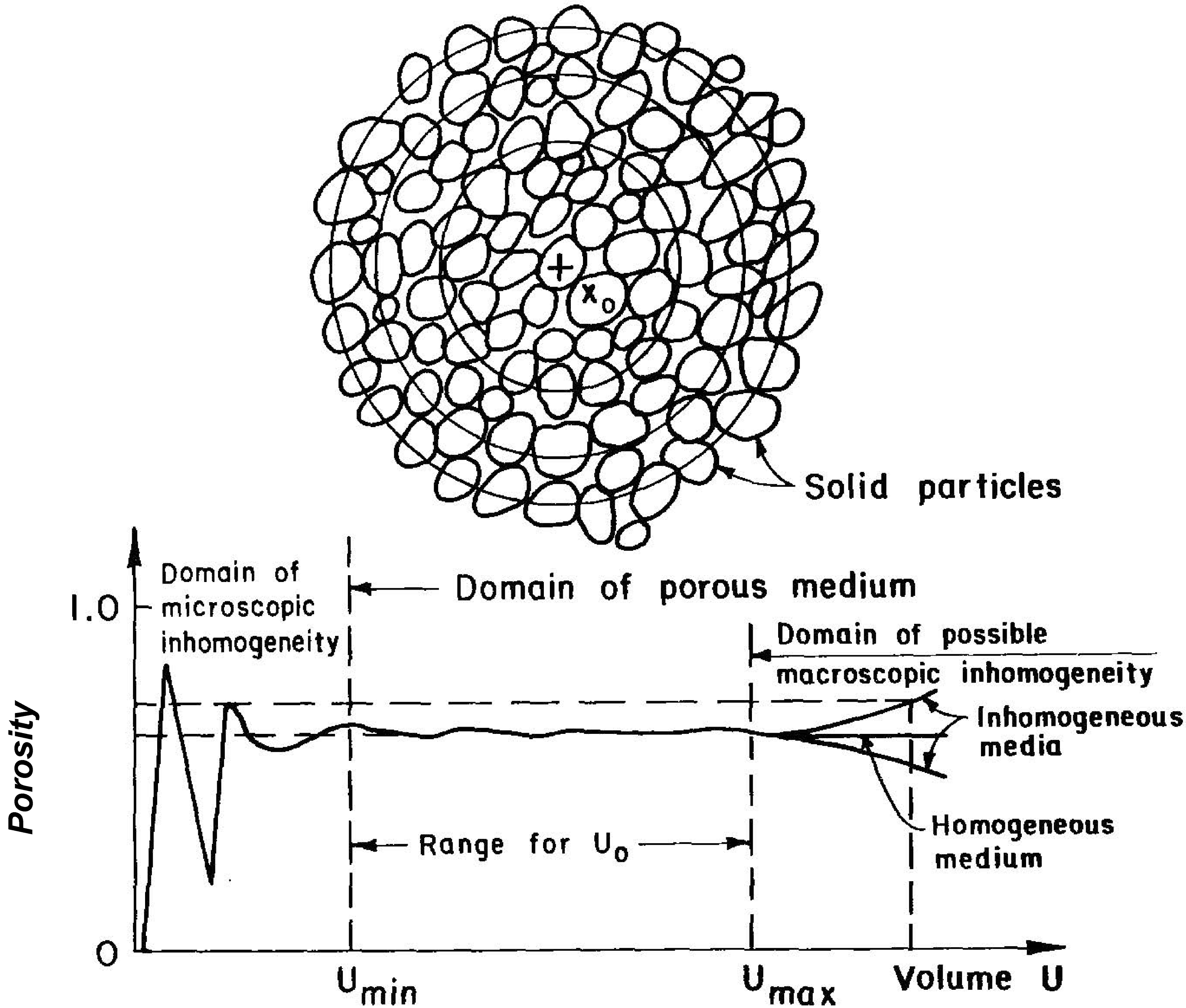
Fundamentals of Fluid Flow in Porous Media

At the end of this module, you will:

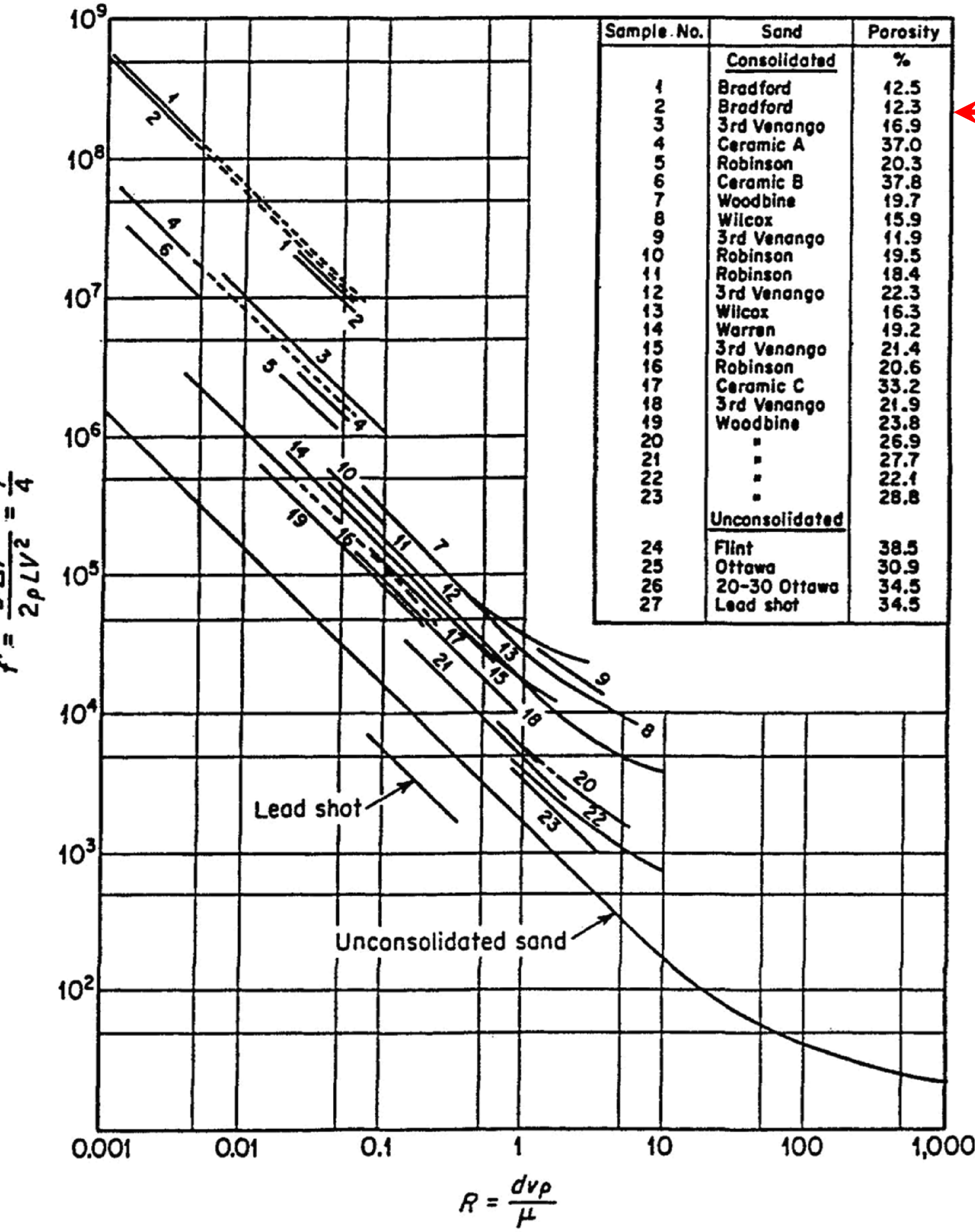
- Be familiar with the concept of a "Representative Elemental Volume."
- Be familiar with the generalized flow behavior (Darcy and Forchheimer Flow).
- Be familiar with the cases of the "ideal-reservoir model:" (radial flow)
  - Diffusivity Equation for the Flow of Slightly Compressible Fluids.
  - Diffusivity Equations for the Flow of Compressible Fluids.
    - Effect of Pressure-Dependent Gas Properties — μz plots
    - Effect of Pressure-Dependent Gas Properties — $(p/\mu z)$ plots
    - Effect of Pressure-Dependent Gas Properties — p_p and t_a
  - Diffusivity Equation for Multiphase Flow
  - Dimensionless Forms of the Diffusivity Equation.
- Be familiar with and be able to use the solutions for the diffusivity equation:
  - Bounded Cylindrical Reservoir (Full Solution)
  - Bounded Cylindrical Reservoir (Pseudosteady-State Flow)
  - Infinite(-Acting) Cylindrical Reservoir (Line-Source Well)
  - Near-Wellbore Damage or Stimulation Effects (Skin Effects)
  - Radial Flow in an Infinite Reservoir With Wellbore Storage.
  - Fractured Wells in Infinite-Acting Reservoirs
- Be familiar with and be able to use the radius of Investigation.
- Be familiar with and be able to use Horner's Approximation.

Representative Elementary Volume



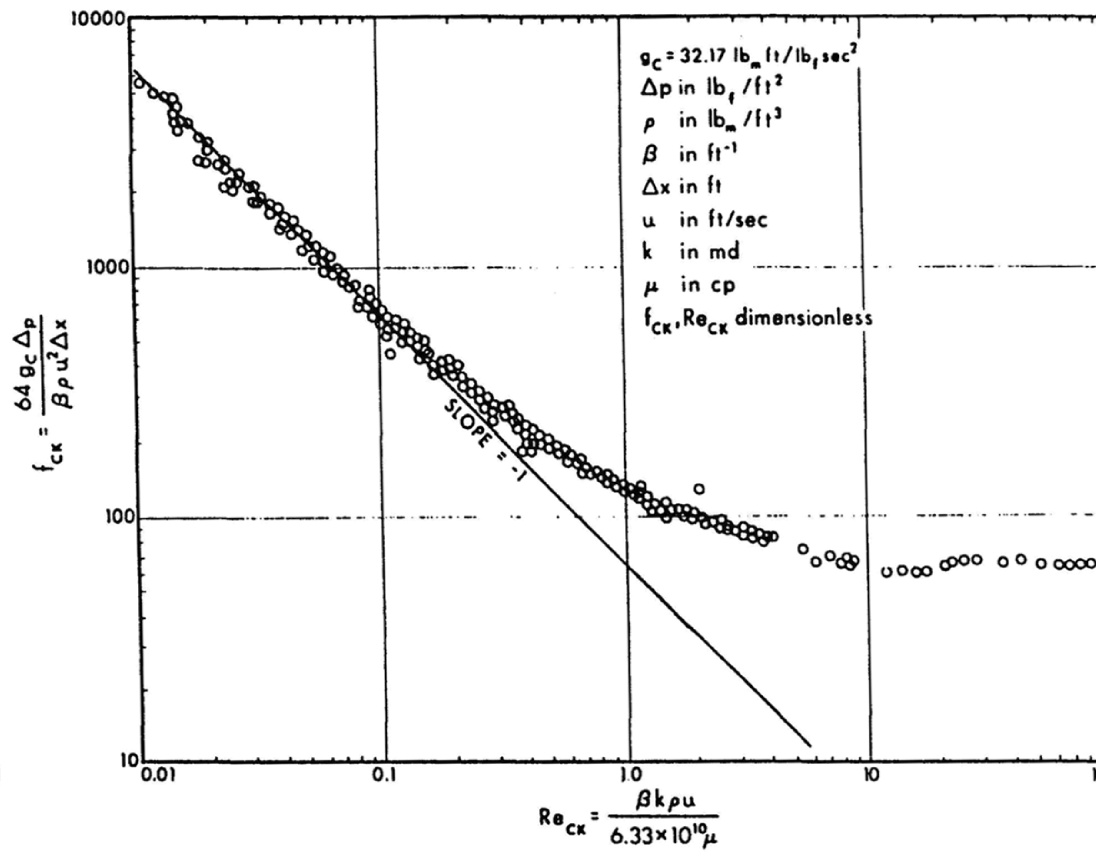
achmat, Y. and Bear, J. "Macroscopic Modelling of Transport
henomena in Porous Media, 1. The Continuum Approach,
ransport in Porous Media, 1:213-240, 1986.

Generalized Flow Behavior — Darcy and Forchheimer Flow



Fancher, Lewis, and Barnes (1933)
Experimental Study of Fluid Flow in Porous Media

Cornell and Katz Unified Flow Relation for Generalized Flow in Porous Media



Amyx, J.W., Bass, D.M., Jr., and Whiting, R.L.: "Petroleum Reservoir Engineering," McGraw-Hill Book Co., New York (1960).

Fancher, G.H., Lewis, J.A., and Barnes, K.B.: "Some Physical Characteristics of Oil Sands," Pa. State College, Min. Ind. Exp. Sta. Bull. 12 (1933), 65-171.

Cornell, D., and Katz, D.L.: "Flow of Gases Through Consolidated Porous Media," Ind. and Eng. Chem. (1953), 45, 2145-2152.

Summary of Diffusivity Equations

Diffusivity Equation for the Flow of Slightly Compressible Fluids: (Liquid Case)

$$\nabla^2 p = \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t}$$

Where: μc_t is assumed constant.

