

Open-Web Expanded Beams and Girders

1. DESIGN CONCEPT

Dramatic savings can be obtained from an often forgotten design concept. The open-web expanded beam has already paid substantial dividends for various engineering firms. It should be considered on many more projects.

The opening up of a rolled beam increases its section modulus and moment of inertia, results in greater strength and rigidity. The reduction in beam weight has a chain effect on savings throughout the structure.

The open-web expanded beam is made economically by flame cutting a rolled beam's web in a zig-zag pattern along its centerline. See Figure 1. One of the two equal halves is then turned end for end and arc welded to the other half. The result is a deeper beam, stronger and stiffer than the original.

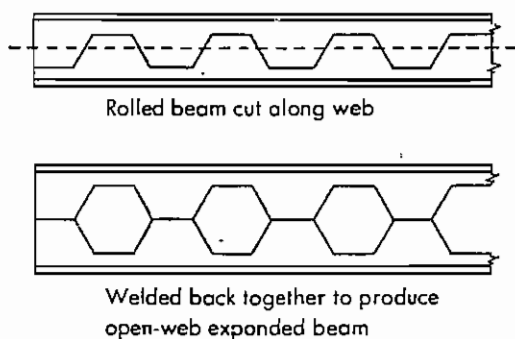


FIG. 1 Result: a deeper beam, stronger and stiffer than the original. Design starts with a lighter beam for immediate savings in material and handling costs. It often eliminates need for heavy built-up beam.

Starting the design with a lighter rolled beam realizes immediate savings in material and handling costs. There is no waste material with this method. It often eliminates the need for a heavy built-up beam.

In the design of buildings, the web opening is frequently used for duct work, piping, etc. which conventionally are suspended below the beam. See Figure 2. On this basis for equivalent strength, open-web expanded beams usually permit a reduction in the distance between ceiling below and floor above and thus provides savings in building height.

Oxygen flame cutting of the light beam web is

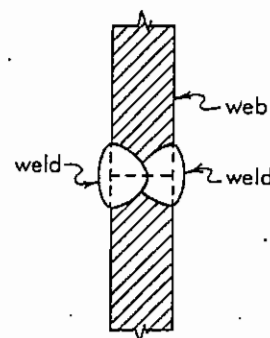


FIG. 2 Use semi-automatic arc welding to rejoin the two halves. A 100% fully penetrated butt weld can often be made with a single pass on each side of web without beveling.

relatively easy on a template-equipped machine.

The use of semi-automatic arc welding to rejoin the two halves enables good, sound welds to be made faster, more economically. Welding is confined to a portion of the web's total length. A 100% fully penetrated butt weld can usually be made with a single pass on each side of the web, without prior beveling of the edges. See Figure 2.

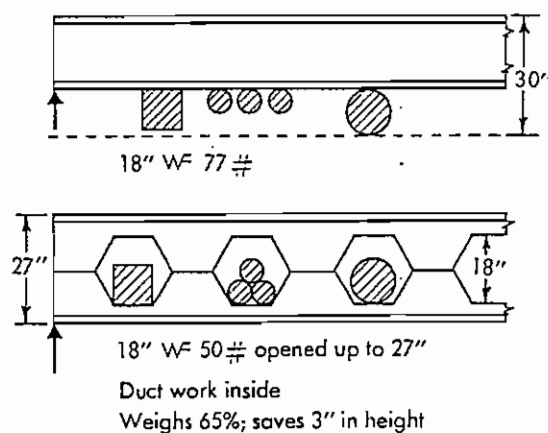


FIG. 3 Opening in web used for duct work, piping, etc., normally suspended below beam. For equivalent strength, open-web expanded beam usually reduces distance between ceiling below and floor above.

Cutting the zig-zag pattern along a slight angle to the beam axis results in a tapered open-web expanded beam. See Figure 4. This has many applications in roof framing, etc.

4.7-2 / Girder-Related Design

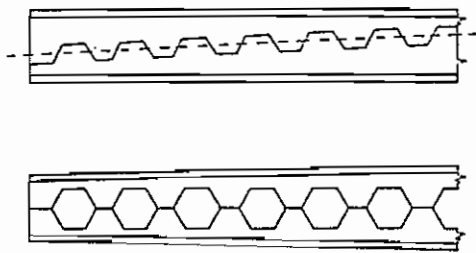


FIG. 4 Cutting the zig-zag pattern along an axis at slight angle to the beam results in tapered open-web expanded beam. This has many applications in roof framing, etc.

Two open-web expanded beams can sometimes be nested together to form a column having a high moment of inertia about both its x-x and y-y axes. See Figure 5.

2. GEOMETRY OF CUTTING PATTERN

The zig-zag cutting pattern and the resulting geometry of the web cut-out help determine properties of the section.

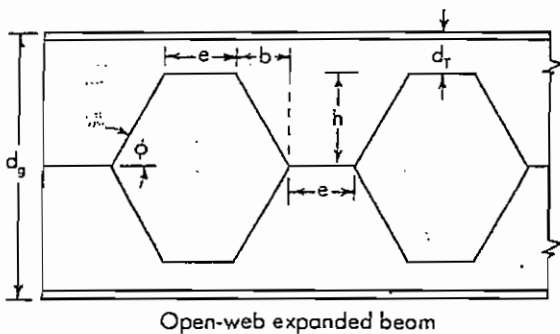
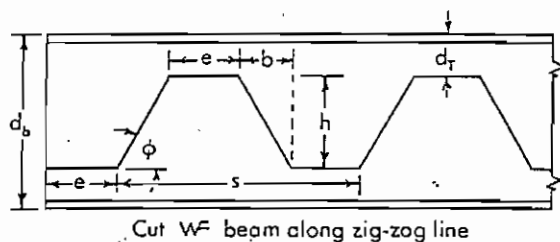


FIGURE 6

$$\tan \phi = \frac{h}{b}$$

$$b = \frac{h}{\tan \phi}$$

$$d_s = d_b + h$$

or

$$d_r = \frac{d_b - h}{2}$$

$$s = 2(b + e)$$

In general, the angle (ϕ) will be within about 45° minimum and about 70° maximum, with 45° and 60° being most commonly used. This angle must be

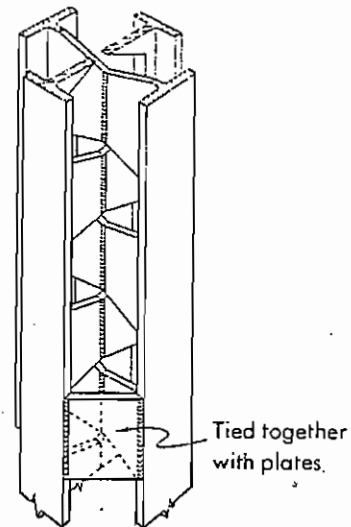


FIG. 5 Two open-web expanded beams can sometimes be nested together to form a column having a high moment of inertia about both its x-x and y-y axes.

sufficient to keep the horizontal shear stress along the web's neutral axis from exceeding the allowable; see Figure 7.

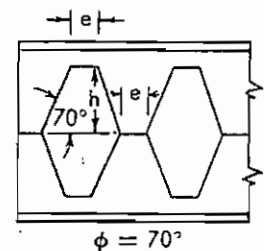
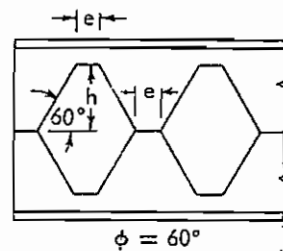
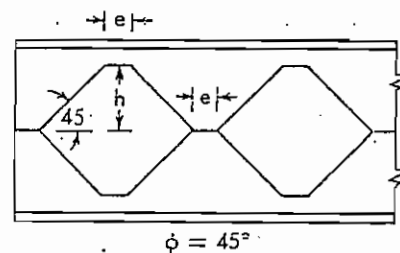


FIGURE 7

The distance (e) may be varied to provide the proper web opening for duct work, etc., and/or the proper distance for welding between openings. See Figure 8. However, as this distance (e) increases, the bending stress within the Tee section due to the applied shear force (V) increases. Thus, there is a limit to how large (e) may be.

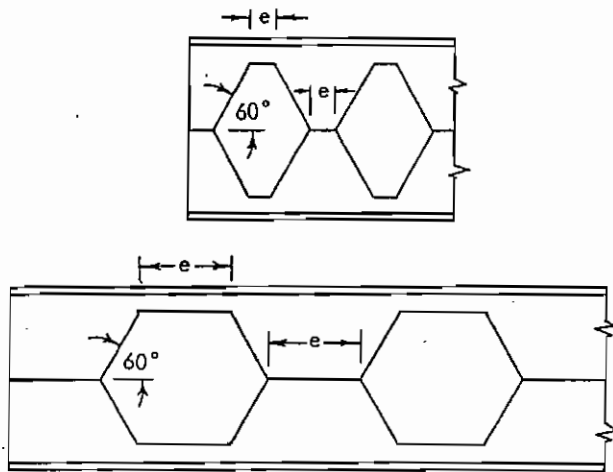


FIGURE 8

3. RESISTANCE TO APPLIED FORCES

Since the beam flanges carry most of the bending load, the loss of web area is not much of a problem as far as moment is concerned. However, shear (V) is carried by the web, and must be considered.

At each web opening, two Tee sections act as members of a frame in resisting vertical shear forces.

At midspan b , Figure 9, the shear (V) is minimum and may have little effect on the beam's strength. Approaching the support in the region of high shear a , the bending stress produced by this shear on the shallow Tee section must be added to the conventional bending stress from the applied beam load.

The bending moment due to shear is diagrammed in Figure 10. Usually, the point of inflection in top and

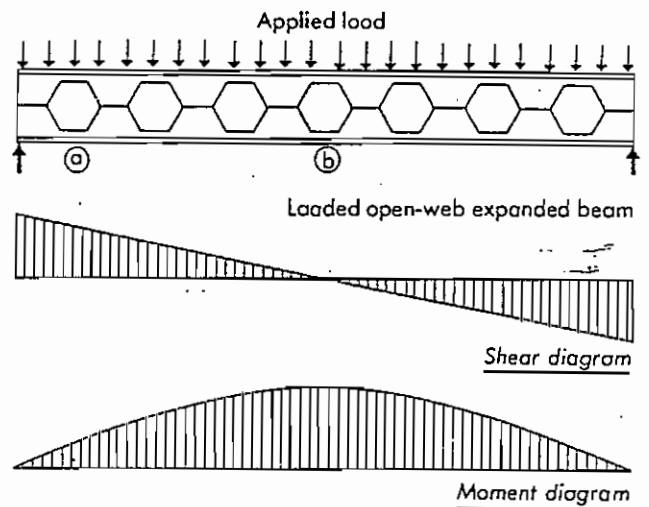


FIGURE 9

bottom Tee sections due to the moment produced by shear, is assumed to be at mid-section of the opening ($e/2$). It is further assumed that the total vertical shear (V) at this point is divided equally between these two Tee sections, since they are of equal depth.

