Tanks, Bins and Hoppers

1. SCOPE

This is a broad classification, covering many types of containers. However, principles and formulas relating to their design are best discussed as a single group. Some of these containers have flat surfaces; some have curved surfaces; some have both. Some carry steam, gasses, or pressurized fluids that exert uniform pressure in all directions; others carry bulk materials such as grain, the weight of which exerts a varying horizontal pressure against the side walls.

The first requisite of a container is that it be tight. It must have sufficient strength to withstand the internal pressure to which it is subjected. In arc-welded construction, the joints are made as tight and strong as the plates joined. In large tanks built up from a number of plates or sheets, butt welds are customarily specified.

Many containers must be designed and fabricated according to the minimum requirements of certain codes, for example ASME. Most containers have thin shells in comparison to their diameters and come under the classification of thin-wall shells.

Types of Containers Flat and/or Curved Surfaces		
tanks	drums	chutes
vats	bins	stacks
hoppers	silos	pipe and piping systems and many others

2. ELEMENTS OF THE CONTAINER

The surfaces of any container must withstand pressure of some type, so it would be well to consider the strength and stiffness of various shapes and forms of plates under uniform pressure.

In analysis of a given container, the designer explodes it into its various elements and applies the corresponding formulas.

Flat Surfaces of Containers

3. GENERAL

Some containers are of box construction, made up entirely of flat surfaces. Other containers, many tanks for example, consist of a cylinder closed at each end by a flat plate.

Table 1 presents design formulas applicable to various flat plates subjected to internal pressure.

Problem 1

Determine the required plate thickness of the following tank to hold water, Figure 1.

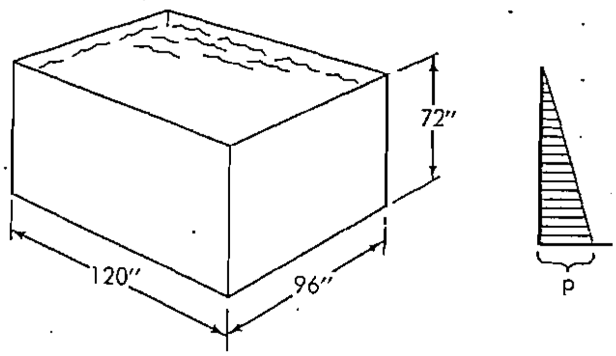


FIGURE 1

Since the varying pressure against side walls is due to the weight of a liquid:

$$p = .4336\ H\ s$$
$$= .4336(6)(1)$$
$$= 2.6 \text{ psi}$$

where:

H = the maximum height of the liquid, in feet

s = the specific gravity of the liquid

It is necessary to consider only the longest side plate, having the greatest span between supports:

120″. The top edge is free, the other three are supported. This is recognized as condition 4D in Table 1.

Since the ratio of plate height to width is—

$$\frac{a}{b} = \frac{72}{120} = .6$$

values are estimated from Table 1 to be—

$$\beta = .14 \qquad \text{and } \gamma = .030$$

Then the required plate thickness is derived from the maximum stress formula:

$$\sigma_{max} = \frac{\beta\, p\, b^2}{t^2}$$

or, assuming an allowable stress of 20,000 psi—

$$t^2 = \frac{\beta\, p\, b^2}{\sigma}$$

$$= \frac{(.14)(2.6)(120)^2}{20{,}000}$$

$$= .262$$

$$\therefore\ t = \sqrt{.262}$$

$$= .512'', \text{ or use } \tfrac{1}{2}'' \text{ ⅊}$$

Checking the deflection of this plate—

$$\Delta_{max} = \frac{\gamma\, p\, b^4}{E\, t^3}$$

$$= \frac{(.030)(2.6)(120)^4}{(30 \times 10^6)(.5)^3}$$

$$= 4.3''$$

Since this deflection would be excessive, a stiffening bar must be added along the top edge of the tank to form a rectangular frame, Figure 2.

