Rigid-Frame Knees (Elastic Design)

1. GENERAL REQUIREMENTS

The knee is an important part of a rigid frame and some thought should be given to its design.

The knee of any rigid frame must be capable of—

1. Transferring the end moment from the beam into the column.
2. Transferring the vertical shear at the end of the beam into the column.
3. Transferring the horizontal shear of the column into the beam.

A knee differs from the usual straight beam in these respects:

1. The neutral axis shifts toward the inner flange, causing an increase in the usual bending forces at this point.
2. Axial flange forces must change direction, causing radial forces to be set up.

2. EVALUATION OF KNEE TYPES

Figure 1 illustrates the five principal types of knees for rigid frames.

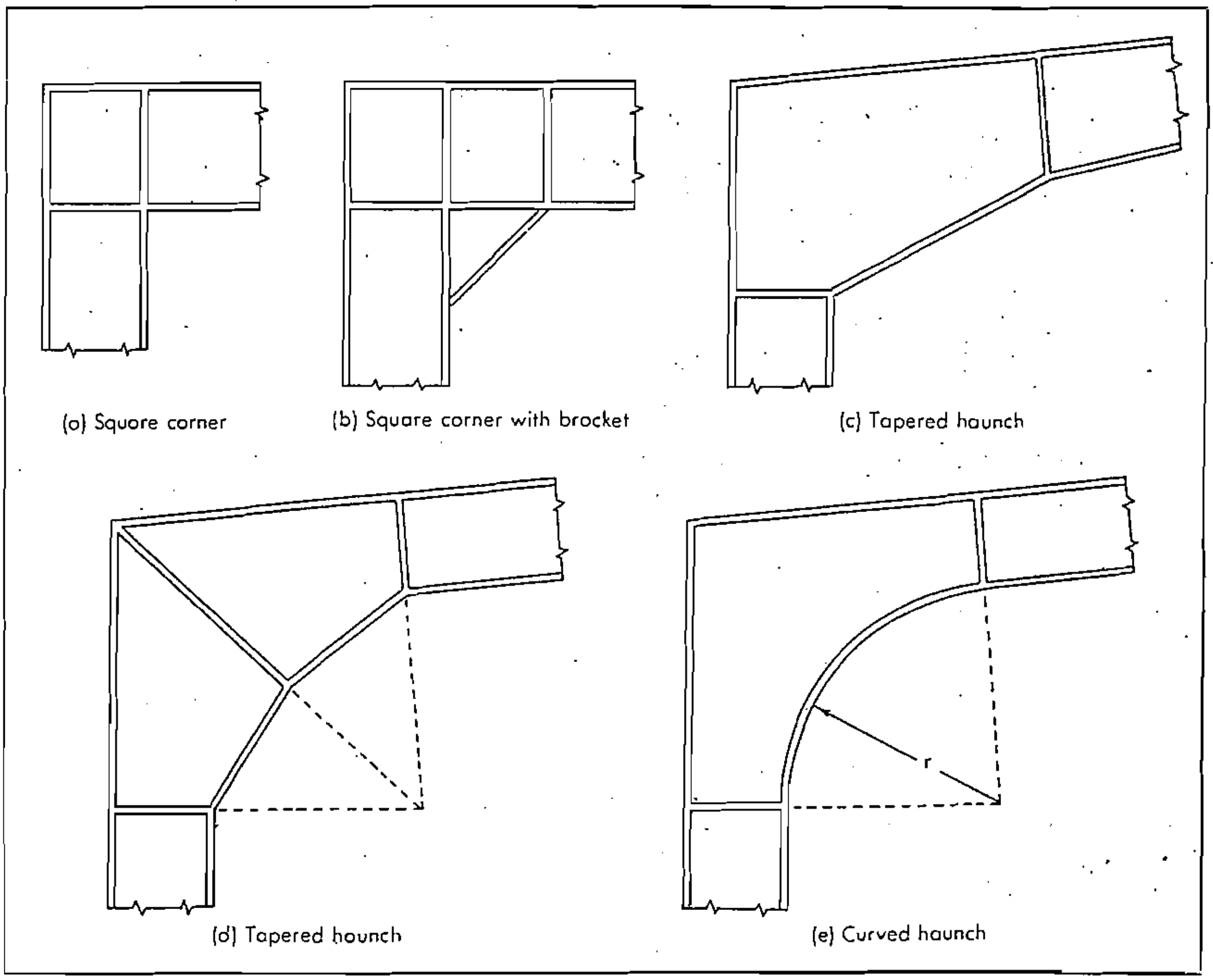


FIGURE 1

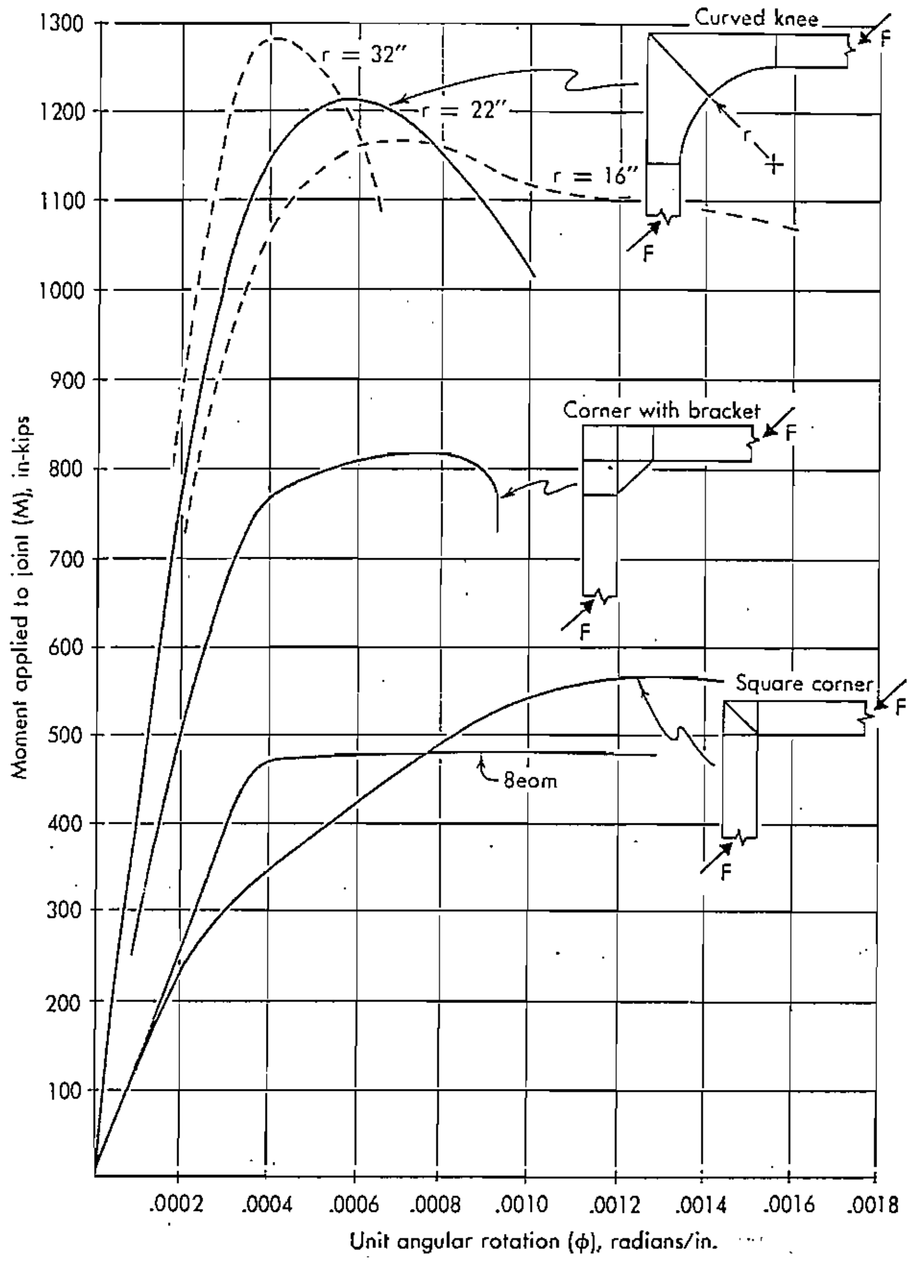


FIGURE 2

It might be thought that the simple square type of knee connection would naturally be as rigid as the connecting members, since it is a continuation of the same section. In many cases, this is true. However, stress causes strain, and the accumulation of strain over a distance results in a movement of some kind: deflection, angular movement, etc. This means that the sharp corner of this joint increases the stress in this region by several times. This stress concentration results in a higher strain and, therefore, greater movement in this local region.

With the square type of knee in which just flange stiffeners are added, it is difficult to exceed the stiffness of the member. In most cases it will just equal the member, and in some cases it will be less.

Figure 2 shows moment-rotation curves of various knee connections.* The vertical axis is the applied moment; the horizontal axis is the resulting rotation of the connection. The vertical height of the curve represents the maximum or ultimate strength of the connection. The slope of the straight portion of the curve represents the stiffness of the connection, with the more nearly vertical curves being the stiffer. The right-hand extremity of the curve represents the rota-

* Figure 2 adapted from "Connections for Welded Continuous Portal Frames", Beedle, Topractsoglon, and Johnston; AWS Journal; Part I July 1951, Part II August 1951, and Part III November 1952.

