Designing for Fatigue Loads

1. ENDURANCE LIMIT

When the load on a member is constantly varying in value, or is repeated at relatively high frequency, or constitutes a complete reversal of stresses with each operating cycle, the material's endurance limit must be substituted for the ultimate strength where called for by design formulas.

Under high load values, the variable or fatigue mode of loading reduces the material's effective ultimate strength as the number of cycles increases. At a given high stress value, the material has a definite service or fatigue life, expressed as N cycles of operations. Conversely, at a given number of service cycles the material has a definite allowable fatigue strength.

The endurance limit is the maximum stress to which the material can be subjected for a given service life.

2. NATURE OF FATIGUE LOADING

Fatigue failure is a progressive failure over a period of time which is started by a plastic movement within a localized region. Although the average unit stresses across the entire cross-section may be below the yield point, a non-uniform distribution of these stresses may cause them to exceed the yield point within a small area and cause plastic movement. This eventually produces a minute crack. The localized plastic movement further aggravates the non-uniform stress ditribution, and further plastic movement causes the crack to progress. The stress is important only in that it causes the plastic movement.

Any fatigue test usually shows considerable scatter in the results obtained. This results from the wide range of time required before the initial crack develops in the specimen. Once this has occurred, the subsequent time to ultimate failure is fairly well confined and proceeds in a rather uniform manner.

The designer when first encountering a fatigue loading problem will often use the material's endurance limit or fatigue strength value given in his engineering handbook, without fully considering what this value represents and how it was obtained. This procedure could lead to serious trouble.

There are many types of fatigue tests, types of loading, and types of specimens. Theoretically the fatigue value used by the designer should be determined in a test that exactly duplicates the actual service conditions. The sample used should preferably be identical to the member, the testing machine should reproduce the actual service load, and the fatigue cycle and frequency should be the same as would be encountered in actual service. For example, if the problem is a butt weld in tension, the allowable fatigue strength used in the design must come from data obtained from loading a butt weld in axial tension on a pulsating type of fatigue testing machine, with the same range of stress.

3. ANALYZING THE FATIGUE LOAD

Figure 1 illustrates a typical fatigue load pattern, the curve representing the applied stress at any given moment of time.

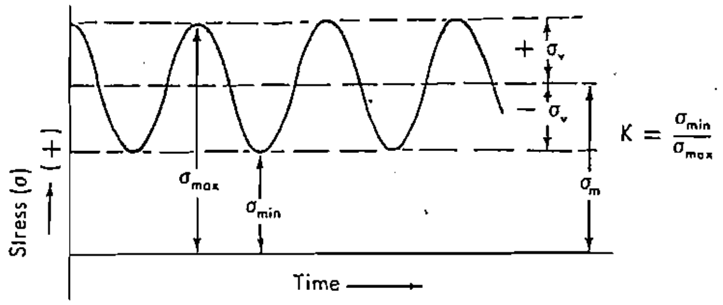
There are two ways to represent this fatigue load:

1. As a mean or average stress (σ_m) with a superimposed variable stress (σ_v).

2. As a stress varying from maximum value (σ_{max}) to a minimum (σ_{min}). Here, the cycle can be represented by the ratio—

$$K = \frac{\sigma_{min}}{\sigma_{max}}$$

FIGURE 1



One approach to this problem is to let the variable stress (σ_v) be the ordinate and the steady or mean stress (σ_m) be the abscissa. When the mean stress (σ_m) is zero, see Figure 2, the varible stress (σ_v) becomes the value for a complete reversal of stress (σ_r). This value would have to be determined by experimental testing, and becomes point *b* in the diagram. When there is no variation in stress, i.e. a steady application of stress, σ_v becomes zero, and the maximum resulting mean stress (σ_m) is equal to the ultimate stress for a steady load (σ_u); this becomes point *a*.

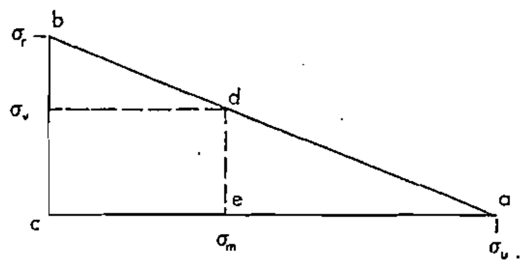


FIGURE 2

where:

σ_r = fatigue strength for a complete reversal of stress

σ_v = variable stress which is superimposed upon steady stress

σ_u = ultimate strength under steady load (Some set σ_u equal to the yield strength, σ_y)

σ_m = mean stress (average stress)

A line connecting points *b* and *a* will indicate the relationship between the variable stress (σ_v) and the mean stress (σ_m) for any type of fatigue cycle, for a given fatigue life (N). This straight line will yield conservative values; almost all of the test data will lie just outside of this line.