Diffusivity Equations for the Flow of Compressible Fluids: (Dry Gas Case)

$$\nabla \bullet \left[\frac{p}{\mu_g z} \nabla p \right] = \frac{\phi c_t}{k} \frac{p}{z} \frac{\partial p}{\partial t}$$

General Formulation

$$\nabla^2 p_p = \frac{\phi \mu_g c_t}{k} \frac{\partial p_p}{\partial t}$$

Pseudopressure Formulation

$$\nabla^2 p_p = \frac{\phi}{k} (\mu_g c_t)_{p_n} \frac{\partial p_p}{\partial t_a}$$

Pseudopressure-Pseudotime Formulation

Diffusivity Equation for Multiphase Flow:

$$\nabla^2 p = \phi \frac{c_t}{\lambda_t} \frac{\partial p}{\partial t}$$

Where:

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}$$

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_f$$

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Diffusivity Equations for the Flow of Compressible Fluids — Dry Gas Case

$$\nabla \bullet \left[\frac{p}{\mu_g z} \nabla p \right] = \frac{\phi c_t}{k} \frac{p}{z} \frac{\partial p}{\partial t}$$

General Formulation

$$\nabla^2 p_p = \frac{\phi \mu_g c_t}{k} \frac{\partial p_p}{\partial t}$$

Pseudopressure Formulation

$$\nabla^2 p_p = \frac{\phi}{k} (\mu_g c_t)_{p_n} \frac{\partial p_p}{\partial t_a}$$

Pseudopressure-Pseudotime Formulation

$$p_{pg} = \left[\frac{\mu_g z}{p} \right]_{p_n} \int_{p_{base}}^{p} \frac{p}{\mu_g z} dp$$

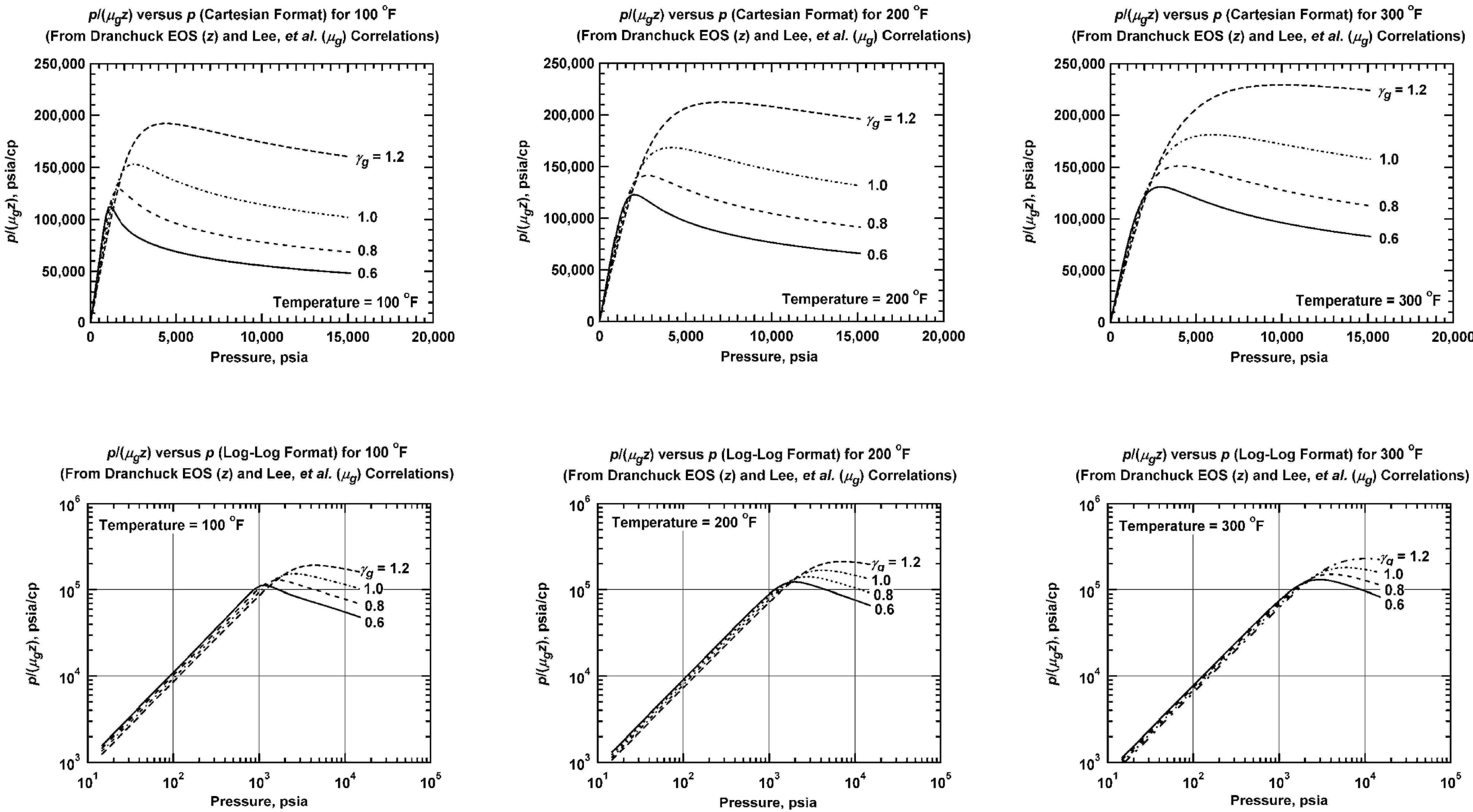
Pseudopressure Definition

$$t_a = [\mu_g c_t]_n \int_0^t \frac{1}{\mu_g (p) c_t (p)} dt$$

Pseudotime Formulation

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Behavior of $p/(\mu_g z)$ vs. p — Dry Gas Case



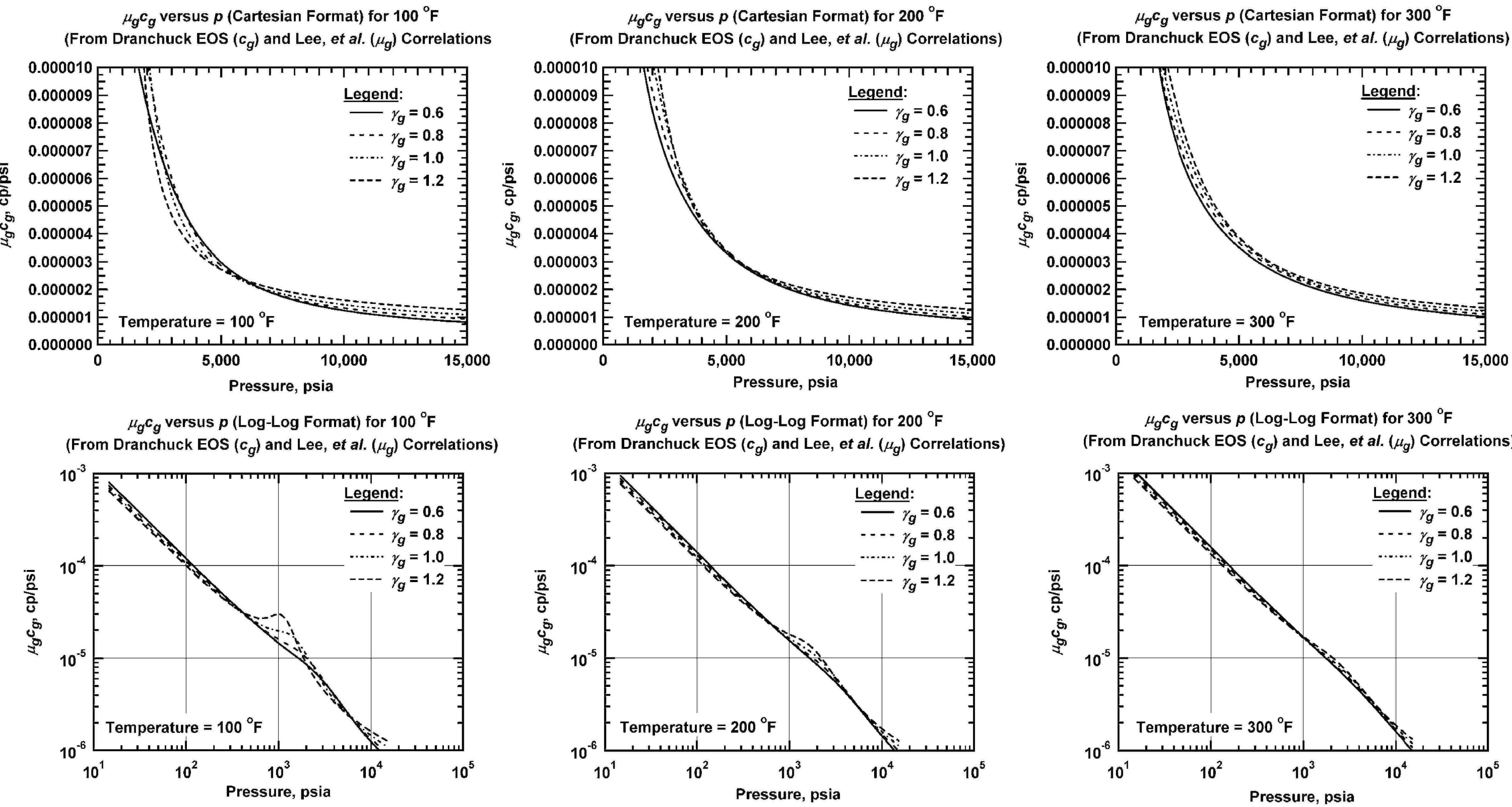
<u>"Dry Gas" PVT Properties</u>: ($p/(\mu_g z)$ vs. p)

- Concept: If ($p/\mu_g z$) = constant, pseudopressure NOT required.
- ($p/\mu_g z$) is NEVER constant, ***pseudopressure is required***.
- If ($p/\mu_g z$) = constant → "p" approximation (never valid).

$$p_{pg} = \left[\frac{\mu_g z}{p}\right]_{p_n} \int_{p_{base}}^{p} \frac{p}{\mu_g z} d$$

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Behavior of $\mu_g c_g$ vs. p — Dry Gas Case



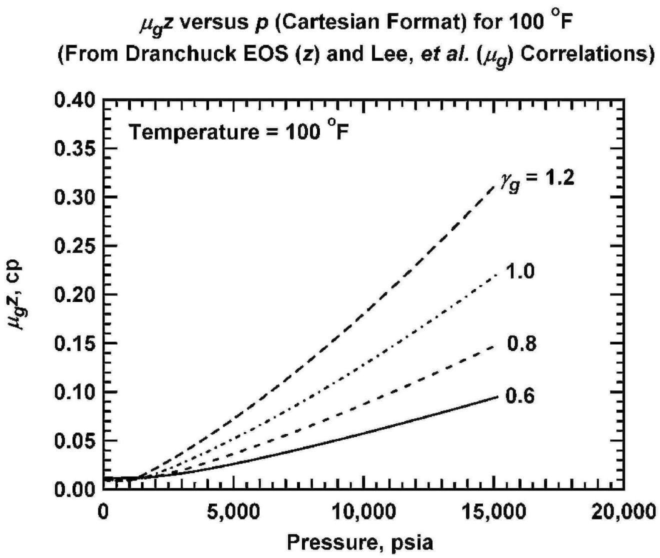
<u>"Dry Gas" PVT Properties</u>: ($\mu_g c_g$ vs. p)

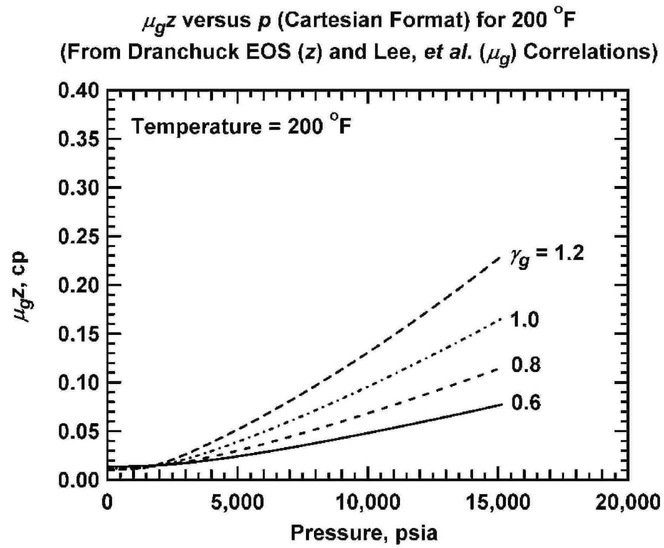
- Concept: If $\mu_g c_g$ = constant, pseudotime NOT required.
- Obvious that $\mu_g c_g$ is NEVER constant, *pseudotime required*.
- This is a major limitation for buildup test analysis.