Actually, the design and stress behavior of an open-web expanded beam or girder is very similar to that of a Vierendeel truss. The primary design considerations are as follows:

1. The top and bottom portions of the girder are subjected to compression and tension bending stresses from the main bending moment, $\sigma_b = M/S_b$. There must be a continuity of these sections throughout the girder length to transfer these stresses. In addition, the compression portion must be checked for lateral sup-

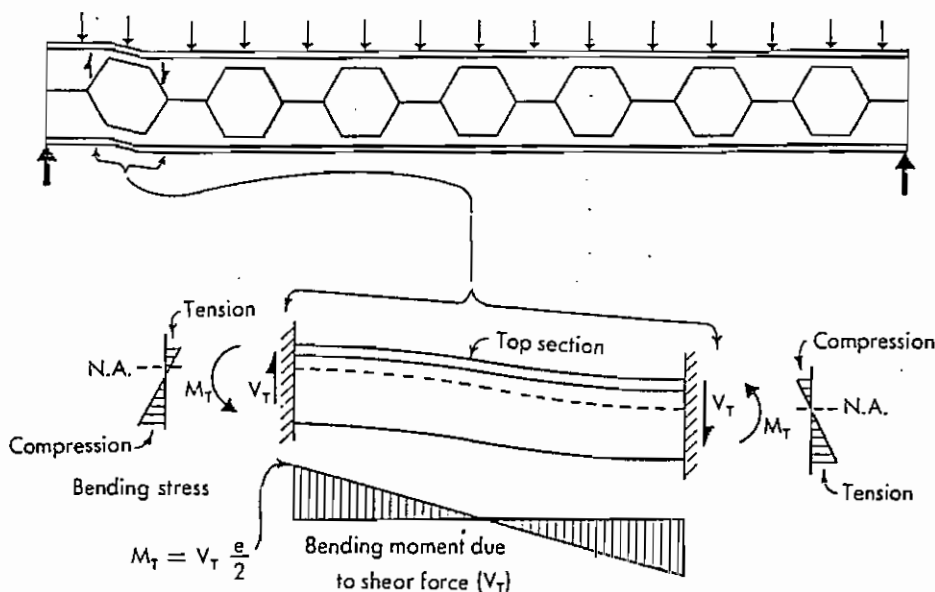


FIGURE 10

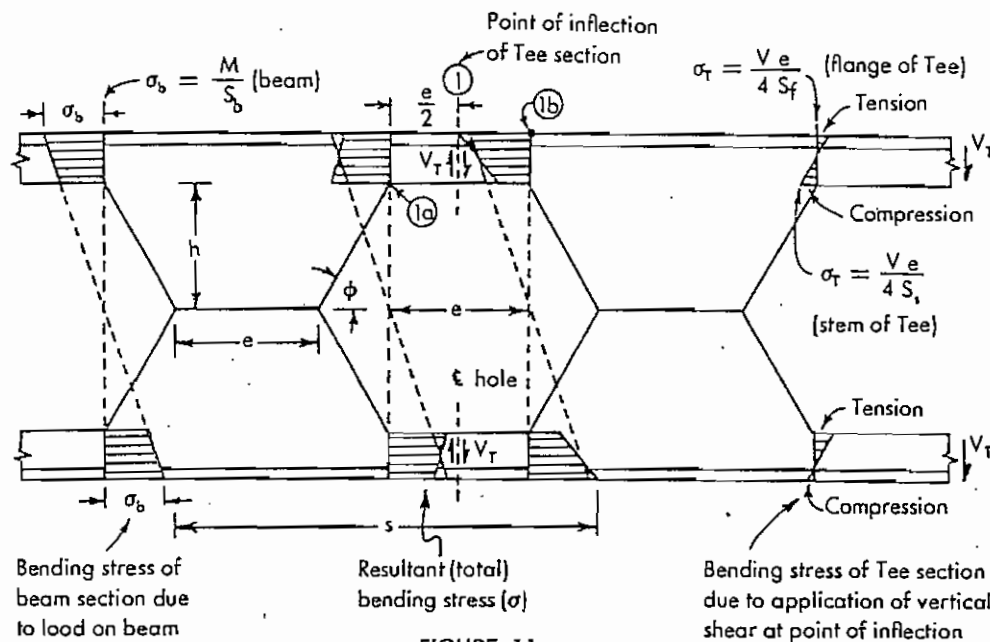


FIGURE 11

port, minimum width-to-thickness ratio, and allowable compressive stress; see the left end of Figure 11.

2. The vertical shear (V) in the girder is carried by the web, and produces vertical shear stresses in the web section, both in the solid portion of the web, and in the stem of the Tee section of the open portion.

3. In the open portion of the web, the vertical shear (V) is divided equally between the top and bottom Tee sections (assuming same depth of Tee sections). Assuming the shear is applied at the mid-opening, it will produce a bending moment on the cantilevered Tee section; see the right-hand end of Figure 11. The resulting secondary bending stresses

$$\sigma_T = \frac{V e}{4 S}$$

must be added to those of the main bending moment, Item 1. If needed, a flange may be added around the inside of the web opening to give the Tee sections added strength.

4. The horizontal shear force (V_h) applied at the solid portion of the web along the girder's neutral axis may subject this portion to buckling. See Figure 20. The resulting compressive bending stress on this unreinforced web section is important because of the possibility of this web section buckling under this stress.

5. The solid portion of the web may transfer a vertical axial force (compressive or tensile) equal to one-half of the difference between the applied vertical shears (V_1) and (V_2) at the end of any given unit panel of the girder. See Figure 27.

6. There should be 100% web depth at the points

of support. Bearing stiffeners may be needed at the ends of the girder where reactions are applied.

4. TOTAL BENDING STRESS IN THE GIRDER

The main bending stress (σ_b) Item 1, acting on a section where the open Tee section starts, is assumed to increase linearly to a maximum at the outer fiber. To this stress must be added or subtracted, depending upon signs, the secondary bending stress (σ_T), Item 3. See central portion of Figure 11.

At point (1a)

Secondary bending stress at stem of Tee due to vertical shear (V) at Section (1), added to main bending stress at stem of Tee due to main moment (M) at Section (1a):

$$\sigma_{1a} = \frac{M_{1a} h}{I_g} + \frac{V_1 e}{4 S_s} \dots \dots \dots (1a)$$

At point (1b)

Secondary bending stress at flange of Tee due to vertical shear (V) at Section (1), added to the main bending stress at flange of Tee due to main moment (M) at Section (1b):

$$\sigma_{1b} = \frac{M_{1b} d_g}{I_g} + \frac{V_1 e}{4 S_f} \dots \dots \dots (1b)$$

Research at the University of Texas* indicated these main bending stresses in the Tee section do not increase linearly to a maximum at the outer fiber of the flange, but in some cases the reverse is true; the stress along the stem of the Tee section is higher than that at the outer fiber of the flange. For this reason, in their analysis, they calculated the bending force $F = M/d$ using the moment (M) on the girder at Section (1),

* "Experimental Investigations of Expanded Steel Beams", by M. D. Altfillisch; Thesis; Aug. 1952.

"Stress Distribution in Expanded Steel Beams", by R. W. Ludwig; Thesis; Jan. 1957.

"An Investigation of Welded Open Web Expanded Beams", by Altfillisch, Cooke, and Toprac; AWS Journal, Feb. 1957, p 77-s.

DEFINITIONS OF SYMBOLS

- d = Distance between neutral axes of Tee section
- d_b = Depth of original beam
- d_g = Depth of expanded girder
- e = Length of Tee section, also length of solid web section along neutral axis of girder.
- h = Height of cut, or distance of expansion
- A_T = Cross-sectional area of Tee section
- I_g = Moment of inertia of open section of expanded girder
- S_f = Section modulus of flange of Tee section
- S_s = Section modulus of stem of Tee section

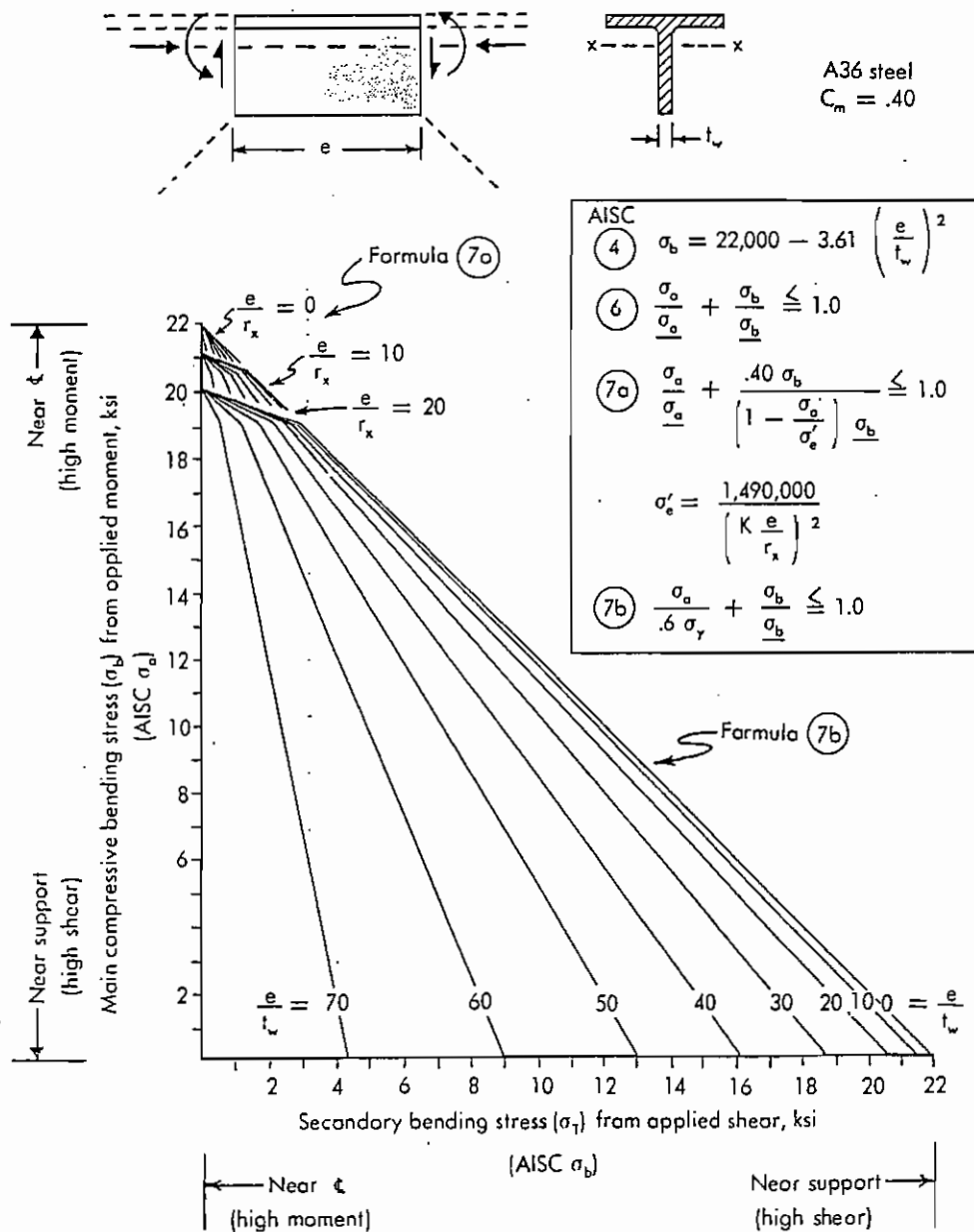


FIGURE 12

the point of inflection of the Tee section. This is convenient because it is the same section at which we assume the vertical shear (V) is applied for the secondary bending stress. They also assume this force (F) is uniformly distributed across the Tee section.

This simplifies the calculations, since for a given unit panel only one section must be considered for both the applied moment (M) and the applied shear (V). This is Section (1) at the point of inflection of the Tee section. Also, only one total bending stress is required for this section—the maximum secondary bending stress at the stem added to the average main bending stress. It does not require calculating at two different points—the stem at Section (1a) and the flange at Section (1b).

$$\sigma_1 = \frac{M_1}{d A_T} + \frac{V_1 e}{4 S_s} \dots \dots \dots (2)$$

since $F = \frac{M_1}{d}$ and

$$\sigma = \frac{F}{A_T} \quad \text{or} = \frac{M_1}{d A_T}$$

The main bending stress (σ_b) and secondary bending stress (σ_T) may be considered according to AISC Interaction Formulas 6, 7a, and 7b. These are shown graphically in Figure 12. (Note that AISC refers to main bending stress as σ_u and to secondary bending stress as σ_b .)

Buckling Due to Axial Compression

The Tee section, because it is subjected to axial compression, also must be checked against buckling according to AISC Sec 1.9.1. See Figure 13, and see Table 1 of limiting ratios for steels of various yield strengths.

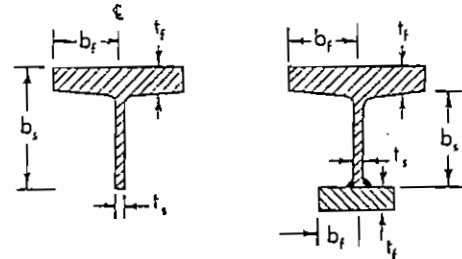


FIGURE 13

Tee Section
Unstiffened

$$\frac{b_f}{t_f} \leq \frac{3000}{\sqrt{\sigma_y}}$$

$$\frac{b_s}{t_s} \leq \frac{4000}{\sqrt{\sigma_y}}$$

Tee Section Stiffened by
Flange Welded Around
Web Opening

$$\frac{b_f}{t_f} \leq \frac{3000}{\sqrt{\sigma_y}}$$

$$\frac{b_s}{t_s} \leq \frac{8000}{\sqrt{\sigma_y}}$$

Number of Points to Check Along Girder's Length

It will be desirable to check the proposed design at only a limited number of points to determine initially whether it will work.

