Essential Random Vibration Fatigue for Structural Dynamic Applications

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What are we talking about Today?

- The extent of fe-safe Random Vibration Solution
- The underpinning assumptions
- Workflows
- Multiple Channels Example
- Fatigue Methods

What is Power Spectral Density?

$$G(f) = 2\frac{X_k^* X_k}{\delta f}$$

Spectral denotes the frequency domain

Power quantifies the contribution of every frequency

Those magnitudes are the Fourier Coefficients, determined by Fast Fourier Transform.

Density is related to the contribution of a band of frequencies in the frequency bin

The size of the frequency bin is the frequency resolution

$$\delta f = \frac{f_s}{N} = \frac{\textit{Sampling Freq}}{\textit{Buffer Size}}$$

What is Random?

- **Randomness** is associated with **Uncertainty.**

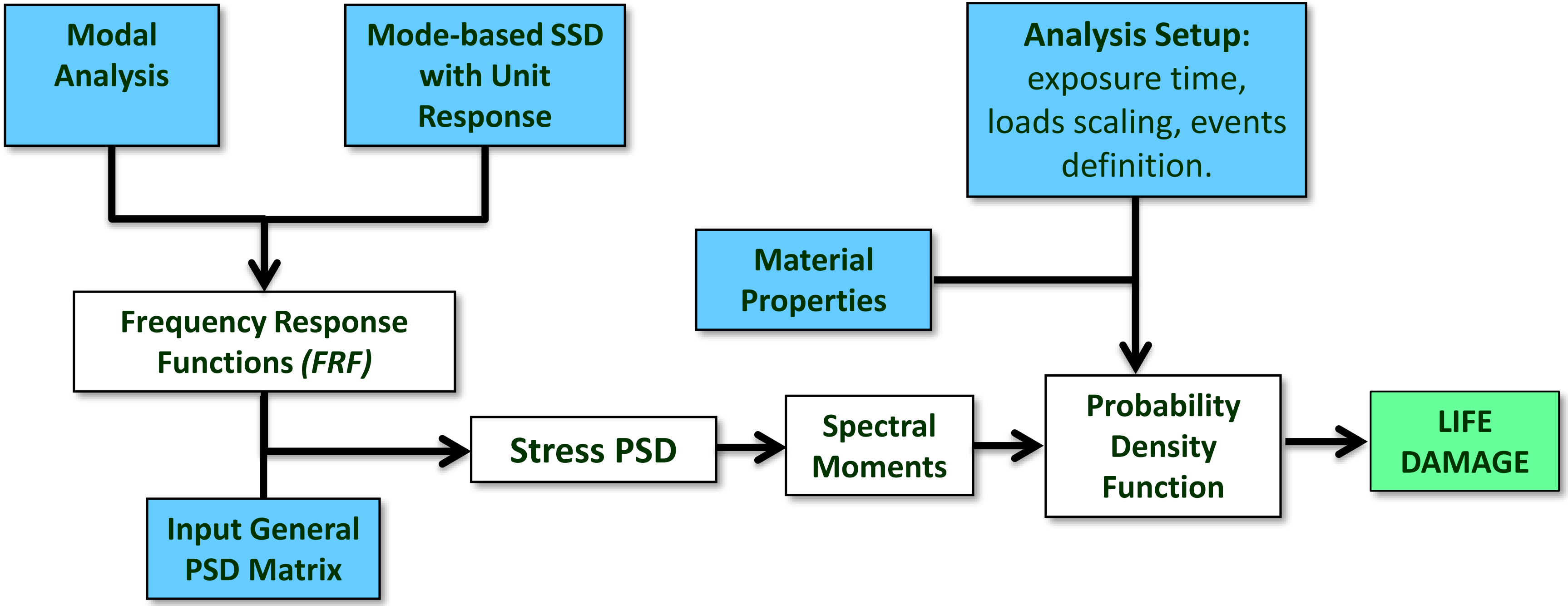
- The precise value of a variable can not be forecast but the space of possible outcomes.

- In fe-safe "Random Vibration Fatigue" the **randomness** is in the **loading**. The input statistically represents the loads.

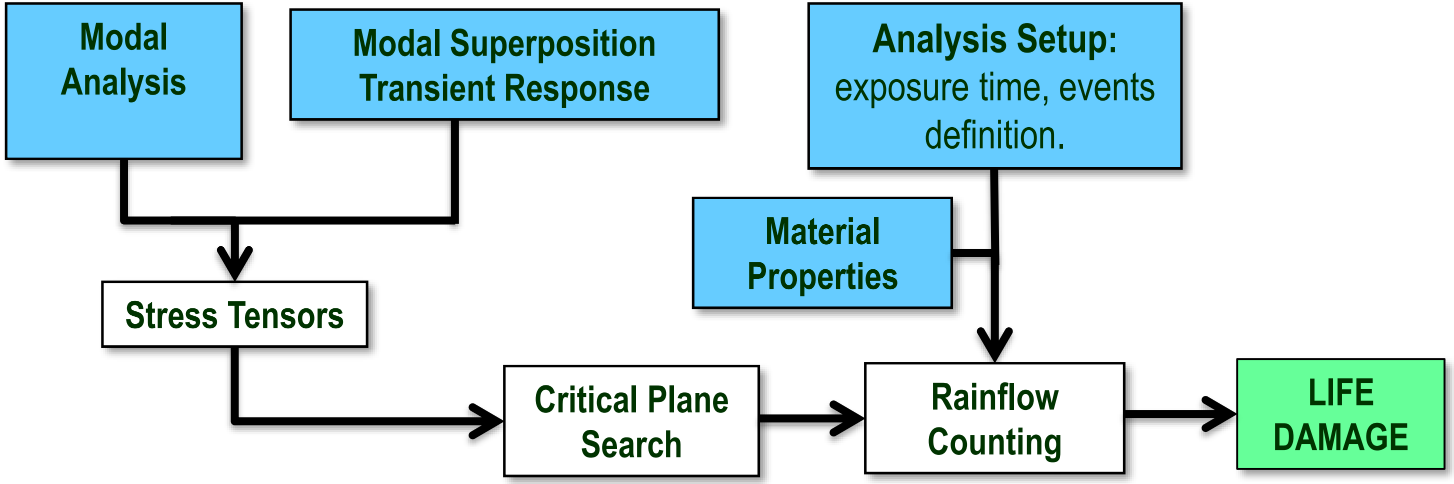
- The output ***is not*** Random.

Frequency Domain and Time Domain **Workflows**

Fatigue Analysis Flowchart (Frequency Domain)

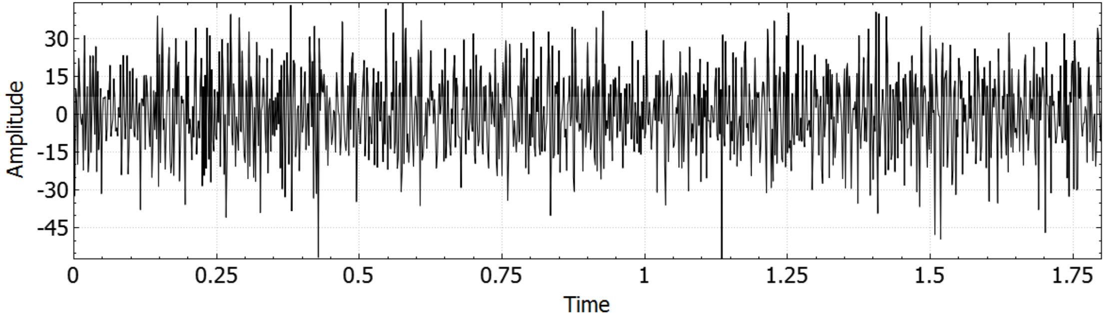


Fatigue Analysis Flowchart (Time Domain)

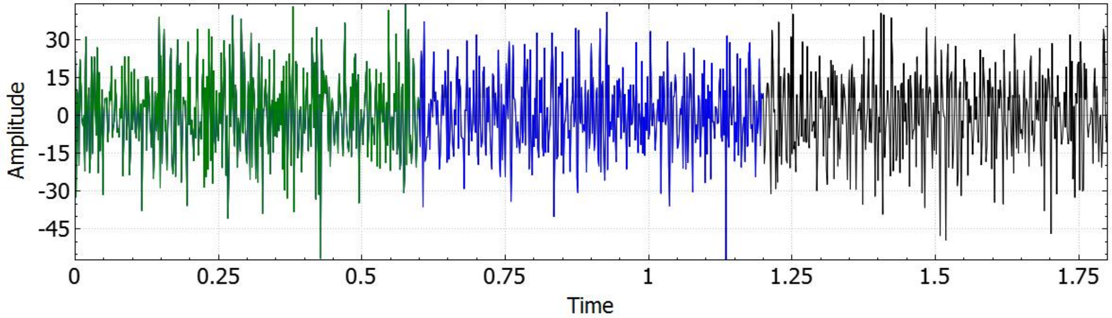


How fe-safe evaluates PSDs?

A given signal



is divided in a number of segments

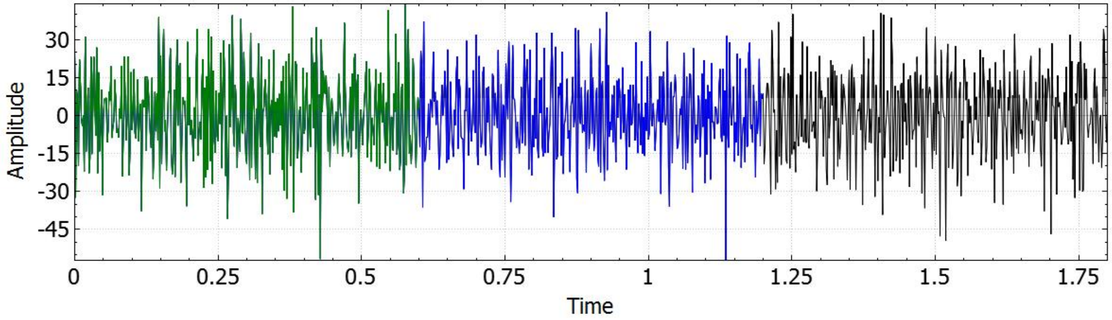


Since you can't perform the FFT of the given signal in just one go.

How fe-safe evaluates PSDs?

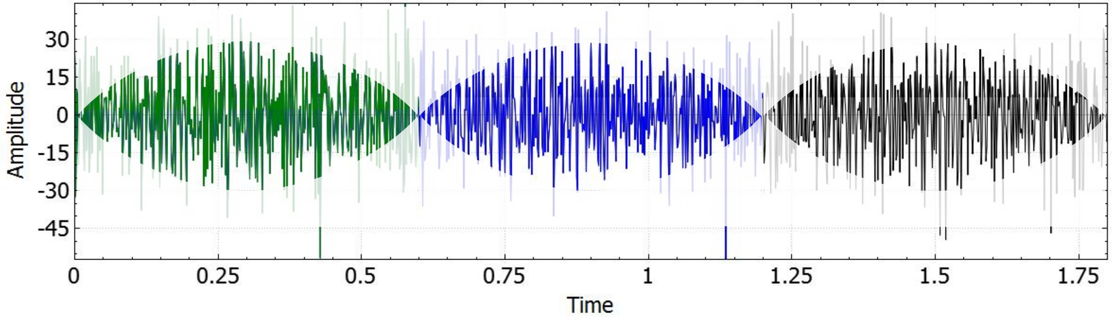
Buffer = 2^n

These segments are not periodic!



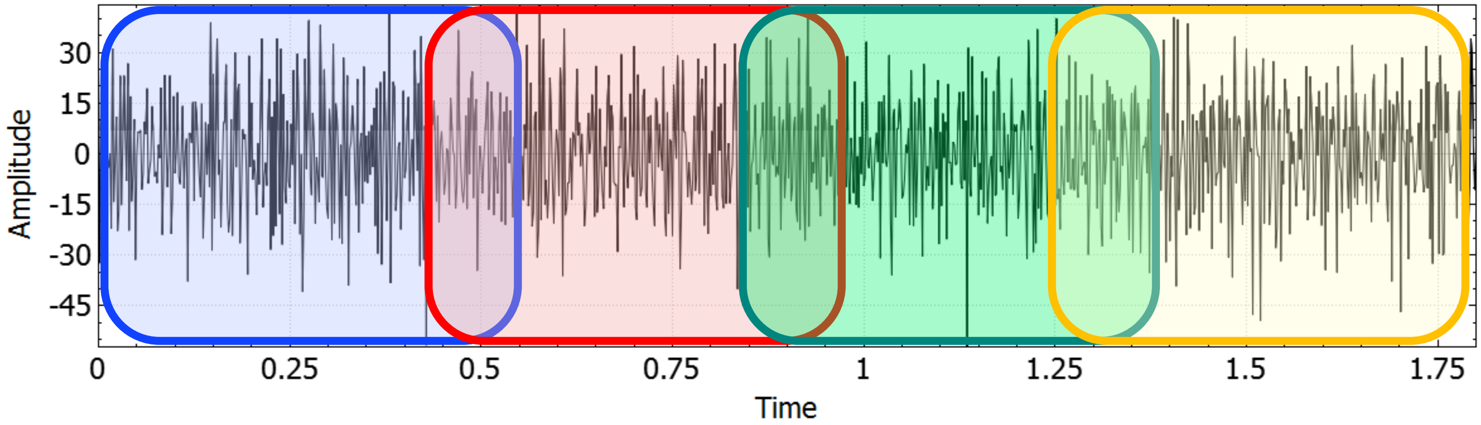
What about these ones?

