Designing for Impact Loads

1. NATURE OF IMPACT LOADING

Impact loading results not only from actual impact (or blow) of a moving body against the member, but by any sudden application of the load (Fig. 1). It may occur in any of the following methods:

1. *A direct impact*, usually by another member or an external body moving with considerable velocity, for example:
   (a) A pile driver hammer striking the top of a pile.
   (b) The die striking the workpiece in a drop forge press or punch press.
   (c) A large rock dropped from a height onto a truck.
2. *A sudden application of force*, without a blow being involved.
   (a) The sudden creation of a force on a member as during the explosive stroke in an engine, the ignition or misfire of a missile motor when mounted on a test stand.
   (b) The sudden moving of a force onto a member, as when a heavy loaded train or truck moves rapidly over a bridge deck, or a heavy rock rolls from the bucket of a shovel onto a truck without any appreciable drop in height.
3. *The inertia of the member* resisting high acceleration or deceleration.
   (a) Rapidly reciprocating levers.
   (b) A machine subject to earthquake shocks or explosives in warfare.
   (c) The braking of a heavy trailer.

2. APPROACH TO DESIGN PROBLEM

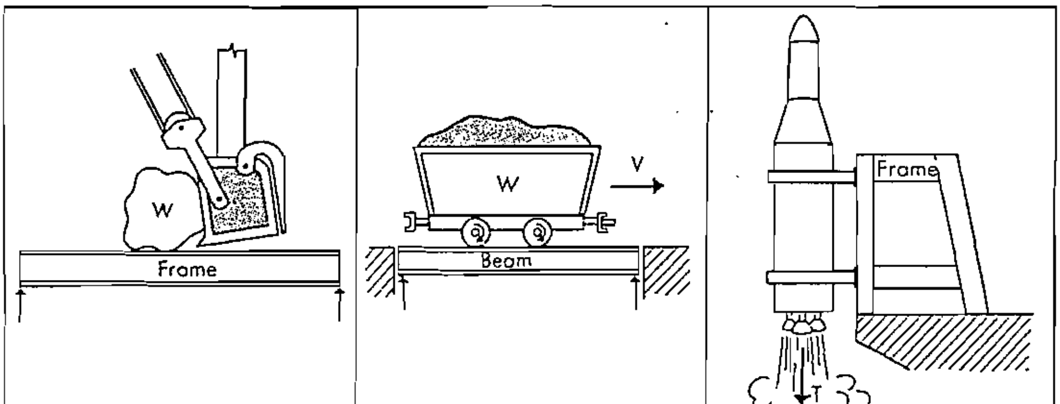
In many cases it is difficult to evaluate impact forces quantitatively. The analysis is generally more qualitative and requires recognition of all of the factors involved and their inter-relationship.

The designer can follow one of two methods:

1. Estimate the maximum force exerted on the resisting member by applying an impact factor. Consider this force to be a static load and use in standard design formulas.

2. Estimate the energy to be absorbed by the resisting member, and design it as an energy-absorbing member.

The properties of the material and the dimensions of the resisting member that give it maximum resistance to an energy load, are quite different from those that give the member maximum resistance to a static load.



Heavy rock rolled from shovel onto frame without any initial drop in height: $h = 0$, $F = 2\,W$	Fast moving, loaded wagon passing over supporting beam: F = between W and 2 W	Sudden ignition of missile; or missile misfires and then re-ignites $F = 2\,T$ (thrust)

FIG. 1 Types of impact loading.

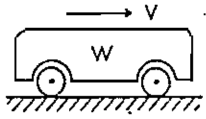
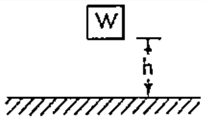
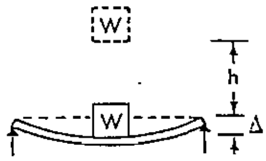
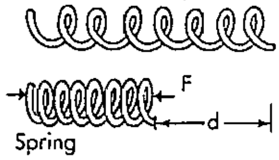
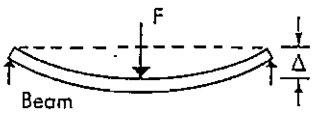
KINETIC ENERGY (E_k) is the amount of work a body can do by virtue of its motion.		$E_k = \frac{W V^2}{2 g}$
POTENTIAL ENERGY (E_p) is the amount of work a body can do by virtue of its position.		$E_p = W h$
If the supporting member is flexible and deflects, this additional movement must be considered as part of the total height the body can fall.		$E_p = W(h + \Delta)$
It is also the amount of work a body can do by virtue of its state of strain or deflection.		$E_p = \frac{F d}{2}$
		$E_p = \frac{F \Delta}{2}$

FIG. 2 Formulas for kinetic energy and potential energy.

