GUIDE FOR

BUCKLING AND ULTIMATE STRENGTH ASSESSMENT FOR OFFSHORE STRUCTURES (LRFD VERSION)

JULY 2016

American Bureau of Shipping
Incorporated by Act of Legislature of
the State of New York 1862

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Foreword

This Guide provides criteria for assessing the structural strength related Ultimate Limit States for the classification of specific types of Offshore Structures to be designed using the Load and Resistance Factor Design (LRFD) criteria issued by ABS.

The LRFD criteria are an alternative to Working Stress Design (WSD) criteria, or as it may also be called, Allowable Strength (or Stress) Design (ASD) criteria. In this Guide, when WSD is mentioned, it refers to both ASD and WSD.

At the time of issuance of this Guide, the ABS criteria in the alternative LRFD format for specific types of Offshore Structures are presented in the ABS *Guide for Load and Resistance Factor Design (LRFD) Criteria for Offshore Structures*. It is intended that, when other LRFD-based Rules and Guides are issued by ABS, they will make reference to this Guide, but users should confirm the applicability of this Guide to other types of Offshore Structures. In case of conflict between the criteria contained in this Guide and ABS Rules or Guides for classification, the latter have precedence.

The criteria presented in this Guide may also apply in other situations such as the certification or verification of a structural design for compliance with the Regulations of a Governmental Authority. However, in such a case, the criteria specified by the Governmental Authority should be used, but they may not produce a design that is equivalent to one obtained from the application of the criteria contained in this Guide. Where the mandated technical criteria of the cognizant Governmental Authority for certification differ from those contained herein, ABS will consider the acceptance of such criteria as an alternative to those given herein so that, at the Owner or Operator's request, both certification and classification may be granted to the Offshore Structure.

This Guide becomes effective on the first day of the month of publication.

Users are advised to check periodically on the ABS website www.eagle.org to verify that this version of this Guide is the most current.

We welcome your feedback. Comments or suggestions can be sent electronically by email to rsd@eagle.org.

GUIDE FOR

BUCKLING AND ULTIMATE STRENGTH ASSESSMENT FOR OFFSHORE STRUCTURES (LRFD VERSION)

CONTENTS

SECTION 1 Introduction

1 General

The Load and Resistance Factor Design (LRFD) based criteria in this Guide are developed primarily through calibration exercises to the existing Work Stress Design (WSD) based criteria in the ABS *Guide for Buckling and Ultimate Strength Assessment for Offshore Structures (Buckling Guide)* and reference to other comparable standards such as ISO 19902, ISO 19905-1, AISC Specification (LRFD) and API RP 2A-LRFD. This Guide is not to be taken as altering the WSD-based criteria in the ABS *Guide for Buckling and Ultimate Strength Assessment for Offshore Structures,* which is meant to be used with WSD-based classification requirements.

It is acknowledged that new methods and criteria for design are constantly evolving. For this reason, ABS may accept the use of an alternative technical approach that is demonstrated to produce an acceptable level of safety.

3 Scope of This Guide

This Guide provides criteria to be applied to the following structural steel components or assemblages:

- Individual structural members (i.e., discrete beams and columns) [see Section 2]
- Plates, stiffened panels and corrugated panels [see Section 3]
- Stiffened cylindrical shells [see Section 4]

For Tubular Joints and requirements related to buckling analysis by the finite element method, refer to the WSD-based criteria as specified in Section 5 and Appendix 1 of the *Buckling Guide* or recognized alternative criteria applicable to these topics.

5 Tolerances and Imperfections

The buckling and ultimate strength of structural components are highly dependent on the amplitude and shape of the imperfections introduced during manufacture, storage, transportation and installation.

Typical imperfections causing strength deterioration are:

- Initial distortion due to welding and/or other fabrication-related process
- Misalignments of joined components

In general, the effects of imperfections in the form of initial distortions, misalignments and weld-induced residual stresses are implicitly incorporated in the buckling and ultimate strength formulations. Because of their effect on strength, it is important that imperfections be monitored and repaired, as necessary, not only during construction, but also in the completed structure to ensure that the structural components satisfy tolerance limits. The tolerances on imperfections to which the strength criteria given in this Guide are considered valid are listed, for example, in IACS Recommendation No. 47 “Shipbuilding and Repair Quality Standard”. Imperfections exceeding such published tolerances are not acceptable unless it is shown using a recognized method that the strength capacity and utilization factor of the imperfect structural component are within proper target safety levels.

7 Gross Scantling

The buckling and ultimate strength formulations provided in this Guide are intended to be used along with the gross scantling of structural components.

9 Loadings

The criteria of this Guide are applied to the strength related Ultimate Limit States of the structure. Usually two primary Loading Conditions are considered for strength design situations after the Offshore Structure is installed. These are referred to as:

i) *Design Operating Condition* (DOC) and *Design Environmental Condition* (DEC) for Floating Production Installations; and

ii) *Static Loading Condition* and *Combined Loading Condition* for Mobile Offshore Drilling Units.

These two loading conditions are associated with the Ultimate Limit States *ULS-a* and *ULS-b* in the ABS *Guide for Load and Resistance Factor Design (LRFD) Criteria for Offshore Structures (LRFD Guide)*, in which the *Temporary Conditions* that a Floating Offshore Installation will experience during transport and installation at the operating site are also referenced to *ULS-a* and *ULS-b*.

In addition, the Accidental Limit State *ALS* is specified in the *LRFD Guide* for the *Redundancy Condition* for Column-Stabilized Units and the *Damaged Condition* for buoyant hull structures of Floating Production Installations.

The *LRFD Guide* also provides definitions of the various load categories to be considered in design, such as those produced by gravity and the environment; and the load factors to be applied to each load category for the loading conditions mentioned above.

Determination of the magnitudes of each load component and each load effect (i.e., stress, deflection, internal boundary condition, etc.) are to be performed using recognized calculation methods and/or test results and are to be fully documented and referenced. As appropriate, the effects of stress concentrations, secondary stress arising from eccentrically applied loads and member displacements (i.e., P-Δ effects) and additional shear displacements and shear stress in beam elements are to be suitably accounted for in the analysis.

The buckling and ultimate strength formulations in this Guide are applicable to static/quasi-static loads. Dynamic (e.g., impulsive) loads, such as those that may result from impact and fluid sloshing, can induce ‘dynamic buckling’, which, in general, is to be dealt with using an appropriate nonlinear analysis.

11 Resistance Factors

The buckling and ultimate strength equations in this Guide provide an estimate of the representative strength, R_n, of the considered components while achieving the lowest standard deviation when compared with nonlinear analyses and mechanical tests.

In general, the required or design strength, R_μ, is to be less than or equal to the representative strength, R_n, divided by a resistance factor, γ_R; or $R_\mu \leq R_n/\gamma_R$.

Unless otherwise indicated in this Guide, the value of γ_R is to be taken as 1.05.

13 Terms and References

13.1 Terminology

In LRFD criteria, particular values of load and resistance quantities are used. It is necessary to be aware of the terminology associated with these values. For the purpose of this Guide, the following terms and definitions apply.

13.1.1 Characteristic Value

A value of load or resistance established with respect to a prescribed probability of not being violated by an unfavorable value.

13.1.2 Design Value

A value of load or resistance obtained from a representative value that has been modified by the appropriate load or resistance factors.

13.1.3 Factored Load

The representative value of a load multiplied by a load factor, which may be greater than, less than, or equal to 1.0.

13.1.4 Factored Resistance

The representative value of a resistance divided by a resistance factor usually greater than 1.0.

13.1.5 Limit State

A state beyond which the structure, or some part of the structure, no longer fulfils specified design criteria.

13.1.6 Load Effect

An effect of a load on the structure, for example, internal forces, internal moments, stresses, strains, rigid body motions, and elastic deformations.

13.1.7 Nominal Value

A value of load or resistance that may not have a statistical basis. It is based on applicable experience or is obtained from a recognized reference standard or recognized code (i.e., criteria mandated by the cognizant governmental authority).

13.1.8 Representative Value

A value of load or resistance that is used to verify the limit state. The representative value can be a characteristic or nominal value or other rationally determined value of the variable. The representative value of a load can relate to upper or lower bound characteristic values.

13.1.9 Resistance

The capacity of a structural component, the cross-section of a component, or a structural detail to withstand the effects of loads.

13.3 Abbreviations

ABS	American Bureau of Shipping
AISC	American Institute of Steel Construction
API	American Petroleum Institute
ASD	Allowable Stress (or Strength) Design
IACS	International Association of Classification Societies
ISO	International Organization for Standardization
LRFD	Load and Resistance Factor Design
WSD	Working Stress Design

13.5 References

References are made in this Guide to other criteria issued by ABS and other organizations. Unless otherwise noted, the applicable edition of a reference is the one officially issued and available on the date the Agreement for Classification is accepted by ABS. Where a particular edition or date associated with a reference is given, it means that particular edition is relevant to the topic being presented in this Guide. Use of a later edition may be permitted upon consultation with ABS. ABS may consider at its discretion, upon the request of the Owner, the application of other appropriate alternative methods and recognized codes of practice.

i) ABS *Guide for Buckling and Ultimate Strength Assessment for Offshore Structures (Buckling Guide)*

ii) ABS *Guide for Load and Resistance Factor Design (LRFD) Criteria for Offshore Structures (LRFD Guide)*

iii) AISC Specification (LRFD), *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, 2010 (Only the LRFD formatted criteria are relevant to this Guide)

iv) API RP 2A-LRFD, *Recommended Practice for Planning Designing and Constructing Fixed Offshore Platforms – Load and Resistance Factor Design*, 1997

v) ABS *Rules for Building and Classing Floating Production Installations (FPI Rules)*

vi) IACS, *Shipbuilding and Repair Quality Standards, Recommendation No. 47*

vii) ISO 19902, *Petroleum and natural gas industries – Fixed steel offshore structures*, 2007

viii) ISO 19905-1, *Petroleum and natural gas industries – Site-specific assessment of mobile offshore units – Part 1: Jack-ups*, 2012

SECTION 2 Individual Structural Members

1 General

This Section provides criteria for individual structural members. The types of members considered in this Section are tubular and non-tubular members with uniform geometric properties along their entire length and made of a single material. The criteria given in the following subsections account for the compactness of member section specified in 2/1.7. Recognizing that the compactness limits specified by various standards are slightly different, ABS will accept alternative recognized standards, such as the AISC Specification (LRFD), API RP 2A-LRFD, ISO 19902 and ISO 19905-1.