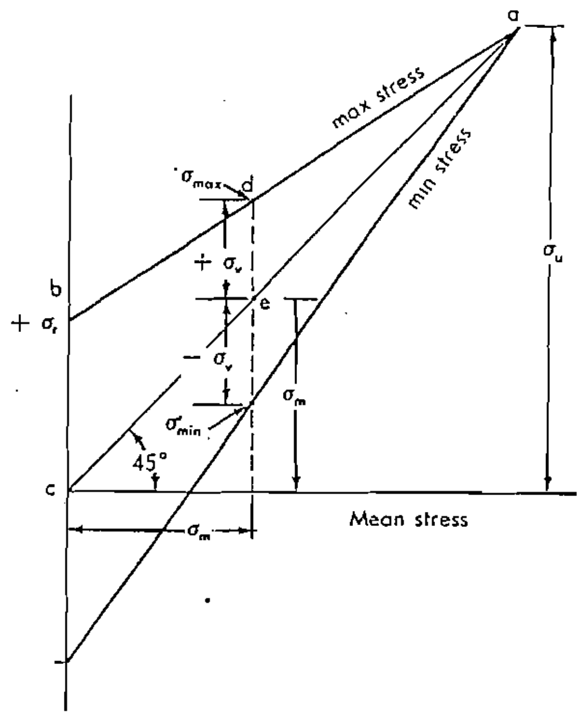


FIGURE 3

From similar triangles it is found that—

$$\frac{\sigma_v}{\sigma_r} + \frac{\sigma_m}{\sigma_u} = 1$$

A Goodman diagram, Figure 3, is constructed from Figure 2 by moving point *a* vertically to a height equal to σ_u; in other words, line a-c now lies at a 45° angle.

It can be shown by similar triangles that the same relationship holds:

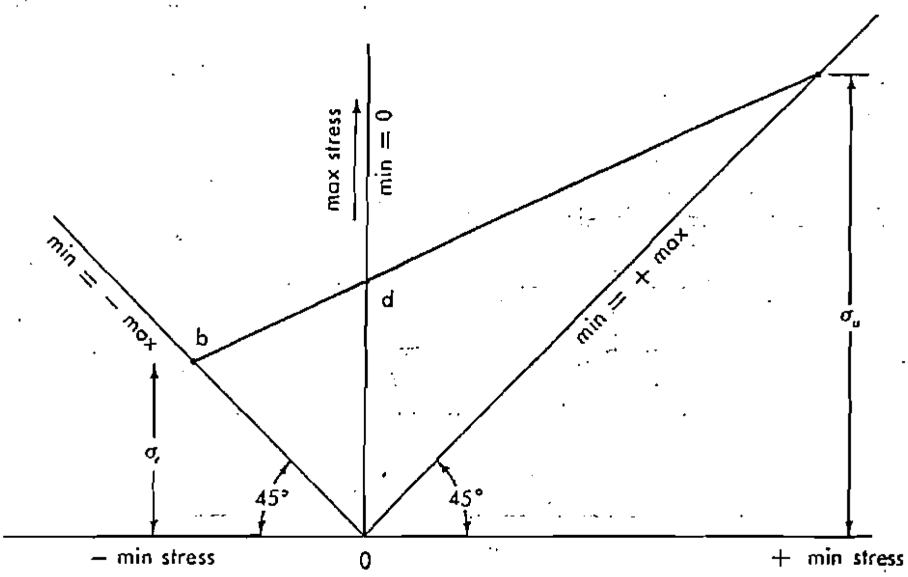


FIGURE 4

$$\frac{\sigma_v}{\sigma_r} + \frac{\sigma_m}{\sigma_u} = 1$$

The Goodman diagram of Figure 3 may be modified so that the ordinate becomes the maximum stress (σ_{max}) and the abscissa becomes the minimum stress (σ_{min}); see Figure 4. It can be proved that all three diagrams yield the same results. The American Welding Society (Bridge Specification) uses this last type of diagram to illustrate their fatigue data test results.

If the maximum stress (σ_{max}) lies on line a-b, this value is found to be—

$$\sigma_{max} = \frac{2\ \sigma_r\ \sigma_u}{\sigma_u + \sigma_r - K(\sigma_u - \sigma_r)}$$

where $K = \frac{\sigma_{min}}{\sigma_{max}}$

The next diagram, Figure 5, is constructed with the values for complete reversal (σ_r) and the ultimate strength (σ_u) for butt welds in tension. The fatigue data from test results are also plotted. Notice the values lie on or slightly above these straight lines for service life (N) of 100,000 cycles and that of 2 million cycles.

These "dependable values" have been reduced to some extent below the minimum values obtained in the test. A factor of safety is applied to obtain allowable values; these are shown by dotted lines. This is expressed as a formula along with a value which should not be exceeded. In this case, the maximum allowable is 18,000 psi. This formula represents the slanting line, but a maximum value must be indicated so that it is not carried too far.

Figure 6 illustrates several types of fatigue cycles, with corresponding K values to be used in the fatigue strength formulas.

4. ALLOWABLE MAXIMUM STRESS

Fatigue strength formulas, for determining the allowable maximum stress for a given service life of N cycles, are presented in Table 1 for A7 mild steel, A373 and A36 steels, in Table 2 for A441 steel, and in Table 3 for T-1, quenched and tempered high yield strength steel.

Required fatigue life or number of cycles will vary but usually starts at several hundred thousand cycles. It is assumed that by the time the value of several million cycles is reached, the fatigue strength has

