Intro to Quantum Programming & Alogirthms

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1 Introduction

1.1 What is a Quantum systeme

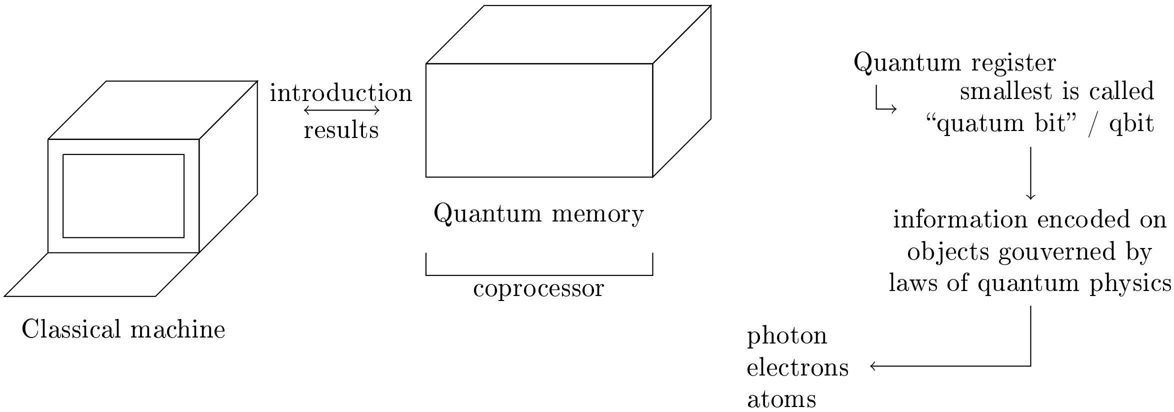


Figure 1: Quantum system diagram

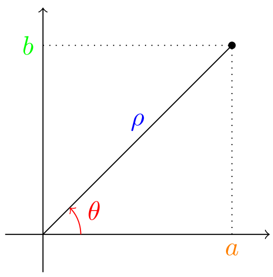
Classically, to encode a <u>bit</u> of information you need :

item an object	coin	magnet
two stats : - dinstinguishable, - that can be set	head / tails	north/south

Quantum : the same !

Object	Photon	electron
pair of stats	polarisation V / H	spin up / down
other pair of states	one photon no photon	
another	in Fiber A in Fiber B	

1.2 Complex numbers



θ : phases
ρ : magnetude

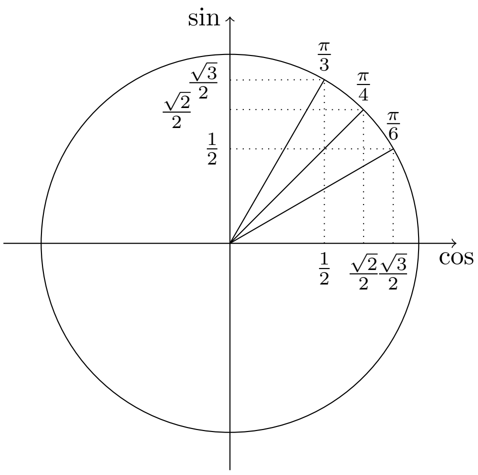
Figure 2: Complex number representation in diagram

$$\begin{aligned} \alpha &= \underbrace{a + b}_{reals} i \\ &= \rho(\cos\theta + i\sin\theta) \\ &= \rho e^{i\theta} \end{aligned}$$

is imaginary number $i^2 = -1$

Absolute value : $|x| = \rho = \sqrt{a^2 + b^2}$
Some properties :

- $\overline{a + bi} = a - bi$
- $\overline{e^x} = e^{\overline{x}}$
- $|\alpha^2| = \rho \times \rho = \alpha \times \overline{\alpha}$



1.3 Hilber Spaces

A state of a quantum system is a vector in a Hilbert Space(Finite dimenssional)
Two states are distinguishable, it means that they are orhtogonal

vector space :

- pick a set (Finite) call it a "a basis" $\mathcal{B} = \{e_i\}_{i \in I}$
- a vector is a linerar combination $\alpha_0 e_0 + \alpha_1 e_1 + \ldots + \alpha_{n-1} e_{n-1}$ $\alpha_i \in \mathcal{I}$
- Scalar product : input 2 vector
  $\langle v|w \rangle = \sum_i \alpha_i \alpha_i'$

