TD n°2

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An object A in category $\mathcal{C}$ is called terminal when there exists exactly one morphism $C \to A$ for any object C of $\mathcal{C}$

1. What is a terminal object in an ordered set seen as an ordered category.
2. Describe a terminal object of **Set** and**Top** and Graph, the category graph and graph homomorphism.
3. Show that if A is terminal and $f : A \to B$ is an isomorphism then B is terminal too.
4. Suppose that A and B are terminal object in $\mathcal{C}$ show that exists an isomorphism $A \xrightarrow{f} B$ is unique.

Correction :

1. A terminal object in an ordered set is the maximum.
2. Any singleton in **Set** or in **Top** and the graph for the category graph.
3. Let $C \in \mathcal{C}$ and g the unique morphism from C to A. Then $f \circ g$ is a morphism from C to B. Let $h \in Hom(C, B)$, then $f \circ h \in Hom(C, A) = \{g\}$. So we have $h = f \circ g$.

$$\begin{array}{ccccc} Hom(C,A) & \xrightarrow{\quad Hom(C,f) \quad} & Hom(C,B) & \xrightarrow{\quad Hom(C,f^{-1}) \quad} & Hom(C,A) \\ C \xrightarrow{g} A & \longmapsto & C \xrightarrow{g} A \xrightarrow{f} B & \longmapsto & C \xrightarrow{g} A \xrightarrow{f} B \xrightarrow{f^{-1}} A \end{array}$$

4. Let $f : A \to B$ and $g : B \to A$ (unique). We have $g \circ f : A \to A = Id_A$ (A is terminal) and $f \circ g : B \to B$ (B is terminal). We show that g is the inverse of f so f is an isomorphism and it is unique.