Vector Spaces and Subspaces

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Spaces of Vectors

Definition 1 (Vector Space)**.** The space R^n consists of all column vectors v with n components.

Definition 2 (Subspace)**.** A subspace of a vector space is a set of vectors that satisfies two requirements. If u and v are vectors in the subspace and c is any scalar, then

(i) $u + v$ is in the subspace

(ii) cu is in the subspace

Column space of A

Definition 3 (Column space)**.** The column space of a matrix A, $\mathrm{C}(A)$ is the vector space made up of all linear combinations of the columns of A.

The system of linear equations $Ax = b$ is solvable exactly when vector b is in the *column space* of A.

Nullspace of A

Definition 4 (Nullspace)**.** The *nullspace* of a matrix A, $\mathrm{N}(A)$ is the vector space consists of all solutions x to the equation $Ax = 0$.

The *rank* of an $m \times n$ matrix A is the number of pivot columns, so the number of free columns is $n - r$, which is the number of *special solutions* and the *dimension* of the *nullspace*.