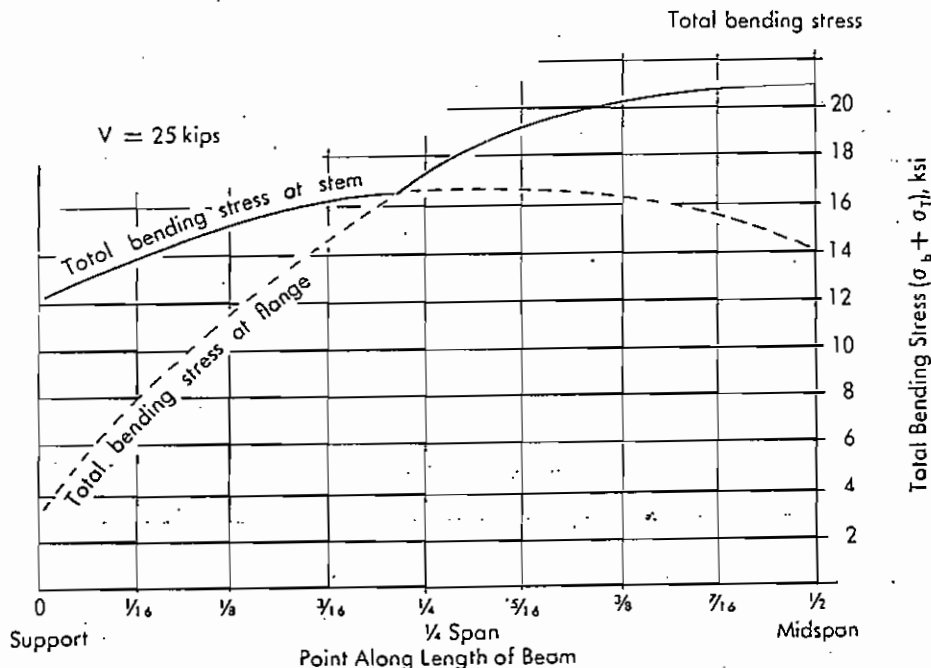


FIGURE 14

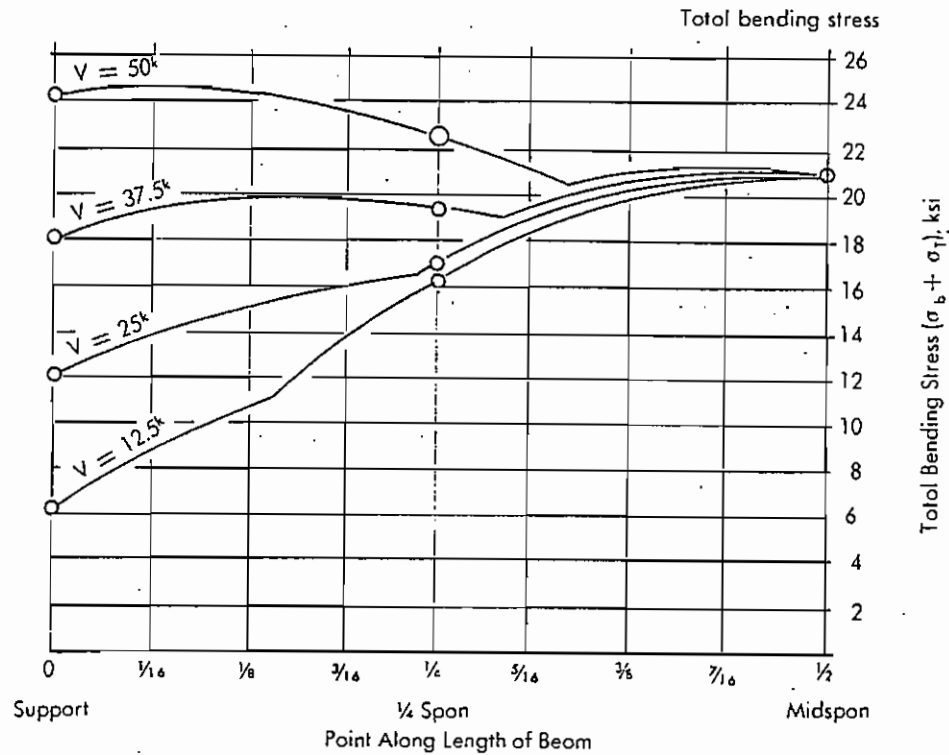


FIGURE 15

Referring to Figure 11, notice the bending stress (σ_b) from the applied moment is assumed to be maximum at the outer fibers of the flange. The bending stress (σ_T) from the applied shear is greatest at the stem of the Tee because its section modulus (S_s) is less than the section modulus at the outer flange (S_f). For this reason, combinations of bending stresses must be considered at the outer fibers of the flange as well as the stem of the Tee.

In Figure 14, the total bending stresses at the outer fiber of the flange as well as at the stem of the Tee section are plotted along the length of the beam. This data is from a typical design problem. In this case, the vertical shear at the support is $V = 25$ kips.

In Figure 15, the example has been reworked with different span lengths, and with different applied uniform loading so that the bending moment (and the bending stress due to this moment) remains the same. The shorter spans require an increased load, hence increased shear (V). The longer spans require a lower load, hence decreased shear (V).

Notice in Figure 15, that for short beams with higher shear force relative to bending moment, this curve for the total bending stress (moment and shear) will rise on the left-hand side, and the point of maximum stress will move to the left, or near the support. Of course there is a limit to how short and how high the vertical shear (V) may be, because this type of open web construction does weaken the web for shear. For

TABLE 1—Limiting Ratios of Section Elements Under Compression

	$\frac{3,000}{\sqrt{\sigma_y}}$	$\frac{4,000}{\sqrt{\sigma_y}}$	$\frac{8,000}{\sqrt{\sigma_y}}$
33,000	16	22	44
36,000	16	21	42
42,000	15	20	39
45,000	14	19	38
46,000	14	19	37
50,000	13	18	36
55,000	13	17	34
60,000	12	16	33
65,000	12	16	31

very high shear loads, the opening in the expanded web would defeat its purpose, and a standard solid web beam or girder should be used. For longer spans, with relatively lower shear force to bending moment, this curve will lower, shifting the point of maximum stress to the right, or near the midspan.

An alternate method to finding the bending stress directly from the applied moment (M) is to convert the moment (M) into a concentrated force (F) applied at the center of gravity of the Tee section and assume it to be uniformly distributed across the section. See Figure 16.

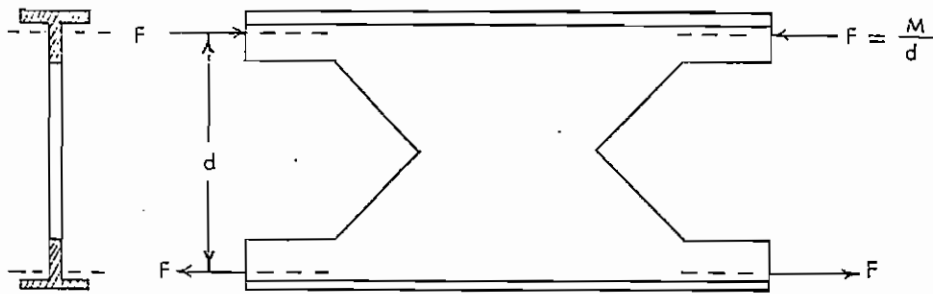


FIGURE 16

Then:

$$\sigma_b = \frac{F}{A_r} = \frac{M}{A_r d} \quad (3)$$

This bending stress is the same at the outer flange of the Tee section as well as the inner stem. It is now only necessary to add the greater bending stress from the applied shear (V) of the Tee section. Therefore, the smaller section modulus at the stem of the Tee section will be used, and only one set of total stress values will be considered.

In Figure 17, the applied moment (M) has been converted into a concentrated force (F) applied at the center of gravity of the Tee section and assumed to be uniformly distributed across the section.

This illustrates that the point of maximum combin-

ation of bending stresses due to applied shear and applied moment lies somewhere between 1) the support (region of high vertical shear) and 2) the midspan (region of high bending moment). This point of maximum stress is indicated in Figure 17 by an arrow.

Unless the beam is examined as in Figure 17 for the maximum stress all the way between the support and midspan, it would be well to check a third point in addition to the support and midspan. A convenient point would be at $\frac{1}{4}$ span.

5. HORIZONTAL SHEAR STRESS

There are three methods of checking the horizontal shear stress along the beam's neutral axis (N.A.):

1. Use the conventional formula for shear stress,

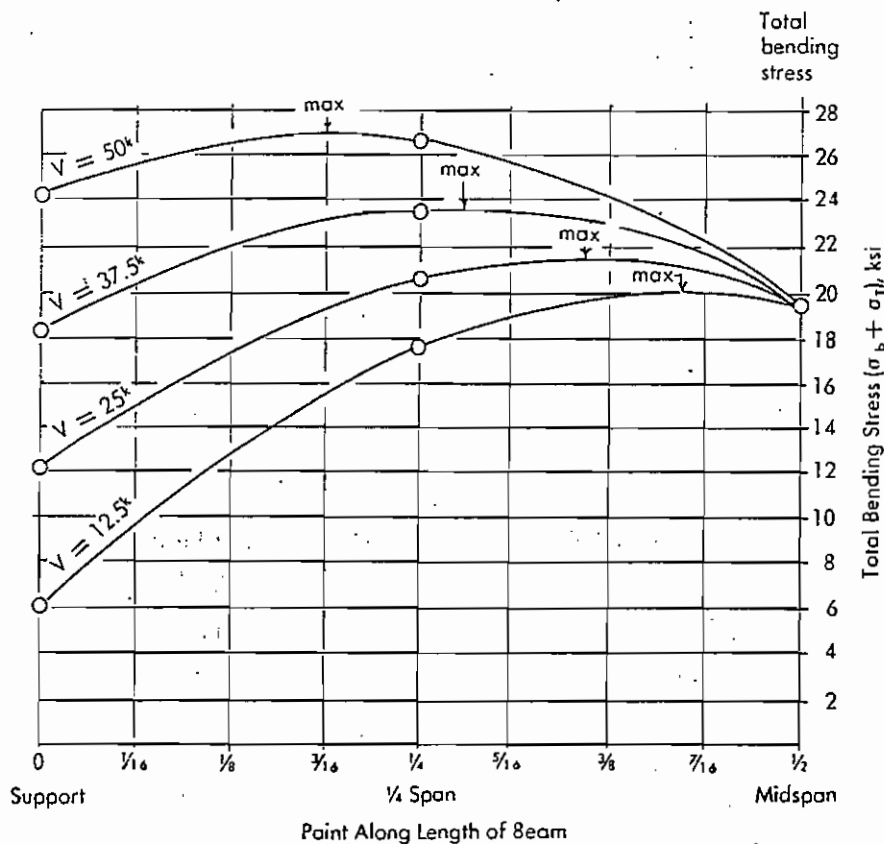


FIGURE 17

assuming the web to be solid ($\tau = \frac{V a y}{I t}$). Then increase this stress by the ratio of overall web segment to net web segment (s/e) to account for only a portion (e/s) of the web along the neutral axis being solid.

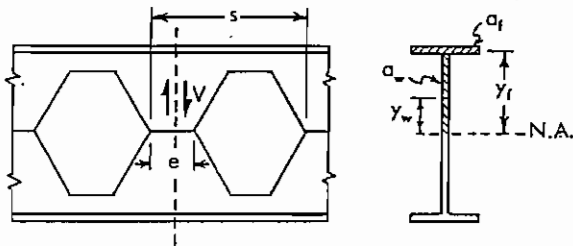


FIGURE 18

$$\tau_n = \frac{V a y}{I t} \left(\frac{s}{e} \right) = \frac{V (a_t y_t + a_w y_w)}{I t_w} \left(\frac{s}{e} \right) \quad (4)$$

2. Treat a top segment of the beam as a free body acted upon by the bending moment force. The difference in this force from one end of the segment to the other is transferred out as horizontal shear along the neutral axis into the similar section below. This horizontal shear force is then divided by the net area of the solid portion of the web section along the neutral axis. See Figure 19.

