Deflection of Curved Beams

1. AREA MOMENT METHOD FOR CURVED CANTILEVER BEAM

In Sect. 2.5, Figures 20 to 23, the area moment method was used to find the deflection of a straight cantilever beam of variable section. This same method may be extended to a curved cantilever beam of variable section.

As before, the beam is divided into 10 segments of equal length (s) and the moment of inertia (I_n) is determined for each segment. See Figure 1.

The moment applied to any segment of the beam is equal to the applied force (P) multiplied by the distance (X_n) to the segment, measured from and at right angles to the line passing through and in the same direction as the load (P).

This moment (M_n) applied to the segment causes it to rotate (θ_n), and—

$$\boxed{\theta_n = \frac{M_n}{E\ I_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The resulting deflection (Δ_n) at the point of the beam where the deflection is to be determined is equal to the angle of rotation of this segment(θ_n) multiplied by the distance (Y_n) to the segment, measured from and at right angles to the line passing through and in the same direction as the desired deflection(Δ).

$$\boxed{\Delta_n = \frac{M_n\ Y_n\ s}{E\ I_n} = \frac{P\ X_n\ Y_n\ s}{E\ I_n}} \quad \ldots\ldots\ldots\ldots (2)$$

The distances (X_n and Y_n) and the moment of inertia (I_n) are determined for each of the 10 segments and placed in table form. In most cases, the deflection to be determined is in line with the applied force so that these two distances are equal and the formula becomes—

$$\boxed{\Delta_n = \frac{P\ X_n^2\ s}{E\ I_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The values of X_n^2/I_n are found and totaled. From this the total deflection (Δ) is found:

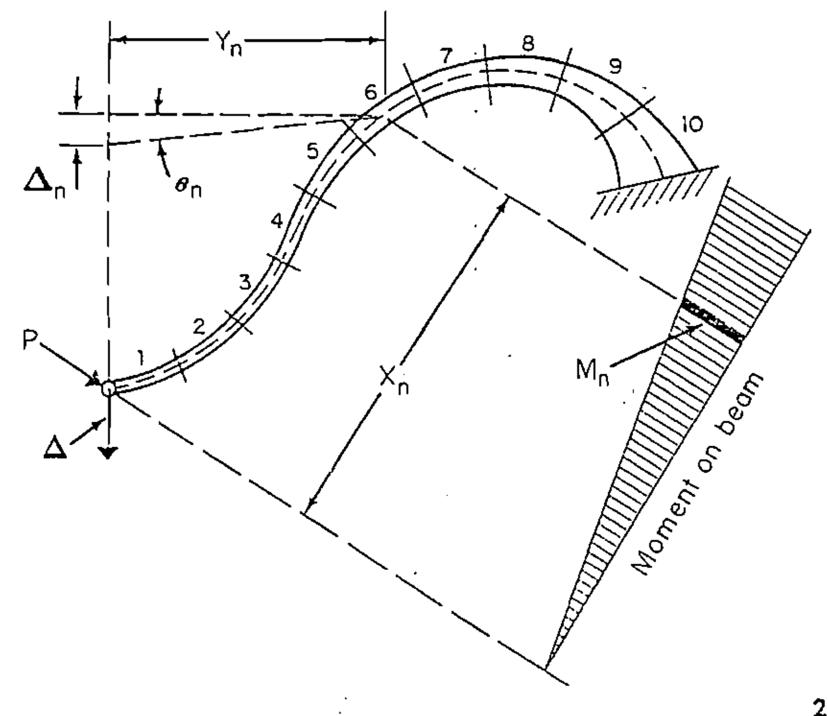


FIG. 1 To find deflection of curved cantilever beam of variable section, first divide it into segments of equal length.

$$\boxed{\Delta = \frac{P\ s}{E} \sum \frac{X_n^2}{I_n}} \quad \text{.......................(4)}$$

A symmetrical beam forming a single continuous arc, for example, is comparable to two equal cantilever beams connected end to end. Thus, the prediction of deflection in a curved beam can be approached in a manner similar to finding the deflection in a straight cantilever beam.

Problem 1

The total vertical deflection (Δ) is needed on a curved beam that will carry a maximum load (P) of 100,000 lbs. See Figure 2. Given the segment length (s) = 10″ and the various values of X_n and I_n, complete the computation.