Tank with Top Edge Stiffener

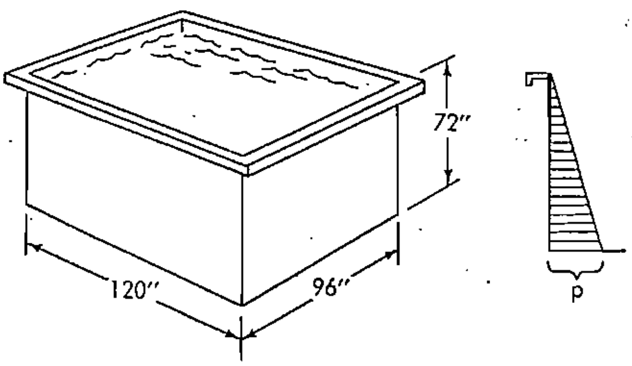


FIGURE 2

The modified tank now satisfies the condition 5A on Table 1, because the critical plate is supported on all four edges.

The ratio of plate height to width still being .6, values are estimated from Table 1 to be—

$$\beta = .102 \qquad \text{and } \gamma = .0064$$

Since the same maximum stress formula applies—

$$t^2 = \frac{\beta\, p\, b^2}{\sigma}$$

$$= \frac{(.102)(2.6)(120)^2}{20{,}000}$$

$$= .191$$

$$\therefore\ t = \sqrt{.191}$$

$$= .437'', \text{ or use } \tfrac{7}{16}'' \text{ ⅊}$$

Checking the deflection of this plate—

$$\Delta_{max} = \frac{\gamma\, p\, b^4}{E\, t^3}$$

$$= \frac{(.0064)(2.6)(120)^4}{(30 \times 10^6)(.4375)^3}$$

$$= 1.37''$$

It might be advisable to go back to the ½″ plate thickness, still using the top edge stiffener, in which case the bending stress and deflection would be reduced to—

$$\sigma_{max} = 15{,}300 \text{ psi} \qquad \text{and } \Delta_{max} = .92''$$

There is another method of determining the bending stress and deflection. A description of this follows immediately.

Considering Plate Section as a Beam

A narrow section of the tank's side panel (width, m = 1″) can be considered as a beam, Figure 3, using formulas taken from Reference Section 8.1 on Beam Diagrams.

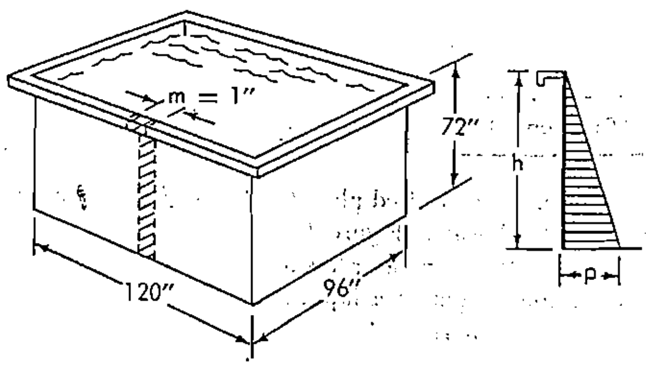
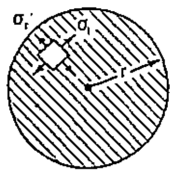


FIGURE 3

TABLE 1—Stress and Deflection, Flat Plates*
Subjected to Internal Pressure (p), psi

CIRCULAR PLATE



(1A) Edges supported; uniform load

At center:

$$(\text{max})\ \sigma_r = \sigma_t = -\frac{1.24\ p\ r^2}{t^2}$$

$$\Delta_{max} = -\frac{.695\ p\ r^4}{E\ t^3}$$

(1B) Edges fixed; uniform load

At center:

$$\sigma_r = \sigma_t = -\frac{.488\ p\ r^2}{t^2}$$

$$\Delta_{max} = -\frac{.1705\ p\ r^4}{E\ t^3}$$

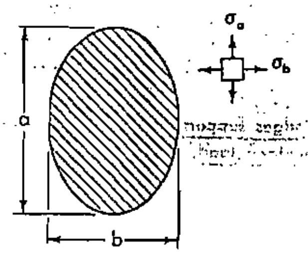
At edge:

$$(\text{max})\ \sigma_r = \frac{3\ p\ r^2}{4\ t^2}$$

$$\sigma_t = \frac{.225\ p\ r^2}{t^2}$$

ELLIPTICAL PLATE

$$\alpha = \frac{b}{a}$$



(2A) Edges supported; uniform load

At center:

$$(\text{max})\ \sigma_b = -\frac{.3125\ (2 - \alpha)\ p\ b^2}{t^2}$$

$$(\text{approx})\ \Delta_{max} = \frac{(.146 - .1\ \alpha)\ p\ b^4}{E\ t^3}$$

(2B) Edges fixed; uniform load

At center:

$$\sigma_a = -\frac{.075\ p\ b^2\ (10\ \alpha^2 + 3)}{t^2\ (3 + 2\ \alpha^2 + 3\ \alpha^4)}$$

$$\sigma_b = -\frac{.075\ p\ b^2\ (3\ \alpha^2 + 10)}{t^2\ (3 + 2\ \alpha^2 + 3\ \alpha^4)}$$

