Finite Element Method, Unit 6

Pressure Boundary Conditions

$$W(\chi,\vec{v}) = \int_V \underline{\underline{F}}\,\underline{\underline{S}} : \delta\underline{\underline{F}}\, dV = 0 \qquad \delta F^i{}_j = v^i{}_{,j}$$

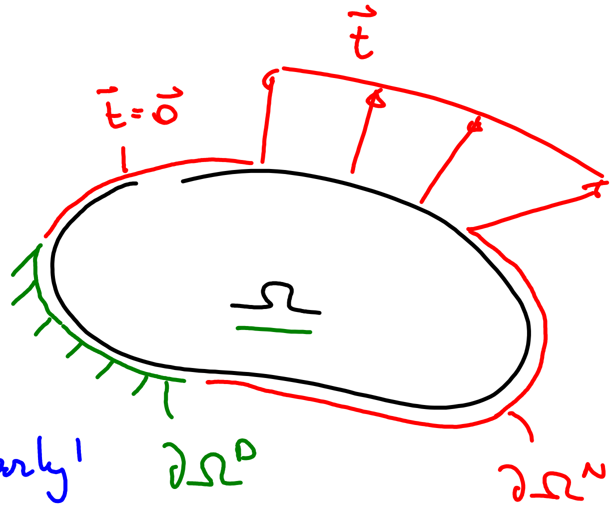
$$W(\chi_k + \Delta\vec{u}_{k+1}, \vec{v}) \approx W(\chi_k,\vec{v}) + DW(\chi_k,\vec{v})[\Delta\vec{u}_{k+1}] \stackrel{!}{=} 0$$

$$DW(\chi_k,\vec{v})[\Delta\vec{u}_{k+1}] = -W(\chi_k,\vec{v})$$

$$\xrightarrow{\text{FEM}} \quad A_{ijkl}\,\Delta\hat{u}_{kl} = F_{ij}$$

External virtual work

$$\int_\Omega \sigma_{ij}\, v^i{}_{,j}\, dv = \int_{\partial\Omega^N} t_i\, v_i\, dA$$



pressure: $\vec{t} = -p\,\vec{n}$

depends on χ nonlinearly!

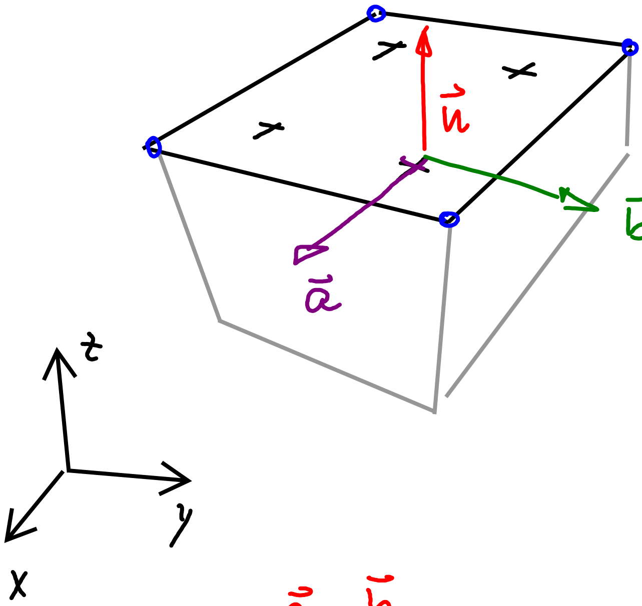
$$W(\chi,\vec{v}) = \underbrace{\int_{\Omega^k} \underline{\underline{F}}\,\underline{\underline{S}} \cdot \delta\underline{\underline{F}}\, dV}_{W^{int}} + \underbrace{\int_{\partial\Omega^k} p\,\vec{v}\cdot\vec{n}\, dA}_{W^{ext}} \stackrel{!}{=} 0$$

