

Finite Element Method, Unit 5

Newton-Raphson in FEM

Time loop \rightarrow increase Dirichlet & Neumann BC
step by step

in each time step do one NR iteration

$$W(x, \vec{v}) \stackrel{!}{=} 0$$

FOR $k = 1: \text{max-newton-steps}$

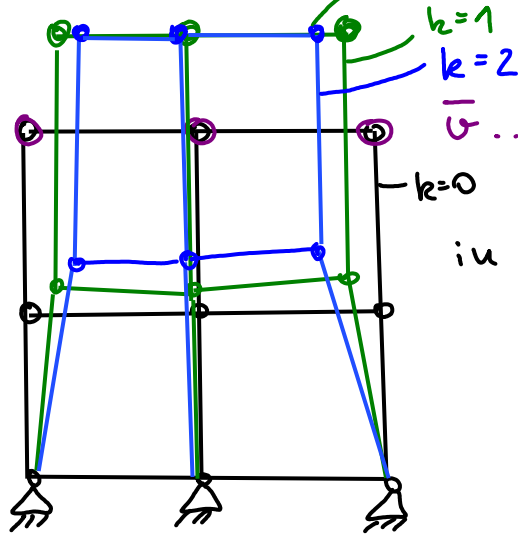
$$W(x_k, \vec{v}) + \underbrace{DW(x_k, \vec{v})[\Delta \vec{u}_{k+1}]}_{\text{linear in } \Delta \vec{u}_{k+1}} \stackrel{!}{=} 0$$

$$DW(x_k, \vec{v})[\Delta \vec{u}_{k+1}] = -W(x_k, \vec{v})$$

FEM \rightarrow

$$\underbrace{A_{ijkl}}_{\substack{\checkmark \\ \text{Newton-Tangent}}} \underbrace{\Delta u_{kl}}_{\text{increment}} = - \underbrace{F_{ij}}_{\text{Residue}}$$

Dirichlet - BC



DRB fulfilled exactly!

$k=1$
 $k=2$

\bar{v} ... vertical displacement prescribed

$k=0$

in each time step:

- set BC \bar{v} at beginning of time step
- perform first NR - step
- set DRB to zero after first NR - step