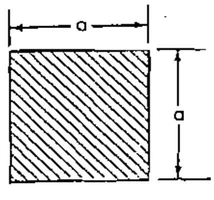
$$\Delta_{max} = -\frac{.1705\ p\ b^4}{E\ t^3\ (6 + 4\ \alpha^2 + 6\ \alpha^4)}$$

At edge:

$$(\text{Span a})\ \sigma_a = \frac{1.5\ p\ b^2\ \alpha^2}{t^2\ (3 + 2\ \alpha^2 + 3\ \alpha^4)}$$

$$(\text{max})\ (\text{Span b})\ \sigma_b = \frac{1.5\ p\ b^2}{t^2\ (3 + 2\ \alpha^2 + 3\ \alpha^4)}$$

SQUARE PLATE



(3A) Edges supported (and held down); uniform load

At center:

$$(\text{max})\ \sigma_a = -\frac{.2670\ p\ a^2}{t^2}$$

$$\Delta_{max} = \frac{.0443\ p\ a^4}{E\ t^3}$$

(3B) Edges fixed; uniform load

At center:

$$\sigma_a = -\frac{.166\ p\ a^2}{t^2}$$

$$\Delta_{max} = -\frac{.0138\ p\ a^4}{E\ t^3}$$

At midpoint of each edge:

$$(\text{max})\ \sigma_a = +\frac{.308\ p\ a^2}{t^2}$$

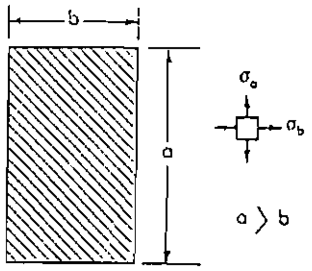
*After Roark, "Formulas for Stress and Strain".

Table 1 continued on following page

Table 1 continued

RECTANGULAR PLATES

$$\alpha = \frac{b}{a}$$



(4A) Edges supported; uniform load

At center:

$$\sigma_a = -\frac{p\,b^2\,(.225 + .382\,\alpha^2 - .320\,\alpha^3)}{t^2}$$

$$(\text{max})\ \sigma_b = -\frac{.75\,p\,b^2}{t^2\,(1 + 1.61\,\alpha^3)} \text{ or } = \frac{\beta\,p\,b^2}{t^2}$$

$$\Delta_{max} = -\frac{.1422\,p\,b^4}{E\,t^3\,(1 + 2.21\,\alpha^3)} \text{ or } = -\frac{\gamma\,p\,b^4}{E\,t^3}$$

(4B) Edges fixed; uniform load

At center:

$$\sigma_a = -\frac{.054\,p\,b^2\,(1 + 2\alpha^2 - \alpha^4)}{t^2}$$

$$\sigma_b = -\frac{.75\,p\,b^2}{t^2\,(3 + 4\,\alpha^4)}$$

$$\Delta_{max} = -\frac{.0284\,p\,b^4}{E\,t^3\,(1 + 1.056\,\alpha^5)} \text{ or } = -\frac{\gamma\,p\,b^4}{E\,t^3}$$

At midpoint of long edges:

$$(\text{max})\ \sigma_b = \frac{.5\,p\,b^2}{t^2\,(1 + .623\,\alpha^6)} \text{ or } = \frac{\beta\,p\,b^2}{t^2}$$

At midpoint of short edges:

$$\sigma_a = \frac{.25\,p\,b^2}{t^2}$$

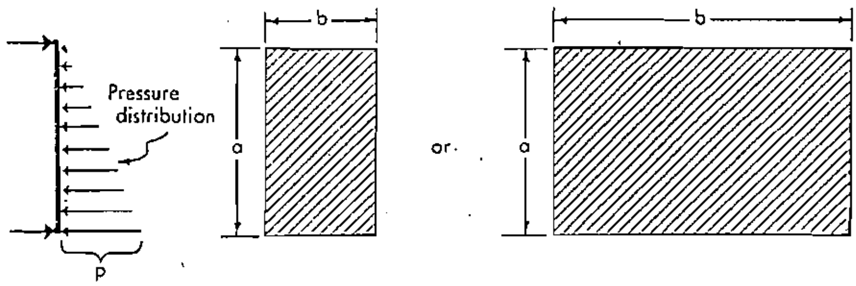
See the following sub-tables for values of β and γ:

$\frac{a}{b}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	∞
						FOR EDGES SUPPORTED						
β	.2874	.3318	.3756	.4158	.4518	.4542	.5172	.5448	.5688	.5910	.6102	.7500
γ	.0443	.0530	.0616	.0697	.0770	.0843	.0906	.0964	.1017	.1064	.1106	.1422
						FOR EDGES FIXED						
β	.3078	.3486	.3834	.4122	.4356	.4252	.4680	.4794	.4872	.4932	.4974	
γ	.0138	.0164	.0188	.0209	.0226	.0240	.0251	.0260	.0267	.0272	.0277	

(4C) All edges supported; varying load
Load increasing uniformly from zero at one edge to a maximum of (p) psi at opposite edge (triangular load)

$$\sigma_{max} = \frac{\beta\,p\,b^2}{t^2}$$

$$\Delta_{max} = \frac{\gamma\,p\,b^4}{E\,t^3}$$



The following values apply to Condition 4C;

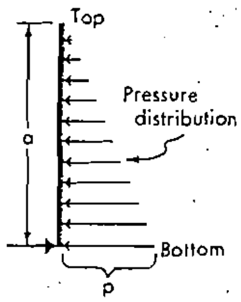
$\frac{a}{b}$	.25	.286	.333	.4	.5	.667	1.0	1.5	2.0	2.5	3.0	3.5	4.0
β	.024	.031	.041	.056	.080	.116	.16	.26	.32	.35	.37	.38	.38
γ	.00027	.00046	.00083	.0016	.0035	.0083	.022	.042	.056	.063	.067	.069	.070

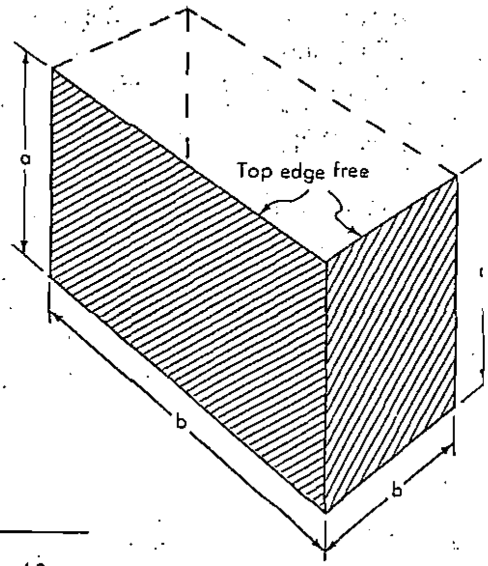
Table 1 continued on facing page

Table 1 continued

(4D) Top edge free, other three edges supported; varying load

Load increasing uniformly from zero at top edge to a maximum of (p) psi at bottom edge (triangular load)





$$\sigma_{max} = \frac{\beta\ p\ b^2}{t^2}$$

$$\Delta_{max} = \frac{\gamma\ p\ b^4}{E\ t^3}$$

The following values apply to Condition 4D;

$\frac{a}{b}$	.5	.667	1.0	1.5	2.0	2.5	3.0	3.5	4.0
β	.11	.16	.20	.28	.32	.35	.36	.37	.37
β	.026	.033	.040	.050	.058	.064	.067	.069	.070

Since the maximum bending moment here is—

$$M_{max} = .0642\ p\ h^2\ m \text{ (with h expressed in inches)}$$
$$= .0642\ (2.6)(72)^2(1)$$
$$= 865 \text{ in.-lbs}$$
$$\sigma_{max} = \frac{M}{s} = \frac{M\ 6}{t^2}$$
$$= 20{,}800 \text{ psi}$$

instead of the 15,300 psi obtained by considering the entire plate width; and—

$$\Delta_{max} = \frac{.0625\ p\ h^4\ m}{E\ I}$$
$$= \frac{.0625(2.6)(72)^4(1)(12)}{(30 \times 10^6)(.5)^3}$$
$$= 1.39''$$

instead of the .92″ obtained by considering the entire plate width.

