\quad KE = \mathbf{36\ J}$

b. $x = ?$ $\quad E = KE + PE = 36\text{ J}$

(when $v = 0$) $\quad$ but KE = 0 at the turn-around point

$$\text{so } PE = \tfrac{1}{2}kx^2 = 36\text{ J}$$

$$x^2 = \frac{2(36\text{ J})}{k}$$

$$x^2 = \frac{72\text{ J}}{800\text{ N/m}}$$

$$x^2 = 0.09\text{ m}^2$$

$$x = 0.30\text{ m} = \mathbf{30\ cm}$$

What are the period and the frequency?

If you look at the graph in figure 6.19, you will notice that the curve repeats itself regularly. The **period** T is the repeat time, or the time taken for one complete cycle. It is usually measured in seconds. You can think of the period as the time between adjacent peaks or valleys on the curve. A slowly oscillating system has a long period, and a rapidly oscillating system has a short period.

Suppose the period of oscillation for a certain spring and mass is half a second. There are then two oscillations each second, which is the **frequency** of oscillation. The frequency f is the number of cycles per unit time, and it is found by taking the reciprocal of the period, $f = 1/T$. A rapidly oscillating system has a very short period and thus a high frequency. The unit commonly used for frequency is the *hertz,* which is defined as one cycle per second.

What determines the frequency of the spring-mass system? Intuitively, we expect a loose spring to have a low frequency of oscillation and a stiff spring to have a high frequency. This is indeed the case. The mass attached to the spring also has an effect. Larger masses offer greater resistance to a change in motion, producing lower frequencies.

The period and frequency of oscillation of a pendulum depend primarily on its length, measured from the pivot point to the center of the bob. To measure the period, you usually measure the time required for several complete swings and then divide by the number of swings to get the time for one swing.

Simple experiments with a ball on a string will give you an idea of how the period and frequency change with length. Try it and see if you can find a trend (see home experiment 1). The motion is regular—you can keep time by the swing of a pendulum or the motion of a mass on a spring.

Will any restoring force produce simple harmonic motion?

When a mass attached to a spring is moved to either side of equilibrium, the spring exerts a force that pulls or pushes the mass back toward the center. We call such a force the **restoring force.** In this case, it is the elastic force exerted by the spring. In any oscillation, there must be some such restoring force.

As discussed in section 6.3, the spring force is directly proportional to the distance x of the mass from its equilibrium position ($F = -kx$). The spring constant k has units of newtons per meter (N/m). Simple harmonic motion results whenever the restoring force has this simple dependence on distance. If the force varies in a more complicated way with distance, we may get an oscillation but not simple harmonic motion, and it will not produce a simple harmonic curve (fig. 6.19).

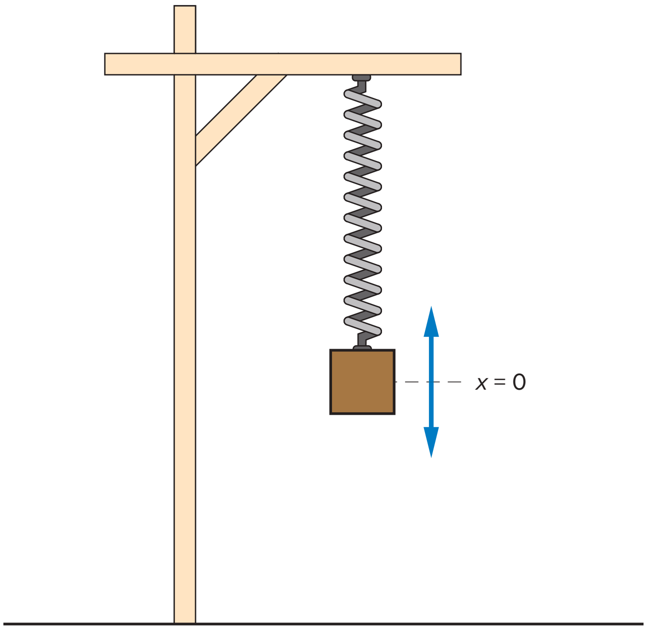


Figure 6.20 A mass hanging from a spring will oscillate up and down with the same period as a horizontal mass-spring system using the same spring and mass.

It is generally easiest to set up a spring-mass system by suspending the spring from a vertical support and hanging a mass on the end of the spring, as in figure 6.20. This arrangement avoids the frictional forces of the tabletop in a horizontal arrangement. In the vertical setup, when the mass is pulled down and released, the system oscillates up and down rather than horizontally. Two forces then act on the mass, the spring force pulling upward and the gravitational force pulling downward.

Because the gravitational force in the vertical setup is constant, it simply moves the equilibrium point lower. The equilibrium point is where the net force is zero—the downward pull of gravity is balanced by the upward pull of the spring. The variations in the restoring force are still provided by the spring. These variations are proportional to the distance from equilibrium just as they are in the horizontal case. This system also meets the condition for simple harmonic motion. The potential energy involved, however, is the sum of the gravitational and elastic potential energies.

Gravity is the restoring force for the simple pendulum. When the pendulum bob is pulled to one side of its equilibrium position, the gravitational force acting on the bob pulls it back toward the center. The part of the gravitational force in the direction of motion is proportional to the displacement, if the displacement from equilibrium is not too large. Thus, for small *amplitudes* of swing, the simple pendulum also displays simple harmonic motion. **Amplitude** is the maximum distance from the equilibrium point.

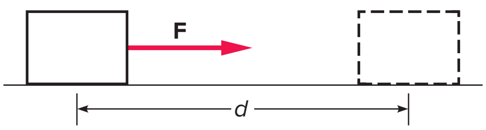
Look around for systems that oscillate. There are many examples, ranging from a springy piece of metal to a ball rolling in a depression of some kind. What force pulls back toward the equilibrium position in each case? Is the motion likely to be simple harmonic motion or a more complicated oscillation? What kind of potential energy is involved? The analysis of vibrations such as these forms an important subfield of physics that plays a role in music, communications, analysis of structures, and other areas.

> Any oscillation involves a continuing interchange of potential and kinetic energies. If there are no frictional forces removing energy from the system, the oscillation will go on indefinitely. A restoring force that increases in direct proportion to the distance from the equilibrium position results in simple harmonic motion, with simple curves (harmonic functions) describing the position, velocity, and acceleration of the object over time.

Summary

The concept of work is central to this chapter. Energy is transferred into a system by doing work on the system, which can result in an increase in either the kinetic energy or the potential energy of the system. If no additional work is done on the system, the total energy of the system remains constant. This principle of conservation of energy allows us to explain many features of the behavior of the system.

1 Simple machines, work, and power. Work is defined as force times the distance involved in moving an object. Only the portion of the force in the direction of the motion is used. In simple machines, work output cannot exceed work input. Power is the rate of doing work.

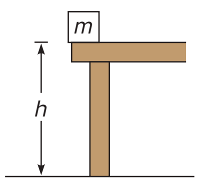


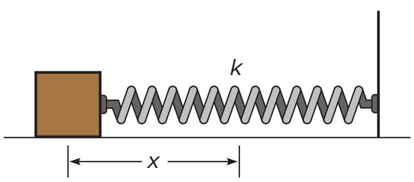
$W = Fd, \qquad P = \frac{W}{t}$

2 Kinetic energy. The work done by the net force acting on an object is used to accelerate the object, and the object gains kinetic energy. Kinetic energy is equal to one-half the mass of the object times the square of its speed. Negative work removes kinetic energy.

$KE = \frac{1}{2}mv^2$

3 Potential energy. If work done on an object moves the object against an opposing conservative force, the potential energy of the object is increased. Two types of potential energy were considered: gravitational potential energy and elastic potential energy.

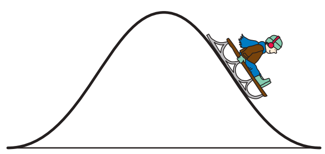




$PE = mgh$
(gravitational)

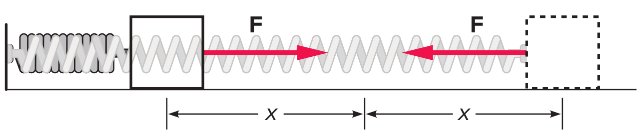
$PE = \frac{1}{2}kx^2$
(elastic)

4 Conservation of energy. If no work is done on a system, the total mechanical energy (kinetic plus potential) remains constant. This principle of conservation of energy explains the behavior of many systems that involve exchanges of kinetic and potential energy. The system can be analyzed by energy accounting.



$E = KE + PE = \text{constant}$

5 Springs and simple harmonic motion. The motions of a simple pendulum and of a mass on a spring both illustrate the principle of conservation of energy, but they involve different kinds of potential energy. They are also examples of simple harmonic motion, which results whenever the restoring force is proportional to the distance of the object from its equilibrium position.



$F = -kx$

> The mastery quizzes in Connect will help with your conceptual understanding of many of the ideas we have introduced.

Key Terms

Simple machine, 103
Mechanical advantage, 104
Work, 104
Power, 106
Kinetic energy, 106
Negative work, 107
Potential energy, 108
Gravitational potential energy, 108
Elastic force, 109
Spring constant, 109
Elastic potential energy, 109
Conservative forces, 110
Conservation of energy, 110
Simple harmonic motion, 116
Period, 117
Frequency, 117
Restoring force, 117
Amplitude, 117

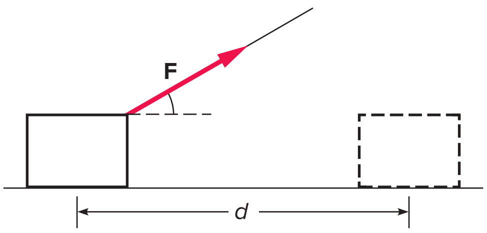
Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available in Connect

Q1. Equal forces are used to move blocks A and B across the floor. Block A has twice the mass of block B, but block B moves twice the distance moved by block A. Which block, if either, has the greater amount of work done on it? Explain.

Q2. A man pushes very hard for several seconds on a heavy rock, but the rock does not budge. Has the man done any work on the rock? Explain.

Q3. A string is used to pull a wooden block across the floor without accelerating the block. The string makes an angle to the horizontal as shown in the diagram.
a. Does the force applied via the string do work on the block? Explain.
b. Is the total force or just a portion of the force involved in doing work? Explain.



Q3 Diagram

Q4. In the situation pictured in question 3, if there is a frictional force opposing the motion of the block, does this frictional force do work on the block? Explain.

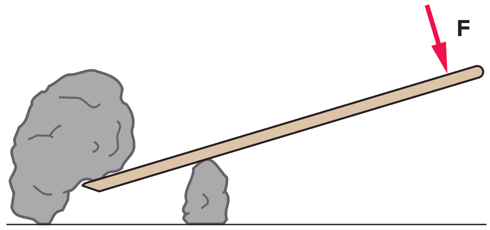
Q5. In the situation pictured in question 3, does the normal force of the floor pushing upward on the block do any work? Explain.

Q6. A ball is being twirled in a circle at the end of a string. The string provides the centripetal force needed to keep the ball moving in the circle at constant speed. Does the force exerted by the string on the ball do work on the ball in this situation? Explain.

*Q7. A man slides across a wooden floor. What forces act on the man during this process? Which, if any, of these forces do work on the man? Explain.

Q8. A woman uses a pulley arrangement to lift a heavy crate. She applies a force that is one-fourth the weight of the crate but moves the rope a distance four times the height that the crate is lifted. Is the work done by the woman greater than, equal to, or less than the work done by the rope on the crate? Explain.

Q9. A lever is used to lift a rock, as shown in the diagram. Will the work done by the person on the lever be greater than, equal to, or less than the work done by the lever on the rock? Explain.



Q9 Diagram

*Q10. A crate on rollers is pushed up an inclined plane into a truck. The pushing force required is less than half the force that would be needed to lift the crate straight up into the truck. Does the inclined plane serve as a simple machine in this situation? Explain.

Q11. A boy pushes his friend across a skating rink. Because the frictional forces are very small, the force exerted by the boy on his friend's back is the only significant force acting on the friend in the horizontal direction. Is the change in kinetic energy of the friend greater than, equal to, or less than the work done by the force exerted by the boy? Explain.

Q12. A child pulls a block across the floor with force applied by a horizontally held string. A smaller frictional force also acts upon the block, yielding a net force on the block that is smaller than the force applied by the string. Does the work done by the force applied by the string equal the change in kinetic energy in this situation? Explain.

Q13. If there is just one force acting on an object, does its work necessarily result in an increase in kinetic energy? Explain.

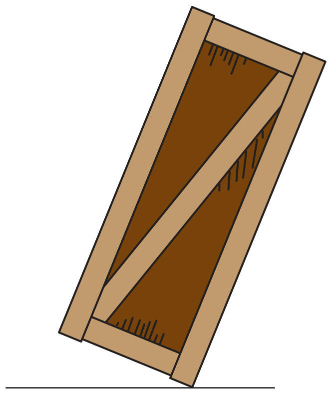
Q14. Two balls of the same mass are accelerated by different net forces such that one ball gains a velocity twice that of the other ball in the process. Is the work done by the net force acting on the faster-moving ball twice that done on the slower-moving ball? Explain.

Q15. A box is moved from the floor up to a tabletop but gains no speed in the process. Is there work done on the box, and if so, what has happened to the energy added to the system?

Q16. When work is done to increase the potential energy of an object without increasing its kinetic energy, is the *net* force acting on the object greater than zero? Explain.

Q17. Is it possible for a system to have energy if nothing is moving in the system? Explain.

Q18. Suppose that work is done on a large crate to tilt the crate, so that it is balanced on one edge, as shown in the diagram, rather than sitting squarely on the floor as it was at first. Has the potential energy of the crate increased in this process? Explain.



Q18 Diagram

*Q19. Which has the greater potential energy: a ball that is 10 feet above the ground or a ball with the same mass that is 20 feet above the bottom of a nearby 50-foot-deep well? Explain.

Q20. When a bow and arrow are cocked, a force is applied to the string in order to pull it back. Is the energy of the system increased? Explain.

Q21. Suppose the physics instructor pictured in figure 6.15 gives the bowling ball a push as she releases it. Will the ball return to the same point or will her chin be in danger? Explain.

Q22. A pendulum is pulled back from its equilibrium (center) position and then released.
  a. What form of energy is added to the system prior to its release? Explain.
  b. At what points in the motion of the pendulum after release is its kinetic energy the greatest? Explain.
  c. At what points is the potential energy the greatest? Explain.

Q23. For the pendulum in question 22, when the pendulum bob is halfway between the high point and the low point in its swing, is the total energy kinetic energy, potential energy, or both? Explain.

Q24. Is the total mechanical energy conserved in the motion of a pendulum? Will it keep swinging forever? Explain.

Q25. A sports car accelerates rapidly from a stop and "burns rubber." (See everyday phenomenon box 6.1.)
  a. What energy transformations occur in this situation?
  b. Is energy conserved in this process? Explain.

Q26. A man commutes to work in a large SUV. (See everyday phenomenon box 6.1.)
  a. What energy transformations occur in this situation?
  b. Is mechanical energy conserved in this situation? Explain.
  c. Is energy of all forms conserved in this situation? Explain.

Q27. Suppose we burn a barrel of oil just to warm our hands on a cold day. (See everyday phenomenon box 6.1.)
  a. From the standpoint of physics, is energy conserved in this process? Explain.
  b. Why is this a bad idea from an economic or environmental standpoint? Explain.

*Q28. A bird grabs a clam, carries it in its beak to a considerable height, and then drops it on a rock below, breaking the clam shell. Describe the energy conversions that take place in this process.

Q29. In pole vaulting, is the elastic potential energy stored in the pole the only type of potential energy involved? Explain. (See everyday phenomenon box 6.2.)

Q30. If one pole-vaulter can run faster than another, will the faster runner have an advantage in the pole vault? Explain. (See everyday phenomenon box 6.2.)

Q31. A mass attached to a spring, which in turn is attached to a wall, is free to move on a friction-free, horizontal surface. The mass is pulled back and then released.
  a. What form of energy is added to the system prior to the release of the mass? Explain.
  b. At what points in the motion of the mass after its release is its potential energy the greatest? Explain.
  c. At what points is the kinetic energy the greatest? Explain.

Q32. Suppose that the mass in question 31 is halfway between one of the extreme points of its motion and the center point. In this position, is the energy of the system kinetic energy, potential energy, or a combination of these forms? Explain.

*Q33. A spring gun is loaded with a rubber dart. The gun is cocked, then fired at a target on the ceiling. Describe the energy transformations that take place in this process.

Q34. Suppose that a mass is hanging vertically at the end of a spring. The mass is pulled downward and released to set it into oscillation. Is the potential energy of the system increased or decreased when the mass is lowered? Explain.

*Q35. A sled is given a push at the top of a hill. Is it possible for the sled to cross a hump in the hill that is higher than its starting point under these circumstances? Explain.

*Q36. Can work done by a frictional force ever increase the total mechanical energy of a system? (Hint: Consider the acceleration of an automobile.) Explain.

Q37. Suppose a pulley system is used to lift a heavy crate, but the pulleys have rusted and there are frictional forces acting on the pulleys. Will the useful work output of this system be greater than, equal to, or less than the work input? Explain.

Exercises

For the exercises in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8\ \text{m/s}^2$ for the acceleration due to gravity.

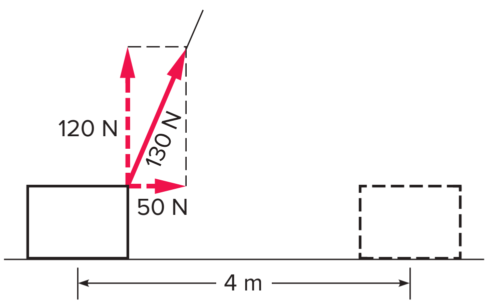
E1. A horizontally directed force of 40 N is used to pull a box a distance of 1.5 m across a tabletop. How much work is done by the 40-N force?

E2. A woman does 210 J of work to move a table 1.4 m across the floor. What is the magnitude of the force that the woman applied to the table if this force is applied in the horizontal direction?

E3. A force of 80 N used to push a chair across a room does 320 J of work. How far does the chair move in this process?

E4. A rope applies a horizontal force of 180 N to pull a crate a distance of 2 m across the floor. A frictional force of 70 N opposes this motion.
a. What is the work done by the force applied by the rope?
b. What is the work done by the frictional force?
c. What is the total work done on the crate?

E5. A force of 130 N is used to drag a crate 4 m across a floor. The force is directed at an angle upward from the crate, so that the vertical component of the force is 120 N and the horizontal component is 50 N, as shown in the diagram.
a. What is the work done by the horizontal component of the force?
b. What is the work done by the vertical component of the force?
c. What is the total work done by the 130-N force?



E5 Diagram

E6. A net force of 60 N accelerates a 4-kg mass over a distance of 15 m.
a. What is the work done by this net force?
b. What is the increase in kinetic energy of the mass?

E7. A 0.3-kg ball has a velocity of 20 m/s.
a. What is the kinetic energy of the ball?
b. How much work would be required to stop the ball?

E8. A box with a mass of 7.0 kg is lifted (without acceleration) through a height of 1.5 m, in order to place it on the shelf of a closet.
a. What is the increase in potential energy of the box?
b. How much work was required to lift the box to this position?

E9. A leaf spring in an off-road truck with a spring constant *k* of 87.6 kN/m (87,600 N/m) is compressed a distance of 6.2 cm (0.062 m) from its original, unstretched position. What is the increase in potential energy of the spring (in kJ)?

E10. To stretch a spring a distance of 0.30 m from the equilibrium position, 180 J of work is done.
a. What is the increase in potential energy of the spring?
b. What is the value of the spring constant *k* of the spring?

E11. A 5-kg rock is going to be moved either vertically or horizontally.
a. What is the increase in the potential energy of the rock when lifting it to a height of 1.8 m without acceleration?
b. What is the increase in kinetic energy to accelerate the same rock horizontally from rest to a speed of 6 m/s?
c. Which requires more work: lifting the rock 1.8 m or accelerating it horizontally to a speed of 6 m/s?

E12. At the low point in its swing, a pendulum bob with a mass of 0.15 kg has a velocity of 3 m/s.
a. What is its kinetic energy at the low point?
b. What is its kinetic energy at the high point?
c. What is its potential energy at the high point (assuming the potential energy at the low point was zero)?
d. Ignoring air resistance, how high will the bob swing above the low point before reversing direction?

E13. A 0.40-kg mass attached to a spring is pulled back horizontally across a table, so that the potential energy of the system is increased from zero to 150 J. Ignoring friction, what is the kinetic energy of the system after the mass is released and has moved to a point where the potential energy has decreased to 60 J?

E14. A sled and rider with a combined mass of 70 kg are at the top of a hill that rises 9 m above the level ground below. The sled is given a push, providing an initial kinetic energy at the top of the hill of 1200 J.
a. Choosing a reference level at the bottom of the hill, what is the potential energy of the sled and rider at the top of the hill?
b. After the push, what is the total mechanical energy of the sled and rider at the top of the hill?
c. If friction can be ignored, what will be the kinetic energy of the sled and rider at the bottom of the hill?

E15. A roller-coaster car has a potential energy of 400,000 J (400 kJ) and a kinetic energy of 130,000 J (130 kJ) at point A in its travel. At the low point of the ride, the potential energy is zero, and 60,000 J (60 kJ) of work has been done against friction since it left point A. What is the kinetic energy of the roller coaster at this low point (in kJ)?

E16. A roller-coaster car with a mass of 900 kg starts at rest from a point 22 m above the ground. At point B, it is 8 m above the ground. [Express your answers in kilojoules (kJ).]
a. What is the initial potential energy of the car?
b. What is the potential energy at point B?
c. If the initial kinetic energy was zero and the work done against friction between the starting point and point B is 30,000 J (30 kJ), what is the kinetic energy of the car at point B?

E17. A 300-g mass lying on a frictionless table is attached to a horizontal spring with a spring constant of 500 N/m. The spring is stretched a distance of 46 cm (0.46 m).
a. What is the initial potential energy of the system?
b. What is the kinetic energy of the system when the mass returns to the equilibrium position after being released?

E18. The time required for one complete cycle of a mass oscillating at the end of a spring is 0.40 s. What is the frequency of oscillation?

E19. The frequency of oscillation of a pendulum is 16 cycles/s. What is the period of oscillation?

Synthesis Problems

For the synthesis problems in this chapter (and subsequent chapters), use the more accurate value of $g = 9.8$ m/s^2 for the acceleration due to gravity.

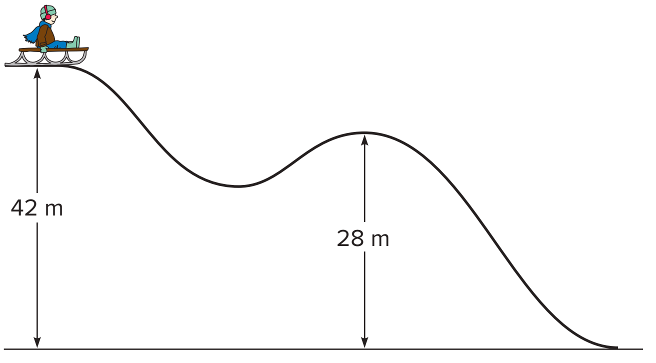
SP1. Suppose that two horizontal forces are acting upon a 0.38-kg wooden block as it moves across a laboratory table: a 6-N force pulling the block and a 2-N frictional force opposing the motion. The block moves a distance of 1.7 m across the table.
a. What is the work done by the 6-N force?
b. What is the work done by the net force acting upon the block?
c. Which of these two values should you use to find the increase in kinetic energy of the block? Explain.
d. What happens to the energy added to the system via the work done by the 6-N force? Can it all be accounted for? Explain.
e. If the block started from rest, what are its kinetic energy and velocity at the end of the 1.7-m motion?

SP2. As described in example box 6.2, a 120-kg crate is accelerated by a net force of 96 N applied for 4 s.
a. What is the acceleration of the crate from Newton's second law?
b. If it starts from rest, how far does it travel in the time of 4 s? (See section 2.5 in chapter 2.)
c. How much work is done by the 96-N net force?
d. What is the velocity of the crate at the end of 4 s?
e. What is the kinetic energy of the crate at this time? How does this value compare to the work computed in part c?

SP3. A slingshot consists of a rubber strap attached to a Y-shaped frame, with a small pouch at the center of the strap to hold a small rock or other projectile. The rubber strap behaves much like a spring. Suppose that for a particular slingshot a spring constant of 700 N/m is measured for the rubber strap. The strap is pulled back approximately 30 cm (0.3 m) prior to being released.
a. What is the potential energy of the system prior to release?
b. What is the maximum possible kinetic energy that can be gained by the rock after release?
c. If the rock has a mass of 40 g (0.04 kg), what is its maximum possible velocity after release?
d. Will the rock actually reach these maximum values of kinetic energy and velocity? Does the rubber strap gain kinetic energy? Explain.

SP4. Suppose that a 300-g (0.30-kg) mass is oscillating at the end of a spring on a horizontal surface that is essentially friction-free. The spring can be both stretched and compressed and has a spring constant of 400 N/m. It was originally stretched a distance of 12 cm (0.12 m) from its equilibrium (unstretched) position prior to release.
a. What is its initial potential energy?
b. What is the maximum velocity that the mass will reach in its oscillation? Where in the motion is this maximum reached?
c. Ignoring friction, what are the values of the potential energy, kinetic energy, and velocity of the mass when the mass is 6 cm from the equilibrium position?
d. How does the value of velocity computed in part c compare to that computed in part b? (What is the ratio of the values? This is interesting, because even though you are midway between the equilibrium point and the maximum displacement, the velocity is much closer to the maximum value you found in part b.)

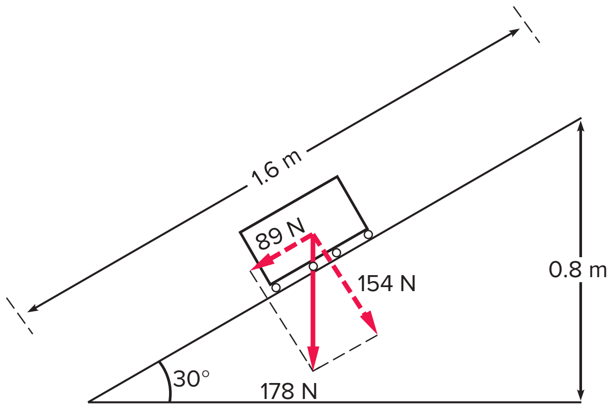
SP5. A sled and rider with a total mass of 50 kg are perched at the top of a hill, as pictured in the diagram. The top of this hill is 42 m above the low point in the path of the sled. A second hump in the hill is 28 m above this low point. Suppose we also know that approximately 3600 J of work is done against friction as the sled travels between these two points.
a. Will the sled make it to the top of the second hump if no kinetic energy is given to the sled at the start of its motion? Explain.
b. What is the maximum height that the second hump can be in order for the sled to reach the top, assuming that the same work against friction will be involved and that no initial push is provided? Explain.



