Top Connecting Plates For Simple Beams and Wind Bracing

1. DESIGN PLATE TO BE STRESSED AT YIELD

A top connecting plate if designed to be stressed at its yield will provide a flexible connection, suitable for a simple beam and easily adapted to carry the additional moment due to wind.

Since this flexibility is due to plastic yielding of the plate, the portion of its length which is to yield should be at least 1.2 times its width.

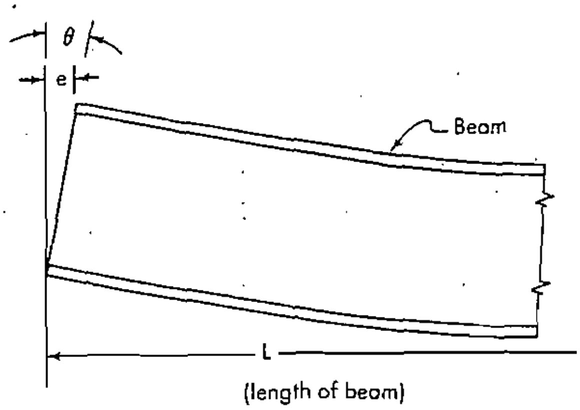


FIGURE 1

The plate should be capable of plastically yielding a distance equivalent to the movement of the end of the top beam flange as it rotates under load if the connection were to offer no restraining action (AISC Sec. 1.15.4); see Figure 1. For a simply supported beam, uniformly loaded, this maximum movement (e) would be:

$$\boxed{e = \frac{2\,\sigma\,(12\,L)}{3\,E} = \frac{\sigma\,L}{3{,}600{,}000}} \qquad \cdots\cdots\cdots\cdots (1)$$

where:

e = movement, in inches

L = length of beam, feet

The graph in Figure 2 illustrates what this movement would be as a function of beam length, under various load conditions.

There is no problem in detailing a top plate to safely yield this much, providing there are no notches which might act as stress risers and decrease the plate's strength. Any widening of the plate for the connecting welds must be done with a smooth transition in width.

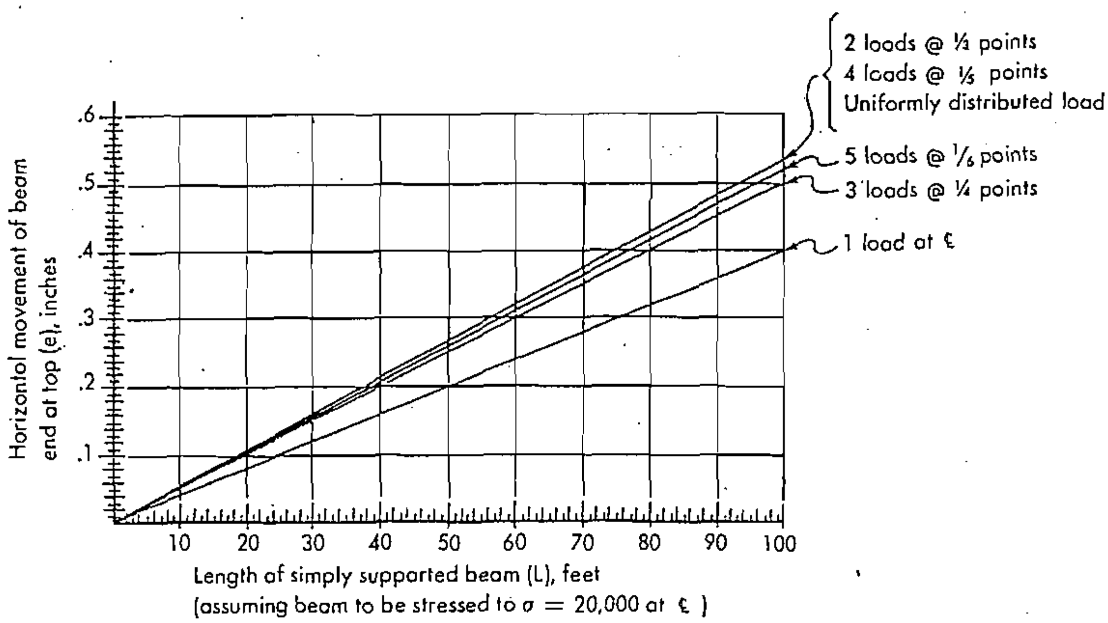


FIGURE 2

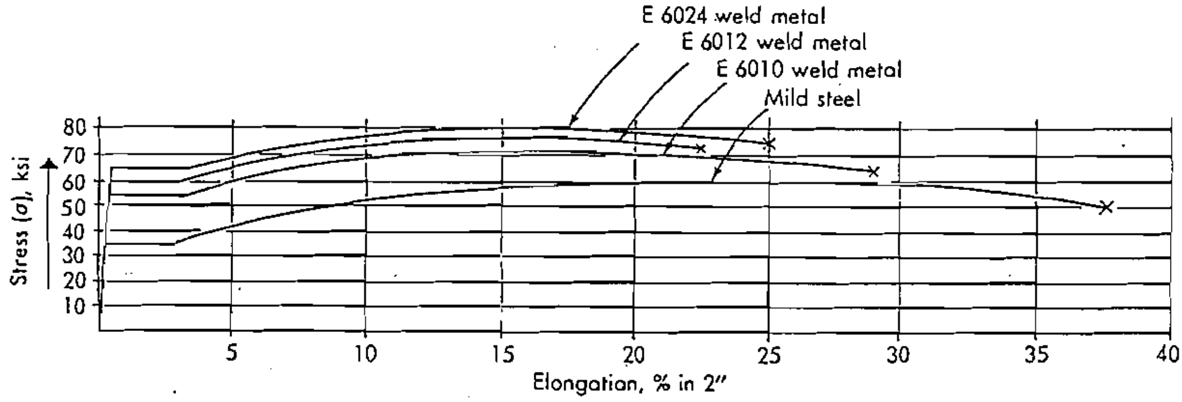


FIG. 3 Stress-strain diagram for weld metal and beam plate.

ASTM specifies the following minimum percent of elongation as measured in an 8" gage length for structural steels:

A7	21%
A373	21%
A36	20%
A242	18%
A441	18%

This minimum value of 20% for A36 steel would represent a total elongation of 20% × 8" = 1.6" within the 8" length.

Notice in Figure 2 that a simply supported beam, uniformly loaded, with a span of 20 feet would rotate inward about .106", so that this particular beam would utilize only 1/15 of the capacity of this top plate to yield.

Figure 3, a stress-strain diagram, shows that a mild steel base plate will yield and reach maximum elongation before its welds reach this yield point.

The test specimen in Figure 4 shows that ample plastic elongation results from the steel tensile specimen necking down and yielding. This is similar to the behavior of a top connecting plate which yields plastically under load.

2. TOP PLATE FOR SIMPLE BEAMS

There is some question as to what value should be used for the end moment in the design of the top plate for simple beams. Any top plate will offer some restraint, and this will produce some end moment. Lehigh researchers originally suggested assuming simple beam construction (AISC Type 2) to have an end restraint of about 20%. On this basis, the end moment for a uniformly loaded beam would be:

$$M_e = (.20)\,\frac{W\,L}{12} = \frac{W\,L}{60}$$

and this is 13.3% of the beam's resisting moment.

Heath Lawson ("Standard Details for Welded Building Construction", AWS Journal, Oct. 1944, p. 916) suggests designing the top plate (simple beam construction) for an end moment of about 25% of the beam's resisting moment. This would correspond to an end restraint of about 37.5%, which approaches the range of "semi-rigid" connections.

In Figure 5 the end of the top connecting plate is beveled and groove welded directly to the column, the groove weld and adjacent plate being designed to develop about 25% of the restraining moment of the

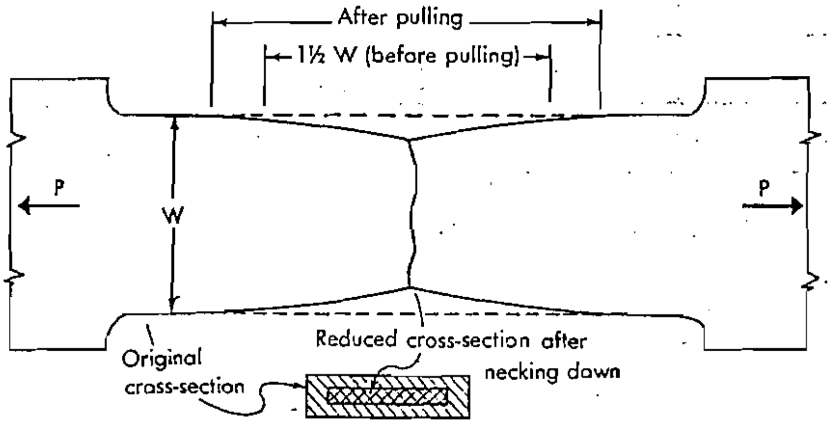


FIGURE 4

beam using the standard allowable bending stress. The standard bending stress allowed here would be limited to $\sigma = .60\ \sigma_y$. (Type 2, simple framing).

Just beyond the groove weld section, the plate is reduced in width so that the same load will produce a localized yield stress (σ_y). The length of this reduced section should be at least 1.2 times its width to assure ductile yielding.

This plate is attached to the beam flange by means of a continuous fillet weld across the end and returning a sufficient distance on both sides of the plate to develop the strength of the groove weld at standard allowables:

A7, A373 Steels; E60 Welds	
$f = 9600\ \omega$ lbs/linear in.	...(2)
A36, A441 Steels; E70 Weld	
$f = 11{,}200\ \omega$ lbs/linear in.	

3. TOP PLATE FOR WIND BRACING

Wind moments applied to simple beam connections present an additional problem. Some means to transfer these wind moments must be provided in a connection which is designed to be flexible. Any additional restraint in the connection will increase the end moment resulting from the gravity load. AISC Sec 1.2 provides for two approximate solutions, referred to hereafter as Method 1 and Method 2.

In tier buildings, designed in general as Type 2 construction, that is with beam-to-column connections (other than wind connections) flexible, the distribution of the wind moments between the several joints of the frame may be made by a recognized empirical method provided that either:

Method 1. The wind connections, designed to resist the assumed moments, are adequate to resist the moments induced by the gravity loading and the wind loading at the increased unit stresses allowable, or

Method 2. The wind connections, if welded and if designed to resist the assumed wind moments, are so designed that larger moments induced by the gravity loading under the actual condition of restraint will be relieved by deformation of the connection material without over-stress in the welds.