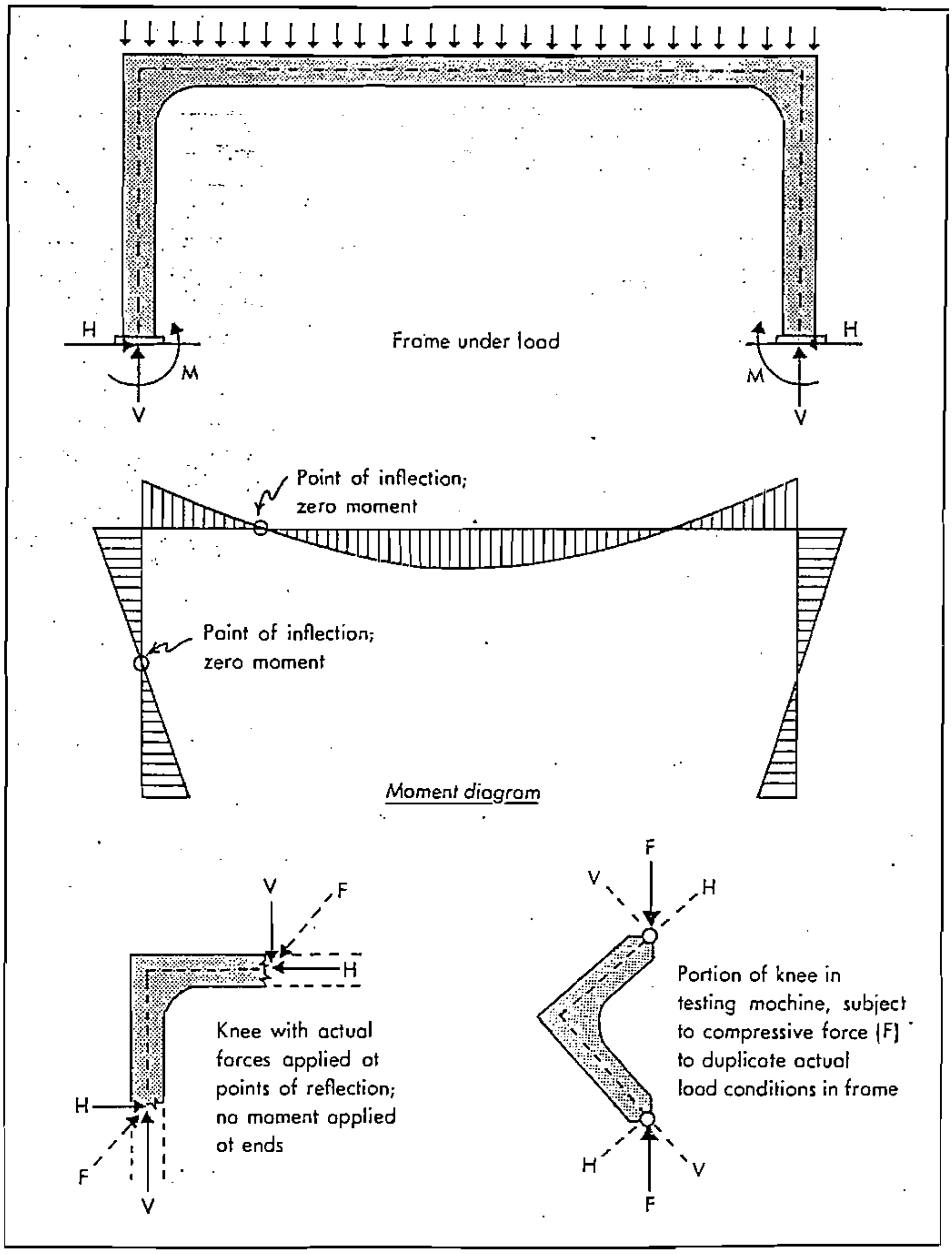


FIGURE 3

tional capacity of the connection.

Notice that the square-corner knee is the most flexible. It falls slightly short of the beam itself, but it does have the greatest rotational capacity. Tapered haunch knees (not shown here) and those with the additional bracket have greater stiffness and higher moment capacity, but less rotational capacity. The curved knees are the most rigid, have the highest moment capacity, and have a rotational capacity somewhere between the simple square corner and the haunched knee. As the radius of curvature of this inner flange is increased, the stiffness and moment capacity increase slightly, with slightly lower rotational capacity. Another purpose of the haunched and curved knees is to move the connection to the beam back into a region of lower moment so that the beam will not be overstressed in bending.

The dimensions of the test knee are so chosen that they extend out to the point of inflection (zero moment) of an actual frame; Figure 3.

In this manner, the testing machine applies a compressive force (F) which becomes the component of the two forces V (vertical) and H (horizontal) which would actually be applied to the knee at the frame's point of inflection.

3. SHEAR IN CONNECTION WEB

An axial force (tensile or compressive) can transfer sideways out of one element of a member as shear. For example, the tensile force from the beam flange will transfer down through the connection web as shear into the supporting column; Figure 4.

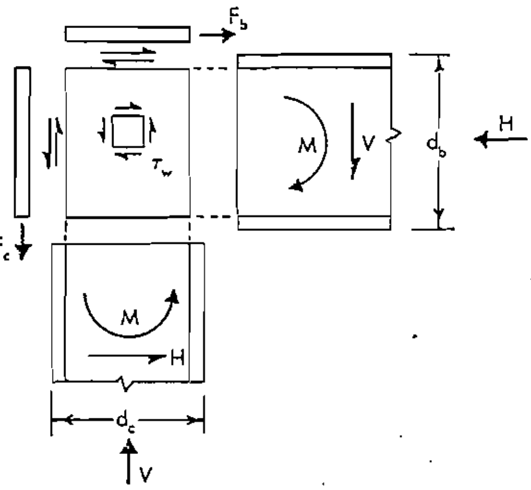


FIGURE 4

where the flange force in the beam is—

$$F_b = \frac{M}{d_b}$$

and the flange force in the column is—

$$F_c = \frac{M}{d_c}$$

Assuming this flange force (F) is transferred as shear into the connection web within the distance equal to the depth of the connecting member, the resulting shear stress within this connection web is—

$$\boxed{\tau = \frac{F_b}{t_w\ d_c} = \frac{F_c}{t_w\ d_b}} \qquad \text{(1)}$$

If this shear stress exceeds the allowable for the web, it must be reduced by increasing the web thickness within the connection area. Or, a pair of diagonal stiffeners must be added to transfer some of this flange force as a diagonal component.

One method of detailing this connection is to calculate the portion of the flange force which may be transferred as shear within the web by stressing it to the allowable. Then, diagonal stiffeners are detailed to transfer whatever flange force remains.

Another method is to assume that the shortening of the diagonal stiffener under the compression component is equal to the diagonal shortening of the web due to the shear stress. From this, the resulting shear stress (τ_w) in the web and the compressive stress (σ_s) in the diagonal stiffener may be found for any given set of conditions.

Derivation of Stress Values

The final diagonal dimension (d_1) of the web, due to shear action on the web, will be—

$$d_1^2 = d_b^2 + d_c^2 - 2\ d_b\ d_c \cos\ (90° - \gamma)$$

but

$$\cos\ (90° - \gamma) = \cos 90° \cos\ (\gamma) + \sin 90° \sin\ (\gamma)$$
$$= \sin \gamma$$

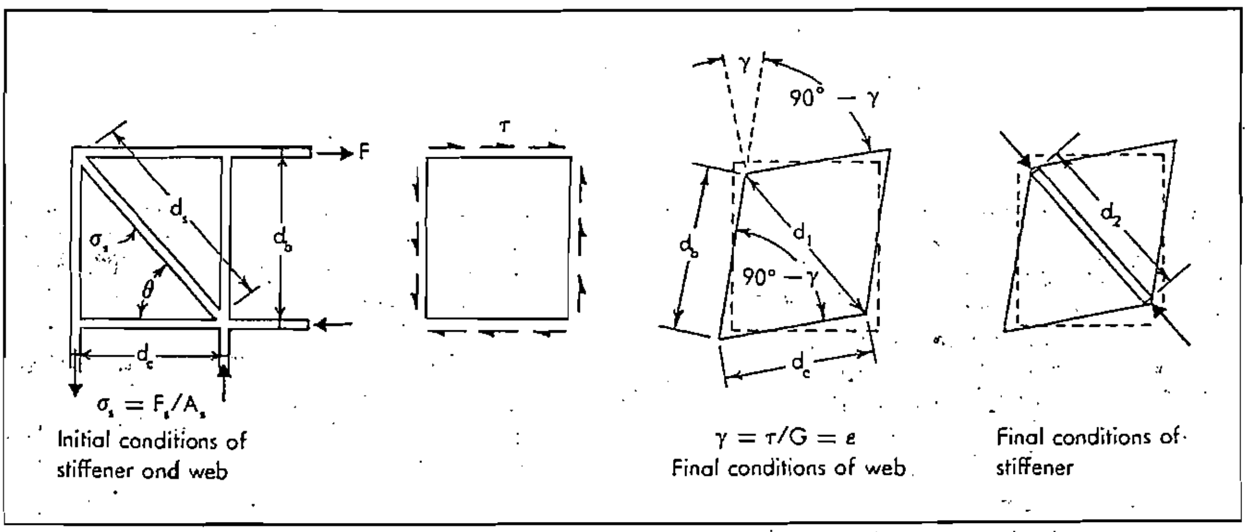


FIGURE 5

For small strains (ϵ_s) and angles (γ)—

$$\sin(\gamma) = \tan(\gamma) = \epsilon_s$$

Hence:

$$d_1^2 = d_b^2 + d_c^2 - 2\, d_b\, d_c \frac{\tau}{E_s} \quad \text{and}$$

$$d_1 = \sqrt{d_b^2 + d_c^2 - 2\, d_b\, d_c \frac{\tau}{E_s}} \quad \text{but}$$

$$d_b = d_c \tan\theta = d_c \frac{\sin\theta}{\cos\theta}$$

$$\therefore d_1 = \sqrt{d_c^2 \frac{\sin^2\theta}{\cos^2\theta} + d_c^2 - 2\, d_c^2 \frac{\tau}{E_s} \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{d_c}{\cos\theta} \sqrt{1 - 2\frac{\tau}{E_s} \sin\theta \cos\theta}$$

The final dimension of the diagonal stiffener (d_2), due to compression, will be—

$$\epsilon = \frac{\sigma_s}{E}$$

Since the movement—

$$\Delta = \epsilon\, d_s$$

$$\text{so} \quad \Delta = \frac{\sigma_s\, d_s}{E}$$

$$\therefore d_2 = d_s - \Delta = d_s - \frac{\sigma_s\, d_s}{E}$$

$$= d_s\left(1 - \frac{\sigma_s}{E}\right) = \frac{d_c}{\cos\theta}\left(1 - \frac{\sigma_s}{E}\right)$$

Since diagonal stiffener and web are attached, the final dimension of diagonals in each case must be equal:

$$d_1 = d_2 \quad \text{or}$$

$$\frac{d_c}{\cos\theta}\sqrt{1 - 2\frac{\tau}{E_s}\sin\theta\cos\theta} = \frac{d_c}{\cos\theta}\left(1 - \frac{\sigma_s}{E}\right)$$

Squaring both sides:

$$1 - 2\frac{\tau}{E_s}\sin\theta\cos\theta = 1 - 2\frac{\sigma_s}{E} + \frac{\sigma_s^2}{E^2} \quad \text{could neglect this last term}$$

or

$$\frac{\tau}{E_s}\sin\theta\cos\theta = \frac{\sigma_s}{E}$$

Since for steel:

$E = 30{,}000{,}000$ psi

$E_s = 12{,}000{,}000$ psi

$\therefore E = 2.5\, E_s$

and the compressive stress in the diagonal stiffener is—

$$\boxed{\sigma_s = 2.5\, \tau \sin\theta \cos\theta} \quad \text{(2)}$$

Now we go back to the flange force (F) since it causes this load on the connection region.

