

## *Solving $Ax = b$ : Row Reduced Form $R$*

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### *Solvability conditions on $b$*

The system of equations  $Ax = b$  is solvable exactly when  $b$  is in the column space  $C(A)$ .

### *Complete solution to $Ax = b$*

To find complete solution to a solvable system of equation  $Ax = b$

- (i) Find  $x_p$ , a particular solution of  $Ax = b$  by setting all free variables to zero and solve for pivot variables.
- (ii) Find  $x_n$ , general solution to the homogenous equation  $Ax = 0$ .

The general solution to  $Ax = b$  is given by  $x = x_p + x_n$ .

### *Rank of a matrix*

The rank of a matrix is the number of pivots of that matrix. If  $r$  is the rank of an  $m \times n$  matrix  $A$ . Then  $r \leq m$  and  $r \leq n$ .

#### *Full column rank*

If  $r = n$ , then *nullspace* contains only the *zero vector*. If  $Ax = b$  has a solution it is unique, so that there is either 0 or 1 solution.

#### *Full row rank*

If  $r = m$ , the equation  $Ax = b$  can be solved for any  $b$ . Since there are  $n - r = n - m$  free variables, the equation  $Ax = 0$  has  $n - m$  special solutions.

#### *Full row and column rank*

If  $r = m = n$ , then  $A$  is an *invertible* square matrix. The *nullspace* has dimension 0,  $Ax = b$  has a unique solution for every  $b$  in  $\mathbb{R}^m$ .

$$\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$$