Welded Connections for Plastic Design

1. INTRODUCTION TO PLASTIC DESIGN

The allowable stress used on steel structures in bending is .60 σ_y, a percentage of the steel's yield strength (AISC Sec 1.5.1.4). A steel structure designed on this basis may carry an overload as great as 1.67 times the designed load before the most stressed fiber reaches the yield point. Naturally, this does not represent the maximum load-carrying capacity of the structure, nor does it indicate the reserve strength still in the structure.

Plastic design does not make use of the conventional allowable stresses, but rather the calculated ultimate load-carrying capacity of the structure.

With this method, the given load is increased by 1.70 times the given live and dead load for simple and continuous beams, 1.85 times the given live and dead load for continuous frames, and 1.40 times these loads when acting in conjunction with 1.40 times any specified wind or earthquake forces. Then the members are designed to carry this load at their ultimate or plastic strength. Some yielding must take place before this ultimate load is reached; however, under normal working loads, yielding will seldom occur.

For the past 25 years, a considerable amount of research, both in Europe and the United States, has been devoted to the ultimate load-carrying capacity of steel structures.

For about 15 years, extensive work on full-scale structures has been going on at Lehigh University under the joint sponsorship of the Structural Committee of the Welding Research Council and the American Institute of Steel Construction. Much has been learned as a result of this work.

Major Conclusions

The ultimate load-carrying capacity of a beam section is much greater than the load at yield point. For many years, it has been known that a beam stressed at its outer fibers to the yield point still had a considerable amount of reserve strength before final rupture or collapse. Consider Figure 1.

In this graph for A36 steel, the vertical axis is the applied moment (M), the horizontal axis is the resulting angle of rotation (ϕ). Within the elastic limit (B), there is a straight-line relationship. It is assumed that the bending stresses are zero along the neutral axis of the beam and increase linearly until they are maximum at the outer fibers. This is illustrated at the top of the figure. At point (A), the maximum outer fiber bending stress has reached 22,000 psi. At point (B), this stress has reached the yield point, or 36,000 psi, and yielding at the outer fiber starts to take place. In conventional design, this point is assumed to be the ultimate load on the member; however, this curve shows there is still some more reserve strength left in the beam. As the beam is still further loaded, as at (C), the outer fibers are not stressed higher, but the fibers down inside the beam start to load to the yield point, as in (D). At this point, the beam becomes a plastic hinge; in other words, it will undergo a considerable amount of angle change with very little further increase in load.

M_y is the moment yield point (B), and M_p is the

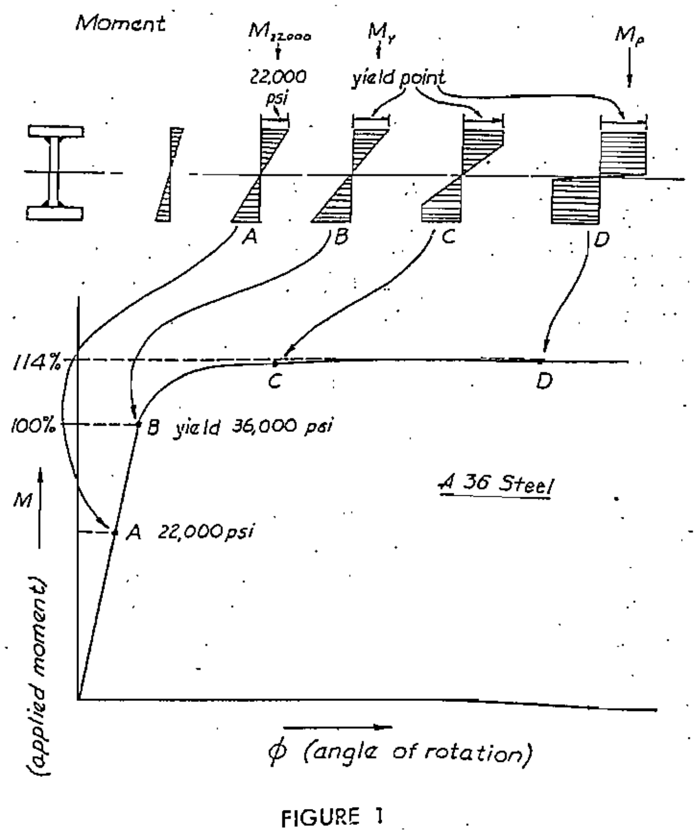


FIGURE 1

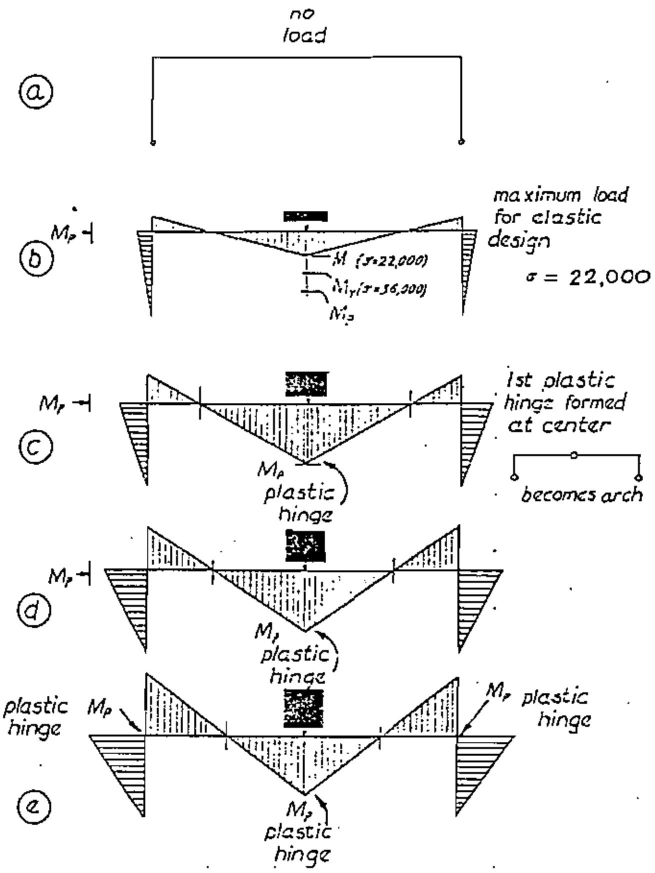


FIGURE 2

plastic moment which causes the beam at point (D) to act as a plastic hinge. For a rectangular cross-section, the plastic moment (M_p) is 1.5 times the moment at yield point (M_y). For the standard rolled WF sections, this plastic moment (M_p) is usually taken as 1.12 times the moment at yield point (M_y). The multiplier varies for other sectional configurations.

Redistribution of moments causes other plastic hinges to form. In Figure 2, a rigid frame with pinned ends is loaded with a concentrated load at midspan. The frame with no load is shown in (a). The frame is loaded in (b) so that its maximum bending stress is 22,000 psi, the allowable. Notice from the bending diagram that the moment at midspan is greater than the moments at the ends or knees of the frame. The three marks at midspan show the moment M where $\sigma = 22{,}000$ psi, or allowable; M_y where $\sigma = 36{,}000$ psi, or yield point; and M_p at plastic hinge. Notice at the left knee how much more the moment can be increased before a plastic hinge is formed.

In (c) the load has been increased until a plastic hinge has been formed at midspan. The knees of the frame in this example have only reached about half of this value. Even though, with conventional thinking, this beam has served its usefulness, it still will not fail because the two knees are still intact and the frame now becomes a three-hinged arch, the other two hinges being the original pinned ends.

Further loading of the frame may be continued, as in (d), with the knees loading up until they become plastic hinges, as in (e). Only when this point is reached would the whole frame fail. This condition is referred to as mechanism; that is, the structure would deform appreciably with only the slightest increase in load.

This entire hinge action takes place in a small portion of the available elongation of the member. In the lower portion of Figure 3 is a stress-strain curve showing the amount of movement which may be used in the plastic range. This may seem large, but it is a very small portion of the whole curve, as shown in the upper portion of the figure, which is carried out to 25% elongation.

The working load is multiplied by a factor of safety (1.85) to give the ultimate load. The design of the structure is based on this ultimate load. In order to establish a proper factor of safety to use in connection with the ultimate load, as found in the plastic method of design, it would be well to consider the loading of a simply supported beam with a concentrated load applied at its midpoint. This is shown in Figure 4. The moment diagrams for this beam are shown for the three loads: the moment M causing a bending stress of 22,000 psi; the moment M_y causing 36,000 psi or yield point; and the moment M_p causing a plastic hinge.

Here, for A36 steel:

(A) Allowable bending stress = 22,000 psi
(B) Yield stress = 36,000 psi = 67% above (A)
(C) Plastic hinge occurs 12% above (B)

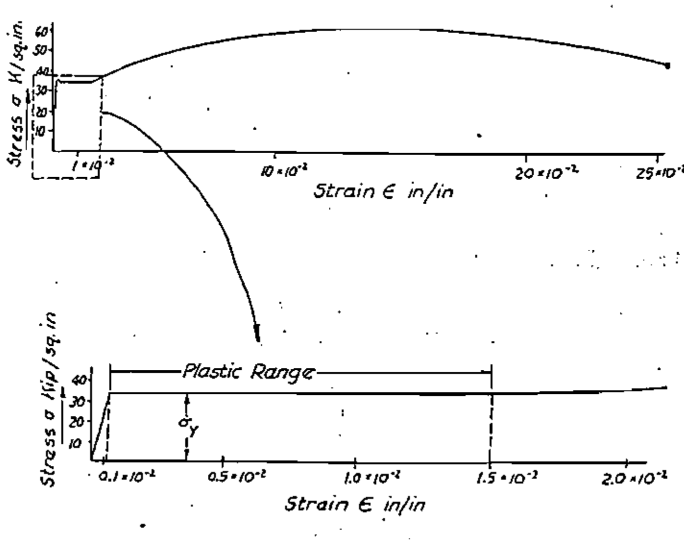


FIGURE 3

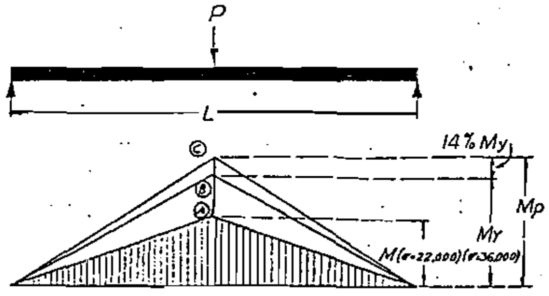


FIGURE 4

Hence:

Ⓒ $= (1.67)(1.12) = 1.88$ of Ⓐ

Thus, the true load factor of safety of the simple beam is 1.88.

In conventional design, it is assumed that the ultimate load is the value which causes the beam to be stressed to its yield point at the point of maximum stress. This would be represented in the figure by the moment at Ⓑ.

In conventional design, if the allowable bending stress is 22,000 psi and the yield point of the (A36) steel is assumed to be 36,000 psi, the designer is actually using a factor of safety of 1.67.

By means of plastic design, the ultimate load is approximately 12% higher (in the case of a WF beam) than the load which causes the yield point to be reached. Therefore, the factor of safety for plastic design on the same basis would be $(1.67)(1.12) = 1.88$.

Example

To illustrate plastic design, a beam will be designed using three different methods: (a) simple beam, (b) elastic design, rigid frame, and (c) plastic design, rigid frame. The beam will have a span of 80′ and carry a concentrated load of 55 kips at midspan. For simplicity the dead load will be neglected.

(a) The simply supported beam is shown in Figure 5 with its moment diagram. The maximum moment formula is found in any beam table. From this, the required section modulus (S) is found to be 600.0 in.³, using an allowable load of 22,000 psi in bending. This beam may be made of a 36″ WF beam which weighs 182 lbs/ft.

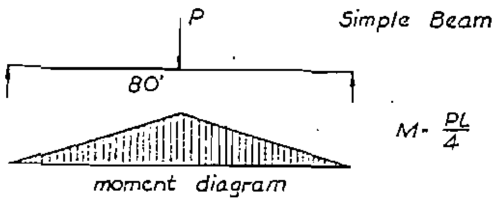


FIGURE 5

Here:

$$M = \frac{P\,L}{4} = \sigma\,S$$

$$S = \frac{P\,L}{4\,\sigma}$$

$$= \frac{(55{,}000)(80 \times 12)}{4(22{,}000)}$$

$$= 600 \text{ in.}^3$$

So, use 36″ WF 182# beam with $S = 621$ in.³

(b) The elastic design, rigid frame is shown in Figure 6. Its span is 80′ and its height is 20′. There are several ways to solve for the bending moments on this frame.

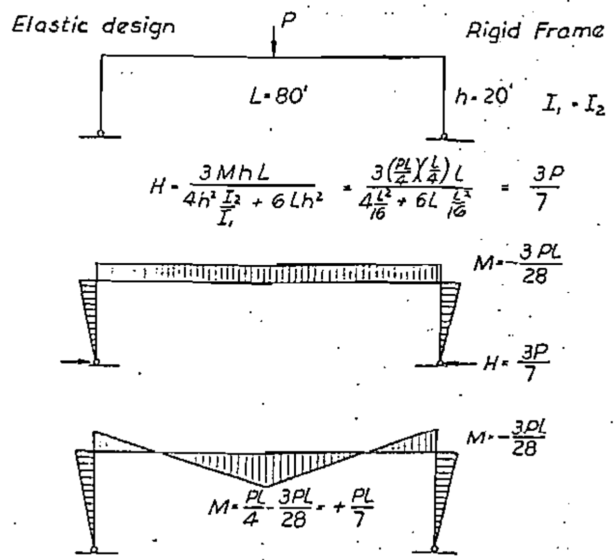


FIGURE 6

In this example the moment at midspan would be—

$$M = \frac{P\,L}{7} = \sigma\,S$$

$$S = \frac{P\,L}{7\,\sigma}$$

$$= \frac{(55{,}000)(80 \times 12)}{7(22{,}000)}$$

$$= 343 \text{ in.}^3$$

So, use a 30″ WF 124# beam with $S = 354.6$ in.³

The redundant or unknown horizontal force at the pinned end of the frame is first found. Then, from this, the moment diagram is drawn and the maximum moment found. The required section modulus (S) of the frame is determined from this maximum moment.

This is found to be 343 in.^{3}, which is 55% of that required for the single beam. This beam could be made of a 30″ WF beam having a weight of 124 lbs/ft.

(c) The plastic design, rigid frame is shown in Figure 7. With this method, the possible plastic hinges are found which could cause a mechanism or the condition whereby the structure beyond a certain stress point would deform appreciably with only the slightest increase in load. These points of plastic hinge, in this example, are at the midpoint and the two ends, and are assigned the value of M_p. An expression is needed from which this value M_p can be found.

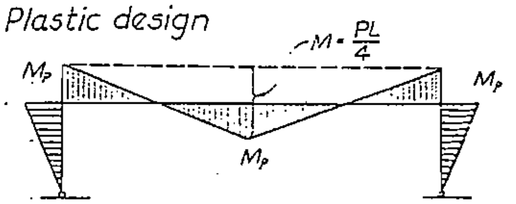


FIGURE 7

Here:

$$M_p + M_p = \frac{P\ L}{4}$$

$$\text{or } M_p = \frac{P_u\ L}{8}$$

$$= \frac{1.85\ P\ L}{8}$$

$$= \frac{1.85(55^k)(80')}{8}$$

$$= 1017.5 \text{ ft-kips}$$

So, use a 27″ WF 114# beam, with plastic moment (M_p) of 1029 ft-kips. (See AISC Manual of Steel Construction, Plastic Section Modulus Table.)

In this case, it is noticed that the altitude of the overall triangle in the moment diagram, which is M_p plus M_p, is also the same as that of the moment diagram of a simply supported beam with a concentrated load at its midspan, Figure 5. This can be found in any beam table. Hence, M_p plus M_p is set equal to $\frac{P\ L}{4}$ using for P the ultimate load which is the working load times 1.85. This works out to $M_p = 1017.5$ ft-kips as the ultimate load plastic moment, at centerline and at the two beam ends.

* * *

Summary of Advantages

As a summary, here are some of the advantages of plastic design:

1. More accurately indicates the true carrying capacity of the structure.
2. Requires less steel than conventional simple beam construction and, in most cases, results in a saving over the use of conventional elastic design of rigid frames.
3. Requires less design time than does elastic design of rigid framing.
4. Result of years of research and testing of full-scale structures.
5. Has the backing of the American Institute of Steel Construction.

2. DESIGN REQUIREMENTS OF THE MEMBER

Loads (AISC Sec. 2.1)

The applied loads shall be increased by the following factor:

1.70 live and dead loads on simple and continuous beams
1.85 live and dead loads on continuous frames
1.40 loads acting in conjunction with 1.40 times any wind and earthquake forces

Columns (AISC Sec. 2.3)

Columns in continuous frames where side-sway is not prevented shall be proportioned so that:

$$\boxed{\frac{2\ P}{P_y} + \frac{L}{70\ r} \leqq 1.0 \quad \text{(AISC formula 20)}} \quad \ldots\ldots\ldots\ldots (1)$$

or

$$\boxed{\frac{L}{r} \leqq 70 - 140\ \frac{P}{P_y}} \quad \ldots\ldots\ldots\ldots (2)$$

where:

L = unbraced length of column in the plane normal to that of the continuous frame

r = radius of gyration of column about an axis normal to the plane of the continuous frame

See the nomograph, Figure 8, for convenience in reading the limiting value of L/r directly from the values of P and P_y.

The AISC formulas (21), (22), and (23) give the effective moment (M_m), which a given shape is capable of resisting in terms of its full plastic moment (M_p) when it supports an axial force (P) in addition to its moment. See Table 1.

The maximum axial load (P) shall not exceed .60 P_y or .60 σ_y A_c, where A_c = cross-sectional area of the column.

FIGURE 8—Limiting Slenderness Ratio of Columns in Continuous Frames (Plastic Design), Sidesway Permitted.

