Efficient Plate Girders

1. GENERAL REQUIREMENTS

Every plate girder must have several properties:

1. Sufficient strength, as measured by its section modulus (S).
2. Sufficient stiffness, as measured by its moment of inertia (I).
3. Ability to carry the shear forces applied to it, as measured by its web area (A_w).
4. Ability to withstand web buckling, as indicated by the empirical relationship of the web depth to web thickness—

$$\boxed{K = \frac{d_w}{t_w}} \qquad (1)$$

In some cases, the depth (d) must be held within a certain maximum value.

Also, the choice of flange and web plates should not result in any unusual fabricating difficulties.

An "efficient" girder will satisfy all of these requirements with the minimum weight.

An "economical" girder will satisfy these same requirements and in addition will be fabricated for the least cost for the whole structure. This may not necessarily be the lowest weight design.

Most structural texts suggest a method of girder design in which some assumption is made as to the depth, usually from $\frac{1}{10}$ to $\frac{1}{12}$ of the girder length (a minimum of $\frac{1}{25}$). Knowing the web depth, the web thickness is then found. This is kept above the value required for web area (A_w) to satisfy the shear forces and also to insure that the ratio $K = d_w/t_w$ will be below the proper value.

Table 1 lists the AASHO (Bridge) limiting values of $K = d_w/t_w$ for common materials, with or without transverse stiffeners.

2. DESIGN APPROACH

It might be well to investigate the efficient girder design on the basis of minimum weight. If done simply, it would offer a good guide or starting point in any design of a girder. An estimate of weight that is obtained quickly would allow the designer to deviate from the efficient depth to a more shallow girder when necessary. He would then balance off the additional weight with any advantages of the altered design, such as increased head room, less fill at bridge approaches, etc.

In order to simplify the derivation of the efficient girder, it will be necessary to assume the depth of the web plate (d_w) is also the distance between the centers of gravity of the two flange plates as well as the overall depth of the girder. See Figure 1.

In the case of welded plate girders where the thickness of flange plates is very small compared to the girder's depth, this assumption doesn't introduce very much of an error while greatly simplifying the procedure and resulting formulas.

The moment of inertia of the girder section is—

$$I = 2\,A_f \left(\frac{d_w}{2}\right)^2 + \frac{t_w d_w^3}{12}$$

$$= \frac{A_f\,d_w^2}{2} + \frac{d_w^4}{12\,K} \text{ and}$$

$$S = \frac{I}{d/2} = A_f\,d_w + \frac{d_w^3}{6\,K} \text{ or}$$

$$A_f = \frac{S}{d_w} - \frac{d_w^2}{6\,K} \text{ also}$$

$$A_w = t_w\,d_w = \frac{d_w^2}{K}$$

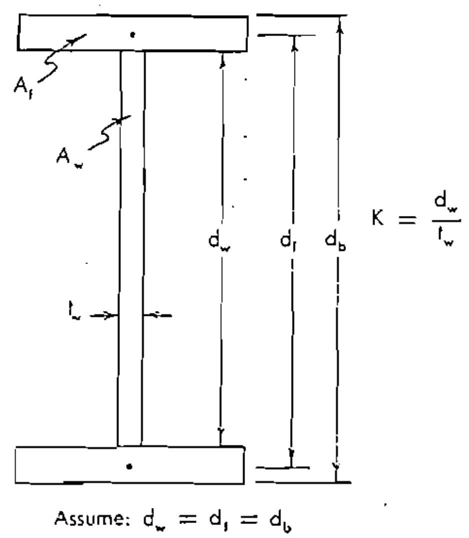


FIG. 1 Girder description.