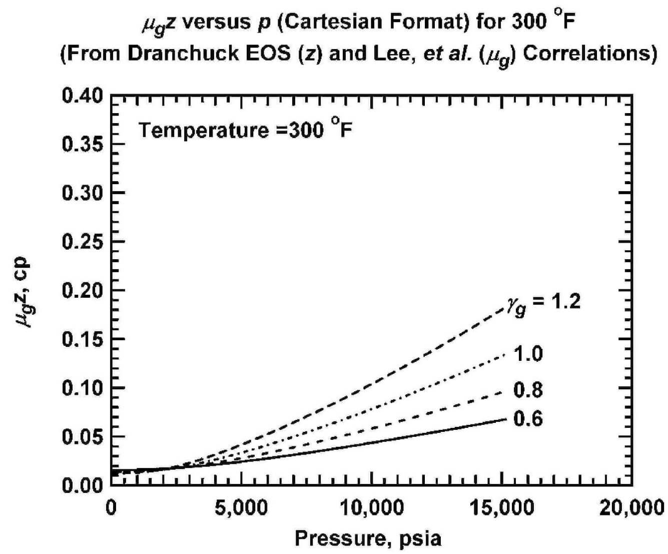
$$t_a = [\mu_g c_t]_n \int_0^t \frac{1}{\mu_g(p) c_t(p)} dt$$

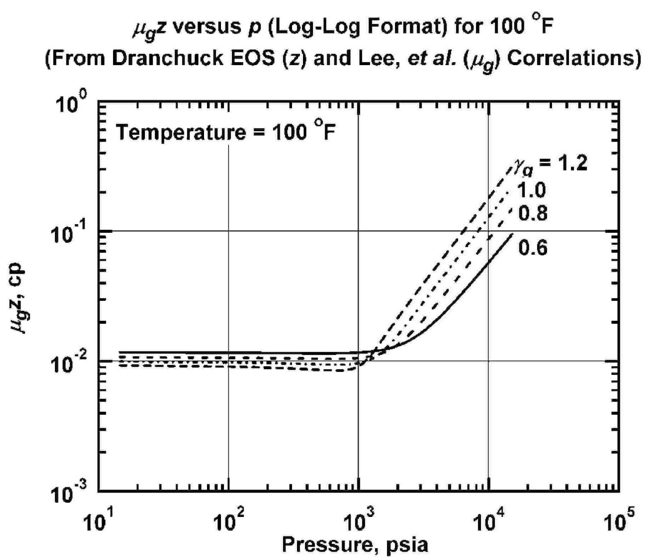
lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

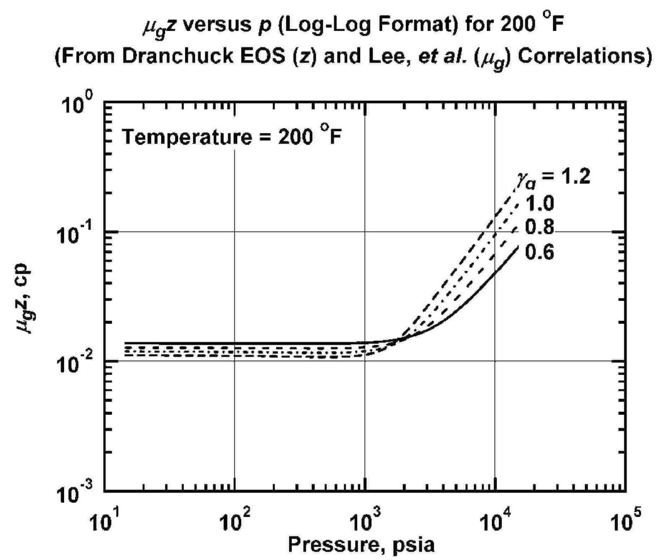
Behavior of $\mu_g z$ vs. p — Dry Gas Case

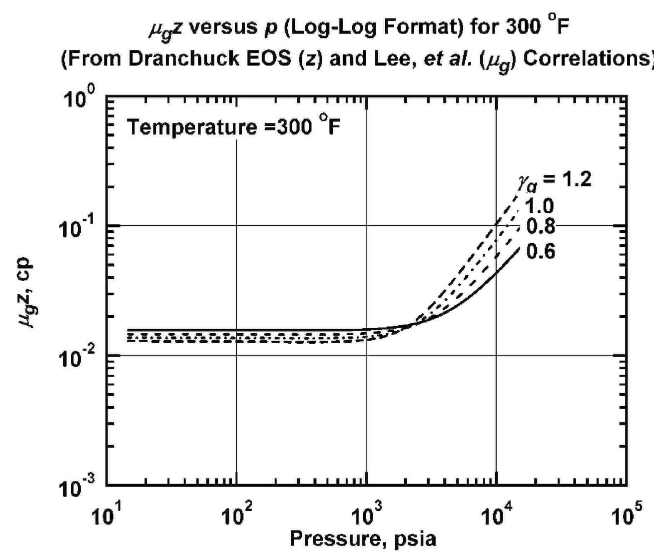












<u>"Dry Gas" PVT Properties</u>: ($\mu_g z$ vs. p)

- **Concept: ($\mu_g z$) = constant, valid only for p<2000 psia.**
- **Basis for the "pressure-squared" approximation (*i.e.*, use of p^2 variable).**
- **The plots indicate that ($\mu_g z$) ≈ constant for p<2000 psia, so p^2 is (generally) valid.**

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Diffusivity Equations for the Flow of Compressible Fluids — p^2 Form (Dry Gas)

$$\nabla^2(p^2) - \frac{\partial}{\partial p^2}[\ln(\mu_g z)]\nabla(p^2)^2 = \frac{\phi \mu_g c_t}{k}\frac{\partial}{\partial t}(p^2)$$

p^2 Form — Full Formulation

$$\nabla^2(p^2) = \frac{\phi \mu_g c_t}{k}\frac{\partial}{\partial t}(p^2)$$

p^2 Form — Approximation
(assumes $\mu_g z$ = constant)

<u>"Dry Gas" Diffusivity Equation</u>: (p^2 form)

- **Assuming ($\mu_g z$) = constant for p<2000 psia, then the ln[$\mu_g z$] derivative term is zero.**
- **Therefore, the p^2 form of the gas diffusivity equation can be used for p<2000 psia.**
- **However; there are NOT many gas reservoirs which are discovered at p<2000 psia.**

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Diffusivity Equation for Multiphase Flow

Gas Equation:

$$\nabla \bullet \left[\left[\frac{k_g}{\mu_g B_g} + R_{so} \frac{k_o}{\mu_o B_o} + R_{sw} \frac{k_w}{\mu_w B_w} \right] \nabla p \right] = \frac{\partial}{\partial t} \left[\phi \left[\frac{S_g}{B_g} + R_{so} \frac{S_o}{B_o} + R_{sw} \frac{S_w}{B_w} \right] \right]$$

Oil Equation:

$$\nabla \bullet \left[\frac{k_o}{\mu_o B_o} \nabla p \right] = \frac{\partial}{\partial t} \left[\phi \frac{S_o}{B_o} \right]$$

Water Equation:

$$\nabla \bullet \left[\frac{k_w}{\mu_w B_w} \nabla p \right] = \frac{\partial}{\partial t} \left[\phi \frac{S_w}{B_w} \right]$$

Multiphase Equation:

$$\nabla^2 p = \phi \frac{c_t}{\lambda_t} \frac{\partial p}{\partial t}$$

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w}$$

Compressibility Terms:

$$c_o = -\frac{1}{B_o} \frac{dB_o}{dp} + \frac{B_g}{B_o} \frac{dR_{so}}{dp}$$

$$c_w = -\frac{1}{B_w} \frac{dB_w}{dp} + \frac{B_g}{B_w} \frac{dR_{sw}}{dp}$$

$$c_g = -\frac{1}{B_g} \frac{dB_g}{dp}$$

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_f$$

Diffusivity Equation for Multiphase Flow: (*p* form)

- **This is the so-called "Perrine-Martin" form.**
- **Requires (very) serious assumptions:** ($\nabla S_o \nabla p$, $\nabla S_w \nabla p$, and $\nabla p \nabla p = \nabla^2 p$ are neglected)
- **Implies that multiphase behavior can be treated just like single-phase behavior.**

lasingame, T.A. Diffusivity Equation Lectures, Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Dimensionless Forms of the Diffusivity Equation

The Dimensionless Diffusivity Equation:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D}$$

Initial and Boundary Conditions:

- **Initial Condition:**

$$p_D(r_D, t_D \le 0) = 0 \qquad \textbf{(uniform pressure)}$$

- **Inner Boundary Condition:**

$$\left[r_D \frac{\partial p_D}{\partial r_D} \right]_{r_D = 1} = -1 \qquad \textbf{(constant rate)}$$

- **Outer Boundary Conditions:**

— **"Infinite-Acting" Reservoir**

$$p_D(r_D \to \infty, t_D) = 0$$

— **"No-Flow" Boundary**

$$\left[r_D \frac{\partial p_D}{\partial r_D} \right]_{r_D = r_{eD}} = 0$$

— **Constant Pressure Boundary**

$$p_D(r_{eD}, t_D) = 0$$

To simplify in your mind…

$$\frac{\partial^2 y}{\partial x^2} + \frac{1}{x}\frac{\partial y}{\partial x} = \frac{\partial y}{\partial t}$$

It's a second order partial differential equation.

We need:

- **An initial condition.**
- **An inner boundary condition.**
- **An outer boundary condition.**

OK, we have all these … now how do we solve this differential equation.

Short Answer: Take PETE 620
Long Answer:

- **Transform approach. (yes)**
- **Separation of Variables. (no)**
- **Something else. (maybe?)**

Blasingame, T.A. "Reservoir Flow Petroleum Solutions Lecture 2 -- Solution of the Radial Diffusivity Equation," Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs (1992).

Solutions of the Radial Flow Diffusivity Equation

Analytical (Bounded Reservoir) Full Solution: (Muskat, 1930's)

$$p_D(\hat{r}_D, t_D) = -\ln[\hat{r}_D] - \frac{3}{4} + \frac{\hat{r}_D^2}{2} + 2\pi t_{DA} - 2\sum_{n=1}^{\infty} \frac{J_0(X_n \hat{r}_D)}{X_n^2 J_0^2(X_n)} \exp(-X_n^2 \pi t_{DA})$$

(where $\hat{r}_D = r/r_e$ and X_n are the positive roots of $J_1(X_n) = 0$)

Approximate (Bounded Reservoir) Full Solution: (Horner, 1950's)

$$p_D(r_D, r_{eD}, t_D) \approx$$

$$\frac{1}{2}\mathrm{E}_1\left[\frac{r_D^2}{4t_D}\right] - \frac{1}{2}\mathrm{E}_1\left[\frac{r_{eD}^2}{4t_D}\right] + 2\frac{t_D}{r_{eD}^2}\exp\left[\frac{-r_{eD}^2}{4t_D}\right] + \left[\frac{r_D^2}{2r_{eD}^2} - \frac{1}{4}\right]\exp\left[\frac{-r_{eD}^2}{4t_D}\right]$$

Infinite-Acting Radial Solution: (Various Sources (heat and water literature))

$$p_D(r_D = 1, t_D) = \frac{1}{2}\mathrm{E}_1\left[\frac{1}{4t_D}\right] \approx \frac{1}{2}\ln\left[\frac{4}{e^{\gamma}} t_D\right]$$

This is the "base" solution for all pressure transient analysis plots, plotting functions, analyses, etc. for the case of vertical <u>unfractured</u> wells.

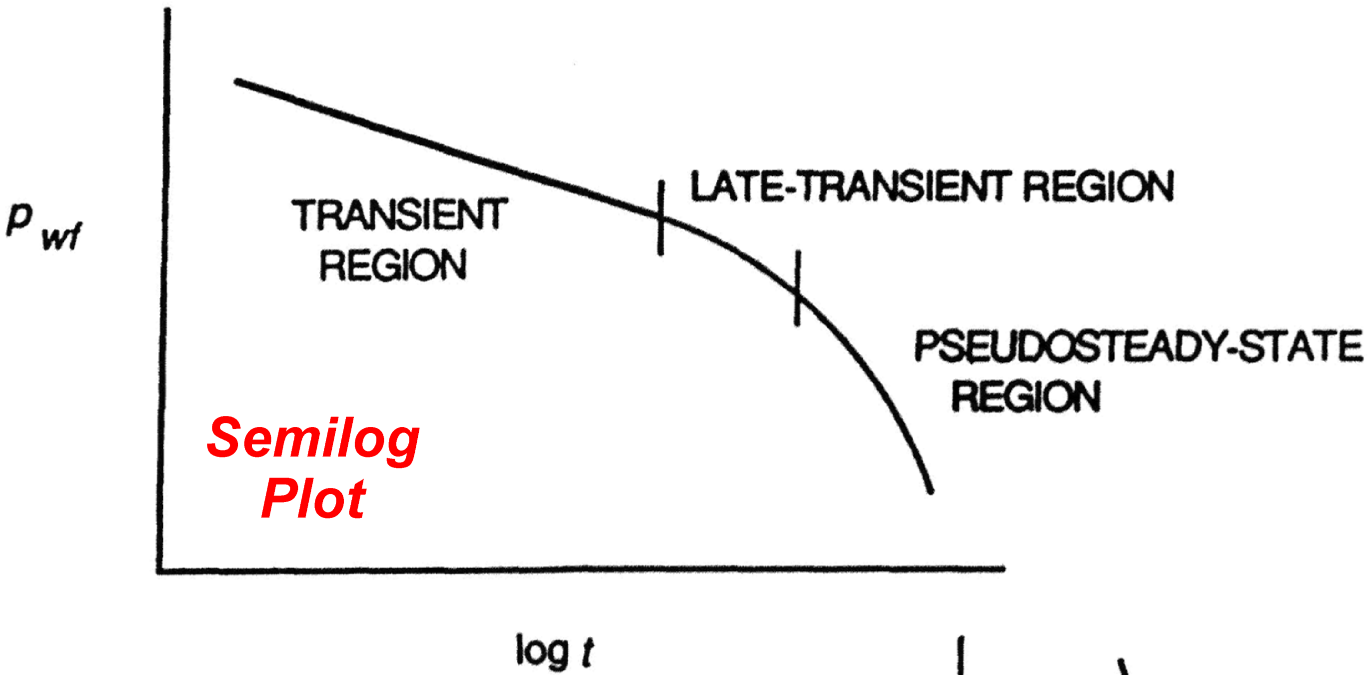
Dimensionless Variables: (Field Units)