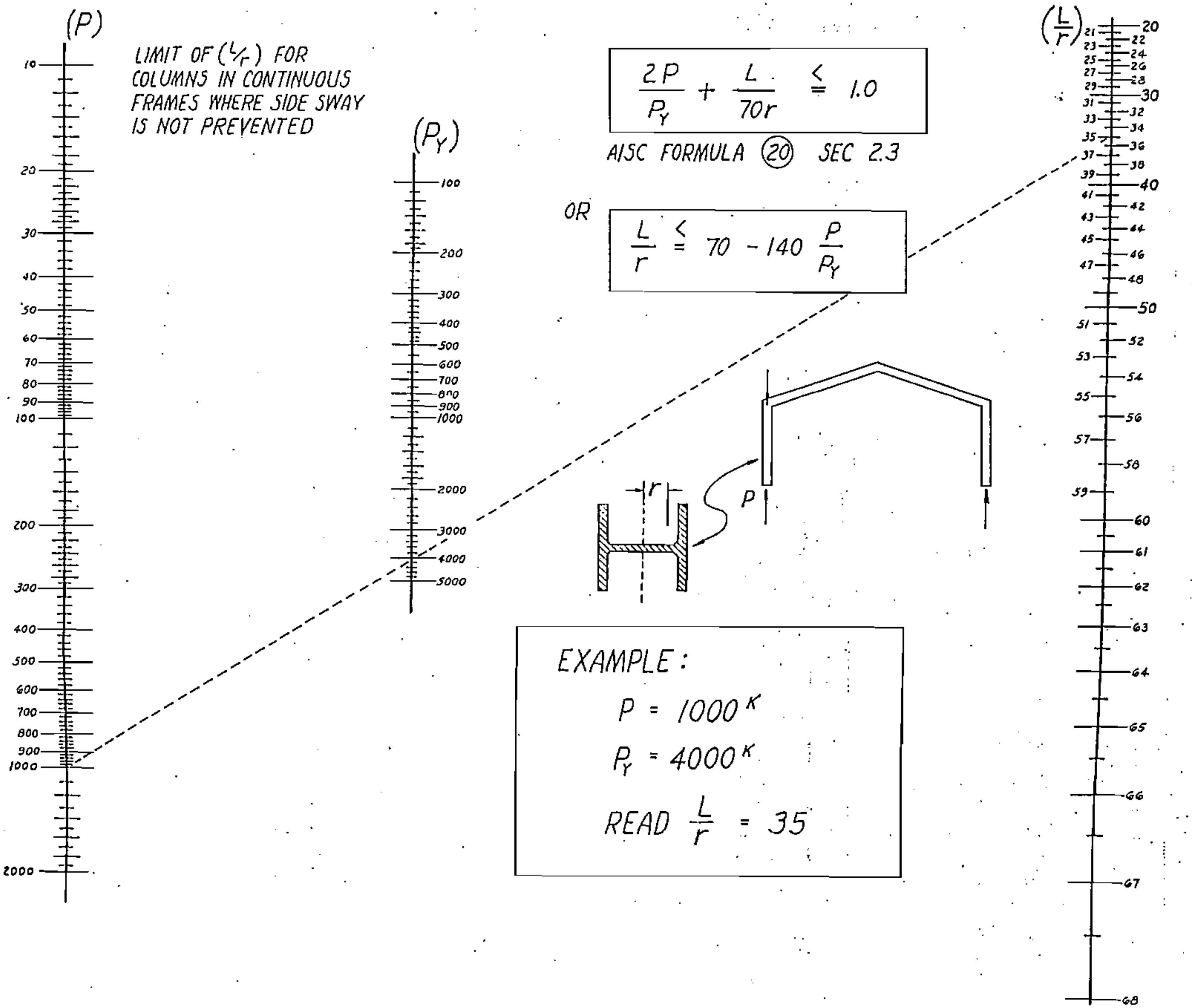
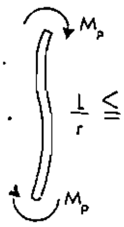
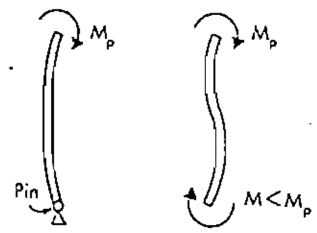


TABLE 1—Allowable End Moments Relative To Full Plastic Moment of Axially-Loaded Members

Case 1	Case 2	Case 3
		
when $P/P_y \leqq 0.15$ $\boxed{M_o = M_P}$ when $P/P_y > 0.15$ $\boxed{\frac{M_o}{M_P} \leqq 1.18 - 1.18\left(\frac{P}{P_y}\right) \leqq 1.0}$ AISC formula (21)	$\boxed{\frac{M_o}{M_P} \leqq B - G\left(\frac{P}{P_y}\right) \leqq 1.0}$ AISC formula (22) when $\frac{L}{r} < 60$ and $\frac{P}{P_y} < .15$ then $\boxed{M_o = P_P}$	$\boxed{\frac{M_o}{M_P} \leqq 1.0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2}$ AISC formula (23)

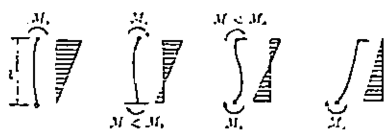
Notes: See Tables 2-33, 3-33, 2-36 and 3-36 for values of B, G, K and J

TABLE 2-33 (AISC Table 4-33)

FOR 33 KSI SPECIFIED YIELD POINT STEEL

Formula (22)

$$\frac{M_o}{M_p} = B - G\left(\frac{P}{P_y}\right)$$



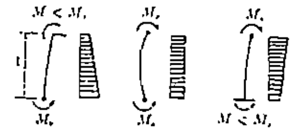
L/r	B	G	L/r	B	G	L/r	B	G
16	1.140	1.172	51	1.164	1.271	86	1.201	1.616
17	1.140	1.174	52	1.165	1.276	87	1.202	1.633
18	1.141	1.177	53	1.165	1.281	88	1.204	1.651
19	1.141	1.179	54	1.166	1.286	89	1.205	1.669
20	1.142	1.182	55	1.167	1.292	90	1.206	1.688
21	1.142	1.184	56	1.168	1.297	91	1.207	1.707
22	1.143	1.187	57	1.169	1.303	92	1.209	1.726
23	1.143	1.189	58	1.170	1.310	93	1.210	1.746
24	1.144	1.191	59	1.171	1.316	94	1.211	1.767
25	1.145	1.194	60	1.172	1.323	95	1.213	1.788
26	1.145	1.196	61	1.173	1.330	96	1.214	1.810
27	1.146	1.198	62	1.174	1.337	97	1.215	1.832
28	1.146	1.200	63	1.175	1.344	98	1.217	1.855
29	1.147	1.203	64	1.176	1.352	99	1.218	1.879
30	1.146	1.205	65	1.177	1.360	100	1.220	1.903
31	1.148	1.207	66	1.178	1.369	101	1.221	1.928
32	1.149	1.209	67	1.179	1.377	102	1.222	1.953
33	1.150	1.212	68	1.180	1.386	103	1.223	1.979
34	1.150	1.215	69	1.181	1.396	104	1.225	2.006
35	1.151	1.217	70	1.182	1.406	105	1.227	2.033
36	1.152	1.220	71	1.183	1.416	106	1.228	2.061
37	1.152	1.222	72	1.184	1.426	107	1.230	2.090
38	1.153	1.225	73	1.186	1.437	108	1.231	2.119
39	1.154	1.228	74	1.187	1.448	109	1.233	2.149
40	1.155	1.231	75	1.188	1.460	110	1.234	2.179
41	1.155	1.234	76	1.189	1.472	111	1.236	2.211
42	1.156	1.237	77	1.190	1.485	112	1.237	2.243
43	1.157	1.240	78	1.191	1.497	113	1.239	2.275
44	1.158	1.243	79	1.192	1.511	114	1.240	2.309
45	1.159	1.247	80	1.194	1.524	115	1.242	2.343
46	1.159	1.251	81	1.195	1.538	116	1.243	2.378
47	1.160	1.254	82	1.196	1.553	117	1.245	2.414
48	1.161	1.258	83	1.197	1.568	118	1.247	2.450
49	1.162	1.262	84	1.198	1.584	119	1.248	2.487
50	1.163	1.267	85	1.200	1.600	120	1.250	2.525

TABLE 3-33 (AISC Table 5-33)

FOR 33 KSI SPECIFIED YIELD POINT STEEL

Formula (23)

$$\frac{M_o}{M_p} = 1.0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2$$



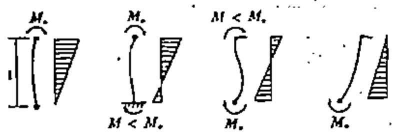
L/r	K	J	L/r	K	J	L/r	K	J
1	.434	.753	41	1.015	.149	81	1.824	−.738
2	.449	.736	42	1.032	.133	82	1.850	−.769
3	.463	.720	43	1.048	.116	83	1.877	−.801
4	.478	.703	44	1.064	.0998	84	1.903	−.833
5	.492	.687	45	1.081	.0832	85	1.930	−.866
6	.506	.671	46	1.097	.0663	86	1.958	−.900
7	.520	.655	47	1.114	.0492	87	1.986	−.934
8	.534	.640	48	1.131	.0318	88	2.014	−.969
9	.548	.624	49	1.148	.0143	89	2.042	−1.004
10	.562	.609	50	1.166	−.0036	90	2.071	−1.041
11	.576	.594	51	1.183	−.0217	91	2.101	−1.077
12	.590	.579	52	1.201	−.0401	92	2.130	−1.115
13	.604	.564	53	1.219	−.0588	93	2.161	−1.153
14	.619	.549	54	1.237	−.0777	94	2.191	−1.192
15	.633	.534	55	1.256	−.0970	95	2.222	−1.231
16	.647	.519	56	1.274	−.117	96	2.254	−1.272
17	.661	.504	57	1.293	−.137	97	2.286	−1.313
18	.675	.490	58	1.312	−.157	98	2.318	−1.354
19	.689	.475	59	1.332	−.177	99	2.350	−1.397
20	.703	.461	60	1.351	−.198	100	2.384	−1.440
21	.717	.447	61	1.371	−.220	101	2.417	−1.484
22	.731	.432	62	1.391	−.241	102	2.451	−1.529
23	.746	.418	63	1.411	−.263	103	2.486	−1.575
24	.760	.403	64	1.432	−.286	104	2.521	−1.621
25	.774	.389	65	1.452	−.309	105	2.556	−1.668
26	.789	.374	66	1.473	−.332	106	2.592	−1.716
27	.803	.360	67	1.495	−.356	107	2.628	−1.765
28	.818	.345	68	1.516	−.380	108	2.665	−1.814
29	.832	.331	69	1.538	−.404	109	2.703	−1.865
30	.847	.316	70	1.560	−.429	110	2.741	−1.916
31	.862	.301	71	1.582	−.455	111	2.779	−1.968
32	.877	.287	72	1.605	−.481	112	2.818	−2.021
33	.892	.272	73	1.628	−.507	113	2.857	−2.077
34	.907	.257	74	1.652	−.534	114	2.897	−2.123
35	.922	.242	75	1.675	−.562	115	2.937	−2.185
36	.937	.227	76	1.699	−.590	116	2.978	−2.242
37	.953	.211	77	1.721	−.618	117	3.020	−2.300
38	.968	.196	78	1.748	−.647	118	3.062	−2.358
39	.984	.180	79	1.773	−.677	119	3.104	−2.417
40	1.000	.165	80	1.799	−.707	120	3.147	−2.478

TABLE 2-36 (AISC Table 4-36)

FOR 36 KSI SPECIFIED YIELD POINT STEEL

Formula (22)

$$\frac{M_o}{M_p} = B - G\left(\frac{P}{P_y}\right)$$



l/r	B	G	l/r	B	G	l/r	B	G
16	1.137	1.173	51	1.163	1.285	86	1.203	1.893
17	1.137	1.176	52	1.164	1.291	87	1.204	1.713
18	1.138	1.179	53	1.165	1.298	88	1.206	1.734
19	1.139	1.182	54	1.166	1.303	89	1.207	1.755
20	1.139	1.184	55	1.166	1.309	90	1.208	1.777
21	1.140	1.187	56	1.167	1.316	91	1.210	1.799
22	1.140	1.189	57	1.168	1.323	92	1.211	1.822
23	1.141	1.192	58	1.170	1.330	93	1.213	1.846
24	1.142	1.194	59	1.171	1.337	94	1.214	1.870
25	1.142	1.196	60	1.172	1.345	95	1.215	1.895
26	1.143	1.199	61	1.173	1.354	96	1.217	1.921
27	1.143	1.201	62	1.174	1.362	97	1.218	1.947
28	1.144	1.204	63	1.175	1.371	98	1.220	1.974
29	1.145	1.206	64	1.176	1.380	99	1.221	2.002
30	1.145	1.209	65	1.177	1.390	100	1.223	2.030
31	1.146	1.211	66	1.178	1.400	101	1.224	2.059
32	1.147	1.214	67	1.179	1.410	102	1.226	2.089
33	1.148	1.216	68	1.180	1.421	103	1.227	2.120
34	1.148	1.219	69	1.181	1.432	104	1.229	2.151
35	1.149	1.222	70	1.183	1.444	105	1.231	2.183
36	1.150	1.225	71	1.184	1.456	106	1.232	2.216
37	1.151	1.228	72	1.185	1.468	107	1.234	2.249
38	1.151	1.231	73	1.186	1.481	108	1.235	2.283
39	1.152	1.234	74	1.187	1.494	109	1.237	2.318
40	1.153	1.237	75	1.189	1.509	110	1.239	2.354
41	1.154	1.241	76	1.190	1.522	111	1.240	2.391
42	1.155	1.244	77	1.191	1.537	112	1.242	2.429
43	1.155	1.248	78	1.192	1.552	113	1.244	2.467
44	1.156	1.252	79	1.194	1.568	114	1.245	2.506
45	1.157	1.256	80	1.195	1.584	115	1.247	2.546
46	1.158	1.260	81	1.196	1.601	116	1.249	2.567
47	1.159	1.265	82	1.197	1.618	117	1.250	2.628
48	1.160	1.270	83	1.199	1.636	118	1.252	2.671
49	1.161	1.275	84	1.200	1.654	119	1.254	2.714
50	1.162	1.280	85	1.201	1.673	120	1.256	2.759

If L/r > 120, the ratio of axial load (P) to plastic load (P_y) shall be—

$$\boxed{\frac{P}{P_y} \leqq \frac{8700}{(L/r)^2}} \qquad \text{(AISC formula 24)} \qquad \ldots\ldots\ldots\ldots (3)$$

Shear (AISC Sec. 2.4)

Webs of columns, beams, and girders not reinforced by a web doubler plate or diagonal stiffeners shall be so proportioned that:

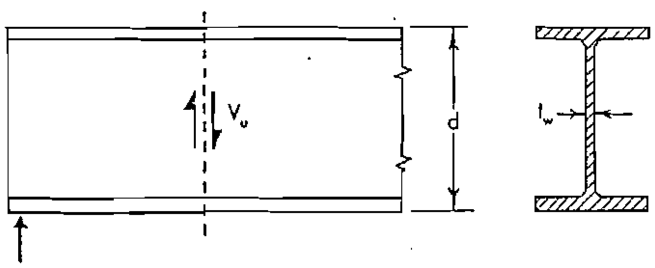


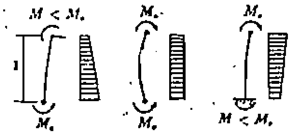
FIGURE 9

TABLE 3-36 (AISC Table 5-36)

FOR 36 KSI SPECIFIED YIELD POINT STEEL

Formula (23)

$$\frac{M_o}{M_p} = 1.0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2$$



l/r	K	J	l/r	K	J	l/r	K	J
1	.435	.753	41	1.036	.137	81	1.904	−.817
2	.450	.736	42	1.053	.121	82	1.932	−.851
3	.464	.719	43	1.070	.104	83	1.961	−.886
4	.479	.702	44	1.087	.0867	84	1.990	−.922
5	.494	.686	45	1.105	.0692	85	2.020	−.958
6	.508	.670	46	1.122	.0516	86	2.050	−.996
7	.523	.654	47	1.140	.0336	87	2.080	−1.034
8	.537	.638	48	1.158	.0154	88	2.111	−1.072
9	.552	.622	49	1.176	−.0031	89	2.142	−1.112
10	.566	.607	50	1.195	−.0219	90	2.174	−1.152
11	.581	.591	51	1.213	−.0411	91	2.206	−1.193
12	.595	.576	52	1.232	−.0605	92	2.239	−1.234
13	.610	.561	53	1.251	−.0803	93	2.272	−1.277
14	.624	.546	54	1.271	−.100	94	2.306	−1.320
15	.639	.531	55	1.290	−.121	95	2.340	−1.364
16	.653	.516	56	1.310	−.142	96	2.375	−1.409
17	.668	.501	57	1.330	−.163	97	2.410	−1.455
18	.682	.486	58	1.351	−.185	98	2.445	−1.501
19	.697	.472	59	1.371	−.207	99	2.482	−1.549
20	.711	.457	60	1.392	−.229	100	2.518	−1.597
21	.726	.442	61	1.413	−.252	101	2.555	−1.646
22	.741	.428	62	1.435	−.275	102	2.593	−1.696
23	.755	.413	63	1.456	−.299	103	2.631	−1.747
24	.770	.398	64	1.478	−.323	104	2.670	−1.799
25	.785	.384	65	1.501	−.348	105	2.709	−1.852
26	.800	.369	66	1.523	−.373	106	2.749	−1.906
27	.815	.354	67	1.546	−.399	107	2.789	−1.960
28	.830	.340	68	1.570	−.425	108	2.830	−2.016
29	.845	.325	69	1.593	−.452	109	2.871	−2.073
30	.860	.310	70	1.617	−.479	110	2.914	−2.130
31	.876	.295	71	1.641	−.507	111	2.956	−2.189
32	.891	.280	72	1.666	−.535	112	2.999	−2.248
33	.907	.265	73	1.691	−.564	113	3.043	−2.309
34	.922	.249	74	1.716	−.593	114	3.087	−2.371
35	.938	.234	75	1.742	−.623	115	3.132	−2.433
36	.954	.218	76	1.768	−.654	116	3.176	−2.497
37	.970	.202	77	1.794	−.685	117	3.224	−2.562
38	.987	.186	78	1.821	−.717	118	3.271	−2.627
39	1.003	.170	79	1.848	−.750	119	3.318	−2.694
40	1.020	.154	80	1.976	−.783	120	3.366	−2.762

Assuming depth of web = .95 d (depth of member), the shear on web section at ultimate load is—

$$V_u = t_w(.95\ d)\ \tau_y$$

$$= t_w(.95\ d)\ \frac{\sigma_y}{\sqrt{3}}$$

or

$$\boxed{V_u \leqq .55\ \sigma_y\ t_w\ d} \qquad \ldots\ldots\ldots\ldots (4)$$

Minimum Width-to-Thickness Ratios (AISC Sec. 2.6)

When subjected to compression involving plastic hinge rotation under ultimate loading, section elements shall be so proportioned that:

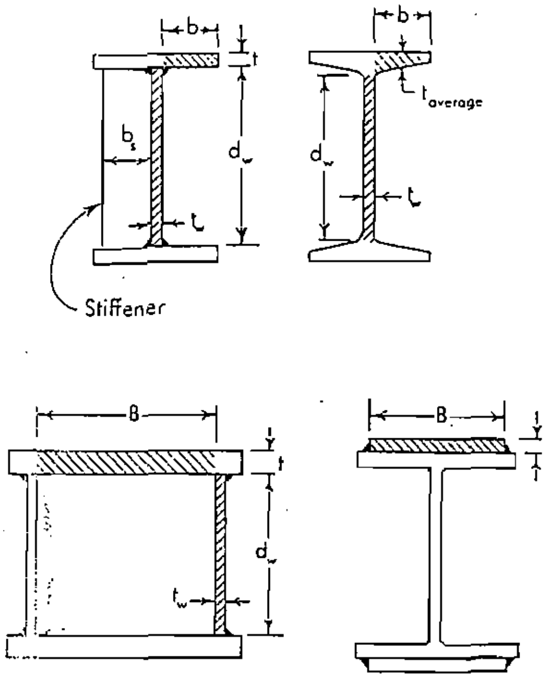


FIGURE 10

$$\boxed{\frac{b}{t} \text{ or } \frac{b_s}{t_s} \leqq 8\tfrac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\boxed{\frac{B}{t} \leqq 32} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\boxed{\begin{array}{l} \dfrac{d}{t_w} \leqq 70 \\ \text{If } P = 0 \end{array}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and when beam or girder is subjected to axial force (P) and plastic bending moment (P_y) at ultimate load,

$$\boxed{\begin{array}{c} \dfrac{d_w}{t_w} \leqq 70 - 100\,\dfrac{P}{P_y} \geqq 43 \\ \text{(AISC formula 25)} \end{array}} \quad \ldots\ldots\ldots\ldots (8)$$

See nomograph, Figure 11, for convenient direct reading of d_w/t_w ratio from values of P and P_y.

Lateral Bracing (AISC Sec. 2.8)

Plastic hinge locations associated with all but the last failure mechanism shall be adequately braced to resist lateral and torsional displacement.

Laterally unsupported distance (L_{cr}) from such braced hinged locations to the nearest adjacent point on the frame similarly braced shall be—

$$\boxed{\begin{array}{c} L_{cr} \leqq \left[\,60 - 40\,\dfrac{M}{M_p}\,\right] r_y \\ \text{(AISC formula 26)} \end{array}} \quad \ldots\ldots\ldots\ldots (9)$$

but need not be less than $\boxed{35\ r_y}$

where:

r_y = radius of gyration of member about its weak axis

M = the lesser of the moments at the ends of the unbraced segment

$\dfrac{M}{M_p}$ = the end moment ratio, positive when the segment is bent in single curvature and negative when bent in double curvature

In the usual square frame, plastic hinges would ultimately form at maximum negative moments at the corners, and at the maximum positive moment near the center of the span. However, a tapered haunch may develop a plastic hinge at the corner and also at the point where the haunch connects to the straight portions of the rafter or column because of the reduced depth of the member. These also become points where lateral bracing must be provided.

3. BASIC REQUIREMENTS OF WELDED CONNECTIONS

Connections are an important part of any steel structure designed according to plastic design concepts. The connection must allow the members to reach their full plastic moments with sufficient strength, adequate rotational ability, and proper stiffness. They must be capable of resisting moments, shear forces, and axial loads to which they would be subjected by the ultimate loading. Stiffeners may be required to preserve the flange continuity of interrupted members at their junction with other members in a continuous frame.

