Matrix Basics

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Some basic results

Some important results

(i) $(AB)^T = B^T A^T$.

(ii) $(AB)^{-1} = B^{-1}A^{-1}$.

(iii) If A is invertible $(A^T)^{-1} = (A^{-1})^T$.

(iv) For a symmetric matrix A, it's inverse A^{-1} is also symmetric.

Matrix multiplication

If we subdivide A and B into blocks that match properly, we can write the product $AB = C$ in terms of products of the blocks:

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$$

Here $C_1 = A_1B_1 + A_2B_3$.

Elimination = Factorization

When there are no row exchanges, elimination leads to the form

$$A = LU$$

The matrix U is *upper triangular* with *pivots* on the diagonal. The matrix L is *lower triangular* and has *ones* on the diagonal.

When row exchanges are required, elimination leads to

$$PA = LU$$

The matrix P is a *permutation matrix*, which has the rows of I in some order.

Some important properties of *Permutation matrices*:

(i) $P^{-1} = p^T$

(ii) Permutation matrices form a multiplicative group. Order of this group is $n!$

Cost of elimination

For an $n \times n$ matrix A, the cost of factoring A into LU is on the order of n^3.