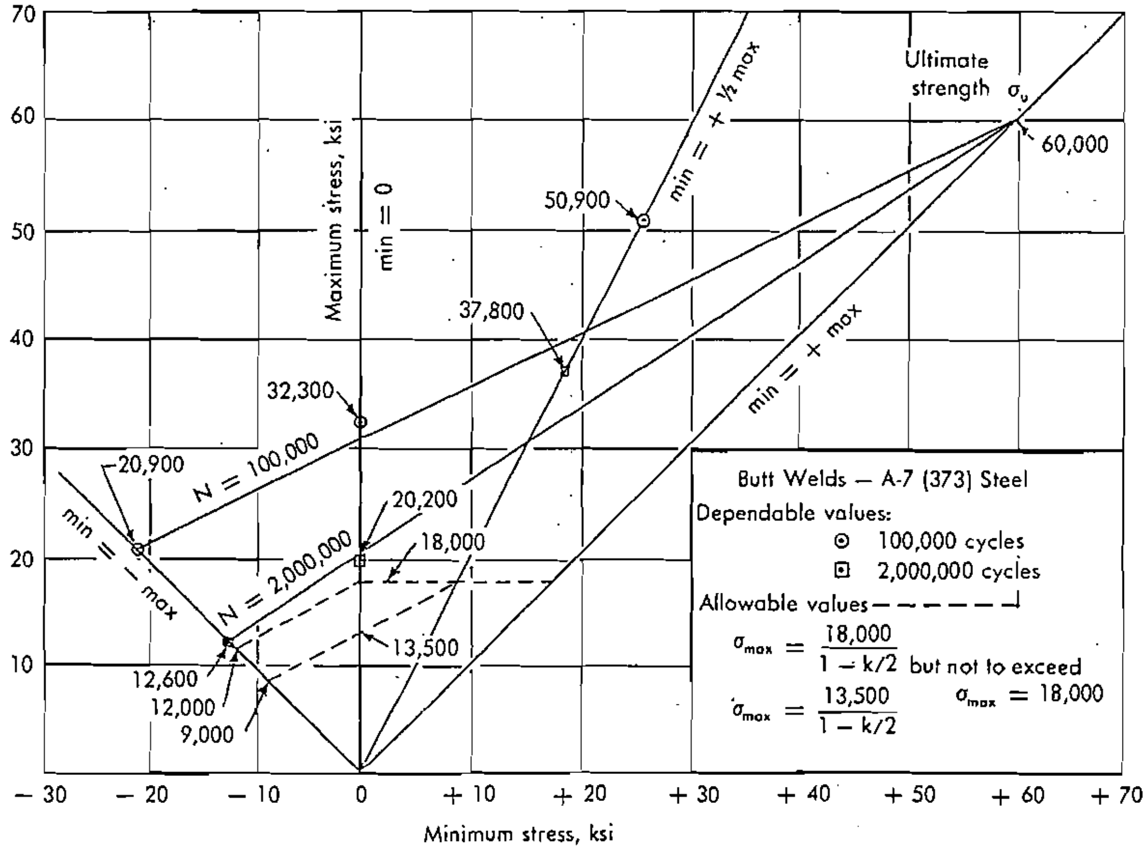


FIGURE 5

leveled off and further stress cycles would not produce failure. For any particular specimen and stress cycle there is a relationship between the fatigue strength (σ) and fatigue life (N) in number of cycles before failure. The following empirical formula may be used to convert fatigue strengths from one fatigue life to another:

$$\sigma_a = \sigma_b \left(\frac{N_b}{N_a}\right)^k$$

where:

σ_a = fatigue strength for fatigue life N_a
σ_b = fatigue strength for fatigue life N_b
N_a = fatigue life for fatigue strength σ_a
N_b = fatigue life for fatigue strength σ_b

The constant (k) will vary slightly with the specimen; however, 0.13 has been widely used for butt welds and 0.18 for plate in axial loading (tension and/or compression).

The curve in Figure 7 illustrates the general increase in fatigue life when the applied fatigue stress is reduced. As an example, in this case, reducing the fatigue stress to 75% of its normal value will in general increase the fatigue life about nine times.

Problem 1

Test data indicates a fatigue life of N_a = 1,550,000 cycles when the member is stressed to σ_a = 30,000 psi. What would be the fatigue strength at a life of 2,000,000 cycles?

TABLE 1—Allowable Fatigue Stress
For A7, A373 and A36 Steels and Their Welds

	2,000,000 cycles	600,000 cycles	100,000 cycles	But Not to Exceed
Base Metal In Tension Connected By Fillet Welds But not to exceed	① $\sigma = \frac{7500}{1 - 2/3\,K}$ psi P_t	③ $\sigma = \frac{10,500}{1 - 2/3\,K}$ psi P_t	⑤ $\sigma = \frac{15,000}{1 - 2/3\,K}$ psi P_t	$\frac{2\,P_t}{3\,K}$ psi
Base Metal Compression Connected By Fillet Welds	② $\sigma = \frac{7500}{1 - 2/3\,K}$ psi	④ $\sigma = \frac{10,500}{1 - 2/3\,K}$ psi	⑥ $\sigma = \frac{15,000}{1 - 2/3\,K}$ psi	P_c psi $\frac{P_c}{1 - \frac{K}{2}}$ psi
Butt Weld In Tension	⑦ $\sigma = \frac{16,000}{1 - \frac{8}{10}K}$ psi	⑪ $\sigma = \frac{17,000}{1 - \frac{7}{10}K}$ psi	⑮ $\sigma = \frac{18,000}{1 - \frac{K}{2}}$ psi	P_t psi
Butt Weld Compression	⑧ $\sigma = \frac{18,000}{1 - K}$ psi	⑫ $\sigma = \frac{18,000}{1 - .8K}$ psi	⑯ $\sigma = \frac{18,000}{1 - \frac{K}{2}}$ psi	P_c psi
Butt Weld In Shear	⑨ $\tau = \frac{9,000}{1 - \frac{K}{2}}$ psi	⑬ $\tau = \frac{10,000}{1 - \frac{K}{2}}$ psi	⑰ $\tau = \frac{13,000}{1 - \frac{K}{2}}$ psi	13,000 psi
Fillet Welds ω = Leg Size	⑩ $f = \frac{5100\,\omega}{1 - \frac{K}{2}}$ lb/in.	⑭ $f = \frac{7100\,\omega}{1 - \frac{K}{2}}$ lb/in.	⑱ $f = \frac{8800\,\omega}{1 - \frac{K}{2}}$ lb/in.	8800 ω lb/in.