AISC Sec. 1.5.6 permits allowable stresses to be increased ⅓ above the values provided in Sec 1.5.1 (steel), and 1.5.3 (welds), when produced by wind or seismic loading acting alone or in combination with the design dead and live loads, on condition that the required section computed on this basis is not less than that required for the design dead and live load and impact, if any, computed without the ⅓ stress increase, nor less than that required by Sec. 1.7, (repeated loading) if it is applicable. Since we are discussing Type 2 construction (simple framing) the initial basic allowable stress is $.60\ \sigma_y$, not $.66\ \sigma_y$.

Method 1

The top plate (Fig. 6) is designed to carry the force resulting from the end moment caused by the combination of the gravity and wind moments, and at a ⅓ increase in the standard stress allowable (or $\sigma = .80\ \sigma_y$). This ⅓ increase may also be applied to the connecting welds (AISC Sec. 1.5.3, & 1.5.6). The fillet welds connecting the lower flange of the beam to the seat angle must be sufficient to transfer this same load.

The top plate must have the ability to yield plastically if overloaded (last paragraph of AISC Sec. 1.2).

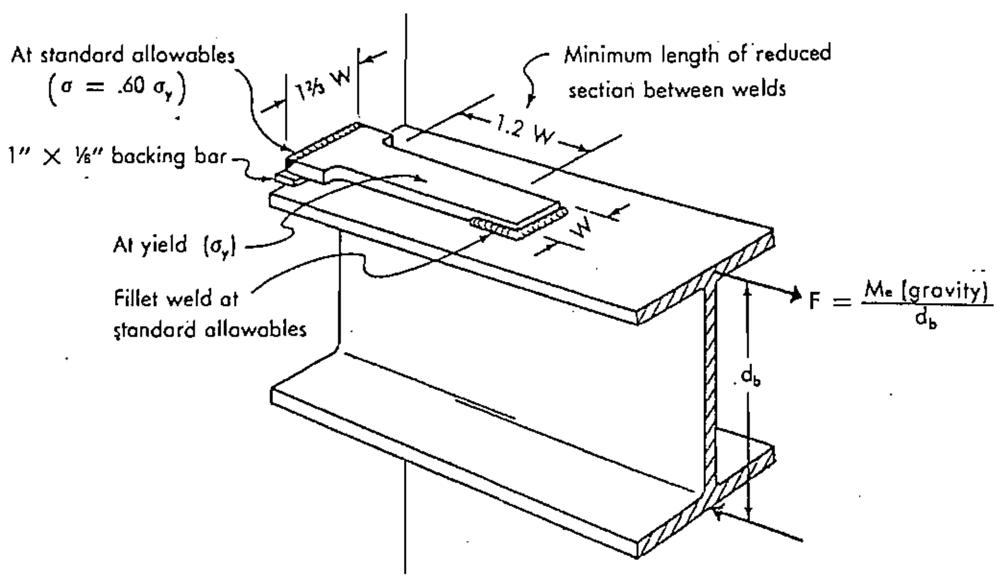


FIGURE 5

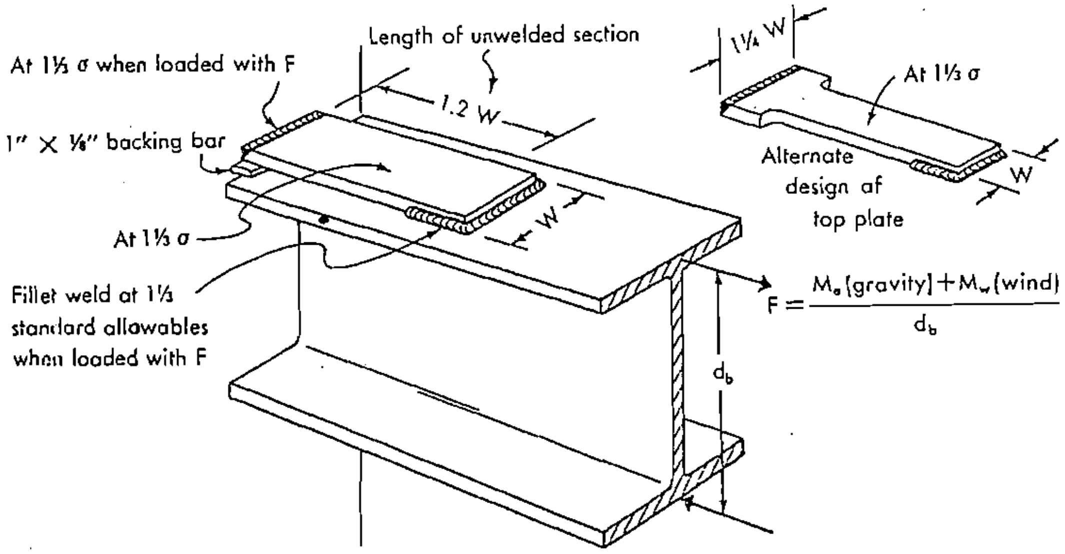


FIGURE 6

In the alternate design of the top plate shown at upper right in Figure 6, the reduced section (W) is designed for the force resulting from the end moment caused by the combination of the gravity and wind moments at a ⅓ increase in the standard allowables. It will reach yield at a 25% increase in load (F). The wider section at the groove weld (1¼ W) will reach 1⅓ σ or .80 σ_y when the reduced section has reached this yield value.

Method 2

The top plate (Fig. 7) is designed to carry the force resulting from the wind moment (M_w) using a ⅓ increase in the standard allowables:

$$\sigma = (1⅓)\ .60\ \sigma_y = .80\ \sigma_y.$$

The top plate must be capable of yielding plastically to relieve larger moments induced by gravity loading, figuring the connecting welds at standard allowables.* This is the same method for figuring the connecting welds of top connecting plates for simply supported beams without wind loads.

The reduced section will reach yield stress (σ_y) at a 25% increase in load (F). The wider section at the groove weld (1⅔ W) will reach standard allowables (.60 σ_y) at this time.

In case there should be a reversal in wind moment, the top plate must be thick enough to safely withstand any compressive load without buckling.

It is recommended that the top plate's thickness be held to at least $\frac{1}{24}$ of its length (L) between welds. This will provide a slenderness ratio (L/r) of 83; and corresponds to about 80% of the allowable compressive strength for a short column (L/r ratio of 1).

* This weld allowable by AISC is not clear; AISC simply says welds shall not be overstressed when plate is at yield.

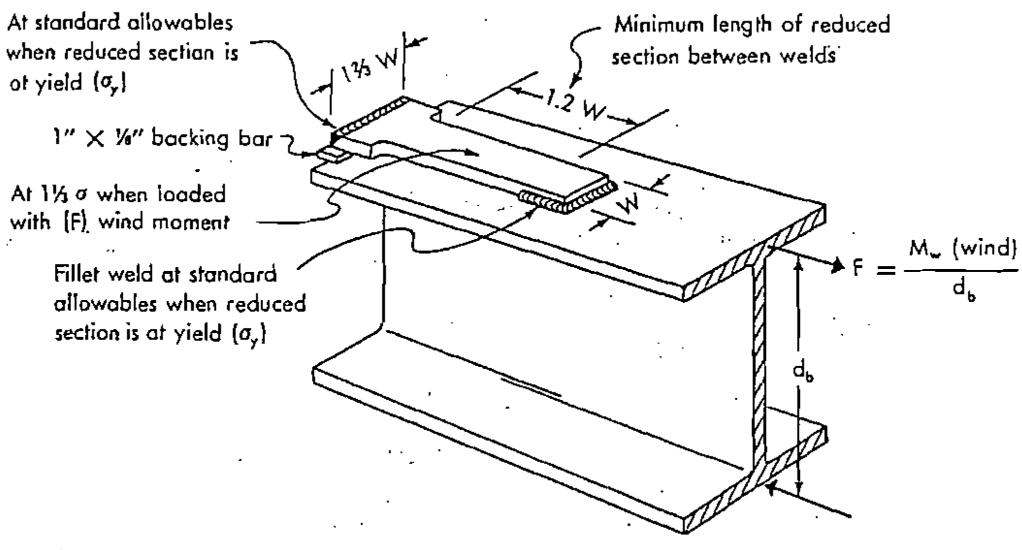


FIGURE 7

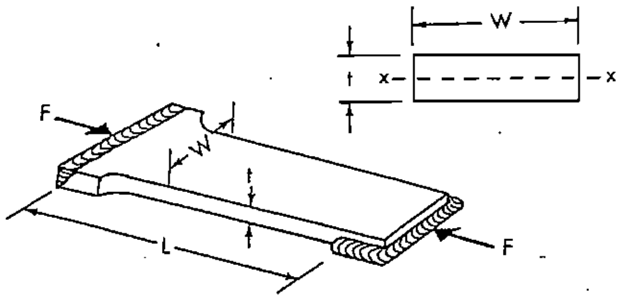


FIGURE 8

Where:

$$I_x = \frac{W\ t^3}{12} \text{ and}$$

$$A = W\ t$$

radius of gyration

$$r = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{W t^3}{12\ W t}} = \frac{t}{2\sqrt{3}}$$

$$= .289\ t$$

slenderness ratio

$$\frac{L}{r} = \frac{(24\ t)}{(.289\ t)}$$

$$= 83$$

$$\therefore \boxed{t > \frac{L}{24}}$$

4. EXAMPLE OF TOP PLATE DESIGN— WITH WIND MOMENT

A 14" WF 38# beam is simply supported and loaded uniformly with 296 lbs/in. on a 15-ft span. Based on these beam-load conditions, the maximum bending moment at center is M = 1200 in.-kips. Use A36 steel and E70 welds. Wind moment on each end is M_w = 600 in.-kips.

Beam conditions here: (See Figure 9.)