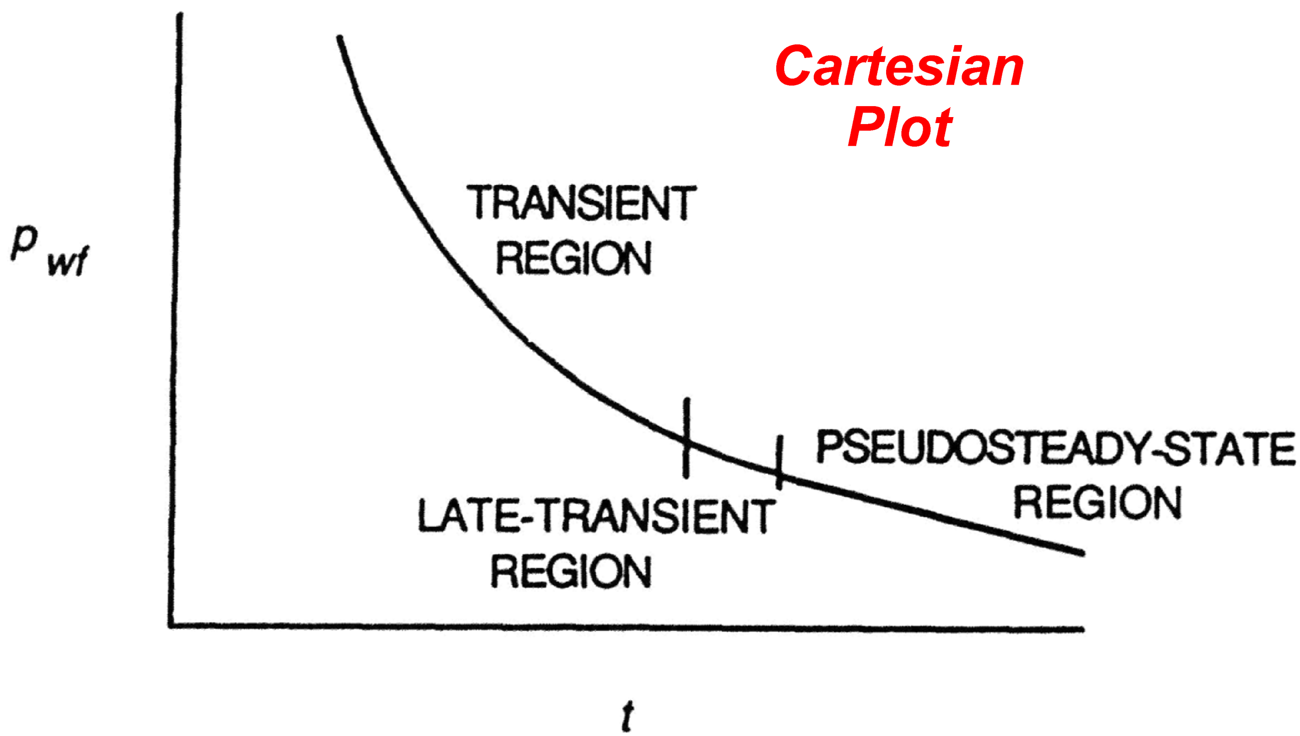
$$t_D = 2.637\mathrm{x}10^{-4} \frac{k}{\phi \mu c_t r_w^2} t \qquad t_{DA} = 2.637\mathrm{x}10^{-4} \frac{k}{\phi \mu c_t A} t \ (t \text{ in hr, } A \text{ in ft}^2)$$

$$p_D = \frac{1}{141.2} \frac{kh}{qB\mu}(p_i - p_r)$$

Solutions of the Radial Flow Diffusivity Equation

Time-Pressure Schematic Plots





ee, W.J. and Wattenbarger, R.A.: Gas Reservoir
ngineering, SPE (1996)

Pseudosteady-State Flow Solutions (*Bounded Circular Reservoir*)

Steady-State Flow Solution: (no practical use, other than lab calculations)

$$p_r = p_w + 141.2 \frac{qB\mu}{kh} \ln\left[\frac{r}{r_w}\right]$$

Pseudosteady-State Solution: (in terms of the *pressure at any radius*)

$$p_r - p_{wf} = 141.2 \frac{qB\mu}{kh}\left[\frac{r_e^2}{(r_e^2 - r_w^2)} \ln\left[\frac{r}{r_w}\right] - \frac{1}{2}\frac{(r^2 - r_w^2)}{(r_e^2 - r_w^2)} + s\right]$$

Pseudosteady-State Solution: (in terms of the *average reservoir pressure*)

$$\overline{p} = p_{wf} + 141.2 \frac{qB\mu}{kh}\left[\ln\left[\frac{r_e}{r_w}\right] - \frac{3}{4} + s\right]$$

Pseudosteady-State Solution: (in terms of the *initial reservoir pressure*)

$$p_{wf} = p_i - 141.2 \frac{qB\mu}{kh}\left[\ln\left[\frac{r_e}{r_w}\right] - \frac{3}{4} + s\right] - 5.615 \frac{qB}{V_p c_t} t$$

Fractured Wells in Infinite-Acting Reservoirs

(Full Solution) Fractured Well in an Infinite-Acting Reservoir: (heat literature)

$$p_D[\,|x_D| \le 1, y_D = 0, t_{Dxf}\,] = \frac{\sqrt{\pi\, t_{Dxf}}}{2}\left[\operatorname{erf}\left[\frac{(1-x_D)}{2\sqrt{t_{Dxf}}}\right] + \operatorname{erf}\left[\frac{(1+x_D)}{2\sqrt{t_{Dxf}}}\right]\right]$$

$$+\frac{(1-x_D)}{4}E_1\left[\frac{(1-x_D)^2}{4t_{Dxf}}\right] + \frac{(1+x_D)}{4}E_1\left[\frac{(1+x_D)^2}{4t_{Dxf}}\right]$$

(Short Time) Fractured Well in an Infinite-Acting Reservoir: (heat literature)

$$p_D(t_{Dxf}) = \sqrt{\pi\, t_{Dxf}}$$

(Long Time) Fractured Well in an Infinite-Acting Reservoir:

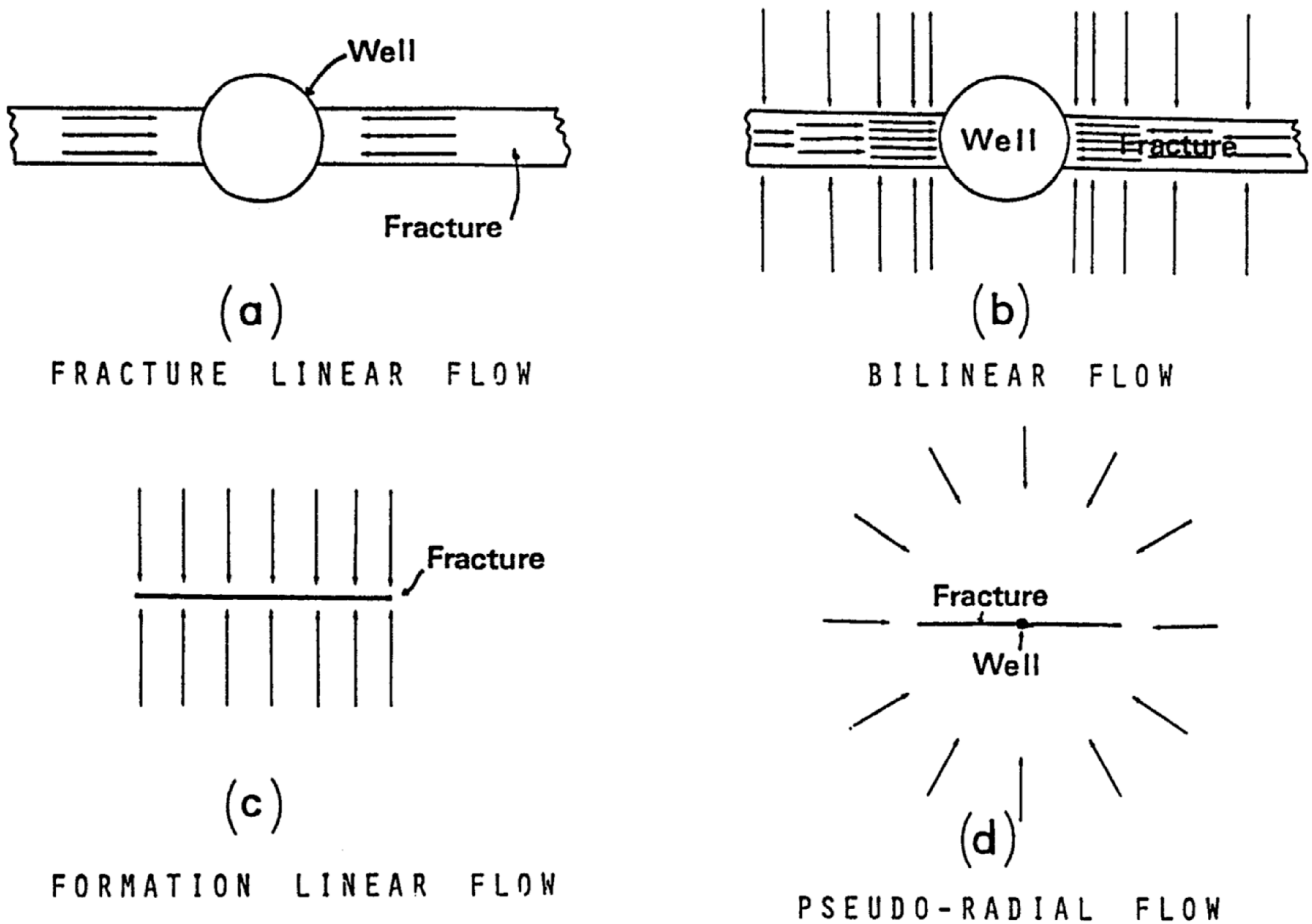
$$p_D(t_{Dxf}) = 1/2\ [\ln(t_{Dxf}) + 2.20000]\ \text{(pseudoradial flow: <1\% error, } t_{Dxf} > 10)$$

Dimensionless Variables: (Field Units)

$$t_{Dxf} = 2.637\text{x}10^{-4}\frac{k}{\phi\mu c_t x_f^2}\,t \qquad p_D = \frac{1}{141.2}\frac{kh}{qB\mu}(p_i - p_r)$$

Fractured Wells in Infinite-Acting Reservoirs

Flow Regimes in Fractured Wells



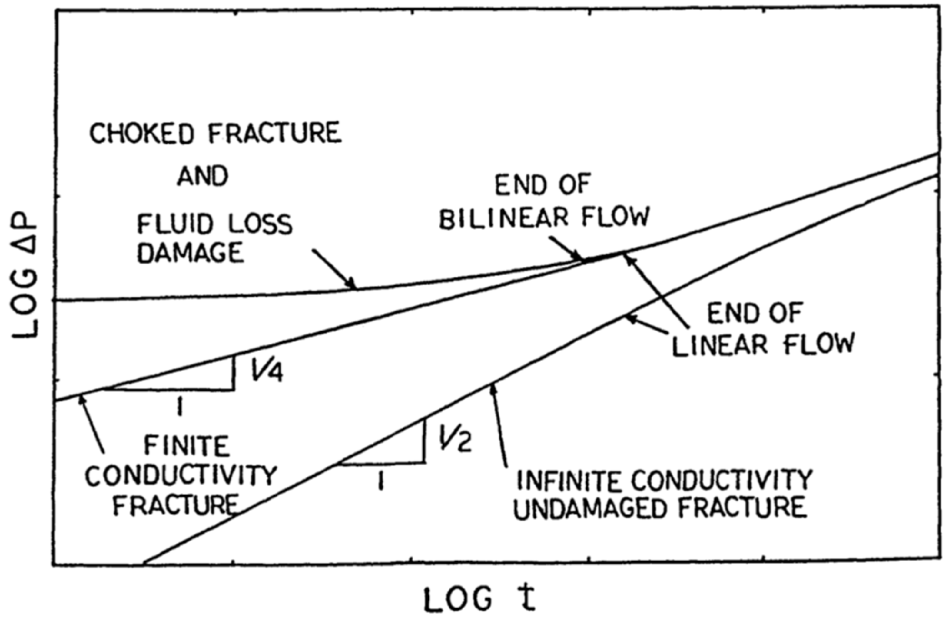
<u>**Discussion: Flow Regimes**</u>

- **FRACTURE LINEAR flow DOES NOT EXIST (a few seconds at most).**
- **FORMATION linear flow → High fracture conductivity.**
- **BILINEAR flow → Low fracture conductivity.**

Cinco-Ley, H., Samaniego-V., F.: "Transient Pressure Analysis for Fractured Wells," JPT (September 1981) 1749-1766.

Fractured Wells in Infinite-Acting Reservoirs

Fracture Damage Comparison



Cinco-Ley, H., Samaniego-V., F.: "Transient Pressure Analysis: Finite Conductivity Fracture Case Versus Damaged Fracture Case," paper SPE 10179 presented at the 1981 SPE Annual Technical Conference and Exhibition, San Antonio, TX, 5-7 Oct.