The behavior of structural members is influenced by a variety of factors, including sectional shape, material characteristics, boundary conditions, loading types and parameters and fabrication methods.

1.1 Geometries and Properties of Structural Members

A structural member with a cross section having at least one axis of symmetry is considered. The geometries and properties of some typical cross sections are illustrated in Section 2, Table 1. For sections which are not listed in Section 2, Table 1, the required geometric properties are to be calculated based on other recognized standards as mentioned in Subsection 2/1.

1.3 Loads and Load Effects

This Section includes the criteria for any of the following loads and load effects:

- Axial force in longitudinal direction, P
- Bending moment, M
- Shear force, V
- Hydrostatic pressure, q
- Combined axial tension and bending moment
- Combined axial compression and bending moment
- Combined axial tension, bending moment and hydrostatic pressure
- Combined axial compression, bending moment and hydrostatic pressure

The load directions depicted in Section 2, Figure 1 are positive.

FIGURE 1
Load Application on a Tubular Member

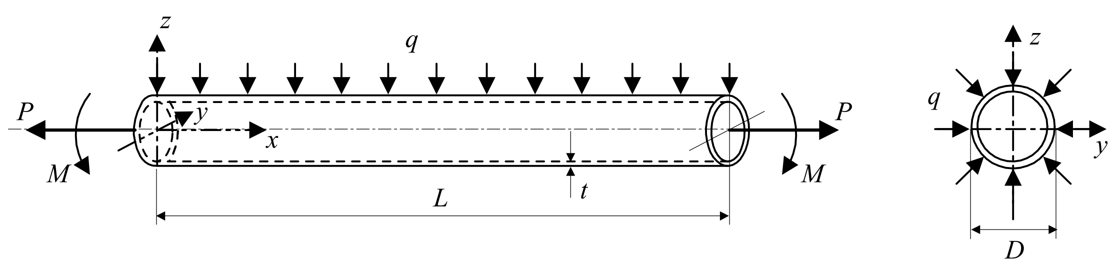


TABLE 1
Geometries, Properties and Compact Limits of Structural Members

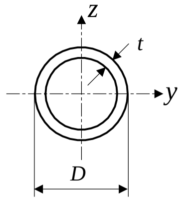
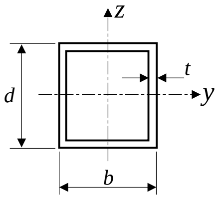
Geometry	*Sectional Shape*	*Geometrical Parameters*	*Axis*	*Properties**	*Compact Limits*
1. Tubular member		D = Outer diameter t = Thickness	N/A	$A = \pi[D^2 - (D - 2t)^2]/4$ $I_y, I_z = \pi[D^4 - (D - 2t)^4]/64$ $K = \pi(D - t)^3 t/4$ $I_o = \pi[D^4 - (D - 2t)^4]/32$ $\Gamma = 0$	$\frac{D}{t} \le \frac{E}{9\sigma_0}$
2. Square or rectangular hollow section		b = Flange width d = Web depth t = Thickness	Major y-y Minor z-z	$A = 2(bt_f + dt_w)$ $I_y = d^2(3bt_f + dt_w)/6$ $I_z = b^2(bt_f + dt_w)/6$ $K = \dfrac{2b^2d^2}{\left(\dfrac{b}{t_f} + \dfrac{d}{t_w}\right)}$ $I_o = I_y + I_z$ $\Gamma = \dfrac{b^2d^2}{24}\dfrac{(dt_f - bt_w)^2}{bt_f + dt_w}$	$\frac{b}{t_f}, \frac{d}{t_w} \le 1.5\sqrt{\frac{E}{\sigma_0}}$

TABLE 1 (continued)
Geometries, Properties and Compact Limits of Structural Members

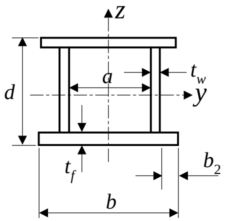
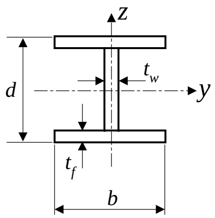
Geometry	*Sectional Shape*	*Geometrical Parameters*	*Axis*	*Properties**	*Compact Limits*
3. Welded box shape		d = Web depth t_w = Web thickness b = Flange width t_f = Flange thickness b_2 = Outstand	Major *y-y* Minor *z-z*	$A = 2(bt_f + dt_w)$ $I_y = d^2(3bt_f + dt_w)/6$ $I_z = (b^3t_f + 3a^2dt_w)/6$ $a = b - 2\,b_2$ $K = \dfrac{2a^2d^2}{\left(\dfrac{a}{t_f} + \dfrac{d}{t_w}\right)}$ $I_o = I_y + I_z$ $\Gamma = \dfrac{(b^3d^2t_f - a^2d^3t_w)^2}{24(b^3d^2t_f + a^2d^3t_w)}$	$\dfrac{a}{t_f}, \dfrac{d}{t_w} \le 1.5\sqrt{\dfrac{E}{\sigma_0}}$ $\dfrac{b_2}{t_f} \le 0.4\sqrt{\dfrac{E}{\sigma_0}}$
4. W-shape		d = Web depth t_w = Web thickness b = Flange width t_f = Flange thickness	Major *y-y* Minor *z-z*	$A = 2bt_f + dt_w$ $I_y = d^2(6bt_f + dt_w)/12$ $I_z = b^3t_f/6$ $K = (2bt_f^3 + dt_w^3)/3$ $I_o = I_y + I_z$ $\Gamma = d^2b^3t_f/24$	$\dfrac{d}{t_w} \le 1.5\sqrt{\dfrac{E}{\sigma_0}}$ $\dfrac{b}{t_f} \le 0.8\sqrt{\dfrac{E}{\sigma_0}}$

TABLE 1 (continued)
Geometries, Properties and Compact Limits of Structural Members

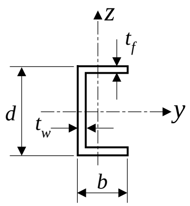
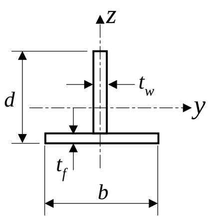
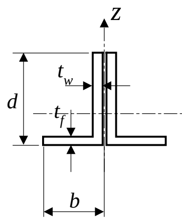
Geometry	*Sectional Shape*	*Geometrical Parameters*	*Axis*	*Properties**	*Compact Limits*
5. Channel		d = Web depth t_w = Web thickness b = Flange width t_f = Flange thickness d_{cs} = Distance of shear center to centroid	Major y-y Minor z-z	$A = 2bt_f + dt_w$ $I_y = d^2(6bt_f + dt_w)/12$ $I_z = d^3t_w(bt_f + 2dt_w)/3A$ $K = (2bt_f^3 + dt_w^3)/3$ $I_o = I_y + I_z + A\,d_{cs}^2$ $\Gamma = \dfrac{d^2b^3t_f(3bt_f + 2dt_w)}{12(6bt_f + dt_w)}$	$\dfrac{d}{t_w} \le 1.5\sqrt{\dfrac{E}{\sigma_0}}$ $\dfrac{b}{t_f} \le 0.4\sqrt{\dfrac{E}{\sigma_0}}$
6. T-bar		d = Web depth t_w = Web thickness b = Flange width t_f = Flange thickness d_{cs} = Distance of shear center to centroid	Major y-y Minor z-z	$A = bt_f + dt_w$ $I_y = d^3t_w(4bt_f + dt_w)/12A$ $I_z = b^3t_f/12$ $K = (bt_f^3 + dt_w^3)/3$ $I_o = I_y + I_z + A\,d_{cs}^2$ $\Gamma = (b^3t_f^3 + 4d^3t_w^3)/144$	$\dfrac{d}{t_w} \le 0.4\sqrt{\dfrac{E}{\sigma_0}}$ $\dfrac{b}{t_f} \le 0.8\sqrt{\dfrac{E}{\sigma_0}}$

TABLE 1 (continued)
Geometries, Properties and Compact Limits of Structural Members

Geometry	*Sectional Shape*	*Geometrical Parameters*	*Axis*	*Properties**	*Compact Limits*
7. Double angles		d = Web depth t_w = Web thickness b = Flange width t_f = Flange thickness d_{CS} = Distance of shear center to centroid	Major y-y Minor z-z	$A = 2(bt_f + dt_w)$ $I_y = d^3t_w(4bt_f + dt_w)/3A$ $I_z = 2b^3t_f/3$ $K = (2bt_f^3 + 8dt_w^3)/3$ $I_o = I_y + I_z + A\,d_{cs}^2$ $\Gamma = (b^3t_f^3 + 4d^3t_w^3)/18$	$\frac{d}{t_w} \le 0.4\sqrt{\frac{E}{\sigma_0}}$ $\frac{b}{t_f} \le 0.4\sqrt{\frac{E}{\sigma_0}}$

