(1) By using the curve interpolation method, we use a cubic parametric polynomial ($f(u) = au^3 + bu^2 + cu + d$) to fit four control points P0, P1, P2, and P3. This cubic polynomial curve passes through the four control points P0, P1, P2 and P3 when parameter u=0, 1/3, 2/3, 1 respectively. Find a matrix M which satisfies the condition f(u) = UMP
(where U= $\begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$, P= $\begin{bmatrix} P0 & P1 & P2 & P3 \end{bmatrix}^T$, f(u) is a vector which has three components (x(u), y(u), z(u)); Each control point Pi $(i = 0,1,2,3)$ is a vector representing x, y, z coordinate as (Pi_x, Pi_y, Pi_z)).

Answer: $M=\begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{1}{27} & \frac{1}{9} & \frac{1}{3} & 1 \\ \frac{8}{27} & \frac{4}{9} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}$

P0= d

P1 = (1/27)a + (1/9)b + (1/3)c +d

P2 = (8/27) a + (4/9)b + (2/3)c + d

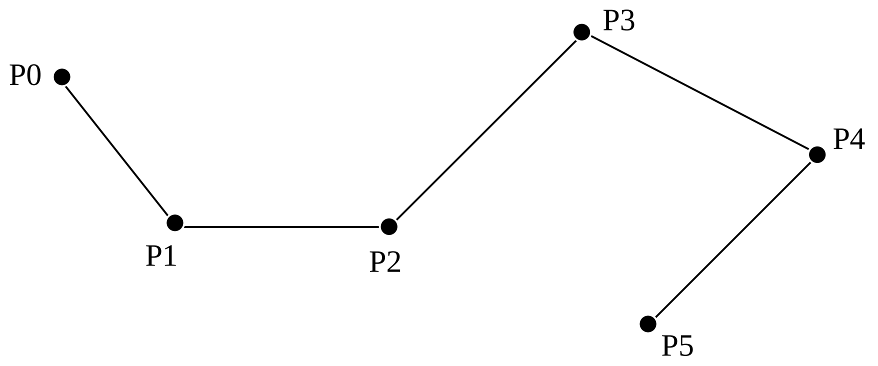
P3 = a + b + c +d

So : $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{1}{27} & \frac{1}{9} & \frac{1}{3} & 1 \\ \frac{8}{27} & \frac{4}{9} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} p0 \\ p1 \\ p2 \\ p3 \end{bmatrix}$

$$f(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

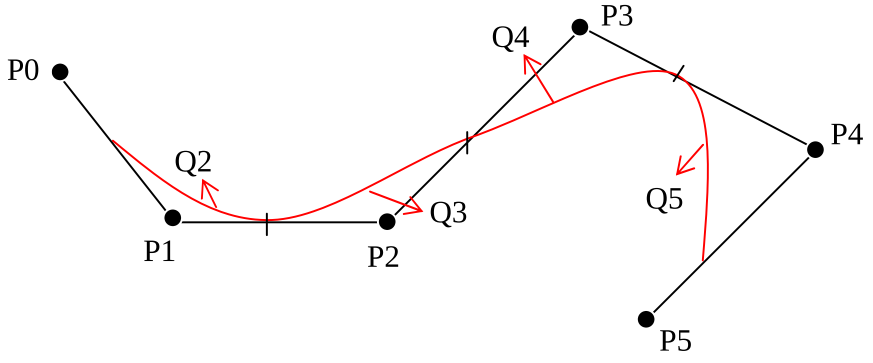
(2) The uniform B-spline curves can be used to approximate the curve segments which are determined by a number of control points. Given six control points as shown below, use the B-spline curves to approximate the six control points in the following case: the degree of the B-spline polynomial is 2.

Indicate how many curve segments there are in this case;
Sketch the B-Spline curves on the following graph and label the curve segments.



