રાષ્ટ્રાપ્ટ મન્નું મધાર્ક છું મહે

Buckling of Plates

1. CAUSES OF BUCKLING

Buckling of flat plates may be experienced when the plate is excessively stressed in compression along opposite edges, or in shear uniformly distributed around all edges of the plate, or a combination of both. This necessitates establishment of values for the critical buckling stress in compression ($\sigma_{\rm cr}$) and in shear ($\tau_{\rm cr}$).

2. BUCKLING OF PLATES IN EDGE COMPRESSION

The critical compressive stress of a plate when subject to compression (σ_{cr}) can be found from the following:

 $\sigma_{cr} = \frac{k \pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \qquad (1)$

FIGURE 1

where:

E = modulus of elasticity in compression (Steel = 30,000,000 psi)

t = thickness of plate, inches

b = width of plate, inches

a = length of plate, inches

v = Poisson's ratio (for steel, usually = 0.3)

 k = constant; depends upon plate shape b/a and support of sides. See Tables 1 and 3.

If the resulting critical stress (σ_{cr}) from this formula is below the proportional limit (σ_{ν}) , buckling is said to be elastic and is confined to a portion of the plate away from the supported side; this does not mean complete collapse of the plate at this stress. This is

represented by the portion of the curve C to D in Figure 2. If the resulting value (σ_{cr}) is above the proportional limit (σ_{p}) , indicated by the portion of the curve A to C, buckling is said to be inelastic. Here, the tangent modulus (E_{t}) must be used in some form to replace Young's or secant modulus (E) in the formula for determining σ_{cr} .

This problem can be simplified by limiting the maximum value of the critical buckling stress ($\sigma_{\rm cr}$) to the yield strength ($\sigma_{\rm r}$). However, the value of the critical buckling stress ($\sigma_{\rm cr}$) may be calculated if required.

Above the proportional limit (σ_n) , the ratio $E = \sigma/\epsilon$ is no longer constant, but varies, depending upon

TABLE 1—Compression Load on Plate

Support (long plotes)	Values for Plate Factor (k) to be Used in Formula	Critical Stress on Plate to Couse Buckling (o'er)
Jupponee of the state of the st	k = 0.425	$\sigma'_{ m cr} = \sigma_{ m cr}$
fixed	k = 1,277	$\sigma'_{er} = \sigma_{er}$
Supported	k = 4.00	$\sigma'_{er} = \sigma_{er}$
supported	k = 5.42	$\sigma'_{er} = \sigma_{er}$
hied ← ← ← ← ← ← ← ← ← ← ← ← ← ← ← ← ← ← ←	k = 6.97	$\sigma'_{er} = \sigma_{er}$

-Bleich, "Buckling Strength of Metal Structures," p. 330

the type of steel (represented by its stress-strain diagram) and the actual stress under consideration (position on the stress-strain diagram). See Figure 3.

Above the proportional limit $(\sigma_{\mathfrak{p}})$, the modulus of elasticity (E) must be multiplied by a factor (λ) to give the tangent modulus (E_t). The tangent modulus (E_t) is still the slope of the stress-strain diagram and E_t = σ/ϵ , but it varies.

If it is assumed that the plate is "isotropic" (i.e., having the same properties in both directions x and y), the critical buckling formula becomes—

where:
$$\lambda = \frac{E_t}{E}$$

$$\sigma_{\rm er} = \frac{\pi^2 \to \lambda}{12(1-v^2)} \left(\frac{t}{b}\right)^2 k \qquad (2)$$

If it is assumed that the plate has "anisotropic" behavior (i.e. not having the same properties in both directions x and y), the tangent modulus (E_t) would be used for stresses in the x direction when the critical stress (σ_{cr}) is above the proportional limit (σ_p) . However, the modulus of elasticity (E) would be used in the y direction because any stress in this direction would be below the proportional limit (σ_p) . In this case, the above formula #2 would be conservative and

the following would give better results:

$$\sigma_{\rm cr} = \frac{\pi^2 E \sqrt{\lambda}}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 k \quad . \tag{3}$$

For steel, this becomes—

$$\sigma_{\rm er} = 2.710 \times 10^7 \sqrt{\lambda} \left(\frac{\rm t}{\rm b}\right)^2 {\rm k} \qquad \dots (4)$$

If the critical buckling stress (σ_{er}) is less than the proportional limit (σ_{p}) then $\lambda = E_{t}/E = 1$ and formula #4 could be used directly in solving for critical stress (σ_{er}).