This method of isolating a 1″ strip of the panel and considering it as a beam will indicate greater bending stress and deflection than actually exists. The reason is that the stiffening effect of the surrounding panel has been neglected for simplicity.

The previous method of considering the entire panel is recommended for its accuracy and for a more efficient design wherever it can be applied.

Adding Another Stiffener

When a panel is divided into two parts by a large stiffener, it becomes a continuous panel, triangularly loaded with a rather high negative moment at the stiffener which acts as a support. There is no simple formula for this; therefore the method of considering a 1″ strip will be used, and of course will result in a slightly greater stress value than actually exists.

The plate thickness in the tank being considered can probably be reduced by adding such a stiffener around the middle of the tank, Figure 4.

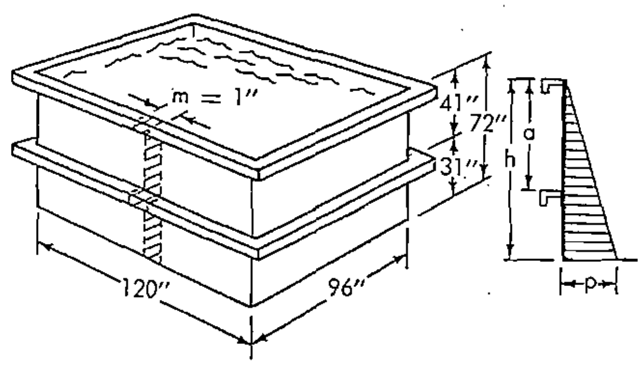


FIGURE 4

The first step is to locate the stiffener at the height which will produce the minimum bending moment in the panel, both above and below the stiffener.

(Again use formulas from Reference Section 8.1 on Beam Diagrams.) This dimension (a), the distance between the two stiffeners, is—

$$a = .57\ h = .57(72) = 41''$$

Then, at the middle stiffener—

$$M_{max} = .0147\ p\ h^2\ m$$
$$= .0147\ (2.6)(72)^2(1)$$
$$= 198\ \text{in.-lbs}$$

Trying $\frac{5}{16}''$ ℄

$$\sigma_{max} = \frac{M}{S} = \frac{M\ 6}{t^2}$$
$$= \frac{(198)6}{(\frac{5}{16})^2}$$
$$= \underline{12{,}200\ \text{psi}} \qquad \text{OK}$$

Container Surfaces Formed By A Figure of Revolution

4. STRESSES IN SHELL

The various container shapes illustrated in Table 2 are formed by a figure of revolution.

In any of these containers, the internal pressure (p) along with the weight of the gas, liquid or other media within the container produces three types of tensile stresses in the container's shell. These are:

1. σ_{mp} = tensile stress in the direction of a meridian. (A meridian is the curve formed by the intersection of the shell and a plane through the longitudinal axis of the container.) This stress is referred to as longitudinal stress.

2. σ_{cp} = tensile stress in the direction of a tangent to a circumference. (A circumference is the curve formed by the intersection of the shell and a plane perpendicular to the longitudinal axis of the container.) This stress is referred to as tangential or circumferential stress but is commonly called the hoop stress.

3. σ_{rp} = tensile stress in the radial direction.

For containers having relatively thin shells (generally considered as less than 10% of the mean radius) and no abrupt change in thickness or curvature, the radial tensile stress (σ_{rp}) and any bending stress may be neglected.