A basic requirement is that the web of the resulting connection must provide adequate resistance against buckling from (a) *Shear*—the diagonal compressive force resulting from shear forces applied to the web from the connecting flanges, which in turn are stressed by the end moment of the member, and (b) *Thrust*—any concentrated compressive force applied at the edge of the web from an intersecting flange of a member, this force resulting from the end moment of that member. See Figure 12.

In addition to meeting the above requirements, the connection should be so designed that it may be economically fabricated and welded.

Groove welds and fillet welds shall be proportioned

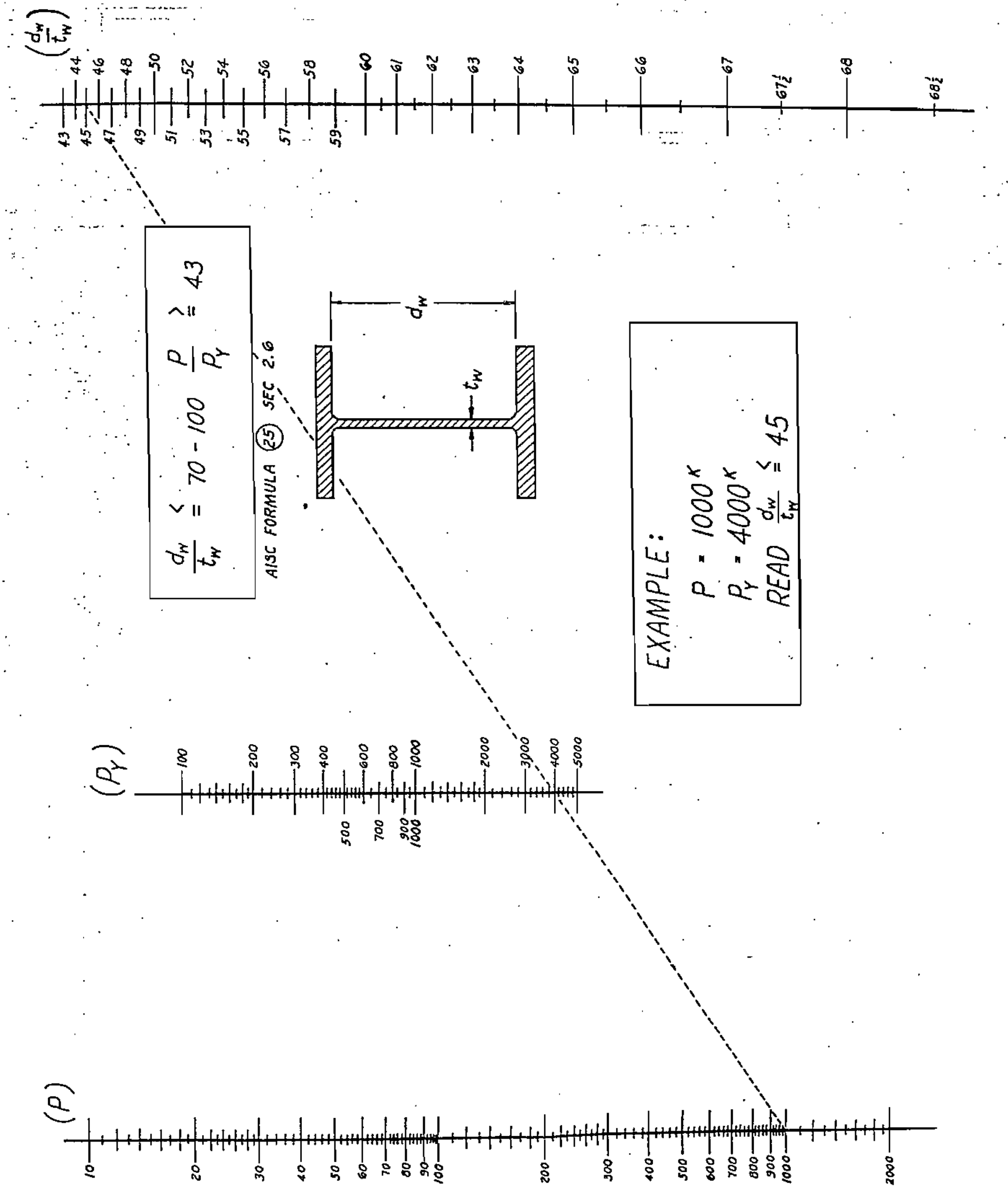


FIGURE 11—Proportioning Web of Beam or Girder Under Axial Load and Plastic Moment.

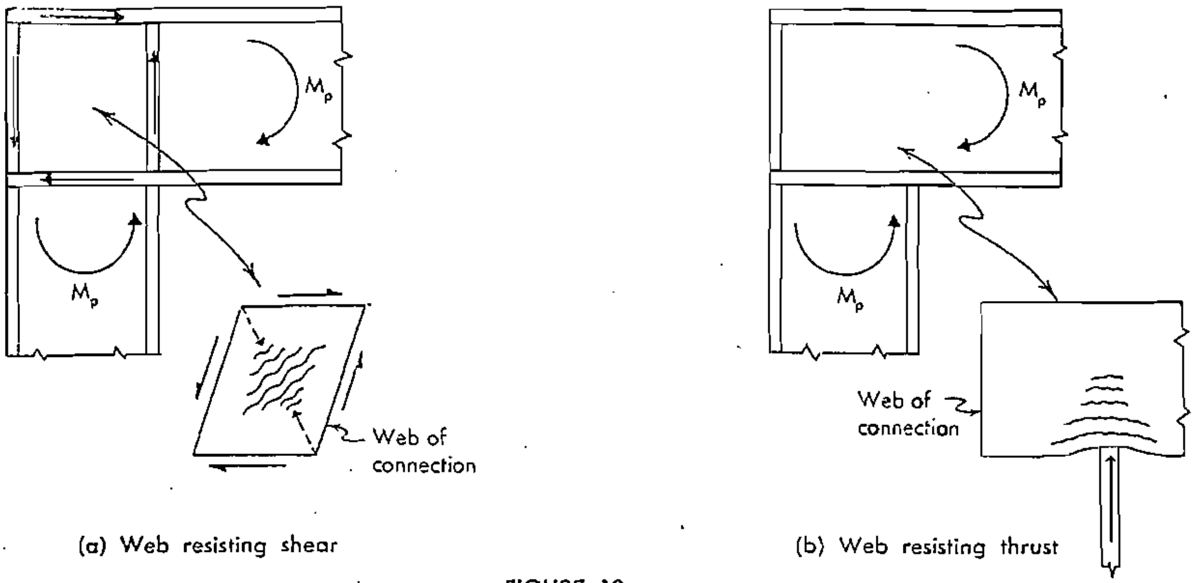


FIGURE 12

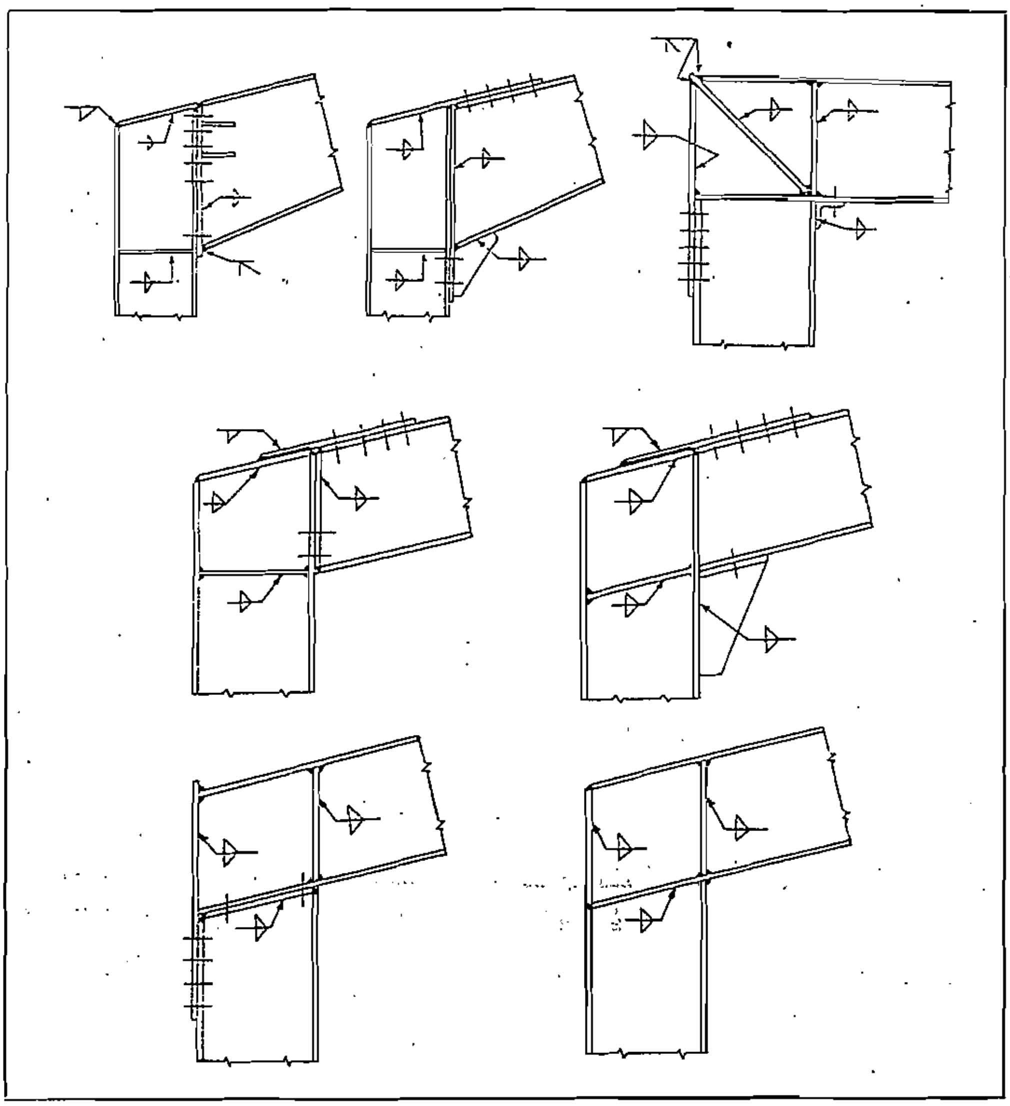


FIGURE 13

to resist the forces produced at ultimate load, using an increase of 1.67 over the standard allowables (AISC Sec. 2.7).

Following pages cover first the design of simple two-way rectangular corner connections, tapered haunches, and curved haunches. Next, the design of beam-to-column connections, whether three-way or four-way, is dealt with.

Analysis and design of a particular connection may not always be as simple as those illustrated on these pages. Figure 13 shows some other typical welded connections.

4. STRAIGHT CORNER CONNECTIONS

Web Resisting Shear

The forces in the flanges of both members at the connection resulting from the moment (M_p) are transferred into the connection web as shear (V).

Some of the vertical shear in the beam (V_b) and the horizontal shear in the column (V_c) will also be transferred into the connection web. However, in most cases these values are small compared to those resulting from the applied moment. Also, in a simple corner connection, these are of opposite sign and tend to reduce the actual shear value in the connection.

In this analysis, only the shear resulting from the applied moment is considered in the web of the connection.

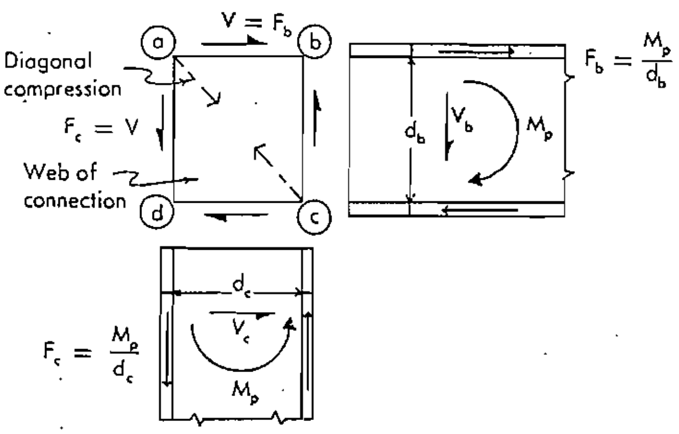


FIGURE 14

The minimum web thickness required to assure that the web of the connection does not buckle from the shear forces set up by the moment applied to the connection (M_p), may be found from the following:

unit shear force applied to connection web

$$\nu = \frac{V}{d} = \frac{F_b}{d_c}\left(\text{also} = \frac{F_c}{d_b}\right) = \frac{M_p}{d_b\ d_c}$$

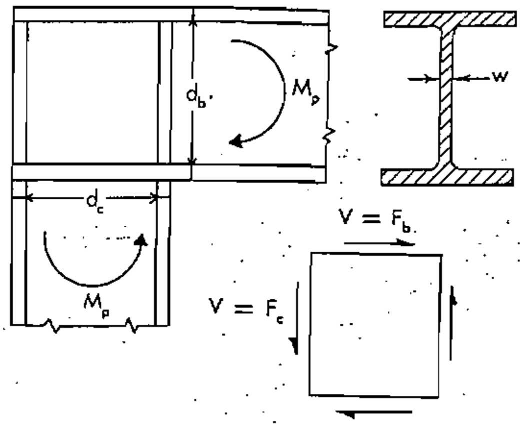


FIGURE 15

resulting shear stress in connection web

$$\tau = \frac{\nu}{w} = \frac{M_p}{w\ d_b\ d_c}$$

The values for the shear stress at yield (τ_y) may be found by using the Mises criterion for yielding—

$$\sigma_{cr} = \sqrt{\sigma_x^2 - \sigma_x\ \sigma_y + \sigma_y^2 + 3\ \tau_{xy}^2}$$

In this application of pure shear, σ_x and $\sigma_y = 0$ and setting the critical value (σ_{cr}) equal to yield (σ_y), we obtain—

$$\sigma_y = \sqrt{3\ \tau_{xy}^2} \qquad \text{or}$$

$$\tau_{xy} = \frac{\sigma_y}{\sqrt{3}}$$

Hence,

$$\tau = \frac{M_p}{w\ d_b\ d_c} = \frac{\sigma_y}{\sqrt{3}}$$

or

$$\boxed{w_r \geqq \frac{\sqrt{3}\ M_p}{d_b\ d_c\ \sigma_y}} \qquad \text{(10)}$$

The nomograph, Figure 16, will facilitate finding this required web thickness.

In the above:

M_p = plastic moment at connection, in.-lbs

d_b = depth of beam, in.

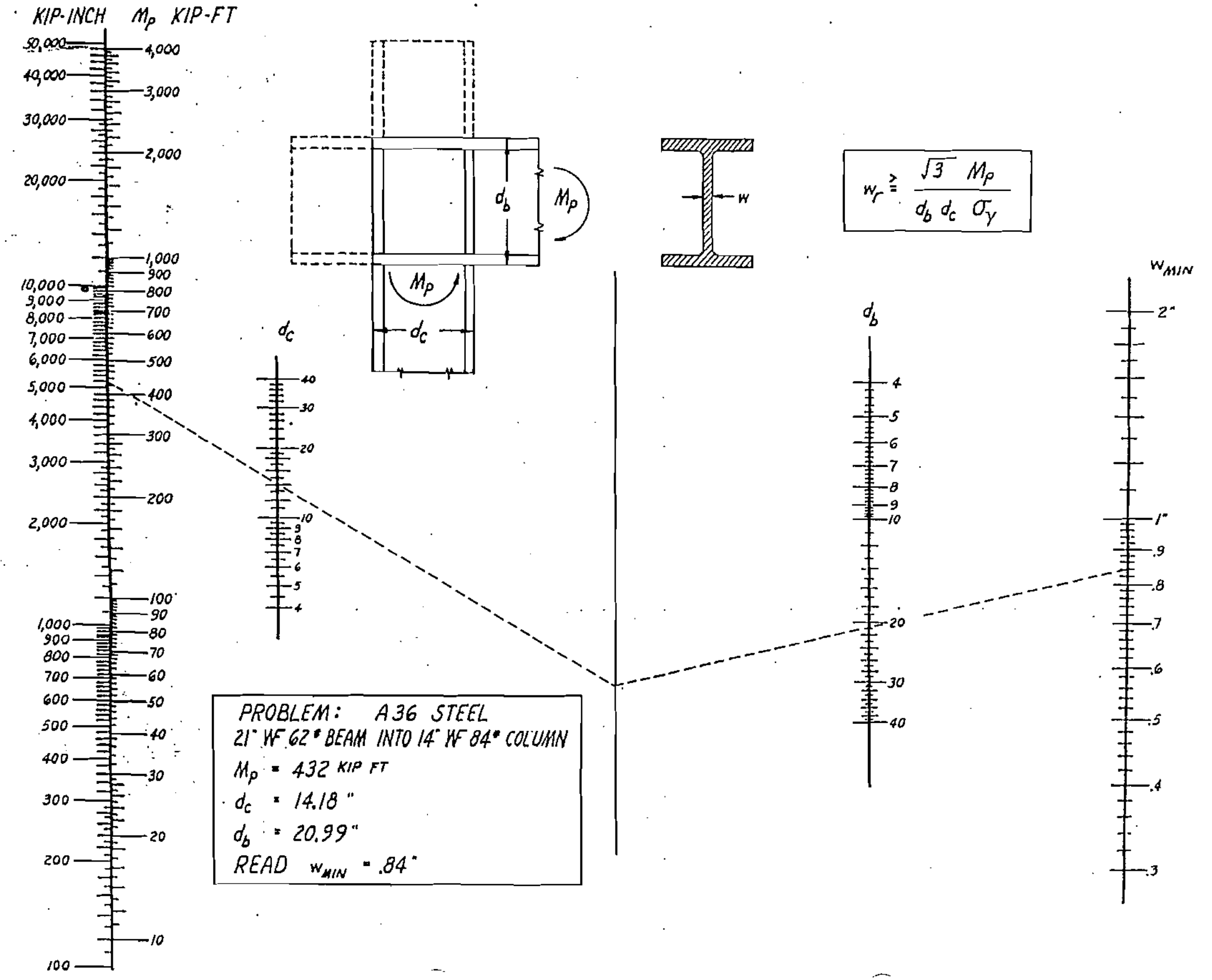
d_c = depth of column, in.-lbs

w = actual web thickness in connection area, in.

w_r = required web thickness in connection area, in.

σ_y = yield strength of steel, psi

FIGURE 16—Thickness of Connection Web to Resist Shear From Unbalanced Plastic Mo...



AISC uses an effective depth of the beam and column as 95% of their actual depths to allow for the presence of plastic strain in the flanges, due to concurrent bending. Applying this reduction to both the depth of the beam (d_b) and the column (d_c), and also expressing the applied plastic moment (M_p) in ft-lbs rather than in.-lbs, this formula becomes:

$$\boxed{w_r = \frac{23{,}000\ M_p}{d_b\ d_c\ \sigma_y}} \quad \ldots\ldots(11)$$

Here M_p = plastic moment, *ft.-lbs*

For most wide flange (WF) sections, the web thickness (w) will be less than the required value (w_r) above, and some form of stiffening will be required.

Web Doubler Plate

A web doubler plate, or a pair, may be used to bring the total web thickness up to the minimum (w_r) obtained above.

Welds should be arranged at the edges of doubler plates so as to transfer the shear forces directly to the boundary stiffeners and flanges.

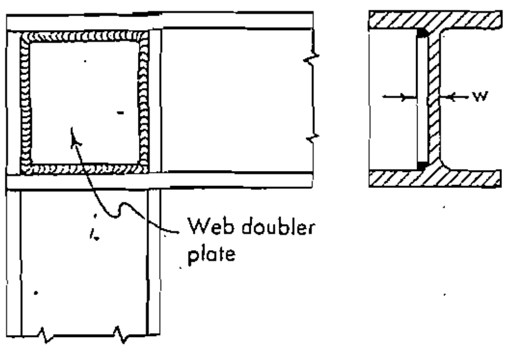


FIGURE 17

Diagonal Stiffeners

A symmetrical pair of diagonal stiffeners may be added to this connection to prevent the web from buckling. These stiffeners resist enough of the flange force (F) that the resulting shear (V) applied to this web is reduced sufficiently to prevent buckling.

