9-MAVZU	BUL FUNKSIYALARINI O‘ZGARUVCHILAR BO‘YICHA YOYILMASI. MDNSH VA MKNSH

{0,1} Bul algebrasidagi xy kon’yunksiya amali oddiy arifmetikadagi 0 va 1 sonlar ustidagi ko‘paytma amaliga mos keladi. Ammo 0 va 1 sonlarni qo‘shish natijasi {0,1} to‘plam doirasidan chetga chiqadi. Shuning uchun I.I.Jegalkin (3.8.1869-28.3.1947) 2 moduliga asosan qo‘shish amalini kiritadi (I.I.Jegalkin 30-yillarning boshida Moskva davlat universitetida birinchi bo‘lib matematik mantiq bo‘yicha ilmiy seminar tashkil etgan). x va y mulohazalarning 2 moduli bo‘yicha qo‘shishni $x+y$ sifatida belgilaymiz va u quyidagi chinlik jadvali bilan beriladi:

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	0

Chinlik jadvalidan ko‘rinib turibdiki, $x+y=\overline{x\leftrightarrow y}$. Mantiq algebrasidagi ko‘paytma va 2 moduli bo‘yicha qo‘shish mantiq amallari uchun kommutativ, assotsiativ va distributiv arifmetik qonunlar o‘z kuchini saqlaydi.

Bul algebrasidagi asosiy mantiqiy amallarni kiritilgan arifmetik amallar orqali quyidagicha ifodalash mumkin:

1. $\bar{x}=x+1$; 2. $x\wedge y=xy$; 3. $x\vee y=xy+x+y$; 4. $x\rightarrow y=xy+x+1$;
5. $x\leftrightarrow y=x+y+1$.

2 moduli bo‘yicha qo‘shish amalining ta’rifiga asosan $x+x=0$ va $xx=x$ $(x^n=x)$.

Bul funksiyalarining o‘zgaruvchilar bo‘yicha yoyilmasi

Teorema 1. E_2 *to'plamdagi har bir* $f(x_1,\dots,x_n)$ *funksiya quyidagi ko'rinishda ifodalanishi mumkin:*

1. $f(x_1,\dots,x_n)=x_1\cdot f(1,x_2,\dots,x_n)\vee \bar{x}_1\cdot f(0,x_2,\dots,x_n)$,
2. $f(x_1,\dots,x_n)=x_1\cdot f(1,x_2,\dots,x_n)\oplus \bar{x}_1\cdot f(0,x_2,\dots,x_n)$,
3. $f(x_1,\dots,x_n)=(x_1\vee f(0,x_2,\dots,x_n))\,\&\,(\bar{x}_1\vee f(1,x_2,\dots,x_n))$.

Isbot. Ushbu formulalarni isbotlash uchun

1) Agar $x_1=0$ bo'lsa, unda

$$f(0,x_2,\dots,x_n)=0\vee f(0,x_2,\dots,x_n)=f(0,x_2,\dots,x_n).$$

2) Agar $x_1=1$ bo'lsa, unda

$$f(1,x_2,\dots,x_n)=1\,\&\,f(1,x_2,\dots,x_n)\vee 0\,\&\,f(0,x_2,\dots,x_n)=f(1,x_2,\dots,x_n).$$

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

Teorema 2. *Har bir* E_2 *dan olingan* $f(x_1,\dots,x_n)$ *funksiyani quyidagicha ifodalash mumkin, bunda* $\forall\kappa, 1\le k\le n$ *:*

1. $f(x_1,\dots,x_n)=\underset{(\sigma_1,\dots,\sigma_k)}{V}\; x_1^{\sigma_1}\,\&\dots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$,

bunda $x^{\sigma}=\begin{cases} x,\ agar\ \sigma=1;\\ \bar{x},\ agar\ \sigma=0;\end{cases}$ $\quad \overset{n}{\underset{i=1}{V}} x_i = x_1\vee x_2\vee\dots\vee x_n.$

2. $f(x_1,\dots,x_n)=\sum\limits_{(\sigma_1,\dots,\sigma_k)} x_1^{\sigma_1}\,\&\dots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$, bunda

