Quantifying Liquidity Provision Costs for Automated Market Makers: An Adaptation of the LVR Model to OLAS-ETH pool

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Introduction

In this article we introduced an approach to quantify the cost of providing Liquidity to Automated Market Makers, specifically tailored to the OLAS-ETH pool (cf. Uniswap_v2:OLAS-ETH). The approach is an adaptation of the Loss-Versus-Rebalancing (LVR) model introduced in Automated Market Making and Loss-Versus-Rebalancing. Specifically, here OLAS is treated as a risky asset and ETH as the numéraire against which we price OLAS.

This model offers insights into the impact of liquidity, price fluctuations, and volatility on the net loss experienced by liquidity providers, and the protocol itself, in decentralized exchanges. Notably, the analysis suggests that the current liquidity depth[1] can be considered already past the point of maximal loss. A further analysis could be done to explore the optimal range of liquidity depth of the OLAS-ETH pool. However, it's crucial to acknowledge that this conclusion is based on specific model assumptions that do not perfectly mirror real-world conditions.

LRV formula

For a gentle introduction of the LVR model and its application to quantifying liquidity provision costs, we recommend referring to the introductory article at LVR: Quantifying Liquidity Provision Costs for Automated Market Makers. For a comprehensive understanding we recommend the article Automated Market Making and Loss-Versus-Rebalancing.

In essence, the LVR model's formula for Uniswap v2 (as detailed on page 18 in Automated Market Making and Loss-Versus-Rebalancing) calculates the loss l per unit of time as a fraction of marked-to-market pool value V as:

$$l(\sigma, P)\,/V(P) = \sigma^2/8,$$

[1] Current liquidity depth i.e. 36’222.3 obtained as the square root of the OLAS reserve times the ETH reserve at the date 05-09-23

where σ is the volatility of the risky asset token. This loss is directly proportional to the square of the volatility.

Additionally, the value of the pool V can be computed as follows:

$$V(\sigma) = 2\sqrt{P}L,$$

where P represents the OLAS spot price (defined as $EthReserve/OlasReserve$) and we use $L = \sqrt{OlasReserve \cdot EthReserve}$ a measure of liquidity.

NetLoss function

To added the net loss experienced by liquidity providers, and specifically within the Autonolas protocol, we formulate the NetLoss function as follows:

$$NetLossAutonolas(L, \sigma, Volume, P) = L_A/L \cdot \sigma^2/8 - 0.003 \cdot L_A \cdot Volume/(2 \cdot \sqrt{P} \cdot L^2),$$

where L_A is the amount of the liquidity owned by Autonolas.

It is it worth mentioning that the value L_A exclusively serve to accurately define the NetLoss function within the context of the Autonolas protocol. Our analysis remains independent of this specific value. Therefore, we will now examine the net loss function as defined below:

$$NetLoss(L, \sigma, Volume, P) = \sigma^2/(L \cdot 8) - 0.003 \cdot Volume/(2 \cdot \sqrt{P} \cdot L^2)$$

Analysis

By setting the derivative of the NetLoss function equal to zero, we identify stationary points. The derivative is provided in the following:

$$d\,NetLoss(L)/dL = - L_A \cdot \sigma^2/(8 \cdot L^2) + 0.003 \cdot L_A \cdot Volume/(\sqrt{P} \cdot L^3)$$

When we set the derivative to zero

$$d\,NetLoss(L)/dL = 0$$

we find the following value

$$L* = (8 \cdot 0.003) \cdot Volume/(\sqrt{P} \cdot \sigma^2) = 0.024 \cdot Volume/(\sqrt{P} \cdot \sigma^2)$$

It is easy to show that this $L*$ is a maximum point (as the second derivative evaluated in that point is negative).

Hence $L^* = 0.024 \cdot Volume/(\sqrt{P} \cdot \sigma^2)$ is the point of maximum loss.

Considering the following data

- the weekly moving average of volume in ETH, i.e. ~520.1 ETH (cf. https://dune.com/queries/2997249/4974102 at the following date 04-09-23) ,
- the OLAS weekly moving average price with respect to ETH as a numéraire, i.e. ~0.000601 (cf. https://dune.com/queries/2999114/4978934 at the following date 04-09-23), and
- the OLAS realized volatility, i.e. 21.68%

we determine that the maximum loss point, L^* , has a value of ~10'838.01.