14" WF 38# beam

$b = 6.776''$

$d_b = 14.12''$

$t_f = .513''$

$S = 54.6\ in.^3$

If there were no wind load, the above connection might be designed for about 25% of the present (gravity) moment as a simply supported beam:

$$M_g = .25\ M_{\text{℄}}$$

$$= .25\ (1200)$$

$$= 300 \text{ in.-kips on connection at each end}$$

$$F = \frac{M_g}{d_b}$$

$$= \frac{(300)}{(14.12)}$$

$$= 21.3 \text{ kips}$$

The reduced section of the top plate is designed to carry this force at yield stress (σ_y):

$$A_p = \frac{F}{\sigma_y}$$

$$= \frac{(21.3 \text{ kips})}{(36{,}000 \text{ psi})}$$

$$= .59 \text{ in.}^2$$

or use a 1¾" x ⅜" plate

$A_p = .656\ in.^2 > .59\ in.^2$ <u>OK</u>

Connecting Welds at Standard Allowables

For the groove weld to the column flange, this plate is widened to 1⅔W, or—

width = 1⅔ (1¾)

= 2.9" <u>or use 3.0"</u>

For the fillet welds to the beam flange, use 5/16" fillets at an allowable force of—

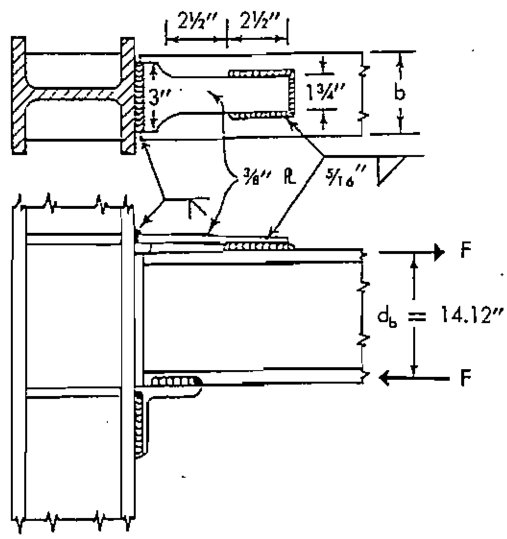


FIGURE 9

$$f_w = 11{,}200\ \omega$$
$$= 11{,}200\ (5/16)$$
$$= 3500 \text{ lbs per linear inch}$$

The length of this weld is—

$$L_w = \frac{F}{f_w}$$
$$= \frac{(.656 \text{ in.}^2)(36{,}000 \text{ psi})}{(3500 \text{ lbs/in.})}$$
$$= \underline{6.74''}$$

This would be 1¾" across the end, and 2½" along the sides.

Applying Method 1 for Additional Wind Moment

This connection will now be designed for the additional wind moment of $M_w = 600$ in.-kips, using Method 1.

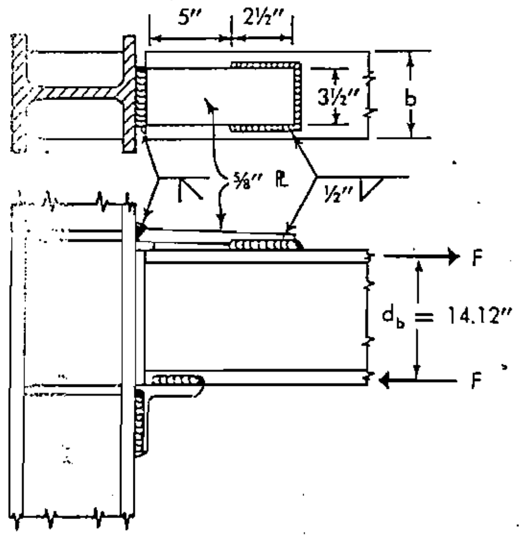


FIGURE 10

Beam conditions here:

14" WF 38# beam
$b = 6.776''$
$d_b = 14.12''$
$t_f = .513''$
$S = 54.6 \text{ in.}^3$

Total moment on the connection is—

$$M = M_g + M_w$$
$$= 300 \text{ in.-kips} + 600 \text{ in.-kips}$$
$$= 900 \text{ in.-kips}$$

Force on top plate is—

$$F = \frac{M}{d_b}$$
$$= \frac{(900 \text{ in.-kips})}{(14.12'')}$$
$$= 63.8 \text{ kips}$$

The top plate is designed for this force at ⅓ higher allowables:

$$A_p = \frac{F}{1\frac{1}{3}\ \sigma}$$
$$= \frac{(63.8 \text{ kips})}{1\frac{1}{3}\ (22{,}000 \text{ psi})}$$
$$= 2.18 \text{ in.}^2$$

or <u>use a 3½" x ⅝" plate</u>

$A_p = 2.19 \text{ in.}^2 > 2.18 \text{ in.}^2$ <u>OK</u>

The connecting welds are figured at ⅓ higher allowables:

For the fillet welds at the beam flange, use ½" fillets. The standard allowable force *is* $f_w = 11{,}200\ \omega = 11{,}200\ (½) = 5600$ lbs per linear inch.

The length of this weld is—

$$L_w = \frac{F}{1\frac{1}{3}\ f_w}$$
$$= \frac{(63.8 \text{ kips})}{1\frac{1}{3}\ (5600)}$$
$$= \underline{8.54''}$$

This weld length would be distributed 3½" across the end, and 2½" along the side edges of the top plate.

The above connection may be cut from bar stock without the necessity of flame cutting any reduced section in it. This is a good connection and is in widespread use. The connecting groove weld and fillet welds are strong enough to develop the plate to yield plastically if necessary due to any accidental overload of the connection.

Some engineers prefer to widen this plate at the groove weld so that if the plate should have to reach yield stress, the connecting welds would be stressed only up to the wind allowable or ⅓ higher, hence $\sigma = .80\ \sigma_y$.

Accordingly, the plate is widened here to 1¼W = 1¼ (3½) = 4⅜".

(See Figure 11.)

The length of the fillet weld, using ½" fillet welds and allowable of $f_w = 5600$ lbs/in., would be—

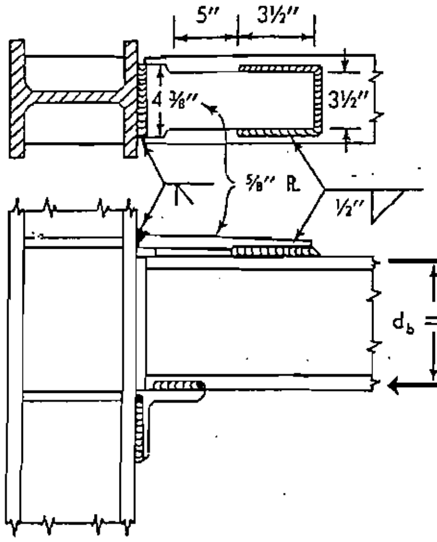


FIGURE 11

$$L_w = \frac{F}{1⅓\ f_w} \quad \text{reduced section at yield } (\sigma_y) \text{ and fillet weld at ⅓ higher allowable}$$

$$= \frac{(2.19 \text{ in.}^2)(36{,}000 \text{ psi})}{1⅓\ (5600)}$$

$$= \underline{10.55''}$$

This would be 3½" across the end, and 3½" along the side edges of the plate.

Applying Method 2 for Additional Wind Moment

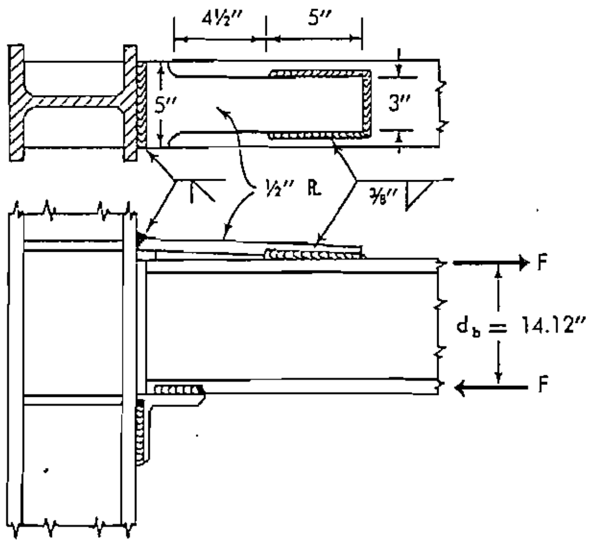


FIGURE 12

Temporarily ignoring the gravity load, the top plate is designed to carry the wind load, $M_w = 600$ in.-kip on each end.

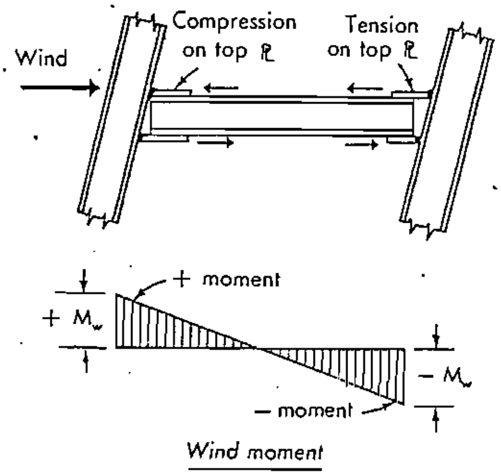


FIGURE 13

$$F = \frac{M_w}{d_b}$$

$$= \frac{(600 \text{ in.-kips})}{(14.12'')}$$

$$= 42.5 \text{ kips}$$

The reduced section of the plate is designed to carry this at ⅓ higher allowable:

$$A_p = \frac{F}{1⅓\ \sigma}$$

$$= \frac{(42.5 \text{ kips})}{1⅓\ (22{,}000)}$$

$$= 1.45 \text{ in.}^2$$

or use 3" by ½" plate

$$A_p = 1.50 \text{ in.}^2 > 1.45 \text{ in.}^2 \qquad \underline{\text{OK}}$$

The plate must now be modified so that larger moments induced by the gravity loading can be relieved by plastic yielding of the top plate, designing the connecting welds at standard allowables.

The plate is widened at the groove weld to 1⅔ W = 1⅔ (3) = 5.0".