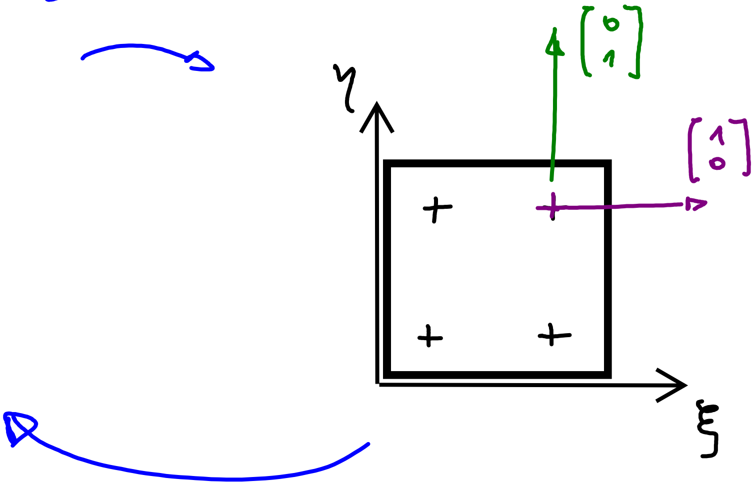
$$W = W^{int} + W^{ext} \;\rightarrow\; DW^{int} + DW^{ext} = -(W^{int} + W^{ext})$$

$$\xrightarrow{\text{FEM}} \quad A_{ijkl}\,\Delta\hat{u}_{kl} = F_{ij}$$

Normal vector $\vec{n}$

$$\vec{\xi} = t(\vec{x})$$





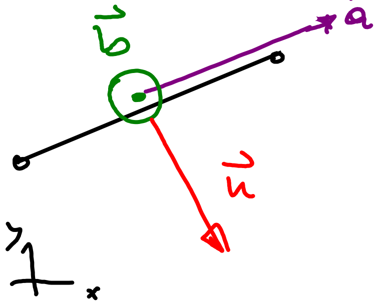
$$\vec{x} = t^{-1}(\vec{\xi}) = \underline{\underline{\hat{x}}}\;\underline{N}$$

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{\|\vec{a} \times \vec{b}\|}$$

$$J^{-1} = \underline{\underline{\hat{x}}}\; d\underline{N}$$

$$J^{-1} = \left[\underbrace{\frac{\partial \vec{x}}{\partial \xi}}_{\vec{a}} \;\; \underbrace{\frac{\partial \vec{x}}{\partial \eta}}_{\vec{b}} \right] \qquad \det J^{-1} = \left\| \frac{\partial \vec{x}}{\partial \xi} \times \frac{\partial \vec{x}}{\partial \eta} \right\| = \|\vec{a} \times \vec{b}\|$$

2D : $\vec{a} = \begin{bmatrix} \partial x/\partial\xi \\ \partial y/\partial\xi \\ 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$W^{ext}(x, \vec{v}) = \int_{\partial\Omega^k} \rho \vec{v} \cdot \vec{n}\, dA = \int_{\partial\Omega^{ref}} \rho \vec{v} \frac{\vec{a} \times \vec{b}}{\cancel{\det(J^{-1})}} \underbrace{\cancel{\det J^{-1}}\, dA^{ref}}_{dA}$$

$$= \int_{\partial\Omega^{ref}} \rho \vec{v} \cdot (\vec{a} \times \vec{b})\, dA$$

$$\vec{a} = J^{-1}(:,1)$$

$$= \int_{\partial\Omega^{ref}} \rho v^j \varepsilon_{jkl} a_k b_l\, dA$$

$$\vec{b} = J^{-1}(:,2)$$

$$F^{ext}_{ij} = \int_{\partial\Omega^{ref}} p \, N^j \, \varepsilon_{ikl} \, a_k b_l \, dA^{ref}$$

$$\Lambda^{ext}_{ijkl} = \int_{\partial\Omega^{ref}} p \, N^j \, \varepsilon_{imn} \left(N^l_{,1} \, b_n \, \delta_{km} + N^l_{,2} \, a_m \, \delta_{kn} \right) dA^{ref}$$