Bridge Plate Girders With Variable Depth

1. TYPES OF HAUNCHED GIRDERS

It has been pointed out* that the sloping bottom flange of the parabolic haunch has a vertical component of its compressive force and this will reduce the shear stress (τ_{xy}) in the girder web in this region. In addition, the concave compression flange produces a radial compressive stress (σ_y) in the web depending on the radius of curvature of the flange.

In contrast, the fish belly haunch provides no appreciable reduction in shear in the critical portion of the web near the support. This is because the slope of the bottom flange is small in that area. Also, the convex compressive flange produces a radial tensile stress (σ_y) in the web, which is greater than the radial compressive stress in the parabolic haunch. This is because of the sharper curvature of the fish belly haunch.

It is seen by observation of the Huber-Mises formula that both of these factors will result in the yield criterion (σ_{cr}) having a lower value in the case of the parabolic haunch. This result compared with the yield strength of the steel (in uniaxial tension) would indicate a higher factor safety.

(Huber-Mises Formula)

$$\boxed{\sigma_{cr} = \sqrt{\sigma_x^2 - \sigma_x\,\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}}$$

Haunched girders do not present much increase in cost for welded construction for longer spans. The web plates are normally trimmed by flame cutting, so that a gradual curve would add little to the cost. In most cases the curved flange plates can be added without prior forming; the flat flange plates are simply pulled into place against the curved web. Although the transverse stiffeners would vary in length, this should be no problem. The flange can still be automatically fillet welded to the web by placing the web in the horizontal position. The portable automatic welder would then ride against the curved flange.

* "Design of the Bridge Over the Quinnipiac River" by Roman Wolchuk.

2. NEED FOR MODIFIED SHEAR FORCE VALUE

The horizontal force (F_h) in the sloping flange is equal to the bending moment at that section divided by the vertical distance between the two flanges:

$$F_h = \frac{M}{d}$$

Or, this force may be found by multiplying the flange area by the bending stress in the flange using the section modulus of the girder. This method will produce a more accurate value.

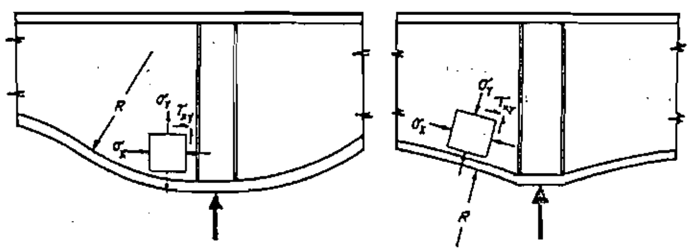
From this value, the actual force in the flange (F_x) may be found, as well as the vertical component (F_v) of this force:

$$F_x = \frac{F_h}{\cos\theta} = \frac{M}{d\cos\theta} \quad \text{and}$$

$$F_v = F_h \tan\theta = \frac{M}{d}\tan\theta$$

This vertical component (F_v) acting along with the shear force in the web resists the external shear (V) at this section.

Modified shear is the resulting shear force in the web after the vertical component of the flange force (F_v) is substracted or added, depending upon whether it acts in the same direction or opposite direction as the shear in the web.



Fish Belly Haunch　　　Parabolic Haunch

FIGURE 1

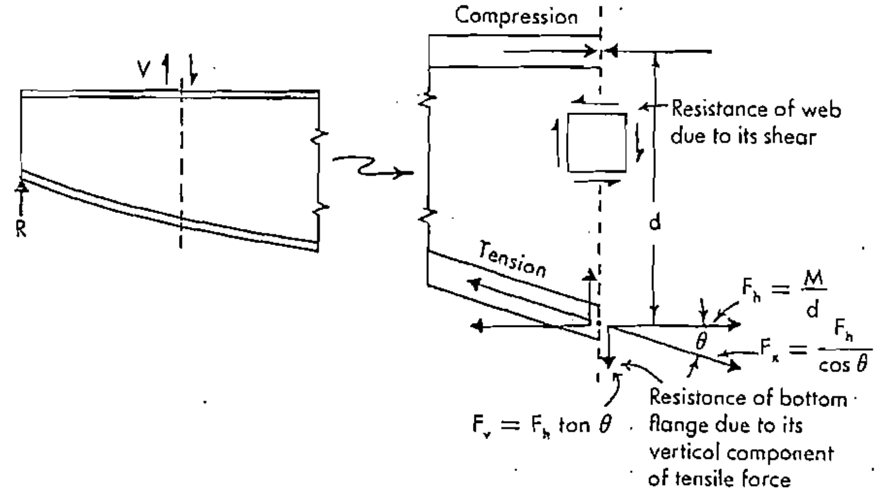


FIGURE 2

Simply Supported Girder Straight or Curved Bottom Flange

See Figure 2.