For the connecting fillet welds to the beam flange, use ⅜" fillets:

$$f_w = 11{,}200\ \omega$$

$$= 11{,}200\ (⅜)$$

$$= 4200 \text{ lbs per linear inch}$$

The length of this weld is—

$$L_w = \frac{F}{f_w} = \frac{(1.5 \text{ in.}^2)(36{,}000 \text{ psi})}{(4200)}$$

$$= \underline{12.9''}$$

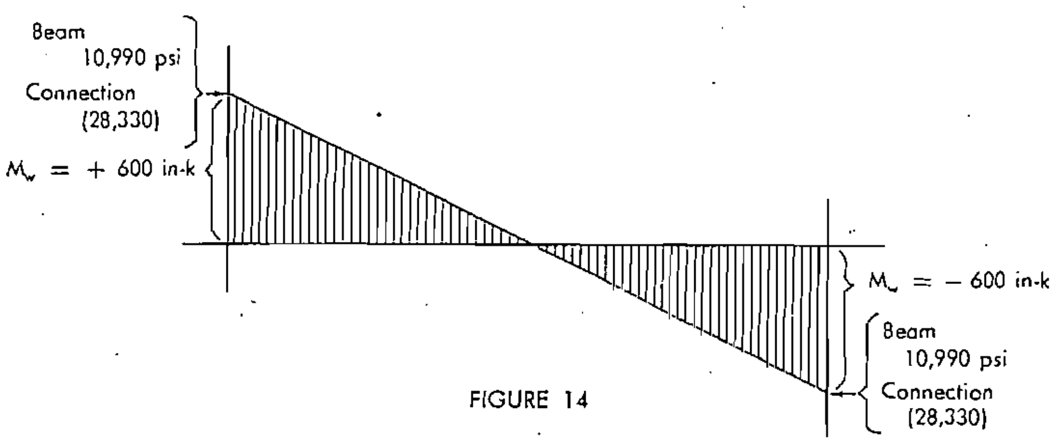


FIGURE 14

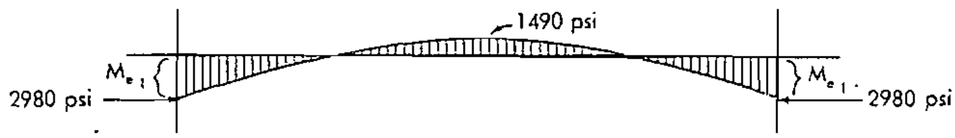


FIGURE 15

This would be 3″ of weld across the end, and 5″ along each side.

5. EXAMINING THIS EXAMPLE

To better understand how this wind connection operates, this example will be examined, using Method 2.

1. The connection is first designed for the wind moment of M_w = 600 in.-kip at ⅓ increase in the standard allowables applied to each end of the beam.

The wind moment will cause a bending stress in the beam of—

$$\sigma_b = \frac{M_w}{S_b}$$

$$= \frac{(600 \text{ in.-kips})}{(54.6 \text{ in.}^3)}$$

$$= 10{,}990 \text{ psi}$$

(See Figure 14.)

The corresponding stress in the top connecting plate is—

$$\sigma_p = \frac{M}{d_b\ A_p}$$

$$= \frac{(600 \text{ in.-kips})}{(14.12)(1.5)}$$

$$= 28{,}330 \text{ psi}$$

Note that the connection will not yield until a stress of 36,000 psi is reached.

$$\text{Let } K = \frac{S_p}{S_b} = \frac{\sigma_b}{\sigma_p}$$

$$= \frac{10{,}990}{28{,}330}$$

$$= .388$$

2. Now the gravity load can be gradually added, treating the beam as having fixed ends, until the right-hand connection reaches yield stress. This would be an additional stress in the connecting plate of: 36,000 − 28,330 = 7670 psi. This would correspond to a stress in the beam end of: (.388)(7670 psi) = 2980 psi.

(See Figure 15.)

Since the allowable moment on this end connection resulting from gravity load is (treated as a fixed end beam)—

$$M_{e1} = \frac{w_1\ L^2}{12} \text{ also } = \sigma_p\ A_p\ d$$

the portion of the gravity load to be added here is—

$$w_1 = \frac{12\ \sigma_p\ A_p\ d}{L^2}$$

$$= \frac{12(7670)(1.5)(14.12)}{(180)^2}$$

$$= 60.2 \text{ lbs/in.}$$

The stress in this beam end due to gravity load is then added to the initial wind moment diagram:

(See Figure 16.)

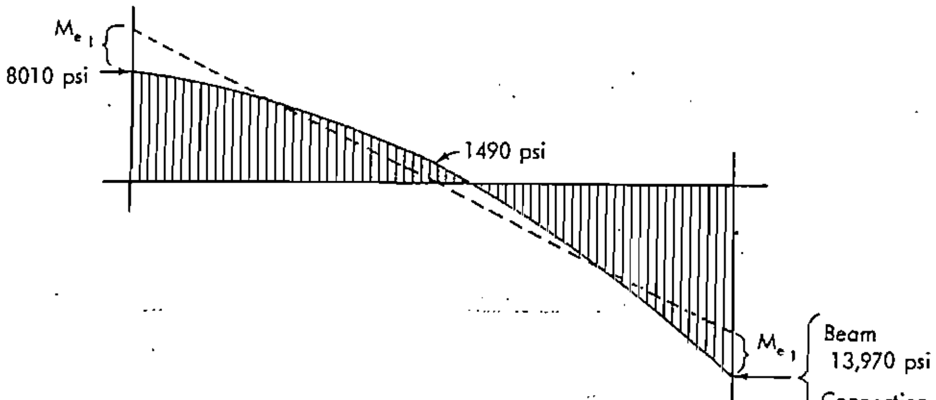


FIGURE 16

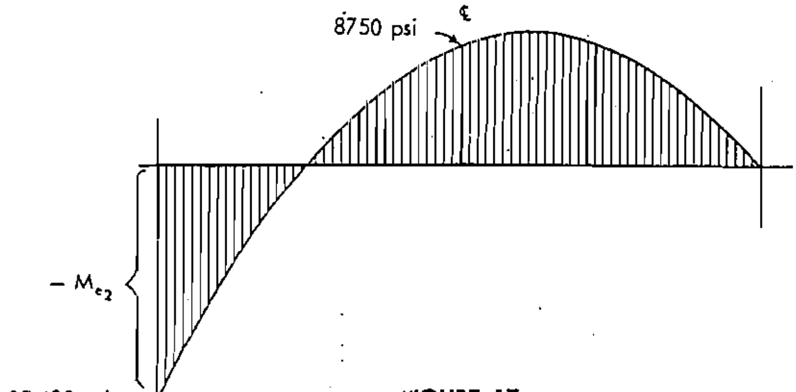


FIGURE 17

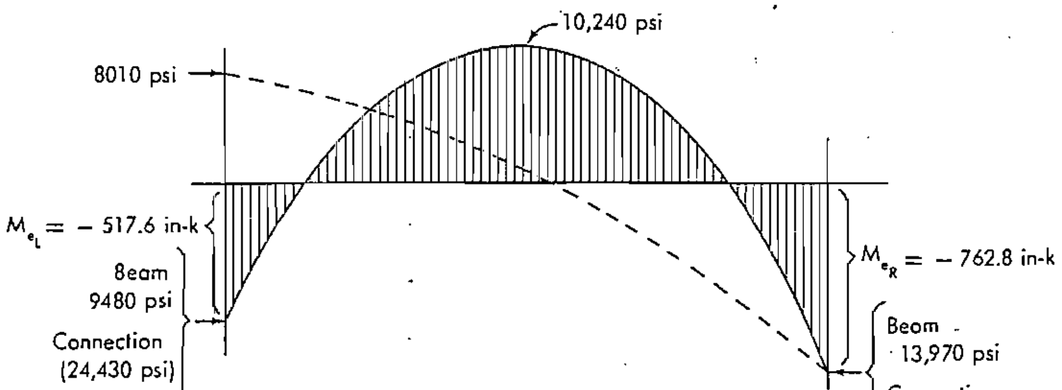


FIGURE 18

At this point, the right-hand connection reaches yield stress ($\sigma_y = 36{,}000$ psi) even though the beam end is stressed to only $\sigma = 13{,}970$ psi.

3. The remainder of the gravity load ($w_2 = w - w_1 = 296 - 60.2 = 235.8$ lbs/in.) can now be applied, treating the beam as having one fixed end on the left and simply supported on the right. See Figure 17.

The resulting end moment here is—

$$M_{e2} = \frac{w_2 \, L^2}{8} = \frac{(235.8)(180)^2}{8}$$

$$= 955 \text{ in.-kip}$$

or a bending stress of

$$\sigma_{b2} = \frac{M_{e2}}{S_b} = \frac{(955 \text{ in.-kips})}{(54.6 \text{ in.}^3)}$$

$$= 17{,}490 \text{ psi}$$

Also since $M_{℄} = \dfrac{w_2 \, L^2}{16}$

$$\sigma_b \text{ at } ℄ = \tfrac{1}{2}\,(17{,}490) = 8750 \text{ psi}$$

These stresses are then added to the previous moment diagram; Figure 18.

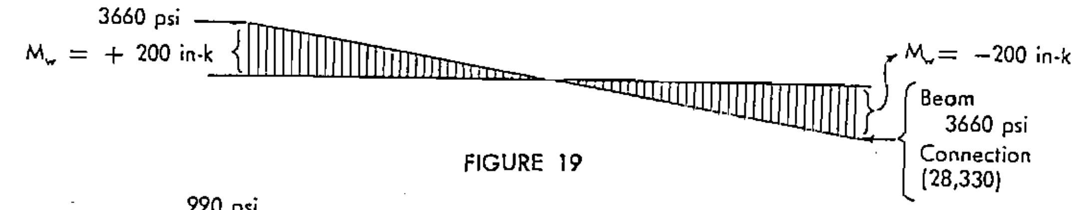


FIGURE 19

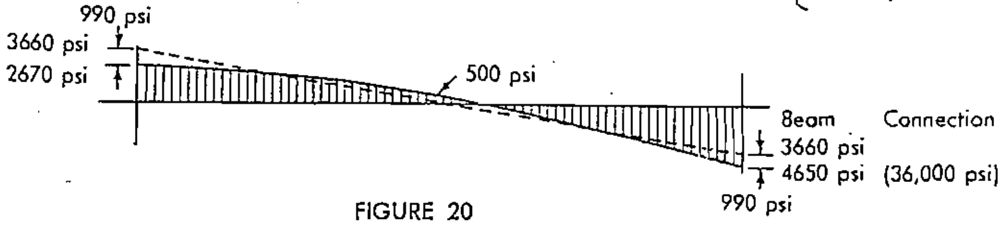


FIGURE 20

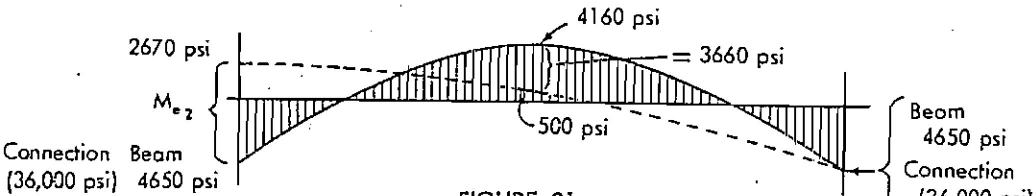


FIGURE 21

6. SMALL WIND MOMENT

A lower design wind moment will not require as large a top connecting plate. The smaller plate will yield sooner and it is possible that the final gravity load would cause both end connections to yield.