* The formulations for the properties are derived assuming that the section is thin-walled (i.e., thickness is relatively small) where:

A = cross sectional area, cm^2 (in^2)

I_y = moment of inertia about y-axis, cm^4 (in^4)

I_z = moment of inertia about z-axis, cm^4 (in^4)

K = St. Venant torsion constant for the member, cm^4 (in^4)

I_0 = polar moment of inertia of the member, cm^4 (in^4)

Γ = warping constant, cm^6 (in^6)

1.5 Failure Modes

Failure modes for a structural member are categorized as follows:

- *Flexural buckling.* Bending about the axis of the least resistance.
- *Torsional buckling.* Twisting about the longitudinal (x) axis. It may occur if the torsional rigidity of the section is low, as for a member with a thin-walled open cross section.
- *Lateral-torsional buckling.* Synchronized bending and twisting. A member which is bent about its major axis may buckle laterally.
- *Local buckling.* Buckling of a plate or shell element that is a local part of a member

1.7 Cross Section Classification

The cross section may be classified as:

i) *Compact.* A cross section is compact if all compressed components comply with the limits in Section 2, Table 1. For a compact section, the local buckling (plate buckling and shell buckling) can be disregarded because yielding precedes buckling.

ii) *Non-Compact.* A cross section is non-compact if any compressed component does not comply with the limits in Section 2, Table 1. For a non-compact section, the local buckling (plate or shell buckling) is to be taken into account.

Note: If a design is being pursued using Plastic Analysis, more stringent compactness criteria than those given in this Guide may be required. These (referred to as "Class 1 Proportions" in some standards) allow plastic hinge formation and rotations for bending moment redistribution as required for Plastic Analysis.

1.9 Adjustment Factor

Strength formulations given in subsequent subsections below may entail adjustment factors as follows.

For axial tension and bending:

$$\psi_1 = 1.0$$

For axial compression (column buckling or torsional buckling):

$$\psi_2 = 0.87 \qquad \text{if } \sigma_{EA} \le P_r\sigma_0$$

$$= 1 - 0.13\sqrt{P_r\sigma_0 / \sigma_{EA}} \qquad \text{if } \sigma_{EA} > P_r\sigma_0$$

where

σ_{EA} = elastic buckling stress, as defined in 2/3.3, N/cm^2 (kgf/cm^2, lbf/in^2)

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

For compression (local buckling of tubular members):

$$\psi_3 = 0.833 \qquad \text{if } \sigma_{Ci} \le 0.55\sigma_0$$

$$= 0.629 + 0.371\sigma_{Ci}/\sigma_0 \qquad \text{if } \sigma_{Ci} > 0.55\sigma_0$$

where

σ_{Ci} = critical local buckling stress, representing σ_{Cx} for axial compression, as specified in 2/9.1, and $\sigma_{C\theta}$ for hydrostatic pressure, as specified in 2/3.9, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

3 Members Subjected to a Single Action

3.1 Axial Tension

Tubular or non-tubular members subjected to axial tensile forces, P_{ut} due to factored loads are to satisfy the following equation:

$$P_{ut} \le \psi_1 P_t/\gamma_R$$

where

P_t = representative value of axial tensile strength N (kgf, lbf)

= $\sigma_0 A$

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

A = cross sectional area, cm^2 (in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined in 2/1.9

3.3 Axial Compression

Tubular or non-tubular members subjected to axial compressive forces, P_{uc}, due to factored loads may fail by local buckling or flexural-torsional buckling. The following equation is to be satisfied:

$$P_{uc} \le \psi_2 P_c/\gamma_R$$

where

P_c = representative axial compressive strength

= $\sigma_{CA} A$

γ_R = resistance factor as defined in Subsection 1/11

ψ_2 = adjustment factor as defined 2/1.9

σ_{CA} = critical buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \begin{cases} \sigma_{EA} & \text{if } \sigma_{EA} \le P_r \sigma_F \\ \sigma_F \left[1 - P_r (1 - P_r) \dfrac{\sigma_F}{\sigma_{EA}} \right] & \text{if } \sigma_{EA} > P_r \sigma_F \end{cases}$$

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_F = σ_0, specified minimum yield point for a compact section

= σ_{Cx}, local buckling stress for a non-compact section from Subsection 2/9

σ_{EA} = elastic buckling stress, which is the lesser of the solutions of the following quadratic equation, N/cm^2 (kgf/cm^2, lbf/in^2)

$$\frac{I_0}{A}(\sigma_{EA} - \sigma_{E\eta})(\sigma_{EA} - \sigma_{ET}) - \sigma_{EA}^2 d_{cs}^2 = 0$$

$\sigma_{E\eta}$ = Euler buckling stress about minor axis, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\pi^2 E/(kL/r_\eta)^2$

σ_{ET} = ideal elastic torsional buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{EK}{2.6I_0} + \left(\frac{\pi}{kL}\right)^2 \frac{E\Gamma}{I_0}$$

r_η = radius of gyration about minor axis, cm (in.)

$$= \sqrt{I_\eta / A}$$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

A = cross sectional area, cm^2 (in^2)

I_η = moment of inertia about minor axis, cm^4 (in^4)

K = St. Venant torsion constant for the member, cm^4 (in^4)

I_0 = polar moment of inertia of the member, cm^4 (in^4)

Γ = warping constant, cm^6 (in^6)

d_{cs} = difference of centroid and shear center coordinates along major axis, cm (in.)

L = member's length, cm (in.)

k = effective length factor, as specified in Section 2, Table 2. When it is difficult to clarify the end conditions, the nomograph shown in Section 2, Figure 2 may be used. The values of G for each end (A and B) of the column are determined:

$$G = \sum \frac{I_c}{L_c} \Big/ \sum \frac{I_g}{L_g}$$

$\sum \frac{I_c}{L_c}$ is the total for columns meeting at the joint considered and $\sum \frac{I_g}{L_g}$ is the total for restraining beams meeting at the joint considered. For a column end that is supported, but not fixed, the moment of inertia of the support is zero, and the resulting value of G for this end of the column would be ∞. However, in practice, unless the footing was designed as a frictionless pin, this value of G would be taken as 10. If the column end is fixed, the moment of inertia of the support would be ∞, and the resulting value of G of this end of the column would be zero. However, in practice, there is some movement and G may be taken as 1.0. If the restraining beam is either pinned ($G = \infty$) or fixed ($G = 0$) at its far end, refinements may be made by multiplying the stiffness (I_g/L_g) of the beam by the following factors:

Sidesway prevented

Far end of beam pinned = 1.5

Sidesway permitted

Far end of beam pinned = 0.5

Far end of beam fixed = 2.0

TABLE 2
Effective Length Factor

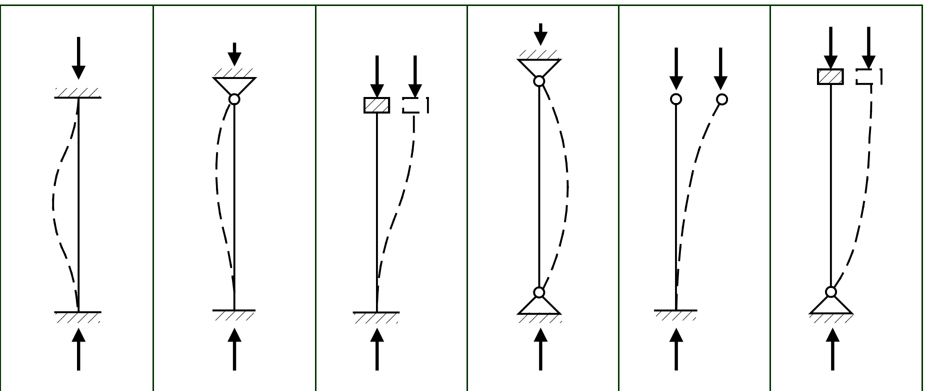
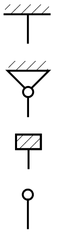
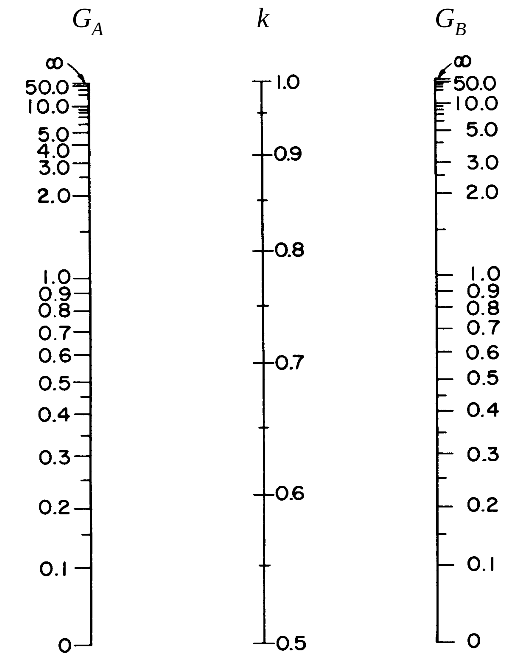
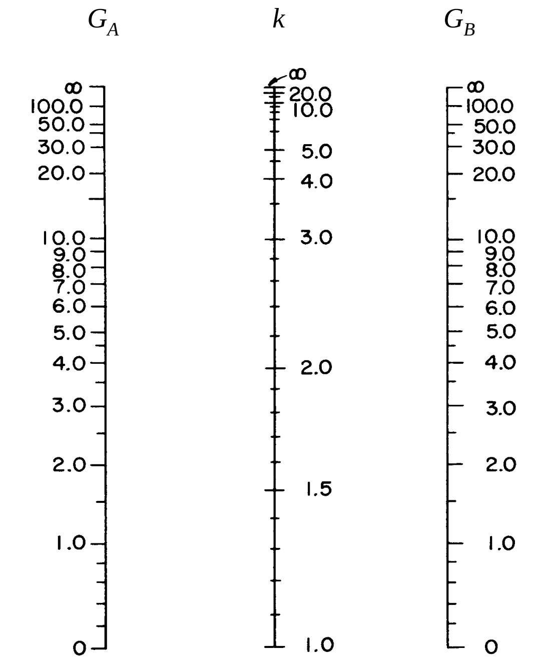
Buckled shape of column shown by dashed line						
Theoretical value	0.50	0.70	1.0	1.0	2.0	2.0
Recommended *k* value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition notation		Rotation fixed. Translation fixed Rotation free. Translation fixed Rotation fixed. Translation free Rotation free. Translation free				

