

DISTRIBUCIONES CONTINUAS

NOMBRE	MOMENTO	ESPERANZA	Varianza
UNIFORME CONTINUA $X \sim U(a, b)$	$* f(x) = \frac{1}{b-a} \quad \forall x \in [a, b]$	$\mu = \int_a^b \frac{x}{b-a} = \frac{b+a}{2}$	$\sigma^2 = E(X^2) = \int_a^b \frac{x^2}{b-a} = \frac{b^3-a^3}{3(b-a)}$ $\sigma^2 = \frac{(b-a)^2}{12}$
GAMMA $X \sim \Gamma(p, a)$	$M(t) = E(e^{tx}) = \frac{a^p}{\Gamma(p)} \int_0^\infty e^{tx} x^{p-1} e^{-ax} dx$ $= \frac{a^p}{\Gamma(p)} \int_0^\infty x^{p-1} e^{-a(x-t/a)} dx = \left[\left(1 - \frac{t}{a}\right)^{-p} = \left(\frac{a}{a-t}\right)^p \right]$ $\Psi(t) = \left(1 - \frac{t}{a}\right)^{-p}$	$N = \alpha_1 = M'(0) = \frac{p}{a}$	$\alpha_2 = M''(0) = \frac{p(p+1)}{a^2}$ $\sigma^2 = \alpha_2 - N^2 = \frac{p(p+1-p)}{a^2} = \frac{p}{a^2}$
EXPONENCIAL $X \sim \text{Exp}(a)$ $X \sim \Gamma(1, a)$	$M(t) = \frac{a}{a-t} \quad t < a$ $\Psi(t) = \left(1 - \frac{t}{a}\right)^{-1} \quad t \in \mathbb{R}$	$N = \alpha_1 = M'(0) = \frac{1}{a}$	$\alpha_2 = M''(0) = \frac{2}{a^2}$ $\sigma^2 = \alpha_2 - N^2 = \frac{1}{a^2}$
CHI-CUADRADO $X \sim \chi^2(n)$ $X \sim \Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$	$M(t) = (1-2t)^{-n/2}$ $\Psi(t) = (1-2t)^{-n/2}$	$\mu = n$	$\sigma^2 = 2n$
BETA $X \sim \text{Beta}(p, q)$	$\alpha_k = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \int_0^1 x^{k+p-1} (1-x)^{q-1} dx = \frac{\beta(p+k, q)}{\beta(p, q)}$	$\alpha_1 = \frac{p}{p+q} = N$	$\alpha_2 = \frac{p(p+1)}{(p+q)(p+q+1)}$ $\sigma^2 = \frac{pq}{(p+q)^2(p+q+1)}$
NORMAL PAFICADA $X \sim N(0, 1)$	$M(t) = e^{\frac{t^2}{2}}$ $\Psi(t) = e^{-\frac{t^2}{2}}$	$N = \alpha_1 = M'(0) = t e^{\frac{t^2}{2}} \Big _0 = 0$	$\sigma^2 = \alpha_2 - N^2 = M''(0) = e^{\frac{t^2}{2}} + t^2 e^{\frac{t^2}{2}} \Big _0 = 1$
NORMAL $X \sim N(\mu, \sigma)$	$Y = \sigma X + \mu$	$N = E(Y) = \sigma \cdot E(X) + E(\mu) = \mu$	$V(Y) = \sigma^2 V(X) = \sigma^2$
T-STUDENT		$N = 0$	$\sigma^2 = \frac{n}{n-2}$
F de FISHER SNEDECOR		$N = \frac{n}{n-2} \quad n \geq 2$	$\sigma^2 = \frac{2n^2(m+n-2)}{m(n-2)(n-4)}$ $n \geq 4$