Consider the same problem as previously but with the wind moment reduced to $M_w = 200$ in.-kip, applied to each end of the beam.

The required top plate is designed for this wind moment:

$$A_p = \frac{M_w}{d_b \, 1\tfrac{1}{3} \, \sigma}$$

$$= \frac{(200 \text{ in.-kips})}{(14.12)\ 1\tfrac{1}{3}\ (22{,}000)}$$

$$= .48 \text{ in.}^2$$

or use a 1" x ½" plate

(This very small top plate is used here only for illustrative purposes.)

$$A_p = .50 \text{ in.}^2 > .48 \text{ in.}^2$$

This moment will cause a bending stress in the beam of—

$$\sigma_b = \frac{M_w}{S_b}$$

$$= \frac{(200 \text{ in.-kips})}{(54.6 \text{ in.}^3)}$$

$$= 3660 \text{ psi} \qquad \text{See Figure 19.}$$

The corresponding stress in the top plate is—

$$\sigma_p = \frac{M_w}{d\ A_p} = \frac{(200 \text{ in.-kips})}{(14.12)\ (.50)}$$

$$= 28{,}330 \text{ psi}$$

$$\text{Let } K = \frac{S_b}{S_p} = \frac{\sigma_b}{\sigma_p}$$

$$= \frac{(3660)}{(28{,}330)}$$

$$= .129$$

A portion of the gravity load is added, treating the beam as having fixed ends, until the right hand connection reaches yield stress. This would be an additional stress in the connection plate of: 36,000 − 28,330 = 7670 psi. This would correspond to a stress in the beam of: (.129) (7670 psi) = 990 psi. See Figure 20.

Since the allowable moment on this end connection resulting from gravity load is—

$$M_{e1} = \frac{w_1 L^2}{12}$$

$$= \sigma_p \, A_p \, d_b$$

the portion of the gravity load to be added here is—

$$W_1 = \frac{12\ \sigma_p\ A_p\ d_b}{L^2} = \frac{12\ (7670)(.50)(14.12)}{(180)^2}$$

$$= 20.1 \text{ lbs/in.}$$

At this point, the right-hand connection reaches yield stress ($\sigma_y = 36{,}000$ psi) even though the end of the beam is stressed to only $\sigma = 4650$ psi.

In this example, if the remainder of the gravity load were applied, the left-hand connection would go over the yield point. For this reason only enough of the gravity load will be added to bring the left-hand connection just to yield, treating the beam as having one fixed end on the left and simply supported on the right. See Figure 21.

To reach yield stress in the left connection, the stress in the beam must increase from 2670 psi compression in upper flange to 4650 psi tension, or 7320 psi.

This would correspond to an applied gravity load of:

$$M_{e2} = \frac{w_2\, L^2}{8}$$

$$= \sigma_b\, S_b$$

$$w_2 = \frac{8\, \sigma_b\, S_b}{L^2}$$

$$= \frac{8(7320 \text{ psi})(54.6 \text{ in.}^3)}{(180)^2}$$

$$= 98.6 \text{ lbs/in.}$$

$$\text{And } M_{\mathbb{C}} = \frac{w_2\, L^2}{16}$$

$$\text{so } \sigma_{\mathbb{C}} = \tfrac{1}{2}(7320)$$

$$= 3660 \text{ psi}$$

This now leaves a gravity load of w_3 to be applied, treating the beam as having simply supported ends since their connections have both reached yield stress.

The remaining gravity load:

$$w_3 = w - w_1 - w_2$$

$$= 296.0 - 20.1 - 98.6$$

$$= 177.3 \text{ lbs/in.}$$

Since:

$$M_{\mathbb{C}} = \frac{w_3\, L^2}{8}$$

$$= \frac{(177.3)(180)^2}{8}$$

$$= 718 \text{ in.-kips}$$

$$\therefore\ \sigma_{\mathbb{C}} = \frac{718}{54.6}$$

$$= 13{,}150 \text{ psi}$$

This stress in the beam is added to the preceding moment diagram; see Figure 22:

The total $\sigma_{\mathbb{C}} = 17{,}310$ psi $< 22{,}000$ psi <u>OK</u>

7. GRAVITY LOAD APPLIED FIRST, THEN WIND LOAD

In the preceding examination of the wind connection, the wind was applied first and then the gravity load. This is the sequence of design followed in Method 2. The cross-sectional area of the top plate is determined by wind only, and then the connecting welds are designed so that larger moments induced by the gravity loading under actual conditions of restraint may cause the plate to yield plastically.

Of course in actual practice, the gravity load is applied first and then the wind may be encountered secondly. The same problem will now be examined in this order of loading.

The beam with the gravity load is considered as simply supported; however, the top plate which must resist the wind moment does restrain the end of the beam to some extent. The larger the plate, the greater the restraint, this will also increase the end moment resulting from the gravity load. It is necessary to get some indication of the restraining action of the connection so that the end moment from the gravity load may be known.

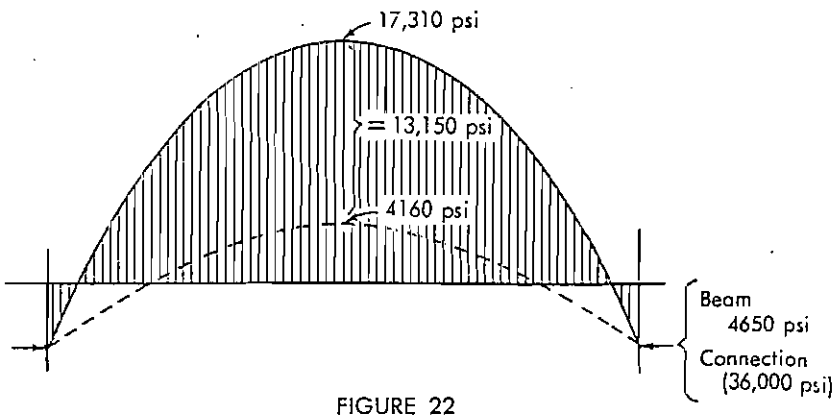


FIGURE 22

To do this, a simple moment-rotation diagram is constructed for both the loaded beam and the connection. The resulting conditions are represented by the point of intersection of these two lines or curves.

In the Lehigh research of connections, the actual test results of moment-rotation of the connections were plotted on this type of diagram; in this example the properties of this top plate connection are computed, and will be fairly accurate since practically all of the movement will occur in the reduced portion of the top plate.

Connection Line

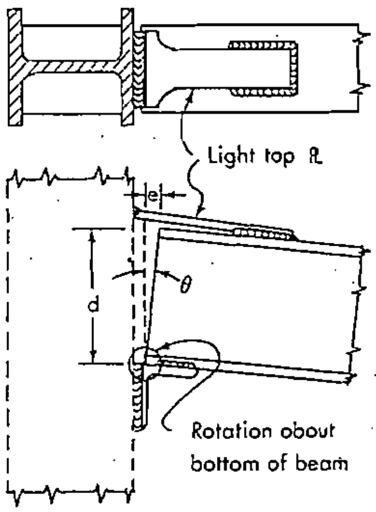


FIGURE 23

$$\boxed{\theta_p = \frac{\sigma_p \, L_p}{E \, d_b}}$$

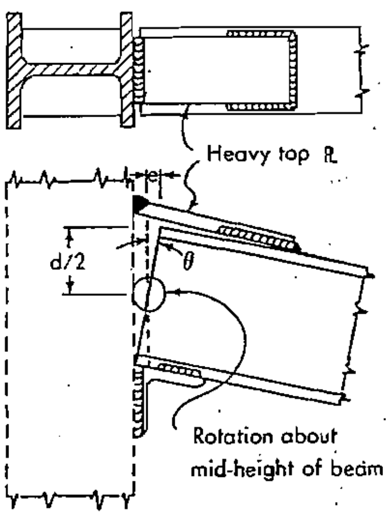


FIGURE 24

$$\boxed{\theta_p = \frac{2 \, \sigma_p \, L_p}{E \, d_b}}$$

where L_p = length of plate section between welds, inches

Since $\theta = \frac{e}{d_b}$ and $e = \epsilon \, L_p$

$$\epsilon = \frac{\sigma}{E}$$

also $\boxed{M_p = \sigma_p \, A_p \, d_b}$

If the bottom of the beam is securely anchored and the top plate is relatively small, Figure 23, rotation may be assumed to occur about a point near the bottom of the beam. As the top plate becomes larger, offering more restraint, this point of rotation moves up. If the top plate has the same size as the beam flange, Figure 24, rotation may be assumed to be at mid-height of the beam.

Since movement (e) depends upon the over-all elastic elongation of the top plate, and for simplicity length (L_p) is shown only as the length of the reduced portion, there is some elongation in the widened section as well as in the reduced section within the fillet welded zone. For this reason the value of the calculated rotation (θ) in this example will be doubled.

Two points will determine the connection line. Since this line passes through the origin or zero load, it is only necessary to have a second point; for simplicity this second point will be a yield conditions.