Stiffeners having a thickness equal to that of the rolled section flange of the beam or column normally will be adequate, although this thickness will be greater than required. The minimum thickness of this stiffener may be found from the following:

The horizontal flange force (F_b) of the beam is resisted by the combined effect of the web shear (V) and the horizontal component of the compressive force (P) in the stiffener.

$$F = V + P \cos \theta$$

where

$$V = w\ d_c\ \tau_y = w\ d_c\ \frac{\sigma_y}{\sqrt{3}}$$

and since

$$F = \frac{M_p}{d_b}$$

$$\therefore \frac{M_p}{d_b} = w\ d_c\ \frac{\sigma_y}{\sqrt{3}} + P \cos \theta \qquad \text{or}$$

$$P = \frac{1}{\cos \theta}\left[\frac{M_p}{d_b} = \frac{w\ d_c\ \sigma_y}{\sqrt{3}}\right]$$

$$\therefore \boxed{A_s = \frac{1}{\cos \theta}\left[\frac{M_p}{d_b\ \sigma_y} - \frac{w\ d_c}{\sqrt{3}}\right]} \quad \ldots(12)$$

where

θ = angle of diagonal stiffener with horizon,

$$\theta = \tan^{-1}\left(\frac{d_b}{d_c}\right)$$

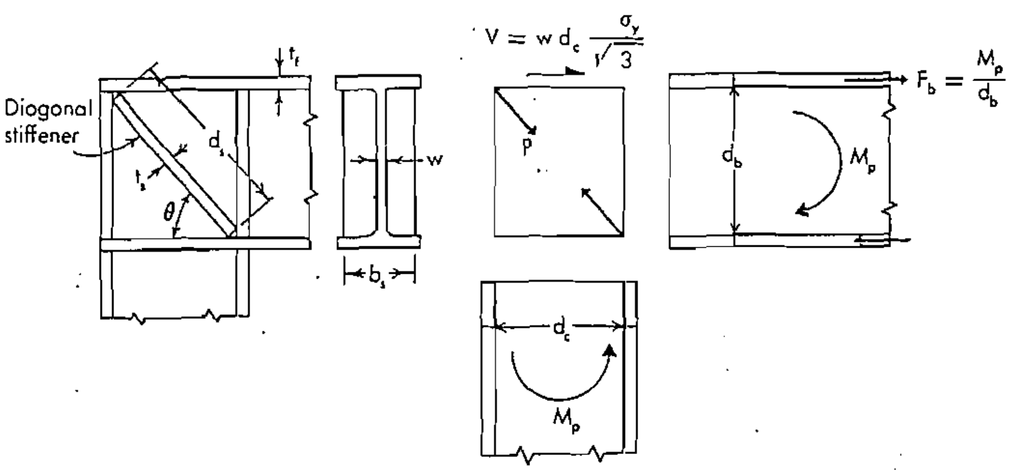


FIGURE 18

A_s = area of a pair of diagonal stiffeners,

$$A_s = b_s\ t_s$$

In the usual detailing of the connection, the required web thickness (w_r) is first found. The actual web thickness (w) of course is known, therefore it would be simpler to change this formula into the following so that the required area of the diagonal stiffener may be found from these two values. (w_r) and (w):

From Formula 10,

$$w_r = \frac{\sqrt{3}\ M_p}{d_b\ \ d_c\ \sigma_y} \qquad \text{or}$$

$$M_p = \frac{w_r\ d_b\ d_c\ \sigma_y}{\sqrt{3}}$$

and substituting this into Formula 12,

$$A_s = \frac{1}{\cos\theta}\left[\frac{M_p}{d_b\ \sigma_y} - \frac{w\ d_c}{\sqrt{3}}\right]$$

and since

$$\cos\theta = \frac{d_c}{d_s}$$

$$\therefore \boxed{A_s = \frac{d_c\ (w_r - w)}{\sqrt{3}\ \cos\theta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

or

$$\boxed{A_s = \frac{d_s\ (w_r - w)}{\sqrt{3}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

or could use

$$t_s = t_f$$

also in all cases

$$\boxed{\frac{b_s}{t_s} \leqq 17} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

For full strength, stiffeners should be welded across their ends with either fillet welds or groove welds, and to the connection web with continuous fillet welds.

Problem 1

To design a 90° connection for a 21″ WF 62# roof girder to a 14″ WF 84# column. Use A36 steel and E70 welds. Load from girder: M_p ultimate plastic moment = 432 ft-kips.

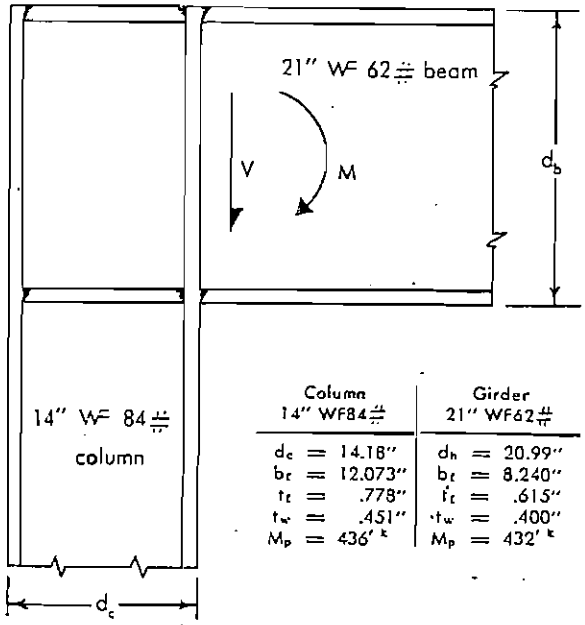


FIGURE 19

The required web reinforcement is determined as follows:

$$w_r \geqq \frac{\sqrt{3}\ M_p}{d_b\ d_c\ \sigma_y}$$

$$\geqq \frac{\sqrt{3}\ (432 \text{ ft-kips} \times 12)}{(20.99'')(14.18'')(36 \text{ ksi})} \qquad \geqq 0.837''$$

web furnished by the 14″ WF 84# column = 0.451″

effective web to be furnished by stiffeners ≧ 0.386″

This reinforcement may be provided by one of two possible types of stiffeners as noted below.

(a) Web Doubler Plate

The additional web plate must be sufficient to develop the required web thickness. The welds should be arranged at the edges so as to transmit the shear forces directly to the boundary stiffeners and flanges. Plate must be .386″ thick, or use a 7/16″ thick plate.

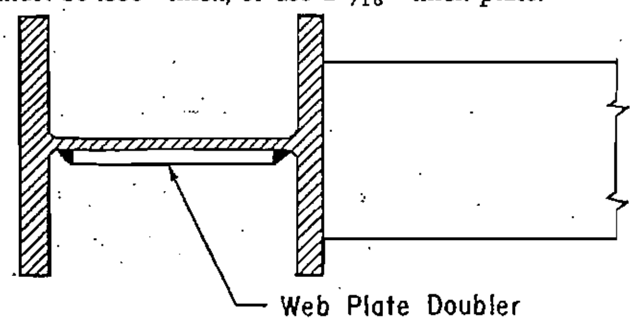


FIGURE 20

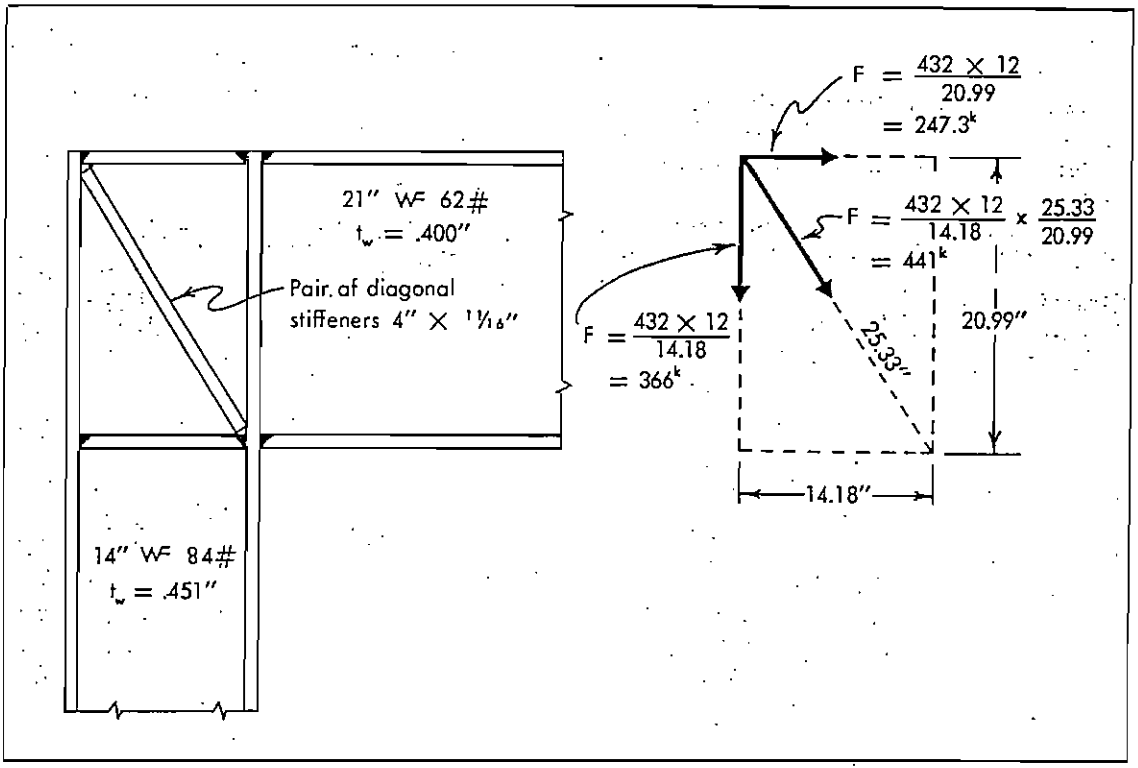


FIGURE 21

(b) Diagonal Stiffener

The diagonal stiffener will resist the diagonal component of the flange load as a compression strut. The flange force to be carried by the stiffener is the portion that exceeds the amount carried by the web. Assuming the bending moment to be carried entirely by the flanges, the compressive force in the diagonal stiffener is computed as in Figure 21.

Multiply this diagonal compressive force of 441 kips by the ratio of the additional thickness needed to that already in the web:

$$441\left(\frac{.386}{.837}\right) = 204 \text{ kips force on diagonal stiffener}$$

or

$$A_s = \frac{P}{\sigma_y}$$

$$= \frac{204 \text{ kips}}{36 \text{ ksi}}$$

$= 5.65$ *in.*^{2} needed in the stiffener

or use a pair of ¾" x 4" stiffeners, $A_s = 6.0 > 5.65$ OK

Now solve this portion of the problem by using Formula 3:

$$A_s = \frac{d_c}{\sqrt{3}\cos\theta}\ (w_r - w)$$

where:

$$\theta = \tan^{-1}\frac{d_b}{d_c}$$

$$= \tan^{-1}\frac{(20.99)}{(14.18)}$$

$$= \tan^{-1} 1.48 \qquad \text{or}$$

$$\theta = 55.93^\circ$$

and

$$\cos 55.93^\circ = .560$$

$$\therefore A_s = \frac{14.18}{\sqrt{3}\,(.560)}\ (.837 - .451)$$

$= 5.65$ *in.*^{2} needed in the stiffener

If $b_s = 8"$, then

$$t_s = \frac{A_s}{b_s}$$

$$= \frac{5.65}{8}$$

$= .707"$ or *use* ¾"

Or use two plates, ¾" x 4", for the diagonal stiffeners. Check their width-to-thickness ratio:

$$\frac{b_s}{t_s} = \frac{8}{3/4} = 10.7 < 17 \qquad \underline{\text{OK}}$$

Weld for Stiffener

Only nominal fillet welding is required between stiffener and connection web to resist buckling. These welds are used simply to hold the stiffeners in position. Welding at terminations of the stiffener should be sufficient to transfer forces.

To develop the full capacity of the stiffener, it may be butt welded to the corners, or full-strength fillet welds may be used.

The required leg size of fillet weld to match the ultimate capacity of the stiffener would be—

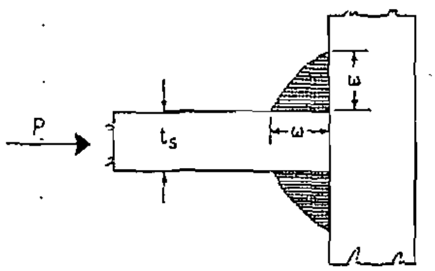


FIGURE 22

E60 Welds & A7, A373 Plate

$$2(9600\ \omega)1.67 = t_s\ 33{,}000$$

$$\omega = 1.03\ t_s$$

$$\boxed{\omega = t_s} \quad \dots\dots\dots\dots (16)$$

E70 Welds & A36 Plate

$$2(11{,}200\ \omega)1.67 = t_s\ 36{,}000$$

$$\omega = .96\ t_s$$

$$\boxed{\omega = t_s} \quad \dots\dots\dots\dots (16)$$

Hence, use ¾" leg fillet welds across the ends of the stiffener.

It may be simpler to make the cross-sectional area of these diagonal stiffeners equal to that of the flange of the member whose web they reinforce.

5. HAUNCHED CONNECTIONS

Haunched connections, Figure 23, are sometimes used in order to more nearly match the moment requirements of a frame. This produces a deeepr section in the region of maximum moment, extending back until the moment is reduced to a value which the rolled section is capable of carrying. In this manner a smaller rolled section may be used for the remainder of the frame. This has been a rather standard practice in the conventional elastic rigid frame.

Haunched knees may exhibit poor rotational ability if the knee buckles laterally before the desired design conditions have been reached.

The haunch connection should be proportioned with sufficient strength and buckling resistance so that a plastic hinge may be formed at the end of the haunch where it joins the rolled member.

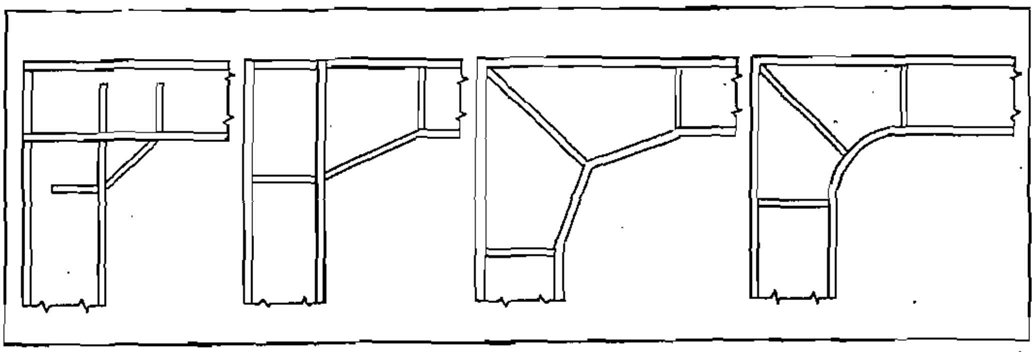
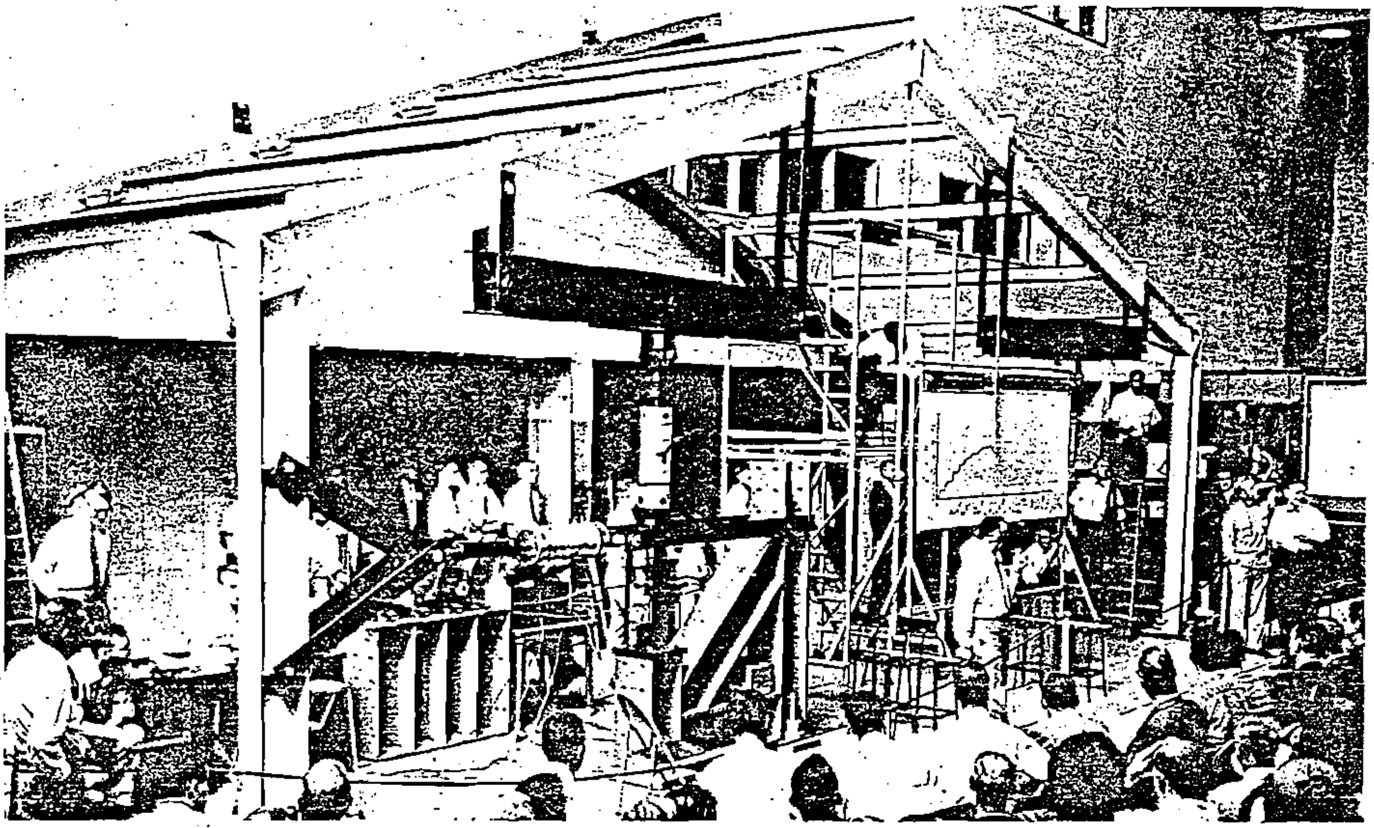
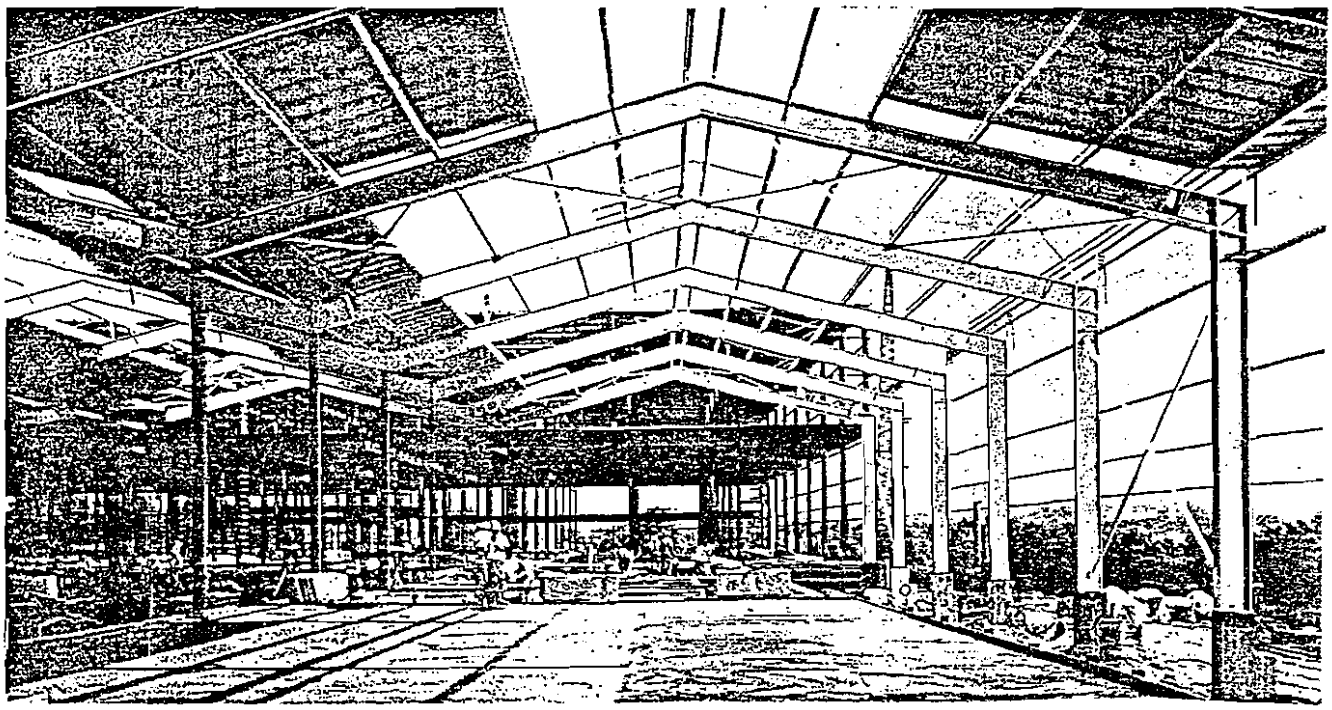


FIGURE 23



Lehigh University's extensive research in plastic design included the testing to destruction of full-scale structures such as this 40' gabled frame.