FIGURE 2
Effective Length Factor



Sidesway Prevented



Sidesway Permittted

FIGURE 2 (continued)
Effective Length Factor

Note: These alignment charts or nomographs are based on the following assumptions:

1. Behavior is purely elastic.
2. All members have constant cross section.
3. All joints are rigid.
4. For columns in frames with sidesway prevented, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.
5. For columns in frames with sidesway permitted, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending
6. The stiffness parameter $L(P/EI)^{1/2}$ of all columns is equal.
7. Joint restraint is distributed to the column above and below the joint in proportion to EI/L for the two columns.
8. All columns buckle simultaneously.
9. No significant axial compression force exists in the restraining beams.

Adjustments are required when these assumptions are violated and the alignment charts are still to be used. Reference is made to ANSI/AISC 360-10, Commentary C2.

3.5 Bending Moment

3.5.1 Non-tubular Members

A non-tubular member subjected to bending moment, M_u due to factored loads is to satisfy the following equation:

$$M_u \leq \psi_1 M_b/\gamma_R$$

Depending on the following conditions, a member may be designed to 2/3.5.1(a), which accounts for a member's capability to develop its plastic moment capacity or 2/3.5.1(b), which accounts for a member's ability to develop its elastic moment capacity.

To apply the plastic criteria in 2/3.5.1(a) the following conditions are to be met, otherwise refer to 2/3.5.1(b):

i) Webs and flanges are to be continuously connected;

ii) Both webs and flanges are to meet the compactness limits given in Section 2, Table 1;

iii) The member is symmetrical about the axis of bending; and

iv) The distance, L_b, between lateral supports of the compression flange or bracing to resist twisting of the cross section is less than or equal to distance, L_r, given by:

$$L_r = 1.76r\sqrt{E/\sigma_0}$$

where

r = radius of gyration about an axis of bending, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

3.5.1(a) Plastic Basis. The following equation is to be satisfied:

$M_u \leq \psi_1 M_b/\gamma_R$

where

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined 2/1.9

M_b = representative value of the bending strength, N-cm (kgf-cm, lbf-in)

= $SM_p \sigma_0$

SM_p = plastic section modulus, cm^3 (in^3)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

3.5.1(b) Elastic Basis. The following equation is to be satisfied:

$M_u \leq \psi_1 M_b/\gamma_R$

where

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined 2/1.9

M_b = representative value of the bending strength, N-cm (kgf-cm, lbf-in)

= $SM_e \sigma_{CB}$

SM_e = elastic section modulus, cm^3 (in^3)

σ_{CB} = critical bending strength to be determined by the critical local or lateral-torsional buckling stress as follows:

The critical lateral-torsional buckling stress is to be obtained from the following equation:

$$\sigma_{C(LT)} = \begin{cases} \sigma_{E(LT)} & \text{if} \quad \sigma_{E(LT)} \leq P_r\sigma_F \\ \sigma_F\left[1 - P_r(1-P_r)\dfrac{\sigma_F}{\sigma_{E(LT)}}\right] & \text{if} \quad \sigma_{E(LT)} > P_r\sigma_F \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

$\sigma_{E(LT)}$ = elastic lateral-torsional buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \sqrt{C}\,\frac{\pi^2 EI_\eta}{SM_e(kL)^2}$$

I_η = moment of inertia about minor axis, as defined in Section 2, Table 1, cm^4 (in^4)

SM_e = section modulus of compressive flange, cm^3 (in^3)

$$= \frac{I_\xi}{\xi_c}$$

I_ξ = moment of inertia about major axis, as defined in Section 2, Table 1, cm^4 (in^4)

ξ_c = distance from major neutral axis to compressed flange, cm (in.)

C = $\dfrac{\Gamma}{I_\eta} + \dfrac{K}{I_\eta}\dfrac{(kL)^2}{2.6\pi^2}$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_F = σ_0, specified minimum yield point for a compact section

= σ_{Cx}, local buckling stress for a non-compact section, as specified in 2/9.3

K = St. Venant torsion constant for the member, cm^4 (in^4)

Γ = warping constant, cm^6 (in^6)

L = member's length, cm (in.)

k = effective length factor, as defined in 2/3.3

3.5.2 Tubular Members

A tubular member subjected to bending moment, M_u, due to factored loads may fail depending on the D/t and slenderness ratios by local buckling or lateral-torsional buckling.

The following equation is to be satisfied:

$M_u \le \psi_1 M_b/\gamma_R$

where

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined 2/1.9

M_b = representative value of the bending strength, N-cm (kgf-cm, lbf-in)

= $SM_e\,\sigma_{CB}$

SM_e = elastic section modulus, cm^3 (in^3)

σ_{CB} = critical bending strength to be obtained from the following equation

$$= \begin{cases} (SM_p/SM_e)\sigma_0 & \text{for} \quad \sigma_0 D/(Et) \le 0.02 \\ [1.038 - 1.90\sigma_0 D/(Et)](SM_p/SM_e)\sigma_0 & \text{for} \quad 0.02 < \sigma_0 D/(Et) \le 0.10 \\ [0.921 - 0.73\sigma_0 D/(Et)](SM_p/SM_e)\sigma_0 & \text{for} \quad \sigma_0 D/(Et) > 0.10 \end{cases}$$

where

SM_e = elastic section modulus, cm^3 (in^3)

= $(\pi/64)[D^4 - (D - 2t)^4]/(D/2)$

SM_p = plastic section modulus, cm^3 (in^3)

= $(1/6)[D^3 - (D - 2t)^3]$

D = outer diameter, cm (in.)

t = thickness, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_0 = specified minimum yield point

For tubular members with $D/t > E/(4.5\sigma_0)$, the local buckling stress is to be determined from 4/3.3.

3.7 Shear Force

Tubular or non-tubular members subjected to shear forces, V_{ut}, due to factored loads are to satisfy the following equation:

$$V_{ut} \le \psi_1 V_t/\gamma_R$$

where

V_t = representative value of shear strength N (kgf, lbf)

= $0.6\sigma_0 A_w$

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

A_w = shear area, cm^2 (in^2); for planar and rolled sections may be taken as the depth of the member times web thickness; for tubular members, use $^1/_2$ cross-sectional area.

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined in 2/1.9

3.9 Hydrostatic Pressure

Tubular members with $D/t \le E/(4.5\sigma_0)$ subjected to external pressure are to satisfy the following equation:

$$\frac{\sigma_\theta}{\psi_3 \sigma_{C\theta}/\gamma_R} \le 1$$

where

σ_θ = hoop stress due to hydrostatic pressure

= $qD/(2t)$

q = factored external pressure

= 1.3 for an Ultimate Limit State (ULS) with no, or operating level, environmental conditions

= 1.1 for an ULS with extreme or severe storm conditions the load factor

$\sigma_{C\theta}$ = critical hoop buckling strength, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\Phi\sigma_{E\theta}$

Φ = plasticity reduction factor

= 1 for $\Delta \le 0.55$

= $\dfrac{0.45}{\Delta} + 0.18$ for $0.55 < \Delta \le 1.6$

= $\dfrac{1.31}{1 + 1.15\Delta}$ for $1.6 < \Delta < 6.25$

= $1/\Delta$ for $\Delta \ge 6.25$

Δ = $\sigma_{E\theta}/\sigma_0$

$\sigma_{E\theta}$ = elastic hoop buckling stress

= $2C_\theta Et/D$

C_θ	=	buckling coefficient	
	=	$0.44t/D$	for $\mu \geq 1.6D/t$
	=	$0.44t/D + 0.21(D/t)^3/\mu^4$	for $0.825D/t \leq \mu < 1.6D/t$
	=	$0.737/(\mu - 0.579)$	for $1.5 \leq \mu < 0.825D/t$
	=	0.80	for $\mu < 1.5$
μ	=	geometric parameter	
	=	$\ell/\left(D\sqrt{2D/t}\right)$	
ℓ	=	length of tubular member between stiffening rings, diaphragms or end connections, cm (in.)	
D	=	outer diameter, cm (in.)	
t	=	thickness, cm (in.)	
E	=	modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel	
σ_0	=	specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)	
γ_R	=	resistance factor as defined in Subsection 1/11	
ψ_3	=	adjustment factor as defined 2/1.9	

For tubular members with $D/t > E/(4.5\sigma_0)$, the criteria of 4/3.5 are to be applied.

