Properties of Sections

1. IMPORTANCE OF SECTION PROPERTY

The basic formulas used in the design of structural members include as one factor the critical property of the material and as another factor the corresponding critical property of the member's cross-section. The property of the section dictates how efficiently the property of the material will be utilized.

The property of section having the greatest importance is the section's area (A). However, most design problems are not so simple that the area is used directly. Instead there is usually a bending aspect to the problem and, therefore, the rigidity factor normally is the section's moment of inertia (I) and the simple strength factor is the section modulus (S).

Another property of section that is of major importance is the section's torsional resistance (R), a modified value for standard sections.

2. AREA OF THE SECTION (A)

The *area* (A) of the member's cross-section is used directly in computations for simple tension, compression, and shear. Area (A) is expressed in square inches.

If the section is not uniform throughout the length of the member, it is necessary to determine the section in which the greatest unit stresses will be incurred.

3. MOMENT OF INERTIA (I)

Whereas a *moment* is the tendency toward rotation about an axis, the *moment of inertia* of the cross-section of a structural member is a measure of the resistance to rotation offered by the section's geometry and size. Thus, the moment of inertia is a useful property in solving design problems where a bending moment is involved.

The moment of inertia is needed in solving any rigidity problem in which the member is a beam or long column. It is a measure of the stiffness of a beam. Moment of inertia is also required for figuring the value of the polar moment of inertia (J), unless a formula is available for finding torsional resistance (R).

The moment of inertia (I) is used in finding the section modulus (S) and thus has a role in solving simple strength designs as well as rigidity designs. The moment of inertia of a section is expressed in inches raised to the fourth power (in.^{4}).

Finding the Neutral Axis

In working with the section's moment of inertia, the *neutral axis* (N.A.) of the section must be located. In a member subject to a bending load for example, the neutral axis extends through the length of the member parallel to the member's structural axis and perpendicular to the line of applied force. The neutral axis represents zero strain and therefore zero stress. Fibers between the neutral axis and the surface to the inside of the arc caused by deflection under load, are under compression. Fibers between the neutral axis and the surface to the outside of the arc caused by deflection under load, are under tension.

For practical purposes this neutral axis is assumed to have a fixed relationship (n) to some reference axis, usually along the top or bottom of the section. In Figure 1, the reference axis is taken through the base line of the section. The total section is next broken into rectangular elements. The moment (M) of each element about the section's reference axis, is determined:

M = area of element multiplied by the distance (y) of element's center of gravity from reference axis of section

The moments of the various elements are then all added together. This summation of moments is next divided by the total area (A) of the section. This gives the distance (n) of the neutral axis from the reference axis, which in this case is the base line or extreme fiber.

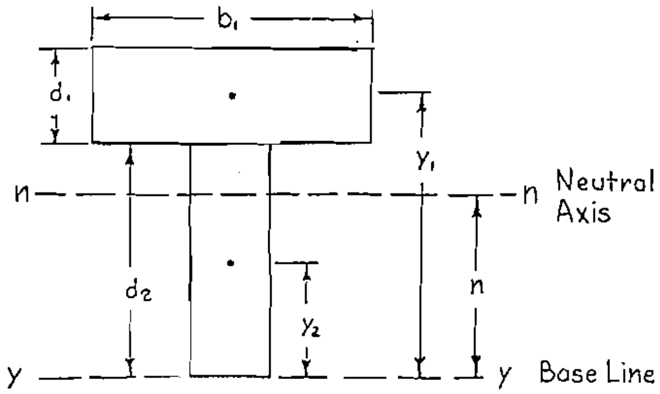


FIGURE 1

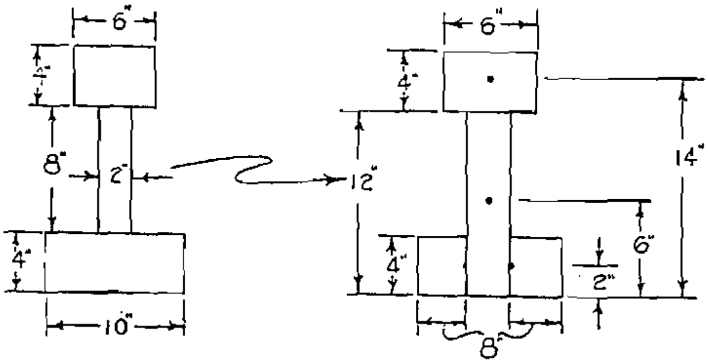


FIGURE 2

Problem 1

The neutral axis of the compound section shown in Figure 2 is located in the following manner:

$$\boxed{n = \frac{\Sigma M}{\Sigma A}} \text{ or } \frac{\text{sum of all moments}}{\text{total area}} \quad \cdots\cdots\cdots\cdots (1)$$

$$= \frac{(4 \cdot 6 \cdot 14) + (2 \cdot 12 \cdot 6) + (4 \cdot 8 \cdot 2)}{(4 \cdot 6) \;+\; (2 \cdot 12) \;+\; (4 \cdot 8)}$$

$$= \frac{336 + 44 + 64}{24 + 24 + 32} = \frac{544}{80}$$

$$= 6.8''$$

Thus, the neutral axis is located 6.8″ above the reference axis or base line and is parallel to it.

Finding the Moment of Inertia

There are various methods to select from to get the value of moment of inertia (I). Four good methods are presented here.

Moment of Inertia for Typical Sections (First Method)

The first method for finding the moment of inertia is to use the simplified formulas given for typical sections. These are shown in Table 1. This method for finding I is the most appropriate for simple sections that cannot be broken down into smaller elements. In using these formulas, be sure to take the moment of inertia about the correct line. Notice that the moment of inertia for a rectangle about its neutral axis is —

$$\boxed{I_n = \frac{bd^3}{12}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

but the moment of inertia for a rectangle about its base line is —

$$\boxed{I_b = \frac{bd^3}{3}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where b = width of rectangle, and
d = depth of rectangle

Moment of Inertia by Elements (Second Method)

In the second method, the whole section is broken into rectangular elements. The neutral axis of the whole section is first found. Each element has a moment of inertia about its own centroid or center of gravity (C.G.) equal to that obtained by the formula shown for rectangular sections. (See Table 1.)

In addition, there is a much greater moment of inertia for each element because of the distance of its center of gravity to the neutral axis of the whole section. This moment of inertia is equal to the area of the element multiplied by the distance of its C.G. to the neutral axis squared.

Thus, the moment of inertia of the entire section about its neutral axis equals the summation of the two moments of inertia of the individual elements.