Then we artificially reduce their amplitudes near the ends using an appropriate function.



Overlapping

fe-safe adopts 10% Overlap



FFT_1 → PSD_1

FFT_2 → PSD_2

FFT_n → PSD_n

Averaged PSD

The Modal Superposition Technique

Global Coordinates $$[M]\{\ddot{z}\}+[C]\{\dot{z}\}+[K]\{z\}=\{F\}$$

Eigen Analysis

Generalized or Modal Masses

$$[\underline{M}]=[X]^T[M][X]$$

Generalized or Modal Stiffness

$$[\underline{K}]=[X]^T[K][X]$$

Generalized or Modal Damping

$$[\underline{C}]=[X]^T[C][X]$$

$$\{Q\}=[X]^T\{F\}$$ *Generalized or Modal Forces*

$$\{z\}=[X]\{q\}$$ *Generalized Displacements*

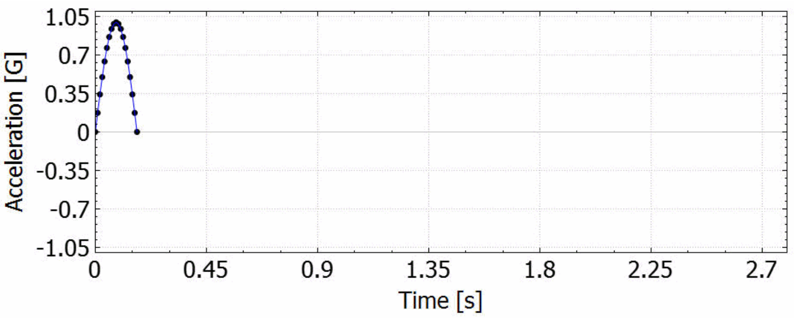
$$\boxed{\{\sigma\}=[X]\{q\}}$$ ***Stress Recovery***

Modal Coordinates $$[\underline{M}]\{\ddot{q}\}+[\underline{C}]\{\dot{q}\}+[\underline{K}]\{q\}=\{Q\}$$ *set of uncoupled equations*

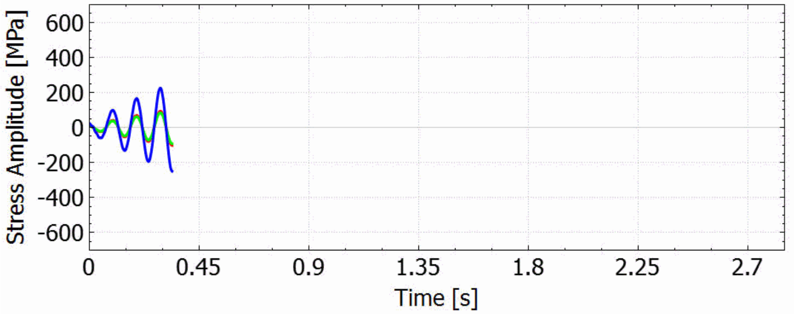
Steady State Dynamics vs. Transient

Time

Unit Acceleration Load

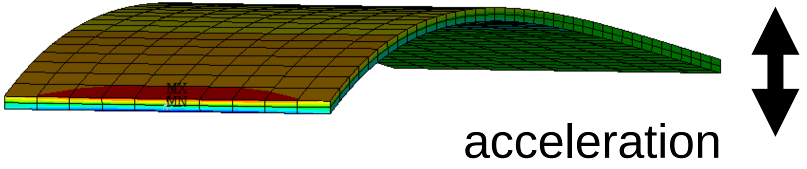


Modal Superposition Transient Analysis

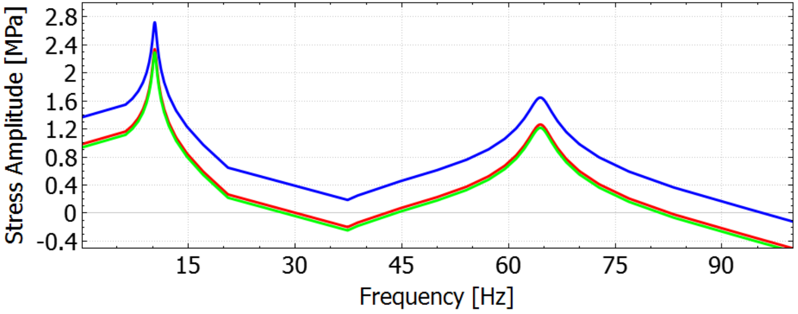


Frequency

Stress Response in a Vibrating Beam



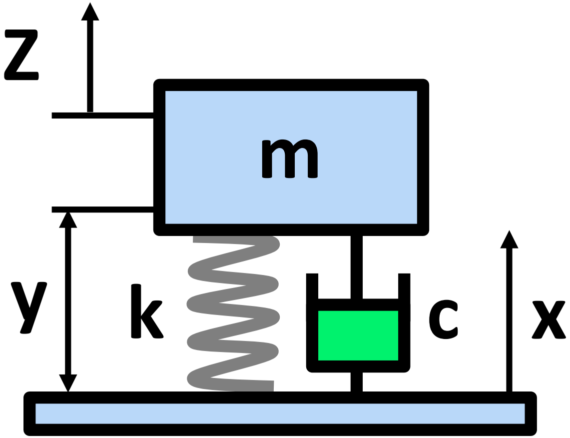
Steady State Dynamic Analysis

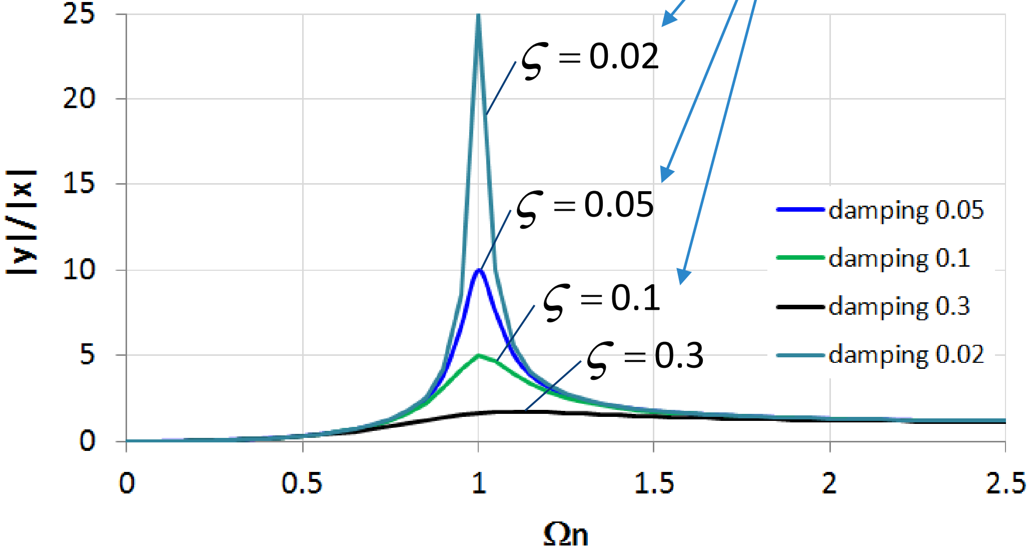


Linear Transfer Function

Damping is tricky!

$$\frac{|y|}{|x|} = \frac{\Omega^2}{\left[\left(1-\Omega^2\right)^2 + \left(2\Omega\varsigma\right)^2\right]^{1/2}} \qquad \Omega = \frac{\omega}{\omega_n}$$





Linear Transfer Function

$$Q^T \cdot G_L \cdot Q = G^\sigma$$

Input PSD Transfer Function Output PSD

$$\frac{G^2}{Hz} \quad \text{x} \quad \left(\frac{MPa}{G}\right)^2 \quad = \quad \frac{MPa^2}{Hz}$$

Dynamic Response Matrix

The first step towards critical planes approach is to assemble the Transition Coefficient Matrix or Dynamic Response Matrix

$$\left[Q(f)\right]_{N\,x\,6}$$

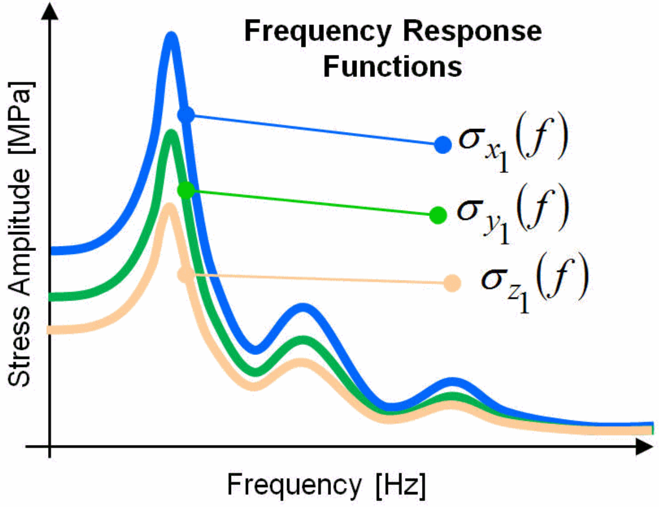
Order of the stress/strain tensor

Number of loading channels

$$Q(f) = \begin{bmatrix} \sigma_{x1} & \sigma_{y1} & \cdots & \sigma_{xz1} \\ \sigma_{x2} & \sigma_{y2} & & \sigma_{xz2} \\ \vdots & & & \vdots \\ \sigma_{xN} & \sigma_{yN} & \cdots & \sigma_{xzN} \end{bmatrix}_{Nx6}$$

Channel 1

Channel N



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$$Q(f) = \begin{bmatrix} \sigma_{x1} & \sigma_{y1} & \cdots & \sigma_{xz1} \\ \sigma_{x2} & \sigma_{y2} & & \sigma_{xz2} \\ \vdots & & & \vdots \\ \sigma_{xN} & \sigma_{yN} & \cdots & \sigma_{xzN} \end{bmatrix}_{Nx6} \begin{array}{l} \text{Channel 1} \\ \\ \text{Channel N} \\ \\ \end{array}$$

$$\left[Q(f)\right]_{Nx6}$$

Order of the stress/strain tensor

Number of loading channels

Linear Transfer Function

Input PSD matrix

$$G_L = \begin{bmatrix} PSD_{11} & & CPSD_{1N} \\ & \ddots & \\ CPSD_{N1} & & PSD_{NN} \end{bmatrix}_{NxN}$$

Transfer Function

$$Q = \begin{bmatrix} \sigma_{x_1} & \sigma_{y_1} & \sigma_{z_1} & \sigma_{xy_1} & \sigma_{yz_1} & \sigma_{xz_1} \\ \vdots & & & & & \vdots \\ \sigma_{x_N} & \sigma_{y_N} & \sigma_{z_N} & \sigma_{xy_N} & \sigma_{yz_N} & \sigma_{xz_N} \end{bmatrix}_{N\,x\,6} \rightarrow$$

Output Stress PSD

$$G^{\sigma} = \begin{bmatrix} G_{11} & \cdots & G_{61} \\ \vdots & \ddots & \vdots \\ G_{16} & \cdots & G_{66} \end{bmatrix}_{6\,X\,6}$$

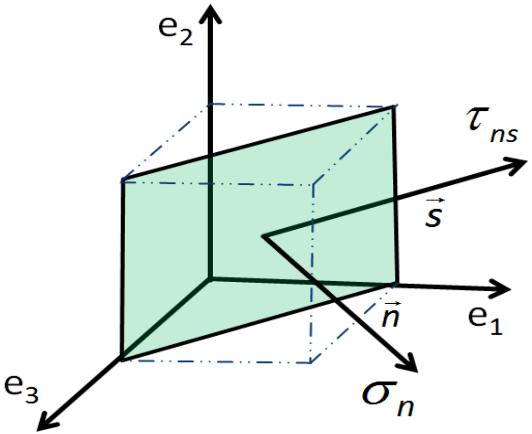
Von Mises Stress PSD

$$G^{\sigma}_{Mises} = Trace\left\{A \cdot G^{\sigma}\right\}$$

Von Mises PSD Matrix

$$A = \begin{bmatrix} 1 & -1/2 & -1/2 & & & \\ -1/2 & 1 & -1/2 & & & \\ -1/2 & -1/2 & 1 & & & \\ & & & 3 & & \\ & & & & 3 & \\ & & & & & 3 \end{bmatrix}$$

Critical Plane Approach



$$G_n^{\sigma} = d_n^T \cdot G^{\sigma} \cdot d_n$$

$$G_s^{\sigma} = d_s^T \cdot G^{\sigma} \cdot d_s$$

$$n = \frac{e_{1x} + e_{3x}}{\sqrt{2}} \vec{i} + \frac{e_{1y} + e_{3y}}{\sqrt{2}} \vec{j} + \frac{e_{1z} + e_{3z}}{\sqrt{2}} \vec{k}$$

