Shear Deflection in Beams

1. NATURE OF SHEAR DEFLECTION

Shear stresses in a beam section cause a displacement or sliding action on a plane normal to the axis of the beam, as shown in the right hand view of Figure 1. This is unlike the deflection resulting from bending in a beam, which is shown in the left hand view of Figure 1.

Normally deflection due to shear in the usual beam is ignored because it represents a very small percentage of the entire deflection. Figure 2 shows that the deflection due to shear increases linearly as the length of the beam increases, whereas the deflection

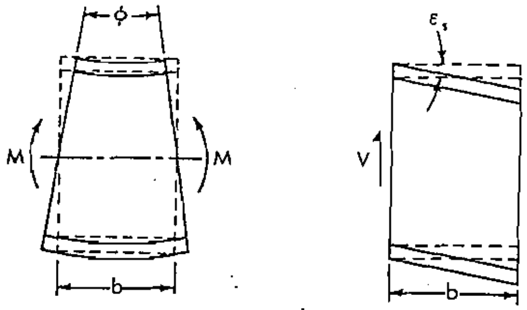


FIG. 1 Deflection in beam caused by bending moment, left, and by shear, right.

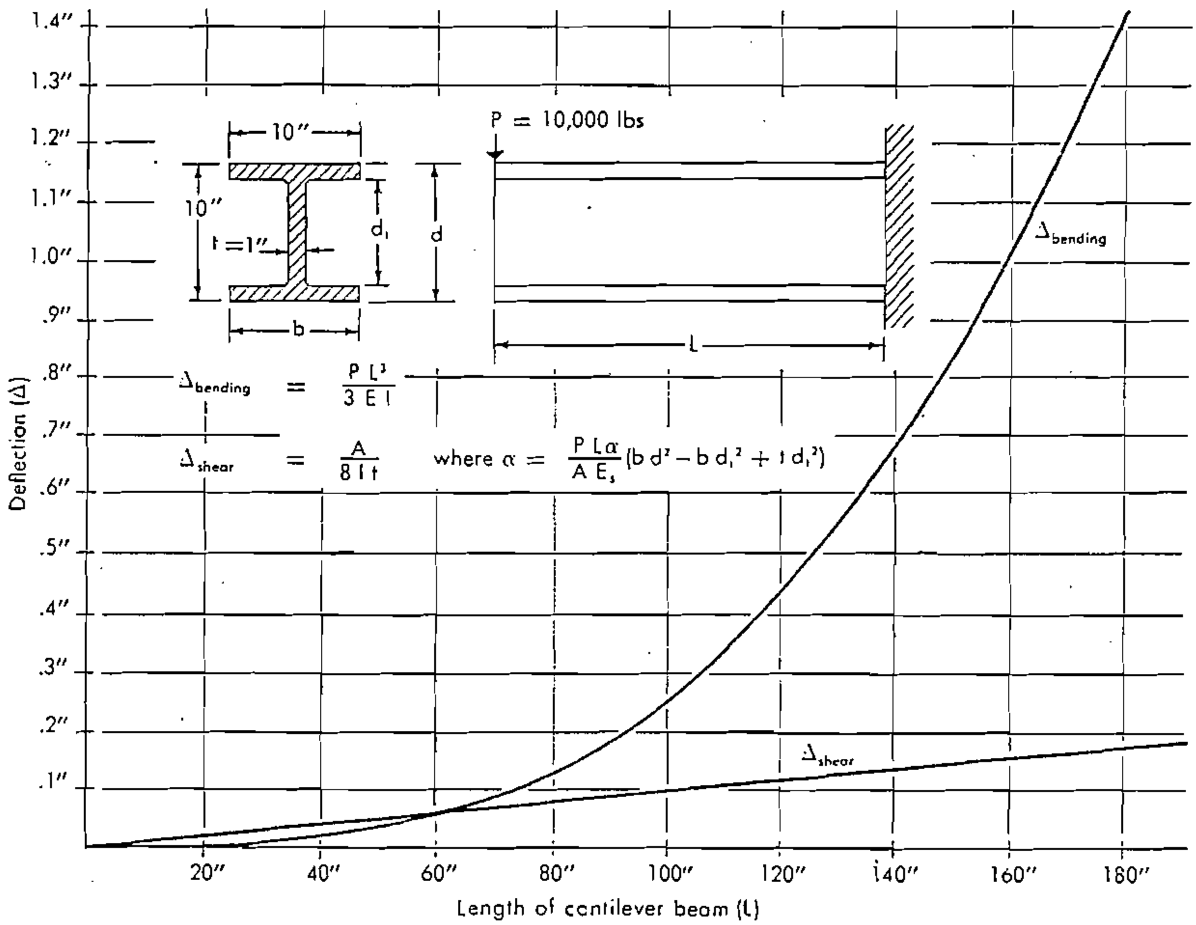


FIG. 2 Deflection caused by shear increases linearly as length of beam, but that caused by bending increases as the third power of beam length.

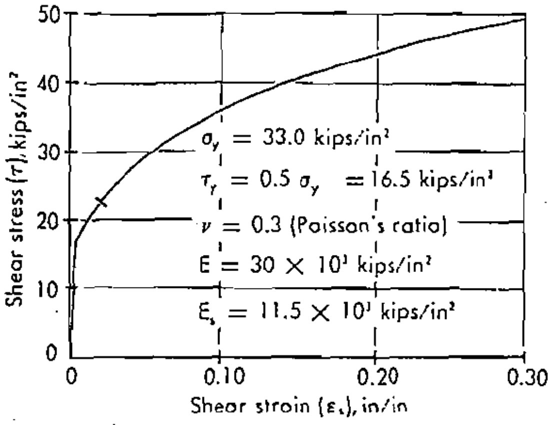


FIG. 3 Shear stress-strain diagram.

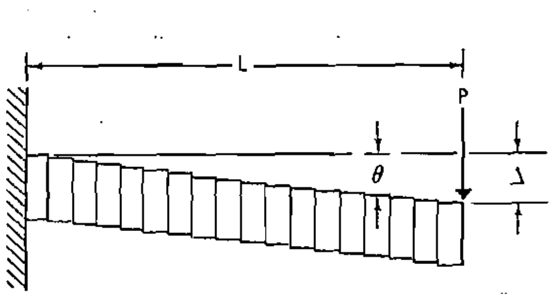
due to bending increases very rapidly as a third power of the length of the beam. For this reason the deflection due to shear is not an important factor except for extremely short spans where deflection due to bending drops off to a very small value.

The deflection due to shear is dependent entirely on the shear distribution across the cross-section of the member and also the value of the shear stress (τ). Figure 3 shows the shear stress-strain diagram which is similar to the usual stress-strain diagram, although the shear yield strength is much lower than the tensile yield strength of the same material. After the shear yield strength is reached, the shear strain (ϵ_s) increases rapidly and the shear strength increases because of strain hardening.

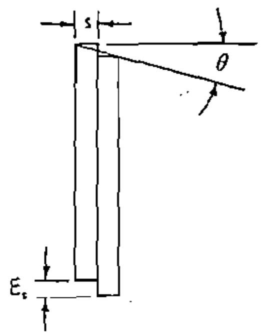
2. DETERMINING SHEAR DEFLECTION

The theory of deflection caused by shear stress is rather simple. However, the actual determination of the shear stresses and their distribution across the beam section (which two factors cause the deflection) is more difficult. In all cases, some kind of a form factor (α) must be determined, and this is simply a matter of expressing the distribution of shear stress throughout the web of the section. Since there is practically no shear stress in the flange area, this particular area has negligible effect on the deflection due to shear (Δ_s).