However, if the critical buckling stress $(\sigma_{\rm er})$ is greater than the proportional limit $(\sigma_{\rm p})$, then $\lambda < 1$ and formula #4 cannot be used directly. It would be better to divide through by $\sqrt{\lambda}$ and express the formula as—

From the value of $\sigma_{cr}/\sqrt{\lambda_s}$, formula #6 will give the value of σ_{cr} . Obtain proper value for the plate factor (k) from Table 1 or 3.

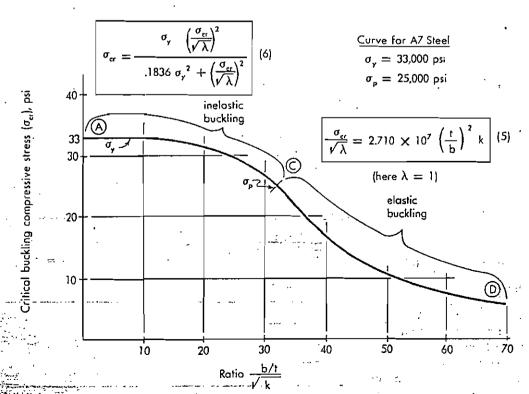


FIG. 2 Buckling stress curve for plates in compression.

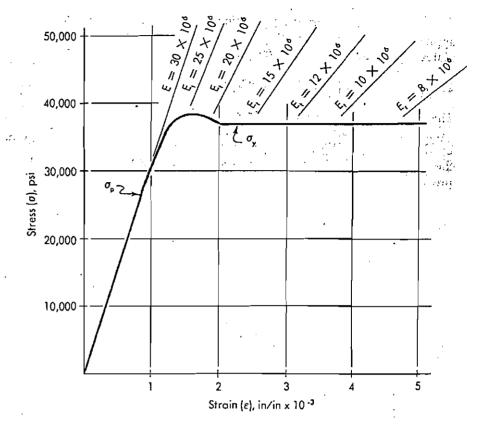


FIG. 3 Stress-strain diagram showing where tangent modulus need be applied to determine critical stress.

Determining Tangent Modulus Factor (λ)

Blaich in "Buckling Strength of Metal Structures", p. 54, gives the following expression for this factor $(\lambda = E_t/E)$:

$$\lambda = \frac{(\sigma_{y} - \sigma_{cr}) \sigma_{cr}}{(\sigma_{y} - \sigma_{p}) \sigma_{p}}$$

where:

 $\sigma_y = \text{yield point}$

 $\sigma_{\rm p}={
m proportional\ limit}$

 $\sigma_{cr} = critical$ buckling stress

If we use a ratio of-

$$\frac{\sigma_{\rm r}}{\sigma_{\rm o}} = 1.32$$
 or $\sigma_{\rm p} = \frac{\sigma_{\rm y}}{1.32}$

the expression becomes—

$$\lambda = \frac{(\sigma_{y} - \sigma_{cr}) \sigma_{cr}}{\left(\sigma_{y} - \frac{\sigma}{1.32}\right) \frac{\sigma}{1.32}}$$
$$= \frac{\sigma_{y} \sigma_{cr} - \sigma_{cr}^{2}}{1836 \sigma_{y}^{2}}$$

 $\cdot \cdot$.1836 $\sigma_{y}^{2} \lambda = \sigma_{y} \sigma_{cr} - \sigma_{cr}^{2}$ or .1836 $\sigma_{y}^{2} \lambda + \sigma_{cr}^{2} = \sigma_{y} \sigma_{cr}$ Then, multiply through by $\frac{\sigma_{\rm cr}}{\lambda}$

.1836
$$\sigma_{\rm r}^2 \sigma_{\rm cr} + \frac{\sigma_{\rm cr}^3}{\lambda} = \frac{\sigma_{\rm r} \sigma_{\rm cr}^2}{\lambda}$$

$$\sigma_{\rm cr} \left(.1836 \ \sigma_{\rm y}^2 + \frac{\sigma_{\rm cr}^2}{\lambda} \right) = \frac{\sigma_{\rm y} \ \sigma_{\rm cr}^2}{\lambda}$$

$$\sigma_{\rm er} = \frac{\sigma_{\rm y} \left(\frac{\sigma_{\rm er.}}{\sqrt{\lambda}}\right)^2}{.1836 \ \sigma_{\rm y}^2 + \left(\frac{\sigma_{\rm er.}}{\sqrt{\lambda}}\right)^2} \qquad (6)$$

