JEGALKIN KO‘PHADI. MONOTON BUL FUNKSIYALARI

MOSINI TOPING

1. $f(x_1,\ldots,x_n)=x_1\cdot f(1,x_2,\ldots,x_n)\vee\bar{x}_1\cdot f(0,x_2,\ldots,x_n)$. 2. $f(x_1,\ldots,x_n)=x_1\cdot f(1,x_2,\ldots,x_n)\oplus\bar{x}_1\cdot f(0,x_2,\ldots,x_n)$. 3. $f(x_1,\ldots,x_n)=(x_1\vee f(0,x_2,\ldots,x_n))\,\&\,(\bar{x}_1\vee f(1,x_2,\ldots,x_n))$.	**K TA O'ZGARUVCHI BO'YICHA YOYILMASI**
1. $f(x_1,\ldots,x_n)=\bigvee\limits_{(\sigma_1,\ldots,\sigma_k)} x_1^{\sigma_1}\,\&\ldots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\ldots,\sigma_k,x_{k+1},\ldots x_n)$. 2. $f(x_1,\ldots,x_n)=\sum\limits_{(\sigma_1,\ldots,\sigma_k)} x_1^{\sigma_1}\,\&\ldots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\ldots,\sigma_k,x_{k+1},\ldots x_n)$. 3. $f(x_1,\ldots,x_n)=\mathop{\&}\limits_{(\sigma_1,\ldots,\sigma_k)}(x_1^{\bar{\sigma}_1}\vee\ldots\vee x_k^{\bar{\sigma}_k}\vee f(\sigma_1,\ldots,\sigma_k,x_{k+1},\ldots x_n))$.	**MUKAMMAL NORMAL SHAKLLAR**
$f(x_1,\ldots,x_n)=\bigvee\limits_{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1}\,\&\ldots\&\,x_n^{\sigma_n}$ $f(x_1,\ldots,x_n)=\sum\limits_{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1}\,\&\ldots\&\,x_n^{\sigma_n}$	**BIR O'ZGARUVCHI BO'YICHA YOYILMASI**

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JEGALKIN KO‘PHADI. MONOTON BUL FUNKSIYALARI

REJA:

- Jegalkin ko‘phadi
- Chiziqli funksiya
- Mantiq algebrasidagi monoton funksiyalar

n ta $x_1,\dots,x_n$ o‘zgaruvchi yordamida inkor amali qatnashmagan elementar kon’yuksiyalar sonini topish talab qilinsin. Shunday elementar kon’yunksiyalar 2^n ta bo’ladi.

Masalan:

1) $n = 2$ bo‘lsa, x_1, x_2: 2) $n = 3$ *bo‘lsa,* x_1, x_2, x_3:

$$\left.\begin{array}{l} x_1 \,\&\, x_2 \\ x_1 \\ x_2 \\ \varnothing \end{array}\right\} 2^2$$

kon’yunksiyalar

$$\left.\begin{array}{c} x_1 \,\&\, x_2 \,\&\, x_3 \\ x_1 \,\&\, x_2 \\ x_1 \,\&\, x_3 \\ x_2 \,\&\, x_3 \\ x_1 \\ x_2 \\ x_3 \\ \varnothing \end{array}\right\} 2^3$$

Shunday qilib, n ta $x_1,\ldots,x_n$ o‘zgaruvchilar yordamida inkor amali qatnashmagan barcha 2^n ta elementar kon’yuksiyalarni $k_1,\ldots,k_{2^n}$ deb belgilash kiritamiz.

Ta’rif-1: $\sum_{i=1}^{2^n} a_i k_i,\ \ bu\ \ yerda\ \ \ a_i \in E_2$

ko‘rinishidagi ko‘phadga Jegalkin ko‘phadi deyiladi.

Teorema-1. *Ixtiyoriy $f(x_1,\ldots,x_n)\in E_2$ bul funksiyasini Jegalkin ko‘phadi ko‘rinishida ifodalash mumkin va u yagonadir.*

Isbot:

$$f(x_1,\ldots,x_n)=\sum_{(\sigma_1,\ldots,\sigma_n)} x_1^{\sigma_1}\,\&\ldots\&\,x_n^{\sigma_n}\,\&\,f(\sigma_1,\ldots,\sigma_n) \qquad (1)$$

(1) formuladagi barcha inkor amallaridan $x^{\sigma} = x + \bar{\sigma}$ tenglik yordamida yo‘qotib yuboramiz. Bu yerda $x^{\sigma} = \begin{cases} x, \; agar \; \sigma = 1; \\ \bar{x}, \; agar \; \sigma = 0. \end{cases}$

Haqiqatdan ham:

$\sigma = 1$ bo‘lsa, $x = x \oplus \bar{1} = x$, agar $\sigma = 0$, bo‘lsa, $\bar{x} = x \oplus \bar{0} = x + 1 = \bar{x}$.

