Analysis of Bending

1. BENDING STRESS

Any force applied transversely to the structural axis of a partially supported member sets up bending moments (M) along the length of the member. These in turn stress the cross-sections in bending.

As shown in Figure 1, the bending stresses are zero at the neutral axis, and are assumed to increase linearly to a maximum at the outer fiber of the section. The fibers stressed in tension elongate; the fibers stressed in compression contract. This causes each section so stressed to rotate. The cumulative effect of this movement is an over-all deflection (or bending) of the member.

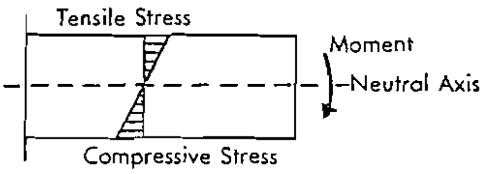


FIGURE 1

The cantilever beam shown in Figure 1 is in tension along the top and in compression along the bottom. In contrast, the relationship of the applied force and the points of support on the member shown in Figure 2 is such that the curve of deflection is inverted, and the member is in tension along the bottom and in compression along the top.

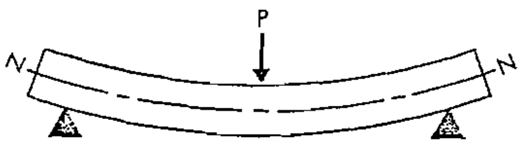


FIGURE 2

Within the elastic range (i.e. below the proportional elastic limit or the yield point), the bending stress (σ_b) at any point in the cross-section of a beam is —

$$\boxed{\sigma_b = \frac{M\ c}{I}} \quad \text{..........} (1)$$

where:

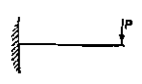
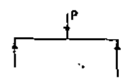
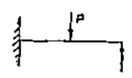
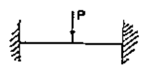
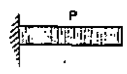
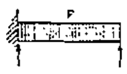
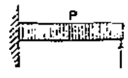
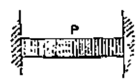
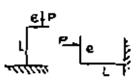
M = bending moment at the section in question, in.-lbs

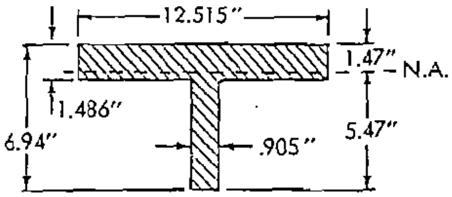
I = moment of inertia of the section, in.^{4}

c = distance from neutral axis to the point at which stress is desired, in.

σ_b = bending stress, may be tension or compression, psi

TABLE 1—Beam Diagrams

Type of Beam	Maximum moment	Maximum deflection	Maximum shear
	$M = PL$ Fixed end	$\Delta = \frac{PL^3}{3EI}$ Free end	$V = P$
	$M = \frac{PL}{4}$ center	$\Delta = \frac{PL^3}{48EI}$ center	$V = \frac{P}{2}$
	$M = \frac{3PL}{16}$ Fixed end	$\Delta = \frac{PL^3}{48EI\sqrt{5}}$	$V = \frac{11}{16}P$
 Guided Free	$M = \frac{PL}{2}$ both ends	$\Delta = \frac{PL^3}{12EI}$ guided end	$V = P$
	$M = \frac{PL}{8}$ center & ends	$\Delta = \frac{PL^3}{192EI}$ center	$V = \frac{P}{2}$
	$M = \frac{PL}{2}$ Fixed end	$\Delta = \frac{PL^3}{8EI}$ Free end	$V = P$
	$M = \frac{PL}{8}$ center	$\Delta = \frac{5PL^3}{384EI}$ center	$V = \frac{P}{2}$
	$M = \frac{PL}{8}$ Fixed end	$\Delta = \frac{PL^3}{185EI}$	$V = \frac{5}{8}P$
 Guided Free	$M = \frac{PL}{3}$ Fixed end	$\Delta = \frac{PL^3}{24EI}$ guided end	$V = P$
	$M = \frac{PL}{12}$ both ends	$\Delta = \frac{PL^3}{384EI}$ center	$V = \frac{P}{2}$
	$M = Pe$ whole beam	$\Delta = \frac{PeL^2}{2EI}$ right angles to force	$V = O$



The bending moment (M) may be determined from standard beam diagrams. Table 1 lists several of these, along with the formulas for bending moment, shear, and deflection. A more complete presentation is included in the Reference Section on Beam Diagrams.

