Discrete Random Variables: Probability Mass Functions

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Basics

Definition 1. A *discrete random variable* is a real-valued function of the outcome of the experiment that can take a finite or countably infinite number of values.

Probability Mass Function

Definition 2. For a discrete random variable X, the *probability mass function* of X, denoted by $p_X(x)$, is the probability of the event $\{X = x\}$ consisting of all outcomes that give rise to a value of X equal to x:

$$p_X(x) = \mathbf{P}(\{X = x\})$$

The Brnoulli Random Variable

Consider the toss of a coin, which comes up a head with probability p, and a tail with probability $1 - p$. The *Bernoulli* random variable takes two values 1 and 0, depending on whether the outcome is a head or a tail:

$$X = \begin{cases} 1 & \text{if a head,} \\ 0 & \text{if a tail.} \end{cases}$$

It's PMF is

$$p_X(k) = \begin{cases} p & \text{if } k = 1, \\ 1 - p & \text{if } k = 0. \end{cases}$$

The Binomial Random Variable

A coin is tossed n times. At each toss, the coin comes up a head with probability p, and a tail with probability $1 - p$, independent of prior tosses. Let X be the number of heads in the n-toss sequence. We refer to X as a *binomial* random variable with *parameters n and p*.

It's PMF is

$$p_X(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \quad k = 0, 1, \ldots, n$$

The Geometric Random Variable

The *geometric* random variable is the number X of tosses needed for a head to come up for the first time. Its PMF is given by

$$p_X(k) = (1-p)^{k-1}p, \quad k = 1, 2, \ldots$$

The Poisson Random Variable

A *Poisson* random variable has a PMF given by

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$$

where λ is a positive parameter characterizing the PMF.