(1) formula quyidagi ko‘rinishga keladi:

$$f(x_1, \ldots, x_n) = \sum_{(\sigma_1, \ldots, \sigma_n)} (x_1 + \bar{\sigma}_1)(x_2 + \bar{\sigma}_2) \ldots (x_n + \bar{\sigma}_n) f(\sigma_1, \ldots, \sigma_n).$$

Hosil bo‘lgan yig‘indidagi o‘zgaruvchilarning birortasida ham inkor amali mavjud emas. Endi qavslarni ochib chiqamiz:

$$f(x_1,\ldots,x_n)=\sum_{i=1}^{2^n} a_i k_i,\ \ a_i\in E_2,$$

k_i – $x_1,\ldots,x_n$ o‘zgaruvchilar yordamida tuzilgan turli elementar kon’yunksiyalar. Shunday qilib, ixtiyoriy bul funksiyasini Jegalkin ko‘phadi yordamida ifodalash mumkinligi isbotlandi.

2) Yagonaligini isboti. Buning uchun n o‘zgaruvchili bul funksiyalari sonini, n o‘zgaruvchili Jegalkin ko‘phadlar soni bilan taqqoslaylik.

Teng kuchli bo‘lmagan n o‘zgaruvchili bul funksiyalari soni 2^{2^n} ta ekanligini bilamiz. Endi biz barcha elementar kon'yunktsiyalarni yozamiz $\{k_1,k_2,\ldots,k_{2^n}\}$, har bir konyunksiya ko‘phadga yo kiradi yoki kirmaydi, shuning uchun bunday ko‘phadlar soni 2^{2^n} bo'ladi.

Xulosa:

1) n o'zgaruvchili bul funksiyalari soni bilan Jegalkin ko'phadlari soni teng ekanligi aniqlandi.

2) Ixtiyoriy funksiyani Jegalkin ko'phadi ko'rinishiga ifodash mumkinligini isbotladik.

3) Har bir Jegalkin ko'phadiga mos keluvchi funksiya mavjud.

Demak, funksiyani ko'phad yordamida ifodalash mumkin va u yagonadir.

Funksiyalarni Jegalkin ko'phadi ko'rinishiga keltirishning bir necha usullari mavjud

I. Chinlik jadvali yordamida funksiyani Jegalkin ko'phadi ko'rinishiga keltirish

(1) formulada $f(\sigma_1,\ldots,\sigma_n)=1$ deb, quyidagi formulani xosil qilamiz:

$$f(x_1,\ldots,x_n)=\sum_{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1}\&\ldots\&x_n^{\sigma_n} \qquad (2)$$

$x^\sigma = x+\bar{\sigma}$ formuladan foydalanib, (2) yig'indidagi barcha inkor amallaridan qutulishimiz mumkin va natijada Jegalkin ko'phadini hosil qilamiz.

II. Noaniq koeffitsientlar usuli

1-teoremaga asosan,

$$f(x_1,\dots,x_n)=\sum_{i=1}^{2^n} a_i k_i,\ \ bu\ \ yerda\ \ \ a_i\in E_2. \qquad (3)$$

(3) formulada noaniq koeffitsientlar a_i bo'lib, ular jami 2^n ta.

Misol. Ushbi funksiyani Jegalkin ko'phadi ko'rinishida ifodalang:

$$f(x_1,x_2,x_3)=(x_1/x_2)+(x_1\wedge x_3)$$

Yechish: Berilgan funksiya uchun noma'lum koeffisientli ko'phad ko'rinishidagi ifodasini izlaymiz:

$$(x_1/x_2)+(x_1\wedge x_3)=ax_1x_2x_3+bx_1x_2+cx_1x_3+dx_2x_3+ex_1+fx_2+gx_3+h$$

