Ship Resistance Simulations with OpenFOAM

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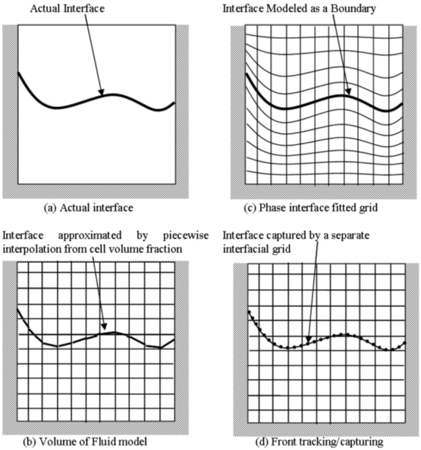
Introduction

- simulate flow around ship moving steadily in calm water
- high Reynolds number 10^6 model scale, 10^9 full scale
- longest wave is $2\pi F^2 L$
- in deep water the Kelvin angle is approximately 19 deg
- seek primarily the force on the body, also position of body

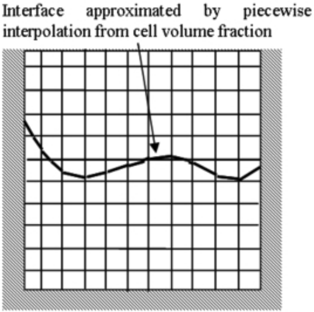
Outline

Interface tracking versus interface capturing



- OpenFOAM has both interface capturing and interface tracking solvers
- most ship hydrodynamics solvers use interface capturing, volume-of-fluid, level set, or a combination, to solve ship hydrodynamics problems.
- Can a ship move (at model or full scale) and not generate a breaking wave?

Volume of Fluid



(b) Volume of Fluid model

- use scalar indicator function to represent the phase of the fluid in each cell (was γ in versions $\leq$ 1.5, is α in versions $\geq$ 1.6).

$$
\begin{aligned}
\alpha &= \alpha(\mathbf{x}, t) && (1)\\
\mu(\mathbf{x}, t) &= \mu_{\text{water}}\alpha + \mu_{\text{air}}(1-\alpha) && (2)\\
\rho(\mathbf{x}, t) &= \rho_{\text{water}}\alpha + \rho_{\text{air}}(1-\alpha) && (3)
\end{aligned}
$$

- The density and viscosity are material properties of the fluids.

$$
\begin{aligned}
\frac{D\alpha}{Dt} &= 0 && (4)\\
\frac{\partial \alpha}{\partial t} + \mathbf{u}\cdot\nabla\alpha &= 0 && (5)\\
\frac{\partial \alpha}{\partial t} + \nabla\cdot\mathbf{u}\alpha &= 0 && (6)
\end{aligned}
$$

Volume-of-fluid with compression

- the α function transitions from 1 to 0 over an infinitesimal thickness. This leads to difficulty in approximating the gradient of α, and results in smearing of the interface.
- One remedy, is to use a modified governing equation. The modification should return solutions of the original equation for the time evolution of the interface, but help by keeping the interface crisp.

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot \mathbf{u}\alpha + \nabla \cdot \mathbf{w}\alpha = 0$$

- **u** is the physical velocity field, and **w** is an artificial velocity field that is directed normal to and towards the interface.

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot \mathbf{u}\alpha + \nabla \cdot \mathbf{w}(\alpha(1-\alpha)) = 0$$

- the user can specify the relative magnitude of the artificial velocity (using `cAlpha`)

Momentum, dynamic pressure

- Full Reynolds-averaged momentum equations for the velocity **U** and pressure P in a fluid with density ρ and dynamic viscosity μ

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{U}\mathbf{U} = -\nabla P + \rho \mathbf{g} + \nabla \cdot \left[(\mu + \mu_t)(\nabla \mathbf{U} + \nabla \mathbf{U}^\top) \right]$$

- Express the pressure in terms of a hydrostatic component, and the remainder or that due to dynamic or non-zero velocity p

$$P = \underbrace{\rho \mathbf{g} \cdot \mathbf{x}}_{\text{hydrostatic}} + \underbrace{p}_{\text{dynamic}}$$

- Governing equation in terms of dynamic pressure

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{U}\mathbf{U} = -\nabla p - \mathbf{g} \cdot \mathbf{x} \nabla \rho + \nabla \cdot \left[(\mu + \mu_t)(\nabla \mathbf{U} + \nabla \mathbf{U}^\top) \right]$$

