

The Design of Hangers and Supports

1. BASIC FORCES AND STRESSES

Designing hangers or brackets for supporting a shell such as a pipe, tank or pressure vessel requires consideration of two important factors:

1. The additional stress of the support forces when combined with the working stress of the shell must not increase the stress in the shell above the allowable limit.

2. The support should not restrain the stressed shell so it becomes too rigid to flex under normal changes in working pressures or loads.

Many types of stresses are involved in any supporting structure. The more common types are the following:

1. The internal pressure of the gas or liquid in the shell, along with its weight, cause tangential (σ_{cp}) and longitudinal (σ_{mp}) tensile stresses in the shell.

2. Any radial force (F_1) acting on a section of the shell causes bending stresses in the ring of the shell (from the bending moment M_r) as well as axial tensile stresses (from the tensile force T), both of which act tangentially to the circumference of the shell.

3. The radial force (F_1) causes radial shear stresses in the shell, and the longitudinal force (F_2) causes longitudinal shear stresses, both adjacent to the hanger. These stresses usually will be low.

After proper analysis of the forces involved, the various stresses must be combined to determine the maximum normal stress (σ_{max} —tensile or compressive) and maximum shear stress (τ_{max}). If the resulting stresses are excessive, a simple study of the individual stresses will indicate what portion of the hanger is under-designed and should be strengthened.

For example, the bending stresses may be excessive, indicating that some type of stiffener ring should be attached to the shell between supports to substantially increase the moment of inertia of the shell section thereby decreasing the bending stress.

The following discussions identify and analyze the effect of various basic stresses and relate them to material thickness and curvature.

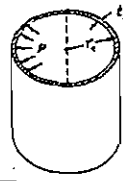
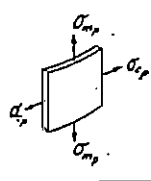
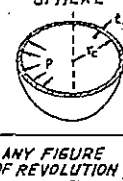
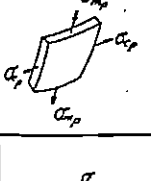

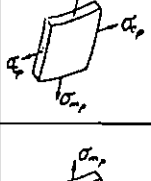
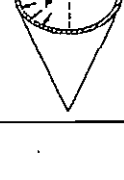

2. STRESSES IN SHELL FROM INTERNAL PRESSURE

As explained more fully in Section 6.5, internal pressure in a shell produces two tensile stresses of importance.

1. σ_{mp} = tensile stress in the direction of the meridian. This is called the longitudinal stress.

2. σ_{cp} = tensile stress in the direction of the tangent to the circumference. This stress is commonly called the hoop stress, but is also referred to as the tangential or circumferential stress.

The tensile stresses σ_{mp} and σ_{cp} can be calculated with the formulas presented in Table 2 of the preceding Section 6.5 and repeated here.

THIN WALL CONTAINERS		
CONTAINER SHAPE	UNIT WALL SEGMENT	TENSILE STRESS FORMULAE
CYLINDER 		$\sigma_{mp} = \frac{p r_c}{2 t_s}$ $\sigma_{cp} = \frac{p r_c}{t_s}$
SPHERE 		$\sigma_{mp} = \sigma_{cp} = \frac{p r_c}{2 t_s}$
ANY FIGURE OF REVOLUTION 		$\frac{\sigma_{cp}}{r_{cp}} + \frac{\sigma_{mp}}{r_{mp}} = \frac{p}{t_s}$ $\sigma_{mp} = \frac{p r_c}{2 t_s}$ $\sigma_{cp} = \frac{p r_c}{t_s} \left(1 - \frac{r_c}{2 r_m} \right)$
CONE 		$\sigma_{mp} = \frac{p r_c}{2 t_s \cos \alpha}$ $\sigma_{cp} = \frac{p r_c}{t_s \cos \alpha}$

3. EFFECT OF HANGER OR SUPPORT WELDED TO SHELL

The force (P) applied to the hanger (see Figure 1) may be resolved into a radial component (F_1) and a longitudinal component (F_2) having the following values:

$$F_1 = P \cos \theta$$

$$F_2 = P \sin \theta$$

where θ is the angle between guy cable or support attached to the shell and the horizontal.

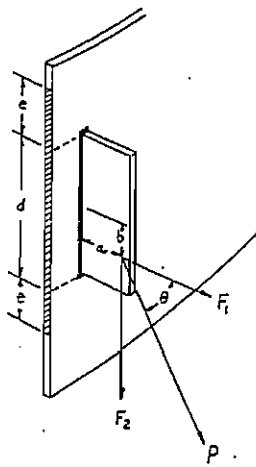


FIGURE 1

If these components are applied at some eccentricity (a and b), they will produce moments applied to the shell section by the hanger and having values:

$$M_1 = b F_1$$

$$M_2 = a F_2$$

Combining these values, observing proper signs, will give the total moment acting on the shell from the hanger:

$$M_h = M_1 + M_2$$

A study of stress distribution in the shell can be resolved into separate analyses of the radial and moment force distributions. Before analyzing these forces, however, the engineer should determine how much shell beyond the hanger is effective in resisting these forces.

