Design of Compression Members

1. INTRODUCTION

The preceding Section 3.1 covers the general Analysis of Compression, along with an evaluation of the methods for determining stress allowables.

This present section deals more specifically with the actual design of columns and other compression members. For purposes of illustration, the term "column" is used quite liberally. This is due partly to much of the material having been originally developed expressly for columns. However, the information is generally applicable to all compression members.

2. RESTRAINT AND EFFECTIVE LENGTH OF MEMBER

Section 3.1 explained how a compression member's slenderness ratio (L/r) relates to its buckling strength. The degree of end restraint on a member results in its having an effective length which may vary considerably from its actual unbraced length. This ratio (K) of effective length to actual unbraced length is used as a multiplier in determining the effective length (L_e) of a compression member.

$$\boxed{L_e = K\,L} \qquad \cdots\cdots (1)$$

where:

L = actual length of the column

L_e = effective length of the column to be used in column formulas

K = effective length factor

Table 1 lists theoretical values of K and the Column Research Council's corresponding recommended values of K for the effective length (L_e) of columns under ideal conditions.

Where End Conditions Can't Be Classified

In actual practice it will be more difficult to classify the end conditions. If classification is doubtful, the Column Research Council recommends the following method based on the relative stiffness of connecting beams and columns.

The stiffness factor of any member is given as I/L, its moment of inertia divided by its length.

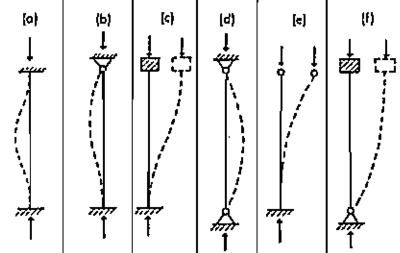
These values are determined for the column or columns in question (I_c/L_c), as well as for any beam or other restraining member lying in the plane in which buckling of the column is being considered (I_g/L_g).

The moments of inertia (I_c and I_g) are taken about an axis perpendicular to the plane of buckling being considered.

The values of G for each end (A and B) of the column are determined:

$$\boxed{G = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}}} \qquad \cdots\cdots (2)$$

TABLE 1—Effective Length (L_e) of Compression Members

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of member is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0*
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10**	2.0

End condition		
	rotation fixed	translation fixed
	rotation free	translation fixed
	rotation fixed	translation free
	rotation free	translation free

*K may be greater than 2.0
**Top end assumed truly rotation free

From "Guide to Design Criteria for Metal Compression Members" 1960, p. 28, Column Research Council

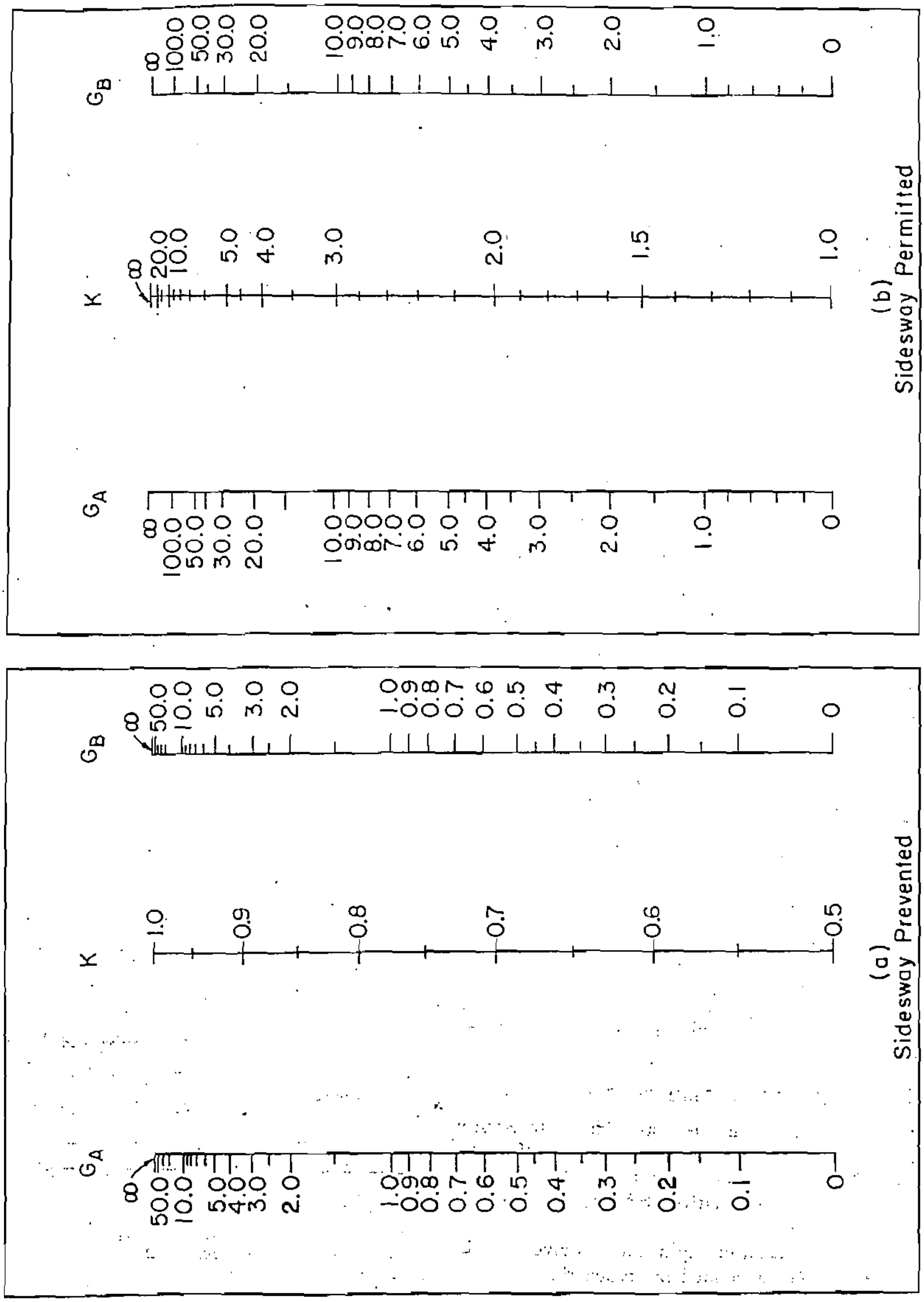


FIGURE 1—Effective Length Factor In Column Design

where:

$\sum \frac{I_c}{L_c}$ = the total for the columns meeting at the joint considered.

$\sum \frac{I_g}{L_g}$ = the total for the beams or restraining members meeting at the joint considered.

For a column end that is supported, but not fixed, the moment of inertia of the support is zero, and the resulting value of G for this end of the column would be ∞. However in practice, unless the footing were designed as a frictionless pin, this value of G would be taken as 10.

If the column end is fixed, the moment of inertia of the support is ∞, and the resulting value of G for this end of the column would be zero. However in practice, there is some movement and G may be taken as 1.0.

If the beam or restraining member is either pinned ($G = \infty$) or fixed against rotation ($G = 0$) at its far end, further refinements may be made by multiplying the stiffness (I/L) of the beam by the following factors:

sidesway prevented

far end of beam pinned = 1.5

far end of beam fixed = 2.0

sidesway permitted

far end of beam pinned = 0.5

For any given column, knowing the values (G_A and G_B) for each end, the nomograph, Figure 1, may be used to determine the value of K so that the effective length (L_e) of the column may be found:

$$L_e = K\,L$$

This nomograph is taken from the Column Research Council's "Guide to Design Criteria for Metal Compression Members", 1960, p. 31. The nomograph was developed by Jackson & Moreland Division of United Engineers and Constructors, Inc.

3. STRENGTH OF COMPRESSION MEMBERS UNDER COMBINED LOADING

A very convenient method of treating combined loadings is the interaction method. (Also see Sect. 2.11, Analysis of Combined Stresses.) Here each type of

Problem 1

Find the effective length factor (K) for column A-B under the following conditions:

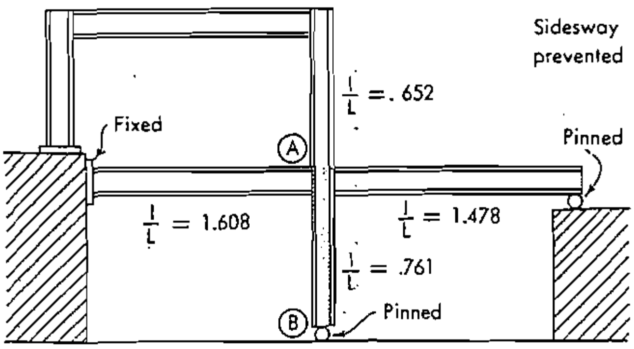


FIGURE 2

Here:

$$G_A = \frac{.652 + .761}{2(1.608) + 1.5(1.478)}$$

$$= .260$$

$$G_B = \infty;\ \text{use } 10.$$

From the nomograph, read K = .76

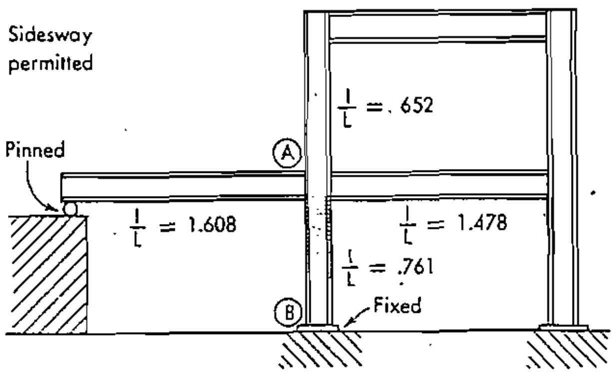


FIGURE 3

Here:

$$G_A = \frac{.652 + .761}{.5(1.608) + 1.478}$$

$$= .620$$

$$G_B = \text{zero; use } 1.0$$

From the nomograph, read K = 1.26

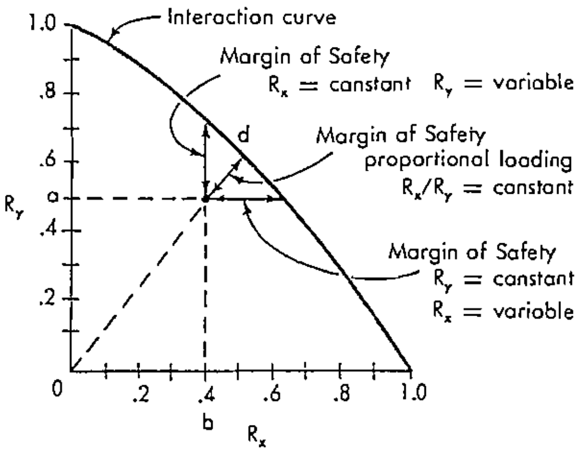


FIGURE 4

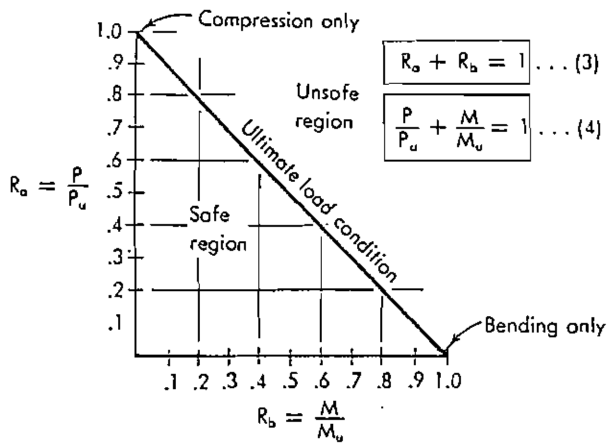


FIGURE 5

load is expressed as a ratio of the actual load to the ultimate load which would cause failure if acting alone.

axial load

$$R_a = \frac{P}{P_u}$$

bending load

$$R_b = \frac{M}{M_u}$$

torsional load

$$R_t = \frac{T}{T_u}$$

In the general example shown in Figure 4, the effect of two types of loads (X and Y) upon each other is illustrated.

The value of $R_y = 1$ at the upper end of the vertical axis is the ultimate value for this type of load on the member when acting alone. The value of $R_x = 1$ at the extreme right end of the horizontal axis is the ultimate value for this type of load on the member when acting alone. These ultimate values are determined by experiment; or when this data is not available, suitable calculations may be made to estimate these values.

The interaction curve is usually determined by actual testing of members under various combined-load conditions. From this, a simple formula is derived to fit the curve and express this relationship.

If points *a* and *b* are the ratios produced by the actual loads, point *c* represents the combination of these conditions. The margin of safety is indicated by how close point *c* lies to the interaction curve. A suitable factor of safety is then applied to these values.

Figure 5 illustrates this for axial compression and bending.

However, the applied bending moment (M_1) causes the column to bend, and the resulting displacement or eccentricity induces a secondary moment from the applied axial force. See Figure 6.

Assume that the moment (M_1) applied to the column is sinusoidal in nature; Figure 7.

A sinusoidal moment applied to a pinned end member results in a sinusoidal deflection curve, whose maximum deflection is equal to—

$$\Delta_1 = \frac{M_1 \, L_b^2}{\pi^2 \, E \, I}$$

Since the critical Euler load is—

$$\boxed{P_e = \frac{\pi^2 \, E \, I}{L_b^2}} \quad \text{.........................(5)}$$

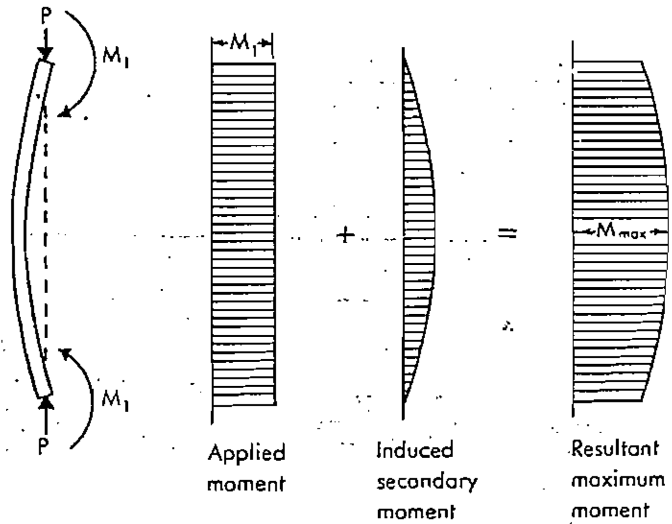


FIGURE 6

this becomes

$$\Delta_1 = \frac{M_1}{P_e}$$

When the axial load (P) is also applied to this deflected column, a secondary moment is induced and this is also sinusoidal in nature, its maximum value being —

$$M_2 = P\ \Delta_1$$

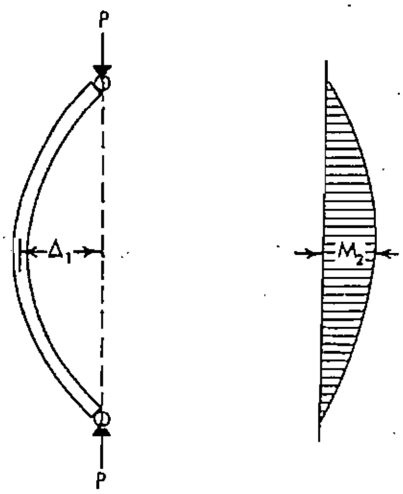


FIGURE 8

This slightly higher moment $(M_2 + M_1)$ will in the same manner produce a slightly greater deflection $(\Delta_2 + \Delta_1)$, etc. Each successive increment in deflection becomes smaller and smaller.