Momentum, viscous stress

- See Henrik Rusche's Thesis, pg 156

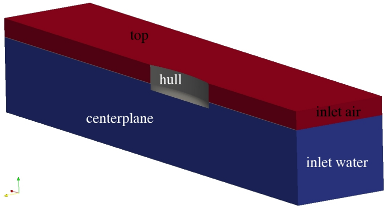
$$\begin{aligned}\nabla\cdot\left[\mu_{\mathrm{eff}}(\nabla\mathbf{U}+\nabla\mathbf{U}^{\top})\right] &= \nabla\cdot(\mu_{\mathrm{eff}}\nabla\mathbf{U})+\nabla\cdot(\mu_{\mathrm{eff}}\nabla\mathbf{U}^{\top})\\ &= \nabla\cdot(\mu_{\mathrm{eff}}\nabla\mathbf{U})+\nabla\mathbf{U}\cdot\nabla\mu_{\mathrm{eff}}+\mu_{\mathrm{eff}}\nabla(\nabla\cdot\mathbf{U})\\ &= \nabla\cdot(\mu_{\mathrm{eff}}\nabla\mathbf{U})+\nabla\mathbf{U}\cdot\nabla\mu_{\mathrm{eff}}\end{aligned}$$

- Final form of the momentum equation:

$$\frac{\partial\rho\mathbf{U}}{\partial t}+\nabla\cdot\mathbf{U}\mathbf{U}=-\nabla p-\mathbf{g}\cdot\mathbf{x}\nabla\rho+\nabla\cdot(\mu_{\mathrm{eff}}\nabla\mathbf{U})+\nabla\mathbf{U}\cdot\nabla\mu_{\mathrm{eff}}$$

- Note, $\nabla\rho$ is zero away from the interface, and VERY large along the interface.

Boundary conditions



- Centerplane: `symmetryPlane`
- Top

$$\begin{aligned} \partial \mathbf{U}/\partial n &= 0 \\ p &= 0 \\ \alpha &= 0 \end{aligned}$$

- Body

$$\begin{aligned} \mathbf{U} &= 0 \\ \partial p/\partial n &= 0 \\ \partial \alpha/\partial n &= 0 \end{aligned}$$

- Inlet

$$\begin{aligned} \mathbf{U} &= \mathbf{U}_\infty \\ \partial p/\partial n &= 0 \\ \alpha &= \begin{cases} 1 & \text{if } z < 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

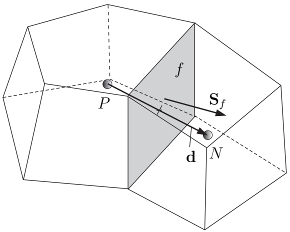
- Outlet

$$\begin{aligned} \partial \mathbf{U}/\partial n &= 0 \\ p &= 0 \\ \partial \alpha/\partial n &= 0 \end{aligned}$$

interFoam

- VOF for interface capturing
- PISO for pressure velocity coupling
- unknowns:

`p_rgh`	p	dynamic pressure
`p`	P	total pressure ($P = p + \rho \mathbf{g} \cdot \mathbf{x}$)
`alpha1`	α	volume fraction
`U`	$\mathbf{U}$	velocity vector
`phi`	$\mathbf{S_f} \cdot \mathbf{U_f}$	velocity flux
`rhoPhi`	$\mathbf{S_f} \cdot \rho_f \mathbf{U_f}$	mass flux
`gh`	$\mathbf{g} \cdot \mathbf{x_P}$	hydrostatic pressure over density at cell center
`ghf`	$\mathbf{g} \cdot \mathbf{x_f}$	hydrostatic pressure over density at face center



`interFoam` algorithm

1. solve transport equation for volume fraction
2. generate linear systems for momentum components U, V, W, using convection and viscous terms only
3. (optional) solve for momentum components using old values of pressure gradient and density gradient
4. form the pressure Poisson equation, and solve (may loop over this for non-orthogonal correction update)
5. update velocity with pressure gradient
6. update face flux with pressure contribution
7. update turbulence quantities

- PISO loop over steps 4-6.

Momentum prediction

- Total momentum equation:

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{UU} = -\nabla p - \mathbf{g} \cdot \mathbf{x} \nabla \rho + \nabla \cdot (\mu_{\text{eff}} \nabla \mathbf{U}) + \nabla \mathbf{U} \cdot \nabla \mu_{\text{eff}}$$

- in prediction, form linear systems using convection and viscous terms only:

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{UU} - \nabla \cdot (\mu_{\text{eff}} \nabla \mathbf{U}) - \nabla \mathbf{U} \cdot \nabla \mu_{\text{eff}} \quad = \quad 0$$

$$\Downarrow$$

$$[A]_U \{U\} = \{b_U\}$$

$$[A]_V \{V\} = \{b_V\}$$

$$[A]_W \{W\} = \{b_W\}$$

- if you “solve” for momentum prediction:

$$\{U\} = [A]_U^{-1} \cdot [\{b_U\} - \nabla p \cdot \mathbf{i} - \mathbf{g} \cdot \mathbf{x} \nabla \rho \cdot \mathbf{i}]$$