SP5 Diagram

SP6. Suppose you wish to compare the work done by pushing a box on rollers up a ramp to the work done if you lift the box straight up to the same final height.
a. What work is required to lift a 178-N box (about 40 lb) up to a table that is 0.8 m off the floor?

b. Let's assume you also have a ramp available that makes an angle of 30° with the horizontal, as shown in the diagram. The ramp is 1.6 m long. The weight of the box (178 N) is due to the Earth's pulling on the box. This 178 N is a force directed straight down. If you push it up a ramp, you are doing work against only the component of this weight along the ramp, which is 89 N, as shown in the diagram. How much work does it require to push the box up the ramp, assuming no friction?
c. Which situation (pushing up the ramp or lifting straight up) requires more work?
d. Which situation requires more force?
e. For which situation is the distance moved greater?
f. What is the change in the gravitational potential energy of the box for each situation?
g. What advantage, if any, is there to using the ramp? Explain.

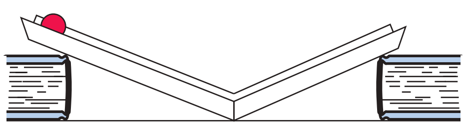


SP6 Diagram

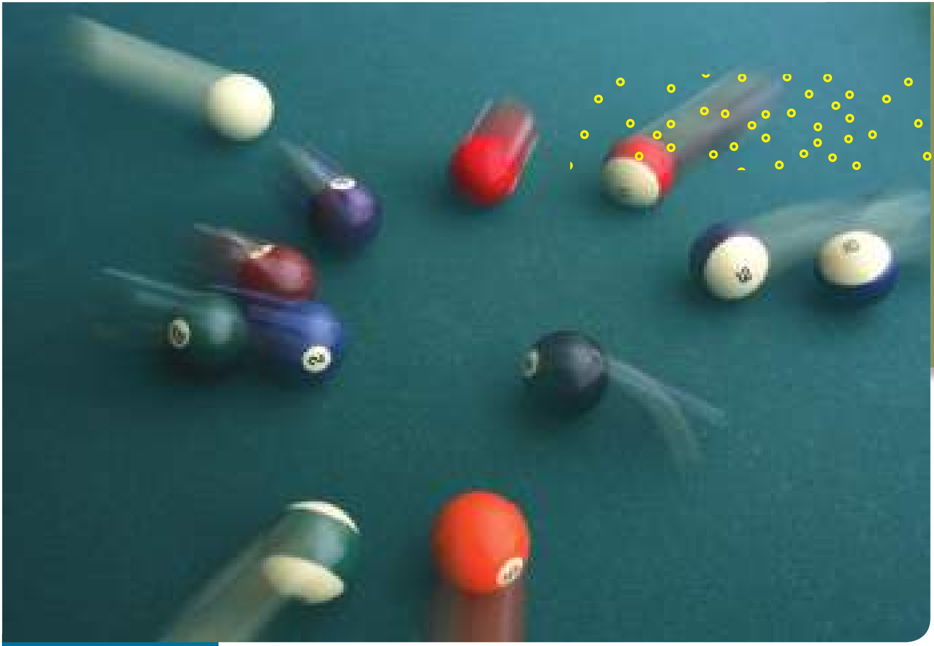
Home Experiments and Observations

HE1. You can easily construct a simple pendulum by attaching a ball to a string (with tape or a staple) and fixing the other end of the string to a rigid support. (A pencil taped firmly to the end of a desk or table will do nicely.)
a. The frequency of oscillation can be measured by timing the swings. The usual method is to use a watch to measure the time required for 10 or more complete swings. The period T (the time required for 1 swing) is then the total time divided by the number of swings counted, and the frequency f is just $1/T$.
b. How does the frequency change if you vary the length of your pendulum? (Try at least three different lengths.)

HE2. A ramp for a marble or small steel ball can be made by bending a long strip of cardboard into a V-shaped groove. Two such ramps can be placed end to end, as pictured in the diagram, to produce a track in which the marble will oscillate.
a. Can you measure a frequency of oscillation for this system? Does this frequency depend upon how high up the ramp the marble starts?
b. How high up the second ramp does the marble go? Is more energy lost per cycle in this system than for a pendulum?


HE2 Diagram

HE3. The height to which a ball bounces after being dropped provides a measure of how much energy is lost in the collision with the floor or other surface. A small portion of the energy is lost to air resistance as the ball is moving, but most is lost in the collision.
a. Trying a number of different balls that you may have available, test the height of the bounce using the same height of release for all of the balls tested. Which ball loses the most energy, and which the least?
b. Can you explain why many balls return to a higher height than a marble will? What characteristics of the balls tested give the best bounce?
c. For a ball that bounces several times, does the period (time between bounces) change with each bounce? Does the bouncing ball undergo simple harmonic motion?



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CHAPTER 7

Momentum

Chapter Overview

In this chapter, we explore momentum and impulse and examine the use of these concepts in analyzing events such as a collision between two opposing athletes. The principle of conservation of momentum is introduced and its limits explained. A number of examples shed light on how these ideas are used, particularly conservation of momentum. Momentum is central to all of these topics—it is a powerful tool for understanding a lot of life's sudden changes.

Chapter Outline

1. **Momentum and impulse.** How can rapid changes in motion be described using the ideas of momentum and impulse? How do these ideas relate to Newton's second law of motion?
2. **Conservation of momentum.** What is the principle of conservation of momentum, and when is it valid? How does this principle follow from Newton's laws of motion?
3. **Recoil.** How can we explain the recoil of a rifle or shotgun using momentum? How is this similar to what happens in firing a rocket?
4. **Elastic and inelastic collisions.** How can collisions be analyzed using conservation of momentum? What is the difference between an elastic and an inelastic collision?
5. **Collisions at an angle.** How can we extend momentum ideas to two dimensions? How does the game of pool resemble automobile collisions?

The word *momentum* is overused by sports announcers to mean changes in the flow of a game. The "old mo" that announcers talk about bears only a metaphorical relationship to the physical concept of momentum. There are plenty of real examples of changes in momentum for us to consider in both the world of sports and the world more generally.

Consider the collision between a hard-charging fullback and a defensive back on the football field (fig. 7.1). If they meet head-on, the velocity of the fullback is sharply reduced, although the two players might continue moving briefly in the original direction of the fullback's velocity. If the defensive back is moving before the collision, his velocity also changes abruptly. There must be strong forces at work to produce these accelerations, but these forces act for only an instant. How do we use Newton's laws to analyze this event?

Momentum, impulse, and conservation of momentum figure in any discussion of collisions. The total momentum of the fullback and defensive back is involved in predicting what will happen after the collision. How is momentum defined, and what does conservation of momentum have to do with Newton's laws? How is conservation of momentum useful in predicting what happens in collisions? These questions will be addressed as we examine a variety of collisions and other high-impact events.

Figure 7.1 A collision between a running back and a defensive back (in red, center of the photograph). How will the two players move after the collision? *Source: U.S. Air Force photo by John Van Winkle*

7.1 Momentum and Impulse

Imagine a baseball heading toward the catcher's mitt when its flight is rudely interrupted by the impact of a bat. In a very short time, the velocity of the ball changes direction and it is accelerated in the direction opposite its original flight. Similar changes happen when a tennis racket hits a ball or when a ball bounces off a wall or the floor. In many everyday situations, a brief impact causes a rapid change in an object's velocity.

The forces responsible for such rapid changes in motion can be large, but they act for very short times and are difficult to measure. Not only are they brief, but they may change rapidly *during* the collision.

What happens when a ball bounces?

Consider the seemingly simple example of dropping a tennis ball. The ball is initially accelerated downward by the gravitational force. When it reaches the floor, its velocity quickly changes in direction, and the ball heads back up toward you (fig. 7.2). There must be a strong force exerted on the ball by the floor during the short time they are in contact. This force provides the upward acceleration necessary to change the direction of the ball's velocity.

If we used a high-speed camera to catch the action during the time the ball is in contact with the floor, we would see that the ball's shape is distorted (fig. 7.3). The ball behaves like a spring, first compressing as it moves downward, then expanding (springing back) as it begins to move upward. A quick test (squeezing the ball with your hands) will persuade you that a strong force is required to distort the ball.

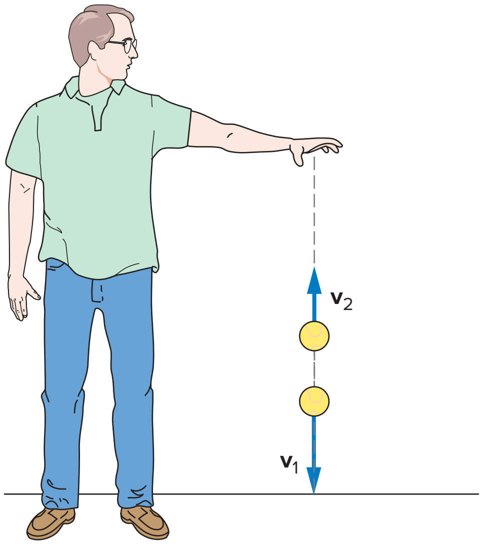


Figure 7.2 A tennis ball bouncing off the floor. There is a rapid change in the direction of the velocity when the ball hits the floor.

What we have, then, is *a strong force acting for a very brief time,* producing a rapid acceleration that quickly changes the ball's velocity from a downward direction to an upward one. The magnitude of the ball's velocity decreases rapidly to zero and then increases equally rapidly in the opposite direction. This all happens in a time so short that we would miss it if we blinked.

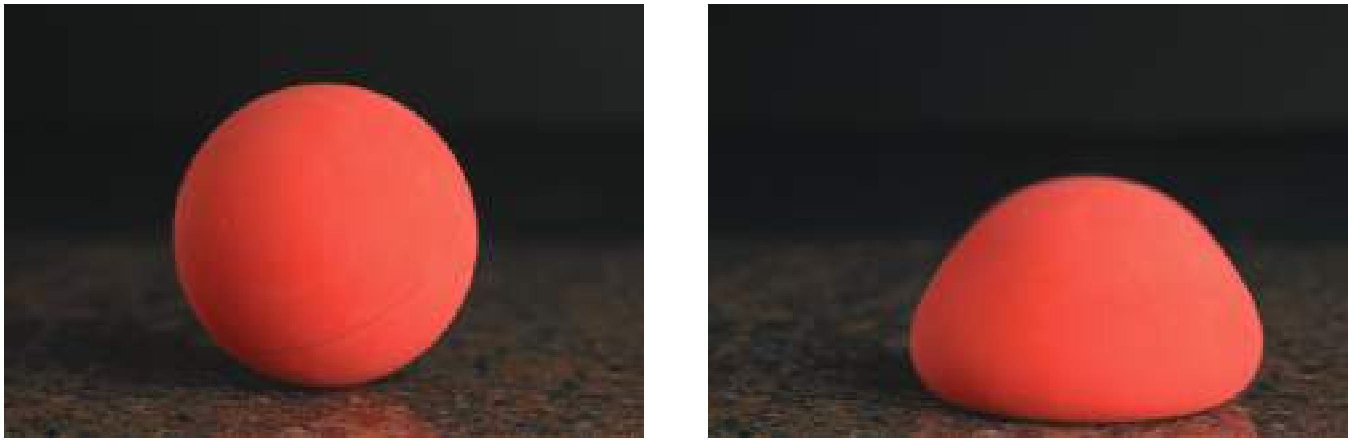


Figure 7.3 Two high-speed photographs of a ball hitting a tabletop, one just before the impact and the other after the impact. The ball is compressed like a spring. *(both): Ted Kinsman/Science Source*

How can we analyze such rapid changes?

We have described the collision of the ball with the floor using force and acceleration, and we could also use Newton's second law to predict how the velocity actually changes. The problem with this approach is that the time of the interaction is very short, and the force itself varies during this short time, so it is hard to describe the collision accurately. It is more productive to look at the *total change in motion* in this brief interaction.

We introduced Newton's second law in chapter 4 using the expression $\mathbf{F}_{\text{net}} = m\mathbf{a}$. The acceleration $\mathbf{a}$ is the *rate of change* in velocity, which can be expressed as the change in velocity $\Delta\mathbf{v}$ divided by the time interval Δt required to produce that change. The time interval is important: The shorter the time for a given change in velocity, the larger the acceleration and the force needed to produce this change.

We can restate Newton's second law as

$$\mathbf{F}_{\text{net}} = m\left(\frac{\Delta\mathbf{v}}{\Delta t}\right)$$

expressing the acceleration in terms of the change in velocity. Multiplying both sides of this equation by the time interval Δt recasts the second law as

$$\mathbf{F}_{\text{net}}\Delta t = m\Delta\mathbf{v}$$

While this is still Newton's second law, rewriting it offers us a different way of looking at events. This new view is more convenient for describing the overall change in motion.

What are impulse and momentum?

Impulse shows up as the quantity on the left side of the recast second law, $\mathbf{F}_{net}\Delta t$. **Impulse** is the force acting on an object multiplied by the time interval over which the force acts. If the force varies during this time interval, and it often does, we must use the *average* value of the force over this time interval.

> Impulse is the average force multiplied by its time interval of action:
>
> impulse $= \mathbf{F}\Delta t$

Because force is a vector quantity, impulse is also a vector in the direction of the average force.

How a force changes the motion of an object depends on both the size of the force and how long the force acts. The stronger the force, the larger the effect, and the longer the force acts, the greater its effect. Multiplying the two factors together to get the impulse shows the overall effect of the force.

On the right side of our recast second law, $m\Delta\mathbf{v}$ is the mass of the object multiplied by the change in velocity produced by the impulse. This product is the change in the *quantity of motion,* to use Newton's own term. We now call this product the *change in linear momentum* of the object, where **linear momentum** is defined thus:

> Linear momentum is the product of the mass of an object and its velocity, or
>
> $\mathbf{p} = m\mathbf{v}$

The symbol $\mathbf{p}$ is often used for linear momentum. If the mass of the object is constant, the change in linear momentum is the mass times the change in velocity, or $\Delta\mathbf{p} = m\Delta\mathbf{v}$.

At this point, we are dealing only with motion in a straight line, so we will drop the term *linear* and just refer to $\mathbf{p}$ as the momentum. In section 8.4, we will introduce angular momentum, and we will then be sure to distinguish between angular and linear momentum.

Like velocity, momentum is a vector and has the same direction as the velocity vector. Two different objects traveling in the same direction can have different masses and velocities but still have the same momentum. For example, a 7-kg bowling ball moving with the relatively slow speed of 2 m/s would have a momentum of 14 kg·m/s. On the other hand, a tennis ball with a mass of just 0.07 kg, moving with the much larger velocity of 200 m/s, has the same momentum as the bowling ball, 14 kg·m/s (fig. 7.4).

Using these definitions of impulse and momentum, we can state our recast form of Newton's second law as

$$\begin{aligned} \textit{impulse} &= \text{change in momentum} \\ &= \Delta\mathbf{p} \end{aligned}$$

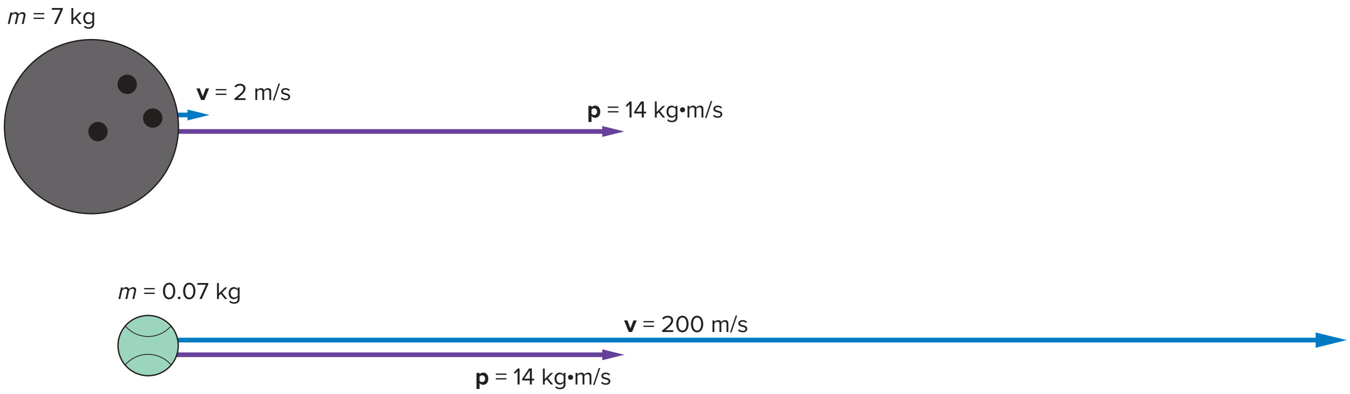


Figure 7.4 A bowling ball and a tennis ball with the same momentum. The tennis ball with its smaller mass must have a much larger velocity.

This statement of the second law is sometimes called the **impulse-momentum principle:**

> The impulse acting on an object produces a change in momentum of the object that is equal in both magnitude and direction to the impulse.

This principle is not a new law but another way of expressing Newton's second law of motion. It is particularly useful for looking at collisions.

How do we apply the impulse-momentum principle?

The impulse-momentum principle applies to almost any collision. Whacking a golf ball with a golf club is a good example (fig. 7.5). The impulse delivered by the golf club produces a change in the golf ball's momentum, also described in example box 7.1. Note that the units of impulse (force multiplied by time, or N·s) must equal those of momentum (mass times velocity, or kg·m/s).

Does the momentum of the bouncing tennis ball we discussed earlier change when it hits the floor? Even if the ball loses no energy in its collision with the floor and bounces back with the same speed and kinetic energy it had just before hitting the floor, the momentum changes, because its direction changes. The momentum decreases to zero as the ball comes to a momentary halt, and it changes again as the ball gains momentum in the opposite direction (fig. 7.6). The total change in momentum is larger than the change that would happen if the tennis ball stopped and did not bounce.

When the tennis ball bounces back with the same speed, the total change in momentum is *twice* the value of the momentum just before the ball hits the floor. Its final momentum is $m\mathbf{v}$, where the direction of $\mathbf{v}$ is upward, but its *initial* momentum was $-m\mathbf{v}$, because the initial velocity was directed downward. We find the change in momentum by subtracting

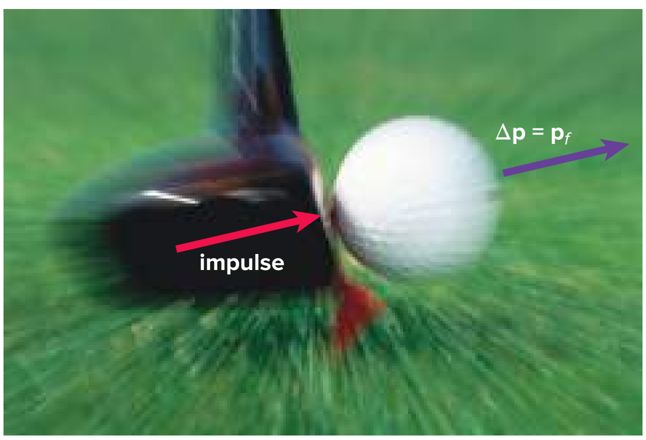


Figure 7.5 An impulse is delivered to the golf ball by the head of the club. If the initial momentum of the ball is zero, the final momentum is equal to the impulse delivered. *Stephen Marks/Getty Images*

Example Box 7.1

Sample Exercise: The Momentum and Impulse of Golf

A golf club exerts an average force of 500 N on a 0.1-kg golf ball, but the club is in contact with the ball for only a hundredth of a second.

a. What is the magnitude of the impulse delivered by the club?
b. What is the change in velocity of the golf ball?

a. $F = 500\ \text{N}$ $\text{impulse} = F\Delta t$
$\Delta t = 0.01\ \text{s}$ $= (500\ \text{N})(0.01\ \text{s})$
$\text{impulse} = ?$ $= \mathbf{5\ N{\cdot}s}$

b. $m = 0.1\ \text{kg}$ $\text{impulse} = \Delta\mathbf{p} = m\Delta\mathbf{v}$

$\Delta v = ?$

$$\Delta v = \frac{\text{impulse}}{m} = \frac{5\ \text{N}\cdot\text{s}}{0.1\ \text{kg}} = \mathbf{50\ m/s}$$

Because the golf ball started at rest, this change in velocity equals the velocity of the ball as it leaves the face of the club. The direction of this velocity is the same as the impulse of the force exerted by the club.

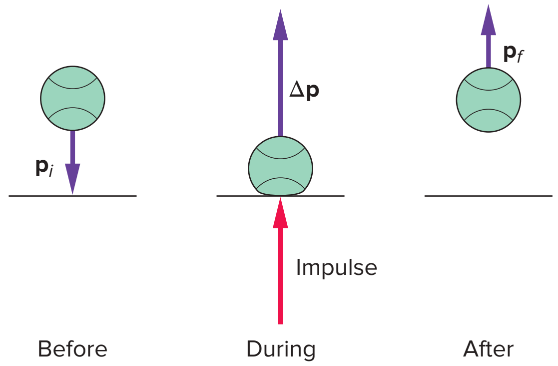


Figure 7.6 The impulse exerted by the floor on the tennis ball produces a change in its momentum.

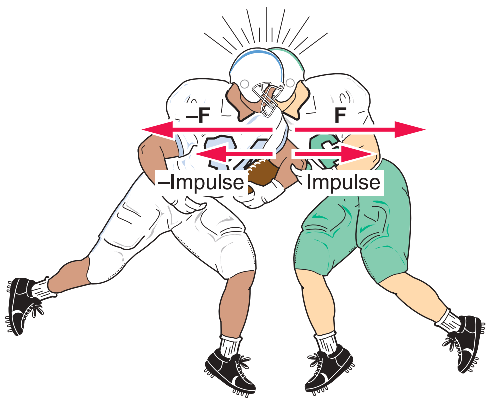


Figure 7.7 Two American football players colliding. The impulses acting on the two players are equal in magnitude but opposite in direction.

the initial value from the final value: $m\mathbf{v} - (-m\mathbf{v}) = 2m\mathbf{v}$. The impulse required to produce this change in momentum is twice as large as what is needed simply to stop the ball.

There are many practical lessons involving impulse and change in momentum. Why does it help to pull your hand back as you catch a hard-thrown ball? When you pull your hand back, you lengthen the time interval Δt. This reduces the average force that your hand must exert on the ball, because impulse is the product of the force and the time interval ($\mathbf{F}\Delta t$). If the time interval is longer, the force can be smaller yet still produce the same impulse and change in momentum. It hurts less this way! A padded dashboard or an air bag similarly lessens injury to passengers by increasing the time interval required to bring them to a halt in a collision. Another practical lesson involving impulse and change in momentum is described in everyday phenomenon box 7.1.

Everything we have done in this section is just another way of working with Newton's second law of motion. In fact, by dividing both sides of the impulse-momentum principle by the time interval Δt, Newton's second law can be expressed in the form that most nearly captures the meaning of Newton's original statement of the second law, $\mathbf{F}_{net} = \Delta\mathbf{p}/\Delta t$. In words, this form of the second law says that the net force acting on an object is equal to the rate of change in momentum of the object. This form covers a wider range of situations than the more familiar $\mathbf{F}_{net} = m\mathbf{a}$.

Momentum and impulse are most useful for evaluating events such as collisions, where powerful forces act briefly to produce striking changes in the motion of objects. The impulse-momentum principle states that the change in momentum is equal to the impulse. This is a different way of stating Newton's second law. The impulse, the product of the average force and the time interval that it is applied, allows us to predict the change in momentum of the object. Large impulses yield large changes in momentum.

7.2 Conservation of Momentum

How do impulse and momentum help to explain the collision between the athletes mentioned in the chapter introduction? The conditions described in section 7.1 are certainly present. The defensive back exerts a sizable but brief force on the fullback (fig. 7.7), and the momentum of both players changes rapidly in the collision.

The principle of **conservation of momentum** provides the key to understanding such a collision. This principle arises when we apply Newton's *third law* to impulse and changes in momentum. Conservation of momentum allows us to predict many features of collisions without requiring detailed knowledge of the forces of impact.

Why and when is momentum conserved?

Let's take a more detailed look at the head-on collision between the hard-charging fullback and the defensive back. To simplify the situation, we assume that the two players meet in midair and that after the collision they move together, with the fullback held in the tackle of the defensive back (fig. 7.7). What happens when they collide?

During the collision, the defensive back exerts a strong force on the fullback, and by Newton's third law, the fullback exerts a force equal in magnitude but opposite in direction on the defensive back. Because the time interval of action Δt is the same for both forces, the impulses $\mathbf{F}\Delta t$ must also be equal in magnitude but opposite in direction. From the impulse-momentum principle (Newton's second law), changes in the momentum $\Delta\mathbf{p}$ for each player are also equal in magnitude but opposite in direction.

If the two players experience equal but oppositely directed momentum changes, the *total change* in momentum of the two players together is zero. We look at the overall *system* and define the total momentum of that system as the sum of the momentum values of the two players. There is no

Everyday Phenomenon Box 7.1

The Egg Toss

The Situation. Have you ever competed to see how far you and a partner can successfully throw and catch a raw egg? The most successful technique involves moving your hand back as the egg lands in it. Why does this technique reduce the likelihood of the egg's breaking? What does this have to do with momentum and impulse? How can your physics knowledge reduce the chances of a raw egg bath? The same principles apply when catching water balloons.

ISHARA S.KODIKARA/AFP/Getty Images

The Analysis. When you throw an egg or a balloon toward your partner, you apply a force and give it momentum. When your partner catches it and brings it to a stop, the object experiences a change in momentum. That change in momentum is equal to the impulse, as discussed earlier in the text. The impulse is also equal to the product of the force of the hand on the object multiplied by the time that force is applied. Therefore, by the impulse-momentum principle, the change in momentum is equal to the force of the hand on the egg multiplied by the time that force is applied: $\Delta p = F\Delta t$.

Unlike a golf swing or a bat hitting a baseball, the idea here is not to increase the change in momentum; the idea is to minimize the force applied to the egg, so that it won't break. It is the strength of the force that will break the egg. The larger the force applied by your hand in catching the egg, the more likely it is you will be splattered.

