CLASSIFICATION NOTES

No. 30.1

BUCKLING STRENGTH ANALYSIS OF BARS AND FRAMES, AND SPHERICAL SHELLS

APRIL 2004

DET NORSKE VERITAS

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FOREWORD

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Amendments and corrections

The main changes are:

This interim edition of Classification Note 30.1 supersedes the July 1995 edition.

The following table shows those Sections that have been deleted, as they are now covered by Recommended Practices and an Offshore Standard:

Deleted Section	*New Reference*
Section 3 - Stiffened flat plates	DNV-RP-C201
Section 4 - Stiffened circular cylindrical shells	DNV-RP-C202
Section 5 - Unstiffened conical shells	DNV-RP-C202
Section 7 - Tolerances	DNV-OS-C401

Sections: 1, 2 and 6 remain unchanged.

It is planned to improve the quality of the figures at the next major revision.

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CONTENTS

1. Buckling of Structures

1.1 Introduction

1.1.1 In the rules for classification of ships (henceforth referred to as the rules), it is required that structural stability shall be provided for the structure as a whole and for each structural member.

1.1.2 There are basically two ways in which a structure may lose its stability. The type of instability shown in Fig.1.l a is known as *snap-through* buckling and is characterized by a load-deflection curve as indicated. The structure collapses when the load is increased beyond the *limit point.*

The other type of instability shown in Fig. l.l b is known as *classical* or *bifurcation* buckling. For relatively small loads, the equilibrium state of the structure is called the *prebuckling state* or the *fundamental state*. When the load is increased a bifurcation point is reached, at which another solution to the equilibrium equations exists. Beyond the bifurcation point the prebuckling path is unstable. The *post buckling* behaviour then depends on the characteristics of the *secondary path*.

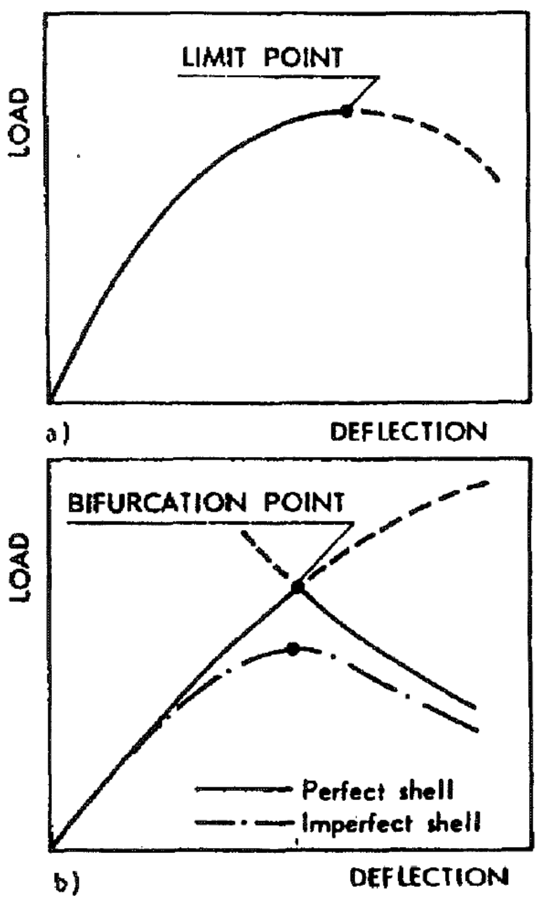


Fig. 1.1
Types of instability

1.1.3 However, if the structure contains an *initial geometric imperfection* in the shape of the buckling mode, the load-displacement curve may be as indicated in Fig. 1.1b. It is seen that an imperfect structure may lose its stability at a limit point that corresponds to a lower load than the bifurcation point of the perfect structure.Whether the bifurcation-point load of the perfect structure is close to the limit-point load of the imperfect structure depends on the shape of the secondary path of the perfect structure.

1.1.4 Because geometric imperfections of various shapes are inevitable in fabricated structures, actual instabilities may be expected to occur at limit points rather than at bifurcation points.

1.2 Buckling strength analysis

1.2.1 Buckling strength analyses are to be based on the characteristic buckling strength for the most unfavourable buckling mode.

1.2.2 The characteristic buckling strength is to be based on the lower 5th percentile of test results. In lieu of more relevant information or more refined analysis, characteristic buckling strength values may be obtained from this Note.

1.2.3 The general procedure for buckling strength analysis according to this Note may be described as follows:

— The state of stress in the structure under consideration is characterized by a *reference stress*, σ. This may be one single stress component, or a defined "equivalent" stress.
— The buckling strength of the structure is defined as the critical value of the reference stress, σ_{cr}.
— The critical stress may be defined relative to the yield stress, σ_F, in such a way that the ratio σ_{cr}/σ_F is determined as a function of the reduced slenderness parameter, λ. A typical buckling strength curve is shown in Fig. 1.2.
— The most general definition of structural slenderness is the reduced slenderness:

$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_E}}$$

where σ_E is the elastic buckling stress. In general σ_E may be determined from classical buckling theory, but for structures which are sensitive to imperfections in the elastic range, σ_E should be modified in such a way that imperfections within specified tolerances are accounted for.
— Typical buckling strength curves are characterized by a plateau, $\sigma_{cr}/\sigma_F = 1.0$, for values of λ less than λ_o, see Fig. 1.2. In such cases it may be concluded that buckling is not relevant when $\lambda < \lambda_o$. For a number of cases it has been found convenient to derive explicit slenderness limitations based on this criterion.

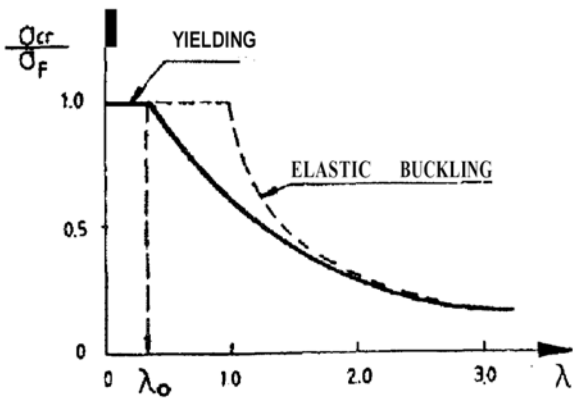


Fig. 1.2
Typical non-dimensional buckling curve

1.3 Usage factor

1.3.1 The buckling stress analysis given in this Note is based on the allowable usage factor method.

1.3.2 The usage factor, η is defined as the ratio between the actual value of the reference stress due to design loading and the critical value of the reference stress, i.e.:

$$\eta = \frac{\sigma}{\sigma_{cr}}$$

1.3.3 The maximum allowable value of the usage factor, η_p is defined in the rules. In general η_p depends on:

— loading condition
— type of structure
— slenderness of structure.

1.4 Fabrication tolerances

1.4.1 The buckling strength of most structures depends on size and shape of geometric imperfections. In general the effect of imperfections is only implicitly incorporated in the formulae for characteristic strength. This means that it has been assumed that the imperfections do not exceed certain limits. These limits are specified in Sec.5 of this Note.

