"Dragonplate" tube, estimated lamina properties from micromechanics

Fiber and Matrix Properties (INPUT)

	Property	Source	
Modulus	$E_{1f} := 34 \cdot 10^6 \text{psi}$	DragonPlate Data Sheet	NOTE THIS FIBER IS LIKELY T650 OR VERY SIMILAR
	$E_{2f} := 2.1 \cdot 10^6 \text{psi}$	By analogy to T650 fibers	
	$E_m := 4.4\text{GPa}$	Kollar&Springer. Approx value for structural epoxies	
Poisson Ratio	$\nu_{12f} := 0.27$	By analogy to T650 fibers	
	$\nu_{23f} := 0.74$	By analogy to P-30X fibers	
	$\nu_m := 0.35$	Kollar&Springer. Approx value for structural epoxies	
Density	$\rho_f := 1770 \frac{\text{kg}}{\text{m}^3}$	By analogy to T650 fibers	
	$\rho_m := 1260 \frac{\text{kg}}{\text{m}^3}$	Kollar&Springer. Approx value for structural epoxies	
CTE	$\alpha_{1f} := -0.6 \cdot \frac{10^{-6}}{\text{K}}$	By analogy to T650 fibers	
	$\alpha_{2f} := 9 \cdot \frac{10^{-6}}{\text{K}}$	By analogy to T650 fibers	
	$\alpha_m := 61 \cdot \frac{10^{-6}}{\text{K}}$	By analogy to EX-1515	
WEAVE	weave_reduction_factor := 5%	Braided shape affects stiffness in-plane somewhat, don't know how much. For weaves this value typically 5-15%, depends on crimp.	

Prepreg / Cure Properties (INPUT)

$M_m := 50\%$ matrix mass fraction, from "Wf" value in DragonPlate datasheet

$V_v := 1.0\%$ void fraction, 0.5% to 2% is typical, depending on processing

$\text{FAW} := 150 \frac{\text{gm}}{\text{m}^2}$ fiber areal weight. No indication of this in DragonPlate datasheet, but value at left is plausible. See note on cured ply thickness (CPT) below. Better if you can get FAW info from DragonPlate.

Ply Properties (CALCULATED, NO BLEED)

$$V_f := \frac{1 - V_v}{1 + \frac{\rho_f}{\rho_m} \cdot \frac{M_m}{1 - M_m}} = 41.2 \cdot \%$$

$$V_m := 100\% - V_f - V_v = 57.8 \cdot \%$$

$$\text{CPT\_estimate} := \frac{\text{FAW}}{\rho_f \cdot V_f} = 206 \cdot \mu\text{m}$$

note, this "cured ply thickness" estimate would agree with the 0.040" thickness of dragonplate tube, when modeled as equivalent to 5 uni plies [-45/+45/0/+45/-45]

Fiber and Matrix Shear Moduli (CALCULATED)

$$G_{12f} := \frac{E_{1f}}{2\cdot\left(1+\nu_{12f}\right)} = 92.3\cdot GPa \qquad G_{23f} := \frac{E_{2f}}{2\cdot\left(1+\nu_{23f}\right)} = 4.2\cdot GPa \qquad G_m := \frac{E_m}{2\cdot\left(1+\nu_m\right)} = 1.6\cdot GPa$$

Lamina Properties (CALCULATED, per Kollar & Springer)

$$E_{11} := (1 - \text{weave\_reduction\_factor})\cdot\left(E_{1f}\cdot V_f + E_m\cdot V_m\right) = 94.1\cdot GPa$$

$$E_{22} := \left[\frac{\sqrt{V_f}}{E_{2f}\sqrt{V_f} + E_m\cdot\left(1-\sqrt{V_f}\right)} + \frac{1-\sqrt{V_f}}{E_m}\right]^{-1} = 7.1\cdot GPa$$

$$E_{33} := E_{22} = 7.1\cdot GPa$$

$$G_{12} := \left[\frac{\sqrt{V_f}}{G_{12f}\sqrt{V_f} + G_m\cdot\left(1-\sqrt{V_f}\right)} + \frac{1-\sqrt{V_f}}{G_m}\right]^{-1} = 4.3\cdot GPa$$

$$G_{23} := \left[\frac{\sqrt{V_f}}{G_{23f}\sqrt{V_f} + G_m\cdot\left(1-\sqrt{V_f}\right)} + \frac{1-\sqrt{V_f}}{G_m}\right]^{-1} = 2.4\cdot GPa$$