Problem 2

Having already located the neutral axis of the section in Figure 2, the resulting moment of inertia of the section (detailed further in Fig. 3) about its neutral axis is found as follows:

$$I_n = \frac{6 \cdot 4^3}{12} + (6 \cdot 4 \cdot 7.2^2) + \frac{2 \cdot 8^3}{12} +$$

$$(2 \cdot 8 \cdot 1.2^2) + \frac{10 \cdot 4^3}{12} + (10 \cdot 4 \cdot 4.8^2)$$

$$= 32 + 1244 + 85.3 + 23 + 53.3 + 921.6$$

$$= \underline{2359 \text{ in.}^4}$$

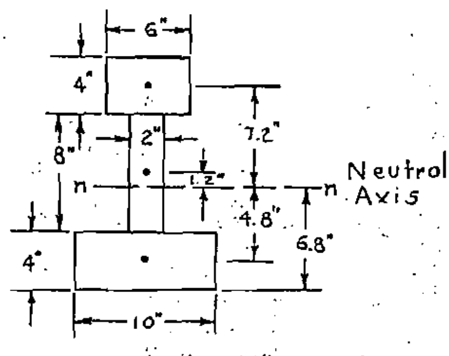


FIGURE 3

Moment of Inertia by Adding Areas (Third Method)

With the third method it is possible to figure moment of inertia of built-up sections without first directly making a calculation for the neutral axis.

This method is recommended for use with built-up girders and columns because the designer can stop briefly as a plate is added to quickly find the new moment of inertia. If this value is not high enough, he simply continues to add more plate and again checks this value without losing any of his previous calculations. Likewise if the value is too high, the designer may deduct some of the plates and again check his result. This is done in the same manner as one using an adding machine, whereby you can stop at any time during adding and take a sub-total, and then proceed along without disrupting the previous figures.

Using the parallel axis theorem for shifting the axis for a moment of inertia, the moment of inertia of the whole section about the reference line y-y is —

$$\boxed{I_y = I_n + A\ n^2} \quad \dots\dots (4)$$

or

$$\boxed{I_n = I_y - A\ n^2} \quad \dots\dots (5)$$

$$\text{Since } n = \frac{\text{total moments about base}}{\text{total area}} = \frac{M}{A}$$

$$\text{and of course } n^2 = \frac{M^2}{A^2}$$

Substituting this back into equation (5):

$$I_n = I_y - \frac{A\ M^2}{A^2}$$

Note: neutral axis (n) has dropped out

Thus:

$$\boxed{I_n = I_y - \frac{M^2}{A}} \quad \dots\dots (6)$$

where:

I_n = moment of inertia of whole section about its neutral axis, n-n

I_y = sum of the moments of inertia of all elements about a common reference axis, y-y

M = sum of the moments of all elements about the same reference axis, y-y

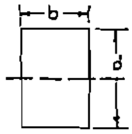
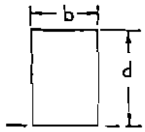
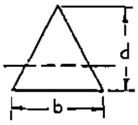
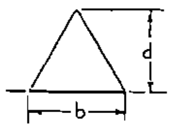
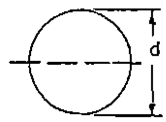
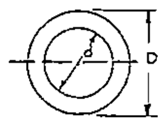
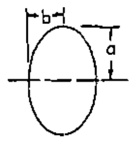
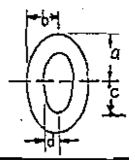
A = total area, or sum of the areas of all elements of section

Although I_y for any individual element is equal to its area (A) multiplied by the distance squared from its center of gravity to the reference axis (y^2), each element has in addition a moment of inertia (I_g) about its own center of gravity. This must be added in if it is large enough, although in most cases it may be neglected:

$$\boxed{I_n = I_y + I_g - \frac{M^2}{A}} \quad \dots\dots (7)$$

The best way to illustrate this method is to work a problem.

TABLE 1—Properties of Standard Sections

	Moment of Inertia I	Section Modulus S	Radius of Gyration r
	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{d}{\sqrt{12}}$
	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	$\frac{d}{\sqrt{3}}$
	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d}{\sqrt{18}}$
	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{d}{\sqrt{6}}$
	$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{d}{4}$
	$\frac{\pi}{64}(D^4 - d^4)$	$\frac{\pi}{32}\frac{(D^4 - d^4)}{D}$	$\frac{\sqrt{D^2+d^2}}{4}$
	$\frac{\pi a^3 b}{4}$	$\frac{\pi a^2 b}{4}$	$\frac{a}{2}$
	$\frac{\pi}{4}\left(a^3b - c^3d\right)$	$\frac{\pi(a^3b - c^3d)}{4a}$	$\frac{1}{a}\sqrt{\frac{a^3b - c^3d}{ab - cd}}$

Problem 3

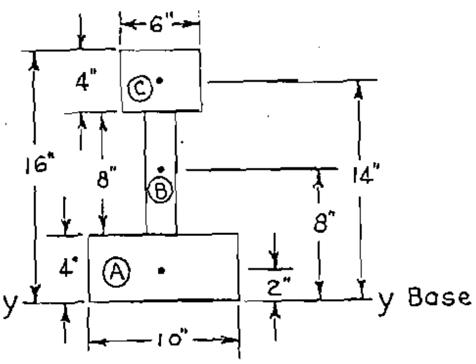


FIGURE 4

The base of this section will be used as a reference axis, y-y. Every time a plate is added, its dimensions are put down in table form, along with its distance (y) from the reference axis. No other information is needed. It is suggested that the plate section size be listed as width times depth (b × d); that is, its width first and depth last.

Plate	Size	Distance y	$A = b \cdot d$ in.^{2}	$M = A \cdot y$ in.^{3}	$I_y = Ay^2 = My$ in.^{4}	$I_g = \frac{bd^3}{12}$ in.^{4}
Ⓐ	10"x4"	2"				
Ⓑ	2"x8"	8"				
Ⓒ	6"x4"	14"				
Total						

The above table has been filled out with all of the given information from the plates. The rest of the computations are very quickly done on slide rule or calculator and placed into the table. Notice how easy and fast each plate is taken care of.

Starting with plate A, 10" is multiplied by 4" to give an area of 40 sq. in. This value is entered into the table under A. Without resetting the slide rule, this figure for A is multiplied by (distance y) 2" to give 80 inches cubed. This value for the element's moment is placed under M in the table. Without resetting the slide rule, this figure for M is multiplied by (distance y) 2" again to give 160 inches to the fourth power. This value for the element's moment of inertia about the common reference axis y-y is recorded under (I_y) in the table.

If the moment of inertia (I_g) of the plate about its own center of gravity appears to be significant, this value is figured by multiplying the width of the plate by the cube of its depth and dividing by 12. This value for I_g is then placed in the extreme right-hand column, to be later added in with the sum of I_y. Thus,

$$I_g = \frac{bd^3}{12}$$

$$= \frac{10 \cdot 4^3}{12}$$

$$= 53.3 \text{ in.}^4$$

Usually the value of I_g is small enough that it need not be considered. In our example, this value of 53.3 could be considered, although it will not make much difference in the final value. The greater the depth of any element relative to the maximum width of the section, the more the likelihood of its I_g value being significant.