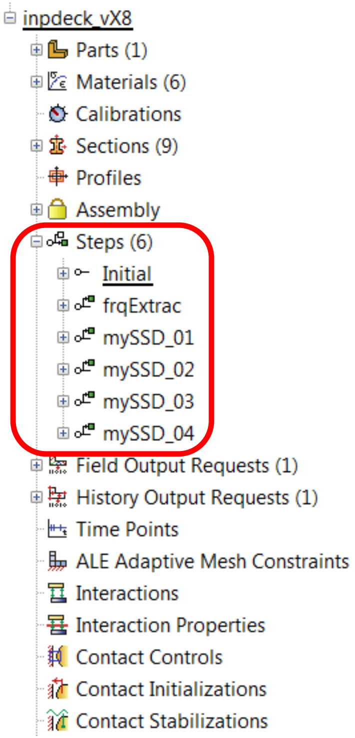
$$s = \frac{e_{1x} - e_{3x}}{\sqrt{2}} \vec{i} + \frac{e_{1y} - e_{3y}}{\sqrt{2}} \vec{j} + \frac{e_{1z} - e_{3z}}{\sqrt{2}} \vec{k}$$

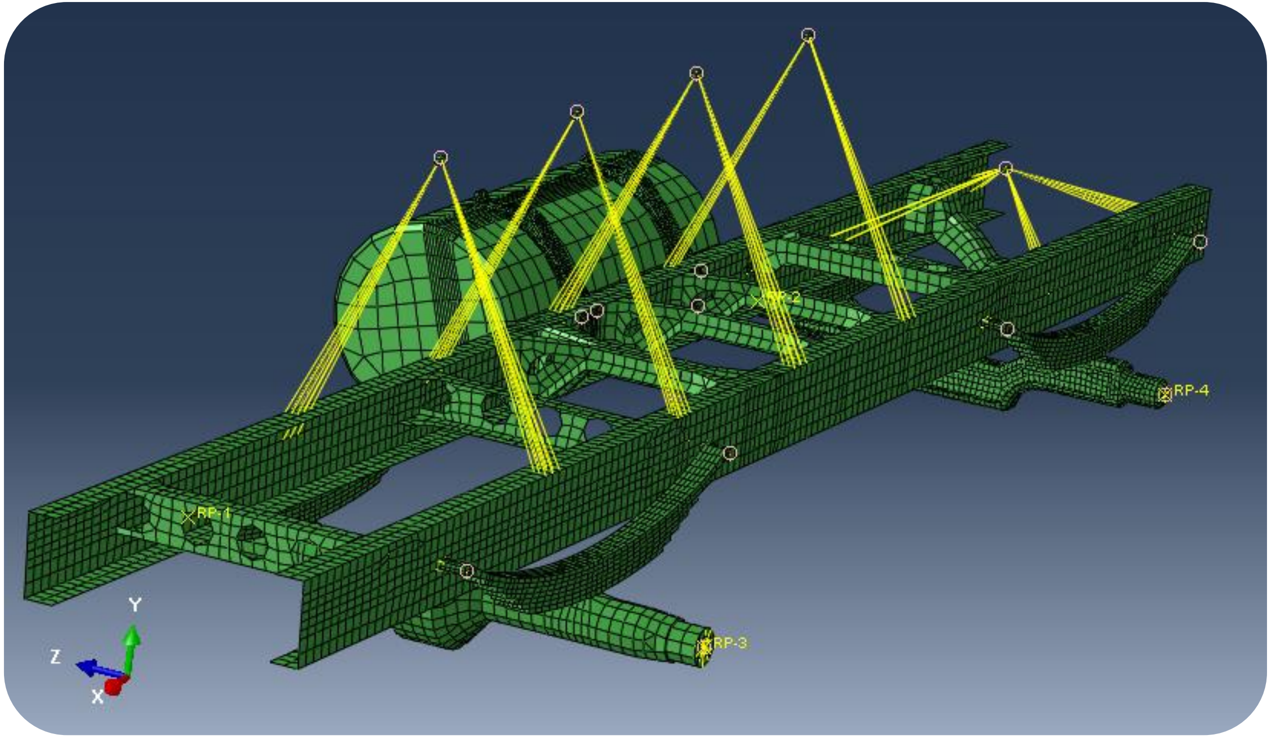
$$d_s = \left[\frac{\left(e_{1x}^2 - e_{3x}^2\right)}{2} \quad \frac{\left(e_{1y}^2 - e_{3y}^2\right)}{2} \quad \frac{\left(e_{1z}^2 - e_{3z}^2\right)}{2} \quad \left(e_{1x}e_{1y} - e_{3x}e_{3y}\right) \quad \left(e_{1y}e_{1z} - e_{3y}e_{3z}\right) \quad \left(e_{1x}e_{1z} - e_{3x}e_{3z}\right) \right]^T$$

$$d_n = \left[\frac{\left(e_{1x} + e_{3x}\right)^2}{2} \quad \frac{\left(e_{1y} + e_{3y}\right)^2}{2} \quad \frac{\left(e_{1z} + e_{3z}\right)^2}{2} \quad \left(e_{1x} + e_{3x}\right)\left(e_{1y} + e_{3y}\right) \quad \left(e_{1y} + e_{3y}\right)\left(e_{1z} + e_{3z}\right) \quad \left(e_{1x} + e_{3x}\right)\left(e_{1z} + e_{3z}\right) \right]^T$$

