

## Matrix Basics

Gilbert Strang

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### Some basic results

Some important results

- (i)  $(AB)^T = B^T A^T$ .
- (ii)  $(AB)^{-1} = B^{-1} A^{-1}$ .
- (iii) If  $A$  is invertible  $(A^T)^{-1} = (A^{-1})^T$ .
- (iv) For a symmetric matrix  $A$ , its inverse  $A^{-1}$  is also symmetric.

### Matrix multiplication

If we subdivide  $A$  and  $B$  into blocks that match properly, we can write the product  $AB = C$  in terms of products of the blocks:

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$$

Here  $C_1 = A_1 B_1 + A_2 B_3$ .

### Elimination = Factorization

When there are no row exchanges, elimination leads to the form

$$A = LU$$

The matrix  $U$  is *upper triangular* with *pivots* on the diagonal. The matrix  $L$  is *lower triangular* and has *ones* on the diagonal.

When row exchanges are required, elimination leads to

$$PA = LU$$

The matrix  $P$  is a *permutation matrix*, which has the rows of  $I$  in some order.

Some important properties of *Permutation matrices*:

- (i)  $P^{-1} = P^T$
- (ii) Permutation matrices form a multiplicative group. Order of this group is  $n!$

*Cost of elimination*

For an  $n \times n$  matrix  $A$ , the cost of factoring  $A$  into  $LU$  is on the order of  $n^3$ .