Designing for Torsional Loading

1. NATURE OF TORSIONAL LOADING

Torsional loading is the application of a force that tends to cause the member to twist about its structural axis.

Torsion is usually referred to in terms of torsional moment or torque (T), which is basically the product of the externally applied force and the moment arm or force arm. The moment arm is the distance of the centerline of rotation from the line of force and perpendicular to it. This distance often equals the distance from the member's center of gravity to its outer fiber (radius of a round shaft, for example), but not always.

The principal deflection caused by torsion is measured by the angle of twist, or by the vertical movement of one corner of the frame.

Steel, in rolled structural shapes or built-up sections, is very efficient in resisting torsion. With steel, torsionally rigid sections are easily developed by the use of stiffeners.

Here are the three basic rules for designing structural members to make the best use of steel where torsional loads are a problem:

1. Use closed sections where possible.
2. Use diagonal bracing.
3. Make rigid end connections.

2. POLAR MOMENT OF INERTIA

When a round shaft is subjected to a twisting or torsional moment (torque), the resulting shear stress in the shaft is—

$$\boxed{\tau = \frac{T\,c}{J}} \quad \ldots\ldots\ldots\ldots (1)$$

where:

τ = shear stress, psi

c = distance from center of section to outer fiber

T = torque, in.-lbs.

J = polar moment of inertia of section, in.^{4}
$= I_x + I_y = 2I$

The angular twist of a round shaft is—

$$\boxed{\theta = \frac{T\,L}{E_s\,J}} \quad \ldots\ldots\ldots\ldots (2)$$

where:

θ = over-all angular twist of shaft, in radians
(1 radian = 57.3° approx.)

L = length of shaft, in inches

E_s = modulus of elasticity in shear
(steel E_s = 12,000,000 psi)

In most cases, the designer is interested in holding the torsional moment within the material's elastic limit. Where the torsional strength of a round shaft *is* required (i.e. the stress it can take without failure), the polar section modulus is J/c, and the allowable torque is thus—

$$T = \tau_u \frac{J}{c}$$

where, lacking test data, the ultimate shear strength of steel (τ_u) is assumed to be in the order of 75% of the material's ultimate tensile strength.

The above three formulas are true for solid round or tubular round shafts. For non-circular sections the shear stresses are not uniform, and therefore the standard torsional formulas no longer hold.

3. TORSIONAL RESISTANCE

Values of torsional resistance (R)—stiffness factor—have been established for various standard sections and provide more reliable solutions to torsional rigidity problems. Values of R are expressed in inches to the fourth power.

Table 1 shows the formulas for shear stress and torsional resistance of various sections. The formulas for solid rectangular sections call for values, of α and β, which are derived from the ratio of section width (b) to depth (d), as shown in the table.

Actual tests show that the torsional resistance (R) of an open section made up of rectangular areas, nearly equals the sum of the torsional resistances of all the individual rectangular areas. For example, the torsional resistance of an I beam is approximately

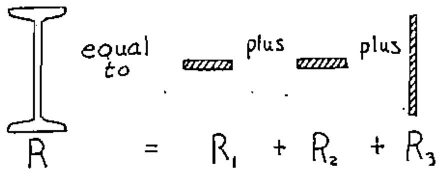


FIGURE 1

equal to the sum of the torsional resistances of the two flanges and web (Fig. 1).

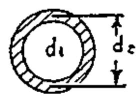
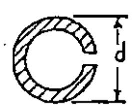
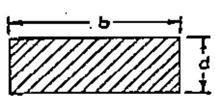
Figure 2 shows the results of twisting an I beam made of three equal plates. Calculated values of twist by using the conventional polar moment of inertia (J) and the torsional resistance (R) are compared with the actual results. This shows greater accuracy by using torsional resistance (R).

This means that the torsional resistance of a flat

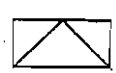
Angle of twist		
all loadings identical	t = .055	t = .055
Conventional method J polar moment of Inertia	.065°	.007°
Method using R Torsional Resistance	21.8°	7.3°
Actual Twist	22°	9.5°

FIGURE 2

TABLE 1—Torsional Properties of Various Sections

Section	Shear Stress	(for steel) R-torsional Resistance
	$\tau = \frac{16\,T}{\pi d^3}$	$R = .0982\,d^4$
	$\tau = \frac{16\,T\,d_2}{\pi (d_2^4 - d_1^4)}$	$R = .0982\,(d_2^4 - d_1^4)$
	$\tau = \frac{3\,T}{\pi d\,t^2}$	$R = 1.0472\,t^3 d$
	$\tau = \frac{4.8\,T}{d^3}$	$R = .1406\,d^4$
	$\tau = \frac{T}{\alpha\,b\,d^2}$	$R = \beta\,b\,d^3$

for solid rectangular sections	$\frac{b}{d}$ = 1.00	1.50	1.75	2.00	2.50	3.00	4.00	6	8	10	∞
α	.208	2.31	.239	.246	.258	.267	.282	.299	.307	.313	.333
β	.141	.196	.214	.229	.249	.263	.281	.299	.307	.313	.333

Use this for diagonal bracing	 single brace	R=3.54 I	I of diagonal brace
	 double brace	R=10.6 I	

	Angle of twist a	b	c	d	e
all loadings identical	t = .060, $3\frac{1}{2}$	t = .060, $2\frac{5}{8}$, $\frac{3}{4}$	t = .060, 1.15	t = .060	t = .060
Conventional method J polar moment of inertia	.01°	.006°	.04°	.04°	.045°
Method using R Torsional Resistance	9.5°	9.7°	10°	.04°	.06°
Actual Twist	9°	9.5°	11°	too small to measure	too small to measure

FIGURE 3

plate is approximately the same whether it is used as such or is formed into an angle, channel, open tube section, etc. This is illustrated in Figure 3. Samples of different sections made of 16-gage steel are subjected to torsion. The flat section twists 9°. The same piece of steel formed into a channel (b) twists 9½°. When rolled into a tube with an open beam (c), it twists 11°.

When the same section is made into a closed section (d) by placing a single tack weld in the middle of the open seam, the torsional resistance increases several hundred times. When the tube becomes a closed section, the torsional stresses are distributed more evenly over the total area, thus permitting a greater load.

Notice the error in using polar moment of inertia (J) for the angle of twist of open sections, and the good agreement by using torsional resistance (R).

> Design Rule No. 1: USE CLOSED SECTIONS WHERE POSSIBLE

The solid or tubular round closed section is best for torsional loading since the shear stresses are uniform around the circumference of the member.

Next to a tubular section, the best section for resisting torsion is a closed square or rectangular tubular section.

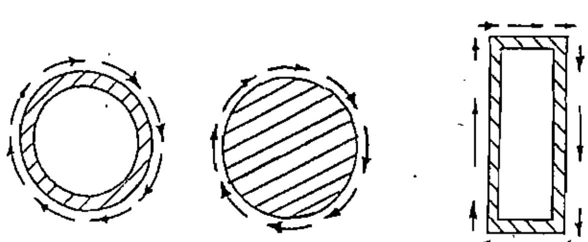


FIGURE 4

Table 2 provides formulas for determining the torsional resistance (R) of various closed tubular sections. It also provides the basic formulas for determining the shear stress (τ) at any given point along the sidewall of any closed section regardless of configuration or variation of thickness, and for determining the section's torsional resistance (R).

The poorest sections for torsional loading are open sections, flat plates, angle sections, channel sections, Z-bar sections, T-bar sections, I-beam sections, and tubular sections which have a slot.

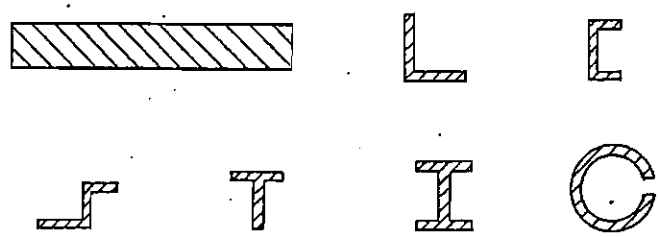


FIGURE 5

After the R values of all areas in a built-up section have been added together, their sum is inserted into the following formula or a modification of it:

$$\boxed{\theta = \frac{T\,L}{E_s\,R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Torque (T) in in.-lbs may be obtained from one of the formulas in Table 3, such as—

$$T = \frac{63{,}000 \times HP}{RPM}$$

or $T = P\,e$

where:

HP = horsepower

RPM = speed of revolution

P = applied force, lbs

e = moment arm of force (the perpendicular distance from the center of rotation to the line of force)

Problem 1

As an example, consider the torsional resistance of a closed round tube and one that is slotted. The tube has an O.D. of 4″, and I.D. of 3″, a length of 100″, and is subjected to a torque of 1000 in.-lbs.

Case 1

From Table 1, the torsional resistance of the closed round tube is found to be—

$$R = 0.0982\ (d_2^4 - d_1^4)$$
$$= 0.0982\ (4^4 - 3^4)$$
$$= \underline{17.19 \text{ in.}^4}$$

and the angular twist is—

$$\theta = \frac{T\ L}{E_s\ R} = \frac{(1000)(100)}{(12 \times 10^6)17.19}$$
$$= 0.000485 \text{ radians, or } \underline{0.0278°}$$

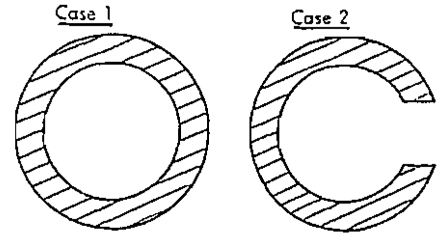
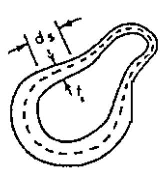


FIGURE 6

Case 2

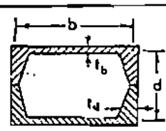
From Table 1, the torsional resistance of the slotted round tube is found to be—

TABLE 2—Torsional Resistance (R) of Closed Tubular Sections



$$\theta = \frac{T\ L}{E_s\ R}$$
$$R = \frac{4[A]^2}{\int \frac{ds}{t_s}}$$
$$\tau_s = \frac{T}{2[A]t_s}$$
$$f = \frac{T}{2A}$$

[A] = area enclosed within mean dimensions.
d_s = length of particular segment of section
t_s = average thickness of segment at point (s)
τ_x = shear stress at point (s)
R = torsional resistance, in^4
E_s = modulus of elasticity in shear (steel = 12,000,000)
θ = angular twist (radians)
L = length of member (inches)
f = unit shear force

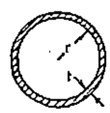


$$\int \frac{d_s}{t_s} = \frac{2_b}{t_b} + \frac{2_d}{t_d}$$

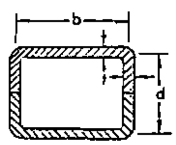
$$R = \frac{4[A]^2}{\int \frac{d_s}{t_s}} = \frac{4(bd)^2}{\frac{2b}{t_b} + \frac{2d}{t_d}} = \frac{2\ b^2\ d^2}{\frac{b}{t_b} + \frac{d}{t_d}}$$

stress at ℄ of b:

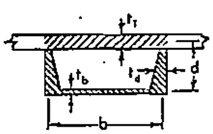
$$\tau_b = \frac{T}{2[A]t_b} = \frac{T}{2\ bd\ t_b}$$



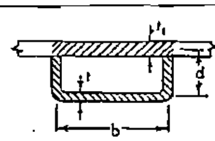
$$R = 2\ \pi\ r^3\ t$$



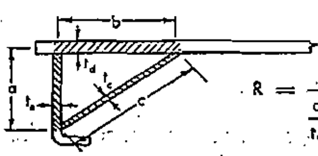
$$R = \frac{2\ t\ b^2\ d^2}{b + d}$$



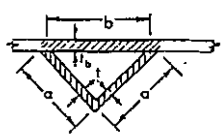
$$R = \frac{4\ b^2\ d^2}{\frac{b}{t_b} + \frac{2d}{t_d} + \frac{b}{t_1}}$$



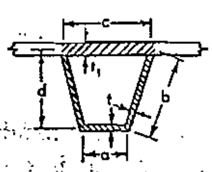
$$R = \frac{4\ b^2\ d^2}{\frac{b + 2d}{t} + \frac{b}{t_1}}$$



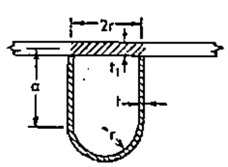
$$R = \frac{a^2\ b^2}{\frac{a}{t_a} + \frac{b}{t_b} + \frac{c}{t_c}}$$



$$R = \frac{a^4}{\frac{2a}{t} + \frac{b}{t_b}}$$



$$R = \frac{(a + c)^2\ d^2}{\frac{a + 2b}{t} + \frac{c}{t_1}}$$



$$R = \frac{4r^2\left(\frac{\pi\ r}{2} + 2a\right)^2}{\frac{2a + \pi\ r}{t} + \frac{2\ r}{t_1}}$$

$$R = 1.0472\ t^3\ d$$
$$= 1.0472\ (\tfrac{1}{2})^3\ 3\tfrac{1}{2}$$
$$= \underline{0.459\ \text{in.}^4}$$

and the angular twist is—

$$\theta = \frac{T\ L}{E_s\ R}$$
$$= \frac{(1000)(100)}{(12 \times 10^6).459}$$
$$= 0.018 \text{ radians, or } \underline{1.04°}$$

Thus, the tube without the slot is many times more rigid than the slotted tube.