The final values would be —

$$\Delta_{max} = \frac{M_{max}}{P_e}$$

since

$$M_{max} = M_1 + P\ \Delta_{max} \quad \text{then}$$

$$M_{max} = M_1 + P\left(\frac{M_{max}}{P_e}\right) \quad \text{or}$$

$$M_{max} = \frac{M_1}{1 - \dfrac{P}{P_e}}$$

Accommodating Increased Moment Due to Deflection

This increase in the moment of the bending load caused by deflection is easily taken care of in the basic interaction formula by an amplification factor (k):

$$k = \frac{M_{max}}{M_1}$$

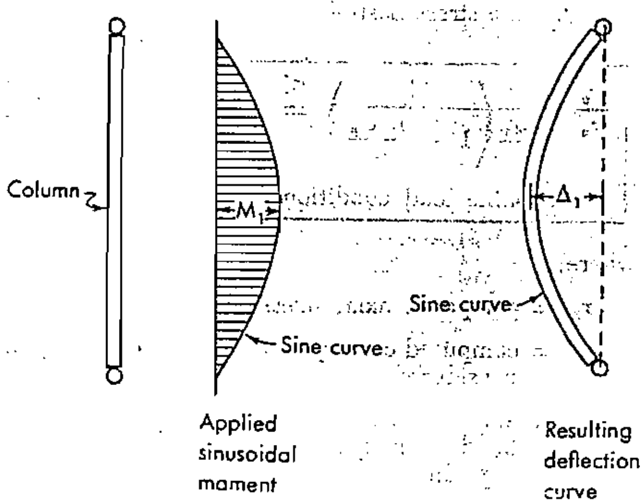


FIGURE 7

$$\therefore\ \boxed{k = \frac{1}{1 - \dfrac{P}{P_e}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The interaction Formula #4 then becomes —

$$\boxed{\begin{array}{c}\dfrac{P}{P_u} + \dfrac{M_1}{M_u}\left(\dfrac{1}{1 - \dfrac{P}{P_e}}\right) = 1 \\ \text{(ultimate load condition)}\end{array}} \quad \ldots\ldots\ldots\ldots (7)$$

Each ultimate load condition factor in the above formula is equal to the corresponding factor for working conditions multiplied by the factor of safety (n); or

$$\frac{n\ P_w}{n\ P_A} + \frac{n\ M_w}{n\ M_A}\left(\frac{1}{1 - \dfrac{n\ P_w}{P_e}}\right) \leqq 1 \quad \text{and}$$

$$\frac{P_w}{P_A} + \frac{M_w}{M_A}\left(\frac{1}{1 - \dfrac{n\ P_w}{P_e}}\right) \leqq 1$$

where: subscript $_w$ is for working loads
subscript $_A$ is for allowable loads

Notice:

$$P_e = \frac{\pi^2\ E\ I}{{L_b}^2} = \frac{\pi^2\ E\ A}{\left(\dfrac{L_b}{r_b}\right)^2}$$

so: $$\sigma_e = \frac{\pi^2\ E}{\left(\dfrac{L_b}{r_b}\right)^2}$$

Or, on a stress basis —

$$\boxed{\begin{array}{c}\dfrac{\sigma_a}{\underline{\sigma_a}} + \dfrac{\sigma_b}{\underline{\sigma_b}}\left(\dfrac{1}{1 - \dfrac{n\ \sigma_a}{\sigma_e}}\right) \leqq 1 \\ \text{(allowable load condition)}\end{array}} \quad \text{.............(8)}$$

where:

σ_a = computed axial stress

σ_b = computed compressive bending stress at point considered

$\underline{\sigma_a}$ = allowable axial stress permitted if there is no bending moment; use largest (L/r) ratio, regardless of plane of bending

$\underline{\sigma_b}$ = allowable compressive bending stress permitted if there is no axial force. (AISC Sec. 1.5.1.4)

The AISC Specification Sec. 1.6.1 uses the same amplification factor. They use the term (F'_e) which is the Euler stress (σ_e) divided by the factor of safety (n). The term (σ'_e) is used here in place of AISC's (F'_e).

$$\sigma'_e = \frac{\sigma}{n} = \frac{\pi^2\ E}{\left(\dfrac{L_b}{r_b}\right)^2 n}$$

$$= \frac{149{,}000{,}000}{\left(\dfrac{L_b}{r_b}\right)^2} = \left(\frac{12{,}210}{\dfrac{L_b}{r_b}}\right)^2$$

AISC uses E = 29,000,000 psi and n = 1.92 in the above.

Here:

r_b = radius of gyration about an axis normal to the plane of bending.

L_b = actual unbraced length of column in the plane of bending

TABLE 2—Euler Stress Divided By Factor of Safety

$$\textit{Values of } \sigma'_e = \frac{149{,}000{,}000}{\left(\dfrac{KL_b}{r_b}\right)^2} = \left(\frac{12{,}210}{\dfrac{KL_b}{r_b}}\right)^2$$

For All Grades of Steel *AISC 1963*

$\frac{KL_b}{r_b}$		1	2	3	4	5	6	7	8	9
20		338,130	308,090	281,880	258,890	238,590	220,580	204,550	190,200	177,310
30	165,680	155,170	145,620	136,930	128,990	121,730	115,060	108,930	103,270	98,040
40	93,200	88,710	84,530	80,650	77,020	73,640	70,470	67,510	64,730	62,110
50	59,650	57,330	55,150	53,090	51,140	49,300	47,560	45,900	44,440	42,840
60	41,430	40,070	38,790	37,570	36,410	35,290	34,240	33,220	32,250	31,320
70	30,440	29,580	28,770	27,990	27,240	26,510	25,820	25,150	24,510	23,890
80	23,300	22,730	22,180	21,650	21,130	20,640	20,160	19,700	19,260	18,830
90	18,410	18,010	17,620	17,240	16,880	16,530	16,180	15,850	15,530	15,210
100	14,910	14,620	14,340	14,060	13,730	13,530	13,280	13,020	12,800	12,570
110	12,340	12,120	11,900	11,690	11,490	11,290	11,100	10,910	10,730	10,550
120	10,370	10,200	10,030	9,870	9,710	9,560	9,410	9,260	9,110	8,970
130	8,840	8,700	8,570	8,440	8,320	8,190	8,070	7,960	7,840	7,730
140	7,620	7,510	7,410	7,300	7,200	7,100	7,010	6,910	6,820	6,730
150	6,640	6,550	6,460	6,380	6,300	6,220	6,140	6,060	5,980	5,910
160	5,830	5,760	5,690	5,620	5,550	5,490	5,420	5,360	5,290	5,230
170	5,170	5,110	5,050	4,990	4,930	4,880	4,820	4,770	4,710	4,660
180	4,610	4,560	4,510	4,460	4,410	4,360	4,320	4,270	4,230	4,180
190	4,140	4,090	4,050	4,010	3,970	3,930	3,890	3,850	3,810	3,770
200	3,730									

L_b = actual unbraced length of column in the plane of bending
r_b = radius of gyration about the axis of bending

According to AISC Sec. 1.5.6, this value (σ'_e) may be increased ⅓ for wind loads.

Table 2 lists the values of σ'_e (Euler stress divided by factor of safety) for $\frac{KL_b}{r_b}$ ratios from 20 to 200. These values apply for all grades of steel, but are based on the conservative factor of safety = 1.92.

The derivation of the amplification factor has been based on a member with pinned ends and a sinusoidal moment applied to it. In actual practice these conditions will vary; however this factor will be reasonably good for most conditions. AISC Sec. 1.6.1 applies a second factor (C_m) to adjust for more favorable conditions of applied end moments or transverse loads.

applied end moments

$$\boxed{C_m = 0.6 + 0.4\,\frac{M_1}{M_2} \geqq 0.4} \quad \cdots\cdots (9)$$

applied transverse loads

$$\boxed{C_m = 1 + \psi\,\frac{\sigma_a}{\sigma'_e}} \quad \cdots\cdots (10)$$

where:

M_1 and M_2 are end moments applied to the column.

$M_1 \leqq M_2$, and the ratio (M_1/M_2) is positive when the column is bent in a single curve and negative when bent in reverse curve.

$$\psi = \frac{\pi^2\,\Delta\,E\,I}{M\,L^2} - 1$$

(see Table 3 for values ψ and C_m for several load conditions)

Here:

Δ = maximum deflection due to transverse load

L = actual length of member also used in deflection (Δ) calculation

M = maximum moment between supports due to transverse load

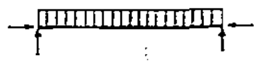
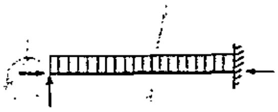
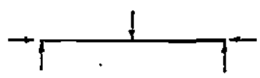
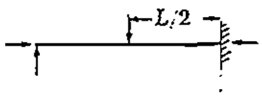
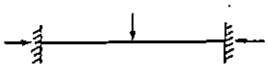
AISC Formulas For Checking

When

$$\boxed{\frac{\sigma_a}{\underline{\sigma_a}} \leqq .15}$$

the influence of the amplification factor is generally small and may be neglected. Hence the following formula will control:

TABLE 3—Value of ψ for Several Load Conditions

Case	ψ	C_m
	0	1.0
	—0.3	$1 - .3\frac{\sigma_a}{\sigma'_e}$
	—0.4	$1 - .4\frac{\sigma_a}{\sigma'_e}$
	—0.2	$1 - .2\frac{\sigma_a}{\sigma'_e}$
	—0.4	$1 - .4\frac{\sigma_a}{\sigma'_e}$
	—0.6	$1 - .6\frac{\sigma_a}{\sigma'_e}$

AISC 1963 Commentary

$$\boxed{\begin{array}{c}\dfrac{\sigma_a}{\underline{\sigma_a}} + \dfrac{\sigma_b}{\underline{\sigma_b}} \leqq 1.0 \\ \text{(AISC Formula 6)}\end{array}}$$

When

$$\boxed{\frac{\sigma_a}{\underline{\sigma_a}} > .15}$$

the amplification factor must be used.

Formula #8 now becomes—

$$\boxed{\begin{array}{c}\dfrac{\sigma_a}{\underline{\sigma_a}} + \dfrac{C_m\ \sigma_b}{\left(1 - \dfrac{\sigma_a}{\sigma'_e}\right)\underline{\sigma_b}} \leqq 1.0 \\ \text{(AISC Formula 7a)}\end{array}} \quad \cdots\cdots (11)$$

This formula provides a check for column stability.

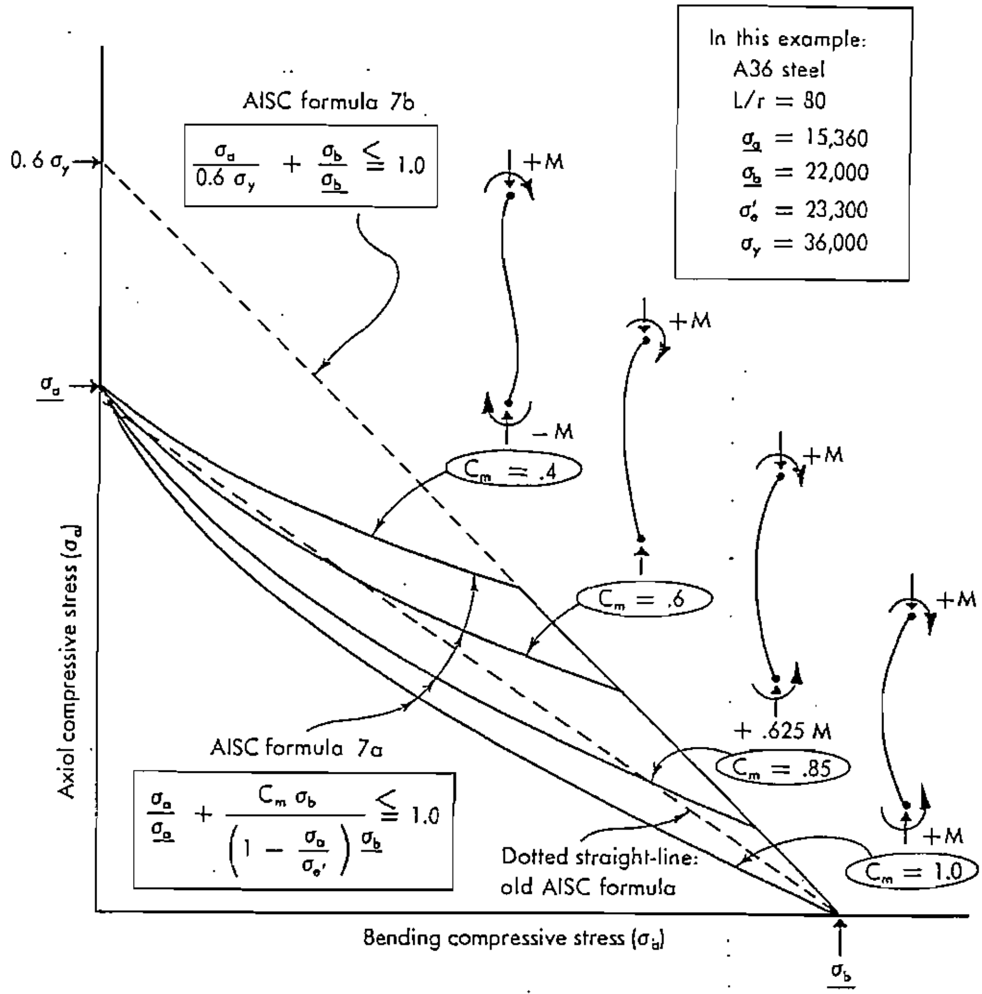


FIGURE 9

It is an attempt to estimate the total bending stress in the central portion of the column and to hold the axial compressive stress down to a safe level.

As L/r increases, this formula will reduce the axial load carrying capacity of the column. This is because the Euler stress (σ_e) decreases as L/r increases.

As C_m increases, caused by a less favorable condition of applied end moments or transverse forces, Formula #11 will reduce the axial load carrying capacity of the column.

The end of the member also must satisfy the straight-line interaction formula:

$$\frac{\sigma_a}{0.6\,\sigma_y} + \frac{\sigma_b}{\underline{\sigma_b}} \leqq 1.0 \qquad \text{(AISC Formula 7b)} \qquad (12)$$

In this formula, the allowable for compression ($\underline{\sigma_a}$) is for a column having a slenderness ratio of L/r = 0, hence $\underline{\sigma_a} = .60\,\sigma_y$.

This formula provides a check for the limiting stress at the ends of the column, and as such applies only at braced points.

Figure 9 is an example of the relationship of AISC Formulas 7a and 7b in the design of a specific member, under various loading conditions.

For bending moments applied about both axes of the column, these formulas become:

$$\frac{\sigma_a}{\underline{\sigma_a}} + \frac{\sigma_{bx}}{\underline{\sigma_{bx}}} + \frac{\sigma_{by}}{\underline{\sigma_{by}}} \leqq 1.0 \qquad \text{(AISC Formula 6)} \qquad (13)$$

$$\frac{\sigma_a}{\underline{\sigma_a}} + \frac{C_{mx}\,\sigma_{bx}}{\left(1 - \frac{\sigma_a}{\sigma'_{ex}}\right)\underline{\sigma_{bx}}} + \frac{C_{my}\,\sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma'_{ey}}\right)\underline{\sigma_{by}}} \leqq 1.0 \qquad \text{(AISC Formula 7a)} \qquad (14)$$

$$\frac{\sigma_a}{.60\,\sigma_y} + \frac{\sigma_{bx}}{\underline{\sigma_{bx}}} + \frac{\sigma_{by}}{\underline{\sigma_{by}}} \leqq 1.0 \qquad \text{(AISC Formula 7b)} \qquad (15)$$

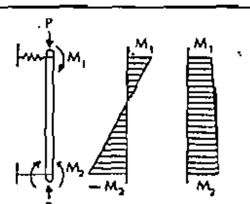
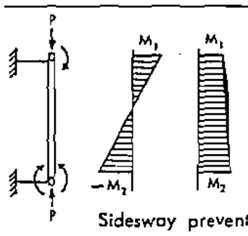
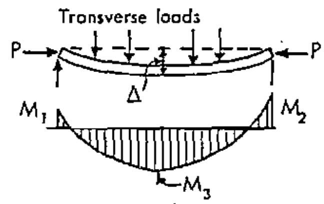
4. DESIGN OUTLINES

The design procedure is simplified by following the appropriate outline in Tables 4, 5, or 6. Table 4 applies to compression members under combined loading (interaction problems). Table 5 applies to open-sectioned members under compression in bending. Table 6 applies to box members under compression in bending. Each of these tables categorize the member-load conditions which must be satisfied, and then presents the required formulas with which to determine the allowable compressive stress.