3. INERTIA FORCES

Inertia is the property of a member which causes it to remain at rest or in uniform motion unless acted on by some external force. Inertia force is the resisting force which must be overcome in order to cause the member to accelerate or decelerate, equal but opposite to—

$$F = \frac{W_m}{g} a$$

where:

W_m = weight of member, lbs

a = acceleration or deceleration of member, in./sec² or ft/sec²

g = acceleration of gravity (386.4 in./sec² or 32.2 ft/sec²

4. IMPACT FORCES

A moving body striking a member produces a force on the member due to its deceleration to a lower velocity or perhaps to zero velocity:

$$F = \frac{W_b a}{g}$$

where:

W_b = weight of body, lbs

a = deceleration of body, in./sec² or ft/sec²

g = acceleration of gravity (386.4 in./sec² or 32.2 ft/sec²)

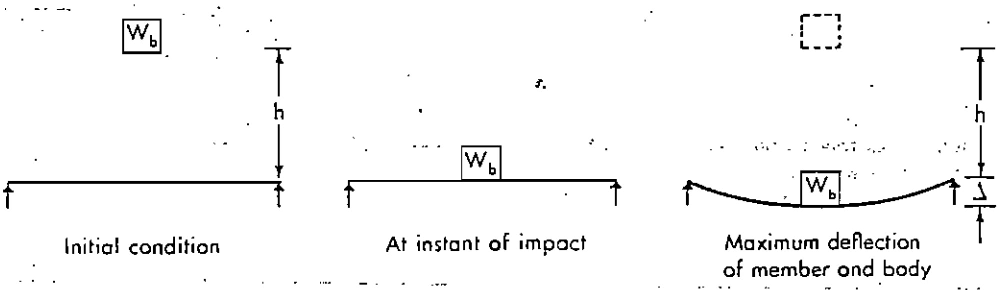


FIG. 3 Effect of member's inertia.

Fortunately the member will deflect slightly and allow a certain time for the moving body (W_b) to come to rest, thereby reducing this impact force (F).

Since the time interval is usually unknown, the above formula cannot be used directly to find the force (F). However, it is usually possible to solve for this force by finding the amount of kinetic energy (E_k) or potential energy (E_p) that must be absorbed by the member (Fig. 2).

This applied energy (E_k) or (E_p) may then be set equal to the energy (U) absorbed by the member within a given stress (σ), see Table 2.

5. POTENTIAL ENERGY OF FALLING BODY ON MEMBER

(See Figure 3)

Potential energy of falling body (W_b):

$$E_p = W_b\ (h + \Delta)$$

Potential energy received by deflected member:

$$E_p = \frac{F\ \Delta}{2}$$

Then:

$$W_b\ h + W_b\ \Delta = \frac{F\ \Delta}{2}$$

but $K = \dfrac{F}{\Delta}$ being the spring constant of the beam

$$W_b\ h + W_b\ \frac{F}{K} = \frac{F^2}{2K}$$

and $F^2 - 2\ W_b\ F - 2\ K\ W_b\ h = 0$

or $$\boxed{F = W_b + \sqrt{W_b^2 + 2\ K\ W_b\ h}}$$

or since $V = \sqrt{2\ g\ h}$

$$\boxed{F = W_b + \sqrt{W_b^2 + \frac{K\ W_b\ V^2}{g}}}$$

If the body (W_b) is suddenly applied to the member without any appreciable drop in height ($h = 0$), the maximum force due to impact is twice that of the applied load (W_b):

$$F = 2\ W_b$$

Thus, it is common practice to apply an impact factor to a load and design as though it were a steady load. As the weight of the supporting member (W_m) increases, this impact factor of (2) becomes less.

In a similar manner, it is possible to express the resultant impact deflection in terms of steady load deflection.

$$\boxed{\Delta = \Delta_{st} + \sqrt{\Delta_{st}^2 + 2\ h\ \Delta_{st}}}$$

or $$\boxed{\Delta = \Delta_{st} + \sqrt{\Delta_{st}^2 + \frac{\Delta_{st}\ V^2}{g}}}$$

Again, if $h = 0$, then $\Delta = 2\ \Delta_{st}$

6. EFFECT OF MEMBER'S INERTIA

If the weight (W_m) of the supporting member is relatively high, some of the applied energy will be absorbed because of the inertia of the member to movement. A good example is the effect of the mass of

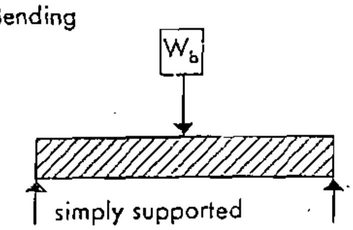
TABLE 1—Basic Laws Used in Analysis of Impact