Furthermore, based on the weekly moving average of liquidity, i.e. ~33'486.98 (cf. https://dune.com/queries/2999114/4978934 at the following date 04-09-23), our analysis suggests that we are already situated on the downwards slope of the NetLoss curve, as illustrated in Fig. 1.

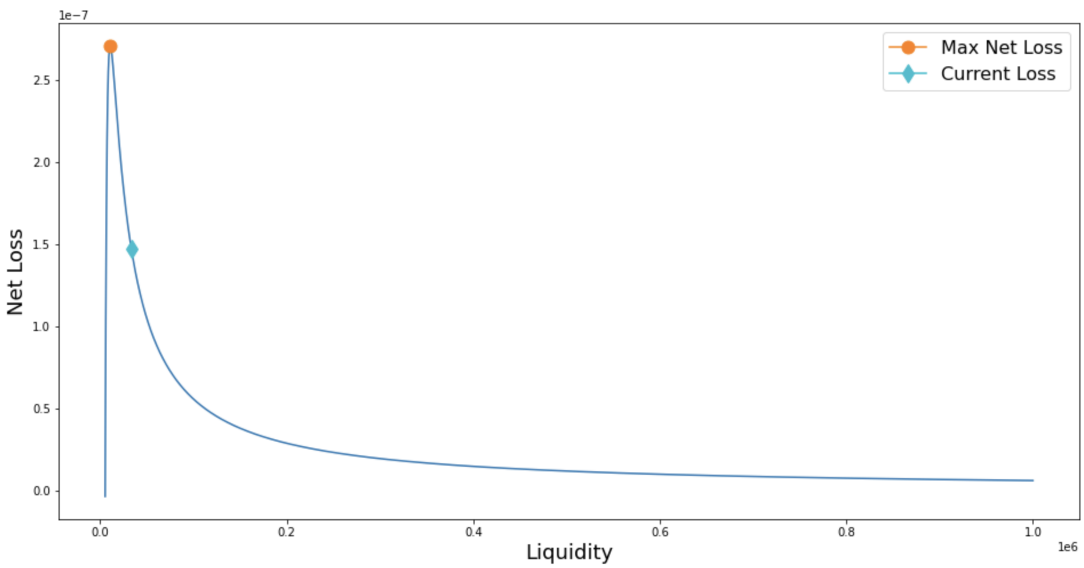

Fig. 1: NetLoss curve as a function of the liquidity L

Comparative statics

Figure 2 provides a visual representation of how the NetLoss function reacts to variations of the weekly moving average of the OLAS spot price, both when it increases or decreases. Notably, when the OLAS spot price increases by a certain percentage, the ETH spot price simultaneously decreases by an equivalent percentage.

Additionally, Figure 2 illustrates the diminishing trend of NetLoss curves as the liquidity increases. Furthermore, it showcases the current loss value - calculated by using the aforementioned weekly moving averages for volume, OLAS spot price, and liquidity - positioned on the downwards slope of the NetLoss curves. Remarkably, this current loss value is already positioned in such a way the liquidity has a more pronounced impact on NetLoss curves than minor fluctuations in prices. Finally, in Fig. 3 it is possible to see the curve of the maximum NetLoss values achievable by altering the weekly moving average of the OLAS spot price.