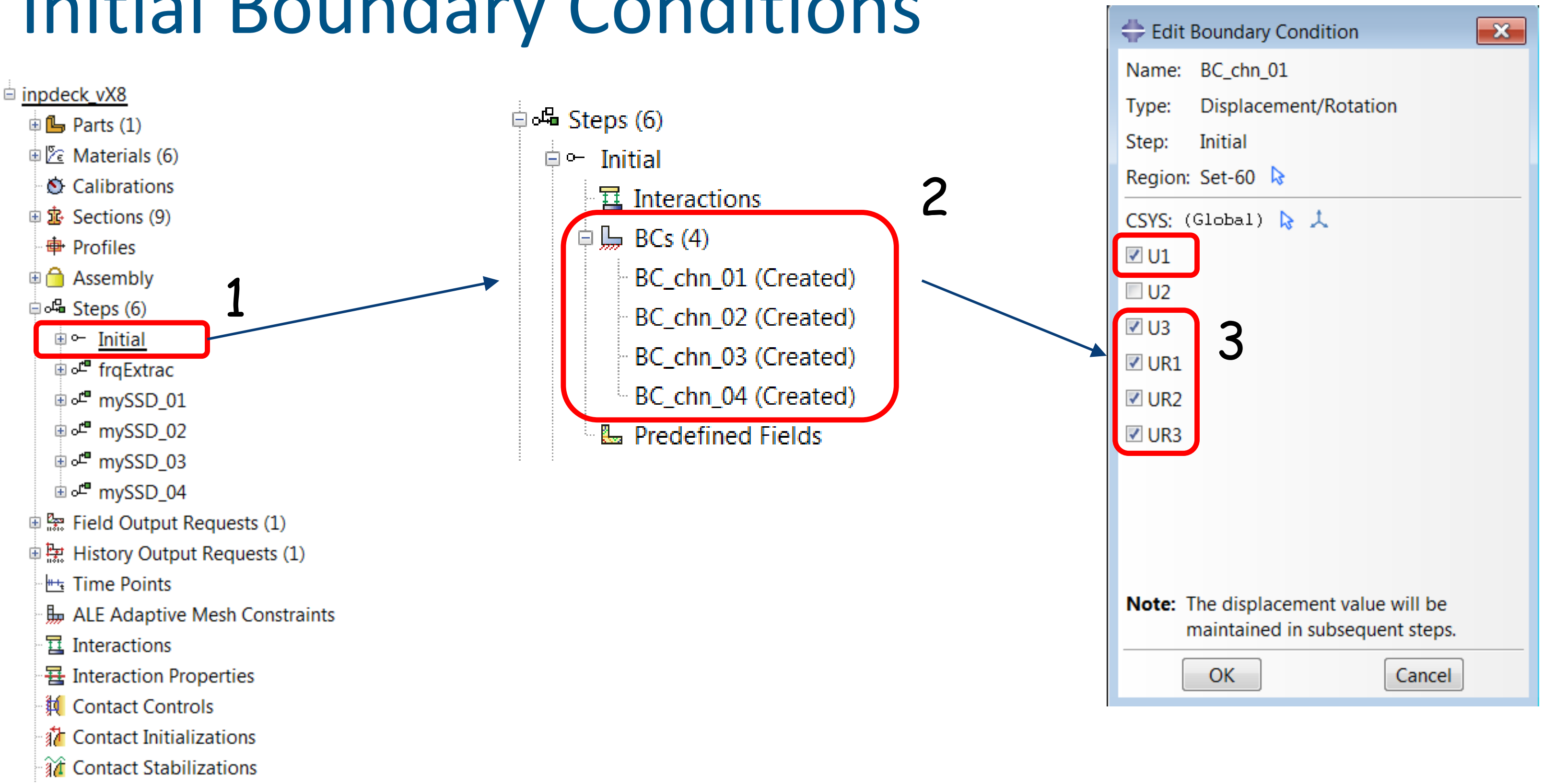
Multiple Channels
Truck Chassis Example

The Truck Chassis Model

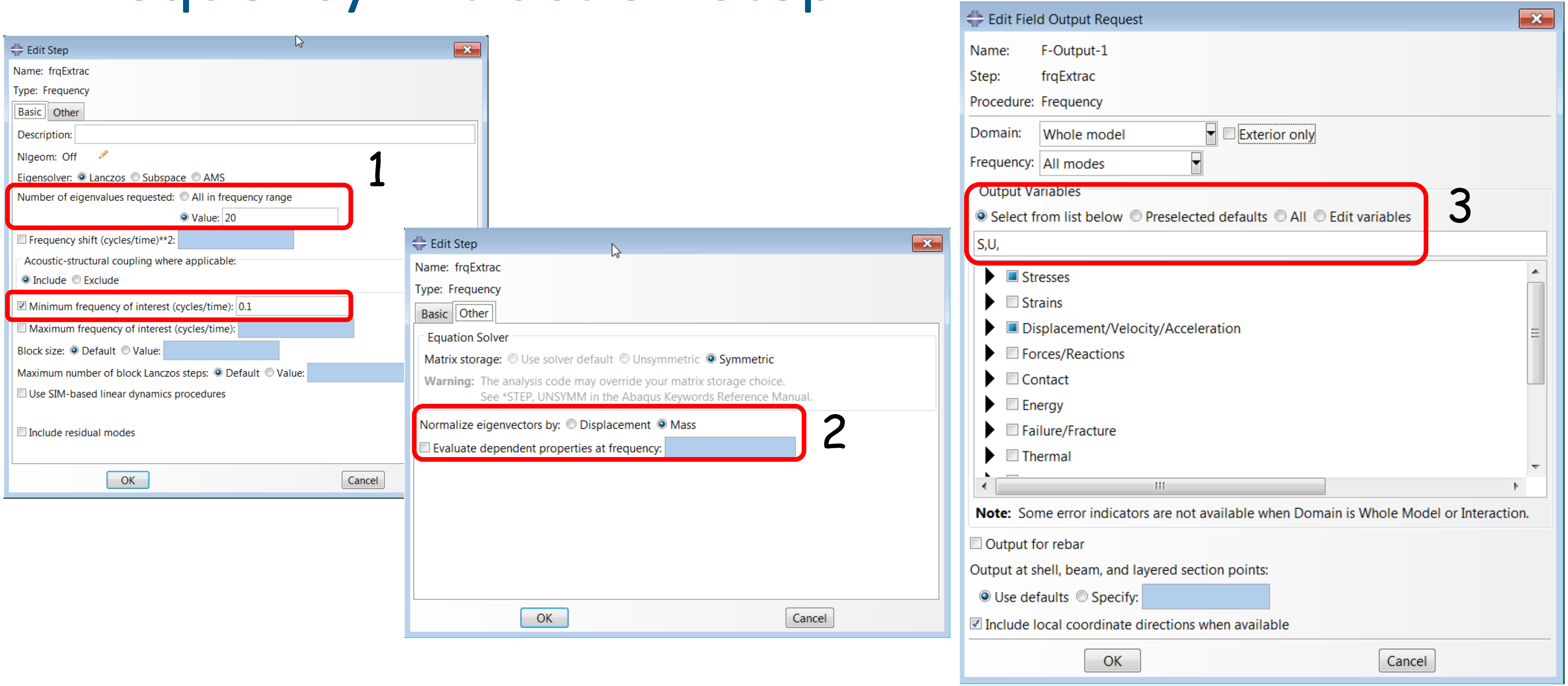




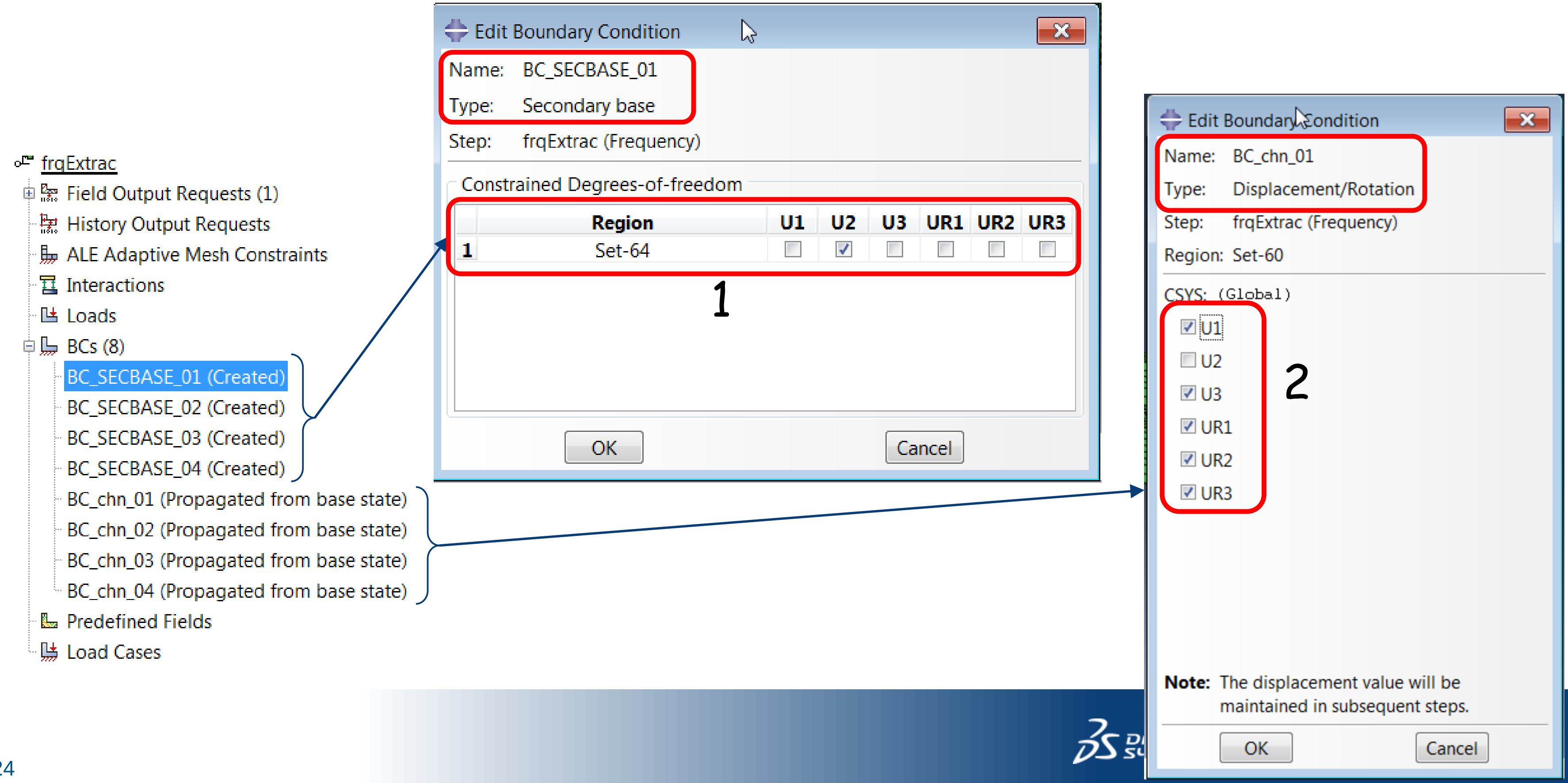
Initial Boundary Conditions



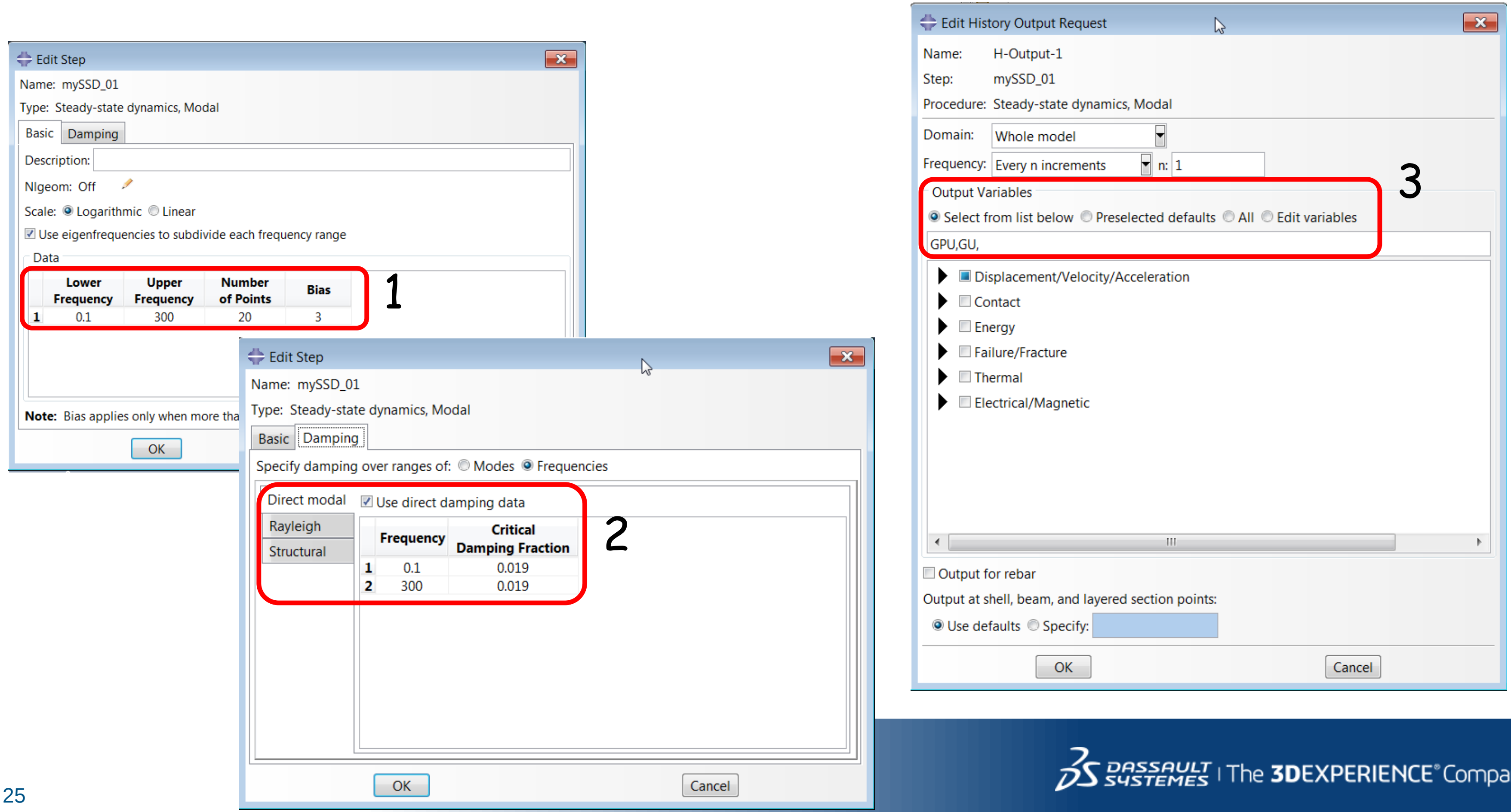
Frequency Extraction Step



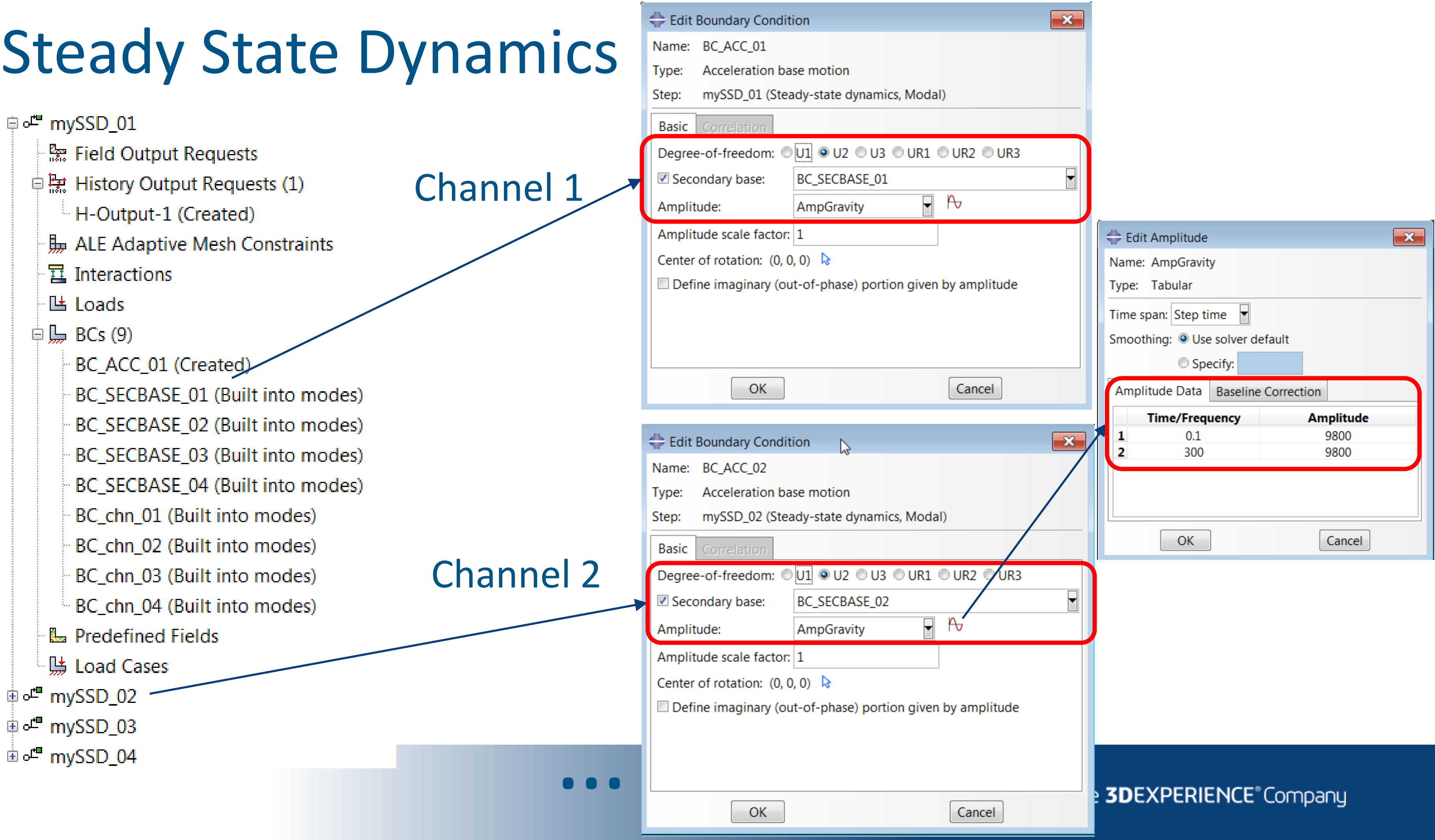
Frequency Extraction Step



Steady State Dynamics

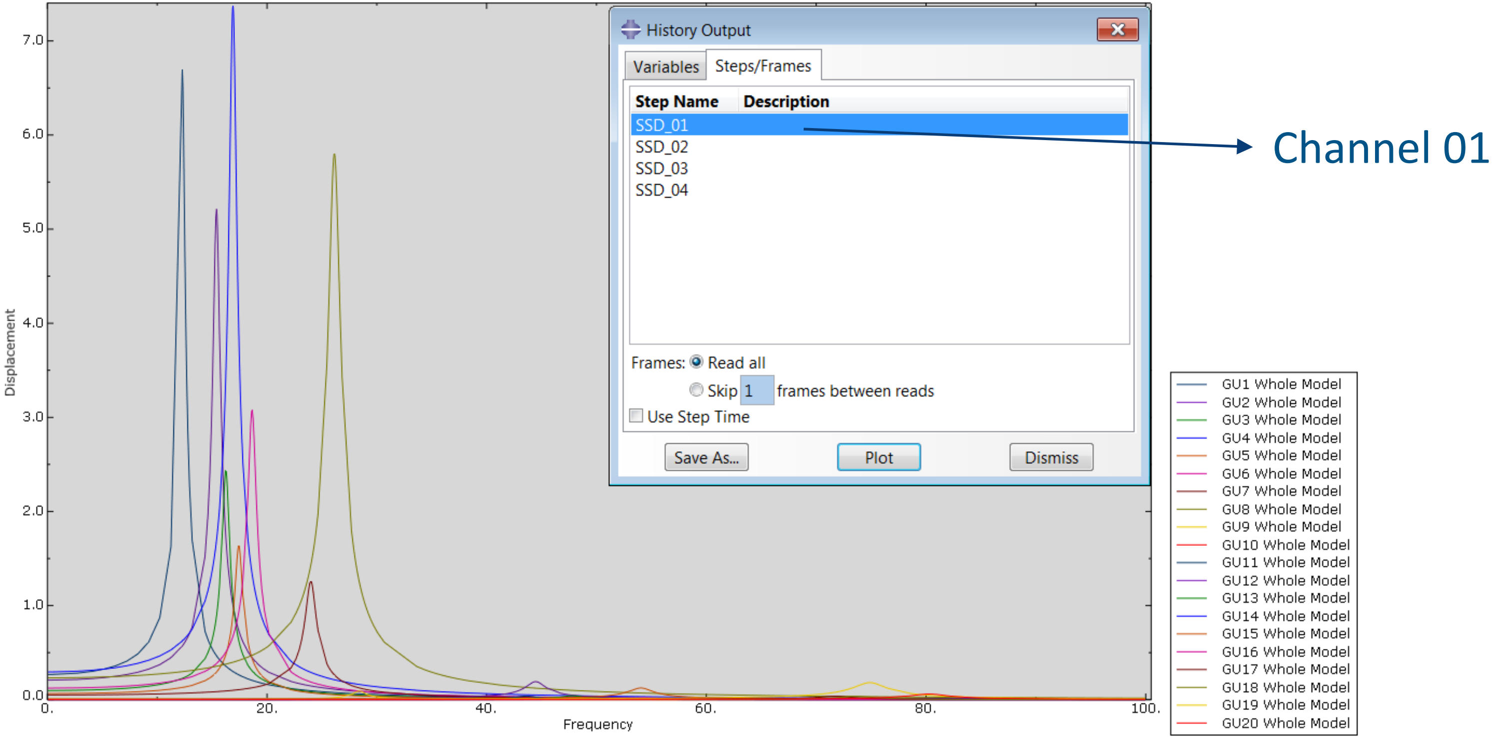


Steady State Dynamics