Once the egg leaves the thrower's hand, it has a certain momentum that stays the same as it travels from the thrower to the catcher. We say this momentum is fixed. The impulse required to reduce this momentum to zero is therefore also fixed; it does not change. Because $F\Delta t$ is fixed, if you wish to decrease the force applied to the egg, the time interval Δt involved in the catch must be increased. We say that force and time are inversely proportional to each other. As one increases, the other decreases *in proportion. F* is proportional to $1/\Delta t$. (*F is proportional to the inverse of* Δt.)

The idea of inverse proportionality comes up frequently in everyday life but is not always recognized or understood. To take a simple example, let's say you have \$2.00 to spend on candy. If you buy candy pieces that cost 10¢ each, you will get 20 pieces of candy. If, instead, you buy candy pieces that cost 25¢ each, you can buy only 8 pieces of candy. The more expensive the candy, the fewer pieces can be purchased, a fact that most of us can easily recognize.

In this example, the amount of money you have to spend is fixed, and it must equal the total cost of the candy, which is the product of the price per piece times the number of pieces. The number of pieces of candy you can buy and the price of each piece of candy are therefore inversely proportional to one another. As one increases, the other must decrease in proportion.

As we have already indicated, inverse proportionality is involved in catching the egg. The impulse is fixed, so force and time are inversely proportional to one another. As you increase one (say, the time to stop it), the other (the force applied) decreases. Your objective is, then, to make the time involved in stopping the egg as long as possible.

How can this be done? You can increase the time involved in the catch substantially by moving your hand back as the egg reaches it. This should be done as smoothly as possible, so that the velocity (and momentum) of the egg decreases to zero much more gradually than if your hand were stationary. Using this technique, the average force applied to the egg can be made much smaller than that involved in a sudden stop. With luck, this force will be small enough that the egg will not break!

This same principle has many applications. Air bags in cars reduce the force applied to your head in a collision by increasing the time it takes your head to come to a stop. Gym floors have more "give" to them to reduce the force on players' knees when jumping and landing on the floor. Dropping a wineglass on a carpeted floor will make a mess by staining the carpet, but it is far less likely to break the glass than if it is dropped onto a concrete floor. In all of these examples, the force has decreased, because the time it takes to stop the object has increased. You may not be able to measure the decrease in time with a stopwatch, because the times involved are very short, but you can see the evidence of the increased time in the result.

The next time you have a picnic, take some raw eggs or water balloons and apply the impulse-momentum principle with your friends and family. With water balloons, good technique will keep you dry, but sometimes it is more fun to get wet.

change in the momentum of this system because the changes in the momentums of the parts cancel one another. The total momentum of the system is *conserved.*

To reach this conclusion, we ignored external forces (produced by other objects) acting on the two players and assumed that the only significant forces were their own forces of interaction. The forces they exert on one another are *internal* to the system consisting of both players. The principle of conservation of momentum can therefore be stated as follows:

> If the net external force acting on a system of objects is zero, the total momentum of the system is conserved.

The forces of interaction between the objects in a system are internal forces whose effects on the total momentum cancel one another, because of Newton's third law of motion. Different portions of the system can exchange momentum without affecting the total momentum of the system. If there is a net *external* force acting on the system because of interaction with some object that is not part of the system, the entire system will be accelerated—and the momentum of the system will change.

Conservation of momentum and collisions

Using the principle of conservation of momentum, what information can we obtain about the results of a collision (like the one between two football players)? If we know the masses of the players and their initial velocities, we can find how fast and in what direction the players will move after they collide. We do not need to know anything about the details of the strong forces involved in the collision itself.

The sample exercise in example box 7.2 treats a head-on collision between a fullback and a defensive back using realistic numerical values. The fullback has a mass of 100 kg (equivalent to a weight of about 220 lb) and is moving straight downfield with a velocity of 5 m/s through the hole created by his linemen. The somewhat smaller defensive back charges up to meet him with a velocity in the opposite direction of −4 m/s (fig. 7.8). The minus sign indicates direction: We have chosen the fullback's direction of motion to be positive.

The total momentum of the system before the collision in example box 7.2 is found by adding the initial momentum of the fullback to the momentum of the defensive back, taking into account the difference in sign. If we assume that both players' feet leave the ground just before the collision (so that there are no frictional forces between their feet and the ground), momentum should be conserved in the collision. The total momentum of the two players moving together after the collision has the same value it had immediately before the collision (fig. 7.9).

Example Box 7.2

Sample Exercise: A Head-on Collision

A 100-kg fullback moving straight downfield with a velocity of 5 m/s collides head-on with a 75-kg defensive back moving in the opposite direction with a velocity of −4 m/s. The defensive back hangs on to the fullback, and the two players move together after the collision.

a. What is the initial momentum of each player?
b. What is the total momentum of the system?
c. What is the velocity of the two players immediately after the collision?

a. *fullback:*

$m = 100\text{ kg}$ $\quad$ $p = mv$

$v = 5\text{ m/s}$ $\quad$ $= (100\text{ kg})(5\text{ m/s})$

$p = ?$ $\quad$ $= \mathbf{500\ kg{\cdot}m/s}$

defensive back: $\quad$ $p = mv$

$m = 75\text{ kg}$ $\quad$ $= (75\text{ kg})(-4\text{ m/s})$

$v = -4\text{ m/s}$ $\quad$ $= \mathbf{-300\ kg{\cdot}m/s}$

b. $p_{\text{total}} = ?$

$$\begin{aligned} p_{\text{total}} &= p_{\text{fullback}} + p_{\text{defensive back}} \\ &= 500\text{ kg·m/s} + (-300\text{ kg·m/s}) \\ &= \mathbf{200\ kg{\cdot}m/s} \end{aligned}$$

c. $v = ?$ (for both players after the collision)

$m = 100\text{ kg} + 75\text{ kg}$ $\quad$ $p = mv$

$= 175\text{ kg}$

$$\begin{aligned} v &= \frac{p_{\text{total}}}{m} \\ &= \frac{200\text{ kg·m/s}}{175\text{ kg}} \\ &= \mathbf{1.14\ m/s} \end{aligned}$$

The positive value of the momentum after the collision means that the two players are traveling in the direction of the fullback's initial motion. The fullback had a larger initial momentum than the defensive back, so his direction of motion prevails when the two values are added. The defensive back will be carried backward briefly before the two players hit the turf.

> Conservation of momentum results when the changes in momentum of different parts of a system cancel each other by Newton's third law. If there are no external forces acting on the system, its total momentum is conserved. The principle applies to all sorts of situations involving collisions and explosions or other forms of brief but forceful interaction between objects.

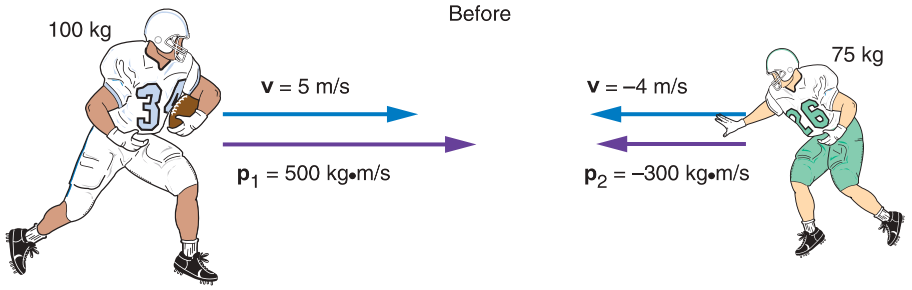


Figure 7.8 The two players before the collision, with velocity and momentum vectors for each.

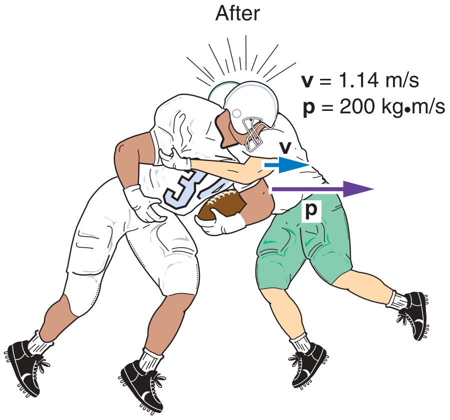


Figure 7.9 The two players after the collision, with velocity and momentum vectors indicated.

Figure 7.10 Two skaters of different masses prepare to push off against one another. Which one will gain the larger velocity?

7.3 Recoil

Why does a shotgun slam against your shoulder when fired, sometimes with painful consequences? How can a rocket accelerate in empty space when there is nothing there to push against but itself? These are examples of the phenomenon of recoil, a common part of everyday experience. Conservation of momentum is the key to understanding recoil.

What is recoil?

Imagine two ice skaters facing each other and pushing against each other with their hands (fig. 7.10). The frictional forces between their skates and the ice are presumably very small, so we can neglect them. The upward normal force and the downward force of gravity cancel each other, too, because we know that there is no acceleration in the vertical direction. The net external force acting on the system of the two skaters is effectively zero, and conservation of momentum should apply.

How do we apply conservation of momentum in this situation? Because neither skater is moving before the push-off, the initial total momentum of the system is zero. If momentum is conserved, the total momentum of the system after the push-off will also be zero. How can the total momentum be zero when at least one of the skaters is moving? Both skaters must move with momentum values equal in magnitude but opposite in direction $\mathbf{p}_2 = -\mathbf{p}_1$. The momentum of the second skater $\mathbf{p}_2$ must be opposite that of the first skater $\mathbf{p}_1$. When added together to find the total momentum of the system, these individual values will cancel each other to produce a total momentum of zero.

After the push-off, the two skaters move in opposite directions with momentum vectors equal in magnitude (fig. 7.11), but their velocities are not of equal magnitude. Because momentum is mass times velocity ($\mathbf{p} = m\mathbf{v}$), the skater with the smaller mass must have the larger velocity

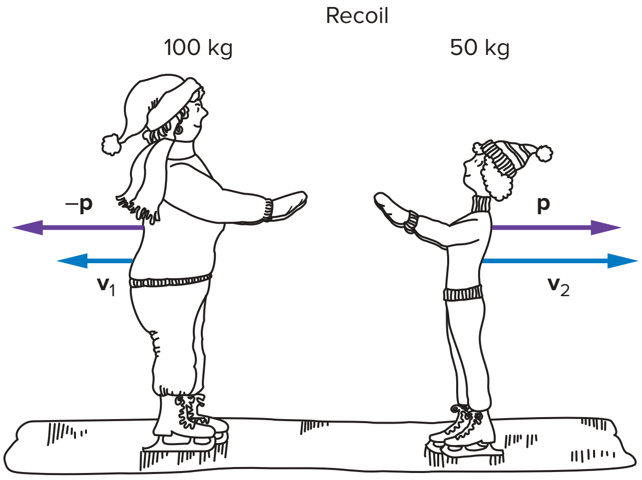


Figure 7.11 The two skaters after pushing off, with the velocity and momentum vectors indicated.

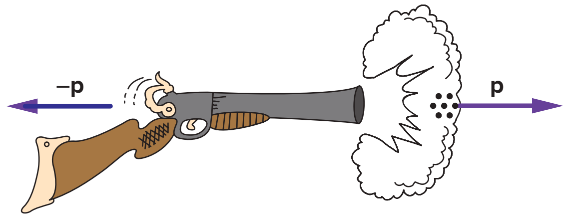


Figure 7.12 The shot and the shotgun have equal but oppositely directed momentums after the gun is fired.

to yield the same magnitude of momentum as the larger skater. Suppose the smaller skater's mass is just half the mass of the larger skater. The smaller skater's velocity will then be twice as large as that of the larger skater after pushing off.

The ice skaters illustrate the basics of **recoil.** A brief force between two objects causes the objects to move in opposite directions. The lighter object attains the larger velocity to equalize the magnitudes of the momentums of the two objects. The total momentum for the system after the push-off equals zero, the value of the momentum of the system before the push-off if the objects were initially at rest. The total momentum of the system is conserved and does not change.

Example Box 7.3

Sample Question: Is Momentum Conserved When Shooting a Shotgun?

Question: When a shotgun is held firmly against your shoulder, is the momentum of the system conserved?

Answer: It depends on how you define the system. If the system is defined as just the shotgun and the pellets, there is then a strong external force exerted on the system by the shoulder of the shooter. Because the condition for conservation of momentum is that the net external force acting on the system be zero, the momentum of this system is not conserved.

If we included the shooter and the Earth in our system, then momentum would be conserved because all of the forces would be internal to this system. The change in the momentum of the Earth would be imperceptible, however.

Recoil of a shotgun

If you have ever fired a shotgun without holding it firmly against your shoulder, you have probably had a painful experience of recoil. What happened? The explosion of the powder in a shotgun causes the shot to move very rapidly in the direction of the gun's aim. If the gun is free to move, it will recoil in the opposite direction, with a momentum equal in magnitude to the momentum of the shot (fig. 7.12).

Even though the mass of the shot is considerably less than that of the shotgun, the momentum of the shot is quite large as a result of its large velocity. If the external forces acting on the system can be ignored, the shotgun recoils with a momentum equal in magnitude to the momentum of the shot. The recoil velocity of the shotgun will be smaller than the shot's velocity because of the larger mass of the gun, but it is still sizable. As the gun slams back against your shoulder, you will know that it has recoiled.

How can you avoid a bruised shoulder? The trick is to hold the gun firmly against your shoulder. (See example box 7.3.) Your own mass then becomes part of the system. This increased mass will produce a smaller recoil velocity, even if you happen to be standing on ice with no frictional forces between your feet and the Earth. More important, the shotgun will not move against your shoulder.

How does a rocket work?

The firing of a rocket is another example of recoil. The exhaust gases rushing out of the tail of the rocket have both mass and velocity and, therefore, momentum. If we ignore external forces, the momentum gained by the rocket in the forward direction will equal in magnitude the momentum of the exhaust gases in the opposite direction (fig. 7.13). Momentum is conserved, just as in our other examples of recoil. The rocket and the exhaust gases push against each other, and Newton's third law applies.

The difference between a rocket and our earlier examples of the skaters and the shotgun is that firing a rocket is usually a continuous process. The rocket gains momentum gradually rather than in a single, short blast. The mass of the

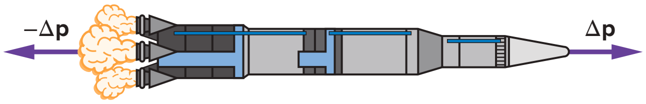


Figure 7.13 If a short blast is fired, the rocket gains momentum equal in magnitude but opposite in direction to the momentum of the exhaust gases.

rocket also changes as fuel is consumed and gases exhaust from the rocket engines. Computation of the final velocity becomes more difficult than for the skaters. For a brief blast of the rocket, though, the same analysis can be used.

Recoil works in empty outer space: The two objects need only push against each other, just as with the skaters and the shotgun. Rocket engines can be used for space travel, unlike the propeller engines or jet engines used on airplanes. Airplane engines depend on the presence of the atmosphere, both as a source of oxygen used in burning fuel and as something to push against. An airplane propeller pushes against the air, and the air, by Newton's third law, pushes against the propeller. This interaction accelerates the airplane. A rocket, on the other hand, is self-contained. It exerts a force on its own exhaust gases, and by the third law, the exhaust gases exert a force on the rocket.

> During recoil, objects push against each other, moving in opposite directions. If external forces can be neglected, momentum is conserved. The total momentum before and after the interaction equals zero. After the interaction, the two objects move away with equal but oppositely directed momentum vectors that cancel each other. Recoil is one of many kinds of brief interaction to which conservation of momentum applies.

7.4 Elastic and Inelastic Collisions

As the example involving football players showed, collisions are one of the most fruitful areas for applying conservation of momentum. Collisions involve large forces of interaction acting for very brief times, and they produce dramatic changes in the motion of the colliding objects. Because the forces of interaction are so large, any external forces acting on the system usually are unimportant by comparison: Momentum is conserved.

Different kinds of collisions produce different results. Sometimes the objects stick together and sometimes they bounce apart. What distinguishes these different cases, and what do the terms *elastic, inelastic,* and *perfectly inelastic* mean when applied to collisions? Is energy conserved as well as momentum? Railroad cars, bouncing balls, and billiard balls help illustrate the differences.

What is a perfectly inelastic collision?

The easiest type of collision to analyze is one where two objects collide head-on and stick together after the collision, like the two football players discussed earlier. Because they stuck together and moved as one object after the collision, we had just one final velocity to contend with.

A collision in which the objects stick together after collision is called a **perfectly inelastic collision.** The objects do not bounce at all. If we know the total momentum of the system before the collision (and external forces are ignored), we can readily compute the final momentum and velocity of the now-joined objects.

Coupling railroad cars are another example of this type of collision. Example box 7.4 uses conservation of momentum to predict the final momentum and velocity of coupled railroad cars from knowledge of the momentum of the system before the collision. The process is much the same as the one used to predict the final velocity of the football players in section 7.2. In both cases, the separate objects move as one following the collision.

In example box 7.4, the total mass of the coupled cars after the collision is five times that of car 5, so the final velocity of the coupled cars must be one-fifth that of car 5 to conserve momentum. The momentum of the system immediately after the collision is equal to that just before the collision, but the velocities have changed. The "final" velocity we calculated is valid immediately after the collision. As the cars continue to move following the collision, frictional forces will gradually decelerate them until they come to rest.

Is energy conserved in collisions?

Is the kinetic energy after the railroad cars collide equal to the original kinetic energy of car 5 in the example in example box 7.4? Using the relationship $KE = \frac{1}{2}mv^2$ introduced in chapter 6, we can compute the kinetic energy before and after the collision. The original kinetic energy of car 5 is 810 kJ. (A kilojoule, kJ, is a thousand joules.) Immediately after the collision, the kinetic energy of the five cars moving together is just 162 kJ. (You can check these values; see exercise E15.) A portion of the original kinetic energy is lost in any perfectly inelastic collision.

If we put a large spring on the front of the moving railroad car and allowed it to bounce off the other four cars rather than coupling, we will find that a greater portion of the kinetic energy is retained in the collision. When the objects bounce, the collision is either *elastic* or only *partially inelastic,* rather than perfectly inelastic. The distinction is based on energy. An **elastic collision** is one in which *no* energy is lost. A **partially inelastic collision** is one in which *some* energy is lost, but the objects do not stick together. The greatest portion of energy is lost in the *perfectly inelastic collision,* when the objects stick.

Example Box 7.4

Sample Exercise: When Railroad Cars Couple

Four railroad cars, all with the same mass of 20,000 kg, sit on a track, as shown in the drawing. A fifth car of identical mass approaches them with a velocity of 9 m/s (to the right). This car collides and couples with the other four cars.

a. What is the initial momentum of the system?
b. What is the velocity of the five coupled cars after the collision?

a. $m_5 = 20{,}000 \text{ kg}$
$v_5 = 9 \text{ m/s}$
$p_{\text{initial}} = ?$

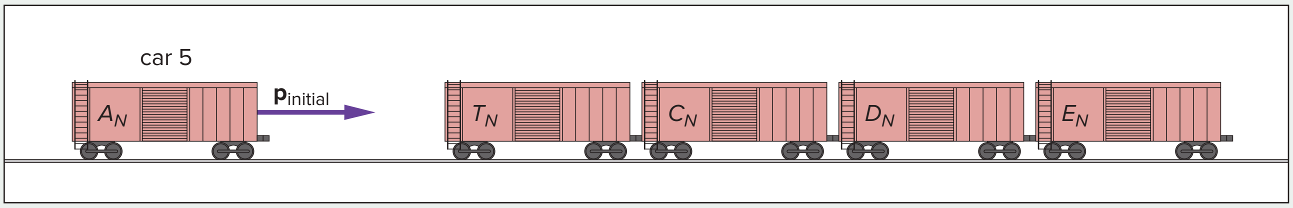
$$\begin{aligned} p_{\text{initial}} &= m_5 v_5 \\ &= (20{,}000 \text{ kg})(9 \text{ m/s}) \\ &= \mathbf{180{,}000\ kg \cdot m/s} \end{aligned}$$

(before the collision)

b. $m_{\text{total}} = 100{,}000 \text{ kg}$
$p_{\text{final}} = p_{\text{initial}}$
$v_{\text{final}} = ?$

$$\begin{aligned} v_{\text{final}} &= \frac{p_{\text{final}}}{m_{\text{total}}} \\ &= \frac{180{,}000 \text{ kg} \cdot \text{m/s}}{100{,}000 \text{ kg}} \\ &= \mathbf{1.8\ m/s} \end{aligned}$$

(for the five cars after the collision)



A railroad car approaches four others at rest on the track. What is the velocity of the cars after they couple?

In most collisions, some kinetic energy is lost, because the collisions are not perfectly elastic. Heat is generated, the objects may be deformed, and sound waves are created, all of which involve conversions of the kinetic energy of the objects to other forms of energy. Even if the objects bounce, we cannot assume that the collision is elastic. More likely, the collision will be partially inelastic, implying that some of the initial kinetic energy has been lost.

A ball bouncing off a floor or wall with no decrease in the magnitude of its velocity is an example of an elastic collision. Because the magnitude of the velocity does not change (only the direction changes), the kinetic energy does not decrease. No energy has been lost. More likely, of course, some energy will be lost in such a collision, and the magnitude of the ball's velocity after the collision will be a little smaller than before.

The opposite extreme to an elastic collision of a ball with the wall is a perfectly inelastic collision in which the ball sticks to the wall. In this case, the velocity of the ball after the collision is zero. So is its kinetic energy. All of the kinetic energy is lost (fig. 7.14).

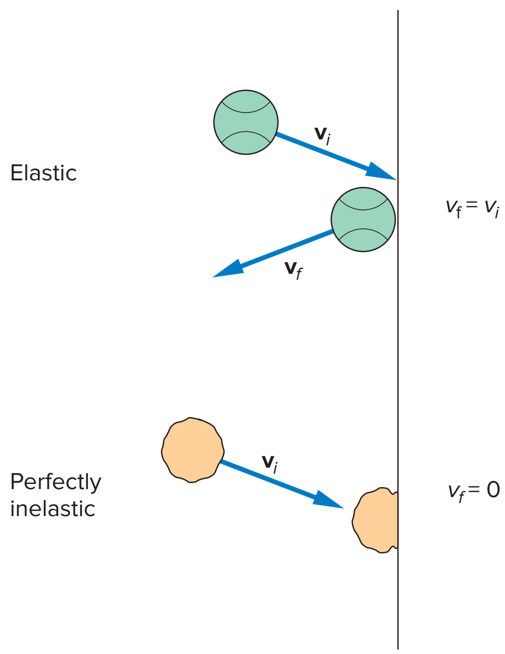


Figure 7.14 An elastic collision and a perfectly inelastic collision of a ball with a wall. The ball sticks to the wall in the perfectly inelastic collision.

What happens when billiard balls bounce?

Very little energy is lost when billiard balls collide with each other. (Time spent playing pool can be justified as a form of experimental physics. Your intuition about elastic collisions can be improved in the process!) The collisions are basically elastic. Both momentum and kinetic energy are conserved in most collisions of billiard balls.

When colliding objects, such as billiard balls, bounce off each other, we must deal with two final velocities rather than one. We can readily compute the total momentum of the system before and after the collision from our knowledge of the initial momentum values of the objects. More information is needed to determine the individual velocities of the objects after the collision, however, because one value is not enough to determine two unknown velocities. (This is why the case of the perfectly inelastic collision, where objects stick together, is particularly easy to analyze.) In an elastic collision, conservation of energy provides the additional information.

For billiard balls, the simplest case is the white cue ball colliding head-on with a second ball that is not moving before it is hit (the 11 ball in fig. 7.15). What happens? If spin is a minor factor in the collision, the cue ball stops dead on impact, and the 11 ball moves forward with a velocity equal to that of the cue ball before the collision. If the 11 ball acquires the same velocity that the cue ball had before the collision, it also has the same momentum $m\mathbf{v}$ as the initial momentum of the cue ball, because both balls have the same mass. Momentum is conserved.

Kinetic energy is also conserved. The cue ball had a kinetic energy of $\frac{1}{2}mv^2$ before the collision. After the collision, the velocity and kinetic energy of the cue ball are zero, but the 11 ball now has a kinetic energy of $\frac{1}{2}mv^2$, because its mass and speed are the same as the cue ball before the collision. Given the equal masses of the two balls, the only way that both momentum and kinetic energy can be conserved is for the cue ball to stop and the 11 ball to move forward with the same momentum and kinetic energy that the cue ball had before the collision. This effect is familiar to any pool player.

The same phenomenon is involved in the familiar swinging-ball demonstration often seen as a decorative toy on mantels or desktops (fig. 7.16). A row of steel balls hangs by threads from a metal or wooden frame. If one ball is pulled back and released, the collision with the other balls results in a single ball from the other end of the chain flying off with the same velocity as the first ball just before the collision. Both momentum and kinetic energy are conserved.

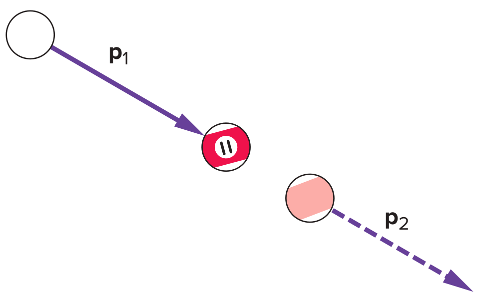


Figure 7.15 A head-on collision between the white cue ball and the 11 ball initially at rest. The cue ball stops, and the 11 ball moves forward.

Figure 7.16 The swinging-ball apparatus provides an example of collisions that are approximately elastic. *Mike Kemp/RubberBall/SuperStock*

If two balls on one side are pulled back and released, two balls fly off from the opposite side of the row of balls after the collision. Again, both momentum and kinetic energy are conserved by this result. You can explore a variety of other combinations. It can be entertaining as well as addictive.

Collisions between hard spheres, such as billiard balls or the steel balls in the swinging-ball apparatus, will generally be more or less elastic. Most collisions involving everyday objects, though, are inelastic to some degree. Some kinetic energy is lost. Momentum will be conserved, however, as long as our concern is with the values of momentum and velocity immediately before and after the collision.

> Conservation of momentum is the primary tool used in understanding collisions. External forces can be ignored for the brief time of the collision, when the collision forces are dominant, and the law of conservation of momentum applies. Kinetic energy can also be conserved if the collision is elastic, as it is approximately for billiard balls or other hard spheres. Most collisions involving familiar objects are partially inelastic and involve some loss of energy. The greatest proportion of energy is lost in perfectly inelastic collisions where objects stick together.