Normally there is no interest in knowing what the bending stresses are somewhere inside a beam. Usually the bending stress at the outer fiber is needed because it is of maximum value. In an unsymmetrical section, the distance c must be taken in the correct direction across that portion of the section which is in tension or that portion which is in compression, as desired. Ordinarily only the maximum stress is needed and this is the stress at the outer fiber under tension, which rests at the greater distance c from the neutral axis.

Problem 1

A standard rolled "T" section (ST-6" wide flange, 80.5 lbs) is used as a beam, 100" long, supported on each end and bearing a concentrated load of 10,000 lbs at the middle. Find the maximum tensile and maximum compressive bending stresses.

Figure 3 shows the cross-section of this beam, together with its load diagram.

Referring to Table 1, the formula for the bending moment of this type of beam is found to be—

$$M = \frac{PL}{4} \text{ and therefore}$$

$$= \frac{(10{,}000)(100)}{4}$$

$$= 250{,}000 \text{ in.-lbs}$$

Since the bottom portion of the beam is stressed in tension, substituting appropriate known values into the formula:

$$\sigma_t = \frac{M\ c}{I}$$

$$= \frac{(250{,}000)(5.47)}{(6.26)}$$

$$= \underline{21{,}845 \text{ psi}} \text{ (tension)}$$

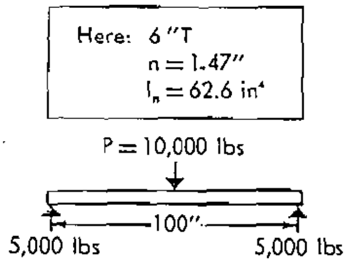


FIGURE 3

The top portion of the beam being in compression,

$$\sigma_c = \frac{M\ c}{I}$$

$$= \frac{(250{,}000)(1.47)}{62.6}$$

$$= \underline{5{,}870 \text{ psi}} \text{ (compression)}$$

Problem 2

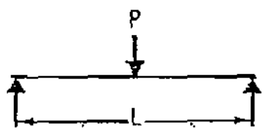


FIGURE 4

Find the maximum deflection of the previous beam under the same loading. From the beam diagrams, Table 1, the appropriate formula is found to be—

$$\Delta_{max} = \frac{P\ L^3}{48\ E\ I} \text{ and therefore}$$

$$= \frac{(10{,}000)(100)^3}{48(30 \times 10^6)(62.6)}$$

$$= \underline{.111''}$$

2. HORIZONTAL SHEAR STRESS

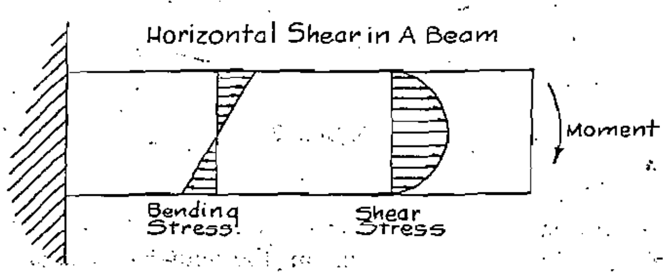


FIGURE 5

In addition to pure bending stresses, horizontal shear stress is often present in beams, Figure 5. It depends on vertical shear and only occurs if the bending moment varies along the beam. (Any beam, or portion of the beam's length, that has uniform bending moment has no vertical shear and therefore no horizontal shear).

Unlike bending stress, the horizontal shear stress is zero at the outer fibers of the beam and is maximum at the neutral axis of the beam. It tends to cause one part of the beam to slide past the other.