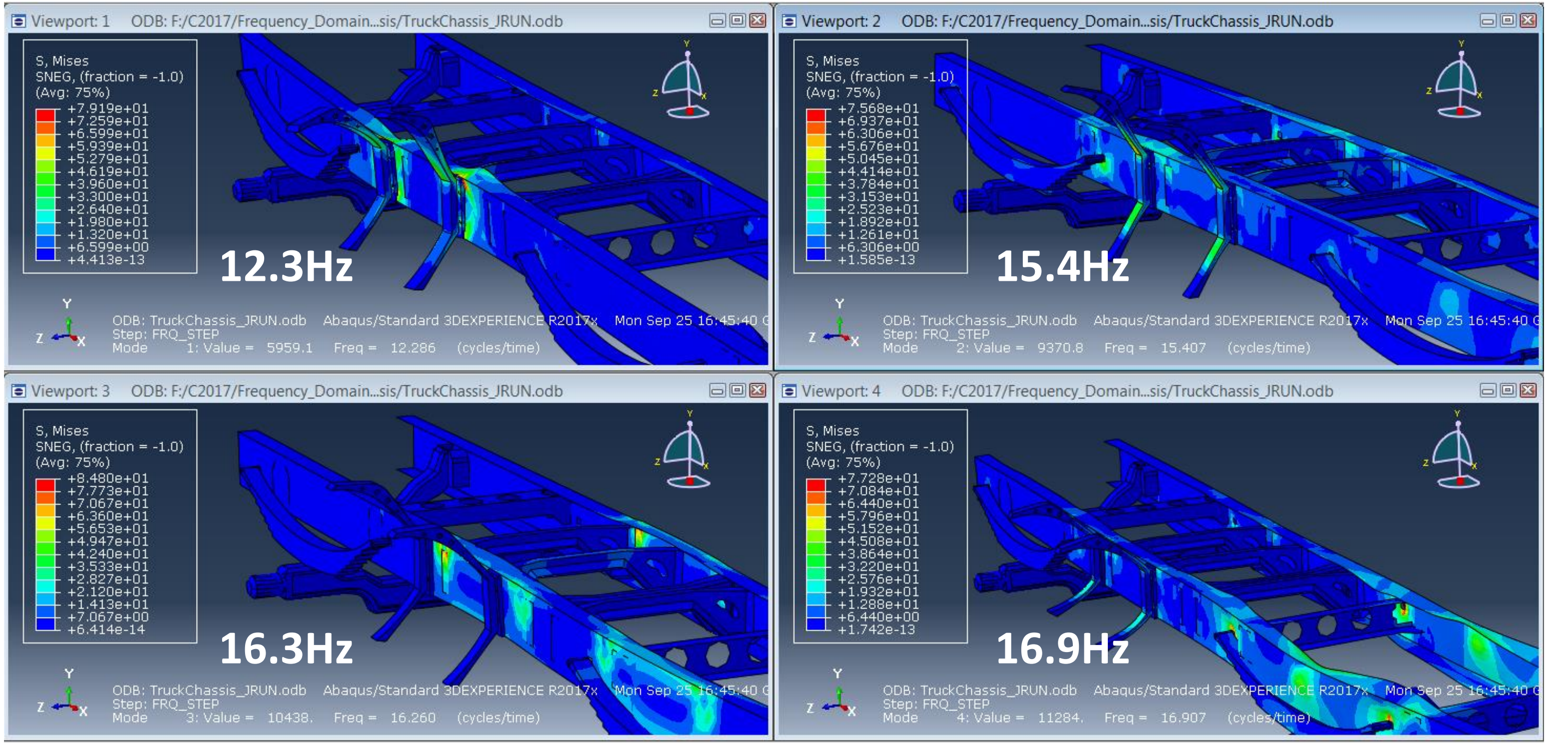
ABAQUS Results

Modal Coordinates or Generalized Displacements

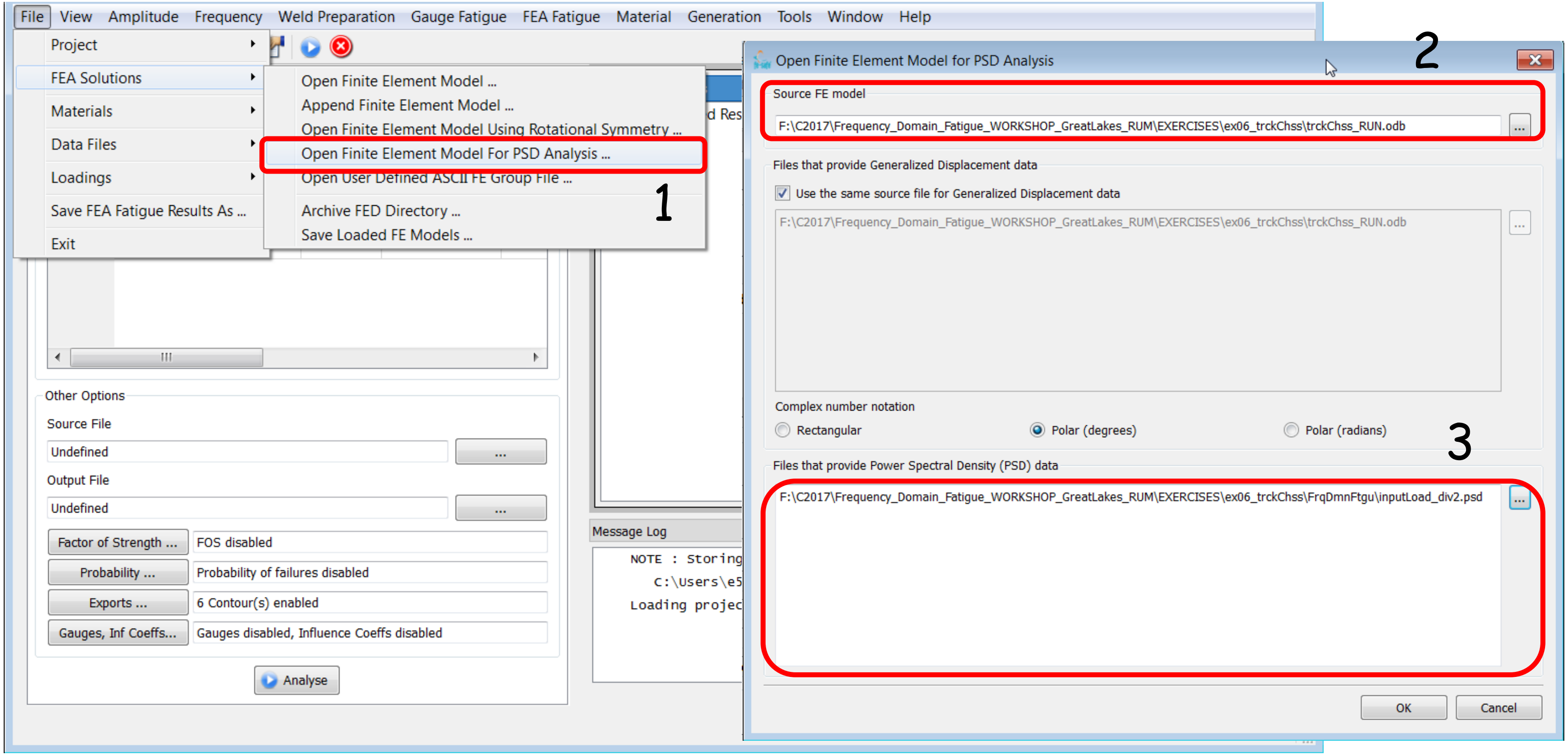


ABAQUS Results

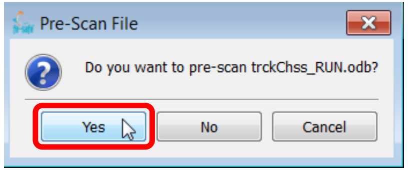
Modal Stresses

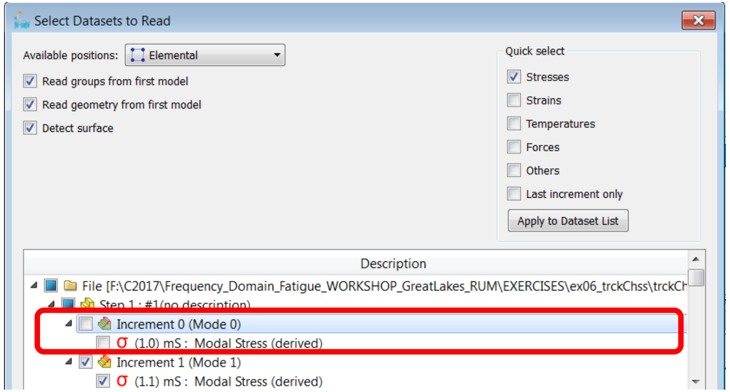


fe-safe Random Vibration

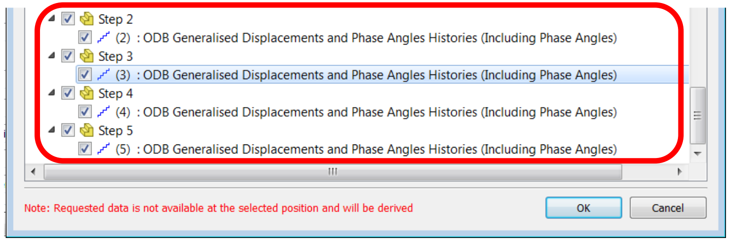


Pre-scanning selections



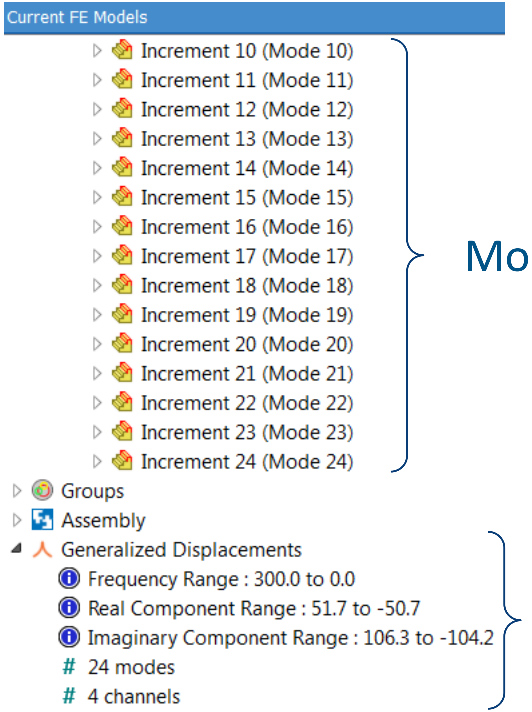


Make sure Mode 0 is unselected.