Adapted from AWS Bridge Specifications. K = min/max
P_c = Allowable unit compressive stress for member.
P_t = Allowable unit tensile stress for member.

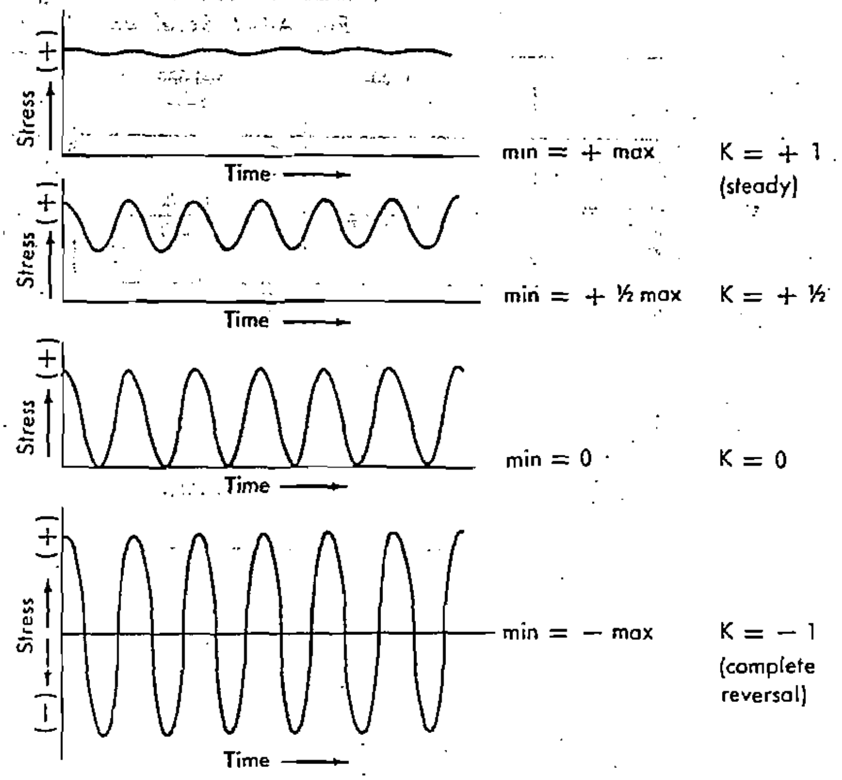


FIGURE 6

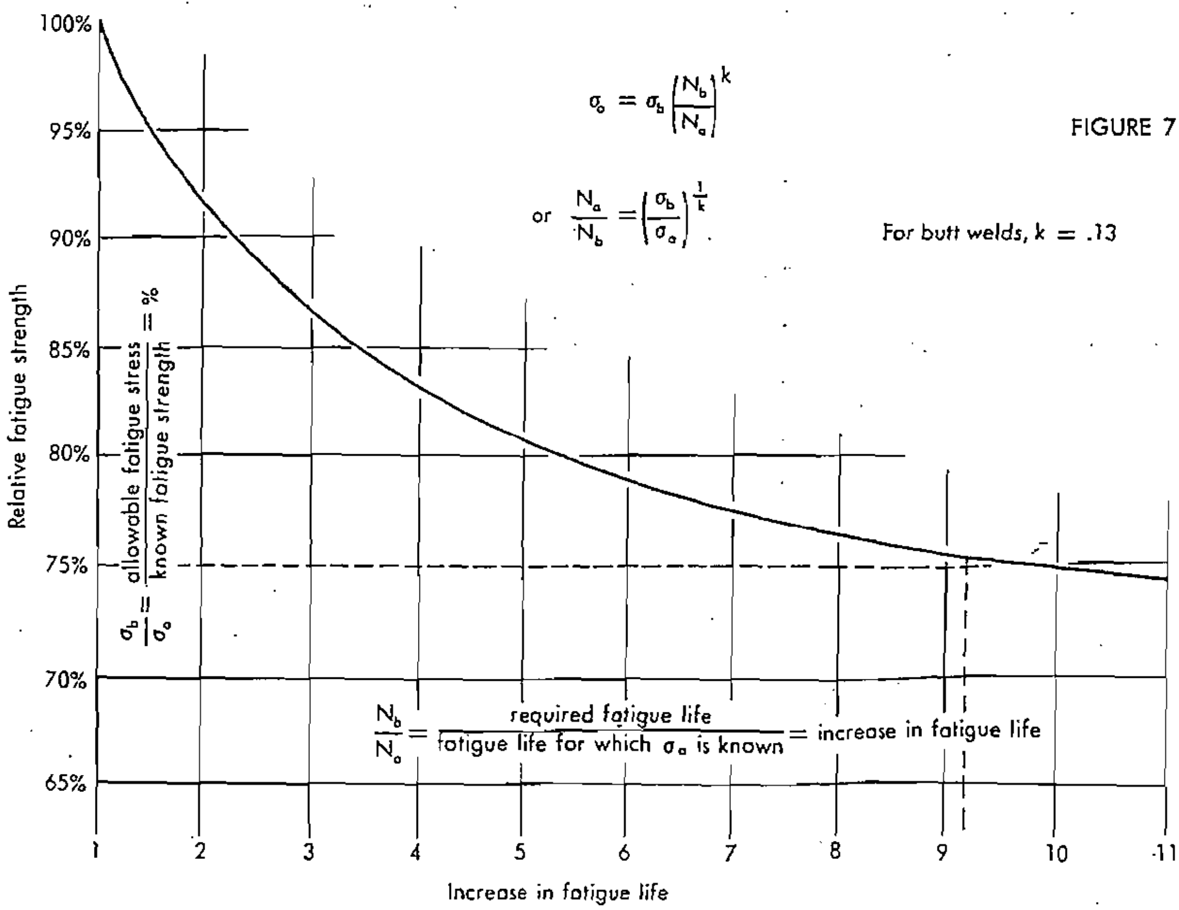


FIGURE 7

TABLE 2—Allowable Fatigue Stress
For A441 Steel and Its Welds

	2,000,000 cycles	600,000 cycles	100,000 cycles	But Not to Exceed
Base Metal In Tension Connected By Fillet Welds	(1) $\sigma = \frac{7500}{1 - 2/3\,R}$ psi	(3) $\sigma = \frac{10,500}{1 - 2/3\,R}$ psi	(5) $\sigma = \frac{15,000}{1 - 2/3\,R}$ psi	$\frac{2\,P_c}{3\,R}$ psi P_t psi
Base Metal Compression Connected By Fillet Welds	(2) $\sigma = \frac{7500}{1 - 2/3\,R}$ psi	(4) $\sigma = \frac{10,500}{1 - 2/3\,R}$ psi	(6) $\sigma = \frac{15,000}{1 - 2/3\,R}$ psi	$\frac{P_c}{1 - \frac{1}{2}R}$ psi P_c psi
Butt Weld In Tension	(7) $\sigma = \frac{16,000}{1 - .8\,R}$ psi	(11) $\sigma = \frac{19,000}{1 - .7\,R}$ psi	(15) $\sigma = \frac{24,000}{1 - \frac{1}{2}R}$ psi	P_t psi
Butt Weld Compression	(8) $\sigma = \frac{24,000}{1 - 1.7\,R}$ psi	(12) $\sigma = \frac{24,000}{1 - R}$ psi	(16) $\sigma = \frac{24,000}{1 - \frac{1}{2}R}$ psi	P_c psi
Butt Weld In Shear	(9) $\sigma = \frac{9000}{1 - \frac{1}{2}R}$ psi	(13) $\sigma = \frac{10,000}{1 - \frac{1}{2}R}$ psi	(17) $\sigma = \frac{13,000}{1 - \frac{1}{2}R}$ psi	13,000 psi