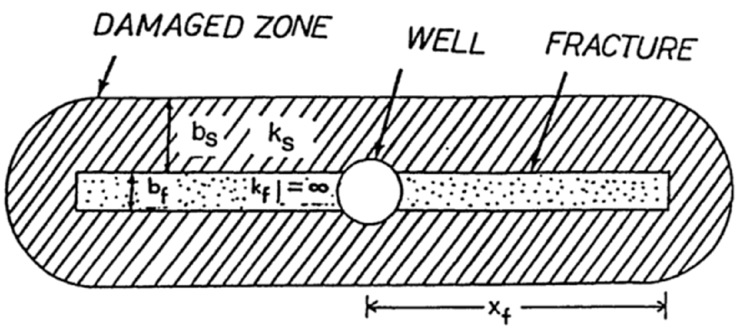


Fig. 8 – Infinite conductivity vertical fracture with fluid loss damage

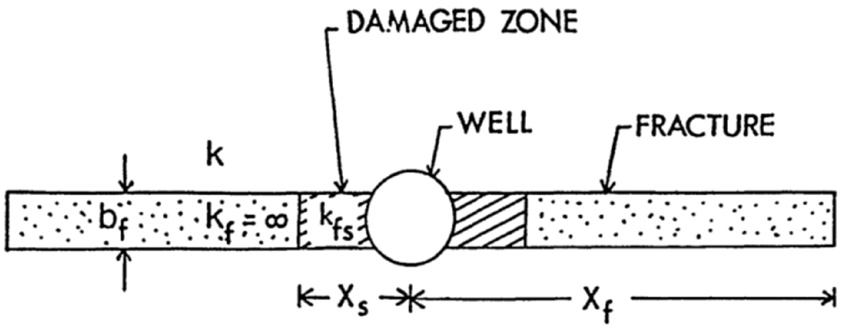


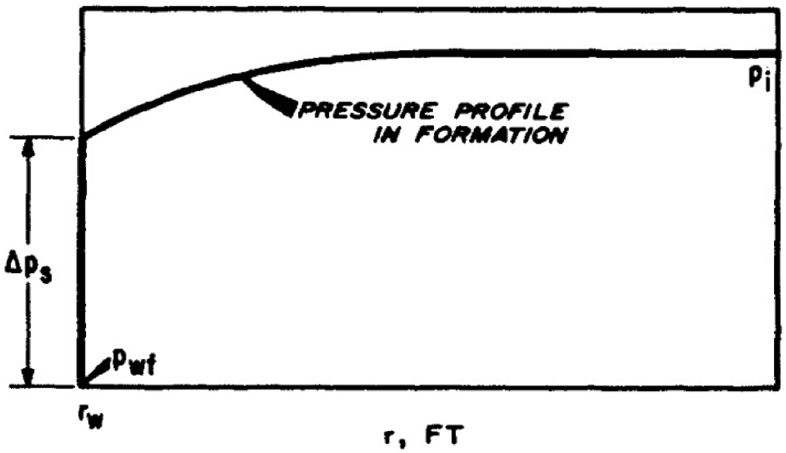
Fig. 5 – Infinite conductivity choked vertical fracture

Discussion: Fracture Damage Comparison

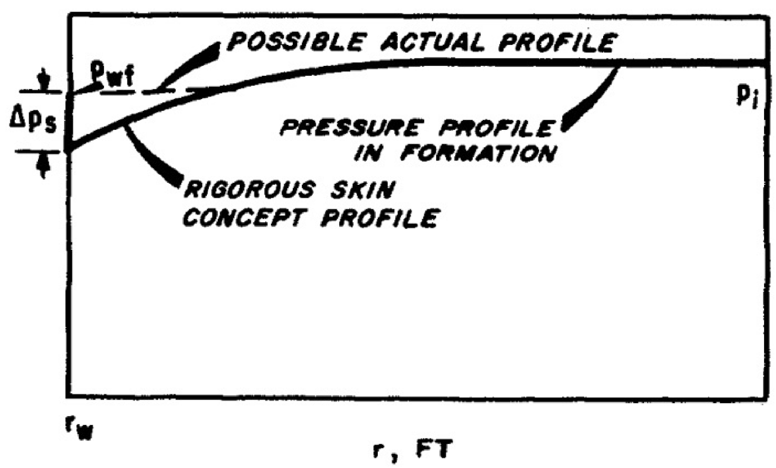
- Argument: Finite conductivity can be modeled as damage... (false!)
- "Fluid loss" damage is now referred to as "fracture face" skin.
- "Choked fracture" damage = constant skin factor. (false!)

Near-Wellbore Damage or Stimulation Effects (Skin Effects)

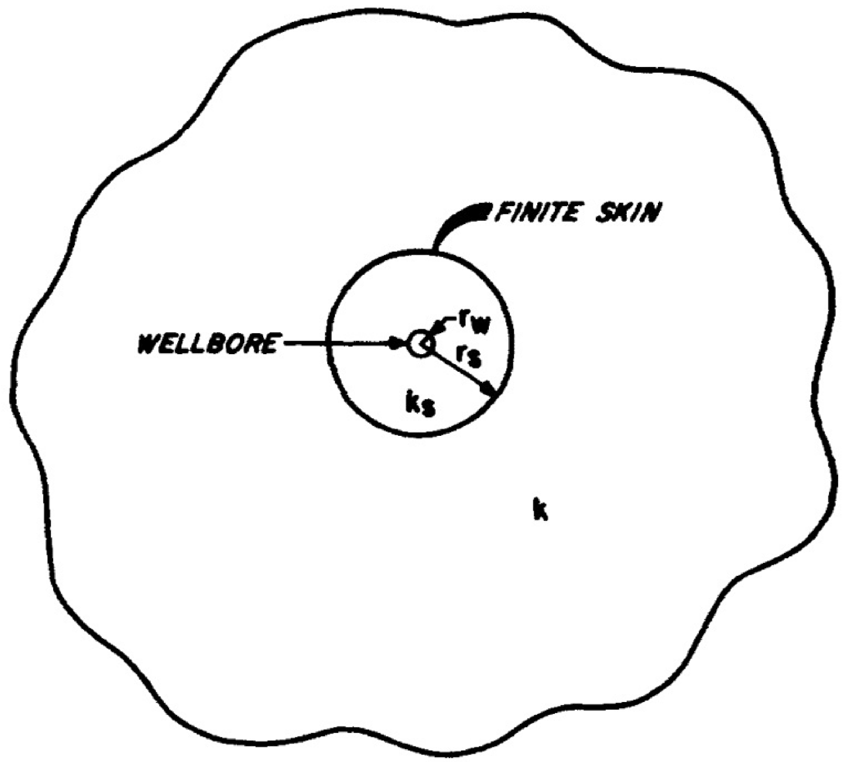
Earlougher, R.C. Jr.: Advances in Well Test Analysis, Monograph Series, SPE, Dallas (1977) 5.



Pressure distribution around a well with a positive skin factor.



Pressure distribution around a well with a negative skin factor.

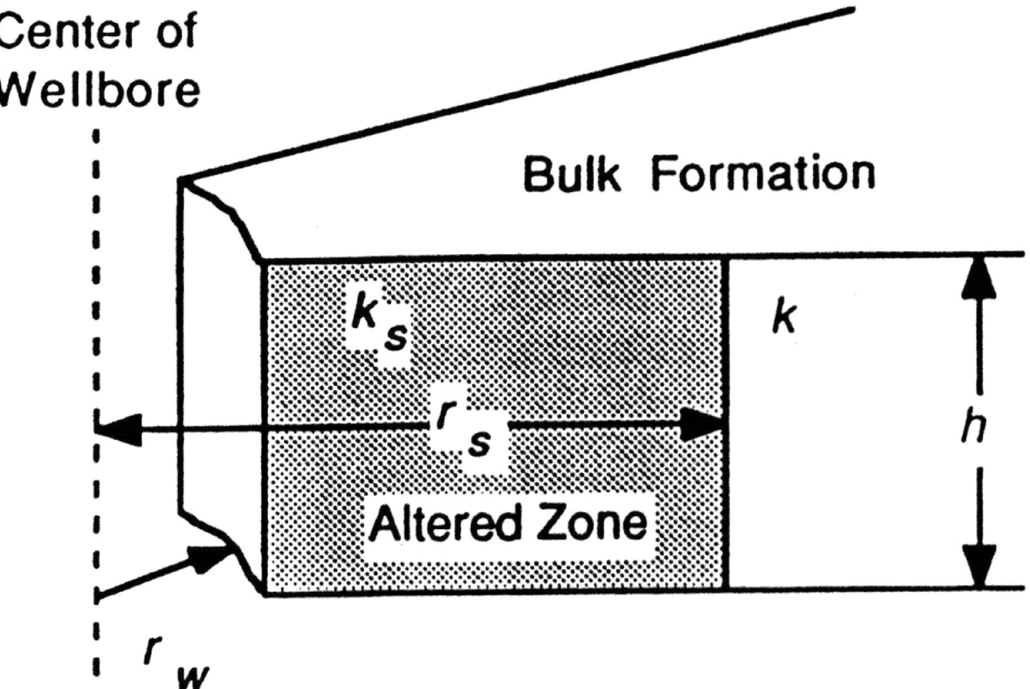


Skin zone of finite thickness.

Discussion: Skin Factor Concept (Unfractured Wells)

- Finite skin concept → zone of "altered" permeability near the well.
- Infinitesimal skin concept → mathematical convenience.
- Negative skin has mathematical (and physical) limitations.

Radial Flow "Skin Factor" (used to represent non-ideal behavior)



Two-region reservoir model of altered zone near the wellbore.

$$\Delta p_s = 141.2\frac{qB\mu}{k_s h}\ln\left[\frac{r_s}{r_w}\right] - 141.2\frac{qB\mu}{kh}\ln\left[\frac{r_s}{r_w}\right]$$

$$= 141.2\frac{qB\mu}{k_s h}\left[\frac{k}{k_s}-1\right]\ln\left[\frac{r_s}{r_w}\right]$$

Using the definition of dimensionless pressure

$$p_D = \frac{1}{141.2}\frac{kh}{qB\mu}(p_i - p_{wf})$$

Which yields

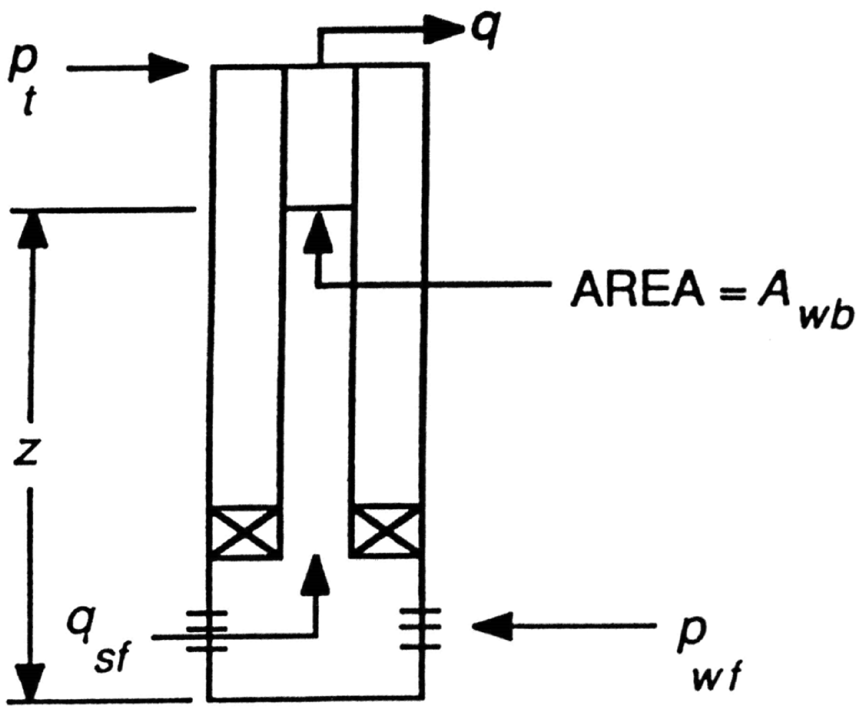
$$\Delta p_{sD} = \left[\frac{k}{k_s}-1\right]\ln\left[\frac{r_s}{r_w}\right]$$

where

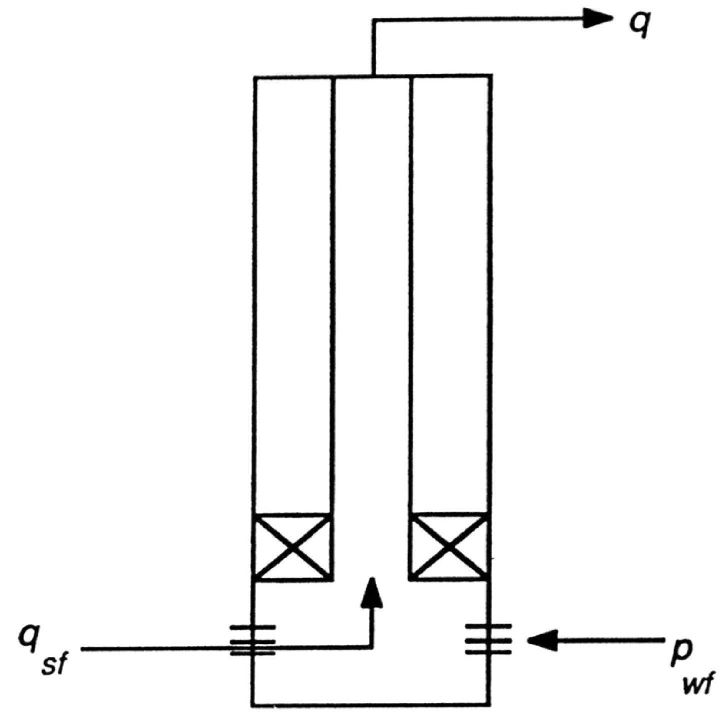
$$\Delta p_{sD} = \frac{1}{141.2}\frac{kh}{qB\mu}\Delta p_s$$

Lee, W.J. and Wattenbarger, R.A.: Gas Reservoir Engineering, SPE (1996).

