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Combining Elastic and Plastic Fatigue Damage in Coiled Tubing

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Abstract

Coiled tubing is a continuous pipe that, having been coiled around a reel for storage, can be deployed and used as a pipeline or riser. During deployment as a riser, the coiled tubing is unspooled from the reel, run into the water and connected to the wellhead. This process plastically strains the pipe causing plastic, or low cycle, fatigue damage. When the coiled tubing is connected to the wellhead, the environmental loading causes elastic stress cycles, resulting in elastic, or high cycle, fatigue damage.

There are numerous methods to determine the fatigue life from either plastic or elastic cycling; however, there is little data within the industry on how the fatigue damage from elastic and plastic cycles combine.

This paper presents the experimental work conducted to show the combined fatigue life of coiled tubing that has been plastically and elastically cycled. The data shows that the combined fatigue life can be lower than the summation of the plastic and elastic fatigue damages using Miner's rule. Existing theory suggests that the combined fatigue life could be as low as 10% of the Miner's rule of fatigue damages; however, the experimental data indicates that a more appropriate value is closer to 75% of the Miner's rule fatigue damage.

Fatigue Damage

Overview

Fatigue is the localized damage process of a component by cyclic loading. Fatigue can be described as consisting of three processes, Lee et al (2005):

1. Crack initiation
2. Crack propagation
3. Final fracture.

During cyclic loading localized plastic deformation occurs in areas of high stress concentration such as a notch, weld pore, or discontinuities between grain boundaries. (1.) The plastic deformation induces permanent damage in the component and a crack develops (crack initiation). (2.) As the component experiences an increasing number of load cycles, the length of the crack increases (crack propagation). (3.) After a certain number of load cycles the crack will cause the component to fail (final fracture).

Fatigue Damage and Cycle Ratio

Fatigue damage, or damage, is defined as the length of the crack in a component, compared to the crack size that causes complete failure. Damage therefore has a value between zero (no damage) and one (failure). Fatigue damage is non-linearly proportional to the number of load cycles the component experiences. An example relationship between fatigue damage and cycle ratio; the number of cycles occurred divided by the number of cycles to failure is shown in Figure 1. This shows that if a component experiences load cycles with constant amplitude, the crack length grows slowly at first, and then accelerates, so that the fastest crack growth occurs nearest to the component failure, a fatigue damage of one. The equation relating cycle ratio to fatigue damage from Lee et al (2005) is given below:

$$D = \left(\frac{n}{N}\right)^{\frac{2}{3}N^{0.4}} \tag{1}$$

where

D	fatigue damage (ratio of actual to failure crack length)
n/N	the cycle ratio, where:
n	number of cycles that have occurred
N	number of cycles to failure

The implications of the non-linear relationship between fatigue damage and cycle ratio are that

- fatigue damage is dependent on the sequence of the load cycles
- fatigue damage is complex and simplifications are required to enable modeling.

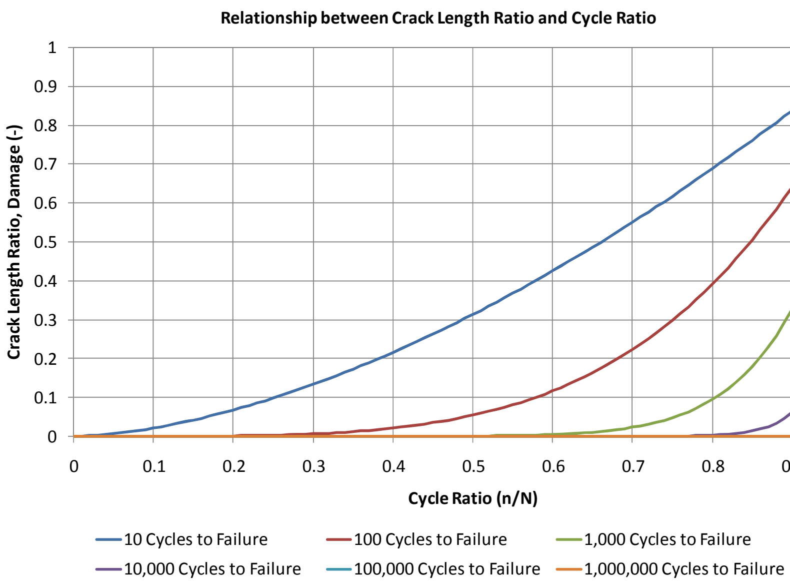

Figure 1: Example relationship between damage and cycle ratio.