1.4.2 A fabricated structure with imperfections exceeding the limits given in Chapter 7 of this Note should only be accepted if the actual usage factor with respect to buckling is found to be small compared to the allowable usage factor, or if it can be proved by adequate methods that the buckling strength of the imperfect structure is sufficient.

2. Bars and Frames

2.1 Introduction

2.1.1 This chapter treats the buckling of bars and frames. Depending on the loading condition, a bar may be referred to as follows:

Column	bar subject to pure compression
Beam	bar subject to pure bending
Beam-column	bar subject to simultaneous bending and compression.

2.1.2 Buckling modes for bars are categorized as follows (see Fig. 2.1):

Flexural buckling of columns: bending about the axis of least resistance.

Torsional buckling of columns: twisting without bending.*Flexural-torsional buckling of columns*: simultaneous twisting and bending.

Lateral-torsional buckling of beams: simultaneous twisting and bending.

Local buckling: buckling of a thin-walled part of the cross-section (plate-buckling, shell-buckling).

2.1.3 The buckling mode which corresponds to the lowest buckling load is referred to as the critical buckling mode.

2.1.4 Flexural buckling may be the critical mode of a slender column of doubly symmetrical cross-section or one which is not susceptible to, or is braced against twisting.

2.1.5 Torsional buckling may be the critical mode of certain open, thin-walled short columns in which shear centre and centroid *coincide* (doubly-symmetrical I-shapes, anti-symmetrical Z-shapes, cruciforms etc.).

2.1.6 Flexural-torsional buckling may be the critical mode of columns whose shear centre and centroid *do not coincide* and which are torsionally weak (thin-walled open sections in contrast to closed thick-walled or solid shapes). It should be emphasized that flexural-torsional buckling analysis is only needed when it is physically possible for such buckling to occur.

2.1.7 Lateral-torsional buckling may be the critical mode when a beam is subjected to bending about its strong axis and not braced against bending about the weak axis.

2.1.8 In this Note it is assumed that the cross-section of the member under consideration has at least one axis of symmetry (Z axis). Members with arbitrary cross-sections are subject to special considerations.

2.1.9 The following symbols are used without a specific definition in the text where they appear:

A = cross-sectional area.
E = Young's modulus.
G = shear modulus ($G = E/2\,(1 + \nu)$)
I = moment of inertia.
σ_F = yield stress of the material as defined in the rules.
ν = Poisson's ratio.

2.2 Characteristic buckling resistance

2.2.1 The characteristic buckling resistance of a compression member, σ_{cr}, is determined by use of the reduced slenderness, λ.

2.2.2 The reduced slenderness, λ, is defined by:

$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_E}}$$

where σ_E is the elastic buckling stress for the buckling mode under consideration.

2.2.3 A compression member is defined as “stocky” if the reduced slenderness with respect to the critical column buckling mode is less than 0,2.

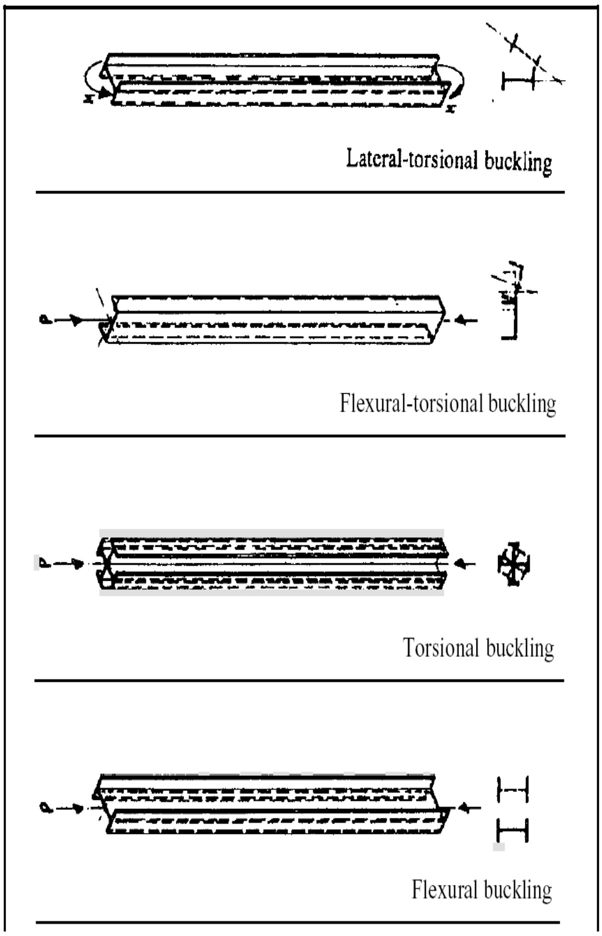


Fig. 2.1
Buckling modes of columns and beams

2.2.4 Non-dimensional buckling curves are given in Fig.2.2. For computations σ_{cr}/σ_F may be obtained form:

— If $\lambda \leq \lambda_o$

$$\frac{\sigma_{cr}}{\sigma_F} = 1.0$$

If $\lambda > \lambda_o$

$$\frac{\sigma_{cr}}{\sigma_F} = \frac{1 + \mu + \lambda^2 - \sqrt{(1 + \mu + \lambda^2)^2 - 4\lambda^2}}{2\lambda^2}$$

where

$\mu = \alpha(\lambda - \lambda_o)$

The coefficients α and λ_o are given in Table 2-1.

Table 2-1 Numerical values of λ_o and α

Curve	λ_o	α
a	0.2	0.20
b	0.2	0.35
c	0.2	0.5
d	0.2	0.65
e	0.6	0.35

2.2.5 Fig. 2.2 shows the assignment of commonly used structural sections to column curves “a”, “b” or “c”. Curve “d” is a non-dimensional buckling curve for sniped plate stiffeners which is referred to in 3.4.5. Curve “e” applies to lateral-torsional buckling of beams.

2.2.6 The yield stress to be used is that of the most highly compressed part of the cross section during buckling. The governing thicknesses in each case are shown in Fig. 2.3.