Segment	X_n	I_n	$\frac{X_n^2}{I_n}$
1	5″	119 in.^{4}	.21
2	15	216	1.04
3	23	358	1.48
4	29	550	1.53
5	32	800	1.28
6	32	800	1.28
7	29	550	1.53
8	23	358	1.48
9	15	216	1.04
10	5	119	.21

$$\sum \frac{X_n^2}{I_n} = \underline{11.08}$$

$$\Delta = \frac{P\ s}{E} \sum \frac{X_n^2}{I_n}$$

$$= \frac{100{,}000 \times 10}{30{,}000{,}000}\ 11.08$$

$$= \underline{0.369''}$$

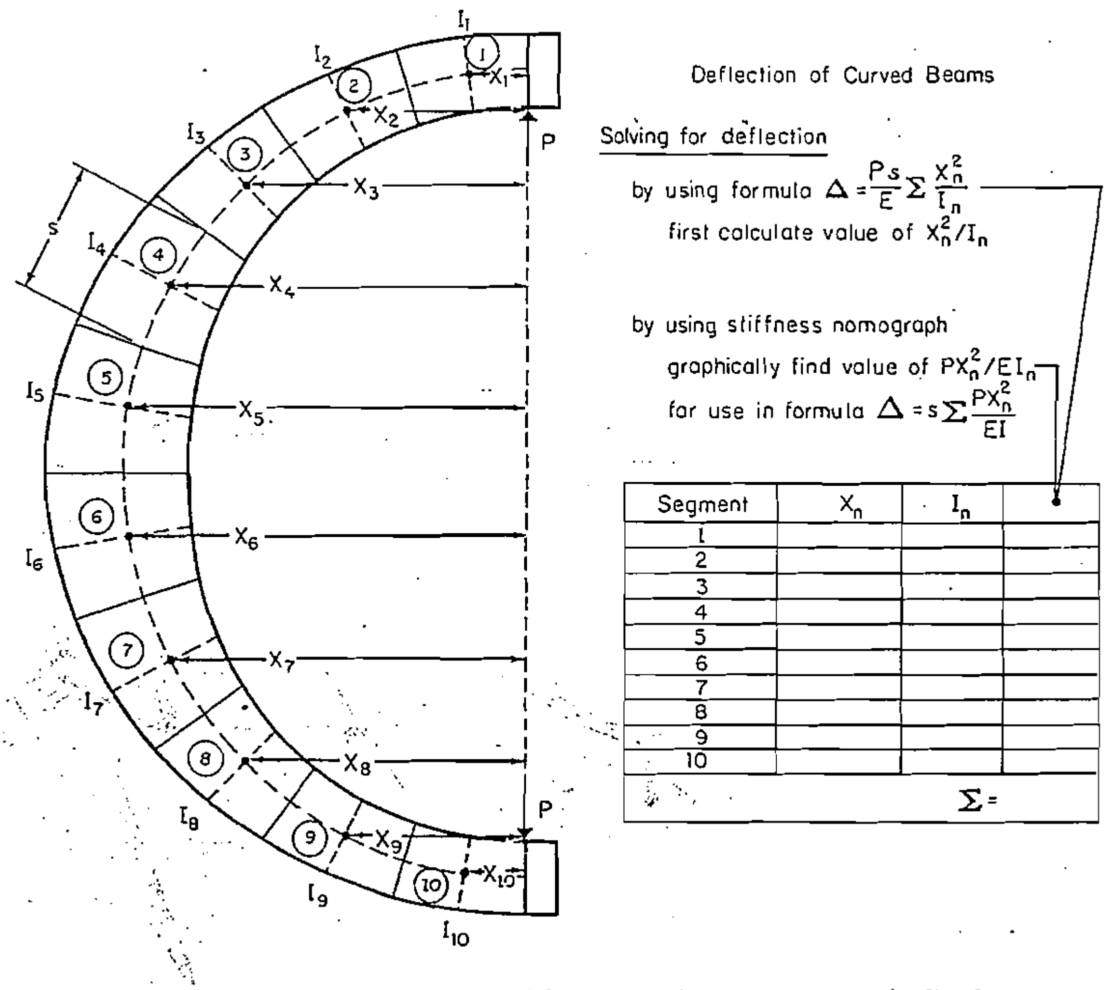
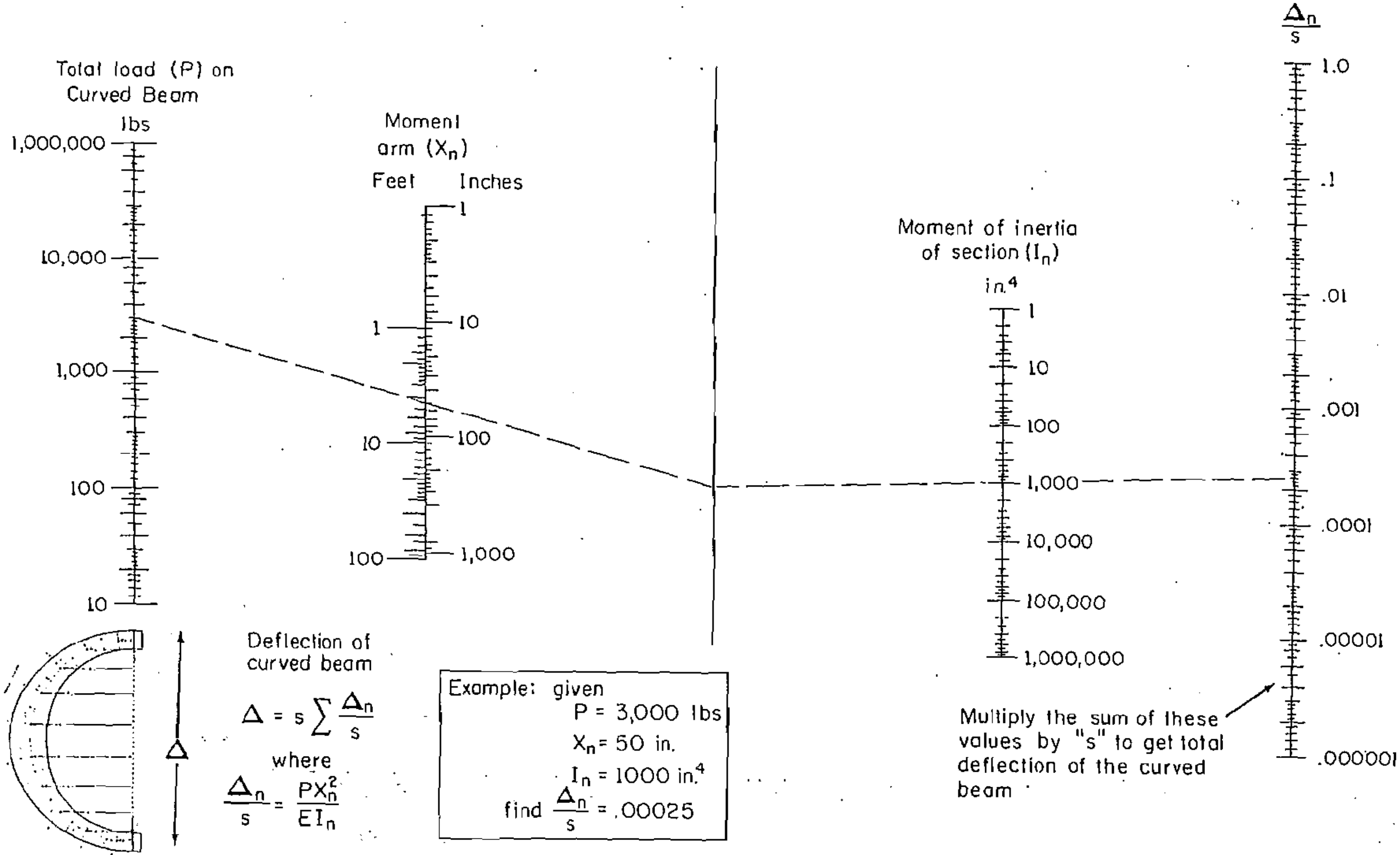