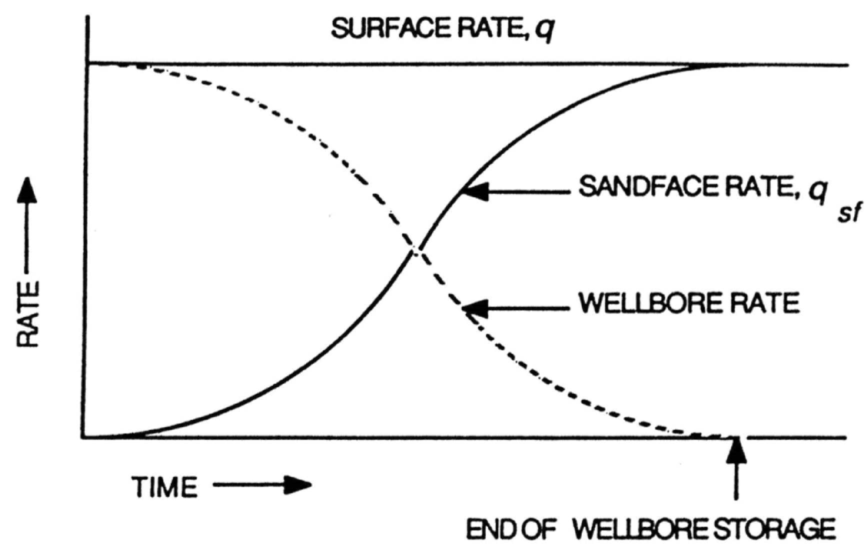
Radial Flow in an Infinite(-Acting) Reservoir With Wellbore Storage



Wellbore with moving liquid/gas interface.



Wellbore with single-phase fluid in the wellbore.



Effect of wellbore storage on sandface flow rate.

Lee, W.J. and Wattenbarger, R.A.: Gas Reservoir Engineering, SPE (1996).

From: Lee, W.J. and Wattenbarger, R.A.: Gas Reservoir Engineering, SPE (1996).

Radial Flow in an Infinite(-Acting) Reservoir With Wellbore Storage

Governing Equations

General Rate Relation

$$q_D = \frac{q_{sf}}{q} = 1 - C_D\left[\frac{dp_{wD}}{dt_D} - \frac{dp_{tD}}{dt_D}\right]$$

$$(q_{sf} - q)B = 24\, C_s\left[\frac{dp_{wf}}{dt} - \frac{dp_{tf}}{dt}\right]$$

The definition of C_s for a fluid filled wellbore

$$C_s = c_{wb}\, V_{wb}$$

The definition of C_s for a well with a rising or falling liquid level

$$C_s = \frac{144}{5.615}\,\frac{A_{wb}}{\rho(g/g_c)}$$

Pressure Relations (Early Times)

$$p_{wD} = \frac{t_D}{C_D}$$

Drawdown

$$p_{wf} = p_i - \frac{qB}{24C_s}\, t$$

Buildup

$$p_{ws} = p_{wf}(\Delta t = 0) + \frac{qB}{24C_s}\,\Delta t$$

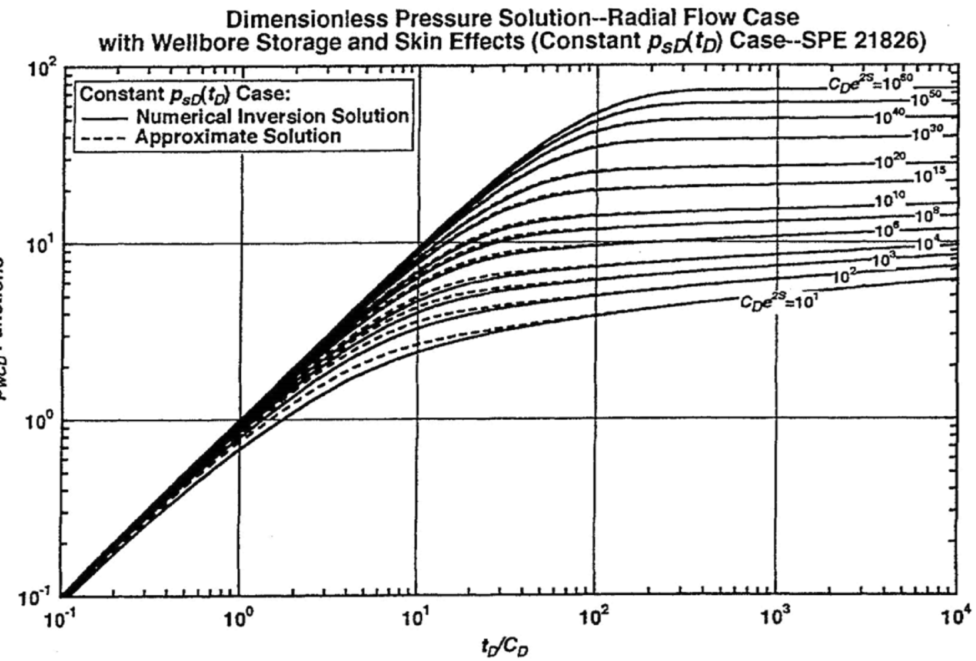
Laplace Domain Identity

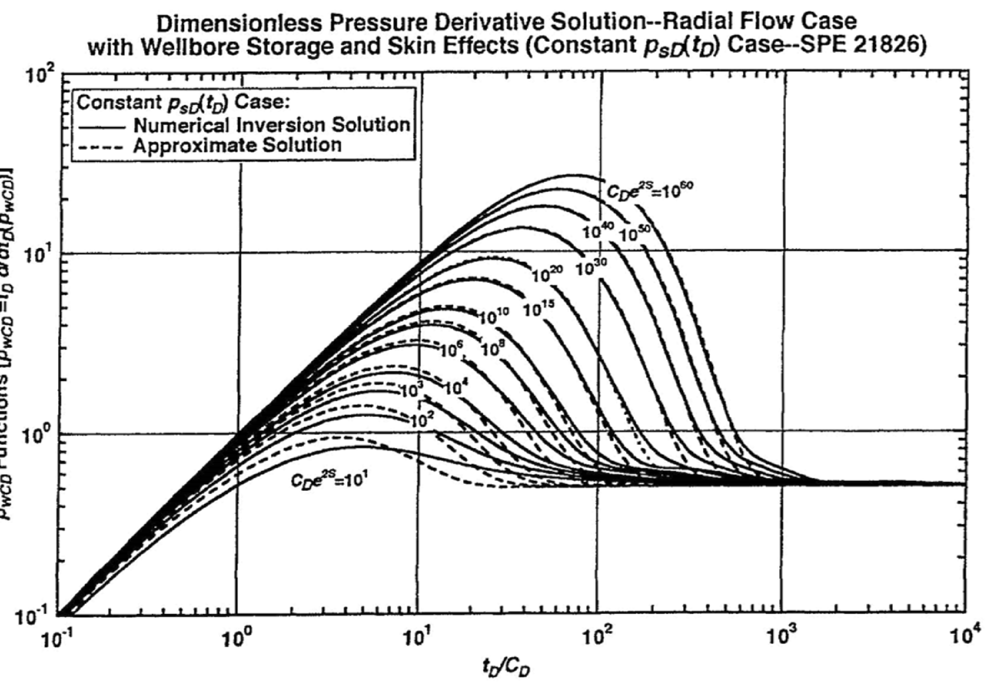
$$\bar{p}_{wD}(u) = \frac{1}{\dfrac{1}{\bar{p}_{sD}(u)} + u^2\, C_D}$$

lasingame, T.A. (1994): "Concepts and Applications in Wellbore Storage Distortion," Course otes for Petroleum Engineering 324 Well Performance, Texas A&U University.

Radial Flow in an Infinite(-Acting) Reservoir With Wellbore Storage

Dimensionless Pressure Relation: Constant Approximation for $p_{sD}(t_D)$





Approximate Solution:

$$p_{wCD}(t_D,s,C_D) = p_{sD}(t_D)\left[1-\exp\left[\frac{-t_D}{p_{sD}(t_D)C_D}\right]\right]$$

$p_{wCD}(t_D)$ = the "wellbore storage" solution.

$p_{sD}(t_D)$ = the "non-wellbore storage" solution.

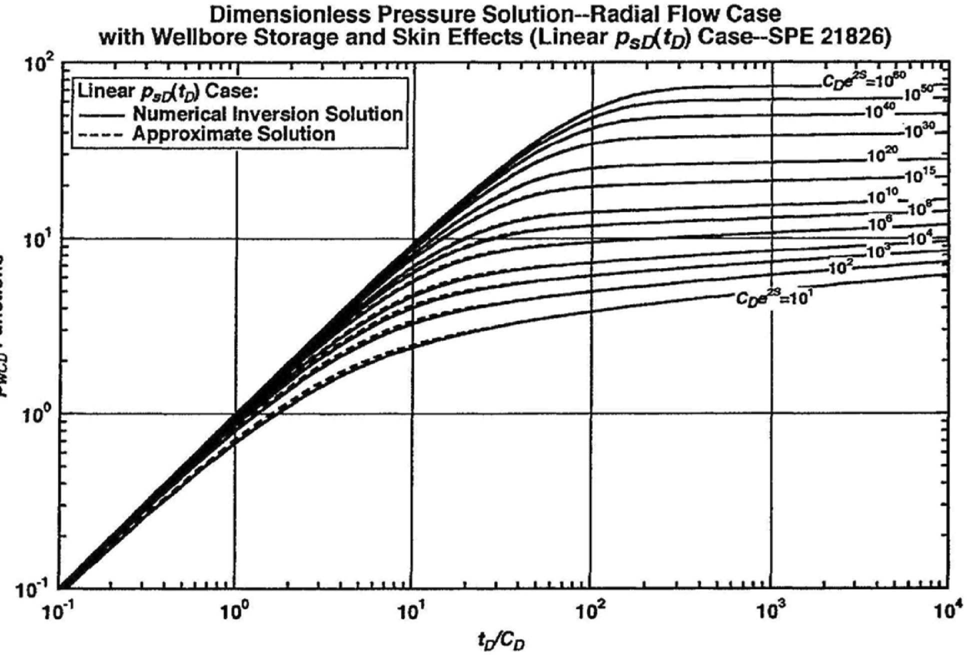
t_D = dimensionless time.

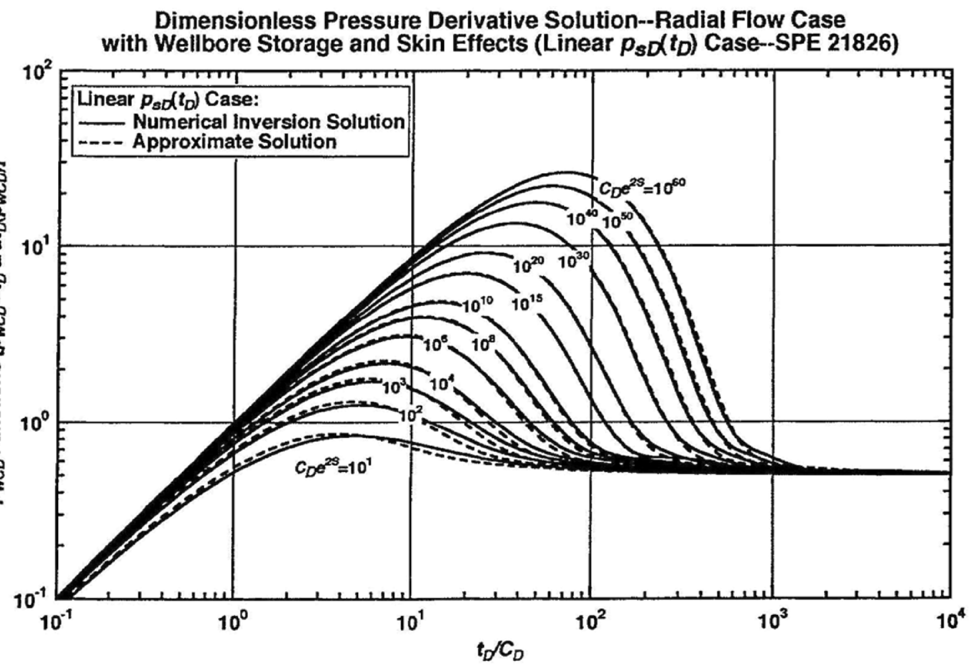
C_D = the dimensionless wellbore storage coefficient.

lasingame, T.A. (1994): "Concepts and Applications in Wellbore Storage Distortion," Course otes for Petroleum Engineering 324 Well Performance, Texas A&U University.

Radial Flow in an Infinite(-Acting) Reservoir With Wellbore Storage

Dimensionless Pressure Relation: Linear Approximation for $p_{sD}(t_D)$





Approximate Solution:

$$p_{wCD}(t_D,s,C_D)=\frac{1}{p'_{sD}(t_D)^2}\frac{[p_{sD}(t_D)-p'_{sD}(t_D)]}{\left[\dfrac{1}{p'_{sD}(t_D)}+\dfrac{1}{t_D/C_D}\right]^2}$$

$$\times\left[1-\exp\left[-\frac{t_D}{C_D}p'_{sD}(t_D)\frac{\dfrac{1}{p'_{sD}(t_D)}+\dfrac{1}{t_D/C_D}}{[p_{sD}(t_D)-p'_{sD}(t_D)]}\right]\right]$$

$$+\frac{1}{\dfrac{1}{p'_{sD}(t_D)}+\dfrac{1}{t_D/C_D}}$$

Where:

$p_{wCD}(t_D)$ = the "wellbore storage" solution.