TABLE 2—Container Surfaces Formed By A Figure of Revolution

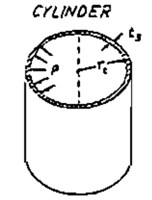
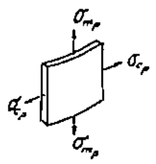
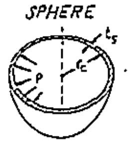
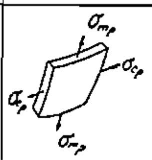
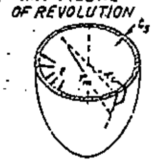
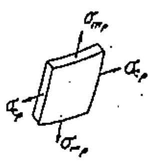
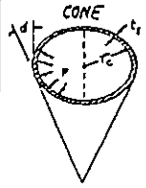
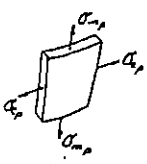
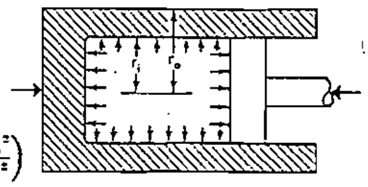
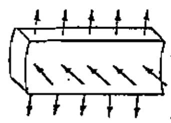
THIN WALL CONTAINERS		
CONTAINER SHAPE	UNIT WALL SEGMENT	TENSILE STRESS FORMULAE
CYLINDER 		$\sigma_{mp} = \frac{p\,r_c}{2\,t_s}$ $\sigma_{cp} = \frac{p\,r_c}{t_s}$
SPHERE 		$\sigma_{mp} = \sigma_{cp} = \frac{p\,r_c}{2\,t_s}$
ANY FIGURE OF REVOLUTION 		$\frac{\sigma_{cp}}{r_{cp}} + \frac{\sigma_m}{r_m} = \frac{p}{t_s}$ $\sigma_{mp} = \frac{p\,r_c}{2\,t_s}$ $\sigma_{cp} = \frac{p\,r_c}{t_s}\left(1 - \frac{r_c}{2\,r_m}\right)$
CONE 		$\sigma_{mp} = \frac{p\,r_c}{2\,t_s \cos\alpha}$ $\sigma_{cp} = \frac{p\,r_c}{t_s \cos\alpha}$

TABLE 3—Stresses in Thick-Wall Cylinders

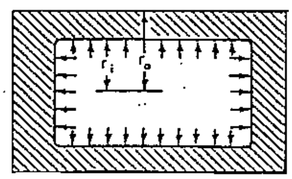
Uniform internal radial pressure only (p), psi



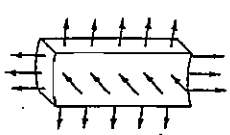
$$\sigma_{mp} = 0$$
$$\sigma_{cp} = p\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$
(max at inner surface)
$$\sigma_{rp} = p$$
(max at inner surface)



Uniform internal pressure in all directions



$$\sigma_{mp} = p\left(\frac{r_i^2}{r_o^2 - r_i^2}\right)$$
$$\sigma_{cp} = p\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$
(max at inner surface)
$$\sigma_{rp} = p$$
(max at inner surface)



The biaxial tensile stresses (σ_{mp}) and (σ_{cp}) in thin-wall containers can be calculated with the basic formulas shown in Table 2, where:

t_s = thickness of shell, in.
r_c = mean radius of a circumference of the shell, in.
r_m = mean radius of the meridian of the shell, in.
p = internal pressure, psi

5. THICK-WALLED CONTAINERS

In thin-walled containers, the hoop stress is assumed to be uniformly distributed across the shell thickness without serious error occurring in stress calculations. However, in a thick-walled container generated by a figure of revolution the decreasing variance of hoop stress from the inner surface to the outer surface of the shell wall must be considered.

Table 3 presents formulas for calculating the stresses in two common thick-walled cylinders. In the first condition, the internal pressure parallel to the structural (longitudinal) axis is balanced by the external force against the moving piston and by the resistance of the cylinder's support, and the resultant longitudinal stress (σ_{mp}) is zero. In the second condition, there is a longitudinal stress (σ_{mp}).

Unfired Pressure Vessels

6. ASME CODE—SECTION 8

Any pressure container of any importance undoubtedly must conform to the minimum requirements of the ASME, so it would be well to use ASME Section 8 "Unfired Pressure Vessels" as a guide. In general this covers containers for pressures exceeding 15 psi up to a maximum of 3,000 psi, and having a diameter exceeding 6".

Table 4 presents the formulas for calculating the minimum required wall thickness of cylindrical shells and spherical shells, where:

p = internal pressure, psi
σ_a = allowable stress (See ASME Sec. 8, par USC-23)
E = joint efficiency (See ASME Sec. 8, par UW-12)

Table 5 presents the formulas for calculating the minimum required thickness of various types of heads. Turn to next page for Table 5.