The table will now be filled out for plates B and C as well:

Plate	Size	Distance y	$A = b \cdot d$ in.^{2}	$M = A \cdot y$ in.^{3}	$I_y = Ay^2 = My$ in.^{4}	$I_g = \frac{bd^3}{12}$ in.^{4}
Ⓐ	10"x4"	2"	40.0	80.0	160.0	53.3
Ⓑ	2"x8"	8"	16.0	128.0	1024.0	85.3
Ⓒ	6"x4"	14"	24.0	336.0	4704.0	32.0
Total			80.0	544.0	5888.0	170.6
					6058	

$$I_n = I_y + I_g - \frac{M^2}{A}$$

$$= 5888 + 170.6 - \frac{(544)^2}{80} = 6059 - 3700$$

$$= 2359 \text{ in.}^4$$

and $n = \frac{M}{A} = \frac{544}{80}$

$$= 6.8'' \text{ (up from bottom)}$$

A recommended method of treating M^2/A on the slide rule, is to divide M by A on the rule. Here we have 544 divided by 80 which gives us 6.8. This happens to be the distance of the neutral axis from the base reference line. Then without resetting the slide rule, multiply this by 544 again by just sliding the indicator of the rule down to 544 and read the answer as 3700. It is often necessary to know the neutral axis, and it can be found without extra work.

Problem 4

To show a further advantage of this system, assume that this resulting moment of inertia (2359 in.^{4}) is not large enough and the section must be made larger. Increasing the plate size at the top from 6″ × 4″ to 8″ × 4″ is the same as adding a 2″ × 4″ area to the already existing section. See Figure 5. The previous column totals are carried forward, and properties of only the added area need to be entered. I_n is then solved, using the corrected totals.

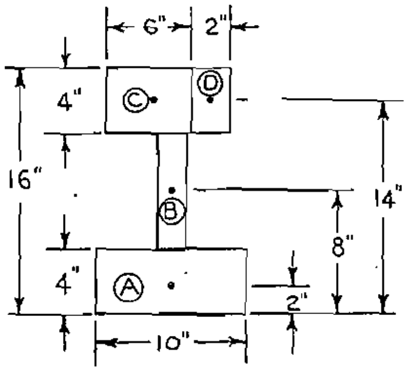


FIGURE 5

Plate	Size	Distance y	$A = b \cdot d$ in.²	$M = A \cdot y$ in.³	$I_y = Ay^2 = My$ in.⁴	$I_g = \frac{bd^3}{12}$ in.⁴
Previous Section		——	80.0	544.0	5888.0	170.6
New D	2″x4″	14″	8.0	112.0	1568.0	10.6
Total			88.0	656.0	7456.0	181.2
					7637	

$$I_n = I_y + I_g - \frac{M^2}{A}$$

$$= 7637 - \frac{(656)^2}{88}$$

$$= 2747 \text{ in.}^4$$

$$\text{and } n = \frac{M}{A} = \frac{656}{88}$$

$$= 7.45'' \text{ (up from bottom)}$$

Moment of Inertia of Rolled Sections (Fourth Method)

The fourth method is the use of steel tables found in the A.I.S.C. handbook and other steel handbooks. These values are for any steel section which is rolled, and should be used whenever standard steel sections are used.

Positioning the Reference Axis

The designer should give some thought to positioning the reference axis (y-y) of a built-up section where it will simplify his computations.

The closer the reference axis (y-y) is to the final neutral axis (N.A.), the smaller will be the values of (I_y and I_g) and M^2/A. Hence, the more accurate these values will be if a slide rule is used.

If the reference axis (y-y) is positioned to lie through the center of gravity (C.G.) of one of the elements (the web, for example), this eliminates any subsequent work on this particular element since $y = 0$ for this element.

If the reference axis (y-y) is positioned along the base of the whole section, the distance of the neutral axis ($n = M/A$) from the reference axis (y-y) then automatically becomes the distance (c_b) from the neutral axis to the outer fiber at the bottom.

The following problem illustrates these points.

Problem 5

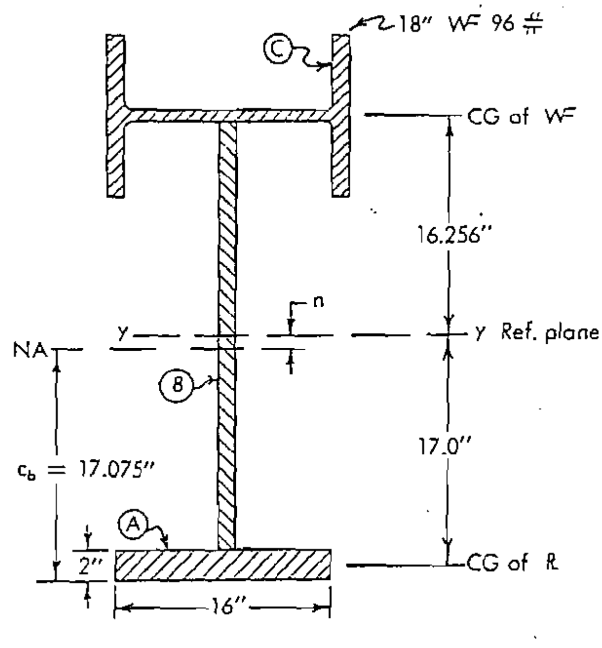


FIGURE 6

It is very easy to incorporate a rolled section into a built-up member, for example this proposed column to resist wind moments. See Figure 6. Find the moment of inertia of the whole section about its neutral axis (I_n) and then find its section modulus (S).

Choosing reference axis (y-y) through the center of gravity (C.G.) of the web plate B makes $y = 0$, and thus eliminates some work for B.

Properties of the standard 18″ WF 96# section are given by the steel handbook as—

$$A = 28.22 \text{ in.}^2 \quad I_y = 206.8 \text{ in.}^4 \quad t_w = .512''$$

The handbook value of $I_y = 206.8$ in.^{4} can be inserted directly into the following table, for the I_g of this WF section C .

By adding areas and their properties:

	Size	y	A	M	I_y	I_g
A	16″ x 2″	−17.0″	32.00	−544.00	+9248.0	+10.7
B	1″ x 32″	0	32.00	0	0	+2730.7
A	18 WF 96#	+16.256″	28.22	+458.74	+7456.62	+206.8
	Total		92.22	− 85.26	+19,652.8	

moment of inertia about neutral axis

$$I_n = I_y + I_g - \frac{M^2}{A}$$

$$= (19{,}652.8) - \frac{(-85.26)^2}{(92.22)}$$

$$= 19{,}574 \text{ in.}^4$$

distance of neutral axis from reference axis

$$n = \frac{M}{A}$$

$$= \frac{(-85.26)}{(92.22)}$$

$$= -.925'' \text{ from axis y-y}$$

distance from N.A. to outer fiber

$$c_b = 18.00 - .925$$

$$= 17.075''$$

section modulus (see Topic 4 which follows)

$$S = \frac{I_n}{c_b}$$

$$= \frac{(19{,}574 \text{ in.}^4)}{(17.075'')}$$

$$= 1146 \text{ in.}^3$$

4. SECTION MODULUS (S)

The *section modulus* (S) is found by dividing the moment of inertia (I) by the distance (c) from the neutral axis to the outermost fiber of the section:

$$\boxed{S = \frac{I}{c}} \quad \ldots\ldots (8)$$

Since this distance (c) can be measured in two directions, there are actually two values for this property, although only the smaller value is usually available in tables of rolled sections because it results in the greater stress. If the section is symmetrical, these two values are equal. Section modulus is a measurement of the strength of the beam in bending. In an unsymmetrical section, the outer face having the greater value of (c) will have the lower value of section modulus (S) and of course the greater stress. Since it has the greater stress, this is the value needed.