TABLE 2-Shear Load on Plate

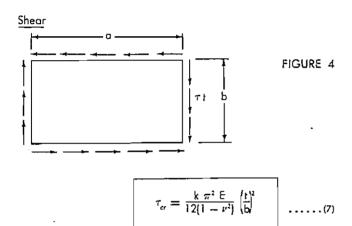
Support	Volues for Plote Factor (k) to be Used in Formula	Critical Stress on Plote to Cause Buckling (7'cr)
σ = o/b improned b b b c c c c c c c	$k = 5.34 + \frac{4}{\alpha^2}$	$\tau'_{\rm cr} = \frac{\sigma_{\rm cr}}{\sqrt{3}}$
a = a/b fired b b	$k = 8.98 + \frac{5.6}{\alpha^2}$	$\tau'_{\rm er} = \frac{\sigma_{\rm er}}{\sqrt{3}}$

2.12-4 / Load & Stress Analysis

See Figure 2 for curves representing these formulas applied to the critical buckling compressive stress of plates of A7 steel ($\sigma_r = 33,000 \text{ psi}$).

3. BUCKLING OF PLATES UNDER SHEAR

The critical buckling shearing stress (τ_{cr}) of a plate when subject to shear forces (τt) may be expressed by the formula in Figure 4 (similar to that used for the critical buckling stress for plates in edge compression).



where:

E = modulus of elasticity in compression (Steel = 30,000,000 psi)

t = thickness of plate, inches

b = width of plate, inches

a = length of plate, inches (a is always the larger of the plate's dimensions)

v = Poisson's ratio (for steel, usually = 0.3)

k = constant; depends upon plate shape b/a and edge restraint, and also accounts for the modulus of elasticity in shear (E_s). See Tables 2 and 3.

It is usual practice to assume the edges simply supported.

Shear yield strength of steel (τ) is usually considered as $\frac{1}{\sqrt{3}}$ of the tensile yield strength (σ_{τ}), or .58 σ_{τ} . Since

$$\tau_{\rm cr} = \frac{\sigma_{\rm cr}}{\sqrt{3}}$$

$$\sigma_{\rm cr} = \frac{\sqrt{3} \text{ k } \pi^2 \text{ E}}{12(1 - \nu^2)} \left(\frac{\text{t}}{\text{b}}\right)^2$$

TABLE 3—Critical Stress for Rectangular Plates Supported On 4 Sides
(Applies to Web of Girders Between Stiffeners and
to Web of Frame Knee Between Stiffeners)

Load	Values far Plate Factor (k) to be Used in Formulas 3, 4, 5, and 6	Critical Stress $ au'_{er}$ and $ au'_{er}$
. Compression $\alpha = \sigma/b \qquad \sigma_1 = \sigma_2 \qquad \sigma_3$	when $\alpha \ge 1$ $k = 4$ when $\alpha \le 1$ $k = \left(\alpha + \frac{1}{\alpha}\right)^2$	<i>d'</i> cr = ^d et
Compresson $a = \sigma/b \qquad a_1 = 0$ $a = \sigma/b \qquad a_2 = 0$	when $\alpha \ge 1$ $k = 7.7$ when $\alpha \le 1$ $k = 7.7 + 33 (1 \rightarrow \alpha)2$	$\sigma'_{cr} = \sigma_{cr}$
Completion $\sigma_1 = -\sigma_2$ $\sigma = a/b$ $\sigma_2 = -\sigma_3$ $\sigma_3 = -\sigma_3$ $\sigma_4 = -\sigma_3$ $\sigma_5 = -\sigma_5$ $\sigma_7 = -\sigma$	when $\alpha \ge \%$ $k = 24$ when $\alpha \le \%$ $k = 24 + 73 (\% - \alpha)^2$	$\sigma_{\rm ec} = \sigma_{\rm er}$
Shear α = σ/b τ β β β β β β β β β β β β β β β β β β	when $a \ge 1$ $k = \sqrt{3} \left(5.34 + \frac{4}{a^2} \right)$ when $a \le 1$ $k = \sqrt{3} \left(4 + \frac{5.34}{a^2} \right)$	$r'_{ m cr} = rac{\sigma_{ m cr}}{\sqrt{3}}$

Since the plate constant (k) can be adjusted to contain the $\sqrt{3}$ factor, this becomes—

$$\sigma_{\rm cr} = \frac{k \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

As before in the buckling of plates by compres-

sion, in the inelastic range the critical stress (σ_{cr}) exceeds the proportional limit (σ_p) , and the tangent modulus (E_t) is introduced by the factor $(\lambda = E_t/E)$. Therefore, formulas #5 and #6 would be used also in the buckling of plates by shear.

Proper values for the plate factor (k) are obtained from Table 2, for pure shear load, and Table 3, for shear load combined with compression.