Here the external shear is—

$$V = A_w \tau_W + \frac{M}{d} \tan \theta$$

and the modified shear is—

$$V' = A_w \tau_W$$

$$= V - \frac{M}{d} \tan \theta$$

In this case the vertical component is subtracted from the web shear.

Continuous Parabolic Haunched Girder

See Figure 3.

Here the external shear is—

$$V = A_w \tau_W + \frac{M}{d} \tan \theta$$

and the modified shear is—

$$V' = A_w \tau_W$$

$$= V - \frac{M}{d} \tan \theta$$

In this case the vertical component is subtracted from the web shear.

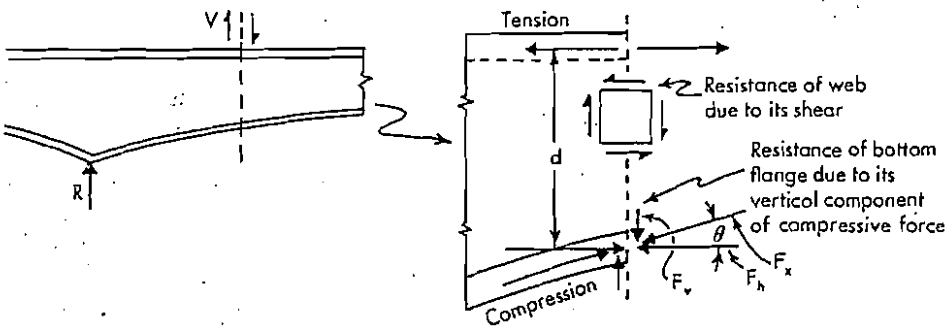


FIGURE 3

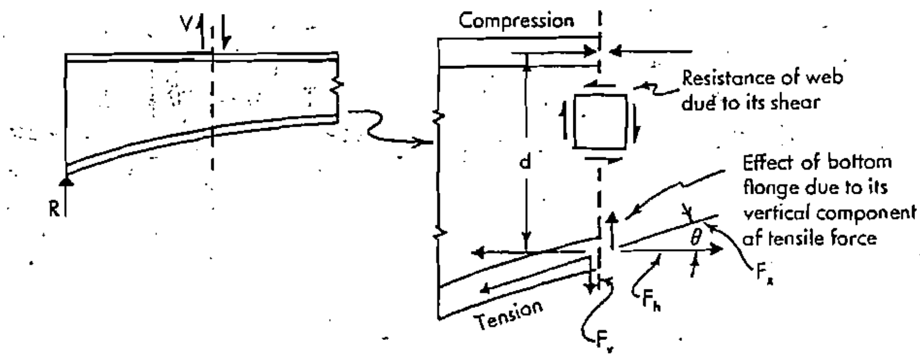


FIGURE 4

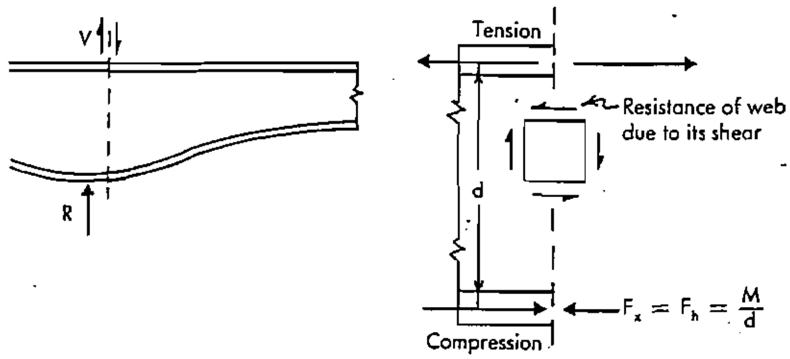


FIGURE 5

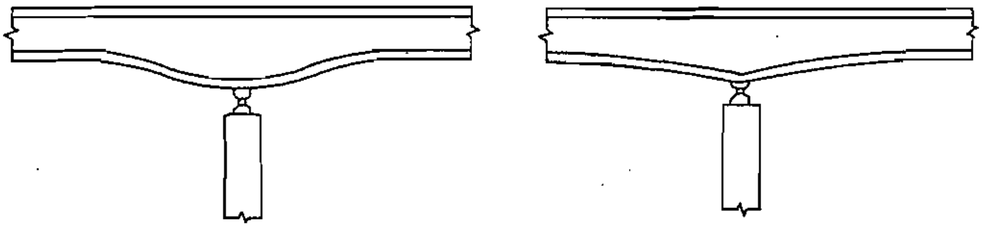


FIGURE 6

Fish Belly Haunch　　Parabolic Haunch

Simply Supported Haunched Girder

See Figure 4.

Here the external shear is—

$$V = A_W\ \tau_w - \frac{M}{d} \tan \theta$$

and the modified shear is—

$$V' = A_W\ \tau_w$$

$$= V + \frac{M}{d} \tan \theta$$

In this case the vertical component is added to the web shear.

Continuous Fish Belly Haunched Girder

See Figure 5.

Here the external shear is—

$$V = A_W\ \tau$$

In this case the flange force has no vertical component; hence, there is no reduction of shear in the web.

Problem 1

Check the haunched girder section (at point of support) shown in Figure 7, to determine the difference between the fish belly haunch and the parabolic haunch in the area of the compression flange near the support.

See Figure 6.

Conditions include the following:

Use of A441 steel

$M = 55{,}000$ ft-kips

$V = 1200$ kips

$I_x = 3{,}979{,}000$ in.^{4}

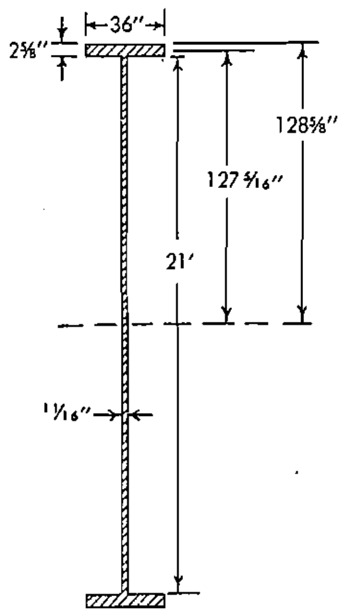


FIGURE 7

Analysis of Parabolic Haunch

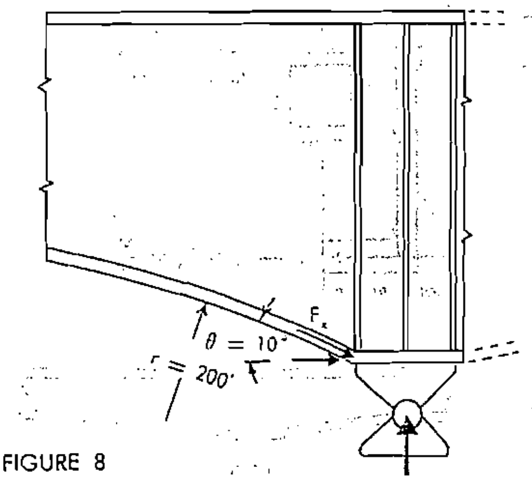


FIGURE 8

average bending stress in lower flange

$$\sigma_f = \frac{Mc}{I}$$

$$= \frac{(55{,}000'^{k} \times 12)(127\tfrac{5}{16}'')}{3{,}979{,}000}$$

$$= 21{,}150 \text{ psi compression}$$

flange forces

$$F_h = \sigma_f A_f$$

$$= (21{,}150)(2\tfrac{5}{8} \times 36)$$

$$= 2{,}000 \text{ kips}$$

$$F_v = F_h \tan\theta$$

$$= (2000)(.1763)$$

$$= 353 \text{ kips}$$

$$F_x = \frac{F_h}{\cos\theta}$$

$$= \frac{2000}{.9848}$$

$$= 2030 \text{ kips}$$

shear stress in web

Since the external shear is—

$$V = A_w\,\tau_w + F_v \text{ or}$$

$$A_w\,\tau_w = V - F_v \text{ and}$$

$$\tau_w = \frac{V - F_v}{A_w}$$

$$= \frac{1200 - 353}{(252 \times 1\tfrac{1}{16})}$$

$$= 4890 \text{ psi}$$

stress in web at lower flange at support

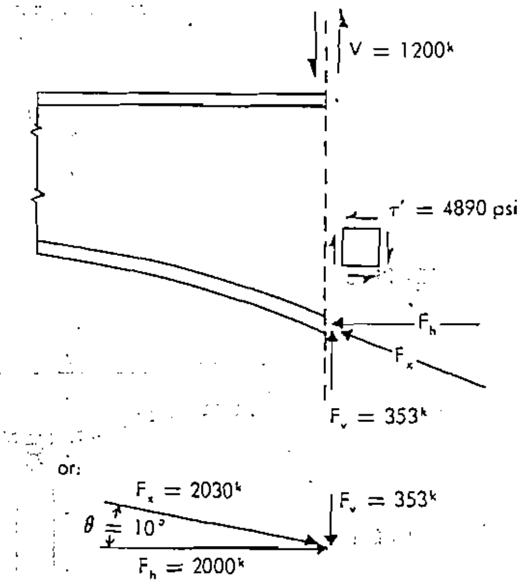


FIGURE 9

$$\sigma_h = \frac{M\,c}{I}$$

$$= \frac{(55{,}000 \times 12)(126)}{(3{,}979{,}000)}$$

$$= 20{,}900 \text{ psi, compression}$$

These stresses in Figure 10, left-hand side, must now be rotated 10° to line up with the sloping flange in order that the radial compressive stress may be added. This is shown on the right-hand side of Figure 10. This may be analyzed by one of two methods:

1. Graphically, using Möhr's circle of stress: (Fig. 11)
   a) Draw the given stresses (σ_x', σ_y', and τ') at the two points (a') and (b')
   b) Construct a circle through these two points
   c) Rotate clockwise through an angle of 2θ or 20°
   d) Read the new stresses (σ_x, σ_y, and τ)
2. Analytically; work is performed as follows:

$$k = \frac{\sigma_x' + \sigma_y'}{2}$$

$$= \frac{20{,}900}{2}$$

$$= 10{,}450$$

$$\tan\alpha = \frac{\tau'}{\tfrac{1}{2}(\sigma_x' + \sigma_y')}$$

$$= \frac{4890}{\tfrac{1}{2}(20{,}900)}$$

$$= .4680$$

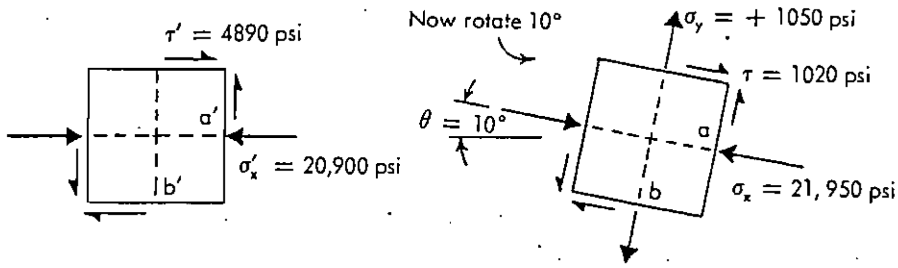


FIGURE 10

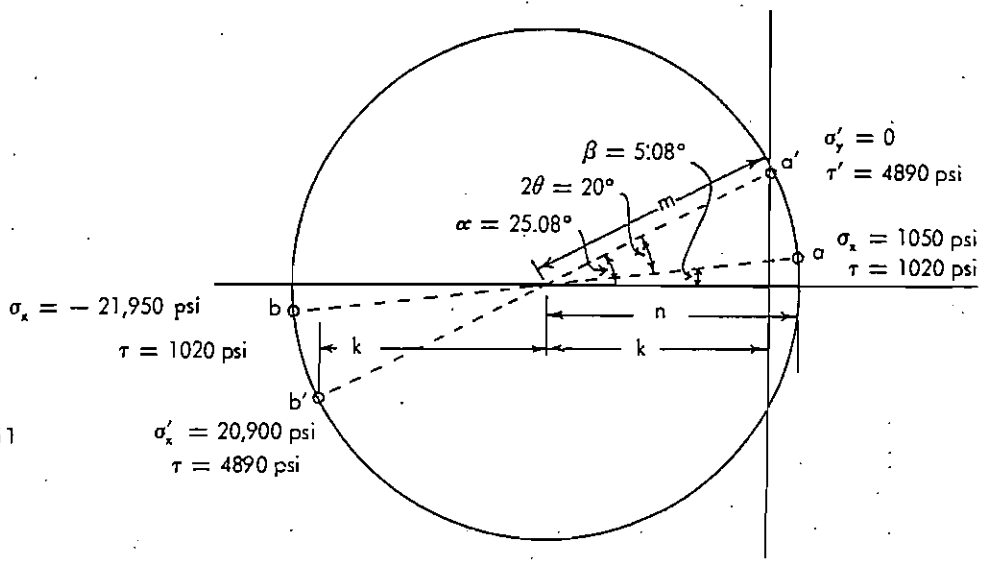


FIGURE 11

$$\alpha = 25.08°$$

$$\beta = 25.08° - 20°$$
$$= 5.08°$$

$$\sin \beta = .0886$$

$$\cos \beta = .9961$$

$$m = \sqrt{k^2 + (\tau')^2}$$
$$= \sqrt{(10,450)^2 + 4890^2}$$
$$= 11,540$$

$$\tau = m \sin \beta$$
$$= (11,540)(.0886)$$
$$= 1020 \text{ psi}$$

$$n = m \cos \beta$$
$$= (11,540)(.9961)$$
$$= 11,500 \text{ psi}$$

$$\sigma_x = k + n$$
$$= (10,450) + (11,500)$$
$$= 21,950 \text{ psi, compression}$$

$$\sigma_y = k - n$$
$$= (10,450) - (11,500)$$
$$= 1050 \text{ psi, tension}$$