With some typical sections it is not necessary to solve first for moment of inertia (I). The section modulus can be computed directly from the simplified formulas of Table 1.

In many cases, however, the moment of inertia (I) must be found before solving for section modulus (S). Any of the previously described methods may be applicable for determining the moment of inertia.

Problem 6

Using a welded "T" section as a problem in finding the section modulus, its neutral axis is first located, Figure 7.

Using the standard formula (#1) for determining the distance (n) of the neutral axis from any reference axis, in this case the top horizontal face of the flange:

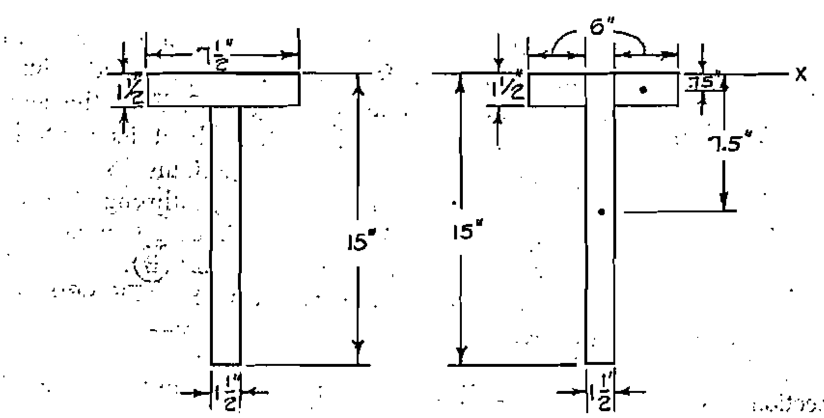


FIGURE 7

$$n = \frac{M}{A} = \frac{\text{Sum of moments}}{\text{Total area of section}}$$
$$= \frac{(6 \cdot 1.5 \cdot 0.75) + (15 \cdot 1.5 \cdot 7.5)}{(6 \cdot 1.5) \quad + \quad (15 \cdot 1.5)}$$
$$= \frac{6.75 + 168.75}{9.0 + 22.5}$$
$$= 5.56''$$

Next, the section's moment of inertia is determined, using the elements method (Figure 8):

$$I_n = \frac{6 \cdot 1.5^3}{12} + (6 \cdot 1.5 \cdot 4.81^2) + \frac{1.5 \cdot 15^3}{12} +$$
$$(1.5 \cdot 15 \cdot 1.94^2)$$
$$= 1.69 + 208.22 + 421.87 + 84.68$$
$$= 716.5 \text{ in.}^4$$

This value is slightly higher than the required $I = 700$ in.^{4} because depth of section was made $d = 15''$ instead of 14.9″.

Finally, the section modulus (S) is determined:

$$S = \frac{I}{c} = \frac{716.5}{9.44}$$
$$= 75.8 \text{ in.}^3$$

5. RADIUS OF GYRATION (r)

The *radius of gyration* (r) is the distance from the neutral axis of a section to an imaginary point at which the whole area of the section could be concentrated and still have the same moment of inertia. This property is used primarily in solving column problems. It is found by taking the square root of the moment of inertia divided by the area of the section and is expressed in inches.

$$\boxed{r = \sqrt{\frac{I}{A}}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (9)$$

6. POLAR MOMENT OF INERTIA (J)

The *polar moment of inertia* (J) equals the sum of any two moments of inertia about axes at right angles to each other. The polar moment of inertia is taken about an axis which is perpendicular to the plane of the other two axes.

$$\boxed{J = I_x + I_y} \quad \dots\dots\dots\dots\dots\dots\dots\dots (10)$$

Polar moment of inertia is used in determining the polar section modulus (J/c) which is a measure

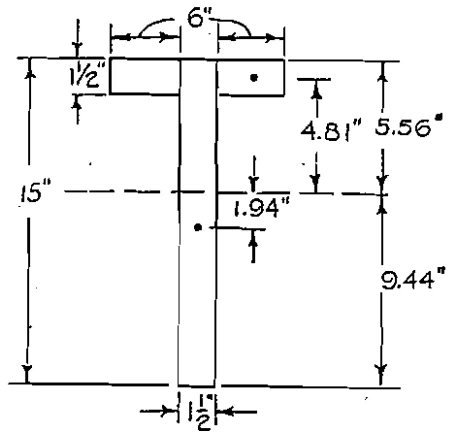


FIGURE 8

of strength under torsional loading of round solid bars and closed tubular shafts.

7. TORSIONAL RESISTANCE (R)

Torsional resistance (R) has largely replaced the less accurate polar moment of inertia in standard design formula for angular twist of open sections. It should be employed where formulas have been developed for the type of section. These are given in the later Section 2.10 on Torsion.

8. PROPERTIES OF THIN SECTIONS

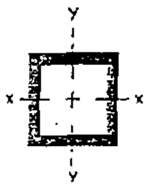
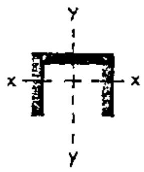
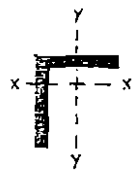
Because of welding, increasingly greater use is being found for structural shapes having thin cross-sections. Thin sections may be custom roll-formed, rolled by small specialty steel producers, brake-formed, or fabricated by welding. Properties of these sections are needed by the designer, but they are not ordinarily listed among the standard rolled sections of a steel handbook. Properties of thin sections customarily are found by the standard formulas for sections.

With a thin section, the inside dimension is almost as large as the outside dimension; and, in most cases, the property of the section varies as the cubes of these two dimensions. This means dealing with the difference between two very large numbers. In order to get any accuracy, it would be necessary to calculate this out by longhand or by using logarithms rather than use the usual slide rule.

To simplify the problem, the section may be "treated as a line", having no thickness. The property of the "line", is then multiplied by the thickness of the section to give the approximate value of the section property within a very narrow tolerance. Table 2 gives simplified formulas for nine properties of six different cross-sections. In this table: d = mean depth, b = mean width of the section, and t = thickness.