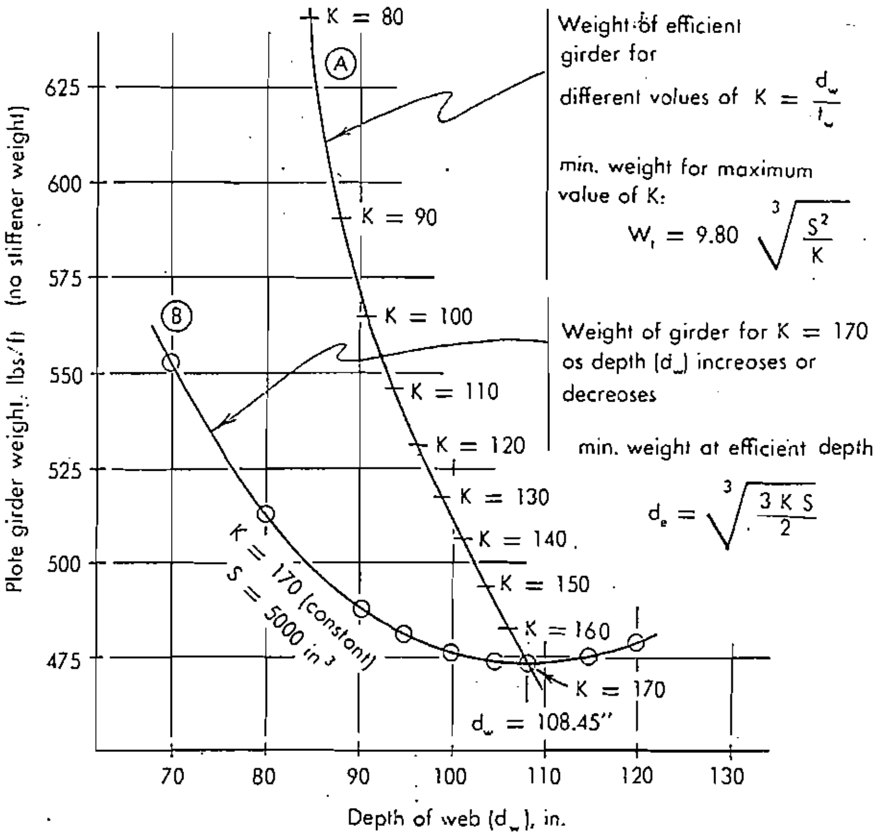


FIG. 2 Relationship of efficient girder weight and depth for given requirements (here, $S = 5{,}000$ in.³).

stiffeners.

It is seen that larger values of K result in lower weight (W_t) and increased depth (d_w) of girder. Conversely, lower values of K will produce heavier and more shallow girders. This represents the lowest weight design for any given value of K.

Assuming the weight of stiffeners will be 20% of the web weight, and since in the efficient girder, the web represents half of the girder weight, the stiffeners would increase the girder weight by 10%, or—

$$\boxed{W_t = 9.80 \sqrt[3]{\frac{S^2}{K}}} \quad \ldots\ldots\ldots (7)$$

which is the weight of girder including weight of stiffeners.

Effect of Changing Dimensions

In an efficient girder the depth of which is determined by Formula 2—

$$d_w = \sqrt[3]{\frac{3\ K\ S}{2}}$$

the weight decreases as the ratio (K) increases; hence use as large a K ratio as is possible (see Table 1). Once the flange area (A_f) is determined, the actual profile of the flange (thickness to width) has almost no effect on the resulting girder weight (W_t).

Occasionally the girder depth may be restricted because of head room or some other reason. The shallow-depth web then must be thicker in order to make up the web area required for the shear forces; in this case, it may be possible to further increase the web thickness, very slightly, to arrive at 1/60 of its clear depth and thus eliminate the transverse stiffeners. If this is the case, the decision not to use stiffeners should be made at the start of the design rather than later. For example, See Figure 3.

Here on the left side, the efficient girder using stiffeners ($K = 170$) weighs 188 lbs/linear ft. Taking this same design and increasing the web thickness to 1/60 of its depth to eliminate the stiffeners, would increase its weight to 328 lbs/linear ft, or 1.74 times. On the other hand if the efficient depth is first determined using no stiffeners ($K = 60$), the weight is increased to only 243 lbs/linear ft, or 1.29 times. In this particular case, the design which eliminated the stiffeners at the start (right-hand girder) weighs only 74% as much as the design which eliminated the stiffeners after the depth was determined (center girder).

The graph in Figure 4 shows the direct effect of changing web depth. Changing the combination of flange dimensions, but using same depth of web (d_w)