5 Members Subjected to Combined Loads

5.1 Axial Tension and Bending Moment

Members subjected to combined axial tension and bending moment are to satisfy the following equations at all cross-sections along their length:

For tubular members:

$$\frac{P_{ut}}{\psi_1 \sigma_0 A/\gamma_R} + \frac{1}{\psi_1/\gamma_R}\left[\left(\frac{M_{uby}}{M_{CBy}}\right)^2 + \left(\frac{M_{ubz}}{M_{CBz}}\right)^2\right]^{0.5} \leq 1$$

For rolled or fabricated-plate sections:

$$\frac{P_{ut}}{\psi_1 \sigma_0 A/\gamma_R} + \frac{M_{uby}}{\psi_1 M_{CBy}/\gamma_R} + \frac{M_{ubz}}{\psi_1 M_{CBz}/\gamma_R} \leq 1$$

where

P_{ut}	=	axial tensile force due to factored loads from 2/3.1, N (kgf, lbf)
M_{uby}	=	bending moment due to factored loads from 2/3.5 about member *y*-axis, N-cm (kgf-cm, lbf-in)
M_{ubz}	=	bending moment due to factored loads from 2/3.5 about member *z*-axis, N-cm (kgf-cm, lbf-in)
M_{CBy}	=	critical bending moment corresponding to member's *y*-axis from 2/3.5, N-cm (kgf-cm, lbf-in)
M_{CBz}	=	critical bending moment corresponding to member's *z*-axis from 2/3.5, N-cm (kgf-cm, lbf-in)

A = cross sectional area, cm^2 (in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ_1 = adjustment factor as defined 2/1.9

5.3 Axial Compression and Bending Moment

Members subjected to combined axial compression and bending moment are to satisfy the following equation at all cross sections along their length:

For tubular members:

When $P_{uc}/P_{CA} > 0.15$:

$$\frac{P_{uc}}{\psi_2 P_{CA}/\gamma_R} + \frac{1}{\psi_1/\gamma_R}\left[\left(\frac{1}{M_{CBy}}\,\frac{C_{my}\,M_{uby}}{1-\dfrac{P_{uc}}{\psi_2 P_{Ey}/\gamma_R}}\right)^2 + \left(\frac{1}{M_{CBz}}\,\frac{C_{mz}M_{ubz}}{1-\dfrac{P_{uc}}{\psi_2 P_{Ez}/_R}}\right)^2\right]^{0.5} \le 1$$

When $P_{uc}/P_{CA} \le 0.15$:

$$\frac{P_{uc}}{\psi_2 P_{CA}/\gamma_R} + \frac{1}{\psi_1/\gamma_R}\left[\left(\frac{M_{uby}}{M_{CBy}}\right)^2 + \left(\frac{M_{ubz}}{M_{CBz}}\right)^2\right]^{0.5} \le 1$$

For rolled or fabricated-plate sections:

When $P_{uc}/P_{CA} > 0.15$:

$$\frac{P_{uc}}{\psi_2 P_{CA}/\gamma_R} + \frac{1}{\psi_1/\gamma_R}\left(\frac{1}{M_{CBy}}\,\frac{C_{my}\,M_{uby}}{1-\dfrac{P_{uc}}{\psi_2 P_{Ey}/\gamma_R}} + \frac{1}{M_{CBz}}\,\frac{C_{mz}M_{ubz}}{1-\dfrac{P_{uc}}{\psi_2 P_{Ez}/\gamma_R}}\right) \le 1$$

When $P_{uc}/P_{CA} \le 0.15$:

$$\frac{P_{uc}}{\psi_2 P_{CA}/\gamma_R} + \frac{1}{\psi_1/\gamma_R}\left(\frac{M_{uby}}{M_{CBy}} + \frac{M_{ubz}}{M_{CBz}}\right) \le 1$$

where

P_{uc} = axial compressive force due to factored loads from 2/3.3, N (kgf, lbf)

M_{uby} = bending moment due to factored loads from 2/3.5 about member *y*-axis, N-cm (kgf-cm, lbf-in)

M_{ubz} = bending moment due to factored loads from 2/3.5 about member *z*-axis, N-cm (kgf-cm, lbf-in)

P_{CA} = critical axial compressive force from 2/3.3, N (kgf, lbf)

M_{CBy} = critical bending strength corresponding to member *y*-axis from 2/3.5, N-cm (kgf-cm, lbf-in)

M_{CBz} = critical bending strength corresponding to member *z*-axis from 2/3.5, N-cm (kgf-cm, lbf-in)

P_{Ey} = Euler buckling force corresponding to member y-axis, N (kgf, lbf)

= $\pi^2 EA/(k_y L/r_y)^2$

P_{Ez} = Euler buckling force corresponding to member z-axis, N (kgf, lbf)

= $\pi^2 EA/(k_z L/r_z)^2$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

A = cross sectional area, cm^2 (in^2)

r_y, r_z = radius of gyration corresponding to the member y- and z-axes, cm (in.)

k_y, k_z = effective length factors corresponding to member y- and z-axes from 2/3.3

C_{my}, C_{mz} = moment factors corresponding to the member y- and z-axes, as follows:

i) For compression members in frames subjected to joint translation (sidesway):

$C_m = 0.85$

ii) For restrained compression members in frames braced against joint translation (sidesway) and with no transverse loading between their supports:

$C_m = 0.6 - 0.4M_1/M_2$

but not less than 0.4 and limited to 0.85, where M_1/M_2 is the ratio of smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

iii) For compression members in frames braced against joint translation in the plane of loading and subject to transverse loading between their supports, the value of C_m may be determined by rational analysis. However, in lieu of such analysis, the following values may be used.

For members whose ends are restrained:

$C_m = 0.85$

For members whose ends are unrestrained:

$C_m = 1.0$

γ_R = resistance factor as defined in Subsection 1/11

ψ_1, ψ_2 = adjustment factors as defined 2/1.9

7 Tubular Members Subjected to Combined Loads with Hydrostatic Pressure

The effect of hydrostatic pressure on member strength can be ignored when the following condition is met. In such a case the criteria given in Subsection 2/5 apply to the member.

The condition, under which hydrostatic pressure can be ignored for a specific member is as follows:

$$(D/t)_{max} \le C/\, h_w^{0.335}$$

where

D = outer diameter, cm (in.)

t = thickness, cm (in.)

C = 211 (314), when h_w is in m (ft)

h_w = effective head of water applicable to the member (i.e., depth below the water surface including wave amplitude and the equivalent water head resulting from applicable soil pressure), m (ft)

To illustrate, some limiting $(D/t)_{max}$ values are listed in the table below.

Effective Head of Water h_w	*Maximum Diameter to Thickness Ratio* $(D/t)_{max}$
43 m (141 ft)	60.0
50 m (164 ft)	56.9
75 m (246 ft)	49.7
100 m (328 ft)	45.1
125 m (410 ft)	41.9
150 m (492 ft)	39.4
200 m (656 ft)	35.8

If a member's D/t exceeds the limiting value $(D/t)_{max}$, the criteria given below in this Subsection are to be used. The following criteria are based on stress rather than strength.

When applying the stress based criteria below in this Subsection, the following conditions are to be observed:

i) Due consideration is to be given to *capped-end* loads on a structural member subjected to hydrostatic pressure.

ii) The equations in this subsection do not apply unless the criteria of 2/3.9 are satisfied first.

iii) Additional criteria may need to apply when $D/t \geq E/4.5\sigma_0$, see Section 4.

7.1 Axial Tension, Bending Moment and Hydrostatic Pressure

When member longitudinal tensile stresses and hoop compressive stresses occur simultaneously, the following interaction equation is to be satisfied where the maximum tensile stress combination is applied:

$$\left(\frac{\sigma_a + \sigma_b - 0.5\sigma_\theta}{\psi_1\sigma_0/\gamma_R}\right)^2 + \left(\frac{\sigma_\theta}{\psi_3\sigma_{C\theta}/\gamma_R}\right)^2 + 2\nu\left|\frac{\sigma_a + \sigma_b - 0.5\sigma_\theta}{\psi_1\sigma_0/\gamma_R}\right|\frac{\sigma_\theta}{\psi_3\sigma_{C\theta}/\gamma_R} \leq 1$$

where

σ_a = absolute value of acting axial stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_b = absolute value of acting resultant bending stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = absolute value of hoop compression stress (see 2/3.9 for the load factor applicable to hydrostatic pressure), N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{C\theta}$ = critical hoop buckling strength (see 2/3.9)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

ν = Poisson's ratio, 0.3 for steel

γ_R = resistance factor as defined in Subsection 1/11

ψ_1, ψ_3 = adjustment factors as defined 2/1.9

7.3 Axial Compression, Bending Moment and Hydrostatic Pressure

When longitudinal compressive stresses and hoop compressive stresses occur simultaneously, the following two equations are to be satisfied where the first equation reflects the maximum compressive stress combination:

$$\frac{\sigma_a + 0.5\sigma_\theta}{\psi_2 \sigma_{Cx}/\gamma_R} + \frac{\sigma_b^*}{\psi_1 \sigma_{CB}/\gamma_R} \le 1 \quad \text{and} \quad \frac{\sigma_\theta}{\psi_3 \sigma_{C\theta}/\gamma_R} \le 1$$

where

σ_a = absolute value of acting axial stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = absolute value of hoop compression stress (see 2/3.9 for the load factor applicable to hydrostatic pressure), N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CB} = critical bending strength, as defined in 2/3.5.3

$\sigma_{C\theta}$ = critical hoop buckling stress, as defined in 2/3.9

σ_{Cx} = critical axial strength

= $\sigma_0\left[1.64 - 0.23(D/t)^{0.25}\right]$ for $D/t \ge 60$

= σ_0 for $D/t < 60$

σ_b^* = resultant compressive bending stress including the effects of member end moment and column amplification effect

$$= \frac{C_m \sigma_b}{\left(1 - \dfrac{\sigma_a}{\psi_2\, \sigma_E/\gamma_R}\right)}$$