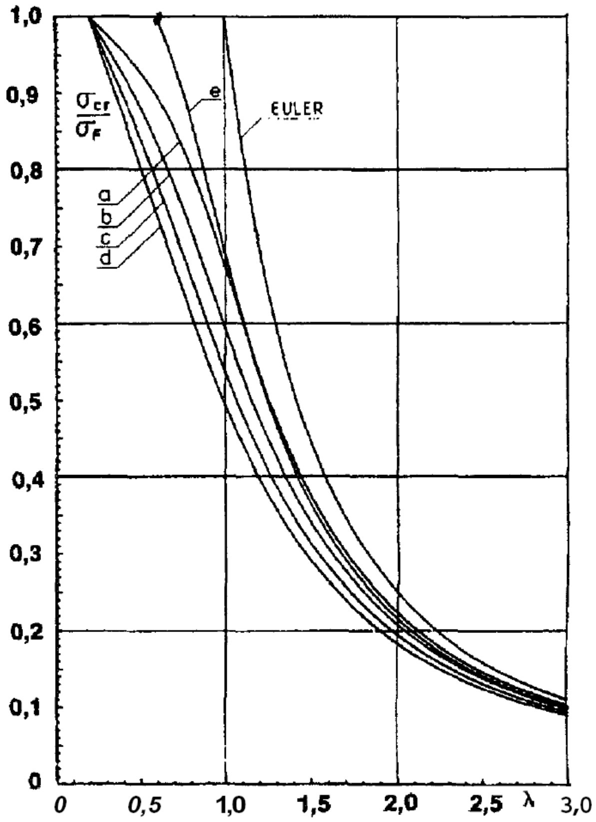
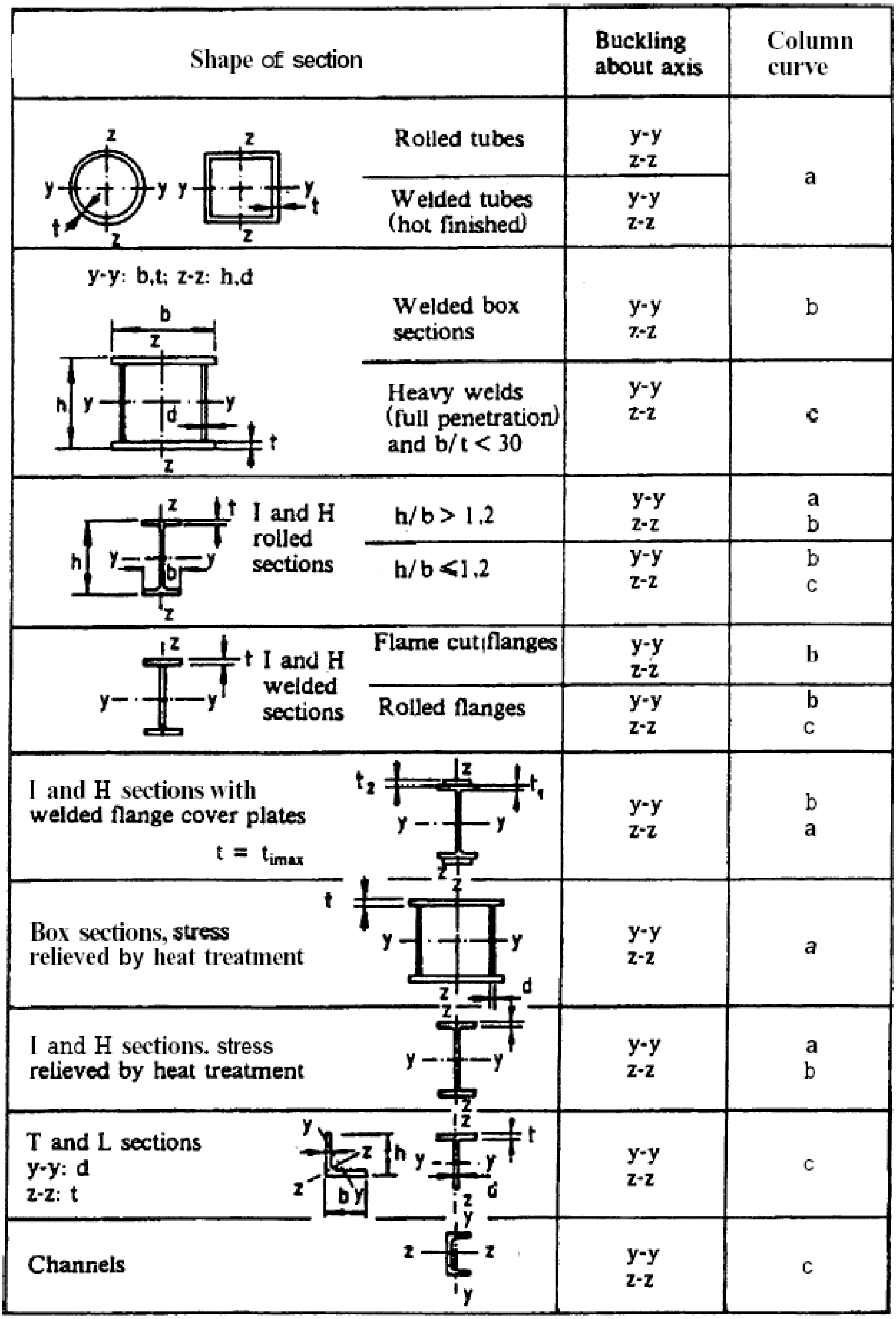


Fig. 2.2
Non-dimensional buckling curves



Shape of section		Buckling about axis	Column curve
	Rolled tubes	y-y z-z	a
	Welded tubes (hot finished)	y-y z-z	a
y-y: b,t; z-z: h,d	Welded box sections	y-y z-z	b
	Heavy welds (full penetration) and b/t < 30	y-y z-z	c
I and H rolled sections	h/b > 1,2	y-y z-z	a b
I and H rolled sections	h/b ≤ 1,2	y-y z-z	b c
I and H welded sections	Flame cut flanges	y-y z-z	b
I and H welded sections	Rolled flanges	y-y z-z	b c
I and H sections with welded flange cover plates $t = t_{imax}$		y-y z-z	b a
Box sections, stress relieved by heat treatment		y-y z-z	a
I and H sections. stress relieved by heat treatment		y-y z-z	a b
T and L sections y-y: d z-z: t		y-y z-z	c
Channels		y-y z-z	c

Fig. 2.3
Column selection chart

2.2.7 A section may be considered as "compact" for the purpose of this Note if the reduced slenderness with respect to local buckling of any part of the section is less than:

- 0.7 for plane parts of the cross section
- 0.5 for curved parts of the cross section.

Methods for evaluation of the reduced slenderness with respect to local buckling are given in the subsequent chapters. In cases where only the axial stress component is different from zero or of any significance for local buckling, the cross section may be considered as compact if the following requirements are satisfied (see Fig. 2.4):

Outstands:

$$\frac{f}{t} \le 0.4 \sqrt{\frac{E}{\sigma_F}}$$

Compressed flange in a box girder:

$$\frac{a}{t} \le 1.35 \sqrt{\frac{E}{\sigma_F}}$$

Web plate with linear distribution of axial stresses:

$$\frac{h}{d} \le (3.35 - \sqrt{2(1+\psi)}) \sqrt{\frac{E}{\sigma_F}} \quad \text{for } -1 \le \psi \le 1$$

Tubular cross sections:

$$\frac{D}{t} \le \frac{E}{9\sigma_F}$$

Shear buckling of the web plate at a position with only shear stresses may be disregarded if:

$$\frac{h}{d} \le 2.0 \sqrt{\frac{E}{\sigma_F}}$$

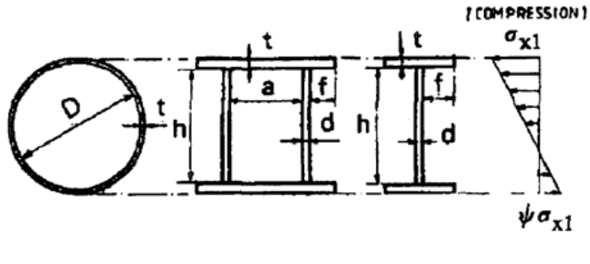


Fig. 2.4
Cross sectional parameters

2.2.8 In cases where the geometric proportions are such that local instability may occur, the yield stress must be substituted by the characteristic local buckling stress. Local instability need not be considered for "compact" sections as defined in 2.2.7.

