A Time Delay Interferometry Rosetta Stone

Michele Vallisneri

Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109[*]

This table compares and interfaces all the notations used for TDI observables in the literature.

reference	geometry	inter-spacecraft measurement	intra-spacecraft measurement (on same bench of y with same indexes)	link vectors	armlengths
Vallisneri 2005		y_{slr} (frequency)	z_{slr}	n_l ($l > 0$ ccw, $l < 0$ cw)	L_l ($l > 0$ ccw, $l < 0$ cw)
AET 1999			n/a		
TAE 2000, ETA 2000, AET 2001 Hogan & Bender 2001, Prince et al. 2002, TDI whitepaper 2002, AET 2003	$1 \to 2 \to 3$ cw	$y_{\lvert l\rvert r}$ (frequency)	$z_{\lvert l\rvert r}$	$n_{\lvert l\rvert}$ (along ccw)	$L_{\lvert l\rvert}$ (no distinction between ccw and cw)
TEA 2002, Tinto et al. 2003		$s_{\lvert l\rvert r}$ (phase)	$\tau_{\lvert l\rvert r}$ (phase)		
Dhurandhar et al 2002		$U_1 = y_{3\text{-}21}$, $U_2 = y_{1\text{-}23}$ $U_3 = y_{2\text{-}13}$, $V_1 = \text{-}y_{231}$ $V_2 = \text{-}y_{312}$, $V_3 = \text{-}y_{123}$		$n_{\lvert l\rvert}$ (along ccw)	$L_{\lvert l\rvert}$ (no distinction between ccw and cw)
Cornish & Rubbo 2003, Rubbo, Cornish, & Poujade 2004		Φ_{sr} (phase)	n/a	$r_{sr} = n_{sr}$	
Hellings 2001	$1 \to 2 \to 3$ ccw	y_{sr} (phase)		n/a	L_{sr} (oriented)
Cornish & Hellings 2003		y_{sr} (frequency)			
Królak et al. 2004				$n_{\lvert l\rvert}$ (along ccw)	L (equal arms)
Shaddock et al. 2003[a], TEA 2004[b]	$1 \to 2 \to 3$ cw	s_{sr} (phase)	τ_{sr} (phase)	n/a	$L_{\lvert l\rvert} = L_{\text{-}l}$, $L_{\lvert l\rvert'} = L_l$ (unprimed cw, primed ccw)
Shaddock 2004					L_{sr} (oriented)

The TDI Rosetta Table. A comparison of the phase-measurement and LISA geometry conventions used in the literature on TDI. In the cited references, A, E, and T refer to J. W. Armstrong, F. B. Estabrook, and M. Tinto. Notations are described with respect to the usage of this paper, with $s \equiv$ sending spacecraft, $l \equiv$ armlink, $r \equiv$ receiving spacecraft; "cw" and "ccw" refer to the progression of spacecraft or link indexes, as seen when looking at the LISA constellation from above (from ecliptic latitude 90° N); when indexes are shown in absolute values, only positive values are used. Tinto and Armstrong 1999 (not included in this table) has $y_1 \equiv$ two-way ccw ($a \to b \to a$), $y_2 \equiv$ two-way cw ($a \to c \to a$).
[a] The semicolon ordered-delay notation was introduced in Shaddock et al. 2003 and TEA 2004.
[b] TEA 2004 uses $n_{\lvert l\rvert}$ to denote link vectors; it is ambiguous from the context whether these are ccw or cw.

[*]Electronic address: Michele.Vallisneri@jpl.nasa.gov

[1] J. W. Armstrong, F. B. Estabrook, and M. Tinto, *Astrophys. J.* **527**, 814 (1999).
[2] J. W. Armstrong, F. B. Estabrook, and M. Tinto, *Class. Quant. Grav.* **18**, 4059 (2001).
[3] J. W. Armstrong, F. B. Estabrook, and M. Tinto, *Class. Quant. Grav.* **20**, S283 (2003).
[4] N. Cornish and R. W. Hellings, *Class. Quant. Grav.* **20**, 4851 (2003).
[5] N. J. Cornish and L. J. Rubbo, *Phys. Rev. D* **67**, 022001 (2003); erratum, *ibid.*, 029905 (2003).
[6] S. V. Dhurandhar, K. R. Nayak, and J.-Y. Vinet, *Phys. Rev. D* **65**, 102002 (2002).
[7] F. B. Estabrook, M. Tinto, and J. W. Armstrong, *Phys. Rev. D* **62**, 042002 (2000).
[8] R. W. Hellings, *Phys. Rev. D* **64**, 022002 (2001).
[9] C. J. Hogan and P. L. Bender, *Phys. Rev. D* **64**, 062002 (2001).
[10] A. Królak, M. Tinto, and M. Vallisneri, *Phys. Rev. D* **70**, 022003 (2004).
[11] T. A. Prince, M. Tinto, S. L. Larson, and J. W. Armstrong, *Phys. Rev. D* **66**, 122002 (2002).
[12] L. J. Rubbo, N. J. Cornish, and O. Poujade, *Phys. Rev. D* **69**, 082003 (2004). See also LISA Simulator v. 2.0, `www.physics.montana.edu/lisa`.
[13] D. A. Shaddock, *Phys. Rev. D* **69**, 022001 (2004).
[14] D. A. Shaddock, M. Tinto, F. B. Estabrook, and J. W. Armstrong, *Phys. Rev. D* **68**, 061303(R) (2003).
[15] M. Tinto and J. W. Armstrong, *Phys. Rev. D* **59**, 102003 (1999).
[16] M. Tinto, J. W. Armstrong, and F. B. Estabrook, *Phys Rev. D* **63**, 021101(R) (2000).
[17] M. Tinto, F. B. Estabrook, and J. Armstrong, "Time Delay Interferometry White Paper", whitepaper for the LISA Mission Science Office (May 2002), `www.srl.caltech.edu/lisa`.
[18] M. Tinto, F. B. Estabrook, and J. W. Armstrong, *Phys. Rev. D* **65**, 082003 (2002).
[19] M. Tinto, F. B. Estabrook, and J. W. Armstrong, *Phys. Rev. D* **69**, 082001 (2004).
[20] M. Tinto, D. A. Shaddock, J. Sylvestre, and J. W. Armstrong, *Phys Rev. D* **67**, 122003 (2003).
[21] M. Vallisneri, *Phys. Rev. D*, in print (2005); `gr-qc/0407102`.