Problem 2

Two 6″ × 2″ × 10½-lb channels are to be used in making a 100″-long frame, which will be subjected to a torque of 1000 in.-lbs. In what relationship to each other will these channels offer the greatest resistance to twist?

Case 1

These two channels when separated but fastened together by end plates do not have much torsional resistance.

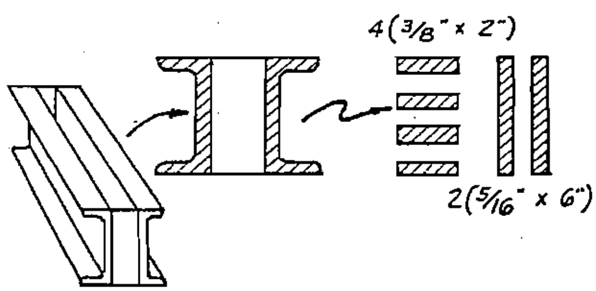


FIGURE 7

From Table 1, the value of R for each of the flanges is found to be—

$$R_1 = 0.0306\ \text{in.}^4$$

and that of each web is—

$$R_2 = 0.0586\ \text{in.}^4$$

and thus the total angular twist is—

$$\theta = \frac{1000 \times 100}{(12 \times 10^6)\ (4 \times .0306 + 2 \times .0586)}$$
$$= 0.0348 \text{ radians, or } \underline{2.0°}$$

TABLE 3—Formulas for Determining Safe Torque Under Various Conditions

Based on tangential load:

$$T = P\ e$$

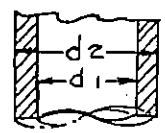
Based on horsepower transmitted:

$$T = \frac{63{,}030 \times HP}{RPM}$$

Based on strength of shaft:

$$T = \frac{.19635\ S_s\ (d_2^4 - d_1^4)}{d_2}$$

where $S_s = 15{,}000$



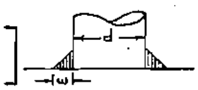
$$T = \frac{2945\ d_2^4 - d_1^4}{d_2}$$

Based on safe twist of shaft (.08°/ft):

$$T = 137\ (d_2^4 - d_1^4)$$

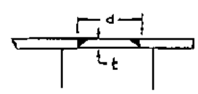
Based on fillet weld leg size around shaft or hub:

$$T = \frac{3781}{d + \omega}\left[(d + \omega)^4 - d^4\right]$$



Based on butt weld size around hub:

$$T = 20{,}420\ d^2\ t$$



Case 2

When these two channels are securely fastened back to back, there is suitable resistance to any slip or movement due to horizontal shear. Here the two webs are considered as one solid web, and the top and bottom flanges are considered solid.

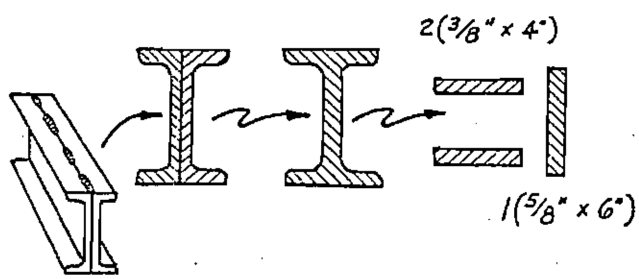


FIGURE 8

From Table 1, the value of R for each of the two composite flanges is found to be—

$$R_1 = 0.066 \text{ in.}^4$$

and that of the composite web is—

$$R_2 = 0.459 \text{ in.}^4$$

and thus the total angular twist is—

$$\theta = \frac{1000 \times 100}{(12 \times 10^6)\ (2 \times .066 + .459)}$$

$$= 0.0141 \text{ radians, or } \underline{0.81^\circ}$$

which is much less than in Case 1.

Case 3

If these two channels were welded toe to toe to form a box section, the torsional resistance would be greatly increased.

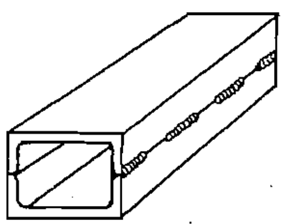


FIGURE 9

From Table 2, the value of R for a box section is found to be—

$$R = \frac{2b^2\ d^2}{\frac{b}{t_b} + \frac{d}{t_d}}$$

where:

$b = 6 - ⅜ = 5.625''$

$d = 4 - \frac{5}{16} = 3.6875''$

$$= \frac{2(5.625)^2(3.6875)^2}{\frac{5.625}{5/16} + \frac{3.6875}{⅜}}$$

$$= 30.91 \text{ in.}^4$$

and the angular twist is—

$$\theta = \frac{1000 \times 100}{(12 \times 10^6)\ 30.91}$$

$$= 0.00027 \text{ radians, or } \underline{0.015^\circ}$$

which is far less than in Case 2, which in turn was much better than Case 1.

Torsional Resistance Nomograph

A panel or other member may be sufficiently resistant to deflection by bending, and yet have very low torsional resistance.

The nomograph, Figure 10, permits the designer to quickly find the torsional resistance of a proposed design. The total torsional resistance of a built-up design equals the sum of the resistances offered separately by the members.

On this nomograph:

Line 1 = Type of section, or element of a built-up section. Observe caution as to meaning of letter symbols. For a solid rectangular section use the ratio of width (a) divided by thickness (b); for a hollow rectangular section use width (b) divided by depth (c).
Line 2 = Dimension (a), in.
Line 3 = Pivot line
Line 4 = Dimension (b), in.
Line 5 = Torsional resistance of the section (R), in.^{4} These values for each element are added together to give the total torsional resistance of the section, and the resistances of the sections are added to give the total torsional resistance of the frame or base. This is used in the design formula for angular twist, or in the next nomograph, Figure 14.

In the case of a member having a built-up cross-section, such as a T or I beam, read the Figure 10 nomograph for the R value of each element or area making up the section. Start at vertical Line 1 in the nomograph, using the scale to the right of it that expresses the rectangular element's a/b ratio. In the case of solid squares or rounds, and closed or open round tubes, go directly to the point on the scale indicated by the visual representation of the cross-section.

Notice that the meaning of *a* and *b* varies. In the case of a rectangular element, *a* is the longer dimension; but in the case of a hollow rectangle, *a* is the wall or plate thickness. The value of *a* or *b* on Lines 1, 2 and 4 must correspond, according to the type of section or element for which torsional resistance (R) is sought.

For hollow rectangular sections (of uniform wall or plate thickness), use the scale along the left of vertical Line 1 that expresses the ratio b/c. Here b = the section's width and c = its depth.

4. MAXIMUM SHEAR STRESS IN BUILT-UP SECTIONS

The maximum shear stress of a rectangular section in torsion lies on the surface at the center of the long side.

For the maximum shear stress on a narrow rectangular section or section element—

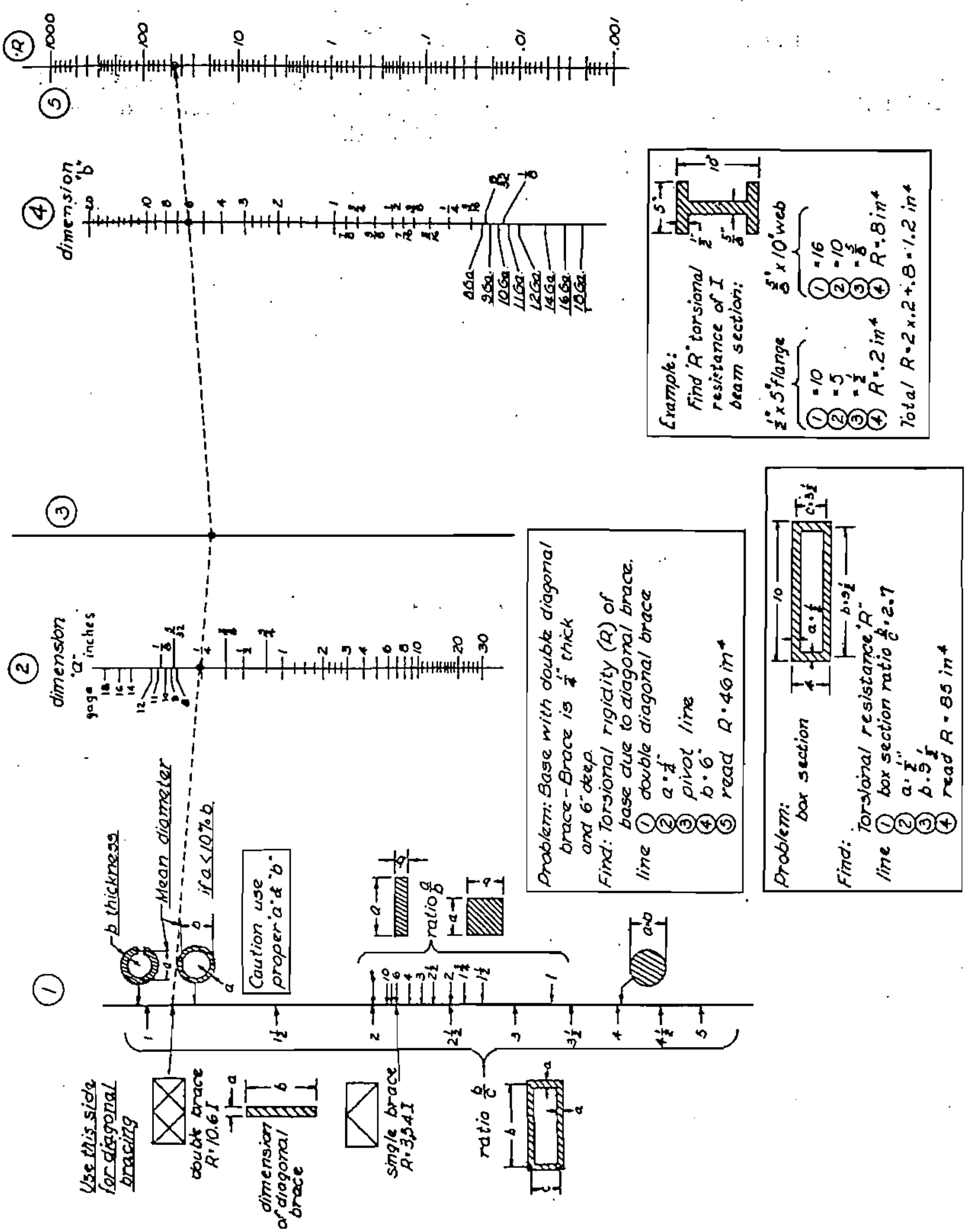


FIGURE 10—TORSIONAL RESISTANCE OF MEMBER

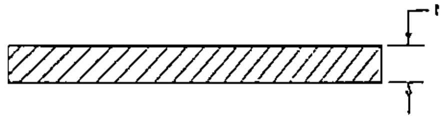


FIGURE 11

$$\tau = \phi \, t \, E_s = \frac{T\,t}{R}$$

where:

ϕ = unit angular twist of whole section (each element twists this amount), in radians/linear inch of member

t = thickness of rectangular section

R = torsional resistance of entire member, not necessarily just this one flat element

This formula can be used for a flat plate, or the flat plate of a built-up section not forming a closed section (i.e. channel, angle, T- or I-beam section).

In such a built-up open section, the unit angular twist (ϕ) of the whole member is first found:

$$\phi = \frac{\theta}{L}$$

and then the maximum shear stress in the specific rectangular element.