Plastic design of this 8-acre rubber plant simplified mathematical analysis of the structure and moment distribution. Two results: a uniform factor of safety and a saving of 140 tons of structural steel.

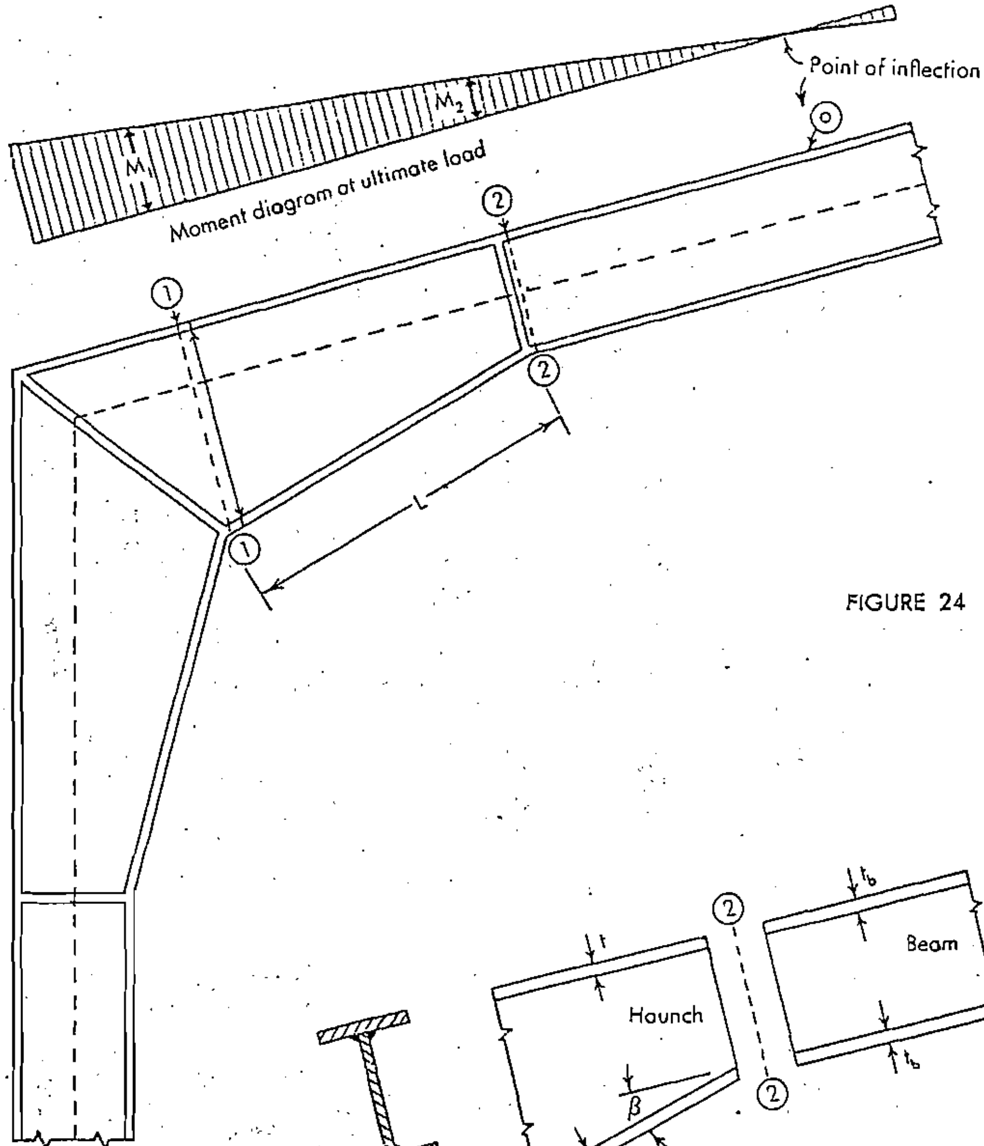


FIGURE 24

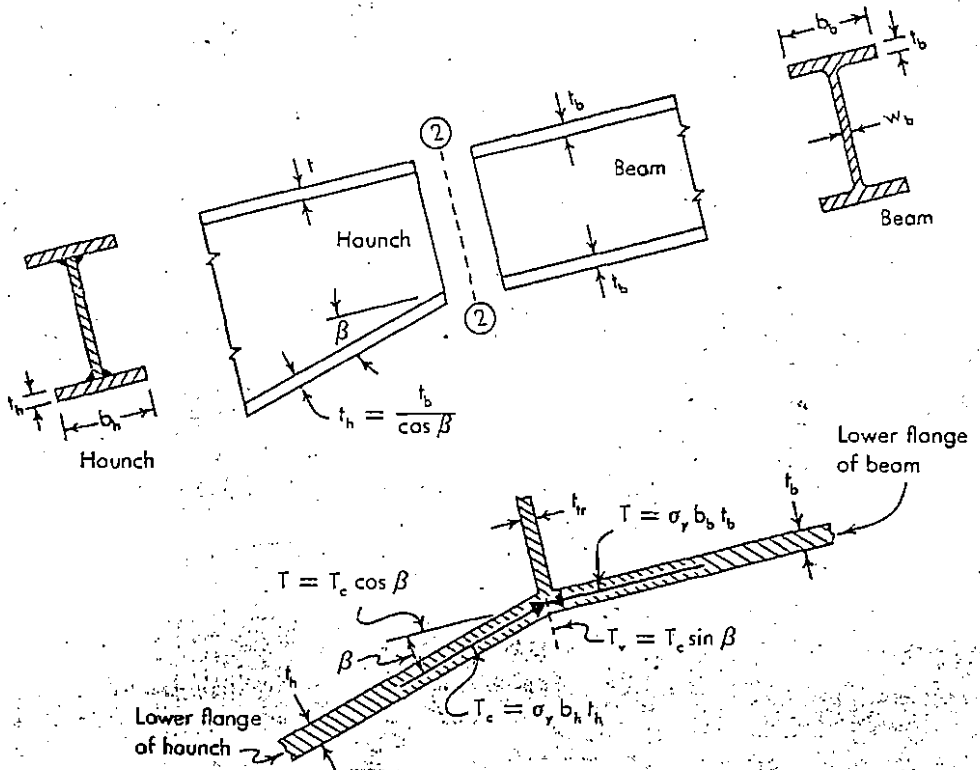


FIGURE 25

A. TAPERED HAUNCH CONNECTIONS

(See Figures 24 and 25, facing page)

Thickness of Top Flange and Web of Haunch

The thickness of the top flange and the web of the haunch should be at least equal to the thickness of the rolled beam to which it connects.

Thickness of Lower Flange of Haunch

The lower flange of the haunch must be increased in thickness so that when it is stressed to the yield point (σ_y), its horizontal component will be equal to the force in the lower beam flange stressed to yield.

The force in the sloping lower flange of the haunch at the plastic moment (M_p) is—

$$T_c = \sigma_y\ b_h\ t_h$$

The component of this force (T_c) in line with and against the force in the beam flange is—

$$T = T_c \cos \beta$$
$$= \sigma_y\ b_h\ t_h \cos \beta$$

and this must match the force (T) in the lower flange of the rolled beam, or:

$$T = \sigma_y\ b_h\ t_h \cos \beta \text{ must equal } T = \sigma_y\ b_b\ t_b$$

Assuming the same flange width for the haunch as the beam, i.e. $b_h = b_b$, gives—

$$\boxed{t_h = \frac{t_b}{\cos \beta}} \qquad (17)$$

Transverse Stiffeners

$$T_r = T_c \sin \beta$$

or $\quad \sigma_y\ b_{tr}\ t_{tr} = \sigma_y\ b_h\ t_h \sin \beta$

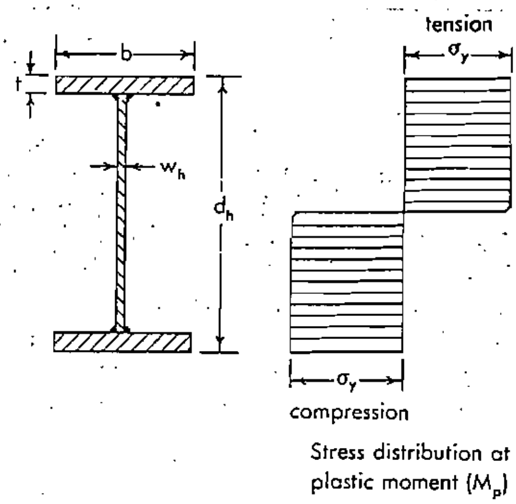
Assuming the same flange width for the stiffener as the beam, i.e. $b_{tr} = b_b$, gives—

$$\boxed{t_{tr} = t_h \sin \beta} \qquad (18)$$

AISC suggests making the total area of these stiffeners not less than ¾ of the haunch flange area (AISC Commentary p 37, item 4).

Required Haunch Section

Section (1-1), in the region of high moment, should be checked. The two flanges may vary in thickness, so for simplicity and a conservative value use the upper flange's thickness. Since this is the tension flange, it will be same or thinner than the lower (compression) flange. It can be shown that the plastic section modulus (Z) of an I section is:



Stress distribution at plastic moment (M_p)

FIGURE 26

resisting plastic moment of section

$$M_p = 2\ b\ t\ \sigma_y \left(\frac{d_h - t}{2}\right) + 2\ w_h \left(\frac{d_h - 2\ t}{2}\right)\left(\frac{d_h - 2\ t}{4}\right)$$

since

$$\boxed{Z = \frac{M_p}{\sigma_y}} \qquad (19)$$

$$\boxed{Z = b\ t\ (d_h - t) + \frac{w_h}{4}\ (d_h - 2\ t)^2} \qquad (20)$$

This increased plastic section modulus may be obtained by:

1. Increasing the depth (d_h) and holding the flange area constant, or
2. Increasing the flange thickness (t) and holding the depth (d_h) constant.

By assuming that $(d_h - t)$ is equal to $(d_h - 2\ t)$, and solving for the expression $(d_h - 2\ t)$, it is found from the above formula that:

$$\boxed{d_h = 2\sqrt{\frac{b^2\ t^2}{w_h^2} + \frac{Z}{w_h}} + 2\ t\left(1 - \frac{b}{w_h}\right)} \qquad (21)$$

From this, the required depth (d_h) of the haunch may be found for any value of plastic section modulus (Z).

The haunch section must be able to develop the plastic moment at any point along its length:

$$\boxed{M_p = Z\ \sigma_y} \quad \text{(22)}$$

or at any section (x-x)—

$$\boxed{\sigma_x = \frac{M_p}{Z} \leqq \sigma_y} \quad \text{(23)}$$

Usually just the two ends of the haunch must be checked. This would be section (1-1) at the haunch point (H), and section (2-2) at the connection to the rolled beam. The latter finding will also dictate the required section modulus of the straight beam, since its highest moment will occur at section (2-2).

Beedle* points out that if the moment is assumed to increase linerally from the point of inflection (O) to the haunch point (H), and the distance (O-R) from the point of inflection to the end of the rolled beam is 3 d, then the critical section will always be along (2-2) if the angle β of the taper is greater than 12°; if this angle is less than 12°, then section (1-1) must *also* be checked.

Lateral Stability

Bracing should be placed at the extremities and the common intersecting points of the compression flange.

* "Plastic Design of Steel Frames" Lynn S. Beedle: John S. Wiley & Sons, publishers.

The commentary of the AISC specifications sets the following limits for lateral bracing.

The taper of the haunch may be such that the resulting bending stress at plastic loading, when computed by using the plastic modulus (Z), is approximately at yield (σ_y) at both ends ① & ②. If this is the case, then limit the unbraced length (L_h):

$$\boxed{L_h \leqq 6\ b_h} \quad \text{(24)}$$

or as an alternate, increase the thickness of the haunch flanges by the factor:

$$\boxed{t = t_h \left[1 + 0.1 \left(\frac{L_h}{b_h} - 6 \right) \right]} \quad \text{(25)}$$

If the bending stress at one end is approximately at yield (σ_y), using the plastic modulus (Z), and at the other end is less than yield ($\sigma_x < \sigma_y$) when using the secton modulus (S), limit the unbraced length (L_h):

$$\boxed{L_h \leqq (17.5 - 0.40\ \sigma_x)\ b_h} \quad \text{(26)}$$

but

$$\boxed{L_h \geqq 6\ b_h}$$

If the bending stress computed on the basis of section modulus (S) is less than yield ($\sigma_x < \sigma_y$) at all transverse sections of the haunch from ① to ②, then check to see that greatest computed stress:

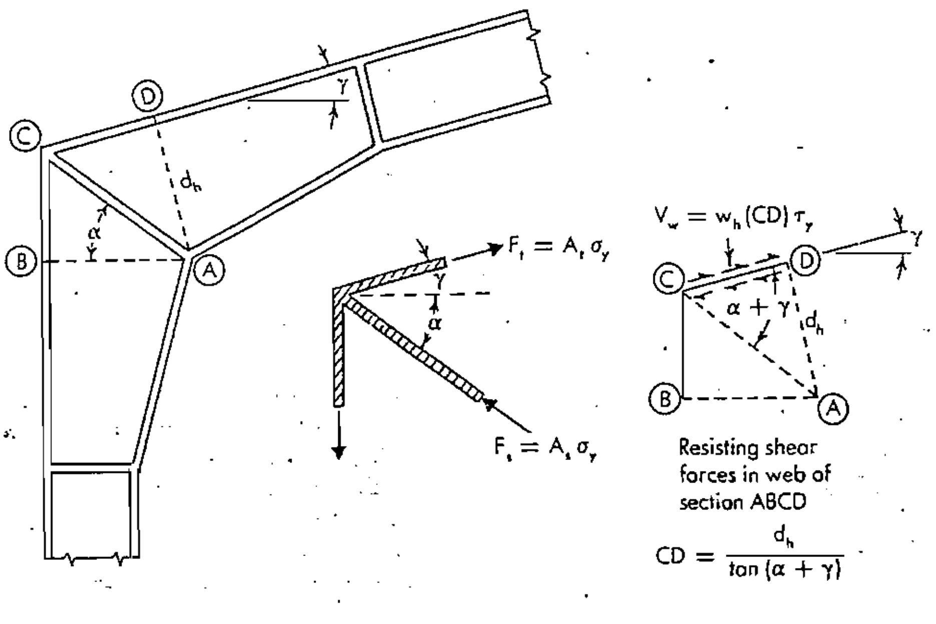


FIGURE 27

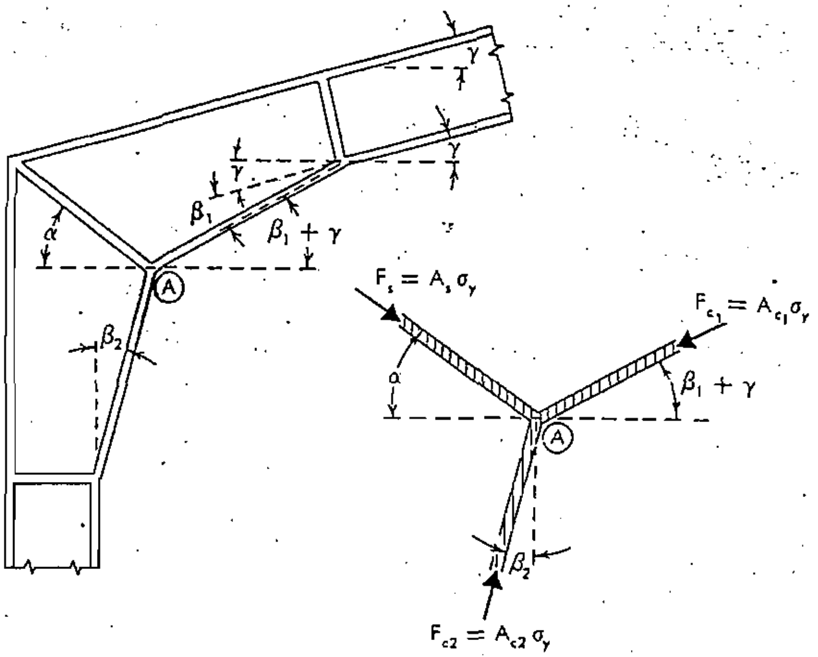


FIGURE 28

$$\boxed{\sigma_y \leqq \frac{(12 \times 10^6)1.67}{\frac{L_h\, d_{max}}{A_c}}} \quad \text{.....(27)}$$

Diagonal Stiffeners

The tapered haunch has an extra-large web area in the bend of the knee. This is subject to buckling, and should be strengthened by diagonal stiffeners. The required stiffener section area should be figured from the compressive force on the web diagonal resulting from the larger of two forces: (a) the tensile forces on the outer flange of the haunch at point (C), and (b) the compressive forces on the inner flange of the haunch at point (A)

(1) Based on tensile forces at (C)

The compressive force in the diagonal stiffener is found by taking the sum of the horizontal components of the forces in the outer flanges and setting them equal to zero. See Figure 27.

$$+ A_t\, \sigma_y \cos\gamma - \left(\frac{w_h\, d_h}{\tan(\alpha + \gamma)}\right)\left(\frac{\sigma_y}{\sqrt{3}}\right) \cos\gamma - A_s\, \sigma_y \cos\alpha = 0$$

or

$$A_s = A_t\left(\frac{\cos\gamma}{\cos\alpha}\right) - \left(\frac{w_h\, d_h \cos\gamma}{\sqrt{3}\tan(\alpha + \gamma)\cos\alpha}\right)$$

$$\boxed{A_s = \frac{\cos\gamma}{\cos\alpha}\left[A_t - \frac{w_h\, d_h}{\sqrt{3}\tan(\alpha + \gamma)}\right]} \quad .(28)$$

where:

A_t = area of top (tension) flange of haunch

A_s = total area of a pair of diagonal stiffeners

(2) Based on compressive forces at (A)

The compressive force in the diagonal stiffener is found in a similar manner as before; the horizontal components of the forces in the inner flanges are set in equilibrium. See Figure 28.

$$+ A_s\, \sigma_y \cos\alpha + A_{c2}\, \sigma_y \sin\beta_2 - A_{c1}\, \sigma_y \cos(\beta_1 + \gamma) = 0$$

or

$$\boxed{A_s = \frac{A_{c1}\cos(\beta_1 + \gamma) - A_{c2}\sin\beta_2}{\cos\alpha}} \quad \text{....(29)}$$

If $A_c = A_{c1} = A_{c2}$, this becomes—

$$\boxed{A_s = \frac{A_c}{\cos\alpha}\left[\cos(\beta_1 + \gamma) - \sin\beta_2\right]} \quad .(30)$$

(3) When outer (tensile) flanges form right angle

If the beam and column are at right angles to each other, $\gamma = 0$. See Figure 29.

and $\beta = \beta_1 = \beta_2$

$\alpha = 45°$

$A_c = A_{c1} = A_{c2}$

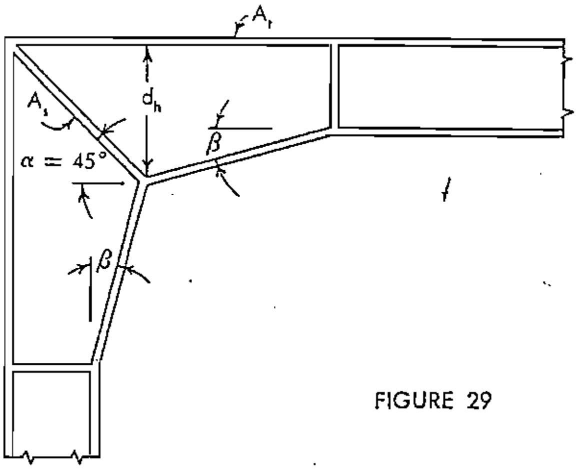


FIGURE 29

Then the preceding two formulas reduce to the following:

based on tensile forces in outer flanges and shear resistance of web

$$\boxed{A_s \geqq \sqrt{2}\, A_t - 0.82\, w_h\, d_h} \quad \ldots\ldots\ldots\ldots (31)$$

based on compressive forces in inner flange

$$\boxed{A_s \geqq \sqrt{2}\, A_c\, (\cos \beta - \sin \beta)} \quad \ldots\ldots\ldots (32)$$

also

$$\boxed{\frac{b_s}{t_s} \leqq 17} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

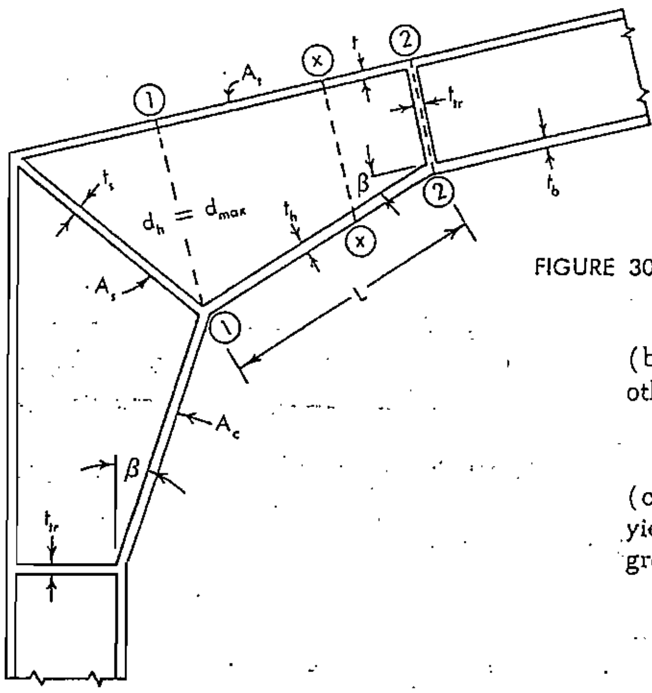


FIGURE 30

The modified formulas above may also be used for convenience in finding the stiffener requirement of gable frames, but will provide a more conservative value.