TABLE 3—Critical Stress for Rectangular Plates Supported On 4 Sides — Continued —

(Applies to Web of Girder Between Stiffeners and to Web of Frame Knee Between Stiffeners)

Load	Yolues for Plate Factor (k) to be Used in Formulas 3, 4, 5, and 6	Critical Stress τ' _{er} and σ' _{er}
Compression and shear $\sigma_i = \sigma_i$ $\frac{\tau}{\sigma_i} = \frac{\sigma_i}{\sigma_i}$	when $\sigma \ge 1$ $k = 2 \eta^2 \beta \sqrt{\beta^2 + 3} \left[-1 + \sqrt{1 + \frac{4}{\beta^2 \eta^2}} \right]$ where $\eta = \frac{4}{3} + \frac{1}{a^2}$	$\tau'_{er} = \frac{\sigma_{er}}{\sqrt{\beta^2 + 3}}$
$a = c \cdot b \beta = \sigma_1/\tau$	when $\frac{1}{2} \le a \le 1$ $k = \frac{\eta^2}{2} \left(a + \frac{1}{a^2} \right)^{\frac{2}{\beta}} \sqrt{\beta^2 + 3} \left[-1 + \sqrt{1 + \frac{4}{\beta^2 + \eta^2}} \right]$ where $\eta = \frac{4 a^2 + 5.34}{(a^2 + 1)^2}$	$\sigma'_{\rm cr} = \frac{\beta \ \sigma_{\rm cr}}{\sqrt{\beta^2 + 3}}$
Lompressian and shear $\sigma_2 = 0$	when $a \ge 1$ $k = 3.85 \ \eta^2 \beta \sqrt{\beta^2 + 3} \left[-1 + \sqrt{1 + \frac{4}{\beta^2 \ \eta^2}} \right]$ where $\eta = \frac{5.34 + 4/a^2}{7.7}$	$\tau'_{\rm cr} = \frac{\sigma_{\rm cr}}{\sqrt{\beta^2 + 3}}$
$\alpha = \alpha'b$ $\beta = \sigma_1/\tau$	when $1/2 \le \alpha \le 1$ $k = 3.85 \ \eta^2 \beta \sqrt{\beta^2 + 3} \left[-1 + \sqrt{1 + \frac{4}{\beta^2 \ \eta^2}} \right]$ where $\eta = \frac{4 + 5.34/\alpha^2}{7.7 + 33 \ (1 - \alpha)^3}$	$\sigma'_{\rm cr} = \frac{\beta \sigma_{\rm cr}}{\sqrt{\beta^2 + 3}}$
Compression and shear $a_1 = -a_1$ $a_2 = -a_3$	when $a \ge 1$ $k = 24 \ \eta \sqrt{\beta^2 + 3} \ \sqrt{\frac{1}{1 + \beta^2 \ \eta^2}}$ where $\eta = \frac{2}{9} + \frac{1}{6 \ a^2}$	$\tau'_{\rm cr} = \frac{\sigma_{\rm cr}}{\sqrt{\beta^2 + 3}}$
$\alpha = 0$ b $\beta = 0,/\tau$	when $\frac{1}{2} \le a \le 1$ $k = 24 \eta \sqrt{\beta^2 + 3} \sqrt{\frac{1}{1 + \beta^2 \eta^2}}$ where $\eta = \frac{1}{6} + \frac{2}{9 a^2}$	$\sigma'_{\rm cr} = \frac{\beta \ \sigma_{\rm cr}}{\sqrt{\beta^2 + 3}}$

4. SUMMARY FOR DETERMINING CRITICAL BUCKLING STRESS OF PLATE

1. The value of the plate factor (k) to be used in formula #5 comes from Tables 1, 2 or 3, adapted from "Buckling Strength of Metal Structures", Bleich, pp. 330, 395, 410.

2. Solve for $\sigma_{cr}/\sqrt{\lambda}$ from formula #5.

a. If $\sigma_{cr}/\sqrt{\lambda} = \sigma_p$, this is the value of σ_{cr} , so go to step 4.

b. If $\sigma_{\rm cr}/\sqrt{\lambda} > \sigma_{\rm p}$, go to step 3.

3. Insert this value $(\sigma_{\rm cr}/\sqrt{\lambda})$ into formula #6, and solve for the critical buckling stress $(\sigma_{\rm cr})$.

4. After the critical stress $(\sigma_{\rm cr})$ has been deter-

mined, the critical buckling stress of the given plate $(\sigma'_{re} \text{ or } \tau'_{er})$ is determined from the relationship shown in the right-hand column of Tables 1, 2, or 3.

5. BUCKLING STRESS CURVES (Compression)

In regard to plates subjected only to compression or only to shear, H. M. Priest and J. Gilligan in their "Design Manual for High Strength Steels" show the curve patterns, Figure 5 (compression) and Figure 10 (shear). They have divided the buckling curve into three distinct portions (A-B, B-C, and C-D), and have lowered the critical stress values in the elastic buckling region by 25% to more nearly conform to actual test results.