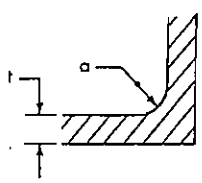


FIGURE 12

Shear stresses tend to concentrate at re-entrant corners. In this case, the maximum stress value should be used and is—

$$\tau_{max} = \tau\left(1 + \frac{t}{4a}\right)$$

where a = inside corner radius.

Problem 3

A 6″ × 2″ × 10½-lb channel is subjected to a torque of T = 1000 in.-lbs. Find the shear stress along the web. See Figure 13.

Applying the formula for rectangular sections from Table 1, find the torsional resistance of each of the two identical 2″ × ⅜″ flanges (R_f) and of the 6″ × 5/16″ web (R_2):

$$R_f = .0306 \text{ in.}^4$$

$$R_2 = .0586 \text{ in.}^4$$

$$\therefore\ R = 2R_1 + R_2 = 2\,(.0306) + .0586 = .1208 \text{ in.}^4$$

Then:

$$\tau = \frac{t\,T}{R} = \frac{5/16 \times 1000}{.1208} = \underline{2{,}580 \text{ psi}}$$

Problem 4

Two 6″ × 2″ × 10½-lb channels are welded toe to toe, to form a short box section. This is subjected to a torque of T = 100,000 in.-lbs. Find the horizontal shear stress at the toes and the amount of groove welding required to hold these channels together for this torsional load. See Figure 14.

From Table 2, the shear stress at mid-length of the short side is found to be—

$$\tau = \frac{T}{2\,[A]\,t}$$

where:

$b = 6 - ⅜ = 5.625″$

$d = 4 - 5/16 = 3.6875″$

$[A] = bd$

$$= \frac{100{,}000}{2(5.625 \times 3.687)⅜} = 6420 \text{ psi}$$

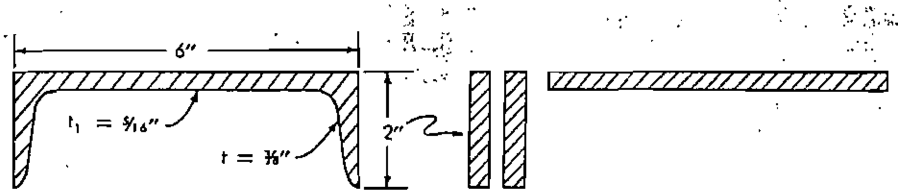
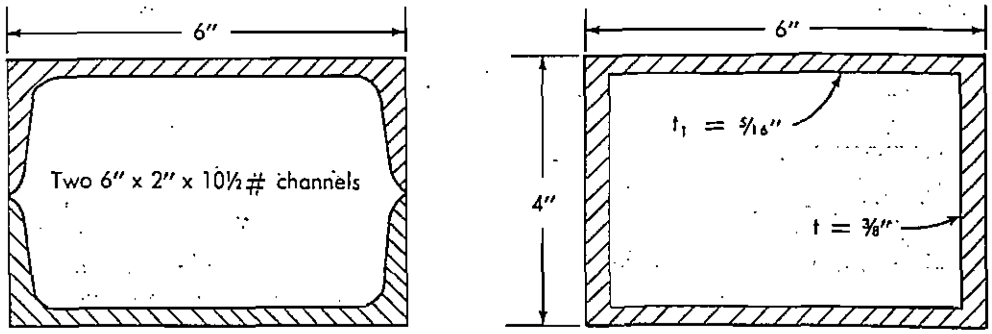


FIGURE 13

FIGURE 14



The horizontal shear force is then—

$$f = \tau\ t$$
$$= 6420 \times .375$$
$$= 2410 \text{ lbs/linear inch}$$

Since weld metal is good for 13,000 psi in shear, the throat or depth of the continuous butt weld must be—

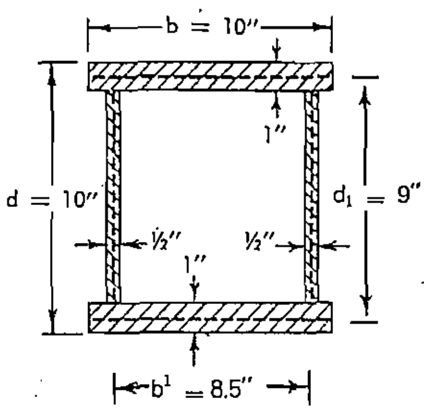
$$f = \tau_{weld}\ t$$
$$2410 = 13{,}000\ t$$
$$\text{or } t = \frac{2410}{13{,}000}$$
$$= .185'' \text{ or } \underline{3/16''}$$

The groove weld connecting the channels must have a throat depth of at least 3/16". Of course, if the torsional load is applied suddenly as an impact load, it would be good practice to add a safety factor to the computed load. This would then necessitate a deeper throat for the butt weld.

Problem 5

Check the following built-up spandrel beam supporting a wall 12′ high, made of 4" of limestone and 9" of brick. The beam's span is 20′, and the dead load of the wall is applied 6" off the beam's centerline.

FIGURE 15



4" limestone + 9" brick = 140 lbs/sq ft

Since the wall is 12′ high, this is a load of 1680 lbs/linear ft or 140 lbs/linear in. Or, use w = 150 lbs/lin in. to include beam weight.

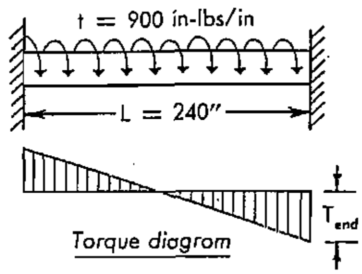
bending resistance (moment of inertia)

$$I_x = \frac{(10)(10)^3}{12} - \frac{(9)(8)^3}{12}$$
$$= 449.3 \text{ in.}^4$$

torsional resistance

$$R = \frac{2\ b_1{}^2\ d_1{}^2}{\frac{b_1}{t_b} + \frac{d_1}{t_d}}$$
$$= \frac{2\ (8.5)^2(9)^2}{\frac{(8.5)}{(1)} + \frac{(9)}{(½)}}$$
$$= 442 \text{ in.}^4$$

The eccentricity of the dead load applies torque to the beam. From torsional member diagrams in Reference Section 8.2:



uniform torque

$$t = 150 \text{ lbs/in.} \times 6''$$
$$= 900 \text{ in.-lbs/in.}$$

angular twist at center of beam

$$\theta_{\mathbb{C}} = \frac{t\ L^2}{8\ E_s\ R} = \frac{(900)(240)^2}{8(12 \times 10^6)(442)}$$
$$= .00122 \text{ radians (or .07°)}$$

torque at end

$$T = \frac{t\ L}{2}$$

$$= \frac{(900)(240)}{2}$$

$$= 108{,}000 \text{ in.-lbs}$$

torsional shear stress

$$\tau = \frac{T}{2\ [A]\ t_s}$$

where:

t_s = thickness of single web

$$= \frac{(108{,}000)}{2(8.5 \times 9)½}$$

$$= 1410 \text{ psi}$$

unit shear force from torque

$$f_t = \tau\ t$$

$$= (1410)(½)$$

$$= 700 \text{ lbs/in.}$$

unit shear force along N.A. from bending

$$V = w\ L/2$$

$$= (150)(120)$$

$$= 18{,}000 \text{ lbs}$$

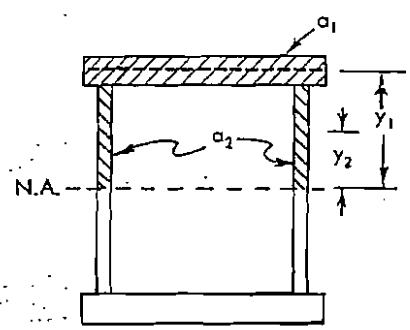


FIGURE 16

$$f_b = \frac{V\ (a_1\ y_1 + a_2\ y_2)}{I\ n}$$

$$= \frac{(18{,}000)(10 \times 4.5 + 1 \times 2.0)}{(449.3)(2 \text{ webs})}$$

$$= 860 \text{ lbs/in.}$$

total unit shear force on beam web (each)

$$f_s = f_t + f_b$$

$$= (700) + (860)$$

$$= 1560 \text{ lbs/in.}$$

total shear stress

$$\tau = \frac{f_s}{t_s}$$

$$= \frac{(2050)}{(½)}$$

$$= 4100 \text{ psi} \quad \underline{\text{OK}}$$

Then to determine the required size of fillet weld between flange and web:

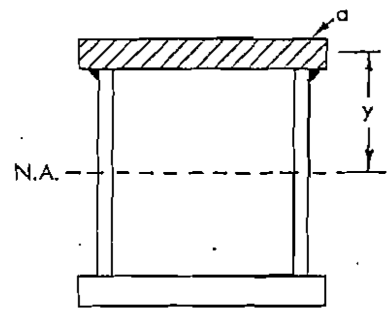


FIGURE 17

unit shear force at weld from bending

$$f_b = \frac{V\ a\ y}{I\ n}$$

$$= \frac{(18{,}000)(10)(4½)}{(449.3)(2)}$$

$$= 900 \text{ lbs/in.}$$

unit shear force at weld from torque

$$f_t = 700 \text{ lbs/in.}$$

total unit shear force at weld

$$f_s = f_t + f_b$$

$$= (700) + (900)$$

$$= 1600 \text{ lbs/in.}$$

required leg size of fillet weld (E70)

$$\omega = \frac{\text{actual force}}{\text{allowable force}}$$

$$= \frac{(1600)}{(11{,}200)}$$

$$= .143'' \text{ or } 3/16''$$

However, because of the 1″ flange, AWS Bldg. 212, AWS Bridge 217 and AISC 1.17.4 would require a 5/16″.

5. BUILT-UP FRAMES

The principles of torsion which determine the best sections for resisting twist apply to built-up frames. Just as the torsional resistance of the section is equal to the total of the resistances of its individual areas, so is the torsional resistance of a frame approximately equal to the total resistance of its individual parts.

The torsional resistance of the frame whose longitudinal members are two channels would be approximately equal to twice the torsional resistance of each channel section; Figure 18. The distance between these members for purpose of this example is considered to have no effect. Since the closed section is best for resisting twist, the torsional resistance of this frame could be greatly increased by making the channels into rectangular box sections through the addition of plate.

Problem 6

A frame is made of two 6" standard pipes, spaced 24" between centers, and having a length of 60". This frame supports a 10-hp motor running at 1800 rpm and driving a pump. Find the approximate twist of the frame under the load.

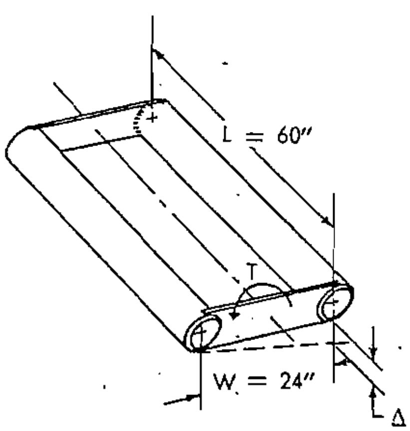


FIGURE 19

The 6" standard pipe has O.D. = 6.625" and I.D. = 6.065". In finding the torsional resistance of each tube:

$$R = .0982\ (d_2^4 - d_1^4)$$
$$= .0982\ (6.625^4 - 6.065^4)$$
$$= 56.30 \text{ in.}^4$$

The torque is easily found:

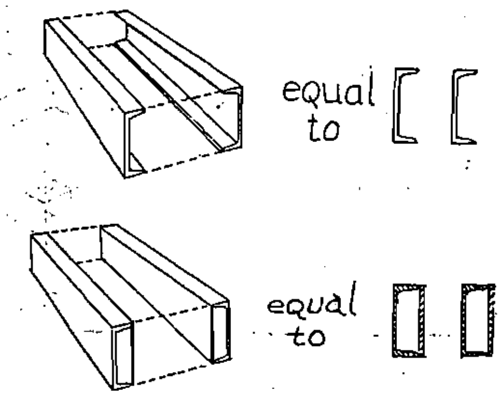


FIGURE 18

$$T = \frac{63{,}030 \times HP}{RPM}$$
$$= \frac{63{,}030 \times 10}{1800}$$
$$= 350 \text{ in.-lbs}$$

Then, adding together the R of each tube, the angular twist is:

$$\theta = \frac{T\ L}{E_s\ R}$$
$$= \frac{350 \times 60}{(12 \times 10^6)\ (2 \times 56.30)}$$
$$= 0.0000156 \text{ radians, or } 0.00089°$$

Maximum deflection in the frame is the vertical displacement (Δ), which is the product of angular twist (θ) and frame width (W) between centers:

$$\Delta = \theta\ W$$
$$= 0.0000156 \times 24''$$
$$= 0.00037''$$

6. DEFLECTION OF BUILT-UP FRAMES

In analyzing the resistance and strength of a built-up frame against twisting, consider the torque applied as two forces in the form of a couple at each end of the frame. In this manner, it is seen that these same forces apply a torque transverse to the frame as well as longitudinal to it.