σ_b = absolute value of acting resultant bending stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

C_m = moment factors corresponding, as defined in 2/5.3

σ_E = Euler buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\pi^2 E\, r^2/(kL)^2$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

k = effective length factor, as defined in 2/3.3

L = member's length, cm (in.)

r = radius of gyration of the cross section of the tubular member, cm (in.)

$$= \sqrt{\frac{I_t}{A_t}}$$

I_t = moment of inertia of the cross section of the tubular member; if the cross section is variable along the length, the minimum value is to be used, cm^4 (in^4)

A_t = cross sectional area of the tubular member; if the cross section is variable along the length, the minimum value is to be used, cm^2 (in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ_1, ψ_2, ψ_3 = adjustment factors as defined 2/1.9

Depending on the magnitude of the maximum compressive stress combination, the following equation should also be satisfied:

$$\frac{\sigma_x - 0.5\sigma_{\theta a}}{\sigma_{aa} - 0.5\sigma_{\theta a}} + \left(\frac{\sigma_\theta}{\sigma_{\theta a}}\right)^2 \le 1 \quad \text{if } \sigma_x > 0.5\sigma_{\theta a}$$

where

σ_x = maximum compressive stress combination

= $\sigma_x + \sigma_b^* + 0.5\sigma_\theta$

σ_{aa} = $\psi_3\sigma_{Ex}/\gamma_R$

$\sigma_{\theta a}$ = $\psi_3\sigma_{E\theta}/\gamma_R$

σ_{Ex} = elastic buckling stress, as defined in 2/9.1

$\sigma_{E\theta}$ = elastic hoop buckling stress, as defined in 2/3.9

γ_R = resistance factor as defined in Subsection 1/11

ψ_3 = adjustment factors as defined 2/1.9; note that ψ_3 may have different values for σ_{aa} and $\sigma_{\theta a}$

9 Local Buckling

For a member with a non-compact section, local buckling may occur before the member as a whole becomes unstable or before the yield point of the material is reached. Such behavior is characterized by local distortion of the cross section of the member. When a detailed analysis is not available, the equations given below may be used to evaluate the local buckling stress of a member with a non-compact section.

9.1 Tubular Members Subjected to Axial Compression

Local buckling stress of tubular members with $D/t \le E/(4.5\sigma_0)$ subjected to axial compression may be obtained from the following equation:

$$\sigma_{Cx} = \begin{cases} \sigma_{Ex} & \text{if} \quad \sigma_{Ex} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1 - P_r)\dfrac{\sigma_0}{\sigma_{Ex}}\right] & \text{if} \quad \sigma_{Ex} > P_r\sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Ex} = elastic buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $0.6Et/D$

D = outer diameter, cm (in.)

t = thickness, cm (in.)

For tubular members with $D/t > E/(4.5\sigma_0)$, the local buckling stress is to be determined from 4/3.3.

9.3 Non-tubular Members Subjected to Compression and Bending Moment

The critical local buckling of a member with rolled or fabricated plate section may be taken as the lowest local buckling stress of the plate components comprising the section. The local buckling stress of the component is to be obtained from the following equation with respect to uniaxial compression and in-plane bending moment:

$$\sigma_{Cx} = \begin{cases} \sigma_{Ex} & \text{if} \quad \sigma_{Ex} \le P_r \sigma_0 \\ \sigma_0 \left[1 - P_r (1 - P_r) \dfrac{\sigma_0}{\sigma_{Ex}} \right] & \text{if} \quad \sigma_{Ex} > P_r \sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_0 = specified minimum yield point, N/cm² (kgf/cm², lbf/in²)

σ_{Ex} = elastic buckling stress, N/cm² (kgf/cm², lbf/in²)

$$= C_1 \begin{cases} \dfrac{8.4}{\kappa + 1.1} & \text{for} \quad 0 \le \kappa \le 1 \\ 7.6 - 6.4\kappa + 10\kappa^2 & \text{for} \quad -1 \le \kappa < 0 \end{cases}$$

E = modulus of elasticity, 2.06×10^7 N/cm² (2.1×10^6 kgf/cm², 30×10^6 lbf/in²) for steel

ν = Poisson's ratio, 0.3 for steel

s = depth of unsupported plate component

t = thickness of plate component

k_s = buckling coefficient, as follows:

i) For a plate component with all four edges simply supported, the buckling coefficient is to be obtained from following equation:

$$k_s = \begin{cases} \dfrac{8.4}{\kappa + 1.1} & \text{for} \quad 0 \le \kappa \le 1 \\ 7.6 - 6.4\kappa + 10\kappa^2 & \text{for} \quad -1 \le \kappa < 0 \end{cases}$$

where

κ = ratio of edge stresses, as defined in Section 2, Figure 3

= $\sigma_{a\min}/\sigma_{a\max}$

ii) For a plate component with other boundary conditions, the buckling coefficient is obtained from Section 2, Table 3

FIGURE 3
Definition of Edge Stresses

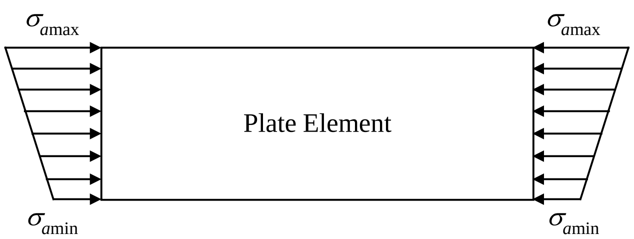
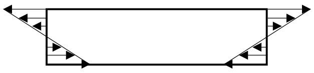
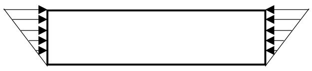


TABLE 3
Minimum Buckling Coefficients under Compression and Bending Moment, k_s *

Loading	*Top Edge Free*		*Bottom Edge Free*	
	Bottom Edge Simply Supported	*Bottom Edge Fixed*	*Top Edge Simply Supported*	*Top Edge Fixed*
$\sigma_{amin}/\sigma_{amax} = 1$ (Uniform compression) 	0.42	1.33	0.42	1.33
$\sigma_{amin}/\sigma_{amax} = -1$ (Pure Bending) 	—	—	0.85	2.15
$\sigma_{amin}/\sigma_{amax} = 0$ 	0.57	1.61	1.70	5.93

* *Note:* k_s for intermediate value of $\sigma_{amin}/\sigma_{amax}$ may be obtained by linear interpolation.

SECTION 3 Plates, Stiffened Panels and Corrugated Panels

1 General

The formulations provided in this Section are to be used to assess the Buckling and Ultimate Strength of plates, stiffened panels and corrugated panels for Ultimate Limit State checks related to strength. The strength criteria are given in terms of stress as is usual practice for continuous structure such as plating.

The criteria provided in this Section are intended to be applied to various types of offshore structures that have design criteria that have been issued by ABS in an LRFD format. Such structures include Floating Production Installations (FPIs) of the TLP and SPAR types, but it is not in the scope of this Guide to use the criteria with ship-type FPIs. In this latter case, see Section 5A-4-2 of the *FPI Rules*.

The design criteria apply also to stiffened panels for which the moment of inertia for the transverse girders is greater than the moment of inertia of the longitudinal stiffeners. It is not in the scope of this Guide to use the criteria for orthotropically stiffened plate panels.

Alternatively, the buckling and ultimate strength of plates, stiffened panels or corrugated panels may be determined based on either appropriate, well-documented experimental data or on a calibrated analytical approach. When a detailed analysis is not available, the equations provided in this section shall be used to assess the buckling strength.

1.1 Geometry of Plate, Stiffened Panel and Corrugated Panels

Flat rectangular plates and stiffened panels are depicted in Section 3, Figure 1. Stiffeners in the stiffened panels are usually installed equally spaced, parallel or perpendicular to panel edges in the direction of dominant load and are supported by heavier and more widely-spaced 'deep supporting members' (i.e., girders). The given criteria apply to a variety of stiffener profiles, such as flat-bar, built up T-profiles, built up inverted angle profiles and symmetric and non-symmetric bulb profiles. The section dimensions of a stiffener are defined in Section 3, Figure 2. The stiffeners may have strength properties different from those of the plate.

