Numerical Simulation and Modelling of Incompressible Flow

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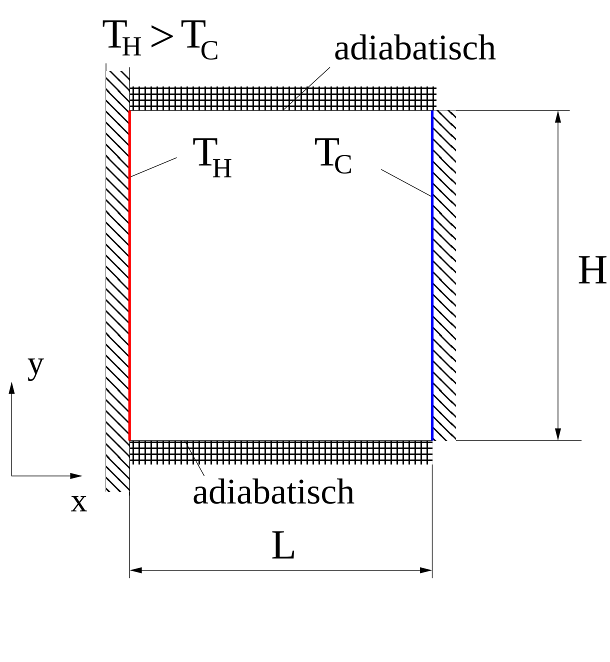
Natural convective flow

Consider the incompressible laminar flow inside an infinitely long cavity with rectangular cross section. The motion is driven by buoyancy, due to temperature-dependent change of density met with the considered non-isothermal flow conditions. The cooled right side wall is kept at given constant wall temperature T_C, while the heated left wall is kept on constant wall temperature T_H. The upper and lower walls are thermally isolated (adiabatic). All material properties, such as kinematic viscosity ν, thermal diffusivity a, and thermal expansion coeffficient $\beta = -\frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial T}\Big|_p$ are assumed constant and given. The aspect ratio be $H/L = 2$. Assuming the so called Boussinesqu approximation, the problem is governed by the following equations:

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_M) \\
\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)
\end{aligned}
$$

with $\quad T_M = (T_H + T_C)/2$



Tasks:

- Nondimensionalize the equations using L, $V_C = \sqrt{g\beta(T_H - T_C)L}$, $\bar{\rho}V_C^2$, and $T_H - T_C$ as reference quantities for length, velocity, pressure, and temperature, respectively. The obtained non-dimensional formulation should solely depend on the parameters

$$Re = V_C L/\nu, \quad Pr = \nu/a$$

- Finite-Volume-Discretization second order in space and time.
For enforcing continuity the Poisson equation for the pressure should be solved with an indirect (iterative) solver.

- Simulate the following cases until a converged (stationary) state is reached:

  i) $Re = 100$, $Pr = 1$
  ii) $Re = 100$, $Pr = 10$
  iii) $Re = 200$, $Pr = 1$

- Evaluate the Nusselts number for all cases:

$$Nu = \frac{Q_w L}{\lambda(T_H - T_C)H}, \quad \text{with} \quad Q_w = -\int_0^H \lambda \left.\frac{\partial T}{\partial x}\right|_{x=0} dy$$

- Documentation of the work in a report. As for the results, show streamlines, contours of u, v and T.