	Linear	Angular
Mass	(1) $M = \dfrac{W}{g}$	(10) $I = \dfrac{W}{g} r^2$ r = radius of gyration
Force	(2) F	(11) $T = F\ d$ d = perpendicular distance from center of rotation to line of force
Velocity	(3) $V = \dfrac{d}{t}$	(12) $\omega = \dfrac{\theta}{t} = 2\pi RPM = \dfrac{V}{r}$ r = radius of point for which ω is to be found
Acceleration	(4) $a = \dfrac{V - V_o}{t}$	(13) $\alpha = \dfrac{\omega - \omega_o}{t}$
Force of Impact	(5) $F = \dfrac{W}{g} a$	(14) $T = I\alpha$
Impulse	(6) F t	(15) T t
Momentum	(7) $\dfrac{W}{g} V$	(16) $I\ \omega$
Kinetic Energy	(8) $\dfrac{W}{2\ g} V^2$	(17) $\dfrac{I\ \omega^2}{2}$
Work	(9) F d	(18) $T\ \theta$

TABLE 2—Impact Formulas for Common Member-Load Conditions

Energy stored in member, may be set equal to kinetic energy

(1) Bending



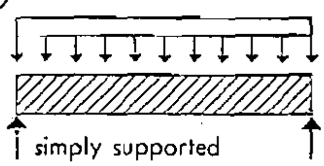
simply supported
concentrated load
uniform section

$$U = \frac{\sigma_y^2 I L}{6 E c^2}$$

$$U = \frac{\sigma_y^2 A L}{6 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .1667)

(6) Bending



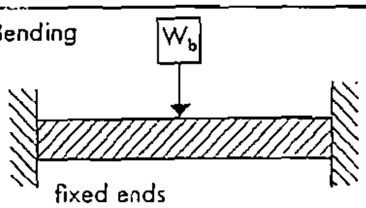
simply supported
uniform load
uniform section

$$U = \frac{4 \sigma_y^2 I L}{15 E c^2}$$

$$U = \frac{4 \sigma_y^2 A L}{15 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .2667)

(2) Bending



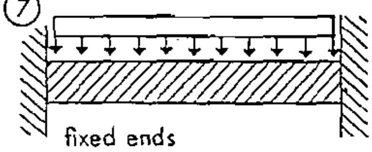
fixed ends
concentrated load
uniform section

$$U = \frac{\sigma_y^2 I L}{6 E c^2}$$

$$U = \frac{\sigma_y^2 A L}{6 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .1667)

(7) Bending



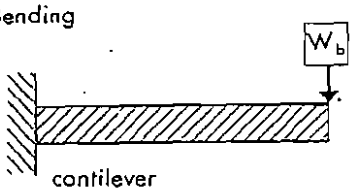
fixed ends
uniform load
uniform section

$$U = \frac{\sigma_y^2 I L}{10 E c^2}$$

$$U = \frac{\sigma_y^2 A L}{10 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .1000)

(3) Bending



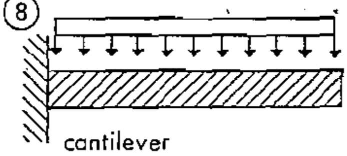
cantilever
concentrated load
uniform section

$$U = \frac{\sigma_y^2 I L}{6 E c^2}$$

$$U = \frac{\sigma_y^2 A L}{6 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .1667)

(8) Bending



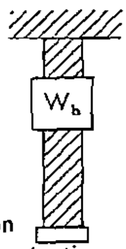
cantilever
uniform load
uniform section

$$U = \frac{\sigma_y^2 I L}{10 E c^2}$$

$$U = \frac{\sigma_y^2 A L}{10 E}\left(\frac{r}{c}\right)^2$$

(Coefficient = .1000)

(4) Tension

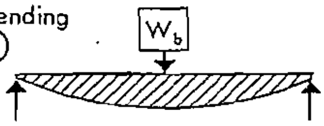


axial
tension
uniform section

$$U = \frac{\sigma_y^2 A L}{2 E}$$

(Coefficient = .500)

(9) Bending

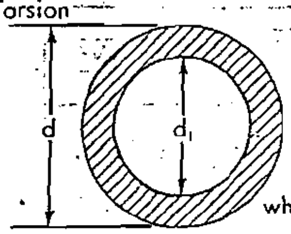


simply supported
concentrated load
variable section so σ = constant value

$$U = \frac{\sigma_y^2 I L}{3 E c^2}$$

(Coefficient = .3333)

(5) Torsion



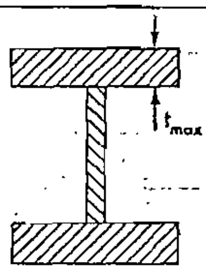
round shaft

$$U = \frac{\sigma_y^2 (d^2 + d_1^2) A L}{4 E_s d^2}$$

where E_s = shear modulus of elasticity

(Coefficient = .250)

(10) Torsion



open section

$$U = \frac{\sigma_y^2 R L}{2 E_s t_{max}}$$

where R = torsional resistance

(Coefficient = .500)

a concrete bridge deck in reducing the impact forces transferred into the member supporting it.