2.2.9 The requirements given in 2.2.7 are not sufficient to secure development of full plastic hinges, which is a basic assumption in connection with plastic design methods.

2.2.10 A compression member which may be defined as both "stocky" and "compact" is not susceptible to buckling.

2.3 Column buckling

2.3.1 For members which are not susceptible to local buckling, there are three different buckling modes to be considered:

- flexural buckling
- torsional buckling
- flexural-torsional buckling.

The characteristic buckling stress, σ_{acr}, for members subjected to pure compression is the buckling stress corresponding to the critical buckling mode.

2.3.2 For members which may fail by flexural buckling, see 2.1.4, the buckling stress is obtained from Fig. 2.2 with λ defined by:

$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_E}} = \frac{\lambda_k}{\pi}\sqrt{\frac{\sigma_F}{E}}$$

$$\sigma_E = \frac{\pi^2 E}{\lambda_k^2} = \text{Euler stress}$$

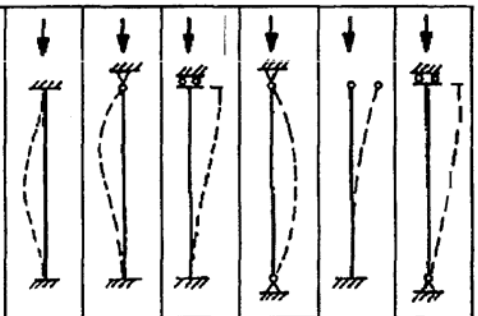
$$\lambda_k = \frac{l_e}{i}$$

$$l_e = Kl = \text{effective length}$$

$$i = \sqrt{\frac{I}{A}} = \text{radius of gyration.}$$

Recommended values for K are given in Table 2-2 for a number of cases. For compression members in frames, see 2.6.

Table 2-2 Effective length factors. Theoretical values and recommended values when ideal conditions are approximated.

Buckled shape of column						
Theoretical value	0,5	0,7	1,0	1,0	2,0	2,0
Recommended design value	0,6	0,8	1,2	1,0	2,1	2,0

2.3.3 For members which may fail by torsional buckling, see 2.1.5, the buckling stress is obtained from Fig. 2.2 curve "e" with λ defined by:

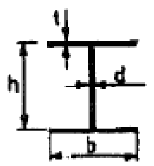
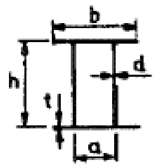
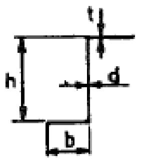
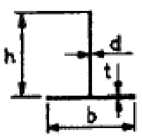
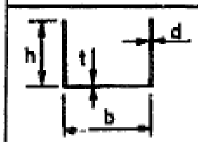
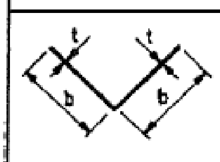
$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_{ET}}}$$

$$\sigma_{ET} = \frac{GI_T}{I_p} + \frac{\pi^2 EC_W}{I_p l_e^2} = \text{elastic torsional bucklin stress}$$

l_e = K = effective length with respect to warping
I_p = polar moment of inertia about the shear centre
I_T = St. Venant torsional constant
C_W = warping constant.

The parameters I_T and C_W are given in Table 2.3 for commonly used cross sections.

Table 2-3 Cross sectional properties

Doubly-symmetrical sections	
	$I_T = \frac{1}{3}(2bt^3 + hd^3)$ $C_W = \frac{h^2 b^3 t}{24}$
	$I_T = \frac{2a^2h^2}{\frac{a}{t} + \frac{h}{d}} + \frac{2t^3(b-a)}{3}$ $C_W = \frac{(h^2 I_1 - a^2 I_2)^2}{2(h^2 I_1 + a^2 I_2)}$ where $I_1 = \frac{1}{12} t b^3$ $I_2 = \frac{1}{12} d h^3$
Point-symmetrical sections	
	$I_T = 2\pi r^3 t$ $C_W = 0$
	$I_T = \frac{1}{3}(2bt^3 + hd^3)$ $C_W = \frac{h^2 b^3 t (bt + 2hd)}{12(2bt + hd)}$
Singly-symmetrical sections	
	$I_T = \frac{1}{3}(2bt^3 + hd^3)$ $C_W = \frac{b^3 t^3 + 4h^3 d^3}{144}$
	$I_T = \frac{1}{3}(bt^3 + 2hd^3)$ $C_W = \frac{b^2 h^3 d(3hd + 2bt)}{12(6hd + bt)}$
	$I_T = \frac{2bt^3}{3}$ $C_W = \frac{b^3 t^3}{18}$

2.3.4 In lieu of more accurate analysis, σ_{ET} may be taken as:

$$\sigma_{ET} = \frac{\pi^2 E I_f}{A_e l_e^2}$$

I_f = moment of inertia of flange, see 2.5
A_e = effective cross sectional area, see Fig. 2.5.

(This simplified approach yields for doubly-symmetrical H- and I-shape sections:

$$\sigma_{ET} \sim \sigma_E$$

where σ_E is the Euler stress for lateral buckling about the weak axis. This result is also obtained from 2.3.3 under the assumption that $I_T = 0$ and $I_p = A\,(h/2)^2$.)

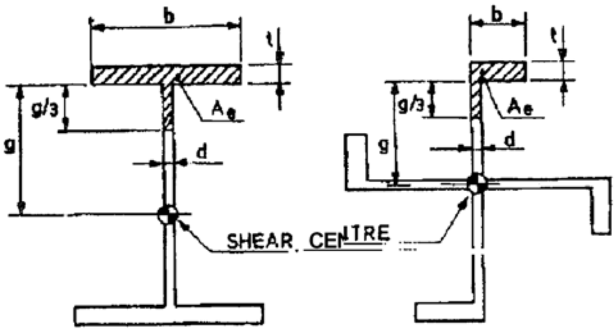


Fig. 2.5
Cross-sectional properties to be used for simple evaluation of the torsional buckling strength

a) $I_f = \frac{1}{12} t b^3$

b) $I_f = \frac{1}{12} t b^3 \left(\frac{1 + 4\left(g\frac{d}{b}\right)t}{1 + \left(g\frac{d}{b}\right)t} \right)$

2.3.5 For members with one axis of symmetry (z-axis) and which may fail by flexural-torsional buckling, see 2.1.6, the buckling stress is obtained from Fig. 2.2 curve “b” with λ defined by:

$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_{EFT}}}$$

$$\sigma_{EFT} = \frac{1}{2\beta}\left[(\sigma_E + \sigma_{ET}) - \sqrt{(\sigma_E + \sigma_{ET})^2 - 4\beta\sigma_E\sigma_{ET}}\right]$$

$$\beta = 1 - \frac{z_o^2 A}{I_p}$$

σ_E = Euler stress for buckling about the z-axis.
σ_{EF} = elastic torsional buckling stress.
σ_{EFT} = elastic flexural-torsional buckling stress.
A = cross sectional area.
I_p = polar moment of inertia about the shear centre.
z_o = distance from centroid to shear centre along the z-axis.