$$G_{13} := G_{12} = 4.3\cdot GPa$$

$$\nu_{12} := \nu_{12f}\cdot V_f + \nu_m\cdot V_m = 0.314$$

$$\nu_{23} := \frac{E_{22}}{2\cdot G_{23}} - 1 = 0.484$$

$$\nu_{13} := \nu_{12} = 0.314$$

Density

$$\rho := \rho_f\cdot V_f + \rho_m\cdot V_m = 1457\frac{kg}{m^3}$$

Thermal Properties

$$\alpha_1 := \frac{V_f\cdot E_{1f}}{E_{11}}\cdot\alpha_{1f} + \frac{V_m\cdot E_m}{E_{11}}\cdot\alpha_m = 1.03\times10^{-6}\,\frac{1}{K}$$

$$\alpha_2 := V_f\cdot\alpha_{2f} + V_m\cdot\alpha_m + V_f\cdot\nu_{12f}\cdot\left(\alpha_{1f}-\alpha_1\right) + V_m\cdot\nu_m\cdot\left(\alpha_m-\alpha_1\right) = 5.09\times10^{-5}\,\frac{1}{K}$$

$$\alpha_3 := \alpha_2 = 5.09\times10^{-5}\,\frac{1}{K}$$

Approximations of In-Plane Properties

$$p := \left(1 - \nu_{12}{}^{2} \cdot \frac{E_{22}}{E_{11}}\right)^{-1} = 1.007$$

$$Q_{11} := p \cdot E_{11} = 9.48 \times 10^{10}\,\text{Pa} \qquad Q_{66} := G_{12} = 4.336 \times 10^{9}\,\text{Pa}$$

$$Q_{22} := p \cdot E_{22} = 7.171 \times 10^{9}\,\text{Pa} \qquad Q_{12} := p \cdot \nu_{12} \cdot E_{22} = 2.249 \times 10^{9}\,\text{Pa}$$

$$\theta := \begin{pmatrix} -45 \\ 45 \\ 0 \\ 45 \\ -45 \end{pmatrix}^{\circ}$$

$$a := \cos(\theta)$$

$$b := \sin(\theta)$$

$$Qbar_{11} := \overrightarrow{\left[a^{4} \cdot Q_{11} + b^{4} \cdot Q_{22} + 2 \cdot a^{2} \cdot b^{2} \cdot Q_{12} + a^{2} \cdot b^{2} \cdot (4 \cdot Q_{66})\right]}$$

$$Qbar_{22} := \overrightarrow{\left[b^{4} \cdot Q_{11} + a^{4} \cdot Q_{22} + 2 \cdot a^{2} \cdot b^{2} \cdot Q_{12} + a^{2} \cdot b^{2} \cdot (4 \cdot Q_{66})\right]}$$

$$Qbar_{12} := \overrightarrow{\left[a^{2} \cdot b^{2} \cdot Q_{11} + a^{2} \cdot b^{2} \cdot Q_{22} + \left(a^{4} + b^{4}\right) \cdot Q_{12} - a^{2} \cdot b^{2} \cdot (4 \cdot Q_{66})\right]}$$

$$Qbar_{66} := \overrightarrow{\left[\left[4 \cdot a^{2} \cdot b^{2} \cdot Q_{11} + 4 \cdot a^{2} \cdot b^{2} \cdot Q_{22} - 8 \cdot a^{2} \cdot b^{2} \cdot Q_{12} + \left(a^{2} - b^{2}\right)^{2} \cdot (4 \cdot Q_{66})\right] \cdot \frac{1}{4}\right]}$$

$$Qbar_{16} := \overrightarrow{\left[\left[2 \cdot a^{3} \cdot b \cdot Q_{11} - 2 \cdot a \cdot b^{3} \cdot Q_{22} - 2 \cdot \left(a^{3} \cdot b - a \cdot b^{3}\right) \cdot Q_{12} - \left(a^{3} \cdot b - a \cdot b^{3}\right) \cdot (4 \cdot Q_{66})\right] \cdot \frac{1}{2}\right]}$$

$$Qbar_{26} := \overrightarrow{\left[\left[2 \cdot a \cdot b^{3} \cdot Q_{11} - 2 \cdot a^{3} \cdot b \cdot Q_{22} + 2 \cdot \left(a^{3} \cdot b - a \cdot b^{3}\right) \cdot Q_{12} + \left(a^{3} \cdot b - a \cdot b^{3}\right) \cdot (4 \cdot Q_{66})\right] \cdot \frac{1}{2}\right]}$$