FIGURE 1
Typical Stiffened Panel

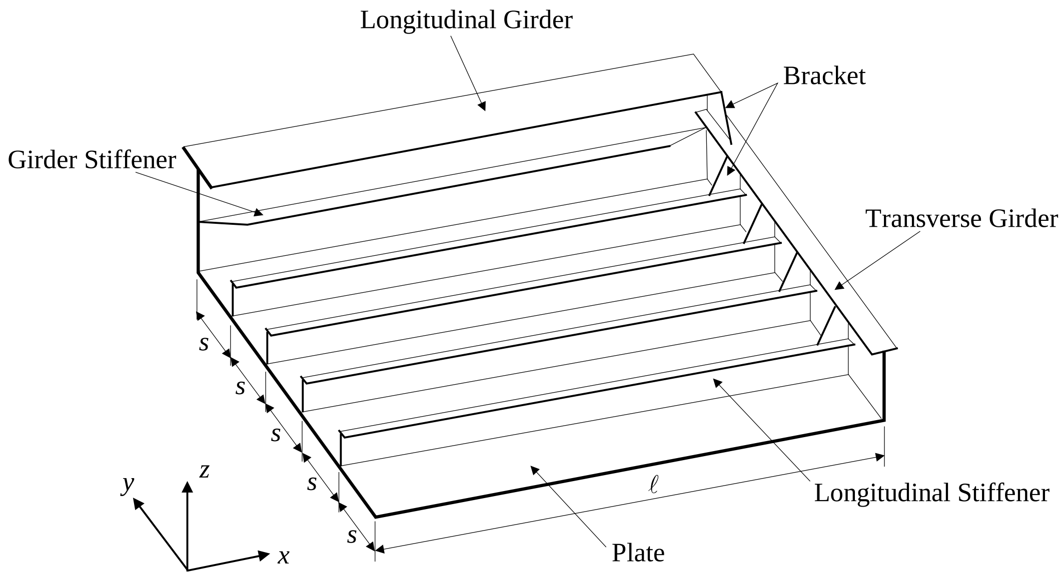
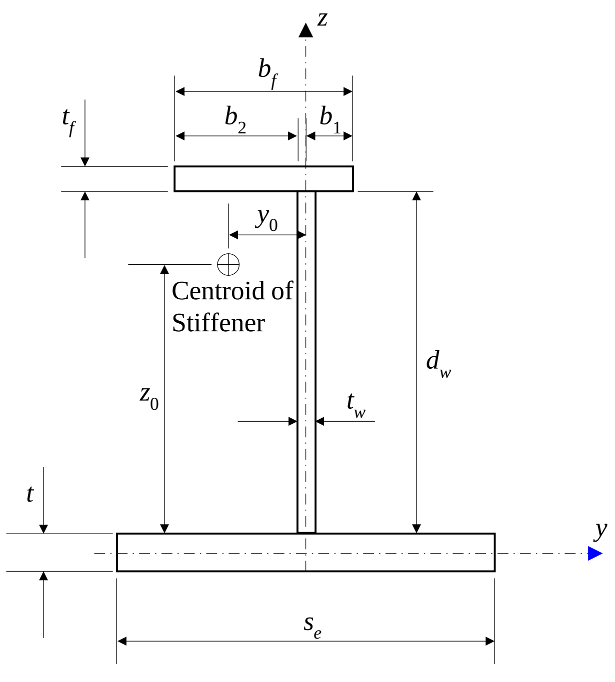


FIGURE 2
Sectional Dimensions of a Stiffened Panel



Corrugated panels, as depicted in Section 3, Figure 3**,** are self-stiffened and are usually corrugated in one direction, supported by stools at the two ends across the corrugation direction. They may act as watertight bulkheads or, when connected with fasteners, they are employed as corrugated shear diaphragms. The dimensions of corrugated panels are defined in Section 3, Figure 4. The buckling strength criteria for corrugated panels given in Subsection 3/11 are applicable to corrugated panels with corrugation angle, ϕ, between 57 and 90 degrees.

FIGURE 3
Typical Corrugated Panel

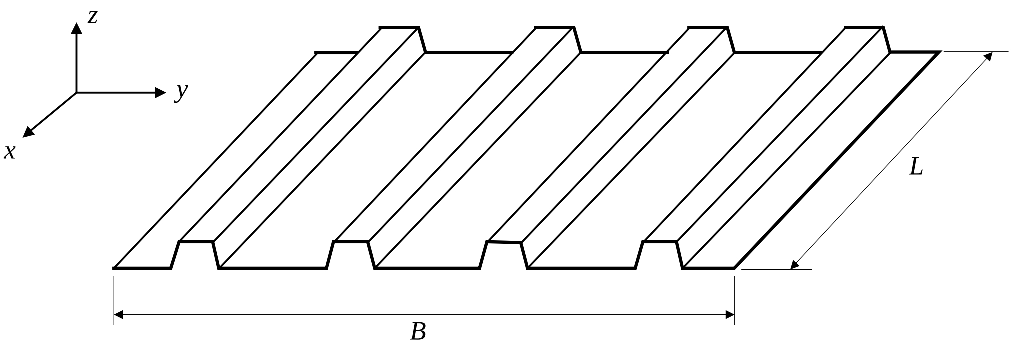
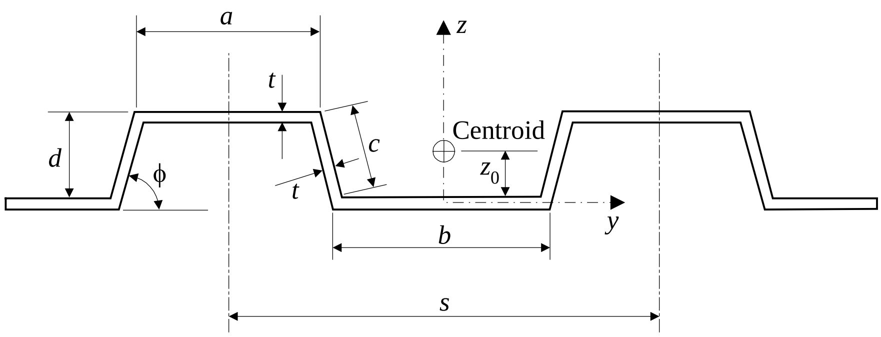


FIGURE 4
Sectional Dimensions of a Corrugated Panel



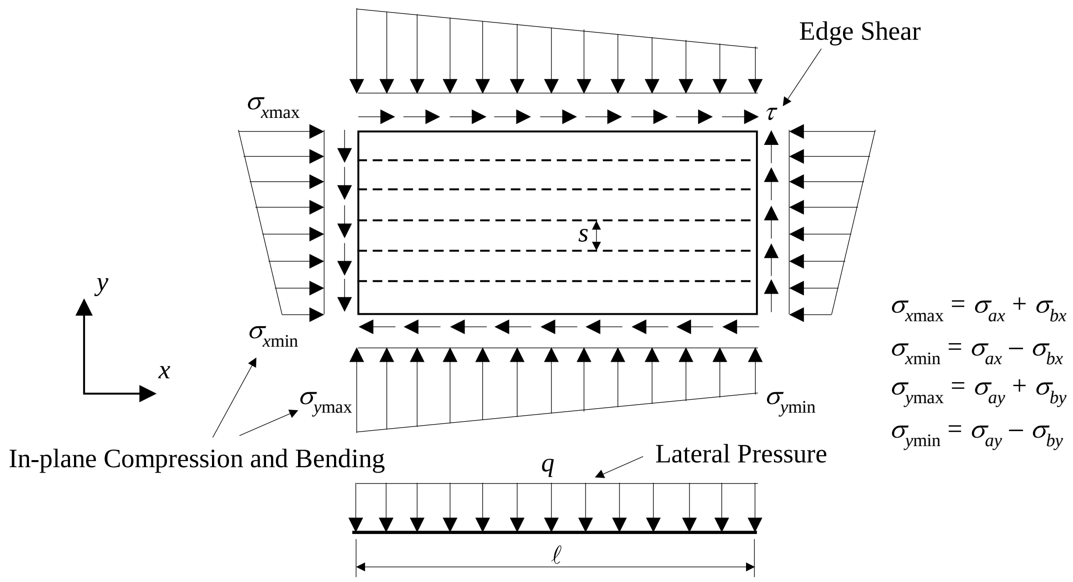
1.3 Load Application

The plate and stiffened panel criteria account for the following load and load effects. The symbols for each of these loads are shown in Section 3, Figure 5.

- Uniform in-plane compression, σ_{ax}, σ_{ay} *
- In-plane bending, σ_{bx}, σ_{by}
- Edge shear, τ
- Lateral loads, q
- Combinations of the above

* *Note*: If uniform stress σ_{ax} or σ_{ay} is tensile rather than compressive, it may be set equal to zero.

FIGURE 5
Primary Loads and Load Effects on Plate and Stiffened Panel



1.5 Buckling Control Concepts

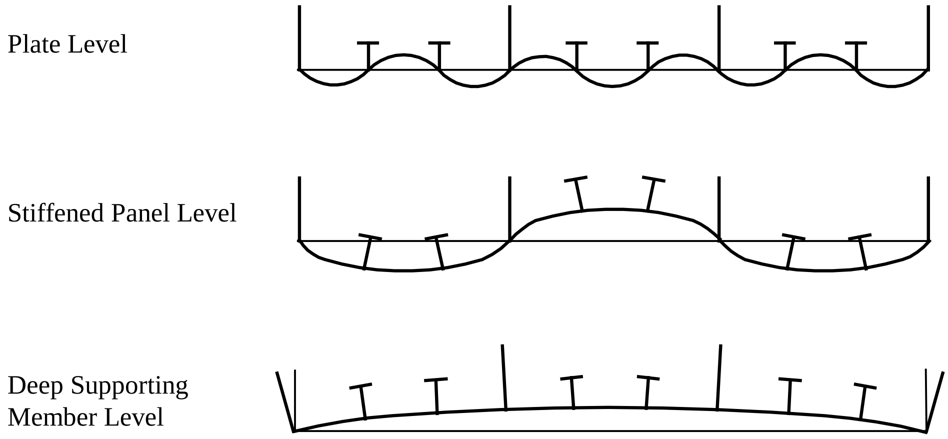
The failure of plates and stiffened panels can be sorted into three levels, namely, the plate level, the stiffened panel level and the entire grillage level, which are depicted in Section 3, Figure 6. An offshore structure is to be designed in such a way that the buckling and ultimate strength of each level is greater than its preceding level (i.e., a well-designed structure does not collapse when a plate fails as long as the stiffeners can resist the extra load they experience from the plate failure). Even if the stiffeners collapse, the structure may not fail immediately as long as the girders can support the extra load shed from the stiffeners.

The buckling strength criteria for plates and stiffened panels are based on the following assumptions and limits with respect to buckling control in the design of stiffened panels, which are in compliance with ABS recommended practices.

