# Tapered Girders

## 1. FABRICATION AND USE

The use of tapered girders has become widespread, especially in the framing of roofs over large areas where it is desirable to minimize the number of interior columns or to eliminate them altogether. They permit placing maximum girder depth where it is needed, while reducing the depth considerably at points where it is not needed.

Tapered girders are fabricated either 1) by welding two flange plates to a tapered web plate, or 2) by cutting a rolled WF beam lengthwise along its web at an angle, turning one half end for end, and then welding the two halves back together again along the web. See Figure 1.

### *Camber When Required*

Camber can be built into the tapered girder when required. When the girder is made from WF beams, each half is clamped into the proper camber during assembly. Then the butt joint along the web is groove welded while the girder is held in this shape. Since the weld along the beam web lies along the neutral axis, no bending or distortion will result from welding, and the girder will retain the shape in which it is held during welding.

When the girder is made of two flange plates and a tapered web, the proper camber can be obtained by simply cutting the web to the proper camber outline. The flange plates during assembly are then pulled tightly against the web, into the proper camber. The four fillet welds joining the flanges to the web are balanced about the neutral axis of the girder and as a result there should be no distortion problem.

### *Application of Tapered Girders*

When the tapered girders are used with the sloping flange at the top, their taper in both directions from the ridge will provide the slope needed for drainage. By varying the depth at the ends of successive girders, the deck can be canted to drain toward roof boxes in the valleys between adjacent gabled spans and at flanking parapet walls.

For flat roofs, the girders are inverted, with their tapered flange down. There are many combinations of roof framing systems possible. For example, on a three-span design, the central span can use the tapered flange up, forming the slope of the roof; the two adjacent spans use the tapered flange down to provide a flat roof, but tilted to continue the same slope as the central section.

The problem of lateral support for the top compression flange of tapered girders is no different than with other beams and girders. Generally the roof deck is sufficiently rigid to function as a diaphragm, and it's only necessary to attach the deck to the top flange. There's apparently no advantage in designing with a reduced stress allowable, in accordance with AISC Formulas 4 or 5, in order to permit a greater distance between bracing points at the top flange.

Where tapered girders are critical, Section 5.11 on Rigid Frame Knees goes into more detail relative to stresses (elastic design).

Because of the reduced depth at the ends of the

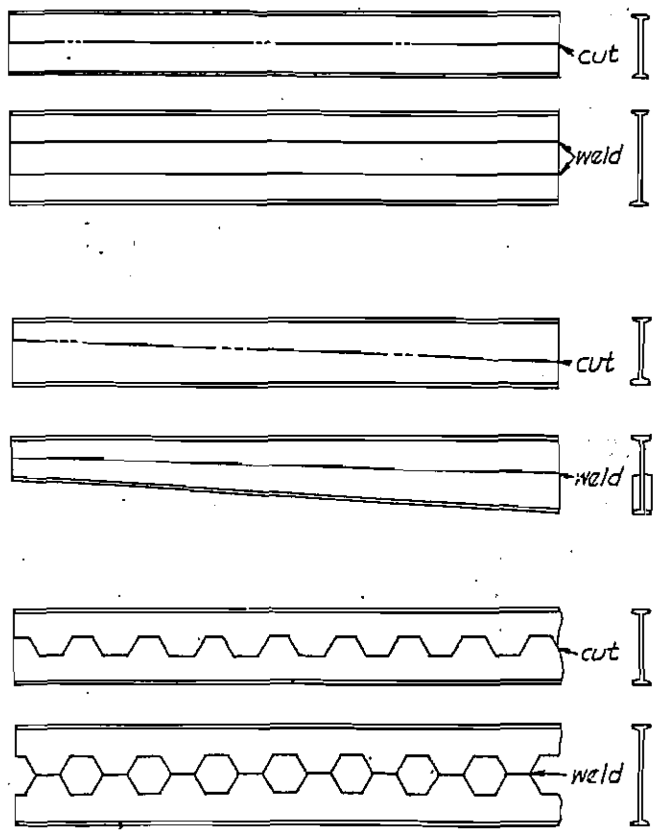



FIGURE 1

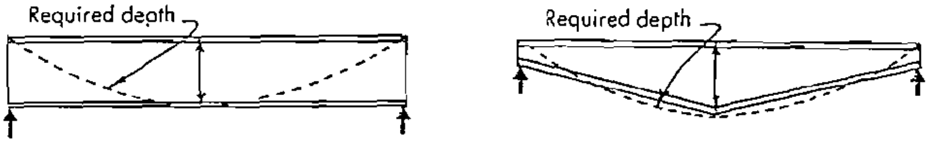



FIGURE 2

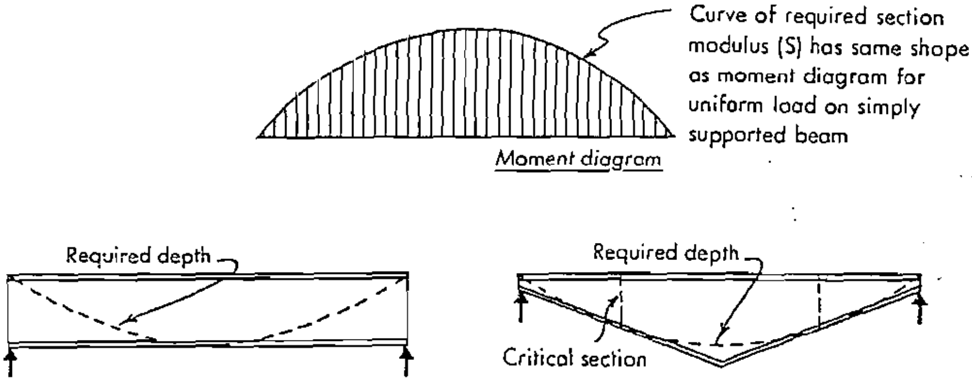



FIGURE 3

tapered girders, their connection to supporting columns may offer little resistance to horizontal forces. For this reason, some knee braces may be required unless the roof deck or a positive system of bracing in the plane of the roof is stiff enough to transmit these forces to adequately braced walls.