2.3.6 The usage factor for members subjected compression is defined by:

$$\eta = \frac{\sigma_a}{\sigma_{acr}}$$

The maximum allowable value of the usage factor, η_p, is defined in the rules (Type 3 structure).

2.4 Lateral-torsional buckling of beam

2.4.1 A beam which is subjected to bending about its strong axis (y-axis) and not restrained against buckling about the weak axis (z-axis) may fail by lateral-torsional buckling. Failure takes place when the largest compression stress reaches a critical value, σ_{bcr}, which is given by:

$$\sigma_{bcr} = \sigma_v$$

The lateral torsional buckling stress, σ_v, may be obtained from curve "e" in Fig. 2.2 by use of the reduced slenderness with respect to lateral-torsional buckling, λ_v.

2.4.2 The reduced slenderness with respect to lateral-torsional buckling is defined by:

$$\lambda_v = \sqrt{\frac{\sigma_F}{\sigma_{EV}}}$$

$$\sigma_{EV} = \frac{\pi^2 E I_z c}{Z_{yc} l_e^2} = \text{elastic lateral-torsional buckling stress}$$

l_e = Kl = *effective length with respect to warping*
Z_{yc} = section modulus with respect to compression flange
I_z = moment of inertia about the weak axis
c = parameter depending on geometric proportions, bending moment distribution and position of load with respect to the neutral axis.

2.4.3 For a beam with constant bending moment (bending moments applied at the ends):

$$c^2 = \frac{C_w}{I_z} + \frac{I_T}{I_z}\frac{l_e^2}{2\pi^2(1+\nu)}$$

C_w = warping constant
I_T = St. Venant torsional constant.

The parameters C_w and I_T are shown in Table 2.3 for commonly used cross sections.

2.4.4 In lieu of more accurate analysis, σ_{EV} may be taken as:

$$\sigma_{EV} = \frac{\pi^2 E I_{zc} h}{Z_{yc} l_e^2}$$

I_{zc} = moment of inertia of the compression flange (for doubly-symmetrical H- and I-shape sections $I_{zc} = I_z/2$)
h = web height.

2.4.5 The simplified approach given in 2.4.4 yields for doubly-symmetrical H- and I-shape sections:

$$\sigma_{EV} \sim \sigma_E$$

where σ_E is the Euler stress for lateral buckling about the weak axis. This result is also obtained from 2.4.2 and 2.4.3 under the assumption that $I_T = 0$ and $Z_{yc} = Ah / 2$ (see also 2.3.4).

2.4.6 Lateral-torsional buckling need not be considered if:

$$l_v < 0.6 \text{ or } l_e < l_{eo}$$

$$l_{eo} = 0.55b\sqrt{\frac{A_c h}{Z_{yc}}\frac{E}{\sigma_F}}$$

l_e = laterally unsupported length
A_c = cross sectional area of compression flange
b = width of compression flange.

2.4.7 Lateral supports of the compression flange are to be designed for 2% of the total compression force that exists in the compression flange.

2.4.8 The usage factor for members subjected to pure bending is defined by:

$$\eta = \frac{\sigma_b}{\sigma_{bcr}}$$

The maximum allowable value of the usage factor, η_p, is defined in the rules (Type 2 structure).

2.5 Buckling of beam-columns

2.5.1

In lieu of more refined analysis, the usage factor for members subjected to compression and bending may be taken as:

$$\eta = \frac{\sigma_a}{\sigma_{acr}} + \frac{\alpha\sigma_b}{\left(1 - \frac{\sigma_a}{\sigma_E}\right)\sigma_{bcr}}$$

σ_a = Axial stress due to compression.
σ_b = Effective axial stress due to bending.

Bending about weak (z-axis) or strong axis (y-axis) see options for σ_{bcr}.

For compression members which are braced against joint translation, σ_b is the maximum bending stress within the middle third of the length of the member, see Fig. 2.6.

σ_{acr} = Characteristic buckling stress for axial compression as defined in 2.3.
σ_E = Euler buckling stress always about weak axis (z-axis).
σ_{bcr} = Characteristic buckling stress for pure bending as defined in 2.4.1. If bending about weak axis (z-axis) then $\sigma_{bcr} = \sigma_F$.
α = Coefficient depending on type of structure and reduced slenderness according to the rules. Reduced slenderness as calculated critical for σ_{acr}.

2.5.2

For doubly symmetrical H- and I-shape and rectangular box sections which are subjected to simultaneous axial compression and bending about two axes, the usage factor may be taken as:

$$\eta = \frac{\sigma_a}{\sigma_{acr}} + \frac{\alpha\sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma_E}\right)\sigma_{bcr}} + \frac{\alpha\sigma_{bz}}{\left(1 - \frac{\sigma_a}{\sigma_E}\right)\sigma_F}$$

σ_{by} = effective axial stress due to bending about strong axis (y-axis).
σ_{bz} = effective axial stress due to bending about weak axis (z-axis).

Otherwise notation as under 2.5.1.

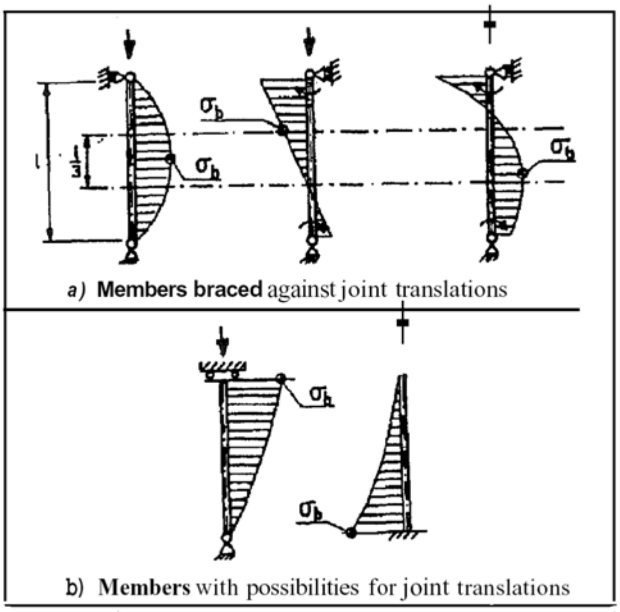


Fig. 2.6
Effective bending stress for beam-columns

2.5.3 The maximum allowable value of the usage factor, η_p, is defined in the rules (Type 3 structure).

2.5.4 When the buckling analyses of a beam-column has been carried out by use of an effective bending stress, σ_b, which is less than the maximum bending stress, it is necessary to evaluate the usage factor with respect to yielding at the position of maximum bending stress.

2.6 Buckling of frames

2.6.1 Effective length factors for columns in a frame may be determined by computing the critical load for the complete frame or for a portion of the frame. The physical significance of the effective length, $l_e = Kl$, is illustrated in Fig. 2.7. For the case shown in this figure, the value of K exceeds 2.0.