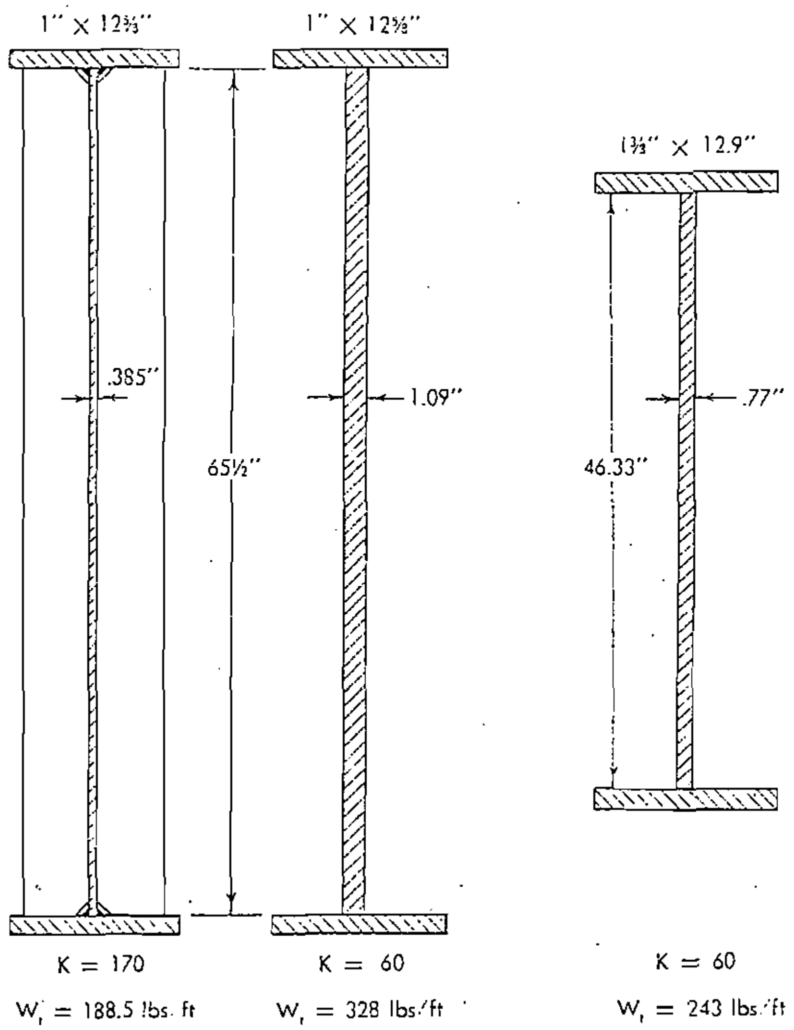


FIG. 3 Efficient girder with stiffeners (left) weighs less. Merely increasing the web thickness to eliminate stiffeners (center) results in greater weight than again designing on basis of efficient depth (right).

and required section modulus (S), does not change the girder weight very much. The thinner and wider flanges result in a very slight reduction in girder weight.

If at any time in the design, the web area (A_w) falls below the required shear-carrying capacity (V), the design becomes dictated by the shear requirements. In this case, a given web area (A_w) must be maintained and the value of K held as high as possible for minimum girder weight.

Weight of the efficient girder depends on:

1. Value of K used (the lower values produce heavier girders), and

2. How far the actual depth deviates from the efficient depth.

3. DESIGN OUTLINE

The following is a guide to the design of an efficient girder. This would represent a starting point for the final girder design.

Given these requirements:

$$S = \frac{M}{\sigma}$$

$$A_w = \frac{V}{\tau}$$

$$K \leqq \frac{d_w}{t_w} \qquad \text{(see Table 1)}$$

Start with Method A, and continue unless it is determined that the web area of the proposed girder does not equal or exceed the given required value. In this case, Method B represents a short detour to be taken in the design procedure.

Method A

1. As a starting point for web depth, use—

$$d_w = \sqrt[3]{\frac{3\,K\,S}{2}}$$

This may occasionally exceed the depth permitted by architectural considerations, in which case the latter

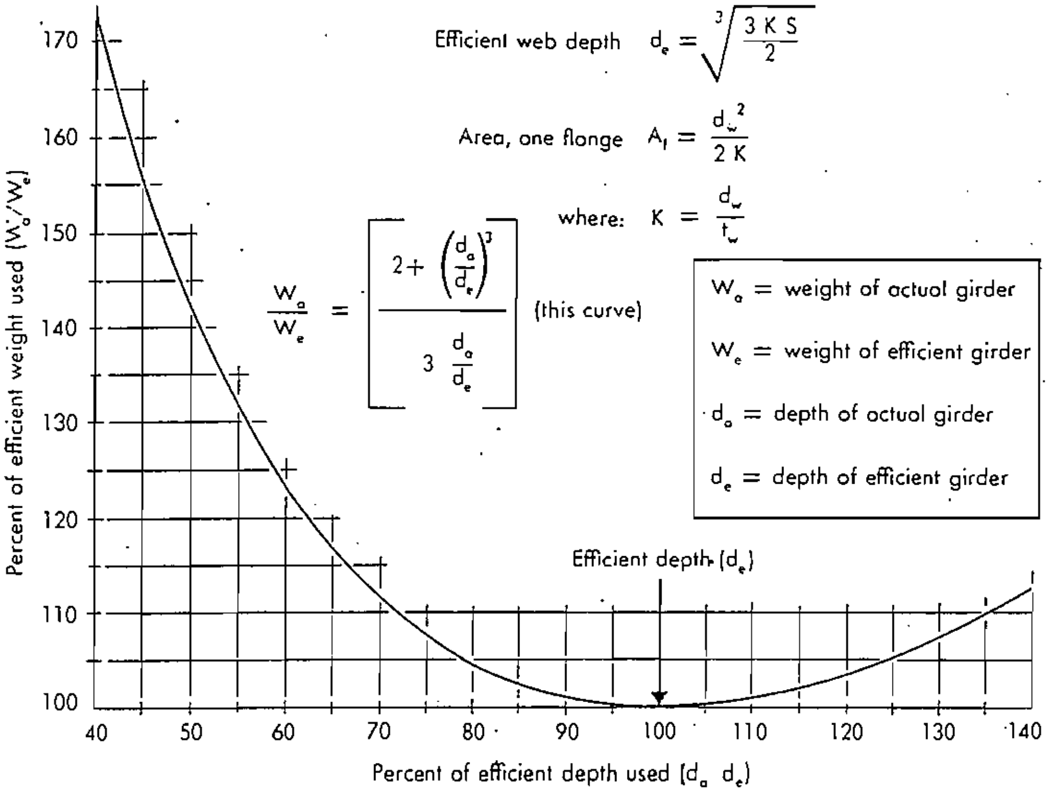


FIG. 4 Effect of changing web depth on girder weight.

must be used.