At first glance, there appears to be quite a weight saving in tapered girder; however, this is not always as great as it might seem:

First, the flange area remains the same; the only weight saving is in the web. See Figure 2.

Second, the depth of the tapered girder at midspan must be increased over that of the conventional straight beam to be sufficient at the critical section (about ¼ span). This is necessary to develop the required section modulus along the full length of the tapered girder.

This will slightly offset the initial weight saving in the web. See Figure 3.

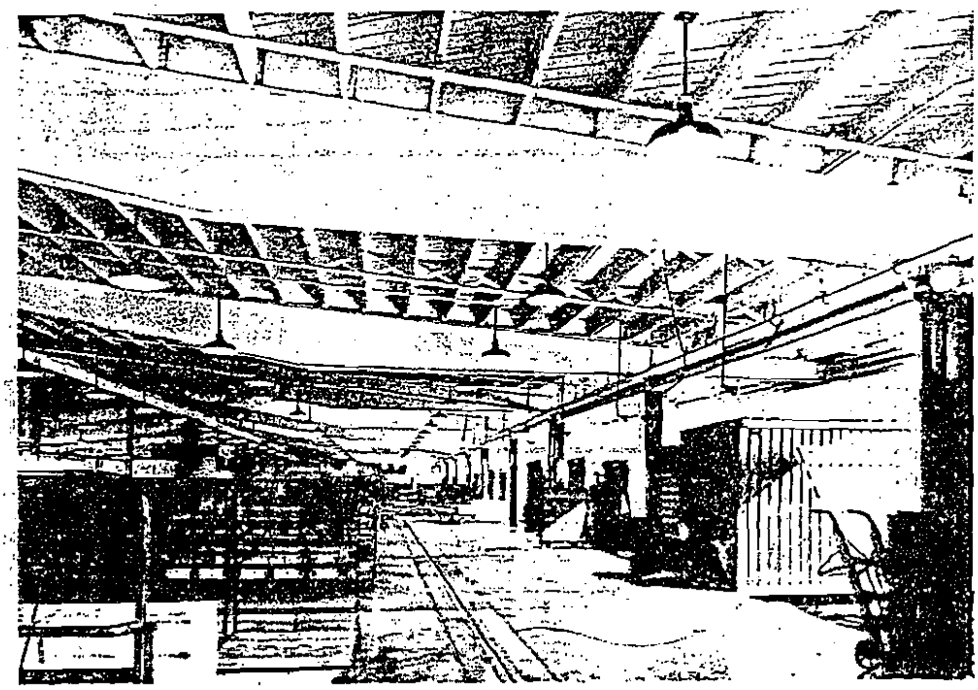


FIG. 4 For flat roofs, tapered girders are used inverted, with tapered flange downward. Frequently the girder is tilted to provide a slope to the roof or roof section.

## 2. DETERMINING CRITICAL DEPTH AND SLOPE

The critical depth section of a tapered girder is that section in which the actual depth of the girder just equals the minimum depth required for the moment. It would be the highest stressed section of the girder in bending.

In the case of a uniformly loaded, simply supported girder, its sloping flange must be tangent to the required-depth curve at this point in order for the beam to have sufficient depth along its length.

Setting the slope of the tapered girder flange so that the critical section is located at the ¼ span will result in about the minimum web weight. See Figure 5.