$$A\_t_{11} := \frac{\sum Qbar_{11}}{\text{length}(\theta)} = 43.7 \cdot \text{GPa} \qquad A\_t_{12} := \frac{\sum Qbar_{12}}{\text{length}(\theta)} = 18.28 \cdot \text{GPa}$$

$$A\_t_{22} := \frac{\sum Qbar_{22}}{\text{length}(\theta)} = 26.2 \cdot \text{GPa} \qquad A\_t_{66} := \frac{\sum Qbar_{66}}{\text{length}(\theta)} = 20.36 \cdot \text{GPa}$$

$$\Delta := A\_t_{11} \cdot A\_t_{22} - A\_t_{12}{}^{2}$$

$$\boxed{QUASI\_E_{1} := \frac{\Delta}{A\_t_{22}} = 31.0 \cdot \text{GPa}} \qquad QUASI\_E_{1} = 4.49 \times 10^{6} \cdot \text{psi}$$

$$\boxed{QUASI\_E_{2} := \frac{\Delta}{A\_t_{11}} = 18.6 \cdot \text{GPa}} \qquad QUASI\_E_{2} = 2.69 \times 10^{6} \cdot \text{psi}$$

$$\boxed{QUASI\_\nu_{12} := \frac{A\_t_{12}}{A\_t_{22}} = 0.698}$$

$$\boxed{QUASI\_G_{12} := A\_t_{66} = 20.4 \cdot \text{GPa}} \qquad QUASI\_G_{12} = 2.95 \times 10^{6} \cdot \text{psi}$$

Matt, here are some sample calcs

$$D := \begin{pmatrix} 1.5 \\ 2 \\ 3.0 \end{pmatrix} \text{in} \qquad t := \begin{pmatrix} 0.040 \\ 0.045 \\ 0.085 \end{pmatrix} \text{in} \qquad r := \frac{D}{2}$$

MASS AND TWIST

$$L := 18\text{in}$$

$$J := \frac{\pi}{32} \cdot \left[D^4 - (D - 2t)^4 \right] = \begin{pmatrix} 0.098 \\ 0.264 \\ 1.655 \end{pmatrix} \cdot \text{in}^4$$

$$\boxed{\text{mass} := \rho\,\pi \cdot \left[r^2 - (r - t)^2 \right] \cdot L = \begin{pmatrix} 0.079 \\ 0.119 \\ 0.335 \end{pmatrix} \text{kg}}$$

$$\tau := 3100\text{in} \cdot \text{lbf}$$

$$\boxed{\text{inertia} := \frac{1}{2} \cdot \overrightarrow{\left[\text{mass} \cdot \left[r^2 + (r - t)^2 \right] \right]} = \begin{pmatrix} 27.1 \\ 73.3 \\ 459.0 \end{pmatrix} \text{kg} \cdot \text{mm}^2}$$

$$\gamma := \frac{\tau \cdot r}{J \cdot \text{QUASI\_G}_{12}} = \begin{pmatrix} 0.008 \\ 0.004 \\ 0.001 \end{pmatrix}$$

$$\boxed{\text{twist} := \frac{\gamma \cdot L}{r} = \begin{pmatrix} 11.06 \\ 4.10 \\ 0.65 \end{pmatrix} \cdot \text{deg}}$$

SOME QUICK STRAIN CHECKS
(far from complete failure analysis, but showing a couple of key checks)

$$\varepsilon_{1ut} := \frac{600\text{ksi}}{E_{1f}} = 0.018$$

Calculating from dragonplate datasheet stated fiber strength

$$\boxed{\text{FOS\_45deg\_plies} := \frac{\varepsilon_{1ut}}{\gamma} = \begin{pmatrix} 2.19 \\ 4.44 \\ 18.55 \end{pmatrix}}$$

Factor of safety against tensile failure of the 45 degree plies

$$\gamma_{12u} := 0.013$$

from Dharan reader

$$\boxed{\text{FOS\_0deg\_plies} := \frac{\gamma_{12u}}{\gamma} = \begin{pmatrix} 1.62 \\ 3.27 \\ 13.66 \end{pmatrix}}$$

Factor of safety against shearing the zero degree ply