Pressure correction

- start with semi-discrete momentum equation

$$[A]_U \{U\} = [\{b_U\} - \nabla p \cdot \mathbf{i} - \mathbf{g} \cdot \mathbf{x} \nabla \rho \cdot \mathbf{i}]$$

- look at equation for a single cell

$$a_P \mathbf{U}_P + \sum a_N \mathbf{U}_N = \mathbf{b}_P - \nabla p - \mathbf{g} \cdot \mathbf{x} \nabla \rho$$

- calculate the velocity without $\nabla \rho$ and ∇p

$$\mathbf{U}_P^\star = a_P^{-1} (\mathbf{b}_P - \sum a_N \mathbf{U}_N)$$

- interpolation of gradients is bad! (Rhie-Chow). Face flux using starred velocity

$$\phi^\star = \mathbf{U}_f^\star \cdot \mathbf{S}_f$$

- now the flux with the density gradient:

$$\phi' = \phi^\star - \mathbf{g} \cdot \mathbf{x}_f \frac{\partial \rho}{\partial n} a_{P,f}^{-1} |\mathbf{S}_f|$$

Pressure correction, cont.

- use the continuity equation to find pressure that makes the velocity discretely divergence free.

$$\nabla \cdot \mathbf{U} = \sum \mathbf{U}_f \cdot \mathbf{S}_f = \sum \phi = 0$$

- ϕ' will not satisfy continuity because it is a numerical approximation, and it does not contain the pressure gradient term. Return to the momentum equation for a single cell, and note the use of the starred velocity.

$$\mathbf{U}_P = \mathbf{U}^\star - a_P^{-1} \nabla p - a_P^{-1} \mathbf{g} \cdot \mathbf{x} \nabla \rho$$

- insert into continuity

$$\nabla \cdot a_P^{-1} \nabla p = \sum \phi'$$

- after solving for p, then update the face flux and velocity

$$\begin{aligned} \phi &= \phi^\star - \nabla \cdot a_{P,f}^{-1} \nabla p_f \\ \mathbf{U} &= \mathbf{U}^\star - \mathbf{g} \cdot \mathbf{x}_f - \nabla p \end{aligned}$$

Time-step size

- Courant number in simple terms:

$$C_o = U\frac{\Delta t}{\Delta x}$$

- For arbitrary polyhedral finite volume:

$$C_o = \frac{\mathbf{U}\cdot\mathbf{S_f}}{\mathbf{d}\cdot\mathbf{S_f}}\Delta t$$

- **d** is the vector from pole center to neighbor center
- if we solve implicit equations, what is an acceptable time step size based on the Courant number?

PISO-settings

- `momentumPredictor`: relatively small additional expense $\rightarrow$ recommended
- `nCorrectors`: this is to loop over pressure system, also known as PISO loops. For strict time accuracy, minimum of 2. Calm-water resistance, 1 should do.
- `nNonOrthogonalCorrectors`: due to small time step, and use of `nCorrectors`, this may be set to 0 in most cases. Perhaps for initial time steps on bad grids a few may help.
- `nAlphaCorr`: loop over α equation. For time-dependent flows 1-2. For steady flow like calm-water resistance, 0.
- `nAlphaSubCycles`: this reduces the time-step size for the explicit integration of the α transport equation. As you increase the time-step size for the total system of equations, and if you need time accuracy (maybe retain stability), increase the number of sub-cycles according.
- `cAlpha`: the compression term in the advection of α is scaled by this parameter. Set to zero to deactivate compression. Set to 1 as a nominal value.

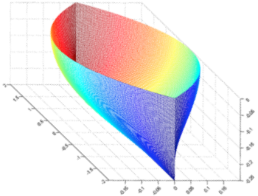
Discretization Settings

- time: `Euler`, Courant number restriction leads to small time steps, first order accuracy is fine for calm-water resistance.
- gradient: linear
- divergence: upwind to aid in convergence. `vanLeer` is second-order away from extrema. `limitedLinearV` may be less diffusive than `vanLeer`
- Laplacian: `Gauss linear corrected`. Second-order, with correction for non-orthogonal part.