The properties of this critical section are—

$$I = 2\,A_f \left(\frac{d_f}{2}\right) + \frac{t_w\,d_w}{12}$$

$$S = \frac{I}{d_b^2} = \frac{A_f\,d_f^2}{d_b} - \frac{t_w\,d_w}{6\,d_b}$$

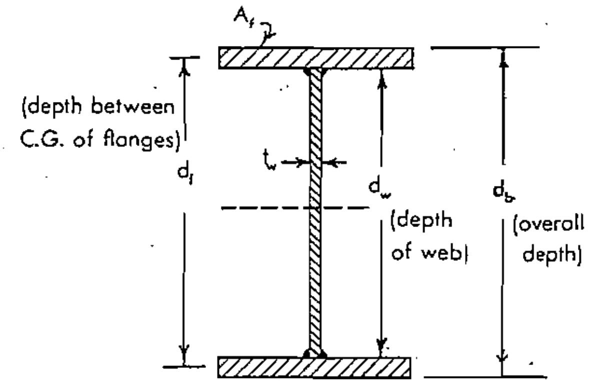



FIGURE 6

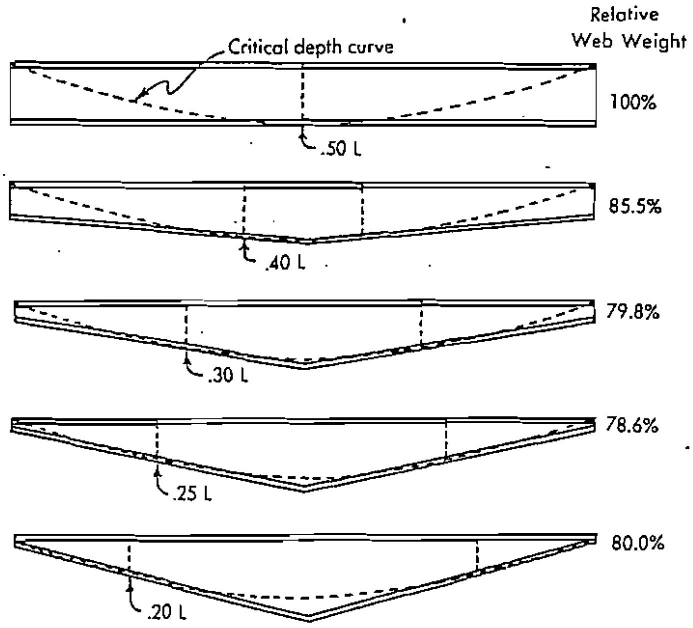



FIGURE 5

This formula for section modulus can be simplified with little loss in accuracy, by letting—

$$d_w = d_f = d_b$$

$$\therefore \boxed{S = A_f\,d_w + \frac{t_w\,d_w^2}{6}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If the section modulus required to resist the bending moment is known, the required beam depth (d) is solved for:

$$d_w^2 + \frac{6\,A_f\,d_w}{t_w} - \frac{6\,S}{t_w} = 0$$

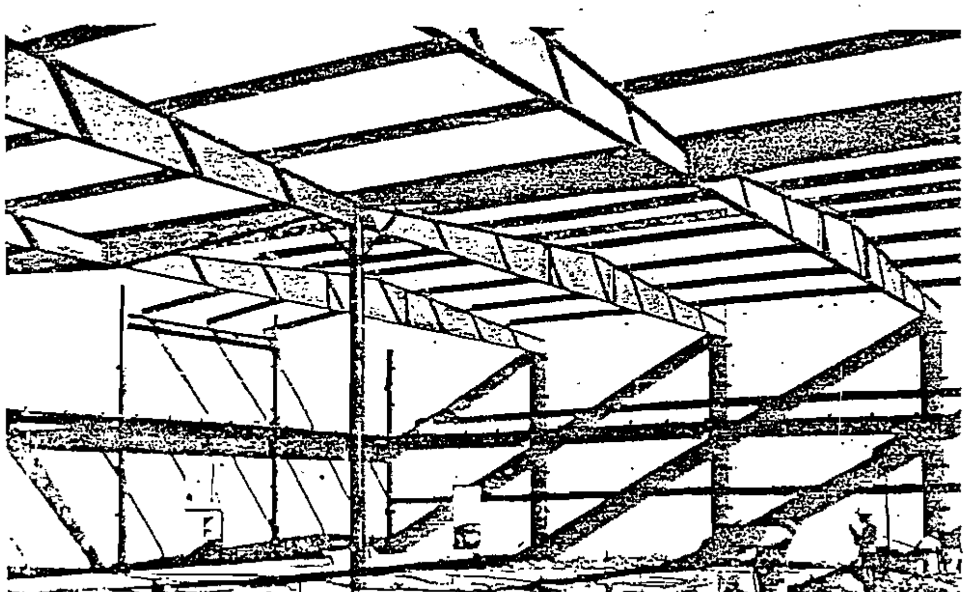
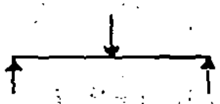
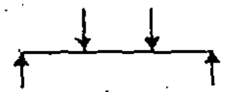
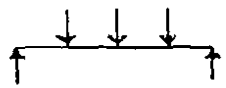
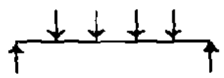
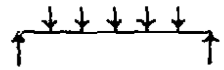
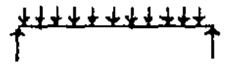


FIG. 7 Tapered girders used with the tapered flange at the top provide for roof drainage in both directions from the ridge. Multi-span designs often call for combinations of girders having tapered flange up and others having tapered flange down.