Funksiyaning qiymatlar jadvalida noma'lum koeffisientlarni aniqlaymiz:

x_1	x_2	x_3	$(x_1 / x_2) + (x_1 \wedge x_3)$	$ax_1x_2x_3 + bx_1x_2 + cx_1x_3 + dx_2x_3 +$ $+ex_1 + fx_2 + gx_3 + h$	
0	0	0		h	
0	0	1		$g+h$	
0	1	0		$f+h$	
0	1	1		$d+f+g+h$	
1	0	0		$e+h$	
1	0	1		$c+e+g+h$	
1	1	0		$b+e+f+h$	
1	1	1		$a+b+c+d+e+f+g+h$	

x_1	x_2	x_3	$(x_1/x_2)+(x_1 \wedge x_3)$	$ax_1x_2x_3+bx_1x_2+cx_1x_3+dx_2x_3+$ $+ex_1+fx_2+gx_3+h$	
0	0	0	1	h	$h=1$
0	0	1	1	$g+h$	$g=0$
0	1	0	1	$f+h$	$f=0$
0	1	1	1	$d+f+g+h$	$d=0$
1	0	0	1	$e+h$	$e=0$
1	0	1	0	$c+e+g+h$	$c=1$
1	1	0	0	$b+e+f+h$	$b=1$
1	1	1	1	$a+b+c+d+e+f+g+h$	$a=0$

$$f(x_1,x_2,x_3)=(x_1/x_2)+(x_1 \wedge x_3)=x_1 \cdot x_2+x_1 \cdot x_3+1$$

III. Superpozitsiyalar metodi.

Asosiy mantiqiy amallarni algebraik amallar (kon'yunksiya, Jegalkin yig'indi) yordamida ifodalay olishimizni inobatga olib, ixtiyoriy funksiyani kerakli almashtirishlar bajarib Jegalkin yig'indisi ko'rinishda ifodalashimiz mumkin.

Masalan. $x \vee y = xy + x + y$ va $\bar{x} = x+1$ formulalardan:

1) $x \vee \bar{y} = x\bar{y} + x + \bar{y} = x(y+1) + x + y + 1 = xy + x + x + y + 1 = xy + y + 1;$

2) $\bar{x} \vee y = \bar{x}y + \bar{x} + y = (x+1)y + x + 1 + y = xy + y + x + 1 + y = xy + x + 1;$

3) $\bar{x} \vee \bar{y} = \bar{x}\,\bar{y} + \bar{x} + \bar{y} =$

$= (x+1)(y+1) + x + 1 + y + 1 = xy + y + x + x + y + 1 = xy + 1.$

KARNO KARTASI USULI.

A	BC			
	00	01	11	10
0	1	0	0	1
1	1	0	1	0

A	BC			
	00	01	11	10
0	0	1	1	0
1	0	1	0	1

A	BC			
	00	01	11	10
0	0	1	1	0
1	0	1	0	1

A	BC			
	00	01	11	10
0	0	1	1	0
1	0	1	0	1

A	BC			
	00	01	11	10
0	0	0	0	0
1	0	0	1	1

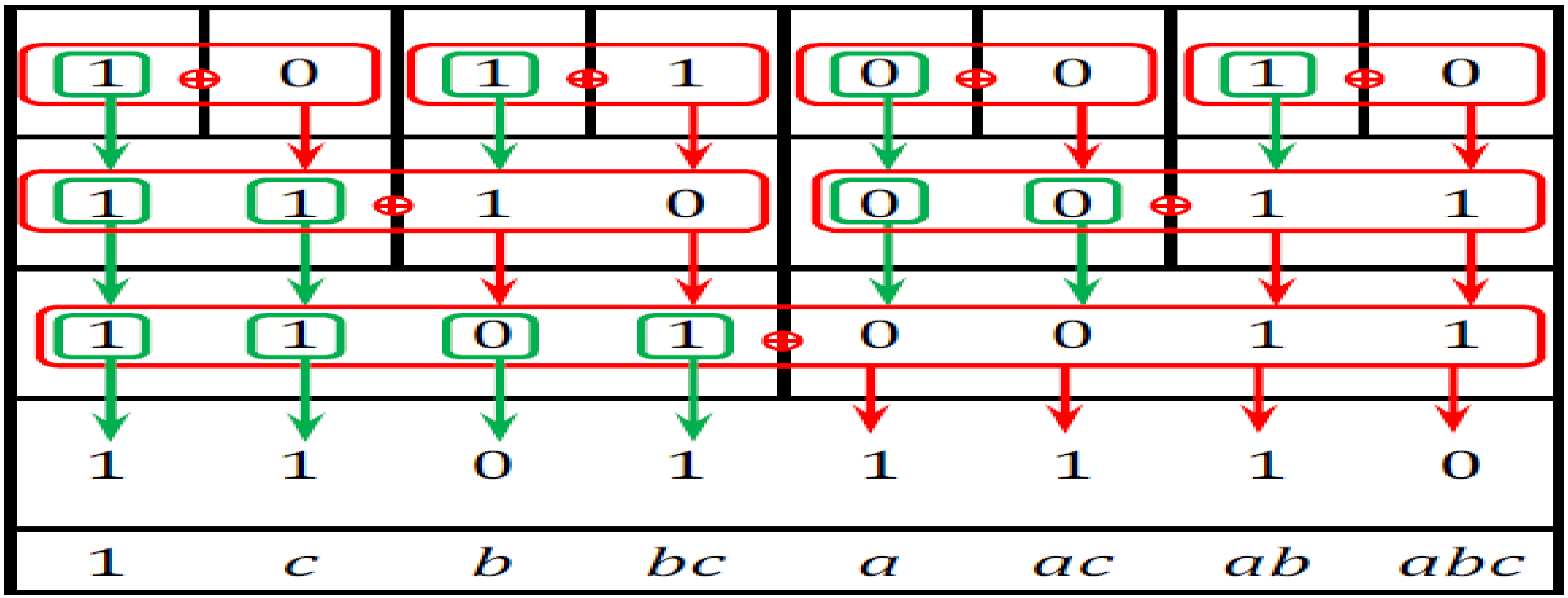
A	BC			
	00	01	11	10
0	0	0	0	0
1	0	0	0	0

