TD n°1

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1 Automata

Exercise 1.1 – Let $\mathcal{A}$ an automata, give an automata $\mathcal{A}_*$ such that $(L_\mathcal{A})^* = L_{\mathcal{A}_*}$

Correction 1.1

Let $\mathcal{A} = (Q, \Sigma, \delta, I, F)$, we want to construct $\mathcal{A}_*$:

- We add all initial states to the finite states for recognizing the empty word.
- And we define δ'

$$\delta' = \begin{cases} q, a & \rightarrow & \delta(q,a) & a \in \Sigma, q \in Q \\ f, \varepsilon & \rightarrow & I & \forall f \in F \end{cases}$$

So at the end we have $\mathcal{A}_* = (Q, \Sigma, \delta', I, F \cup I)$

Exercise 1.2 – Show that $(a|b)^* = (a^*b^*)^*$

Correction 1.2

We will proceed by double inclusion :

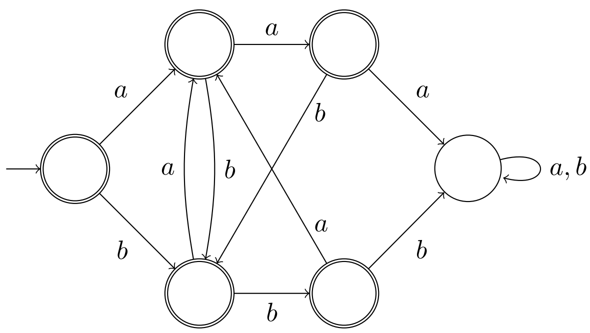
$\subseteq$: we will proof this inclusion by induction on the length of $w \in (a|b)^*$.

- If $|w| = 0$, so $w = \varepsilon \in (a^*b^*)^*$
- By induction hypothesis we know that all words w of the length n is in $(a^*b^*)^*$. We have two case for the words of length $n+1$:
  - $w \cdot a$, $a \in (a|b)$ so by induction hypothesis we have $w \cdot a \in (a|b)^*(a|b) = (a|b)^*$.
  - $w \cdot b$ same.

$\supseteq$: We know that $(a|b)^* = \Sigma^*$ so all words $w \in (a^*b^*)^*$ is on $(a|b)^*$ because $w \in \Sigma^*$

Exercise 1.3 – Give a DFA on $\Sigma = \{a, b\}$ recognizing all words having no more than two consecutive occurrences of the same letter.

Correction 1.3



Exercise 1.4 – We define the duplication of a word by : $dd(\varepsilon) = \varepsilon$ and $\forall x \in \Sigma : dd(v \cdots x) = dd(v) \cdots xx$.
Example : on $\Sigma = \{a, b\}$, $dd(aba) = aabbaa$. We suppose that L is a rational language.
Show that $dd(L) = \{dd(v) | v \in L\}$ is rational to.

Correction 1.4

Let $\mathcal{A}_L = \{Q, \Sigma, \delta, I, F\}$ an automaton who recognize L.
We construct $\mathcal{A}_{dd(L)} = \{Q', \Sigma, \delta', I, F'\}$ the automaton who recognize $dd(L)$:

- $Q' = Q \uplus Q = \{(i, q) | q \in Q, i \in \{0, 1\}\}$
- $F' = \{(q, 1) | q \in F\} \cup \{(q, 0), | q \in I \cap F\}$
- $$\delta' = \begin{cases} (q,1), c & \rightarrow & \delta(q,c) & c \in \Sigma \\ (q,0), c & \rightarrow & (q,1) & c \in \Sigma \end{cases}$$

Exercise 1.5 – Given an automaton $\mathcal{A}$, give a (non-deterministic) automaton recognizing L^R the mirror language of $L_{\mathcal{A}}$.

Correction 1.5

Let $\mathcal{A} = \{Q, \Sigma, \delta, I, F\}$ an automaton who recognize L. So we have $\{Q, \Sigma, \delta', F, I\}$ with :

$$\delta'(q, x) = \{q' | q \in \delta(q', x)\}$$

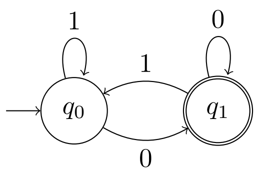
an automaton who recognize L^R.

Exercise 1.6 – Let $\Sigma = \{0, 1\}$ an alphabet.

- We consider a language L_2 the set of binary words representing a multiple of two. This language is recognizable ?
- Same question for L_3 the set of binary words representing a multiple of three.
- What about the L_6 language for binary words representing a multiple of 6 ?

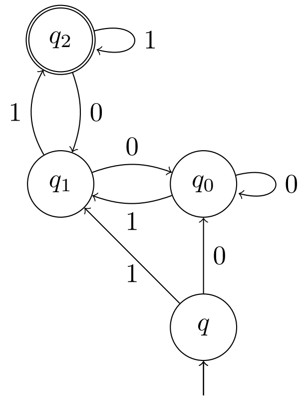
Correction 1.6

- For L_2 we just need to recognize the words which end in 0 :



- For L_3 we have 3 ($n \bmod 3 \in \{0, 1, 2\}$).
  For each edge (k the number read) :
  - If we read a zero, new result : $2k$
  - If we read a zero, new result : $2k + 1$

state	read	next
q_0	0	$0 \times 2 \bmod 3 = 0$
	1	$0 \times 2 + 1 \bmod 3 = 1$
q_1	0	$1 \times 2 \bmod 3 = 2$
	1	$1 \times 2 + 1 \bmod 3 = 0$
q_2	0	$2 \times 2 \bmod 3 = 1$
	1	$2 \times 2 + 1 \bmod 3 = 2$



- For L_6, we can use the same construction (7 states).

Exercise 1.7 – A Dyck language is the set D of well-parenthesized words on an alphabet $\{\mathtt{(}, \mathtt{)}\}$. For example, the word `(()())(()(()))` is well parenthesized.

This property can be formally defined :

- For any prefix u of w, the number of) in u is less than the number of (
- There are as many (as there are) in the word

Show that D is not a regular language.

Correction 1.7

Assume that D is regular.
We have the word $w = \mathtt{(}^p\mathtt{)}^p$ with $p \leq 1$. We pose $x = \varepsilon$, $y = \mathtt{(}^p$ and $z = \mathtt{)}^p$.
The conditions are well verified : $|xy| \leq p$ and $|y| \geq 1$.
If we consider the word xy^2z. This word has $2p$ `(` and p `)`. So xy^2z is not in D.
By the pumping lemma D is not regular.