At yield:

$$\theta_p = \frac{\sigma_p \, L_p}{E \, d_b}$$

$$= \frac{(36{,}000 \text{ psi})(4.5'')}{(30 \times 10^6)(14.12'')}$$

$$= .382 \times 10^{-3} \text{ radians}$$

This value will be doubled because of elastic elongation of other portions of the plate:

$$\theta_p = .764 \times 10^{-3} \text{ radians}$$

and:

$$M_p = \sigma_p \, A_p \, d_b$$

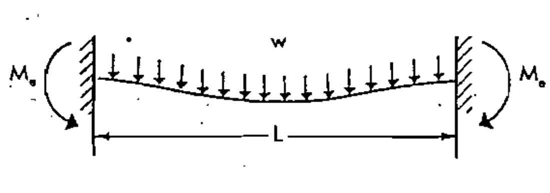
$$= (36{,}000 \text{ psi})(1.5 \text{ in.}^2)(14.12'')$$

$$= 762 \text{ in.-kips}$$

Beam Line—Gravity load, uniformly loaded

It is necessary to have two points to determine this beam line on the moment-rotation chart:

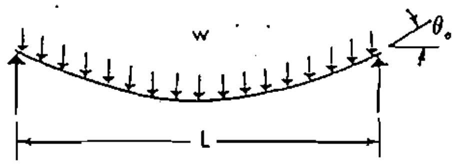
(a) the end moment (M_e) if fully restrained



$$\boxed{M_e = \frac{w \, L^2}{12} = \frac{2 \, M_{℄}}{3}}$$

$$\therefore M_e = \frac{2\,M_{\mathbb{C}}}{3} = \frac{2(1200)}{3} = 800 \text{ in.-kips}$$

(b) the end rotation (θ_e) if simply supported



$$\boxed{\theta_e = \frac{w\,L^3}{24\,E\,I} = \frac{M_e\,L}{2\,E\,I}}$$

where L = length of beam in inches

$$\therefore \theta_e = \frac{M_e\,L}{2\,E\,I} = \frac{(800)(180)}{2(30 \times 10^6)(385.3)} = 6.24 \times 10^{-3} \text{ radians}$$

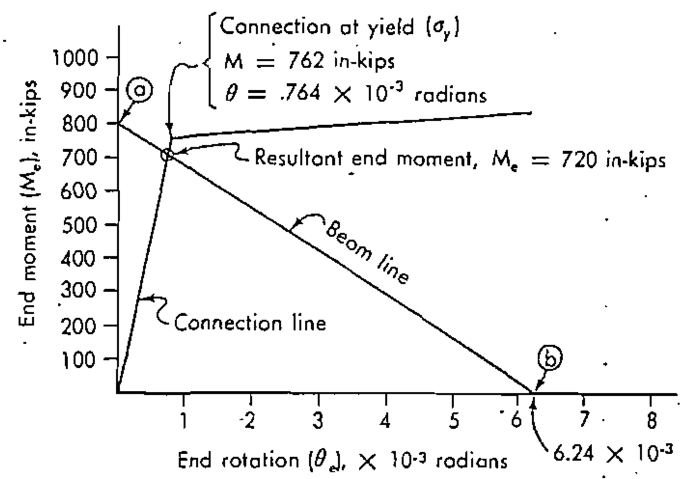


FIGURE 25

With the gravity load only on the beam, this would indicate that the end moments would be M_e = 720 in.-kip. This would leave:

$$M_{\mathbb{C}} = 1200 - 720 = 480 \text{ in.-kips}$$

This would correspond to a bending stress at the end of the beam of—

$$\sigma = \frac{M_e}{S_b} = \frac{(720 \text{ in.-kips})}{(54.6'')} = 13{,}200 \text{ psi}$$

See Figure 26.

The stress at centerline of the beam would be—

$$\sigma = \frac{M_{\mathbb{C}}}{S_b} = \frac{(480 \text{ in.-kips})}{(54.6'')} = 8800 \text{ psi}$$

As before $K = \frac{\sigma_b}{\sigma_p} = .388$ so that the stress in the connecting plate would be—

$$\sigma_p = \frac{13{,}200 \text{ psi}}{.388} = 34{,}020 \text{ psi}$$

Now the wind load is gradually applied equally to both ends until the right-hand connection reaches yield. This would occur when the stress in the connecting plate is increased from 34,020 psi to 36,000 psi, or an increase of 1980 psi. This would correspond to a wind moment of—

$$M_{w1} = \sigma_p\,A_p\,d_b = (1980 \text{ psi})(1.5 \text{ in.}^2)(14.12'') = 42.0 \text{ in.-kips}$$

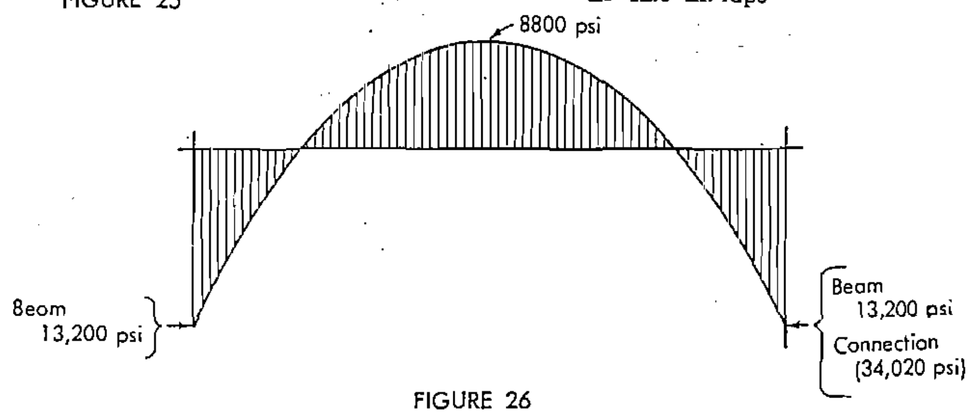


FIGURE 26

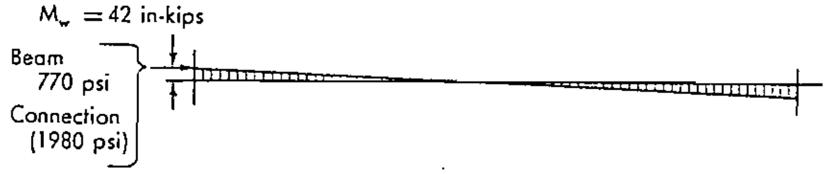


FIGURE 27

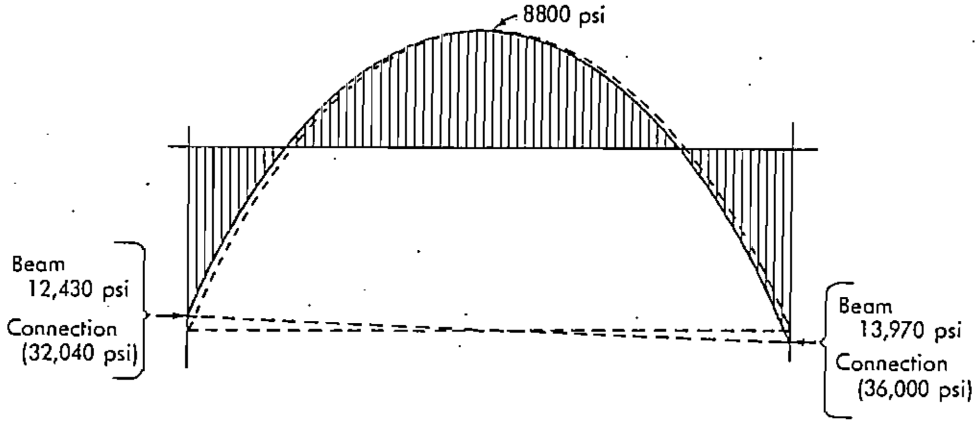


FIGURE 28

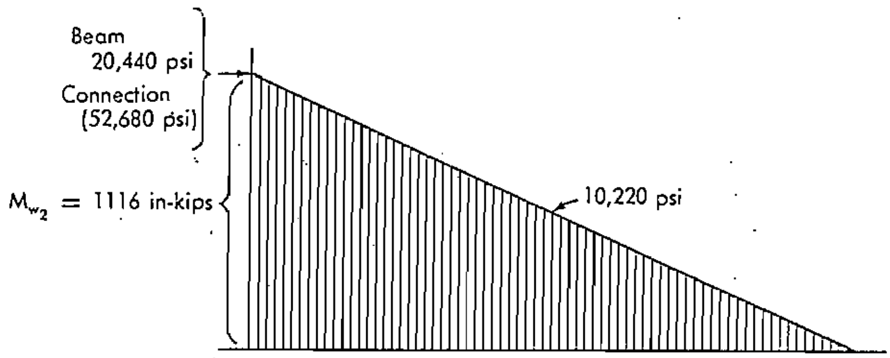


FIGURE 29

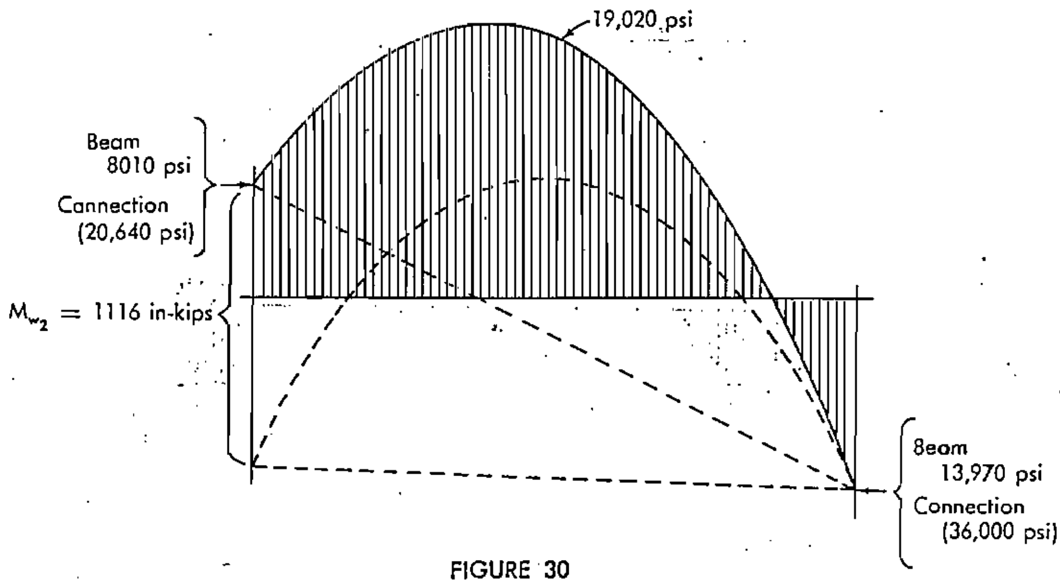


FIGURE 30

And stress in the beam is—

$$\sigma_b = (.388)(1980)$$
$$= 770 \text{ psi} \qquad \text{See Figure 27.}$$

Adding this wind moment diagram to the initial gravity moment diagram gives Figure 28.

