Graph Algorithms
TD1 : Graph Colouring

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1 Some properties of colouring

Exercice 1.1 – What is the chromatic number of an even cycle C_{2n} ? Of an odd cycle C_{2n+1}

Exercice 1.2 – Show that a graph is bipartite if and only if it contains no odd cycle.

Exercice 1.3 – Show that for every graph G, there exists an order on the vertices such that the greedy algorithm applied in this order returns a colouring with $\chi(G)$ colours.

Exercice 1.4 – Prove that $\chi(G) \geq |V(G)|/\alpha(G)$, for every graph G.

2 Interval graphs

Given a set of intervals $\mathcal{I} = \{I_1, \dots, I_n\}$ where $I_i = [a_i, b_i]$ for every $1 \geq i \geq n$, the interval graph associated whit $\mathcal{I}$ is the graph $G = (V, E)$ where $V = \{1, \dots, n\}$ and $ij \in E$ iff I_i and I_j intersect, i.e. $a_i \leq b_j$ and $a_j \leq b_i$, for every $i \leq i, j \leq n$.

Exercice 2.1 – Show that in an interval graph, there exists a simplicial vertex, i.e. a vertex v such that $N[v]$ induces a clique.

Exercice 2.2 – Write an algorithm that computes an optimal proper colouring of an interval graph G. You may assume that we know the intervals. The goal complexity is $\mathcal{O}(n \ln n + m)$.

Exercice 2.3 – We now want to write an algorithms which computes a proper colouring of any graph G, and use $\chi(G)$ colours if G is an interval graph (so in particular we don't know the intervals if this is the case). Show that this can be done with the greedy colouring algorithm applied with a reverse degeneracy ordering.