TABLE 4—Design Outline for Compression Members Under Combined Loading *(Interaction Problems)*

If $\boxed{\frac{\sigma_a}{\underline{\sigma}_a} \leqq .15}$ check $\boxed{\frac{\sigma_a}{\underline{\sigma}_a} + \frac{\sigma_b}{\underline{\sigma}_b} \leqq 1}$ (AISC Formula 6) using $\sigma_b = \frac{M}{S}$

If $\boxed{\frac{\sigma_a}{\underline{\sigma}_a} > .15}$

Category (A)	Category (B)	Category (C)
Columns in frames with computed moments maximum at the ends with no transverse loading, and sidesway is permitted. Here the lateral stability of the frame depends upon the bending stiffness of its members.	Columns with computed moments maximum at the ends with no transverse loading, and sidesway is prevented	Compression members with additional transverse loads; for example a compressive chord of a truss with transverse loading between supports (panel points).
 Sidesway permitted	 Sidesway prevented	 No translation of joints
$C_m = 0.85$	$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geqq 0.4$	$C_m \doteq 1 + \psi \frac{\sigma_a}{\sigma'_e}$ $\psi = \frac{\pi^2 \Delta E I}{M_3 L^2} - 1$ Δ = max deflection due to transverse loading M_3 = max moment between supports due to trans. loading Use KL in computing $\underline{\sigma}_a$ Use L_b in computing moments (M)
Check #11 and #12 using $\sigma_b = \frac{M_2}{S}$	Check #11 and #12 using $\sigma_b = \frac{M_2}{S}$	Check #11 using $\sigma_b = \frac{M_3}{S}$ Check #12 using $\sigma_b = \frac{M_2}{S}$

$$\frac{\sigma_a}{\underline{\sigma}_a} + \frac{C_m \, \sigma_b}{\left(1 - \frac{\sigma_a}{\sigma'_e}\right) \underline{\sigma}_b} \leqq 1.0 \tag{11}$$

(AISC Formula 7a)

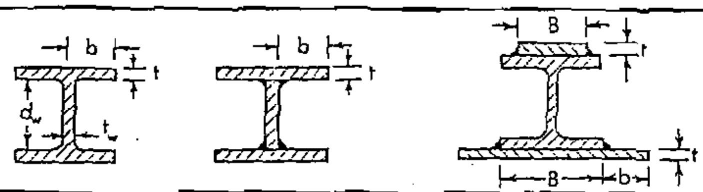
$$\frac{\sigma_a}{.6\ \sigma_y} + \frac{\sigma_b}{\underline{\sigma}_b} \leqq 1.0 \tag{12}$$

(AISC Formula 7b)

$\underline{\sigma}_a$, $\underline{\sigma}_b$ and $.60\ \sigma_y$ may be increased $\frac{1}{3}$ for wind (AISC Sec 1.5.6)

TABLE 5—Design Outline for Compression Members Under Compression In Bending

Members Which Are Symmetrical About An Axis In Plane of Bending And Having Some Lateral Support of Compression Flange



Compression elements which are not "compact" but meet the following AISC Sec 1.9 requirements $b/t \leqq \frac{3000}{\sqrt{\sigma_y}}$ * $B/t \leqq \frac{8000}{\sqrt{\sigma_y}}$ *	If in addition, lateral support of compression flange does not exceed: A7, A373, A36 steels 13 b_f Other stronger steels $\frac{2300\ b_f}{\sqrt{\sigma_y}}$ or $\frac{20{,}000{,}000\ A_f}{d\ \sigma_y}$ (in.)
Having an axis of symmetry in the plane of its web: AISC 1.5.1.4.5 ④ $\underline{\sigma_b} = \left[1.0 - \frac{\left(\frac{L}{r}\right)^2}{2C_c^2\ C_b}\right].6\ \sigma_y$ when $\frac{L}{r} \leqq 40$ don't need AISC Formula 4 ⑤ $\underline{\sigma_b} = \frac{12{,}000{,}000}{\frac{L\ d}{A_f}}$ Use the larger value of ④ or ⑤ but $\leqq .60\ \sigma_y$	and compression elements meet the following AISC Sec 1.5.1.4.1 "compact section" requirements: $b/t \leqq \frac{1600}{\sqrt{\sigma_y}}$ ‡ $B/t \leqq \frac{6000}{\sqrt{\sigma_y}}$ $\frac{d_w}{t_w} \leqq \frac{13{,}300}{\sqrt{\sigma_y}}\left(1 - 1.43\ \frac{\sigma_a}{\underline{\sigma_a}}\right)$ but need not be less than $\frac{8000}{\sqrt{\sigma_y}}$ $\underline{\sigma_b} = .66\ \sigma_y$ † (1.5.1.4.1)

* This ratio may be exceeded if the bending stress, using a width not exceeding this limit, is within the allowable stress.
† For "compact" columns (AISC Sec. 1.5.1.4.1) which are symmetrical about an axis in the plane of bending, with the above lateral support of its compression flange and $\sigma_a = .15\ \sigma_a$ use 90% of the moments applied to the ends of the column if caused by the gravity loads of the connecting beams.
‡ For rolled sections, an upward variation of 3% may be tolerated.

In Tables 5 and 6:

- L = unbraced length of the compression flange
- b_f = width of compression flange
- d = depth of member treated as a beam
- r = radius of gyration of a Tee section comprising the compression flange plus 1/6 of the web area; about an axis in the plane of the web. For shapes symmetrical about their x axis of bending, substitution of r_y of the entire section is conservative
- A_f = area of the compression flange

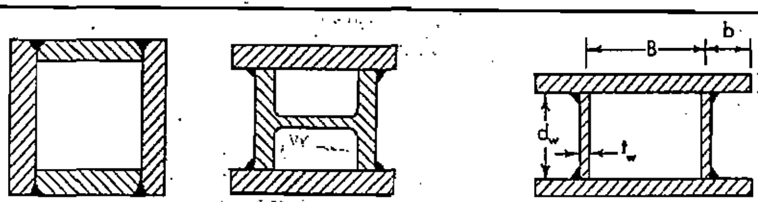
M_1 is the smaller and M_2 the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where M_1/M_2 is the ratio of end moments. This ratio is positive when M_1 and M_2 have the same sign, and negative when they have different signs. When the bending moment within an unbraced length is larger than that at both ends of this length, the ratio shall be taken as unity.

$$C_b = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + .3\left(\frac{M_1}{M_2}\right)^2$$

$$C_c = \sqrt{\frac{2\ \pi^2\ E}{\sigma_y}}$$

(but not more than 2.3 can conservatively be taken as 1.0)

TABLE 6—Design Outline for Box Members Under Compression In Bending
Members Which Are Symmetrical About An Axis In Plane of Bending



No AISC limit on lateral support of compression flange because box section is torsionally rigid	And, if lateral support does not exceed: A7, A373, A36 steels $13\ b_f$ Other stronger steels $\dfrac{2400\ b_f}{\sqrt{\sigma_y}}$ or $\dfrac{20{,}000{,}000\ A_f}{d\ \sigma_y}$ (in.)
Compression elements which are not "compact" but meet the following AISC Sec 1.9 requirements (1.5.1.4.3) $b/t = \dfrac{3000}{\sqrt{\sigma_y}}$ * $B/t = \dfrac{8000}{\sqrt{\sigma_y}}$ *	And comparison elements meet the following AISC Sec 1.5.1.4.1 "compact section" requirements: $b/t \leqq \dfrac{1600}{\sqrt{\sigma_y}}$ ‡ $B/t \leqq \dfrac{6000}{\sqrt{\sigma_y}}$ $\dfrac{d_w}{t_w} \leqq \dfrac{13{,}300}{\sqrt{\sigma_y}}\left(1 - 1.43\ \dfrac{\sigma_a}{\sigma_a}\right)$ but need not be less than $\dfrac{8000}{\sqrt{\sigma_y}}$
$\sigma_b = .60\ \sigma_y$	$\sigma_b = .66\ \sigma_y$ †

Note: All notes from Table 5 apply equally to this Table 6.

TABLE 6A

		yield strength of steel 33,000	36,000	42,000	45,000	46,000	50,000	55,000	60,000	65,000	90,000*	95,000*	100,000*
Allowable bending stress	$\sigma = .60\ \sigma_y$	20,000	22,000	25,000	27,000	27,500	30,000	33,000	36,000	39,000	54,000	57,000	60,000
	$\sigma = .66\ \sigma_y$	22,000	24,000	28,000	29,500	30,500	33,000	36,500	39,500	43,000	59,400	62,700	66,000
Width-to-thickness ratio not to exceed:	$\dfrac{1600}{\sqrt{\sigma_y}}$	8.8	8.4	7.8	7.5	7.5	7.2	6.8	6.5	6.3	5.3	5.2	5.1
	$\dfrac{3000}{\sqrt{\sigma_y}}$	16.5	15.8	14.6	14.1	14.0	13.4	12.8	12.2	11.8	10.0	9.7	9.5
	$\dfrac{6000}{\sqrt{\sigma_y}}$	33.0	31.6	29.2	28.3	28.0	26.8	25.6	24.5	23.5	20.0	19.5	19.0
	$\dfrac{8000}{\sqrt{\sigma_y}}$	44.0	42.1	39.0	37.7	37.3	35.8	34.1	32.6	31.4	26.6	25.9	25.3
	$\dfrac{13{,}300}{\sqrt{\sigma_y}}$	73.2	70.0	64.8	62.6	62.0	59.5	56.7	54.3	52.2	44.4	43.1	42.1
Lateral support of compression flange of "compact" sections not to exceed:	$\dfrac{2400}{\sqrt{\sigma_y}}$	$13.2b_f$	$12.6b_f$	$11.7b_f$	$11.3b_f$	$11.2b_f$	$10.7b_f$	$10.2b_f$	$9.8b_f$	$9.4b_f$	$8.0b_f$	$7.8b_f$	$7.6b_f$
	$\dfrac{20{,}000{,}000\ A_f}{\sigma_y\ d}$	$606\ \dfrac{A_f}{d}$	$555\ \dfrac{A_f}{d}$	$476\ \dfrac{A_f}{d}$	$444\ \dfrac{A_f}{d}$	$435\ \dfrac{A_f}{d}$	$400\ \dfrac{A_f}{d}$	$364\ \dfrac{A_f}{d}$	$333\ \dfrac{A_f}{d}$	$308\ \dfrac{A_f}{d}$	$222\ \dfrac{A_f}{d}$	$210\ \dfrac{A_f}{d}$	$200\ \dfrac{A_f}{d}$
	$C_c = \sqrt{\dfrac{2\ \pi^2\ E}{\sigma_y}}$	131.7	126.1	116.7	112.8	111.6	107.0	102.0	97.7	93.8	79.8	77.6	75.7
1.18.2.3: max. longitudinal spacing between intermittent fillet welds attaching compression flange to girders	$S \leqq \dfrac{4000}{\sqrt{\sigma_y}}\ t \leqq 12''$	22.0t	21.0t	19.5t	18.9t	18.7t	17.9t	17.1t	16.3t	15.7t	13.3t	13.0t	12.6t

*Quenched & Tempered Steels: yield strength at 0.2% offset
Round off to nearest whole number

5. BUILT-UP COMPRESSION MEMBERS

The basic requirements of welds on built-up compression members, as specified by AISC, are summarized by Figures 10, 11, 12, and 13.

Welding at the ends of built-up compression members bearing on base plates or milled surfaces (AISC 1.18.2.2):

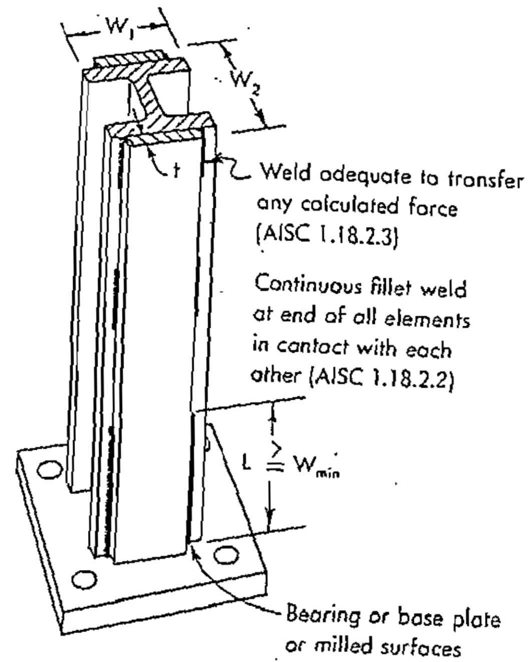


FIGURE 10

Plate in contact with a shape (AISC 1.18.2.3):

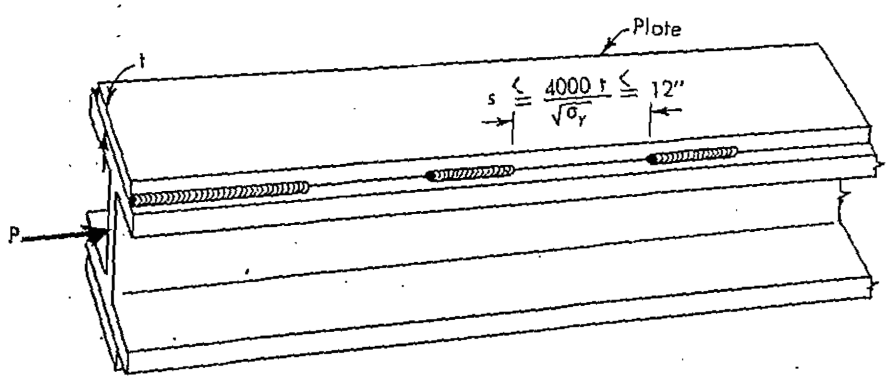


FIGURE 11

Two rolled shapes in contact with each other (AISC 1.18.2.3):

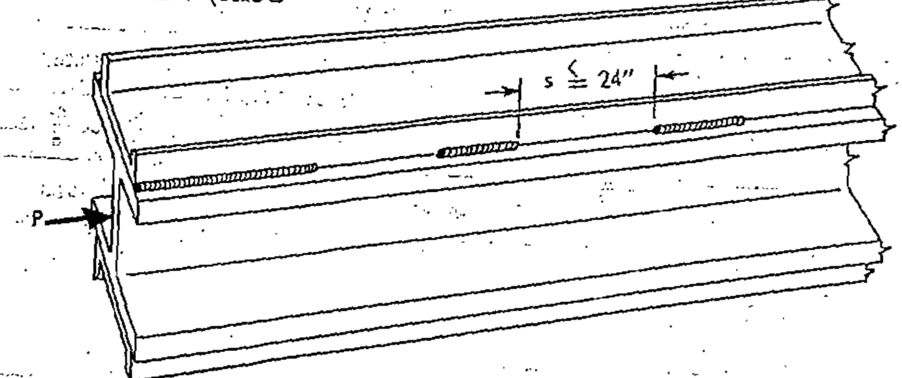


FIGURE 12

Two or more rolled shapes separated by intermittent fillers (AISC 1.18.2.4):

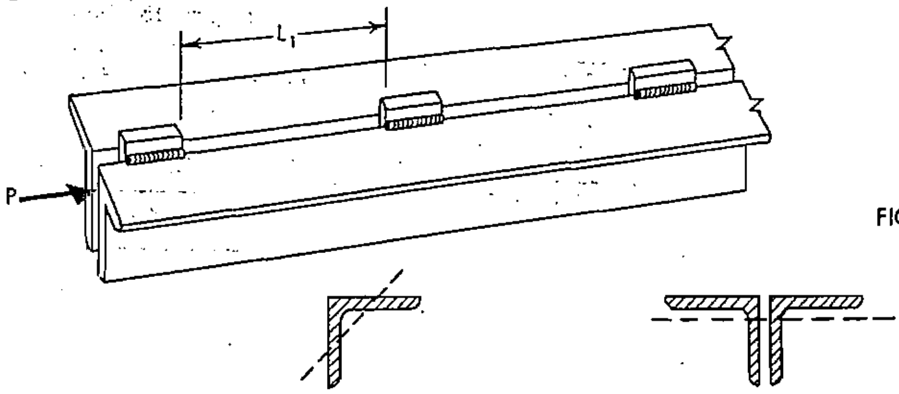


FIGURE 13

$$\left(\frac{L_1}{r}\right) \text{ of either member } \leqq \left(\frac{L}{r}\right) \text{ of whole member}$$

Tie Plates and Lacing

Main compression member built-up from plates or shapes and carrying a calculated force:

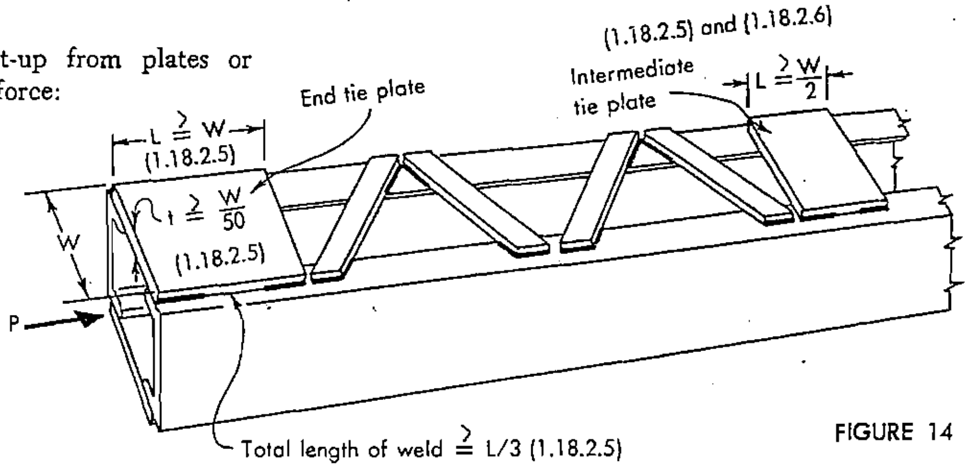


FIGURE 14

The spacing of lacing must be such (AISC 1.18.2.6) that—

$$\left(\frac{S}{r_1}\right) \text{ of element } \leqq \left(\frac{L}{r}\right) \text{ of whole member}$$

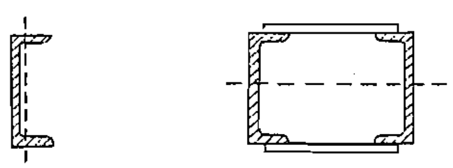
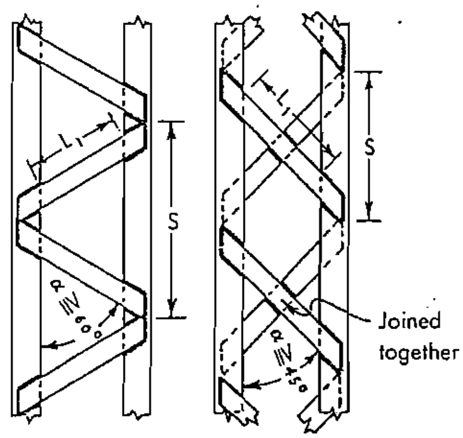


FIGURE 15



Single Bracing

Double Bracing

FIGURE 16

When the spacing between intermittent welds S > 15″, preferably use double bracing or braces made from angles (AISC 1.18.2.6).