FIG. 2 For deflection of simple curved beam, use Eq. 4 or nomograph, Fig. 3.

FIGURE 3—Deflection of Curved Beam
(Stiffness Nomograph)



2. SIMPLIFICATION USING NOMOGRAPH

By using the stiffness nomograph, Figure 3, the computation can be considerably shortened with no significant loss of accuracy. The nomograph is based on the modified formula:

$$\boxed{\Delta = s \sum \frac{P\ X_n^2}{E\ I_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Readings are obtained from the nomograph for PX_n^2/EI_n for each segment and entered in the last column of the table. These are then added and their sum multiplied by s to give the total vertical deflection.

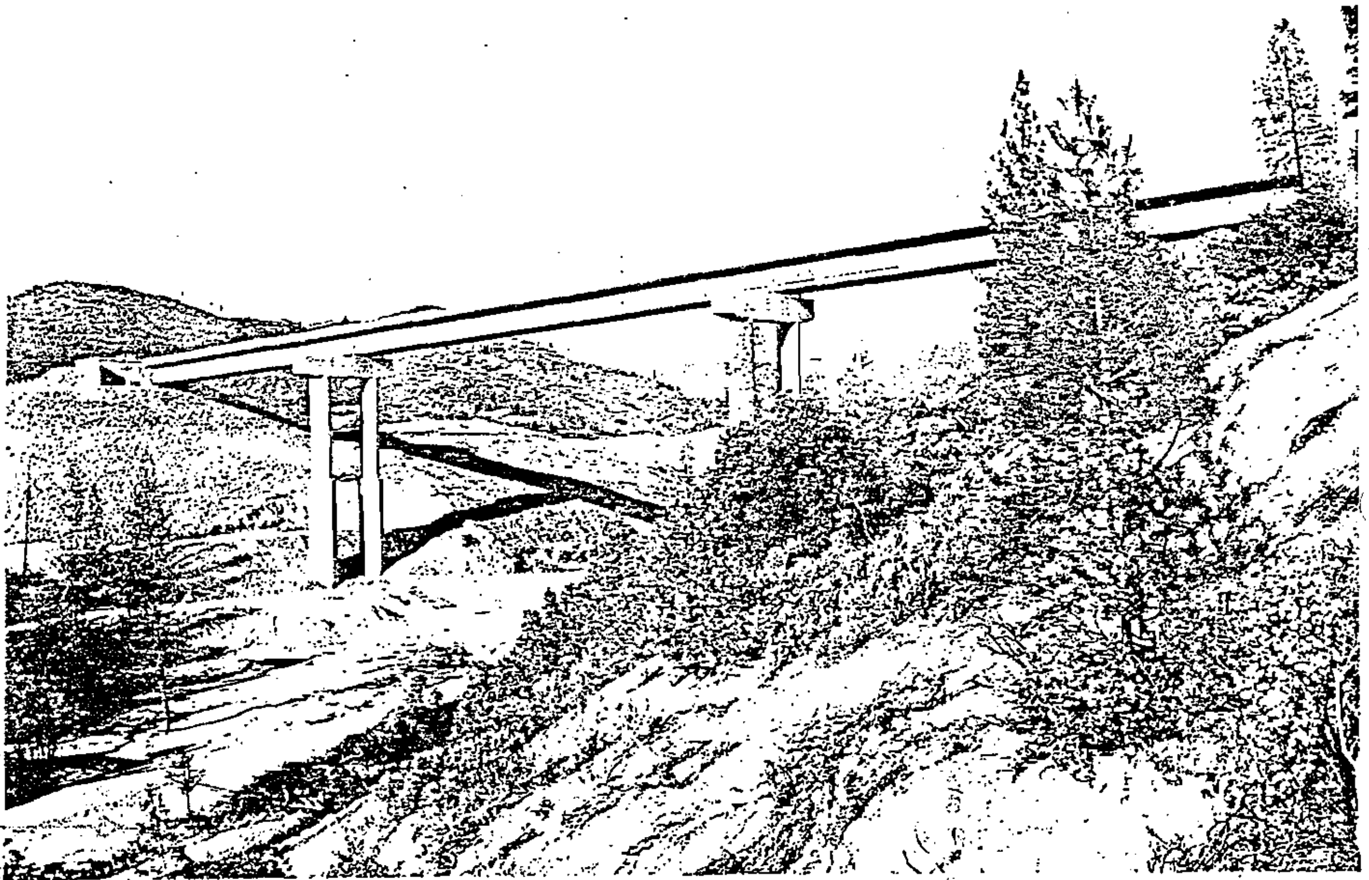
Segment	X_n	I_n	$\frac{P\ X_n^2}{E\ I_n}$
1	5	119	.0006
2	15	216	.0036
3	23	358	.0048
4	29	550	.0050
5	32	800	.0043
6	32	800	.0043
7	29	550	.0050
8	23	358	.0048
9	15	216	.0036
10	5	119	.0006

$$\sum \frac{P\ X_n^2}{E\ I_n} = \underline{.0366}$$

Problem 2

Use the same beam example as in Problem 1, the same values for P, s, X_n and I_n, and the same form of table. Complete the computation.

$$\Delta = s \sum \frac{P\ X_n^2}{E\ I_n}$$
$$= 10 \times .0366$$
$$= \underline{0.366''}$$



Engineers of the Whiskey Creek Bridge in No. California specified that the 300′ welded steel girders across each span utilize three types of steel in order to meet stress requirements economically while maintaining uniform web depth and thickness and uniform flange section. High strength quenched and tempered steel was prescribed for points of high bending moment, A-373 where moments were low, and A-242 elsewhere.