radial force of lower compression flange against web

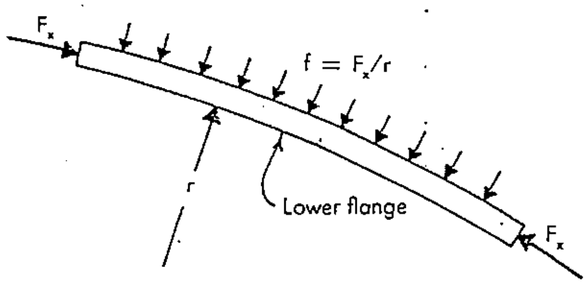
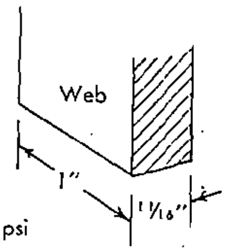


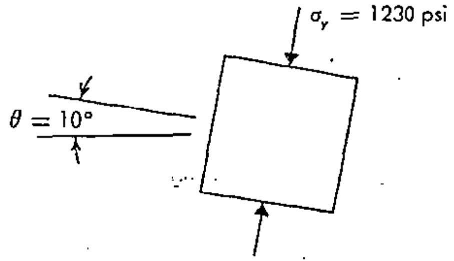
FIGURE 12

$$f = \frac{F_x}{r}$$
$$= \frac{(2030)}{(200 \times 12)}$$
$$= 846 \text{ lbs/linear in.}$$

resultant radial compressive stress in web

$$\sigma_y = \frac{(846)}{(1 \times {}^{11}\!/_{16})}$$
$$= 1230 \text{ psi, compression}$$





This produces the final stress condition of:

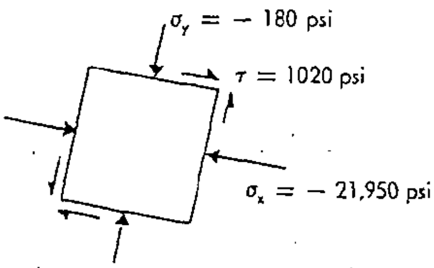


FIGURE 13

critical stress

Using the Huber-Mises formula:

$$\sigma_{cr} = \sqrt{\sigma_x^2 - \sigma_x\,\sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2}$$
$$= \sqrt{(-21{,}950)^2 - (-21{,}950)(-180) + (-180)^2 + 3(1020)^2}$$
$$= \underline{22{,}000 \text{ psi}}$$