Values indicated on this typical curve are for ASTM A-7 (mild) steel, having a yield strength of 33,000 psi.

The buckling curve (dashed line) of Figure 2 has been superimposed on the Priest-Gilligan curve for comparison.

TABLE 4—Buckling Stress Formulas (Compression)

Portion of Curve	Factor b∕t √k	Critical Buckling Compressive Stress (σ_{cr}) Determined by
A to B	0 to $\frac{3820}{\sqrt{\sigma_y}}$	$\sigma_{cr} = \sigma_{y}$
B to C	$\frac{3820}{\sqrt{\sigma_y}} \text{ to } \frac{5720}{\sqrt{\sigma_y}}$	$\sigma_{er} = 1.8 \sigma_r - n \frac{b/t}{\sqrt{k}}$ where: $n = \frac{\sqrt{\sigma_r^3}}{4770}$
C to D	$\frac{5720}{\sqrt{\sigma_y}}$ and over	$\sigma_{\rm cr} = \left[\frac{4434}{\left(\frac{b/t}{\sqrt{k}}\right)}\right]^2$

The horizontal line (A to B) is the limit of the yield strength (σ_y) . Here σ_{cr} is assumed equal to σ_y . The curve from B to C is expressed by-

$$\sigma_{\rm cr} = 1.8 \ \sigma_{\rm r} - n \ \frac{(b/t)}{\sqrt{k}} \quad \middle| \quad {\rm where:} \ n = \frac{\sqrt{\sigma_{\rm r}^3}}{4770}$$

The curve from C to D is 75% of the critical buckling stress formula, Figure 1, or:

$$\sigma_{cr} = .75 \frac{k \pi^2 E}{12 (1 - \nu^2)} \left(\frac{t}{b}\right)^2$$
$$= \left[\frac{4434}{\frac{b/t}{\sqrt{k}}}\right]^2$$

All of this is expressed in terms of the factor

$$\sigma_{cr} = \sigma_{r}$$

$$33,000$$

$$\sigma_{cr} = 1.8 \sigma_{r} - n \frac{b/t}{\sqrt{k}}$$
See Table 4.

$$\sigma_{cr} = 1.8 \sigma_{r} - n \frac{b/t}{\sqrt{k}}$$

$$\sigma_{cr} = \frac{4434}{\left(\frac{b/t}{\sqrt{k}}\right)}$$
Fig. 2

$$\sigma_{cr} = \frac{4434}{\left(\frac{b/t}{\sqrt{k}}\right)}$$

$$10,000$$

$$10,000$$

$$10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70$$

Rotio

FIG. 5 Buckling stress curves

for plates in edge compression.

Critical buckling compressive stress (σ_{cr}) for A-7 steel having σ_{r}

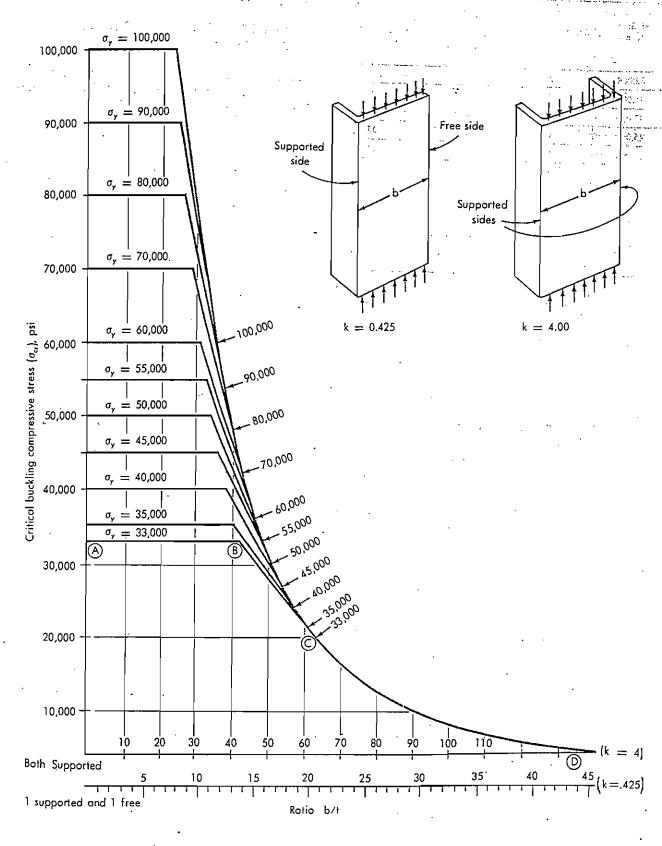


FIG. 6 Buckling stress curves (plates in edge compression) for various steels.