Plastic and Elastic Cycling

Fatigue damage is caused by load cycling. Three types of load cycling are considered as follows:

- Low cycle, which are large stress/strain cycles which require less than 10^3 cycles to fail a component.
- High cycle, which are generally cycles within the elastic range of the material and require from 10^3 to 10^7 cycles to fail a component.
- Fatigue limit, which are low stress cycles that are considered non-damaging. These stress levels are considered within the endurance limit of the component.

Plastic load cycling is considered to be low cycle fatigue. Elastic load cycling is considered to be high cycle fatigue.

Plastic load cycling causes crack initiation, where a (micro) crack is formed due to the high concentrated loading and deformation of the material. Further plastic load cycles tear the material growing the crack by crack propagation.

Elastic load cycling causes crack propagation, where an existing crack is grown. If no crack exists the elastic cycling is considered to cause little fatigue damage.

Predicting Number of Cycles to Failure

The number of cycles to failure for a load cycle can be determined using either stress or strain. Typically stress is used for elastic cycling and strain is used for plastic load cycles. A brief overview of these methods is given below. Further details can be found in Lee et al (2005).

Stress-Based Fatigue Damage

In riser systems, where the stress ranges are in the elastic region, fatigue damage may be calculated using S-N curves, where for a given stress range the number of cycles to failure can be estimated. An example S-N curve is given in Figure 2. The equation for the S-N curve is given below:

$$N = k\sigma^{-m} \tag{2}$$

where

k	fatigue curve constant
σ	stress range (peak to peak)
m	fatigue curve exponent

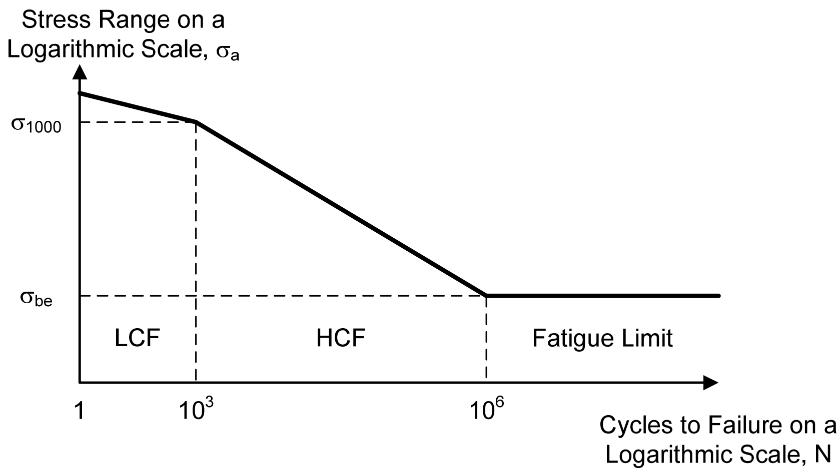

Figure 2: Example S-N curve, after Lee et al (2005).

The stress levels that correspond to the 10^3 and 10^6 cycles to failure in Figure 2 can be estimated from the formula below Lee et al (2005). However S-N curves are generally derived from experimental data and the fatigue constant and exponent are published in design codes such as DNV-RP-C203.

$$\sigma_{1000} = 0.9\; Cr\; \sigma_u \tag{3}$$
$$\sigma_{be} = C_M\; Cr\; \sigma_u \tag{4}$$

where

σ_{1000}	stress range at 1,000 cycles to failure
σ_{be}	stress range at 1,000,000 cycles to failure
σ_u	ultimate stress of the material
C_r	reliability factor, 0.868 for 95% reliability (2 standard deviations)
C_M	material type ranging from 0.26 for martensite to 0.55 for ferrite steel

Strain-Based Fatigue Damage

Coiled tubing fatigue may be calculated using the strain-life method, where the number of reversals to failure is determined for strain amplitudes (‘strain’ is used, as the amplitude of the load reversals is above the elastic limit of the material). One load cycle is equal to two load reversals. The Morrow (1965) equation, presented in Lee et al (2005) used for strain-life fatigue is given below:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{5}$$

where

ε_a	strain amplitude
σ_f'	fatigue strength coefficient
E	Young’s modulus
Nf	strain reversals
b	fatigue strength exponent, usually varying between -0.04 and -0.15 for metals
ε_f'	fatigue ductility coefficient
c	fatigue ductility exponent, usually varying between -0.3 and -1.0 for metals

Combining Fatigue Damage

Overview

A number of methods exist that can be used to combine fatigue damage from different load cycles. The three being considered here are

- Miner's linear damage rule,
- Double linear damage rule,
- Power law damage rule.

