Project: Retailer Distribution Operation Optimization

Submitted in partial fulfillment of the requirements of the course

BANA 7020 Optimization Models

By

Adrian Valles

Nirupam Sharma

Shashank Badre

MS Business Analytics Candidates 2018

Department of Operations, Business Analytics & Information Systems

Carl H Lindner School of Business: University of Cincinnati

Introduction and problem statement

This executive report presents our solution to the distribution operation optimization of a small online retailer. This report is part of the final project of the Optimization Models class (BANA 7020), which is a required class in the MS Business Analytics program.

A small online retailer wants to optimize its distribution operation. At the end of each day, the retailer has a set of orders that must be delivered the next day. The retailer can use a shipping company (e.g., USPS, Fedex, UPS) to deliver the goods. The firm can also contract a delivery truck and plan a route to visit the costumers, or any combination of shipping companies and delivery truck. Each order has a time window in which it can be served. If an order is served using a shipping company, then the third-party shipping company will satisfy the time window requirement. However, if an order is fulfilled using the delivery truck, then the corresponding customer should be visited within its stablished time window.

Through this report, and using FICO Xpress optimization software, we will present the analytical formulation of the model as well as the solution that minimizes the total cost of the distribution operation, given the following assumptions:

- There are 25 orders that must be satisfied which are randomly generated with x and y coordinates between 0 and 100. The Distribution center is located at the origin (0,0).
- The cost of the delivery truck is equal to $1 for each unit of distance traveled in the delivery tour. The truck must start and end at the depot. The truck starts at the depot at 7am and travels at a constant speed of 50 distance units/hr.
- Columns TW_a and TW_b in **Table 1** show the lower and upper time window for each order. Time windows are given in a 24-hour format, i.e., time windows for "Order1" are from 2pm to 9pm (or from 14 to 21 hours).
- Columns USPS, Fedex, and UPS present the cost (in dollars) of delivering an order using the corresponding shipping company.

OrderID	x coordinate	y coordinate	TW_a	TW_b	USPS	Fedex	UPS
Order1	38	41	14	21	30	20	55
Order2	58	10	13	23	50	55	60
Order3	66	98	10	20	50	55	60
Order4	16	89	13	19	65	50	70
Order5	48	61	8	16	55	45	75
Order6	56	47	10	20	30	50	35
Order7	34	90	10	16	70	65	75
Order8	13	76	8	15	65	45	45
Order9	31	99	9	19	20	60	55
Order10	40	20	10	17	75	20	35
Order11	58	55	9	15	35	75	35
Order12	59	32	13	15	25	15	55
Order13	54	27	10	19	30	75	70
Order14	75	99	10	19	70	30	70
Order15	96	81	8	15	60	50	20
Order16	6	28	14	24	35	70	30
Order17	52	40	11	14	20	75	65
Order18	12	61	11	14	15	65	15
Order19	34	22	13	16	45	50	25
Order20	17	42	8	16	25	45	30
Order21	23	61	8	17	40	15	45
Order22	35	1	8	12	35	65	40
Order23	97	70	13	15	45	70	75
Order24	18	40	9	13	65	40	30
Order25	67	33	13	19	50	70	40
Order26	0	0	7	24	0	0	0
Order27	0	0	7	24	0	0	0

***Table 1**: Location, time and cost by Order ID*

We have added 2 new observations to **table 1**, Orders 26 and 27, that represent the depot at point (0,0). Order 26 represents the starting point for the truck and Order 27 represents the ending point. Because the truck starts at the depot at 7am, the time window for Orders 26 and 27 goes from 7 to 24 given the 24-hour format. This action taken is key for our analytical representation of the program, which is shown next.

Analytical representation: Linear mixed integer program

The analytical form is the general mathematical expression that solves the problem. It is composed of parameters, variables, objective function and constraints.