There now is left a wind moment of 600 − 42 = 558 in.-kip to be applied to each end, but since the right-hand connection has reached yield stress, the remaining moment of 2 x 558 = 1116 in.-kip must be added to the left end of the beam.

$$\sigma_b = \frac{M}{S_b}$$
$$= \frac{1116 \text{ in.-kips}}{54.6}$$
$$= 20{,}440 \text{ psi}$$

and $\sigma_p = \dfrac{20{,}440}{.388}$

= 52,680 psi (compression) to be added to the *32,040 psi in tension already in the left-hand* connecting plate

Adding this last wind moment diagram to the diagram in Figure 28 gives the final diagram, Figure 30.

8. ALTERNATE GRAPHICAL SOLUTION

This same example can be illustrated in a slightly different manner. The right-hand connection and beam end is on the right of Figure 31; the left-hand connection and its beam end is on the left.

As before, the beam line with gravity load only is constructed for both ends. This beam line represents the moment at the end caused by the gravity load, the actual value of the moment depends on the effect of the connection.

A wind moment would be represented by a horizontal line through the actual value of the moment. It would not be influenced by the connection unless it exceeds the yield of the connection; then the portion of the wind moment carried would be limited by the yield of the connection. Any wind moment superimposed on the gravity load will shift the beam line vertically up or down depending on the sign of the wind moment.

By observation, the right-hand connection can be

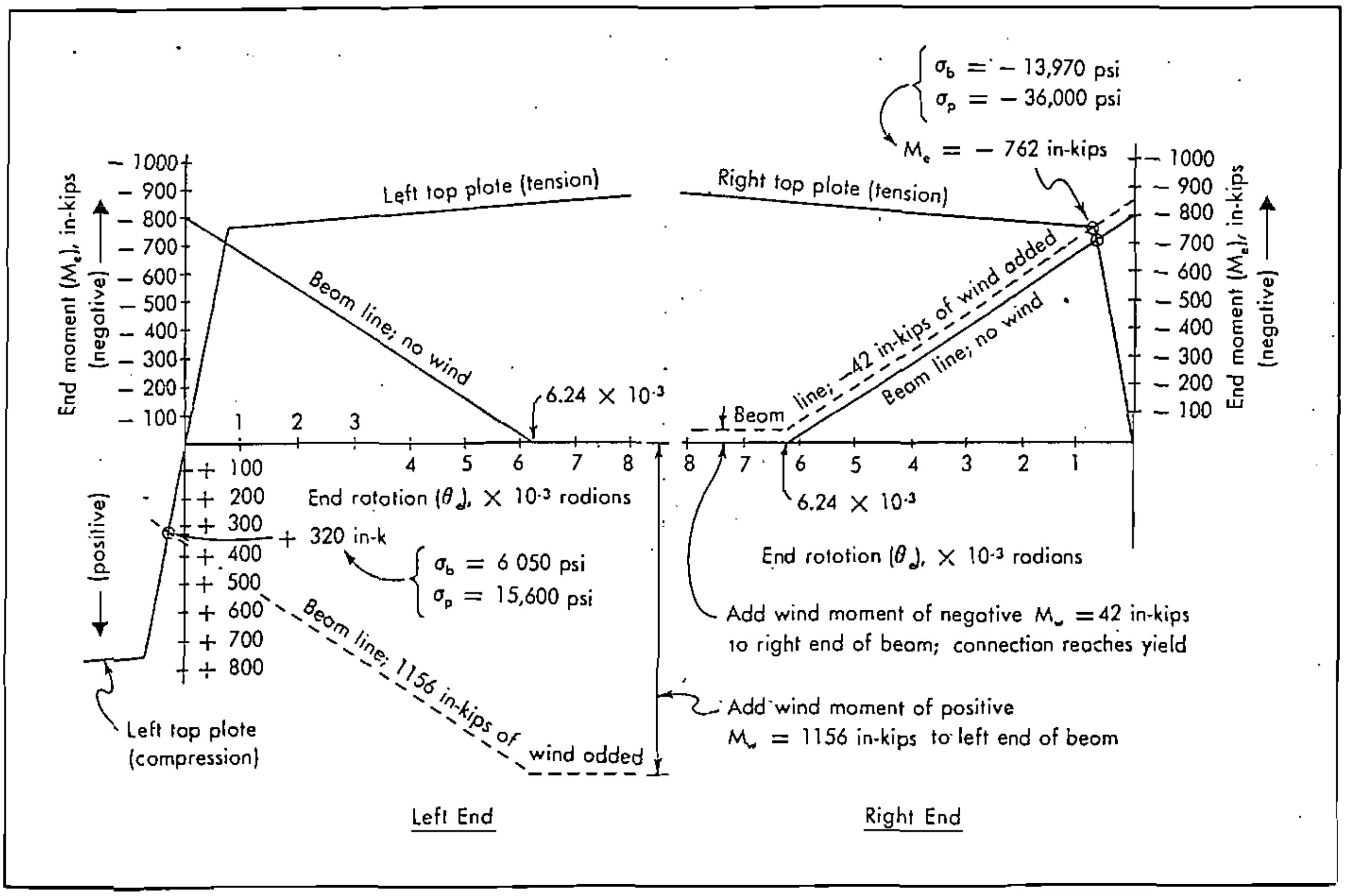


FIGURE 31

increased another 42.0 in.-kip from wind, then it will reach yield and no further moment can be applied. Since the applied wind moment was 600 in.-kip on each end, this will leave a balance of 2 x 600 in.-kip — 42 in.-kip = 1156 in.-kip to be carried entirely by the left-hand connection.

To do this, the beam line on the left of Figure 31 will be lowered vertically + 1156 in.-kip; see the dotted line. This will intersect the connection curve (extended into the positive moment region) at an end moment of M_e = 320 in.-kip.

This will correspond to a bending stress in the beam end of 6050 psi, and in the connection plate of 15,600 psi. In this case, the connection curve had to be extended downward into the positive moment region in order to intersect the new beam line. This indicates a + moment and reverses the stress in the plate, now compression, and the bottom of the beam connection is now in tension.

The previous examination of this problem indicated a bending stress in the left end of the beam of σ_b = 8010 psi; this examination indicates a stress of σ_b = 6050 psi. Why should there be a difference? The previous examination stopped after the first end moment due to gravity load was determined and then for simplicity from then on considered the connection as perfectly rigid, whereas this examination considered the elastic properties of the connecting plate all the way through the problem. This last approach would be a little more accurate.

This same problem was previously worked with a reduced wind moment of M_w = 200 in.-kip applied to each end. Figure 32 shows how this can be worked graphically. This is an interesting problem since the lower wind moment requires a smaller top plate, with ⅓ the cross-sectional area, hence ⅓ the strength, and the gravity load caused the plate to yield plastically at both ends even before any wind load is applied. This is represented by the black dot where the beam line (without wind) intersects with the connection curve.

When the wind moment is added, the right connection is already at yield and can carry no additional moment, therefore the entire wind moment of 2 x 200 in.-kip = 400 in-kip must be carried by the left-hand connection. Accordingly the beam line is lowered vertically a distance of 400 in.-kip; see the dashed line. As this is lowered, the resulting moment (M_e) and rotation (θ_e) of the connection (black dot) slide down parallel to the elastic portion of the connection line until it intersects with this new beam line (white dot).

In Figure 33 these final conditions representing the beam with gravity load and wind load are represented with black dots. If the wind were now removed, the left beam line moves upward 200 in.-kip and the right beam line moves down 200 in.-kip, the new conditions being represented by the white dots. For a complete reversal of wind, this operation is again repeated and is represented by the broken lines.