Fillet Welds ω = leg size	(10) $f = \frac{5100\,\omega}{1 - \frac{1}{2}R}$ lb/in.	(14) $f = \frac{7100\,\omega}{1 - \frac{1}{2}R}$ lb/in.	(18) $f = \frac{8800\,\omega}{1 - \frac{1}{2}R}$ lb/in.	* $f = 10,400\,\omega$ lb/in.

Adopted from AWS Bridge Specifications.
* if SAW-1, use 8800
R = min/max load
P_t = Allowable unit tensile stress for member.
P_c = Allowable unit compressive stress for member.

TABLE 3—Allowable Fatigue Stress
For Quenched and Tempered Steels of High Yield Strength and Their Welds

	2,000,000 cycles	600,000 cycles	100,000 cycles	But Not to Exceed
Base Metal In Tension—Not Adjacent to Welds	(1) $\sigma = \frac{29,000}{1 - .65\,K}$ psi	(2) $\sigma = \frac{33,000}{1 - .60\,K}$ psi	(3) $\sigma = \frac{39,500}{1 - .50\,K}$ psi	$\sigma = 54,000$ psi
Butt Weld In Tension	(4) $\sigma = \frac{16,500}{1 - .80\,K}$ psi	(5) $\sigma = \frac{21,000}{1 - .75\,K}$ psi	(6) $\sigma = \frac{31,000}{1 - .60\,K}$ psi	$\sigma = 54,000$ psi
Fillet Weld ω = leg size	(7) $f = \frac{6,360\,\omega}{1 - .80\,K}$ lbs/in.	(8) $f = \frac{9,900\,\omega}{1 - .75\,K}$ lbs/in.	(9) $f = \frac{14,500\,\omega}{1 - .60\,K}$ lbs/in.	$f = 26,160\,\omega$ lbs/in.

Above values adopted from "The Fabrication and Design of Structures of T-1 Steel" by Gilligan and England, United States Steel Corporation.