7.5 Collisions at an Angle

What happens when objects such as billiard balls or automobiles collide at an angle, rather than head-on? Some interesting applications of conservation of momentum arise when motion is not confined to a straight line. It becomes more apparent that momentum is a vector when objects are free to move in two dimensions. Balls on a pool table, cars colliding at an intersection, and football players tackling one another all provide interesting examples.

An inelastic, two-dimensional collision

Two football players, originally traveling at right angles to one another, collide and stick together, as in figure 7.17. What will be their direction of motion after the collision? How do we apply conservation of momentum in this two-dimensional case? In figure 7.17, we assume that the two players have the same masses and initial speeds as in our earlier example (section 7.2), but we no longer have a head-on collision. The momentum of the defensive back is now directed across the field as the fullback heads downfield.

Because momentum is a vector, we need to add the individual momentum vectors of the fullback and the defensive back to get the total momentum of the system before the collision. This can be done most readily by using a vector diagram with the vectors drawn to scale. The vectors can then be added graphically, as we have done before. As shown in figure 7.18, the total momentum of the system before the collision is the hypotenuse of the right triangle formed by adding the other two momentum vectors.

If momentum is conserved in the collision, the total momentum of the two players after the collision will equal the total momentum before the collision. Because the two players move together after the collision, they will travel in the direction of the total momentum vector shown in figure 7.18.

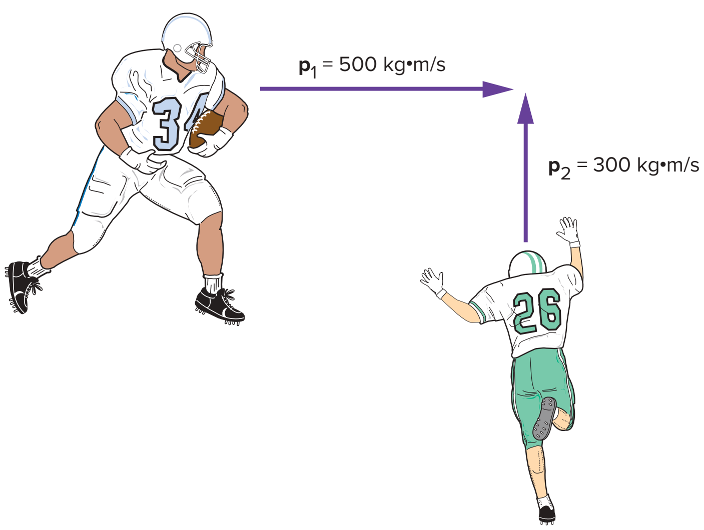


Figure 7.17 The fullback and the defensive back approaching each other at a right angle.

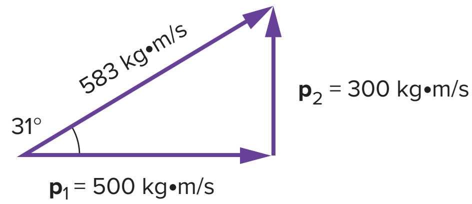


Figure 7.18 The total momentum of the two football players prior to the collision is the vector sum of their individual momentums.

(See appendix C for a review of vector addition.) The direction of motion of both players changes as a result of the collision. The larger momentum of the fullback before the collision dictates that the final direction of motion is more downfield than across the field, but it is some of both. This result makes intuitive sense if you imagine yourself as one of the players.

The final direction of motion of the two football players after the collision depends on their momentum values before the collision. If the defensive back were bigger or moving faster than we assumed initially, he would have a larger momentum, and his tackle would cause a more impressive change in the direction of the fullback's motion. On the other hand, if the defensive back is small and moving slowly, his effect on the fullback's direction will be small. Adjusting the length of the momentum arrow $\mathbf{p}_2$, the momentum of the defensive back in figure 7.18, will illustrate these changes.

Everyday phenomenon box 7.2 describes a similar situation. Two cars approaching an intersection at right angles to one another collide and stick together. Working backward from information about the final direction of travel, the investigator can draw conclusions about the initial velocities of the two cars. Conservation of momentum is extremely important in accident analysis.

What happens in elastic, two-dimensional collisions?

When billiard balls collide, they do not stick together after the collision: When objects bounce, we have to contend with two final velocities with different directions. Although many real collisions are like this, analysis is more complicated for them than for the perfectly inelastic examples we have discussed. More information is needed to predict the final velocities. If we know that the collision is elastic, however, conservation of kinetic energy can provide that additional information.

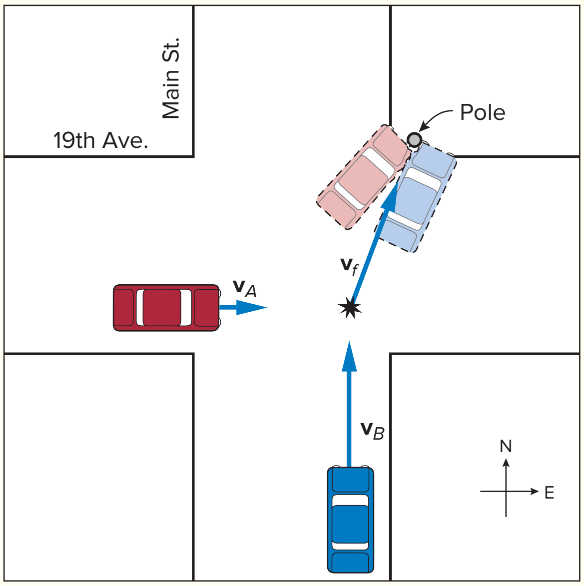
Experimental physics on the pool table comes through again with an interesting example (and one of practical value

Everyday Phenomenon Box 7.2

An Automobile Collision

The Situation. Officer Jones is investigating an automobile collision at the intersection of Main Street and 19th Avenue. Driver A was traveling east on 19th Avenue when she was struck from the side by driver B, who was traveling north on Main Street. The two cars stuck together after the collision and ended up against the lamppost on the northeast corner of the intersection.

Both drivers claim they started up just as the light in their direction changed to green and then collided with the other driver, who was running a red light and speeding. There are no other witnesses. Which driver is telling the truth?

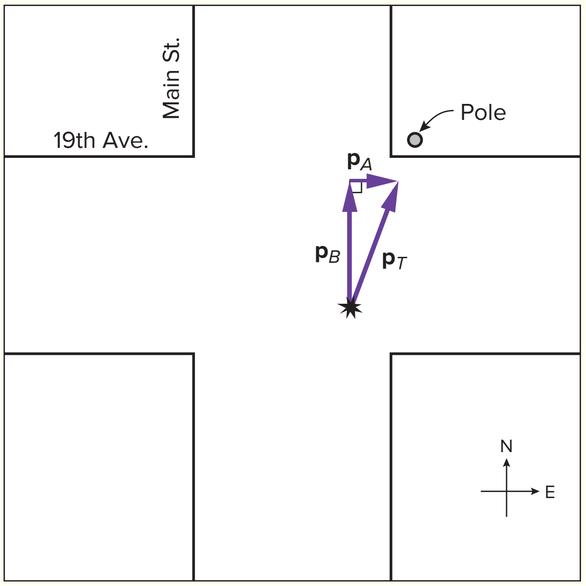


The collision at Main Street and 19th Avenue.

The Analysis. Officer Jones, having taken a physics course during college and being trained in the art of accident investigation, makes these observations:

1. The point of impact is well marked by the shards of glass from the headlights of B's car and other debris. Officer Jones indicates this point on the diagram in her accident report form.
2. The direction the two cars are traveling after the impact is also obvious. (She indicated this by a line drawn from the point of impact to the cars' final resting spot.)
3. Both cars have about the same mass (both are compacts of roughly the same vintage and size).
4. Conservation of momentum should determine the direction of the momentum vector after the collision.

After sketching the diagram and noting the direction of the final momentum vector, Officer Jones concludes that B is lying. Why? The final momentum vector must be equal to the sum of the initial momentum vectors of the two cars before the collision. Because the cars were traveling at right angles to one another, the two initial momentum vectors form the sides of a right triangle whose hypotenuse is the total momentum of the system. The diagram clearly shows that the momentum of B's car must have been considerably larger than the momentum of A's car.



Officer Jones's accident report contains a vector diagram derived from conservation of momentum.

Because both drivers claimed to have just started from a complete stop after the traffic signal changed, the driver with the larger velocity before the collision is not telling it like it was. Driver B is the one who had the larger velocity and, so, was presumably speeding through the red light. Driver B is thus cited by Officer Jones.

to any pool player). Suppose the cue ball strikes the stationary 11 ball at an angle (off-center), as in figure 7.19. What happens to the two balls after the collision? The combined effects of conservation of momentum and conservation of kinetic energy lead to a unique result well known to serious pool players.

The initial momentum of the system is simply that of the cue ball, the only one moving. Its direction is indicated by the arrow labeled $\mathbf{p}_i$ in figure 7.19 and in the drawings in figure 7.20. The force of interaction (and the impulse) between the two balls is along a line joining the centers of the balls at the point of impact. The 11 ball moves off along this line because the force of contact pushes it in that direction.

The total momentum of the system after the collision must still be in the direction of the initial momentum, because momentum is conserved in the collision. Conservation of momentum also restricts the possible momentum and direction of the cue ball's motion after the collision (fig. 7.20). The momentum vectors of the two balls after the collision are added here to give the total momentum of the system, $\mathbf{p}_{\text{total}}$, which must be equal in both magnitude and direction to the initial momentum of the system.

Because the collision is elastic, the initial kinetic energy of the cue ball, $\frac{1}{2}mv^2$, must also equal the sum of the kinetic energies of the two balls after the collision. Because the masses of the two balls are equal, conservation of kinetic energy in the collision requires that*

$$v^2 = (v_1)^2 + (v_2)^2$$

where v is the speed of the cue ball before the collision, and v_1 and v_2 are the speeds of the two balls afterward. The velocity vectors form a triangle like the one formed by the momentum vectors in figure 7.20. If the sum of the squares of the two sides equals the square of the third side of the triangle, this triangle must be a right triangle, according to the Pythagorean theorem from plane geometry. If the velocity vectors form a right triangle, so do the momentum vectors, which have the same directions as the corresponding velocity vectors.

Conservation of momentum requires that the momentum vectors add to form a triangle, but conservation of kinetic energy dictates that it be a *right* triangle. The cue ball will move off at a right angle (90°) to the direction of motion of the 11 ball after the collision. In playing pool, this is an important piece of intelligence if you are planning your next shot. Conservation of momentum *and* conservation of kinetic energy determine the shot.

If you have a pool table handy, test these conclusions using a variety of impact angles. (Marbles or steel balls are a suitable substitute.) You may not get perfect 90° angles after the collision: The collision is not perfectly elastic, and spin can sometimes be a factor. The angle between the two final velocities, though, will usually be within a few degrees of a right angle. Seeing is believing, however, so give it a try.

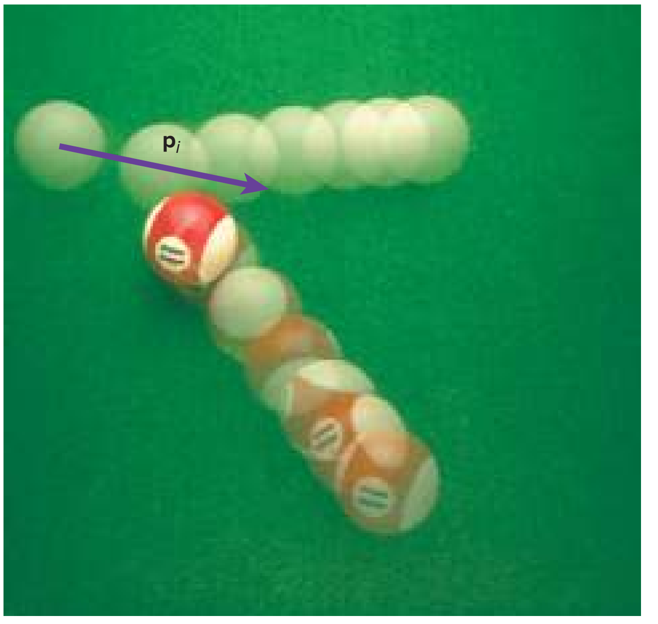


Figure 7.19 The cue ball is aimed at a point off-center on the second ball to produce an angular collision. *Richard Megna/ Fundamental Photographs, NYC*

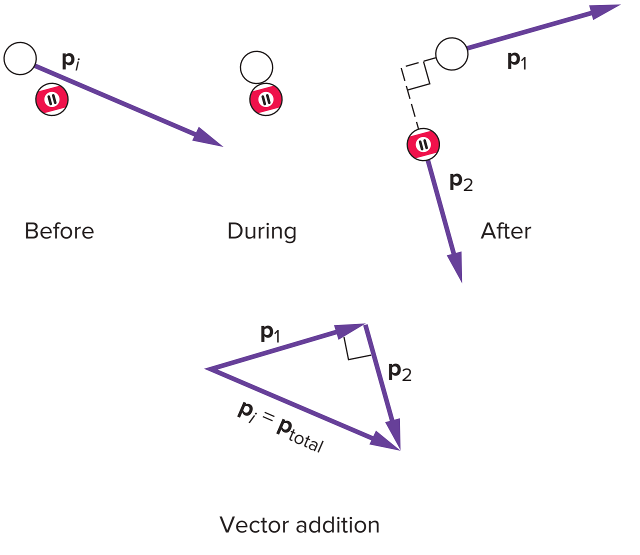


Figure 7.20 The momentum vectors of the two balls after the collision add to give the total (initial) momentum of the system. The paths of the two balls are approximately at right angles after the collision.

*Conservation of kinetic energy requires that $\frac{1}{2}mv^2 = \frac{1}{2}m(v_1)^2 + \frac{1}{2}m(v_2)^2$, but the mass and the factor of $\frac{1}{2}$ can be divided out of the equation.

Conservation of momentum requires that the direction of the momentum vector be conserved, as well as its size. When collisions occur at an angle, this requirement restricts the directions and velocities of the resulting motions. If the collision is elastic, as with billiard balls, conservation of energy adds another restriction. If you can imagine the direction and magnitude of the original momentum vector, you will have some sense of the outcome. These conservation laws are powerful predictors of what happens when people, billiard balls, cars, and even subatomic particles or stars collide.

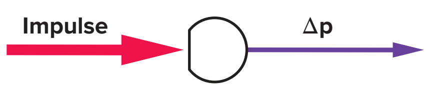
Debatable Issue

Statistics on automobile accidents provide convincing evidence that the use of seat belts (with shoulder harnesses) reduces the number of serious injuries and fatalities in automobile accidents. On the other hand, there are some cases in which being thrown from a car in an accident actually improves your chances of survival—if the car catches fire, for example. Given this, should individuals be able to choose not to use seat belts without legal consequences?

Summary

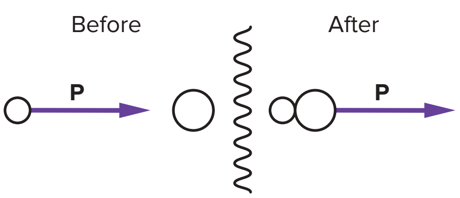
In this chapter, we recast Newton's second law in terms of impulse and momentum to describe interactions between objects, such as collisions, that involve strong interaction forces acting over brief time intervals. The principle of conservation of momentum, which follows from Newton's second and third laws, plays a central role.

1 **Momentum and impulse.** Newton's second law can be recast in terms of momentum and impulse, yielding the statement that the net impulse acting on an object equals the change in momentum of the object. Impulse is defined as the average force acting on an object multiplied by the time interval during which the force acts. Momentum is defined as the mass of an object times its velocity.



$\mathbf{F}_{net}\Delta t = \Delta\mathbf{p}, \quad \mathbf{p} = m\mathbf{v}$

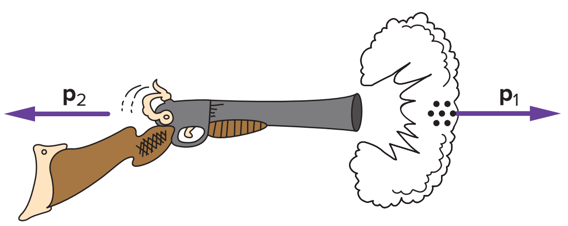
2 **Conservation of momentum.** Newton's second and third laws combine to yield the principle of conservation of momentum: If the net external force acting on a system is zero, the total momentum of the system is a constant.



If $\mathbf{F}_{external} = 0$

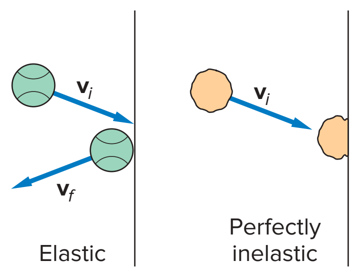
$\mathbf{P}_{total}$ = constant

3 **Recoil.** If an explosion or push occurs between two objects initially at rest, conservation of momentum dictates that the total momentum after the event must still be zero if there is no net external force. The final momentum vectors of the two objects are equal in size but opposite in direction.

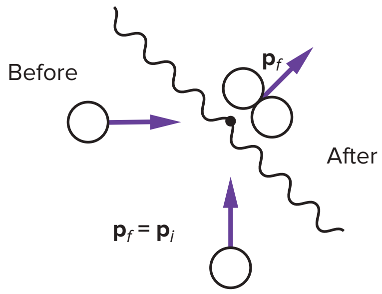


$\mathbf{p}_2 = -\mathbf{p}_1$

4 **Elastic and inelastic collisions.** A perfectly inelastic collision is one in which the objects stick together after the collision. If external forces can be ignored, the total momentum is conserved. An elastic collision is one in which the total kinetic energy is also conserved.



5 **Collisions at an angle.** Conservation of momentum is not restricted to one-dimensional motion. When objects collide at an angle, the total momentum of the system before and after the collision is found by adding the momentum vectors of the individual objects.



Impulse and momentum are evident in so many everyday phenomena. It is easy to become confused about which properties stay the same and which change, especially in collisions. The study hints in Connect should help clarify these ideas.

Key Terms

Impulse, 126
Linear momentum, 126
Impulse-momentum principle, 127
Conservation of momentum, 128
Recoil, 132
Perfectly inelastic collision, 133
Elastic collision, 133
Partially inelastic collision, 133

Study Hint

Except for the examples involving impulse, most of the situations described in this chapter highlight the principle of conservation of momentum. The basic ideas used in applying conservation of momentum are these:

1. External forces are assumed to be much smaller than the very strong forces of interaction in a collision or other brief event. If external forces acting on the system can be ignored, momentum is conserved.
2. The total momentum of the system before the collision or other brief interaction $\mathbf{p}_{\text{initial}}$ is equal to the momentum after the event $\mathbf{p}_{\text{final}}$. Momentum is conserved and does not change.
3. Equality of momentum before and after the event can be used to obtain other information about the motion of the objects.

For review, look back at how these three points are used in each of the examples in this chapter. The total momentum of the system before and after the event is always found by adding the momentum values of the individual objects as *vectors*. You should be able to describe the magnitude and direction of this total momentum for each of the examples.

Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available in Connect

Q1. Does the length of time that a force acts on an object have any effect on the strength of the impulse produced? Explain.

Q2. Two forces produce equal impulses, but the second force acts for a time twice that of the first force. Which force, if either, is larger? Explain.

Q3. Is it possible for a baseball to have as large a momentum as a much more massive bowling ball? Explain.

Q4. Are impulse and force the same thing? Explain.

Q5. Are impulse and momentum the same thing? Explain.

Q6. If a ball bounces off a wall so that its velocity coming back has the same magnitude it had prior to bouncing,
 a. Is there a change in the momentum of the ball? Explain.
 b. Is there an impulse acting on the ball during its collision with the wall? Explain.

Q7. Is there an advantage to following through when hitting a baseball with a bat, thereby maintaining a longer contact between the bat and the ball? Explain.

Q8. What is the advantage of a padded dashboard compared to a rigid dashboard in reducing injuries during collisions? Explain using momentum and impulse ideas.

Q9. What is the advantage of an air bag in reducing injuries during collisions? Explain using impulse and momentum ideas.

*Q10. If an air bag inflates too rapidly and firmly during a collision, it can sometimes do more harm than good in low-velocity collisions. Explain using impulse and momentum ideas.

Q11. If you catch a baseball or softball with your bare hand, will the force exerted on your hand by the ball be reduced if you pull your arm back during the catch? Explain. (See everyday phenomenon box 7.1.)

Q12. Suppose you move your hand forward to meet the egg when performing the egg-toss game. Will this be more or less likely to break the egg than moving your hand backward? Explain. (See everyday phenomenon box 7.1.)

Q13. A truck and a bicycle are moving side by side with the same velocity. Which, if either, will require the larger impulse to bring it to a halt? Explain.

Q14. Is the principle of conservation of momentum always valid, or are there special conditions necessary for it to be valid? Explain.

Q15. A ball is accelerated down a fixed inclined plane under the influence of the force of gravity. Is the momentum of the ball conserved in this process? Explain.

Q16. Two objects collide under conditions where momentum is conserved. Is the momentum of each object conserved in the collision? Explain.

Q17. Which of Newton's laws of motion are involved in justifying the principle of conservation of momentum? Explain.

*Q18. A compact car and a large truck have a head-on collision. During the collision, which vehicle, if either, experiences
a. The greater force of impact? Explain.
b. The greater impulse? Explain.
c. The greater change in momentum? Explain.
d. The greater acceleration? Explain.

Q19. A fullback collides midair and head-on with a lighter defensive back. If the two players move together following the collision, is it possible that the fullback will be carried backward? Explain.

Q20. Two ice skaters, initially at rest, push off one another. What is the total momentum of the system after they push off? Explain.

Q21. Two shotguns are identical in every respect (including the size of shell fired) except that one has twice the mass of the other. Which gun will tend to recoil with greater velocity when fired? Explain.

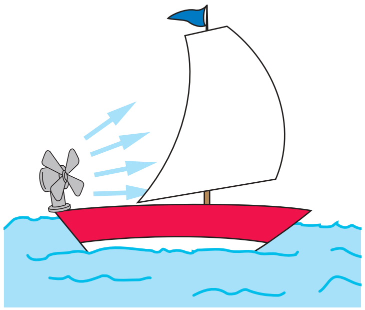
*Q22. When a cannon rigidly mounted on a large boat is fired, is momentum conserved? Explain, being careful to clearly define the system being considered.

Q23. Is it possible for a rocket to function in empty space (in a vacuum) where there is nothing to push against except itself? Explain.

Q24. Suppose you are standing on a surface that is so slick you can get no traction at all in order to begin moving across this surface. Fortunately, you are carrying a bag of oranges. Explain how you can get yourself moving.

Q25. Suppose an astronaut in outer space suddenly discovers that the tether connecting her to the space station is cut and she is slowly drifting away from the shuttle. Assuming that she is wearing a tool belt holding several wrenches, how can she move herself back toward the space station? Explain.

*Q26. Suppose that on a perfectly still day, a sailboat enthusiast decides to bring along a powerful battery-operated fan in order to provide an air current for his sail, as shown in the diagram.
a. What are the directions of the change in momentum of the air at the fan and at the sail?
b. What are the directions of the forces acting on the fan and on the sail due to these changes in momentum?
c. Would the sailor be better off with the sail furled (down) or unfurled (up)? Explain.



Q26 Diagram

Q27. A skateboarder jumps on a moving skateboard from the side. Does the skateboard slow down or speed up in this process? Explain, using conservation of momentum.

Q28. A railroad car collides and couples with a second railroad car that is standing still. If external forces acting on the system are ignored, is the velocity of the system after the collision equal to, greater than, or less than that of the first car before the collision? Explain.

Q29. Is the collision in question 28 elastic, partially inelastic, or perfectly inelastic? Explain.

Q30. If momentum is conserved in a collision, does this indicate conclusively that the collision is elastic? Explain.

Q31. A ball bounces off a wall with a velocity whose magnitude is less than it was before hitting the wall. Is the collision elastic? Explain.

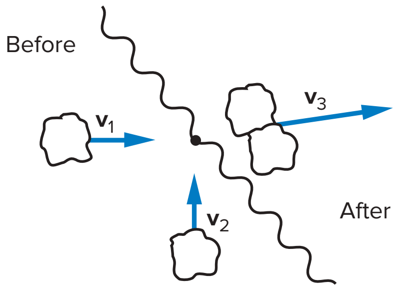
Q32. A ball bounces off a wall that is rigidly attached to the Earth.
a. Is the momentum of the ball conserved in this process? Explain.
b. Is the momentum of the entire system conserved? Explain, being careful to clarify how you are defining the system.

Q33. A cue ball strikes an 8 ball of equal mass, which is initially at rest. The cue ball stops and the 8 ball moves forward with a velocity equal to the initial velocity of the cue ball. Is the collision elastic? Explain.

Q34. Two lumps of clay traveling through the air in opposite directions collide and stick together. Their momentum vectors prior to the collision are shown in the diagram. Sketch the momentum vector of the combined lump of clay after the collision, making the length and direction appropriate to the situation. Explain your result.

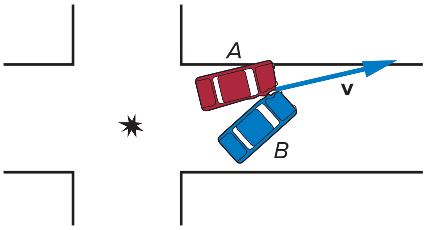
Q34 Diagram

Q35. Two lumps of clay, of equal mass, are traveling through the air at right angles to each other with velocities of equal magnitude. They collide and stick together. Is it possible that their velocity vector after the collision is in the direction shown in the diagram? Explain.



Q35 Diagram

Q36. Two cars of equal mass collide at right angles to each other in an intersection. Their direction of motion after the collision is as shown in the diagram. Which car had the greater velocity before the collision? Explain. (See everyday phenomenon box 7.2.)



Q36 Diagram

Q37. A car and a small truck traveling at right angles to each other with the same speed collide and stick together. The truck's mass is roughly twice the car's mass. Sketch the direction of their momentum vector immediately after the collision. Explain your result. (See everyday phenomenon box 7.2.)

*Q38. A cue ball strikes a glancing blow against a second billiard ball initially at rest. Sketch the situation, indicating the magnitudes and directions of the momentum vectors of each ball before and after the collision.

Exercises

E1. An average force of 4800 N acts for a time interval of 0.003 s on a golf ball.
  a. What is the magnitude of the impulse acting on the golf ball?
  b. What is the change in the golf ball's momentum?