The flange force of the beam is equal to the shear force carried by the web plus the horizontal component of the compressive force carried by the diagonal stiffener.

$$F = \tau\, t_w\, d_c + \sigma_s\, A_s \frac{d_c}{d_n} \quad \text{or}$$

$$\boxed{F = \tau\, t_w\, d_c + \sigma_s\, A_s \cos\theta} \quad \text{(3)}$$

Substituting (2) into (3) gives—

$$F = \tau\, t_w\, d_c + (2.5\, \tau \sin\theta\cos\theta)\, A_s \cos\theta$$

$$= \tau\, [t_w\, d_c + 2.5\, A_s \sin\theta \cos^2\theta]$$

or, the shear stress in the connection web is—

$$\boxed{\tau = \frac{F}{t_w\, d_c + 2.5\, A_s \sin\theta\cos^2\theta}} \quad \text{(4)}$$

Also, from (2)—

$$\tau = \frac{\sigma_s}{2.5 \sin\theta\cos\theta}$$

Substituting this into (3)—

$$F = \left(\frac{\sigma_s}{2.5\sin\theta\cos\theta}\right) t_w\, d_c + \sigma_s\, A_s \cos\theta$$

$$= \sigma_s\left(\frac{t_w\, d_c}{2.5\sin\theta\cos\theta} + A_s\cos\theta\right)$$

or, the compressive stress in the diagonal stiffener is—

$$\boxed{\sigma_s = \frac{F}{\dfrac{t_w\, d_c}{2.5\sin\theta\cos\theta} + A_s\cos\theta}} \quad \text{(5)}$$

Some knees are more complex than those described here and analysis must consider factors that are covered more adequately in Section 2.12, Buckling of Plates.

Problem 1

To check stiffener requirements on the square knee connection shown in Figure 6, for the loads indicated. A36 steel and E70 welds are used.

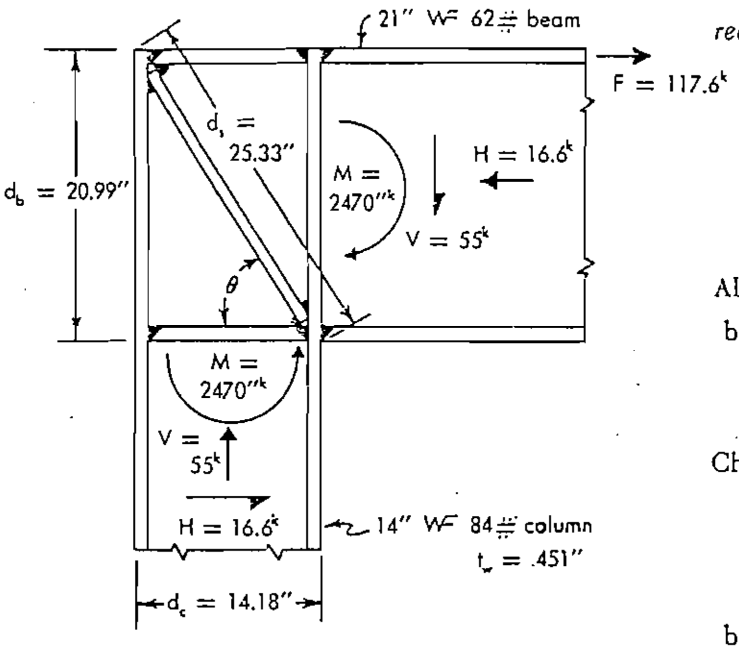


FIGURE 6

Here:

$$\sin\theta = \frac{20.99}{25.33} = .829$$

$$\cos\theta = \frac{14.18}{25.33} = .561$$

$$\tan\theta = \frac{20.99}{14.18} = 1.480$$

flange force on the beam

$$F = \frac{M}{d_b} = \frac{(2470)}{(20.99)} = 117.6 \text{ kips}$$

Method 1

horizontal component carried by web in shear

$$F_w = \tau\, t_w\, d_c = (14{,}500)(.451)(14.18) = 92.8 \text{ kips}$$

This leaves (117.6 — 92.8 =) 14.8 kips to be carried by the horizontal component of the compressive force on the diagonal stiffener.

compressive force on stiffener

$$F_s = 14.8\left(\frac{25.83}{14.18}\right) = 26.4 \text{ kips}$$

required sectional area of stiffeners

$$A_s = \frac{F_s}{\sigma_s} = \frac{(26.4)}{(22.0)} = 1.2 \text{ in.}^2 \text{ (pair)}$$

Also required:

$$b_s/t_s = 17$$

Hence, use a pair of ½″ x 3″ diagonal stiffeners.

Checking this size against the requirements:

$$A_s = 2 \times ½″ \times 3″ = 3.0 \text{ in.}^2 > 1.2 \text{ in.}^2 \quad \text{OK}$$

$$b_s/t_s = \frac{2 \times 3″}{½″} = 12 < 17 \quad \text{OK}$$

Method 2 Plastic Design (See Sect. 5.12)

required thickness of connection web

$$w_r = \frac{\sqrt{3}\, M_p}{\sigma_y\, d_b\, d_c} = \frac{\sqrt{3}\,\sigma_y\, Z}{\sigma_y\, d_b\, d_c} = \frac{\sqrt{3}\,(144.1)}{(20.99)(14.18)} = .837″$$

This exceeds the actual web thickness of t_w = .451″, so stiffening is required.

required area of diagonal stiffeners

$$A_s = \frac{d_s}{\sqrt{3}}(w_r - w_c) = \frac{(25.33)}{\sqrt{3}}(.837) - .451) = 5.64 \text{ in.}^2 \text{ (pair)}$$

Use a pair of ¾″ x 4″ diagonal stiffeners.

Checking this size against the requirements:

$$A_s = 2 \times ¾″ \times 4″ = 6.0 \text{ in.}^2 > 5.64 \text{ in.}^2 \quad \text{OK}$$

$$b_s/t_s = \frac{2 \times 4″}{¾″} = 10.7 < 17 \quad \text{OK}$$

Method 3 Start with a pair of ½" x 3" diagonal stiffeners and, assuming both diagonals contract the same amount under load, check stresses in web and stiffener.

shear stress in web

$$\tau = \frac{F}{t_w\ d_c + 2.5\ A_s \sin\theta \cos^2\theta}$$

$$= \frac{117.6}{(.451)(14.18) + 2.5\ (3.0)(.829)(.561)^2}$$

$$= \underline{14{,}080 \text{ psi}}$$

compressive stress in diagonal

$$\sigma_s = \frac{F}{\dfrac{t_w\ d_c}{2.5 \sin\theta \cos\theta} + A_s \cos\theta}$$

$$= \frac{117.6}{\dfrac{(.451)(14.18)}{2.5\ (.829)(.561)} + (3.0)(.561)}$$

$$= \underline{14{,}200 \text{ psi}}$$

As a matter of interest, increasing the size of the diagonal stiffener to ¾" x 4" would decrease these stresses to—

$$\tau = 11{,}400 \text{ psi}$$

$$\sigma_s = 13{,}250 \text{ psi}$$

4. COMPRESSIVE FORCES IN CONNECTION WEB

An axial force is able to change its direction if suitable resisting components of force are available.

In the square or tapered haunch, this abrupt change in direction of the compressive flange force is accomplished by means of a diagonal stiffener; Figure 7(b).

In the curved haunch, this change in direction of the axial force is uniform along the curved edge of the flange and results from radial compressive forces in the web; Figure 7(a).

The force in the inner flange of the knee is greater than the force in the outer flange because it has a smaller radius of curvature. Usually this inner flange is the compression flange; therefore, this is the region to be checked for stiffening requirements using the following formula for radial compressive forces in the web.

$$\boxed{f_r = \frac{F_c}{r_i} \text{ lbs/linear in. of web}} \quad \ldots\ldots\ldots\ldots(6)$$

In this case, the unit radial force (f_r) is a function of the compressive force (F_c) in the flange and the radius of curvature (r_i) of the flange.

This action is similar to the radial pressure applied *to the rim of a pulley by the tensile forces in the belt.*

As the radius of curvature decreases, these forces increase.

As this change in direction of the flange becomes more abrupt, as in a square or tapered haunch, these radial forces are concentrated into a single force. And, they must be resisted by a diagonal stiffener; Figure 5(b).

The axial force in the flange is assumed to be uniformly distributed across the width, therefore the *radial pressure or stress is—*

$$\boxed{\sigma_r = \frac{F_c}{r_i\ b_f} = \frac{\sigma\ t_f}{r_i} = \frac{f_r}{b_f}} \quad \ldots\ldots\ldots\ldots(7)$$

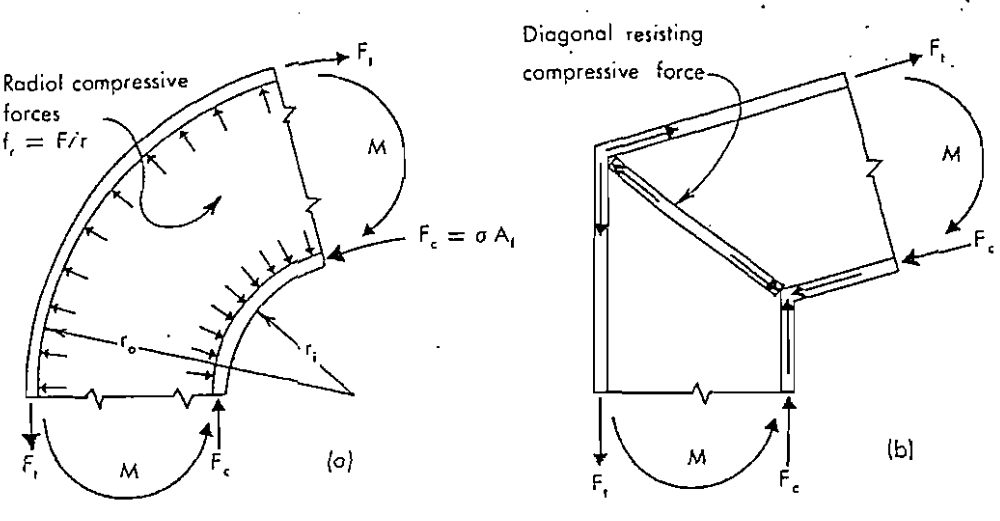


FIGURE 7

When applied to the flange, this radial stress will load any cross-section as a cantilever beam, since it is supported only along its centerline by the web; Figure 8.

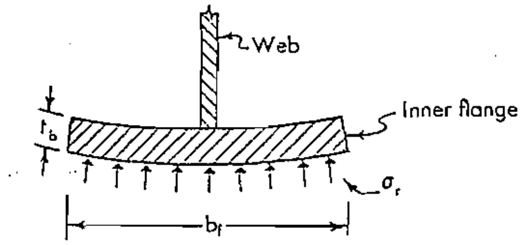


FIG. 8 Cross-section of lower flange and web.

