

# CS436/CS580L Homework 1

Fall 2018, Xi Peng

## Instruction:

(a) Release: Sep 16, 2018.

**(b) Due: Oct 7, 2018.**

(c) 10 points for each question.

(d) All Bishop exercises should follow the Bishop Link on the course

website: <https://sites.google.com/site/xipengcshomepage/teaching/fall2018>

## Probability:

### Question 1.

- (a) Suppose that A is an event such that  $P(A) = 0$  and that B is any other event. Prove that A and B are independent events.
- (b) Soldier A and Soldier B are practicing shooting. The probability that A would miss the target is 0.2 and the probability that B would miss the target is 0.5. The probability that both A and B would miss the targets is 0.1.
  - What is the probability that at least one of the two will miss the target?
  - What is the probability that exactly one of the two soldiers will miss the target?
- (c) A box contains three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. Then we randomly select one card from this box, and we can know the color of the selected cards upper side. If this side is green, what is the probability that the other side of the card is also green?

**Question 2.** Bishop Exercises 1.11;

**Question 3.** Bishop Exercises 1.13;

## Linear Algebra:

**Question 4.** Bishop Exercises 1.14;

## Expectation:

### Question 5.

- (a) If an integer between 100 and 200 is to be chosen at random, what is the expected value?
- (b) A rabbit is playing a jumping game with friends. She starts from the origin of a real line and moves along the line in jumps of one step. For each jump, she flips a coin. If heads, she would jump one step to the left (i.e. negative direction). Otherwise, she would jump one step to the right. The chance of heads is  $p$  ( $0 \leq p \leq 1$ ). What is the expected value of her position after  $n$  jumps? (assume each step is in equal length and assume one step as one unit on the real line)
- (c) Suppose that the random variable  $X$  has a uniform distribution on interval  $[0, 1]$ . Random variable  $Y$  has a uniform distribution on the interval  $[4, 10]$ .  $X$  and  $Y$  are independent. Suppose a rectangle is to be constructed for which the lengths of two adjacent sides are  $X$  and  $Y$ . What is the expected value of the area of this rectangle?

(d) Suppose that  $X$  is a random variable. if  $E(X) = \mu$ ,  $\text{Var}(X) = \sigma^2$ , then what is the value of  $E[X(X - 1)]$ ?

**Question 6.** Bishop Exercises 2.8;

**Question 7.** Bishop Exercises 2.12;

### Normal Distribution:

#### Question 8.

Suppose  $X$  has a normal distribution with mean 1 and variance 4. Find the value of the following:

-  $P(X \leq 3)$

-  $P(|X| \leq 2)$

### Bayesian Inference:

**Question 9.** In a certain day care class, 30 percent of the children have grey eyes, 50 percent of the children have blue eyes, and the other 20 percent eyes are in other colors. One day they play a game together. In the first run, 65 percent of the grey eye kids were selected into the game, 82 percent of the blue eye kids selected in, and 50 percent of the kids with other colors were chosen. If a child is selected at random from the class, and we know that he was not in the first run game, what is the probability that he has blue eyes?

(Provide detailed solving process to get the final value)

**Question 10.** Given following data records:

Tid	Home	Marital Status	Taxable Income	Cheat (Lable)
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

(a) Predict the class label of the instance  $X = (\text{Home}=\text{Yes}, \text{Married}, \text{Income}=70\text{K})$

(b) Predict the class label of the instance  $X = (\text{Home}=\text{No}, \text{Single}, \text{Income}>80\text{K})$

(Provide detailed solving process to get the final result)