Summary of Tapered Haunch Requirements

$$w_h \geqq w_{beam}$$

$$t_h \geqq \frac{t_b}{\cos \beta}$$

Based on load from tension flange—

$$A_s \geqq \sqrt{2}\, A_t - 0.82\, w_h d_h$$

Based on load from compression flange—

$$A_s \geqq \sqrt{2}\; A_c\, (\cos \beta - \sin \beta)$$

also $\frac{b_s}{t_s} \leqq 17$

$$t_{tr} \geqq t_h \sin \beta = \frac{b_h}{17}$$

$$t_{tr}\, b_{tr} \geqq \tfrac{3}{4}\, t_b b_b$$

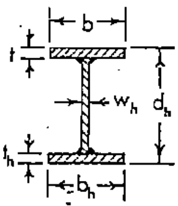
$$Z_h = b\, t\, (d_h - t) + \frac{w_h}{4}\, (d_h - 2\, t)^2 \geqq \frac{M_p}{\sigma_y}$$

Check lateral stability of compression flange

(a) if both ends of haunch ① or ② are stressed to yield (σ_y) using Z

$$L_h \leqq 6\, b_h$$

or increase $t = t_h \left[1 + 0.1 \left(\frac{L_h}{b_h} - 6 \right) \right]$



Section 1-1

(b) if one end is stressed to yield (σ_y) using Z, and other end is stressed below yield ($\sigma_x < \sigma_y$) using S

$$L_h \leqq (17.5 - 0.40\, \sigma_x)\, b_h \geqq 6\, b_h$$

(c) if entire haunch from ① to ② is stressed below yield ($\sigma_x < \sigma_y$) using S. Here, check to see that greatest commuted stress:

$$\sigma_x \leqq \frac{(12 \times 10^6) 1.67}{\dfrac{L_h\, d_{max}}{A_c}}$$

B. CURVED HAUNCH CONNECTIONS

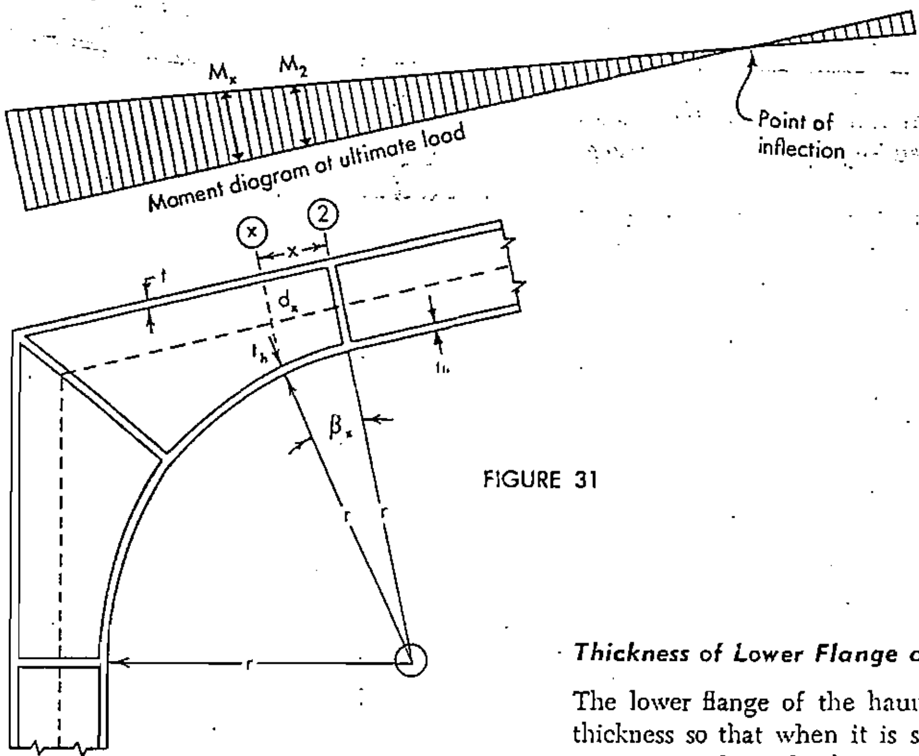


FIGURE 31

Here:

β = angle between tangents of given section and beam flange

r = radius of curvature of inner flange

d_x = depth of curved haunch at any section (x-x)

$= d_2 + r(1 - \cos \beta_x)$

$x = r \sin \beta_x$

It is seen in Figure 31 that the moment resulting from ultimate loading gradually increases out to the corner of the haunch. However, the depth of the haunch and therefore its bending stress also increases toward the corner, so that the critical section (x-x) within the haunch will occur at some distance (x) or some angle (β_x) from section 2-2. For most curved haunches, this angle (β_x) will be about 12°.

Thickness of Top Flange and Web of Haunch

The thickness of the top flange and of the web of the haunch should be at least equal to these features of the rolled beam to which it connects. If bending stress at ②, $\sigma_2 = \dfrac{M_2}{S} < \sigma_y$, then the outer flange thickness of the haunch (t) does not have to exceed the beam flange thickness (t_b) (AISC Commentary).

Thickness of Lower Flange of Haunch

The lower flange of the haunch must be increased in thickness so that when it is stressed to yield (σ_y), its component along the beam axis is equal to the force in the lower beam flange when stressed to yield.

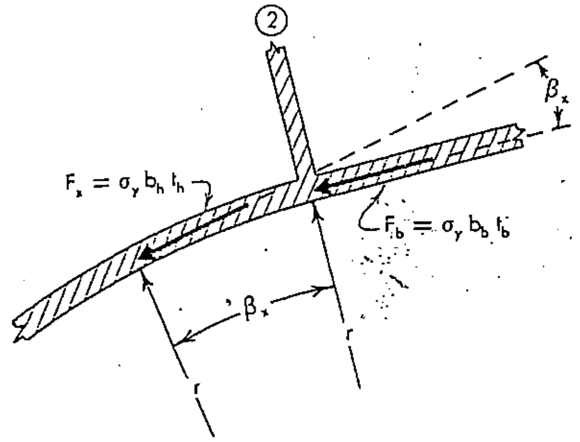


FIGURE 32

$$F_x = \frac{F_b}{\cos \beta_x}$$

$$\sigma_y\ b_h\ t_h = \frac{\sigma_y\ b_b\ t_b}{\cos \beta_x}$$

or

$$\boxed{t_h \geq \frac{t_b}{\cos \beta_x}} \quad \text{.......................... (34)}$$

As in the tapered haunch, the plastic section modulus (Z) at any given point (X) is:

$$\boxed{Z_x = b_h\, t_h\, (d_x - t_h) + \frac{w_h}{4}(d_x - 2\, t_h)^2} \quad \ldots (35)$$

For any given depth (d_x), the plastic section modulus (Z_x) may be increased by increasing the flange thickness (t_h).

Assuming the web thickness and flange width of the curved haunch is at least equal to that of the beam, the required thickness of the lower flange would be:

$$Z_x = b_h\, t_h\, (d_x - t_h) + \frac{w_h}{4}(d_x - 2\, t_h)^2$$

$$Z_x = b_h d_x t_h - b_h t_h^2 + \frac{w_h d_x^2}{4} - w_h d_x t_h + w_h t_h^2$$

$$t_h^2\,(b_h - w_h) - t_h\, d_x\,(b_h - w_h) - \frac{w_h\, d_x^2}{d} + Z_x = 0$$

$$\boxed{t_h = \frac{d_x}{2} - \sqrt{\frac{\frac{d_x^2\, b_h}{4} - Z_x}{b_h - w_h}}} \quad \ldots\ldots\ldots\ldots (36)$$

The AISC Commentary (Sec. 2.7) recommends that the thickness of this inner flange of the curved haunch should be—

$$\boxed{t_h \geqq (1 + m)\, t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

where values for (m) come from the graph, Figure 33.

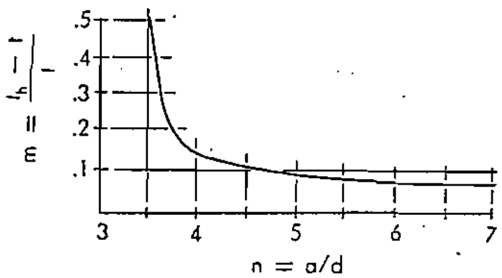


FIGURE 33

Here:

a = distance from point of inflection (M = 0) of the column to the point of plastic moment (M_p) in the haunch

d = depth of column section

In order to prevent local buckling of the curved inner flange, limit the radius of curvature to—

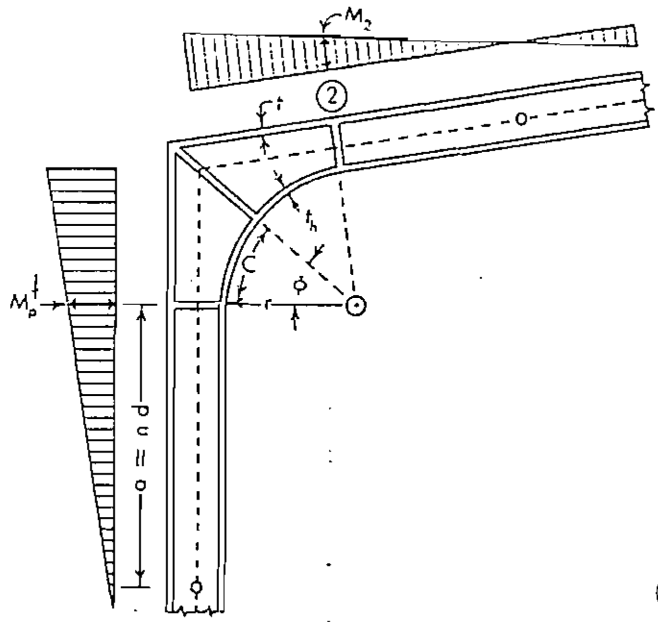


FIGURE 34

$$\boxed{r \leqq 6\, b_h}^{*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

This is based on a 90° knee (outer flanges form a right angle), which is the most conservative.

The radius of curvature may be increased above this limit if additional points of support are added to decrease the critical arc length (C).

The unbraced length between points of lateral support must be held to—

$$\boxed{C \leqq 6\, b_h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

where

$C = r\,\phi$

ϕ = radian measure

If the unbraced length (C) exceeds this limit, the thickness of the curved inner flange must be increased by—

$$0.1\left(\frac{C}{b_h} - 6\right) t_h$$

or the final thickness will be—

$$\boxed{t_h\left[\,1 + 0.1\left(\frac{C}{b_h} - 6\right)\right]} \quad \ldots\ldots\ldots\ldots (40)$$

An alternate method would be to increase the width of the inner flange (b_h) to a minimum of C/6

* ASCE Commentary on Plastic Design in Steel, p. 116.

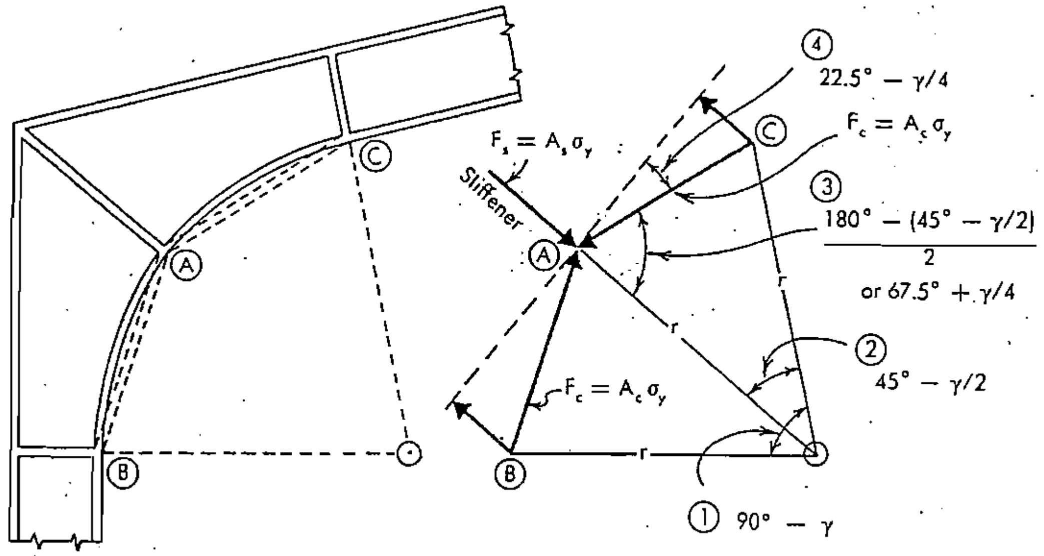


FIGURE 35

without decreasing the original flange thickness (t_h):

$$\boxed{b_h \geqq \frac{C}{6}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Diagonal Stiffeners

(1) Based on compressive forces at Ⓐ

An approximate value of the compressive force applied to the diagonal stiffener as a result of the compressive forces in the curved inner flange may be made by treating the curved haunch as a tapered haunch. See Figure 35.

$$A_s\ \sigma_y = 2\ A_c\ \sigma_y\ \sin\ (22.5° - \gamma/4)$$

or

$$\boxed{A_s \geqq 2\ A_c\ \sin\left(\frac{90° - \gamma}{4}\right)} \quad \ldots\ldots\ldots\ldots (42)$$

(2) Based on tensile forces at Ⓒ

The compressive force in the diagonal stiffener is found by taking the horizontal components of these tensile flange forces, and setting them equal to zero. See Figure 36.

$$A_t\ \sigma_y\ \cos\gamma - \frac{w_h\ d_h}{\tan(\alpha + \gamma)}\ \frac{\sigma_y}{\sqrt{3}} \cos\gamma - A_s\ \sigma_y\ \cos\alpha = 0$$

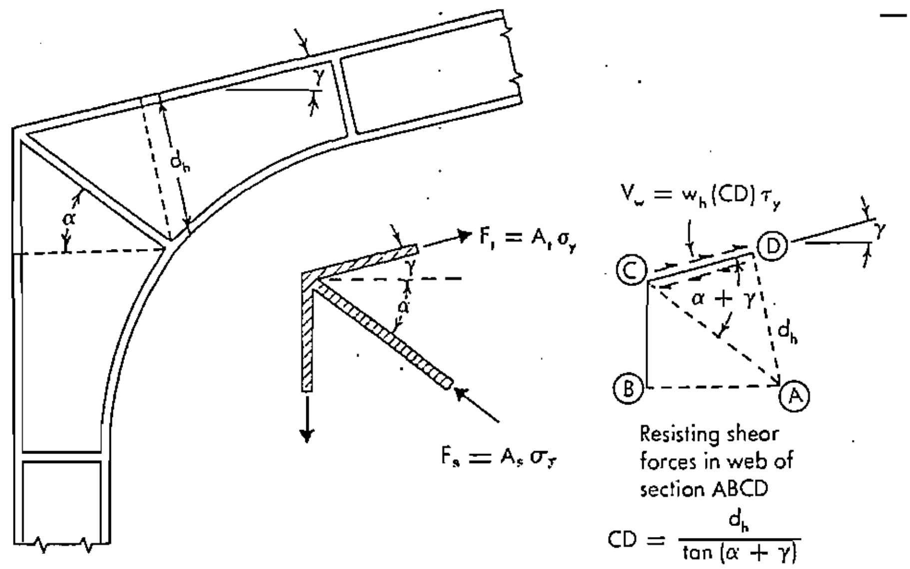


FIGURE 36

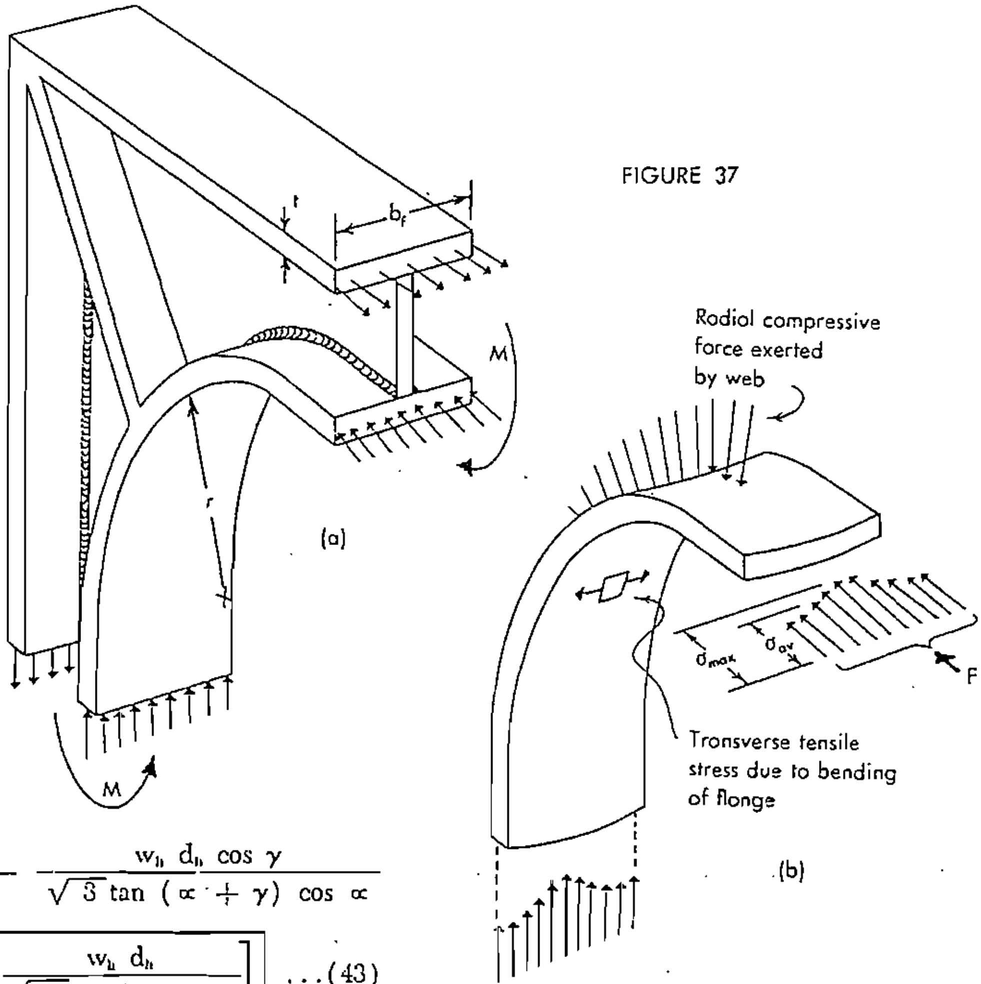


FIGURE 37

or

$$A_s = A_t \frac{\cos \gamma}{\cos \alpha} - \frac{w_h \, d_h \cos \gamma}{\sqrt{3} \tan(\alpha + \gamma) \cos \alpha}$$

$$\boxed{A_s \geqq \frac{\cos \gamma}{\cos \alpha} \left[A_t - \frac{w_h \, d_h}{\sqrt{3} \, \tan(\alpha + \gamma)} \right]} \quad \ldots (43)$$

where:

A_t = area of top (tension) flange of haunch

A_s = total area of a pair of diagonal stiffeners

Radial Support of Lower Flange

The radial components of force in the curved inner flange tend to push the flange in toward the web, and to bend the flange as shown in Figure 37(b). Because of the slight yielding of the outer edge of the flange, there is a non-uniform distribution of the flange stress (σ), Figure 37(a). This stress is maximum in line with the web. There is also a transverse tensile stress across the outer face of this flange, Figure 37(b).