TABLE 2—Properties of Thin Sections

Where thickness (t) is small, b = mean width, and d = mean depth of section

Section						
I_x	$\frac{td^3(4b+d)}{12(b+d)}$	$\frac{td^2}{12}(6b+d)$	$\frac{td^2}{6}(3b+d)$	$\frac{td^3(2b+d)}{3(b+2d)}$	$\frac{td^3(4b+d)}{12(b+d)}$	$t\pi r^3$
S_x	$\frac{td^2(4b+d)}{6(2b+d)}$ bottom $\frac{td}{6}(4b+d)$ top *	$\frac{td}{6}(6b+d)$ *	$\frac{td}{3}(3b+d)$ *	$\frac{td}{3}(2b+d)$ top $\frac{td^2(2b+d)}{3(b+d)}$ bottom *	$\frac{td}{6}(4b+d)$ top $\frac{td^2(4b+d)}{6(2b+d)}$ bottom *	$t\pi r^2$
I_y	$\frac{tb^3}{12}$	$\frac{tb^3}{6}$	$\frac{tb^2}{6}(b+3d)$	$\frac{tb^2}{12}(b+6d)$	$\frac{tb^3(b+4d)}{12(b+d)}$	——
S_y	$\frac{tb^2}{6}$	$\frac{tb^2}{3}$	$\frac{tb}{3}(b+3d)$ *	$\frac{tb}{6}(b+6d)$ *	$\frac{tb^2(b+4d)}{6(b+2d)}$ right side $\frac{tb}{6}(b+4d)$ left side *	——
I_{xy}	0	0	0	0	$\frac{tb^2d^2}{4(b+d)}$	0
R	$\frac{t^3}{3}(b+d)$	$\frac{t^3}{3}(2b+d)$	$\frac{2tb^2d^2}{b+d}$	$\frac{t^3}{3}(b+2d)$	$\frac{t^3}{3}(b+d)$	$2t\pi r^3$
r_x max. or min.	$\frac{\sqrt{\frac{d^3(4b+d)}{12}}}{b+d}$	$\sqrt{\frac{d^2(6b+d)}{12(2b+d)}}$	$\sqrt{\frac{d^2(3b+d)}{12(b+d)}}$	$\frac{\sqrt{\frac{d^3}{3}(2b+d)}}{(b+2d)}$		0.7071 r
NA	$\frac{d^2}{2(b+d)}$ down from top			$\frac{d^2}{b+2d}$ down from top	$\frac{d^2}{2(b+d)}$ down from top $\frac{b^2}{2(b+d)}$	——
r_y min. or max.	$\sqrt{\frac{b^3}{12(b+d)}}$	$\sqrt{\frac{b^3}{6(2b+d)}}$	$\sqrt{\frac{b^2(b+3d)}{12(b+d)}}$	$\sqrt{\frac{b^2(b+d)}{12(b+2d)}}$		——

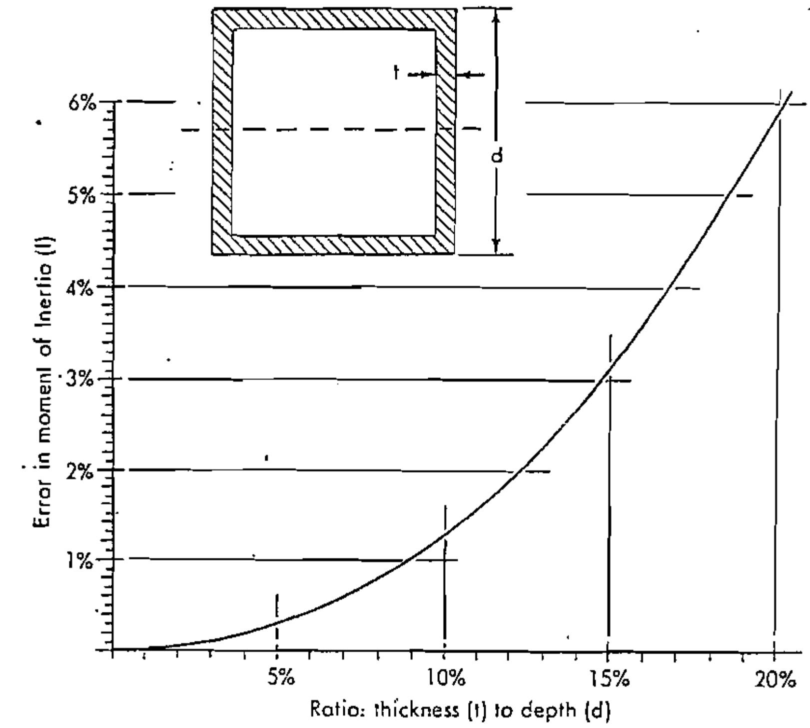
(* = add t/2 to c for S)

The error in calculating the moment of inertia by this Line Method versus the conventional formula is represented by the curve in Figure 9, using a square tubular section as an example. As indicated, the error increases with the ratio of section thickness (t) to depth (d).

An excellent example of the savings in design time offered by use of the Line Method exists as (column) Problem 4 in Section 3.1.

Table 3 gives the most important properties of additional thin sections of irregular but common configurations.

FIG. 9 Possible error in using Line Method is minimal with low ratio of section thickness to depth.



For additional formulas and reference tables, see "Light Gage Cold-Formed Steel Design Manual" 1962, American Iron & Steel Institute.

9. SHEAR AXIS AND SHEAR CENTER

Since the bending moment decreases as the distance of the load from the support increases, bending force f_1 is slightly less than force f_2, and this difference $(f_2 - f_1)$ is transferred inward toward the web by the longitudinal shear force (f_s). See Figure 10.

$$\boxed{f_s = f_s' + \frac{P\,a\,y}{I_x}} \quad \text{.....................(11)}$$

This force also has an equal component in the transverse direction. A transverse force applied to a beam sets up transverse (and horizontal) shear forces within the section. See Figure 11.

In the case of a symmetrical section, A, a force (P) applied in line with the principal axis (y-y) does not result in any twisting action on the member. This

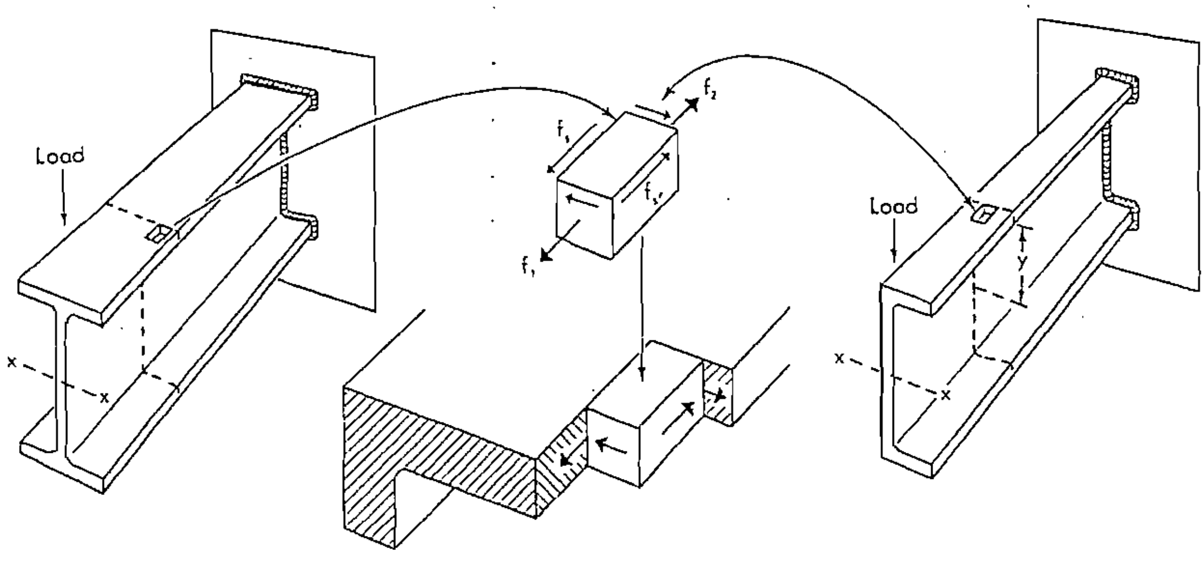
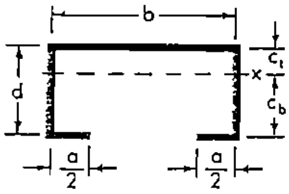