By substitution:

$$V_h = \frac{M_2 - M_1}{d} \text{ which acts along distance } (e).$$

This horizontal shear force is then divided by the net area of the solid web section ($e t_w$) to give the shear stress:

$$\tau_n = \frac{M_2 - M_1}{d e t_w} \quad (5)$$

3. Using the same free body, Figure 19, take moments about point (y):

$$\frac{V_1}{2} \left(\frac{s}{2} \right) + \frac{V_2}{2} \left(\frac{s}{2} \right) - V_h \frac{d}{2} = 0$$

$$\text{or } V_h = (V_1 + V_2) \left(\frac{s}{2d} \right)$$

Assuming that $\frac{V_1 + V_2}{2} = V_x$, the average vertical shear at this point, this becomes—

$$V_h = V_x \left(\frac{s}{d} \right)$$

$$\text{and } \tau_n = \frac{V_h}{t_w e} \quad (6)$$

6. WEB BUCKLING DUE TO HORIZONTAL SHEAR FORCE

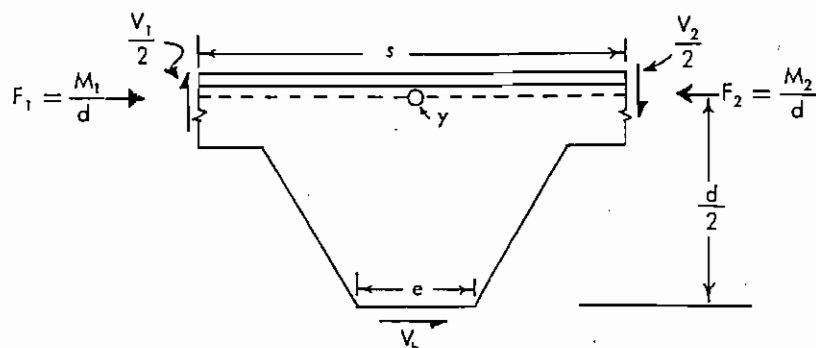
The web of a *conventional plate girder* may have to have transverse intermediate stiffeners to keep it from buckling due to the diagonal compressive stresses resulting from the applied shear stresses. If stiffeners are used, the girder will have a higher carrying capacity. This is because the web, even though at the point of buckling, is still able to carry the diagonal tensile stresses, while the stiffener will transfer the compressive forces. The web of the girder then functions as the web of a truss.

However, in the *open-web expanded girder*, treated as a *Vierendeel truss*, the open portion prevents any tension acting in the web. Therefore, a transverse stiffener on the solid web section will not function as the vertical compression member for truss-like action.

Since this solid portion of the web is isolated to some extent, the horizontal shear force (V_h) applied along the neutral axis of the beam will stress this web portion in bending.

The simplest method of analysis would be to consider a straight section (n), Figure 20. However, the resulting bending stress acting vertically would somehow have to be resolved about an axis parallel to the

FIGURE 19



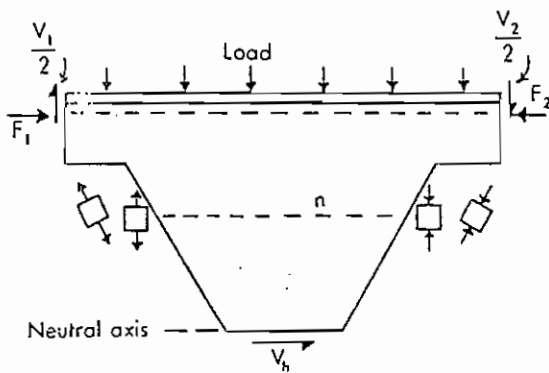


FIGURE 20

sloping edge of this tapered web section.

One method by which tapered beams and knees are analyzed is the Wedge Method, originally proposed by W. R. Osgood and later modified by H. C. Olander (ASCE Transaction paper 2698, 1954). With this method, Figure 21, the non-parallel sides are extended out to where they intersect; this becomes point O. From this point as a center, an arc is drawn through the wedge section representing the section (a) to be considered. The section modulus of this curved section is determined.

The actual forces and moments applied to the member are then transferred out to point O. The horizontal force (V_h) will cause a moment at point O.

It can be shown that these forces and moments acting at point O cause the bending stresses on the curved section (a) of the wedge; see Figure 22.

Moment acting on curved section (a):

$$\begin{aligned} M &= V_h \rho - M_o \\ &= V_h \rho - V_o f \\ &= V_h (\rho - f) \end{aligned}$$

Radial bending stress on this curved section (a):

$$\sigma_r = \frac{M}{S} = \frac{V_h (\rho - f)}{S} \quad \dots \dots \dots (7)$$

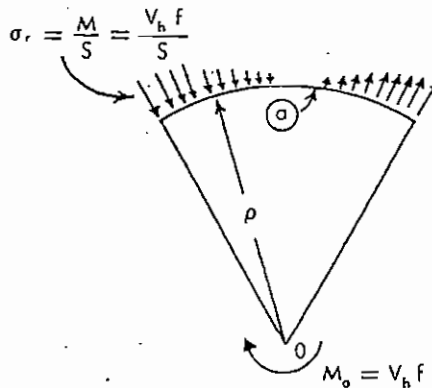
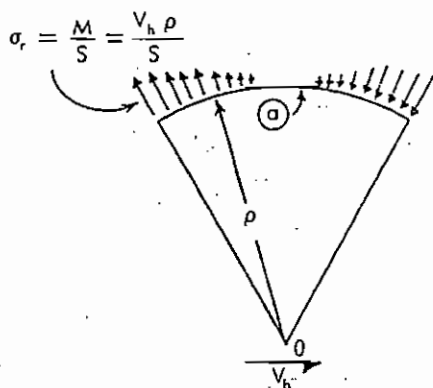


FIGURE 22

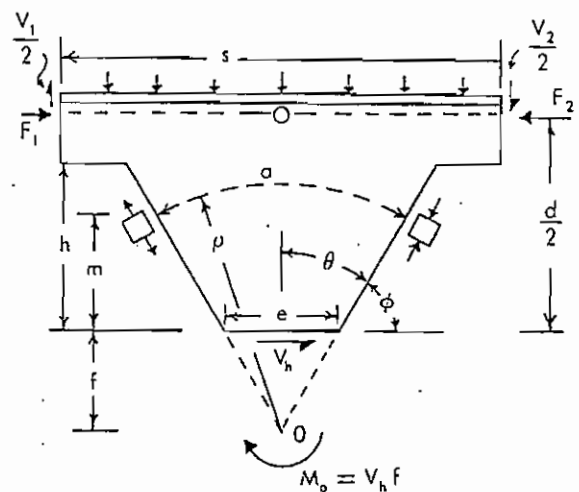


FIGURE 21

where

$$f = \frac{e}{2 \tan \theta} \quad \dots \dots \dots (8)$$

Since:

$$\rho = \frac{m + f}{\cos \theta} \text{ or}$$

$$\rho = \frac{m}{\cos \theta} + \frac{e}{2 \sin \theta} \quad \dots \dots \dots (9)$$

Since:

$$a = 2 \pi \rho \frac{\theta}{2 \pi}$$

$$= 2 \rho \theta \text{ and}$$

$$S = \frac{t_w a^2}{6}$$

$$S = \frac{2}{3} t_w \rho^2 \theta^2$$

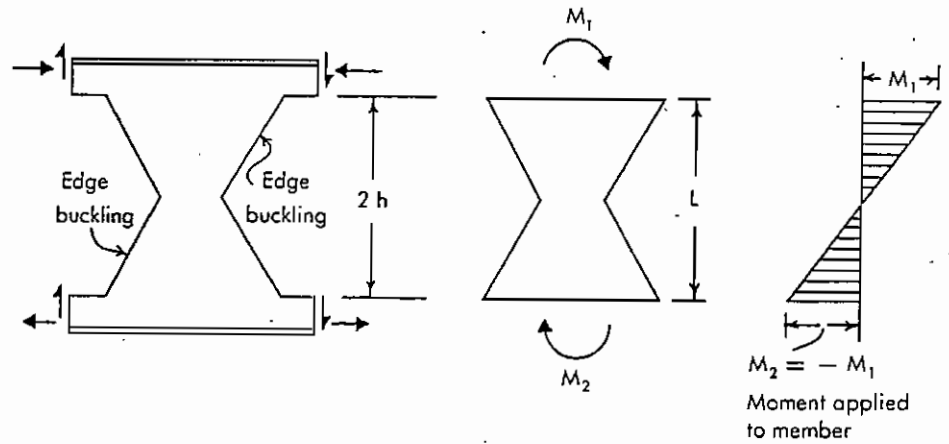


FIGURE 23

Therefore, the radial bending stress along curved section (a):

$$\sigma_r = \frac{3 V_h (\rho - f)}{2 t_w \rho^2 \theta^2} \dots \dots \dots (10)$$

It can be shown that the curved section (a) having the greatest bending stress (σ_r) occurs at a distance of:

$$m = \frac{e}{\tan \theta} \left(\cos \theta - \frac{1}{2} \right) \leq h \dots \dots \dots (11)$$

This value of (m) will be less than (h) and may be used in the following Formula 12 if (e) does not exceed these values—

$$\begin{aligned} \text{for } \theta = 45^\circ, e &\leq 4.83 h \\ \text{for } \theta = 30^\circ, e &\leq 1.58 h \end{aligned}$$

For most designs, this would be true and Formula 12 could be used directly without first solving for (m) in Formula 11.

This value of (m) for the position of the greatest bending stress may be inserted back into Formula 10, and the following will give the greatest bending stress along (a):

$$\sigma_r (\max) = \frac{3 V_h \tan \theta}{4 t_w e \theta^2} \dots \dots \dots (12)$$

The next step is to determine the allowable compressive bending stress (σ). If the above bending stress in the solid portion of the web (σ_r) is excessive, it might be possible to increase the distance (e). However, this will also increase the length of the Tee

section, resulting in increase of the secondary bending stress in the Tee section (σ_T). As an alternative to increasing distance (e), it would be possible to stiffen the outer edge of this wedge portion of the web by adding a flange around the edge of the hole in the web in the particular panel which is overstressed.

Allowable Compressive Bending Stress

There are two suggestions for determining the allowable compressive bending stress along the sloping edge of the wedge section of the web:

1. Treat this section as a prismatic member and apply AISC Sec. 1.5.1.4.5 Formula 4; see Figure 23. AISC Formula (4) for allowable compressive stress:

$$\sigma = \left[1.0 - \frac{(L/r)^2}{2 C_c^2 C_b} \right] .60 \sigma_y$$

where

$$C_b = 1.75 - 1.05 \left(\frac{M_1}{M_2} \right) + .3 \left(\frac{M_1}{M_2} \right)^2 \leq 2.3$$

and

$$C_c = \sqrt{\frac{2 \pi^2 E}{\sigma_y}}$$

See additional notes, Section 3.1.

Since $M_1 = -M_2$ in the above formula, $C_b = 2.83$; but since it cannot exceed 2.3 therefore $C_b = 2.3$ and AISC Formula (4) becomes—

$$\sigma = \left[1.0 - \frac{10.434}{C_c^2} \left(\frac{h}{t_w} \right)^2 \right] .60 \sigma_y \dots \dots (13)$$

See Table 2 for values of Formula 13 for various steels.

2. As an alternate method, treat this as a canti-

4.7-12 / Girder-Related Design

TABLE 2—Allowable Compressive Stress On Wedge Section of Open-Web Girder For Various Steels

Steel's Yield Strength σ_y	Allowable Compressive Stress σ
36,000	22,000 — 14.44 $\left(\frac{h}{t_w}\right)^2$
42,000	25,000 — 19.15 $\left(\frac{h}{t_w}\right)^2$
45,000	27,000 — 22.14 $\left(\frac{h}{t_w}\right)^2$
46,000	27,500 — 23.04 $\left(\frac{h}{t_w}\right)^2$
50,000	30,000 — 27.34 $\left(\frac{h}{t_w}\right)^2$
55,000	33,000 — 33.10 $\left(\frac{h}{t_w}\right)^2$
60,000	36,000 — 39.35 $\left(\frac{h}{t_w}\right)^2$
65,000	39,000 — 46.27 $\left(\frac{h}{t_w}\right)^2$

lever beam, and measure its unsupported length (L) from the point of inflection (e) to the support; see Figure 24.

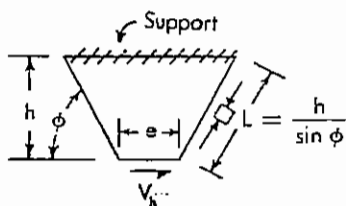


FIGURE 24

where:

$$r = \frac{t_w}{\sqrt{12}} = .29 t_w$$

Consider the outer fiber of this cantilever as an element in compression. Using the resulting (L/r) ratio, determine the allowable compressive stress from the AISC tables.