The unit radial force (f_r) acting on the curved inner flange from the axial compressive force (F_c) within the flange, Figure 38, is—

$$f_r = \frac{F_c}{r} \text{ (lbs/cir inch)}$$

Treating a 1″ slice of this flange supported by the web of the haunch as a cantilever beam and uniformly loaded with this unit radial force (f_r), Figure 39:

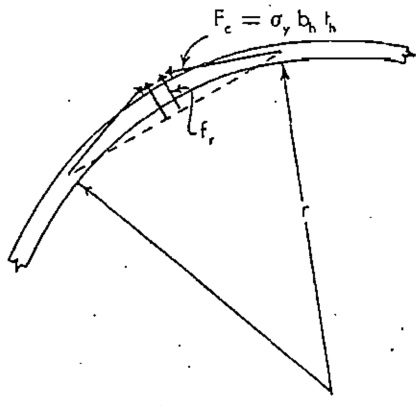


FIGURE 38

$$Z = \frac{t_h^2}{4}$$

$$f_r = \frac{F_c}{r} = \frac{\sigma_y \, b_h \, t_h}{r}$$

or unit load (p) on section:

$$p = \frac{\sigma_y \, t_h}{r}$$

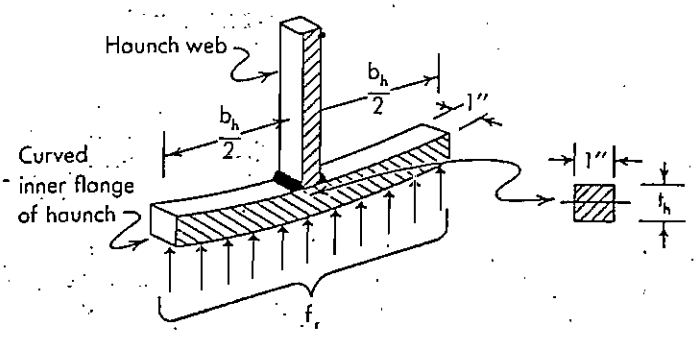


FIGURE 39

$$M = \frac{p}{2}\left(\frac{b}{2}\right)^2$$

$$M = \frac{\sigma_y\ t_h}{2\ r}\left(\frac{b_h}{2}\right)^2 = \frac{\sigma_y\ t_h\ b_h^2}{8\ r}$$

also

$$M \leqq \sigma_y\ Z \leqq \frac{\sigma_y\ t_h^2}{4}$$

$$\frac{b_h^2}{8\ r} \leqq \frac{t_h}{4}$$

or

$$\boxed{\frac{b_h^2}{r\ t_h} \leqq 2} \qquad \text{.......................(44)}$$

Therefore limit the ratio of flange width to thickness (b_f/t_x) of the curved inner flange to the following, whichever is the smaller:

$$\boxed{\frac{b_h}{t_h} \leqq \frac{2\ r}{b_h} \leqq 17} \qquad \text{.......................(45)}$$

Provide stiffeners at and midway between the two points of tangency. Make the total cross-sectional area of the pair of diagonal stiffeners at their midpoint not less than ¾ of the inner curved flange area.

Summary of Curved Haunch Requirements

thickness of outer flange (t) $\geqq t_b$

web of haunch $(w_h) \geqq w_b$

thickness of curved inner flange $(t_h) \geqq \dfrac{t_b}{\cos\beta}$

$$= (1 + m)\ t$$

(based on tensile flange)

$$A_s \geqq \frac{\cos\gamma}{\cos\alpha}\left[A_t - \frac{w_h\ d_h}{\sqrt{3}\ \tan(\alpha + \gamma)}\right]$$

(based on compressive flange)

$$A_s \geqq 2A_c \sin\left(\frac{90 - \gamma}{4}\right) \qquad \text{and}$$

$$A_s \geqq \tfrac{3}{4}\ A_c$$

If bending stress at ② $\sigma_2 = \dfrac{M_2}{S} < \sigma_y$, then

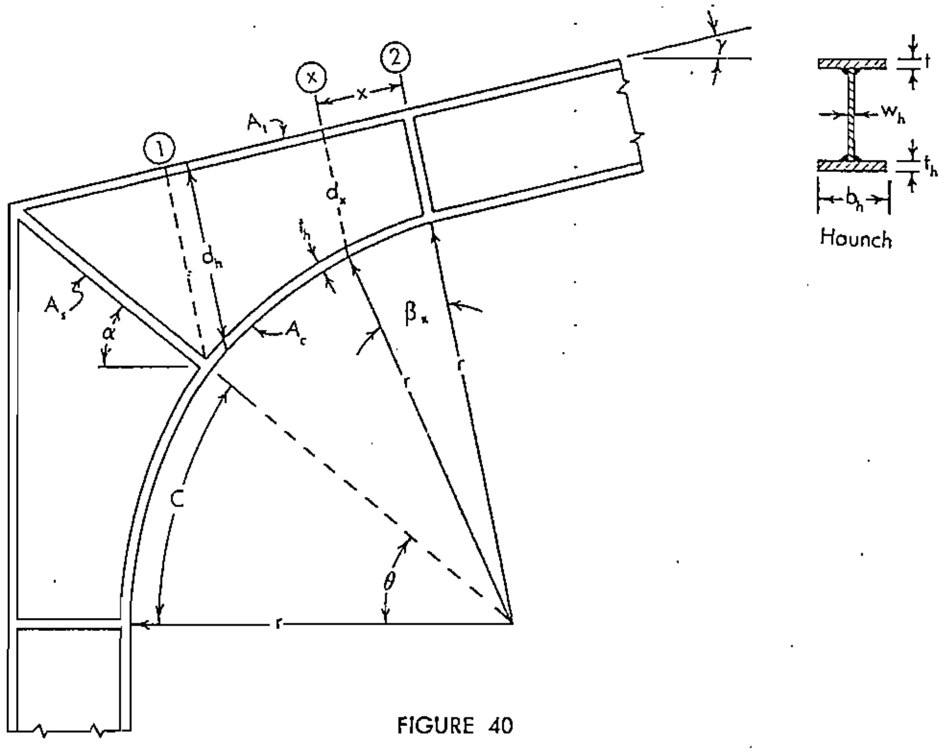


FIGURE 40

outer flange thickness (t) does not have to exceed beam flange (t_b).

$$r \leqq 6\ b_b$$

Otherwise, use additional lateral support to decrease arc length (C).

Assume critical section (x-x) at—

$$\beta_x = 12°$$

then

$$Z_x = b_b\ t_b\ (d_x - t_b) + \frac{w_b}{4}\ (d_x - 2\ t_b)^2$$

and

$$Z_x \geqq \frac{M_x}{\sigma}$$

$$C \leqq 6\ b_b$$

where:

$C = r\ \phi$

ϕ = radian measure

Otherwise, increase the thickness of the curved flange to—

$$t_b \left[1 + 0.1 \left(\frac{C}{b} - 6\right)\right]$$

or increase the width of the curved inner flange to—

$$b_b \geqq \frac{C}{6}$$

without decreasing the flange thickness.

$$\frac{b_b}{t_x} \leqq \frac{2\ r}{b_b} \leqq 17$$

6. BEAM-TO-COLUMN CONNECTIONS (Multiple Span)

Web Resisting Shear

When the moments in two beams framing into an interior column differ by a larger amount, this difference in moment will cause large shear forces to act on the connection web. The web must be checked to see if it has sufficient thickness; if not, it must be reinforced with either a web doubler plate or diagonal stiffeners. (See Figure 41.)

horizontal shear applied on connection web

along top portion

$$= F_2 - F_1 - V_4$$

$$= \frac{M_2}{d_2} - \frac{M_1}{d_1} - V_4$$

shear resisted by connection web

along top portion

$$= w\ d_c\ \tau_y$$

$$= w\ d_c\ \frac{\sigma_y}{\sqrt{3}} \qquad \text{or}$$

$$w\ d_c\ \frac{\sigma_y}{\sqrt{3}} = \frac{M_2}{d_2} - \frac{M_1}{d_1} - V_4$$

or

$$\boxed{w_r = \frac{\sqrt{3}}{d_c\ \sigma}\left[\frac{M_2}{d_2} - \frac{M_1}{d_1} - V_4\right]} \quad \ldots\ldots\ldots (46)$$

where:

V_4 = horizontal shear force in the column above the connection, lbs

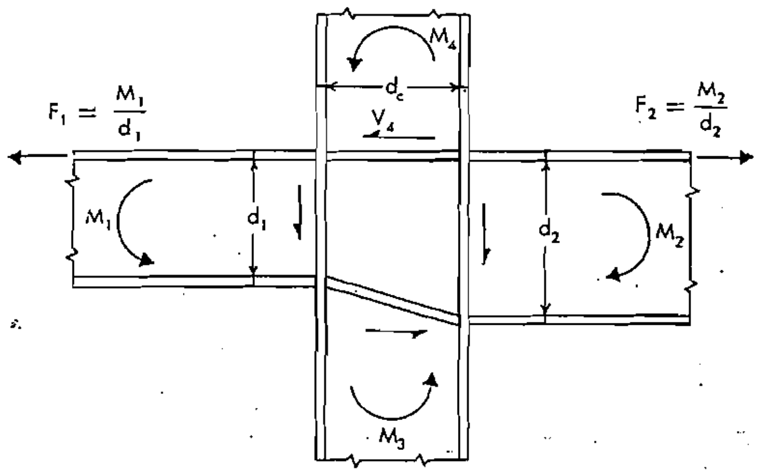
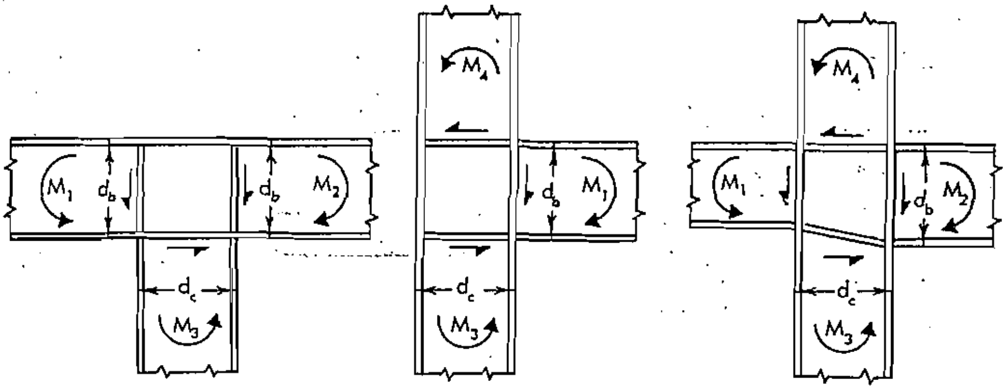


FIGURE 41



$M = M_1 - M_2 = M_3$ $\quad$ $M = M_1 = M_3 + M_4$ $\quad$ $M = M_1 - M_2 = M_3 + M_4$

FIGURE 42

M_1 and M_2 = moments in beams (1) and (2), in.-lbs.

d_c = depth of column, in.

d_1 and d_2 = depth of beams (1) and (2)

w = *thickness of connection web, in.*

If it is assumed that:

1. the column height (h) has a point of inflection at mid-height,

2. the depth of the larger beam (d_2) is 1/15 of the column height (h), or less,

3. the yield strength of the steel is $\sigma_y = 33{,}000$ psi, and

4. the unbalanced moment (M) is expressed in foot-kips,

this formula will reduce to the following:

$$\boxed{w_r = \frac{19{,}400\ M}{d_b\ d_c\ \sigma_y}} \quad \text{.......................} (47)$$

The method of determining the value of M is illustrated in Figure 42.

Web Resisting Thrust

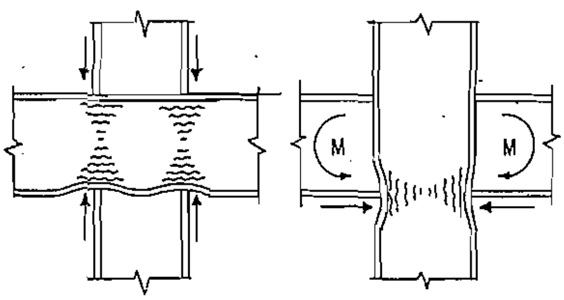


FIGURE 43

Stiffeners are quite often required on members in line with the compression flanges which act against them, to prevent crippling of the web where the concentrated compressive force is applied.

Where a beam supports a column, or a column *supports a beam, on just one flange, the stiffeners on* its web need only extend just beyond its neutral axis.

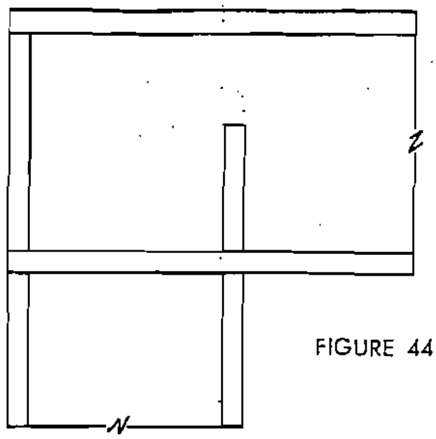


FIGURE 44

The following formulas will indicate when stiffeners are required, and also the necessary size of these stiffeners:

1. Web stiffeners are required adjacent to the beam tension flange if—

$$\boxed{t_c < 0.4\ \sqrt{A_f}} \quad \text{.......................} (48)$$

2. Web stiffeners are required adjacent to the beam compression flange if—

$$\boxed{w_c \geqq w_r} \quad \text{...........................} (49)$$

where:

$$w_r = \frac{A_f}{t_b + 5\ K_c}$$

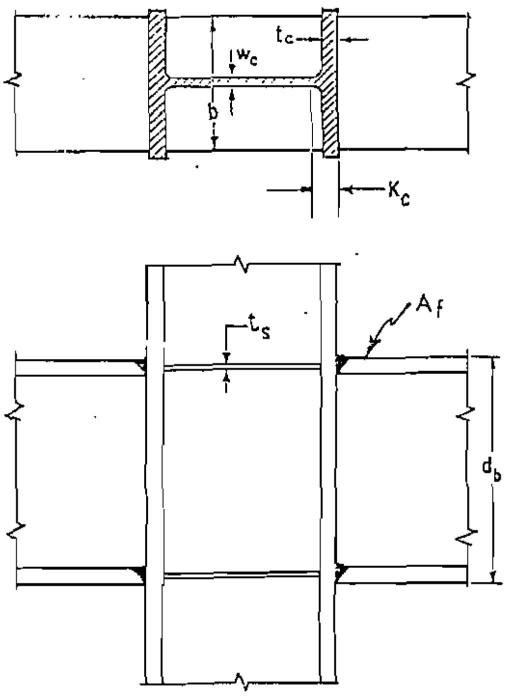


FIGURE 45

If horizontal flange plate stiffeners are used, Figure 45, their dimensions are found from the following:

$$\boxed{t_s \geqq \frac{A_f - w_c\,(t_b + 5\,K_c)}{b_s}} \qquad \text{(50)}$$

or

$$\boxed{t_s \geqq \frac{A_f}{b_s}\left[1 - \frac{w_c}{w_r}\right]} \qquad \text{(51)}$$

also

$$\boxed{t_s \geqq \frac{b_s}{17}} \qquad \text{(52)}$$

where:

$A_f = b_b \times t_b$

w_r = required thickness of connection web

w_c = actual thickness of column web; here actual thickness of connection web

(See Section 5.7 on Continuous Connections for further explanation.)

If vertical plate stiffeners are used, Figure 46, they should be proportioned to carry the excess of beam flange force over that which the column web is able to carry. It is assumed the beam flange extends almost the full width of the column flanges, and that the stiffeners are only half as effective, since they lie at the outer edge of the flange.

$$\boxed{t_s \geqq \frac{A_f}{t_b + 5\,K_c} - w_c} \qquad \text{(53)}$$

or

$$\boxed{t_s \geqq w_r - w_c} \qquad \text{(54)}$$

also

$$\boxed{t_s \geqq \frac{d_c}{30}} \qquad \text{(55)}$$

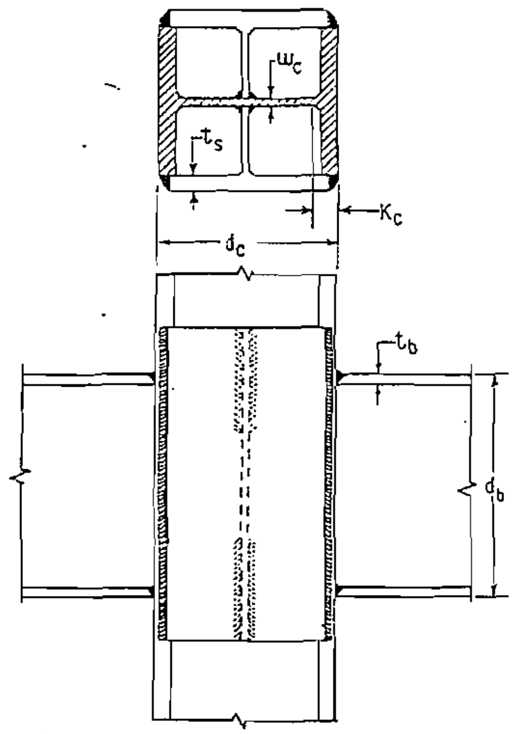


FIGURE 46

(See Section 5.7 on Continuous Connections for further explanation.)

The nomograph, Figure 47, may be used to find the distance $(t_b + 5\,K_c)$ over which the concentrated force from the beam flange spreads out into the column web. In the case of a built-up column, use the flange thickness (t_c) and find the distance $(t_b + 5\,t_c)$ from the nomograph.

This value of $(t_b + 5\,K_c)$ or $(t_b + 5\,t_c)$ can then be used in finding the required web thickness (w_r) from the nomograph, Figure 48.

FIGURE 47—Spread of Flange Thrust Into Column Web.