This helps to show that the over-all resistance against twisting is the sum of the resistances of all the members, longitudinal as well as transverse. It is usually more convenient to express the resulting angular twist in terms of vertical deflection of the frame corner which receives the vertical load.

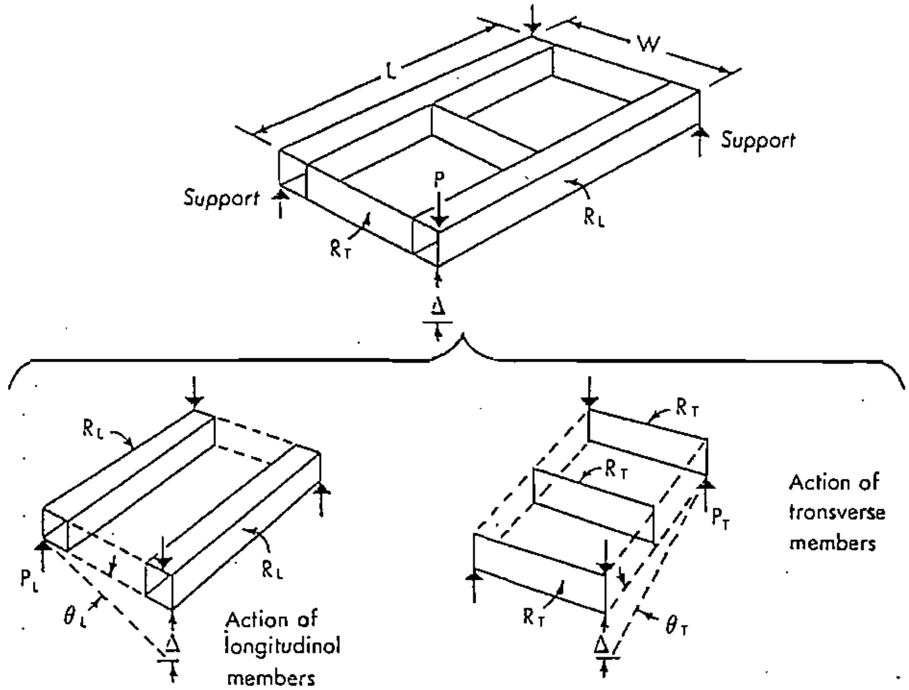


FIGURE 20

The longitudinal members are now considered to make up a frame of their own. When the vertical force (P_L) applied at the corner reaches the proper value, the frame will deflect vertically the given distance (Δ) and each longitudinal member will twist (θ_L). The same separate analysis is also made of the transverse members.

By observation we find—

$$\Delta = \theta_L W = \theta_T L$$

Then:

$$\theta_L = \frac{\Delta}{W} \text{ and } \theta_T = \frac{\Delta}{L}$$

Using the common formula for angular twist—

$$\theta_L = \frac{T_L\, L}{E_s\, n_L\, R_L} \text{ and } \theta_T = \frac{T_T\, W}{E_s\, n_T\, R_T}$$

and substituting for θ_L and θ_T —

$$\frac{\Delta}{W} = \frac{T_L\, L}{E_s\, n_L\, R_L} \text{ and } \frac{\Delta}{L} = \frac{T_T\, W}{E_s\, n_T\, R_T}$$

Then:

$$T_L = \frac{\Delta\, E_s\, n_L\, R_L}{W\, L} \text{ and } T_T = \frac{\Delta\, E_s\, n_T\, R_T}{W\, L}$$

Since the applied torque is—

$$T_L = P_L\, W \text{ and } T_T = P_T\, L$$

$$\therefore P_L = \frac{T_L}{W} \text{ and } P_T = \frac{T_T}{L}$$

and substituting for P_L and P_T —

$$P_L = \frac{\Delta\, E_s\, n_L\, R_L}{W^2\, L} \text{ and } P_T = \frac{\Delta\, E_s\, n_T\, R_T}{W\, L^2}$$

Since the external force (P) applied at the corner is the sum of these two forces:

$$P = P_L + P_T = \frac{\Delta\, E_s\, n_L\, R_L}{W^2\, L} + \frac{\Delta\, E_s\, n_T\, R_T}{W\, L^2}$$

$$= \frac{\Delta\, E_s}{W\, L}\left(\frac{n_L\, R_L}{W} + \frac{n_T\, R_T}{L}\right)$$

$$\therefore \boxed{\Delta = \frac{P\, L\, W}{E_s}\left[\frac{1}{\dfrac{n_L\, R_L}{W} + \dfrac{n_T\, R_T}{L}}\right]} \quad \ldots\ldots\ldots(4)$$

where:

L = length of whole frame, in.

W = width of whole frame, in.

R_L = torsional resistance of longitudinal member, $in.^4$

R_T = torsional resistance of transverse member, $in.^4$

n_L = number of longitudinal members

n_T = number of transverse members

P = load applied at corner, lbs

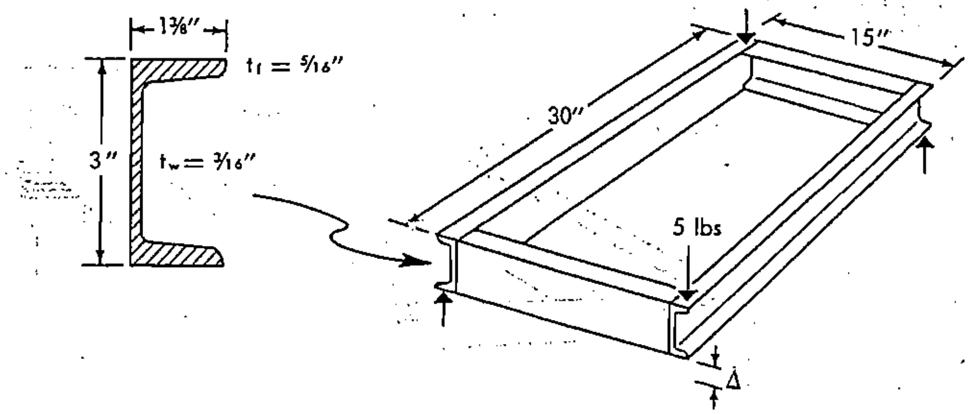


FIGURE 21

E_s = modulus of elasticity in shear (steel: 12×10^6), psi

Δ = vertical deflection, in.

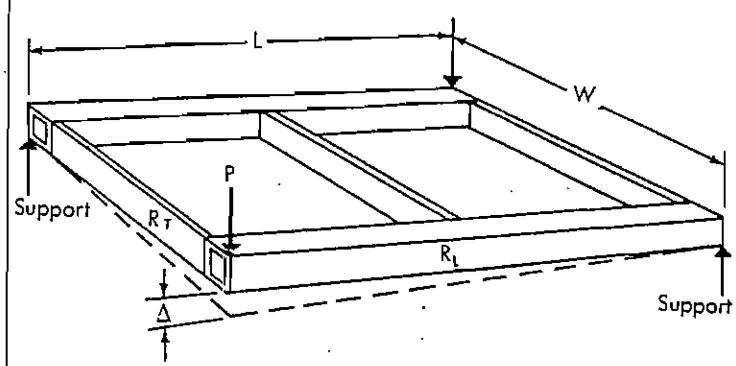
It can be seen that the torque on a given member is actually produced by the transverse forces supplied by the cross members attached to them. These same forces subject the cross members to bending. In other words, the torque applied to a member equals the end moment of the cross member attached to it. There is some deflection due to bending of all the members, and this would slightly increase the over-all deflection of the frame. For simplicity this has been neglected in this analysis.

Problem 7

To illustrate the use of the preceding deflection formula, consider a small elevator frame 15″ wide and 30″ long, made of standard 3″ channel, Figure 21. Find the

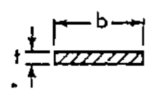
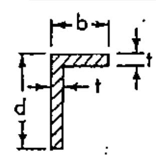
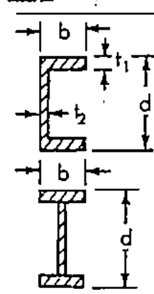
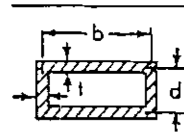
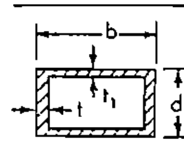
TABLE 4—Torsional Resistance of Frame and Various Sections

Deflection of Frame Under Torsional Load



$$\Delta = \frac{P\,L\,W}{E_s}\left[\frac{1}{\dfrac{n_L\,R_L}{W} + \dfrac{n_T\,R_T}{L}}\right]$$

Torsional Resistance of Common Sections

	$R = \dfrac{b\,t^3}{3}$
	$R = \dfrac{(b + d)t^3}{3}$
	$R = \dfrac{2\,b\,t_1^3 + d\,t_2^3}{3}$
	$R = \dfrac{2\,t\,b^2\,d^2}{b + d}$
	$R = \dfrac{2\,t\,t_1\,(b - t)^2\,(d - t_1)^2}{b\,t + d\,t_1 - t^2 - t_1^2}$

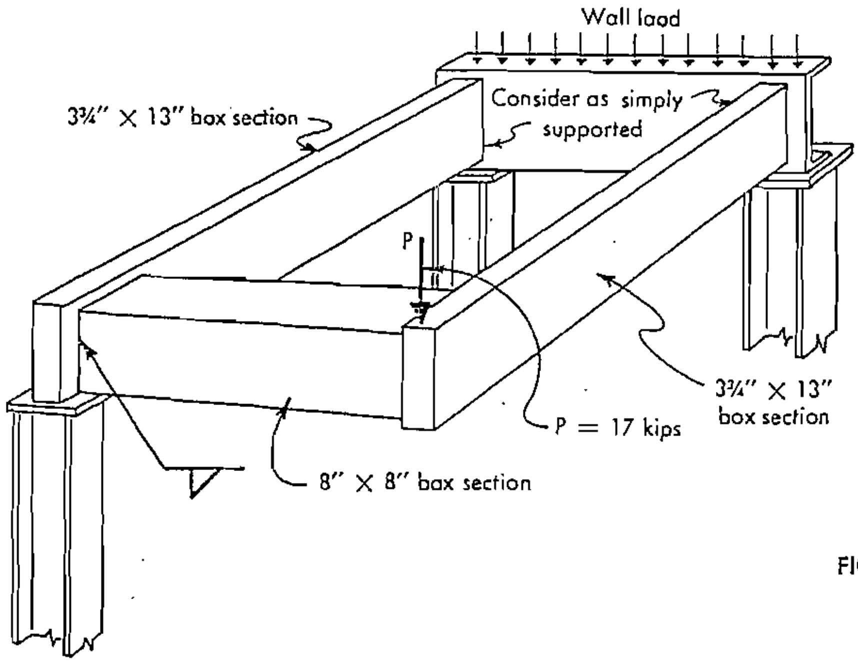


FIGURE 22

vertical deflection of the unsupported corner when under a load of 5 lbs.

