Orthogonal vectors and subspaces

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Orthoganality of the Four Subspaces

Definition 1 (Orthogonal subspaces)**.** Subspace S is said to be *orthogonal* to a subspace T, if every vector in S is orthogonal to every vector in T.

Remark 1. Every vector x in the *nullspace* of A is perpendicular to every row of A. The *nullspace* $\mathrm{N}(A)$ and the *row space* $\mathrm{C}(A^T)$ are orthogonal subspaces of R^n.

Remark 2. Every vector y in the *nullspace* of A^T is perpendicular to every column of A. The *nullspace* $\mathrm{N}(A^T)$ and the *column space* $\mathrm{C}(A)$ are orthogonal subspaces of R^m.

Orthogonal Complements

Definition 2 (Orthogonal complement)**.** The *orthogonal complement* of a subspace V contains every vector that is perpendicular to V. This orthogonal subspace is denoted by $V^{\perp}$.

Theorem 1 (Fundamental Theorem of Linear Algebra)**.** $\mathrm{N}(A)$ is the orthogonal complement of the row space $\mathrm{C}(A^T)$(in R^n). $\mathrm{N}(A^T)$ is the orthogonal complement of the column space $\mathrm{C}(A)$(in R^m).