PERMUTATION AND COMBINATION

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1) How many words can be formed by re-arranging the letters of the word ASCENT such that A and T occupy the first and last position respectively.

a.) 5! b.) 4! c.) 6! - 2! d.) $\frac{6!}{2!}$

A _ _ _ _ T

4! ways

2.) There are 2 brothers among a group of 20 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two brothers?

a.) 2 * 19! b.) 18! * 18 c.) 19! * 18 d.) 2 * 18!

20 person → 2 Brother
20 person → 18 person → 1 must at middle → 17 person B/w Brothers.

B_1 M B_2

No. of ways to choose 1 person To sit at middle = $^{18}C_1$

No. of ways of arranging 2 Brothers = 2!

No. of ways of Remaing 17 person = 17!

$$^nC_r = \frac{n!}{(n-r)!\,(r)!}$$

$$17! \times 2! \times {}^{18}C_1$$

$$17! \times 2! \times \frac{18!}{17! \times 1!}$$

$$\frac{17! \times 2! \times 18 \times 17!}{17! \times 1}$$

$$= 18 \times 17! \times 2!$$

$$= 18! \times 2!$$

$$= 18! \times 2$$

3.) There are 12 Yes (or) No questions. How many ways can these be answered.

a.) 1024 b.) 2048 c.) 4096 d.) 144

Each question has 2 option and totally 12 question

$2^n \Rightarrow n = 12 \Rightarrow 2^{12} = 4096$

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4.) How many ways can 4 prizes be given away to 3 boys, if each boy is eligible for all the prizes?

a.) 256 b.) 12 c.) 81 d.) None of these.

4 prizes ⟶ 3 boys

3^4 ways = 81 ways

5.) A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. In how many can they travel?

a.) 9 b.) 26 c.) 126 d.) 3920

car 1 = 5 person

car 2 = 4 person

Totally = 8 person.

1st car = $8C_5 = \frac{8!}{5! \times 3!} = \frac{5! \times 6 \times 7 \times 8}{5! \times 6} = 56$ ways

2nd car = $8C_4 = \frac{8!}{4! \times 4!} = \frac{5 \times 6 \times 7 \times 8 \times 4!}{4! \times 4!} = \frac{5 \times 6 \times 7 \times 8}{4 \times 4} = 70$ ways

56 ways + 70 ways = 126 ways.

6.) How many numbers are there between 100 and 1000 such that at least one of their digit is 6?

a.) 648 b.) 258 c.) 654 d.) 252

⇒ 3 digit number

⇒ 4 digit number

0 - 9 ⇒ 10 numbers

Numbers b/w 1000 to 100 = 900

900 - 648 = 252 ways

8 9 9

0 - 9 ⇒ 9 ways

Remove 6

Remove 1 and 6

Numbers do not have 6

8 × 9 × 9 = 648 ways.

7.) How many ways can 10 letters be posted in 5 boxes, if each of the post boxes can take more than 10 letters?

a.) 5^{10} b.) 10^5 c.) $10P_5$ d.) $10C_5$

Each of the Box can have 10 letter capacity.

Totally 5 boxes

5^{10} ways

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8.) In how many ways can the letters of the word Education be be arranged so that the relative position of the vowels and consonants remains the same as in the word Education?

a.) $\frac{9!}{4}$ b) $\frac{9!}{4! * 5!}$ c.) $4! * 5!$ d.) None of These.

1	2	3	4	5	6	7	8	9
✓		✓		✓		✓	✓	
E	D	U	C	A	T	I	O	N

5 VOWELS AND 4 CONSONANT

9.) In how many ways can 15 people be seated around two round tables with seating capacities of 7 and 8 people.

a.) $\frac{15!}{8!}$ b) $7! * 8!$ c.) $(15C_8) * 6! \times 7!$ d.) $2 * (15C_7) * 6! * 7$