A	BC			
	00	01	11	10
0	0	0	0	0
1	0	0	0	0

A	BC			
	00	01	11	10
0	0	0	0	0
1	0	0	0	0

$$P = 1 \oplus C \oplus AB$$

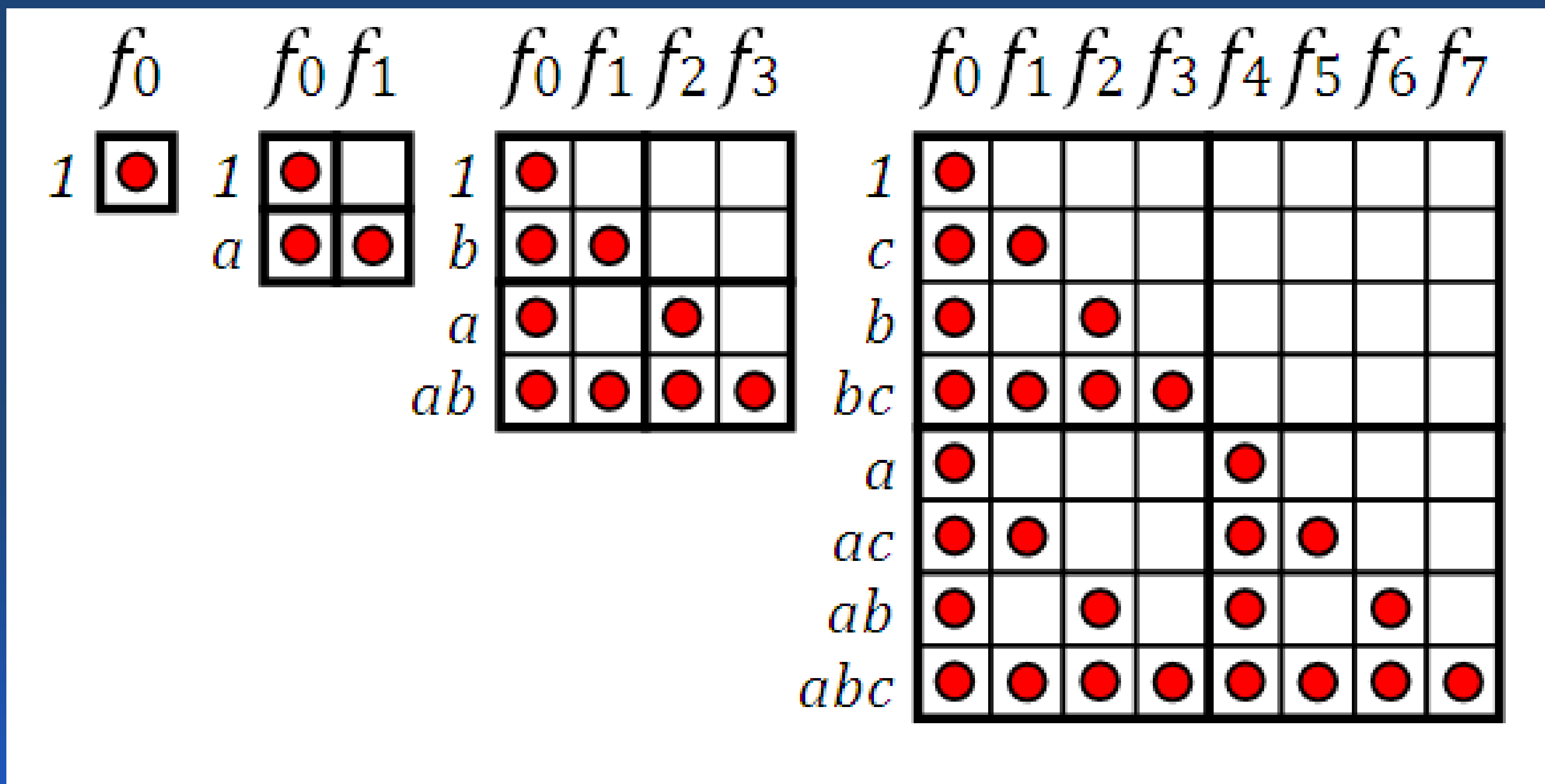
PASKAL USULI



$$f(a, b, c) = 1 \oplus a \oplus c \oplus ab \oplus ac \oplus bc$$

$\oplus$ — *побитная операция «Исключающее ИЛИ»*

YIG'INDI USULI.



Ta'rif-2. $x_{i_1} + x_{i_2} + \ldots + x_{i_k} + a$ *ko'rinishidagi funksiya chiziqli funksiya deb aytiladi. Bu yerda* $a \in E_2 = \{0,1\}$.

Chiziqli funksiyaning ifodasidan ko'rinib turibdiki, n argumentli chiziqli funksiyalar soni 2^{n+1} ga teng va bir argumentli funksiyalar doimo chiziqli funksiya bo'ladi.

Jegalkin ko'phadi ko'rinishidagi har bir funksiyaning argumentlari soxta emas argumentlar bo'ladi. Haqiqatan ham, x_1 shunday argument bo'lsin. U vaqtda ixtiyoriy $f(x_1,...,x_n)$ funksiyani quyidagi ko'rinishda yozish mumkin:

$$f(x_1,...,x_n) = x_1\varphi(x_2,...,x_n) + \psi(x_2,...,x_n).$$

Bu yerda φ funksiyasi aynan 0 ga teng emas, aks holda x_1 argument f funksiyaning (ko'phadning) argumentlari safiga qo'shilmasdi.

Endi $x_2,\dots,x_n$ argumentlarning shunday qiymatlarini olamizki, $\varphi=1$ bo‘lsin. U vaqtda f funksiyaning qiymati x_1 argumentning qiymatiga bog‘liq bo‘ladi. Demak, x_1 soxta argument emas.