For single bracing:

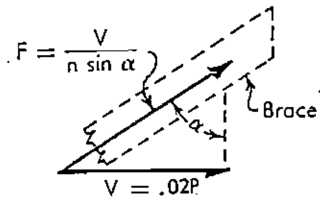
$$\left(\frac{L_1}{r_1}\right) \leqq 140$$

For double bracing:

$$\left(\frac{L_1}{r_1}\right) \leqq 200$$

Design lacing bar for axial compressive force (F):

$$F = \frac{V}{n \sin \alpha}$$



(AISC 1.18.2.6)

where:

n = number of bars carrying shear (V)

Determine allowable compressive stress ($\underline{\sigma_a}$) from one of the following two formulas:

If $\left(\frac{L_1}{r}\right) \leqq 120$*

$$\boxed{\underline{\sigma_a} = \frac{\left[1 - \frac{\left(\frac{KL_1}{r}\right)^2}{2\ C_c^2}\right]}{F.S.}\ \sigma_y} \text{*} \quad \ldots\ldots\ldots\ldots(16)$$

(AISC Formula 1)

(Use Tables 6 through 14, Section 3.1)

If $\left(\frac{L_1}{r}\right) > 120$*

$$\boxed{\underline{\sigma_a} = \frac{\underline{\sigma_a} \text{ from Form. \#15}}{1.6 - \frac{1}{200}\left(\frac{L_1}{r}\right)}} \text{*} \quad \ldots\ldots\ldots\ldots(17)$$

(AISC Formula 3)

here K = 1

On continuous cover plates with access holes (AISC 1.18.2.7):

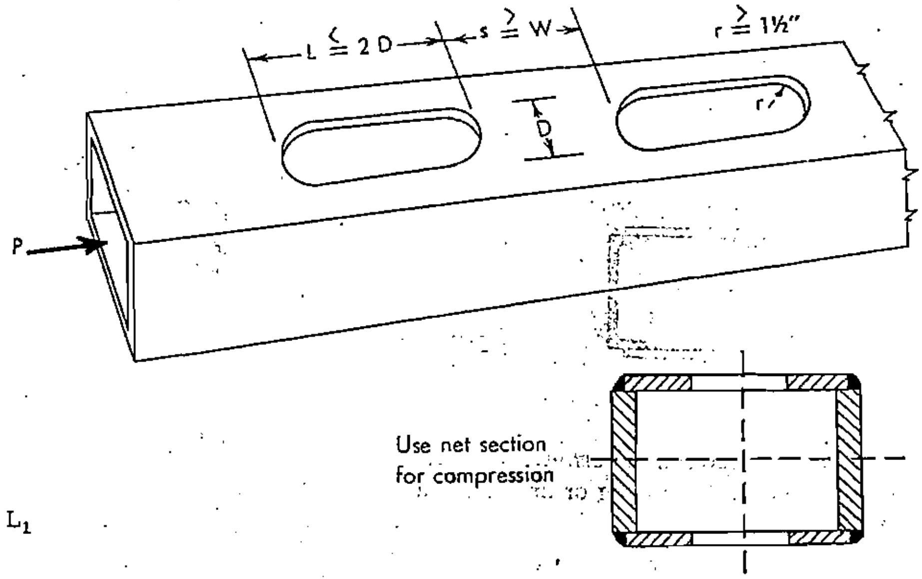


FIGURE 17

* For double brace, use .70 L_1

Typical Built-Up Compression Members

Figure 18 shows a number of examples of compression members built up from common shapes by means of welded construction. As indicated in lower views, perforated plates are often substituted for lacing bars for aesthetic effect.

Problem 2

To check the design of the following built-up section for the hoist of a boom. The 15′ column is fabricated from A36 steel by welding four 4″ x 3½″ x ½″ angles together with lacing bars.

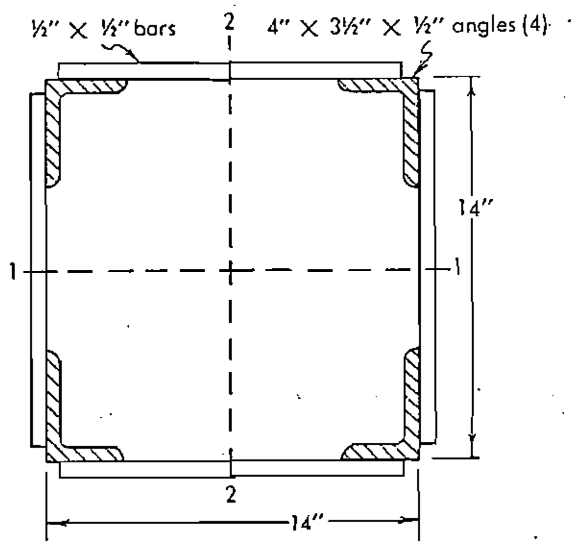
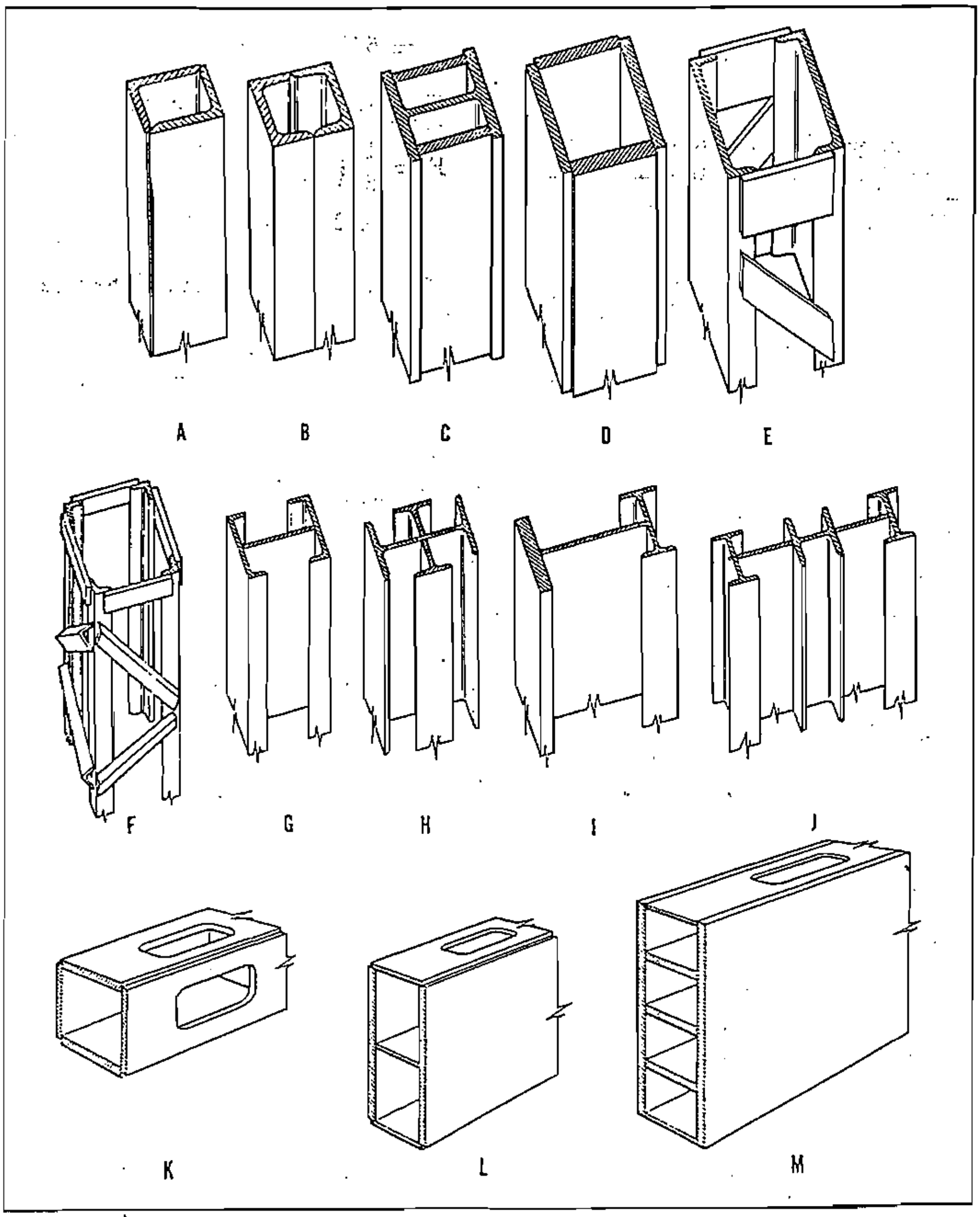


FIGURE 19

FIGURE 18—Typical Built-Up Compression Members



properties of each corner angle

$A = 3.5$ in.^{2}

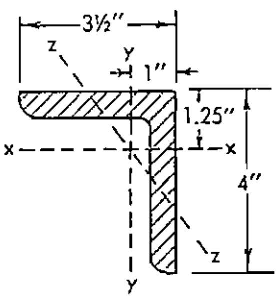
$r_z = .72''$

$I_x = 5.3$ in.^{4}

$I_y = 3.8$ in.^{4}

$x = 1.0''$

$y = 1.25''$



moment of inertia of built-up section about axis 1-1

$$I_1 = 4(3.5)(5.75)^2 + 4(5.3) = 484 \text{ in.}^4$$

moment of inertia of built-up section about axis 2-2

$$I_2 = 4(3.5)(6)^2 + 4(3.8) = 519 \text{ in.}^4$$

least radius of gyration

$$r_1 = \sqrt{\frac{I_1}{A}}$$

$$= \sqrt{\frac{(484)}{4(3.5)}}$$

$$\doteq 5.89''$$

slenderness ratio

$$\frac{L}{r} = \frac{(15')(12)}{(5.89)}$$
$$= 30.6$$

Then from Table 7 in Sect. 3.1, the allowable compressive stress is $\sigma_c = 19{,}900$ psi and the allowable compressive load is —

$$P = \sigma_c\ A$$
$$= (19{,}900)(14)$$
$$= 278.6 \text{ kips}$$

Check slenderness ratio of single 4" x 3½" x ½" angle between bracing:

$$\frac{L}{r_z} = \frac{(16.2)}{(.72)}$$
$$= 22.4 < 30.6 \quad \underline{\text{OK}}$$

(AISC Sec. 1.18.2.6)

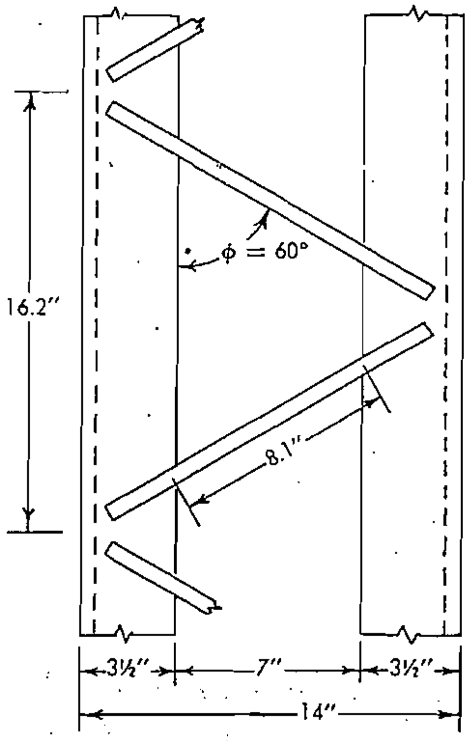


FIGURE 20

Design of Lacing Bars

AISC specifies that lacing bars be proportioned to resist a shearing force normal to the axis of the member and equal to 2% of the total compressive force on the member (Sec. 1.18.2.6):

$$V = 2\%\ P$$
$$= (.02)(278.6^k)$$
$$= 5.57^k \quad (2 \text{ bars})$$

The axial force on each bar is—

$$F = \frac{1}{2}\left(\frac{5.57}{.866}\right)$$
$$= 3.22^k$$

The unsupported length of the lacing bar between connecting welds is —

$$L = \frac{14'' - (2 \times 3\tfrac{1}{2}'')}{.866}$$
$$= 8.1''$$

The least radius of gyration of the ½" x ½" bar is obtained thusly —

$$A = \tfrac{1}{4} \text{ in.}^2$$
$$I = \frac{(\tfrac{1}{2})(\tfrac{1}{2})^3}{12}$$
$$= \frac{1}{192}$$
$$r = \sqrt{\frac{I}{A}}$$
$$= \sqrt{\left(\frac{1}{192}\right)\left(\frac{4}{1}\right)}$$
$$= .144$$

And the slenderness ratio of the lacing bars is —

$$\frac{L}{r} = \frac{(8.1)}{(.144)}$$
$$= 56.3 < 140 \quad \underline{\text{OK}} \text{ single lacing}$$

(AISC Sec. 1.18.2.6)

From Table 7 in Sect. 3.1, the allowable compressive stress on the bas is —

$$\underline{\sigma_c} = 17{,}780 \text{ psi}$$

The allowable compressive force on the bar is —

$$F = \sigma_c\ A$$
$$= (17{,}780)(.25)$$
$$= 4.45^k > 3.22^k \quad \underline{\text{OK}}$$

If each end of each bar is connected to the angles by two 1½" long $\frac{3}{16}$" (E70) fillet welds, this will provide an allowable force of —

$$F = 2 \times 1\tfrac{1}{2} \times 2100 \text{ lbs/in} = 6.3^k > 4.45^k \quad \underline{\text{OK}}$$

Problem 3

A multi-story building, having no interior columns, has a typical welded built-up column with the section shown in Figure 21.

A36 steel and E70 welds are employed.

The following three load conditions are recognized:

Case A	Case B	Case C
dead and live loads no wind	dead and live loads with wind in y-y direction	dead and live loads with wind in x-x direction
P = 2500 kips M_x = 250 ft-kips M_y = 0	P = 2700 kips M_x = 2200 ft-kips M_y = 0	P = 2800 kips M_x = 250 ft-kips M_y = 1200 ft-kips

properties of the 14″ WF 426# section

$A = 125.25 \text{ in.}^2$

$I_x = 6610.3 \text{ in.}^4$

$I_y = 2359.5 \text{ in.}^4$

moment of inertia about x-x

Let reference axis be a-a here

Parts	d	A	M	I_x	I_g
20 x 4	− 19.0	80.0	− 1520.0	+ 28,880	107
1½ x 34	0	51.0	0	0	4913
14 WF 426#	+ 17.94	125.25	+ 2247.0	+ 40,310	2360
Total		256.25	+ 727	+ 76,570	

$$I_x = 76{,}570 - \frac{+\ 727^2}{256.25}$$

$$= 74{,}507 \text{ in.}^4$$

$$NA = \frac{+\ 727}{256.25}$$

$$= +\ 2.84'' \quad \text{(from a-a)}$$

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$= \sqrt{\frac{(74{,}507)}{(256)}}$$

$$= 17.05''$$

moment of inertia about y-y

$$I_y = \frac{4 \times 20^3}{12} + \frac{34\ (1\tfrac{1}{2})^3}{12} + 6610$$

$$= 9287 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{(9287)}{(256)}}$$

$$= 6.03''$$

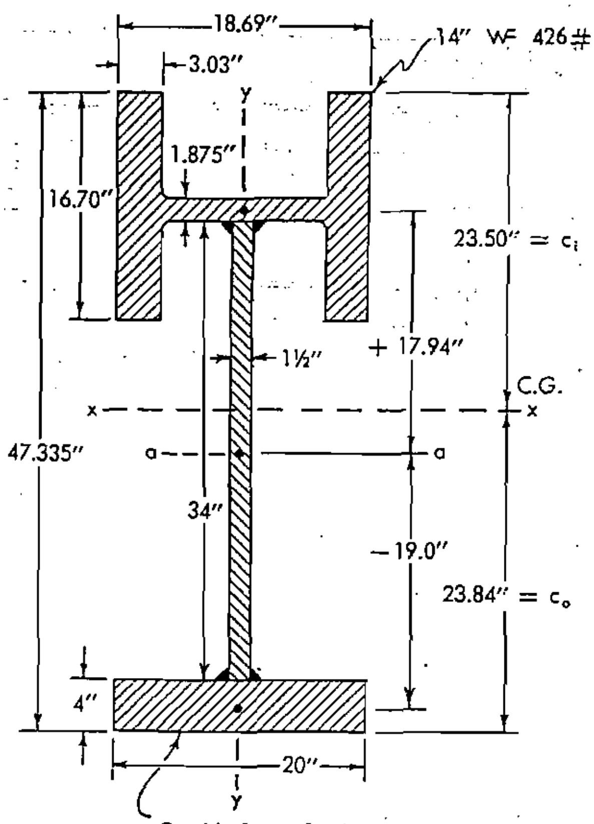


FIGURE 21

Allowable Stresses

The various axial compressive stresses and bending stresses on the built-up column are checked according to Formulas #11 and 12 (AISC Sec. 1.6.1, Formulas 6, 7a, and 7b).

When wind loads are included, the basic allowable stresses are increased by ⅓, provided the resulting section is not less than that required for dead load, live loads, and any impact (AISC Sec. 1.5.6).

Compression members are considered "compact" when symmetrical about an axis in the plane of bending, with lateral support of the column's compression flange not exceeding a distance equal to 13 times its width (A36 steel) (AISC Sec. 1.5.1.4.1). For "compact" columns, the engineer can use just .90% of moments applied to ends of the column if caused by gravity loads on connecting beams (no wind loads) and $\sigma_a \leqq .15\ \underline{\sigma_a}$ (AISC Sec 1.5.1.4.1).