2. For web thickness, use

$$t_w = \frac{d_w}{K}$$

3. Check the resulting values for

$$K = \frac{d_w}{t_w}$$

$$A_w = d_w \; t_w$$

Try to use values of t_w and d_w that will provide the highest allowable value of K. If resulting A_w equals or exceeds the given required value, proceed to Step 4 of Method A; if not, jump to Step 3A of Method B.

Method A cont'd

4. Now compute the web's moment of inertia:

$$I_w = \frac{t_w \; d_w^3}{12}$$

5. Select a flange thickness and compute the distance from the entire section's neutral axis to the outer fiber (c), and then compute c_f:

$$c = \frac{d_w}{2} + t_f$$

$$c_f = \frac{d_w + t_f}{2}$$

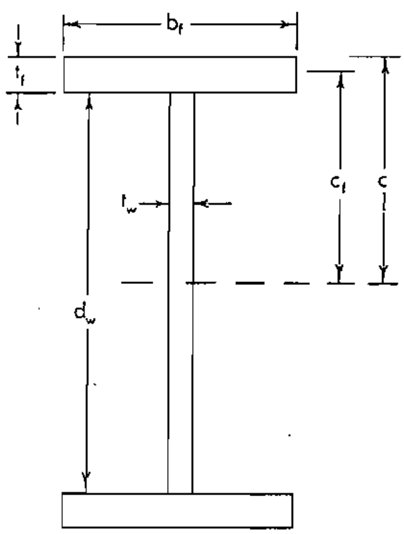
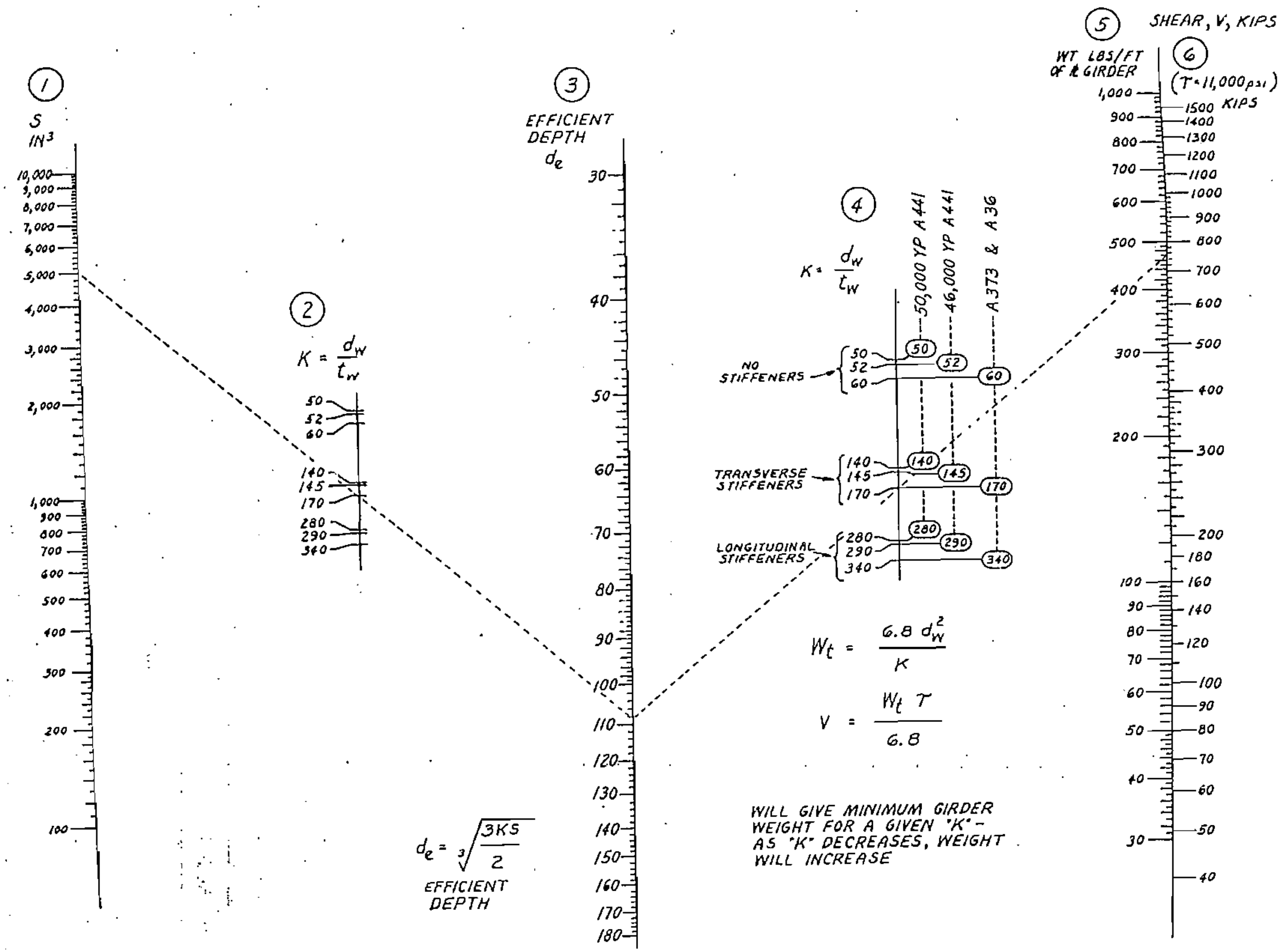


FIG. 5. Girder description.