TABLE 5-Factors for Buckling Formulas

Yield Strength of Steel or pai	$\left(\frac{b/t}{\sqrt{k}}\right) \text{ for Point B}$ $= \frac{3820}{\sqrt{\sigma_r}}$	$\frac{\left(\frac{b/t}{\sqrt{k}}\right) \text{ for Point C}}{\frac{5720}{\sqrt{\sigma_r}}}$	$n = \frac{\sqrt{\sigma_y^3}}{4770}$
33,000	21,0	31.5	1260
35,000	20.4	30.6	1370
40,000	19.1	28.6	1680
45,000	18.0	27.0	2000
50,000	17.1	. 25.6	2340
55,000	16.3	24.4	2700
60,000	15.6	23.4	3080
70,000	14.4	21.6	3470
80,000	13.5	20.2	4740
90,000	12,7	19.1	5660
100,000	12.1	18.1	6630

TABLE 6-Limiting Values of b/t (Code)

Side Conditions	Yield Strength o _y psi	AISC	AASHO	AREA
One simply supported; the other free	33,000	13 & 16	12	12
	50,000	11 & 13	_	
Both simply "supported	33,000	44	40 .	40
	50,000	'36	34	32

AISC—American Institute of Steel Construction
AASHO—American Association of State Highway Officials
AREA—American Roilway Engineers Association

Factors needed for the formulas of curves in Figure 5, for steels of various yield strengths, are given in Table 5.

Figure 6 is just an enlargement of Figure 5, with additional steels having yield strengths from 33,000 psi to 100,000 psi.

For any given ratio of plate width to thickness (b/t), the critical buckling stress (σ_{cr}) can be read directly from the curves of this figure.

6. FACTOR OF SAFETY

A suitable factor of safety must be used with these values of b/t since they represent ultimate stress values for buckling.

Some structural specifications limit the ratio b/t to a maximum value (point B) at which the critical buckling stress (σ_{cr}) is equal to the yield strength (σ_r) . By so doing, it is not necessary to calculate the buckling stress. These limiting values of b/t, as specified by several codes, are given in Table 6.

In general practice, somewhat more liberal values

of b/t are recognized. Table 7, extended to higher yield strengths, lists these limiting values of b/t.

7. EFFECTIVE WIDTH OF PLATES IN COMPRESSION

The $20'' \times 4'''$ plate shown in Figure 7, simply supported along both sides, is subjected to a compressive load.

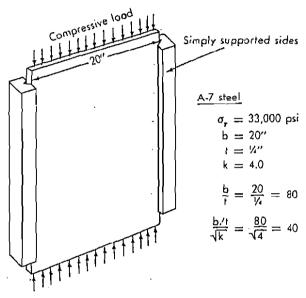


FIGURE 7

Under these conditions, the critical buckling compressive stress ($\sigma_{\rm cr}$) as found from the curve ($\sigma_{\rm r} = 33,000~{\rm psi}$) in Figure 6 is—

$$\sigma_{
m cr} = 12,280$$
 psi

TABLE 7—Usual Limiting Values of b/t

/osual cimiting falaci	
One Edge Simply Supported; the Other Edge Free	Both Edges Simply Supported
13.7	42.0
13.3	40.8
12.5	38.2
11.7 + ^.	36.0
2. 11.1	34.2
10.6	32.6
10.1	31.2
9.4	28.8
8,8	27.0
8.3	25.4
7.9	24.2
	One Edge Simply Supported; the Other Edge Free 13.7 13.3 12.5 11.7 11.1 10.6 10.1 9.4 8,8 8.3

This value may also be found from the formulas in Table 4.

Since the ratio $\frac{b/t}{\sqrt{k}}$ is 40.0 and thus exceeds the value of 31.5 for point C, the following formula must be used—

$$\sigma_{\text{er}} = \left[\frac{4434}{\frac{\text{b/t}}{\sqrt{\text{k}}}}\right]^2 = \left[\frac{4434}{40}\right]^2$$
$$= 12,280 \text{ psi}$$

At this stress, the middle portion of the plate would be expected to buckle, Figure 8. The compressive load at this stage of loading would be—

$$P = A \sigma = (20" \times 44") 12,280$$

= 61,400 lbs

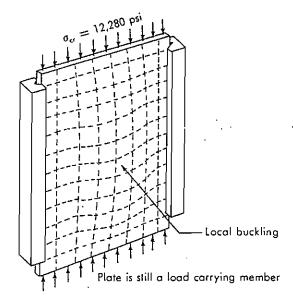


FIGURE 8

The over-all plate should not collapse since the portion of the plate along the supported sides could still be loaded up to the yield point (σ_y) before ultimate collapse.