If the section is not "compact", AISC Formulas 4 and 5 must be used to determine the allowable compressive bending stress ($\underline{\sigma_{bx}}$ and $\underline{\sigma_{by}}$).

check for lateral support

L_c = maximum unbraced length of compression flange for "compact" section

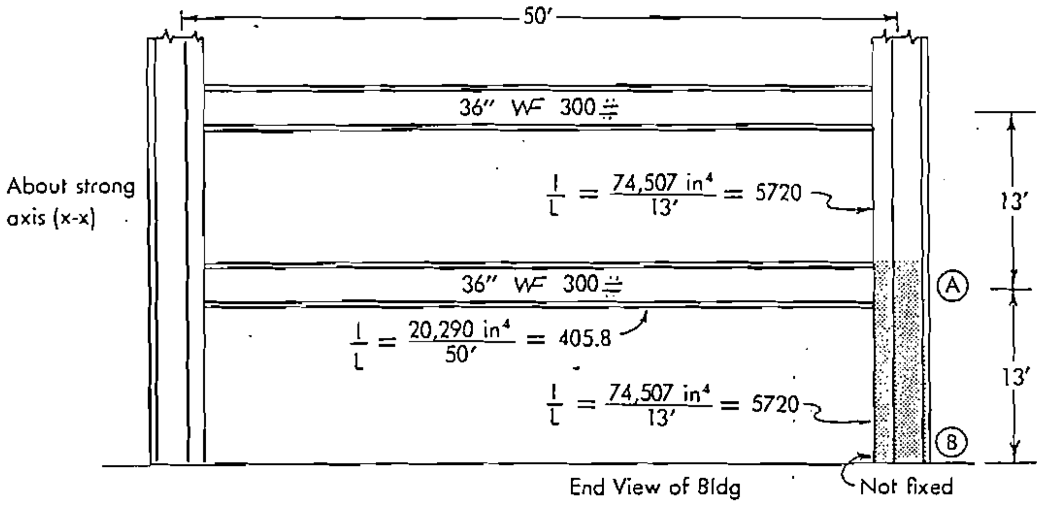


FIGURE 22 (a)

$$L_{cx} = 13\ b_{fx}$$
$$= 13(18\tfrac{3}{4}'')$$
$$= 244'' \text{ or } 20.3' > 13' \quad \text{OK}$$
$$L_{cy} = 13\ b_{fy}$$
$$= 13(16\tfrac{3}{4}'')$$
$$= 218'' \text{ or } 18.2' > 13' \quad \text{OK}$$

check for "compact" section
flange half, width to thickness

(a) outer flange plate

$$\frac{b_f}{t_f} = \frac{10''}{4''}$$
$$= 2.5 < \frac{1600}{\sqrt{\sigma_y}} \quad \text{or } 8.4 \quad \text{OK}$$

(b) inner WF section

$$\frac{b_f}{t_f} = \frac{8.35''}{3.03''}$$
$$= 2.75 < \frac{1600}{\sqrt{\sigma_y}} \quad \text{or } 8.4 \quad \text{OK}$$

check web depth to web thickness

$$\text{Actual } \frac{d_w}{t_w} = \frac{34''}{1\tfrac{1}{2}} = 22.6$$

$$\text{Allowable } \frac{d_w}{t_w} \leq \frac{13{,}300}{\sqrt{\sigma_y}}\left(1 - 1.43\ \frac{\sigma_a}{\sigma_a}\right)$$

but need not be less than $\frac{8000}{\sqrt{\sigma_y}}$

$$\frac{d_w}{t_w} \leq 70\left(1 - 1.43 \times \frac{9{,}760}{17{,}970}\right) \leq 17.3$$

but need not be less than 42.1

$$42.1 > 22.6 \quad \text{OK}$$

Therefore it is a "compact" section and following can be used:

$$\underline{\sigma_{bx}} = \underline{\sigma_{by}} = .66\ \sigma_y \text{ or } 24{,}000 \text{ psi}$$

Euler stress (σ'_{ex}) *and* (σ'_{ey})

About strong axis (x-x):

$$\frac{K_x L_x}{r_x} = \frac{569''}{17.05''} = 33.4$$

From Table 2, read $\sigma'_{ex} = 133{,}750$ psi.

About weak axis (y-y):

$$\frac{K_y L_y}{r_y} = \frac{328''}{6.03''} = 54.4$$

From Table 2, read $\sigma'_{ey} = 50{,}400$ psi.

allowable axial compressive stress

$$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}}$$
$$= \frac{2(5720)}{1(\ 406)}$$
$$= 28.2$$
$$G_B = \infty \text{ or } 10$$

Sidesway being permitted, from the nomograph (Fig. 1):

$$K = 3.65 \text{ and}$$
$$L_e = K\ L$$
$$= (3.65)(13' \times 12'')$$
$$= 569''$$

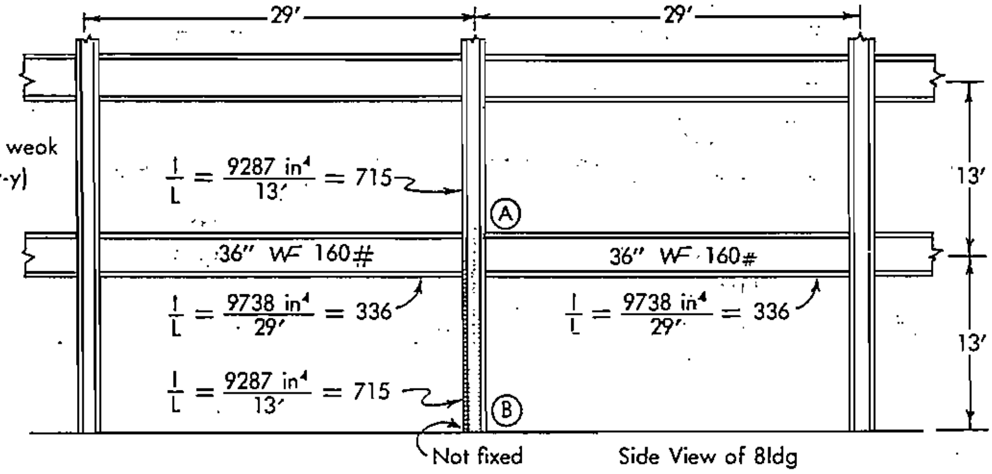


FIGURE 22 (b)

$$\frac{L_e}{r_x} = \frac{(569'')}{(17.05'')}$$

$$= 33.4$$

$$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}}$$

$$= \frac{2(715)}{2(336)}$$

$$= 2.13$$

$$G_B = \infty \text{ or } 10$$

Sidesway being permitted, from the nomograph (Fig. 1):

$$K = 2.1 \text{ and}$$

$$L_e = K\ L$$

$$= 2.1\ (13' \times 12'')$$

$$= 328''$$

$$\frac{L_e}{r_y} = \frac{(328'')}{(6.03'')}$$

$$= 54.4$$

This value of $r_y = 54.4$ governs, and from Table 7 in Sect. 3.1 (A36 steel)

$$\sigma_a = 17{,}970 \text{ psi}$$

Column Analysis

The following three analyses of the column (Cases A, B, and C) are for columns with computed moments maximum at the ends with no transverse loading and with sidesway being permitted.

This would be category A on Table 4. In this case ($C_m = .85$) for both axes (x-x) and (y-y).

CASE A Dead and Live Loads; No Wind

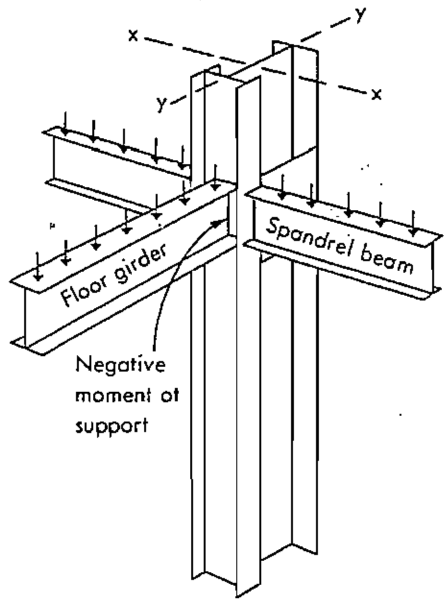


FIGURE 23

applied loads

$$P = 2500 \text{ kips}$$

$$M_x = 250 \text{ ft-kips}$$

$$M_y = 0$$

applied stresses

$$\sigma_a = \frac{P}{A}$$

$$= \frac{(2500 \times 1000)}{(256.25)}$$

$$= 9760 \text{ psi}$$

$$\sigma_{bx} = \frac{M_x\ c}{I_x}$$

$$= \frac{(250 \times 1000 \times 12)(23.50)}{(74{,}507)}$$

$$= 947 \text{ psi (max at } 4'' \times 20'' \text{ flange ℄)}$$

$\sigma_{by} = 0$

If $\frac{\sigma_a}{\underline{\sigma_a}} = .15$, $.9M_x$ can be used (Sec 1.5.1.4.1); but in this case, $\frac{\sigma_a}{\underline{\sigma_a}} = \frac{9760}{17{,}970} = .54$ $= .54 > .15$ so full value of M_x must be used.

allowable stresses

$\underline{\sigma_a} = 17{,}970$ psi

Since it is a "compact" section laterally supported within 13 times its compression flange width (Sec 1.5.1.4.1):

$\underline{\sigma_{bx}} = \underline{\sigma_{by}} = .66\ \sigma_y = 24{,}000$ psi

$\sigma'_{ex} = 133{,}750$ psi

$0.60\ \sigma_y = 22{,}000$ psi

checking against Formula #14 (AISC 7a)

$$\frac{\sigma_a}{\underline{\sigma_a}} + \frac{C_{mx}\ \sigma_{bx}}{\left(1 - \frac{\sigma_a}{\sigma'_{ex}}\right)\underline{\sigma_{bx}}} + \frac{C_{my}\ \sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma'_{ey}}\right)\underline{\sigma_{by}}} \leqq 1$$

Here $C_m = .85$ because sidesway is permitted

$$\frac{(9760)}{(17{,}970)} + \frac{(.85)(947)}{\left(1 - \frac{9760}{133{,}750}\right)(24{,}000)}$$

$$= .579 < 1.0 \quad \underline{\text{OK}}$$

checking against Formula #15 (AISC 7b)

$$\frac{\sigma_a}{0.6\ \sigma_y} + \frac{\sigma_{bx}}{\underline{\sigma_{bx}}} + \frac{\sigma_{by}}{\underline{\sigma_{by}}} \leqq 1$$

$$\frac{(9760)}{(22{,}000)} + \frac{(947)}{(24{,}000)} = .482 < 1.0 \quad \underline{\text{OK}}$$

<u>CASE B</u> Dead and Live Loads; Wind in Y Direction

applied loads

$P = 2700$ kips $\quad M_x = 2200$ ft-kips $\quad M_y = 0$

applied stresses

$$\sigma_a = \frac{P}{A} = \frac{2700 \times 1000}{256.25} = 10{,}520 \text{ psi}$$

$$\sigma_{bx} = \frac{M_x\ c}{I_x}$$

$$= \frac{(2200 \times 1000 \times 12)(23.50)}{74{,}507}$$

$$= 8330 \text{ psi (max at } 4'' \times 20'' \text{ flange ℄)}$$

We cannot use .9 M_x, because wind loading is involved; hence full value of M_x must be used.

$\sigma_{by} = 0$

allowable stresses

$\underline{\sigma_a} = 17{,}970 \times 1.33$	Wind in addition (Sec 1.5.6)
$\underline{\sigma_{bx}} = 24{,}000 \times 1.33$	Wind in this direction (Sec 1.5.6)
$\underline{\sigma_{ex}} = 133{,}750 \times 1.33$	Wind in this direction (Sec 1.6.1 and 1.5.6)

checking against Formula #14 (AISC 7a)

$$\frac{\sigma_a}{\underline{\sigma_a}} + \frac{C_{mx}\ \sigma_{bx}}{\left(1 - \frac{\sigma_a}{\sigma'_{ex}}\right)\underline{\sigma_{bx}}} + \frac{C_{my}\ \sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma_{e}'{}_{x}}\right)\underline{\sigma_{by}}} \leqq 1.0$$

$$\frac{(10{,}520)}{(17{,}970 \times 1.33)}$$

$$+ \frac{(.85)(8330)}{\left(1 - \frac{10{,}520}{133{,}750 \times 1.33}\right)(24{,}000 \times 1.33)}$$

$$= .676 < 1.0 \quad \underline{\text{OK}}$$

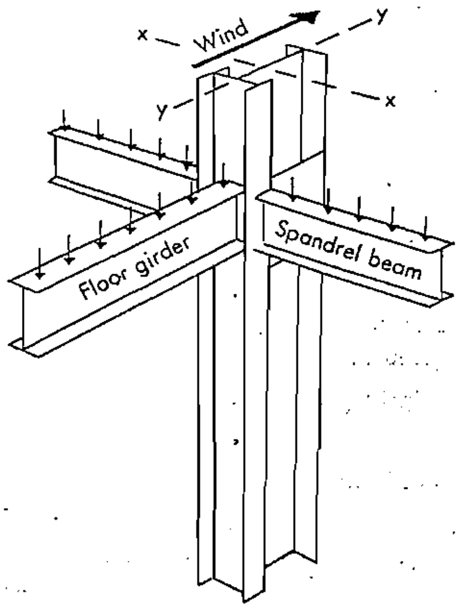


FIGURE 24

checking against Formula #15 (AISC 7b)

$$\frac{\sigma_a}{0.6\ \sigma_y} + \frac{\sigma_{bx}}{\underline{\sigma_{bx}}} + \frac{\sigma_{by}}{\underline{\sigma_{by}}} \leqq 1.0$$

$$\frac{(10{,}520)}{(22{,}000 \times 1.33)} + \frac{(8330)}{(24{,}000 \times 1.33)}$$

$$= .621 < 1.0 \quad \underline{\text{OK}}$$

<u>CASE C</u> Dead and Live Loads; Wind in X Direction

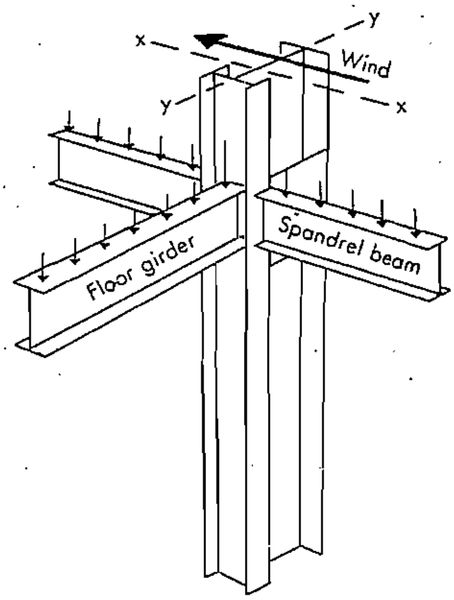


FIGURE 25

applied loads

$P = 2800$ kips
$M_x = 250$ ft-kips
$M_y = 1200$ ft-kips

applied stresses

$$\sigma_a = \frac{P}{A}$$

$$= \frac{(2800 \times 1000)}{(256.25)}$$

$$= 10{,}920 \text{ psi}$$

$$\sigma_{bx} = \frac{M_x\ c}{I_x}$$

$$= \frac{(250 \times 1000 \times 12)(23.50)}{(74{,}507)}$$

$= 947$ psi (max at 4″ × 20″ flange ℞)

$$\sigma_{by} = \frac{M_y\ c}{I_y}$$

$$= \frac{(1200 \times 1000 \times 12)(9.35)}{(9286)}$$

$= 14{,}500$ psi (max at flange of WF section)

$$\text{or} = \frac{(1200 \times 1000 \times 12)(10.0)}{(9286)}$$

$= 15{,}500$ psi (max at outer edge of 4″ × 20″ ℞)

We cannot use .9 M, because wind loading is involved; hence full value of (M_x) and (M_y) must be used.