The horizontal shear stress at any point in the cross-section of a beam, Figure 6, is—

$$\boxed{\tau = \frac{V\,a\,y}{I\,t}} \quad \cdots\cdots (2)$$

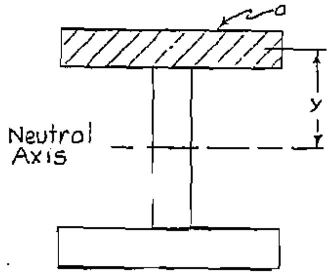


FIGURE 6

where:

V = external vertical shear on beam, lbs

I = moment of inertia of whole section, in.^{4}

t = thickness of section at plane where stress is desired, in.

a = area of section beyond plane where stress is desired, in.^{2}

y = distance of center of gravity of area to neutral axis of entire section, in.

Problem 3

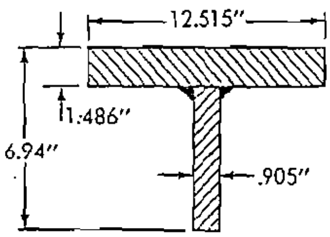


FIGURE 7

Assume that the "T" beam in our previous example (Problem 1) is fabricated by welding. Under the same load conditions,

(a) Find the horizontal shear stress in the plane where the web joins the flange.

(b) Then find the size of continuous fillet welds on both sides, joining the web to the flange.

From the beam diagrams, Table 1, the appropriate formula for vertical shear (V) is found to be—

$$V = \frac{P}{2} \text{ and thus}$$

$$= \frac{10{,}000}{2}$$

$$= 5{,}000 \text{ lbs}$$

The following values also are known or determined to be—

$I = 62.6$ in.^{4}

$a = 1.486 \times 12.515 = 18.6$ in.^{2}

$y = 0.727''$

$t = 0.905''$

(a) Substituting the above values into the formula, the horizontal shear stress (τ) is found:

$$\tau = \frac{V\,a\,y}{I\,t}$$

$$= \frac{(5000)(18.6)(0.727)}{(62.6)(0.905)}$$

$$= \underline{1196 \text{ psi}}$$

(b) Since the shear force is borne entirely by the web of the "T", the horizontal shear force (f) depends on the thickness of the web in the plane of interest:

$f = \tau\, t$ and thus

$= 1196 \times 0.905$

$= 1080$ lbs/in.

There are two fillet welds, one on each side of the "T" joining the flange to the web. Each will have to support half of the shear force or 540 lbs/in. and its leg size would be:

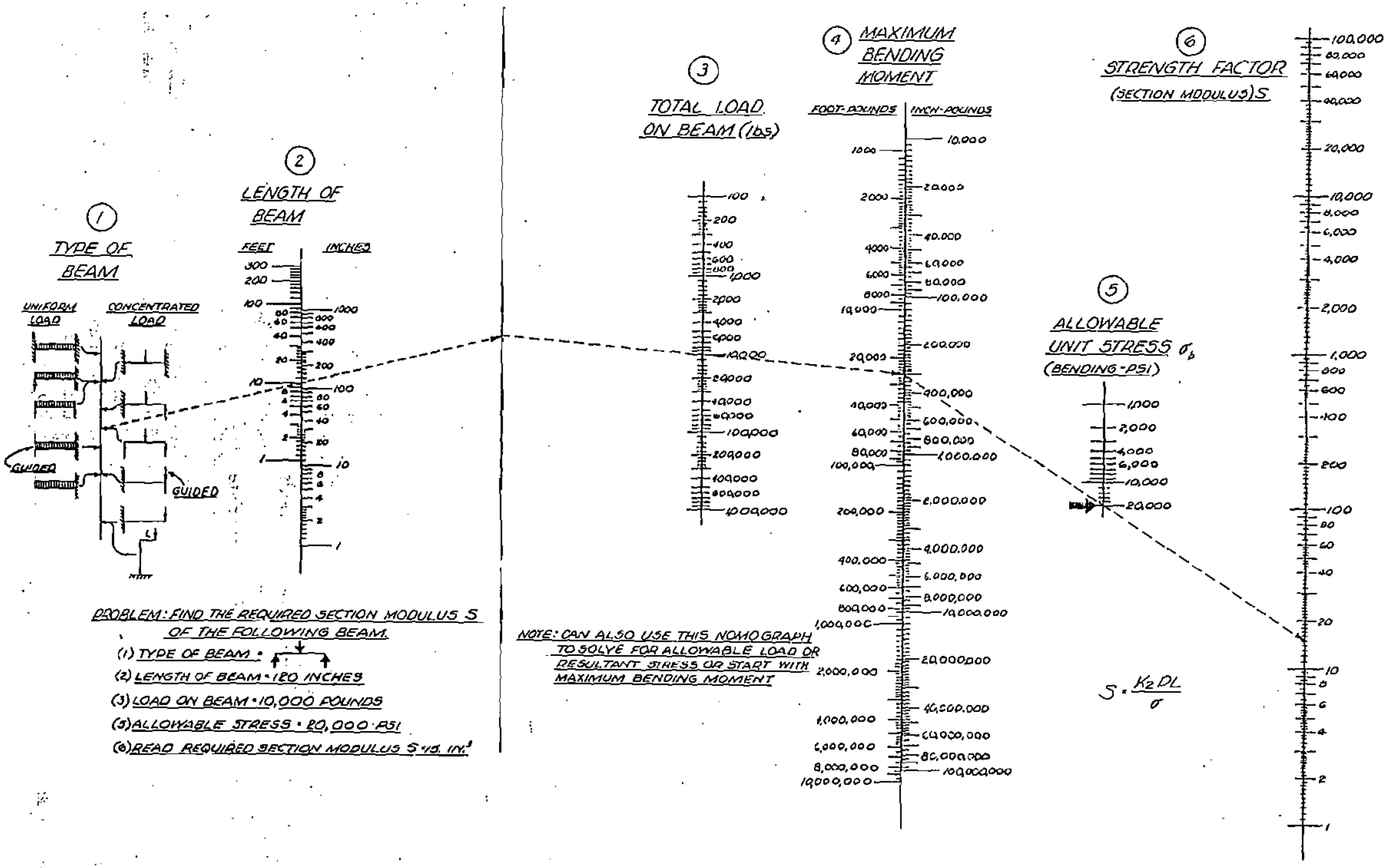
$$\omega = \frac{540}{9600}$$

$$= .056''$$

This would be an extremely small continuous fillet weld. Based upon the AWS, the minimum size fillet weld for the thicker 1.47″ plate would be 5/16″.

If manual intermittent fillet welds are to be used, the percentage of the length of the joint to be welded would be:

FIGURE 8—Required Section Modulus of Beam Under Bending Load
(Strength Nomograph)



$$\% = \frac{\text{calculated leg size of continuous fillet weld}}{\text{actual leg size of intermittent fillet weld used}} \times 100$$

$$= \frac{.056}{5/16} = 18\%$$

A 5/16 ⊿ 3–12 fillet weld would satisfy this requirement because it results in 25% of the length of the joint being welded.