This results in an indicated factor of safety against yielding of—

$$FS = \frac{\sigma_y}{\sigma_{cr}}$$
$$= \frac{(42{,}000)}{(22{,}000)}$$
$$= \underline{1.90}$$

Analysis of Fish Belly Haunch

Now using the same load conditions on the fish belly haunch with the same web and flange dimensions:

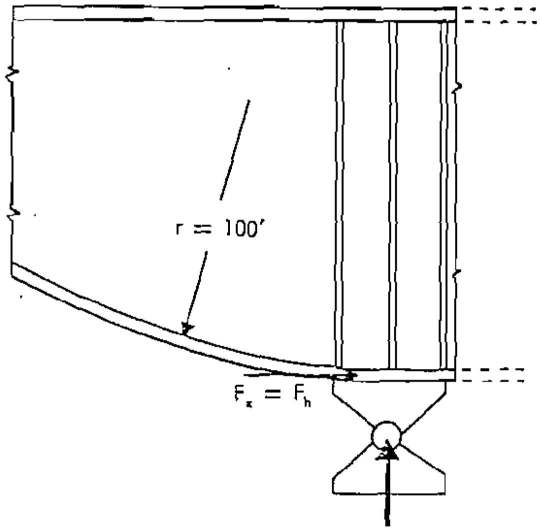


FIGURE 14

At this point: $\sigma_x = \sigma_h$ & $F_x = F_h$

stress in web or lower flange from bending moment

$$\sigma_x = \frac{M\ c}{I}$$
$$= \frac{(55{,}000'^{k} \times 12)(126)}{3{,}979{,}000}$$
$$= 20{,}900 \text{ psi, compression}$$

average stress in lower flange from bending moment

$$\sigma_t = \frac{M\ c}{I}$$
$$= \frac{(55{,}000'^{k} \times 12)(127\,{}^{5}\!/_{16})}{3{,}979{,}000}$$
$$= 21{,}150 \text{ psi}$$

force in lower flange from bending moment

$$F_x = \sigma_t\, A_t$$
$$= (21{,}150)(2\tfrac{5}{8} \times 36)$$
$$= 2000 \text{ kips}$$

radial tensile force of lower compression flange against web

$$f = \frac{F_x}{r}$$
$$= \frac{(2000)}{(100 \times 12)}$$
$$= 1670 \text{ lbs/linear in.}$$

resultant radial tensile stress in web

$$\sigma_y = \frac{f}{t_w}$$
$$= \frac{(1670)}{(1 \times {}^{11}\!/_{16})}$$
$$= 2420 \text{ psi}$$

shear stress in web

$$\tau = \frac{V}{A_w}$$
$$= \frac{(1200)}{(252 \times {}^{11}\!/_{16})}$$
$$= 6930 \text{ psi}$$

combining stresses to find the critical stress

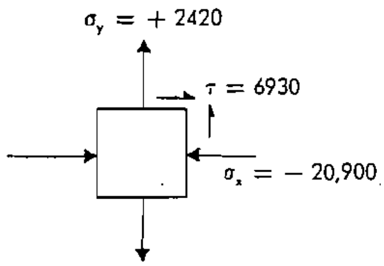


FIGURE 15

Using the Huber-Mises formula:

$$\sigma_{cr} = \sqrt{\sigma_x^2 - \sigma_x\,\sigma_y + \sigma_y^2 + 3\,\tau_{xy}}$$
$$= \sqrt{(20{,}900)^2 - (-20{,}900)(+2460) + (+2460)^2 + 3(6{,}930)^2}$$
$$= \underline{25{,}100 \text{ psi}}$$

This results in an indicated factor of safety against yielding of—

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{cr}}$$
$$= \frac{(42{,}000)}{(25{,}100)}$$
$$= \underline{1.67}$$

It is apparent from this that the parabolic haunch has a slightly lower critical stress and, therefore, a slightly higher factor of safety.