allowable stresses

$\underline{\sigma_a} = 17{,}970 \times 1.33$	Wind in addition (Sec 1.5.6)
$\underline{\sigma_{bx}} = 24{,}000$	No wind in this direction
$\underline{\sigma_{by}} = 24{,}000 \times 1.33$	Wind in this direction (Sec 1.5.6)
$\sigma'_{ex} = 133{,}750$	No wind in this direction
$\sigma'_{ey} = 50{,}400 \times 1.33$	Wind in this direction

checking against Formula #11 (AISC 7a)

$$\frac{\sigma_a}{\underline{\sigma_a}} + \frac{C_m\ \ \sigma_{bx}}{\left(1 - \frac{\sigma_a}{\sigma'_{ex}}\right)\underline{\sigma_{bx}}} + \frac{C_m\ \ \sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma'_{ey}}\right)\underline{\sigma_{by}}} \leqq 1.0$$

$$\frac{(10{,}920)}{(17{,}970 \times 1.33)} + \frac{(.85)(947)}{\left(1 - \frac{10{,}920}{133{,}750}\right)(24{,}000)}$$

$$+ \frac{(.85)(15{,}500)}{\left(1 - \frac{10{,}920}{50{,}400 \times 1.33}\right)(24{,}000 \times 1.33)}$$

$$= .986 < 1.0 \text{ OK}$$

If there is any question about this built-up column section being a "compact" section about the y-y axis, we must use $\underline{\sigma_{by}} = 22{,}000$. This would result in $1.03 > 1.0$. However, this could be overcome by re-adjusting the 4″ × 20″ flange plate down to a distance within the depth of the WF (18.69″). Then $\sigma_{by} = 14{,}500$ and this would result in $.996 < 1.0$ <u>OK.</u>

checking against Formula #11 (AISC 7b)

$$\frac{\sigma_a}{0.6\ \sigma_y} + \frac{\sigma_{bx}}{\underline{\sigma_{bx}}} + \frac{\sigma_{by}}{\underline{\sigma_{by}}} = 1.0$$

$$\frac{(10{,}920)}{(22{,}000 \times 1.33)} + \frac{(947)}{(24{,}000)} + \frac{(15{,}500)}{(24{,}000 \times 1.33)}$$

$$= .898 < 1.0 \quad \underline{\text{OK}}$$

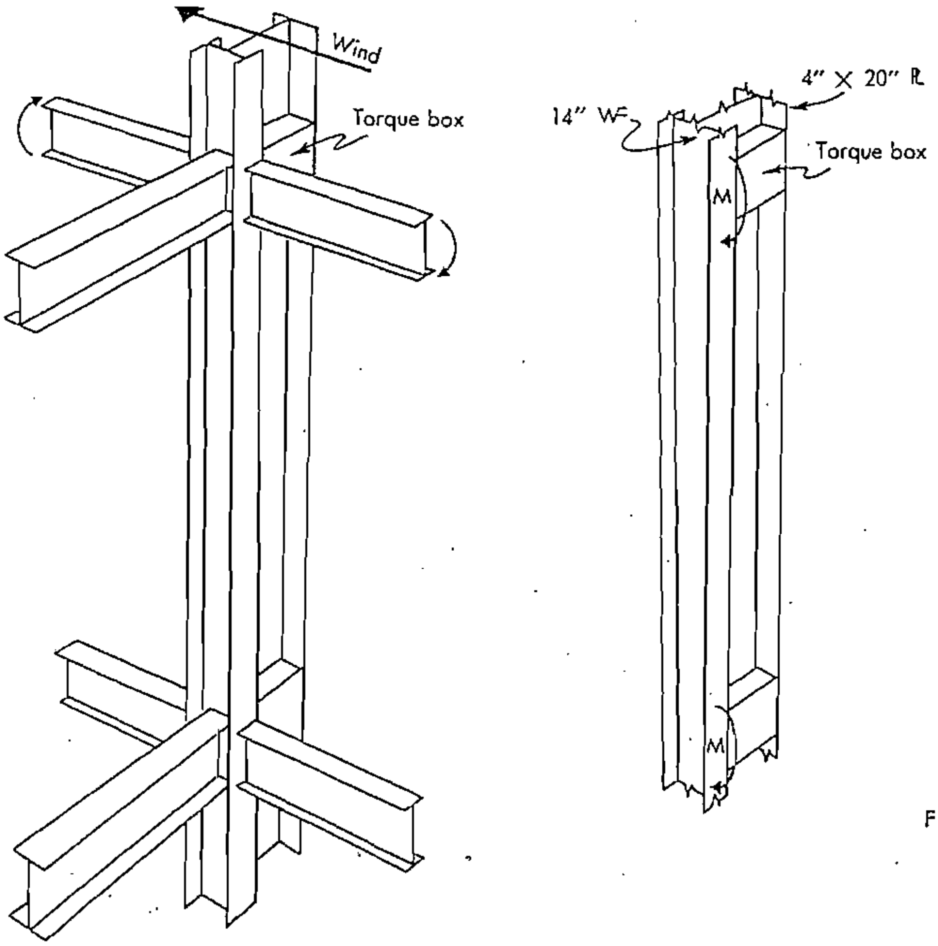


FIGURE 26

Torsion on Built-Up Column

One item left to investigate in the built-up column is the twisting action applied to it. In Case C, the wind in the x-x direction causes a moment of $M_y = 1200$ ft-kips because of the restraint of the spandrel beams.

(1) One way to analyze this problem is to assume that this moment (M_y) is resisted by the elements (the 14" WF section and the 4" × 20" flange plate) of the built-up column in proportion to their moments of inertia about axis y-y. See Figure 26.

Since:

$I_{WF} = 6610$ in.^{4}

$I_{℞} = 2667$ in.^{4}

The moment resisted by the 4" × 20" flange plate is—

$$M_{℞} = \frac{(1200 \text{ ft-kip})(2667)}{(6610 + 2667)}$$

$$= 346 \text{ ft-kips} = 4{,}152{,}000 \text{ in.-lbs}$$

This moment is to be transferred as torque from the 14" WF section to the 4" × 20" plate through a torque box, made by adding ½"-thick plates to the built-up column in line with the beam connections.

This torque box is checked for shear stress; Figure 27.

$$\tau = \frac{T}{2\ t\ b\ d}$$

$$= \frac{(4{,}152{,}000)}{2(\tfrac{1}{2})(18.2)(34.5)}$$

$$= 6600 \text{ psi } \underline{\text{OK}}$$

(2) Another method of checking this twisting action is to consider the moment (M_y) as applying torque to the built-up column. See Figure 28.

This applied moment may be considered as two flange forces: in this case, 411 kips in the upper and the lower flanges of the spandrel beam, but in opposite directions. Since these forces are not applied at the "shear center" of the column, a twisting action will be applied to the column about its longitudinal axis within the region of the beam connection where these forces are applied; there is no twisting action along the length of the column in between these regions.

Since an "open" section such as this built-up

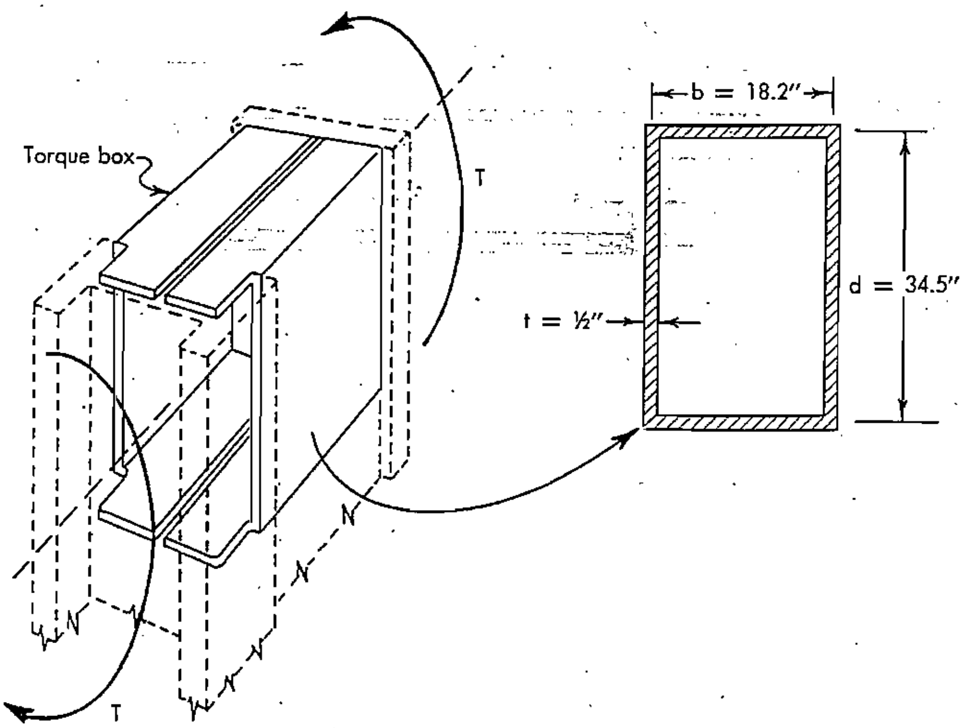


FIGURE 27

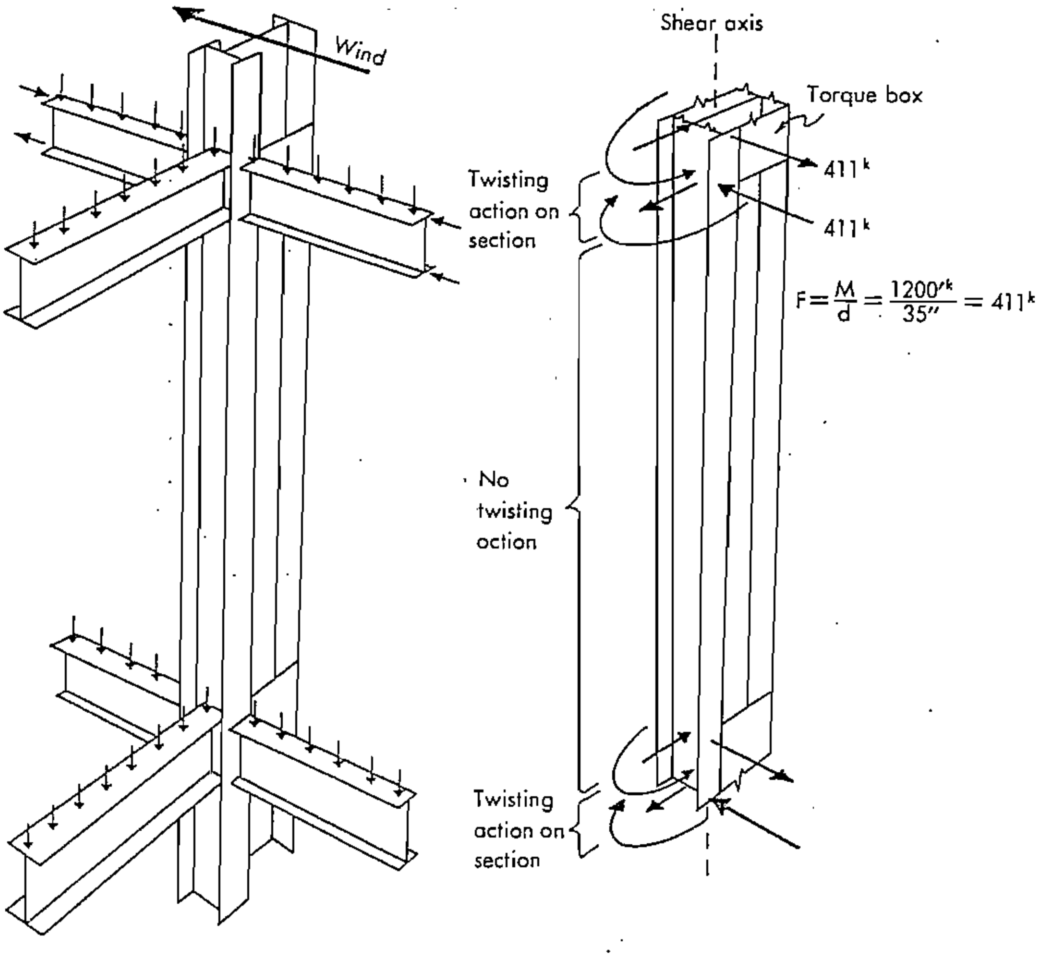


FIGURE 28

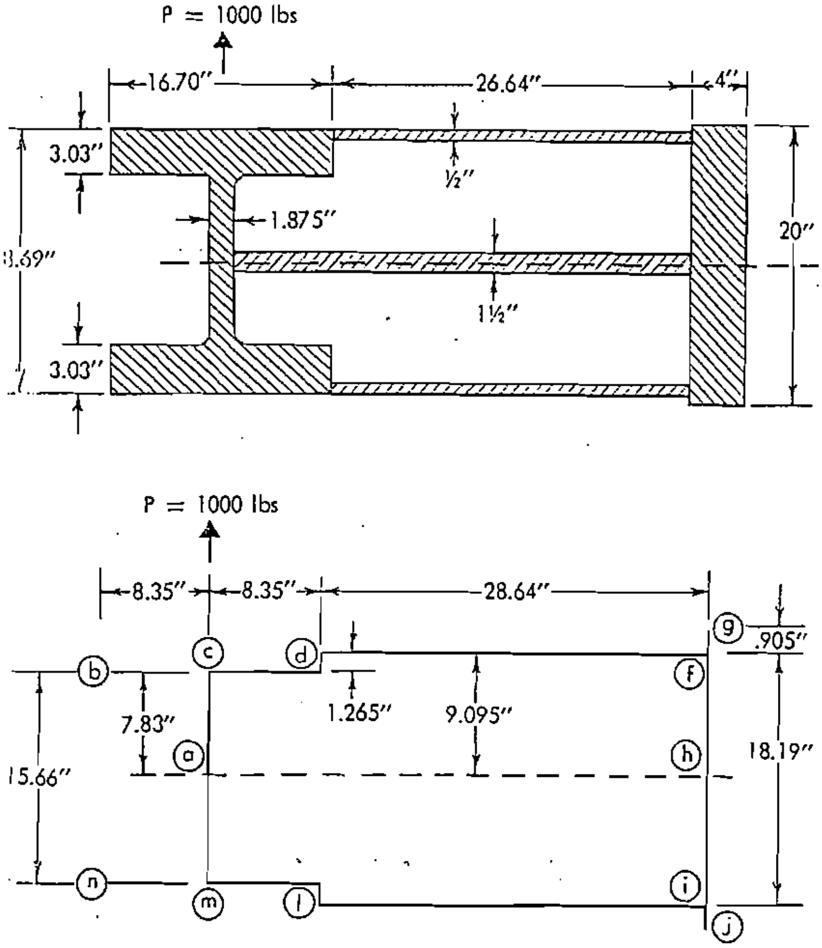


FIGURE 29

column offers very little torsional resistance, two plates will be added within this region to form a closed section about the shear axis to transfer this torque. See Figure 29.

If this torque had to be transferred from one floor to the next, these plates would have to be added the full length of the column. However, this torque is only within the region of the connecting beams which apply these forces, hence plates are only added within this short distance.

$$I_y = 6610.3 + 2(26.64 \times ½)(9.095)^2 + \frac{4 \times 20^3}{12}$$

$$+ 2\,\frac{26.64(½)^2}{12} + \frac{34(1½)^3}{12}$$

$$= 11{,}491 \text{ in.}^4$$

In our analysis of the column under Case C loading conditions, a transverse force of 1 kip was assumed to be applied in line with the web of the WF section of the built-up column (this is the position of the spandrel beams). This cross-section is in the plane of the top flange of the spandrel beam. Just below this, in the plane of the lower flange of the spandrel beam, this 1-kip force will be applied in the opposite direction.

Treating this short section of the built-up column as a beam, the shear forces due to this 1-kip force will be analyzed on the basis of shear flow. In an open section it is not difficult to do this because there is always one or more starting points, the unit shear force at the outer edges always being zero. But in a closed section such as this, it is necessary to assume a certain value (usually zero) at some convenient point, in this case at the midpoint of the web of the WF section. The unit shear forces are then found, starting from this point and working all the way around the section using the general formula—

$$q_2 = q_1 + \frac{V\,a\,y}{I}$$

where:

V = transverse force applied to section (lbs)

I = moment of inertia of built-up section about the axis normal to the applied force (in.^{4})

a = area of portion of section considered (in.^{2})

y = distance between center of gravity of this area and the neutral axis of the built-up section (in.)

q_1 = unit shear force at the start of this area (lbs/in.)

q_2 = unit shear force at the end of this area (lbs/in.)

This work is shown as Computation A. Below, in Figure 30, the total shear force (Q) in the various areas of this section are found; these are indicated by arrows. This work is shown as Computation B. By Computation C, these shear forces are seen to produce an unbalanced moment of 70.519 in-lbs, which if unresisted will cause this section of the column to twist.

In order to counterbalance this moment, a negative moment of the same value is set up by a constant shear force flow of—

$$q = -54.1 \text{ lbs per linear inch}$$

When this is superimposed upon the original shear flow, Figure 30, we obtain the final flow shown in Figure 31. The resulting shear stress (τ) is obtained by dividing the unit shear force (q) by the thickness of the section. Also the values must be increased because the actual force is 411 kips instead of 1 kip, the work and resulting shear stresses are shown as Computation D. See Figure 32 also. These shear stresses seem reasonable.