Using the appropriate formula from Table 4, torsional resistance of the U channel cross-section is—

$$R = \frac{2\ bt_1^3 + dt_2^3}{3} = \frac{2\ bt_f^3 + dt_w^3}{3}$$

$$= \frac{2\ (1.375)(.3125)^3}{3} + \frac{3(\ 1875)^3}{3}$$

$$= .0346\ \text{in.}^4$$

Substituting actual values into formula #4:

$$\Delta = \frac{P\ L\ W}{E_s}\left[\frac{1}{\frac{n_L\ R_L}{W} + \frac{n_T\ R_T}{L}}\right]$$

$$= \frac{(5)(30)(15)}{(12 \times 10^6)}\left[\frac{1}{\frac{2(.0346)}{15} + \frac{2(.0346)}{30}}\right]$$

$$= \underline{.027''}$$

The actual deflection when tested was—

$$\Delta = .030''$$

Problem 8

The structural frame of Figure 22, simply supported at three corners, is designed to support a 17-kip load at its unsupported corner. Here the width between centerlines of the longitudinal members is 34.75″, and the latter are 82″ long. Determine:

a) The approximate vertical deflection of the unsupported corner,

b) the shear stress in longitudinal and transverse members, and

c) the size of the connecting weld between the longitudinal and transverse members.

torsional resistance of longitudinal members

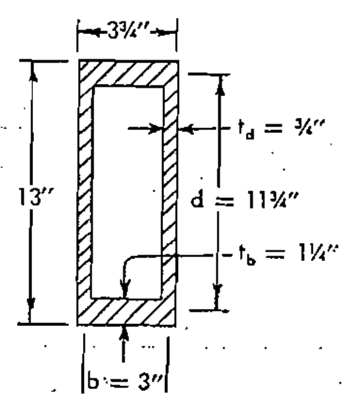


FIGURE 23

$$R_L = \frac{2\ b^2\ d^2}{\frac{b}{t_b} + \frac{d}{t_d}}$$

$$= \frac{2(3)^2\ (11\tfrac{3}{4})^2}{\frac{(3)}{(1\tfrac{1}{4})} + \frac{(11\tfrac{3}{4})}{(\tfrac{3}{4})}}$$

$$= 137.5\ \text{in.}^4$$

torsional resistance of transverse member (only one in this example)

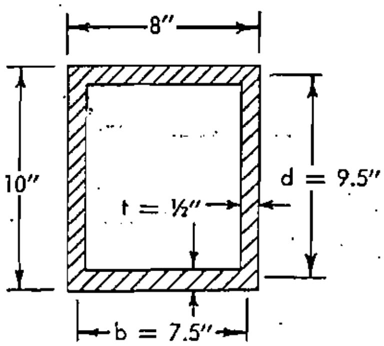


FIGURE 24

$$R_T = \frac{2\, b^2\, d^2}{\frac{b}{t_b} + \frac{d}{t_d}}$$

$$= \frac{2(7.5)^2(9.5)^2}{\frac{(7.5)}{(\frac{1}{2})} + \frac{(9.5)}{(\frac{1}{2})}}$$

$$= 298.3 \text{ in.}^4$$

vertical deflection of frame

$$\Delta = \frac{P\, W\, L}{E_s}\left[\frac{1}{\frac{n_L\, R_L}{W} + \frac{n_T\, R_T}{L}}\right]$$

$$= \frac{(17{,}000)(34\frac{3}{4})(82)}{(12 \times 10^6)}\left[\frac{1}{\frac{(2)(137.5)}{(34\frac{3}{4})} + \frac{(1)(298.3)}{(82)}}\right]$$

$$= .35''$$

shear stress in longitudinal member

The applied torque on only one longitudinal member is —

$$T_L = \frac{\Delta\, E_s\, n_L\, R_L}{W\, L}$$ See formula development, p. 2.10-12

$$= \frac{(35)(12 \times 10^6)(1)(137.5)}{(34\frac{3}{4})(82)}$$

$= 202{,}500$ in.-lbs, each member

The shear stress at midpoint of the longitudinal member, on the short side of its cross-section is —

$$\tau_b = \frac{T_L}{2\,[A]\,t_b}$$

$$= \frac{(202{,}500)}{2(3 \times 11\frac{3}{4})(1\frac{1}{4})}$$

$$= 2300 \text{ psi}$$

and the shear stress at midpoint of the member, on the long side of its cross-section is —

$$\tau_d = \frac{T_T}{2\,[A]\,t_d}$$

$$= \frac{(202{,}500)}{2(3 \times .11\frac{3}{4})(\frac{3}{4})}$$

$$= 3820 \text{ psi}$$

shear stress in transverse member

In a similar manner it is found that the applied torque on the transverse member is —

$$T_T = \frac{\Delta\, E_s\, n_t\, R_t}{W\, L}$$

See formula development, p. 2.10-12

$$= \frac{(.35)(12 \times 10^6)(1)(298.3)}{(34\frac{3}{4})(82)}$$

$= 438{,}500$ in.-lbs

Since the cross-section of the transverse member is a hollow rectangle of uniform thickness, the shear stress at mid-length along either side of the section is —

$$\tau = \frac{T_L}{2\,[A]\,t}$$

$$= \frac{(438{,}500)}{2(7.5 \times 9.5)(\frac{1}{2})}$$

$$= 6160 \text{ psi}$$

size of connecting fillet weld

Treating the weld as a line —

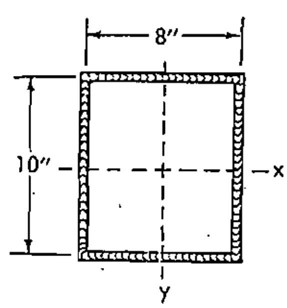


FIGURE 25

$$I_x = \frac{b\, d^2}{2} + \frac{d^3}{6}$$

$$= \frac{(8)(10)^2}{2} + \frac{(10)^3}{6}$$

$$= 566.7 \text{ in.}^3$$

$$I_y = \frac{b^2\ d}{2} + \frac{b^3}{6}$$
$$= \frac{(8)^2(10)}{2} + \frac{(8)^3}{6}$$
$$= 405.3 \text{ in.}^3$$

and the polar moment of inertia is —

$$J_w = I_x + I_y$$
$$= (566.7) + (405.3)$$
$$= 972 \text{ in.}^3$$

Assuming just two vertical welds transfer vertical shear (V), the length of the weld is —

$$L_w = 2 \times 10 = 20''$$

torque on weld

From the standard design formula for torsion —

$$\tau = \frac{T\ c}{J} \text{ lbs/in.}^2 \text{(stress)}$$

the corresponding formula for total weld force is obtained —

$$f_t = \frac{T\ c}{J} \text{ lbs/in. (force per linear inch of weld)}$$

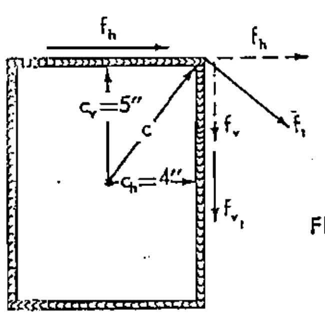


FIGURE 26

The horizontal component of this torque is —

$$f_h = \frac{T\ c_v}{J_w}$$
$$= \frac{(438{,}500)(5)}{(972)}$$
$$= 2250 \text{ lbs/in.}$$

and the vertical component of this torque is —

$$f_v = \frac{T\ c_h}{J_w}$$
$$= \frac{(438{,}500)(4)}{972}$$
$$= 1805 \text{ lbs/in.}$$

vertical shear on weld

Since the vertical shear on the joint is —

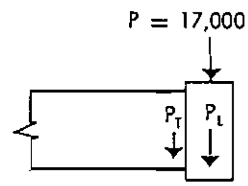


FIGURE 27

$$V = P_T = \frac{T_L}{W}$$
$$= \frac{(202{,}500)}{(34\tfrac{3}{4})}$$
$$= 5825 \text{ lbs}$$

the resultant force on the vertical welds is —

$$\boxed{A_w = L_w}$$

$$f_{v1} = \frac{V}{A_w}$$
$$A_w = L_w$$
$$= \frac{(5825)}{(20'')}$$
$$= 290 \text{ lbs/in.}$$

Notice that, if the load (P) is applied to the end of the transverse member instead of the longitudinal member, the portion going back into the longitudinal member (P_L = 17,000 − 5825 = 11,175 lbs) must be transferred through the connecting weld and the resulting unit force from vertical shear is:

$$f_{v1} = \frac{V}{A_w}$$
$$= \frac{(11{,}175)}{(20)}$$
$$= 560 \text{ lbs/in. instead of } 290 \text{ lbs/in.}$$

moment on weld

Since the bending moment on the joint is—

$$M = T_L$$
$$= 202{,}500 \text{ in.-lbs}$$

the resultant force on the weld is —

$$f_m = \frac{M\ c}{I_x}$$
$$= \frac{(202{,}500)(5)}{(566.7)}$$
$$= 1785 \text{ lbs/in.}$$

Resolving combined forces on weld at point of greatest effect—

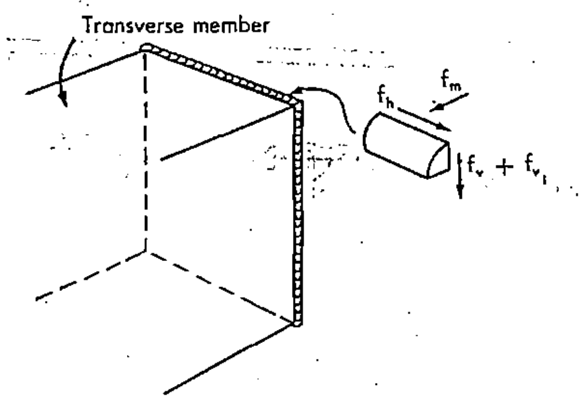


FIGURE 28

$$f_r = \sqrt{f_h^2 + f_m^2 + (f_v + f_{v1})^2}$$
$$= \sqrt{(2250)^2 + (1785)^2 + (1805 + 290)^2}$$
$$= 3560 \text{ lbs/in.}$$

Since 11,200 lbs is the accepted allowable load per linear inch of fillet weld having a 1″ leg size, the minimum leg size for this application is—

$$\omega = \frac{3560}{11{,}200} \leftarrow \text{(E70-weld allowable)}$$
$$= .318''$$

or use 5/16″ fillet weld.

7. BRACING OF FRAMES

The two main stresses on a member under torsional loading are (1) transverse shear stresses and (2) longitudinal shear stresses.

These two stresses combine to produce diagonal tensile and compressive stresses which are maximum at 45°. At 45°, the transverse and longitudinal shear stresses cancel each other. Therefore, there is no twisting stress or action on a diagonal member placed at 45° to the frame.

In a frame made up of flat members, the transverse shear stresses cause the longitudinal members to twist. The longitudinal shear stresses cause the cross braces and end members to twist.

On a diagonal member at 45° to axis of twist, the transverse and longitudinal shear stress components are opposite in direction to each other and cancel out, but in line with this member they combine to produce diagonal tensile and compressive stresses which tend

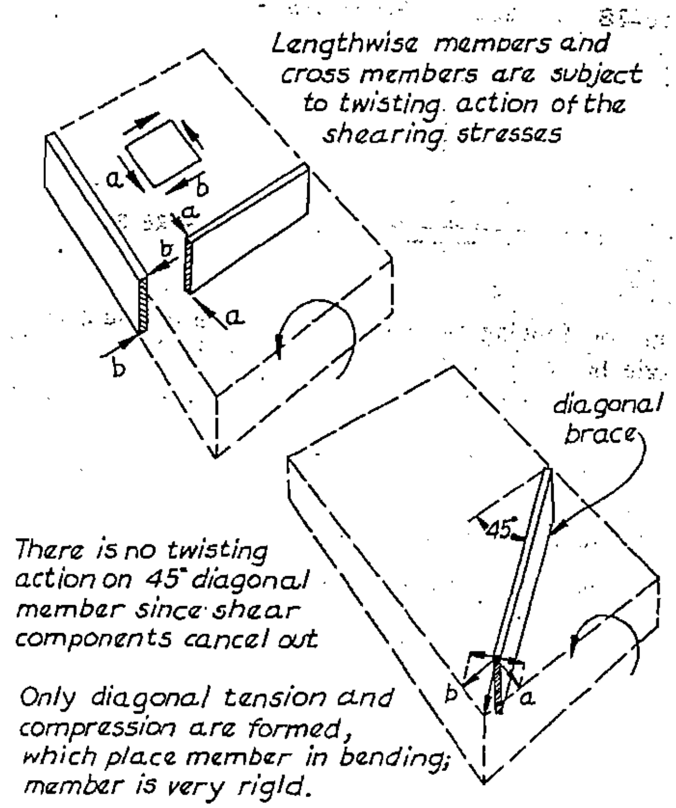


FIGURE 29

to cause bending rather than twisting. See Figure 29.

Since these two shear stresses cancel out, there is no tendency for a diagonal member placed in this direction to twist.

The diagonal tensile and compressive stresses try to cause this diagonal member to bend; but being very resistant to bending, the diagonal member greatly stiffens the entire frame against twisting.

Design Rule No. 2: USE DIAGONAL BRACING

Stiffening the Braces

Previous experience in designing longitudinal side members for bending is now used to design these diagonal members.