$p_{sD}(t_D)$ = the "non-wellbore storage" solution.

$p_{sD}'(t_D)$ = well testing derivative of the "non-wellbore storage" solution.

t_D = dimensionless time.

C_D = the dimensionless wellbore storage coefficient.

lasingame, T.A. (1994): "Concepts and Applications in Wellbore Storage Distortion," Course otes for Petroleum Engineering 324 Well Performance, Texas A&U University.

Radius of Investigation (Transient Radial Flow)

$$p_D(r_D,t_D) = \frac{1}{2}\mathrm{E}_1\left[\frac{r_D^2}{4t_D}\right]$$

$$\approx \frac{1}{2}\ln\left[\frac{4}{e^\gamma}\frac{t_D}{r_D^2}\right] \quad (\gamma = 0.577216....\text{Euler's Constant})$$

Solve for $p_D(r_D,t_D) = 0$, implies that $p(r_{inv},t) = p_i$

$$\frac{1}{2}\ln\left[\frac{4}{e^\gamma}\frac{t_D}{r_{D,inv}^2}\right] = 0$$

$$\frac{4}{e^\gamma}\frac{t_D}{r_{D,inv}^2} = 1$$

$$r_{D,inv}^2 = \frac{4}{e^\gamma}t_D \text{ or } r_{D,inv} = \sqrt{\frac{4}{e^\gamma}t_D}$$

lasingame, T.A. (1994): "Concepts and Applications in Wellbore Storage Distortion," Course
otes for Petroleum Engineering 324 Well Performance, Texas A&U University.

Radius of Investigation (Transient Radial Flow)

$$r_{D,inv} = \sqrt{\frac{4}{e^{\gamma}} t_D}$$

where

$$p_D = \frac{1}{141.2} \frac{kh}{qB\mu} (p_i - p_{wf})$$

$$t_D = 2.637 \times 10^{-3} \frac{k}{\phi \mu c_t r_w^2} t$$

$$r_D = \frac{r}{r_w}$$

Solving using the dimensionless variables yields

$$r_{inv} = \sqrt{\frac{4}{e^{\gamma}} 2.637 \times 10^{-3}} \sqrt{\frac{kt}{\phi \mu c_t}}$$

$$= 2.434 \times 10^{-2} \sqrt{\frac{kt}{\phi \mu c_t}}$$

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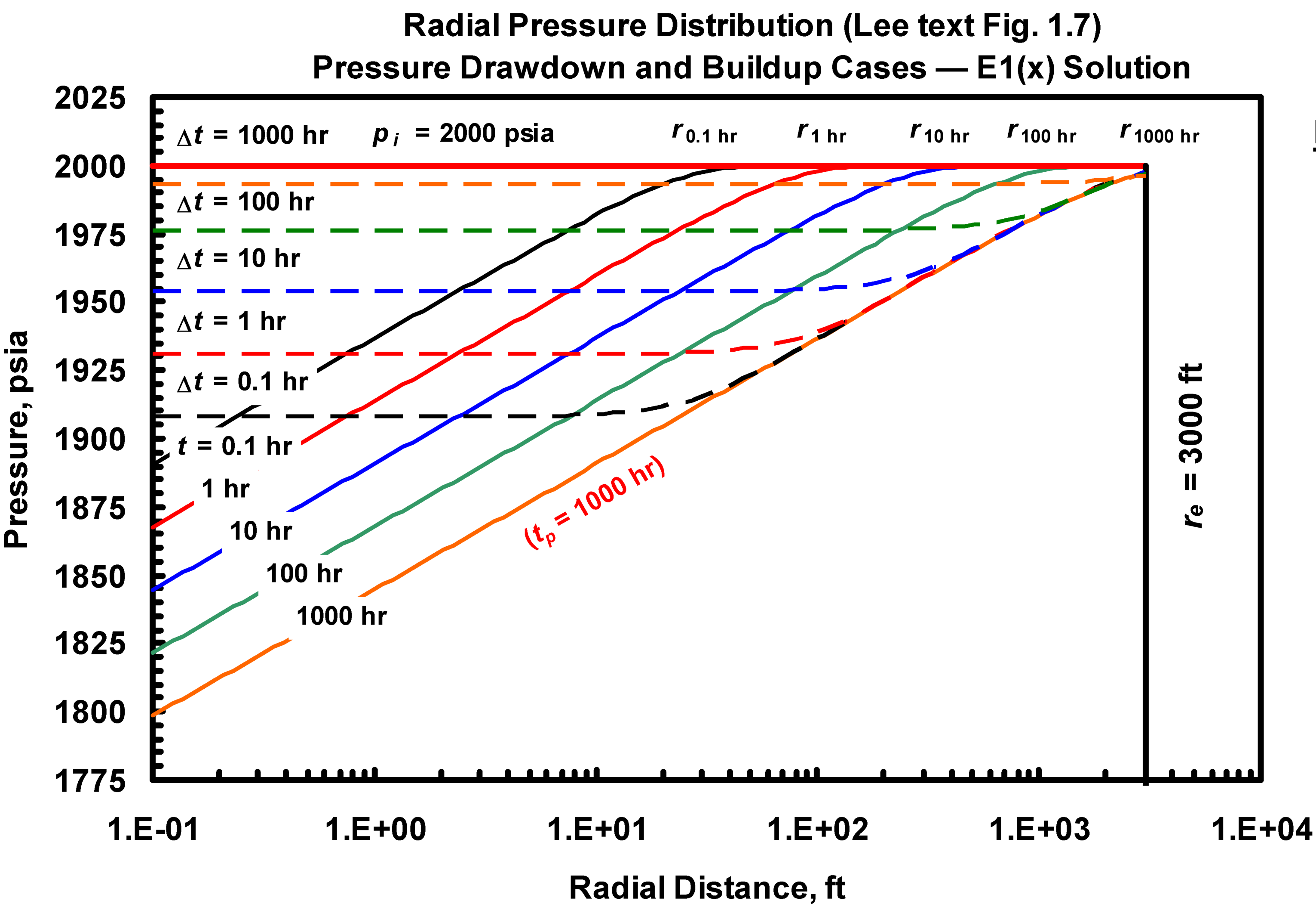
Radius of Investigation Concepts

Pressure Distributions — Transient Flow



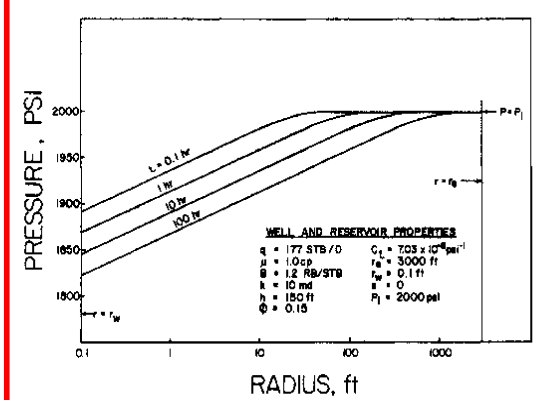


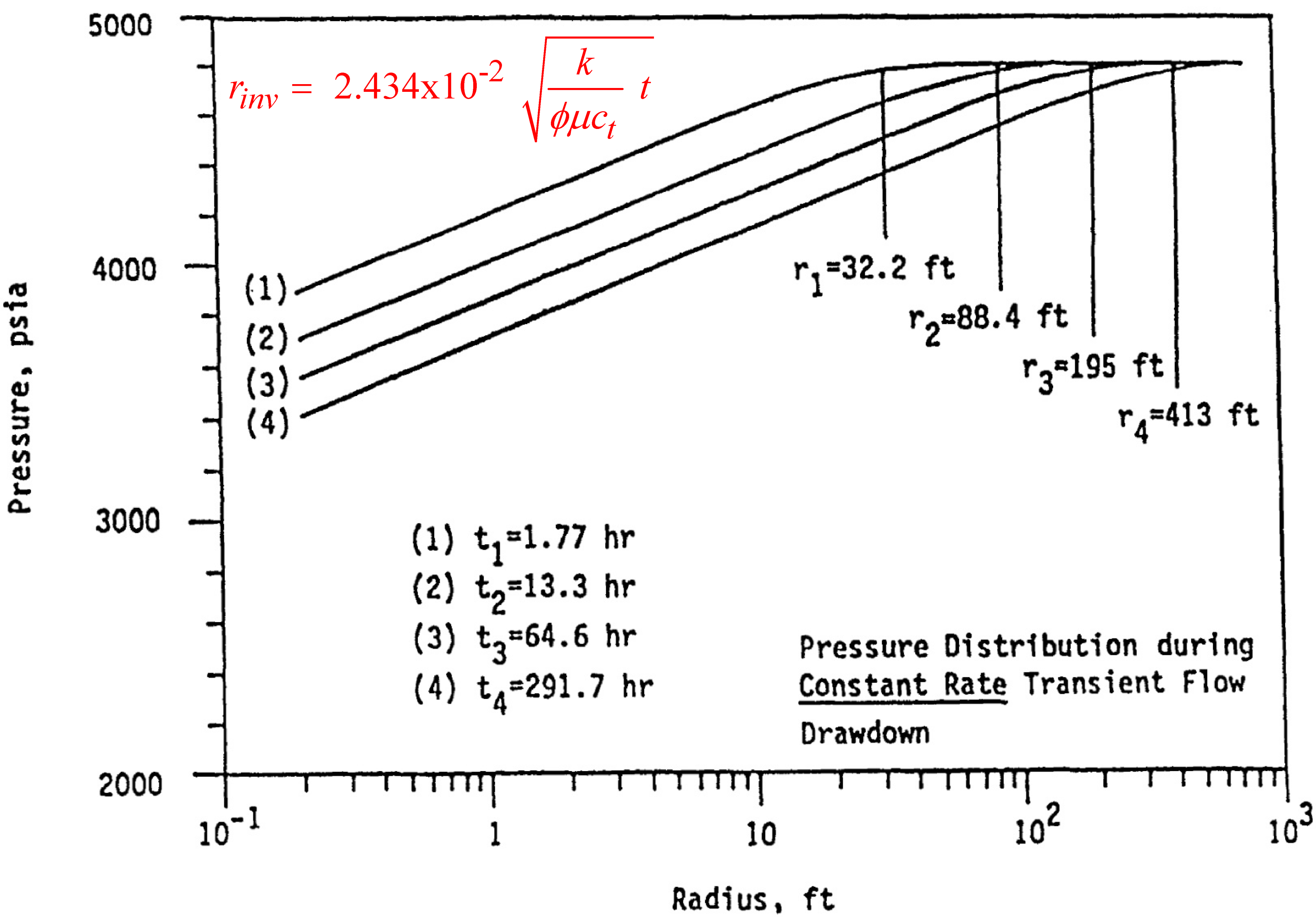
Fig. 1.7 – Pressure distribution in formation near producing well.

Pressure Distributions for Transient Radial Flow

- Note the "radius of investigation." [$p(r,t)$ trend intersection at p_i]
- Note that the buildup pressure trends retrace last drawdown trend (t_p = 1000 hr).
- All measurements are at the wellbore, we cannot "see" in the reservoir.