Answer:

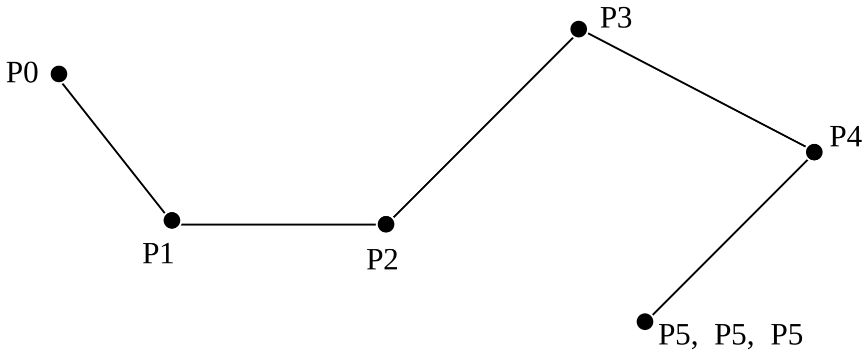
(a) 4 curve segments (see red curves Q2, Q3, Q4, Q5)



(3) If the control point P5 is repeated as shown below. P5 will be used more than once in the evaluation for one curve segment. If we use the uniform cubic B-spline curves to fit the control points: p0, p1, p2, p3, p4, p5, p5, p5.

(a) Indicate the range of the knot values (i.e., 0, … ,m).

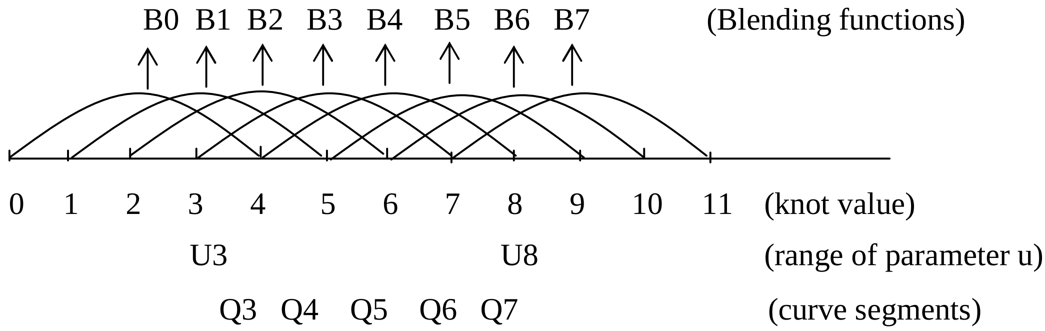
(b) Plot the blending functions within the knot value range (0, … ,m).
(c) Sketch the uniform cubic B-spline curves to fit the control points: p0, p1, p2, p3, p4, p5, p5, p5. Label all the curve segments explicitly.



Answer:

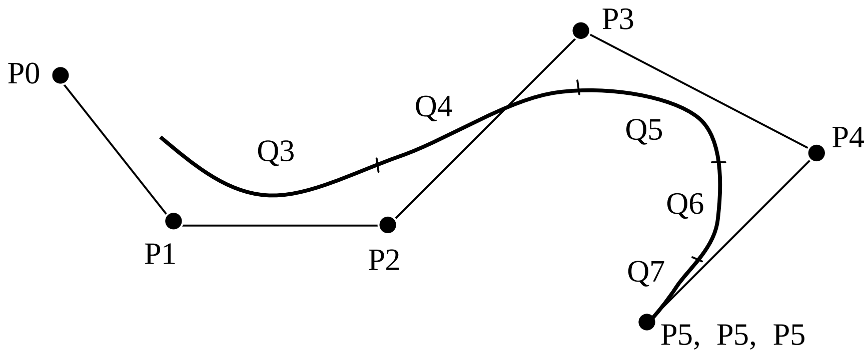
(a) 0, 1, …, m=11;
Total: 8 + 4 = 12 (or 7 + 4 +1 =12)
knot values: 0, 1, …, 11
(number of control point: 0, 1, .., 7), n=7
(degree of polynomial is 3, so d = 3+1 =4)

(b) m= 11
control points: p0, p1, p2, p3, p4, p5, p6, p7
(p5), (p5)
the B-spline curve is defined only in the range from knot value d-1 =3 to n+1 =8



Note: p6=p5, p7=p5

(c)



Note: curve Q7 passes through p5 point at u = U8

Because:

P(u) = B7(u) * p5 + B6(u)*p5 + B5(u)*p5 + B4(u)*p4
B4(u) = 0 at u = U8
So: P(u) = p5 * (B7(u) + B6(u) + B5(u) + B4(u)) at u=U8
B7(u) + B6(u) + B5(u) + B4(u) =1
So: P(u) = p5