It is important that the diagonal members have a high moment of inertia to provide sufficient stiffness so there will be no failure from local buckling, under severe torsional loads.

Since the diagonal brace is not subjected to any twisting action, it is not necessary to use a closed box section.

For short diagonal braces, use a simple flat bar. The top and/or bottom panel of the frame will stiffen this to some extent (Fig. 30). As the unsupported length of the diagonal brace becomes longer, it may become necessary to add a flange (Fig. 31). This is

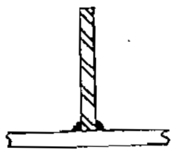


FIGURE 30

done by flanging one edge of the brace or using an angle bar or T section. The flange of the brace may also be stiffened to keep it from buckling.

For open frames with no flat panel, it is better to use a channel or I beam section having two flanges (Fig. 32).

Relative Effectiveness of Bracing

Tests were made on scale models of typical machine frames to illustrate increase in resistance to twist as a result of the diagonal bracing.

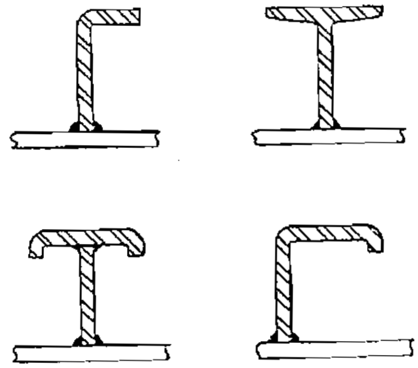


FIGURE 31

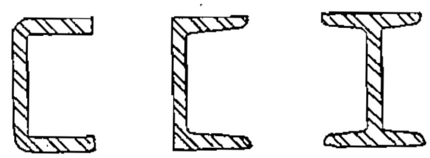


FIGURE 32

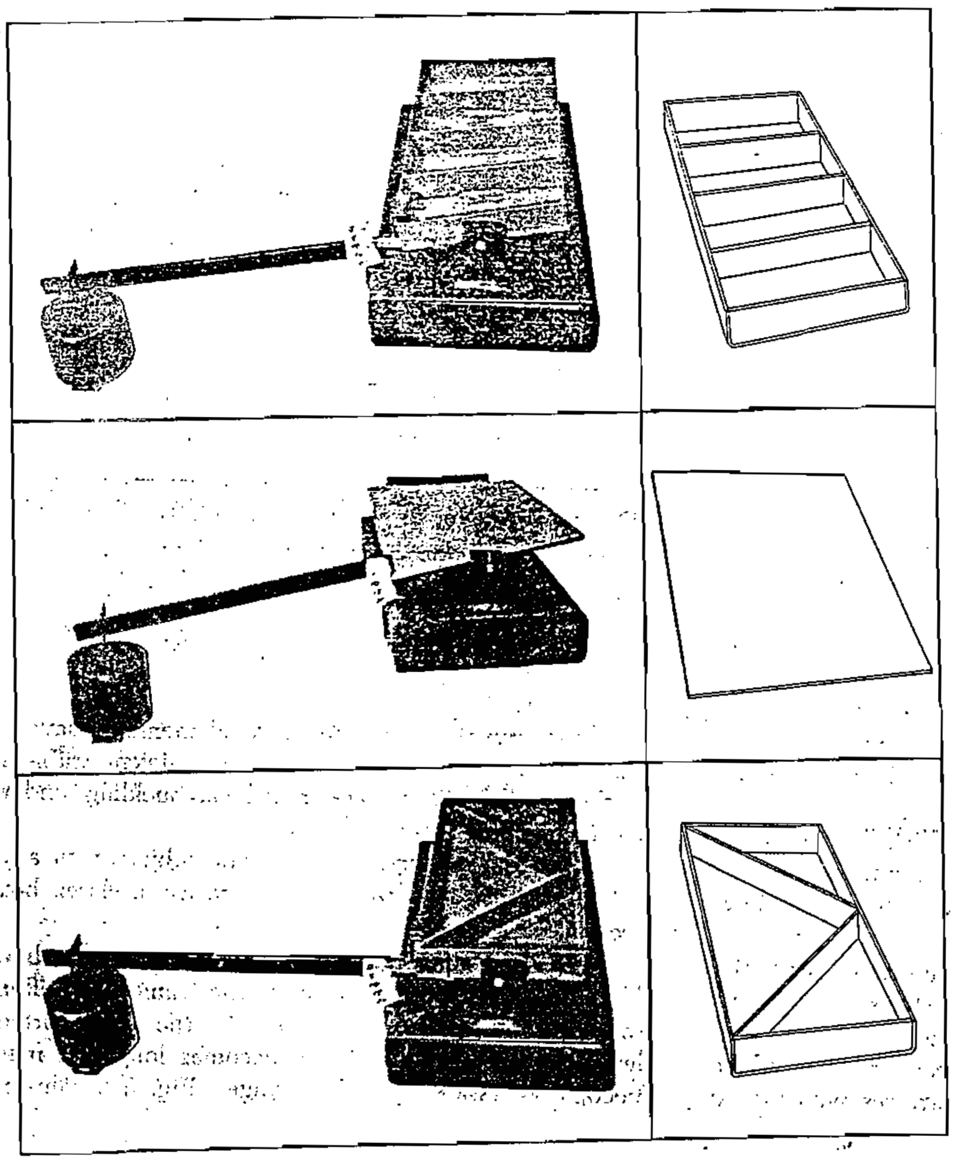


FIGURE 33

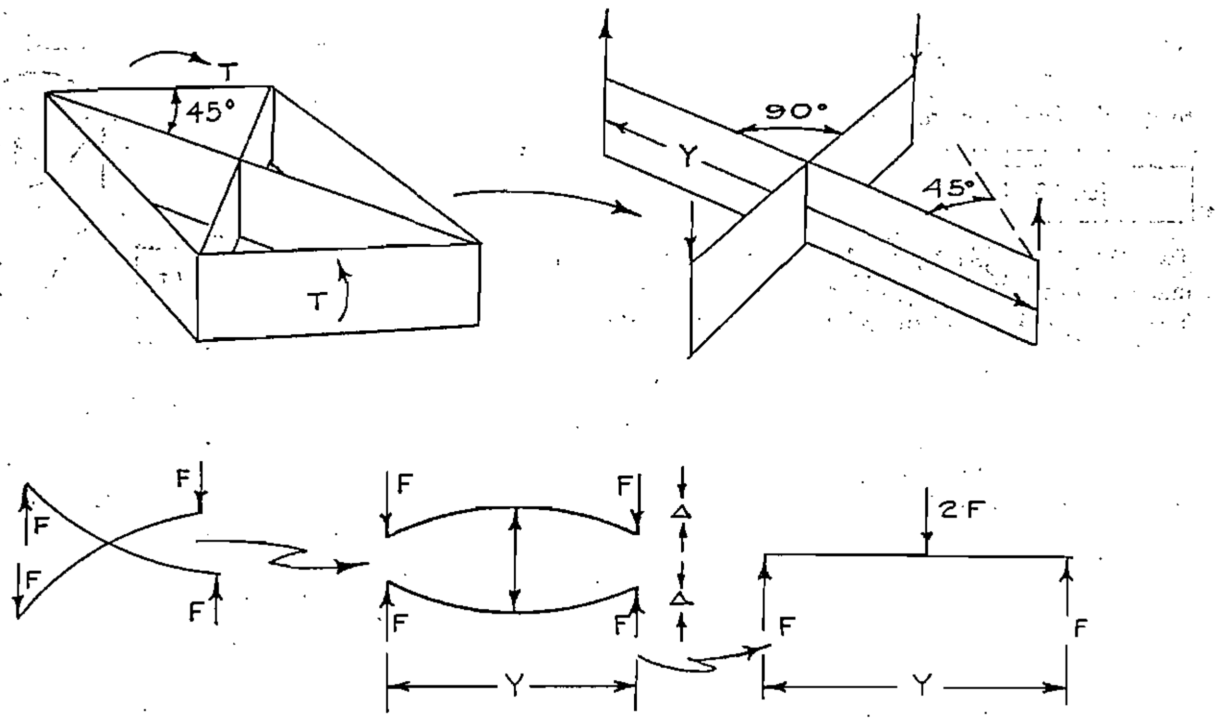


FIGURE 34

The top frame in Figure 33 has conventional cross bracing at 90° to side members. It twisted 9°.

The above frame is little better in resistance to twist than a flat sheet of the same thickness, as shown in the middle. The plain sheet twisted 10°.

The bottom frame has diagonal braces at 45° with side members. It twisted only ¼°. It is 36 times as resistant to twisting as the first frame, yet uses 6% less bracing material.

8. DIAGONAL BRACING (Double)

(See Figure 34)

An approximate indication of the angular twist of a frame using double diagonal bracing (in the form of an X) may be made by the following procedure. Here each brace is treated as a beam.

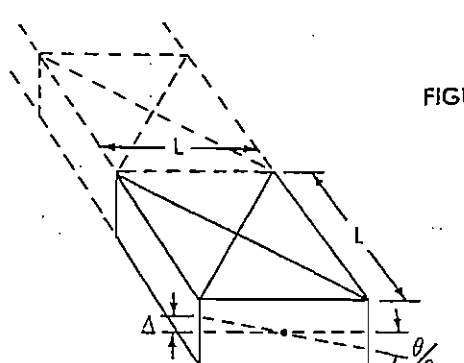


FIGURE 35

$$\Delta = \frac{(2\ F)\ Y^3}{48\ E\ I} \qquad \text{(simply supported)}$$

$$\theta/2 = \frac{\Delta}{½\ L} = \frac{2\ \Delta}{L} = \frac{F\ Y^3}{12\ E\ I\ L}$$

Since $T = F\ L$, then $F = \frac{T}{L}$

$$\therefore\ \theta = \frac{T\ Y^3}{6\ E\ I\ L^2}$$

Since $Y = \sqrt{2}\ L$

$$\theta = \frac{T\ (\sqrt{2})^3\ L^3}{6\ E\ I\ L^2} = \frac{\sqrt{2}\ T\ L}{3\ E\ I}$$

also $\theta = \frac{T\ L}{E_s\ R}$ Hence $\frac{\sqrt{2}\ T\ L}{3\ E\ I} = \frac{T\ L}{E_s\ R}$

and $R = \frac{3\ E\ I}{\sqrt{2}\ E_s} = 5.3\ I$

$E = 30 \times 10^6$
$E_s = 12 \times 10^6$

For fixed ends, $R = 21.2\ I$

For the usual frame, the following is suggested:

$$\boxed{R = 10.6\ I}$$

which appeared in Table 1.

Therefore: For a double diagonal brace use $R = 10.6\ I$ and substitute this value into the standard formula: $\theta = \dfrac{T\ L}{E_s\ R}$

to get the frame's angular twist (radians).

Problem 9

Two ¼″ × 10″ plates, 40″ long, spaced 20″ apart to make a frame 40″ long, are subjected to a torque of T = 1000 in.-lbs. Find the relative angular twist on the frame, when using conventional and diagonal bracing.