Details are given below.

Miner's Linear Damage Rule

Miner's linear damage rule, or Miner's rule, assumes that the relationship between fatigue damage is linearly proportional to the cycle ratio; therefore for a given load, each load cycle causes the same amount of damage. The advantage of this rule is that the sequence of the stress levels is independent of the fatigue damage, so the rule is simple to apply. Experimental data has shown that, for elastic cycling, Miner's linear damage rule provides reasonable correlation with fatigue test data, Lee et a l(2005).

Using Miner's rule, the total fatigue damage from different load cycles is the sum of the individual cycle ratios, as shown below:

$$D_T = \sum \frac{n_i}{N_i} \qquad (6)$$

where

D_T total fatigue damage
n_i number of cycles of that stress range that occurred
N_i number of cycles of that stress range to failure

Miner's linear damage rule is the most widely used of all of the damage rules, and is considered the industry standard method. However, the criticism of Miner's linear damage rule is that it is unreliable and may over-estimate the number of cycles to failure; it is, therefore, considered non-conservative.

Power Law Damage Rule

The power law damage rule was developed by Manson and Halford, discussed in Lee et al (2005), and derived from the relationship between damage and cycle ratio. The method assumes that for a given damage level the cycle ratios from different loading can be determined. This is shown in Figure 3, A, where the damage / cycle ratios are sketched for two different load levels. For a given damage, D, the cycle ratios, n1/N1 and n2/N2, can be determined.

If a two-step load sequence is assumed, the number of cycles from the first load step can be recorded, and the damage for load step one calculated. The equivalent cycle ratio for the second load step can then be determined, and consequently the number of cycles remaining assessed. This is shown in Figure 3, B.

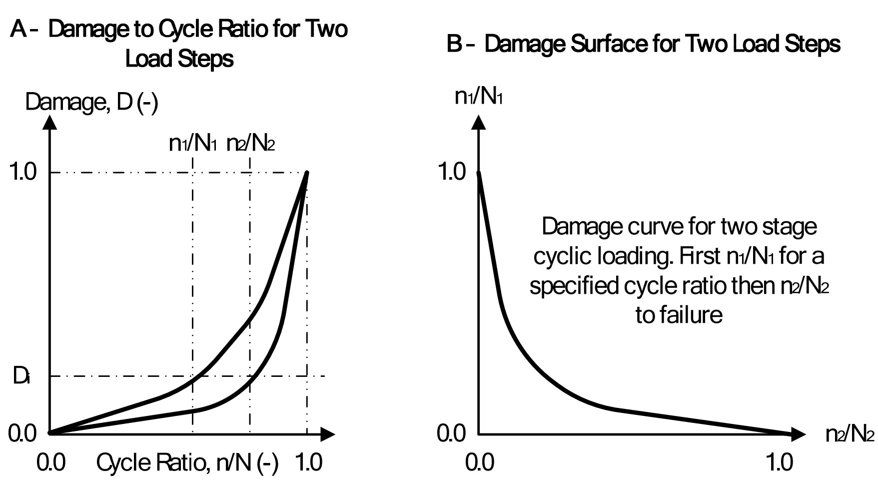

Figure 3: Power law damage rule, after Lee et al (2005)

The same damage level for two stress levels in terms of cycle ratio is as follows:

$$D_i = \left(\frac{n_1}{N_1}\right)^{\frac{2}{3}N_1^{0.4}} = \left(\frac{n_2}{N_2}\right)^{\frac{2}{3}N_2^{0.4}} \quad (7)$$

Rearranging the above equation for the first cycle ratio gives:

$$\frac{n_1}{N_1} = \left[\frac{n_2}{N_2}\right]^{\left(\frac{N_2}{N_1}\right)^{0.4}} \quad (8)$$

This equation presents a method that can be used to calculate the fatigue life of two or more load steps. The fatigue life determined using this method is dependent on the sequence of loading. It can be observed from Figure 2.3, A, that when load step one occurs before load step two, the total cycle ratio at failure would be below one. Conversely, if load step two occurred before load step one the total cycle ratio at failure would be greater than one.