TABLE 4—Wall Thickness of Shells
Subjected to Internal Pressure (p), psi
(ASME-8: Unfired Pressure Vessels)

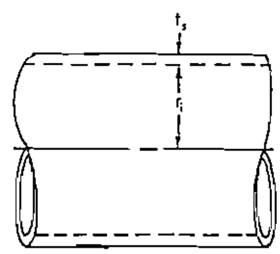
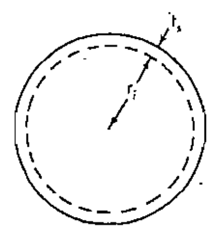
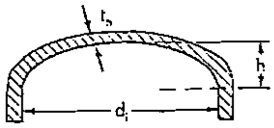
CYLINDRICAL SHELLS (UG-27c and UA-1)	SPHERICAL SHELLS (UG-27d and UA-3)
	
Thin shell — when $t_s < \frac{1}{2} r_i$ and $p < .385\,\sigma_a E$	Thin shell — when $t_s < .356\, r_i$ and $p < .665\,\sigma_a E$
$t_s = \frac{p\, r_i}{\sigma_a E - .6\,p}$	$t_s = \frac{p\, r_i}{2(\sigma_a E - .1\,p)}$
Thick shell — when $t_s > \frac{1}{2} r_i$ and $p > .385\,\sigma_a E$	Thick shell — when $t_s > .356\, r_i$ and $p > .665\,\sigma_a E$
$= r_i(\sqrt{Z} - 1)$	$t_s = r_i(\sqrt[3]{Y} - 1)$
where $Z = \frac{\sigma_a E + p}{\sigma_a Z - p}$	where $Y = \frac{2(\sigma_a E + p)}{2\,\sigma_a E - p}$

TABLE 5—Thickness of Formed Heads
Subjected to Internal Pressure (p) on Concave Side
(ASME-8: Unfired Pressure Vessels)

ELLIPSOIDAL HEAD
(UG-32d and UA-4c)



Standard head — where $h = d_i/4$
(h = minor axis: inside depth of head minus skirt)

$$t_h = \frac{p\,d_i}{2(\sigma_a E - .1\,p)}$$

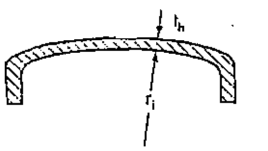
Head of other proportions

$$t_h = \frac{p\,d_i\,K}{2(\sigma_a E - .1\,p)}$$

where:

$$K = \frac{1}{6}\left[2 + \left(\frac{d_i}{2h}\right)^2\right]$$

TORISPHERICAL HEAD
(UG-32e and UA-4d)



Standard head — where $r_k = .06\,r_i$
(r_k = knuckle radius)

$$t_h = \frac{.885\,p\,r_i}{\sigma_a E - .1\,p}$$

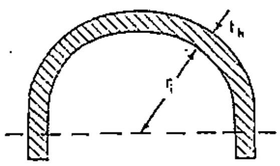
Head of other proportions

$$t_h = \frac{p\,r_i\,M}{2(\sigma_a E - .1\,p)}$$

where:

$$M = \frac{1}{4}\left[3 + \sqrt{\frac{r_i}{r_k}}\right]$$

HEMISPHERICAL HEAD
(UG-32f and UA-3)



Thin head — when $t_h < .356\,r_i$ and $p < .665\,\sigma_a E$

$$t_h = \frac{p\,r_i}{2(\sigma_a E - .1\,p)}$$

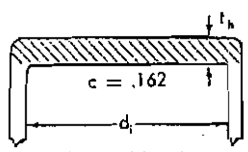
Thick head — when $t_h > .356\,r_i$ and $p > .665\,\sigma_a E$

$$t_h = r_i\left(\sqrt[3]{Y} - 1\right)$$

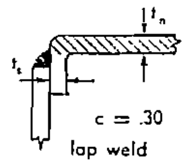
where:

$$Y = \frac{2(\sigma_a E + p)}{2\,\sigma_a E - p}$$

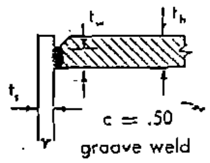
FLAT HEAD (UG-34)



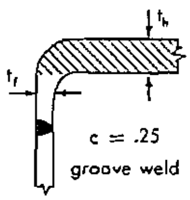
integral head



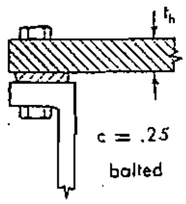
c = .30
lap weld
$r_{min} = 3\,t_c$



c = .50
groove weld



c = .25
groove weld



c = .25
bolted

t_w = twice required thickness of spherical shell or 1.25 t_s and not greater than t_h

$$t_h = d_i\sqrt{\frac{c\,p}{\sigma_a}}$$