# TABLE 1

| | 1 conc. load | 2 conc. loads | 3 conc. loads | 4 conc. loads | 5 conc. loads | uniform load |
|---|---|---|---|---|---|---|
| |  |  |  |  |  |  |
| critical depth at | at $\frac{1}{2}$ L | at load $\frac{1}{3}$ L | at 1st loads $\frac{1}{4}$ L | at 1st loads $\frac{1}{6}$ L | at $\frac{1}{4}$ span $\frac{1}{4}$ L | at $\frac{1}{4}$ span $\frac{1}{4}$ L |
| critical depth $d_w$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{3\ PL}{2t_w\sigma}} - \frac{3A_f}{t_w}$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{2\ PL}{t_w\sigma}} - \frac{3A_f}{t_w}$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{9\ PL}{4t_w\sigma}} - \frac{3A_f}{t_w}$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{2.4\ PL}{t_w\sigma}} - \frac{3A_f}{t_w}$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{13\ PL}{4t_w\sigma}} - \frac{3A_f}{t_w}$ | $\sqrt{\left(\frac{3A_f}{t_w}\right)^2 + \frac{9\ wL^2}{16t_w\sigma}} - \frac{3A_f}{t_w}$ |
| slope $\theta$ | $\frac{1.5\ P}{\sigma(t_w d_w + 3A_f)}$ | $\frac{3\ P}{\sigma(t_w d_w + 3A_f)}$ | $\frac{1.5\ P}{\sigma(t_w d_w + 3A_f)}$ | $\frac{3\ P}{\sigma(t_w d_w + 3A_f)}$ | $\frac{4.5\ P}{\sigma(t_w d_w + 3A_f)}$ | $\frac{.75\ wL}{\sigma(t_w d_w + 3A_f)}$ |
| depth at center line $d_{℄}$ | $d_{℄} = d_w$ | $d_{℄} = d_w + \frac{1}{3} \tan\theta$ | $d_{℄} = d_w + \frac{1}{4} \tan\theta$ | $d_{℄} = d_w + .3\ L \tan\theta$ | $d_{℄} = d_w + \frac{1}{4} \tan\theta$ | $d_{℄} = d_w + \frac{1}{4} \tan\theta$ |
| depth at end $d_e$ | $d_e = d_w - \frac{1}{2} \tan\theta$ | $d_e = d_w - \frac{1}{3} \tan\theta$ | $d_e = d_w - \frac{1}{4} \tan\theta$ | $d_e = d_w - .2\ L \tan\theta$ | $d_e = d_w - \frac{1}{4} \tan\theta$ | $d_e = d_w - \frac{1}{4} \tan\theta$ |

or $\boxed{d_w = \sqrt{\left(\frac{3\ A_f}{t_w}\right)^2 + \frac{6\ S}{t_w}} - \frac{3\ A_f}{t_w}}$ .........(2)

For a simply supported, uniformly loaded, tapered girder—

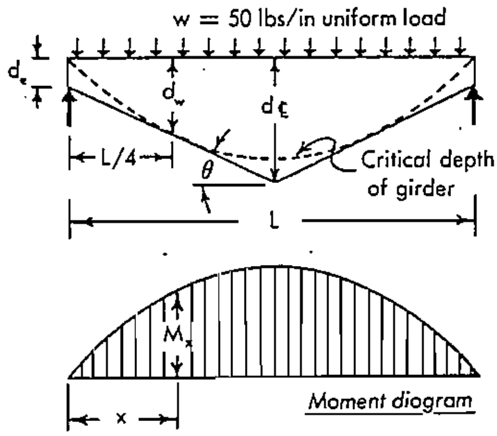



FIGURE 8

$$M_x = \frac{w\ x}{2}\ (L - x)$$

$$S_x = \frac{M_x}{\sigma}$$

$$d_x = \sqrt{\left(\frac{3\ A_f}{t_w}\right)^2 + \frac{6\ S}{t_w}} - \frac{3\ A_f}{t_w}$$

or to find the depth in one step—

$$d_x = \sqrt{\left(\frac{3\ A_f}{t_w}\right)^2 + \frac{3\ w\ x}{t_w\ \sigma}\ (L - x)} - \frac{3\ A_f}{t_w}$$

To find the slope of the critical-depth curve formed by points $d_x$ along the girder length, this expression for depth ($d_x$) is differentiated with respect to the distances ($x$):

$$\theta = \frac{dd_x}{dx} = \frac{\frac{3\ w}{2\ t_w\ \sigma}\ (L - 2_x)}{\sqrt{\left(\frac{3\ A_f}{t_w}\right)^2 + \frac{3\ w\ x}{t_w\ \sigma}\ (L - x)}}$$

It is simpler to find the slope at ¼ span, letting x = L/4:

$$\theta = \frac{\frac{3\ w\ L}{4\ t_w\ \sigma}}{\sqrt{\left(\frac{3\ A_f}{t_w}\right)^2 + \frac{9\ w\ L^2}{16\ t_w\ \sigma}}}$$

Also, at x = L/4:

$\boxed{d_w = \sqrt{\frac{3\ A_f}{t_w} + \frac{9\ w\ L^2}{16\ t_w\ \sigma}} - \frac{3\ A_f}{t_w}}$ ........(3)

$\boxed{\theta = \frac{.75\ w\ L}{\sigma\ (t_w\ d_w + 3\ A_f)}}$ ................(4)

and:

$\boxed{d_℄ = d_w + \frac{L}{4}\ \tan\ \theta}$ ..................(5)

$\boxed{d_e = d_w - \frac{L}{4}\ \tan\ \theta}$ ..................(6)

Since loading on the girder is not always uniform, the above formulas do not always apply. Table 1 summarizes the working formulas to use for various conditions of loading, as well as locating the critical depth.

## 3. CONCENTRATED LOADS

Figure 9 shows the effects of placing multiple loads upon a simply-supported tapered girder. These effects on the bending moment and the critical depth of the girder can be explained as follows:

- In the case of the single concentrated load at midspan, the critical depth section is at midspan, and the maximum slope is $\theta$.
- In the case of 2 equal concentrated loads applied at ⅓ points, the critical depth section is at the points of load application and the maximum slope is $\theta$. Assuming the slope were to pivot about this critical depth section, any slope less than this value would cause the depth at the end to increase at twice the rate at which the depth at centerline is decreasing. Since such a shift would increase the web weight, this maximum slope value of $\theta$ should be used initially.

