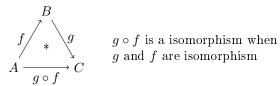
## TD n°1

## Valeran MAYTIE

An isomorphism  $f: A \to B$  such that there exists  $g: B \to A$  such that  $g \circ f = Id_A$  and  $f \circ g = Id_B$ .

- 1. Show that g is unique
- 2. Show that in the following situation



- 3. deduce in (\*) that g is an isomorphism when f and  $g \circ f$  are isomorphism
- 4. deduce in (\*) that f is an isomorphism when g and  $g \circ f$  are isomorphism
- 5. Suppose that in  $A \xrightarrow{f} B \xrightarrow{g} A \xrightarrow{h} B$  one has  $g \circ f = Id_A$  and  $h \circ g = Id_B$  show that f = h in that case
- 6. Characterize the isomorphisms in the category Set of sets and function. Top of topological spaces and continuous functions

## **Correction**:

1. Let  $h: B \to A$  a morphism that  $h \circ f = Id_A$  and  $f \circ h = Id_B$ 

$$h = h \circ Id_B$$
 By neutrality  $= h \circ (f \circ g)$  By associativity  $= Id_A \circ g$  By neutrality By neutrality

We can say that g is unique. We will make note  $f^{-1}$ 

2. We have  $f^{-1}$  and  $g^{-1}$  the inverse of f and g (they are isomorphism)

We will show that  $f^{-1} \circ g^{-1}$  is an inverse of  $g \circ f$ 

$$(f^{-1}\circ g^{-1})\circ (g\circ f)=f^{-1}\circ (g^{-1}\circ g)\circ f$$
 By associativity 
$$=f^{-1}\circ f$$
 
$$=Id_A$$

A similar reasoning can be used to show : (  $g \circ f) \circ (f^{-1} \circ g^{-1}$ 

3. Let  $g' = f \circ (g \circ f)^{-1}$ 

$$g \circ g' = g \circ (f \circ (g \circ f)^{-1})$$

$$= (g \circ f) \circ (g \circ f)^{-1}$$

$$= Id_C$$

$$g' \circ g = (f \circ (g \circ f)^{-1}) \circ g$$

$$= f \circ (g \circ f)^{-1} \circ g \circ f \circ f^{-1}$$

$$= f \circ (g \circ f)^{-1} \circ (g \circ f) \circ f^{-1}$$

$$= f \circ f^{-1}$$

$$= Id_B$$

g is well an isomorphism.

- 4. Roughly the same proof.
- 5. We have:

$$f = (h \circ g) \circ f$$
  $h \circ g = Id_B$   
 $= h \circ (g \circ f)$  By associativity  
 $= h$   $g \circ f = Id_A$ 

This question implies the first question.

6. Homomorphism (= A map between two structures, that preserves the operations of the structures  $f(x \bullet y) = f(x) \bullet f(y)$ )

2