Select the steps that contain the Generalized Displacements

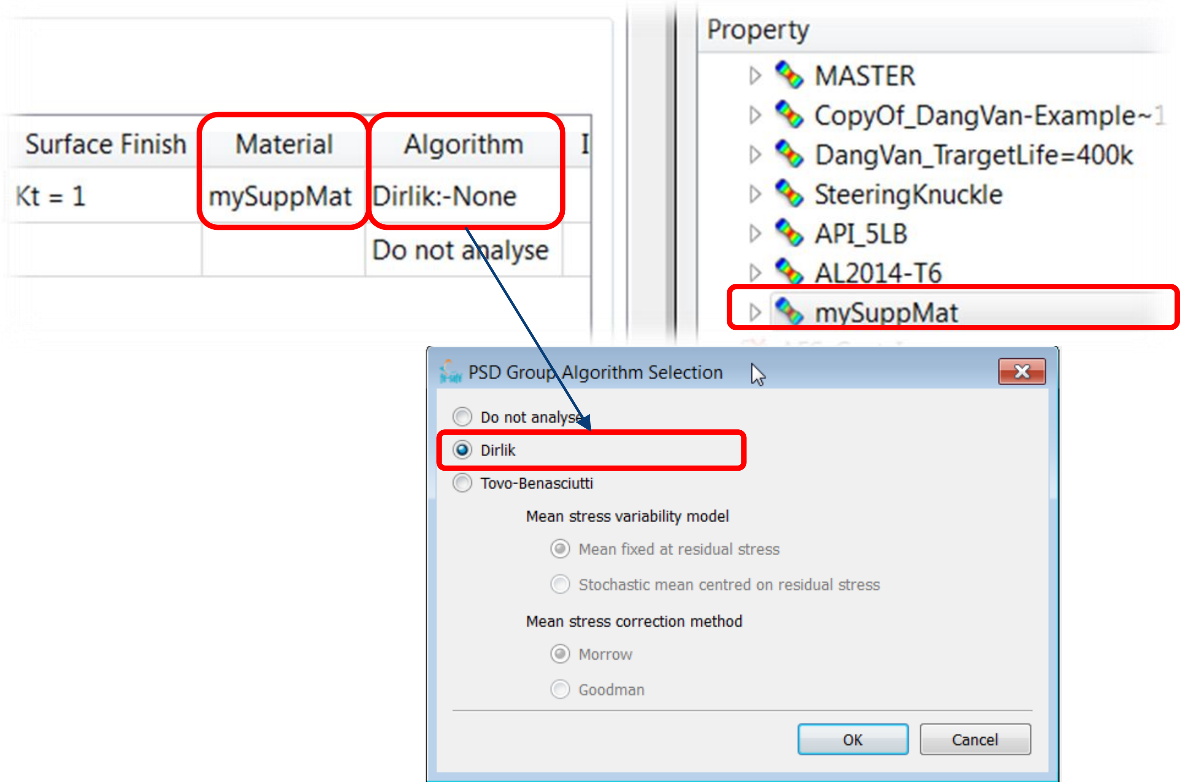
The data loaded in fe-safe



Modal Stresses

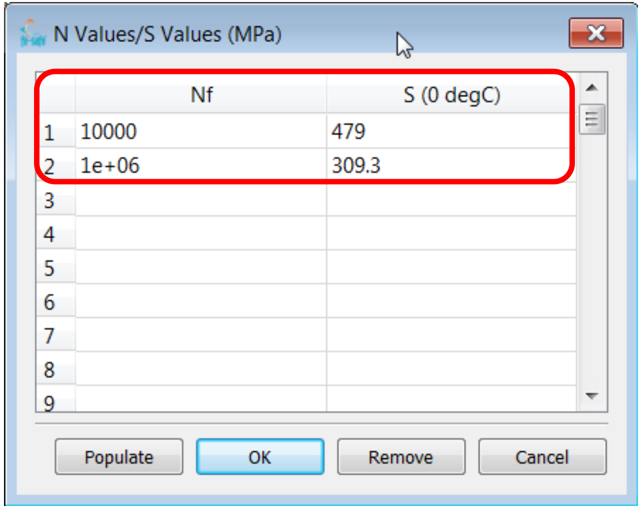
Modal Coordinates
(aka Generalized Displacements)

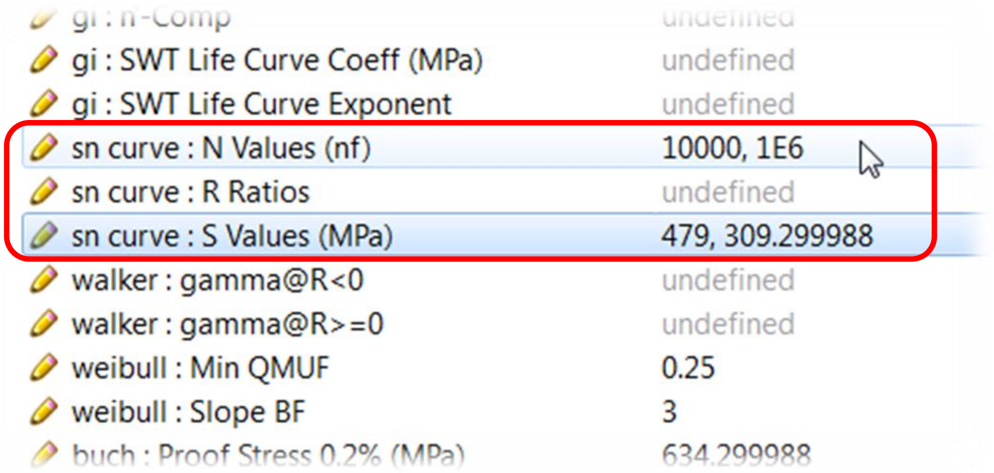
Material and Methods



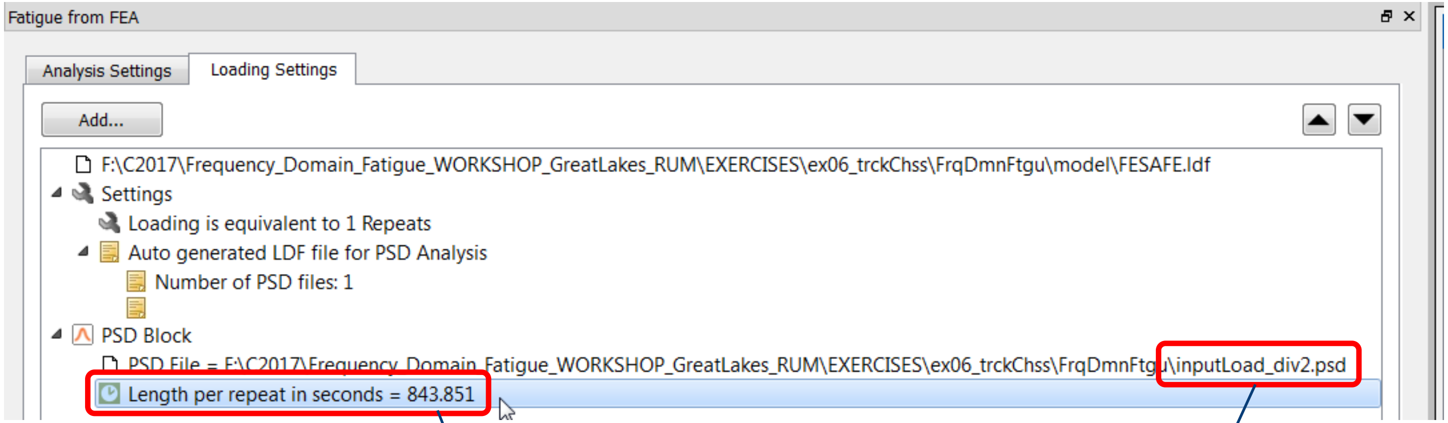
Material and Methods

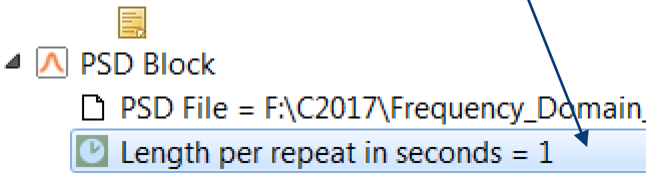
fe-safe Random Vibration is a Stress-Based Approach





Loading Block Definition





Set the length to 1 sec

If the loads change you point to the new file that contemplates the changes. No need to run ABAQUS again.

Interpreting the Input PSD File

Input PSD File

```
Number of channels = 4
#Input sample rate = 598.802
#Input signals' length = 505300
Exposure time(seconds) = 843.851

#Input #0: file F:/C2017/Frequency_Domain_Fatigue_W
#Input #1: file F:/C2017/Frequency_Domain_Fatigue_W
#Input #2: file F:/C2017/Frequency_Domain_Fatigue_W
#Input #3: file F:/C2017/Frequency_Domain_Fatigue_W

#PSD_0_0_frq        PSD_0_0_real
1.16954 0.000112501
2.33907 0.000128473
3.50861 0.000104452
4.67814 0.000171993
5.84768 0.000232986
7.01722 0.00042747
8.18675 0.000990825
9.35629 0.0015742
```

...

Number of Load Channels

Exposure Time

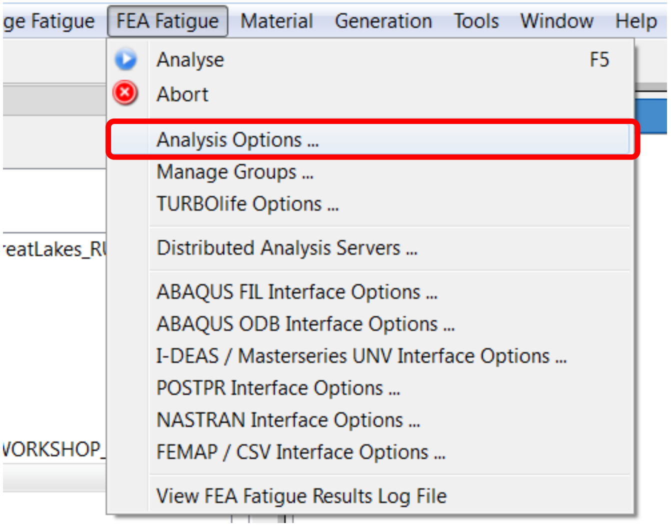
Auto PSDs

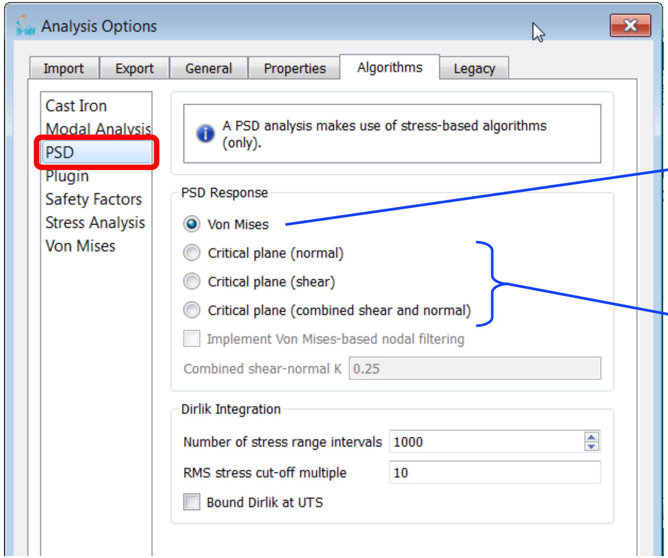
Cross PSDs

...

```
#CPSD_0_2_frq        CPSD_0_2_real        CPSD_0_2_imag
1.16954 0.000103774      3.0678e-07
2.33907 0.000125386      -2.08408e-06
3.50861 6.50101e-05      -1.40455e-05
4.67814 7.82215e-05      -2.2799e-05
5.84768 9.09092e-05      -5.5179e-05
7.01722 9.89627e-05      -0.000138587
8.18675 5.43902e-05      -0.000401876
9.35629 0.000288409      -0.000289905
10.5258 0.00026273       -9.75707e-05
11.6954 -0.000389818     0.00025891
12.8649 -0.000826054     0.00105563
```