3. WELDS CONNECTING SLOPING FLANGE TO WEB

In any girder, the horizontal shear force in the connecting weld between the web and the horizontal flange is found from the following formula:

$$f = \frac{V\,a\,y}{I\,n} \text{ lbs/in.}$$

Approximate value:

$$f_h = \frac{V}{d\,n} \text{ lbs/in.}$$

Where the flange slopes, the modified vertical shear (V′) must be used. The shear component along the slope will be—

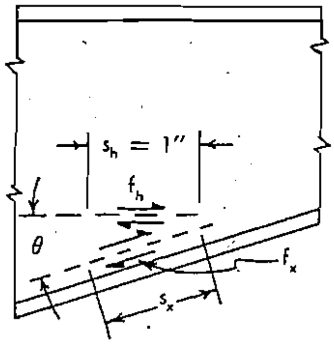
$$f_x = \frac{f_h}{\cos\theta}$$

but the distance along this slope for every horizontal inch is—

$$s_x = \frac{1''}{\cos\theta}$$

so that the shear force on the weld along this sloping flange is obtained from the above formula for the horizontal flange, using the modified value of V′:

FIGURE 16



$$f = \frac{f_x}{s_x} = \frac{\dfrac{f_h}{\cos\theta}}{\dfrac{s_h}{\cos\theta}} = \frac{f_h}{s_h}$$

$$\therefore \boxed{f = \frac{V'\,a\,y}{I\,n} \text{ lbs/in.}}$$

or the approximate:

$$\boxed{f = \frac{V'}{d\,n} \text{ lbs/in.}}$$

where:

f = shear force on weld, lbs/linear in.

V = external shear on the section, lbs.

V′ = modified shear on section if sloping flange, lbs

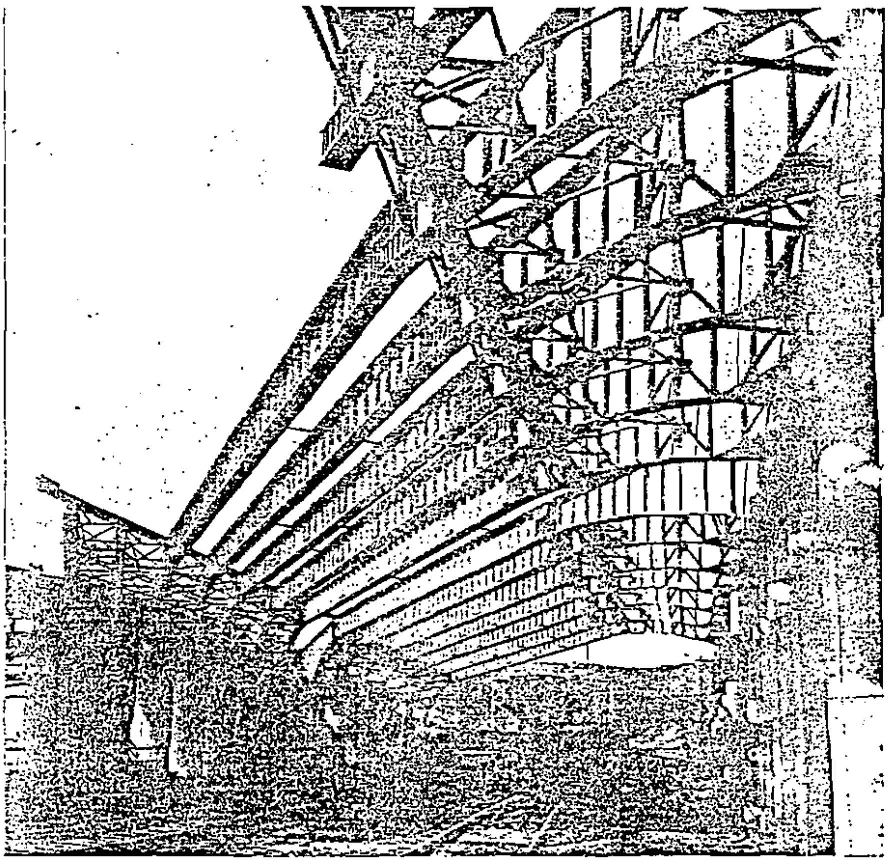
a = area of flange held by weld, in.^{2}

y = vertical distance between center of gravity of flange held by weld, and neutral axis of section, in.