FIG. 6 Efficient Web Depth and Approximate Weight of Plate Girder



6. With this, compute the section's total required moment of inertia:

$$I_t = S\ c$$

7. Now select a flange width from the following:

$$b_f = \frac{I_t - I_w}{2\ t_f\ c_f^2}$$

Since:

$$I_f = 2\ b_f\ t_f\ c_f^2$$

and use the next larger convenient plate width for flange width (b_f).

8. Then check

$$I_f = 2\ b_f\ t_f\ c_f^2 \text{ and}$$

$$I_t = I_f + I_w \text{ and}$$

$$S = \frac{I_t}{c}$$

This final value of section modulus (S) must equal or exceed the value initially stated as a requirement to resist the bending moment.

Method B ***When Shear Governs Design of Girder***

If the web area (A_w) computed back in Step 3 does not equal or exceed the given required amount, take these additional steps before proceeding with Step 4 of Method A.

3A. Calculate the web thickness (t_w) and web depth (d_w) from the required web area (A_w) and required depth-to-thickness ratio (K), using the following formulas:

$$t_w \geqq \sqrt{\frac{A_w}{K}}$$

and

$$d_w \leqq t_w\ K$$

3B. Using this as a guide, adjust the thickness (t_w) and depth (d_w) of the web plate to satisfy the above conditions and also the following:

$$t_w\ d_w \geqq A_w$$

which must equal or exceed the required value of A_w ($= V/\tau$); and

$$\frac{d_w}{t_w} \leqq K$$

which must equal or be less than the maximum allowable value of K.

Having selected d_w and t_w, return to Step 4 of Method A and follow through to completion (Step 8).

Short-Cut Nomographs

The first nomograph, Figure 6, will quickly give the girder's efficient web depth as well as its estimated weight (lbs/lin ft).

On this nomograph:

Line 1 = required section modulus (S)
Line 2 = required ratio of web depth to web thickness (K)
Line 3 = (read:) efficient web depth (d_e)
Line 4 = required ratio of web depth to web thickness (K)
Line 5 = (read:) estimated weight of girder (W_t)
Line 6 = (read:) allowable shear carried by web (V) on the basis of τ = 11,000 psi (bridges)

If the right-hand line 6 should indicate an allowable shear value (V) for the efficient web which is less than the actual value, the girder design must be based on the shear-carrying capacity of the web. This is done by going to the second nomograph, Figure 7:

Here:

Line 1 = actual shear value which must be carried by the web (V)
Line 2 = required ratio of web depth to web thickness (K)
Line 3 = (read:) web thickness to be used (t_w)
Line 4 = required ratio of web depth to web thickness (K)
Line 5 = (read:) web depth to be used (d_w)

The weight of this shear design may be estimated by the third nomograph, Figure 8. Two values of weight are obtained; these must be added together.

Here, for first weight:

Line 1a = required section modulus (S)
Line 2a = web depth (d)
Line 3 = (read:) estimated weight (W_t)

For the second weight:

Line 1b = shear to be carried by web (V)
Line 2b = allowable shear stress (τ)
Line 3 = (read:) estimated weight (W_t)