If the applied energy is expressed in terms of the velocity of the body (V), the reduced velocity (V_e) at instant of impact is—

$$V_e = V\left(\frac{W_b}{W_b + W_e}\right) = \left(\frac{V}{1+\frac{W_e}{W_b}}\right)$$

where:

W_b = weight of the body

W_e = equivalent weight of the member

If the member were compact and concentrated at a point, the entire weight of the member would be effective in reducing the velocity of the body. However, the supporting member is spread out in the form of a beam or frame and therefore only a portion of its weight is effective in moving along with the body and slowing it down. Timoshenko shows the portion of the weight of the member to be used is:

- Simply supported beam with concentrated load at midpoint

$$W_e = .486\ W_m$$

- Cantilever beam with concentrated load at end

$$W_e = .236\ W_m$$

The reduced kinetic energy (E_k) applied to the member causing stress and deflection would be

$$E_k = \frac{(W_b + W_e)\ V_e^2}{2\ g} = \frac{W_b\ V^2}{2\ g}\left(\frac{1}{1 + \frac{W_e}{W_b}}\right)$$

If the applied energy is expressed in terms of the height of fall of the body (h), the reduced velocity (V_e) may be expressed in terms of a reduced effective height (h_e):

$$h_e = \frac{V_e^2}{2\ g}$$

This represents the effective height the body would have to fall in order to have the reduced velocity (V_e) at the instant of impact with the member.

7. ENERGY-ABSORBING CAPACITY OF MEMBER

The allowable energy load, or load that can be absorbed elastically (without plastic deformation) by the member in bending, is basically—

$$\boxed{U = k\ \frac{\sigma_y^2\ I\ L}{E\ c^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where (k) is a constant for a specific type of beam with a specific type of loading. Table 2 shows the application of this formula to various member and load conditions, with numerical values substituted for the (k) factor.

Observation shows that the critical property of the section is $\frac{I}{c^2}$, while that of the material is $\frac{\sigma_y^2}{2E}$.

8. IMPACT PROPERTIES OF MATERIAL

The two most important properties of a material that indicate its ability to absorb energy are obtained from the stress-strain diagram (Fig. 4).

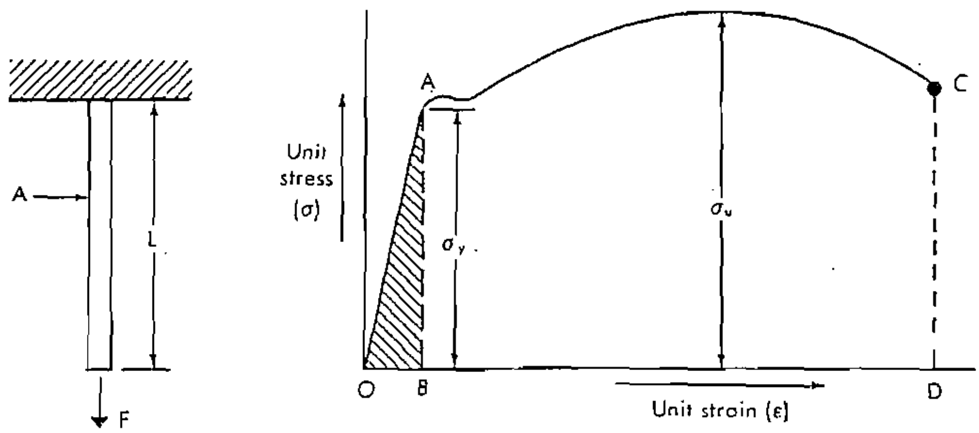


FIG. 4 Stress-strain diagram: basis for material's impact properties.

The *modulus of resilience* (u) of a material is its capacity to absorb energy within its elastic range, i.e. without permanent deformation. This is represented on the tensile stress-strain diagram by the area under the curve defined by the triangle O A B, having its apex A at the elastic limit.

$$\boxed{u = \frac{\sigma_y^2}{2\,E}} \quad \ldots\ldots (2)$$

Since the absorption of energy is actually a volumetric property, the u in (in.-lbs/in.^{3}) = u in psi.

When impact loading exceeds the elastic limit (or yield strength) of the material, it calls for toughness in the material rather than resilience.

The *ultimate energy resistance* (u_u) of a material indicates its toughness or ability to resist fracture under impact loading. This is a measure of how well the material absorbs energy without fracture. A material's ultimate energy resistance is represented on the stress-strain diagram by the total area OACD under the curve. Here point A is at the material's yield strength (σ_y) and point C at its ultimate strength (σ_u). For ductile steel, the ultimate energy resistance is approximately—

$$\boxed{u_u = A_{OACD} = \frac{\sigma_y + \sigma_u}{2}\,\epsilon_u} \quad \ldots\ldots (3)$$

where:

ϵ_u = ultimate unit elongation, in./in.