Parameters:

- x: x coordinates between 0 and 100 of each order.
- y: y coordinates between 0 and 100 of each order.
- TW_a: lower time window for each order.
- TW_b: upper time window for each order.
- Cf: cost (in dollars) of delivering an order using Fedex.
- Cu: cost (in dollars) of delivering an order using UPS.
- Cs: cost (in dollars) of delivering an order using USPS.
- d: distance matrix for each of the orders: d(i,j) := sqrt((x(j)-x(i))^2 + (y(j)-y(i))^2).
- t: time matrix, which is equal to d/50 given that the truck moves at 50 distance units/hr.

Variables:

$$\mathrm{Yij} = \begin{cases} 1 \text{ if arc(i, j)is in tour for truck} \\ 0 \text{ otherwise} \end{cases}$$

$$\mathrm{Fi} = \begin{cases} 1 \text{ if Fedex satisfies order i} \\ 0 \text{ otherwise} \end{cases}$$

$$\mathrm{Ui} = \begin{cases} 1 \text{ if USPS satisfies order i} \\ 0 \text{ otherwise} \end{cases}$$

$$\mathrm{Si} = \begin{cases} 1 \text{ if UPS satisfies order i} \\ 0 \text{ otherwise} \end{cases}$$

Ti = Current arrival time of the truck at node i (24hr-format)

Objective Function:

Min $\sum\sum cd_{ij} . Y_{ij} + \sum (C_{iF} . F_i) + \sum (C_{is} . S_i) + \sum (C_{iu} . U_i)$

Subject to the following **constraints:**

$\sum Y_{ij} + F_i + S_i + U_i = 1$ ∀i belongs to N / {t}

$\sum Y_{ji} + F_i + S_i + U_i = 1$ ∀i belongs to N / {s}

$T_j \geq T_i + t_{ij} - 24(1 - Y_{ij})$

$l_i \leq t_i$

$t_i \leq u_i$

$F_{26} + S_{26} + U_{26} = 0$

$F_{27} + S_{27} + U_{27} = 0$

Y_{ij} , F_i , S_i , U_i are binary 0,1 and T_i is non-negative

Interpretation of the Objective function

Min $\sum cd_{ij}. Yij + \sum (C_{iF} . F_i) + \sum (C_{is} . S_i) + \sum (C_{iu} . U_i)$

- $\sum cd_{ij} .Y_{ij}$ signifies that if an order is delivered using truck, then the cost of delivery is the amount of dollars charged per unit of distance between source i and destination j.
- $\sum (C_{iF} . F_i)$ signifies that if an order is delivered using FedEx services then the cost of delivery is the amount of dollars charged by FedEx for each order.
- $\sum (C_{is} . S_i)$ signifies that if an order is delivered using UPS services then the cost of delivery is the amount of dollars charged by UPS for each order.
- $\sum (C_{iu} . U_i)$ signifies that if an order is delivered using USPS services then the cost of delivery is the amount of dollars charged by USPS for each order.

Interpretation of the constraints

$\sum Y_{ij} + F_i + S_i + U_i = 1$ ∀i belongs to N / {t}

$\sum Y_{ji} + F_i + S_i + U_i = 1$ ∀i belongs to N / {s}

- These constraints specify that each order must be delivered using either truck, Fedex, UPS or USPS. We exclude nodes t and s for "all arcs coming in/out" respectively.
-

$T_j \geq T_i + t_{ij} - 24(1 - Y_{ij})$

- This constraint specifies that if an order is delivered using truck then the time for the delivery of the next order should be greater or equal than the addition of the current delivery time and time required to travel from current delivery location to the next delivery location. If truck is not used, the constraint is always met.

$l_i \leq t_i$

$t_i \leq u_i$

- These constraints specify that each order must be delivered within the acceptable time windows for each specific order.

$F_{26} + S_{26} + U_{26} = 0$

$F_{27} + S_{27} + U_{27} = 0$

- Given that nodes 26 and 27 were created as the starting and point of the truck, they cannot be satisfied by a shipping company.