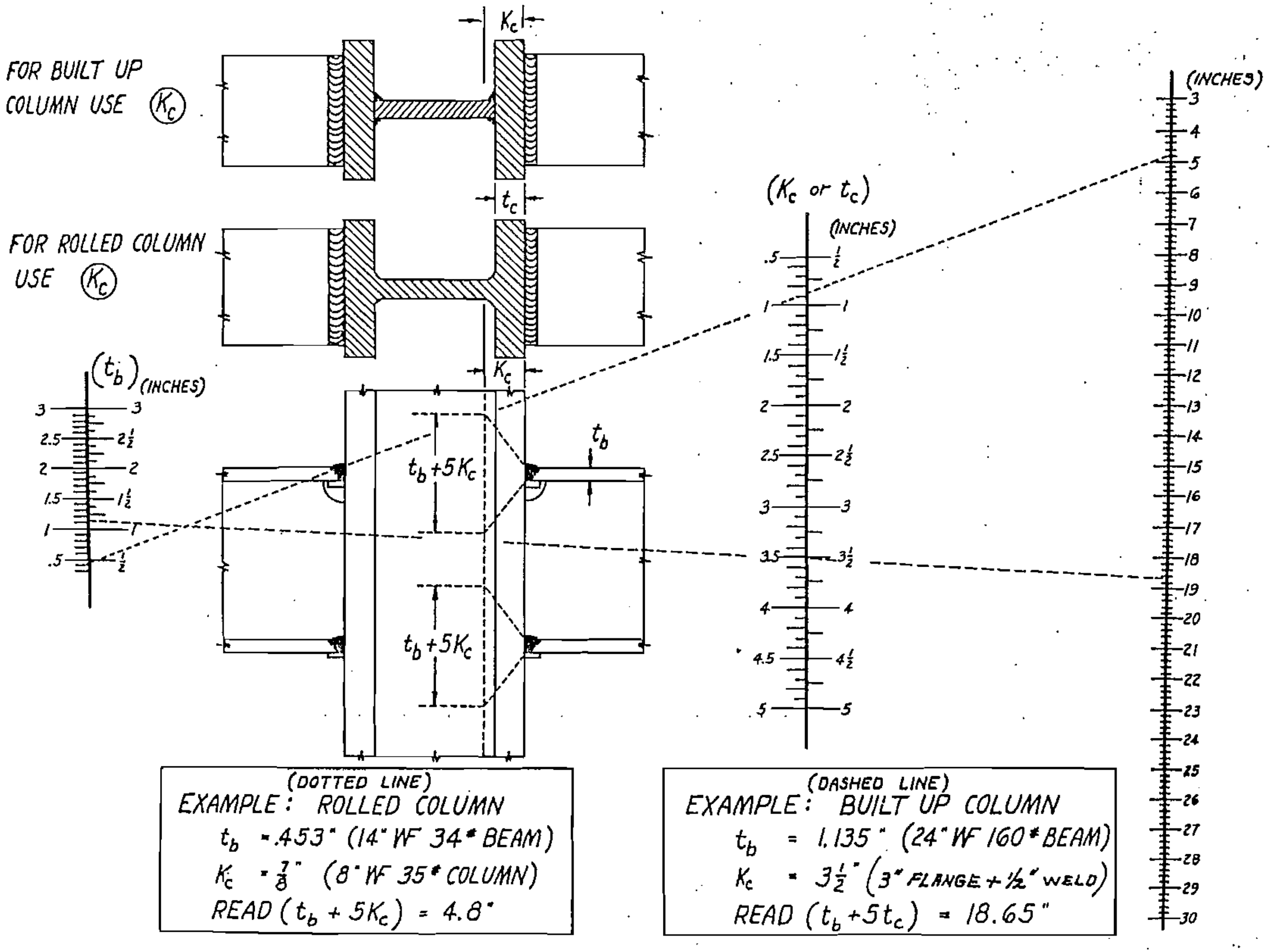
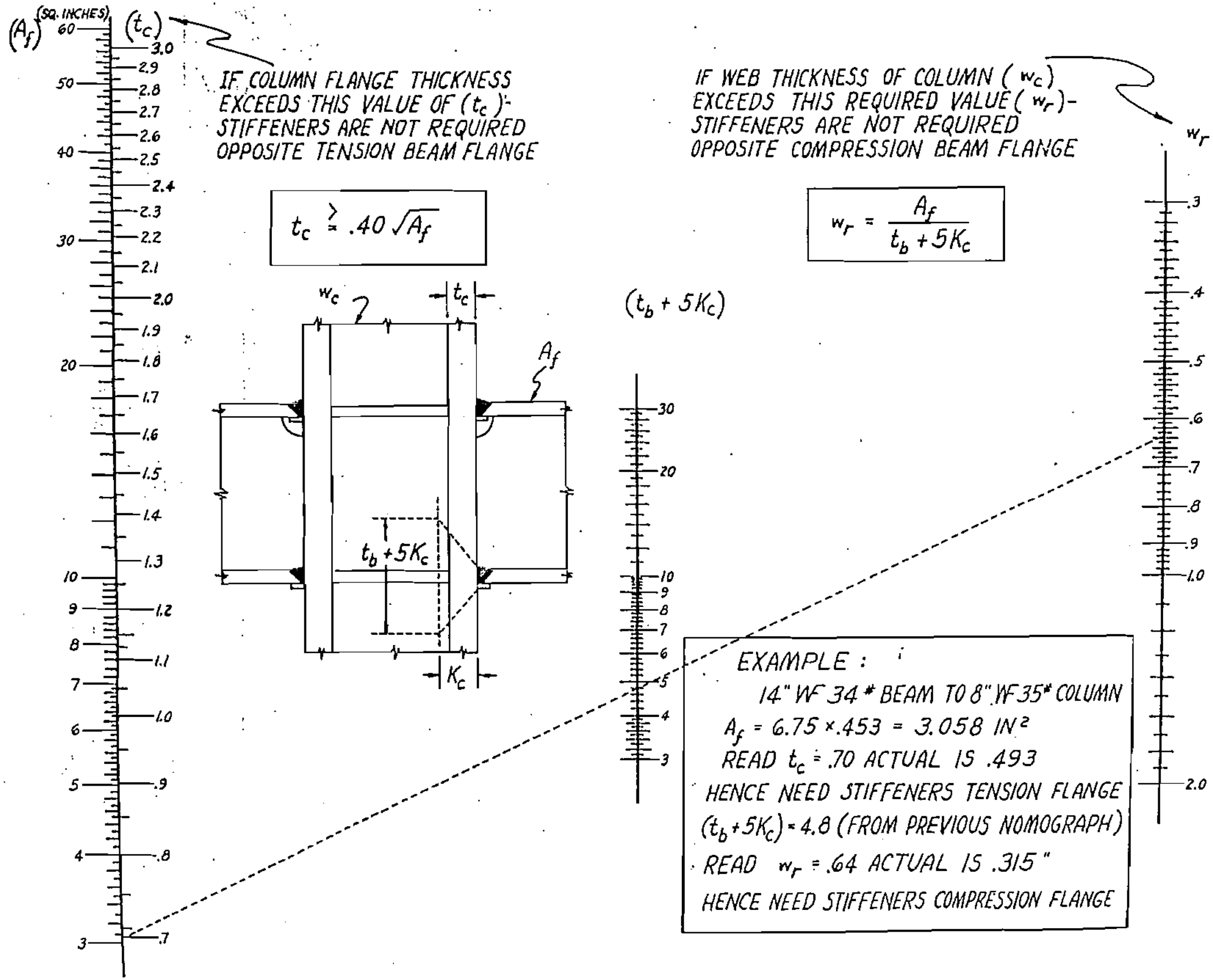


FIGURE 48—Thickness of Connection Web To Resist Thrust of Compression Flange.



Problem 2

Is reinforcement necessary at this interior connection? Moments at ultimate load are shown below. A36 steel and E70 welds.

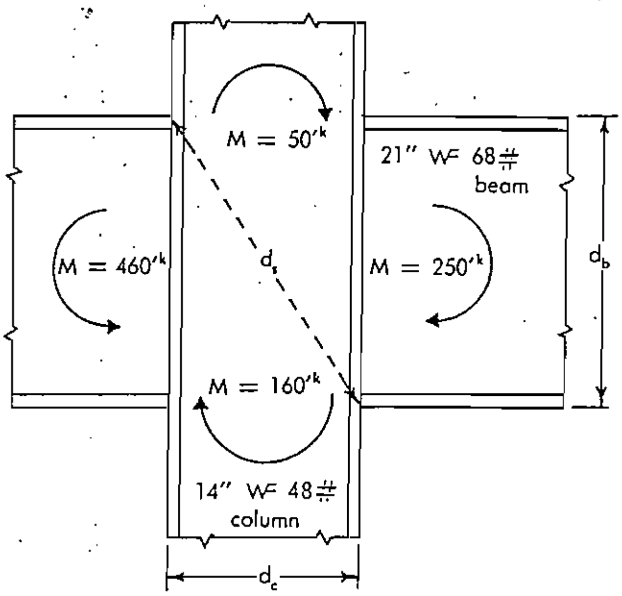


FIGURE 49

beam dimensions

$d_b = 21.13''$

$b_b = 8.27''$

$w_b = .430''$

$t_b = .685''$

column dimensions

$d_b = 13.81''$

$w_c = .339''$

$b_c = 8.031''$

$K_c = 1\tfrac{3}{16}''$

diagonal of connection web

$$d_s = \sqrt{d_b^2 + d_c^2}$$
$$= \sqrt{21.13^2 + 13.81^2}$$
$$= 23.18''$$

Web Resisting Shear

The necessary web thickness will be determined by the AISC requirements for webs in the connection region. The algebraic sums of the clockwise and counter-clockwise moments on opposite sides of the connection are:

$$M = 460 \text{ ft-kips} - 250 \text{ ft-kips}$$
$$= 210 \text{ ft-kips}$$

and

$$M = 160 \text{ ft-kips} + 50 \text{ ft-kips}$$
$$= 210 \text{ ft-kips}$$

required thickness of connection web

$$w_r = \frac{\sqrt{3}\, M}{d_b\, d_c\, \sigma_y}$$
$$= \frac{\sqrt{3}\,(210 \text{ ft-kips} \times 12)}{(21.13)(13.81)(36 \text{ ksi})}$$
$$= .416''$$

Conclusions (Fig. 50)

(a) This required web thickness would be satisfied if the beam were allowed to run through the column. This would give a web thickness of .430". OK

(b) If the column were to run continuous through the beam, as illustrated above, then a ¼" doubler plate would be required in this connection area to make up the difference in thickness.

(c) Another choice would be to use a pair of diagonal stiffeners having the following cross-sectional area:

$$A_s = \frac{d_s\,(w_r - w_c)}{\sqrt{3}}$$
$$= \frac{(23.18)(.416 - .339)}{\sqrt{3}}$$
$$= 1.03 \text{ in.}^2$$

Or use a pair of 3" by ⅜" stiffeners, the area of which checks out as—

$$A_s = \tfrac{3}{8}''\,(2 \times 3'' + .339'')$$
$$= 2.38 \text{ in.}^2 > 1.03 \text{ in.}^2 \quad \underline{\text{OK}}$$

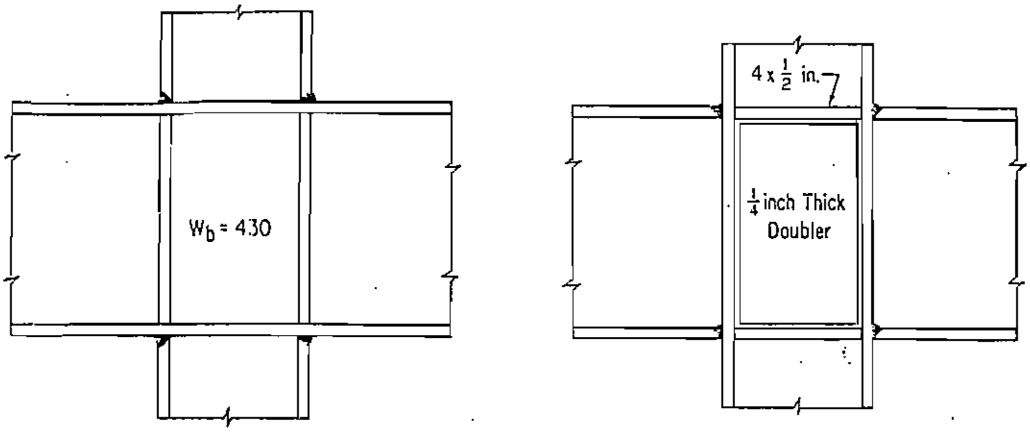
Also, the required thickness is—

$$t_s \geqq \frac{b_s}{17}$$
$$= \frac{2 \times 3''}{17}$$
$$= .35'' < \tfrac{3}{8}'' \quad \underline{\text{OK}}$$

Web Resisting Thrust

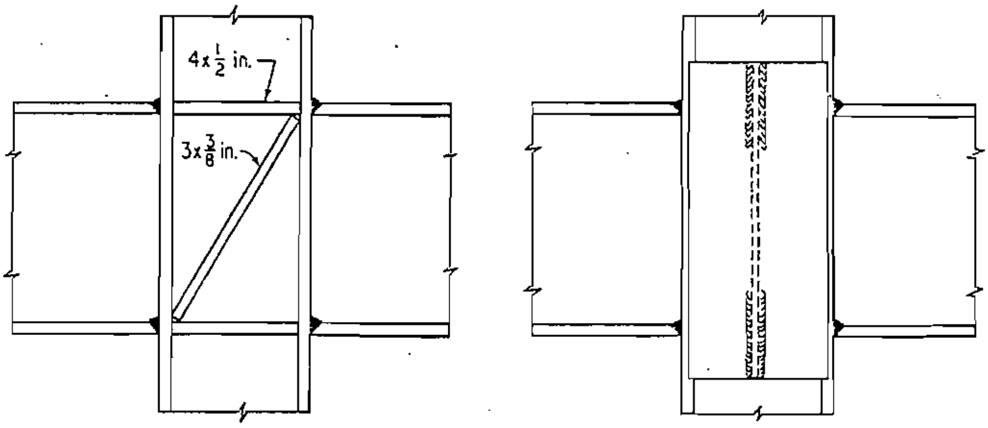
In addition to this, the web of the column must be checked against buckling from the concentrated compressive forces applied by the beam flanges.

If the web thickness exceeds the following value, stiffeners are not needed opposite beam compression flange:



(a) Run beam through column
Add plate stiffeners across beam, in line with column flanges to transfer column load

(b) A 1/4" doubler plate
(d) A pair of 4" x 1 1/2" horizontal flange plate stiffeners



(c) A pair of 3" x 3/8" diagonal stiffeners
(d) A pair of 4" x 1/2" horizontal flange plate stiffeners

(e) A pair of Tee vertical stiffeners cut from 21" WF 112# or 5/8" plate
Tee section also provides the necessary additional web material for this connection.

FIGURE 50

$$w_r = \frac{A_f}{t_b + 5\ K_c}$$

$$= \frac{(8.27'' \times .687'')}{(.685'') + 5(1\tfrac{3}{16}'')}$$

$$= .856''$$

Since $w_c = .339''$, some additional stiffening is required. There are two solutions.

(d) *Horizontal flange plate stiffeners,* the required thickness of which is found from the following formula:

$$t_s \geqq \frac{A_f}{b_s}\left[1 - \frac{W_c}{w_r}\right]$$

$$\geqq \frac{(8.27'' \times .685'')}{(8'')}\left[1 - \frac{(.339'')}{(.856'')}\right]$$

$$\geqq .428''$$

but the following is called for—

$$t_s \geqq \frac{b_s}{17}$$

$$\geqq \frac{(2 \times 4'')}{16}$$

$$\geqq .47''$$

Hence, use a pair of 4" x ½" horizontal plate stiffeners.

(e) *Vertical stiffeners,* the required thickness of which is found from the following formula:

$$t_s \geqq w_r - w_c$$

$$\geqq .856'' - .339''$$

$$\geqq .517''$$

and this checks against the following requirement—

$$t_s \geqq \frac{d_s}{30}$$

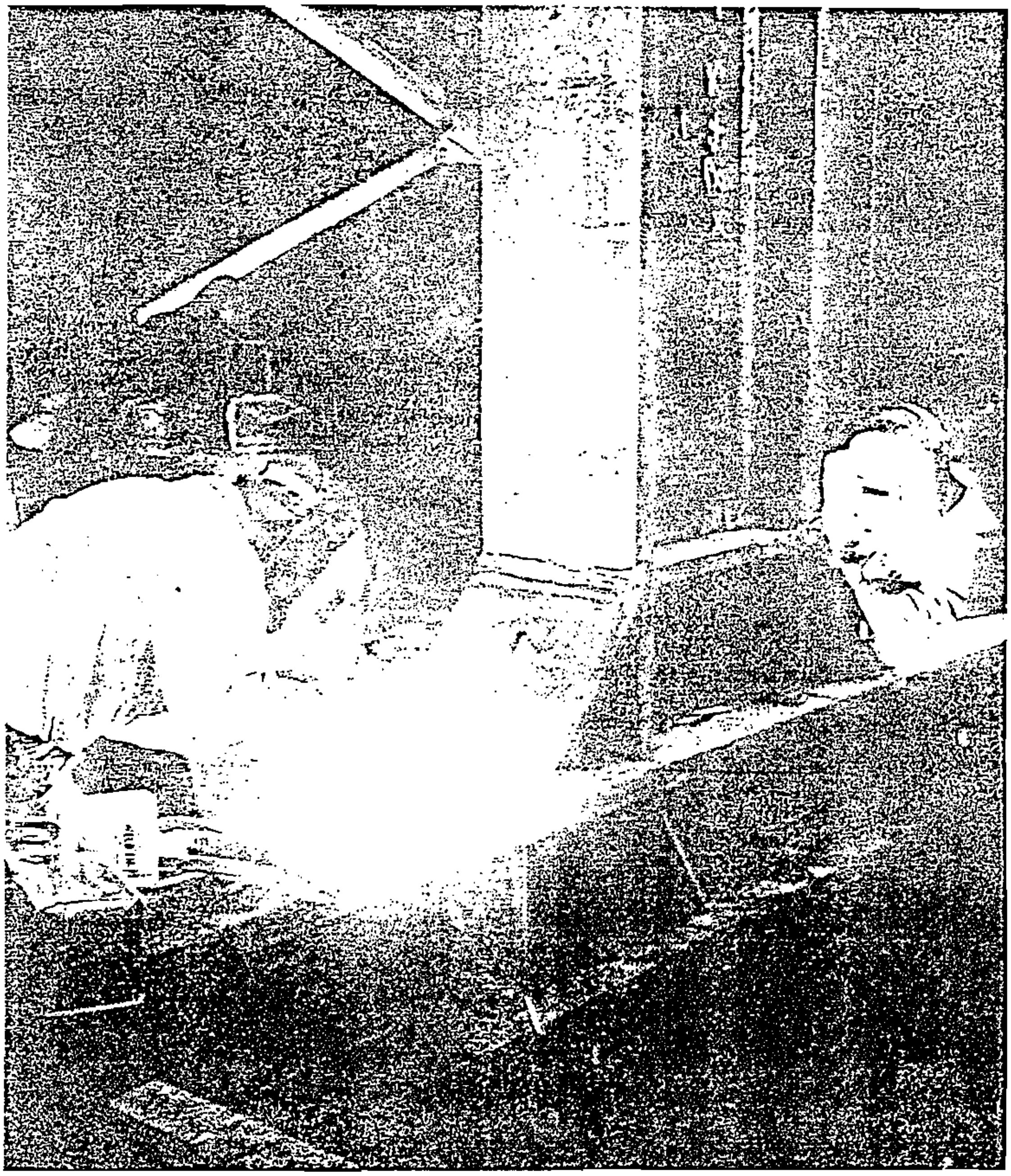
$$\geqq \frac{(13.81'')}{30}$$

$$\geqq .46'' < .517''$$

This T section could be flame cut from a 12" WF 112# section, which has a flange thickness of .865" (we need .517") and a flange width of 13.00" (we need at least 12.625"). Otherwise, it could be fabricated from ⅝" thick plate welded together.

Summary

There are four possible methods of making this connection, Figure 50. Each uses a combination of the preceding solutions to stiffen the connection web so it may safely transmit the shear forces resulting from the unbalanced moment as well as to prevent buckling from the concentrated compressive forces applied by the beam.



Shop-fabricated Vierendeel trusses lowered steel requirements and reduced time for erection of Hamburgers clothing store in Baltimore. Here a weldor is connecting a corner bracket between web member and bottom chord of the truss, using low-hydrogen electrode for root passes.