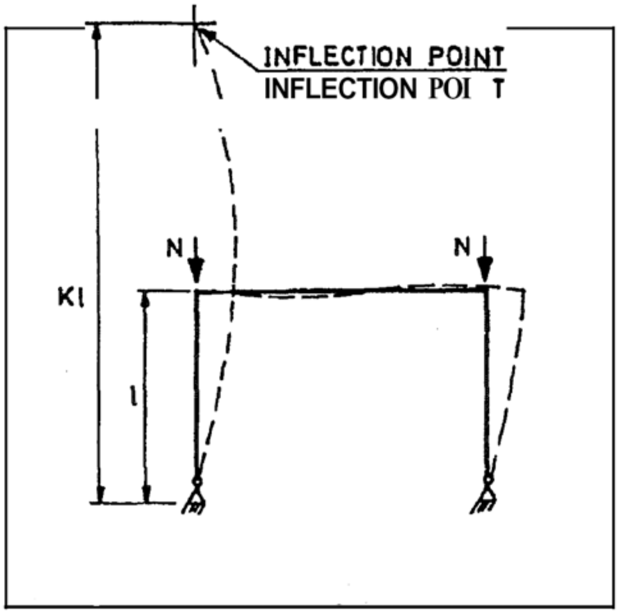


Fig. 2.7
The significance of effective length, K*l*

2.6.2 A procedure for determining K for braced and unbraced rectangular frames is based on the use of the alignment charts, shown in Fig. 2.8. At each end of the compressed member the following parameter is defined:

$$G = \frac{\sum \frac{I_c}{l_c}}{\sum \frac{I_b}{l_b}}$$

Σ indicates summation for all members rigidly connected to that joint and lying in the plane in which buckling of the column is being considered.

I_c, l_c	=	moment of inertia and length of the compressed members (columns)
I_b, l_b	=	moment of inertia and length of the uncompressed members (beams).

2.6.3 Having determined G_A and G_B for end A and end B of the member under consideration, K is obtained by constructing a straight line between the appropriate points on the scales for G_A and G_B.

2.6.4 The alignment charts given in Fig. 2.8 are based on the following assumptions:

— behaviour is elastic
— all members have constant cross section
— all joints are rigid
— for the sidesway prevented case, rotations at the far ends of restraining beams are equal in magnitude but opposite in sense to the joint rotations at the columns ends (single curvature bending), see Fig. 2.8a
— for the sidesway unprevented case, rotations at the far ends of the restraining members are equal in magnitude and in the same sense as the joint rotations at the column ends (reverse curvature bending), see Fig. 2.8b

— the column stiffness parameter $l\sqrt{\frac{P}{EI}}$ must be identical for all columns

— the restraining moments provided by the beams at an end of a column are distributed between the columns in the ratio of the I/l values of the columns

— all columns in the frame buckle simultaneously.

2.6.5 The alignment charts shown in Fig. 2.8 may be used in more general cases, provided that G is determined as:

$$G = \frac{\sum \frac{I_c}{l_c}}{\sum \frac{I_b}{l_b}}$$

where α is a correction factor depending on the boundary conditions at the far end on the beam, see Fig. 2.9.

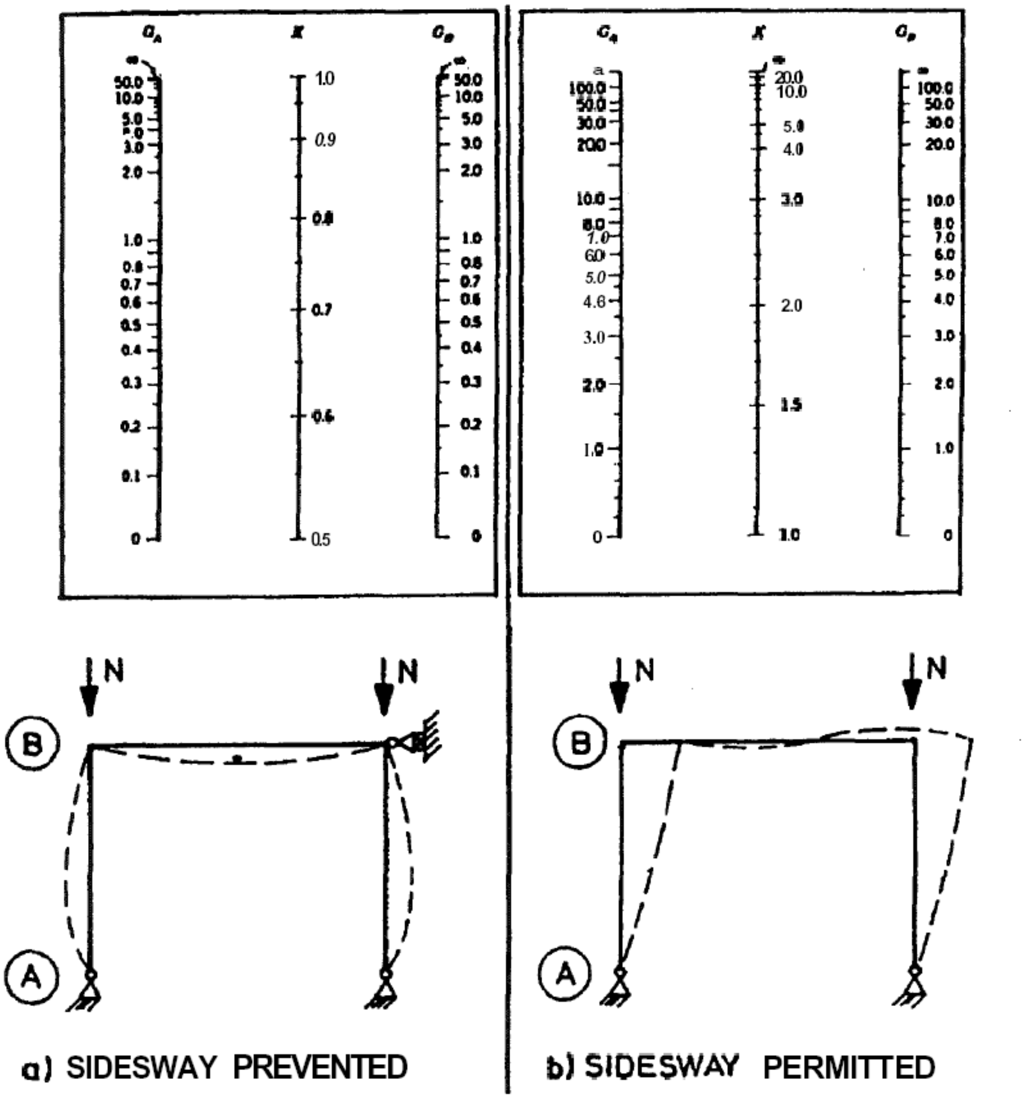


Fig. 2.8
Alignment charts to be used for evaluation of the effective length factor, K

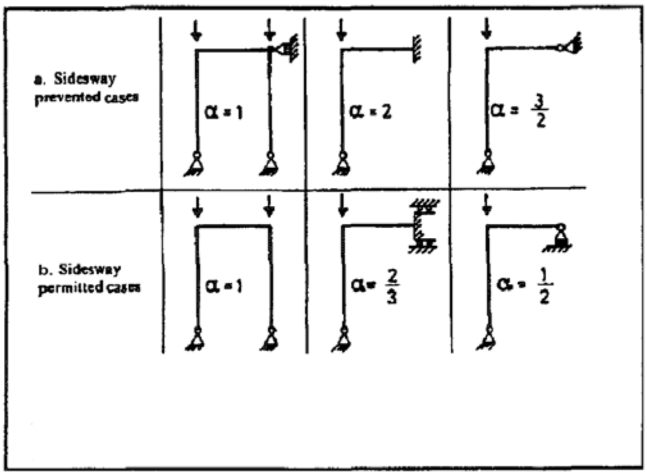


Fig. 2.9
The modification factor α for different boundary conditions

2.6.6
When the moment connections between columns and beams are not fully rigid, G must be determined as:

$$G = \frac{\sum \frac{I_c}{l_c}}{\sum \alpha\beta \frac{I_b}{l_b}}$$

where β is a correction factor depending on the relative joint rigidity:

$$\beta = \frac{1}{1 + \frac{C_b}{C_j}}$$

C_b = beam stiffness parameter
C_j = joint stiffness parameter.