The bending moment along the centerline of the beam flange due to this radial load will be:

$$M = \frac{\sigma_r}{2}\left(\frac{b_f}{2}\right)^2 = \frac{\sigma_r\ b_f^2}{8}$$

$$= \frac{\sigma\ t_f\ b_f^2}{r_i\ 8}$$

Also:

$$M = \sigma_t\ S$$

$$= \frac{\sigma_t\ t_f^2}{6} \quad \text{or}$$

Where:

$$S = \frac{1''\ t_f^2}{6}$$

$$\frac{\sigma_t\ t_f^2}{6} = \frac{\sigma\ t_f\ b_f^2}{8\ r_i} \quad \text{and}$$

$$\sigma_t = \frac{3}{4}\ \frac{\sigma\ b_f^2}{r_i\ t_f}$$

From this relationship, it is seen that in order to hold the transverse tensile stress (σ_t) to a value not exceeding the axial compressive stress of the flange (σ), the following must be held:

$$\boxed{\frac{b_f^2}{r_i\ t_f} \leqq \frac{4}{3} \text{ or } 1⅓} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

If this value is exceeded, stiffeners would be used between the inner compressive curved flange and web.

b_f = width of flange

t_f = thickness of flange

r_i = radius of curvature of inner flange

σ_t = transverse tensile stress in flange

σ = axial compressive stress in flange

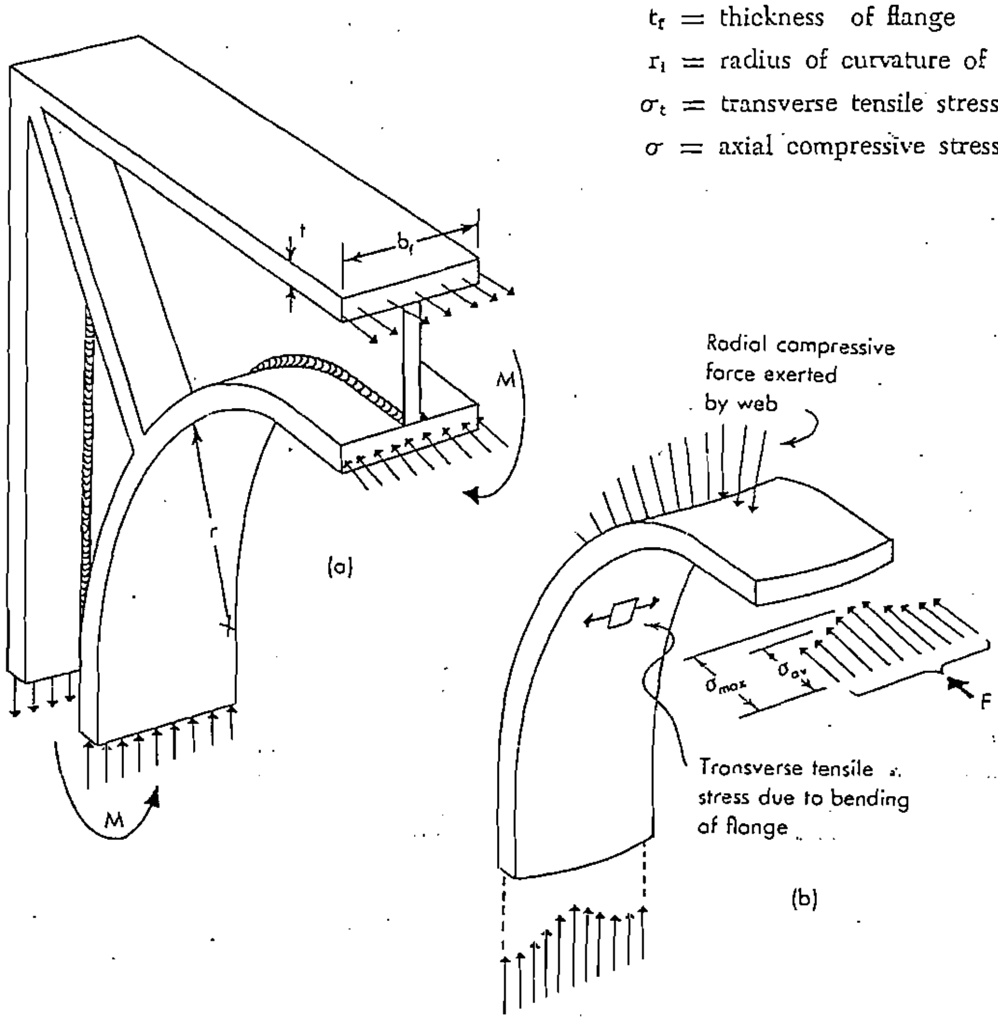


FIGURE 9

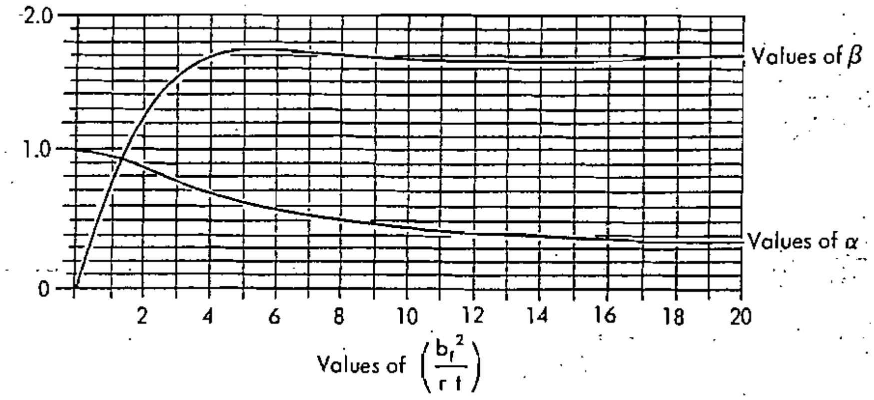


FIGURE 10

This analysis assumes a uniform distribution of stress across the cross-section of the flange.

If this is based on plastic design, the plastic section modulus (Z) is used instead of section modulus (S), where

$$Z = \frac{1''\ t_f^2}{4} \qquad S = \frac{1''\ t_f^2}{6}$$

Then (7) becomes the following:

$$\boxed{\frac{b_b^2}{r_i\ t_b} \leqq 2} \quad \dots\dots\dots\dots\dots\dots\dots\dots (9)$$

Bleich has carried this analysis a little further; see Figure 9.*

Because of the slight yielding of the flange's outer edge, there is a non-uniform distribution of flange stress (σ). This compressive stress is maximum in line with the web. In the following formula, the value of α comes from the graph, Figure 10.

$$\boxed{\sigma_{max} = \frac{\sigma}{\alpha}}^{*} \quad \dots\dots\dots\dots\dots\dots\dots\dots (10)$$

The transverse tensile bending stress (σ_t) in the curved flange is found in the following formula; the value of β comes from the graph, Figure 10.

$$\boxed{\sigma_t = \beta\ \sigma_{max}}^{*} \quad \dots\dots\dots\dots\dots\dots\dots\dots (11)$$

If this value is too high, stiffeners should be welded between this flange and the web. These keep the flange from bending. These stiffeners usually need not extend all the way between flanges, but may be a series of short triangular plates connecting with the curved flange.

The unit radial compressive force (f_r) which acts transverse to the connecting fillet welds between the curved flange and the web is found from—

* From "Design of Rigid Frame Knees" F. Bleich, AISC.

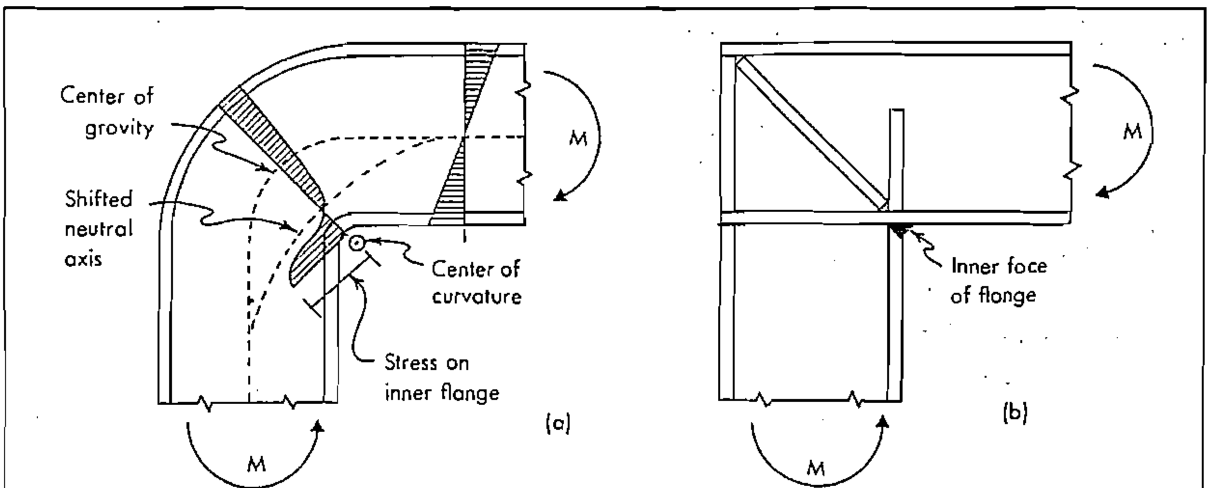


FIGURE 11

$$\boxed{\begin{array}{c} f_r = \dfrac{F}{r} \text{ lbs/linear in.} \\ (2 \text{ welds}) \end{array}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

5. EFFECT OF RADIUS OF CURVATURE ON STRESS IN INNER CORNER

A straight beam has an infinite radius of curvature ($r = \infty$). As the beam becomes curved, this radius decreases, and the neutral axis no longer coincides with the center of gravity, but shifts toward the inner face. See Figure 11 (a).

Because of the shift of the neutral axis, the bending stress in the inner flange increases greatly while the bending stress in the outer flange decreases. This increase at the inner flange becomes more severe as the radius of curvature decreases.

In a square-knee connection, this radius of curvature is provided by only the reinforcement of the bevel groove weld or fillet weld on this inside corner; Figure 11 (b). For this reason, the square knee may not quite develop the full plastic moment of the connecting member unless it is somehow reinforced.

If for some reason a reversal in moment should be applied to the knee and the inner face of the knee is subjected to tension instead of the usual compression, it is important that this be a good sound weld. This is especially true at the surface of the weld. If the knee is loaded up to its plastic moment, the metal within the section below the weld is stressed up to its yield strength. During this time, the weld undergoes a considerable amount of plastic yielding and some strain hardening. The weld metal does have the ability to elongate about 28% as measured in 2″ before failure. However, this zone in which the yielding is confined is very narrow, being the width of the weld. Consequently, the overall movement of the connection due to plastic yielding of the weld is very low, although sufficient.

In this case almost all of the weld's ability to elongate may be used in developing the plastic moment of the connection. Any defect in the weld which would lower its ductility would probably prevent the connection from reaching its plastic moment. The knee could have greater strength and rotational capacity if this inner face were changed to a haunched or curved knee section. In testing these square knees in tension, plastic moment was reached when this weld was of good quality. Fortunately most knees are stressed in compression at this inner corner, without any tendency for this weld to fail.

6. LOCATING SECTION OF HAUNCH TO CHECK

Most theories concerning the strength of knees differ only in the placing of the neutral axis, and in locating the resulting section for determining the section modulus.