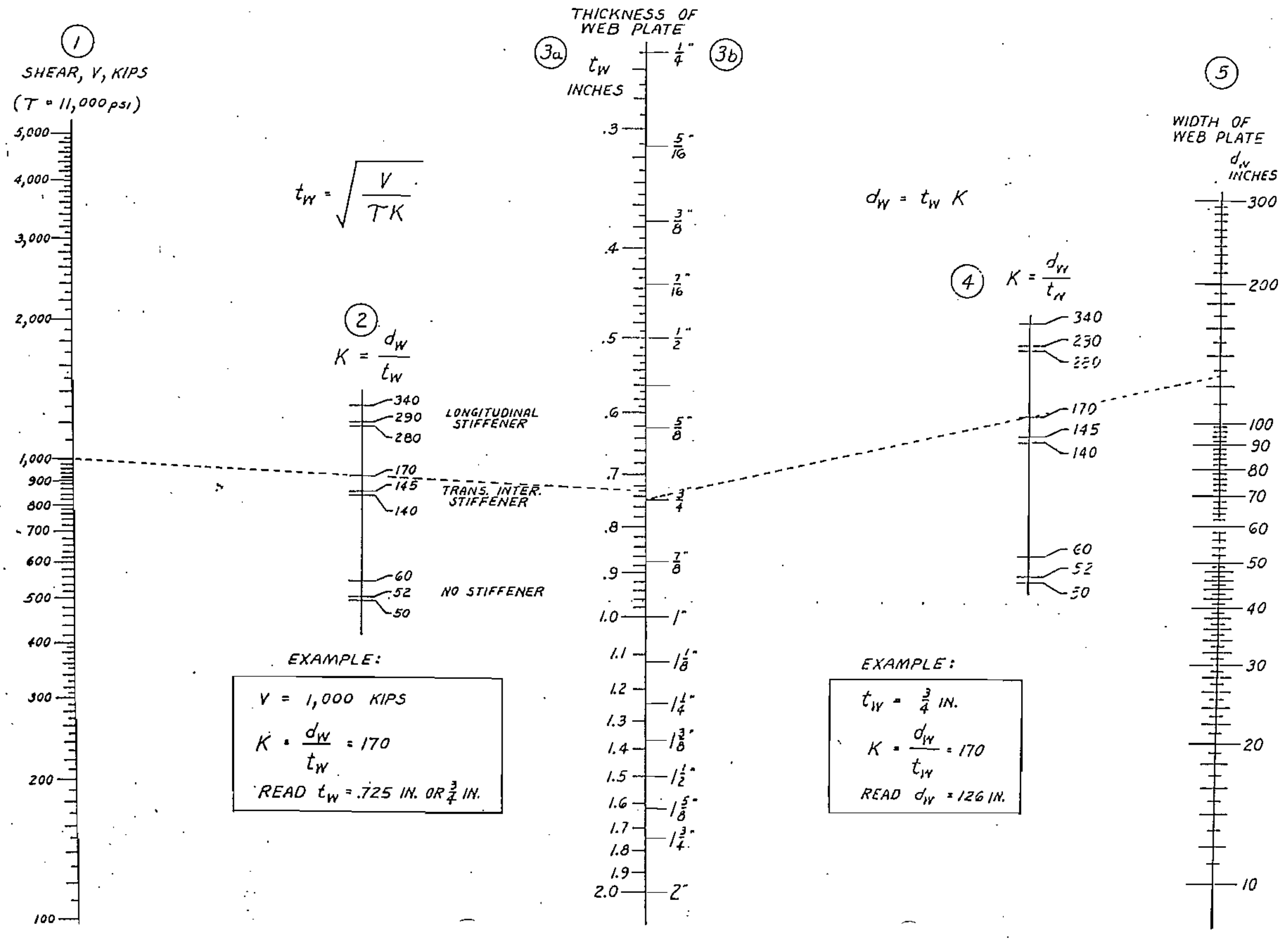
The sum of these two weights still does not include the weights of stiffeners if required.

Problem 1

Design a bridge girder for the following loads:

M = 7500 ft-kips
V = 600 kips

FIG. 7 Required Thickness and Depth of Plate Girder Web
Based on Shear-Carrying Capacity



For A36 steel, AASHO Sec 1.6.75 (see Table 1) requires the K ratio of web depth to thickness (d_w/t_w) to be not more than K = 170 using transverse stiffeners.

Then:

$$S = \frac{M}{\sigma} = \frac{(7500)(12)}{(18 \text{ ksi})} = 5000 \text{ in.}^3$$

$$A_w = \frac{V}{\tau} = \frac{(600)}{(11 \text{ ksi})} = 54.5 \text{ in.}^2$$

Following the suggested outline for designing an efficient girder:

1. $$d_w = \sqrt[3]{\frac{3\ K\ S}{2}} = \sqrt[3]{\frac{3(170)(5000)}{2}} = 108.45''$$

2. $$t_w = \frac{d_w}{K} = \frac{(108.45)}{(170)} = .638''$$

or *use an* $^{11}\!/_{16}''$ *thick web, 110″ deep.*

3. Check these proposed dimensions:

$$K = \frac{d_w}{t_w} = \frac{(110)}{(11/16)} = 160 < 170 \quad \textit{O.K.}$$

$$A_w = t_w\ d_w = (11/16)(110) = 75.6 \text{ in.}^2 > 54.5 \text{ in.}^2 \quad \textit{O.K.}$$

4. $$I_w = \frac{t_w\ d_w^3}{12} = \frac{(11/16)(110)^3}{12} = 76{,}255 \text{ in.}^4$$

5. Let flange thickness be $t_f = 2''$:

$$c = \frac{d_w}{2} + t_f = \frac{(110)}{2} + (2) = 57.0''$$

$$c_f = \frac{d_w + t_f}{2} = \frac{(110) + (2)}{2} = 56.0''$$

6. $$I_t = S\ c = (5000 \text{ in.}^3)(57'') = 285{,}000 \text{ in.}^4$$

7. $$b_f = \frac{I_t - I_w}{2\ t_f\ c_f^2} = \frac{(285{,}000) - (76{,}255)}{2\ (2)\ (56)^2} = 16.65''$$

or use 17.0″ wide × 2″ thick flange plates

8. Then, to find properties of the actual proposed section:

$$I_f = 2\ b_f\ t_f\ c_f^2 = 2\ (17)(2)(56)^2 = 213{,}250 \text{ in.}^4$$

$$I_t = I_f + I_w = (213{,}250) + (76{,}250) = 289{,}500 \text{ in.}^4$$

$$S = \frac{I_t}{c} = \frac{(289{,}500)}{(57)} = 5080 \text{ in.}^3 > 5000 \text{ in.}^3 \quad \textit{OK}$$