E2. What is the momentum of a 1300-kg car traveling with a speed of 27 m/s (60 MPH)?

E3. A bowling ball has a mass of 7 kg and a speed of 1.5 m/s. A major league baseball has a mass of 0.142 kg and a speed of 40 m/s. Which ball has the larger momentum?

E4. A force of 128 N acts on a ball for 0.45 s. If the ball is initially at rest,
  a. What is the impulse on the ball?
  b. What is the final momentum of the ball?

E5. A 0.14-kg ball traveling with a speed of 40 m/s is brought to rest in a catcher's mitt. What is the size of the impulse exerted by the mitt on the ball?

E6. A ball experiences a change in momentum of 64 kg·m/s.
  a. What is the impulse acting on the ball?
  b. If the time of interaction is 0.15 s, what is the magnitude of the average force acting on the ball?

E7. A 75-kg front-seat passenger in a car moving initially with a speed of 23 m/s (50 MPH) is brought to rest by an air bag in a time of 0.3 s.
  a. What is the impulse acting on the passenger?
  b. What is the average force acting on the passenger in this process?

E8. A ball traveling with an initial momentum of 1.7 kg·m/s bounces off a wall and comes back in the opposite direction with a momentum of −1.7 kg·m/s.
  a. What is the change in momentum of the ball?
  b. What impulse would be required to produce this change?

E9. A ball traveling with an initial momentum of 5.1 kg·m/s bounces off a wall and comes back in the opposite direction with a momentum of −4.3 kg·m/s.
  a. What is the change in momentum of the ball?
  b. What impulse is required to produce this change?

E10. A fullback with a mass of 108 kg and a velocity of 3.2 m/s due north collides head-on with a defensive back with a mass of 79 kg and a velocity of 5.6 m/s due south.
  a. What is the initial momentum of each player?
  b. What is the total momentum of the system before the collision?
  c. If they stick together and external forces can be ignored, what direction will they be traveling immediately after they collide?

E11. An ice skater with a mass of 70 kg pushes off against a second skater with a mass of 30 kg. Both skaters are initially at rest.
  a. What is the total momentum of the system after they push off?
  b. If the larger skater moves off with a speed of 2.8 m/s, what is the corresponding speed of the smaller skater?

E12. A rifle with a mass of 3.4 kg fires a bullet with a mass of 7.8 g (0.0078 kg). The bullet moves with a muzzle velocity of 860 m/s after the rifle is fired.
a. What is the momentum of the bullet after the rifle is fired?
b. If external forces acting on the rifle can be ignored, what is the recoil velocity of the rifle?

E13. A rocket ship at rest in space gives a short blast of its engine, firing 60 kg of exhaust gas out the back end with an average velocity of 450 m/s. What is the change in momentum of the rocket during this blast?

E14. A railroad car with a mass of 13,000 kg collides and couples with a second car of mass 20,000 kg that is initially at rest. The first car is moving with a speed of 3.5 m/s prior to the collision.
a. What is the initial momentum of the first car?
b. If external forces can be ignored, what is the final velocity of the two railroad cars after they couple?

E15. For the railroad cars in example box 7.4,
a. What is the kinetic energy of car 5 before the collision?
b. What is the kinetic energy of all five cars just after the collision?
c. Is energy conserved in this collision?

E16. A 4150-kg truck traveling with a velocity of 12 m/s due east collides head-on with a speeding 900-kg car traveling with a velocity of 40 m/s due west. The two vehicles stick together after the collision.
a. What is the momentum of each vehicle prior to the collision?
b. What are the size and direction of the total momentum of the two vehicles after they collide?

E17. For the two vehicles in exercise E16,
a. Sketch to scale the momentum vectors of the two vehicles prior to the collision.
b. Add the two vectors on your sketch graphically.

E18. A car with a mass of 1600 kg traveling with a speed of 25 m/s collides at right angles with a truck with a mass of 3000 kg traveling with a speed of 10 m/s.
a. Sketch to proper scale and direction the momentum vector of each vehicle prior to the collision.
b. Using the graphical method of vector addition, add the momentum vectors to get the total momentum of the system prior to the collision.

E19. Refer to example box 7.2 and figures 7.17 and 7.18. Assume you have a fullback with a mass of 100 kg and an initial velocity of 5.0 m/s, heading due east. A defensive back has a mass of 75 kg and an initial momentum of 4.0 m/s, heading due north. They collide and stick together, with a final momentum of 583 kg m/s, as shown in figure 7.18.
a. What is the final speed, after the collision, of the two football players?
b. How does this compare to the final speed for the head-on collision computed in example box 7.2?

Synthesis Problems

SP1. A fast ball thrown with a velocity of 40 m/s (approximately 90 MPH) is struck by a baseball bat, and a line drive comes back toward the pitcher with a velocity of 65 m/s. The ball is in contact with the bat for a time of just 0.005 s. The baseball has a mass of 142 g (0.142 kg).
a. What is the change in momentum of the baseball during this process?
b. Is the change in momentum greater than the final momentum? Explain.
c. What is the magnitude of the impulse required to produce this change in momentum?
d. What is the magnitude of the average force that acts on the baseball to produce this impulse?

SP2. A bullet is fired into a block of wood sitting on a block of ice. The bullet has an initial velocity of 800 m/s and a mass of 0.007 kg. The wooden block has a mass of 1.3 kg and is initially at rest. The bullet remains embedded in the block of wood afterward.
a. Assuming that momentum is conserved, find the velocity of the block of wood and bullet after the collision.
b. What is the magnitude of the impulse that acts on the block of wood in this process?
c. Does the change in momentum of the bullet equal that of the block of wood? Explain.

SP3. Consider two cases in which the same ball is thrown against a wall with the same initial velocity. In case A, the ball sticks to the wall and does not bounce. In case B, the ball bounces back with the same speed it came in with.
a. In which of these two cases is the change in momentum the largest?
b. Assuming that the time during which the momentum change takes place is approximately the same for these two cases, in which case is the larger average force involved?
c. Is momentum conserved in this collision? Explain.

SP4. A car traveling at a speed of 22 m/s (approximately 50 MPH) crashes into a solid concrete wall. The driver has a mass of 80 kg.
a. What is the change in momentum of the driver as he comes to a stop?
b. What impulse is required in order to produce this change in momentum?
c. How do the application and magnitude of this force differ in two cases: the first, in which the driver is wearing a seat belt, and the second, in which he is not wearing a seat belt and is stopped instead by contact with the windshield and steering column? Will the time of action of the stopping force change? Explain.

SP5. A 1600-kg car traveling due east with a speed of 30 m/s collides head-on with a 4800-kg truck traveling due west with a speed of 15 m/s. The two vehicles stick together after the collision.

a. What is the total momentum of the system prior to the collision?
b. What is the velocity of the two vehicles just after the collision?
c. What is the total kinetic energy of the system before the collision?
d. What is the total kinetic energy just after the collision?
e. Is the collision elastic? Explain.

Home Experiments and Observations

HE1. Take two marbles or steel balls of the same size and practice shooting one into the other. Make these observations:

a. If you produce a head-on collision with the second marble initially at rest, does the first marble come to a complete stop after the collision?
b. If the collision with a second marble occurs at an angle, is the angle between the paths of the two marbles after the collision a right angle (90°)?
c. If marbles of different sizes and masses are used, how do the results of parts a and b differ from those obtained with marbles of the same mass?

HE2. If you have access to a pool table, try parts a and b of the observations in home experiment 1 on the pool table. What effect does putting spin on the first ball have on the collisions?

HE3. If you have both a basketball and a tennis ball, try dropping the two of them onto a floor with a hard surface, first individually and then with the tennis ball placed on top of the basketball before the two are dropped together.

a. Compare the height of the bounce of each ball in these different cases. The case where the two are dropped together may surprise you.
b. Can you devise an explanation for these results using impulse and Newton's third law? (Consider the force between the basketball and the floor, as well as that between the tennis ball and the basketball for the case where they are dropped together.)

HE4. Place a cardboard box on a smooth tile or wood floor. Practice rolling a basketball or soccer ball at different speeds and allowing the ball to collide with the box. Observe the motion of both the box and the ball just after the collision.

a. How do the results of the collision vary for different speeds of the ball (slow, medium, fast)?
b. If we increase the weight of the box by placing books inside, how do the results of the collision change for the cases in part a?
c. Can you explain your results using conservation of momentum?

CHAPTER 8

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Rotational Motion of Solid Objects

Chapter Overview

Starting with a merry-go-round—and making use of the analogy between linear and rotational motion—we first consider what concepts are needed to describe rotational motion. We then turn to the causes of rotational motion, which involve a modified form of Newton's second law. Torque, rotational inertia, and angular momentum are introduced as we proceed. Our goal is to develop a clear picture of both the description and the causes of rotational motion. After studying this chapter, you should be able to predict what will happen in many common examples of spinning or rotating objects, such as ice skaters and divers. The world of sports is rich in examples of rotational motion.

Chapter Outline

1 **What is rotational motion?** How can we describe rotational motion? What are rotational velocity and acceleration, and how are they related to similar concepts used to describe linear motion?

2 **Torque and balance.** What determines whether a simple object, such as a balance beam, will rotate? What is torque, and how is it involved in causing an object to rotate?

3 **Rotational inertia and Newton's second law.** How can Newton's second law be adapted to explain the motion of rotating objects? How do we describe rotational inertia, an object's resistance to changes in rotational motion?

4 **Conservation of angular momentum.** What is angular momentum, and when is it conserved? How do spinning skaters or divers change their rotational velocities?

5 **Riding a bicycle and other amazing feats.** Why does a bicycle remain upright when it is moving but not when it is stationary? Can we treat rotational velocity and angular momentum as vectors?

In neighborhood parks, there are often child-propelled merry-go-rounds, as shown in figure 8.1. A merry-go-round consists of a circular steel platform mounted on an excellent bearing, so that it rotates without much frictional resistance. Once set in rotation, it will continue to rotate. Even a child can start it moving and then jump on (and sometimes fall off). Along with the swings, the slide, and the little animals mounted on heavy-duty springs, the merry-go-round is a popular center of activity in the park.

The motion of a merry-go-round bears both similarities and differences to motions we have already considered. A child sitting on the merry-go-round experiences circular motion, so some of the ideas discussed in chapter 5 come into play. What about the merry-go-round itself, though? It certainly moves, but it goes nowhere. How do we describe its motion?

Rotational motion of solid objects, such as the merry-go-round, is common: The rotating Earth, a spinning skater, a top, and a turning wheel all exhibit this type of motion. For Newton's theory of motion to be broadly useful, it should explain what is happening in rotational motion as well as in linear motion (an object moves from one point to another in a straight line). What causes rotational motion? Can Newton's second law be used to explain such motion?

We will find that there is a useful analogy between the linear motion of objects and rotational motion. The questions just posed can be answered best by making full use of this analogy. Taking advantage of the similarities between rotational motion and linear motion saves space in our mental computers, thus making the learning process more efficient.

Figure 8.1 A merry-go-round is an example of rotational motion. How do we describe and explain this motion?
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8.1 What Is Rotational Motion?

A child begins to rotate the merry-go-round described in the introduction. She does so by holding on to one of the bars on the edge of the merry-go-round as she stands beside it. She begins to push the merry-go-round, accelerating as she goes, until eventually she is running, and the merry-go-round is rotating quite rapidly.

How do we describe the rotational motion of the merry-go-round or that of a spinning ice skater? What quantities would we use to describe how fast they are rotating or how far they have rotated?

Rotational displacement and rotational velocity

How would you measure how fast the merry-go-round is rotating? If you stood to one side and watched the child pass your position, you could count the number of revolutions the child made in a given time, measured with your watch. Dividing the number of revolutions by the time in minutes yields the average rotational speed in revolutions per minute (rpm), a commonly used unit for describing the rate of rotation of motors, Ferris wheels, and other rotating objects.

If you say that the merry-go-round rotates at a rate of 15 rpm, you have described how fast an object is turning. The rate is analogous to speed or velocity, quantities used to describe how fast an object is moving in the case of linear motion. We usually use the term **rotational velocity** to describe this rate of rotation. Revolutions per minute is just one of several units used to measure this quantity.

In measuring the rotational velocity of the merry-go-round, we describe how far it rotates in revolutions or complete cycles. Suppose that an object rotates less than one complete revolution. We can then use a fraction of a revolution to describe how far it has turned, but we might also use an angle measured in degrees. Because there are 360° in one complete revolution or full circle, revolutions can be converted to degrees by multiplying by 360°/rev.

The quantity measuring how far an object has turned or rotated is an angle, often called the **rotational displacement.** It can be measured in revolutions, degrees, or a simple but less familiar unit called the **radian.*** The three units commonly used to describe rotational displacement are summarized in figure 8.2. Rotational displacement is analogous to the distance traveled by an object in linear motion. If we include the direction of travel, this linear distance, which we discussed in chapter 2, is sometimes called the **linear displacement.**

*The radian is defined by dividing the arc length through which the point travels by the radius of the circle on which it is moving. Thus, in figure 8.2, if the point on the merry-go-round moves along the arc length a distance s, the number of radians involved is s/r, where r is the radius. Because we are dividing one distance by another, the radian itself has no dimensions. Also, because the arc length s is proportional to the radius r, it does not matter how large a radius we choose. The ratio of s to r will be the same for a given angle. By definition of the radian, 1 revolution (rev) = 360° = 2π radians, and 1 radian (rad) = 57.3°.

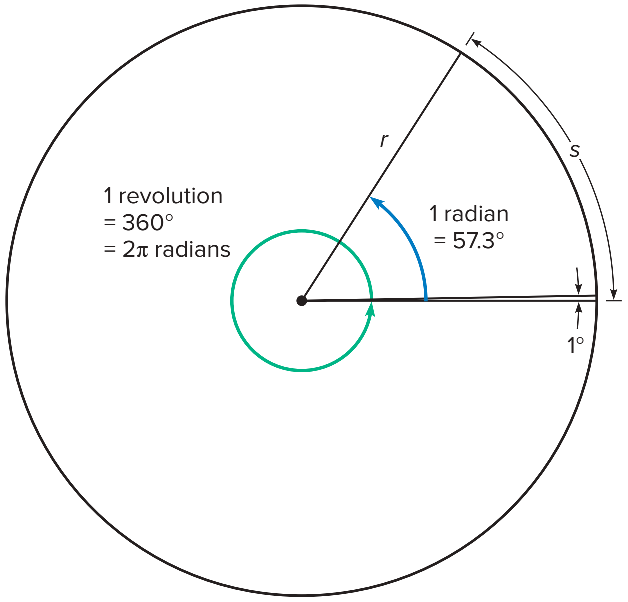


Figure 8.2 Revolutions, degrees, and radians are different units for describing the rotational displacement of the merry-go-round.

The symbols used to describe rotational quantities mainly come from the Greek alphabet. Greek letters are used to avoid confusion with other quantities represented by letters of our ordinary Roman alphabet. The Greek letter theta (θ) is commonly used to represent angles (rotational displacements), and the Greek letter omega (ω) is used to represent rotational velocities.

The quantities we have just introduced for describing the motion of an object such as the merry-go-round can be summarized as follows:

> Rotational displacement θ is an angle showing how far an object has rotated.

Also,

> Rotational velocity ω is the rate of change of rotational displacement. It is found by dividing the change in the rotational displacement by the time taken for this displacement to happen:
>
> $$\omega = \frac{\Delta\theta}{t}$$

In describing rotational velocity, we usually use either revolutions or radians as the measure of rotational displacement. Degrees are less commonly used. Example box 8.1 shows the conversion between revolutions per minute (rpm), radians per minute, and radians per second.

Example Box 8.1

Sample Exercise: Using Radians

Suppose an object has a rotational velocity of 33 revolutions per minute.

a. What is the rotational velocity in radians/minute?
b. What is the rotational velocity in radians/second?

a.

$$\omega = 33\ \frac{\text{rev}}{\text{min}}$$

$$1\ \text{rev} = 2\pi\ \text{radians}$$

$$\omega = 33\ \frac{\cancel{\text{rev}}}{\text{min}} \times \frac{2\pi\ \text{radians}}{1\ \cancel{\text{rev}}} = 66\pi\ \frac{\text{radians}}{\text{min}} = 207.3\ \frac{\text{radians}}{\text{min}}$$

b.

$$\omega = 207.3\ \frac{\text{radians}}{\text{min}}$$

$$1\ \text{min} = 60\ \text{sec}$$

$$\omega = 207.3\ \frac{\text{radians}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{sec}} = 3.5\ \frac{\text{radians}}{\text{sec}}$$

Lines drawn through the units indicate cancellation.

What is rotational acceleration?

In our original description of the child pushing the merry-go-round, the rate of rotation increased as she ran alongside. This involves a *change* in the rotational velocity, which suggests the concept of **rotational acceleration.** The Greek letter alpha (α) is the symbol used for rotational acceleration. It is the first letter in the Greek alphabet and corresponds to the letter a, used to represent linear acceleration.

Rotational acceleration can be defined similarly to linear acceleration (see chapter 2):

> Rotational acceleration is the rate of change in rotational velocity. It is found by dividing the change in rotational velocity by the time required for this change to occur,
>
> $$\alpha = \frac{\Delta\omega}{t}$$

The units of rotational acceleration are rev/s^2 and rad/s^2.

These definitions for both rotational velocity and rotational acceleration actually yield the *average* values of these quantities. To get *instantaneous* values, the time interval t must be made very small, as in the linear-motion definitions of instantaneous velocity and instantaneous acceleration (see sections 2.2 and 2.3). This then yields the rate of change of either displacement or velocity at a given instant in time.

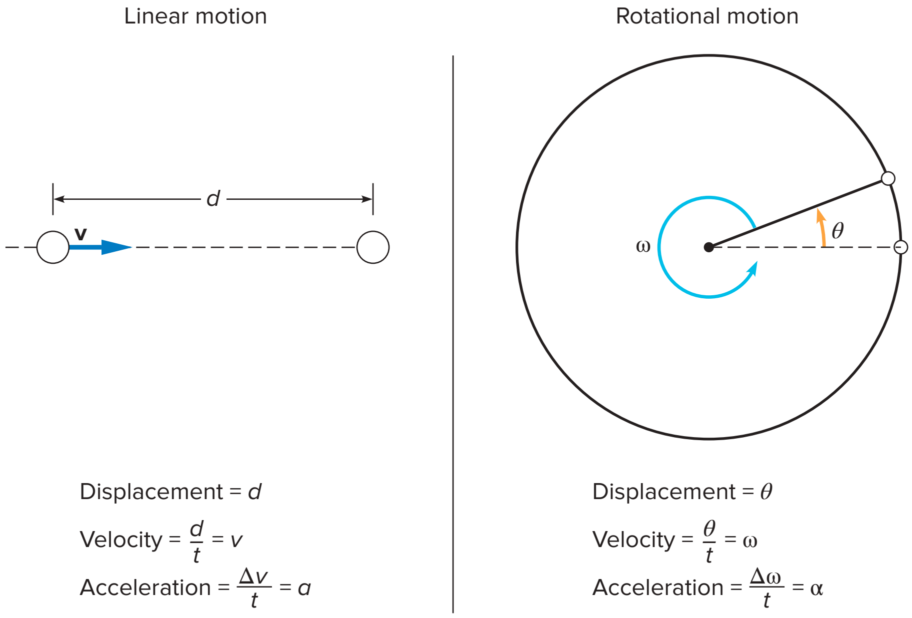


Figure 8.3 There is a close resemblance between quantities used to describe linear motion and those used to describe rotational motion.

You will remember these definitions of rotational displacement, velocity, and acceleration better if you keep in mind the complete analogy that exists between linear and rotational motion. This analogy is summarized in figure 8.3. In one dimension, distance d represents the change in position or *linear* displacement, which corresponds to *rotational* displacement θ. Average velocity and acceleration for linear motion are defined as before, with the corresponding definitions of rotational velocity and acceleration shown on the right side of the diagram in figure 8.3.

Constant rotational acceleration

In chapter 2, we introduced equations for the special case of constant linear acceleration because of its many important applications. By comparing linear and rotational quantities, we can write similar equations for constant rotational acceleration by substituting the rotational quantities for the corresponding linear quantities in the equations developed for linear motion. Table 8.1 shows the results beside corresponding equations for linear motion. Just as v_0 represents the initial linear velocity, ω_0 refers to the initial angular velocity. Example box 8.2 is an application of the equations for constant rotational acceleration.

The merry-go-round in example box 8.2 starts from rest and rotates through nine complete revolutions in 1 minute, a good effort by the person pushing. It is unlikely this rate of acceleration could be sustained much longer than 1 minute. The rotational velocity reached in this time is a little less than a third of a revolution per second, a fast rotational velocity for such a merry-go-round.

Table 8.1

Constant Acceleration Equations for Linear and Rotational Motion

Linear Motion	Rotational Motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$d = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Example Box 8.2

Sample Exercise: Rotating a Merry-Go-Round

Suppose a merry-go-round is accelerated at a constant rate of 0.005 rev/s^2, starting from rest.

a. What is its rotational velocity at the end of 1 min?
b. How many revolutions does the merry-go-round make in this time?

a. $\alpha = 0.005\ \text{rev/s}^2$ $\quad$ $\omega = \omega_0 + \alpha t$

$\omega_0 = 0$ $\quad$ $= 0 + (0.005\ \text{rev/s}^2)\,(60\ \text{s})$

$t = 60\ \text{s}$ $\quad$ $= \mathbf{0.30\ rev/s}$

b. $\theta = ?$ $\quad$ Since ω_0 is equal to zero,

$$\theta = \frac{1}{2}\alpha t^2$$

$$= \frac{1}{2}(0.005\ \text{rev/s}^2)(60\ \text{s})^2$$

$$= \mathbf{9\ rev}$$

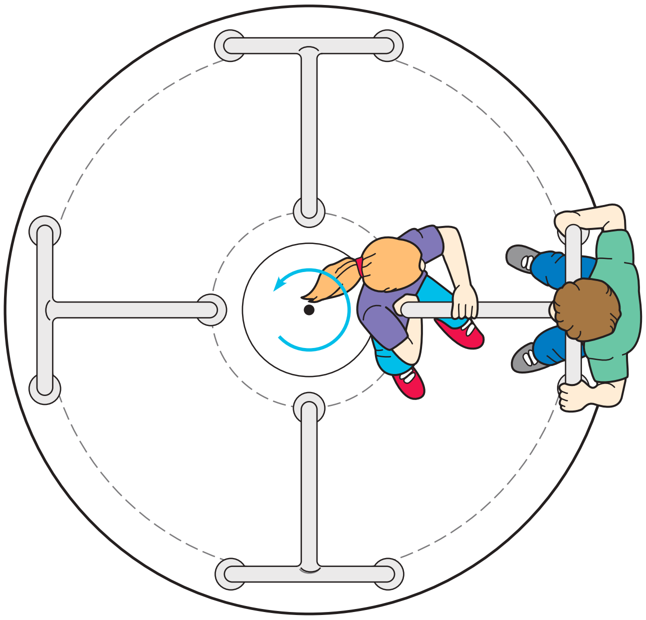


Figure 8.4 The rider near the edge travels a greater distance in 1 revolution than the one near the center.

How are linear and rotational velocity related?

How fast is the rider going when the merry-go-round in example box 8.2 is rotating with a velocity of 0.30 rev/s? The answer to this question depends on where the rider is sitting. He or she will move faster when seated near the edge of the merry-go-round than when seated near the center. The question involves a relationship between the linear speed of the rider and the rotational velocity of the merry-go-round.

In figure 8.4, the positions of the two riders are shown with dashed lines indicating the different radii. The rider seated at the greater distance from the center travels a larger distance in 1 revolution than the rider near the center, because the circumference of his circle is greater. The outside rider is therefore moving with a greater linear speed than the rider near the center.

The farther the rider is from the center, the farther he travels in 1 revolution, and the faster he is moving. The circumference of the circle on which the rider is traveling increases in proportion to the radius of the circle *r,* the distance of the rider from the center. If we express the rotational velocity in radians per second (rad/s), the relationship for the linear speed of the rider takes the form

$$v = r\omega$$

The linear speed v of a rider seated a distance r from the center of a merry-go-round is equal to r times the rotational velocity ω of the merry-go-round. (For this simple relationship to be valid, however, the rotational velocity must be expressed in radians per second rather than revolutions or degrees per second.)

The rate at which the merry-go-round or other object turns will affect how fast a point on the rotating object will move—in other words, its linear speed. Linear speed will depend on the distance from the axis of rotation. A child out at the edge of the merry-go-round will get a bigger thrill from the ride than one more timidly parked near the middle.

Rotational displacement, rotational velocity, and rotational acceleration are the quantities we need to fully describe the motion of a rotating object. They describe how far the object has rotated (rotational displacement), how fast it is rotating (rotational velocity), and the rate at which the rotation is changing (rotational acceleration). These definitions are analogous to similar quantities used to describe linear motion. They tell us how the object is rotating, but not why. Causes of rotation are considered next.

8.2 Torque and Balance

What causes the merry-go-round to rotate in the first place? To get it started, a child has to push it, which involves applying a force. The direction and point of application of force are important to the success of the effort. If the child pushes straight in toward the center, nothing happens. How do we apply a force to produce the best effect?

Unbalanced torques cause objects to rotate. What are *torques,* though, and how are they related to forces? A look at a simple scale or *balance* can help us get at the idea.

When is a balance balanced?

Consider a balance made of a thin but rigid beam supported by a **fulcrum,** or pivot point, as in figure 8.5. If the beam is balanced before we place weights on it, and if we put equal weights at equal distances from the fulcrum, we expect that the beam will still be balanced. By *balanced,* we mean that it will *not* tend to rotate about the fulcrum.

Suppose we wish to balance unequal weights on the beam. To balance a weight twice as large as a smaller weight, would we place the two weights at equal distances from the fulcrum? Intuition suggests that the smaller weight needs to be placed farther from the fulcrum than the larger weight for the system to be balanced, but it may not tell you how much

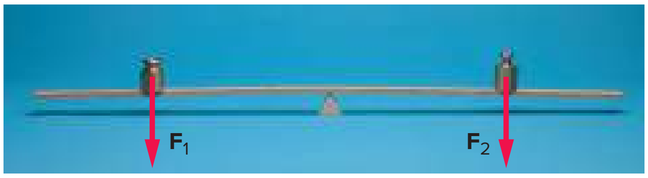


Figure 8.5 A simple balance with equal weights placed at equal distances from the fulcrum.
James Ballard/McGraw-Hill Education

Figure 8.6 A simple balance with unequal weights placed at different distances from the fulcrum. What determines whether the system will be balanced?
James Ballard/McGraw-Hill Education

farther (fig. 8.6). Trial and error with a simple balance will show that the smaller weight must be placed twice as far from the fulcrum as the larger (twice as large) weight.