Mantiq algebrasidagi hamma n argumentli chiziqli funksiyalar to‘plamini L harfi bilan belgilaymiz. Uning elementlarining soni 2^{n-1} ga teng bo‘ladi.

Teorema. *Agar $f(x_1,\dots,x_n)\notin L$ bo‘lsa, u holda undan argumentlari o‘rniga 0 va 1 konstantalarni hamda x va $\bar{x}$ funksiyalarni, ayrim holda f ustiga “–“ inkor amalini qo‘yish usuli bilan $x_1 x_2$ funksiyani hosil etish mumkin.*

Monoton funksiyalar. $0<1$ munosabati orqali $\{0,1\}$ to‘plamini tartiblashtiramiz.

1-ta’rif. *$\alpha=(\alpha_1,...,\alpha_n)$ va $\beta=(\beta_1,...,\beta_n)$ qiymatlar satri bo‘lsin. α qiymatlar satri β qiymatlar satridan shunda va faqat shundagina oldin keladi deb aytamiz, qachon $\alpha\prec\beta$ yoki α va β qiymatlar satri ustma-ust tushsa, u holda $\alpha\prec\beta$ shaklida yozamiz.*

2-ta’rif. *$\alpha=(\alpha_1,...,\alpha_n)$ va $\beta=(\beta_1,...,\beta_n)$ ixtiyoriy qiymatlar satri bo‘lsin. $\alpha\prec\beta$ dan $f(\alpha_1,..., \alpha_n)\le f(\beta_1,..., \beta_n)$ bajarilishi kelib chiqsa, u holda $f(x_1,...,x_n)$ funksiya monoton funksiya deb aytiladi.*

