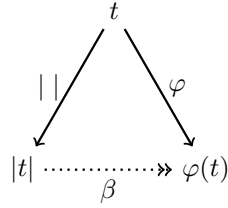


λ-calculus

Valeran MAYTIE

Lemma – $\forall t, |t| \rightarrow_{\beta}^* \varphi(t)$



Proof: We will show our property by induction on the term t .

- Case $t = x$, $|x| = x = \varphi(x)$, so we have $|t| \rightarrow_{\beta}^0 \varphi(t)$.
- Case $t = \lambda x.t_0$, by the induction hypothesis we know that $|t_0| \rightarrow_{\beta}^* \varphi(t_0)$.

$$\begin{aligned}
 |(\lambda x.t_0)| &= \lambda x.|t_0| \\
 &\rightarrow_{\beta}^* \lambda x.\varphi(t_0) && \text{By induction hypothesis} \\
 &= \varphi(\lambda x.t_0)
 \end{aligned}$$

- Case $t = \underline{\lambda} x.t_0$, by the induction hypothesis we know that $|t_0| \rightarrow_{\beta}^* \varphi(t_0)$.

$$\begin{aligned}
 |(\underline{\lambda} x.t_0)| &= \lambda x.|t_0| \\
 &\rightarrow_{\beta}^* \lambda x.\varphi(t_0) && \text{By induction hypothesis} \\
 &= \varphi(\underline{\lambda} x.t_0)
 \end{aligned}$$

- Case $t = t_1 t_2$ where $t_1 \neq \underline{\lambda} x.t_0$, by the induction hypothesis we know that $|t_1| \rightarrow_{\beta}^* \varphi(t_1)$ and $|t_2| \rightarrow_{\beta}^* \varphi(t_2)$.

$$\begin{aligned}
 |t_1 t_2| &= |t_1| |t_2| \\
 &\rightarrow_{\beta}^* \varphi(t_1) \varphi(t_2) && \text{By induction hypothesis} \\
 &= \varphi(t_1 t_2)
 \end{aligned}$$

- Case $t = (\underline{\lambda} x.t_0) t_1$, by the induction hypothesis we know that $|t_0| \rightarrow_{\beta}^* \varphi(t_0)$ and $|t_1| \rightarrow_{\beta}^* \varphi(t_1)$.

$$\begin{aligned}
 |(\underline{\lambda} x.t_0) t_1| &= (\lambda x.|t_0|) |t_1| \\
 &\rightarrow_{\beta}^* (\lambda x.\varphi(t_0)) \varphi(t_1) && \text{By induction hypothesis} \\
 &\rightarrow \varphi(t_0)\{x \leftarrow \varphi(t_1)\} \\
 &= \varphi((\underline{\lambda} x.t_0) t_1)
 \end{aligned}$$

□