The shell with stiffeners can be compared to a curved beam with an extremely wide flange, Figure 1. Von Karman* suggests that an effective width (e) of

* "Analysis of Some Thin-Walled Structures", Von Karman, ASME paper AER-55-19C, Aer Eng, Vol. 5, No. 4, 1933.

RADIAL FORCE (f_a) DISTRIBUTION

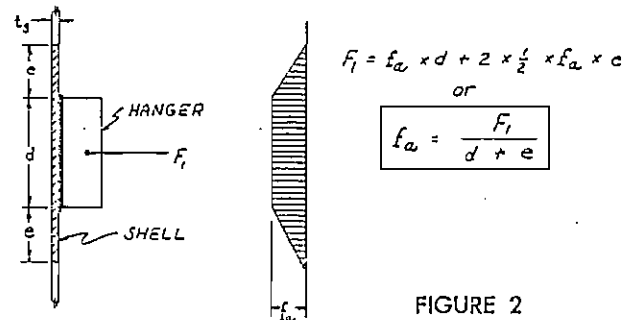


FIGURE 2

the flange on each side of the stiffening web is approximately—

$$e = \frac{\sqrt{t_s r_c}}{2}$$

where:

r_c = radius of shell curvature, inches

t_s = thickness of shell, inches

The value of "e" should be limited to a maximum of $12t_s$.

The radial component (F_1) of the force (P) is applied directly to the shell. It is reasonable to assume that the radial forces applied to the additional shell width (e) would decrease linearly to almost zero at its outer limits. This assumed distribution of radial forces (f_a) due to the radial component (F_1) is sketched in Figure 2.

The value of f_a is equivalent to the force (lbs) on a 1" wide ring of the shell.

The longitudinal component (F_2) of the force (P) because of its eccentricity (a), and the radial component (F_1) because of its eccentricity (b), combine into moment M_h and apply radial forces to the shell having a distribution similar to that of bending forces, i.e. maximum at the outer fibers and zero along the neutral axis. The assumed distribution of the radial forces (f_b) due to the action of the applied moment is indicated in Figure 3.

RADIAL FORCE (f_b) DISTRIBUTION

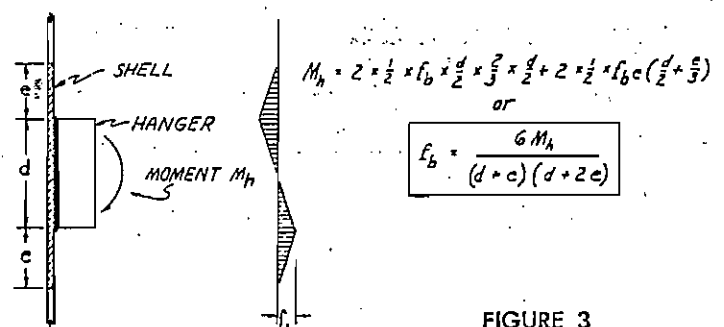


FIGURE 3

The value of f_b is equivalent to the force (lbs) on a 1" wide ring of the shell.

The resulting radial forces applied on the shell must be added, being careful to watch the signs:

$$f_1 = f_a + f_b$$

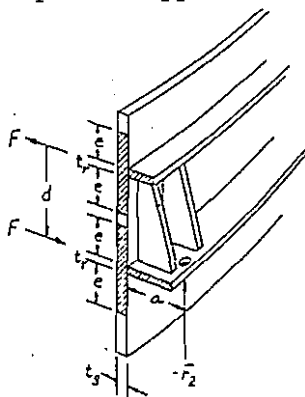
4. EFFECT OF ADDING STIFFENING RING

For additional stiffening of the shell at the support, rings may be welded to the shell. As before, the additional width of the shell on each side of the ring assumed to be effective in resisting these forces is—

$$e = \frac{\sqrt{t_s r_c}}{2}$$

with e not to exceed $12 t_s$ on each side of the ring.

The total radial force (F) applied to this built-up section is the radial force resulting from the longitudinal force (F_2), plus any radial force (F_1) applied at this point of support:



$$F = \frac{F_2 a}{d} + F_1$$

IN THIS CASE SINCE $F_1 = 0$

$$F = \frac{F_2 a}{d}$$

FIGURE 4

After determining the bending moment in this built-up ring resulting from the radial forces at the point of support, the moment of inertia (I) of the section is calculated. The bending stresses are then found and later combined with any other stresses.

