

# Orthogonal matrices

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**Definition 1** (Orthonormal vectors). The vectors  $q_1, q_2, \dots, q_n$  are *orthonormal* if:

$$q_i^T q_j = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases}$$

**Remark 1** (Orthogonal matrix). If the columns of  $[q_1 \dots q_n]$  are *orthonormal*, then  $Q^T Q = I$ . A matrix with orthonormal columns is called an *orthonormal matrix* and if the matrix is square it is called an *orthogonal matrix*. Then  $Q^T = Q^{-1}$ .