Case 1 (Conventional bracing)

Here the torsional resistance of the plate section is known, from Table 4, to be—

$$R = \frac{b\ t^3}{3}$$

$$\therefore R = 2\,\frac{(10)(.25)^3}{3}$$

$$= .104 \text{ in.}^4 \text{ (both sides)}$$

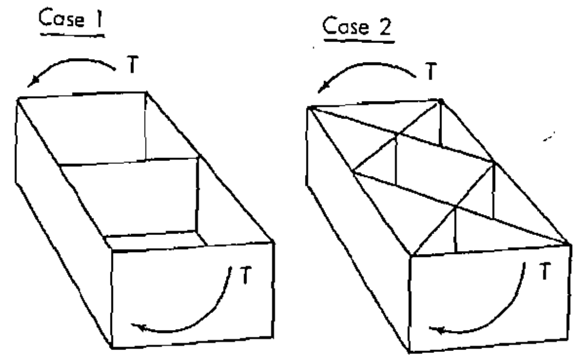


FIGURE 36

The total angular twist is then—

$$\theta = \frac{T\ L}{E_s\ R}$$

$$= \frac{(1000)(40)}{(12 \times 10^6)(.104)}$$

$$= .0321 \text{ radians or } \underline{1.84°}$$

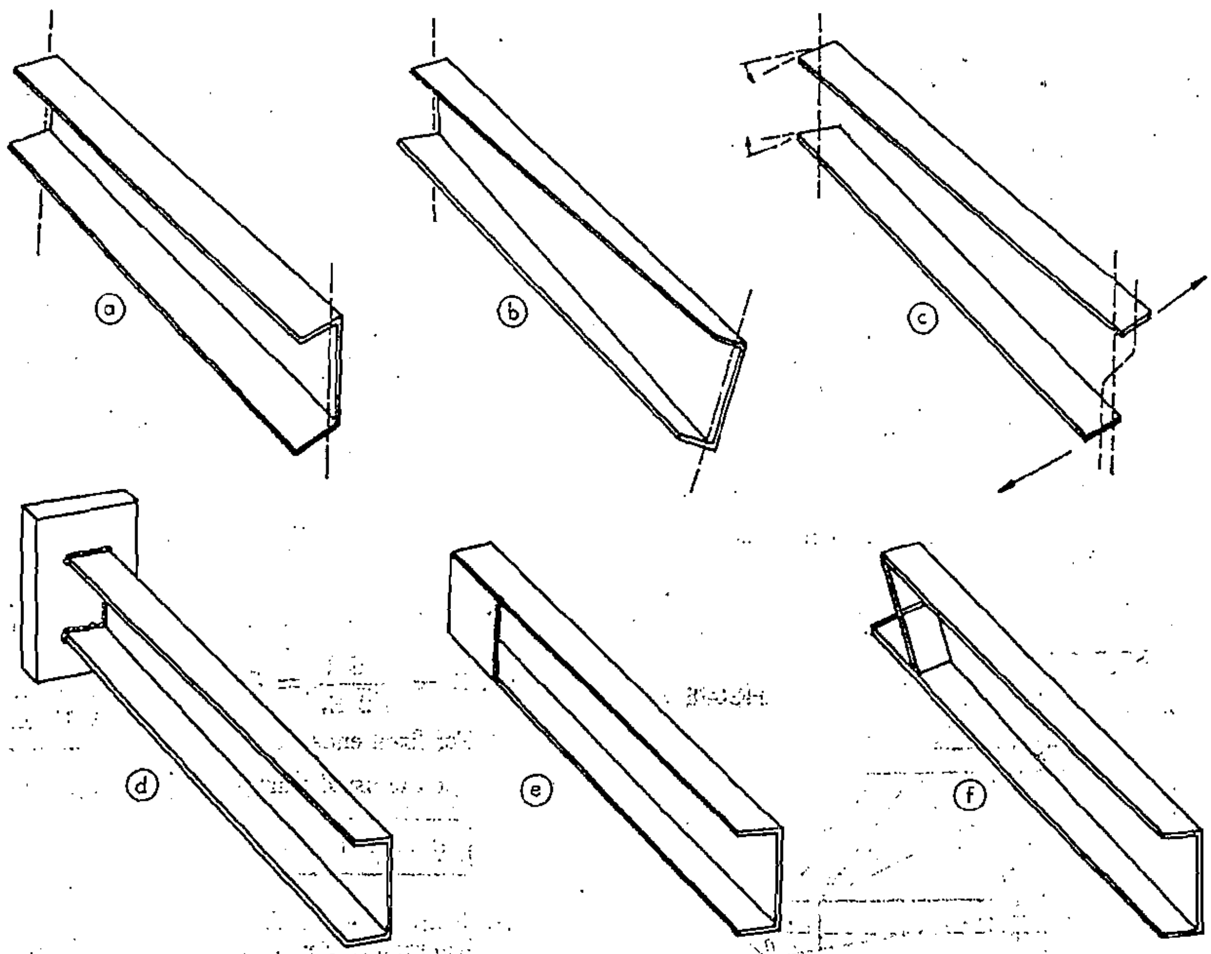


FIGURE 37

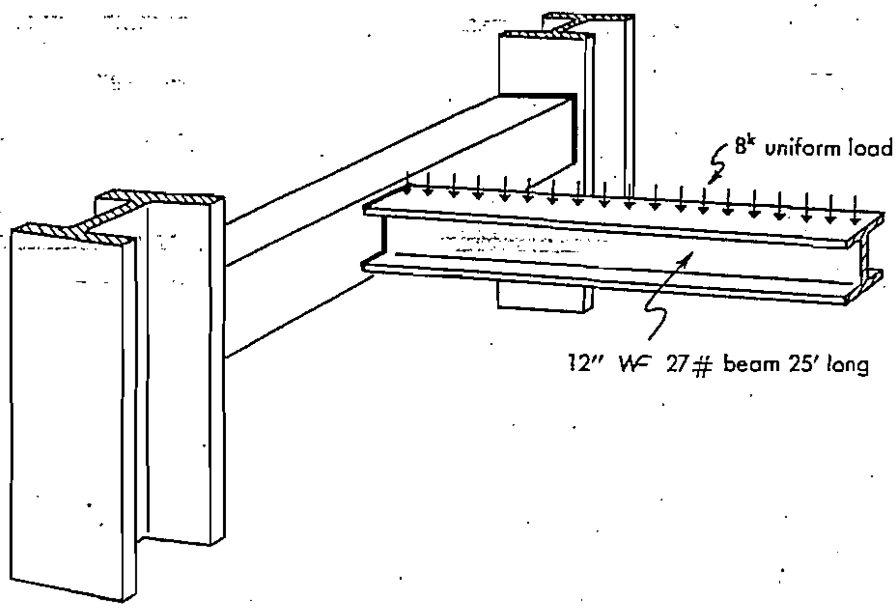


FIGURE 38

Case 2 (Diagonal bracing)

Since this is "double" bracing, the Table 1 formula for this type of frame is used —

$$R = 10.6\ I$$

First find the moment of inertia for the cross-section of a brace, which is a simple rectangle, assuming the brace also is ¼" × 10":

$$I = \frac{b\ d^3}{12}$$

where b = the section width (plate thickness), and d = the section depth

$$I = \frac{.25(10)^3}{12}$$

$$= 20.83 \text{ in.}^4$$

then substituting into the formula for R —

$$R = 10.6\ (20.83)$$

$$= 221 \text{ in.}^4$$

The angular twist on the frame is then —

$$\theta = \frac{T\ L}{E_s\ R}$$

$$= \frac{(1000)(40)}{(12 \times 10^6)(221)}$$

$$= .0000152 \text{ radians or } \underline{.00087°}$$

9. END CONNECTIONS OF TORSION MEMBERS

When a member having an open section is twisted, the cross-section warps (see b, in Fig. 37) if ends of the member are free. The flanges of these members not only twist, but they also swing outward (see c), allowing the member to twist more. If the ends of the flanges can be locked in place in relation to each other, this swinging will be prevented.

Design Rule No. 3: MAKE RIGID END CONNECTIONS

There are several methods of locking the flanges together. The simplest is to weld the end of the member to the supporting member as in (d). If the supporting member is then neither thick enough nor rigid enough, a thin, square plate may be welded to the two flanges at the end of the member (e). Another method is to use diagonal braces between the two flanges at the two ends of the member (f).

Either of these methods reduces the angular twist by about ½.

Members having a box section, when butt welded directly to a primary member, have the fully rigid end connections required for high torsional resistance.

Problem 10

A 12" WF 27-lb beam, 25' long, with a uniformly distributed load of 8 kips, is supported at each end by a box girder. See Figure 38. If the beam is continuously welded to these girders, estimate a) the resulting end

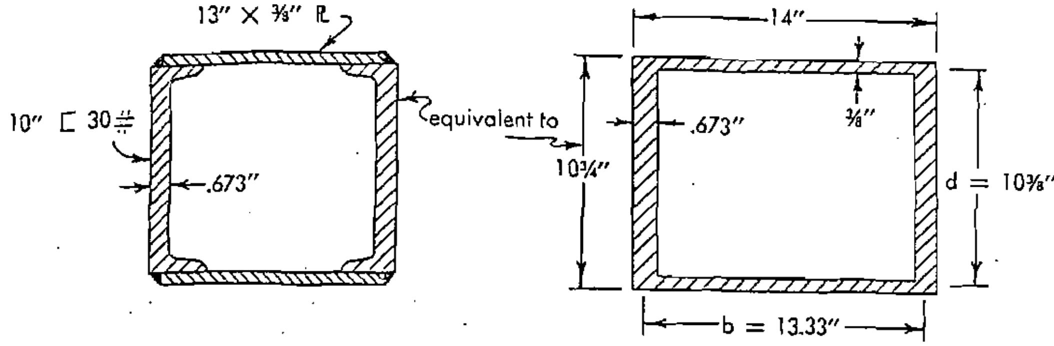


FIGURE 39

moments in the beam, b) the torsional stresses in the girder, and c) the weld size required to hold the box girder together.

torsional resistance of box girder

$$R = \frac{2\ b^2\ d^2}{\frac{b}{t_b} + \frac{d}{t_d}} \qquad \text{(See Figure 39)}$$

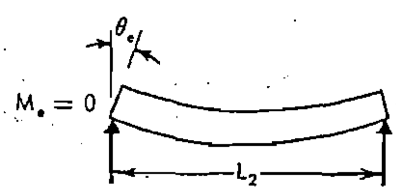
$$= \frac{2(13.33)^2(10\tfrac{3}{8})^2}{\frac{(13.33)}{(\frac{1}{2})} + \frac{(10\tfrac{3}{8})}{(.673)}}$$

$$= 910\ \text{in.}^4$$

Torque in the central section of the box girder support is equal to the end moment of the supporting beam.

end moment of beam

See Sect. 8.1 Beam Formulas.



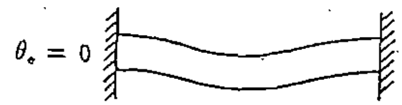
If the beam is simply supported without any end restraint, the end moment (M_e) is zero, and the slope of the beam at the end is —

$$\theta_e = \frac{W\ L_2^2}{24\ E\ I_2}$$

$$= \frac{(8^k)(25' \times 12'')^2}{24(30 \times 10^6)(204.1)}$$

$$= .0049\ \text{radians}$$

Now, if the ends of the beam are so restrained that it cannot rotate, the end moment becomes —



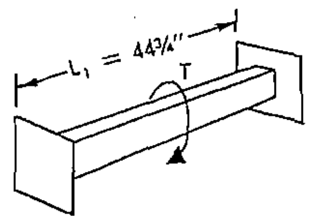
$$M_e = \frac{W\ L_2}{12}$$

$$= \frac{(8^k)(25' \times 12'')^2}{12}$$

$$= 200\ \text{in.-kips}$$

torque on box girder

See Sect. 8.2 Torsional Member Formulas.



Determine what torque must be applied to the central section of the supporting box girder to cause it to rotate the same amount as the end rotation of the supported beam, if simply supported (θ_e = .0049 radians):

$$\theta_{\mathfrak{C}} = \frac{T\ L_1}{4\ E_s\ R}$$

$$\text{or } T = \frac{4\ E_s\ R\ \theta_{\mathfrak{C}}}{L_1}$$

$$= \frac{4(12 \times 10^6)(910)(.0049)}{(44\tfrac{3}{4}'')}$$

$$= 4780\ \text{in.-kips}$$

A moment-rotation chart shows the relationship; see Figure 40. A straight line represents the end moment (M_e) and end rotation (θ_e) of the supported beam

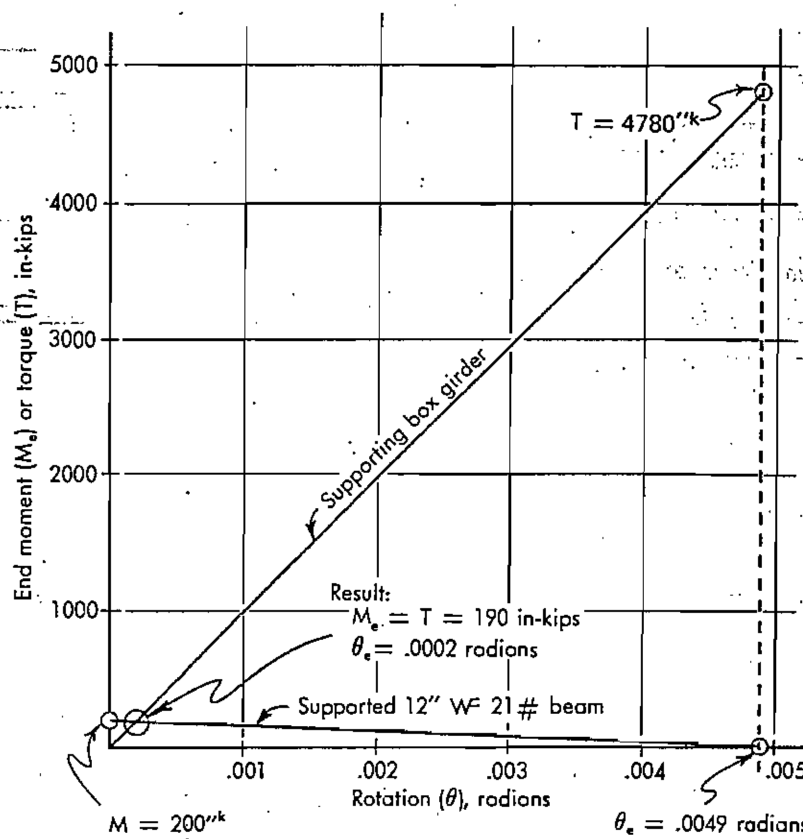


FIGURE 40

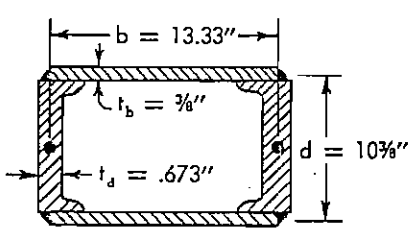
under all conditions of end restraint. A similar straight line, but in the opposite direction, represents the applied torque (T) and angular rotation (θ) at the central section of the supporting box girder.

