λ-calculus

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Halting problem

There is no λ-term H such has $H[t] = T$ if T has a normal form and $H[T] = F$ if T has no normal form.

Let N the set of λ-term that have a normal form.
N is not empty (it contains all the variables) and N is not equal to Λ because it not contains the λ-term Ω.
So we have $\Lambda \backslash N$ non-empty and non-equal to Λ
By Scott's theorem, the set N is not recursively separable. So the λ-term H does not exist.

List in pure λ calculus

We define this useful lambda term :

- $[I] = \lambda x.x$
- $[T] = \lambda x\ y.x$
- $[F] = \lambda x\ y.y$
- $\langle t, u\rangle = \lambda x.x\ t\ u$
- $\pi_1\langle t, u\rangle = \langle t, u\rangle[T]$
- $\pi_2\langle t, u\rangle = \langle t, u\rangle[F]$

We define our integers as follows :

- $[0] = [I]$
- $[S] = \lambda n.\langle [F], n\rangle$
- $[isZ] = \lambda n.\pi_1\ t$
- $[P] = \lambda n.\pi_2\ t$

Finally, we define our lists as follows :

- $[] = \lambda n\ f.\ n$
- $x :: l = \lambda n\ f.\ f\ x\ l$

The list $0 :: 1 :: []$ is represented as follows:

$$\lambda n_0\ f_0.\ f_0\ [0]\ (\lambda n_1\ f_1.\ f_1\ [1]\ (\lambda n\ f.\ n))$$