The power law damage rule is not generally used to calculate fatigue damage as it requires prior knowledge of the load sequence. For systems with multiple load steps, such as risers, the calculation would quickly become complicated.

Double Linear Damage Rule

The double linear damage rule is a linearization of the power law damage rule that assumes that fatigue damage can be represented by two linear relationships; one for crack propagation and one for crack initiation. An example is shown in Figure 4, where the co ordinates of the 'knee' in the double linear damage rule were calculated as follows:

$$\left[\frac{n_1}{N_1}\right]_{knee} = 0.35\left(\frac{N_1}{N_2}\right)^{0.25} \quad (9)$$

$$\left[\frac{n_2}{N_2}\right]_{knee} = 0.65\left(\frac{N_1}{N_2}\right)^{0.25} \quad (10)$$

The advantage of the double linear damage rule is that within the linear systems, or phases, the damage is allowed to be summed using linear theory, such as Miner's rule. This implies that the order of the cycling within each phase is not important. The double linear damage rule can be implemented with relative ease compared to the power law damage rule.

The limitation of the double linear damage rule is that the equations for the knee have been validated with experimental data to an elastic to plastic cycle ratio of 1,000; whereas the elastic to plastic cycle ratio for coiled tubing used as a riser is expected to be around 1,000,000. The double linear damage rule is considered more conservative than Miner's rule.

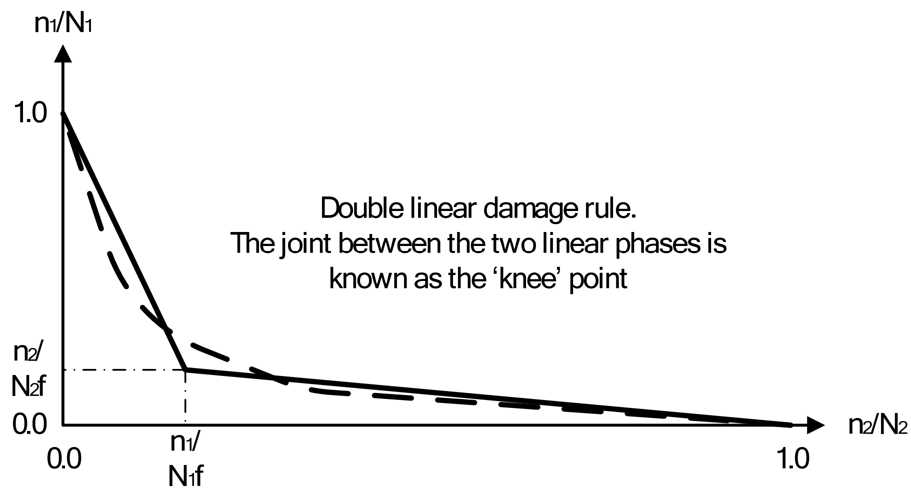

Figure 4: Double linear damage rule, after Lee et al (2005)

Comparison of Methods

The fatigue damage along a piece of coiled tubing is calculated for consecutive deployment consisting of spooling and un-spooling, without straightening, and ten days of connected operations, as can be seen in Table 1. The cumulative fatigue damage is determined using each method; hence the number of deployments estimated, as shown in Table 2. The elastic cycle fatigue damage incorporated a factor of safety of five, representing an inspectable fatigue critical component.

The number of deployments varies from the conservative 4 to the over-estimated 38. This shows that using different methods to combine plastic and elastic fatigue damage changes the results significantly. This indicates that further research is required to validate and calibrate the models for determining the combined fatigue life from elastic and plastic cycles.

Table 1: Assumed fatigue characteristics of coiled tubing

Cycles	Number of Cycles to Failure	Cycles per Deployment	Cycle Ratio per Deployment	Factor of Safety Applied
Plastic	128	3	0.0234	1
Elastic	1.72×10^{8}	86400	5.03×10^{-4}	5

Table 2: Summary of deployments by fatigue method

Method	Fatigue Damage Ratio	Deployments	Comment
Miner’s Linear Damage Rule	100 %	38	Known to be unreliable for combining damage from two load cycles of different orders of magnitude. Results shows maximum number of deployments
Power Law Damage Rule	10 %	4	Standard industry model that produced conservative estimation of number of deployments. Indicates the need to validate the model with test data
Double Linear Damage Rule	15 %	6	Gives a conservative result using an industry standard method.