$\sum\limits_{i=1}^{n} x_i = x_1\oplus x_2\oplus\dots\oplus x_n.$

3. $f(x_1,\dots,x_n)=\underset{(\sigma_1,\dots,\sigma_k)}{\&}\;(x_1^{\bar{\sigma}_1}\vee\dots\vee x_k^{\bar{\sigma}_k}\vee f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n))$;

bunda $\overset{n}{\underset{i=1}{\&}} x_i = x_1\,\&\,x_2\,\&\dots\&\,x_n,\qquad x^{\bar{\sigma}}=\begin{cases} x,\ agar\ \sigma=0;\\ \bar{x},\ agar\ \sigma=1;\end{cases}$

1-tasdiqning isboti: $k-(\alpha_1,\dots,\alpha_k)$ bo'lsa, $f(x_1,\dots,x_n)$ funksiya quyidagi ko'rinishga keladi - $f(\alpha_1,\dots,\alpha_k,x_{k+1},\dots x_n)$.

Ta'rifga asosan,

$$\left.\begin{array}{l} \alpha_1^{\sigma_1}=1 \;\Leftrightarrow\; \alpha_1=\sigma_1 \\ \alpha_2^{\sigma_2}=1 \;\Leftrightarrow\; \alpha_2=\sigma_2 \\ \dots\dots\dots\dots\dots\dots.. \\ \alpha_k^{\sigma_k}=1 \;\Leftrightarrow\; \alpha_k=\sigma_k \end{array}\right\}$$

Bundan esa, $\alpha_1^{\sigma_1} \& \alpha_2^{\sigma_2} \& \ldots \& \alpha_k^{\sigma_k} = 1 \quad \Leftrightarrow \quad \alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$.

$1 \& x = x$ dan f funksiya berilgan formulaga faqat va faqat $\alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$ bo‘lganda o‘rinli.

Bundan, formulaning o‘ng tomoni

$$\underset{(\sigma_1,\ldots,\sigma_k)}{V} x_1^{\sigma_1} \ldots x_k^{\sigma_k} \,\&\, f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n) = f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n)$$ ga teng

chunki qolgan barcha kon’yunksiyalar = 0. Formulaning chap tomonining ham $f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n)$ ko‘rinishga ega chunki, $\alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$.

Demak, $f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n) = f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n)$.

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

MDNSH va MKNSH.

Natija 1. *2-teoremaning 1-tasdiqida* $k = n$ *bo‘lsa, unda yoyilma quyidagi ko‘rinishga ega bo‘ladi:*

$$f(x_1,\ldots,x_n) = \underset{(\sigma_1,\ldots,\sigma_n)}{V} x_1^{\sigma_1} \ldots x_n^{\sigma_n} \cdot f(\sigma_1,\ldots,\sigma_n). \qquad (1)$$

Agar $f \neq 0$, *unda (1) formuladan*

$$f(x_1,\ldots,x_n) = \underset{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=1}}{V} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n}$$ *– Mukammal diz’yunktiv normal shakl (MDNSh).*

Natija 2. *Agar 2-teoremaning 3-tasdiqida* $k = n$ *bo‘lsa, unda unda yoyilma quyidagi ko‘rinishga ega bo‘ladi:*

$$f(x_1,\ldots,x_n) = \underset{(\sigma_1,\ldots,\sigma_n)}{\&} (x_1^{\bar{\sigma}_1} \vee \ldots \vee x_n^{\bar{\sigma}_n} \vee f(\sigma_1,\ldots,\sigma_n)). \qquad (2)$$

Agar $f \neq 1$, *unda (2) formuladan*

$$f(x_1,\ldots,x_n) = \underset{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=0}}{\&} (x_1^{\bar{\sigma}_1} \vee \ldots \vee x_n^{\bar{\sigma}_n})$$ *– Mukammal kon’yunktiv normal shakl (MKNSh).*

Natija 3. *Agar 2-teoremaning 2-tasdiqida* $k = n$ *bo‘lsa, unda unda yoyilma quyidagi ko‘rinishga ega bo‘ladi:*

$$f(x_1,\ldots,x_n) = \sum_{(\sigma_1,\ldots,\sigma_n)} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n} \& f(\sigma_1,\ldots,\sigma_n) \qquad (3)$$

Agar $f \neq 0$, *unda (3) formuladan*

$$f(x_1,\ldots,x_n) = \sum_{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1} \,\&\ldots\&\, x_n^{\sigma_n} \;.$$