Try it yourself using a ruler for the beam and a pencil for the fulcrum. Coins can be used as the weights. Experiments will show that both the weight and the distance from the fulcrum are important. The farther a weight sits from the fulcrum, the more effective it will be in balancing larger weights on the other side of the fulcrum. *Weight times distance from the fulcrum* determines the effect. If this product is the same for weights placed on either side of the fulcrum, the balance will not rotate.

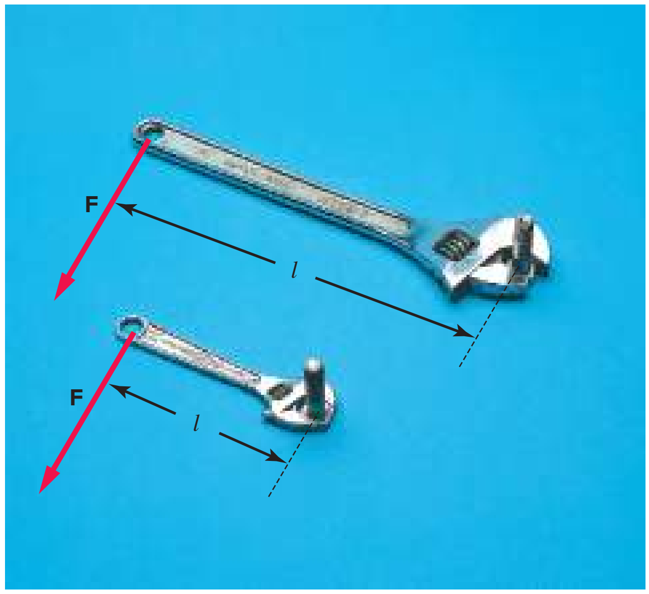


Figure 8.7 A wrench with a long handle is more effective than one with a short handle because of the longer lever arm for the longer wrench.
James Ballard/McGraw-Hill Education

What is a torque?

This product of the force and the distance from the fulcrum—which describes the tendency of a weight to produce a rotation—is called the **torque.** More generally, it is described as the following.

> The torque, τ, about a given axis or fulcrum is equal to the product of the applied force and the lever arm, *l:*
>
> $$\tau = Fl$$
>
> The **lever arm** is the perpendicular distance from the axis of rotation to the line of action of the applied force.

The symbol τ is the Greek letter tau and is commonly used for torque.

The length l is the distance from the fulcrum to the point of application of the force and must be measured in a direction perpendicular to the line of action of the force. This distance is called the *lever arm* or *moment arm* of the force in question. The strength of the torque depends directly on both the size of the force and the length of its lever arm. If the torques produced by weights on either side of the fulcrum of our balance are equal in magnitude, the scale is balanced. It will not rotate.

Most of us have tried to turn a nut with a wrench at some time. We exert the force at the end of the wrench, in a direction perpendicular to the handle (fig. 8.7). The handle is the lever, and its length determines the lever arm. A longer handle is more effective than a shorter one, because the resulting torque is greater.

As the term suggests, *lever arm* comes from our use of levers to move objects. Moving a large rock with a crowbar, for example, involves leverage. The applied force is most effective if it is applied at the end of the bar *and* perpendicular to the bar. The lever arm l is just the distance from the fulcrum to the end of the bar. If the force is applied in some other direction, as in figure 8.8, the lever arm is shorter than it would be if the force were applied perpendicular to the

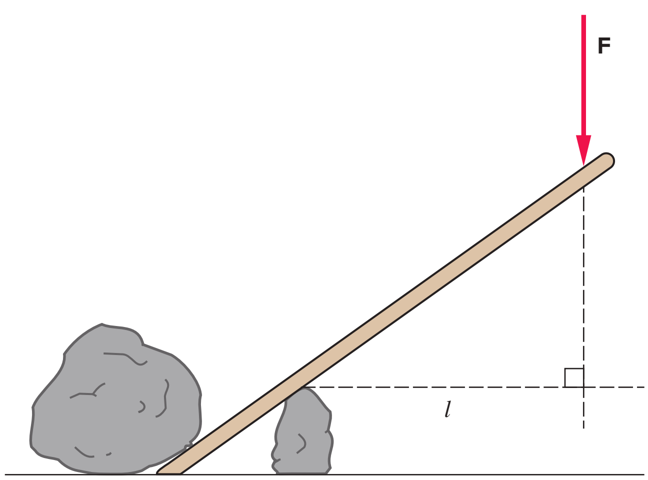


Figure 8.8 When the applied force is not perpendicular to the crowbar, the lever arm is found by drawing a perpendicular line from the fulcrum to the line of action of the force.

bar. The lever arm is found by drawing the perpendicular line from the fulcrum to the line of action of the force, as indicated in figure 8.8.

How do torques add?

The direction of rotation associated with a torque is also important. Some torques tend to produce clockwise rotations and others counterclockwise rotations about a particular axis. For example, the torque due to the heavier weight on the right side of the fulcrum in figure 8.6 will produce a clockwise rotation about the fulcrum if it acts by itself. This is opposed by the equal-magnitude torque of the weight on the left side of the fulcrum, which would produce a counterclockwise rotation. The two torques cancel each other when the system is balanced.

Because torques can have opposing effects, we assign opposite signs to torques that produce rotations in opposite directions. If, for example, we chose to call torques that produce a counterclockwise rotation positive, torques producing clockwise rotations would be negative. (This is the conventional choice—it is unimportant which direction is chosen as positive as long as you are consistent in a given situation.) Identifying the sign of the torque indicates whether it will add to or subtract from other torques.

In the case of the simple balance, the net torque will be zero when the beam is balanced, because the two torques are equal but have opposite signs. The condition for balance, or equilibrium, is that the net torque acting on the system be zero. Either no torques act or the sum of the positive torques equals the sum of the negative torques, canceling one another by adding up to zero.

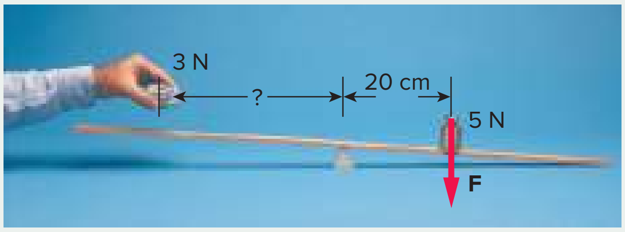
In example box 8.3, we find the distance that a 3-N weight must be placed from a fulcrum to balance a 5-N weight producing a net torque of zero. (Because $\mathbf{W} = m\mathbf{g}$, a 5-N weight has a mass of approximately 0.5 kg, or 500 g.) The units of torque are those of force times distance, Newton-meters (N·m) in the metric system.*

Example Box 8.3

Sample Exercise: Balancing a System

Suppose we have a 3-N weight that we want to balance against a 5-N weight on a beam, which is balanced when no masses are in place. The 5-N weight is placed 20 cm to the right of the fulcrum.

a. What is the torque produced by the 5-N weight?
b. How far would we have to place the 3-N weight from the fulcrum to balance the system?



Where should the 3-N weight be placed on the beam to balance the system?
James Ballard/McGraw-Hill Education

a. $F = 5\text{ N}$ $\qquad \tau = -Fl$

$l = 20\text{ cm} = 0.2\text{ m}$ $\qquad = -(5\text{ N})(0.2\text{ m})$

$\tau = ?$ $\qquad = -1\text{ N·m}$

The minus sign indicates that this torque would produce a clockwise rotation.

b. $F = 3\text{ N}$ $\qquad \tau = Fl$

$l = ?$ $\qquad l = \dfrac{\tau}{F}$

$$= \frac{+1\text{ N}\cdot\text{m}}{3\text{ N}}$$

$$= \mathbf{0.33\ m\ (33\ cm)}$$

What is the center of gravity of an object?

Often, the weight of an object is itself an important factor in whether the object will rotate. How far, for example, could the child in figure 8.9 walk out on the plank without the plank's tipping? The weight of the plank is important in this case, and the concept of *center of gravity* is useful.

The **center of gravity** is the point about which the weight of the object itself exerts no net torque. If we suspended the object from its center of gravity, there would be no net torque at the suspension or support point. The object would be balanced. You can locate the center of gravity of a rodlike

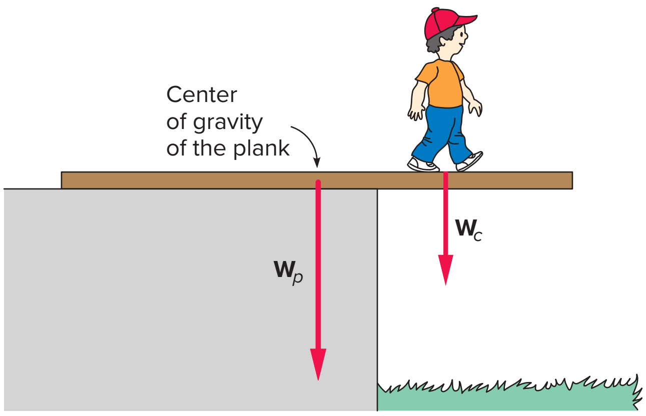


Figure 8.9 How far can the child walk without tipping the plank? The entire weight of the plank, W_p, can be treated as though it is located at the center of gravity.

*Although the product of a Newton-meter (N·m) equals a Joule (J) when we are using it as an energy unit, when a N·m is used as a torque unit, we state it as N·m, not Joules.

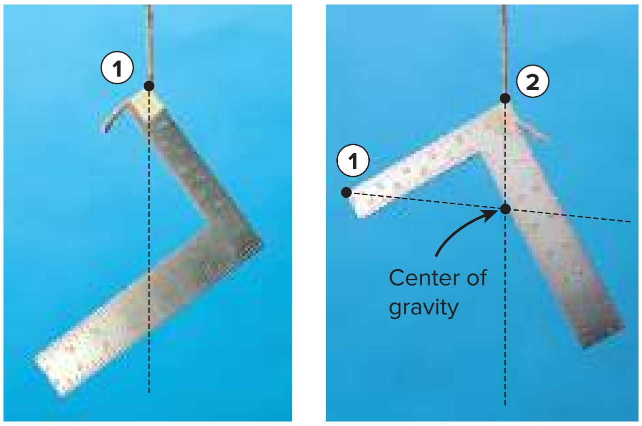


Figure 8.10 Locating the center of gravity of a planar object. The center of gravity does not necessarily lie within the object.
James Ballard/McGraw-Hill Education

object by finding the point where it balances on your finger or other suitable fulcrum. For a more complex two-dimensional (planar) object, you can locate the center of gravity by suspending the object from two different points, drawing a line straight down from the point of suspension in each case, and locating the point of intersection of the two lines, as figure 8.10 illustrates.

In the case of the plank (fig. 8.9), the center of gravity is at the geometric center of the plank, provided that the plank is uniform in density and cut. The pivot point will be the edge of the supporting platform, the point to consider when computing torques. The plank will not tip as long as the counterclockwise torque produced by the weight of the plank about the pivot point is larger than the clockwise torque produced by the weight of the child. The weight of the plank is treated as though it is concentrated at the center of gravity of the plank.

The plank will verge on tipping when the torque of the child about the edge equals the torque of the plank in magnitude. This determines how far the child can walk on the plank before it tips. As long as the torque of the plank about the edge of the platform is larger than the torque of the child, the child is safe. The platform keeps the plank from rotating counterclockwise.

The location of the center of gravity is important in any effort at balancing. If the center of gravity lies below the pivot point, as in the balancing toy in figure 8.11, the toy will automatically regain its balance when disturbed. The center of gravity returns to the position directly below the pivot point, where the weight of the toy produces no torque. In this position, the lever arm for the weight of the clown and bar is zero.

Similarly, the location of your center of gravity is important in performing various maneuvers, athletic or otherwise. Try, for example, touching your toes with your back and heels against a wall. Why is this apparently simple trick impossible for most people to do? Where is your center of gravity relative to the pivot point determined by your feet? Center of gravity and torque are at work here.

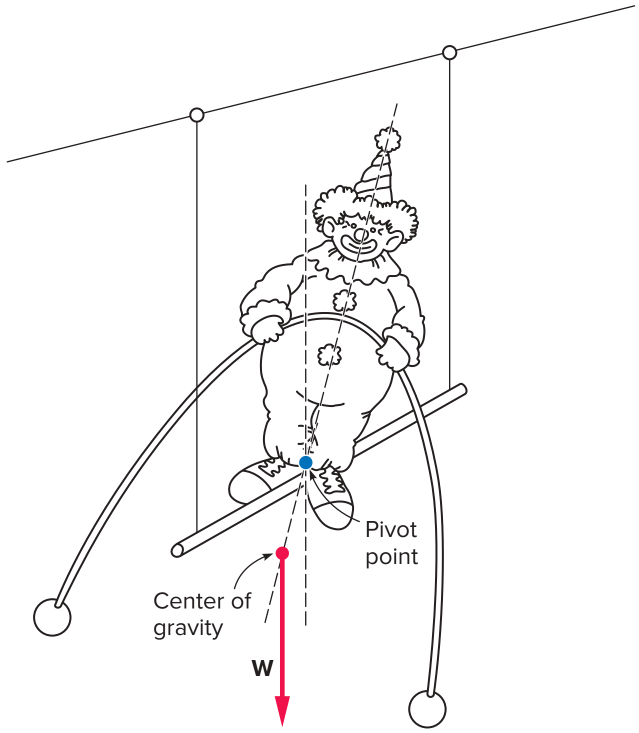


Figure 8.11 The clown automatically returns to an upright position, because the center of gravity is below the pivot point.

> Torques determine whether or not something will rotate. A torque is found by multiplying a force by its lever arm (the perpendicular distance from the axis of rotation to the line of action of the force). If the torque tending to produce a clockwise rotation equals the torque tending to produce a counterclockwise rotation, there is no rotation. If one of these torques is larger than the other, the torque will be unbalanced and the system will rotate.

8.3 Rotational Inertia and Newton's Second Law

When a child starts a merry-go-round rotating by running beside it and applying a force, the force exerted produces a torque about the axle. From our discussion in section 8.2, we know that the net torque acting on an object determines whether or not it will begin to rotate. Can we predict the rate of rotation by knowing the torque?

In linear motion, net force and mass determine the acceleration of an object, according to Newton's second law of motion ($\mathbf{a} = \mathbf{F}/m$). How do we adapt Newton's second law to cases of rotational motion? In this case, *torque* determines the rotational acceleration. A new quantity, the rotational inertia, takes the place of mass.

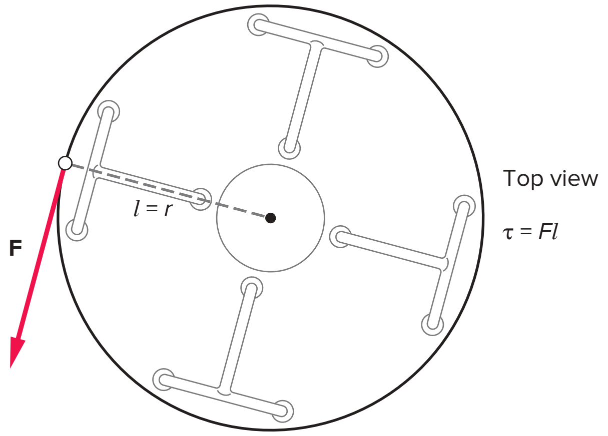


Figure 8.12 The child exerts a force at the rim of the merry-go-round that produces a torque about the axle.
(photo): Stockbyte/Getty Images

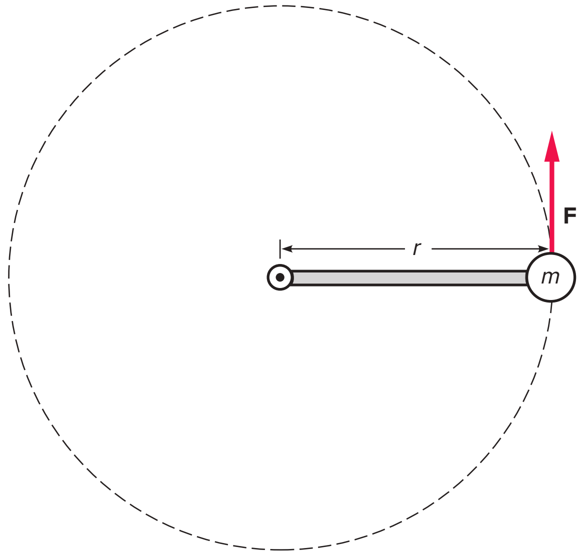


Figure 8.13 A single, concentrated mass at the end of a very light rod is set into rotation by the applied force **F.** Use Newton's second law to find the acceleration.

What is rotational inertia?

Let's return to the merry-go-round. The propulsion system (one energetic child or tired parent) applies a force at the edge of the merry-go-round. The torque about the axle is found by multiplying this force by the lever arm—in this case, the radius of the merry-go-round (fig 8.12). If the frictional torque at the axle is small enough to be ignored, the torque produced by the child is the only one acting on the system. This torque produces the rotational acceleration of the merry-go-round.

How would we find this rotational acceleration? To find the linear acceleration produced by a force acting on an object, we use Newton's second law, $\mathbf{F}_{net} = m\mathbf{a}$. By analogy, we can develop a similar expression for rotational motion, where the torque τ replaces the force and the rotational acceleration α replaces the linear acceleration. But what quantity should we use in place of the *mass* of the merry-go-round?

In linear motion, mass represents the inertia, or resistance to a change in motion. For rotational motion, a new concept is needed, **rotational inertia,** also referred to as the **moment of inertia.** The rotational inertia is the resistance of an object to change in its rotational motion. Rotational inertia is related to the mass of the object but also depends on how that mass is distributed about the axis of rotation.

To get a feeling for a concept, physicists often use the trick of considering the simplest possible situation. For rotational motion, the simplest case is a single, concentrated mass at the end of a very light rod, as in figure 8.13. If a force is applied to this mass in a direction perpendicular to the rod, the rod and mass will begin to rotate about the fixed axis at the other end of the rod.

For the rod and mass to undergo a rotational acceleration, the mass itself must have a linear acceleration. Like riders on a merry-go-round, however, the farther the mass is from the axis, the faster it moves for a given rotational velocity ($v = r\omega$). To produce the same rotational acceleration, a mass at the end of the rod must receive a larger linear acceleration than one nearer the axis. It is harder to get the system rotating when the mass is at the end of the rod than when it is nearer to the axis.

Applying Newton's second law to this situation, we find that the resistance to a change in rotational motion depends on the *square* of the distance of this mass from the axis of rotation. Because the resistance to change also depends on the size of the mass, the rotational inertia of a concentrated mass is

rotational inertia = mass × square of distance from axis

$$I = mr^2$$

where I is the symbol commonly used for rotational inertia, and r is the distance of the mass m from the axis of rotation. The total rotational inertia of an object like the merry-go-round can be found by adding the contributions of different parts of the object lying at different distances from the axis.

Newton's second law modified for rotational motion

By analogy to Newton's second law, $\mathbf{F}_{net} = m\mathbf{a}$, we can state the second law for rotational motion as follows:

> The net torque acting on an object about a given axis is equal to the rotational inertia of the object about that axis times the rotational acceleration of the object, or
>
> $$\tau_{net} = I\alpha$$

To put it differently, the rotational acceleration produced is equal to the torque divided by the rotational inertia, $\alpha = \tau_{net}/I$. The larger the torque, the larger the rotational acceleration, *but* the larger the rotational inertia, the smaller the rotational acceleration. Rotational inertia dictates how hard it is to change the rotational velocity of the object.

To get a feel for these ideas, consider a simple object such as a twirler's baton. A baton consists of two masses at the end of a rod (fig. 8.14). If the rod itself is light, most of the baton's rotational inertia comes from the masses at each end. If you hold the baton at the center, a torque can be applied with your hand, producing a rotational acceleration and starting the baton to rotate.

Suppose we can move these masses along the rod. If we move the masses toward the center of the rod, so that the distance from the center is half the original distance, what happens to the rotational inertia? The rotational inertia decreases to one-fourth of its initial value, ignoring the contribution of the rod. Rotational inertia depends on the *square* of the distance of the mass from the axis. Doubling the distance quadruples the rotational inertia. Halving the distance divides the rotational inertia by four.

The baton will be four times as hard to get to rotate when the masses are at the ends as when they are halfway from the ends. In other words, the torque needed to produce a rotational acceleration will be four times as large when the masses are at the ends as when they are at the intermediate positions. If you had a rod with adjustable masses, you could feel the difference in the amount of torque needed to start it rotating. Try a pencil with lumps of clay for the masses as a substitute.

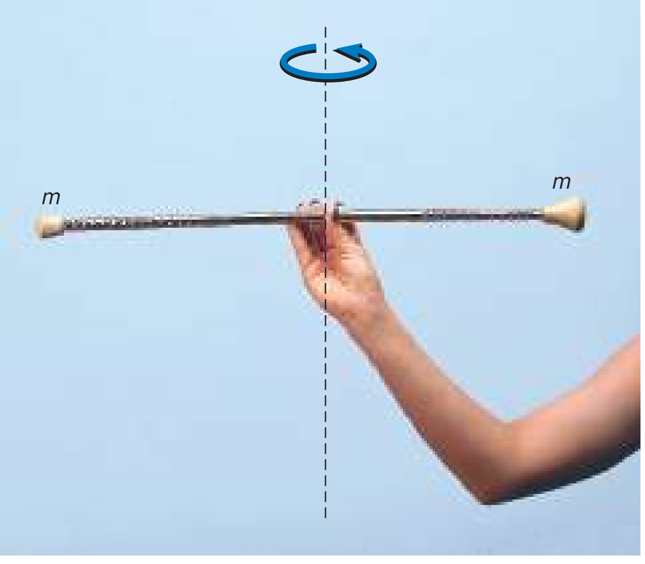


Figure 8.14 The rotational inertia of a baton is determined largely by the masses at the ends.
James Ballard/McGraw-Hill Education

Finding the rotational inertia of the merry-go-round

Finding the rotational inertia of an object like a merry-go-round is more difficult than just multiplying the mass by the square of the radius. Not all of the mass of the merry-go-round is at the outer edge—some of it is closer to the axis and will make a smaller contribution to the rotational inertia. Imagine breaking down the merry-go-round into several pieces, finding the rotational inertia of each piece, and adding together the rotational inertias for all the pieces to get the total.

Results of this process for a few simple shapes are shown in figure 8.15. The equations illustrate the ideas we have

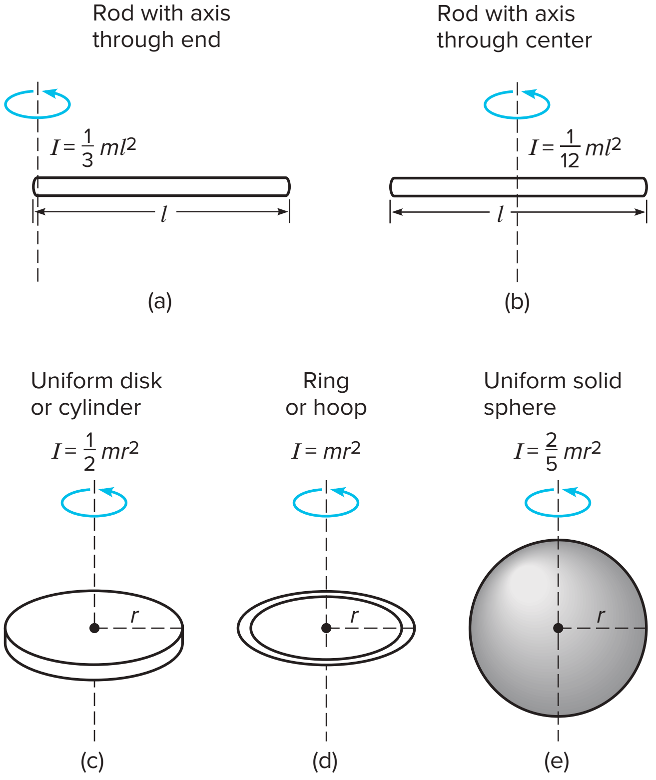


Figure 8.15 Expressions for the rotational inertia of several objects, each with a uniform distribution of mass over its volume. The letter *m* is used to represent the total mass of the object.

Example Box 8.4

Sample Exercise: Turning a Merry-Go-Round and a Rider

A simple merry-go-round has a rotational inertia of 800 kg·m^2 and a radius of 2 m. A child with a mass of 40 kg sits near the edge of the merry-go-round.

a. What is the total rotational inertia of the merry-go-round and the child about the axis of the merry-go-round?

b. What torque is required to give the merry-go-round and child a rotational acceleration of 0.05 rad/s^2?

a. $I_{\text{merry-go-round}} = 800\ \text{kg·m}^2$ $\quad I_{\text{child}} = mr^2$

$m_{\text{child}} = 40\ \text{kg}$ $\quad = (40\ \text{kg})\ (2\ \text{m})^2$

$r = 2\ \text{m}$ $\quad = 160\ \text{kg·m}^2$

The total rotational inertia is

$$\begin{aligned} I_{\text{total}} &= I_{\text{merry-go-round}} + I_{\text{child}} \\ &= 800\ \text{kg·m}^2 + 160\ \text{kg·m}^2 \\ &= \mathbf{960\ kg·m^2} \end{aligned}$$

b. $\alpha = 0.05\ \text{rad/s}^2$ $\quad \tau_{\text{net}} = I\alpha$

$\tau_{\text{net}} = ?$ $\quad = (960\ \text{kg·m}^2)\ (0.05\ \text{rad/s}^2)$

$\quad = \mathbf{48\ N·m}$

discussed. For example, a solid disk has a smaller rotational inertia than a ring of the same mass and radius, because the mass of the disk is, on average, closer to the axis. The location of the axis is also important. A rod has a larger rotational inertia about an axis through one end than about an axis through the middle. When the axis of rotation is at the end of the rod, there is more mass at greater distances from the axis.

Depending on how it is constructed, the merry-go-round might be like a solid disk. A child sitting on the merry-go-round will also affect the rotational inertia. If several children all sit near the edge of the merry-go-round, their rotational inertia makes it more difficult to get the merry-go-round moving. If the children cluster near the center, they provide less additional rotational inertia. If you are feeling tired, have the children sit near the middle. You will save some effort.