$(n-1)!$ → for circle

Table 1 = 7 people

$n=7$ $(7-1)! \Rightarrow 6!$

$15C_7 \Rightarrow$ 7 people sit

Remaining 8 people in table 2

$(8-1)!$

$7!$

$15C_7 \times 6! \times 7!$

Table 2 = 8 people

$n=8$

$(8-1)! = 7!$

$15C_8$ → 8 people sit

Remaining 7 people in table 2

$(7-1)!$

$6!$

$15C_8 \times 6! \times 7!$

Both are correct

10.) If the letters of the word CHASM are ~~are~~ rearranged to form 5 letter words such that none of the word repeat and the results arranged in ascending order as in a dictionary what is the rank of the word CHASM?

a.) 24 b) 31 c.) 32 d.) 30

Totally 5! ways are possible = 120 ways.

5 letter word is Rearranged in Ascending order.

$\frac{120}{5} = 24$ ways.

A = 24 ways
C = 24 ways
H = 24 ways
M = 24 ways
S = 24 ways.

25th word start with CA then 3! ways possible. = 6 ways

31st word start with CHA then 2! ways

then

24 + 6 + 2 ways ⇒ 32 ways.

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11.) How many words of 4 consonants and 3 vowels can be made from 12 consonants and 4 vowels, if all the letters are different.

$^{12}C_4 \times {}^4C_3 \times (4+3)!$ $\quad {}^nC_r = \frac{n!}{(n-r)! \times (r)!}$

$\frac{12!}{8! \times 4!} \times \frac{4!}{3! \times 1!} \times 7!$ → Totally 7 words.

$\frac{8! \times 9 \times 10 \times 11 \times 12}{8! \times 24} \times \frac{3! \times 4}{3! \times 1} \times 7!$

$495 \times 4 \times 7!$

a.) $^{16}C_7 \times 7!$
b.) $^{12}C_4 \times {}^4C_3 \times 7!$
c.) $^{12}C_3 \times {}^4C_4$
d.) $^{12}C_4 \times {}^4C_3$

12.) In how many ways can 5 letters be posted in 3 post boxes, if any number of letters can be posted in all of the three post boxes.

a.) $^5C_3$ b.) $^5P_3$ c.) 5^3 d.) 3^5

3 → Boxes
5 → letters.

13.) How many number of times will the digit '7' be written when listing the integers from 1 to 1000?

a.) 271 b.) 300 c.) 252 d.) 304

I.) Hundred's position =
700 → 799 → 100

II.) Tens' position
70 - 79 ⇒ 10
170 - 179 ⇒ 10
⋮
970 - 979 ⇒ 10
→ 10 × 10 = 100

III.) Ones
1 → 1000 ⇒
0 → 9 ⇒ 1
10 → 19 ⇒ 1
⋮
990 → 999 ⇒ 1
→ 10 × 10 = 100

⇒ 100 + 100 + 100 = 300

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14) There are 6 boxes numbered 1, 2, ... 6. Each box is to be filled up either with a red (or) a green ball in such a way that atleast 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

a) 5 b) 21 c) 33 d) 60

If Each Box has 1 green ball = 6 Boxes

If Box has 2 green ball but numbered of Box is consequtively = (1,2) (2,3) (3,4) (4,5) (5,6) = 5 Boxes

If Box has 3 green ball but numbered of Box is consequtively (1,2,3) (2,3,4) (3,4,5) (4,5,6) = 4 Boxes

If Box has 4 green ball but numbered of Box is consequtively (1,2,3,4) (2,3,4,5) (3,4,5,6) = 3 boxes.

If Box has 5 green ball but numbered of Box is consequtively (1,2,3,4,5) (2,3,4,5,6) = 2 boxes

If Box has 6 (all 5 are) green ball but numbered of Box is consequtively (1,2,3,4,5,6) = 1 Box.

6B + 5B + 4B + 3B + 2B + 1B = 21 Boxes

15) What is the value of $1*1! + 2*2! + 3*3! + \ldots + n*n!$, where n! means n factorial or $n(n-1)(n-2)\ldots 1$

a) $n(n-1(n-1!))$

b) $(n+1!) / (n(n-1))$

c) $(n+1! - n!)$

d) $(n+1! - 1!)$