The beam stiffness parameter is given by:

— For the sidesway prevented case:

$$C_b = 2\alpha \frac{EI_b}{l_b}$$

— For the sidesway permitted case:

$$C_b = 6\alpha \frac{EI_b}{l_b}$$

For the T-joint in-plane bending case shown in Fig. 2.10, the joint stiffness parameter may be taken as:

$$C_j = 0.43ER^3\left(\frac{T}{R} - 0.01\right)^{\left(2.35 - 1.5\frac{r}{R}\right)}$$

For the T-joint out-of-plane bending case shown in Fig. 2.10, the joint stiffness parameter may be taken as:

$$C_j = 0.216\,ER^3\left(1.59 - \frac{r}{R}\right)\left(\frac{T}{R} - 0.02\right)^{\left(2.45 - 1.6\frac{r}{R}\right)}$$

The expressions for C_j are only valid if:

$0.33 < r/R < 0.8$ and $10 < R/T < 30$

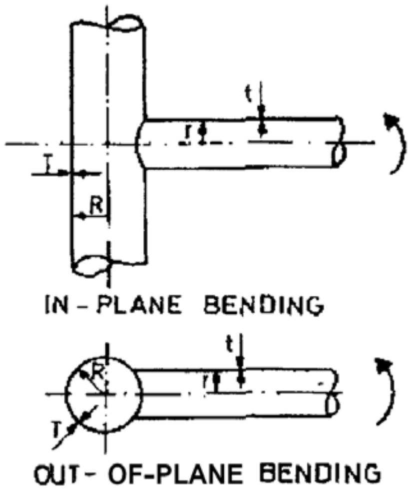


Fig. 10
Tubular T-joints subjected to bending

2.7 Overall buckling of built-up members

2.7.1 A built-up member is here assumed to be composed of two or more sections (chords) separated from one another by intermittent transverse connecting elements (bracings), see Fig. 2.11. It is assumed that all connections are welded.

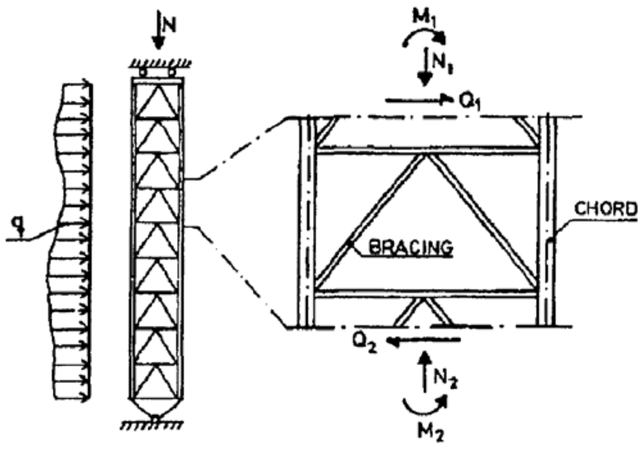


Fig. 11
Built-up compression member

2.7.2 Overall buckling of a built-up member corresponds to flexural buckling of a homogenous member. The characteristic buckling stress may be obtained from Fig. 2.2 with λ defined by:

$$\lambda = \sqrt{\frac{\sigma_F}{\sigma_E}}$$

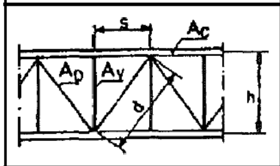
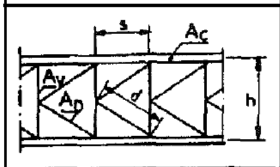
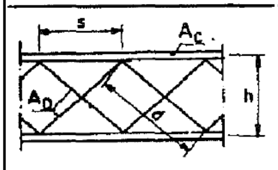
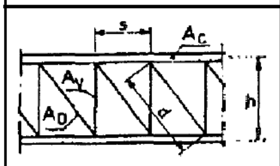
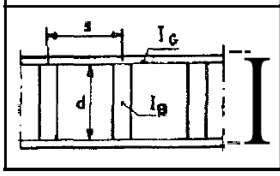
$$\sigma_E = \frac{\pi^2 E}{(1+\omega)\lambda_k^2} = \text{Euler stress}$$

$$\omega = 2\pi^2(1+\upsilon)\frac{I}{l_e^2 A_Q}$$

= shear stiffness modification factor

λ_k	=	l_e = column slenderness of the built-up member regarded as homogenous
i	=	$\sqrt{\frac{I}{A}}$ = radius of gyration
l_e	=	Kl = effective length
A, A_Q, I	=	cross sectional area, effective shear area, and moment of inertia, see Tables 2.4 and 2.5
K	=	effective length factor to be determined in accordance with 2.3.2 or 2.6.3.

Table 2-4 Equivalent shear area of plane built-up members

	Equivalent shear area
	$A_Q = \frac{(1+\nu)sh^2}{\frac{d^3}{2A_D}+\frac{s^3}{3A_C}}$
	$A_Q = \frac{(1+\nu)sh^2}{\frac{d^3}{A_D}+\frac{h^3}{8A_V}+\frac{s^3}{12A_C}}$
	$A_Q = \frac{(1+\nu)sh^2}{\frac{d^3}{A_D}-\frac{s^3}{12A_C}}$
	$A_Q = \frac{(1+\nu)sh^2}{\frac{d^3}{2A_D}+\frac{h^3}{2A_V}+\frac{s^3}{12A_C}}$
	$A_Q = \frac{48(1+\nu)I_G}{s^2\left(1+2\frac{d}{s}\frac{I_G}{I_B}\right)}\left(\frac{d}{h}\right)$

2.7.3 In addition to the overall buckling analysis, it is necessary to carry out buckling analysis for each single element of the built-up member.

2.7.4 If the characteristic buckling stress of a single chord element is less than the yield stress, the overall buckling analysis is to be based on a reduced yield stress equal to the characteristic buckling stress of this chord element.

