The Newton-Raphson Method

Finite Element Method [304.007]

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Contents

Exercise 1: Finding roots of nonlinear scalar functions

This exercise demonstrates how important a good initial guess is for the convergence of the Newton-Raphson method.

Exercise Find the roots of the two functions below. Try different initial guesses x_0. Give good and bad examples for the initial guess for both functions.

a) $f(x) = \arctan(x) \qquad x \in [-4, 4]$

b) $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x^2 - 1 \qquad x \in [-1.5, 4.5]$

You can use the Script `newton_scalar_with_plot.m` as a template for getting a feeling for the function and good initial guesses. This key uses the `pause` command, so you have to click your left mouse button or press $\boxed{\rightarrow}$ to proceed one step.
After finding out a good initial guess, you can then use the script `newton_scalar` as a template for doing the computation with checking convergence.

Exercise 2: Finding roots of nonlinear equation systems

The Newton-Raphson method can be used for solving nonlinear equation systems. In this example, we will calculate the positions and weights of the Gauß integration points. We choose two integration points in the interval $[-1, 1]$. This leads to the following nonlinear equation system:

$$\begin{aligned} w_1 + w_2 &= 2 \\ \xi_1 w_1 + \xi_2 w_2 &= 0 \\ \xi_1^2 w_1 + \xi_2^2 w_2 &= \frac{2}{3} \\ \xi_1^3 w_1 + \xi_2^3 w_2 &= 0\,. \end{aligned}$$

Mind the following:

- You can use the script `newton_scalar.m` as a template.
- The unknown variables are stored in the vector $\boldsymbol{x} = \begin{bmatrix} w_1 & w_2 & \xi_1 & \xi_2 \end{bmatrix}^{\mathrm{T}}$.
- The equation system must be rearranged in order to have the form $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}$.

Give one example each for a bad and a good initial guess.
Hints:

- Replace the function handle for evaluating the value by a function file with a vector of size (4,1) as input and a vector of size (4,1) as output.
- Replace the function handle for evaluating the derivative matrix (= Newton tangent) by a function file with a vector of size (4,1) as input and a matrix of size (4,4) as output. This matrix must contain all partial derivatives.
- Define the initial conditions in a vector $\boldsymbol{x}_0$ of size (4,1).

Conclusion

In continuum mechanics, we want to solve nonlinear partial differential equations with the FE method. Typically, in each step of the Newton-Raphson method, a very large linear equation system has to be solved. A good initial guess and the correct computation of the tangent (derivative matrix) is essential for keeping the computing time as short as possible.

Flowchart of the Newton-Raphson Method

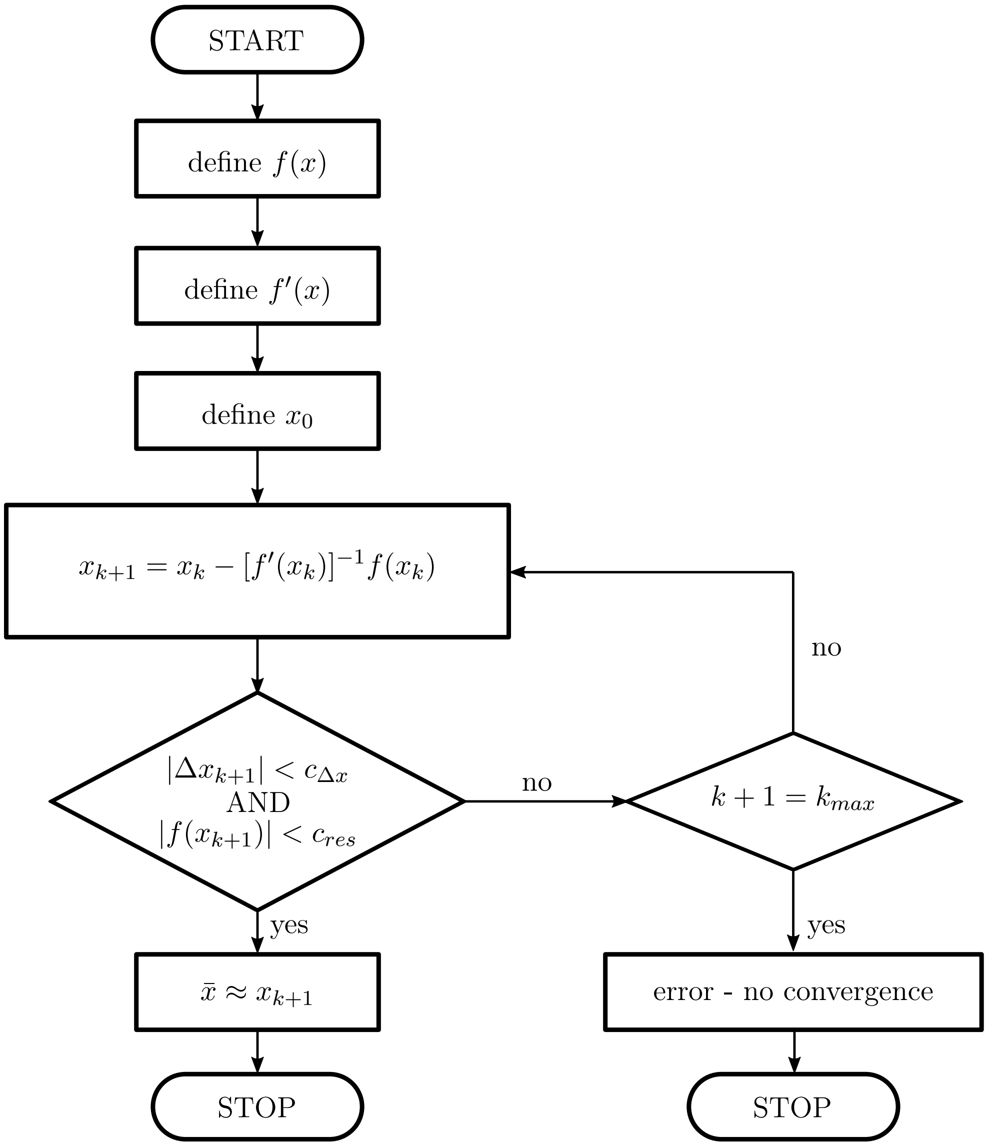

Figure 1: Flowchart of Newton-Raphson method