Example box 8.4 attaches some numbers to these quantities. A child sitting on a merry-go-round is being accelerated by a push at a rate of 0.05 rad/s^2.* A torque of 48 N·m is needed to produce this rotational acceleration. A force of 24 N applied at the edge would have a lever arm of 2 m and produce the necessary torque of 48 N·m, a reasonable force for a child to generate if the child is not too small.

*To use Newton's second law for rotational motion, the rotational acceleration must be stated in radians per second squared. If the rotational acceleration is provided in rev/s^2 or some other angular unit, we convert it to rad/s^2 before proceeding.

Rotational inertia is the resistance to change in rotational motion. It depends on both the mass of the object and the distribution of that mass about the axis of rotation. The rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha$, shows the quantitative relationship between torque, rotational inertia, and rotational acceleration. Torque takes the place of force, rotational inertia replaces mass, and rotational acceleration replaces linear acceleration.

8.4 Conservation of Angular Momentum

Have you ever watched an ice skater go into a spin? She starts the spin with her arms and one leg extended, then brings them in toward her body. As she brings her arms in, the rate of the spin increases; as she extends her arms again, her rotational velocity decreases (fig. 8.16).

The concept of angular, or rotational, momentum is useful in situations like this. The principle of conservation of angular momentum explains a variety of phenomena like the ice skater and includes tumbling divers or gymnasts, as well as the motion of the planets around the sun. How can we use the analogy between linear and rotational motion to understand these ideas?

What is angular momentum?

If you were asked to invent the idea of angular (rotational) momentum, how would you go about it? As discussed in chapter 7, linear momentum is the mass (the inertia) times the linear velocity of an object ($\mathbf{p} = m\mathbf{v}$). An increase in either the mass or the velocity increases the momentum. Because it is a measure of both how much is moving and how fast it is moving, Newton called momentum the *quantity of motion.*

What is momentum's rotational equivalent? In comparing rotational and linear motion, rotational inertia plays the role of mass, while rotational velocity replaces linear

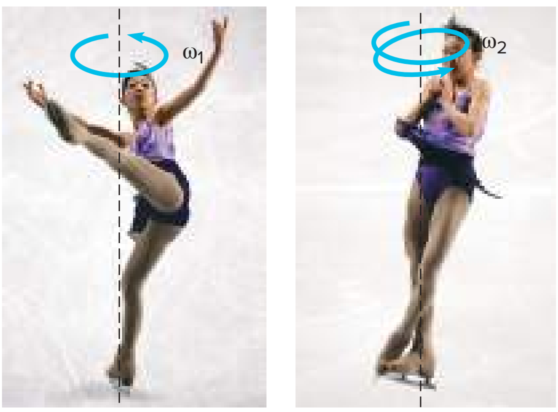


Figure 8.16 The rotational velocity of the skater increases as she pulls her arms and leg in toward her body.
(both): Jeff Haynes/AFP/Getty Images

velocity. By analogy, we can define **angular momentum** as follows:

> Angular momentum is the product of the rotational inertia and the rotational velocity, or
>
> $$L = I\omega$$

where L is the symbol used for angular momentum.

The term *angular momentum* is more common than *rotational momentum,* but either can be used.

Like linear momentum, angular momentum is the product of two quantities: an inertia and a velocity. A bowling ball spinning slowly might have the same angular momentum as a baseball spinning much more rapidly because of the larger rotational inertia I of the bowling ball. With its enormous rotational inertia, the Earth has a huge angular momentum associated with its daily turn about its axis, even though the rotational velocity is small.

When is angular momentum conserved?

We have used the analogy between linear and rotational motion to introduce angular momentum. Can we also use it to state the principle of conservation of angular momentum? In chapter 7, we found that linear momentum is conserved when there is no net external force acting on a system. When would angular momentum be conserved?

Because torque takes the role of force for rotational motion, we can state the principle of **conservation of angular momentum** thus:

> If the net torque acting on a system is zero, the total angular momentum of the system is conserved.

Torque replaces force, and angular momentum replaces ordinary or linear momentum. Table 8.2 lists some important parallels between linear and rotational motion.

Table 8.2

Corresponding Concepts of Linear and Rotational Motion

Concept	Linear Motion	Rotational Motion
Inertia	m	I
Newton's second law	$\mathbf{F}_{net} = m\mathbf{a}$	$\boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$
Momentum	$\mathbf{p} = m\mathbf{v}$	$L = I\omega$
Conservation of momentum	If $\mathbf{F}_{net} = 0$, $\mathbf{p}$ = constant	If $\boldsymbol{\tau}_{net} = 0$, L = constant
Kinetic energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$

Changes in the ice skater's rate of spin

Conservation of angular momentum is the key to understanding what happens when the spinning ice skater increases her rotational velocity by pulling in her arms. The external torque acting on the skater about her axis of rotation is very small, so the condition for conservation of angular momentum exists. Why does her rotational velocity increase?

When the skater's arms and one leg are extended, they contribute a relatively large portion to her total rotational inertia—their average distance from her axis of rotation is much larger than for other portions of her body. Rotational inertia depends on the square of the distance of various portions of her mass from the axis ($I = mr^2$). The effect of this distance is substantial, even though her arms and one leg are only a small part of the total mass of the skater. When the skater pulls her arms and leg in toward her body, their contribution to her rotational inertia decreases, and therefore her total rotational inertia decreases.

Conservation of angular momentum requires that her angular momentum remain constant. Because angular momentum is the product of the rotational inertia and rotational velocity, $L = I\omega$, if I decreases, ω must increase for angular momentum to stay constant. She can slow her rate of spin by extending her arms and one leg again, which she does at the end of the spin. This increases her rotational inertia and decreases her rotational velocity: Angular momentum is conserved. These ideas are illustrated in example box 8.5.

This phenomenon can be explored using a rotating platform or stool with good bearings to keep the frictional torques small (fig. 8.17). In these demonstrations, students can hold masses in their hands, which increase the changes in rotational inertia that happen as the arms are drawn in toward the body. A striking increase in rotational velocity can be achieved!

Example Box 8.5

Sample Exercise: Some Physics of Figure Skating

An ice skater has a rotational inertia of 1.2 kg·m² when her arms are extended and a rotational inertia of 0.5 kg·m² when her arms are pulled in close to her body. If she goes into a spin with her arms extended and has an initial rotational velocity of 1 rev/s, what is her rotational velocity when she pulls her arms in close to her body?

$I_1 = 1.2\ \text{kg}\cdot\text{m}^2$
$I_2 = 0.5\ \text{kg}\cdot\text{m}^2$
$\omega_1 = 1\ \text{rev/s}$
$\omega_2 = ?$

Because angular momentum is conserved

$$L_{final} = L_{initial}$$
$$I_2\omega_2 = I_1\omega_1$$

Dividing both sides by I_2,

$$\begin{aligned}\omega_2 &= (I_1/I_2)\omega_1\\ &= (1.2\ \text{kg}\cdot\text{m}^2/0.5\ \text{kg}\cdot\text{m}^2)(1\ \text{rev/s})\\ &= (2.4)(1\ \text{rev/s})\\ &= \mathbf{2.4\ rev/s}\end{aligned}$$

Figure 8.17 A student holding masses in each hand while sitting on a rotating stool can achieve a large increase in rotational velocity by bringing his arms in toward his body.
James Ballard/McGraw-Hill Education

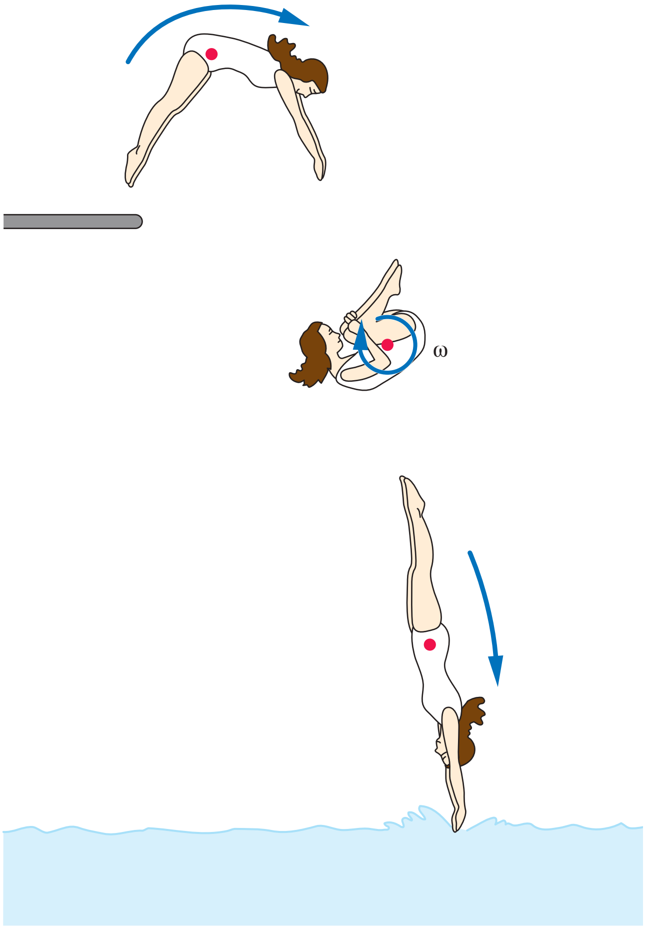


Figure 8.18 The diver increases her rotational velocity by pulling into a tuck position, thus reducing her rotational inertia about her center of gravity.

A similar effect is at work when a diver pulls into a tuck position to produce a spin. In this case, the diver starts with her body extended and a slow rate of rotation about an axis through her body's center of gravity (fig. 8.18). As she goes into a tuck, the rotational inertia about this axis is reduced, and rotational velocity increases. As her dive nears completion, she comes out of the tuck, increasing the rotational inertia and decreasing the rotational velocity. (The torque about the center of gravity due to the gravitational force acting on the diver is zero.)

There are many examples of varying the rotational velocity by changing the rotational inertia. It is much easier to produce a change in the rotational inertia of a body than to change the mass of the body. We simply change the distance of various portions of the mass from the axis of rotation. Conservation of angular momentum provides a quick explanation for these phenomena.

Kepler's second law

Conservation of angular momentum also plays a role in the orbit of a planet about the sun; in fact, it can be used to explain Kepler's second law of planetary motion (see section 5.3). Kepler's second law says that the radius line from the sun to the planet sweeps out equal areas in equal times. The planet moves faster in its elliptical orbit when it is nearer to the sun than when it is farther from the sun.

The gravitational force acting on the planet produces no torque about the sun, because its line of action passes directly through the sun (fig. 8.19). The lever arm for this force is zero, and the resulting torque must also be zero. Angular momentum, therefore, is conserved.

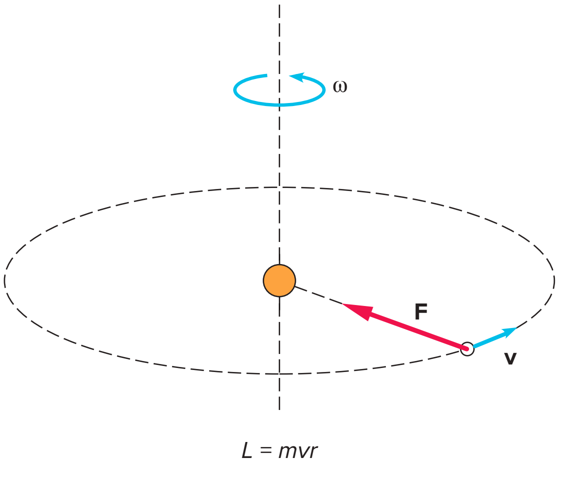


Figure 8.19 The gravitational force acting on the planet produces no torque about an axis through the sun, because the lever arm is zero for this force.

When the planet moves nearer to the sun, its rotational inertia I about the sun decreases. To conserve angular momentum, the rotational velocity of the planet about the sun (and thus its linear velocity*) must increase to keep the product $L = I\omega$ constant. This requirement results in equal areas being swept out by the radius line in equal times. The velocity of the planet must be larger when the radius gets smaller to keep the area being swept out the same.

You can observe a related effect in a simple experiment with a ball on a string. If you let the string wrap around your finger as it rotates, which produces a smaller radius of rotation, the ball will increase its rotational velocity about your finger. The rotational velocity ω increases as the rotational inertia I decreases because of the decreased radius. Angular momentum is conserved. Try it!

Everyday phenomenon box 8.1 provides an example in which angular momentum is conserved at some points in the motion of a yo-yo. At other points, the angular momentum changes under the influence of torques.

> By analogy to linear momentum, angular momentum is the product of the rotational inertia and the rotational velocity. Angular momentum is conserved when the net external torque acting on a system is zero. Decreases in rotational inertia lead to increases in rotational velocity, as demonstrated by the spinning ice skater. A spinning diver, a ball rotating at the end of a string, and a planet orbiting the sun are other examples of this effect.

8.5 Riding a Bicycle and Other Amazing Feats

Have you ever wondered why a bicycle remains upright when it is moving but promptly falls over when not moving? Angular momentum is involved, but some additional wrinkles are needed in the explanation. The direction of angular momentum is an important consideration. How can angular momentum have direction, and how is this direction involved in explaining the behavior of a bicycle, a spinning top, or other phenomena?

Is angular momentum a vector?

Linear momentum is a vector, and the direction of the momentum **p** is the same as that for the velocity **v** of the object. Because angular momentum is associated with a rotational velocity, the question comes down to whether rotational velocities have direction. How would we define the direction of a rotational velocity?

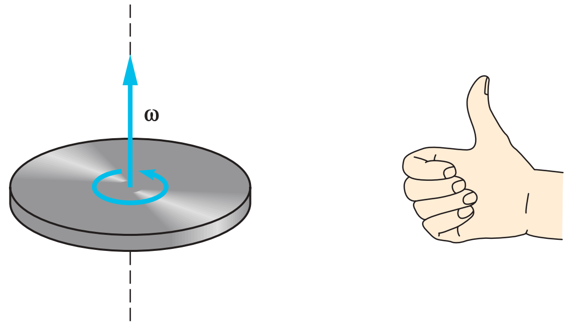


Figure 8.20 The direction of the rotational-velocity vector for the counterclockwise rotation is defined to be upward along the axis of rotation, as indicated by the thumb on the right hand with the fingers curled in the direction of rotation.

If a merry-go-round (or just a disk) is rotating in a counterclockwise direction, as in figure 8.20, how might we indicate that direction with an arrow? The term *counterclockwise* indicates the direction of rotation as seen from a certain perspective, but it does not define a unique direction. To complete our description, we would also have to specify the axis of rotation and our perspective, or viewpoint. An object seen rotating counterclockwise when viewed from above is seen rotating clockwise when viewed from below. We could draw an axis of rotation and a curved arrow around it, as we often do, but it would be more desirable to specify direction with a simple straight arrow.

The usual solution to this problem is to define the direction of the rotational-velocity vector as being along the axis of rotation and in the upward direction for the counterclockwise rotation in figure 8.20. A rule for whether the vector should point up or down along the axis can be defined with the help of your right hand. If you hold your right hand with the fingers curling around the axis of rotation in the direction of the rotation, your thumb points in the direction of the rotational-velocity vector. This is referred to as the **right-hand rule.** If the merry-go-round were rotating clockwise (instead of counterclockwise), your thumb would point down, the direction of the rotational-velocity vector. Try it. Be careful to use your right hand (even if you are lucky enough to be left-handed).

The direction of the angular-momentum vector is the same as the rotational velocity, because $\mathbf{L} = I\omega$. Conservation of angular momentum requires that the *direction* of the angular-momentum vector remain constant, as well as its magnitude.

Angular momentum and bicycles

Most of us have had some experience with riding a bicycle. The wheels of a bicycle acquire angular momentum when the bicycle is moving. Torque is applied to the rear wheel by the pedals and chain to produce a rotational acceleration. If the bicycle is moving in a straight line, the direction of the

*For a compact mass rotating about some axis, the definition of angular momentum reduces to $L = mvr$, where mv is the linear momentum and r is the perpendicular distance from the axis of rotation to the line along which the object is moving at that instant. If r decreases, v must increase to conserve angular momentum.

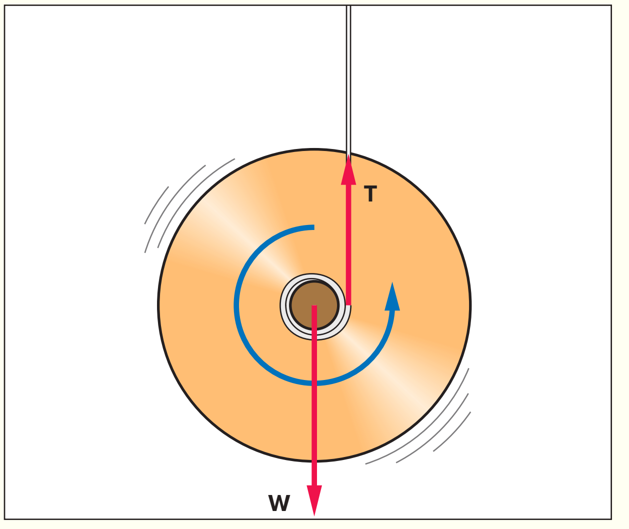
Everyday Phenomenon Box 8.1

Achieving the State of Yo

The Situation. A physics professor noticed that one of his students often carried a yo-yo to class and was proficient at putting the yo-yo through its paces. The professor challenged the student to explain the behavior of the yo-yo using the principles of torque and angular momentum.

In particular, the professor asked the student to explain why the yo-yo sometimes comes back but sometimes can be made to "sleep," or continue to rotate, at the end of the string. What are the differences in these two situations?

A yo-yo will come back to your hand, or with sufficient skill, you can make it "sleep" at the end of its string.
Kelly Redinger/Design Pics



A cut-away diagram showing the forces acting on the yo-yo when it is falling. Its weight and the tension in the string are the only significant forces.

The Analysis. The student carefully examined the yo-yo's construction and how the string is attached. He noticed that the string is not tied tightly to the axle of the yo-yo but ends in a loose loop around the axle instead. When the yo-yo is at the end of its string, the string can slip on the axle. When wound around the axle, on the other hand, the string is less likely to slip.

Usually, the yo-yo is started with the string wound around the axle and looped around the middle finger. When the yo-yo is released from the hand, the string unwinds, and the yo-yo gains rotational velocity and angular momentum. The student reasoned that a torque must be at work, and he drew a force diagram for the yo-yo that looked like the one in the diagram. Two forces act on the yo-yo: its weight acting downward and the tension in the string acting upward.

Because the yo-yo is accelerated downward, the weight must be greater than the tension to produce a downward net force. The weight does not produce a torque about the center of gravity of the yo-yo, though, because its line of action passes through the center of gravity, and the lever arm is zero. The tension acts along a line that is off-center and produces a torque that will cause a counterclockwise rotation about the center of gravity, as in the drawing.

The torque due to the tension in the string produces a rotational acceleration, and the yo-yo gains rotational velocity and angular momentum as it falls. The yo-yo has a sizable angular momentum when it reaches the bottom of the string, and in the absence of external torques to change this angular momentum, it will be conserved. This is what happens when the yo-yo "sleeps" at the bottom of the string: The only torque acting is the frictional torque of the string slipping on the axle, and this will be small if the axle is smooth.

What happens, however, when the yo-yo returns to the student's hand? The yo-yo artist (yo-yoist?) jerks lightly on the string at the instant that the yo-yo reaches the bottom of the string. This jerk provides a brief impulse and upward acceleration of the yo-yo. Because it is already spinning, the yo-yo continues spinning in the same direction, and the string rewinds itself around the axle of the yo-yo. The line of action of the tension in the string is now on the opposite side of the axle, though, and its torque causes the rotational velocity and angular momentum to decrease. The rotation should stop when the yo-yo slips back into the student's hand.

When the yo-yo is rising, the net force acting on the yo-yo is still downward, and the linear velocity of the yo-yo decreases, along with its rotational velocity. The only time a net force acts upward is when the upward impulse is delivered by jerking on the string. The situation is similar to a ball bouncing on the floor—the net force is downward except during the very brief time of contact with the floor. Our ability to affect the nature and timing of the impulse through the string causes the yo-yo either to sleep or to return. This is what the "art of yo" is all about.

Figure 8.21 The angular-momentum vector for each wheel is horizontal when the bicycle is upright.
Bob Coyle/McGraw-Hill Education

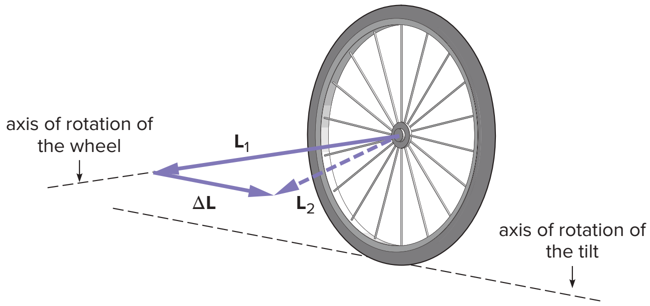


Figure 8.22 The change in angular momentum (Δ**L**) associated with a leftward tilt points straight back, parallel to the line of contact of the tires with the road. This change causes the angular-momentum vector (and the wheel) to turn to the left.

angular-momentum vector is the same for both wheels and is horizontal (fig. 8.21).

To tip the bike over, the direction of the angular-momentum vector must change, and that requires a torque. This torque would normally come from the gravitational force acting on the rider and the bicycle through their center of gravity. When the bicycle is exactly upright, this force acts straight downward and passes through the axis of rotation for the falling bike. This axis of rotation is the line along which the tires contact the road. The torque about this axis will be zero, because the line of action of the force passes through the axis of rotation and the lever arm is zero. The direction and magnitude of the original angular momentum are conserved.

If the bike is not perfectly upright, a gravitational torque acts about the line of contact of the tires with the road. As the bike begins to fall, it acquires a rotational velocity and angular momentum about this axis. By the right-hand rule, the direction of that angular-momentum vector is along the axis and points forward or backward depending on the direction of tilt. If the bike tilts to the left as seen from behind, the change in angular momentum associated with this torque points straight back, as in figure 8.22.

If the bike is standing still, that is all there is to it—the gravitational torque causes the bike to fall. When the bike is moving, however, the change in angular momentum Δ**L** produced by the gravitational torque adds to the angular momentum already present ($\mathbf{L}_1$) from the rotating tires. As shown in figure 8.22, this causes a change in the direction of the total angular-momentum vector ($\mathbf{L}_2$). This change in direction can be accommodated simply by turning the wheel of the bicycle rather than letting the bike fall. We compensate for the effects of the gravitational torque by turning the bicycle toward the direction of the impending fall. The larger the initial angular momentum, the smaller the turn required. The angular momentum of the wheels is a major factor in stabilizing the bicycle.

Study Hint

Visualizing these angular momentum vectors and their changes can be an abstract and difficult task. The effect will seem much more real if you can directly experience it. If a bicycle wheel mounted on a handheld axle (such as that pictured in figure 8.23) is available, try the tilt effect yourself. Grasp the wheel with both hands by the handles on each side and have someone give it a good spin with the wheel in a vertical plane. Then, try tilting the wheel downward to the left to simulate a fall. The wheel will seem to have a mind of its own and will turn to the left, as suggested by figure 8.22.

This result may be surprising—yet all of us who have ridden bicycles take advantage of it routinely. When the bike is moving slowly, sharp turns of the wheel can keep it from falling while you shift your weight. Smaller adjustments suffice when the bike is moving more rapidly. By leaning into a curve, you use the gravitational torque to change the direction of angular momentum, helping to round the curve. Likewise, if you roll a coin along a tabletop, you will see it curve as it begins to fall. The path curves in the direction the coin is tilting.

You can also observe this effect of torque in changing the direction of an angular-momentum vector by holding a bicycle upright on its rear wheel and having a friend spin the front wheel. It is harder to change the direction of this wheel when it is spinning rapidly than when it is spinning slowly or not at all. You will also get the feeling that the wheel has a mind of its own. As you try to tilt the wheel, it will tend to turn in a direction perpendicular to the tilt.

Figure 8.23 A student holds a spinning bicycle wheel while sitting on a stool that is free to rotate. What happens if the wheel is turned upside down?
(both): James Ballard/McGraw-Hill Education

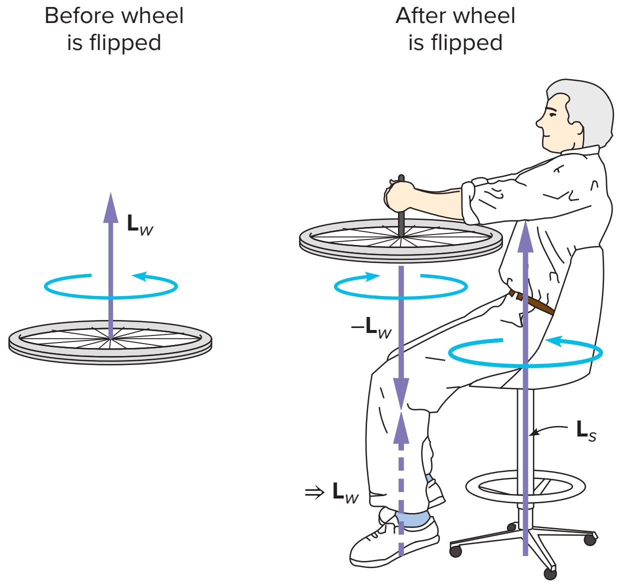


Figure 8.24 The angular momentum of the objects about the rotation axis of the stool, $\mathbf{L}_s$, adds to the angular momentum of the wheel about its own axis, -$\mathbf{L}_w$, to yield the direction and magnitude of the original angular momentum, $\mathbf{L}_w$.

A bicycle tire mounted on a handheld axle is even more effective for sensing the effects of torques applied to the axle. This is a common demonstration apparatus, but usually the tire is filled with steel cable rather than air. The steel cable gives the wheel a larger rotational inertia and a larger angular momentum for a given rate of spin. If you hold the axle on either side while the wheel is spinning in a vertical plane and then try to tip the wheel, you get a sense of what happens when you are riding a bicycle. It also demonstrates how hard it is to change the direction of the angular momentum of a rapidly spinning wheel. Everyday phenomenon box 8.2 discusses how torques are involved in the gear system of a bicycle.

Rotating stools and tops

The handheld bicycle wheel is good for other demonstrations that highlight angular momentum as a vector. If a student holds the wheel with its axle in the vertical direction while sitting on a rotating stool, conservation of angular momentum produces striking results. It is best to start the wheel spinning while holding the stool, so that the stool does not rotate initially. The student then turns the wheel over, as in figure 8.23, reversing the direction of the angular-momentum vector of the wheel.