If more depth is needed at the end because of higher vertical shear, do this by pivoting about this critical depth section. This will result in the least increase in web weight. It can be shown that, under this condition, the resulting depth at centerline will be—

$\boxed{d_℄ = \frac{3\ d_w - d_e}{2}}$ ......................(7)

- In the case of 3 equal concentrated loads applied at ¼ points, the critical depth section will be chosen at ¼ span. The slope of the girder must lie somewhere between $\theta$ and $\phi$. For any angle between these two values, the weight of the web will remain the same

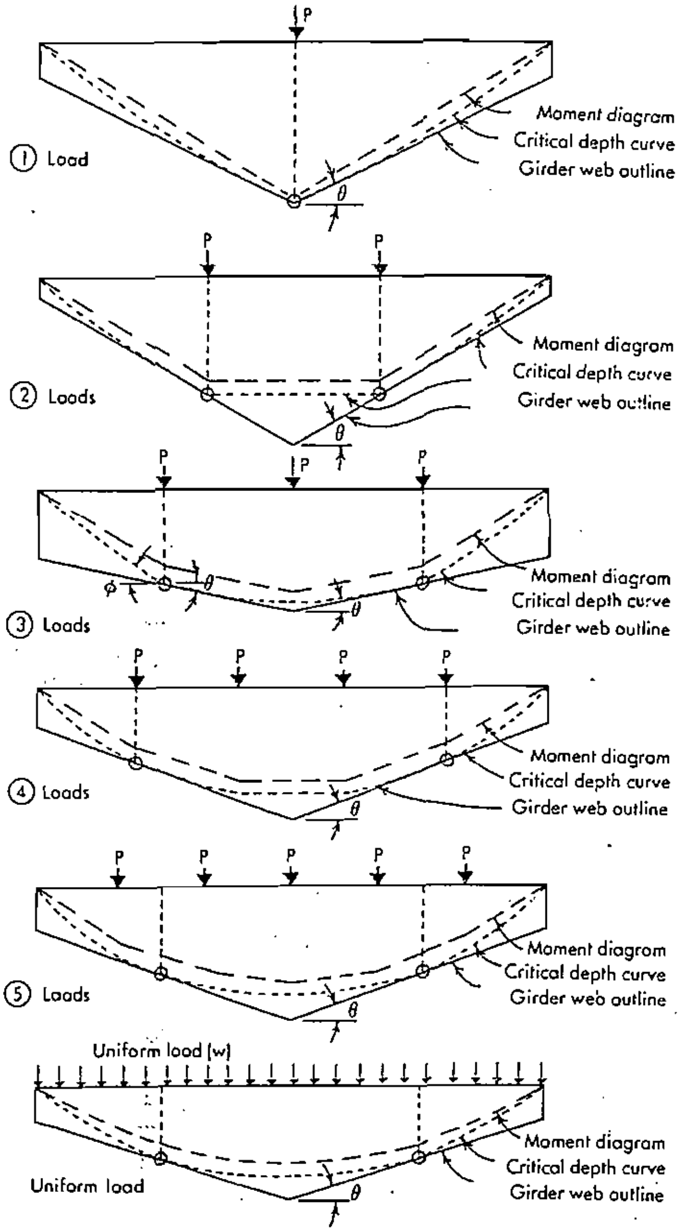



FIGURE 9

because this is pivoting halfway between the end and the centerline. Any change in the web depth at the centerline will change the depth at the end at the same rate, but inversely.

Of the two extreme conditions, it would be better to use the angle θ since this will give a larger value for the depth at the end, which may be needed because of the higher vertical shear value. There would be no advantage in using φ.

- In the case of 4 equal concentrated loads applied at ⅕ points, use the critical depth section at the first load. The section chosen lies closer to the end of the girder and further from the centerline. Because of this, the depth at centerline will change at a faster rate than the depth at the end as the slope is varied. Therefore, for the lowest web weight, keep the depth at centerline as small as possible, hence use the angle θ.
- In the case of 5 equal concentrated loads applied at ⅙ points, the critical depth section will be taken at the ¼ span for convenience. The slope of the girder will be θ.

**Problem 1**

Design a welded tapered girder, with a uniformly-distributed load of 600 lbs/ft; Figure 10. The girder has a length of 50′ and is simply supported. Use A36 steel and E70 welds.

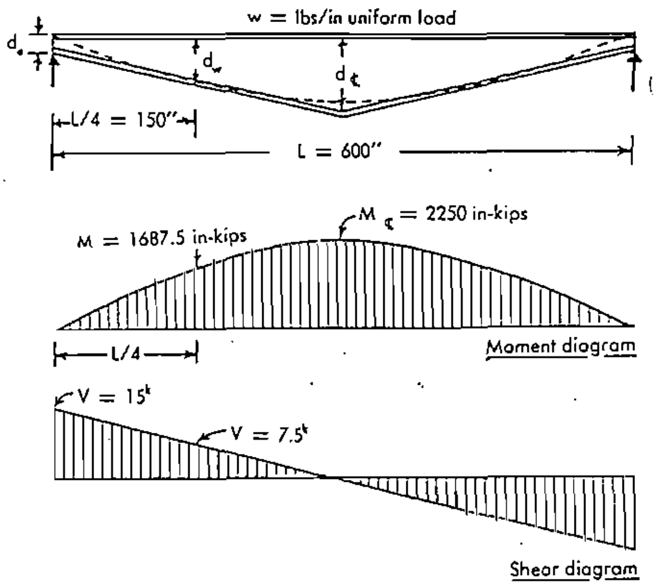



FIGURE 10

The top compression flange of the girder has suitable support.