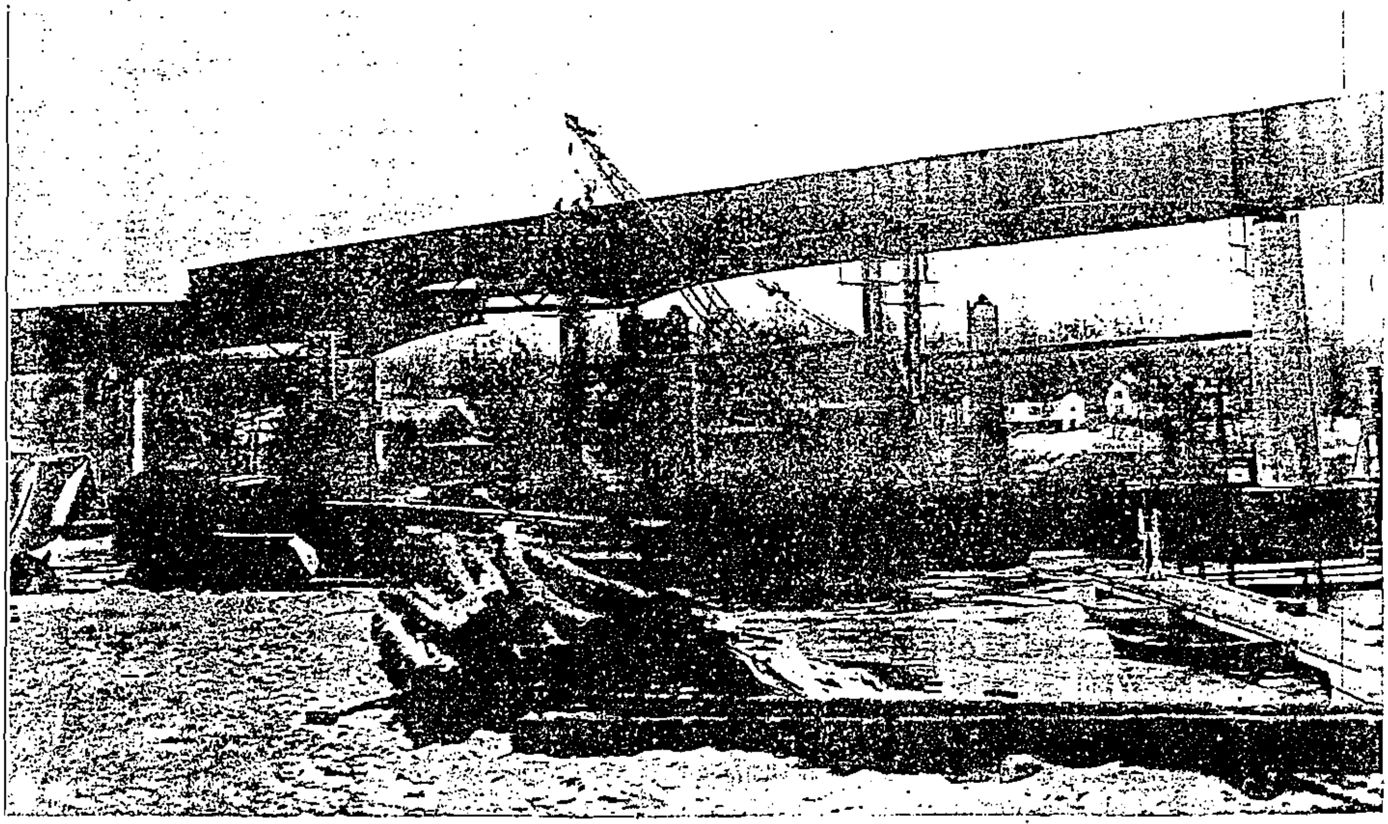
I = moment of inertia of section, in.^{4}

n = number of welds connecting web to flange

d = distance between C.J. of flanges, in.



Erection view of New York State Thruway bridge shows haunched girders. Straightness and true camber of the lower flanges are apparent. Note vertical stiffeners and suspended (235') span bearing surfaces at girder junctions.



Portion of 295' span of bridge on Connecticut Turnpike being settled onto supporting piers. Note continuous parabolic haunched girder construction.