Experiment

Overview

A set of experiments were proposed to examine the number of plastic and elastic cycles to failure of a material representing coiled tubing. The experiments were conducted with the primary purpose of illustrating the effect of combining plastic and elastic cycles, with the intention of conducting further experiments using full-scale coiled tubing sections.

Material Samples

The material tested was ASTM A606 plate, 0.3 in thick master coil. Dog bone shaped samples were machined to ASTM E466, Figure 5, which had a length of approximately 140 mm, were 30 mm wide at each end and tapered to 5.5 mm wide in the middle. The taper had a radius of 100 mm. All edges of the samples were lightly polished to remove any sharp edges.

In the centre of the samples, notches were added to artificially lower the number of cycles to failure. Each 0.25 mm or 0.5 mm notch was cut using an EDM machine. With a 0.25 mm notch, the minimum width of the sample was 6.0 mm. With a 0.5 mm notch, the minimum width of the sample was 6.5 mm. This change was made to ensure that the smallest cross-sectional area at the start of each test was identical.

Strain gauges were attached to each sample and were used during the tests to record axial strain.

Strain and strain cycling was conducted using the tension apparatus at DNV in Singapore (DNV Pte Ltd, 2010).

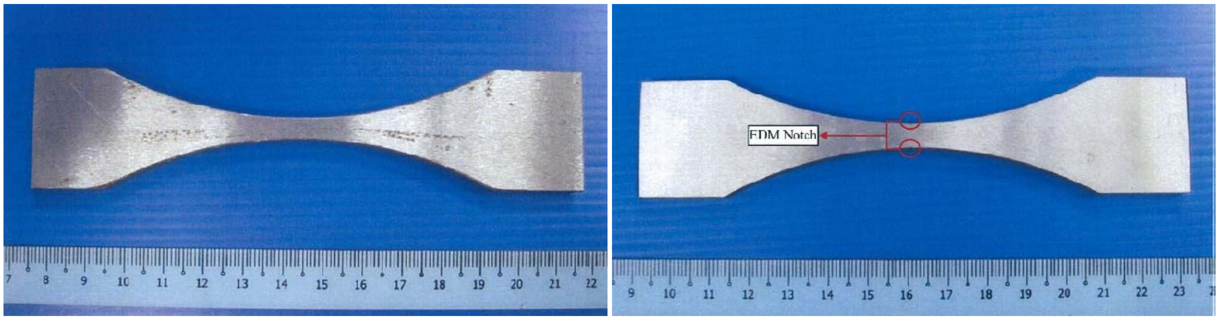

Figure 5: Test samples A; smooth and B; notched.

Test Procedure

Each test was conducted using the following test procedure:

1) Load the specimen in tension to 1.8% strain and then in compression to 0.0% strain by monitoring the strain measured by the strain gauge on the sample. Record stress and strain.
2) Load the sample cyclically from 0.0% to 1.8% then to 0.0% strain and repeat for the desired number of plastic cycles.
3) Cycle the sample with a constant stress range of 300 MPa or 400 MPa until failure. Record number of cycles.

For each notch size (0.0 mm, 0.25 mm and 0.5 mm) the first test conducted was to cycle the sample to failure using just plastic cycling. The second test cycled the sample to failure using only elastic cycling. These tests were repeated to provide an average number of plastic and elastic cycles to failure. Subsequent tests were then conducted; plastically cycling a sample to 10%, 20%, 30%, 50% and 75% of the average number of plastic cycles to failure. Each sample was then elastically cycled to failure.

Experimental Data

For each notch size the average plastic cycles to failure and average elastic cycles to failure for 300 MPa and 400 MPa stress are shown in Table 3. As expected, as the notch size increased the number of cycles to failure decreased. An unnotched sample required on average 520 plastic cycles to failure, while a 0.25 mm notched sample required 1/5 the number of plastic cycles to failure. For the elastic cycles to failure the un-notched samples exceeded 1,000,000 cycles and the tests were stopped. To ensure that the samples failed due to elastic cycling within a reasonable time frame, testing was conducted using the notched samples.