5. EFFECT OF THESE FORCES UPON A SECTIONAL RING OF THE SHELL

Forces (f_1) normal to the shell set up tangential tensile forces (T) and bending moments (M_r) in the ring of the shell, Figure 5.

Stresses σ_{et} and σ_{eb} are added to σ_{vi} to give σ_c = total tangential (or circumferential) stress in a section of the critical shell ring.

The maximum shear stress is equal to $\frac{1}{2}$ the difference of the two principal stresses (σ) having the greatest algebraic difference. See Section 2.11, Topic 2.

The following are typical examples that demonstrate the use of these formulas for calculating the stresses in a shell.

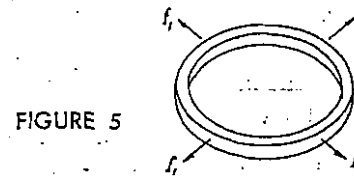


FIGURE 5

NOTE: FOR 1 INCH WIDE RING RADIAL FORCES ARE F_1 ; WHEN SECTION IS BUILT UP AS ILLUSTRATED IN PREVIOUS SKETCH, RADIAL FORCES ARE F .

THESE FORCES AND MOMENTS CAN BE TABULATED FOR VARIOUS SUPPORT CONFIGURATIONS:

TABLE I				
NUMBER OF HANGERS	FORMULA FOR TANGENTIAL TENSILE FORCE $T = K_1 F_1$		FORMULA FOR BENDING MOMENT M_r in ring $M_r = K_2 F_1 r_c$	
	VALUES FOR K_1		VALUES FOR K_2	
	AT HANGERS	HALFWAY BETWEEN HANGERS	AT HANGERS	HALFWAY BETWEEN HANGERS
2	0	0.500	+ 0.318	- 0.182
3	0.289	0.577	+ 0.188	- 0.100
4	0.500	0.707	+ 0.136	- 0.071
6	0.866	1.000	+ 0.089	- 0.045
8	1.207	1.306	+ 0.065	- 0.033
	RESULTING TENSILE STRESS $\sigma_{et} = \frac{T}{A}$		RESULTING BENDING STRESS $\sigma_{eb} = \frac{M_r}{S}$	

where:

A = area of shell ring cross-section or built-up section

S = section modulus of the same section.

Problem 1

Part A: Four hangers are used for guying a smoke stack with its axis in the vertical position, Figure 6.

DATA			
$P = 100 \text{ psi}$	$P = 250 \text{ lb}$	$a = 2 \text{ in.}$	
$r_c = 30 \text{ in.}$	$\theta = 60^\circ$	$b = 0$	
$t_s = \frac{1}{2} \text{ in.}$	$d = 10 \text{ in.}$	$n = 4 \text{ HANGERS}$	

CALCULATING TENSILE STRESS IN SHELL FROM INTERNAL PRESSURE

$$\sigma_{et} = \frac{Pr_c}{t_s} = \frac{(100)(30)}{\frac{1}{2}} = 6,000 \text{ psi}$$

$$\sigma_{mr} = \frac{Pr_c}{2t_s} = \frac{(100)(30)}{(2)(\frac{1}{2})} = 3,000 \text{ psi}$$

MOMENT ON SHELL SECTION FROM FORCES APPLIED TO HANGER

$$F_1 = P \cos \theta = 250 \times .5 = 125 \text{ lb}$$

$$F_2 = P \sin \theta = 250 \times .866 = 217 \text{ lb}$$

$$M_{r1} = a F_2 + b F_1 = 2 \times 217 + 0 \times 125 = 434 \text{ lb-in.}$$

EFFECTIVE SHELL WIDTH "e" EACH SIDE OF HANGER

$$e = \frac{\sqrt{t_s r_c}}{2} = \frac{\sqrt{\frac{1}{2} \times 30}}{2} = 1.94 \text{ or } 2 \text{ in.}$$

CALCULATING RADIAL FORCES APPLIED TO SHELL

$$f_a = \frac{F_1}{d/c} = \frac{125}{10/2} = 10.4 \text{ lb/in. RING OF SHELL}$$

$$f_b = \frac{GM_r}{(d/c)(d/2c)} = \frac{6 \times 434}{(10/2)(10/2)} = 15.5 \text{ lb/in. RING}$$