Design for critical section at ¼ span; check moment at centerline; and check shear stress at centerline and end.

From Section 8.1, Beam Diagrams:

$$M_x = \frac{w\,L}{2}\,(L - x)$$

*at* $x = L/4$

$$M = \frac{3\,w\,L^2}{32}$$

$$= \frac{3(50)(600)^2}{32}$$

$$= 1687.5 \text{ in.-kips}$$

and:

$$S = \frac{M}{\sigma}$$
$$= \frac{(1687.5)}{(22{,}000)}$$
$$= 76.7 \text{ in.}^3$$

To use an "efficient" section (Sect. 4.2, Topic 2), the efficient depth would be—

$$d = \sqrt[3]{\frac{3\ K\ S}{2}}$$

It would be preferable not to have to use transverse intermittent stiffeners. Looking in Section 4.1 on Plate Girders for Buildings, Topic 2, it is seen that these stiffeners are not required if:

a) The ratio $K = \frac{d_w}{t_w}$ is less than 260

b) The shear stress ($\tau$) does not exceed that of AISC Formula 9.

This means the values of K and shear stress ($\tau$) shall fall within the values of the right-hand column of AISC Table 3-36, in Section 4.1, page 25.

Assume a value of $K = 70$ at the end of the girder; here the shear (V) is highest. Assume a value of $K = 170$ at midspan; here the shear (V) is very low. This means at ¼ span (the critical section under consideration) K would fall halfway between these two values, or $K = 120$.

*therefore, the efficient depth*

$$d = \sqrt[3]{\frac{3\ K\ S}{2}}$$
$$= \sqrt[3]{\frac{3(120)(76.7)}{2}}$$
$$= 24.0''$$

*required flange area (efficient section)*

$$A_f = \frac{d^2}{2\ K}$$
$$= \frac{(24)^2}{2(120)}$$

$= 2.4$ in.$^2$ or *use ½″ × 5″ flange,* the area of which is $A_f = 2.5$ in.$^2$

*web thickness*

$$t_w = \frac{d_w}{K}$$
$$= \frac{(24)}{(120)}$$

$= .20''$ or *use a* $\frac{3}{16}''$ *thick plate.* Then—

*ratio of web's depth to thickness*

$$K = \frac{d_w}{t_w}$$
$$= \frac{(24)}{(3/16)}$$
$$= 128$$

And from Table 3-36 in Sect. 4.1; since with no stiffeners $a/d_w = \infty$ (over 3), allowable shear is $\tau = 5000$ psi.

*actual shear stress*

$$\tau = \frac{V}{A_w}$$
$$= \frac{(7.5 \text{ kips})}{(3/16)\ (24)}$$
$$= 1670 \text{ psi} < 5000 \text{ psi} \quad OK$$

*required slope of tapered girder*

$$\theta = \frac{.75\ w\ L}{\sigma\ (t_w d_w + 3\ A_f)}$$
$$= \frac{.75(50)(600)}{(22{,}000)\ (3/16 \times 24 + 3 \times 2.5)}$$
$$= .0852 \text{ radians, or } 4.88°$$

*required depth of web*

$$d_{℄} = d_w + \frac{L}{4} \tan \theta \qquad \tan 4.88° = .08538$$
$$= (24.0) + \frac{(600)}{4}\ (.08538)$$
$$= 24.0 + 12.8$$
$$= 36.8''$$

$$d_e = d_w - \frac{L}{4} \tan \theta$$
$$= (24.0) - (12.8)$$
$$= 11.2''$$

**Check Shear Stress at End**

$$A_w = 3/16\ (11.2)$$
$$= 2.1 \text{ in.}^2$$

$$\tau = \frac{V}{A_w}$$
$$= \frac{(15 \text{ kips})}{(2.1)}$$
$$= 7140 \text{ psi}$$

Here

$$K = \frac{d_w}{t_w}$$

$$= \frac{(11.2)}{(3/16)}$$

$= 60$, and from Table AISC 3-36 in Section 4.1, page 25 it is determined that no stiffeners are required.

***Check Section at Midspan***

$$K = \frac{d_w}{t_w}$$

$$= \frac{(36.8)}{(3/16)}$$

$$= 196 < 260 \quad OK$$

Also, practically no shear here.

$$M_{\mathfrak{L}} = \frac{w\,L^2}{8}$$

$$= \frac{(50)\,(600)^2}{8}$$

$$= 2250 \text{ in.-kips}$$

$$S_{\mathfrak{L}} = A_f\,d_w + \frac{t_w\,d_w^2}{6}$$

$$= (2.5)\,(36.8) + \frac{(3/16)\,(36.8)^2}{6}$$

$$= 134.4 \text{ in.}^3$$

$$\sigma_{\mathfrak{L}} = \frac{M_{\mathfrak{L}}}{S_{\mathfrak{L}}}$$

$$= \frac{(2250 \text{ in.-kips})}{(134.4 \text{ in.}^3)}$$

$$= 16{,}750 \text{ psi} < 22{,}000 \text{ psi} \quad OK$$

**Problem 2** Alternate Design

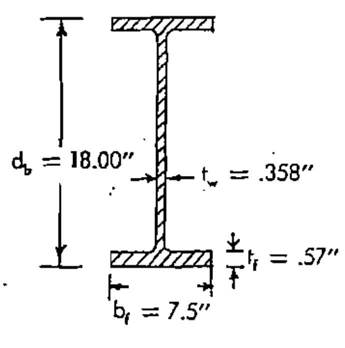



FIGURE 11

To make this tapered girder by splitting a WF rolled beam, and welding back together after reversing one-half end for end.