These two lines are plotted, and where they intersect is the resulting end moment (M_e) or torque (T) and the angular rotation (θ):

$$M_e = T = 190 \text{ in.-kips}$$
$$\theta_e = .0002 \text{ radians}$$

torsional shear stresses in box girder

FIGURE 41



$$\tau_b = \frac{T}{2\,[A]\,t_b}$$
$$= \frac{(190''^{k})}{2(13.33 \times 10\tfrac{3}{8})(\tfrac{3}{8})}$$
$$= 1830 \text{ psi}$$

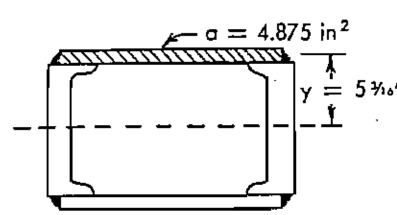
torsional shear force on fillet weld

$$f_1 = \tau_b\, t_b$$
$$= (1830)(\tfrac{3}{8})$$
$$= 690 \text{ lbs/lin in.}$$

which must be transferred by the fillet weld joining the top and bottom plates to the side channels, to make up the box girder.

horizontal shear force on fillet weld due to bending

FIGURE 42



$$I = 2(103.0) + 2(4.875)(5\tfrac{3}{16})^2$$
$$= 468 \text{ in.}^4$$

Half of the 8-kip load goes to each end of the beam, or a 4-kip load is applied to the central section of each box girder. And V = 2 kips.

$$\therefore f_2 = \frac{V\ a\ y}{I\ n}$$

$$= \frac{(2^k)(4.875)(5\tfrac{5}{16})}{(468)(2 \text{ welds})}$$

$$= 54 \text{ lbs/lin in.}$$

total shear force on weld

$$f = f_1 + f_2$$

$$= (690) + (54)$$

$$= 744 \text{ lbs/lin in.}$$

required leg size of fillet weld (E70 welds)

$$\omega = \frac{\text{actual force}}{\text{allowable force}}$$

$$= \frac{744}{11{,}200}$$

$$= .066'' \text{ (continuous)}$$

However, AWS and AISC would require a minimum fillet weld leg size of $\tfrac{3}{16}''$ (See Section 7.4).

If intermittent fillet welds are to be used, the length and spacing of the welds would be—

$$\% = \frac{\text{calculated leg size of continuous weld}}{\text{actual leg size of intermittent weld used}}$$

$$= \frac{(.066)}{(\tfrac{3}{16})}$$

$$= 35\%$$

or use $\tfrac{3}{16}''$ $\bigvee$ $3'' - 8''$

Alternate Design

As a matter of interest, consider the support to be provided by a 10″ WF 39-lb beam.

(See Figure 43)

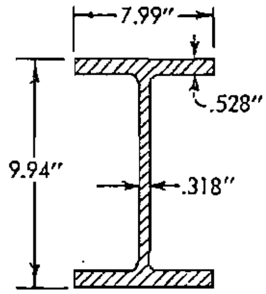


FIGURE 43

torsional resistance of supporting beam

$$R = \frac{2\ b\ t_f^3}{3} + \frac{d\ t_w^3}{3}$$

$$= \frac{2(7.99)(.528)^3}{3} + \frac{(9.94)(.318)^3}{3}$$

$$= 0.89 \text{ in.}^4$$

torque on supporting beam

Determine what torque must be applied to the central section of this supporting beam for it to rotate the same amount as the end rotation of the supported beam, if simply supported ($\theta_e = .0049$ radians):

$$\theta = \frac{T\ L_1}{4\ E_s\ R}$$

or $$T = \frac{4\ E_s\ R\ \theta}{L_1}$$

$$= \frac{4(12 \times 10^6)(0.89)(.0049)}{(44\ \tfrac{3}{4})}$$

$$= 4.67 \text{ in.-kips}$$

The moment-rotation diagram, Figure 44, shows the resulting end moment on the supported beam to be 4.67 in.-kips. Thus, this beam could be connected as a

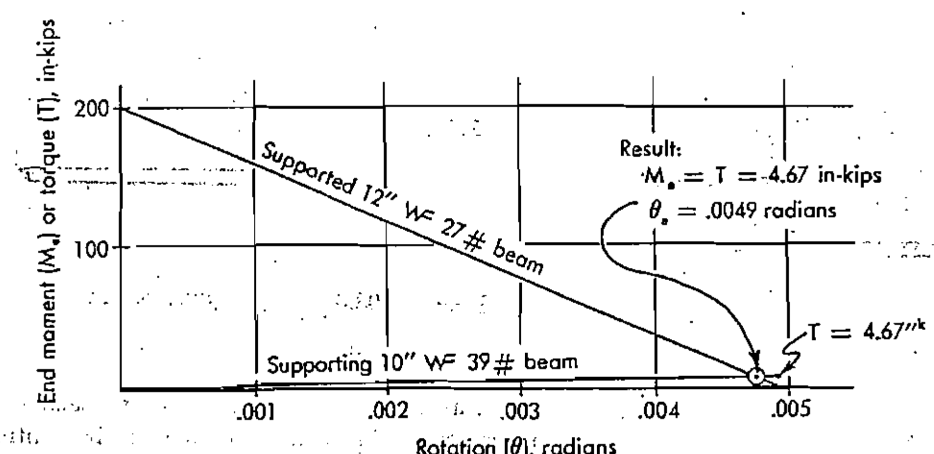


FIGURE 44

simply supported beam with just vertical welds on the web sufficient to carry the 4-kip shear reaction. The end restraint is about 2.3%.

10. MEMBRANE ANALOGY

Membrane analogy is a very useful method to understand the behavior of open sections when subjected to torsion. To make use of this method, holes are cut into a thin plate making the outline of various shaped sections. A membrane material such as soap film is spread over the open surface and air pressure is applied to the film. The mathematical expressions for the slope and volume of this membrane or film covering the openings representing different cross-sections are the same as the expressions for the shear stresses and torsional resistance of the actual member being studied. It is from this type of analysis that formulas for various types of open sections subjected to torsion have been developed and confirmed.

If several outlines are cut into the thin plate and the same pressure applied to each membrane, the following will be true:

1. The volumes under the membranes will be proportional to the torsional resistances of the corresponding sections.

2. The slope of the membrane's surface at any point is proportional to the shear stress of the section at this point.

3. A narrow section (thin plate) has practically the same torsional resistance regardless of the shape of the section it is formed into. Notice a, b, and c in Figure 45. For a given area of section, the volume under the membrane remains the same regardless of the shape of the section.

It is possible to determine the torsional resistance of these open sections by comparing them with a standard circle on this same test plate whose torsional resistance can readily be calculated.

By comparing the membrane of the slotted open tube, (c) in Figure 45, to that of the membrane of the closed tube (e), it is readily seen why the closed tube is several hundred times more resistant to twist, when it is remembered that the volume under the membrane is proportional to the torsional resistance.

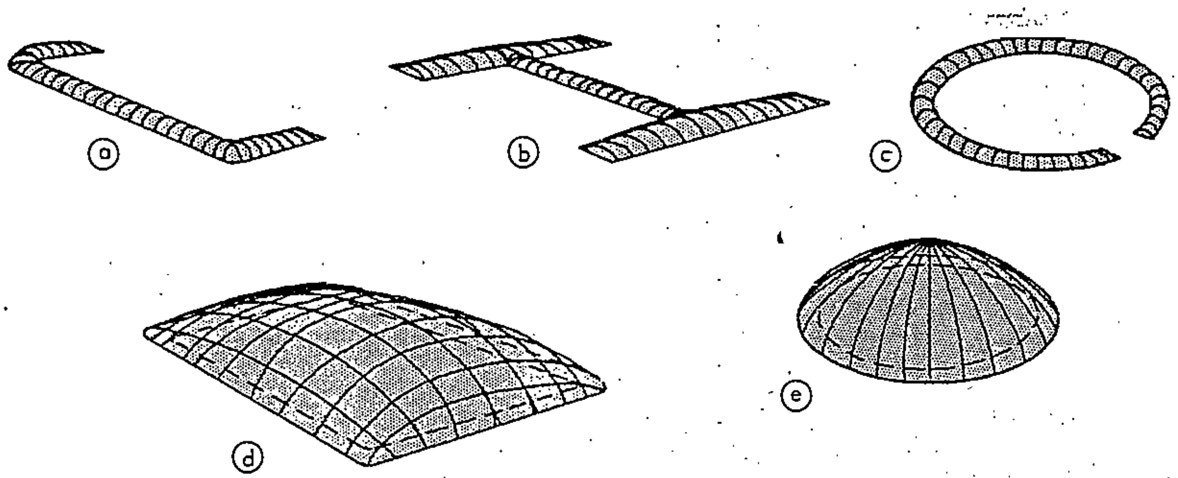
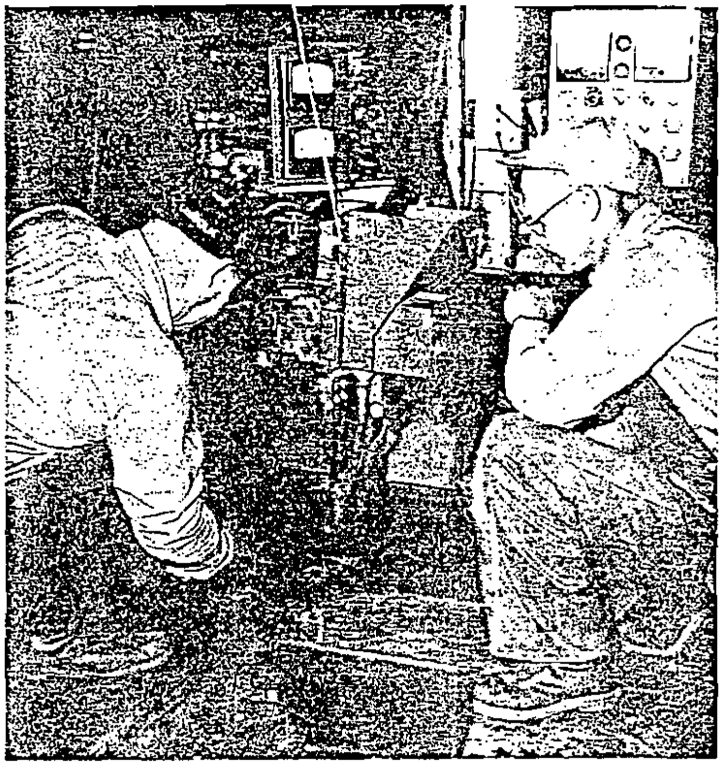


FIGURE 45



Modern structural steel shops are equipped with highly efficient equipment for the welding of fabricated plate girders. Here an automatic submerged-arc welder runs a transverse splice in $\frac{7}{8}''$ web plate to full width, with the aid of a small runout tab previously tacked in place.

This automatic submerged-arc welder mounted on a track-mounted, gantry type manipulator runs a web-to-flange fillet weld the full 84′ girder length. Welding generators travel with the manipulator.