TOTAL RADIAL FORCE

$$f_1 = f_a + f_b = 10.4 + 15.5 = 25.9 \text{ lb/in. RING}$$

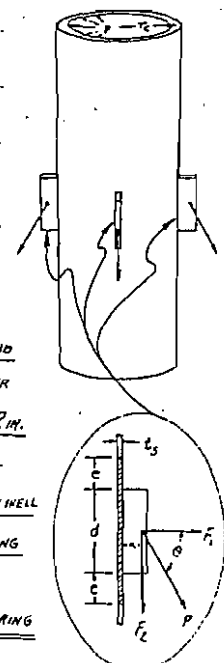
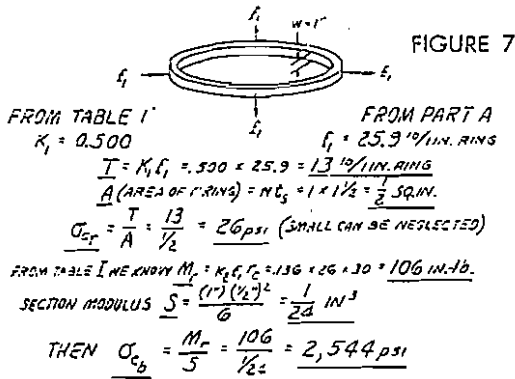


FIGURE 6

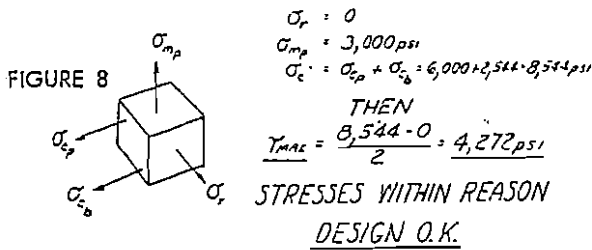
6.6-4 / Miscellaneous Structure Design

Determine the total radial force acting on the shell as a result of the force (P) applied to the hangers.

Part B: With tangential tensile force (T) and bending moment (M_r) per 1" wide ring of this shell resulting from radial forces (f_1) applied to the four hangers, calculate the tensile (σ_{ct}) and bending (σ_{cb}) stresses at the hangers.



Conclusion: Combining these stresses in the outer fiber of the shell adjacent to the hanger shows our analysis of the shear stress (τ_{max}) to be—



Problem 2

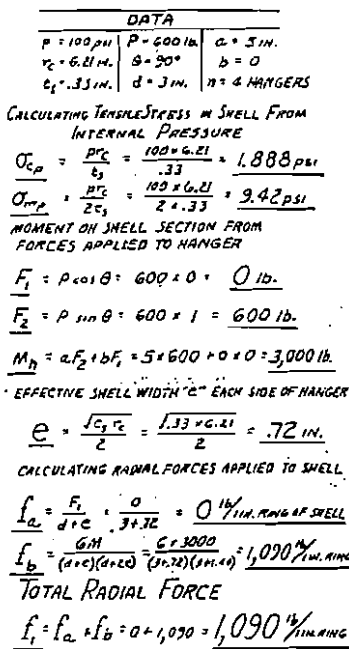
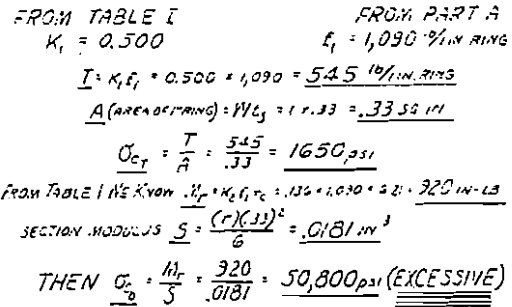


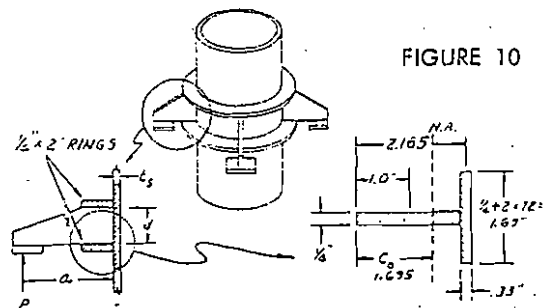
FIGURE 9

Part A: Four hangers are used to support a vertical 12" stand pipe, Figure 9. Determine the total radial force acting on the shell as a result of the force (P) applied to the hangers.

Part B: With tensile force (T) and bending moment (M_r) per 1" wide ring of this shell resulting from radial forces (f_1) applied at the four hangers, calculate the tensile (σ_{ct}) and bending (σ_{cb}) stresses at the hangers.



Since this bending stress in the ring of the shell is excessive, it is necessary to stiffen the shell in this region. To accomplish this, two $1/4" \times 2"$ ring stiffeners are added as illustrated, Figure 10.