Radius of Investigation Concepts

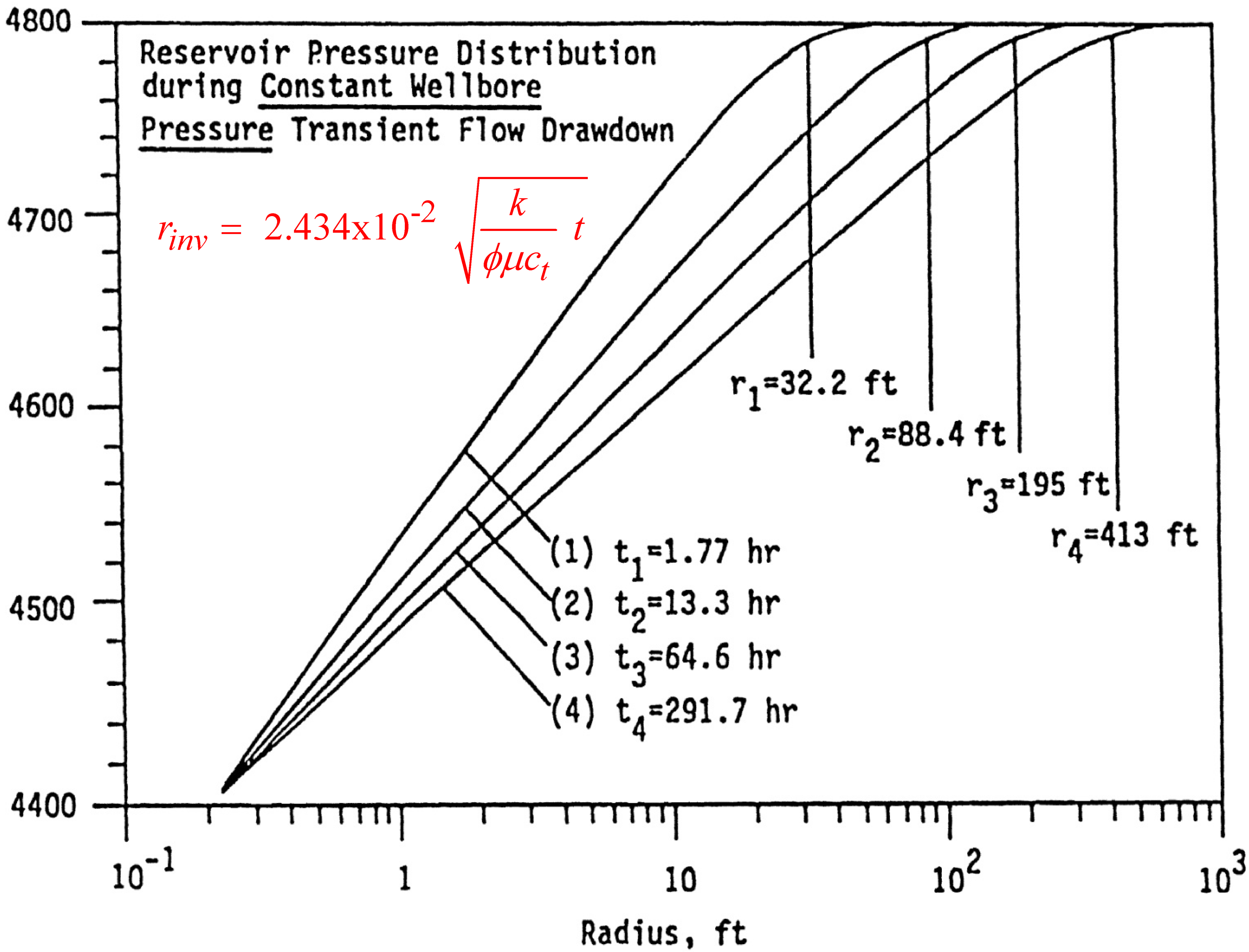
Pressure Distributions — Transient Flow (Constant Rate Case)



lasingame, T.A.: Variable-Rate Analysis: Transient and Pseudosteady-State Methods of
nterpretation and Application, M.S. Thesis, Texas A&M University (1986).

Radius of Investigation Concepts

Pressure Distributions — Transient Flow (Constant Wellbore Pressure Case)

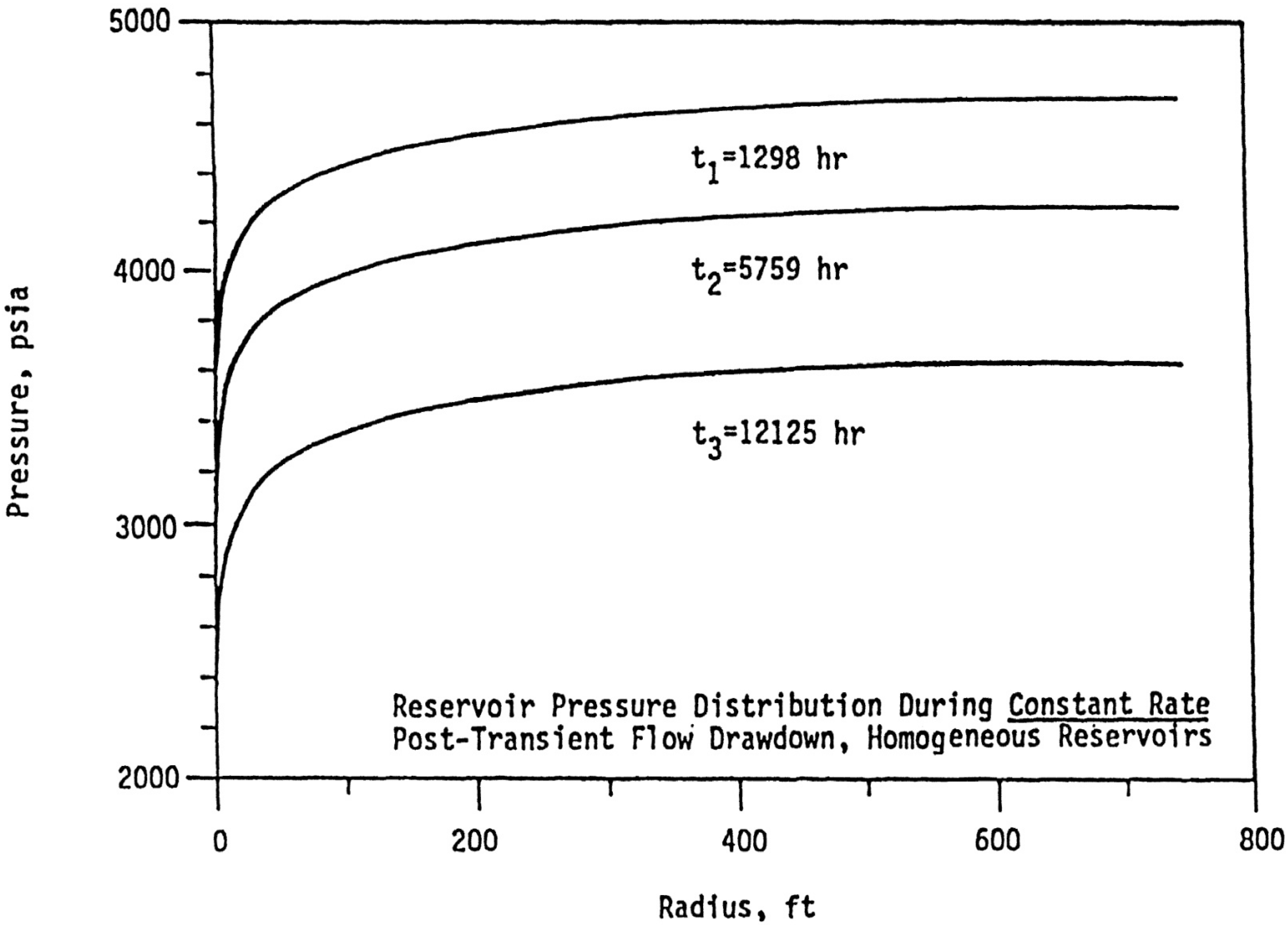


$$r_{inv} = 2.434\text{x}10^{-2}\sqrt{\frac{k}{\phi\mu c_t}t}$$

lasingame, T.A.: Variable-Rate Analysis: Transient and Pseudosteady-State Methods of
terpretation and Application, M.S. Thesis, Texas A&M University (1986).

Radius of Investigation Concepts

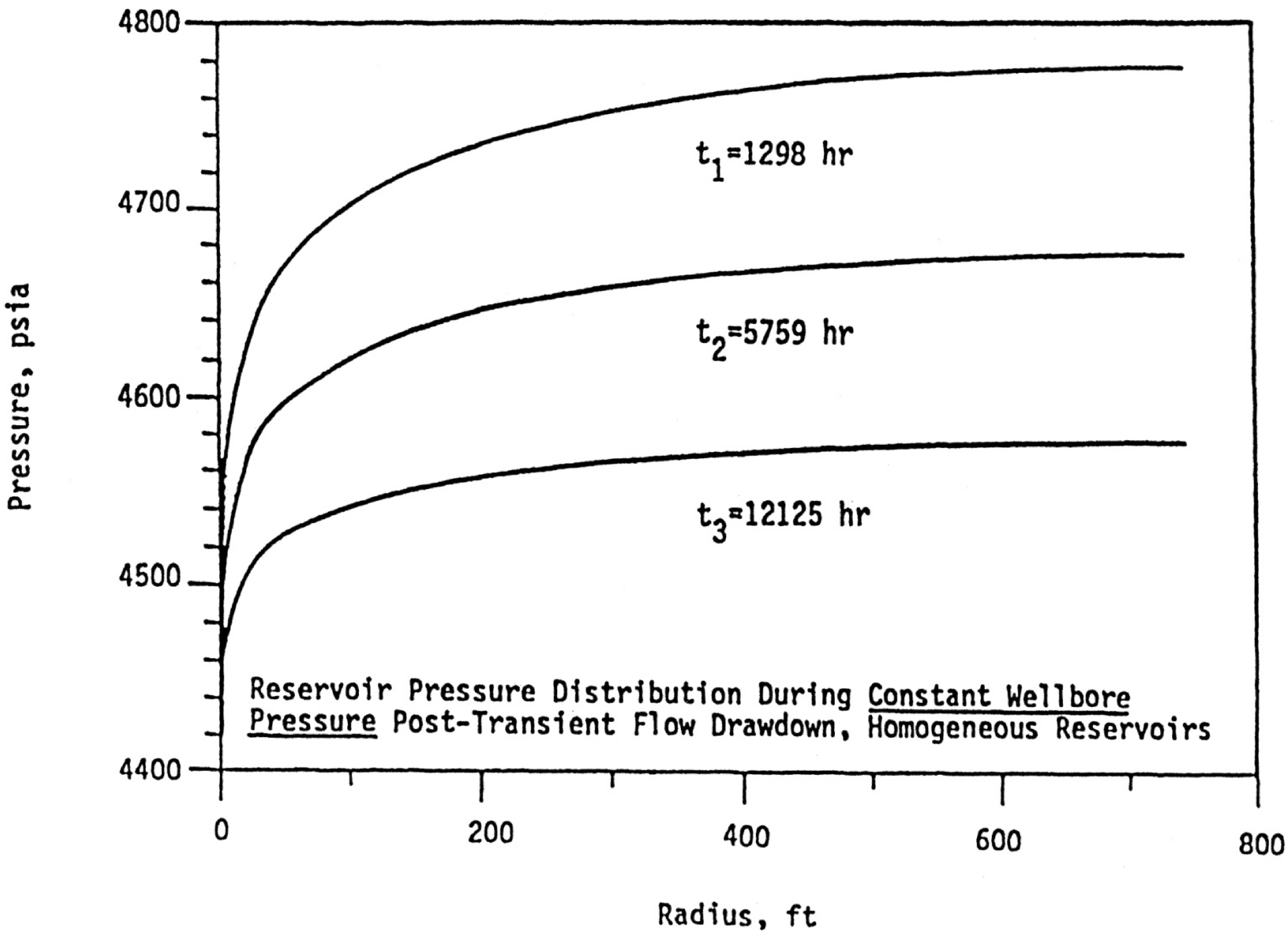
Pressure Distributions — Pseudosteady-State Flow (Constant Rate Case)



Blasingame, T.A.: Variable-Rate Analysis: Transient and Pseudosteady-State Methods of Interpretation and Application, M.S. Thesis, Texas A&M University (1986).

Radius of Investigation Concepts

Pressure Distributions — Pseudosteady-State Flow (Constant Wellbore Pressure Case)



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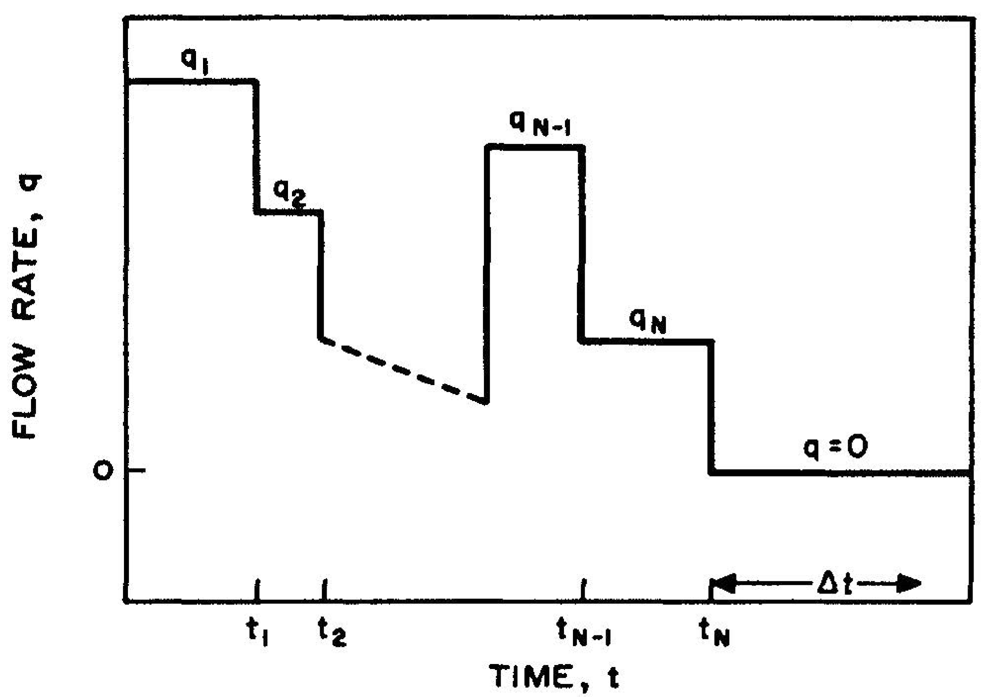
Horner Approximation and Recommendations for Well Rates

Horner's Approximation

$$t_p = 24 \frac{Np}{q_{last}}$$

(from the text) We can offer two helpful guidelines for how long the most recent rate should be in effect. First, if the most recent rate is maintained long enough for the radius of investigation achieved at this rate to reach the drainage radius of the tested well, then Homer's approximation always is sufficiently accurate. This rule is quite conservative, however. Second, for a new well that undergoes a series of rapid rate changes, <u>it is usually sufficient to establish the last constant rate for at least twice as long as the previous rate</u>. When any doubt exists about whether these guidelines are satisfied, the best approach is to use superposition to model the production history.

Schematic of rate variation preceding a pressure buildup test.



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