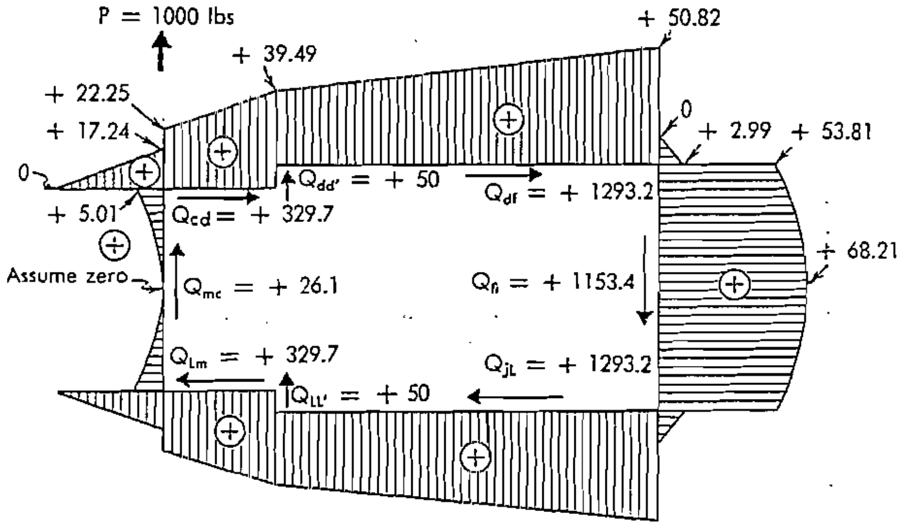


FIGURE 30

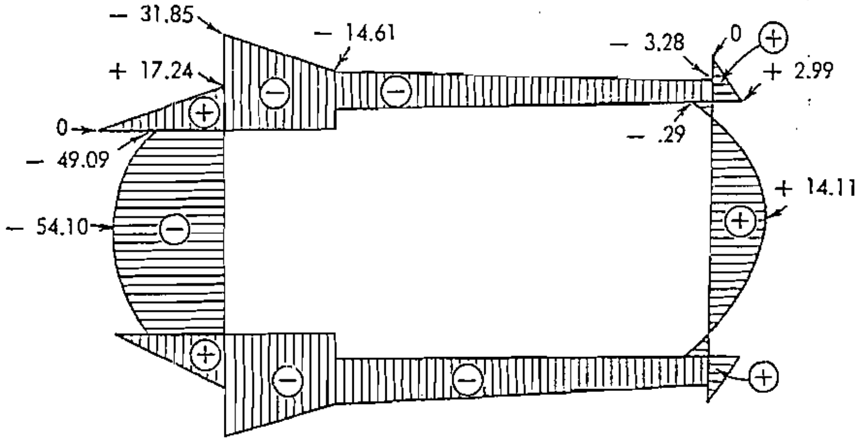


FIGURE 31

Computation A

1. $q_a = 0$	0
2. $q_c = 0 + \dfrac{V\,a\,y}{I} = 0 + \dfrac{(1000)(7.83 \times 1.875)(3.92)}{11{,}491.} = 0 + 5.01 =$	5.01
3. $q_b = 0$	0
4. $q_c' = q_b + \dfrac{V\,a\,y}{I} = 0 + \dfrac{(1000)(8.35 \times 3.03)(7.83)}{11{,}491.} = 0 + 17.24 =$	17.24
5. $q_c'' = q_c + q_c' = 5.01 + 17.24 =$	22.25
6. $q_d = q_c'' + \dfrac{V\,a\,y}{I} = 22.25 + \dfrac{(1000)(8.35 \times 3.03)(7.83)}{11{,}491.} = 22.25 + 17.24 =$	39.49
7. $q_f = q_d + \dfrac{V\,a\,y}{I} = 39.49 + \dfrac{(1000)(28.64 \times 1/2)(9.095)}{11{,}491.} = 39.49 + 11.33 =$	50.82
8. $q_g = 0$	0
9. $q_f' = q_g + \dfrac{V\,a\,y}{I} = 0 + \dfrac{(1000)(.905 \times 4)(9.548)}{11{,}491.} = 0 + 2.99 =$	2.99
10. $q_f'' = q_f' + q_f = 2.99 + 50.82 =$	53.81
11. $q_h = q_f'' + \dfrac{V\,a\,y}{I} = 53.81 + \dfrac{(1000)(9.095 \times 4)(4.548)}{11{,}491.} = 53.81 + 14.40 =$	68.21

Computation B

12. $Q_{ac} = (2/3 \times 0 + 1/3 \times 5.01)\ 15.66 = 26.1\ \#$

13. $Q_{bd} = (1/2 \times 17.24 \times 8.35) + \dfrac{22.25 + 39.49}{2} \times 8.35) = 329.7\ \#$

14. $Q_{dd'} = 39.49 \times 1.265 = 50.0\ \#$

15. $Q_{df} = \dfrac{39.49 + 50.82}{2} \times 28.64 = 1293.2\ \#$

16. $Q_{fh} = (2/3 \times 68.21 + 1/3 \times 53.81)\ 18.19 = 1153.4\ \#$

Check $\Sigma V = 0$

$+ 1000 + 26.1 + 50.0 - 1153.4 + 50.0 = 1126.1 - 1153.4 = -27.3$ OK (Close)

Computation C

Now, take moments about (m)

$M_m = (+329.7)(15.66) - (100)(8.35) + (1293.2)(18.19) + (1153.4)(36.99) = 70{,}519$

The unbalanced moment is 70,519 in-lbs

Make $\Sigma M_m = 0$ a constant shear force flow, which must be added to form a negative moment of $-70{,}519$.

The resulting shear force is —

$$q = \frac{-M}{2[A]} = \frac{-70{,}519}{2(651.7)} = -54.1 \text{ lbs/in.}$$

Where [A] = area enclosed by centerline of web, flanges, and plates
[A] = (15.66)(8.35) + (18.19)(28.64) = 651.7 in²

This gives the true shear flow (Fig. 31).

Computation D

If this force is P = 441,000 lbs, the shear stresses in the section are —

$$(a)\ \tau_a = \frac{q}{t} = \frac{411 \times 54.10}{1.875} = 11{,}850 \text{ psi}$$

$$(c'')\ \tau_c'' = \frac{q}{t} = \frac{411 \times 31.85}{3.03} = 4320 \text{ psi}$$

$$(d)\ \tau_d = \frac{q}{t} = \frac{411 \times 14.61}{\frac{1}{2}} = 12{,}000 \text{ psi}$$

$$(f)\ \tau_f = \frac{q}{t} = \frac{411 \times 3.28}{\frac{1}{2}} = 2690 \text{ psi}$$

$$(h)\ \tau_h = \frac{q}{t} = \frac{411 \times 14.11}{4} = 1450 \text{ psi}$$

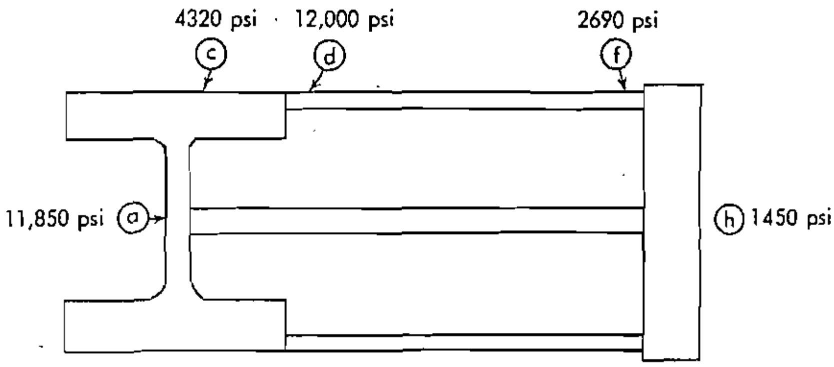


FIGURE 32

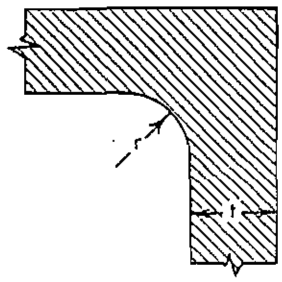


FIGURE 34

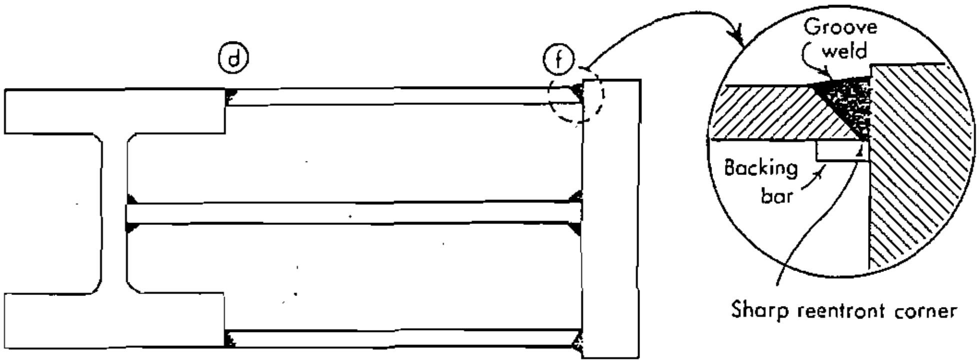


FIGURE 33

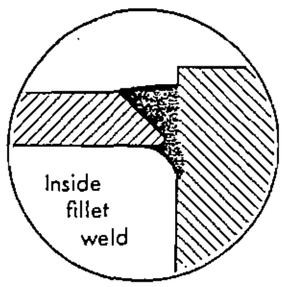


FIGURE 35

Reentrant Corners (Figures 33 and 34)

The only other concern on this built-up construction is the sharp reentrant corner at points (d) and (f).

Timoshenko in "Theory of Elasticity", p. 259, indicates the following shear stress increase for a reentrant corner:

$$\boxed{\tau_{max} = \tau\left(1 + \frac{t}{4r}\right)}$$

In structural steel, any stress concentration in this area probably would be relieved through plastic flow and could be neglected unless fatigue loading were a factor or there were some amount of triaxial stress along with impact loading.

Of course if a fillet weld could be made on this inside corner, it would eliminate this problem. See Figure 35. This is possible in this case, because these plates for the torque box are not very long and the welding operator could reach in from each end to make this weld.

6. SIZE OF WELDS FOR FABRICATED COLUMN

The welds that join the web of a built-up column to its inside WF section and its outside flange plate, are subject to longitudinal shear forces resulting from the changing moment along the length of the column.

As an example, continue with the conditions stated for the preceding Problem 3.

The bending force in the flanges of the girder applied to the column is found by dividing this moment (M_x) by the depth of the girder:

$$F = \frac{M_x}{d}$$

$$= \frac{2200 \text{ ft-kip} \times 12''}{35''}$$

$$= 754 \text{ kips}$$

The point of contraflexure, or zero moment, is assumed at about midheight of the column. The horizontal force at this point, or transverse shear in the column, may be found by dividing half of the moment applied to the column at the connection by about one-half of the column height. This assumes half of applied moment enters upper column and half enters lower column.

$$F_h = \frac{M}{\frac{1}{2} h}$$

$$= \frac{1100 \text{ ft-kip}}{6.5'}$$

$$= 170 \text{ kips}$$

The moment and shear diagrams for the column when loaded with dead and live loads and wind in the y-y direction (Case B) are given in Figure 36.

This shear diagram indicates the transverse shear within the region of the beam connection is $V_2 = 584$ kips, and that in the remaining length of the column is $V_1 = 170$ kips.

The size of the connecting weld shall be determined for the larger shear within the region of the beam connection, and for the lower shear value for the remaining length of the column. The minimum fillet weld size is also dependent on the maximum thickness of plate joined (AWS Building Article 212 a 1, and AISC Sec. 1.17.4).

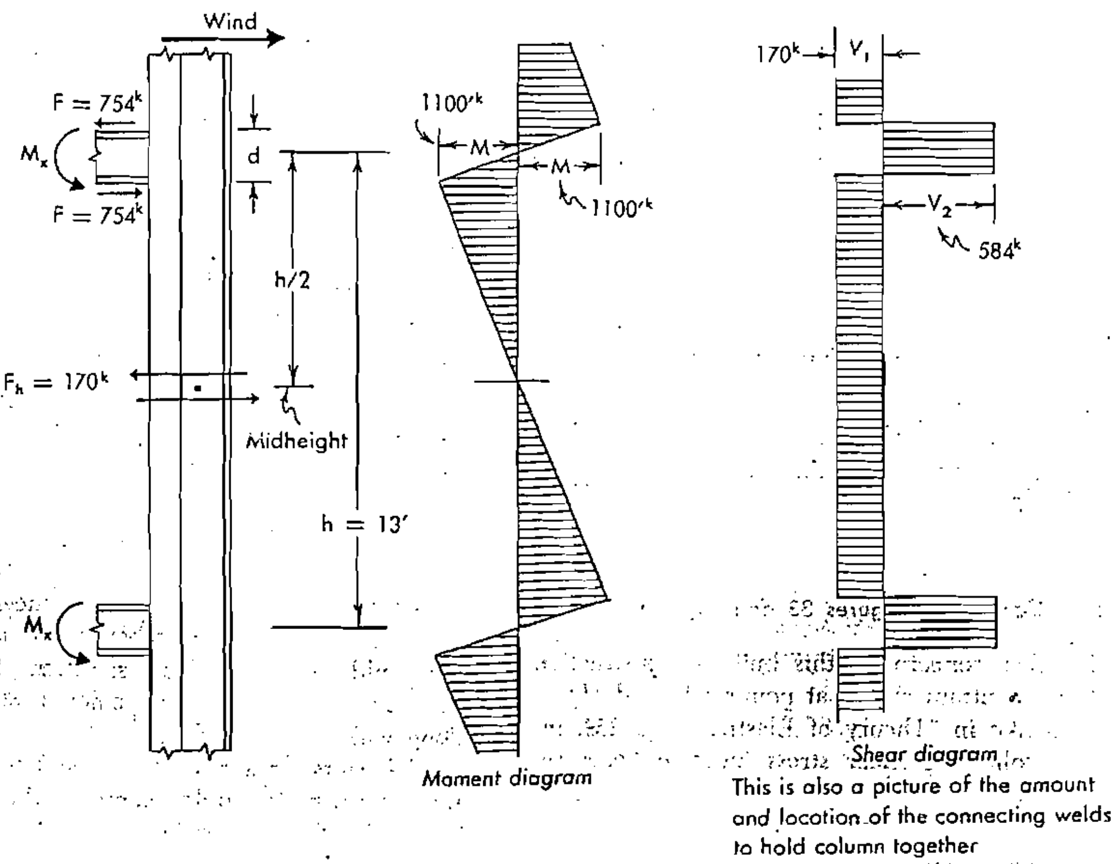


FIGURE 36

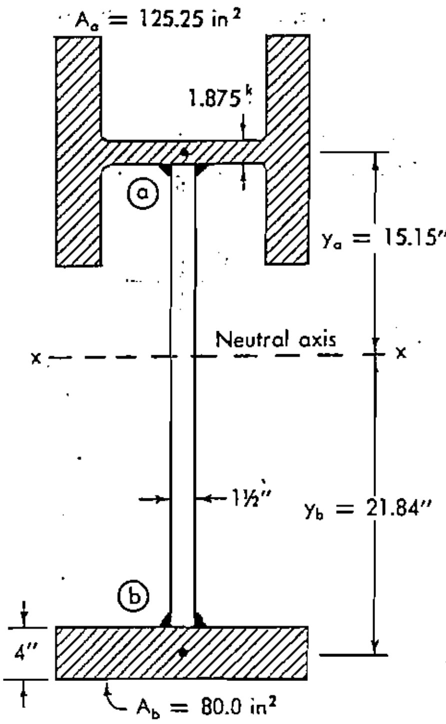


FIGURE 37

where:

$A = 256.25$ in.²

$I_x = 74{,}507$ in.⁴

The following allowable shear force for the fillet weld will be used:

$f = 11{,}200\omega$ (A36 steel and E70 weld metal)

We will not reduce the shear carrying capacity of the fillet weld due to the axial compressive stress on it.

weld (a) in the way of the beam connection

$$f_a = \frac{V_2\ a_a\ y_a}{I\ n}$$

$$= \frac{(584^k)(125.25)(1515)}{(74{,}507)(2 \text{ welds})}$$

$$= 7450 \text{ lbs/in}$$

$$\text{leg size } \omega = \frac{7450}{11{,}200}$$

$$= .665'' \text{ or use } 3/4''$$

weld (a) for the remaining length of the column

$V_1 = 170^k$ or 29% of V_2

hence use 29% of the leg size or .192″. However, the maximum thickness of plate here is 1⅞″, and the minimum size of fillet weld for this thickness is ⅜″ (AWS Bldg Art 212 and AISC Sec. 1.17.4). Hence use $\omega = 3/8''$.

Weld (b) in line with the beam connection

$$f_b = \frac{V_2\ a_b\ y_b}{I\ n}$$

$$= \frac{(584^k)(80)(21.84)}{(74{,}507)(2 \text{ welds})}$$

$$= 6860 \text{ lbs/in.}$$

$$\text{leg size } \omega = \frac{6860}{11{,}200}$$

$$= .612'' \text{ or use } 5/8''$$

weld (b) for the remaining length of the column

$V_1 = 170^k$ or 29% of V_2

hence use 29% of the above leg size, or leg size ω = .178″ or 3/16″; however, the maximum thickness of plate here is 4″ and the minimum size of fillet weld for this thickness is ½″ (AWS Bldg Art 212 and AISC Sec. 1.17.4). Hence use ½″.

When the column is subjected to the dead and live loads and wind in the x-x direction, bending is about the y-y axis. Here the inside and outside portions of the column are continuous throughout the cross-section of the column, and the connecting welds do not transfer any force; hence, the weld size as determined above for Case B would control.

Perhaps weld (a) should be further increased within the region of the beam connection, to transfer the horizontal forces of the beam end moment back into the column web. The horizontal stiffeners in the column at this point, however, would undoubtedly take care of this.