FIGURE 10

TABLE 3—Properties of Typical Irregular Thin Sections

Where thickness (t) is small, b = mean width, and d = mean depth of section

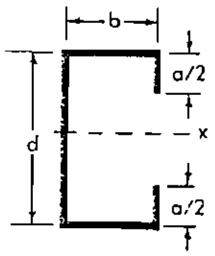


$$I_x = \frac{t\,d^2 \left[\,k\,b^2 + (k+1)^2 \frac{bd}{3} + \frac{d^2}{3}\,\right]}{b\,(k+1) + 2\,d} \qquad k = \frac{a}{b}$$

$$c_b = \frac{d\,(b+d)}{d\,(k+1) + 2\,d} \qquad c_t = \frac{d\,(k\,b+d)}{b\,(k+1) + 2\,d}$$

$$S_b = \frac{t\,d \left[\,k\,b^2 + (k+1) \frac{2\,b\,d}{3} + \frac{d^2}{3}\,\right]}{b+d}$$

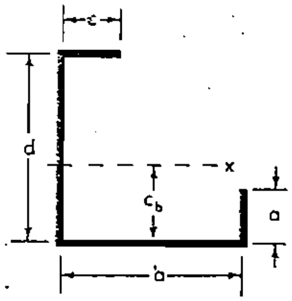
$$S_t = \frac{t\,d \left[\,k\,b^2 + (k+1) \frac{2\,b\,d}{3} + \frac{d^2}{3}\,\right]}{k\,b+d}$$



$$I_x = \frac{t\,d^3}{12}(k^3 - 3\,k^2 + 3\,k + 1) + \frac{t\,b\,d^2}{2}$$

$$k = \frac{a}{d}$$

$$S_x = \frac{t\,d^2}{6}(k^3 - 3\,k^2 + 3\,k + 1) + t\,b\,d$$



$$c_b = \frac{a^2 + 2\,c\,d + d^2}{2\,(a+b+c+d)}$$

$$I_x = \frac{t\,(a^3 + 3\,c\,d^2 + d^3)}{3} - \frac{t\,(a^2 + 2\,c\,d + d^2)^2}{4\,(a+b+c+d)}$$

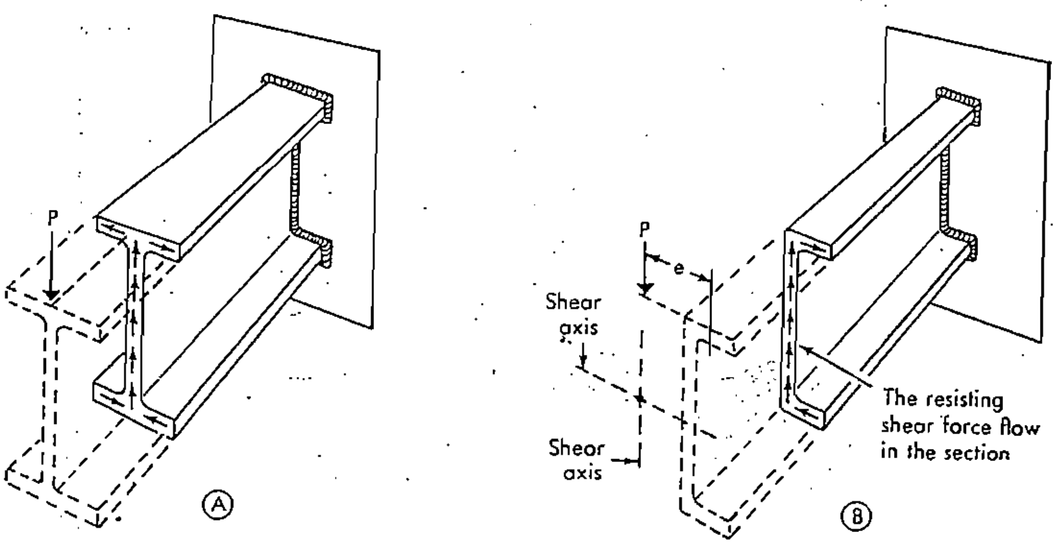


FIGURE 11

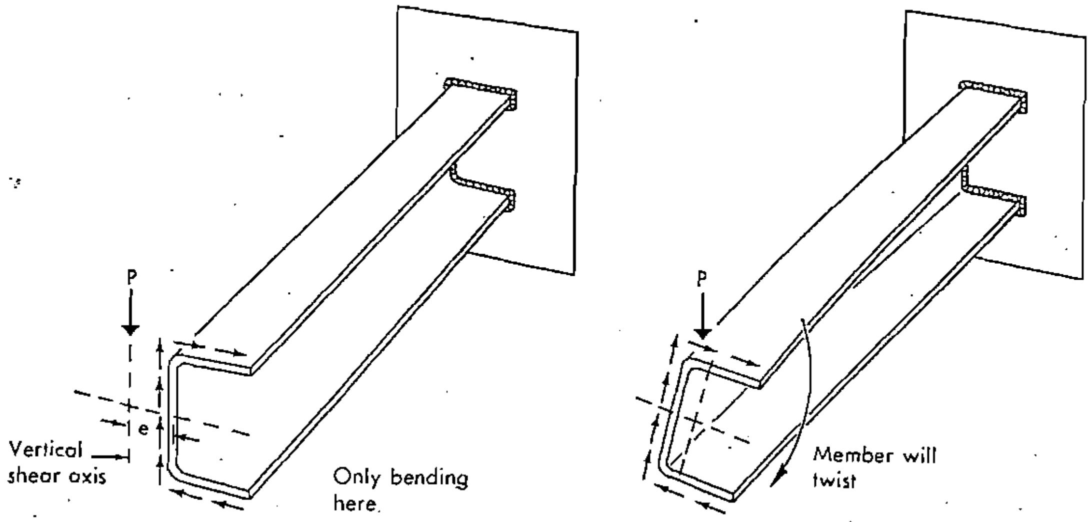


FIGURE 12

is because the torsional moment of the internal transverse shear forces (⟶) is equal to zero.

On the other hand, in the case of an unsymmetrical section, B, the internal transverse shear forces (⟶) form a twisting moment. Therefore, the force (P) must be applied eccentrically at a proper distance (e) along the shear axis, so that it forms an external torsional moment which is equal and opposite to the internal torsional moment of the transverse shear forces. If this precaution is not taken, there will be a twisting action applied to the member which will twist under load, in addition to bending. See Figure 12.

Any axis of symmetry will also be a shear axis.

There will be two shear axes and their intersection forms the shear center (Q).

A force, if applied at the shear center, may be at any angle in the plane of the cross-section and there will be no twisting moment on the member, just transverse shear and bending.