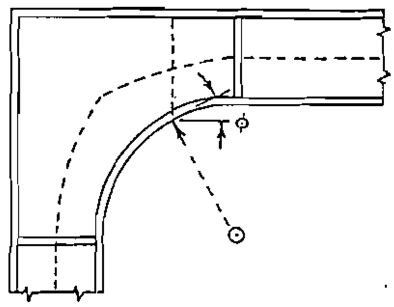


FIGURE 12

One method, Figure 12, uses straight sections normal to the axis of either the beam or column. The section modulus is determined about an axis through the center of gravity of the section. The resulting stress in the inner flange is increased by the factor

$$\frac{1}{\cos^2 \phi}$$

where ϕ is the slope of the flange. Although this method is easy, it might indicate excessively high stresses when the flange has a rather steep slope.

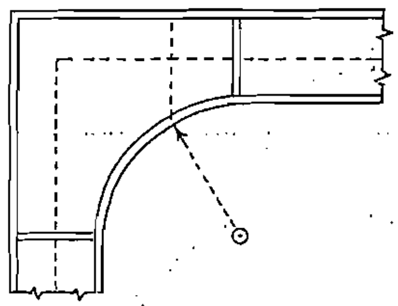


FIGURE 13

Another method, Figure 13, is to extend the centerlines of the beam and column to intersect in the knee. Straight sections are used, and the section modulus is determined about an axis lying on this centerline. This will give conservative values for the stress in the sloping flange. Because of this, no factor is used for the stress on the sloping flange.

A more accurate but longer method, Figure 14, is based on a curved section forming a wedge beam by

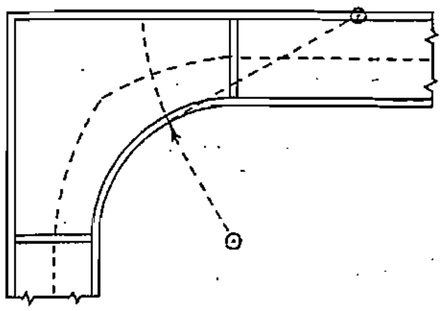


FIGURE 14

W. R. Osgood* and modified by H. C. Olander.**

* "Theory of Flexure for Beams with Nonparallel Extreme Fibers" by W. R. Osgood, ASME Vol. 61, 1939.
** "Stresses in the Corners of Rigid Frames" by H. O. Olander, ASCE Transactions Paper 2698, 1953.

Method of Using a Straight Cross-Section

Dimension of Straight Section
The dimensions of a straight section (A-B) of the haunch may be found from the following:

Here:

$$\boxed{v = r \sin(2\alpha)} \quad \ldots\ldots (13)$$

$$\boxed{d_h = d + r(1 - \cos 2\alpha)} \quad \ldots\ldots (14)$$

Bending Stress in Curved Flange (See Figure 16.)

Here:

$$b = a \cos \phi$$

$$f_b = \frac{f_a}{\cos \phi}$$

$$\sigma_a = \frac{f_a}{a \times 1''} \text{ or } a = \frac{f_a}{\sigma_a\ 1''}$$

$$\sigma_b = \frac{f_b}{b \times 1''}$$

$$= \frac{f_a}{\cos \phi} \times \frac{1}{a \cos \phi}$$

$$= \frac{f_a}{\cos \phi} \times \frac{\sigma_a \times 1''}{f_a \cos \phi} \quad \text{or}$$

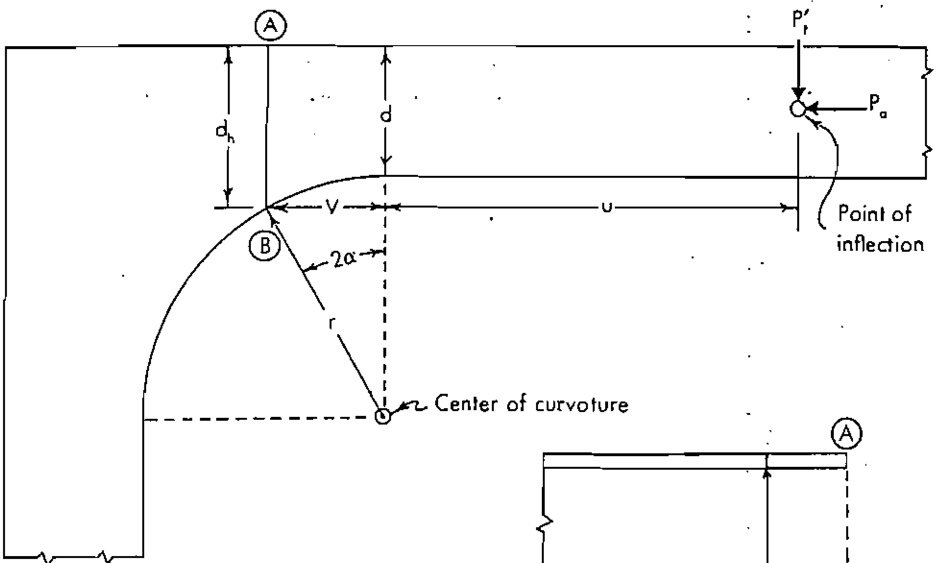


FIGURE 15

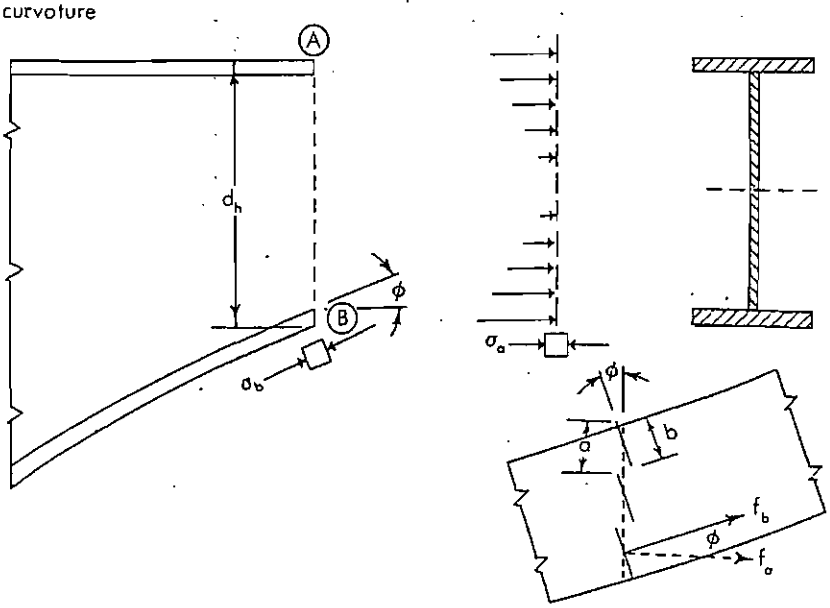


FIGURE 16

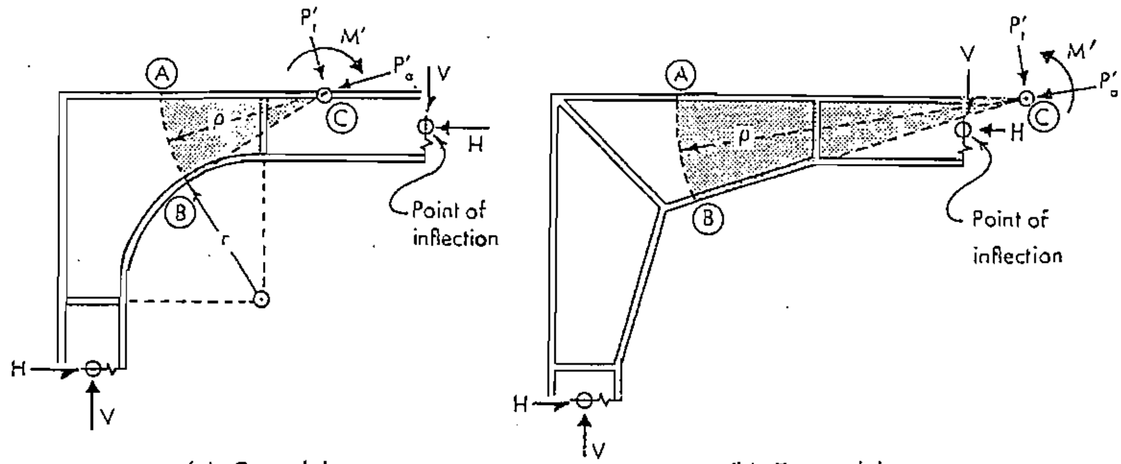


FIGURE 17

$$\boxed{\sigma_b = \frac{\sigma_n}{\cos^2 \theta}} \quad \text{(15)}$$

Wedge Method of Determining Section

The wedge method may be used on any beam section whose flanges are not parallel.

A curved section (A-B) is constructed where the stresses are to be checked. This is normal to both flanges and has a radius (ρ) the center of which lies on the straight flange. See Figure 17.

The transverse force (P_t'), axial force (P_a'), and moment (M') acting at the apex (C) of the wedge are found. See Figure 18.

Here:

$$\boxed{\rho = \frac{d}{\sin(2\alpha)} + \frac{r[1 - \cos(2\alpha)]}{\sin(2\alpha)}} \quad \text{(16)}$$

$$\boxed{n = \frac{d}{\tan(2\alpha)} - \frac{r[1 - \cos(2\alpha)]}{\sin(2\alpha)}} \quad \text{(17)}$$

$$\boxed{m = u - n} \quad \text{(18)}$$

$$\boxed{d_h = \rho\, 2\alpha} \quad \text{(19)}$$

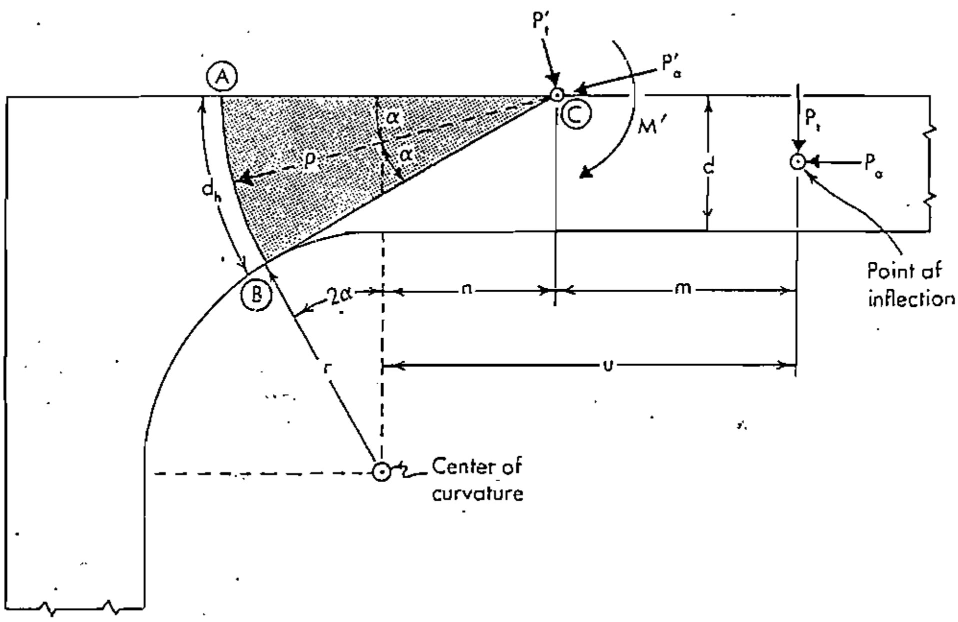


FIGURE 18

transverse force applied to wedge at point C

$$\boxed{P_t' = P_t \cos \alpha - P_a \sin \alpha} \quad \ldots\ldots\ldots\ldots (20)$$