Then, to find the weight of this designed girder:

$$2\ A_f = 2(2'')(17'') = \quad 68.0$$

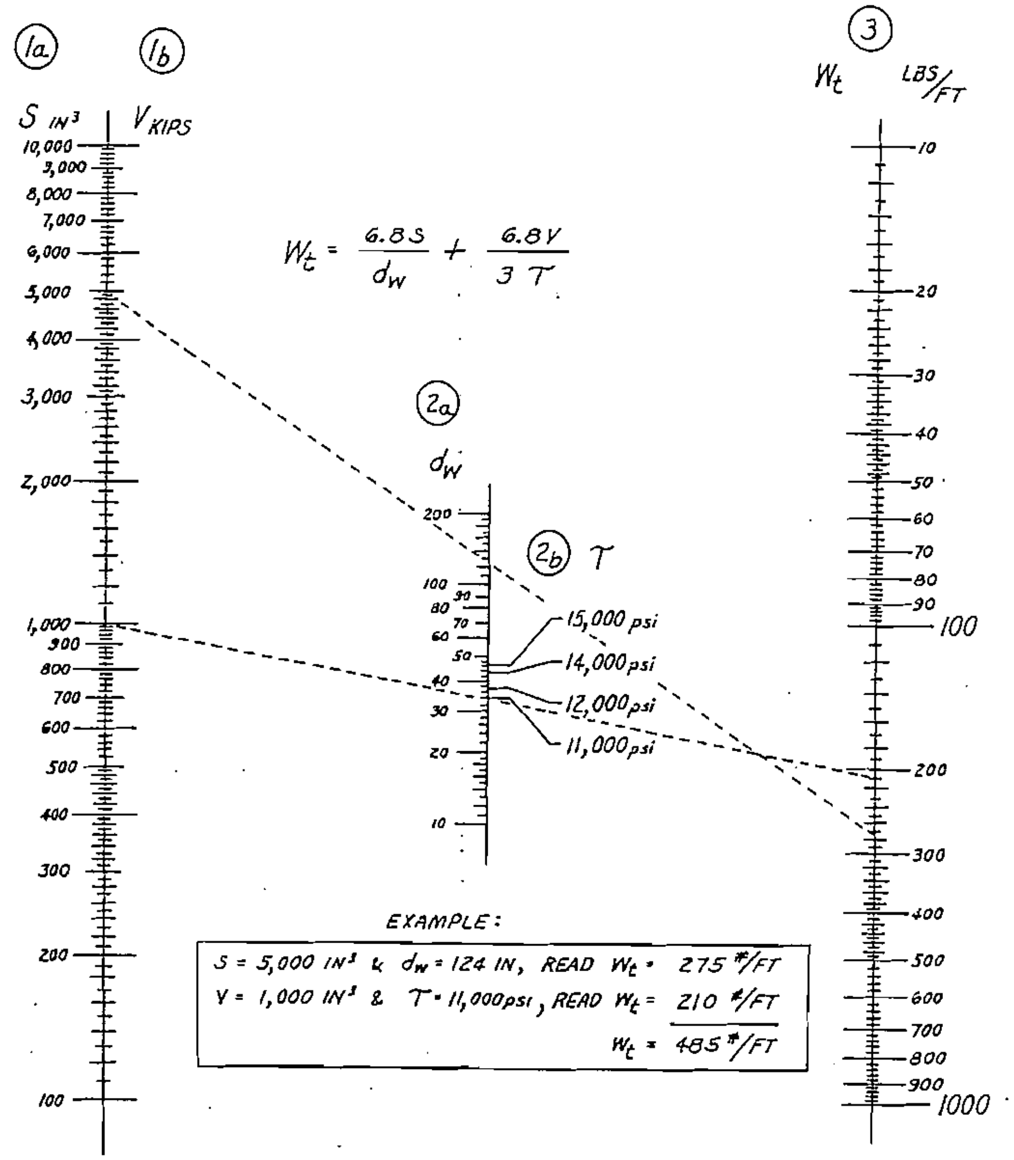
$$A_w = (11/16'')(110'') = \underline{\quad 75.6}$$

$$143.6 \text{ in.}^2$$

$\therefore W_t$ = *488 lbs/lin ft* of girder, on the basis of steel's weighing 3.4 lbs/lin ft/in.^{2} of cross section.

To show that this does result in the minimum girder weight, nine other combinations have been figured, from a web depth of 70″ up to 120″, as shown by Curve B in Figure 2. In the example just worked, the various dimensions were rounded off to the next

FIG. 8 Weight of Plate Girder When Design Is Governed by Shear



size fraction based on available plate. The actual plate girder example using a web depth of 110″ weighed 488 lbs/ft, yet the efficient girder for this same depth should weigh 473 lbs/ft.

Four other combinations of flange dimensions were figured, using the same web depth (d_w = 108.45″), but there was little difference in girder weight. The thinner and wider flanges result in a very slight reduction in weight.