Since the required section modulus of the critical section at ¼ span is—

$$S = 76.7 \text{ in.}^3$$

an 18″ WF 50-lb beam could be used.

*properties of this rolled beam*

$$A_f = (.57)(7.5)$$

$$= 4.27 \text{ in.}^2$$

$$d_w = 18.00 - 2(.57)$$

$$= 16.86''$$

$$S = 89.0 \text{ in.}^3$$

*shear stress at ¼ span*

$$\tau = \frac{V}{A_w}$$

$$= \frac{(7.5 \text{ kips})}{(.358)(16.86)}$$

$$= 1240 \text{ psi} \quad OK$$

*slope of tapered girder*

$$\theta = \frac{.75\,w\,L}{\sigma(t_w\,d_w + 3A_f)}$$

$$= \frac{(.75)(50)(600)}{(22{,}000)(.358 \times 16.96 + 3 \times 4.27)}$$

$$= .05415 \text{ radians or } 3.10^\circ$$

$$\boxed{\begin{aligned} \tan\theta &= .05416 \\ \cos\theta &= .99854 \end{aligned}}$$

*depth of web*

$$d_{\mathfrak{L}} = d_w + \frac{L}{4}\tan\theta$$

$$= (16.96) + \frac{(600)}{4}(.05416)$$

$$= 16.96 + 8.12$$

$$= 25.08$$

$$d_e = d_w - \frac{L}{4}\tan\theta$$

$$= (16.96) - (8.12)$$

$$= 8.84$$

Before going further, check the *shear stress at the end of beam*—

$$A_w = t_w\,d_w$$

$$= (.358)(8.84)$$

$$= 3.17 \text{ in.}^2$$

$$\tau = \frac{V}{A_w}$$
$$= \frac{(15^k)}{(3.17)}$$
$$= 4730 \text{ psi} \quad OK$$

*depth of beam*

$$d_{\mathbb{C}} = d_w + 2(t_f)$$
$$= (25.08) + 2(.57)$$
$$= 26.12''$$
$$d_e = d_w + 2\,(t_f)$$
$$= (8.84) + 2(.57)$$
$$= 9.88''$$

*starting point of cut*

$$d_{\mathbb{C}} = a + d = a + \frac{a}{\cos\theta}$$
$$= a\left(1 + \frac{1}{\cos\theta}\right)$$
$$= a\left(1 + \frac{1}{.99854}\right)$$
$$= 2.0014\ a \text{ and}$$
$$a = \frac{26.12}{2.0014}$$
$$= 13.06''$$

or use the dimension (a = 130″) to determine the starting point for flame cutting the WF beam to prepare a tapered girder.

***Check Girder Section at Midspan***

$$K = \frac{d_w}{t_w}$$
$$= \frac{(25.08)}{(.358)}$$
$$= 78 \quad OK$$

Also, practically no shear here.

$$M_{\mathbb{C}} = 2250 \text{ in.-kips}$$
$$S_{\mathbb{C}} = A_f\, d_w + \frac{t_w\, d_w^2}{6}$$
$$= (.427)(25.08) + \frac{(.358)(25.08)^2}{6}$$
$$= 166.8 \text{ in.}^3$$
$$\sigma_{\mathbb{C}} = \frac{M_{\mathbb{C}}}{S_{\mathbb{C}}}$$
$$= \frac{(2250)}{(166.8)}$$
$$= 13{,}500 \text{ psi} \quad OK$$

## 4. DEFLECTION OF TAPERED GIRDERS

The area-moment method may be used with good results to find the deflection of tapered girders, where no portion of the member has a constant moment of