Fatigue Methods



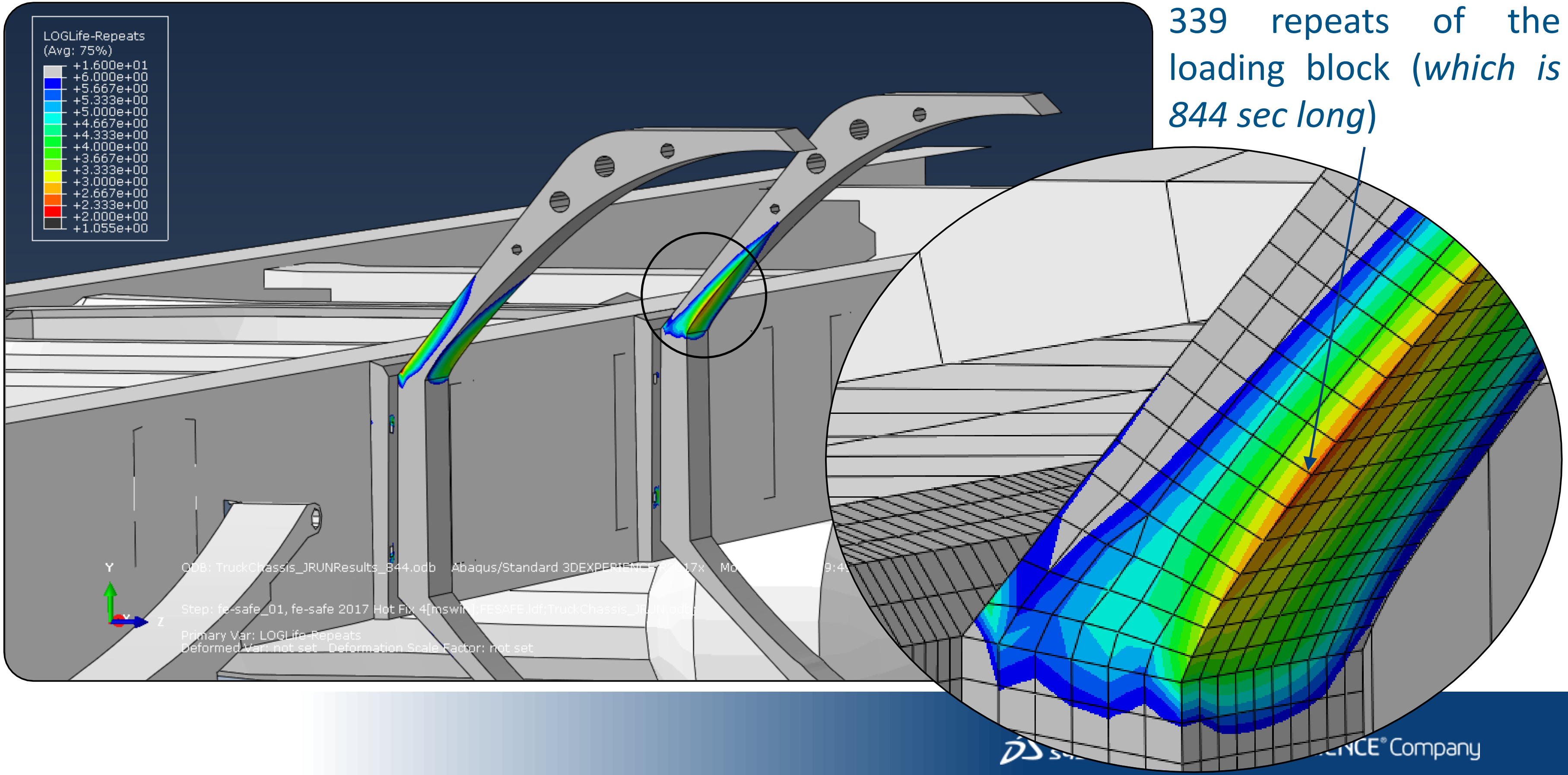


Invariant Method

Critical plane methods

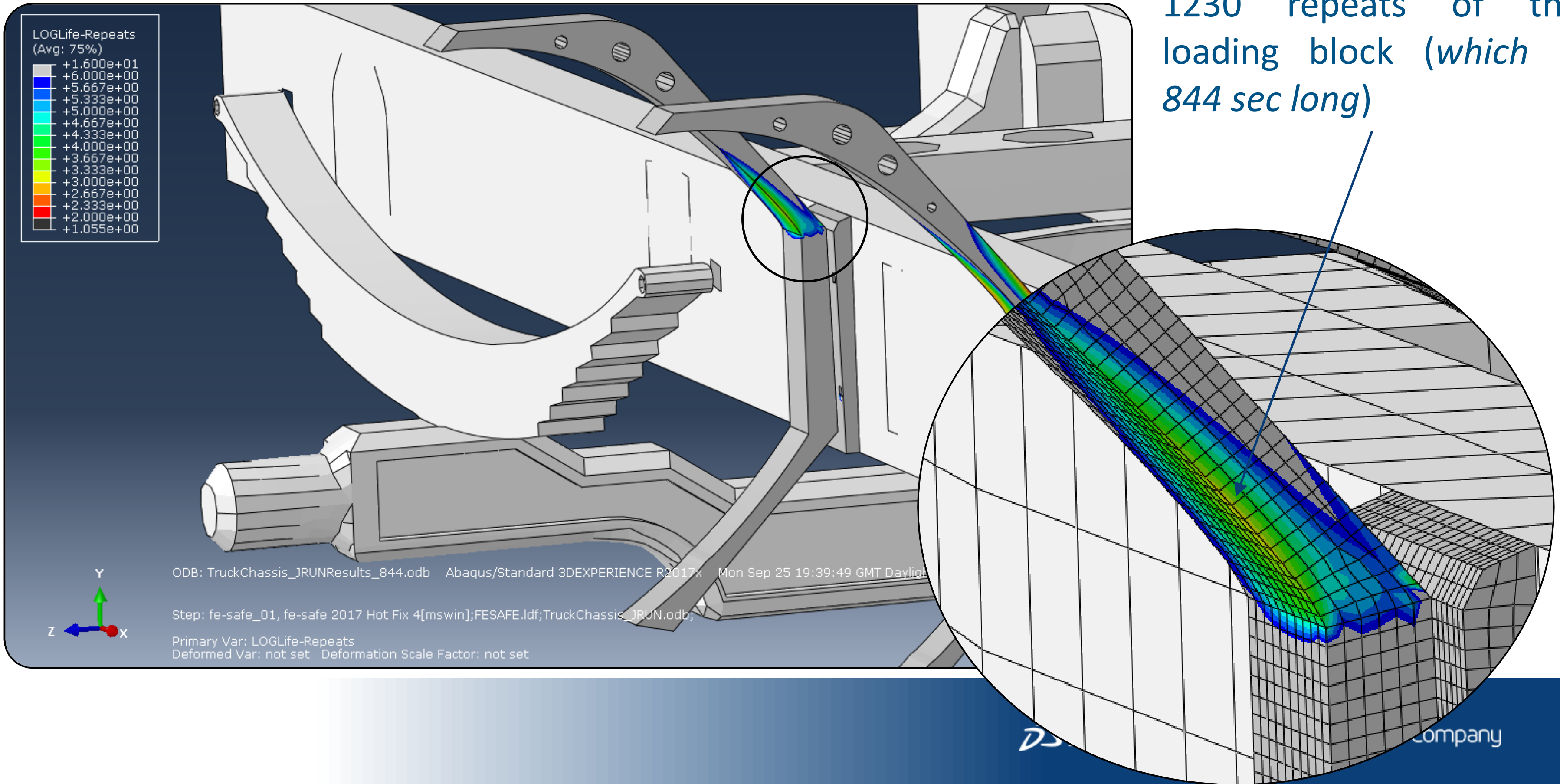
Ready to RUN!

Fatigue Life



339 repeats of the loading block (*which is 844 sec long*)

Fatigue Life



1230 repeats of the loading block (*which is 844 sec long*)

Fatigue Methods

Narrow Band Approach

Bendat

$$D_{BEN} = \frac{n_0^+ T}{k}\left(\sqrt{2m_0}\right)^b \Gamma\left(1+\frac{b}{2}\right)$$

$$\lambda_{2/b} = \int_0^\infty \omega^{2/b} G(\omega)\, d\omega$$

$$N\sigma_a^b = k$$

Single Moment
Lutes and Larsen

$$D_{LAR} = \frac{T}{2\pi k}\left(2\sqrt{2}\right)^b \Gamma\left(1+\frac{b}{2}\right)\left(\lambda_{2/b}\right)^{b/2}$$

$$n_0^+ = \sqrt{\frac{m_2}{m_0}}$$

Ortiz and Chen

$$D_{ORT} = \frac{1}{\gamma}\left(\sqrt{\frac{m_2 m_Q}{m_0 m_{Q+2}}}\right)^m D_{NB}$$

$$\gamma = \sqrt{\frac{m_2^2}{m_0 m_4}}$$

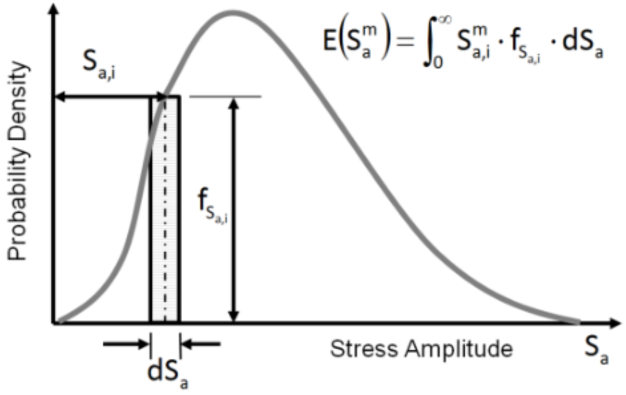
Steinberg
3-Band Method

$$D_{STEIN} = (1/k)\, n_0^+ T\left[0.683\left(2\sqrt{m_0}\right)^b + 0.271\left(4\sqrt{m_0}\right)^b + 0.043\left(6\sqrt{m_0}\right)^b\right]$$

The Dirlik's Method

$$D_{DIR} = \frac{E[P]T}{k} \sum_{0}^{\infty} S_r^b \, p(S_r) \, dS_r$$

$$p(S_r) = \frac{1}{2\sqrt{m_0}} \left[\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}} \right]$$



$$Z = \frac{S_r}{2\sqrt{m_0}} \qquad D_1 = \frac{2(x_m - \gamma^2)}{1+\gamma^2} \qquad D_2 = \frac{1-\gamma-D_1+D_1^2}{1-R} \qquad D_3 = 1 - D_1 - D_2$$

$$E[P] = \sqrt{\frac{m_4}{m_2}} \qquad x_m = \frac{M_1}{M_0}\sqrt{\frac{M_2}{M_4}} \qquad R = \frac{\gamma - x_m - D_1^2}{1-\gamma-D_1+D_1^2} \qquad Q = \frac{1.25(\gamma - D_3 - D_2 R)}{D_1}$$

The Dirlik's Method

$$D_{DIR} = \frac{E[P]T}{k} \sum_{0}^{\infty} S_r^b p(S_r) dS_r$$

$$p(S_r) = \frac{1}{2\sqrt{m_0}} \left[\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}} \right]$$

$$Z = \frac{S_r}{2\sqrt{m_0}} \qquad D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2} \qquad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \qquad D_3 = 1 - D_1 - D_2 \qquad \gamma = \sqrt{\frac{m_2^2}{m_0 m_4}}$$

$$E[P] = \sqrt{\frac{m_4}{m_2}} \qquad x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}} \qquad R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \qquad Q = \frac{1.25(\gamma - D_3 - D_2 R)}{D_1}$$

The Building Blocks

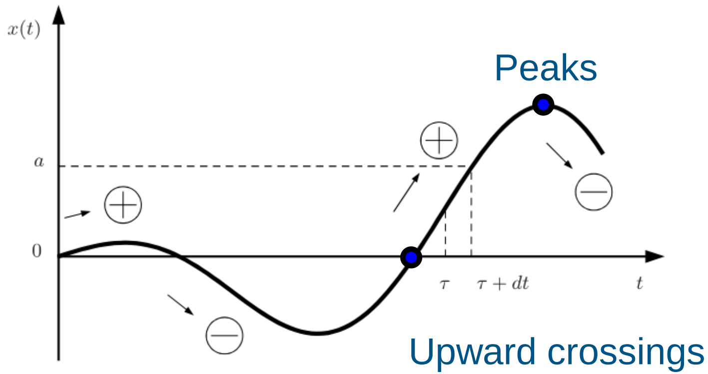
$$m_n = \sum_{k=1}^{N} f_k^n \cdot PSD(k) \cdot \Delta f$$

$\sqrt{m_0}$ Standard deviation

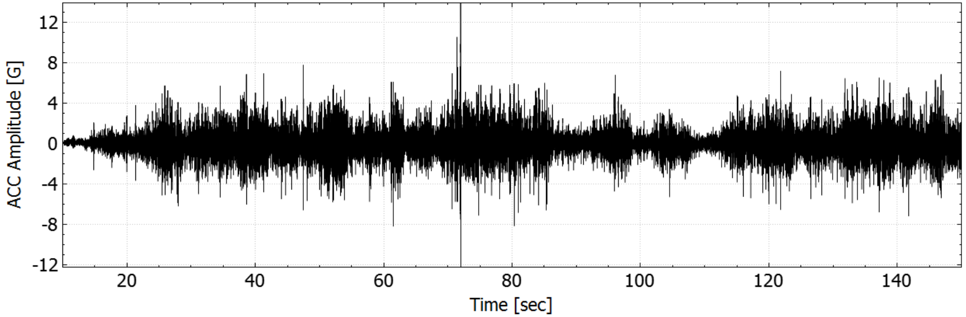
$E[P] = \sqrt{\frac{m_4}{m_2}}$ Peaks per second

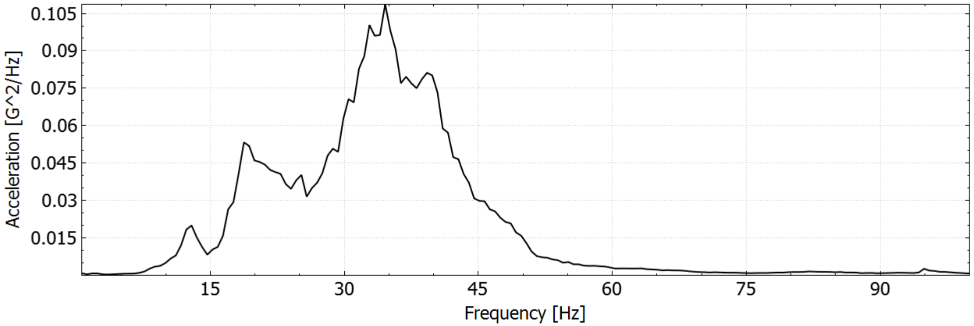
$E[0] = \sqrt{\frac{m_2}{m_0}}$ Upward crossings per second

$\gamma = \sqrt{\frac{m_2^2}{m_0 m_4}}$ bandwidth



Long Signal Example





Final Time = 842.165 sec

Moment_0 = 2.00314
Moment_1 = 70.9763
Moment_2 = 3171.05
Moment_4 = 2.76368e+007

Number Of Peaks/Second = 93
Number Of Zero Crossings/Second = 40
Irregularity Factor = **0.43 (1 = Narrow Band)**

TB Method

according to **Tovo & Benasciutti** Method

$$D_{RFC} = bD_{NB} + \left(1 - b_{app}\right)D_{RC}$$

Narrow band contribution to the damage

Broad band contribution to the damage

Weighting factor

Spectral moments

$$\lambda_i = \int_0^\infty \omega^i S(\omega)\, d\omega$$

Upward mean crossing per second

$$v_+ = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \qquad \alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0 \lambda_2}}$$

Peaks per second

$$v_p = \frac{1}{2\pi}\sqrt{\frac{\lambda_4}{\lambda_2}} \qquad \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0 \lambda_4}}$$

Weighting coefficient

$$b_{app} = \frac{(\alpha_1 - \alpha_2)\left[1.112\left(1 + \alpha_1\alpha_2 - (\alpha_1 + \alpha_2)\right)e^{2.11\alpha_2} + (\alpha_1 + \alpha_2)\right]}{(\alpha_2 - 1)^2}$$

