YOYILMASI. MDNSHVA. MANASIA O'ZGARUVCHILAR BO'YICHA BULFUNKSIVALARINI

 $\{0,1\}$  Bul algebrasidagi xy kon'yunksiya amali oddiy arifmetikadagi 0 va 1 sonlar ustidagi ko'paytma amaliga mos keladi. Ammo 0 va 1 sonlarni qo'shish natijasi  $\{0,1\}$  to'plam doirasidan chetga chiqadi. Shuning uchun I.I.Jegalkin (3.8.1869-28.3.1947) 2 moduliga asosan qo'shish amalini kiritadi (I.I.Jegalkin 30-yillarning boshida Moskva davlat universitetida birinchi bo'lib matematik mantiq bo'yicha ilmiy seminar tashkil etgan). x va y mulohazalarning 2 moduli bo'yicha qo'shishni x+y sifatida belgilaymiz va u quyidagi chinlik jadvali bilan beriladi:

x	у	x+y
0	0	0
0	1	1
1	0	1
1	1	0

Chinlik jadvalidan koʻrinib turibdiki,  $x+y=\overline{x\leftrightarrow y}$ . Mantiq algebrasidagi koʻpaytma va 2 moduli boʻyicha qoʻshish mantiq amallari uchun kommutativ, assotsiativ va distributiv arifmetik qonunlar oʻz kuchini saqlaydi.

Bul algebrasidagi asosiy mantiqiy amallarni kiritilgan arifmetik amallar orqali quyidagicha ifodalash mumkin:

1. 
$$\bar{x} = x + 1$$
; 2.  $x \wedge y = xy$ ; 3.  $x \vee y = xy + x + y$ ; 4.  $x \rightarrow y = xy + x + 1$ ;

5. 
$$x \leftrightarrow y = x + y + 1$$
.

2 moduli bo'yicha qo'shish amalining ta'rifiga asosan x + x = 0 va xx = x ( $x^n = x$ ).

## Bul funksiyalarining oʻzgaruvchilar boʻyicha yoyilmasi

**Teorema 1.**  $E_2$  to 'plamdagi har bir  $f(x_1,...,x_n)$  funksiya quyidagi ko 'rinishda ifodalanishi mumkin:

1. 
$$f(x_1,...,x_n) = x_1 \cdot f(1,x_2,...,x_n) \vee \bar{x}_1 \cdot f(0,x_2,...,x_n)$$
,

2. 
$$f(x_1,...,x_n) = x_1 \cdot f(1,x_2,...,x_n) \oplus \overline{x}_1 \cdot f(0,x_2,...,x_n)$$
,

3. 
$$f(x_1,...,x_n) = (x_1 \lor f(0,x_2,...,x_n)) \& (\overline{x_1} \lor f(1,x_2,...,x_n))$$
.

## Isbot. Ushbu formulalarni isbotlash uchun

1) Agar  $x_1 = 0$  bo'lsa, unda

$$f(0, x_2,...,x_n) = 0 \lor f(0, x_2,...,x_n) = f(0, x_2,...,x_n).$$

2) Agar  $x_1=1$  bo'lsa, unda

$$f(1, x_2, ..., x_n) = 1 & f(1, x_2, ..., x_n) \lor 0 & f(0, x_2, ..., x_n) = f(1, x_2, ..., x_n).$$

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

**Teorema 2.** Har bir  $E_2$  dan olingan  $f(x_1,...,x_n)$  funksiyani quyidagicha ifodalash mumkin, bunda  $\forall \kappa, 1 \le k \le n$ :

$$1. f(x_1,...,x_n) = V x_1^{\sigma_1} \& ... \& x_k^{\sigma_k} \& f(\sigma_1,...,\sigma_k,x_{k+1},...x_n),$$

bunda 
$$x^{\sigma} = \begin{cases} x, \ agar \ \sigma = 1; \\ \overline{x}, \ agar \ \sigma = 0; \end{cases}$$
 
$$\bigvee_{i=1}^{n} x_{i} = x_{1} \lor x_{2} \lor \dots \lor x_{n}.$$

$$2. f(x_1,...,x_n) = \sum_{(\sigma_1,...,\sigma_k)} x_1^{\sigma_1} \& ... \& x_k^{\sigma_k} \& f(\sigma_1,...,\sigma_k,x_{k+1},...x_n), \text{ bunda}$$

$$\sum_{i=1}^{n} x_i = x_1 \oplus x_2 \oplus \dots \oplus x_n.$$

3. 
$$f(x_1,\ldots,x_n) = \underset{(\sigma_1,\ldots,\sigma_k)}{\&} (x_1^{\overline{\sigma_1}} \vee \ldots \vee x_k^{\overline{\sigma_k}} \vee f(\sigma_1,\ldots,\sigma_k,x_{k+1},\ldots x_n)),$$

bunda 
$$\underset{i=1}{\overset{n}{\&}} x_i = x_1 \overset{n}{\&} x_2 \overset{n}{\&} \dots \overset{n}{\&} x_n, \qquad x^{\overline{\sigma}} = \begin{cases} x, \ agar \ \sigma = 0; \\ \overline{x}, \ agar \ \sigma = 1; \end{cases}$$

## MDNSH va MKNSH.

Natija 1. 2-teoremaning 1-tasdiqida k = n boʻlsa, unda yoyilma quyidagi koʻrinishga ega boʻladi:

$$f(x_1, \dots, x_n) = V \qquad x_1^{\sigma_1} \dots x_n^{\sigma_n} \cdot f(\sigma_1, \dots, \sigma_n). \tag{1}$$

Agar  $f \neq 0$ , unda (1) formuladan

$$f(x_1,...,x_n) = V \atop f(\sigma_1,...,\sigma_n) = 1 \atop f(\sigma_1,...,\sigma_n) = 1} x_1^{\sigma_1} \&...\& x_n^{\sigma_n} - Mukammal \ diz'yunktiv \ normal$$

shakl (MDNSh).

Natija 2. Agar 2-teoremaning 3-tasdiqida k = n boʻlsa, unda unda yoyilma quyidagi koʻrinishga ega boʻladi:

$$f(x_1,...,x_n) = \mathcal{X}_{(\sigma_1,...,\sigma_n)}(x_1^{\overline{\sigma}_1} \vee ... \vee x_n^{\overline{\sigma}_n} \vee f(\sigma_1,...,\sigma_n)). \tag{2}$$

Agar  $f \neq 1$ , unda (2) formuladan

$$f(x_1,...,x_n) = \underset{f(\sigma_1,...,\sigma_n)=0}{\&} (x_1^{\overline{\sigma_1}} \vee ... \vee x_n^{\overline{\sigma_n}}) - Mukammal \ kon'yunktiv \ normal \ shakl$$

(MKNSh).

Natija 3. Agar 2-teoremaning 2-tasdiqida k = n boʻlsa, unda unda yoyilma quyidagi koʻrinishga ega boʻladi:

$$f(x_1,...,x_n) = \sum_{(\sigma_1,...,\sigma_n)} x_1^{\sigma_1} \& ... \& x_n^{\sigma_n} \& f(\sigma_1,...,\sigma_n)$$
(3)

Agar  $f \neq 0$ , unda (3) formuladan

$$f(x_1,...,x_n) = \sum_{\substack{(\sigma_1,...,\sigma_n) \\ f(\sigma_1,...,\sigma_n) = 1}} x_1^{\sigma_1} \& ... \& x_n^{\sigma_n}.$$