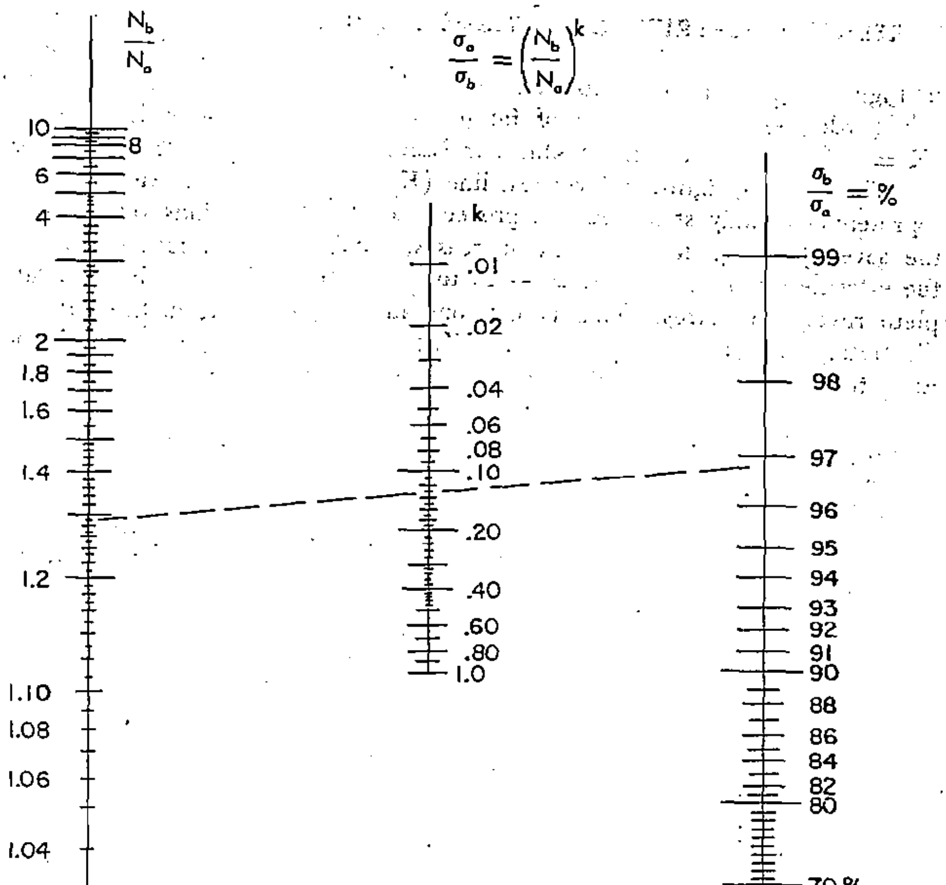


FIGURE 8
FATIGUE NOMOGRAPH

Given: Test data indicates a butt-weld fatigue life of $N_a = 1{,}550{,}000$ cycles when the member is stressed to $\sigma_a = 30{,}000$ psi

Find: The weld's fatigue strength (σ_b) at 2,000,000 cycles (N_b)

$$\frac{N_b}{N_a} = \frac{2{,}000{,}000}{1{,}550{,}000} = 1.29$$

and since the butt weld's k factor is .13, the nomograph indicates

$$\frac{\sigma_b}{\sigma_a} = 96.8\%$$

or $\sigma_b = 30{,}000 \times 96.8\% = 29{,}000$ psi

Since:

$$\frac{\sigma_a}{\sigma_b} = \left(\frac{N_b}{N_a}\right)^k \quad \text{(For butt welds, } k = 0.13\text{) or:}$$

$$\frac{\sigma_b}{\sigma_a} = \left(\frac{N_a}{N_b}\right)^k \quad \text{and:}$$

$$\frac{\sigma_b}{30{,}000} = \left(\frac{1{,}550{,}000}{2{,}000{,}000}\right)^{.13} = (0.775)^{.13}$$

Using logarithms* for the right hand side:

$$\begin{aligned} &= 0.13(\log 0.775) = 0.13(9.88930 - 10) \\ &= 1.285609 - 1.3 \qquad \text{(add 8.7 to left side and} \\ &+ 8.7 \qquad\quad - 8.7 \qquad \text{subtract 8.7 from right side)} \\ &\overline{\quad 9.985609 - 10.0 \quad} \end{aligned}$$

The anti-log of this is 0.96740; hence:

$$\frac{\sigma_b}{30{,}000} = 0.96740$$

$$\sigma_b = 30{,}000 \times 0.96740$$

$$= \underline{29{,}020} \text{ psi at } N_b = 2{,}000{,}000 \text{ cycles)}$$

The nomograph, Figure 8, further facilitates such conversion and permits quickly finding the relative allowable stress for any required fatigue life provided the fatigue strength at some one fatigue life is known and that the constant k value has been established. Conversely, the relative fatigue life can be readily found for any given stress and any constant (k).

* A log-log slide rule could be used to find the value of 0.775 raised to the 0.13 power.

5. RELATIVE SEVERITY OF FATIGUE PROBLEM

In Figure 9, the allowable fatigue stress is the vertical axis (*ordinate*) and the type of fatigue stress cycle ($K = min/max$) is the horizontal axis (abscissa).

The extreme right-hand vertical line ($K = +1$) represents a steady stress. As we proceed to the left, the severity of the fatigue cycle increases; finally at the extreme left-hand axis ($K = -1$) there is a complete reversal of stress. This is just one method of illustrating fatigue stress conditions. The important thing to be noticed here is that actual fatigue strength or allowable fatigue values are not reduced below the steady stress condition until the type of cycle ($K = min/max$) has progressed well into the fatigue type of loading.

In the case of 2 million cycles, the minimum stress must drop down to ½ of the maximum stress before there is any reduction of allowable strength. In the case of 100,000 cycles, the minimum stress can drop to zero before any reduction of allowable strength takes place. Even at these levels, the member and welds would be designed as though they were subjected to a steady load. The stress cycle must extend into a wider range of fluctuation before it becomes necessary to use lower fatigue allowables.

In other words, a fatigue problem occurs only if —

1. Stress is very high,
2. Anticipated service extends for a great number of cycles,
3. Stress fluctuates over a wide range.

And it generally requires all three of these situations occurring simultaneously to produce a critical fatigue condition worthy of consideration.

The allowable fatigue strength values obtained from the formulas in Table 1 take all three of these into consideration, and it is believed they will result in a conservative design.