7. SQUARE AND RECTANGULAR HOT-ROLLED SECTIONS FOR COLUMNS

Square and rectangular tubular shapes are now being hot rolled from A7 (33,000 psi yield) and A36 (36,000 psi yield) steel at about the same price as other hot-rolled sections.

These sections have exceptionally good compressive and torsional resistance. See Tables 7 and 8 for dimensions and properties of stock sizes.

Many engineers feel that the round tubular section is the best for a column since it has a rather high radius of gyration in all directions. This is much better than the standard WF or I sections, which have a much lower radius of gyration about the weaker y-y axis.

Unfortunately the usually higher cost of round tubular sections prohibits their universal use for columns.

However, a square tube is slightly better than the round section; for the same outside dimensions and cross-sectional area the square tube has a larger radius of gyration. This of course would allow higher compressive stresses. Consider the following two sections, 12′ long, made of A36 steel:

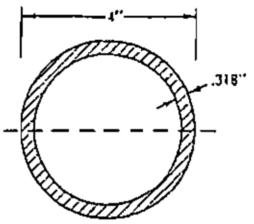


FIGURE 38

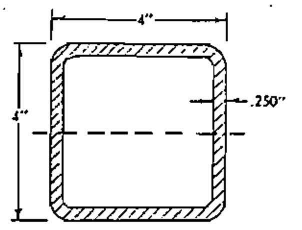


FIGURE 39

3½″ extra-heavy pipe

$A = 3.678$ in.^{2}

$W_t = 12.51$ lbs/ft

$r_{min} = 1.31''$

$$\frac{L}{r} = \frac{(144'')}{(1.31'')} = 110.0$$

$\underline{\sigma_c} = 11{,}670$ psi

$P = (11{,}670)(3.678)$
$= 42.9^k$

4″ × 4″ square tubing

$A = 3.535$ in.^{2}

$W_t = 12.02$ lbs/ft

$r_{min} = 1.503''$

$$\frac{L}{r} = \frac{(144'')}{(1.503'')} = 95.8$$

$\underline{\sigma_c} = 13{,}500$ psi

$P = (13{,}500)(3.535)$
$= 47.6^k$

In this example, the square tube has 3.9% less weight and yet has an allowable load 11% greater. Its radius of gyration is 14.7% greater.

For another example, consider the following A36 section:

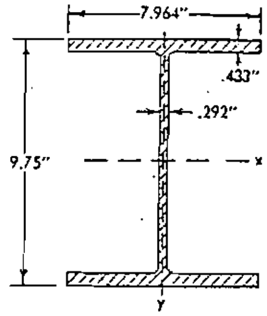


FIGURE 40

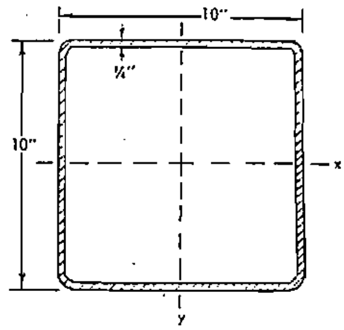


FIGURE 41

10″ WF 33#

$A = 9.71$ in.^{2}

$r_x = 4.20''$

$r_y = 1.94''$

$$\frac{L}{r} = \frac{(144'')}{(1.94'')} = 74.2$$

$\underline{\sigma_c} = 15{,}990$ psi

$P = (15{,}990)(9.71)$
$= 155.0^k$

10″ □ 32#

$A = 9.48$ in.^{2}

$r_{min} = 3.949''$

$$\frac{L}{r} = \frac{(144'')}{(3.95'')} = 36.5$$

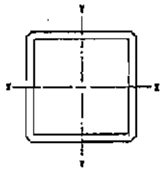
$\underline{\sigma_c} = 19{,}460$ psi

$P = (19{,}460)(9.48)$
$= 184.3^k$

The 32-lb/ft 10″ square tubular section has a radius of gyration which is more than twice that about the weak y-y axis of the 33-lb/ft 10″ WF section. This results in an allowable compressive load 19% greater.

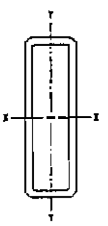
The second advantage to the square and rectangular sections is the flat surface they offer for connections. This results in the simplest and most direct type of joint with minimum preparation and welding. Also by closing the ends, there would be no maintenance problem. It is common practice in many tubular structures not to paint the inside.

TABLE 7
Square Hollow Structural Tubing*



Size, inches	Wall, inches	Weight per foot, pounds	Area of metal, sq. inches	Moment of inertia	Section modulus	Radius of gyration
2x2	.1875	4.31	1.2688	.6667	.6667	.7249
	.250	5.40	1.5890	.7612	.7612	.6921
2½x2½	.1875	5.59	1.6438	1.4211	1.1369	.9298
	.250	7.10	2.0890	1.6849	1.3479	.8921
	.3125	8.44	2.4829	1.8585	1.4868	.8652
3x3	.1875	6.86	2.0188	2.5977	1.7318	1.1344
	.250	8.80	2.5890	3.1509	2.1006	1.1032
	.3125	10.57	3.1079	3.5664	2.3776	1.0712
3½x3½	.1875	8.14	2.3938	4.2904	2.4517	1.3399
	.250	10.50	3.0890	5.2844	3.0196	1.3079
	.3125	12.69	3.7329	6.0826	3.4758	1.2765
4x4	.1875	9.31	2.7383	6.4677	3.2338	1.5369
	.250	12.02	3.5354	7.9880	3.9940	1.5031
	.3125	14.52	4.2720	9.2031	4.5016	1.4677
	.375	16.84	4.9543	10.152	5.0760	1.4315
	.500	20.88	6.1416	11.234	5.6169	1.3524
5x5	.1875	11.86	3.4853	13.208	5.2831	1.9459
	.250	15.42	4.5354	16.595	6.6380	1.9128
	.3125	18.77	5.5220	19.489	7.7955	1.8785
	.375	21.94	6.4543	21.946	8.7784	1.8440
	.500	27.68	8.1416	25.521	10.208	1.7705
6x6	.1875	14.41	4.2383	23.496	7.8322	2.3545
	.250	18.82	5.5354	29.845	9.9482	2.3220
	.3125	23.02	6.7720	35.465	11.822	2.2884
	.375	27.04	7.9543	40.436	13.479	2.2547
	.500	34.48	10.142	48.379	16.126	2.1841
7x7	.1875	16.85	4.9577	37.698	10.771	2.7575
	.250	22.04	6.4817	48.052	13.729	2.7228
	.3125	26.99	7.9389	57.306	16.373	2.6867
	.375	31.73	9.3339	65.544	18.727	2.6499
	.500	40.55	11.927	78.913	22.547	2.5722
8x8	.250	25.44	7.4817	73.382	18.346	3.1318
	.3125	31.24	9.1889	88.095	22.024	3.0963
	.375	36.83	10.834	101.46	25.366	3.0603
	.500	47.35	13.927	124.08	31.021	2.9849
	.625	56.98	16.761	141.41	35.353	2.9046
10x10	.250	37.23	9.4817	147.89	29.578	3.9494
	.3125	39.74	11.589	179.12	35.824	3.9146
	.375	47.03	13.834	208.21	41.642	3.8795
	.500	60.95	17.927	259.81	51.962	3.8069
	.625	73.98	21.761	302.94	60.587	3.7311

TABLE 8
Rectangular Hollow Structural Tubing*



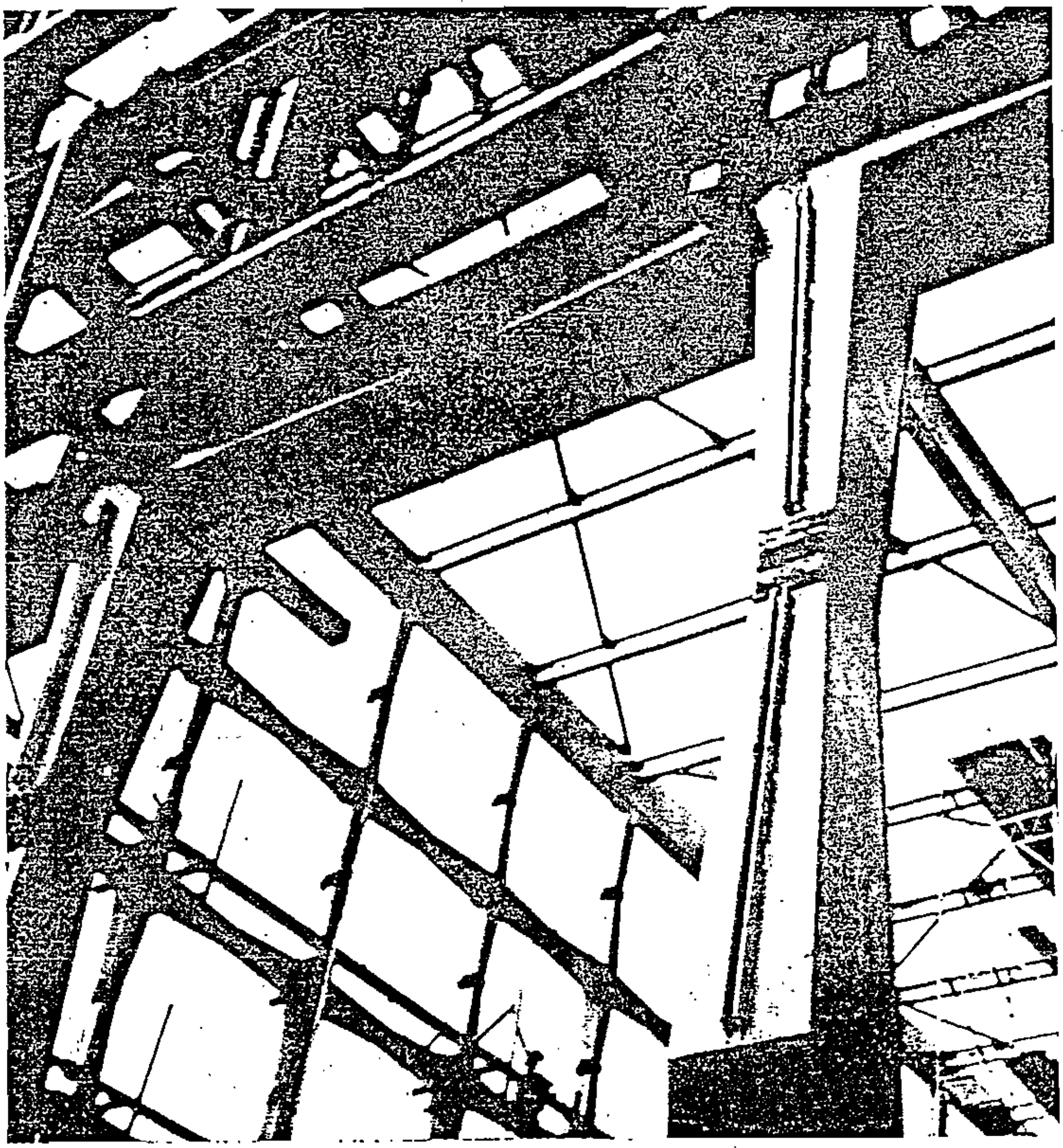
Size, inches: Axis Y-Y	Size, inches: Axis X-X	Wall, inches	Weight per foot, pounds	Area of metal, square inches	Axis X-X: Moment of inertia	Axis X-X: Section modulus	Axis X-X: Radius of gyration	Axis Y-Y: Moment of inertia	Axis Y-Y: Section modulus	Axis Y-Y: Radius of gyration
3	2	.1875	5.59	1.6435	1.8551	1.2367	1.0623	.9758	.9758	.7704
		.250	7.10	2.0890	2.2030	1.4687	1.0269	1.1466	1.1466	.7409
		.3125	8.44	2.4829	2.4327	1.6218	.9898	1.2528	1.2528	.7103
4	2	.1875	6.86	2.0186	3.8654	1.9327	1.3837	1.2849	1.2849	.7978
		.250	8.80	2.5590	4.6593	2.3447	1.3458	1.5321	1.5321	.7692
		.3125	10.57	3.1079	5.3041	2.6520	1.3064	1.7029	1.7029	.7402
4	3	.1875	8.14	2.3938	5.2291	2.6146	1.4780	3.3404	2.2269	1.1813
		.250	10.50	3.0890	6.4459	3.2249	1.4450	4.0988	2.7326	1.1519
		.3125	12.69	3.7329	7.4335	3.7169	1.4112	4.7000	3.1333	1.1221
5	3	.1875	9.31	2.7383	8.8629	3.5452	1.7991	4.0118	2.6746	1.2104
		.250	12.02	3.5354	10.549	4.3797	1.7598	4.9195	3.2797	1.1796
		.3125	14.52	4.2720	12.612	5.0448	1.7182	5.6255	3.7504	1.1475
		.375	16.84	4.9543	13.907	5.5628	1.6754	6.1552	4.1034	1.1146
		.500	20.88	6.1416	15.355	6.1418	1.5812	6.6839	4.4559	1.0432
6	3	.1875	10.58	3.1133	13.991	4.6637	2.1199	4.7545	3.1697	1.2358
		.250	13.72	4.0354	17.435	5.8128	2.0788	5.6675	3.9116	1.2056
		.3125	16.65	4.8970	20.287	6.7622	2.0353	6.7592	4.5061	1.1748
		.375	19.39	5.7043	22.612	7.5373	1.9910	7.4560	4.9706	1.1433
		.500	24.28	7.1416	25.629	8.5431	1.8944	8.2672	5.5115	1.0759
6	4	.1875	11.86	3.4853	17.160	5.7198	2.2179	9.1952	4.5976	1.6236
		.250	15.42	4.5354	21.574	7.1913	2.1810	11.509	5.7544	1.5930
		.3125	18.77	5.5220	25.345	8.4487	2.1424	13.463	6.7313	1.5614
		.375	21.94	6.4543	28.553	9.5178	2.1033	15.097	7.5486	1.5294
		.500	27.66	8.1416	33.213	11.071	2.0198	17.400	8.7002	1.4619
7	5	.1875	14.41	4.2383	29.380	8.3943	2.6329	17.552	7.0210	2.0350
		.250	18.82	5.5354	37.341	10.669	2.5973	22.241	8.8963	2.0045
		.3125	23.02	6.7720	44.396	12.685	2.5604	26.365	10.546	1.9731
		.375	27.04	7.9543	50.645	14.470	2.5233	29.985	11.994	1.9416
		.500	34.48	10.142	60.642	17.326	2.4453	35.688	14.275	1.8759
8	4	.1875	14.41	4.2383	34.826	8.7070	2.8666	11.923	5.9614	1.6772
		.250	18.82	5.5354	44.230	11.058	2.8267	15.030	7.5148	1.6478
		.3125	23.02	6.7720	52.533	13.133	2.7852	17.722	8.8160	1.6177
		.375	27.04	7.9543	59.864	15.966	2.7433	20.042	10.021	1.5874
		.500	34.48	10.142	71.475	17.869	2.6548	23.567	11.784	1.5244
8	6	.1875	16.85	4.9577	45.772	11.443	3.0385	29.549	9.8493	2.4413
		.250	22.04	6.4817	58.362	14.590	3.0007	37.608	12.536	2.4088
		.3125	26.99	7.9389	69.617	17.404	2.9613	44.784	14.928	2.3751
		.375	31.73	9.3339	79.643	19.911	2.9211	51.143	17.048	2.3408
		.500	40.55	11.927	95.916	23.979	2.8358	61.374	20.458	2.2684
10	6	.250	25.44	7.4817	100.35	20.070	3.6623	45.879	15.293	2.4763
		.3125	31.24	9.1889	120.45	24.089	3.6205	54.903	18.301	2.4444
		.375	36.83	10.834	138.69	27.739	3.5783	63.076	21.009	2.4119
		.500	47.35	13.927	169.45	33.896	3.4884	76.541	25.514	2.3443
10	8	.250	28.83	8.4818	124.12	24.824	3.8254	68.403	22.101	3.2284
		.3125	35.49	10.439	149.78	29.957	3.7880	106.57	26.643	3.1916
		.375	41.93	12.334	173.45	34.690	3.7501	123.28	30.821	3.1616
		.500	54.15	15.927	214.64	42.929	3.6711	152.25	38.062	3.0918
12	6	.250	28.83	8.4518	157.33	26.217	4.3065	54.150	18.050	2.5267
		.3125	35.49	10.439	189.65	31.609	4.2624	65.022	21.674	2.4958
		.375	41.93	12.334	219.41	36.589	4.2178	74.909	24.969	2.4644
		.500	54.15	15.927	270.69	45.149	4.1241	91.708	30.569	2.3996

*(1) Tables 7 and 8 are used here by permission of United States Steel Corporation.

(2) Standard sizes listed represent outside dimensions.

(3) These sizes of tubing are normally in stock and available for immediate delivery; other sizes will be stocked or rolled as required.

(4) The weight, area, and other properties given were calculated on the basis of a section with rounded corners and consequently show the actual section properties rather than the idealized version considering the corners as square.



Four all-welded multilayer Vierendeel trusses make up the exposed frame of the beautiful Rare Book Library of Yale University. Weld-fabricated tapered box sections are used in the trusses. Good planning held field welding to a minimum, the trusses being shop built in sections. Here, a cruciform vertical member of the grilled truss is field spliced.