Allowable Shear Stress

From either Formula 13 or the above Method 2, we obtain the allowable compressive bending stress (σ). Since $V_h = \tau t_w e$ and holding the maximum bending stress (σ_r) of Formula 12 to the allowable (σ), we obtain the following—

$$\begin{aligned} \sigma_r &= \frac{3 V_h \tan \theta}{4 t_w e \theta^2} \\ &= \frac{3 \tau t_w e \tan \theta}{4 t_w e \theta^2} \leq \sigma \end{aligned}$$

or:

$$\tau \leq \frac{4 \theta^2}{3 \tan \theta} \sigma \leq .40 \sigma_y \quad \dots \dots \dots (14)$$

Formula 14 for allowable shear stress (τ) has been simplified for various angles of cut (θ); see Table 3.

If the allowable shear stress (τ) in this web section is held within the value shown in Formula 14, no further check of web buckling due to the compressive bending stress will have to be made, nor will this edge have to be reinforced with a flange.

To keep the resulting shear stress within this allowable, either (t_w) or (e) may have to be increased; see Figure 25.

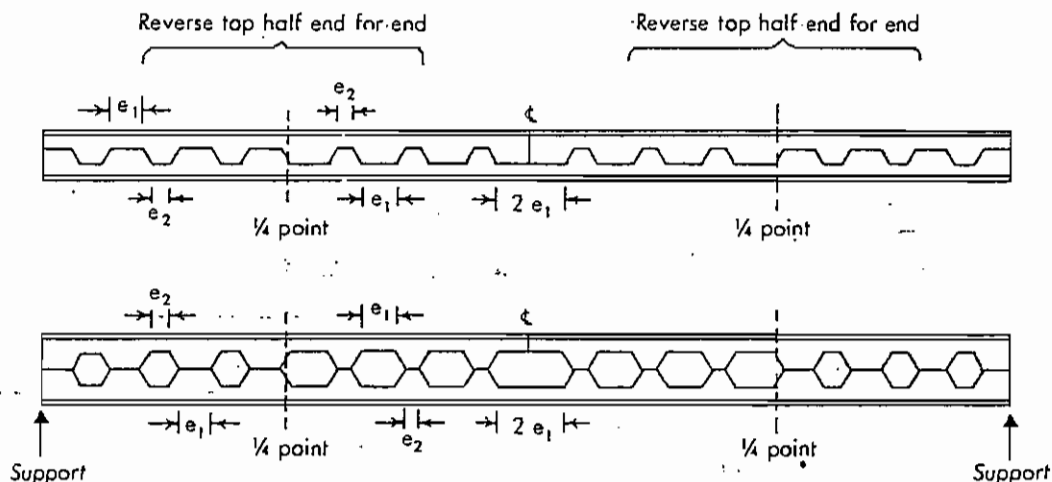


FIGURE 25

Adjusting the Distance of Cut (e)

The distance (e) may be varied to provide the proper strength of the web, or the proper opening for duct work; see Figure 8. However, as this distance (e) increases, the secondary bending stress within the Tee section due to the applied shear force (V) also increases.

In other words, (e) must be sufficiently large to provide proper strength in the web section, yet must be small enough to provide proper bending strength in the Tee section. In both cases, these stresses are caused directly by the applied vertical shear (V) on the member. This becomes more critical near the supports where the shear is the highest. Larger trial WF beam sections are chosen until the value of (e) will satisfy both conditions.

It would be possible to gradually vary the size of the openings from the support to the centerline; however, this would be difficult to fabricate. If this is desired, it might be better to use two dimensions of horizontal cut (e_1) and (e_2), alternating them and reversing their order at the $\frac{1}{4}$ point. See Figure 25. This would allow a larger value of (e_1) for the strength of the web and a smaller value of (e_2) for the strength of the Tee section, near the support in the region of high shear (V). In the central region of the girder between the $\frac{1}{4}$ points where the shear (V) is one-half of this value or less, these values will reverse, resulting in the smaller value of (e_2) for the web and the larger value of (e_1) for the Tee.

The top portion of the cut WF beam would be cut in half and each half turned end for end. This will require a butt groove weld. However, this top section is in compression and the requirement for the weld will not be as severe as though it were in the bottom tensile chord. It might be possible to make this compression butt joint by fillet welding splice bars on each side of the Tee section. This lap joint would transfer the compressive force; the splice bars would apply additional stiffness and therefore a higher allowable compressive stress for this Tee section at midspan.

TABLE 3—Allowable Shear Stress For Various Angles of Cut

$\phi = 45^\circ$	$\theta = 45^\circ$	$\tau_s \leq .8225 \sigma_s$
$\phi = 50^\circ$	$\theta = 40^\circ$	$\tau_s \leq .7745 \sigma_s$
$\phi = 55^\circ$	$\theta = 35^\circ$	$\tau_s \leq .7106 \sigma_s$
$\phi = 60^\circ$	$\theta = 30^\circ$	$\tau_s \leq .6332 \sigma_s$

This cutting pattern results in the hole at the centerline having twice the length as the others. However, this is the region of only high moment (M); there is almost no shear (V). This section should be sufficient if it can develop the required compression from the main bending load.

Stiffening Edge of Wedge Section

The edge of the wedge section of the web may be strengthened against buckling due to the horizontal shear force, by adding a flange around the web opening. See Figure 26.

Here:

$$S = A_f a + \frac{t_w a^2}{6}$$

$$= 2 A_f \rho \theta + \frac{2}{3} t_w \rho^2 \theta^2$$

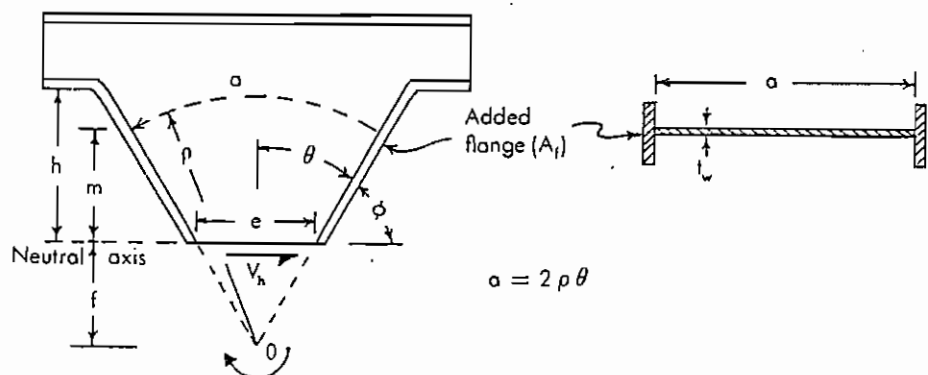
Inserting this into Formula 7, we get—

$$\sigma_r = \frac{V_u (\rho - f)}{2 A_f \rho \theta + \frac{2}{3} t_w \rho^2 \theta^2} \dots \dots \dots (15)$$

It can be shown that the value of (m) for the position of the greatest bending stress is—

$$m = \left(\cos \theta \left[1 + \sqrt{1 + \frac{3 A_f}{f t_w \theta}} \right] - 1 \right) \leq h \quad (16)$$

FIGURE 26



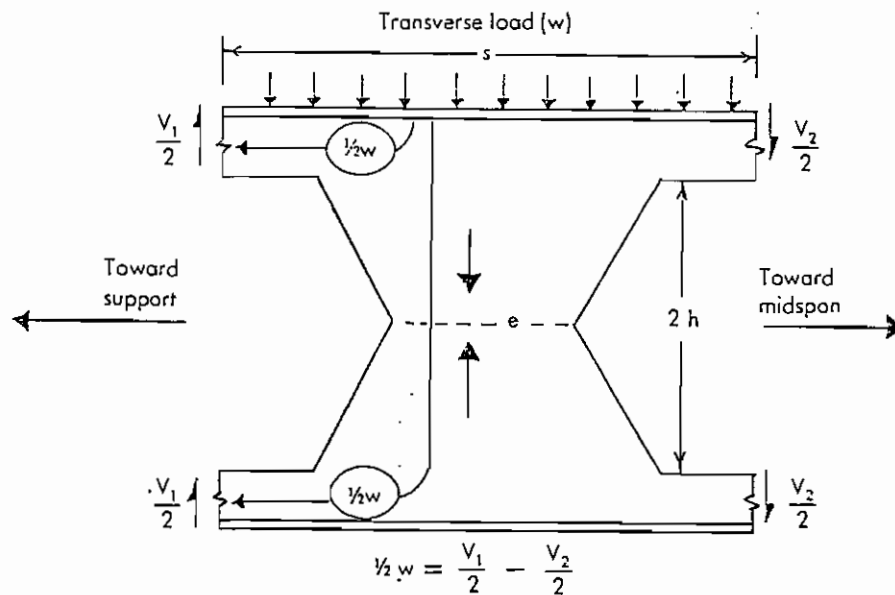


FIGURE 27

This value of (m) could then be used in Formula 12 for the bending stress. This would give the following formula for the greatest bending stress:

$$\sigma_r = \frac{\frac{V_h}{2\theta} \sqrt{1 + \frac{2A_f}{K}}}{K + 2A_f + (K + A_f) \sqrt{1 + \frac{2A_f}{K}}} \quad (17)$$

where:

$$K = \frac{2 t_w \theta}{3 f}$$

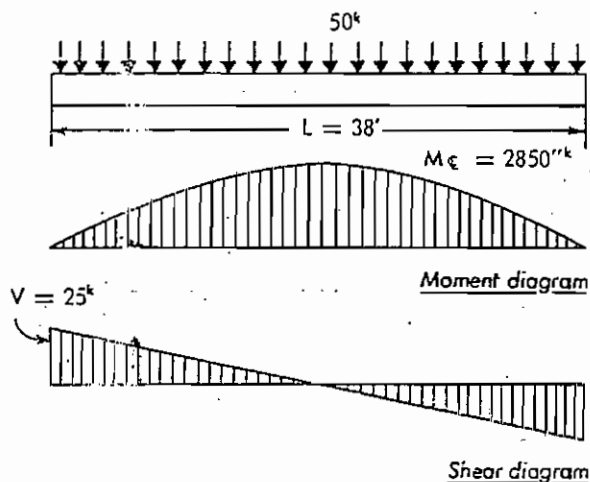


FIGURE 28

7. WEB BUCKLING DUE TO COMPRESSION

Any direct transverse load applied to the upper flange of the open-web girder is carried as vertical shear on the web. See Figure 27. Since this resisting shear is equally divided between the top and bottom Tee section chords, half of this transverse load applied to a unit panel segment of the girder (distance s) must be transferred as compression down through the solid portion (e) of the web into the bottom chord.

If it is felt that this solid web section, acting as a column, cannot handle this force, it could be reinforced with a transverse (vertical) stiffener. Usually this force, one-half of the applied transverse load with the segment (s), is small. Thus, the resulting compressive stress within this web section (e) is low, and stiffening is not usually required.

Compressive stress in web section (e):

$$\sigma = \frac{w}{2 e t_w} = \frac{V_1 - V_2}{2 e t_w} \quad (18)$$

The allowable compressive stress would be found in the AISC tables, using —

$$L = 2 h$$

$$r = \sqrt{\frac{t_w}{12}} = .29 t_w$$

8. GENERAL OUTLINE FOR DESIGN OF OPEN-WEB EXPANDED GIRDER

Design of an open-web expanded girder will be facilitated by following the design outline below. Its application is demonstrated by working a typical design problem: Design an open-web expanded girder with a span of 38 ft to support a uniformly-distributed load of 50 kip. Design on the basis of using A36 steel and E70 welds, and angle of cut $\phi = 45^\circ$. See Figure 28.