Since the absorption of energy is actually a volumetric property, u_u in (in.-lbs/in.^{3}) = u_u in psi.

Impact properties of common design materials are charted in Table 3.

9. IMPACT PROPERTIES OF SECTION

The section property which is needed to withstand impact loads or to absorb energy in bending is I/c^2.

This is very important because as moment of inertia (I) increases with deeper sections, the distance from the neutral axis to the outer fiber (c) increases *as its square*. So, increasing only the depth of a section will increase the section's moment of inertia but with little or no increase in impact property.

For example, suppose there is a choice between these two beams:

Section Property	Beam A 12" WF 65# Beam	Beam B 24" WF 76# Beam
I	533.4 in.^{4}	2096.4 in.^{4}
c	6.06 in.	11.96 in.
Steady load strength $S = \frac{I}{c}$	$\frac{533.4}{6.06} = 88.2$ in.^{3}	$\frac{2096.4}{11.96} = 175$ in.^{3}
Impact load strength $\frac{I}{c^2}$	$\frac{533.4}{(6.06)^2} = 14.5$ in.^{2}	$\frac{2096.4}{(11.96)^2} = 14.6$ in.^{2}

The new beam (B) with twice the depth, has about 4 times the bending stiffness (I), and 2 times the steady load strength (I/c), but for all practical purposes there is no increase in the impact load strength (I/c^2). In this example, there would be no advantage in changing from (A) to (B) for impact.

10. IMPROVING ENERGY ABSORPTION CAPACITY

The basic rule in designing members for maximum energy absorption is to have the maximum volume of the member subjected to the maximum allowable stress. If possible, this maximum stress should be uniform on every cubic inch of the member.

1. For any given cross-section, have the maximum amount of the area stressed to the maximum allowable. In the case of beams, place the greatest area of the section in the higher stressed portion at the outer fibers.

2. Choose sections so the member will be stressed to the maximum allowable stress along the entire length of the member.

For a member subjected to impact in axial tension, specifying a constant cross-section from end to end will uniformly stress the entire cross-section to the maximum value along the full length.

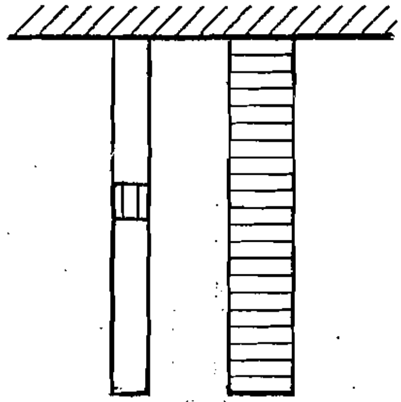
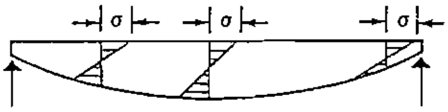


TABLE 3—Impact Properties of Common Design Materials

Material	σ_y Tensile Proportional limit lbs/in.^{2}	σ_u Tensile Ultimate Strength lbs/in.^{2}	E Tensile Modulus of Elasticity lbs/in.^{2}	ϵ_u Ultimate Unit Elongation in./in.	u Tensile Modulus of Resilience in.-lbs/in.^{3}	u_u Toughness - Ultimate Energy Resistance in.-lbs/in.^{3}
Mild Steel	35,000	60,000	30×10^6	0.35	20.4	16,600
Low Alloy (under ¾")	50,000	70,000	30×10^6	.18	41.6	
(¾ to 1½")	46,000	67,000	30×10^6	.19	35.2	
(over 1½ to 4")	42,000	63,000	30×10^6	.19	29.4	
Medium carbon steel	45,000	85,000	30×10^6	0.25	33.7	16,300
High carbon steel	75,000	120,000	30×10^6	0.08	94.0	5,100
T-1 Steel	100,000	115,000 to 135,000	30×10^6	0.18	200.0*	about 19,400
Alloy Steel	200,000	230,000	30×10^6	0.12	667.0	22,000
Gray Cast Iron	6,000	20,000	15×10^6	0.05	1.2	70
Malleable Cast Iron	20,000	50,000	23×10^6	0.10	17.4	3,800

* Based on integrator-measured area under stress-strain curve.

A beam can be designed for constant bending stress along its entire length, by making it of variable depth. Although the cross-section at any point is not uniformly stressed to the maximum value, the outer fiber is stressed to the maximum value for the entire length of the member.



In Table 3 the member in tension (No. 4) has three times the energy-absorption capacity of the simple beam with a concentrated load (No. 1). This is because the tensile member (No. 4) has its entire cross-section uniformly stressed to maximum for its full length. In contrast, the maximum bending stress in beam No. 1 is at the outer fibers only; and this bending stress decreases away from the central portion of the beam, being zero at the two ends.