The effect of the bottom ring will be considered since it will apply radial tensile forces to the built-up ring and shell section. Using the method of finding moment of inertia by adding areas (Sect. 2.2), the properties of this section are as follows:

RING SECTION	A	d	M=Ad	$I_x=Md$	I_y
1.69 x .33	.557	+2.165	1.205	2.61	.005
1/4" x 2.0	.500	+1.0	.500	.50	.167
TOTAL	1.057		1.705	3.282	

THEN MOMENT OF INERTIA ABOUT NEUTRAL AXIS WILL BE

$$I_{NA} = I_x - \frac{M^2}{A} = 3.282 - \frac{1.705^2}{1.057} = 0.532 \text{ in}^4$$

AND NEUTRAL AXIS WILL BE

$$NA = C_b = \frac{M}{A} = \frac{1.705}{1.057} = +1.613 \text{ in.}$$

The radial force (F) acting on the ring section and resulting from the vertical force (P) is—

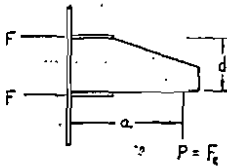
$$F = \frac{F_2 a}{d} = \frac{600 \times 5}{3} = 1000 \text{ lb}$$


FIGURE 11

Part C: Recalculation of the tensile (σ_{ct}) and bending (σ_{cb}) stresses at the hangers yields the following results:

FROM TABLE 1
 $K_1 = 0.500$

THE NEW F
 $F = 1,000$

$$T = K_1 F = 0.500 \times 1,000 = 500 \text{ lb}$$

$$A \text{ (TOTAL FROM TABLE 2)} = 1.057 \text{ sq. in.}$$

$$\sigma_{ct} = \frac{T}{A} = \frac{500}{1.057} = 473 \text{ psi}$$

$$\text{FROM TABLE I WE KNOW } M_r = K_2 F r_c = 1.32 \times 1,000 \times 6.21 = 845 \text{ in.-lb.}$$

$$\text{THEN } \sigma_{cb} = \frac{M_r c}{I} = \frac{845 \times 1.695}{.532} = 2,690 \text{ psi}$$

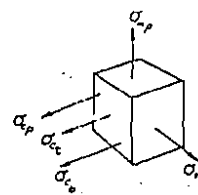
The hoop stress of $\sigma_{ch} = 1,888$ psi in the shell will be assumed to be reduced when considered to be acting over the entire cross-section of the built-up ring section:

$$\sigma_{cp} = 1,888 \times \frac{\text{AREA SHELL IN RING SECTION}}{\text{AREA OF RING SECTION}}$$

$$\sigma_{cp} = 1,888 \times \frac{1.69 \times .33}{1.057}$$

$$\sigma_{cp} = 990 \text{ psi}$$

Combining these stresses in the outer fiber of the lower ring, adjacent to the hanger, we find the maximum shear stress (τ_{max}) to be—



$\sigma_p = 0$ NOTE: THE MERIDIAN TENSILE STRESS OF $\sigma_c = 942$ psi IN THE SHELL ONLY AND NOT IN OUTER PORTION OF LOWER RING.