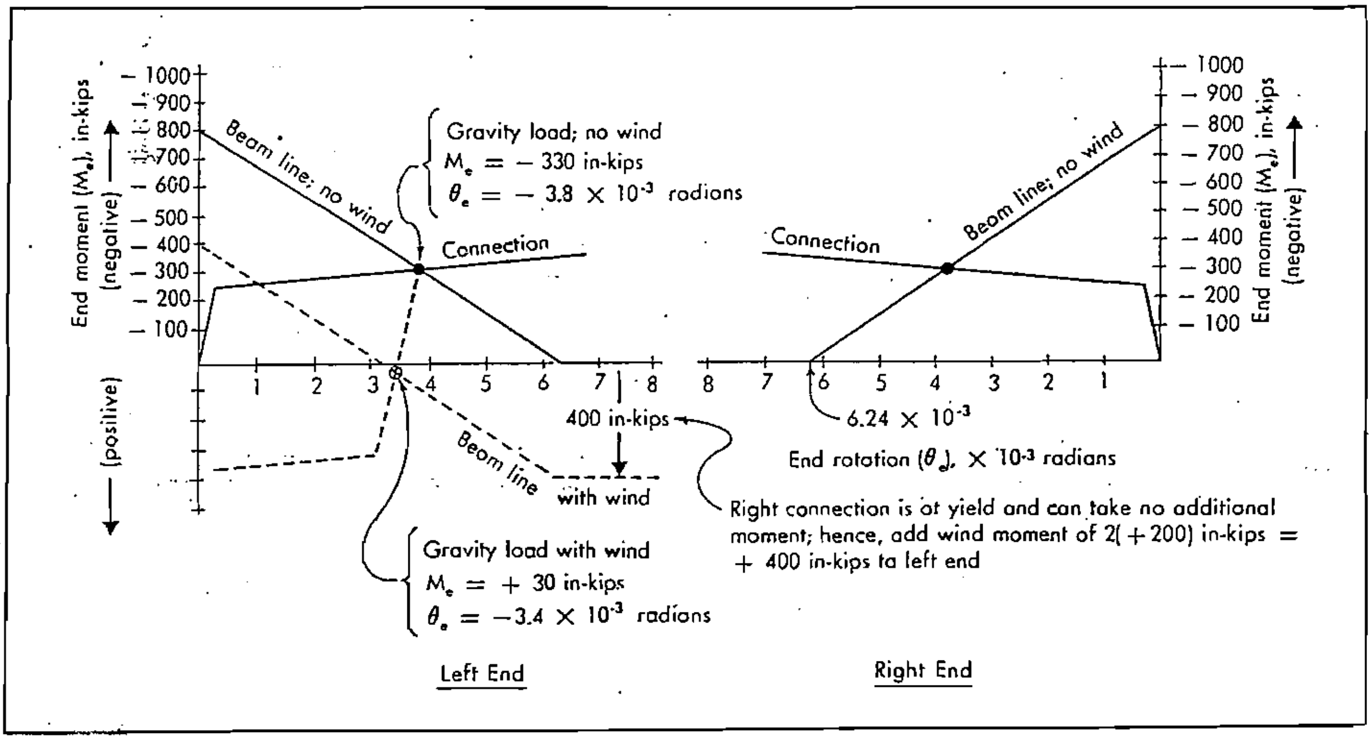


FIGURE 32

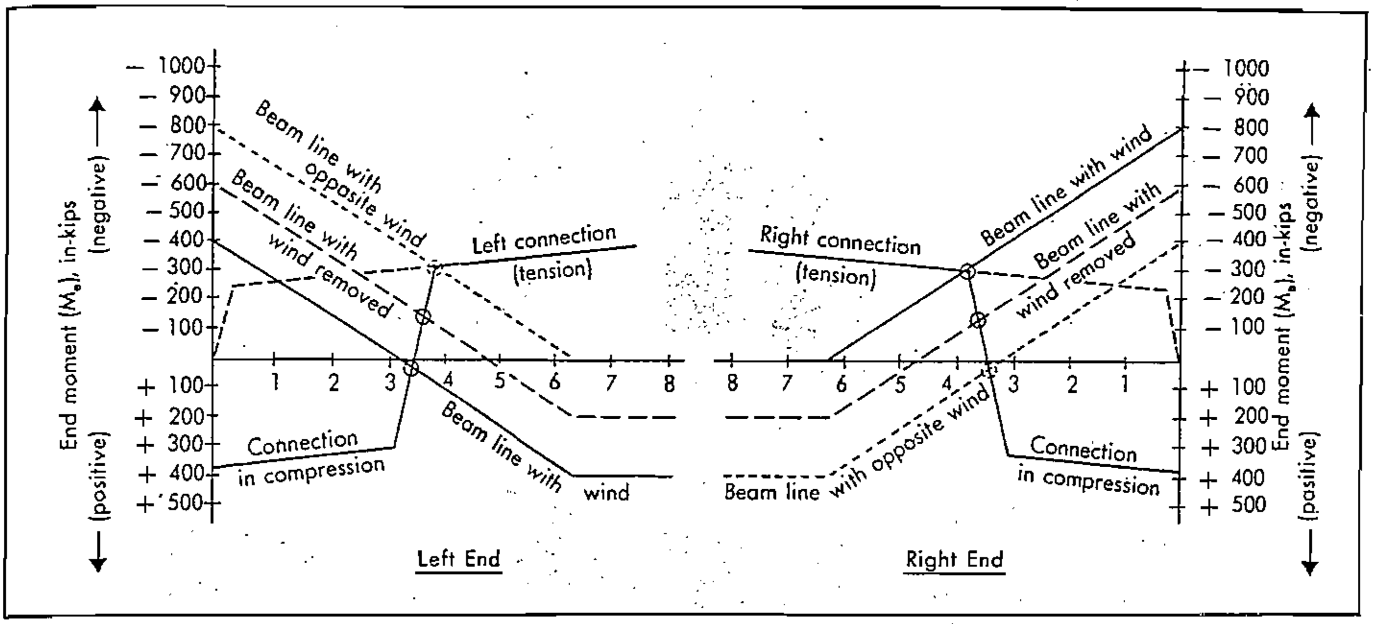
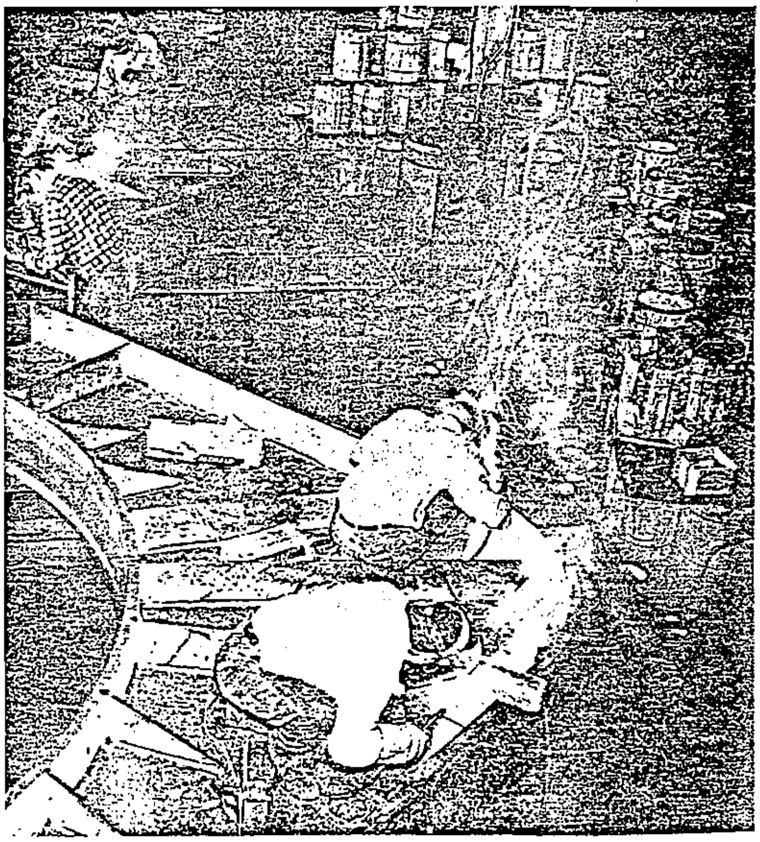
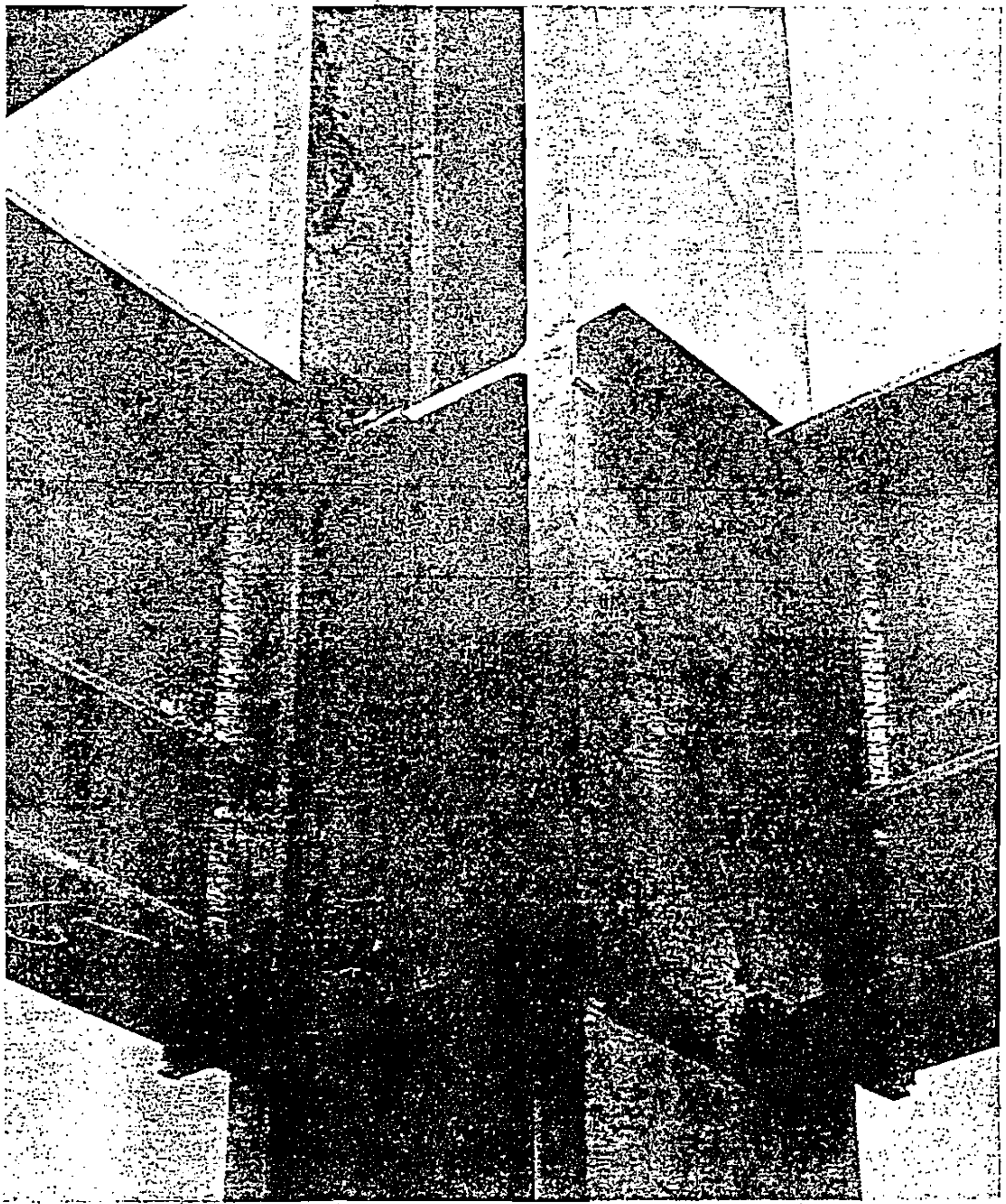


FIGURE 33



Typical scene in structural shop with weldors attaching stiffeners in place on curved knees. Proper use of welding results in significant savings in structural steel weight and in fabricating costs.



Welded continuous connections were used extensively in the Hartford Building in San Francisco. Photo shows the use of short Tee sections welded in place under ends of girders to provide deeper section at the point of maximum negative moment. Note that columns are weld fabricated. The small angle supports steel roof decking.