STEP 1. Determine the expanded girder's required section modulus (S_g) at midspan for the main bending moment:

$$S_g = \frac{M_g}{\sigma}$$

$$S_g = \frac{2850^{rk}}{22,000 \text{ psi}} = 130 \text{ in.}^3$$

STEP 2. For the relationship of the expanded girder's depth to that of the original beam, let—

$$K_1 = \frac{d_g}{d_b}$$

Assume it = about 1.5

STEP 3. Select a trial WF beam having a section modulus of—

$$S_b = \frac{S_g}{K_1}$$

$$S_b = \frac{130}{1.5} = 86.4 \text{ in.}^3 \text{ (use this as a guide)}$$

Try an 18" WF 50# /ft beam, having $S_b = 89.0 \text{ in.}^3$

Now, refigure K_1 using the S_b of the actual selected beam:

$$K_1 = \frac{S_g}{S_b}$$

$$K_1 = \frac{130}{89} = 1.46$$

STEP 4. Determine the height of cut (h) and round off to the nearest inch or fraction of an inch:

$$h = d_b (K_1 - 1)$$

$$h = 18.0 (1.46 - 1) = 8.3" \text{ or use } 8"$$

However, h cannot exceed the following value

in order to keep the vertical shear stress in the stem of the Tee section within the allowable:

$$d_T \geq \frac{V^*}{2 t_w \tau} \text{ where } \tau = .40 \sigma_y$$

$$d_T = \frac{25^k}{2(.358)(14,500)} = 2.41"$$

$$h \leq d_b - 2 d_T$$

$$h = 18 - 2(2.41) = 13.18" > 8" \quad \text{OK}$$

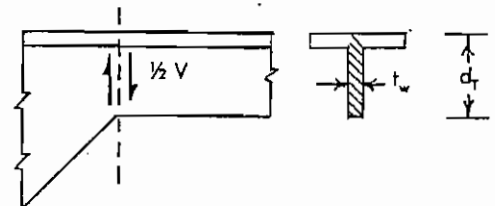


FIGURE 29

STEP 5. Then

$$d_g = d_b + h$$

$$d_g = 18 + 8 = 26"$$

$$d_T = \frac{d_g}{2} - h$$

$$d_T = \frac{26}{2} - 8 = 5"$$

$$d_s = d_T - t_r$$

$$d_s = 5 - .57 = 4.43"$$

STEP 6. Determine the allowable compressive bending stress on wedge section of web, using modified AISC Sec 1.5.1.4.5 Formula (4):

$$(13) \quad \sigma = \left[1.0 - \frac{10.434}{C_e^2} \left(\frac{h}{t_w} \right)^2 \right] .60 \sigma_y$$

where:

$$C_e = \sqrt{\frac{2 \pi^2 E}{\sigma_y}}$$

* Could assume shear (V) is about 95% of maximum shear (at the support) because first panel will be away from the point of support. However, because we are not at the support, there will be some main bending stresses to be added to these secondary bending stresses in the Tee section from applied shear (V). Hence, it would be better to use full value of shear (V).

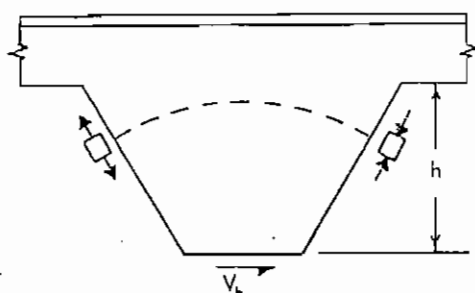


FIGURE 30

or from Table 2 (values for different steels):

$$\sigma = 22,000 - 14.44 \left(\frac{h}{t_w} \right)^2$$

$$\sigma = 22,000 - 14.44 \left(\frac{8}{.358} \right)^2 = 14,810 \text{ psi}$$

The compressive bending stress can be kept within the above allowable value, if the shear stress in this wedge web section is held to—

$$(14) \quad \tau = \frac{4 \theta^2}{3 \tan \theta} \sigma \leq .40 \sigma_y$$

$$\tau = \frac{4(.7854)^2(14,810)}{3(1.00)} = 12,180 \text{ psi which}$$

is less than $.40 \sigma_y = 14,400 \text{ psi}$, so is OK

STEP 7. Estimate the maximum shear stress along the neutral axis of the girder's web section, assuming the web to be solid the full length of the girder:

$$\tau_{max} = 1.16 \tau_{av} = 1.16 \frac{V^*}{t_w d_g}$$

$$\tau_{max} = 1.16 \frac{95\% (25^k)}{(.358)(26)} = 2960 \text{ psi}$$

* Where (V) is the shear at the first wedge section, assume about 95% of the maximum shear (at the support) because the first panel will be away from the point of support. This is all right here because we are working with just one stress (shear); there is no main bending stress to be considered.

The maximum shear stress is equal to about 1.16 times the average shear stress.

STEP 8. Knowing the maximum shear on a solid web section, and the allowable shear for the open web section, we now have the ratio—

$$\frac{e}{s} = \frac{\tau_{max}}{\tau} = K_2$$

Since:

$$\tau = \tau_{max} \frac{s}{e}$$

Then

$$\frac{e}{s} = \frac{\tau_{max}}{\tau} = \frac{2960}{12,180} = .243 = K_2$$

If this ratio (K_2) is reasonably low (up to about $\frac{3}{8}$ "), there is a good chance this trial WF beam may be used.

Since

$$s = 2(e + h \tan \theta), \text{ then}$$

$$e \geq \frac{2 h \tan \theta}{\frac{1}{K_2} - 2}$$

$$e = \frac{2(8)(1.00)}{\frac{1}{.243} - 2} = 7.56'' \text{ or use } 8''$$

Distance (e) is usually constant along the full length of the girder. However, it is possible to vary this distance; in this case there will be two dimensions (e_1) and (e_2). Near the support, (e_1) lies along the neutral axis of the girder determining the width of the solid web section and (e_2) determines the width of the Tee section. See left end of Figure 31.

At the $\frac{1}{4}$ point, the details are reversed, and dimension (e_1) rather than (e_2) will control the secondary bending stress (σ_T). See right end of Figure 31. Since

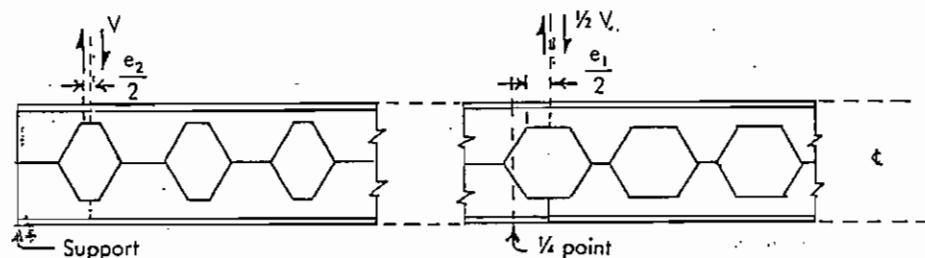


FIGURE 31

the shear (V) at this $\frac{1}{4}$ point is reduced to about half of that at the support, the distance (e_1) may be double that of (e_2) and still not increase the resulting secondary bending stress (σ_T). Therefore, $K_3 = e_2/e_1$ should not be less than $\frac{1}{2}$.

Using the two dimensions (e_1) and (e_2), the above formulas become:

$$\frac{e_1}{s} = \frac{\tau_{\max}}{\tau} = K_2$$

Let

$$K_3 = \frac{e_2}{e_1}$$

Since

$$s = e_1 + e_2 + 2h \tan \theta$$

then

$$e_1 \geq \frac{2h \tan \theta}{\frac{1}{K_2} - 1 - K_3}$$

STEP 9. Now determine the properties of expanded girder:

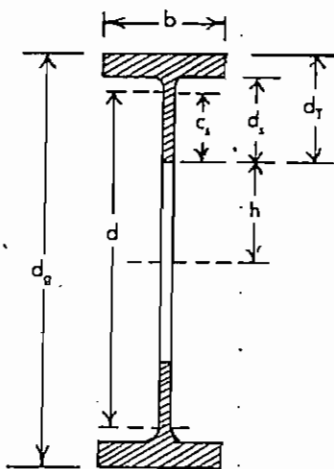


FIGURE 32

$$A_T = A_f + A_s = b t_f + d_s t_w = 5.861 \text{ in.}^2$$

$$M_T = A_f \left(d_s + \frac{t_f}{2} \right) + A_s \frac{d_s}{2} = 23.67 \text{ in.}^3$$

$$I_T = A_f \left(d_s^2 + d_s t_f + \frac{t_f^2}{3} \right) + A_s \frac{d_s^2}{3} = 105.53 \text{ in.}^4$$

From this we get—

$$c_s = \frac{M_T}{A_T} = \frac{23.67}{5.861} = 4.039''$$

$$I_T = I_f - c_s M_T$$

$$I_T = 105.53 - (4.039)(23.67) = 9.93 \text{ in.}^4$$

$$S_s = \frac{I_T}{c_s}$$

$$S_s = \frac{9.93}{4.039} = 2.46 \text{ in.}^3$$

$$d = 2(h + c_s)$$

$$d = 2(8 + 4.039) = 24.077''$$

$$I_g = 2 I_T + \frac{A_T d^2}{2}$$

$$I_g = 2(9.93) + \frac{(5.861)(24.077)^2}{2} = 1719.1 \text{ in.}^4$$

$$S_g = \frac{2 I_g}{d_g}$$

$$S_g = \frac{2(1719.1)}{26} = 132.2 \text{ in.}^3$$

STEP 10. At the support, check the secondary bending stress:

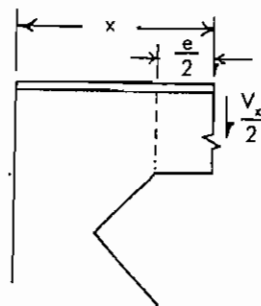


FIGURE 33

$$\sigma_T = \frac{V_s e}{4 S_s}$$

$$\sigma_T = \frac{(25^k)(8)}{4(2.46)} = 20,300 \text{ psi}$$

The allowable compressive bending stress may be found in a similar manner to that of Step 6, except the unsupported length here is (e).

At the support, there is no main bending moment,

TABLE 4—For Various Steels

$\sigma_r = 36,000$	$\sigma = 22,000 - 14.44 \left(\frac{h}{t_w} \right)^2$	$\sigma = 22,000 - 3.61 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 42,000$	$\sigma = 25,000 - 19.15 \left(\frac{h}{t_w} \right)^2$	$\sigma = 25,000 - 4.79 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 45,000$	$\sigma = 27,000 - 22.14 \left(\frac{h}{t_w} \right)^2$	$\sigma = 27,000 - 4.85 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 46,000$	$\sigma = 27,500 - 23.04 \left(\frac{h}{t_w} \right)^2$	$\sigma = 27,500 - 5.76 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 50,000$	$\sigma = 30,000 - 27.34 \left(\frac{h}{t_w} \right)^2$	$\sigma = 30,000 - 6.84 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 55,000$	$\sigma = 33,000 - 33.10 \left(\frac{h}{t_w} \right)^2$	$\sigma = 33,000 - 8.28 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 60,000$	$\sigma = 36,000 - 39.35 \left(\frac{h}{t_w} \right)^2$	$\sigma = 36,000 - 9.84 \left(\frac{e}{t_w} \right)^2$
$\sigma_r = 65,000$	$\sigma = 39,000 - 46.27 \left(\frac{h}{t_w} \right)^2$	$\sigma = 39,000 - 11.57 \left(\frac{e}{t_w} \right)^2$

hence no axial compressive force acting on this Tee section. The allowable stress here is—

$$\sigma = \left[1.0 - \frac{2.609}{C_c^2} \left(\frac{e}{t_w} \right)^2 \right] .60 \sigma_r$$

or, from Table 4 of values for different steels—

$$\sigma = 22,000 - 3.61 \left(\frac{e}{t_w} \right)^2$$

$$\sigma = 22,000 - 3.61 \left(\frac{8}{.358} \right)^2 = 20,200 \text{ psi}$$

STEP 11. At midspan of girder, check the main bending stress:

(as a compressive or tensile stress)

$$(3) \quad \sigma_b = \frac{F}{A_T} = \frac{M_{\pm}}{d A_T}$$

$$\sigma_b = \frac{2850^{\text{rk}}}{(24.08)(5.861)} = 20,200 \text{ psi}$$

or

(as a bending stress)

$$\sigma_b = \frac{M_{\pm}}{S_x}$$

$$\sigma_b = \frac{2850^{\text{rk}}}{132.2} = 21,600 \text{ psi}$$

STEP 12. If the main bending stress (σ_b) in Step 11 is excessive, it may be reduced slightly with a higher

value of (h); however, this will greatly increase the secondary bending stress (σ_T) of Step 10, since it reduces the depth (d_t) of the Tee section. In this case undoubtedly, the WF beam selected cannot be used and a larger WF beam must be tried.

If the main bending stress (σ_b) is within the allowable, but the secondary bending stress (σ_T) in Step 10 exceeds the allowable, (σ_T) may be greatly reduced by decreasing (h) with just a slight increase in (σ_b).

Stresses (σ_b) and (σ_T) may be considered according to AISC interaction formulas (6), (7a) and (7b), shown graphically in Figure 12.

As a matter of interest, Table 5 shows that decreasing (h) results in a large decrease in the secondary bending stress (σ_T) and a slight increase in the main bending stress (σ_b).

If (h) cannot be reduced because (σ_b) is close to the allowable, use two different size holes, (e_1) and (e_2). Provide a larger value of distance (e_1) for the compressive bending stress in the wedge section of the web, but a lower value of (e_2) for the cantilevered Tee section.