6. COMBINED FATIGUE STRESSES

Several formulas are available for this consideration but very little actual testing has been done on this. In many cases there is not very good agreement between the actual test and the formulas.

1. Principal-stress theory —

$$\sigma_e = \frac{\sigma_x + \sigma_y}{2} + \tfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2}$$

2. Maximum shear-stress theory—

$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2}$$

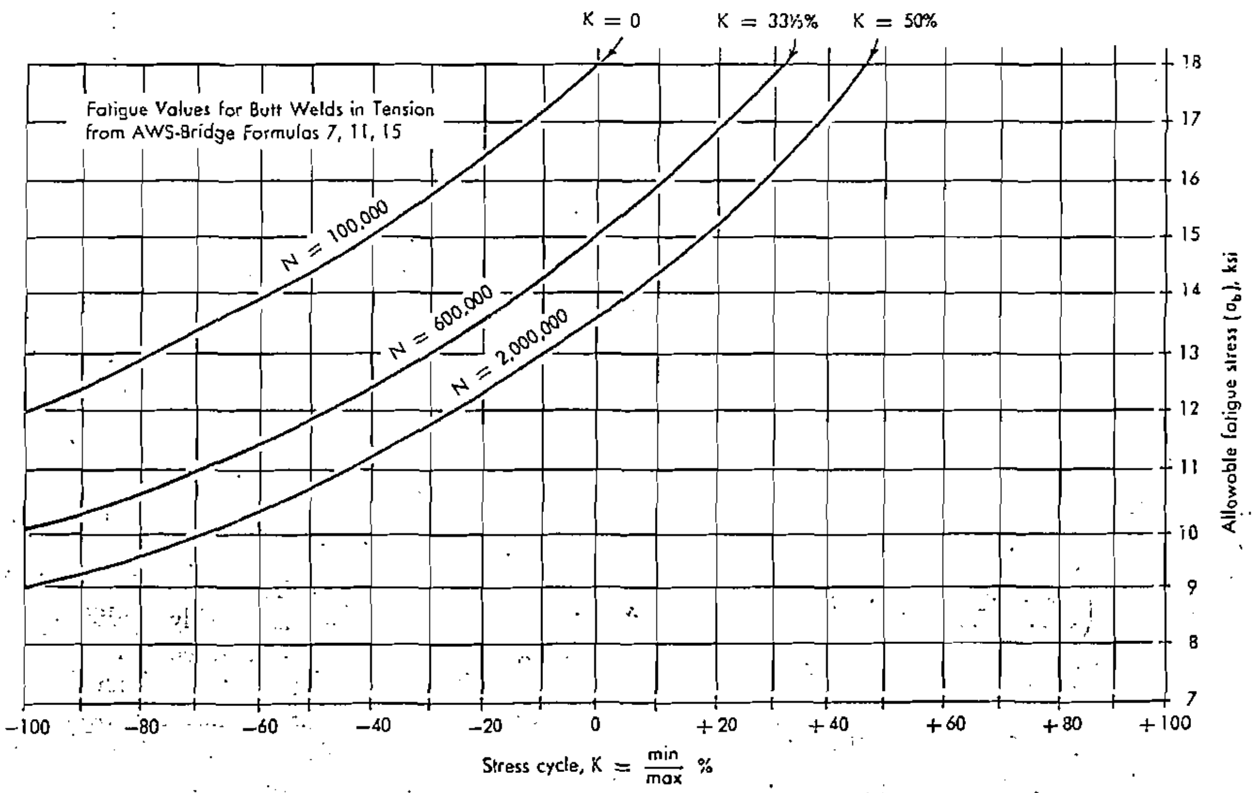


FIG. 9 Severity of fatigue depends on stress value and range of fluctuation, as well as service life.

TABLE 4—Fatigue Strength of Butt Welds
Summary of Results, Using 7/8-In. Carbon-Steel Plates

Description of Specimen	FATIGUE STRENGTH IN 1000's OF PSI					
	TENSION TO AN EQUAL COMPRESSION		0 TO TENSION		TENSION TO TENSION ½ AS GREAT	
	N = 100,000	N = 2,000,000	N = 100,000	N = 2,000,000	N = 100,000	N = 2,000,000
As Welded	22.3	14.4	33.1	22.5	53.3	36.9
Reinforcement On Stress Relieved	21.3	15.1	31.9	23.7		37.6
Reinforcement Machined Off Not Stress Relieved	28.9		48.8	28.4		43.7
Reinforcement Machined Off Stress Relieved	24.5	16.6	49.4	27.8		42.6
Reinforcement Ground Off Not Stress Relieved	26.8		44.5	26.3		
Plain Plate Mill Scale On	27.7	17.1	49.8	31.6		50.0
Plain Plate Mill Scale Machined Off and Surface Polished			59.6			
Butt Weld. Reinforcement and Mill Scale Machined Off and Surface Polished			53.9			

3. Shear-stress-invariant theory—

$$\sigma_e = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2}$$

4. Combined bending and torsion. Findley corrected shear-stress theory for anistropy—

$$\sigma_e = \sqrt{\sigma_x^2 + \left(\frac{\sigma_b}{\tau}\right)^2 \tau_{xy}^2}$$

where σ_b/τ is the ratio of fatigue strength in pure bending to that in pure tension.