This portion of the plate, called the "effective width" can be determined by finding the ratio b/t when (σ_{cr}) is set equal to yield strength (σ_y) or point B.

From Figure 6 we find-

$$\frac{b}{t} = 42.0$$

or from Table 4 we find-

$$\frac{b/t}{\sqrt{k}} = 21.0$$

Since k = 4.0 (both sides simply supported), the ratio—

$$\frac{b}{t} = 21.0\sqrt{k}$$
$$= 42.0$$

Since the plate thickness $t=44^{\prime\prime}$ width, b=42.0~t or $b=10.5^{\prime\prime}$.

This is the effective width of the plate which may be stressed to the yield point (σ_r) before ultimate collapse of the entire plate.

The total compressive load at this state of loading would be as shown in Figure 9.

The total compressive load here would be-

$$P = A_1 \sigma_1 + A_2 \sigma_2$$

= $(10\frac{1}{2} \times \frac{1}{4})(33,000) + (9\frac{1}{2} \times \frac{1}{4})(12,280)$
= $115,800$ lbs

Another method makes no allowance for the central buckled portion as a load carrying member, it being assumed that the load is carried only by the supported portion of the plate. Hence the total compressive load would be—

$$P = A_1 \sigma_1$$

= $(10\frac{1}{2} \times \frac{1}{4})(33,000)$
= 86,600 lbs

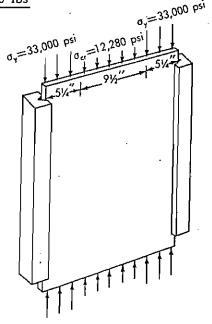


FIGURE 9

2.12-10 / Load & Stress Analysis

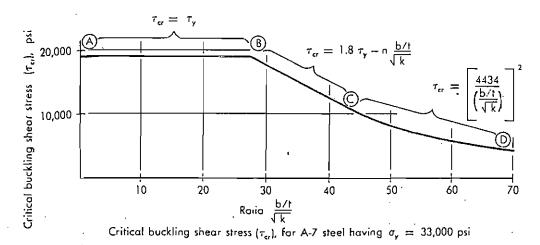


FIG. 10 Buckling stress curves for flate plates in shear.

8. BUCKLING STRESS CURVES (Shear)

The Priest & Gilligan curve, corresponding to Figure 5, when applied to the buckling of plates in shear is shown in Figure 10.

The curve is expressed in terms of $\left(\frac{b/t}{\sqrt{k}}\right)$. See Table 8. Comparison of Figure 10 and Table 8 with Figure 5 and Table 4 reveals the parallelism of critical buckling stress for compression (σ_{cr}) and for shear (τ_{cr}) .

Figure 11 is just an enlargement of Figure 10, with additional steels having yield strengths from 33,000 psi to 100,000 psi. Factors needed for the formulas of curves in Figure 11 are given in Table 9.

For any value of $\left(\frac{b/a}{\sqrt{k}}\right)$ the critical buckling shear stress (τ_{cr}) can be read directly from the curves of this figure.

A suitable factor of safety must be used with these values since they represent ultimate stress values for buckling.

By holding the ratio of $\left(\frac{b/a}{\sqrt{k}}\right)$ to the value at point B, $\tau_{cr} = \tau_r$ and it will not be necessary to compute the critical shear stress (τ_{cr}) . Assuming the edges are simply supported, the value of $k = 5.34 + 4(b/a)^2$ Then using just the three values of b/a as 1 (a square panel), ½ (the length twice the width of panel) and zero (or infinite length), the required b/t value is obtained from Table 10 for steels of various yield strengths. The plate thickness is then adjusted as necessary to meet the requirement.

Notice in Figure 10 and Table 8 that the critical buckling stress in shear is given directly as (τ_{er}) . In Tables 2 and 3 it is given first as (σ_{er}) and then changed to (τ_{er}) .