Notice that decreasing the depth of the beam at its supports, so the maximum bending stress is uniform along the entire length of the beam, doubles the energy-absorbing capacity of the beam. See (1) and (9).

For a steady load, doubling the length of a beam will double the resulting bending stress. However, for an impact load, doubling the length of the beam will reduce the resulting impact stress to 70.7% of the original.

Two identical rectangular beams can theoretically absorb the same amount of energy and are just as strong under impact loading. The section property

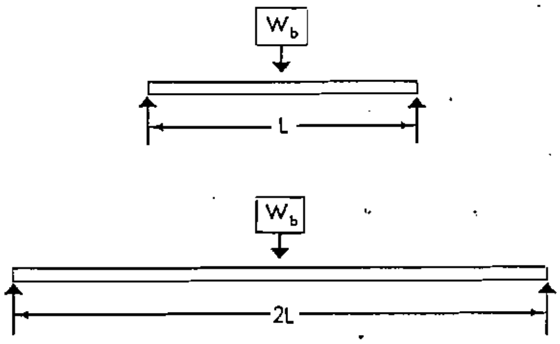


FIGURE 5

which determines this is I/c^2, and this is constant for a given rectangular area regardless of its position.

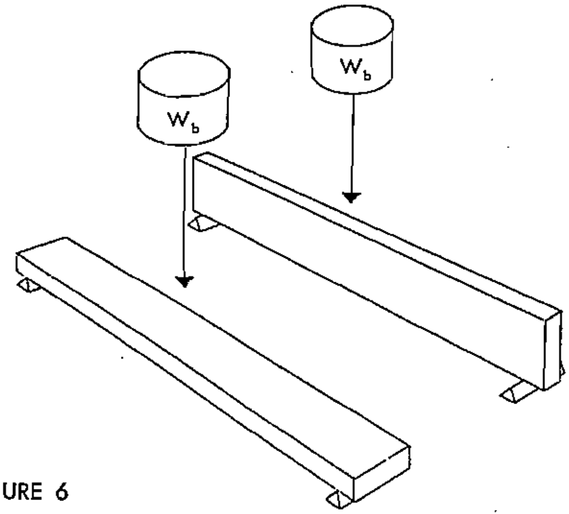


FIGURE 6

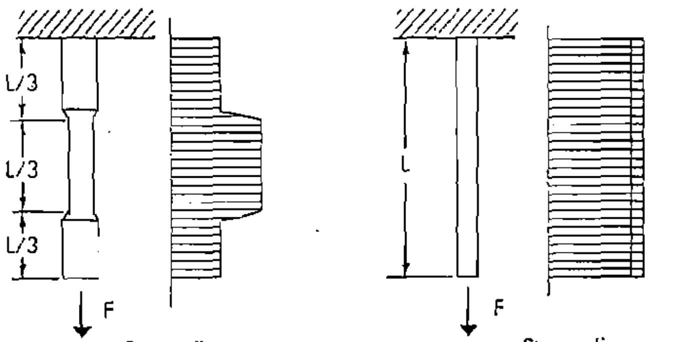


FIGURE 7

The two tensile bars shown in Figure 5 have equal strength under steady loads; yet, the bar on the right, having uniform cross-section, is able to absorb much more energy and can withstand a greater impact load.

Summary

1. The property of the section which will reduce the impact stress in tension is increased volume (AL).

2. The property of the section which will reduce the impact stress in a simple beam is:

$$\text{increased } \sqrt{\frac{I\ L}{c^2}} \text{ or } = \frac{r}{c}\sqrt{A\ L}$$

3. In a simple beam, a decrease in length (L) will decrease the static stress, but will increase the stress due to impact.

4. In a simple tensile bar of a given uniform cross-section, increasing the length (l) will not alter the static stress yet it will decrease the stress due to impact.

11. NOTCH EFFECT ON ENERGY ABSORBING CAPACITY

In Figure 8, diagrams e and f represent the energy absorbed along the length of a member. The total energy absorbed corresponds to the area under this diagram.

Assume the notch produces a stress concentration of twice the average stress (d). Then for the same maximum stress, the average stress will be reduced to ½ and the energy absorbed (f) will be ¼ of the energy absorbed if no notch were present (e). For a stress concentration of three times the average stress, the energy absorbed will be 1/9, etc.

Notched bar impact test results are of limited value to the design engineer, and can be misleading:

(a) The test is highly artificial in respect to severe notch condition and manner of load condition.

(b) The results can be altered over a wide range by changing size, shape of notch, striking velocity, and temperature.

(c) The test does not simulate a load condition likely to be found in service.

(d) The test does not give quantitative values of the resistance of the material to energy loads.