$$\sigma_{cp} = 0$$

$$\sigma_c = \sigma_p + \sigma_{ct} + \sigma_{cb} = 990 + 473 + 2,690$$

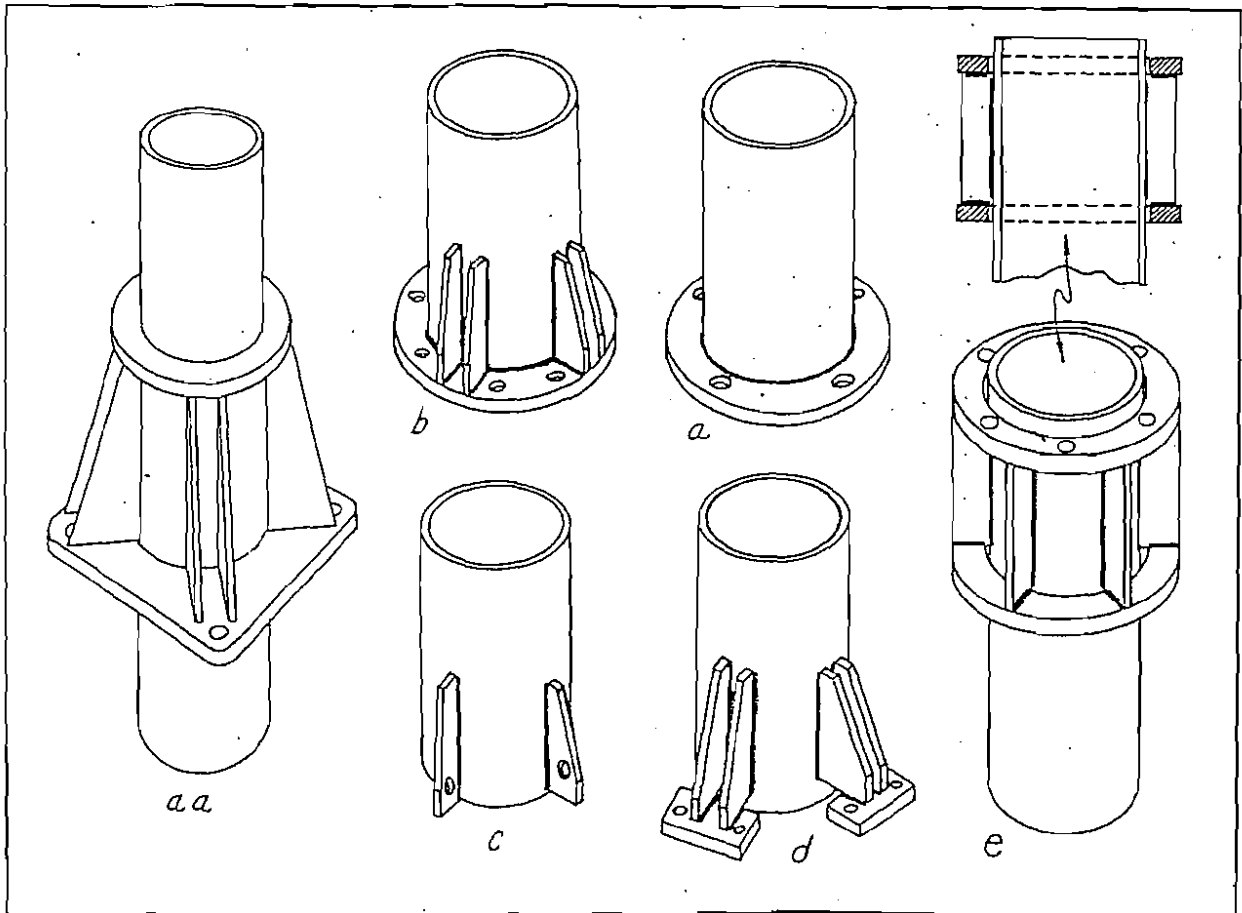
$$\sigma_c = 4,153 \text{ psi}$$

$$\tau_{max} = \frac{4,153 - 0}{2} = 2,070 \text{ psi}$$

STRESS WITHIN REASON
DESIGN O.K.

FIGURE 12

FIG. 13—Typical Hangers and Supports



Problem 3

Part A: What transverse or radial force (F_1) can be applied to the web of this I section through the gusset plate shown? See Figure 14. The resulting bending stresses are to be kept down to a reasonable value, such as $\sigma = 15,000$ psi, since the I section is already under applied load. The gusset plate intersects the web of the I section along a predetermined distance of $d = 10''$.

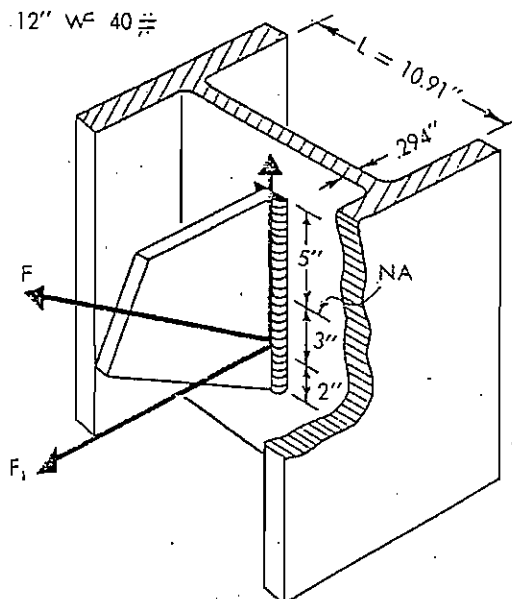


FIGURE 14

The analysis of this problem again stems from Figures 1, 2 and 3 and related text. Here, the gusset plate acts as a hanger.

Considering the web of the I section as a panel, the section flanges act as stiffeners and give the entire section a high moment of inertia about its x-x axis. However, to be conservative assume the width of web beyond the gusset that is effective in resisting the bending moment on the web to have a maximum value of 12 times the web thickness.

effective width of web

$$\begin{aligned} e &= 12 t_w \\ &= 12 (.294'') \\ &= 3.53'' \end{aligned}$$

moment on web due to force on gusset

$$M = F_1 \times 3''$$

tangential forces applied to web (see Fig. 2)

$$f_a = \frac{F_1}{d + e}$$

$$= \frac{F_1}{(10) + (3.53)}$$

$$= .074 F_1 \text{ lbs/in.}$$

$$f_b = \frac{6 M}{(d + e)(d + 2e)}$$

$$= \frac{6 (F_1 \times 3'')}{(10 + 3.53)(10 + 7.06)}$$

$$= .078 F_1 \text{ lbs/in.}$$

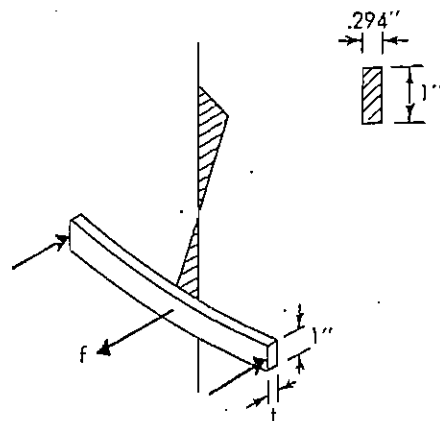
total tangential forces applied to web

$$f = f_a + f_b$$

$$= .074 F_1 + .078 F_1$$

$$= .152 F_1 \text{ lbs/in.}$$

Consider a 1''-wide strip of the web:



section modulus of strip

$$\begin{aligned} S &= \frac{1'' (.294'')^2}{6} \\ &= .0144 \text{ in.}^3 \end{aligned}$$

tangential force on strip

$$\text{Since: } M_x = \frac{f L}{4}$$

$$f = \frac{4 M_x}{L} = \frac{4 \sigma S}{L}$$

$$= \frac{4 (15,000) (.0144)}{(10.91)}$$

$$= 79.2 \text{ lbs/1''-wide strip}$$

But:

$$f = .152 F_1$$

\therefore *allowable tangential force on web*

$$F_1 = \frac{79.2}{.152}$$

$$= 521 \text{ lbs}$$

Part B: What transverse force (F_1) can be applied if it is concentric with the center of gravity of the connection? See Figure 15. There would be no moment (M).

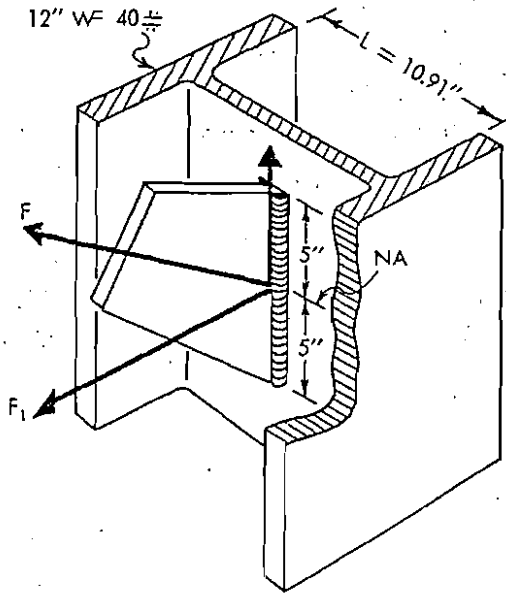


FIGURE 15

Here:

$$M = 0$$

hence:

$$f_b = 0$$

$$\begin{aligned} f_a &= \frac{F_1}{d + e} \\ &= \frac{F_1}{(10) + (3.53)} \\ &= .074 F_1 \end{aligned}$$

Consider a 1"-wide strip of the web. As before:

$$S = .0144 \text{ in.}^3$$

$$M_{\text{E}} = \frac{f L}{4}$$

$$f = 79.2 \text{ lbs/1"-wide strip}$$

But:

$$f = .074 F_1$$

$$\begin{aligned} \therefore F_1 &= \frac{79.2}{.074} \\ &= 1070 \text{ lbs} \end{aligned}$$

General Formula

A general formula, if the transverse force (F_1) is concentric with the center of gravity of the connection, is—

$$F_1 = \frac{4 \sigma t_w^2 (d + e)}{6 L}$$

$$\begin{array}{l} \text{Assume:} \\ e = 12 t_w \end{array}$$

Part C: What transverse force (F_1) can be applied if a stiffener is added to the web section to increase its bending strength? See Figure 16.

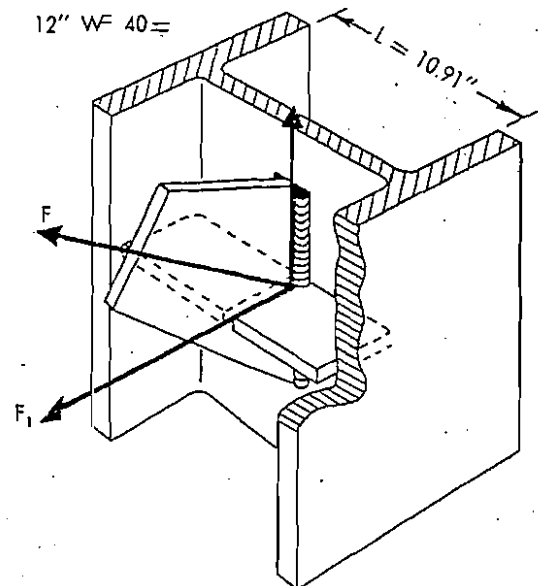


FIGURE 16

The stiffened web will now have a much greater moment of inertia in the direction of tangential force. Although the gusset plate intersects the web of the I for a distance of 10", to be conservative only a portion of this ($b \leq t_s + 2e$) can be considered as resisting the moment on the web.