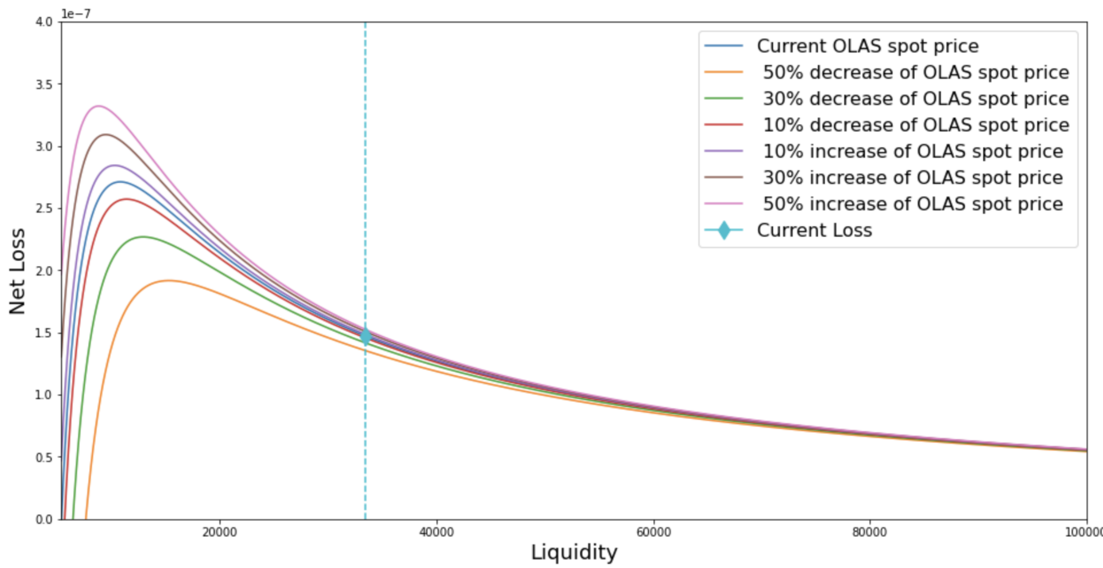

Fig. 2: NetLoss curves over the liquidity using the different OLAS spot prices (respectively, eth spot prices).

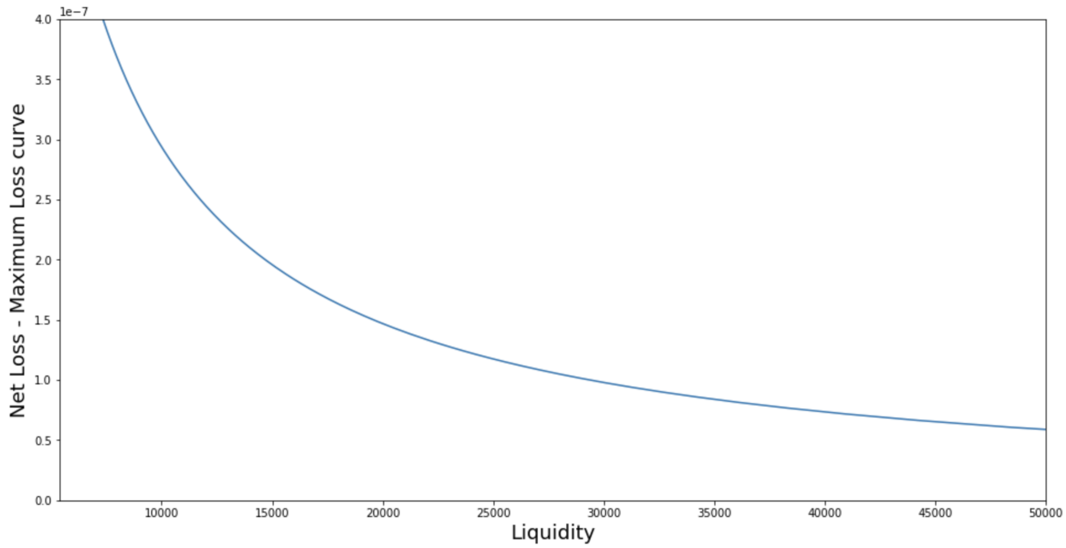

Fig. 3: Maximum NetLoss values over the liquidity starting from an OLAS spot price 50% bigger than the weekly moving average to an OLAS spot price 50% lower than the weekly moving average.

Conclusion

Our analysis suggests that the current liquidity depth may be past the point of maximal loss using this adaptation of the LVR model to the pool. Additionally, it shows that this current loss value is already positioned in such a way liquidity has a more pronounced impact on NetLoss curves than minor fluctuations in prices.

However, it is worth mentioning that conclusions are based on specific model assumptions that may not fully align with real-world conditions. The model assumes the existence of a fee-free trading venue with limitless liquidity for arbitrage opportunities. Given OLAS's limited availability on centralized exchanges, the model may not fully reflect the complexities of real-world trading conditions.

Nonetheless, within the confines of the provided assumptions, further analysis could explore the liquidity value beyond which significant changes would yield minimal loss variations.