5. Combined tensile stresses. Gough suggests—

$$\frac{\sigma_x^2}{\sigma_{ox}^2} + \frac{\sigma_y^2}{\sigma_{oy}^2} = 1$$

where:

σ_{ox} = fatigure strength in (x) direction

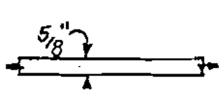
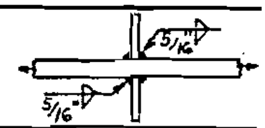
σ_{oy} = fatigue strength in (y) direction

σ_x and σ_y = applied stresses

7. INFLUENCE OF JOINT DESIGN

Any abrupt change of section along the path of stress flow will reduce the fatigue strength. It is not welding that effects a reducing of the fatigue strength but the resultant shape or geometry of the section. It is for this reason that fillet welds have lower fatigue strength. simply because they are used in lap joints and all lap joints including riveted joints have lower fatigue strength.

TABLE 5—Effect of Transverse Attachments On Fatigue Strength

$K = \frac{min}{max.} = -1$			
100,000 cycles	25,800 psi	25,400 psi	22,900 psi
2,000,000 cycles	22,800 psi	18,900 psi	13,100 psi

By means of Table 4, we can see that removing the reinforcement of a butt weld increases its fatigue strength to that of unwelded plate, also that stress relieving the weld has no appreciable effect on its fatigue strength.

Table 5 illustrates the effect of transverse fillet welds upon the fatigue strength of plate; this is ⅝" plate.

The attachment causes an abrupt change in section, and this reduces the fatigue strength of the plate. It is believed these results could be duplicated by machining these joints out of solid plate, without any welding.

8. GUIDES TO DESIGNING FOR FATIGUE LOADING

1. Usually a member is stressed to the full maximum value for only a portion of its fatigue life or cycles. For most of its fatigue life, the member is stressed to a much lower value, and not to its full rated capacity; hence, most fatigue loading is not as severe as it may first appear.

Consider actual stress rather than average stress.

Reduce if possible the range of stress without increasing the maximum or average stress.

2. Fatigue loading requires careful fabrication, smooth transition of sections.

Avoid attachments and openings at locations of high stress.

Avoid sharp corners.

Use simple butt weld instead of lap or T fillet weld.

Grinding the reinforcement off of butt welds will increase the fatigue strength. This weld will have about the same fatigue strength as unwelded plate. Grinding, however, should not be specified unless essential, since it does add to the final unit cost.

Avoid excessive reinforcement, undercut, overlap, lack of penetration, roughness of weld.

Avoid placing weld in an area which flexes.

Stress relieving the weld has no appreciable effect upon fatigue strength.

Difficulties are sometimes caused by the welds being too small, or the members too thin.

3. Under critical loading, place material so that the direction of rolling (of plate in steel mill) is in line with force, because the fatigue strength may be higher in this direction than if placed at right angles with the direction of rolling. See Figure 10.

4. Where possible, form member into shape that it tends to assume under load, and hence prevent the resulting flexial movement.

5. Avoid operating in the critical or resonant frequency of individual member or whole structure to avoid excessive amplitude.

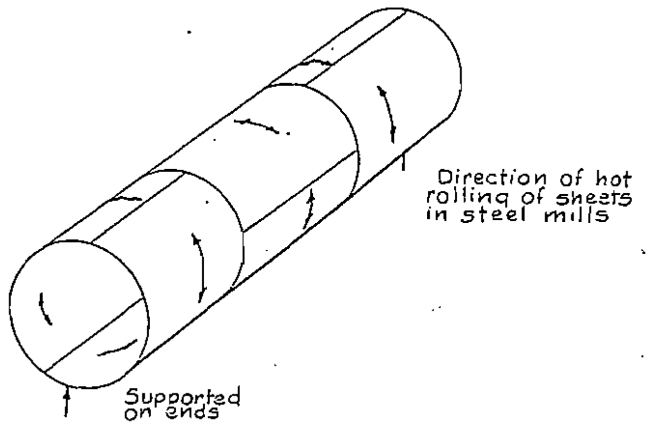
6. Perhaps consider prestressing a beam in axial compression. This will reduce the tensile bending stress and lessen chance for fatigue failure even though the compressive bending stress is increased to some extent.

7. Avoid eccentric application of loads which may cause additional flexing with each application of load.

8. Stiffeners decrease flexibility of panel and result in better fatigue strength, unless they cause a more abrupt change of section.

9. A rigid frame type of structure or statically indeterminate type of structure may be better than a simple structure since the load is shared by other members; hence, the structure is less likely to collapse immediately if a fatigue failure starts in one member.

10. Avoid biaxial and triaxial stresses, avoid restrained internal sections.



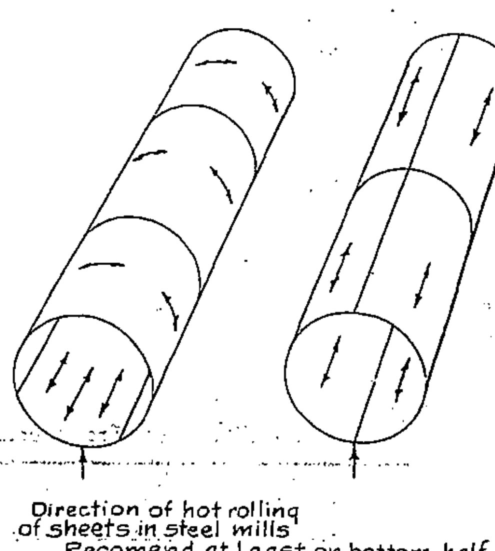


FIG. 10 Grain direction of sheet or plate should be in line with force, for greater fatigue strength.