Table 3: Average number of cycles to failure for plastic or elastic cycles

Notch Size	Average Plastic Cycles to Failure	Average Elastic Cycles to Failure with 300MPa Load	Average Elastic Cycles to Failure with 400MPa Load
0.0 mm	519	+	+
0.25 mm	109	340970	75217
0.5 mm	85	182495	35642

Notes:
\+ Experiments exceeded 1.0 x10^6 cycles and were stopped

The number of plastic and elastic cycles to failure for each notch size and the 300 MPa and 400 MPa elastic cycling are shown in Figure 6. As expected, the general trend shows that as the number of plastic cycles increases, the number of elastic cycles required to fail the samples reduces and nominally follows the power law shape shown in Figure 1 and Figure 4. The equation for the power law, from equation (8), is shown below.

$$\frac{n_e}{N_e} = 1 - \left[\frac{n_p}{N_p}\right]^{\left(\frac{N_p}{N_e}\right)^P} \qquad (11)$$

where

n_e	number of elastic cycles
N_e	number of elastic cycles to failure
n_p	number of plastic cycles
N_p	number of plastic cycles to failure
P	power law exponent

Comparison with Theory

The plastic and elastic cycles to failure were normalized to the average plastic and elastic cycles to failure given in Table 2, and are presented in Figure 7. This shows that when normalized most of the test data follows a similar non-linear trend, which may be described using a power law form, equation (11) with a power, P, of 0.05. However, this value was arbitrarily chosen and does not account for data points where the normalized cycles to failure were above 1.0. A more conservative value for P was 0.15 as this curve forms a conservative trend line where each data point appears above the trend line. For comparison, a linear trend is also shown in Figure 7, which represents Miner's rule.

A comparison of the number of deployments calculated using the power law damage rule with a range of exponents is given in Table 4. This shows that if P was assumed to be 0.15 the number of deployments was 29, which gave a fatigue life of around 75% of the fatigue life generated using Miner's rule.

Table 4: Deployments calculated using power law damage rule with differing powers

Power Law Exponent	Deployments	Comparison to Miner's Rule
Miner's Linear Damage Rule, P = 0.0	38	100%
0.05	37	97.4%
0.15	29	76.3%
0.25	17	44.7%
0.40	4	10.5%