TABLE 5

$h = 7''$	$h = 8''$	$h = 9''$	$h = 10''$
$A_T = 6.040$	$A_T = 5.861$	$A_T = 5.682$	$A_T = 5.503$
$d_T = 5.5$	$d_T = 5.0$	$d_T = 4.5$	$d_T = 4.0$
$S_x = 2.98$	$S_x = 2.46$	$S_x = 1.97$	$S_x = 1.57$
$S_x = 127.96$	$S_x = 132.22$	$S_x = 136.53$	$S_x = 139.17$
$d = 22.82$	$d = 24.08$	$d = 25.32$	$d = 26.54$
$e = 6.0$	$e = 9.0$	$e = 10.5$	$e = 16.5$
$\sigma_T = 12,600$	$\sigma_T = 20,300$	$\sigma_T = 33,300$	$\sigma_T = 65,700$
$\sigma_b = 22,230$	$\sigma_b = 21,600$	$\sigma_b = 21,000$	$\sigma_b = 20,250$

STEP 13. Make any adjustments necessary to facilitate fabrication. See the text immediately following this design outline.

STEP 14. After the girder is detailed, the stresses may be rechecked in view of more exact values of (V_x) and (M_x) since the exact positions of the panels are not known. Also, it may be well to check additional points between the point of support and midspan. See Figure 34 and Table 6.

9. DESIGN MODIFICATION TO FACILITATE FABRICATION

The practical aspects of structural fabrication may mean some adjustment of original girder design is required.

If Same Size Holes Are to be Used

If openings in the web are to be of uniform size for the full length of the girder, that is $e_1 = e_2$, and the open-web expanded girder is to be symmetrical about its centerline, let n = number of unit panels and use as a starting point in measuring a unit panel either:

(a) Centerline of wedge web section.

Figure 35, or

(b) Centerline of open Tee section,

Figure 36

Divide the length of the required girder (L_g) by the length of one unit panel (s) to get the number of units (n). Then reduce (n) to the nearest whole

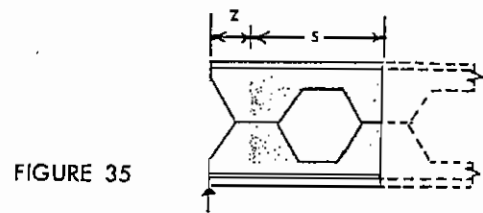


FIGURE 35

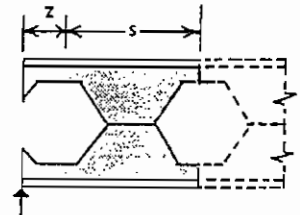


FIGURE 36

number. The distance left over (z) on each side is—

$$z = \frac{L_g - n s}{2}$$

Since the length of the open-web expanded girder is—

$$L_g = n s + 2 z$$

the length of the WF beam to be cut is—

$$L_b = (n + \frac{1}{2}) s + 2 z$$

The extra length of WF beam required is—

$$L_g - L_b = \frac{1}{2} s$$

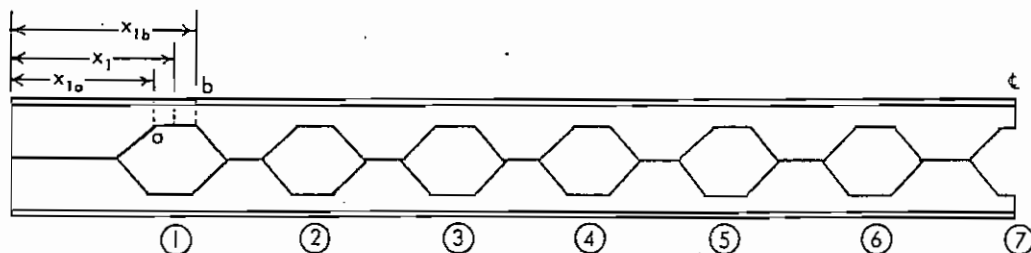


FIGURE 34

TABLE 6

	X_a	X	X_b	V_x	M_a	M_b	SECONDARY		MAIN		TOTAL	
							σ_1 @ stem a	σ_1 @ flange b	σ_2 @ stem a	σ_2 @ flange b	$\sigma = \sigma_1 + \sigma_2$ stem a	$\sigma = \sigma_1 + \sigma_2$ flange b
1	32"	36"	40"	21.05 ^K	744" ^K	911" ^K	-17,100	-4,065	-3,460	-6,880	-20,560	-10,945
2	64"	68"	72"	17.55	1372	1515	-14,270	-3,390	-6,380	-11,450	-20,650	-14,840
3	96"	100"	104"	14.04	1892	2200	-11,420	-2,715	-8,800	-16,620	-20,220	-19,335
4	128"	132"	136"	10.53	2300	2385	-8,560	-2,035	-10,700	-18,030	-19,260	-20,065
5	160"	164"	168"	7.02	2595	2650	-5,710	-1,360	-11,600	-20,000	-17,310	-21,360
6	192"	196"	200"	3.51	2778	2807	-2,860	-680	-12,920	-21,200	-15,780	-21,880
7	224"	228"		0	2849		0	0	-13,250	-21,550	-13,250	-21,550

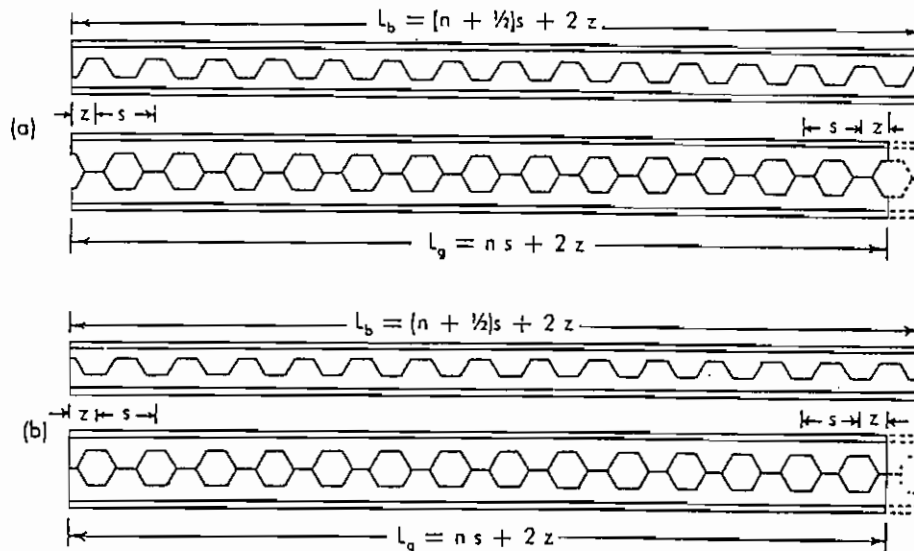


FIGURE 37

In either case (a) or (b), there probably will be a small hole left in the girder at the ends which must be filled. The simplest method is to add one or a pair of web doubler bars or plates at each end to cover and lap over the holes. See Figure 38.

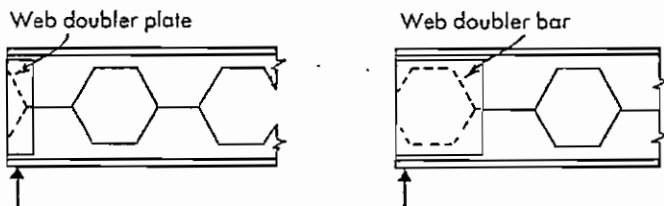


FIGURE 38

If the same size holes are to be used, that is $e_1 = e_2$, and the girder is not to be symmetrical about its centerline, then start a unit panel right at one end of the girder. The other end may have a partial hole in the web which will have to be covered. The only advantage to this method is that just one end will have a hole in the web to be covered. See Figure 39.

It might be possible to adjust the value of (e) so that the panels will fit exactly into the length of the girder (L_g). See Figure 40.

Here:

$$L_g = ns + e$$

$$= e(2n + 1) + 2nh \tan \phi$$

First, determine the number of holes (n) from the following formula and round off to the nearest whole number—

$$n = \frac{L_g - e}{s} = \frac{L_g - e}{2e + 2h \tan \phi} \dots \dots \dots (19)$$

Second, find the required value of (e) from the following formula—

$$e = \frac{L_g - 2nh \tan \phi}{2n + 1} \dots \dots \dots (20)$$

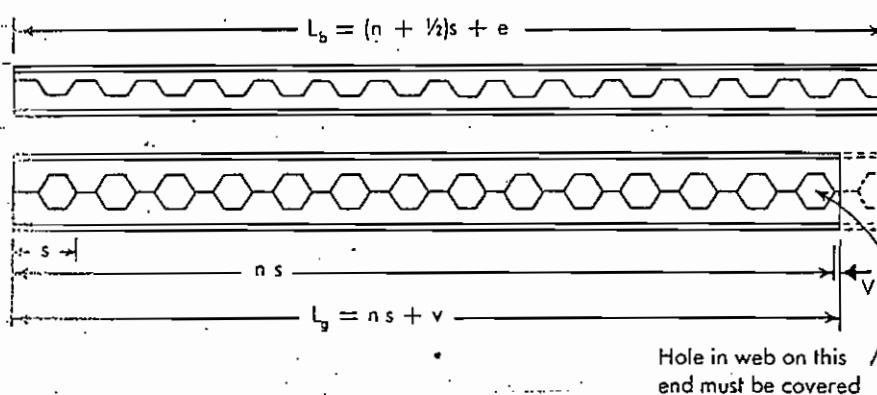
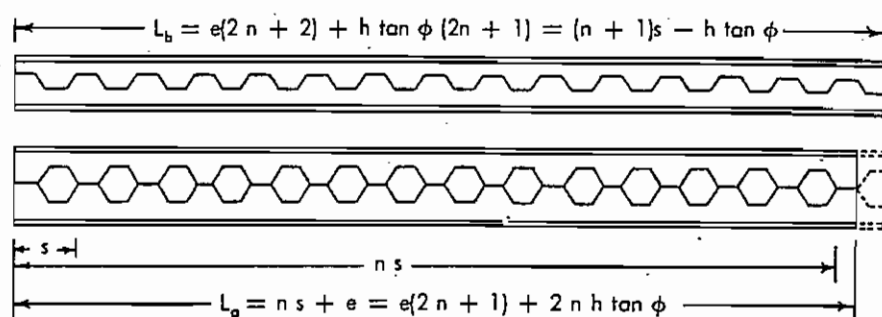


FIGURE 39

FIGURE 40



This adjusted value of (e) cannot be less than that of Step 8 in the design outline, nor exceed the value which would result in an excessive secondary bending stress (σ_T) in Step 10.

If Different Size Holes Are to be Used

If distances (e_1) and (e_2) are not to be the same, and the girder is symmetrical about its centerline, then the following method may be employed.

In order to easily fabricate this type of open-web girder, it is necessary to be able to rotate each top half about the $\frac{1}{4}$ point. This presents two possibilities—case (a) rotation at the $\frac{1}{4}$ point about the larger dimension (e_1), and case (b) rotation at the $\frac{1}{4}$ point about the smaller dimension (e_2). See Figure 41.

Let (n) = number of holes in the web, counting the centerline hole as two holes.

Determine the approximate number of holes from—

$$n = \frac{L}{e_1 (1 + K_3) + h \tan \phi} \quad \dots\dots\dots (21)$$

Case (a). There are an odd number of holes in each half, therefore:

Adjust (n) so it is a multiple of 2 only, and solve for (e_1) from the following—

$$e_1 = \frac{L - (n - 1) h \tan \phi}{n(1 + K_3) + 2(1 - K_3)} \quad \dots\dots\dots (22a)$$

Case (b). There are an even number of holes in each half, therefore:

Adjust (n) so it is a multiple of 4, and solve for (e_1) from the following—

$$e_1 = \frac{L - (n - 1) h \tan \phi}{n(1 + K_3)} \quad \dots\dots\dots (22b)$$

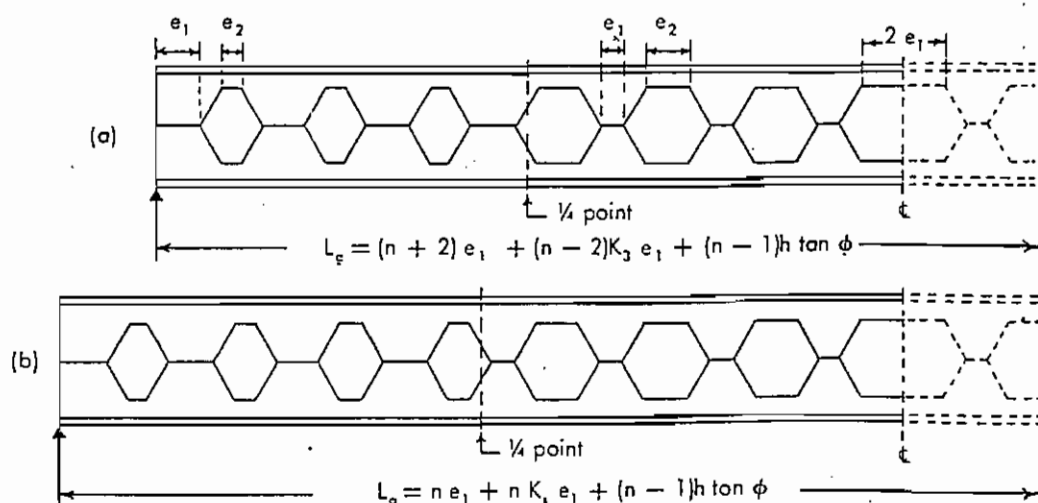
In both case (a) and case (b) this resulting value of (e_1) should not be less than that obtained in Step 8 and that just used in Formula 21 to find (n).