axial force applied to wedge at point C

$$\boxed{P_a' = P_a \cos \alpha + P_t \sin \alpha} \quad \ldots\ldots\ldots\ldots (21)$$

moment about point C

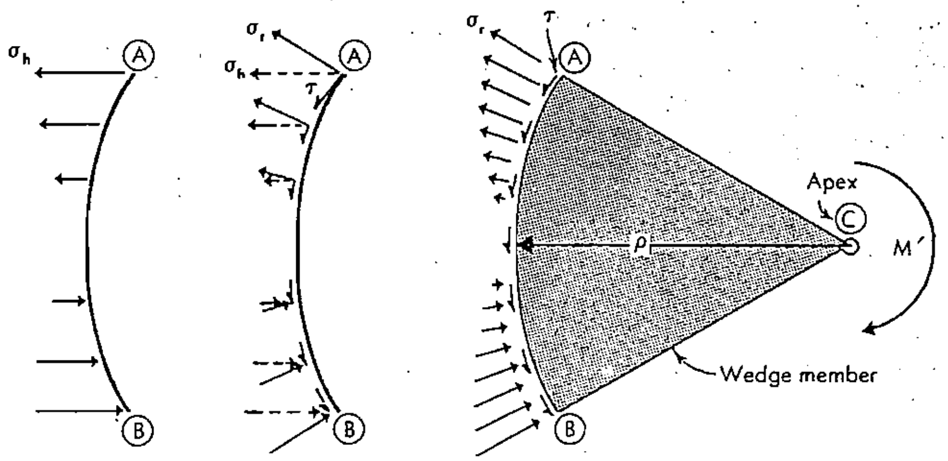
$$\boxed{M' = + P_t\, m + P_a \frac{d}{2}} \quad \ldots\ldots\ldots\ldots (22)$$

These applied forces result in various stresses on the curved haunch section, as described in following paragraphs.

Moment (M') Applied to Wedge Member

The horizontal bending stresses (σ_h) resulting from the applied moment (M'), Figure 19(a), may be replaced with its two components: radial bending stress (σ_r) and tangential shear stress (τ), Figure 19(b). In Figure 19(c) are shown the resulting stresses.

It is seen in taking moments about the apex (C) of the wedge that all of the radial bending stresses pass through this point and cannot contribute to any moment. The tangential shear stresses along the curved section (A-B) acting normal to, and at a distance (ρ)

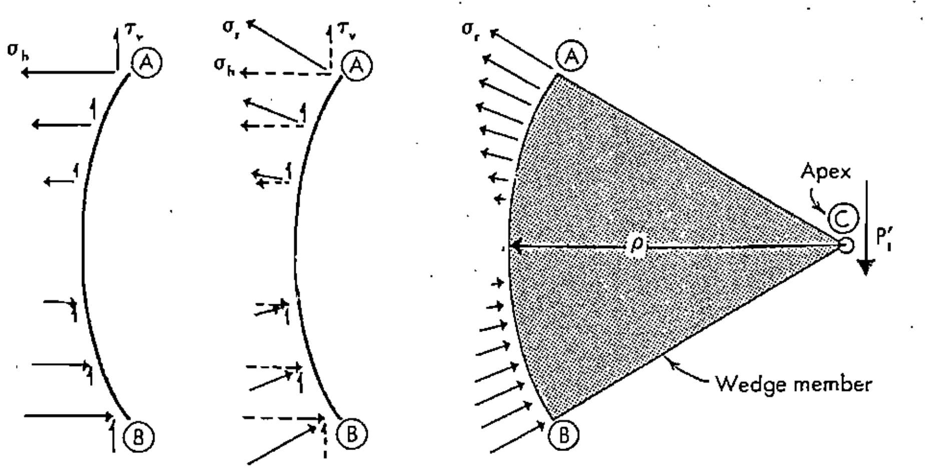


(a) Resisting horizontal bending stress

(b) Components of bending stress

(c) Resisting radial bending stress (σ_r) normal to curved section (A—B); also tangential shear stress (τ)

FIGURE 19



(a) Resisting horizontal bending stress and vertical shear stress

(b) Components of these two stresses

(c) Resisting radial bending stress (σ_r) normal to curved section (A—B)

FIGURE 20

will produce an equal and opposite moment. The value of this tangential shear force (V) acting on this curved section (A-B) may be found from the following:

$$\boxed{V = \frac{M'}{\rho}} \qquad (23)$$

Transverse Force (P_t') Applied to Wedge Member

The applied transverse force (P_t') results in horizontal bending stresses (σ_h) as well as vertical shear stresses; Figure 20(a).

These two stresses may be completely replaced with a single component, radial bending stress (σ_r); Figure 20(b). The results are shown in Figure 20(c). Notice that no tangential shear stresses are present.

Axial Force (P_a') Applied to Wedge Member

The axial force (P_a') applied at the apex of the wedge member, causes radial stresses to occur along the curved section (A-B); Figure 21. There are no tangential shear stresses from this force, because they cancel out.

Summary

The effects of all these forces applied to the wedge member may be summarized as follows:

shear stress on section A-B

$$\boxed{V = \frac{M'}{\rho}}$$

$$\boxed{\tau = \frac{V\,Q}{I\,t} = \frac{V(A_f y_f + A_w y_w)}{I\,t}} \qquad (24)$$

moment applied to section A-B

$$\boxed{M = M' + P_t'\,\rho} \qquad (25)$$

normal stress on inner flange

$$\boxed{\sigma_r = + \frac{P'_a}{A} + \frac{M\,c_i}{I}} \qquad (26)$$

normal stress on outer flange

$$\boxed{\sigma_r = - \frac{P'_a}{A} + \frac{M\,c_o}{I}} \qquad (27)$$

Problem 2

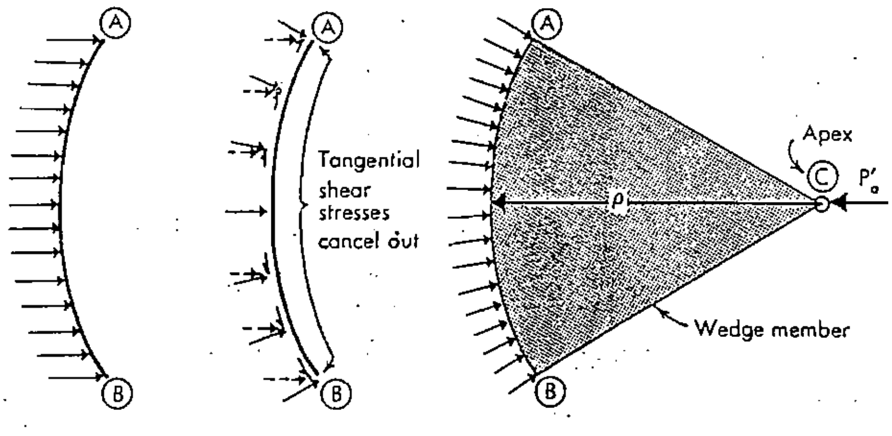
To check stresses and stiffener requirements on the knee connection shown in Figure 22, for the loads indicated. A36 steel and E70 welds are used.

STEP 1: Check Lower Curved Flange (Figure 23)

properties of haunch section (1-1)

Use reference axis (y-y) through centerline of web plate.

Plate	A	y	$M = A \cdot y$	$I_y = M \cdot y$	I_g
3/4" x 10"	7.50	+24.500	+183.75	+4502.	—
1/2" x 48.25"	24.125	0	0	0	4681
1" x 10"	10.000	−24.625	−246.25	+6064	—
Total	41.625		− 62.50	15.247	



(a) Resulting axial stress

(b) Components of axial stress

(c) Resisting radial stresses (σ_r)

FIGURE 21

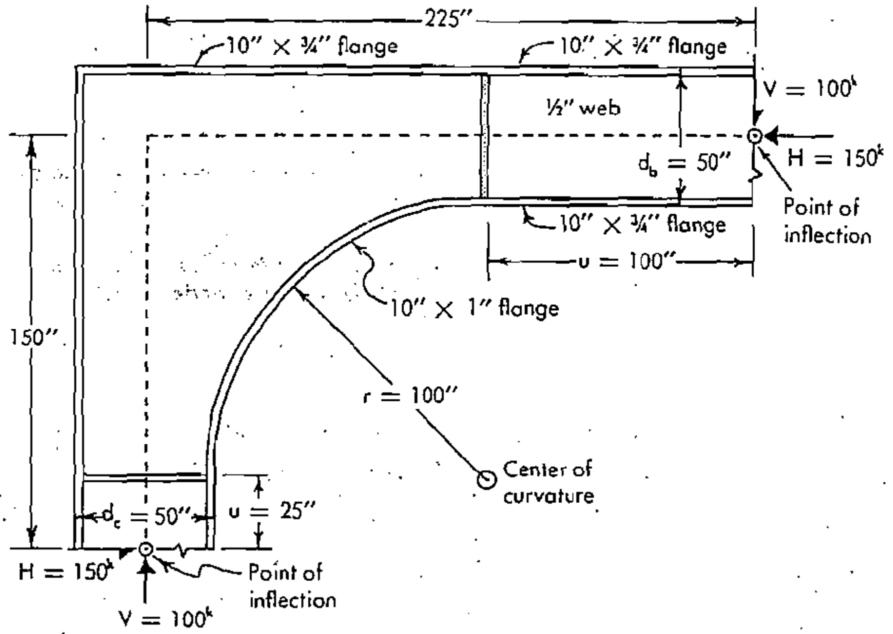


FIGURE 22

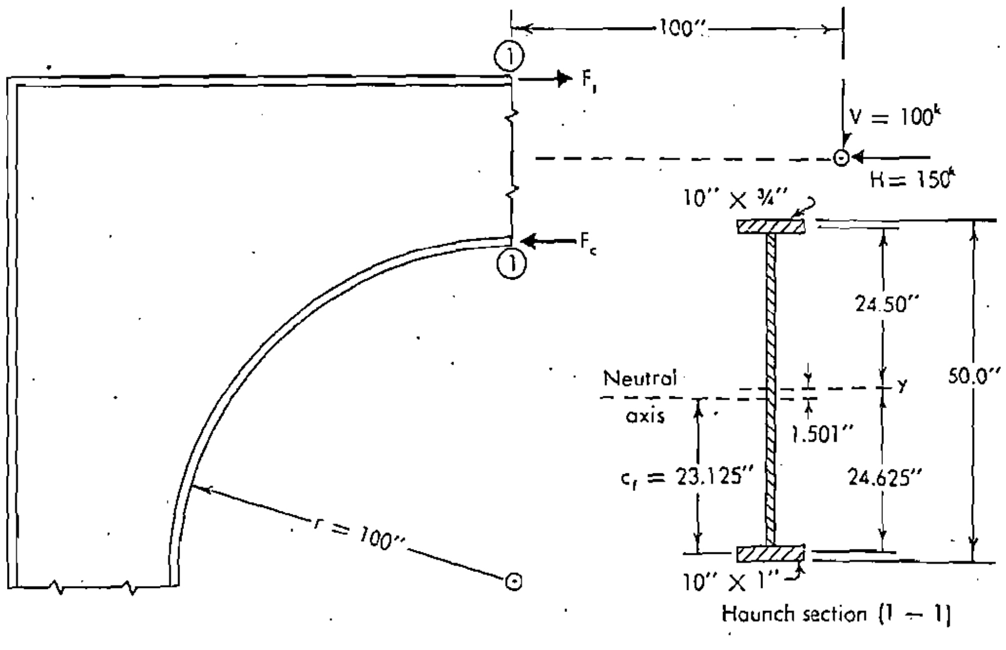


FIGURE 23

$$I_{NA} = I_y + I_g - \frac{M^2}{A}$$

$$= (15{,}247) - \frac{(-62.50)^2}{(41.625)}$$

$$= 15{,}153 \text{ in.}^4$$

$$NA = \frac{M}{A}$$

$$= \frac{(-62.50)}{(41.625)}$$

$$= -1.501$$

$$c_f = 23.125$$

average stress in lower curved flange at (1-1)