The following formulas are valid for several types of beams and loading:



Shear deflection of cantilever beam with concentrated load



$$\theta = \frac{\Delta}{L} = \frac{\epsilon_s}{s}$$

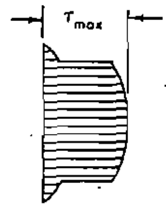
$$\Delta = \theta L$$
(as $s \rightarrow 0$)
$$\theta = \epsilon_s$$

$$E_s = \frac{\tau}{\epsilon_s} \quad \text{or} \quad \epsilon_s = \frac{\tau}{E_s}$$

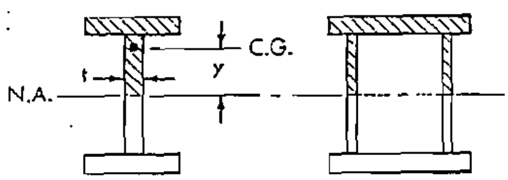
$$\tau_{max} = \tau_{av}\,\alpha = \frac{P\alpha}{A}$$

$$\Delta = \theta L = \epsilon_s L = \frac{\tau L}{E_s} = \frac{P L \alpha}{A E_s}$$

$$\therefore \quad \boxed{\Delta = \frac{P L \alpha}{A E_s}}$$



Shear stress (τ)



$$\text{Form factor } \alpha = \frac{\tau_{max}}{\tau_{aver}} = \frac{\left(\frac{V a y}{I t}\right)}{\left(\frac{V}{A}\right)} = \frac{a y A}{I t}$$

a = area beyond neutral axis
y = distance between center of gravity of this area and neutral axis of entire cross-section
A = total area of section
I = moment of inertia of section
t = total thickness of web
E_s = shear modulus of elasticity
ϵ_s = shear strain
τ = shear stress

FIG. 4 Form factor for shear deflection in built-up beams.

simply supported beam; uniform load (w)

$$\boxed{\Delta_s = \frac{w\ L^2\ \alpha}{8\ A\ E_s}} \quad \text{..........} (1)$$

simply supported beam; concentrated load (P)

$$\boxed{\Delta_s = \frac{P\ L\ \alpha}{4\ A\ E_s}} \quad \text{..........} (2)$$

cantilever beam; uniform load (w)

$$\boxed{\Delta_s = \frac{w\ L^2\ \alpha}{2\ A\ E_s}} \quad \text{..........} (3)$$

cantilever beam; concentrated load (P)

$$\boxed{\Delta_s = \frac{P\ L\ \alpha}{A\ E_s}} \quad \text{..........} (4)$$

where:

P = total load, lbs

A = area of entire section

E_s = modulus of elasticity in shear (steel = 12,000,000 psi)

w = distributed load, lbs/linear in.

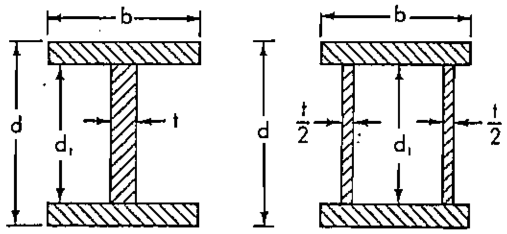


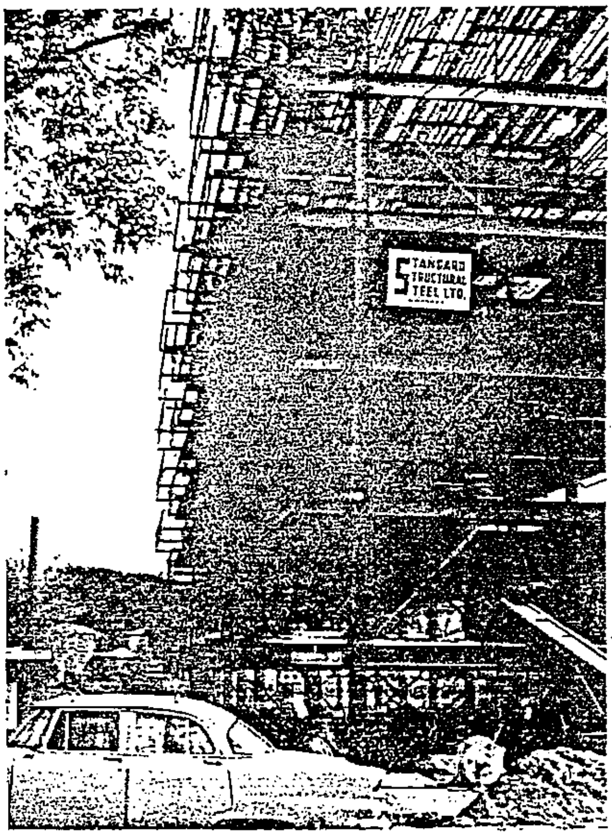
FIG. 5 Beam sections for which Eq. 5 applies.