10. TAPERED OPEN-WEB EXPANDED GIRDERS

Cutting the zig-zag pattern along an axis at a slight angle to the axis of the beam results in a tapered girder. See Figure 42.

In order to have the deeper section at the mid-span, it is necessary to cut the top portion in half and reverse these two top halves. The cut could be made in the lower portion; however this is in tension, and a simpler weld could be made in the compression or top portion.

FIGURE 41



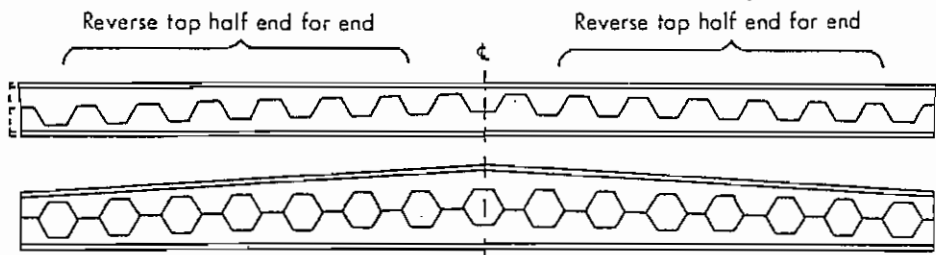


FIGURE 42

In tapered open-web expanded girders, the axial force in the chord which slopes has a vertical component ($F_v = F_h \tan \alpha$); here ($F_h = M/d$).

Whenever this chord changes direction, for example at the midspan of the girder, this vertical component must be considered. It will be carried as shear in the web members back to the support, and in this case has a sign opposite to that of the main shear (V). Hence, its effect is to reduce the shear over most of the girder's length, but to increase it in the midspan region.

The modified shear becomes—

$$V' = A_w \tau = V - F_v = V - \frac{M_h}{d} \tan \alpha$$

This means there is a vertical shift of the initial shear diagram on each half of the girder, so that the central portion to be checked which initially had zero shear ($V = 0$) now has a shear value ($V' = F_v$) as

well as the maximum bending moment. See Figure 43.

A transverse stiffener at the point where the sloping flange changes direction would transfer the vertical component of the flange efficiently into the web. The greater the change in slope, the more important this would become.

If there is a panel opening at this point, the Tee section must resist this vertical component in bending (in this example, the top Tee section). This is similar to the analysis of the secondary bending stress (σ_T) due to the shear applied to the Tee section at midopening where each half behaved as a cantilever beam. See Figure 44. However, in this case, the cantilever beams have fixed ends (at the centerline of the girder); resulting in one-half the bending moment and stress. (This half length Tee section is treated as a beam fixed at one end and guided at the other end, with a concentrated load.)

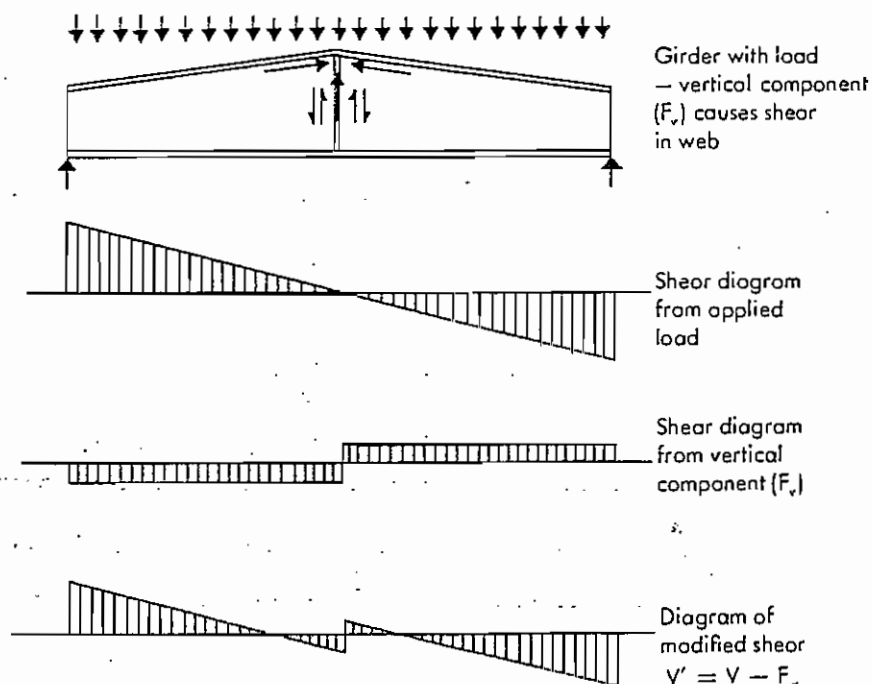


FIGURE 43

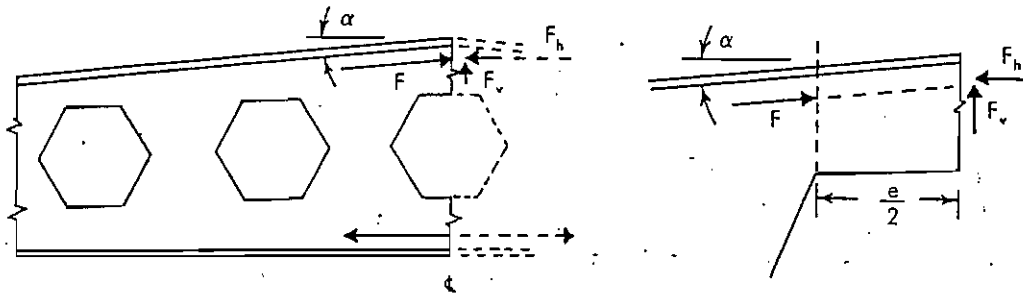
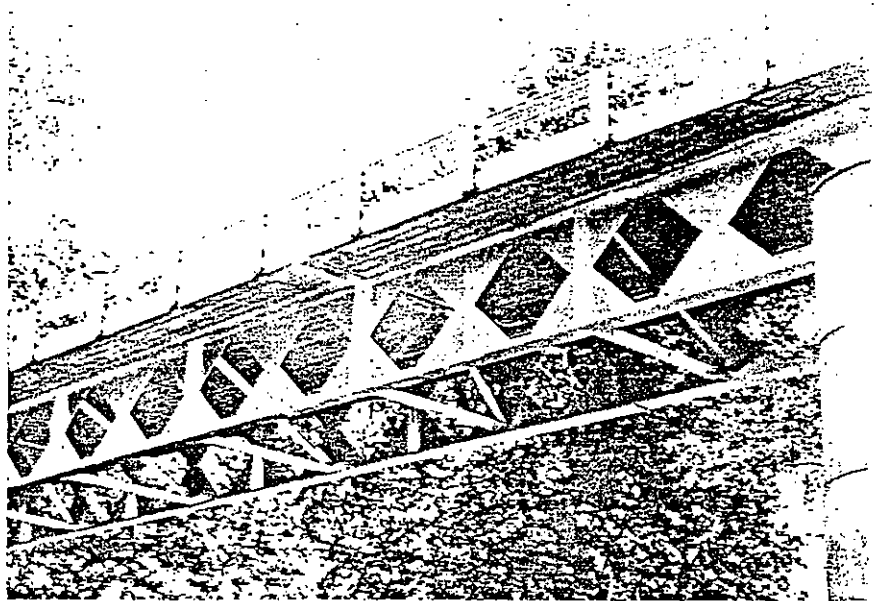
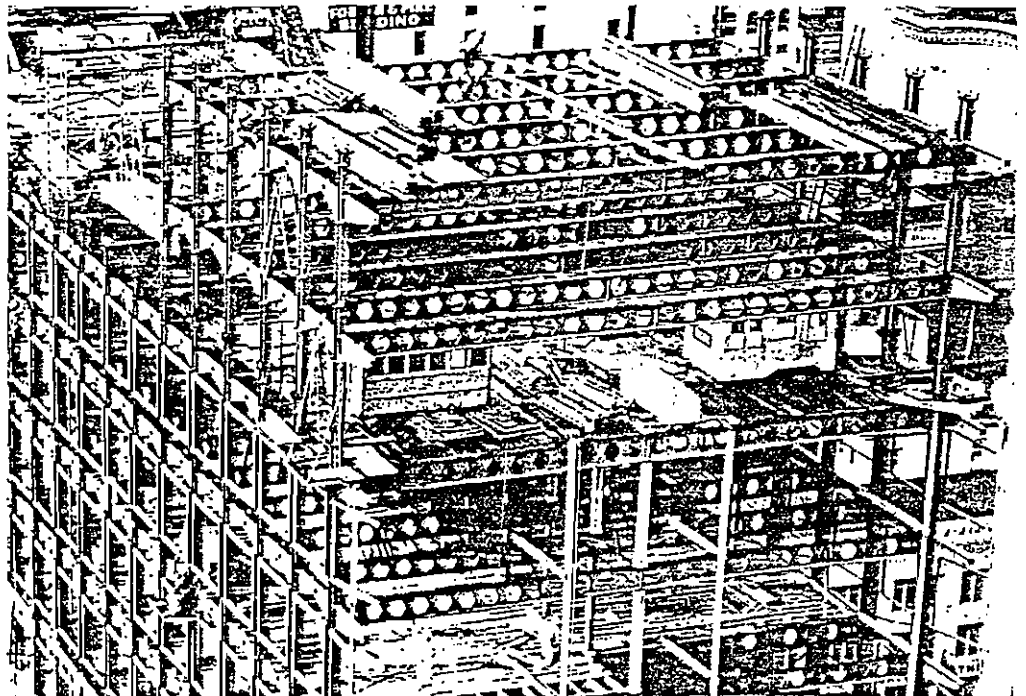


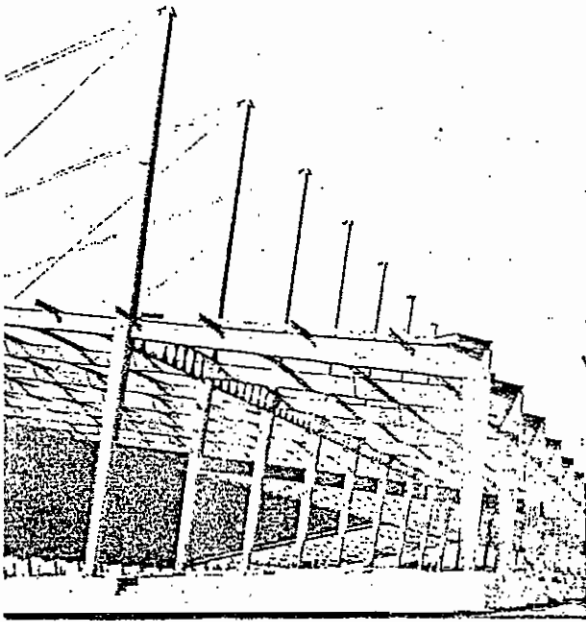
FIGURE 44

The open-web expanded rolled beam is sometimes an economical substitute for a heavy built-up plate girder.



In the 21-story Washington Bldg., open-web expanded beams led to significant savings in construction costs.





Open-web expanded beam serves as longitudinal roof girder in the Tulsa Exposition Center. It provides the needed high moment of inertia, at minimum weight, and eliminates lateral wind bracing. Below, weldor is shown making connections of beam to the tapered box columns.