Following the analysis of a stiffened plate as given in Section 6.6:

Here:

$$e = 3.53''$$

$$A_p = 2.2216 \text{ in.}^2 \text{ (area of effective stiffened portion of web)}$$

$$I_p = .01601 \text{ in.}^4$$

$$A_s = 1.5 \text{ in.}^2 \text{ (area of stiffener section)}$$

$$I_s = 1.125 \text{ in.}^4$$

$$d = 1.647'' \text{ (distance, C.G. of stiffener to C.G. of web)}$$

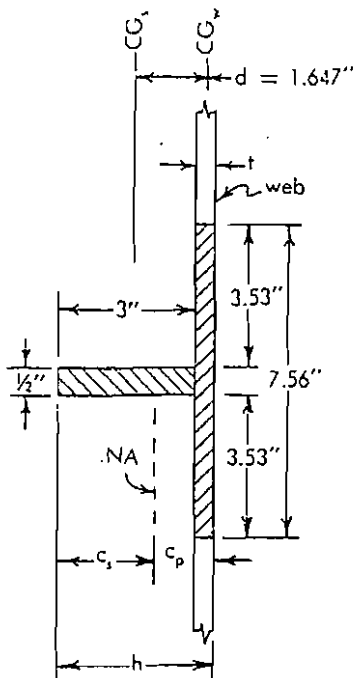


FIGURE 17

moment of inertia of entire section

$$\begin{aligned}
 I &= I_s + I_p + \frac{A_s A_p d^2}{A_s + A_p} \\
 &= (1.25) + (.01601) + \frac{(1.5)(2.216)(1.647)^2}{(1.5) + (2.216)} \\
 &= 3.570 \text{ in.}^4
 \end{aligned}$$

distance of N.A. to outer fiber

$$c_s = h - c_p$$

and since

$$c_p = \frac{A_s d}{A_s + A_p} + \frac{t}{2}$$

$$\begin{aligned}
 \therefore c_s &= h - \frac{t}{2} - \frac{A_s d}{A_s + A_p} \\
 &= (3.294) - (.147) - \frac{(1.5)(1.647)}{(1.5) + (2.216)} \\
 &= 2.483''
 \end{aligned}$$

section modulus of entire section resistant to force (F_1) which is maximum at extreme fiber

$$\begin{aligned}
 S &= \frac{I}{c_s} \\
 &= \frac{(3.570)}{(2.483)} \\
 &= 1.438 \text{ in.}^3
 \end{aligned}$$

and since

$$M = \frac{F_1 L}{4}$$

$$\begin{aligned}
 \text{or } F_1 &= \frac{4 M}{L} = \frac{4 \sigma S}{L} \\
 &= \frac{4 (15,000)(1.438)}{(10.91)} \\
 &= 7920 \text{ lbs allowable tangential force on web}
 \end{aligned}$$

Alternate Location of Stiffener

The web stiffener could be placed on the back side of the web (Fig. 18). However, additional brackets might have to be used to safely transfer the transverse force (F_1) back into the stiffener. Otherwise, both the gusset plate and the stiffener might be overstressed in a localized area where the two intersect (Fig. 19).

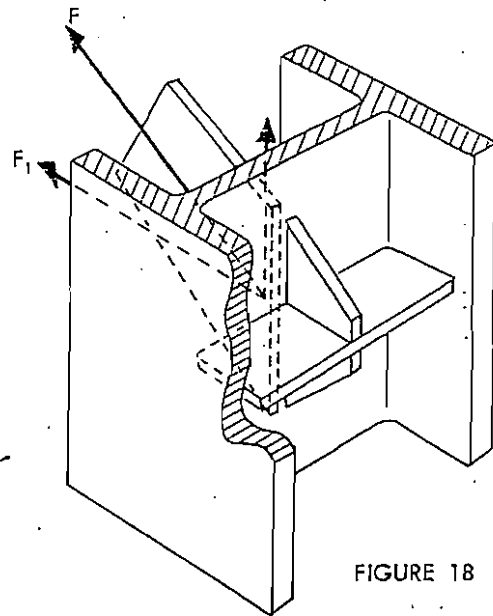


FIGURE 18

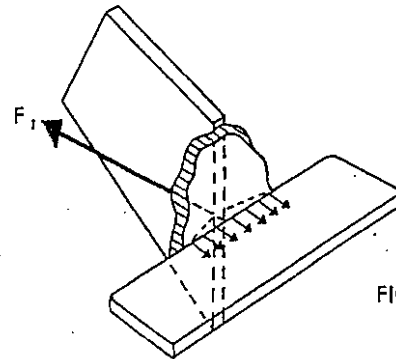


FIGURE 19