The slope of the deflection curve (θ) is equal at each cross-section to the shearing strain (ϵ_s) at the centroid of this cross-section. α is a factor with which the average shearing stress (τ_{av}) must be multiplied in order to obtain the shearing stress (τ_{max}) at the centroid of the cross-sections.

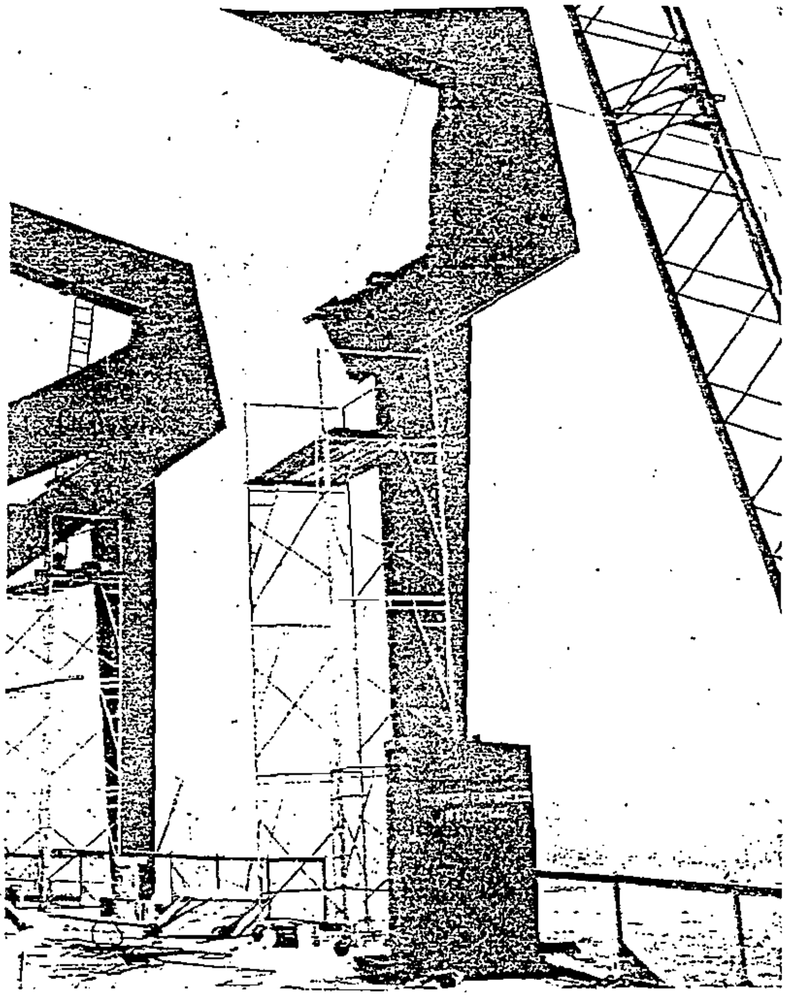
On this basis, the form factor (α) for an I beam or box beam would be:

$$\boxed{\alpha = \frac{A}{8\ I\ t}(b\ d^2 - b\ d_1^2 + t\ d_1^2)} \quad \text{..........} (5)$$

where Figure 5 applies. Don't compute area (A) in this formula because it will cancel out when used in the formulas for shear deflection.



Welding was used extensively in the fabrication and erection of this steel-framed, 8-story, balconized apartment building which features cantilevered cross beams in the upper stories. The building was designed basically as a rigid structure with main beams designed plastically and light X-braces used to accommodate wind moments. The welded steel design cost 16¢/sq ft less than a reinforced concrete building would have.



Both shop and field welding were used extensively in building the Anaheim Stadium, home of the Los Angeles Baseball club—the Angels. The steelwork was designed as an earthquake-resistant frame, with high moment carrying capacity in both directions. Having very good torsional resistance in addition to bending strength in both directions, the tapered box section frames can be located more widely (45′ centers along straight sides) and eliminate the need for conventional cross-bracing between bents.