Problem 2

Consider the same girder in which the shear load is increased to V = 1000 kips. This will illustrate the work to be done where shear (V) would govern the design.

Here:

$$A_w = \frac{V}{\tau} = \frac{(1000)}{(11 \text{ ksi})} = 90.9 \text{ in.}^2$$

Following the suggested outline:

1. $d_w = \sqrt[3]{\frac{3\ K\ S}{2}}$

$= \sqrt[3]{\frac{3(170)(5000)}{2}}$

$= 108.45''$

2. $t_w = \frac{d_w}{K}$

$= \frac{(108.45)}{(170)}$

$= .638''$

In the previous problem, this led to a web 11/16″ × 110″; however—

3. $A_w = t_w\ d_w$

$= (11/16)(110)$

$= 75.6\ \text{in.}^2 < 90.9\ \text{in.}^2$

In this case the $^{11}/_{16}''$ × 110″ web plate has insufficient area to carry the shear load. So, switching to Method B:

3A. $t_w = \sqrt{\frac{A_w}{K}}$

$= \sqrt{\frac{(90.9)}{(170)}}$

$= .732''$

or *use a ¾″-thick web plate.*

$d_w = t_w\ K$

$= (¾)(170)$

$= 127.5''$

or *use a 124″ deep web plate.*

3B. Check:

$K = \frac{d_w}{t_w}$

$= \frac{(124)}{(¾)}$

$= 165.3 < 170$ *OK*

$A_w = t_w\ d_w$

$= (¾)(124)$

$= 93.0\ \text{in.}^2 > 90.9\ \text{in.}^2$ *OK*

Now returning to the basic Method A outline:

4. $I_w = \frac{t_w\ d_w^3}{12}$

$= \frac{(¾)(124)^3}{12}$

$= 119{,}164.\ \text{in.}^4$

5. Let flange thickness be $t_f = 2''$:

$c = \frac{d_w}{2} + t_f$

$= \frac{(124)}{2} + (2)$

$= 64''$

$c_f = \frac{d_f + t_f}{2}$

$= \frac{(124) + (2)}{2}$

$= 63''$

6. $I_t = S\ c$

$= (5000\ \text{in.}^3)(64'')$

$= 320{,}000\ \text{in.}^4$

7. $b_f = \frac{I_t - I_w}{2\ t_f\ c_f^2}$

$= \frac{(320{,}000) - (119{,}164)}{2\ (2)(63)^2}$

$= 12.65''$

or *use 13″ wide x 2″ thick flange plates*

8. Then, to find properties of the actual proposed section:

$I_f = 2\ b_f\ t_f\ c_f^2$

$= 2(13)(2)(63)^2$

$= 206{,}388\ \text{in.}^4$

$I_t = I_f + I_w$

$= (206{,}388) + (119{,}164)$

$= 325{,}550\ \text{in.}^4$

$S = \frac{I_t}{c}$

$= \frac{(325{,}550)}{(64)}$

$= 5090\ \text{in.}^3 > 5000\ \text{in.}^3$ *OK*

Then, to find the weight of this designed plate girder:

$$2\ A_f = 2(2'')(13'') = 52.0$$
$$A_w = (\tfrac{3}{4}'')(120'') = \underline{90.0}$$
$$142.0 \text{ in.}^2$$

$\therefore\ W_t =$ *482.8 lbs/lin ft* of girder

Problem 3

Find the approximate web dimensions and weight for the same girder, using the nomographs, Figures 6, 7 and 8.

1st Nomograph

Given:

$S = 5000 \text{ in.}^3$

$K = \dfrac{d_w}{t_w} = 170$

read:

$d = 108''$

Given:

$K = \dfrac{d_w}{t_w} = 170$

read:

$W_t =$ *470 lbs/ft*

and:

$V =$ *750 kips* allowable

Using an actual depth of 110″ as in Figure 1 would increase this estimated weight to 483 lbs/ft as read on the nomograph. In Problem 1, the weight was computed to be 488 lbs/ft; this slight increase is due to the increase in web thickness from the required .638″ to the next fraction, 11/16″.

2nd Nomograph

If the shear value is increased to V = 1000 kips as in Problem 2, this exceeds the allowable value of 750 kips read from the first nomograph. Therefore, shear governs the design and the second nomograph must be used.

Given:

$V = 1000$ kips

$K = \dfrac{d_w}{t_w} = 170$

read:

$t_w = .725''$ or *use* $\tfrac{3}{4}''$

Given:

$K = \dfrac{d_w}{t_w} = 170$

read:

$d_w = 126''$ or *use 124″*

3rd Nomograph

Given:

$S = 5000 \text{ in.}^3$

$d = 124''$

read:

$W_t = \longrightarrow$ 275 lbs/ft

Given:

$V = 1000$ kips

$\tau = 11{,}000$ psi

read:

$W_t = \longrightarrow$ + 210 lbs/ft

Total = 485 lbs/ft

In Problem 2, the weight was computed to be 482.8 lbs/ft.