Wigley Hull Experiments

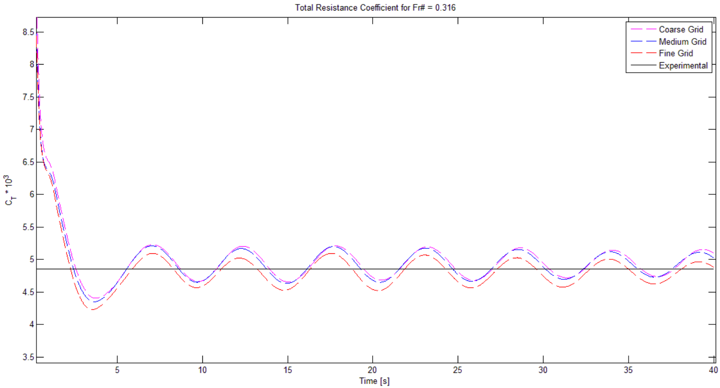
- Well used test data. Body fixed and free to sink and trim. SRI
- $0.08 < F < 0.40$
- $2 \times 10^6 < R < 1 \times 10^7$

Item	Symbol	Value	Unit
Length	L	4.0	m
Beam	B	0.4	m
Draft	T	0.25	m
Wetted Surface	S	2.3796	m^2



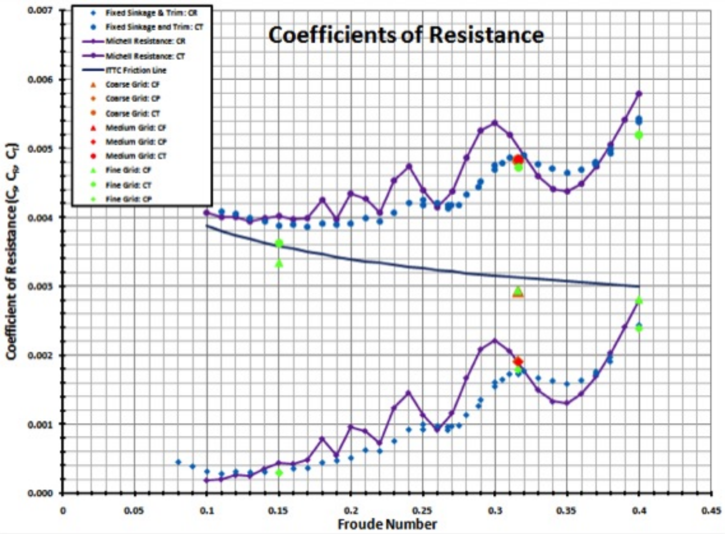
Wigley Hull Computations: Time Integration

- 144K cell coarse grid (Pointwise)



courtesy of Mert Türkol

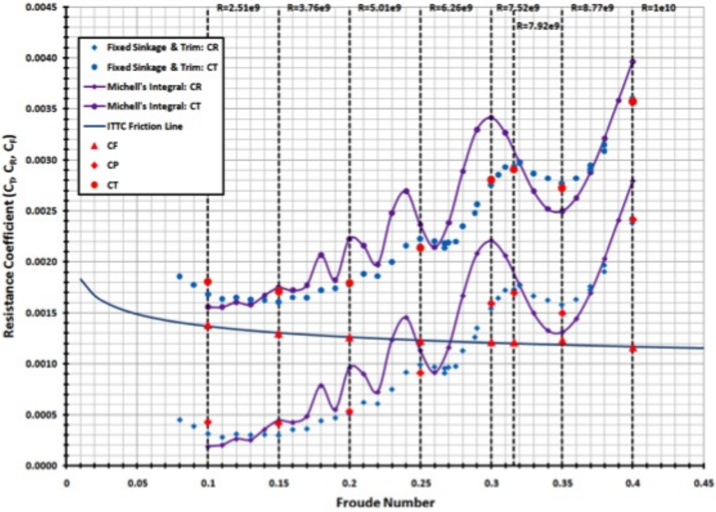
Wigley Hull Computations: Convergence



courtesy of Ensign William Garland

Wigley Hull Computations: Full Scale

- 400 m Ship



Hull Force Library

- control over quantity calculated and the write syntax

$$
\begin{aligned}
\mathbf{F}_p &= \int_S p\mathbf{n}dS \\
\mathbf{M}_p &= \int_S (\mathbf{x}_f - \mathbf{x}_o) \times p\mathbf{n}dS \\
\mathbf{F}_v &= \int_S \bar{\bar{\bar{\tau}}} \cdot \mathbf{n}dS \\
\mathbf{M}_v &= \int_S (\mathbf{x}_f - \mathbf{x}_o) \times \bar{\bar{\bar{\tau}}} \cdot \mathbf{n}dS
\end{aligned}
$$

- column 1:time, 2-4: $\mathbf{F}_p$, 5-7: $\mathbf{F}_p$, 8-10: $\mathbf{M}_v$, 11-13: $\mathbf{M}_v$