2.7.5 Bracing members, see Fig. 2.11, shall be designed to resist the effect of an overall shear force, Q_d, given by:

$$Q_d = Q + Q_o$$

where Q is the maximum shear force due to design loading and Q_o is defined by:

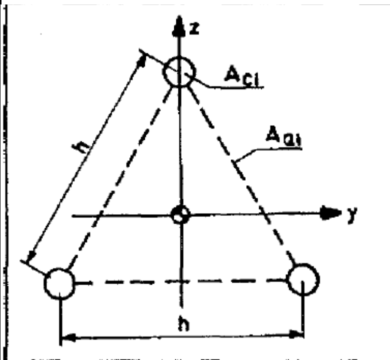
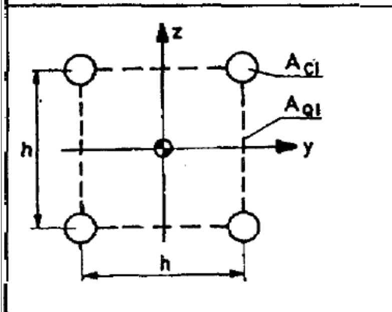
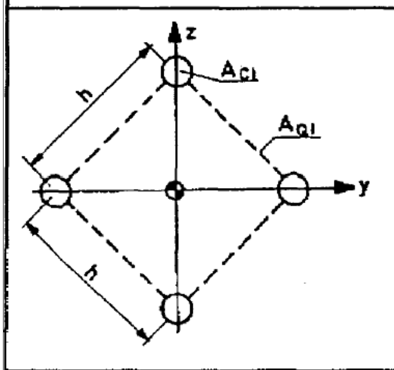
$$Q_o = \pi \frac{P}{P_E - P}\frac{M_{max}}{Kl}\cos\frac{\pi}{Kl}x$$

P	=	average axial force in each leg
P_E	=	Euler buckling stress for the beam column
M_{max}	=	maximum 1st order bending moment i.e. due to:

— lateral load
— eccentric axial load
— initial def. out of straightness

σ_{cr}	=	characteristic overall buckling stress of the built-up member determined in accordance with 2.7.2
x	=	distance from zero bending moment.

Table 2-5 Cross-sectional properties of three-dimensional built-up members

	$A = 3A_{Ci}$ $A_{Qy} = A_{Qz} = \frac{3}{2}A_{Qi}$ $I_y = I_z = \frac{1}{2}A_{Ci}h^2$ $I_T = \frac{1}{4}A_{Qi}h^2$
	$A = 4A_{Ci}$ $A_{Qy} = A_{Qz} = 2A_{Qi}$ $I_y = I_z = A_{Ci}h^2$ $I_T = A_{Qi}h^2$
	$A = 4A_{Ci}$ $A_{Qy} = A_{Qz} = 2A_{Qi}$ $I_y = I_z = A_{Ci}h^2$ $I_T = A_{Qi}h^2$

3. Unstiffened Spherical Shells

3.1 Introduction

3.1.1 This chapter treats the buckling of unstiffened spherical shells and dished end closures.

3.1.2 The following symbols are used without a specific definition in the text where they appear:

E = Young's modulus
N = axial load
p = lateral pressure
r = middle radius of the shell
t = shell thickness
σ_F = yield stress of the material as defined in the rules.

3.2 Stresses

3.2.1 Spherical shells are usually designed to resist lateral pressure. For a complete spherical shell subjected to uniform lateral pressure the state of stress is defined by the principal membrane stresses, σ_1 and σ_2, defined by:

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

3.2.2 For the spherical shell segment shown in Fig. 6.1, the meridional membrane stress is given by:

$$\sigma_\phi = \frac{pr}{2t}\frac{\sin(\phi+\alpha)\sin(\phi-\alpha)}{(\sin\phi)^2} + \frac{N}{2\pi rt(\sin\phi)^2}$$

The circumferential membrane stress is given by:

$$\sigma_\theta = \frac{pr}{t} - \sigma_\phi$$

If the axial force, N, is due to end pressure alone, the stresses are given by:

$$\sigma_\phi = \sigma_\theta = \frac{pr}{2t}$$

These equations are only valid if the edges are reinforced. If the axial force is due to end pressure only, the required cross-sectional area of the reinforcement is:

$$A = \frac{\sin 2\alpha}{2(1-\upsilon)}rt$$

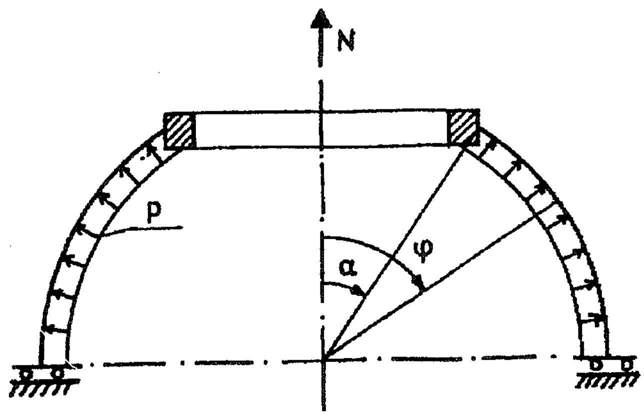


Fig. 6.1
Spherical shell segment

3.3 Shell buckling, general

3.3.1 Buckling of an unstiffened spherical shell occurs when the largest compressive principal membrane stress, σ_1, reaches a critical value, σ_{cr}. The critical stress may be taken as:

$$\sigma_{cr} = \frac{\sigma_F}{\sqrt{1-\psi+\psi^2+\lambda^4}}$$

where

$\psi = \sqrt{\frac{\sigma_2}{\sigma_1}}$ = stress ratio ($-1 \le \psi \le 1$)

$\lambda = \sqrt{\frac{\sigma_F}{\sigma_E}}$ = reduced slenderness

σ_1 = largest compressive principal membrane stress
σ_2 = principal membrane stress normal to σ_1 (compressive or tensile)
σ_E = elastic buckling stress.

3.3.2 The usage factor for shell buckling is defined by:

$$\eta = \frac{\sigma_1}{\sigma_{cr}}$$

The maximum allowable value of the usage factor, η_p, is defined in the rules (Type 5 structure).

3.3.3 The elastic buckling stress σ_E may be taken as:

$$\sigma_E = 0.606\rho E\frac{t}{r}$$

In lieu of more detailed information ρ may be taken as:

$$\rho = \frac{0.5}{\sqrt{1+\frac{1}{100(3-2\psi)}\frac{r}{t}}}$$

3.3.4 For a complete sphere subjected to uniform external pressure the stress ratio is $\psi = 1$, and the expressions given above for σ_{cr} and ρ may be written as:

$$\sigma_{cr} = \frac{\sigma_F}{\sqrt{1+\lambda^4}}$$

$$\rho = \frac{0.5}{\sqrt{1+\frac{\frac{r}{t}}{100}}}$$

3.4 Buckling of dished ends convex to pressure

3.4.1 Hemispherical ends are to be designed as a complete sphere under uniform pressure.

3.4.2 Torispherical ends are to be designed as a complete sphere with radius equal to the crown radius. However, the thickness should not be less than 1.2 times the thickness required for a structure of the same shape subjected to internal pressure.

3.4.3 Ellipsoidal ends are to be designed as a complete sphere with radius equal to r^2/H, where H is the short axis and r is the long axis of the ellipsoid. However, the thickness should not be less than 1.2 times the thickness required for a structure of the same shape subjected to internal pressure.