Tovo & Benasciutti Method

Level Crossing PDF

Weighing Factor b

$$D_{RFC}^{TB} = \frac{\nu}{C}\Bigg[b\int_{-SL}^{0}\int_{0}^{SL+m} S^k p_{LC}(s,m)\,ds\,dm + b\int_{0}^{SL}\int_{0}^{SL-m}\left(\frac{S}{1-m/SL}\right)^k p_{LC}(s,m)\,ds\,dm +$$

$$(1-b)\int_{-SL}^{0} dm \int_{0}^{SL+m} S^k p_{RC}(s,m)\,ds + (1-b)\int_{0}^{SL}\int_{0}^{SL-m}\left(\frac{S}{1-m/SL}\right)^k p_{RC}(s,m)\,ds\,dm \Bigg]$$

Range Count PDF

Range Count Joint Distribution

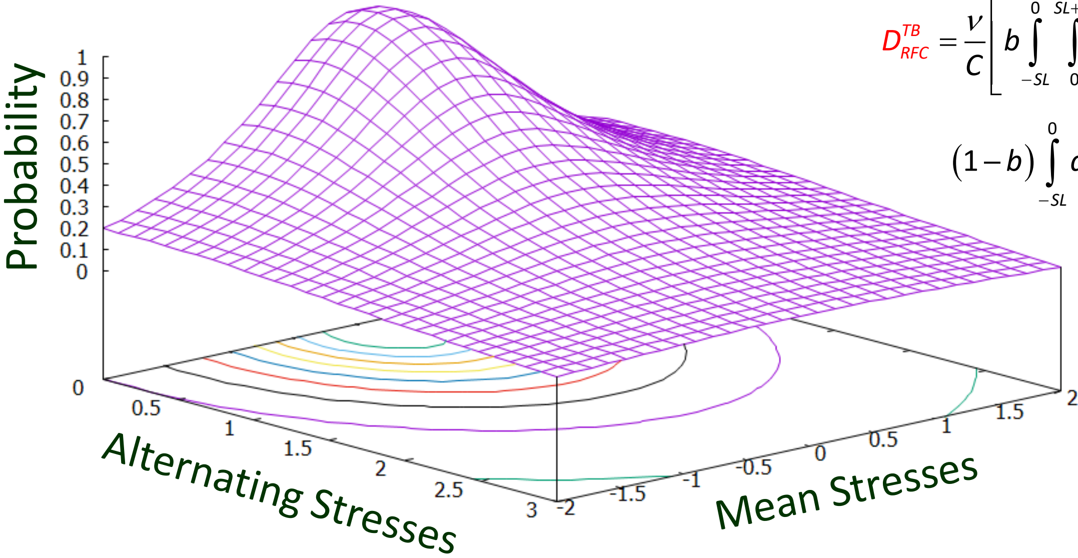


Figure 15

Level Crossing Probability Density Function

$$p_{LC}(s,m) = \begin{cases} \left[\left(p_p(s) - p_v(s)\right)\delta(m-m_c) + p_v(m)\delta(s)\right] & if\ (s+m) > m_c \\ \left[p_p(m)\delta(s)\right] & if\ (s+m) \le m_c \end{cases}$$

$$p_p(x) = \frac{\sqrt{1-\alpha_2^2}}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{(x-m_c)^2}{2\sigma_x^2(1-\alpha_2^2)}} + \frac{\alpha_2(x-m_c)}{\sigma_x^2} e^{-\frac{(x-m_c)^2}{2\sigma_x^2}} \Phi\left(\frac{\alpha_2(x-m_c)}{\sigma_x\sqrt{(1-\alpha_2^2)}}\right)$$

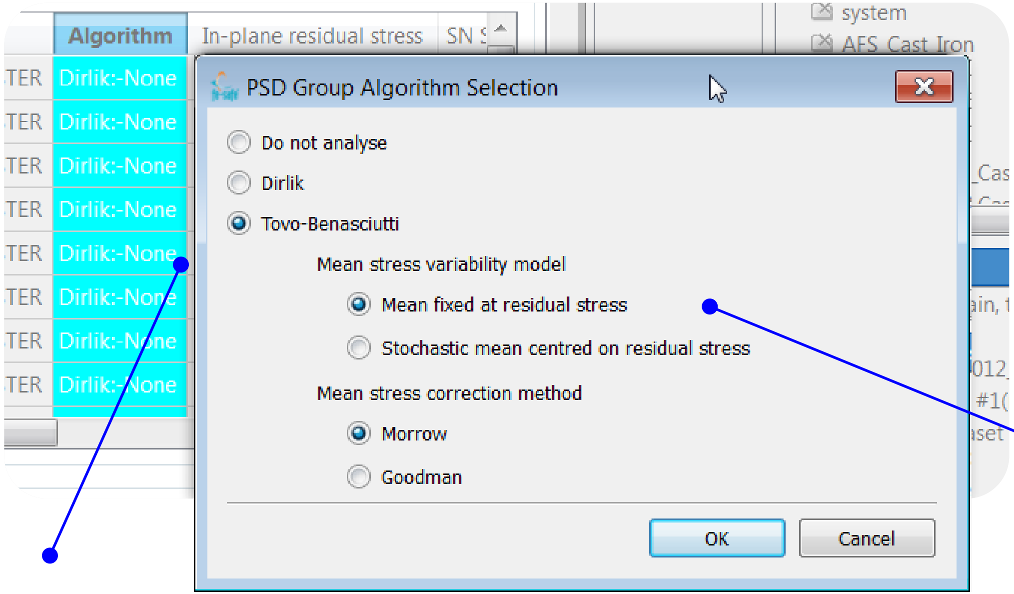
$$p_v(x) = P_p(2m_c - x)$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{\frac{-t^2}{2}}\,dt$$

Range Count Probability Density Function

$$p_{a,m}^{RC}(s,m) = \frac{1}{\sqrt{2\pi\lambda_0(1-\alpha_2^2)}} e^{-\frac{(m-m_c)^2}{2\lambda_0(1-\alpha_2^2)}} \cdot \frac{s}{\lambda_0\alpha_2^2} e^{-\frac{s^2}{2\alpha_2^2\lambda_0}}$$

Mean Stresses



The employed mean stress correction takes the Goodman form for positive mean m:

$$S'_a = S_a\left(1 - \frac{m}{S_L}\right)^{-1}$$

The limit stress can be set to either the stress which gives damage of 1 on the SN curve (Use SN curve intercept in MSC, the default) or the UTS (over conservative).

When the random mean form of Tovo-Benasciutti is used, then as well as integrating the expected damage over the Rayleigh distribution of stress, the (wide band) range counting component is also integrated over the Gaussian distribution of mean stresses. This will produce more damage than using a fixed mean.

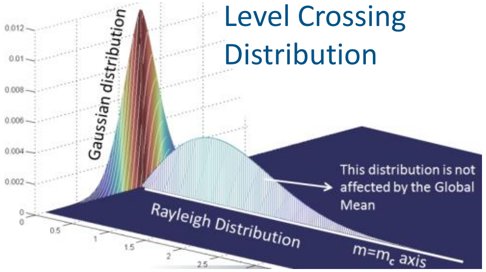
Vibration fatigue

$$D_{RFC} = bD_{NB} + (1-b)D_{BB}$$

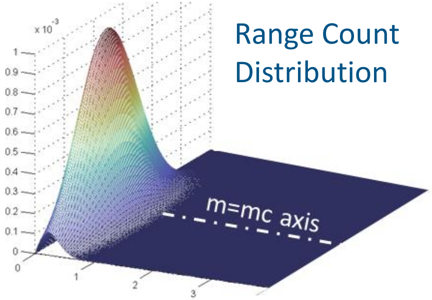
Frequency-Domain Fatigue Analysis

fe-safe 2016:

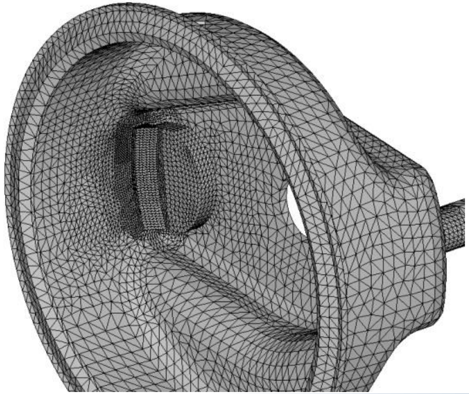
- Tovo and Benasciutti Method
  - Addresses mean and residual stresses
  - Weights damage evaluated by narrow and broad band distributions



Level Crossing Distribution

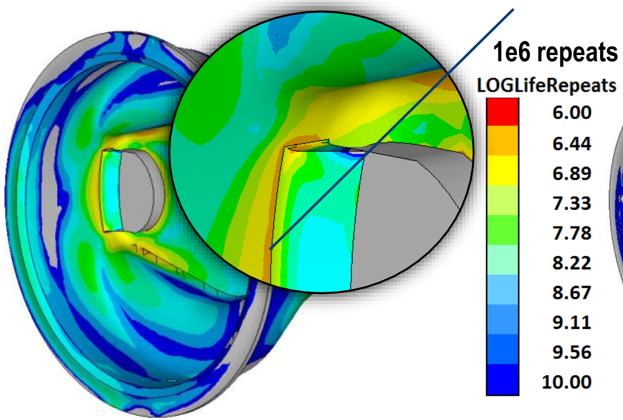


Range Count Distribution



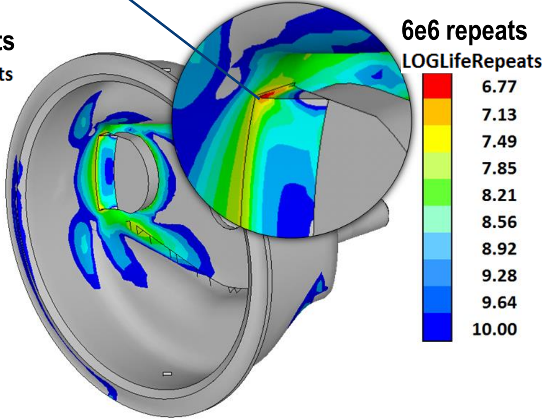
TB Method

failure



non-failure

Dirlik Method



Fatigue Failure

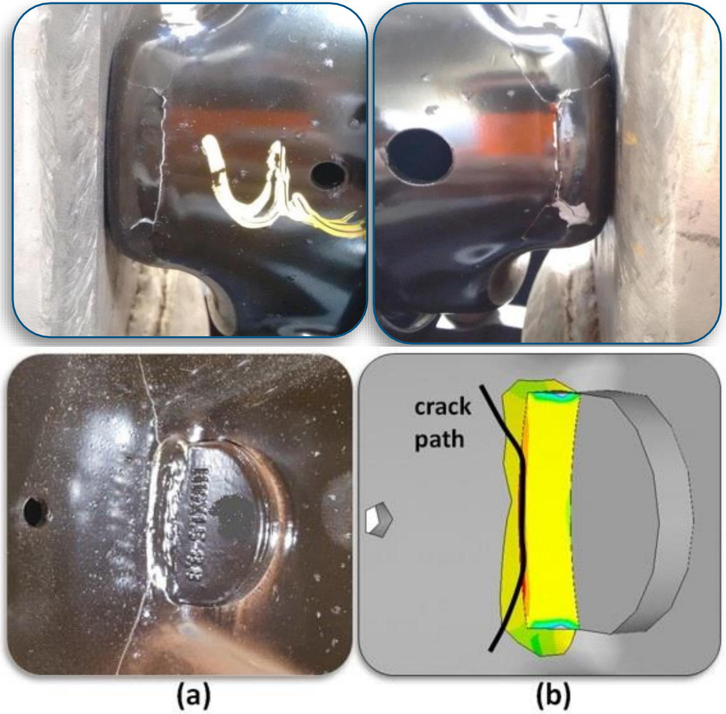


Figure 21

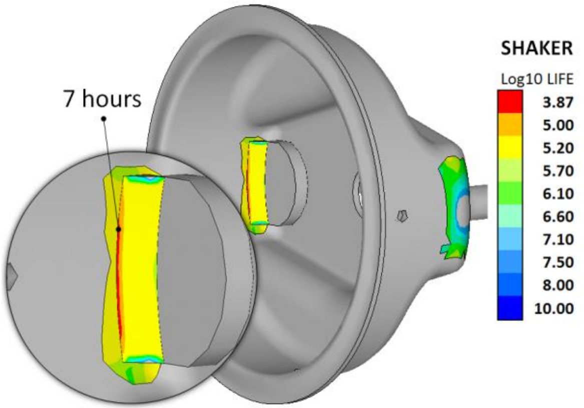


Figure 22

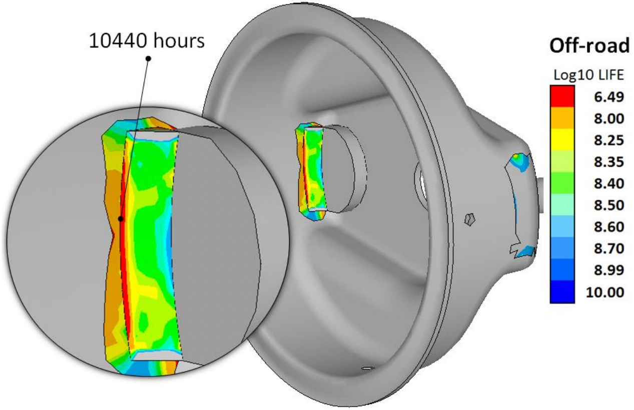


Figure 23

IF WE ™