DISTRIBUCIONES DISCRETAS



NOMBRE	FUNCIÓN CARACTERÍSTICA	MOMENTO	ESPERANZA	VARIANZA
SINGULAR $f(x) = 1$ si $x=a$	$\psi(t) = \sum_x e^{itx} f(x) = e^{ita}$	$M(t) = \sum_x e^{itx} f(x) = e^{ita}$	$N = d_1 = M'(0) = a$	$d_2 = M''(0) = 0$ $\sigma^2 = d_2 - \mu^2 = 0$
UNIFORME $f(x) = \frac{1}{K}$ $x \in [a, b]$	$\psi(t) = \sum_{j=1}^K e^{itx_j} f(x_j) = \frac{1}{K} \sum_{j=1}^K e^{itx_j}$	$M(t) = \sum_{j=1}^K e^{itx_j} f(x_j) = \frac{1}{K} \sum_{j=1}^K e^{itx_j}$	$N = d_1 = M'(0) = \frac{1}{K} \sum_{j=1}^K x_j = \bar{x}$	$d_2 = M''(0) = \frac{1}{K} \sum_{j=1}^K x_j^2$ $\sigma^2 = d_2 - \mu^2 = \frac{1}{K} \sum_{j=1}^K (x_j - \bar{x})^2$
BERNOULLI $f(x) = p^x q^{1-x}$ $X \sim B(1, p)$	$\psi(t) = \sum_{x=0,1} e^{itx} f(x) = f(0) + e^{it} f(1) = q + p e^{it}$	$M(t) = \sum_{x=0,1} e^{itx} f(x) = f(0) + e^{it} f(1) = q + p e^{it}$	$N = d_1 = M'(0) = p$	$d_2 = M''(0) = p$ $\sigma^2 = d_2 - \mu^2 = p - p^2 = p(1-p)$
BINOMIAL $X \sim B(n, p)$	$N = E(X) = E\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n E(X_i) = np$ $\sigma^2 = V(X) = V\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n V(X_i) = npq$			
POISSON $X \sim P(\lambda)$ $f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$	$M(t) = E(e^{tx}) = \sum_x e^{tx} f(x) = \sum_x e^{tx} e^{-\lambda} \frac{\lambda^x}{x!} = e^{-\lambda} \sum_x \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$	$M(t) = E(e^{tx}) = \sum_x e^{tx} f(x) = \sum_x e^{tx} e^{-\lambda} \frac{\lambda^x}{x!} = e^{-\lambda} \sum_x \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$	$N = d_1 = M'(0) = \lambda$	$d_2 = M''(0) = \lambda + \lambda^2$ $\sigma^2 = d_2 - \mu^2 = \lambda$
GEOMÉTRICA $f(x) = (1-p)^{x-1} p$	$M(t) = E(e^{tx}) = \sum_x e^{tx} f(x) = \sum_{x=1}^{\infty} e^{tx} (1-p)^{x-1} p = p e^t (1 + q e^t + \dots) = p e^t \frac{1}{1 - q e^t}$	$M(t) = E(e^{tx}) = \sum_x e^{tx} f(x) = \sum_{x=1}^{\infty} e^{tx} (1-p)^{x-1} p = p e^t (1 + q e^t + \dots) = p e^t \frac{1}{1 - q e^t}$	$N = M'(0) = \frac{1}{p}$	$d_2 = M''(0) = \frac{q}{p^2}$ $\sigma^2 = \frac{q}{p^2}$
BINOMIAL NEGATIVA	$M(t) = \left(\frac{p e^t}{1 - q e^t}\right)^K$ $t < \ln(1/q)$	$M(t) = \left(\frac{p e^t}{1 - q e^t}\right)^K$ $t < \ln(1/q)$	$N = \frac{K(1-p)}{p}$	$\sigma^2 = \frac{K(1-p)}{p^2}$
HIPERGEOMÉTRICA $X \sim H(n, a, b)$	$* P(X=x) = \frac{\binom{a}{x} \binom{b}{n-x}}{\binom{a+b}{n}}$ $x \in [0, n]$ $x \leq a, n-x \leq b$	$x \in [0, n]$ $x \leq a, n-x \leq b$	$N = np$	$\sigma^2 = npq \frac{n-a}{n-1}$