TABLE 8—Buckling Stress Formulas (Shear)

3			
Portion of Curve	Factor $\frac{b/t}{\sqrt{k}}$	Crítical Buckling Shear Stress (τ _{ct}) Determined by	
A to B	0 to $\frac{3820}{\sqrt{\tau_r}}$	$ au_{ m er} = au_{ m f}$	
B to C	$\frac{3820}{\sqrt{\tau_{\gamma}}} \text{ to } \frac{5720}{\sqrt{\tau_{\gamma}}}$	$\tau_{\rm er} = 1.8 \ \tau_{\rm r} - n \ \frac{b/t}{\sqrt{k}}$ where: $n = \frac{\sqrt{\tau_{\rm y}^4}}{4770}$	
· C to D	$\frac{5720}{\sqrt{\tau_y}}$ and over	$\tau_{\rm er} = \left[\frac{\frac{4434}{5/t}}{\frac{5/t}{\sqrt{k}}}\right]^2$	

TABLE 9—Factors for Buckling Formulas (Shear)

Yield Strength of Steel σ_{r-} psi	Corresponding Shearing Yield Strength 7 = .58 0, psi	$\frac{\frac{b/t}{\sqrt{k}} \text{ for point B}}{= \frac{3820}{\sqrt{\tau_r}}}$	$\frac{b/t}{\sqrt{k}} \text{ for point C}$ $= \frac{5720}{\sqrt{\tau_r}}$	$n = \frac{\sqrt[3]{\tau_y^3}}{4770}$
33,000	19,100	27.6	41.4	550
35,000	20,300	27.6	40.2	610
40,000	23,200	25.1	37.6	740
45,000 :	26,100	23.6	35.4	- 880
50,000	29,000	22.4	33.6	1030
55,000	31,900	21.4	32.1	1200
60,000	34,800	20.5	30.7	1360
70,000	40,600	19.0	28.4	1680
80,000	46,400	17,7	26.6	. 2100
90,000	52,200	16,7	25.1	: 2500
100,000	58,000	15.9	23.8	2920

TABLE 10—Maximum Values of b/t

Maximum Values of b/t to Hold τ_{er} to τ_{r} (Panels with simply supported edges)

Tensile Yield Strength σ_y psi	b/o = 1 (square panel)	b/a = ½ (panel with length twice the width)	b/a = 0 (panel with infinite length)
33,000	84.5	69.6	63.9
35,000	82.0	67.6	62.0
40,000	76.7	63.2	58.0
्र ^क 45,000	72.3	59.6	54.7
50,000	68.6	56.5	51.9
55,000	65.4	53.9	49.5
60,000	62.6	51.6	47.4
70,000	58.0	47.8	43.9
80,000	54.2	44.7	41.0
90,000	51.1	42.1	38.7
100,000	48.5	40.0	36.7

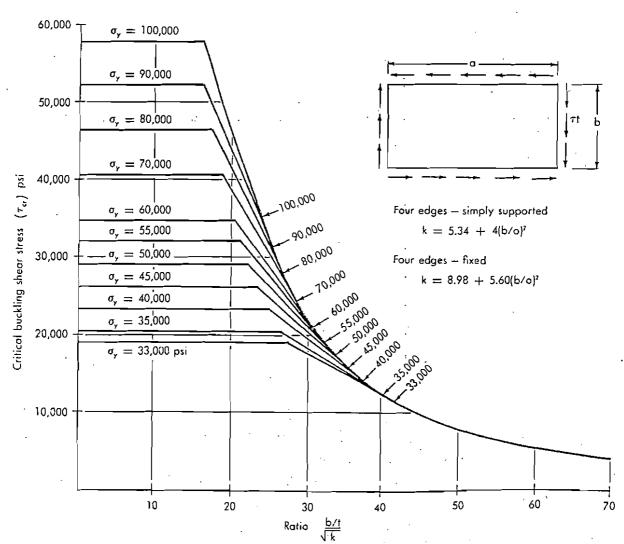
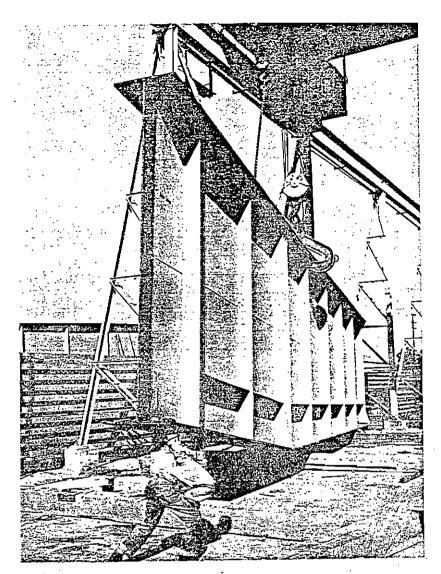


FIG. 11 Buckling stress curves (plates in shear) for various steels.

2.12-12 / Load and Stress Analysis.



United Airlines hangar at San Francisco features double-cantilevered roof over areas into which large jet aircraft are wheeled, nosing up to the 3-story inner "core" for servicing. Center girder section half (at left) is completely shop welded. Large plate girders like this one are stiffened to prevent web buckling due to edge compression. Cantilevered welded plate girders weigh 125 tons.