$$\sigma_t = \frac{P}{A} + \frac{M\ c_t}{I}$$

$$= \frac{(150\text{ kips})}{(41.625)} + \frac{(100'' \times 100\text{ kips})(23.125'')}{(15{,}153\text{ in.}^3)}$$

$$= \underline{18{,}870\text{ psi (compression)}}$$

force in flange

$$F_c = \sigma_t\ A_t$$

$$= (18{,}870)(10)$$

$$= 188.7\text{ kips}$$

radial pressure of flange against web

$$f_r = \frac{F_c}{r}$$

$$= \frac{(188.7)}{(100)}$$

$$= 1.887\text{ kips/in.}$$

radial compressive stress in web

$$\sigma = \frac{f_r}{t_w}$$

$$= \frac{(1887\text{ lbs/in.})}{(\tfrac{1}{2}'')}$$

$$= \underline{3774\text{ psi}}$$

The outer edges of the lower curved flange will tend to bend away from the center of curvature under this radial pressure, and will cause an uneven distribution of flange stress.

The maximum flange stress will be—

$$\sigma_{max} = \frac{\sigma_{av}}{\alpha}$$

and the transverse bending stress in the flange will be—

$$\sigma_t = \beta\ \sigma_{max}$$

The values of α and β are obtained from Figure 10. In this case,

$$\frac{b_t^2}{r\ t} = \frac{(10)^2}{(100)(1)} = 1$$

and we find—

$$\alpha = .96 \qquad \beta = .70$$

Hence:

maximum flange stress

$$\sigma_{max} = \frac{18{,}870}{.96}$$

$$= \underline{19{,}660\text{ psi}}$$

transverse bending stress in flange

$$\sigma_t = \beta\ \sigma_{max}$$

$$= (.70)(19{,}600)$$

$$= \underline{13{,}760\text{ psi}}$$

These stresses are a little high, so radial stiffeners will be added between the lower curved flange and the web.

STEP 2: Check Haunch Section for Bending Stress Using Olander's wedge method and curved section (A-B) (See Figure 24.)

Here:

$\sin 18° = .30902$

$\cos 18° = .95106$

$\tan 18° = .32492$

$\sin 9° = .15643$

$\cos 9° = .98769$

$18° = .31417$ radians

dimensions of wedge section (ABC)

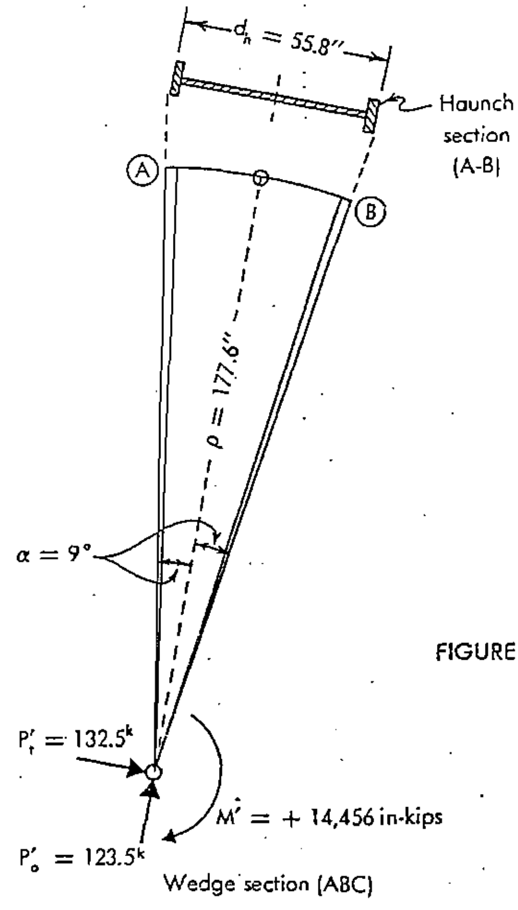


FIGURE 25

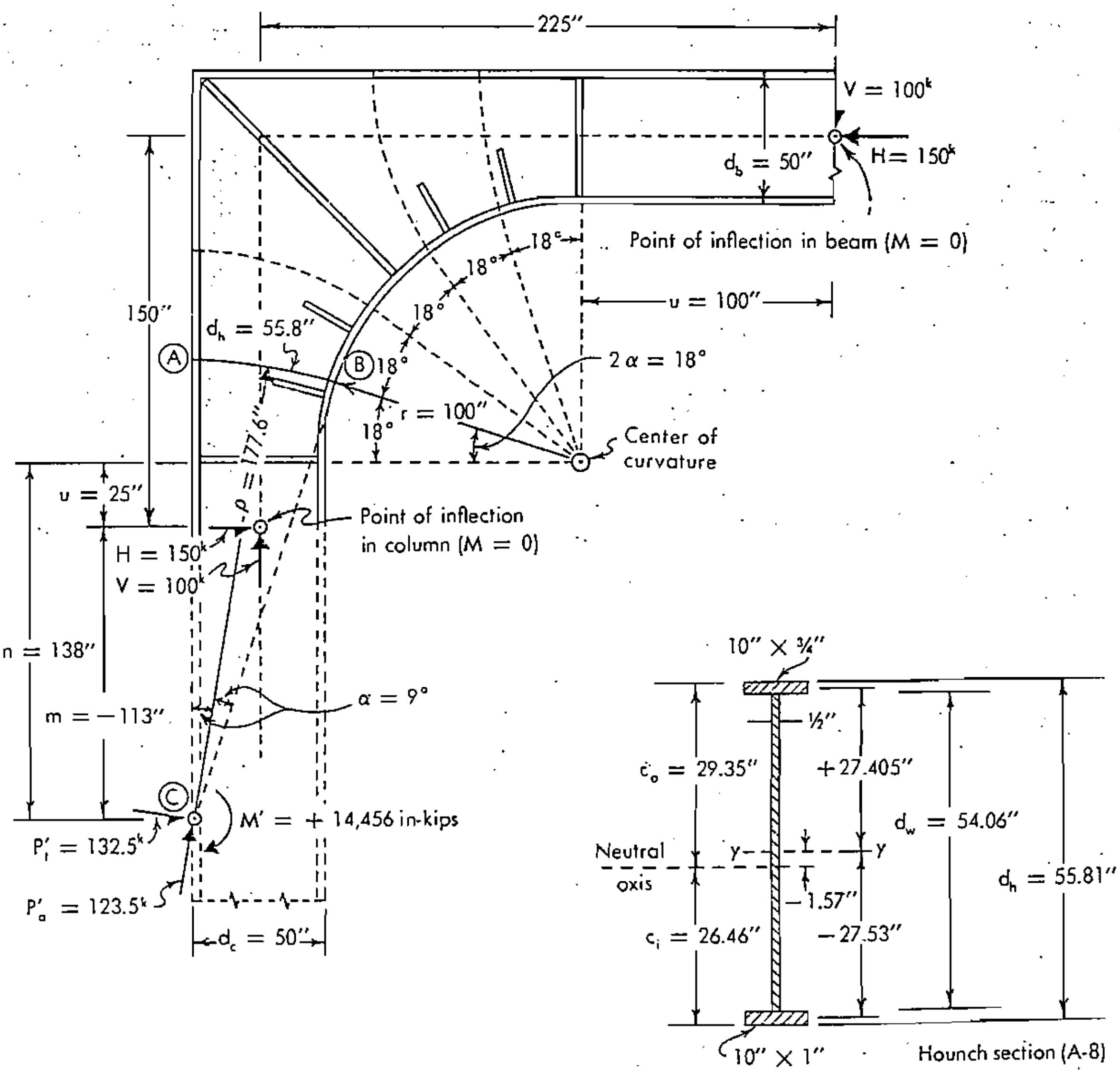


FIGURE 24

$$\rho = \frac{d}{\sin 2\alpha} + \frac{r}{\sin 2\alpha}\,(1 - \cos 2\alpha)$$
$$= \frac{(50)}{(.30902)} + \frac{(100)}{(.30902)}\,(.04894)$$
$$= 161.79 + 15.84$$
$$= 177.63''$$
$$d_h = \rho\, 2\alpha$$
$$= (177.63)(.31417 \text{ radians})$$
$$= 55.81''$$

$$d_w = 55.81'' - \tfrac{3}{4}'' - 1''$$
$$= 54.06''$$
$$n = \frac{d}{\tan 2\alpha} - \frac{r}{\sin 2\alpha}\,(1 - \cos 2\alpha)$$
$$= \frac{(50)}{(.32492)} - (15.84)$$
$$= 138.04''$$
$$u = 25''$$

$$m = u - n$$
$$= 25'' - 138.04''$$
$$= -113.04''$$

properties of haunch section (A-B)

Use reference axis (y-y) through centerline of web plate.

