Solving $Ax = b$: Row Reduced Form R

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Solvability conditions on b

The system of equations $Ax = b$ is solvable exactly when b is in the column space C(A).

Complete solution to $Ax = b$

To find complete solution to a solvable system of equation $Ax = b$

(i) Find x_p, a particular solution of $Ax = b$ by setting all free variables to zero and solve for pivot variables.

(ii) Find x_n, general solution to the homogenous equation $Ax = 0$.

The general solution to $Ax = b$ is given by $x = x_p + x_n$.

Rank of a matrix

The rank of a matrix is the number of pivots of that matrix. If r is the *rank* of an $m \times n$ matrix A. Then $r \leq m$ and $r \leq n$.

Full column rank

If $r = n$, then *nullspace* contains only the *zero vector*. If $Ax = b$ has a solution it is unique, so that there is either 0 or 1 solution.

Full row rank

If $r = m$, the equation $Ax = b$ can be solved for any b. Since there are $n - r = n - m$ free variables, the equation $Ax = 0$ has $n - m$ special solutions.

Full row and column rank

If $r = m = n$, then A is an *invertible* square matrix. The *nullspace* has dimension 0, $Ax = b$ has a unique solution for every b in $\mathbb{R}^m$.

$rank(AB) \leq \min(rank(A), rank(B))$