Solution and Code formulation

Once the objective function, decision variables, constants and constraints were identified, Xpress optimization software was used to solve the analytical expression previously presented.

<u>Code description</u>

The following steps are performed in Xpress to solve the given optimization business problem

1. Parameters are declared to store information present in the data file. For each column in data a separate parameter is defined
2. Variables were declared to store information
   a. Y(i,j): Binary variable to tell whether arc i->j is present in the tour
   b. F(i),U(i),S(i): Binary variable to denote whether the given order is served by FedEx, UPS and USPS
   c. T(i): Continuous variable to define the time at which each order is delivered
3. Data was loaded by first converting orders.xls file into a data.txt file and then the information was loaded into each parameter using array assignment
4. We calculated the distance matrix from the x-y coordinates available in the data
5. Convert the distance matrix into time matrix
6. Define all the constraints
7. Define the objective function and then call the minimization function on the objective function and then print the objective function followed by running the optimization for 30 minutes.

<u>Code used in Xpress</u>

The code used to solve the problem is shown in the next page.

```
!@encoding CP1252
model FinalProject
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

!optional parameters section
parameters
!  SAMPLEPARAM1='c:\test\'
!  SAMPLEPARAM2=false
   PROJECTDIR='' ! for when file is added to project
end-parameters

!sample declarations section
declarations ! under this section we declare the variables

 node = 1..27                               ! Constant: Number of persons/projects
  M = 123456789

        ! parameters
orderID: array(node) of string
  x: array(node) of integer
  y: array(node) of integer
  TW_a: array(node) of integer
  TW_b: array(node) of integer
  Cf: array(node) of integer
  Cs: array(node) of integer
  Cu: array(node) of integer
  d: array(node,node) of real
  t: array(node,node) of real ! this time is how much time it takes to go from i to j and is calculated


   ! variables
  F: array(node) of mpvar
  S: array(node) of mpvar
  U: array(node) of mpvar
  Y: array(node,node) of mpvar ! wwith mpvar we declare the variable. By default
  ! the variables are continous and non negative
  T: array(node) of mpvar


end-declarations

 forall(i in node)
  forall(j in node)
   Y(i,j) is_binary

forall(i in node)
F(i) is_binary

forall(i in node)
S(i) is_binary

forall(i in node)
U(i) is_binary
```

```
! Read from data file
 declarations
  PP = 7
  NP = 27                                  ! Constant: Number of persons/projects
  P = 1..NP                                ! Set (range) of persons
  W = 1..PP                               ! Set (range) of work projects
  PREF: array(P,W) of integer
 end-declarations

 initializations from 'data1.txt'
  PREF
 end-initializations

forall(i in node)
 x(i)  := PREF(i,1)

forall(i in node)
 y(i)  := PREF(i,2)

forall(i in node)
 TW_a(i)  := PREF(i,3)
forall(i in node)
 TW_b(i)  := PREF(i,4)
forall(i in node)
 Cu(i)  := PREF(i,5)
forall(i in node)
 Cf(i)  := PREF(i,6)
forall(i in node)
 Cs(i)  := PREF(i,7)
```

```
! distance matrix should be 27*27 given that we have included start and finish as nodes 26 and 27
 forall(i in node)
  forall (j in node)
   d(i,j)  := sqrt((x(j)-x(i))^2 + (y(j)-y(i))^2)
! I think we now need to make a time matrix based on the distance and speed of 50
forall(i in node)
  forall (j in node)
   t(i,j)  := d(i,j)/50
   ! objetive
 Obj:= (sum(i in node)sum(j in node) d(i,j)*Y(i,j)) +
 (sum(i in node) Cf(i)*F(i))+(sum(i in node) Cs(i)*S(i))+(sum(i in node) Cu(i)*U(i))
! constraints
! in the first constraint we are making sure that all the arrival nodes are
! either satisfied by truck or by a third party
forall(i in node | i<>27 )sum(j in node | j<>i)Y(i,j)+ F(i) +S(i)+U(i)  = 1
! in the first constraint we are making sure that all the departure nodes are
! either satisfied by truck or by a third party
forall(i in node | i<>26)sum(j in node | j<>i)Y(j,i)+ F(i) + S(i) + U(i)  = 1
! next, we need to make sure that the time of arrival is higher than
! the (time of departure at the previous node plus the transportation time)
forall(i in node)
forall(j in node)
T(j) >=  T(i) + t(i,j) - 24*(1-Y(i,j))
! in this constraint we are making sure that the lower limit is lower than the arrival time
forall(i in node)
TW_a(i)  <= T(i)
! arrival time is greater than the upper limit
 forall(i in node)
T(i)  <= TW_b(i)
```