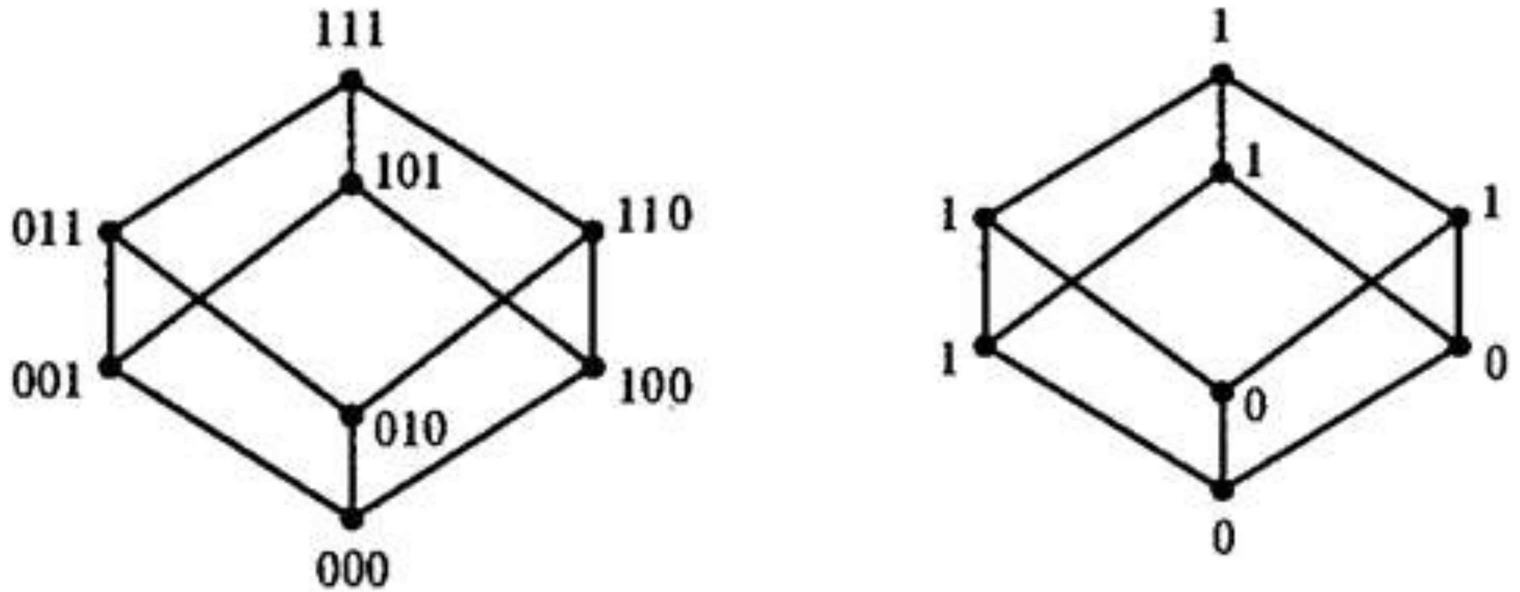
3-ta’rif. *$\alpha\prec\beta$ dan $f(\alpha_1,...,\alpha_n)>f(\beta_1,...,\beta_n)$ munosabat kelib chiqsa, u holda $f(x_1,...,x_n)$ nomonoton funksiya deb aytiladi.*

Asosiy elementar mantiqiy funksiyalardan 0, 1, x, xy, $x\vee y$ funksiyalar monoton funksiyalar bo‘lib, $\bar{x}$, $x\rightarrow y$, $x\leftrightarrow y$, $x+y$ funksiyalar nomonoton funksiyalardir.

MISOL. $(x \vee y \vee z)(x' \vee y \vee z)(x \vee y' \vee z)$. *A*vval qiymatlar jadvalini tuzamiz:

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- Endi har bir qiymatlar satrini va natijasini taqqoslaymiz:
- 000 < 001, 000 < 101, 000 < 010, 000 <110, 000 < 011,
- 000 < 100, 000 < 111, 001 < 011, 001 < 101, 001 < 111,
- 010 < 110, 010 < 111, 010 < 011, 011 < 111, 100 < 101,
- 100 < 110, 100< 111, 101<111, 110< 111, 011 < 111.
- Demak, berilgan funksiyamiz monoton funksiya.


111
101
011
110
001
100
010
000
1
1
1
1
1
0
0
0

RAHMAT