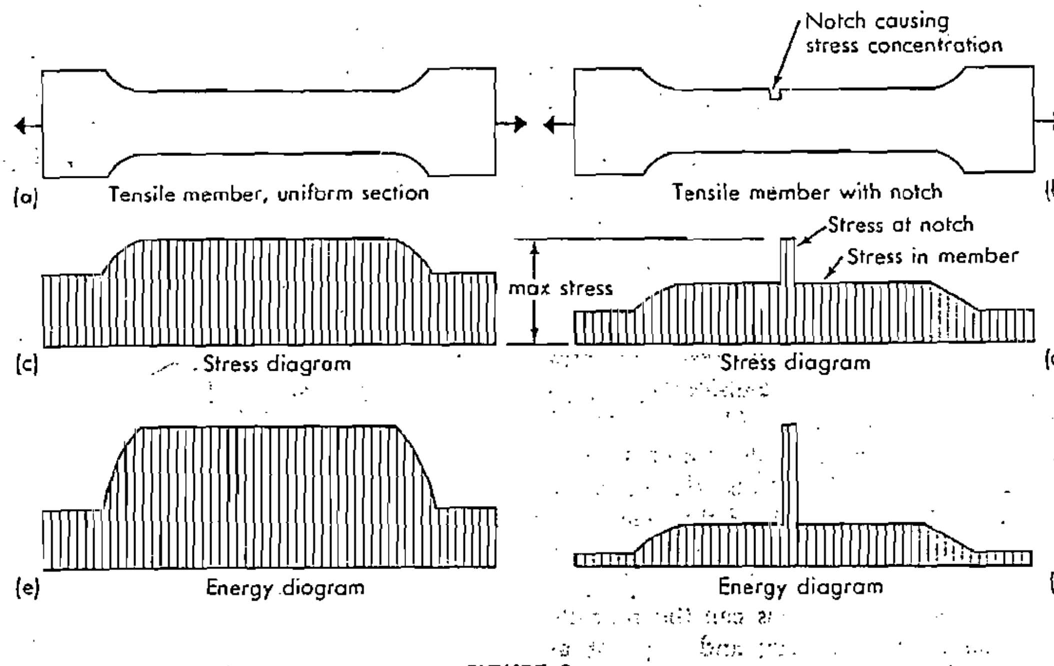


FIGURE 8

12. GUIDES TO DESIGNING FOR IMPACT LOADS

1. Design the member as an energy-absorbing system, that is have the maximum volume of material stressed to the highest working stress; this increases the energy absorbed.
2. For any given cross-section of the member, have the maximum area subjected to the maximum allowable stress; also stress the entire length to this value.
3. The property of the section which will reduce the impact stress in tension is increased volume (A L).
4. The property of the section which will reduce the impact stress in bending is increased I/c^2.
5. Increasing the length (L) of a beam will increase the static stress, but will decrease stress due to impact.
6. Increasing the length (L) of a tensile member of uniform cross-section will not change the static stress, but will decrease stress due to impact.
7. Use the basic formula, or those shown in Table 3, as a guide to select the required property of section and property of material.
8. Select material that has a high modulus of resilience $u = \frac{\sigma_y^2}{2\,E}$. Materials having lower modulus of elasticity (E) generally have lower values of yield strength (σ_y), and this latter value is more important because it is squared. Therefore steels with higher yield strengths have higher values of modulus of resilience and are better for impact loads.
9. The material should be ductile enough to plastically relieve the stress in any area of high stress concentration; and have good notch toughness.
10. The material should have high fatigue strength if the impact load is repeatedly applied.
11. The material should have good notch toughness, and for low temperature service, a low transition temperature.
12. Reduce stress concentrations to a minimum and avoid abrupt changes in section.
13. If possible, place material so that the direction of hot rolling (of plate or bar in steel mill) is in line with impact force.
14. For inertia forces, decrease the weight of the member, while maintaining proper rigidity of the member for its particular use. This means lightweight, well-stiffened members having sufficient moment of inertia (I) should be used.
15. One aid against possible inertia forces caused by the rapid movement of the member due to explosive energy, earthquakes, etc., is the use of flexible supports, to decrease the acceleration and/or deceleration of the member.