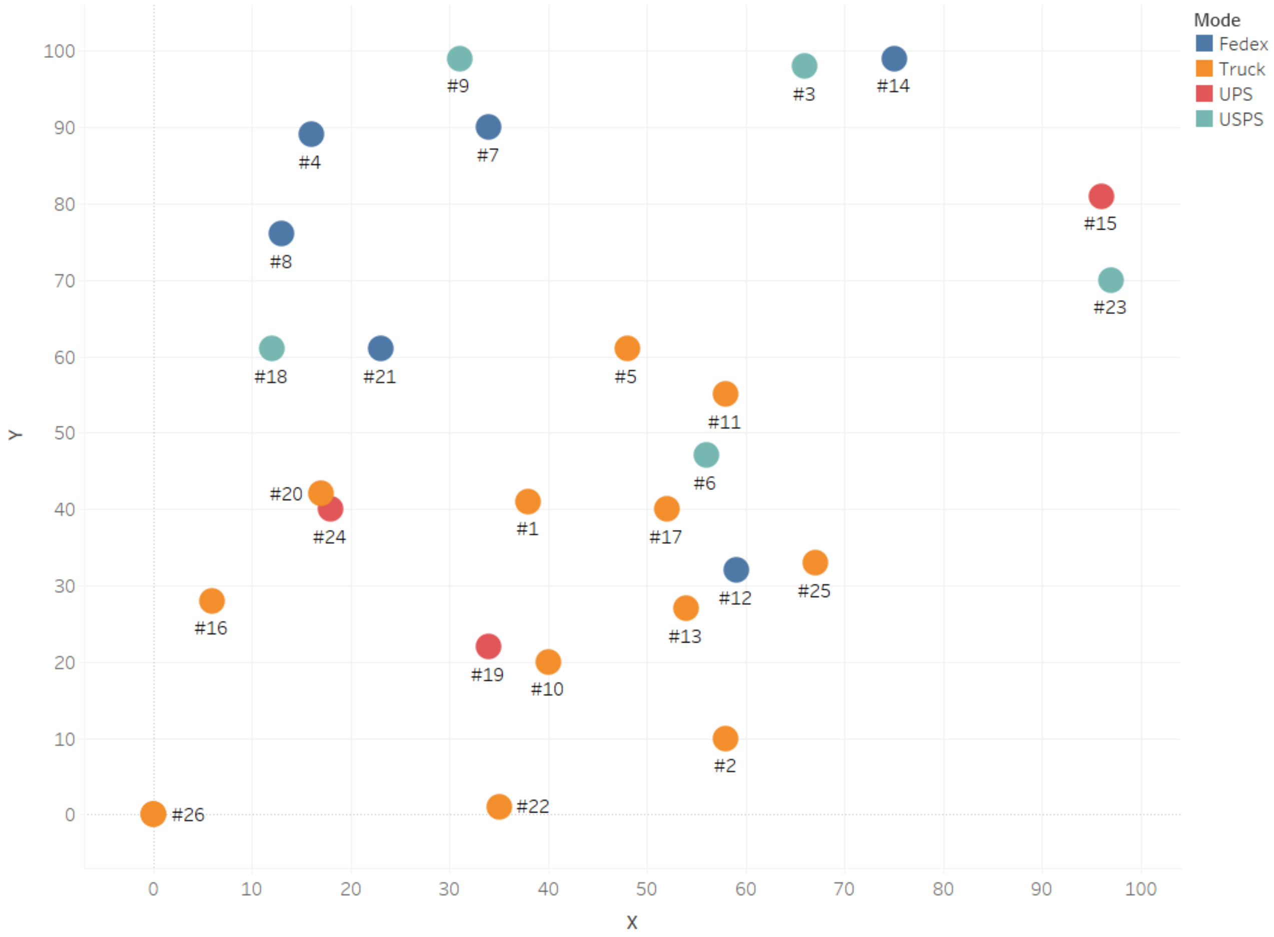
```
! given that we have defined the start as node 26 and end at node 27
! , we just need to make sure that nodes 26 and 27 are not covered by third parties
F(26)+S(26)+U(26)  = 0
F(27)+S(27)+U(27)  = 0
! stoping after 30 minutes
setparam("XPRS_MAXTIME", 1800)
! Solve the problem
minimize(Obj)
! Solution printing


 writeln("Total cost: ", getobjval)

 forall(i in node)
    forall(j in node)
    if(getsol(Y(i,j))=1) then
            writeln(" the arc between ", i, " and ", j, " is covered by truck " )
    end-if
  forall(i in node)
            if(getsol(U(i))=1) then
            writeln(" the shipping company UPS will deliver order ", i )
            end-if
  forall(i in node)
            if(getsol(F(i))=1) then
            writeln(" the shipping company Fedex will deliver order ", i )
            end-if
  forall(i in node)
            if(getsol(S(i))=1) then
            writeln(" the shipping company USPS will deliver order ", i )
            end-if
```