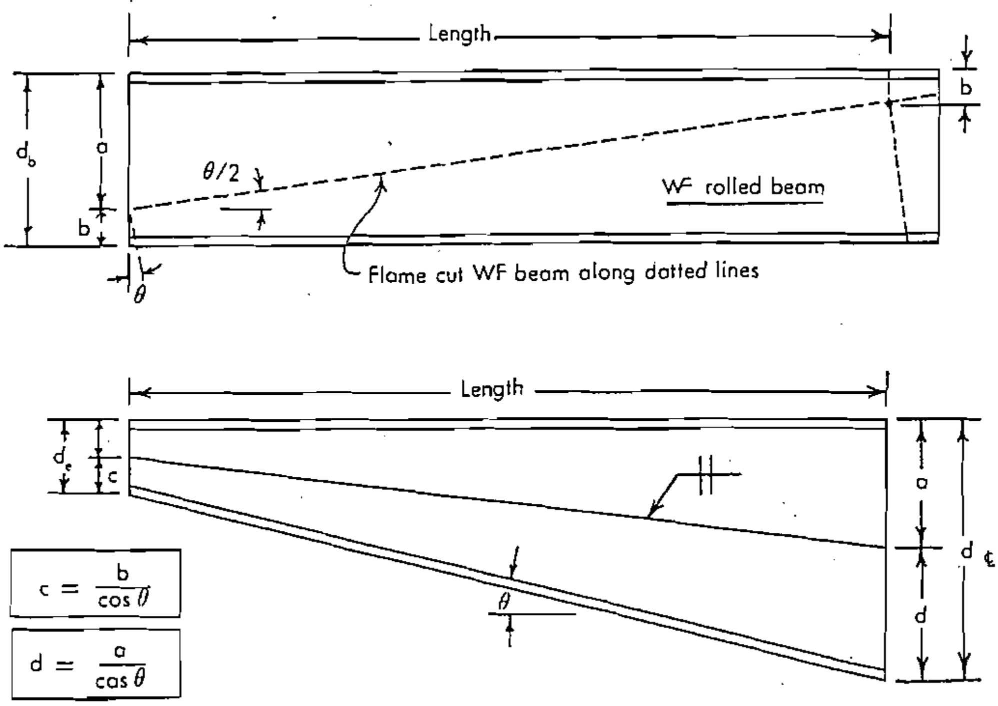



FIG. 12 Turn one-half end for end, and submerged-arc weld this web joint without special edge preparation. Trim ends.

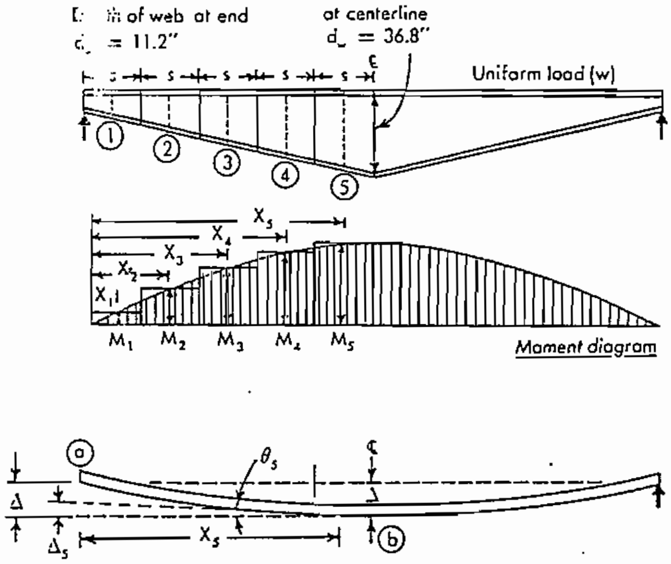



FIGURE 13

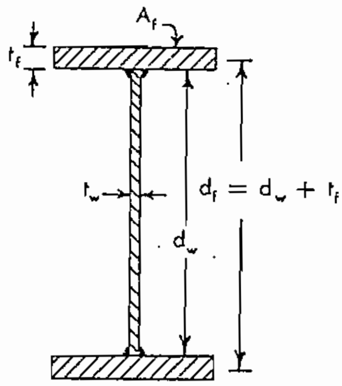



FIGURE 14

For each division, the moment of inertia ($I_n$), moment ($M_n$), and distance to the end ($x$) are determined and listed in table form.

inertia. This method is described under Topics 5 and 7 of Section 2.5 on Deflection by Bending.

**Problem 3**

To compute the deflection of the tapered girder shown in Figure 13. This girder has a uniform load of 50 lbs/in., and a length of 50′ or 600″.

Using the area-moment method, the distance of point (a) from the tangent to point (b) equals the moment of the area under the moment diagram taken about point (a), divided by the EI of the section.

Divide the girder into 10 equal lengths ($s = 60''$ long). The greater the number of divisions, the more accurate the answer will be.

| | x | $d_w$ | $d_f$ | $I_x$ | $M_x$ | $\frac{M_x X}{I_x}$ |
|---|---|---|---|---|---|---|
| ① | 30″ | 13.76″ | 14.26″ | 346.in.⁴ | 427.5 in.-k | 37.2 |
| ② | 90″ | 18.88″ | 19.38″ | 669.in.⁴ | 1147.5 in.-k | 154.6 |
| ③ | 150″ | 24.00″ | 24.50″ | 1117.in.⁴ | 1687.5 in.-k | 226.7 |
| ④ | 210″ | 29.12″ | 29.62″ | 1702.in.⁴ | 2047.5 in.-k | 253.2 |
| ⑤ | 270″ | 34.24″ | 34.74″ | 2439.in.⁴ | 2227.5 in.-k | 246.7 |
| | | | | | Total → | 918.4 |

Here, for each segment:

$$\boxed{I_n = \frac{A_f\, d_f^2}{2} + \frac{t_w\, d_w^3}{12}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Since:

$A_f = 3.0$ in.²

$t_w = \frac{3}{16}''$

The above formula, in this problem, reduces to:

$$I_n = 1.5\, d_f^2 + \frac{d_w^3}{64}$$

Since:

$$\Delta_n = \frac{M_n\, s\, X_n}{E\, I_n}$$

$$\boxed{\Delta = \frac{s}{E}\, \Sigma\, \frac{M_n\, X}{I_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and:

$$\Delta = \frac{(60)}{(30 \times 10^6)}\, 918.4$$

$$= \underline{1.84''}$$