As stated previously, unless forces which are applied transverse to a member also pass through the shear axis, the member will be subjected to a twisting moment as well as bending. As a result, this beam should be considered as follows:

1. The applied force P should be resolved into a force P′ of the same value passing through the shear center (Q) and parallel to the original applied force P. P′ is then resolved into the two components at right angles to each other and parallel to the principal axes of the section.

2. A twisting moment (T) is produced by the applied force (P) about the shear center (Q).

The stress from the twisting moment (T) is computed separately and then superimposed upon the stresses of the two rectangular components of force P′.

This means that the shear center must be located. Any axis of symmetry will be one of the shear axes.

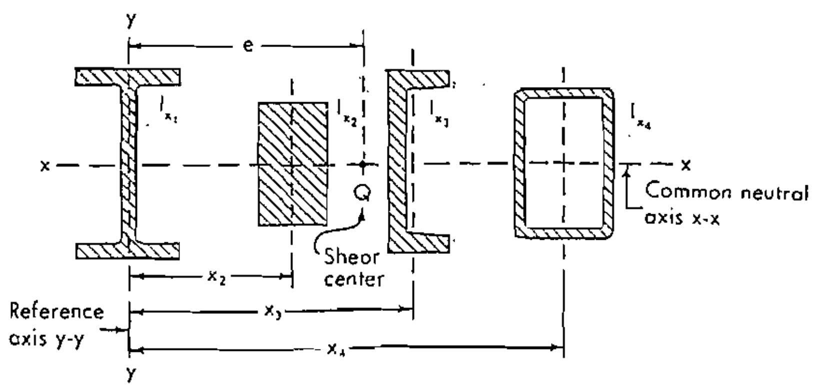
For open sections lying on one common neutral axis (y-y), the location of the other shear axis is—

$$e = \frac{\Sigma\, I_x\, X}{\Sigma\, I_x}$$

Notice the similarity between this and the following:

$$d = \frac{\Sigma\, M}{\Sigma\, A} = \frac{\Sigma\, A\, d}{\Sigma\, A}$$

FIGURE 13



which is used to find the neutral axis of a built-up section.

Just as the areas of individual parts are used to find the neutral axis, now the moments of inertia of individual areas are used to find the shear axis of a composite section, Figure 13. The procedure is the same; select a reference axis (y-y), determine I_x for each member section (about its own neutral axis x-x) and the distance X this member section lies from the reference axis (y-y). The resultant (e) from the formula will then be the distance from the chosen reference axis (y-y) to the parallel shear axis of the built-up section.

Here:

$$e = \frac{I_{x1} X_1 + I_{x2} X_2 + I_{x3} X_3 + I_{x4} X_4}{I_{x1} + I_{x2} + I_{x3} + I_{x4}}$$

or:

$$\boxed{e = \frac{\Sigma I_x X}{\Sigma I_x}} \quad \dots\dots\dots (12)$$

Locating Other Shear Centers

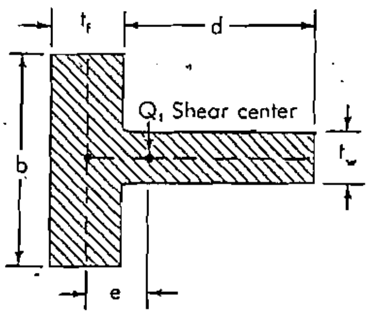


FIGURE 14

Here:

$$e = \frac{\Sigma I_x X}{\Sigma I_x} = \frac{\frac{t_f b^3}{12} \times 0 + \frac{d t_w^3}{12} \times \frac{t_f + d)}{2}}{I_x}$$

$$= \frac{d t_w^3 (t_f + d)}{24 I_x}$$

Normally Q might be assumed to be at the intersection of the centerlines of the web and the flange.

> The James F. Lincoln Arc Welding Foundation also publishes collections of award-winning papers describing the best and most unique bridges, buildings and other structures in which modern arc welding is used effectively.

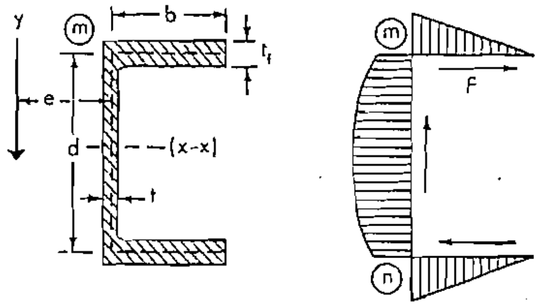


FIGURE 15

Here, at point M:

$$f_s = \frac{V a y}{I_x} = \frac{V (b t_f)(d/2)}{I_x}$$

$$F = ½ f_s b = \frac{V b^2 d t_f}{4 I_x}$$

$$\Sigma M_n = 0 = + F d - V e = 0$$

$$e = \frac{F d}{V} = \frac{V b^2 d^2 t_f}{V 4 I_x}$$

$$= \frac{b^2 d^2 t_f}{4 I_x}$$

or, since areas have a common (x-x) neutral axis:

$$e = \frac{\Sigma I_x X}{\Sigma I_x} = \frac{\frac{t d^3}{12} \times 0 + 2 \times (b t_f)(d/2)^2 \frac{b}{2}}{I_x}$$

$$= \frac{b^2 d^2 t_f}{4 I_x}$$

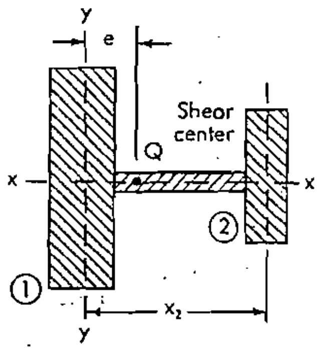


FIGURE 16

Here:

$$e = \frac{\Sigma I_x X}{\Sigma I_x} = \frac{I_{x1} 0 + I_{x2} X_2}{I_{x1} + I_{x2}}$$

$$= \frac{X_2 I_{x2}}{I_x}$$

Figure 17 suggests an approach to locating shear axes of some other typical sections.