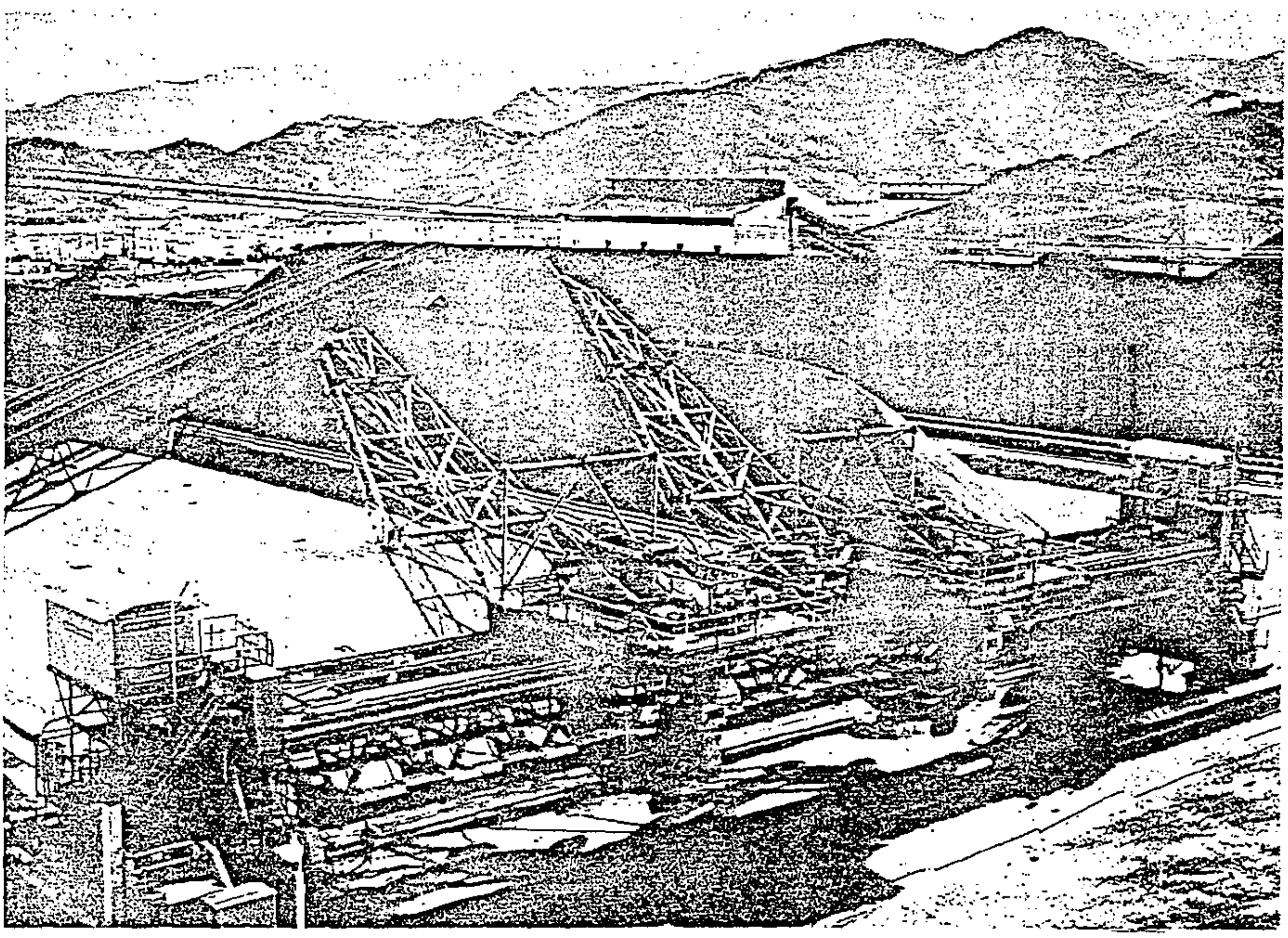
3. QUICK METHOD FOR FINDING REQUIRED SECTION MODULUS (STRENGTH) OR MOMENT OF INERTIA (STIFFNESS)

To aid in designing members for bending loads, the following two nomographs have been constructed. The first nomograph determines the required strength of a straight beam. The second nomograph determines the required stiffness of the beam.

In both nomographs several types of beams are included for concentrated loads as well as uniform loads. The length of the beam is shown both in inches and in feet, the load in pounds. In the first nomograph (Fig. 8) an allowable bending stress (σ_b) is shown and the strength property of the beam is read as section modulus (S). In the second nomograph (Fig. 9) an allowable unit deflection (Δ/L) is shown. This is the resulting deflection of the beam divided by the length of the beam. The stiffness property of the beam is read as moment of inertia (I).

By using these nomographs the designer can quickly find the required section modulus (strength) or moment of inertia (stiffness) of the beam. He can then refer to a steel handbook to choose a steel section that will meet these requirements.

If he wishes to fabricate the section from welded steel, he may use any of the methods for building up a steel section having the required values of section modulus or moment of inertia discussed in Properties of Sections.



More than a carload of welding electrode was employed in the fabrication of this huge bucket-wheel iron ore reclaiming machine at the Eagle Mountain Mine. Steel pipe was used extensively in the 170′ long all-welded truss, of triangular cross-section, that is the main load-carrying member.

FIGURE 9—Required Moment of Inertia of Beam Under Bending Load
(Stiffness Nomograph)