The output obtained from the code for 30 minutes is shown below and it contains the cost of the total distribution, the tour of the truck and what orders are satisfied by each shipping company. The total cost associated with this distribution is **696.2$** and 15 orders are delivered through shipping companies.

```
Total cost: 696.229
 the arc between 1 and 20 is covered by truck
 the arc between 2 and 13 is covered by truck
 the arc between 5 and 1 is covered by truck
 the arc between 10 and 2 is covered by truck
 the arc between 11 and 5 is covered by truck
 the arc between 13 and 25 is covered by truck
 the arc between 16 and 27 is covered by truck
 the arc between 17 and 11 is covered by truck
 the arc between 20 and 16 is covered by truck
 the arc between 22 and 10 is covered by truck
 the arc between 25 and 17 is covered by truck
 the arc between 26 and 22 is covered by truck
 the shipping company UPS will deliver order 15
 the shipping company UPS will deliver order 19
 the shipping company UPS will deliver order 24
 the shipping company Fedex will deliver order 4
 the shipping company Fedex will deliver order 7
 the shipping company Fedex will deliver order 8
 the shipping company Fedex will deliver order 12
 the shipping company Fedex will deliver order 14
 the shipping company Fedex will deliver order 21
 the shipping company USPS will deliver order 3
 the shipping company USPS will deliver order 6
 the shipping company USPS will deliver order 9
 the shipping company USPS will deliver order 18
 the shipping company USPS will deliver order 23
```