Hilbert space is a complex vector space with a scalar product
Let $\mathcal{H}$ be defined by $\{|0\rangle, |1\rangle\}$
A vector in general : $\alpha|0\rangle + \beta|1\rangle$
example of orthogonal vectors :

$$|0\rangle \perp |1\rangle$$
$$|0\rangle + |1\rangle \perp |0\rangle - |1\rangle$$

in a Hilbert space, there is a norm :
$\|v\| = \sqrt{\langle u, v\rangle}$

1.4 Kronecker product

4)
$\mathcal{E}$ and $\mathcal{F}$ two Hilbert spaces
build $\mathcal{E} \otimes \mathcal{F}$
Consider $\mathcal{H} \otimes \mathcal{H}$ a generic elemevector is :
$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
$|00\rangle \perp$ any other basics element
Suspose that $v \perp v'$ what about $v \otimes |0\rangle$ and $v' \otimes |0\rangle$?

1.5 Quantum bit

The state of a quantum bit is a vector in $\mathcal{H} = \{\alpha|0\rangle + \beta|1\rangle\}$

- of norm 1
- modulo a global phase

We say that v and w are relatex by a global phase if exists Θ angle such that $v = e^{i\Theta}w$

$$v = \alpha|0\rangle + \beta|1\rangle$$
$$w = \alpha'|0\rangle + \beta'|1\rangle$$

We can write $v \simeq w$ the state of a qubit is :

- an equivlence class order $\simeq$
- a set of vectors classed under multiplication by a global phase

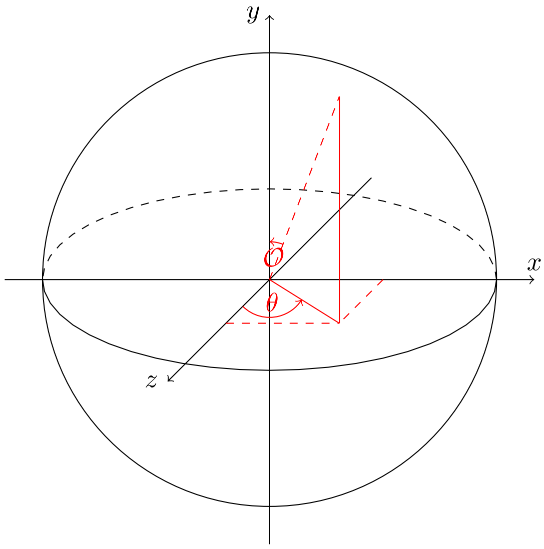
Consider $|0\rangle$, $e^{i\pi/2}|0\rangle$, $-|0\rangle$ are all represent the same qubit because they only differ by an irrelevant global phase.
Each of then is a representative element of the same qubit state.
Consider a qubit in state

$$\begin{aligned}
\alpha|0\rangle + \beta|1\rangle &= \rho_a e^{i\theta_a} + \rho_b e^{i\theta_b}|1\rangle \\
&\simeq e^{-i\theta_a}(\rho_a e^{i\theta_a}|0\rangle + e^{i\theta_b}|1\rangle) \\
&= (\rho_a e^{i\theta_a - i\theta_a}|0\rangle + e^{i\theta_b - i\theta_a}|1\rangle) \\
&= (\rho_a|0\rangle + e^{i\theta}|1\rangle) && \text{with } \theta = \theta_b - \theta_a \\
&= 5)
\end{aligned}$$

a cononical rep element of a qubit state is of the form :

$$cos(O/2)|0\rangle + e^{i\theta} sin(O/2)|1\rangle$$

with $O \in [0, \pi]$ and $\theta \in [0, 2\pi[$



a quantum bit is instanciated by :

- a physical object
- a pair of othogonal states $|0\rangle$: false and $|1\rangle$: true
- a state of the qubit is a superposition of $|0\rangle$ and $|1\rangle$ a linear count of norm 1.

consider the object A and B coding a qubit and another one B coding a qubitstate in $\mathcal{H}$
Now join the 2 systems AB state in $\mathcal{H} \otimes \mathcal{H}$.
classicaly
7)
a 2-qubit system has a state $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
one way to build such a system is to :

- generate two separte states
- join then $(a|0\rangle + b|1\rangle) \otimes (a'|0\rangle + b'|1\rangle) = aa'|00\rangle + ab'|01\rangle + a'b|10\rangle + a'b'|11\rangle$

can I reach $\frac{\sqrt{2}}{2}(|00\rangle + |11\rangle)$? NO ! It is not a separable element.

3-qubit state live in $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$
8) n-qubit