Plate	A	y	$M = A \cdot y$	$I_y = M \cdot y$	I_g
¾" x 10"	7.50	+27.405	+205.54	+5,633	—
½" x 54.06"	27.03	0	0	0	+6583
1" x 10"	10.00	−27.53	−275.30	+7,579	—
Total	44.53		− 69.76	+19,795	

$$I_{NA} = I_y + I_g - \frac{M^2}{A}$$
$$= (19{,}795) - \frac{(-69.76)^2}{(44.53)}$$
$$= 19{,}686 \text{ in.}^4$$
$$NA = \frac{M}{A}$$
$$= \frac{(-69.76)}{(44.53)}$$
$$= -1.567''$$
$$c_o = 29.35'' \qquad c_i = 26.46''$$

Find forces applied at apex (C) of wedge section (ABC):

transverse force at C

$$P_t' = P_t \cos \alpha - P_a \sin \alpha$$
$$= (150)(.98769) - (100)(.15643)$$
$$= 132.5 \text{ kips}$$

axial force at C

$$P_a' = P_a \cos \alpha + P_t \sin \alpha$$
$$= (100)(.98769) + (150)(.15643)$$
$$= 123.5 \text{ kips}$$

moment about C

$$M' = -P_t\, m - P_a \frac{d}{2}$$
$$= (-150)(-113.04'') - (100)\frac{(50)}{2}$$
$$= +14{,}456 \text{ in.-kips}$$

These forces result in the following stresses on the haunch section (A-B) of the wedge (see Figure 26):

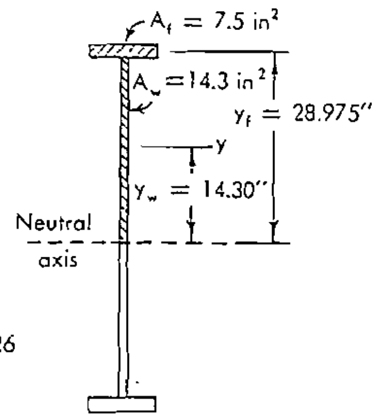


FIGURE 26

shear stresses in section (A-B)

$$V = \frac{M'}{\rho}$$
$$= \frac{(14{,}456)}{(177.63)}$$
$$= 81.35 \text{ kips}$$
$$\tau = \frac{V\,Q}{I\,t_w} = \frac{V[A_f y_f + A_w y_w]}{I\,t_w}$$
$$= \frac{(81.35)(7.5 \times 28.975 + 14.3 \times 14.3)}{(19{,}686)(½)}$$
$$= 1800 \text{ psi}$$

moment applied to section (A-B)

$$M = M' - P_t' \rho$$
$$= (+14{,}456) - (132.5)(177.6)$$
$$= -9082 \text{ in.-kips}$$

normal stress on inner flange

$$\sigma_r = -\frac{P'_a}{A} + \frac{M\,c_i}{I}$$
$$= -\frac{(123.5)}{(44.53)} + \frac{(-9082)(26.46)}{(19{,}686)}$$
$$= \underline{-15{,}000 \text{ psi}}$$

normal stress on outer flange

$$r = -\frac{P'_o}{A} - \frac{M\,c_o}{I}$$
$$= -\frac{(123.5)}{(44.53)} - \frac{(-9082)(29.35)}{(19{,}686)}$$
$$= \underline{+10{,}800 \text{ psi}}$$

As an alternate method *Check Haunch Section for Bending Stress Using Conventional Straight Section (A-B)*.

(See Figure 27.)

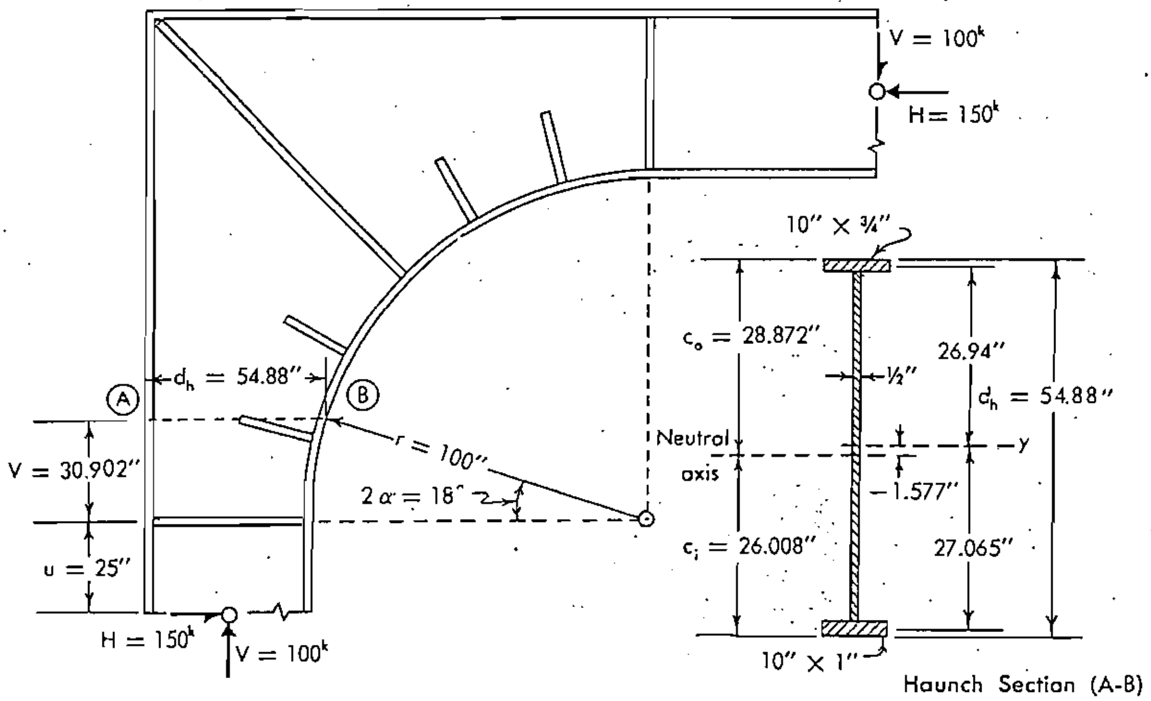


FIGURE 27

Here:

$$v = r \sin(2\alpha)$$
$$= (100)(.30902)$$
$$= 30.902''$$
$$d_h = d + r(1 - \cos 2\alpha)$$
$$= (50) + (100)(.0488)$$
$$= 54.88''$$

properties of haunch section (A-B)

Use reference axis (y-y) through centerline of web.

Plate	A	y	M = A·y	I_y = M·y	I_g
¾" x 10"	7.50	+26.94	+202.05	+5443.2	—
½" x 53.13"	26.565	0	0	0	+6249
1" x 10"	10.000	−27.065	−270.65	+7325.3	—
Total	44.065		− 68.60	19,018	

$$I_{NA} = I_y + I_g - \frac{M^2}{A}$$
$$= (19{,}018) - \frac{(-68.60)^2}{(44{,}065)}$$
$$= 18{,}911 \text{ in.}^4$$
$$NA = \frac{M}{A}$$
$$= \frac{(-68.60)}{(44.065)}$$
$$= -1.557''$$

$$c_o = 28.872'' \qquad c_i = 26.008''$$

moment applied to section

$$M = (150)(55.902) = 8385.3 \text{ in.-kips}$$

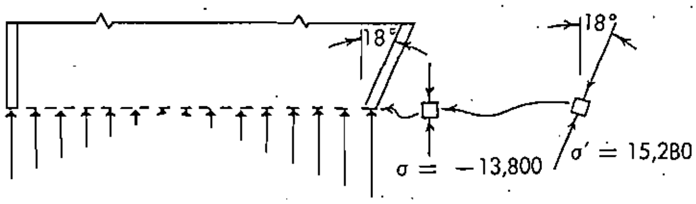


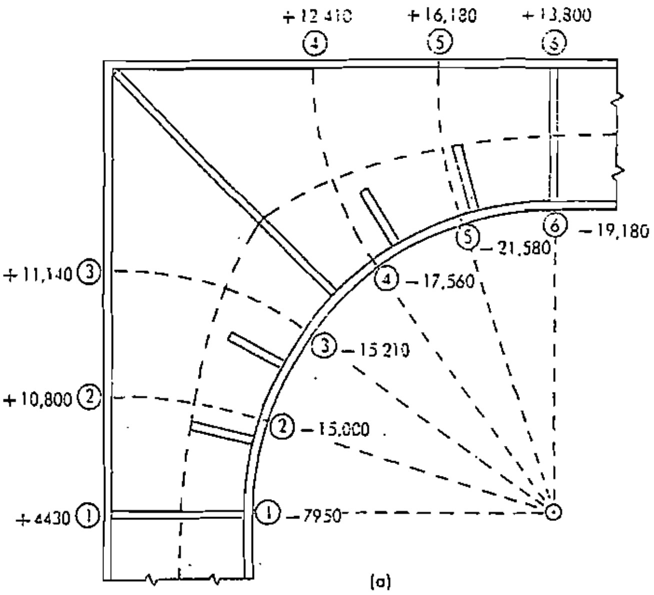
FIGURE 28

tensile bending and axial stress in outer flange

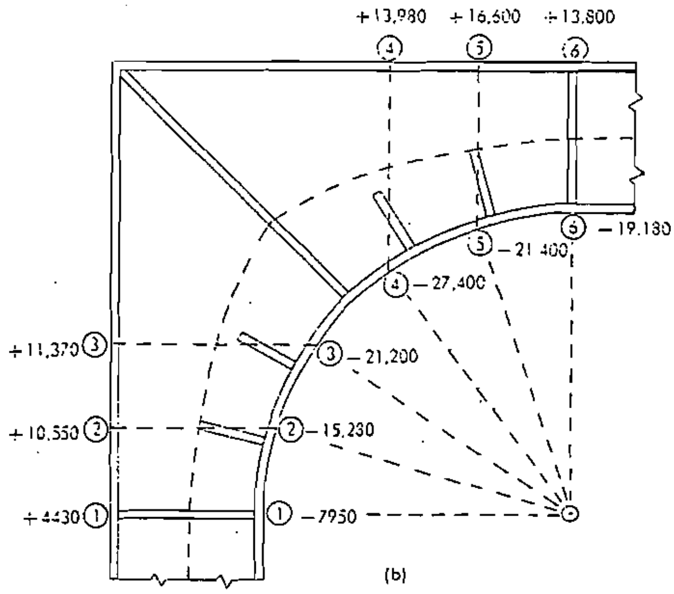
$$\sigma = -\frac{P}{A} + \frac{M\,c_o}{I}$$
$$= -\frac{(100)}{(44.065)} + \frac{(8385.3)(28.872)}{(18{,}911)}$$
$$= +10{,}550 \text{ psi, tension}$$

compressive bending and axial stress normal to section in inner flange

$$\sigma = -\frac{P}{A} - \frac{M\,c_i}{I}$$
$$= -\frac{(100)}{(44.065)} - \frac{(8385.3)(26.008)}{(18{,}911)}$$
$$= -13{,}800 \text{ psi, compression}$$



(a) Bending stresses in haunch using curved wedge sections, based on Olander method



(b) Bending stresses in haunch using conventionol straight sections

FIGURE 29

stress normal to axis of curved flange

$$\sigma' = \frac{\sigma}{\cos^2 2\,\alpha} = \frac{13{,}800}{(.95106)^2} = \underline{15{,}280 \text{ psi, compression}}$$

STEP 4: Summary

Figure 29 summarizes the stresses at several sections of the haunch for both the wedge method and the conventional method using straight sections.

The wedge method gives results that check close with experimental results, although it does require more time. The conventional method using straight sections in which the stress on the inward curved flange is increased to account for the sloping flange is easier. However, note that it does give higher values for the steeper slope.