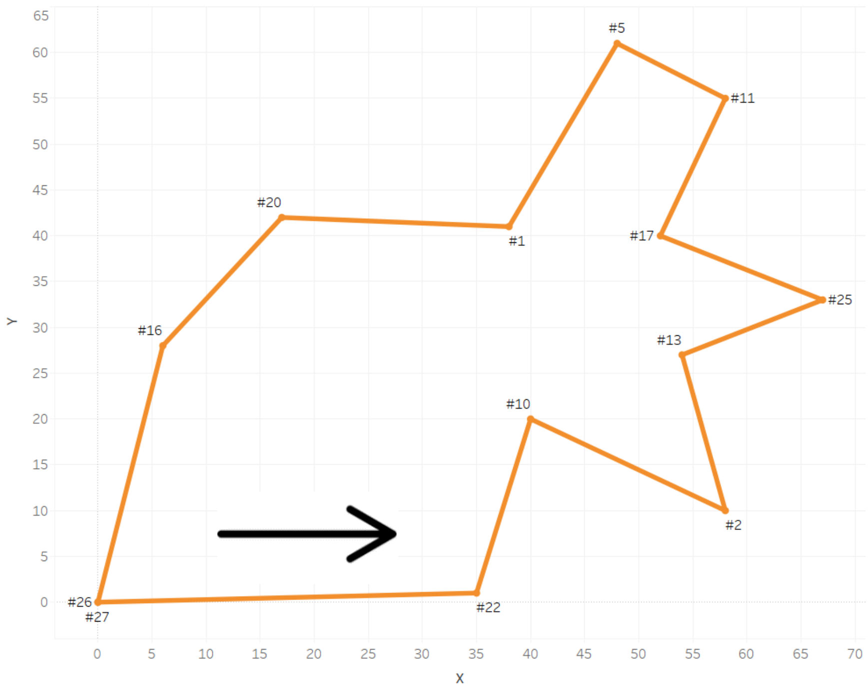
Next, we have made a graphical representation using Tableau where we can see what orders are satisfied by the truck and what orders are satisfied by the shipping companies. In the second plot we can see the tour that the truck follows.

Delivery method for each order



Sum of X vs. sum of Y. Color shows details about Mode. The marks are labeled by Node. Details are shown for Node.

Truck Tour



Sum of X vs. sum of Y. Color shows details about Mode. The marks are labeled by Node. Details are shown for Node. The view is filtered on Mode, which keeps Truck.

Alternative Scenarios

In order to prove the validity of the code and the analytical expression, we have considered 3 different scenarios that would allow us to make sure the solutions obtained are right.

<u>Scenario A: Truck travels at 80 units/hr</u>

We therefore change the speed of the truck from 50 units per hour to 80 units per hour. The following change is done in the code to alter the speed.

```
! New scenario it goes at 80 miles per hour instead of 50 miles/hr
forall(i in node)
  forall (j in node)
   t(i,j) := d(i,j)/80
```

Logically, since the truck travels at a faster speed, we expect that the cost is less our equal than in our original scenario since the number of order satisfied by truck will be higher or equal than in the original scenario.

As we can see in this new scenario, the cost is reduced to **569.1$** and only 2 orders are satisfied by shipping companies.

```
Total cost: 569.113
 the arc between 1 and 16 is covered by truck
 the arc between 2 and 13 is covered by truck
 the arc between 3 and 14 is covered by truck
 the arc between 4 and 7 is covered by truck
 the arc between 5 and 11 is covered by truck
 the arc between 6 and 17 is covered by truck
 the arc between 7 and 3 is covered by truck
 the arc between 8 and 4 is covered by truck
 the arc between 10 and 19 is covered by truck
 the arc between 11 and 20 is covered by truck
 the arc between 12 and 2 is covered by truck
 the arc between 13 and 10 is covered by truck
 the arc between 14 and 23 is covered by truck
 the arc between 16 and 27 is covered by truck
 the arc between 17 and 5 is covered by truck
 the arc between 18 and 8 is covered by truck
 the arc between 19 and 1 is covered by truck
 the arc between 20 and 24 is covered by truck
 the arc between 21 and 18 is covered by truck
 the arc between 22 and 6 is covered by truck
 the arc between 23 and 25 is covered by truck
 the arc between 24 and 21 is covered by truck
 the arc between 25 and 12 is covered by truck
 the arc between 26 and 22 is covered by truck
 the shipping company UPS will deliver order 15
 the shipping company USPS will deliver order 9
```

<u>Scenario B: The truck now charges $3 for each unit of distance traveled in the delivery tour</u>

We keep the speed as 50 and change the cost from 1$ per unit of distance to 3 per unit of distance.