Can you imagine what happens then? To conserve angular momentum, the original direction of the angular-momentum vector must be maintained. The only way this can happen is for the stool with the student volunteer to begin to rotate in the same direction that the wheel was rotating *initially.* The sum of the angular-momentum vector of the wheel about its axis of rotation and the angular-momentum vector of the student, stool, and wheel about the axis of rotation of the stool add to yield the original angular momentum (fig. 8.24). This will be true if the angular momentum gained by the student, stool, and wheel is exactly twice the original angular momentum of the wheel. The student can stop the rotation of the stool by flipping the wheel axis back to its original direction.

The direction of angular momentum and its conservation are important in many other situations. The angular momentum of the helicopter's rotors, for example, is an extremely important factor in helicopter design. The motion of a top also shows fascinating effects. If you have a top, observe what happens to the direction of the angular-momentum vector as the top slows down. As the top begins to totter, the change in direction of the angular-momentum vector causes the rotation axis of the top to rotate (precess) about a vertical line. Does this remind you of what happens with a bicycle wheel?

Angular momentum and its direction are also central to atomic and nuclear physics. The particles that make up atoms have spins, and these spins imply angular momentum. The ways these angular-momentum vectors add are used to explain a variety of atomic phenomena. While the size scales differ enormously, it is useful to recognize the common ground that atoms and nuclei share with bicycle wheels and the solar system.

Like linear momentum, angular momentum is a vector. Its direction is the same as that of the rotational-velocity vector, which is along the axis of rotation, with the right-hand rule specifying which way it points along that axis. Conservation of angular momentum requires that both the magnitude and the direction of the angular-momentum vector be constant (if there are no external torques). Many interesting phenomena can be explained using these ideas, including the stability of a moving bicycle, the motion of a spinning top, and the behavior of atoms and galaxies.

Everyday Phenomenon Box 8.2

Bicycle Gears

The Situation. Most modern bicycles come equipped with the ability to change gears. When we are climbing a hill, we shift into a low gear, making pedaling easier. When we are on level ground or a downgrade, we shift into higher gears, allowing us to cover more ground per turn of the pedal crank.

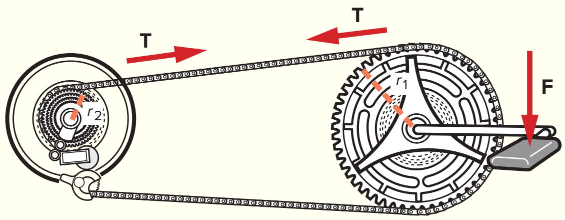
How do these gears work? How does the torque exerted on the rear wheel change when we change gears? What is the advantage of having many different gears? Adding a rotational twist to the concept of simple machines can help in understanding how gears work. Similar ideas apply to an automobile transmission.

There are seven different sprockets on the rear-wheel gear assembly of a 21-speed bicycle. The two smaller sprockets on the pedal wheel lie behind the largest sprocket.
Adam Gault/OJO Images/Getty Images

The Analysis. The photograph above shows the pedal wheel and the rear-wheel gear assembly for a 21-speed bicycle. There are seven sprockets (toothed wheels) of different sizes on the rear-wheel hub. There are also three different sprockets on the pedal wheel, only one of which is fully visible in the photograph. A pulley and lever mechanism (called a derailleur) allows us to move the chain from one sprocket to another. This is controlled by levers mounted on the handlebars that are linked to the derailleurs by cables.

When we pedal the bicycle, we apply a torque to the pedal sprocket by pushing on the pedals. If our feet push perpendicularly to the pedal shaft, then the lever arm is just the length of the shaft. That will be the case when the pedal is in the forward position where we get maximum torque, as shown in the drawing. This maximum-torque position alternates from the left foot to the right foot as the crank turns.

The torque exerted on the pedal chain ring causes the sprocket to accelerate rotationally, provided that this torque is larger than the opposing torque exerted by the tension in the chain pulling back on the sprocket. This tension, in turn, produces a torque on the rear wheel via the rear-wheel sprocket. The size of the torque transmitted to the rear wheel depends on which of the several sprockets is engaged with the chain. A larger sprocket radius yields a larger torque because of the greater lever arm. (The lever arm is equal to the radius of the sprocket engaged.) As with an automobile, the rear wheel pushes against the road surface via friction, and by Newton's third law, the frictional force pushes forward on the bicycle.



A force applied to the pedal produces a torque on the pedal wheel. This torque produces a tension in the chain that exerts a smaller torque on the rear-wheel sprocket due to its smaller radius.

How are simple-machine ideas involved? Suppose the chain is engaged with the smallest sprocket on the rear-wheel assembly. The torque exerted on the rear wheel by the chain is then relatively small because of the small lever arm. The wheel turns several times, however, for each turn of the pedal sprocket. If, for example, the radius of the pedal sprocket is five times that of the rear-wheel sprocket, then the rear wheel turns five times for each turn of the pedal crank. (The circumference of each sprocket [$2\pi r$] is proportional to the radius, and the circumference determines how far the chain must travel for each turn.)

In this case, then, a small torque turns the rear wheel through a large angle, while a larger input torque turns the pedal sprocket through a smaller angle. By analogy to linear work (force times distance moved), rotational work can be defined as the torque times the angle through which the sprocket turns ($\tau\theta$). As for any simple machine, the work output equals the work input, ignoring frictional torques at the wheel axles.

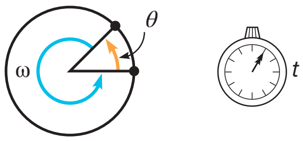
Would the situation we have just described represent a high gear or a low gear? Because we are getting several turns of the rear wheel for each turn of the pedal crank, this is a high gear. For a lower gear, we need to move the chain to a larger sprocket on the rear wheel (or a smaller sprocket on the pedal wheel). This will transmit a larger torque to the rear wheel at the expense of turning the wheel through a smaller angle and moving the bicycle through a smaller distance for each turn of the crank. When we are going uphill, we need this larger torque to overcome the pull of gravity.

For a 21-speed bike, there are three sprocket sizes on the pedal wheel and seven on the rear wheel, allowing 21 (3×7) different ratios between the two sprockets. The advantage of having all these choices is that we can adjust the mechanical advantage of our gear system to the conditions we encounter, thus adjusting the force we need to apply to the pedals to achieve the desired torque. If this force is too large, we will quickly tire. When it is small, however, we may not be taking maximum advantage of the easier riding conditions. We can go faster in a higher gear.

Summary

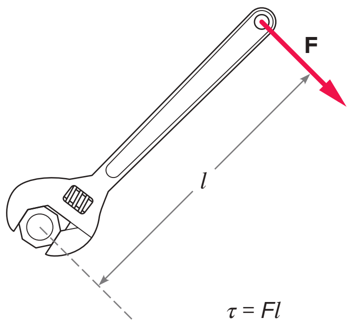
We have considered the rotational motion of a solid object and what causes changes in rotational motion. We have used an analogy between linear motion and rotational motion to develop many of the concepts. The key points are the following:

1 **What is rotational motion?** Rotational displacement is described by an angle. Rotational velocity is the rate of change of that angle with time. Rotational acceleration is the rate of change of rotational velocity with time.



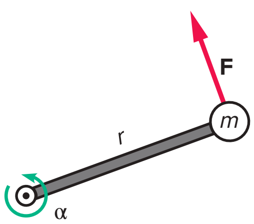
$\omega = \dfrac{\theta}{t}, \qquad \alpha = \dfrac{\Delta\omega}{t}$

2 **Torque and balance.** A torque is what causes an object to rotate. It is defined as a force times the lever arm of the force, which is the perpendicular distance from the line of action of the force to the axis of rotation. If the net torque acting on an object is zero, the object will not change its state of rotation.



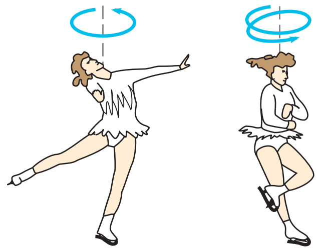
$\tau = Fl$

3 **Rotational inertia and Newton's second law.** In the form of Newton's second law that relates to rotation, torque takes the place of force, rotational acceleration replaces ordinary linear acceleration, and rotational inertia replaces mass. Rotational inertia depends on the distribution of mass about the axis of rotation.



$\tau_{net} = I\alpha, \qquad I = mr^2$

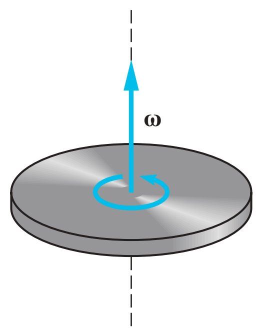
4 **Conservation of angular momentum.** By analogy to linear momentum, angular momentum is defined as the rotational inertia times the rotational velocity. It is conserved when no net external torque acts on the system.



$L = I\omega$

If $\tau_{ext} = 0$, L = constant

5 **Riding a bicycle and other amazing feats.** The directions of the rotational-velocity and angular-momentum vectors are defined by the right-hand rule. These vectors explain the stability of a moving bicycle and other phenomena. If there are no external torques, the direction of the angular momentum is conserved, as well as its magnitude.



The notation used in this chapter may seem like so much Greek to you, but the equations are very similar to the ones you learned earlier with linear motion. It will help if you use the units to identify the variables in the problems. Try the practice problems in Connect. They have complete solutions, which you should look at *after* you try the problems!

Key Terms

Conceptual Questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
Q = sample responses are available in appendix D
Q = sample responses are available in Connect

Q1. Which units would not be appropriate for describing a rotational velocity: rad/min^2, rev/s, rev/h, m/s? Explain.

Q2. Which units would not be appropriate for describing a rotational acceleration: rad/s, rev/s^2, rev/m^2, degrees/s^2? Explain.

Q3. A coin rolls down an inclined plane, gaining speed as it rolls. Does the coin have a rotational acceleration? Explain.

Q4. The rate of rotation of an object is gradually slowing down. Does this object have a rotational acceleration? Explain.

Q5. Is the rotational velocity of a child sitting near the center of a rotating merry-go-round the same as that of another child sitting near the edge of the same merry-go-round? Explain.

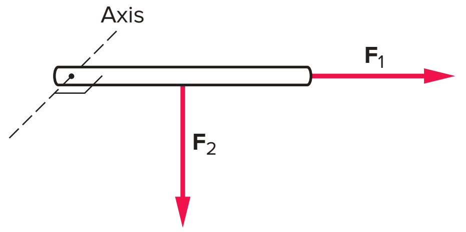
Q6. Is the linear speed of a child sitting near the center of a rotating merry-go-round the same as that of another child sitting near the edge of the same merry-go-round? Explain.

Q7. If an object has a constant rotational acceleration, is its rotational velocity also constant? Explain.

*Q8. A ball rolls down an inclined plane, gaining speed as it goes. Does the ball experience both linear and rotational acceleration? How far does the ball travel in 1 revolution? How is the linear velocity of the ball related to its rotational velocity? Explain.

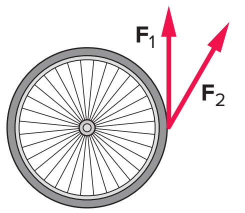
Q9. Which, if either, will produce the greater torque: a force applied at the end of a wrench handle (perpendicular to the handle) or an equal force applied in the same direction near the middle of the handle? Explain.

Q10. Which of the forces pictured as acting upon the rod in the diagram will produce a torque about an axis perpendicular to the plane of the diagram at the left end of the rod? Explain.



Q10 Diagram

Q11. The two forces in the diagram have the same magnitude. Which orientation will produce the greater torque on the wheel? Explain.



Q11 Diagram

Q12. Is it possible to balance two objects of different weights on the beam of a simple balance resting upon a fulcrum? Explain.

*Q13. Is it possible for the net force acting on an object to be zero but the net torque to be greater than zero? Explain. (Hint: The forces contributing to the net force may not lie along the same line.)

Q14. You are trying to move a large rock using a steel rod as a lever. Will it be more effective to place the fulcrum nearer to your hands or nearer to the rock? Explain.

Q15. A pencil is balanced on a fulcrum located two-thirds of the distance from one end. Is the center of gravity of this pencil located at its center point? Explain.

Q16. A solid plank with a uniform distribution of mass along its length rests on a platform with one end of the plank protruding over the edge. How far out can we push the plank before it tips? Explain.

*Q17. A uniform metal wire is bent into the shape of an L. Will the center of gravity for the wire lie on the wire itself? Explain.

Q18. An object is rotating with a constant rotational velocity. Can there be a net torque acting on the object? Explain.

*Q19. A tall crate has a higher center of gravity than a shorter crate. Which will have the greater tendency to tip over if we push near the top of the crate? Explain with a force diagram. Where is the probable axis of rotation?

Q20. Two objects have the same total mass, but object A has its mass concentrated closer to the axis of rotation than object B. Which object will be easier to set into rotational motion? Explain.

Q21. Is it possible for two objects with the same mass to have different rotational inertias? Explain.

Q22. Can you change your rotational inertia about a vertical axis through the center of your body without changing your total weight? Explain.

Q23. A solid sphere and a hollow sphere made from different materials have the same mass and the same radius. Which of these two objects, if either, will have the greater rotational inertia about an axis through its center? Explain.

Q24. Is angular momentum always conserved? Explain.

Q25. A metal rod is rotated first about an axis through its center and then about an axis passing through one end. If the rotational velocity is the same in both cases, is the angular momentum also the same? Explain.

Q26. A child on a freely rotating merry-go-round moves from near the center to the edge. Will the rotational velocity of the merry-go-round increase, decrease, or not change at all? Explain.

***Q27**. Moving straight inward, a large child jumps onto a freely rotating merry-go-round. What effect will this have on the rotational velocity of the merry-go-round? Explain.

Q28. Is it possible for an ice skater to change his rotational velocity without involving any external torque? Explain.

Q29. Suppose you are rotating a ball attached to a string in a circle. If you allow the string to wrap around your finger, does the rotational velocity of the ball change as the string shortens? Explain.

Q30. Does the direction of the angular-momentum vector of the wheels change when a bicycle goes around a corner? Explain.

Q31. Does the direction of the angular momentum vector change when a yo-yo starts to move upward having reached the bottom of the string? Explain. (See everyday phenomenon box 8.1.)

Q32. Can a yo-yo be made to "sleep" if the string is tied tightly to the axle? Explain. (See everyday phenomenon box 8.1.)

Q33. An ice skater is spinning counterclockwise about a vertical axis when viewed from above. What is the direction of her angular-momentum vector? Explain.

Q34. A pencil, balanced vertically on its eraser, falls to the right.
a. What is the direction of its angular-momentum vector as it falls?
b. Is its angular momentum conserved during the fall? Explain.

*Q35. A top falls over quickly if it is not spinning but will stay approximately upright for some time when it is spinning. Explain why this is so.

Q36. When we shift gears on the rear-wheel gear of a bicycle, does the torque applied to that wheel change? Explain. (See everyday phenomenon box 8.2.)

Q37. In what foot position do we exert maximum torque on a bicycle pedal? Explain. (See everyday phenomenon box 8.2.)

Q38. If we move the chain to a larger sprocket on the rear-wheel gear of a bicycle, are we changing to a higher or a lower gear? Explain. (See everyday phenomenon box 8.2.)

Exercises

E1. Suppose a merry-go-round is rotating at the rate of 8 rev/min.
a. Express this rotational velocity in rev/s.
b. Express this rotational velocity in rad/s.

E2. At one time, the rate of rotation for popular music records on a record player was 33 rpm.
a. Express this rotational velocity in rev/s.
b. Through how many revolutions does the record turn in a time of 5 s?

E3. Suppose a disk rotates through 8 revolutions in 5 seconds.
a. What is its displacement in radians in this time?
b. What is its average rotational velocity in rad/s?

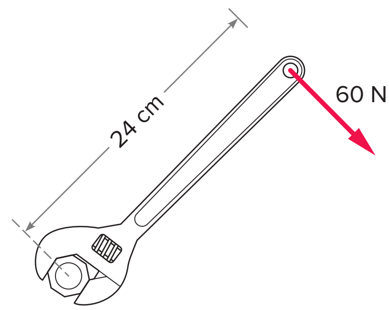
E4. A bicycle wheel is rotationally accelerated at the constant rate of 1.3 rev/s^2.
a. If it starts from rest, what is its rotational velocity after 5 s?
b. Through how many revolutions does it turn in this time?

E5. The rotational velocity of a merry-go-round increases at a constant rate from 0.3 rad/s to 2.1 rad/s in a time of 6 s. What is the rotational acceleration of the merry-go-round?

E6. The rotational velocity of a spinning disk decreases from 14 rev/s to 3 rev/s in a time of 15 s. What is the rotational acceleration of the disk?

E7. Starting from rest, a merry-go-round accelerates at a constant rate of 0.4 rev/s^2.
a. What is its rotational velocity after 6 s?
b. How many revolutions occur during this time?

E8. A force of 60 N is applied at the end of a wrench handle that is 24 cm long. The force is applied in a direction perpendicular to the handle, as shown in this question's diagram.
a. What is the torque applied to the nut by the wrench?
b. What would the torque be if the force were applied halfway up the handle instead of at the end?

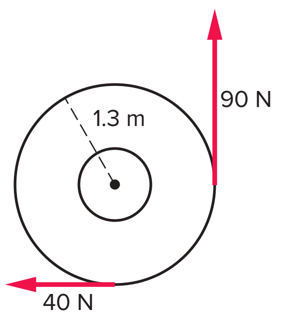


E8 Diagram

E9. A weight of 40 N is located a distance of 8 cm from the fulcrum of a simple balance beam. At what distance from the fulcrum should a weight of 25 N be placed on the opposite side in order to balance the system?

E10. A weight of 8 N is located 12 cm from the fulcrum on the beam of a simple balance. What weight should be placed at a point 5 cm from the fulcrum on the opposite side in order to balance the system?

E11. Two forces are applied to a merry-go-round with a radius of 1.3 m, as shown in this question's diagram. One force has a magnitude of 90 N and the other a magnitude of 40 N.
   a. What is the torque about the axle of the merry-go-round due to the 90-N force?
   b. What is the torque about the axle due to the 40-N force?
   c. What is the net torque acting on the merry-go-round?



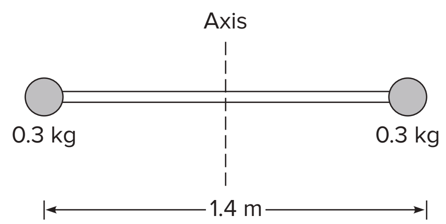
E11 Diagram

E12. A net torque of 93.5 N·m is applied to a disk with a rotational inertia of 11.0 kg·m^2. What is the rotational acceleration of the disk?

E13. A wheel with a rotational inertia of 8.3 kg·m^2 accelerates at a rate of 4.0 rad/s^2. What net torque is needed to produce this acceleration?

E14. A torque of 76 N·m producing a counterclockwise rotation is applied to a wheel about its axle. A frictional torque of 14 N·m acts at the axle.
   a. What is the net torque about the axle of the wheel?
   b. If the wheel accelerates at the rate of 6 rad/s^2 under the influence of these torques, what is the rotational inertia of the wheel?

E15. Two 0.3-kg masses are located at either end of a 1.4-m long, very light and rigid rod, as shown in this question's diagram. What is the rotational inertia of this system about an axis through the center of the rod?



E15 Diagram

E16. A mass of 0.75 kg is located at the end of a very light and rigid rod 60 cm in length. The rod is rotating about an axis at its opposite end with a rotational velocity of 12 rad/s.
   a. What is the rotational inertia of the system?
   b. What is the angular momentum of the system?

E17. A uniform disk with a mass of 7 kg and a radius of 0.4 m is rotating with a rotational velocity of 15 rad/s.
   a. What is the rotational inertia of the disk? (See fig. 8.15.)
   b. What is the angular momentum of the disk?

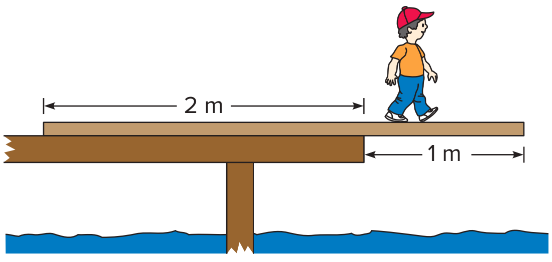
E18. A student, sitting on a stool holds a mass in each hand. When his arms are extended, the total rotational inertia of the system is 5.6 kg·m^2. When he pulls his arms in close to his body, he reduces the total rotational inertia to 1.4 kg·m^2. When he is rotating with his hands held close to his body, his rotational velocity is 48 rpm. If there are no external torques, what is the rotational velocity of the system when he extends his arms?

Synthesis Problems

SP1. A merry-go-round has a radius of 1.5 m and a rotational inertia of 800 kg·m^2. A child pushes the merry-go-round with a constant force of 92 N applied at the edge and parallel to the edge. A frictional torque of 14 N·m acts at the axle of the merry-go-round.
   a. What is the net torque acting on the merry-go-round about its axle?
   b. What is the rotational acceleration of the merry-go-round?
   c. At this rate, what will the rotational velocity of the merry-go-round be after 16 s if it starts from rest?
   d. If the child stops pushing after 16 s, the net torque is now due solely to the friction. What then is the rotational acceleration of the merry-go-round? How long will it take for the merry-go-round to stop turning?

SP2. A 3-m-long plank with a weight of 130 N is placed on a dock with 1 m of its length extended over the water, as in the diagram. The plank is uniform in density, so that the center of gravity of the plank is located at the center of the plank. A boy with a weight of 180 N is standing on the plank and moving out slowly from the edge of the dock.
   a. How far from the pivot point at the edge of the dock is the center of the plank?
   b. What is the torque exerted by the weight of the plank about the pivot point at the edge of the dock? (Treat all the weight as acting through the center of gravity of the plank.)
   c. How far from the edge of the dock can the boy move until the plank is just on the verge of tipping?

d. How can the boy test this conclusion without falling into the water? Explain.



SP2 Diagram

SP3. Several children (having a total mass of 90 kg) are riding on a merry-go-round that has a rotational inertia of 1100 kg·m^2 and a radius of 2.4 m. The average distance of the children from the axle of the merry-go-round is 2.2 m initially, because they are all riding near the edge.

a. What is the rotational inertia of the children about the axle of the merry-go-round? What is the total rotational inertia of the children and the merry-go-round?

b. The children now move inward toward the center of the merry-go-round, so that their average distance from the axle is 0.8 m. What is the new rotational inertia for the system?

c. If the initial rotational velocity of the merry-go-round was 1.3 rad/s, what is the rotational velocity after the children move in toward the center, assuming that the frictional torque can be ignored? (Use conservation of angular momentum.)

d. Is the merry-go-round rotationally accelerated during this process? If so, where does the accelerating torque come from?

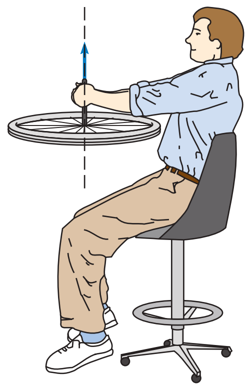
SP4. A student sitting on a stool that is free to rotate, but is initially at rest, holds a bicycle wheel. The wheel has a rotational velocity of 8 rev/s about a vertical axis, as shown in the SP4 diagram. The rotational inertia of the wheel is 2.5 kg·m^2 about its center, and the rotational inertia of the student, wheel, and stool about the rotational axis of the stool is 6 kg·m^2.

a. What is the rotational velocity of the wheel in rad/s?

b. What are the magnitude and direction of the initial angular momentum of the system?

c. If the student flips the axis of the wheel, reversing the direction of its angular-momentum vector, what is the rotational velocity (magnitude and direction) of the student and the stool about their axis after the wheel is flipped? (Hint: See fig. 8.24.)

d. Where does the torque come from that accelerates the student and the stool? Explain.



SP4 Diagram

Home Experiments and Observations

HE1. If there is a park nearby containing a freely rotating child's merry-go-round, take some time with a friend to observe some of the phenomena discussed in this chapter. In particular, make these observations:

a. What is a typical rotational velocity that can be achieved with the merry-go-round? How would you go about measuring this?

b. How long does it take for the merry-go-round to come to rest after you stop pushing? Could you estimate the frictional torque from this information? What other information would you need?

c. If you or your friend is riding on the merry-go-round, what happens to the rotational velocity when you move inward or outward from the axis of the merry-go-round? How do you explain this?

HE2. Create a simple balance using a ruler as the balance beam and a pencil as the fulcrum. (A pencil or pen with a hexagonal cross section is easier to use than one with a round cross section.)

a. Does the ruler balance exactly at its midpoint? What does this imply about the ruler?

b. Using a nickel as your standard, what are the ratios of the weights of pennies, dimes, and quarters to that of the nickel? Describe the process used to find these ratios.

c. Is the distance from the fulcrum necessary to balance two nickels on one side with a single nickel on the opposite side exactly half the distance for the single nickel? How would you account for any discrepancy?

HE3. You can make a simple top by cutting a circular piece of cardboard, poking a hole through the center, and using a short, dull pencil for the post. A short, wooden dowel with a rounded end works even better than a pencil.

a. Try building such a top and testing it. How far up the pencil should the cardboard disk sit for best stability?

b. Observe what happens to the axis of rotation of the top as it slows down. What is the direction of the angular-momentum vector, and how does it change?

c. What happens to the stability of your top if you tape two pennies near the edge on opposite sides of the cardboard disk?

HE4. Try spinning a quarter or other large coin about its edge on a smooth tabletop or other similar surface. Describe the motion that follows, paying particular attention to the direction of the angular-momentum vector.

HE5. As described in the study hint in section 8.5, use a bicycle wheel mounted on a handheld axle (probably available from your friendly physics department) to study the angular momentum vectors and their changes. In particular, make these observations:

a. Grasp the wheel with both hands by the handles on each side and have someone give it a good spin with the wheel in a vertical plane. Then, try tilting the wheel downward to the left to simulate a fall. What direction does the wheel turn? Describe what happens.

b. Grasp the wheel with both hands by the handles on each side and hold it in the vertical orientation. Without spinning the wheel, rotate it into the horizontal plane. Now repeat this, but first have someone give it a good spin with the wheel in the vertical plane. Is it harder or easier to rotate it into the horizontal plane when the wheel is spinning? Explain how this can be related to riding a bicycle.