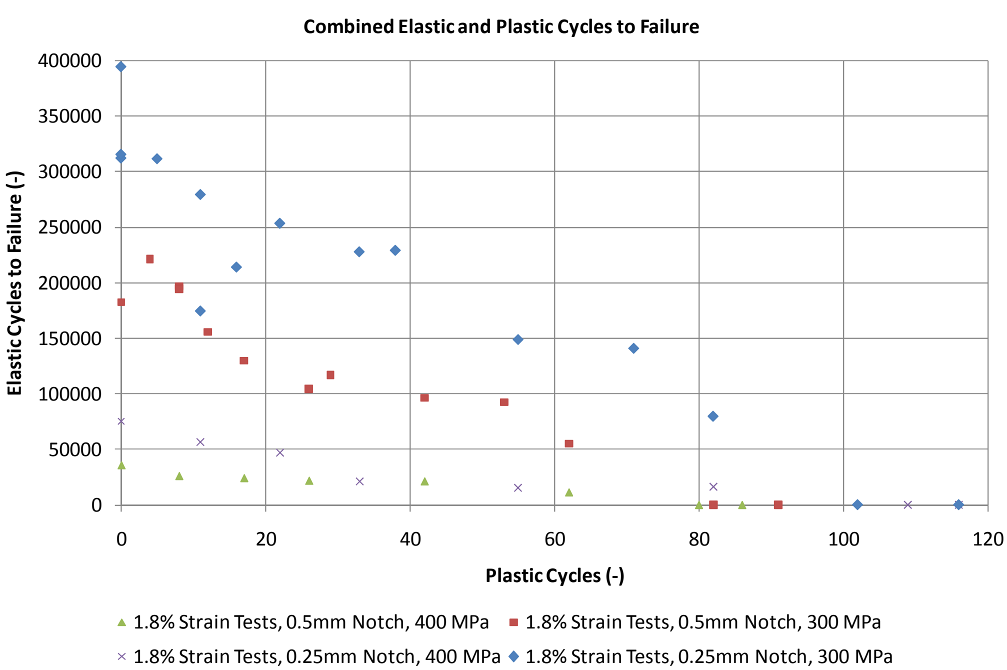

Figure 6: Plot of plastic cycles to failure with elastic cycles to failure

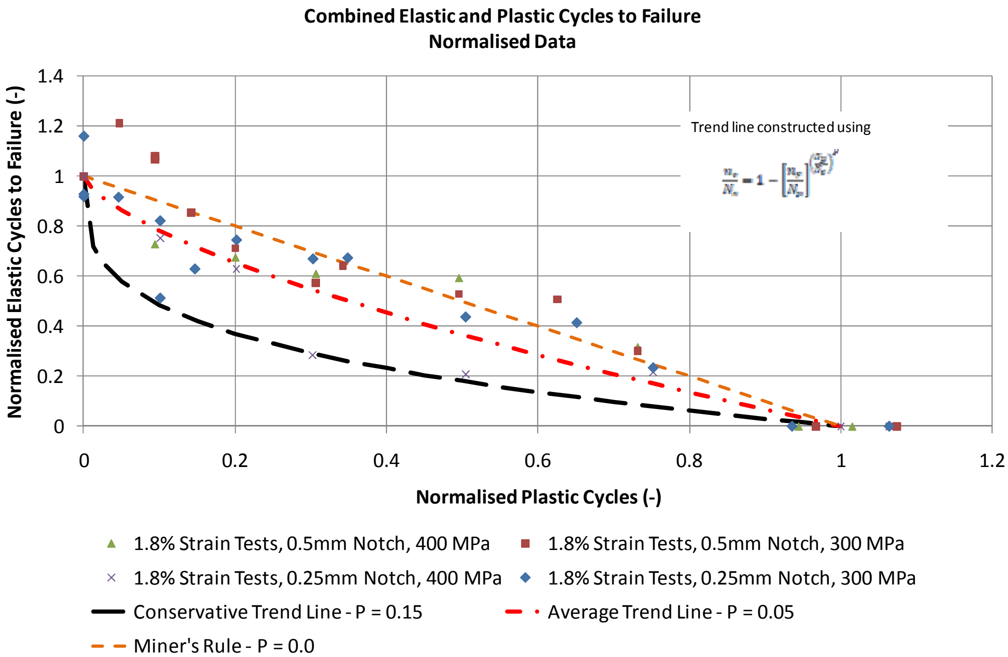

Figure 7: Plot of normalized plastic cycles to failure with normalized elastic cycles to failure

Conclusions

The experiments presented were conducted as a preliminary investigation to determine the fatigue life of coiled tubing that was subject to both plastic and elastic cycles. The work shows that the combined fatigue damage from plastic and elastic cycles is higher than the linear summation of the fatigue damage from both sources. Experimental work conducted confirms that plastic and elastic cycles can be combined using a non-linear relationship. Examination of the experimental data suggests that a power law damage rule provides an appropriate method to determine the overall fatigue damage. The data shows that the published value of P of 0.4 may be too conservative and that a value of 0.15 is more appropriate; however, further work and experiments are required to determine the appropriate power law exponent to use for fatigue design.

The experiments cycled the steel samples assuming that all of the plastic cycles occurred before the elastic cycles. This test sequence was chosen to simplify the testing process; however the cycling experienced by coiled tubing risers would be more appropriately represented as a repeating sequence of plastic and elastic cycles. It is expected that changing the sequence of the plastic and elastic cycles changes the overall fatigue damage, and may improve the overall fatigue life.

Additional areas for further work include examination of how corrosion affects the combined plastic and elastic fatigue damage, performing tests using coiled tubing sections, internal pressure cycling and test coiled tubing sections that contain a bias weld.

References

ASTM E466 (2007) – “Standard Practice for Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials”.

DnV-RP-C203 (2008) – “Fatigue Design of Offshore Steel Structures”.

DNV Pte Ltd (2010) – “Combined Elastic / Plastic Fatigue Tests”, Report number EP013798-12IH2ZK-3, Confidential Report.

Lee, Pan, Hathaway & Barkey (2005) – “Fatigue Testing and Analysis (Theory and Practice)”, Elsevier.