TD n°1

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1 Automata

Exercise 1.1 – Let $\mathcal{A}$ an automata, give an automata $\mathcal{A}_*$ such that $(L_\mathcal{A})^* = L_{\mathcal{A}_*}$

Correction 1.1

Let $\mathcal{A} = (Q, \Sigma, \delta, I, F)$, we want to construct $\mathcal{A}_*$:

- We add all initial states to the finite states for recognizing the empty word.
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So at the end we have $\mathcal{A}_* = (Q, \Sigma, \delta', I, F \cup I)$

Exercise 1.2 –

Correction 1.2

Exercise 1.3 –

Correction 1.3

Exercise 1.4 –

Correction 1.4

Exercise 1.5 –

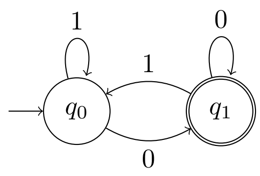
Correction 1.5

Exercise 1.6 – Let $\Sigma = \{0, 1\}$ an alphabet.

- We consider a language L_2 the set of binary words representing a multiple of two. This language is recognizable ?
- Same question for L_3 the set of binary words representing a multiple of three.
- What about the L_6 language for binary words representing a multiple of 6 ?

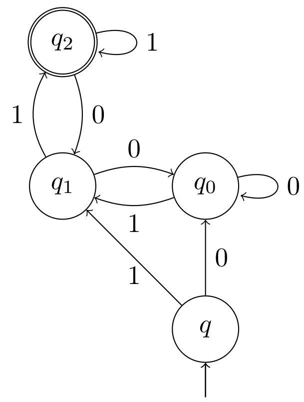
Correction 1.6

- For L_2 we just need to recognize the words which end in 0 :



- For L_3 we have 3 ($n \bmod 3 \in \{0,1,2\}$).
  For each edge (k the number read) :
  - If we read a zero, new result : $2k$
  - If we read a zero, new result : $2k+1$

state	read	next
q_0	0	$0 \times 2 \bmod 3 = 0$
	1	$0 \times 2 + 1 \bmod 3 = 1$
q_1	0	$1 \times 2 \bmod 3 = 2$
	1	$1 \times 2 + 1 \bmod 3 = 0$
q_2	0	$2 \times 2 \bmod 3 = 1$
	1	$2 \times 2 + 1 \bmod 3 = 2$



- For L_6, we can use the same construction (7 states).

Exercise 1.7 – A Dyck language is the set D of well-parenthesized words on an alphabet $\{\texttt{(},\texttt{)}\}$. For example, the word `(()())(()(()))` is well parenthesized.

This property can be formally defined :

- For any prefix u of w, the number of) in u is less than the number of (
- There are as many (as there are) in the word

Show that D is not a regular language.

Correction 1.7

Assume that D is regular.
We have the word $w = \texttt{(}^p\texttt{)}^p$ with $p \leq 1$. We pose $x = \varepsilon$, $y = \texttt{(}^p$ and $z = \texttt{)}^p$.
The conditions are well verified : $|xy| \leq p$ and $|y| \geq 1$.
If we consider the word xy^2z. This word has $2p$ `(` and p `)`. So xy^2z is not in D.
By the pumping lemma D is not regular.