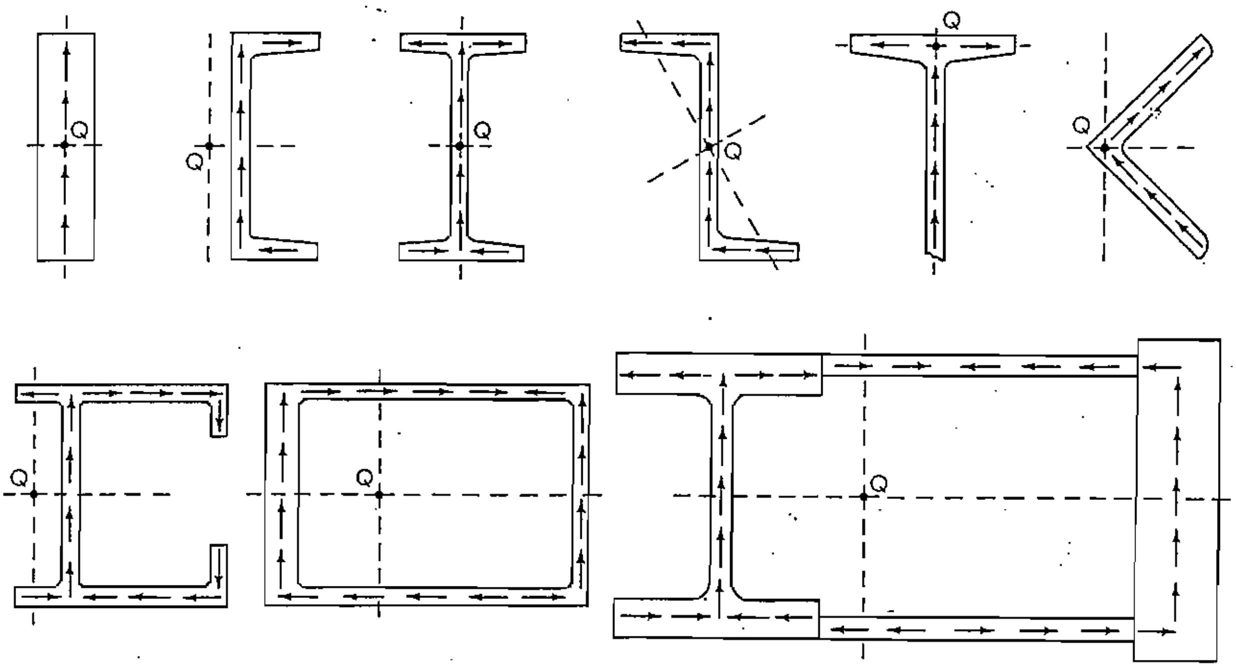
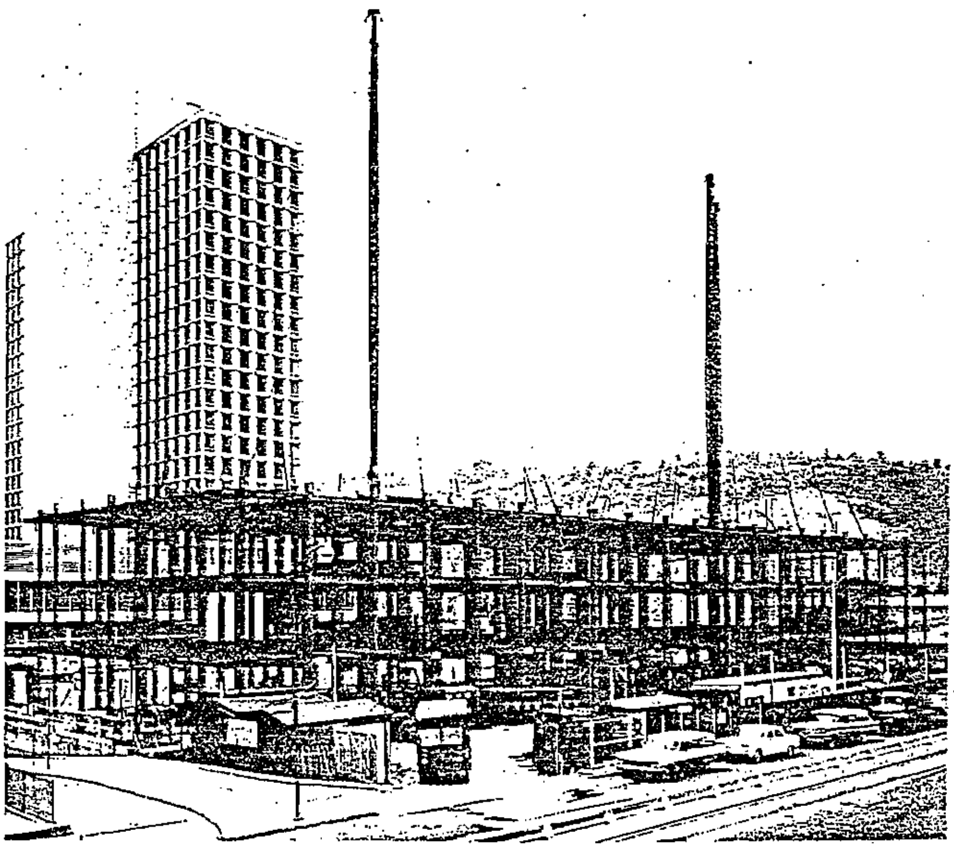
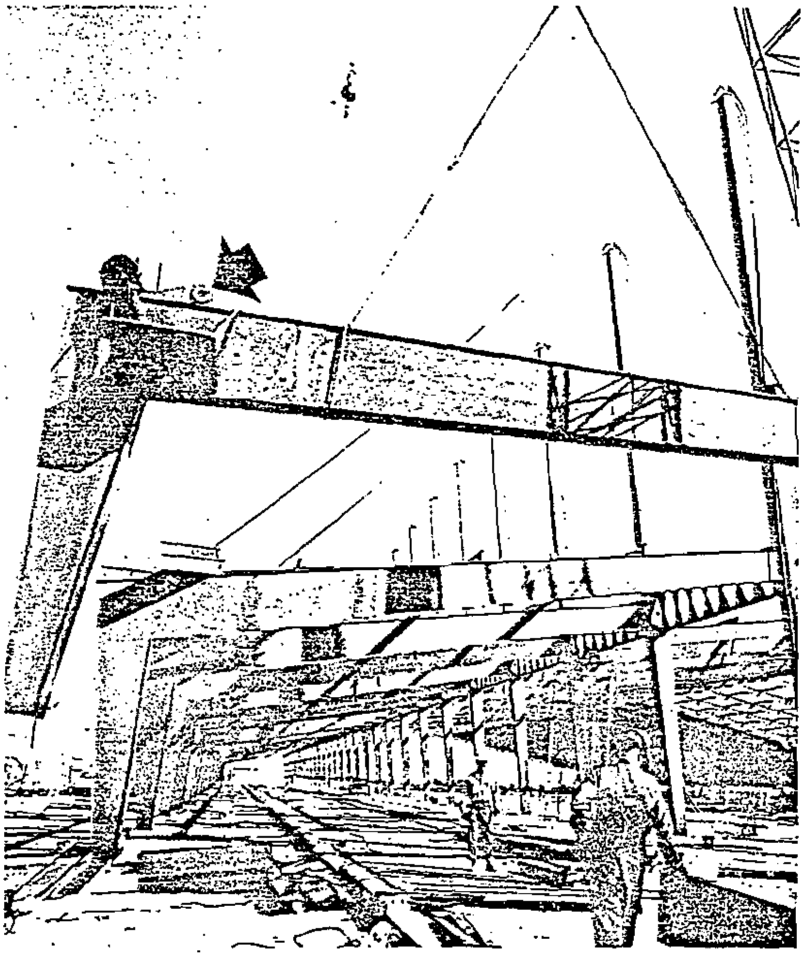


FIGURE 17

Structural steel for Gateway Towers, 26-story Pittsburgh apartment building was erected in tiers of three floors each by two derricks. Shop and field welding combined to facilitate erection; nearly 15 tons of electrode were used.





Eighty-foot hollow steel masts and suspension cables help support the continuous roof framing system of the 404′ x 1200′ Tulsa Exposition Center. Welds holding brackets (arrow) to which cables are anchored are designed to withstand the high tensile forces involved in such a structure.