The following change is done in the code to alter the cost. We are just changing the cost of the truck in the objective function to 3.

```
  ! objetive with cost 3
 Obj:= (sum(i in node)sum(j in node) 3*d(i,j)*Y(i,j)) +
 (sum(i in node) Cf(i)*F(i))+(sum(i in node) Cs(i)*S(i))+(sum(i in node) Cu(i)*U(i))
```

Since the cost of the truck increases, we now expect that the cost of this scenario is higher than the original cost, and that more orders are satisfied by shipping companies.

As we can see in the new output, the cost has increased to 805$ and the track is not satisfying any order, since it's now cheaper to meet all of the demand through shipping companies.

```
Total cost: 805
 the arc between 26 and 27 is covered by truck
 the shipping company UPS will deliver order 15
 the shipping company UPS will deliver order 16
 the shipping company UPS will deliver order 19
 the shipping company UPS will deliver order 24
 the shipping company UPS will deliver order 25
 the shipping company Fedex will deliver order 1
 the shipping company Fedex will deliver order 4
 the shipping company Fedex will deliver order 5
 the shipping company Fedex will deliver order 7
 the shipping company Fedex will deliver order 8
 the shipping company Fedex will deliver order 10
 the shipping company Fedex will deliver order 12
 the shipping company Fedex will deliver order 14
 the shipping company Fedex will deliver order 21
 the shipping company USPS will deliver order 2
 the shipping company USPS will deliver order 3
 the shipping company USPS will deliver order 6
 the shipping company USPS will deliver order 9
 the shipping company USPS will deliver order 11
 the shipping company USPS will deliver order 13
 the shipping company USPS will deliver order 17
 the shipping company USPS will deliver order 18
 the shipping company USPS will deliver order 20
 the shipping company USPS will deliver order 22
 the shipping company USPS will deliver order 23
```

<u>Scenario C: The truck now charges $10 for each unit of distance traveled in the delivery tour</u>

The cost now changes from 1$ per unit of distance to 10$ per unit of distance.

The following change is done in the code to alter the cost. We are just changing the cost of the truck in the objective function to 10

```
  ! objetive using cost of 10
Obj:= (sum(i in node)sum(j in node)10* d(i,j)*Y(i,j))
+ (sum(i in node) Cf(i)*F(i))+(sum(i in node) Cs(i)*S(i))+(sum(i in node) Cu(i)*U(i))
```

Since with a cost of 3$ per unit all the order we already satisfied by shipping companies, we expect the new cost to be of 805$ (same as scenario B) since all the demand will be exclusively covered one more time by shipping companies.

```
Total cost: 805
 the arc between 26 and 27 is covered by truck
 the shipping company UPS will deliver order 15
 the shipping company UPS will deliver order 16
 the shipping company UPS will deliver order 19
 the shipping company UPS will deliver order 24
 the shipping company UPS will deliver order 25
 the shipping company Fedex will deliver order 1
 the shipping company Fedex will deliver order 4
 the shipping company Fedex will deliver order 5
 the shipping company Fedex will deliver order 7
 the shipping company Fedex will deliver order 8
 the shipping company Fedex will deliver order 10
 the shipping company Fedex will deliver order 12
 the shipping company Fedex will deliver order 14
 the shipping company Fedex will deliver order 21
 the shipping company USPS will deliver order 2
 the shipping company USPS will deliver order 3
 the shipping company USPS will deliver order 6
 the shipping company USPS will deliver order 9
 the shipping company USPS will deliver order 11
 the shipping company USPS will deliver order 13
 the shipping company USPS will deliver order 17
 the shipping company USPS will deliver order 18
 the shipping company USPS will deliver order 20
 the shipping company USPS will deliver order 22
 the shipping company USPS will deliver order 23
```

.