API 6A Cylindrical Pressure Vessel Calculation

Calculation for pressure vessels subjected to stress from internal pressure, axial loads and an external bending moment.

Material Data:

$Material := \text{"API"}$

Input "API" for a API Standard Material
Input "NS" for a Non-Standard API Material

$S_y := 80000\,psi$

Specified Minimum Yield Strength @ Design Temperature

$S_{yt} := 80000\,psi$

Specified Minimum Yield Strength @ Test Temperature

$S_{ut} := 95000\,psi$

Specified Ultimate Tensile Strength

Design Stress Intensity at Operating Condition

$$S_m := \left| \begin{array}{l} \min\left(\frac{2}{3}\cdot S_y, \frac{1}{2}\cdot S_{ut}\right) \quad \text{if Material = "NS"} \\ \frac{2}{3}\cdot S_y \quad \text{otherwise} \end{array} \right. = 53333.3\,psi$$

Maximum Stress Intensity at Hydrotest Condition

$$S_t := \left| \begin{array}{l} \min\left(\frac{5}{6}\cdot S_{yt}, \frac{2}{3}\cdot S_{ut}\right) \quad \text{if Material = "NS"} \\ \frac{5}{6}\cdot S_{yt} \quad \text{otherwise} \end{array} \right. = 66666.7\,psi$$

Maximum Combined Primary and Secondary Stress Intensity

$$S_s := \left| \begin{array}{l} \min\left(2\cdot S_y, 1\cdot S_{ut}\right) \quad \text{if Material = "NS"} \\ 2\cdot S_y \quad \text{otherwise} \end{array} \right. = 160000\,psi$$

Pressures:

$$P_w := 15000 \cdot psi$$

Working Pressure

$$P_t := 1.5 \cdot P_w = 22500 \ psi$$

Hydrotest Pressure

Geometrical Data:

$$OD := 10.095 \cdot in = 10.095 \ in$$

Minimum Outer Diameter of Pressure Vessel

$$ID := 7.395 \cdot in$$

Maximum Inside Diameter of Pressure Vessel

Calculation of Variables

$$R_o := \frac{OD}{2} = 5.048 \ in$$

Max Outside Radius

$$t := R_o - \frac{ID}{2}$$

$$t = 1.35 \ in$$

Wall Thickness

$$R_i := \frac{ID}{2}$$

$$R_i = 3.697 \ in$$

Inside Radius

$$A_x := \pi \left(R_o^2 - R_i^2 \right)$$

$$A_x = 37.089 \ in^2$$

Cross-Sectional Area

$$R_m := R_o - \frac{t}{2} = 4.372 \ in$$

Radius at Midsurface

$$\sqrt{R_m \cdot t} = 2.43 \ in$$

Max Distance for Local Membrane Stress

External Loading

$$F_a := 0 lbf$$

Axial Force (negative denotes compression)

$$M := 0 \cdot ft \cdot lbf$$

Bending Moment

Membrane Stresses at Operating Pressure

$$P_{mh} := \frac{P_w \cdot R_i}{t} = 41083.3 \ psi$$

Membrane Hoop Stress

$$P_{mr} := \frac{-P_w}{2} = -7500 \, psi$$

Membrane Radial Stress

$$P_{mz1} := \frac{\left(P_w \cdot \pi \cdot R_i^2 + F_a\right)}{A_x} + 4 \cdot M \cdot \frac{\left(R_i + \frac{t}{2}\right)}{\pi\left(R_o^4 - R_i^4\right)}$$

$$P_{mz1} = 17370.6 \, psi$$

Membrane Axial Stress on Tension Side

$$P_{mz2} := \frac{\left(P_w \cdot \pi \cdot R_i^2 + F_a\right)}{A_x} - 4 \cdot M \cdot \frac{\left(R_i + \frac{t}{2}\right)}{\pi\left(R_o^4 - R_i^4\right)}$$

$$P_{mz2} = 17370.6 \, psi$$

Membrane Axial Stress on Compression Side

Max Stress Intensity at Operating Pressure

$$D_1 := \left|P_{mh} - P_{mr}\right| = 48583.3 \, psi$$

$$D_2 := \left|P_{mh} - P_{mz1}\right| = 23712.8 \, psi$$

$$D_3 := \left|P_{mz1} - P_{mr}\right| = 24870.6 \, psi$$

$$D_4 := \left|P_{mh} - P_{mz2}\right| = 23712.8 \, psi$$

$$D_5 := \left|P_{mz2} - P_{mr}\right| = 24870.6 \, psi$$

$$P_m := max\left(D_1, D_2, D_3, D_4, D_5\right)$$

Allowable

$$S_m = 53333.3 \, psi$$

$$\boxed{P_m = 48583.3 \, psi}$$

Primary Membrane Stress Intensity at Operating Condition

Max Primary Plus Secondary Stress Intensity at Operating Pressure

$$P_s := \frac{2 \cdot P_w \cdot \left(\frac{R_o}{R_i}\right)^2}{\left(\frac{R_o}{R_i}\right)^2 - 1}$$

Equation 4-222 (2) from ref 2

Allowable

$$S_s = 160000 \, psi$$

$$\boxed{P_s = 64741.1 \, psi}$$

Primary Plus Secondary Stress Intensity at Operating Condition

Max Stress Intensity at Hydrotest Pressure

$$P_{mt} := \frac{P_t \cdot R_i}{t} + \frac{P_t}{2}$$

Equation 4-222 (1) from ref 2

Allowable

$S_t = 66666.7 \text{ psi}$ $P_{mt} = 72875 \text{ psi}$ Primary Membrane Stress Intensity at Hydrotest Condition

Conclusions:

$$Stress_{Operating} := if\left(P_m \leq S_m \wedge P_s \leq S_s, \text{"Pass"}, \text{"Fail"}\right)$$ $Stress_{Operating} = \text{"Pass"}$

$$Stress_{Hydrotest} := if\left(P_{mt} \leq S_t, \text{"Pass"}, \text{"Fail"}\right)$$ $Stress_{Hydrotest} = \text{"Fail"}$

References:

1. Design Handbook for API 6A / 16A / 17D Equipment, John H Fowler, Ninth Edition 2011.
2. 2004 ASME Boiler and Pressure Vessel Code, Section VIII Division 2 - Alternative Rules, ASME, New York, 2004
3. American Petroleum Institute, Specification for Wellhead and Christmas Tree Equipment, API Specification 6A, Nineteenth Edition / ISO 10423:2003 (Modified), API, Washington DC, 2004