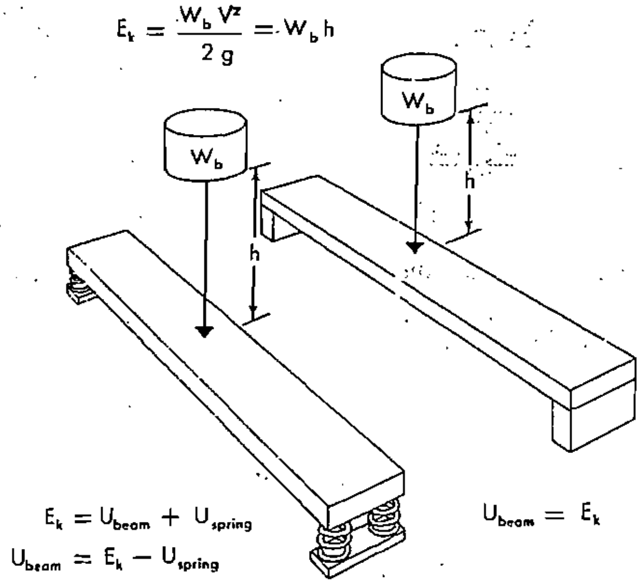


FIGURE 9

Problem 1 ***Accelerating a Load***

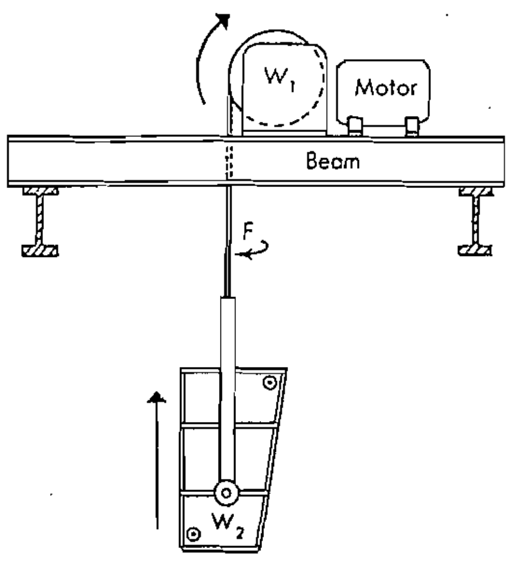


FIGURE 10

Find the load placed on the supporting beam for a hoisting unit in the shaft of a mine if the 5000-lb load (W_2) is accelerated upward to a velocity (V) of 1800 feet per minute in 5 seconds (t). The dead weight of the hoisting unit is 1000 lbs (W_1).

acceleration

$$a = \frac{V_2 - V_1}{t}$$

$$= \frac{\left(\frac{1800}{60}\right) - 0}{(5)}$$

$$= 6 \text{ ft/sec}^2$$

force of acceleration

$$F_a = \frac{W_2}{g} a$$

$$= \frac{(5000)}{(32.2)} (6)$$

$$= 931 \text{ lbs}$$

total load on beam

$$W_1 + W_2 + F_a = (1000) + (5000) + (931)$$

$$= 6931 \text{ lbs}$$

Problem 2 ***Decelerating a Load***

Assume the truck brakes the trailer, because brakes on trailer have failed, and stops from a speed of 60 miles per hour within 15 seconds.

$$V_1 = 60 \text{ MPH} = \frac{(5280)\ (60)}{(3600)}$$

$$= 88 \text{ ft/sec}$$

deceleration

$$a = \frac{V_2 - V_1}{t}$$

$$= \frac{0 - (88)}{(15)}$$

$$= -5.86 \text{ ft/sec}^2$$

force of deceleration

$$F = \frac{W}{g} a$$

$$= \frac{(40{,}000)}{(32.2)} (5.86)$$

$$= 7275 \text{ lbs}$$

The king pin on the fifth wheel, connecting the trailer to the tractor must be designed to transfer this force.

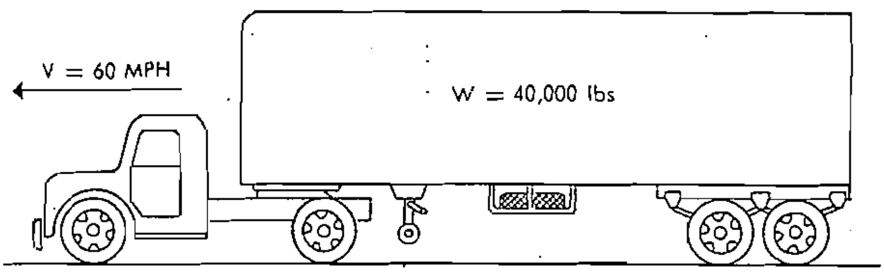


FIGURE 11

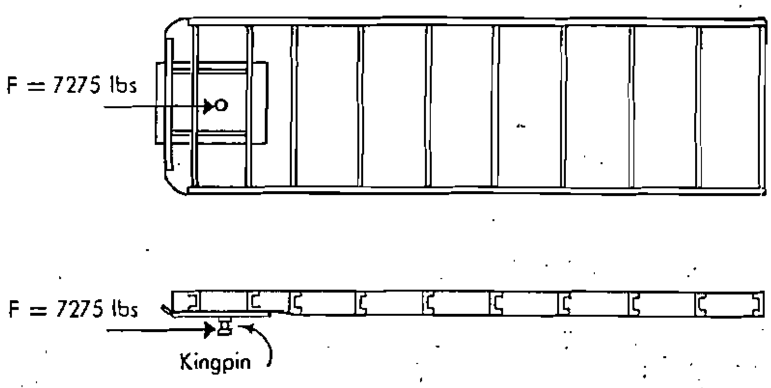


FIGURE 12