- The buckling strength of each stiffener is generally greater than that of the plate panel it supports.
- Stiffeners with their associated effective plating are to have moments of inertia not less than i_0, given in 3/9.1. If not satisfied, the overall buckling of stiffened panel is to be assessed, as specified in 3/5.7.
- The deep supporting members (i.e., girders) with their associated effective plating are to have moments of inertia not less than I_s, given in 3/9.5. If not satisfied, the overall buckling of stiffened panel is also necessary, as given in 3/5.7. In addition, tripping (e.g., torsional/flexural instability) is to be prevented if tripping brackets are provided, as specified in 3/7.7.
- Faceplates and flanges of girders and stiffeners are proportioned such that local instability is prevented (see 3/9.7).
- Webs of girders and stiffeners are proportioned such that local instability is prevented (see 3/9.9).

For plates and stiffened panels that do not satisfy these limits, a detailed analysis of buckling strength using an acceptable method should be submitted for review.

FIGURE 6
Failure Modes ("Levels") of Stiffened Panel



Section 3, Figure 6 illustrates the collapse shape for each level of failure mode. From a reliability point of view, no individual collapse mode can be 100 percent prevented. Therefore, the buckling control concept used in this Subsection is that the buckling and ultimate strength of each level is greater than its preceding level in order to avoid the collapse of the entire structure.

The failure ("levels") modes of a corrugated panel can be categorized as the face/web plate buckling level, the unit corrugation buckling level and the entire corrugation buckling level. In contrast to stiffened panels, corrugated panels will collapse immediately upon reaching any one of these three buckling levels.

1.7 Adjustment Factor

Strength formulations given in subsequent subsections below may entail adjustment factors as follows. The adjustment factor is to take the following value:

$\psi = 1.0$

3 Plate Panels

For rectangular plate panels between stiffeners, buckling is acceptable, provided that the ultimate strength given in 3/3.3 and 3/3.5 of the structure satisfies the specified criteria. Offshore practice demonstrates that only an ultimate strength check is required for plate panels. A buckling check of plate panels is necessary when establishing the attached plating width for stiffened panels. If the plating does not buckle, the full width is to be used. Otherwise, the effective width is to be applied if the plating buckles but does not fail.

3.1 Buckling State Limit

For the Buckling State Limit of plates subjected to in-plane and lateral pressure loads, the following strength criterion is to be satisfied:

$$\left(\frac{\sigma_{x\max}}{\sigma_{Cx}/\gamma_R}\right)^2 + \left(\frac{\sigma_{y\max}}{\sigma_{Cy}/\gamma_R}\right)^2 + \left(\frac{\tau}{\tau_C/\gamma_R}\right)^2 \leq 1$$

where

$\sigma_{x\max}$ = maximum compressive stress in the longitudinal direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{y\max}$ = maximum compressive stress in the transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ = edge shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Cx} = critical buckling stress for uniaxial compression in the longitudinal direction, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Cy} = critical buckling stress for uniaxial compression in the transverse direction, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_C = critical buckling stress for edge shear, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

The critical buckling stresses are specified below.

3.1.1 Critical Buckling Stress for Edge Shear

The critical buckling stress for edge shear, τ_C, may be taken as:

$$\tau_C = \begin{cases} \tau_E & \text{for} \quad \tau_E \le P_r \tau_0 \\ \tau_0 \left[1 - P_r (1 - P_r) \dfrac{\tau_0}{\tau_E} \right] & \text{for} \quad \tau_E > P_r \tau_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

τ_0 = shear strength of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\dfrac{\sigma_0}{\sqrt{3}}$

σ_0 = specified minimum yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_E = elastic shear buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $k_s \dfrac{\pi^2 E}{12(1-\nu^2)} \left(\dfrac{t}{s} \right)^2$

k_s = boundary dependent constant

= $\left[4.0 \left(\dfrac{s}{\ell} \right)^2 + 5.34 \right] C_1$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν = Poisson's ratio, 0.3 for steel

ℓ = length of long plate edge, cm (in.)

s = length of short plate edge, cm (in.)

t = thickness of plating, cm (in.)

C_1 = 1.1 for plate panels between angles or tee stiffeners

= 1.0 for plate panels between flat bars or bulb plates

= 1.0 for plate elements, web plate of stiffeners and local plate of corrugated panels

3.1.2 Critical Buckling Stress for Uniaxial Compression and In-plane Bending

The critical buckling stress, σ_{Ci} ($i = x$ or y), for plates subjected to combined uniaxial compression and in-plane bending may be taken as:

$$\sigma_{Ci} = \begin{cases} \sigma_{Ei} & \text{for} \quad \sigma_{Ei} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{Ei}}\right] & \text{for} \quad \sigma_{Ei} > P_r\sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{Ei} = elastic buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= k_s \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{s}\right)^2$$

For loading applied along the short edge of the plating (long plate):

$$k_s = C_1\begin{cases} \dfrac{8.4}{\kappa + 1.1} & \text{for} \quad 0 \le \kappa \le 1 \\ 7.6 - 6.4\kappa + 10\kappa^2 & \text{for} \quad -1 \le \kappa < 0 \end{cases}$$

For loading applied along the long edge of the plating (wide plate):

$$k_s = C_2\begin{cases} \left[1.0875\cdot\left(1+\dfrac{1}{\alpha^2}\right)^2 - 18\dfrac{1}{\alpha^2}\right]\cdot(1+\kappa) + 24\dfrac{1}{\alpha^2} & \text{for} \quad \kappa < \dfrac{1}{3} \quad \text{and} \quad 1 \le \alpha \le 2 \\ \left[1.0875\cdot\left(1+\dfrac{1}{\alpha^2}\right)^2 - 9\dfrac{1}{\alpha}\right]\cdot(1+\kappa) + 12\dfrac{1}{\alpha} & \text{for} \quad \kappa < \dfrac{1}{3} \quad \text{and} \quad \alpha > 2 \\ \left(1+\dfrac{1}{\alpha^2}\right)^2(1.675 - 0.675\kappa) & \text{for} \quad \kappa \ge \dfrac{1}{3} \end{cases}$$

where

α = aspect ratio

= ℓ/s

κ = ratio of edge stresses, as defined in Section 3, Figure 5*

= $\sigma_{i\min}/\sigma_{i\max}$

* *Note:* There are several cases in the calculation of ratio of edge stresses, κ:

- If uniform stress σ_{ai} ($i = x, y$) < 0 (tensile) and in-plane stress σ_{bi} ($i = x, y$) = 0, buckling check is not necessary, provided edge shear is zero;
- If uniform stress σ_{ai} ($i = x, y$) < 0 (tensile) and in-plane bending stress σ_{bi} ($i = x, y$) $\neq 0$, then $\sigma_{i\max} = \sigma_{bi}$ and $\sigma_{i\min} = -\sigma_{bi}$, so that $\kappa = -1$;
- If uniform stress σ_{ai} ($i = x, y$) > 0 (compressive) and in-plane bending stress σ_{bi} ($i = x, y$) = 0, $\sigma_{i\max} = \sigma_{i\min} = \sigma_i$, then $\kappa = 1$;
- If uniform stress σ_{ai} ($i = x, y$) >0 (compressive) and in-plane bending stress σ_{bi} ($i = x, y$) $\neq 0$, $\sigma_{i\max} = \sigma_{ai} + \sigma_{bi}$, $\sigma_{i\min} = \sigma_{ai} - \sigma_{bi}$ then $-1 < \kappa < 1$.

σ_0 = specified minimum yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν	=	Poisson's ratio, 0.3 for steel		
ℓ	=	length of long plate edge, cm (in.)		
s	=	length of short plate edge, cm (in.)		
t	=	thickness of plating, cm (in.)		
C_1	=	1.1	for plate panels between angles or tee stiffeners	
	=	1.0	for plate panels between flat bars or bulb plates	
	=	1.0	for plate elements, web plate of stiffeners and local plate of corrugated panels	
C_2	=	1.2	for plate panels between angles or tee stiffeners	
	=	1.1	for plate panels between flat bars or bulb plates	
	=	1.0	for plate elements and web plates	

3.3 Ultimate Strength under Combined In-plane Stresses

The ultimate strength for a plate between stiffeners subjected to combined in-plane stresses is to satisfy the following equation:

$$\left(\frac{\sigma_{x\max}}{\sigma_{Ux}/\gamma_R}\right)^2 - \varphi\left(\frac{\sigma_{x\max}}{\sigma_{Ux}/\gamma_R}\right)\left(\frac{\sigma_{y\max}}{\sigma_{Uy}/\gamma_R}\right) + \left(\frac{\sigma_{y\max}}{\sigma_{Uy}/\gamma_R}\right)^2 + \left(\frac{\tau}{\tau_U/\gamma_R}\right)^2 \le 1$$

where

$\sigma_{x\max}$ = maximum compressive stress in the longitudinal direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{y\max}$ = maximum compressive stress in the transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ = edge shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

φ = coefficient to reflect interaction between longitudinal and transverse stresses (negative values are acceptable)

= $1.0 - \beta/2$

σ_{Ux} = ultimate strength with respect to uniaxial stress in the longitudinal direction, N/cm^2 (kgf/cm^2, lbf/in^2)

= $C_x\sigma_o \ge \sigma_{Cx}$

C_x = $2/\beta - 1/\beta^2$ for $\beta > 1$

= 1.0 for $\beta \le 1$

σ_{Uy} = ultimate strength with respect to uniaxial stress in the transverse direction, N/cm^2 (kgf/cm^2, lbf/in^2)

= $C_y\sigma_0 \ge \sigma_{Cy}$

C_y = $C_x \cdot \frac{s}{\ell} + 0.1\left(1 - \frac{s}{\ell}\right)\left(1 + 1/\beta^2\right)^2 \le 1$

τ_U = ultimate strength with respect to edge shear, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\tau_C + 0.5\left(\sigma_0 - \sqrt{3}\tau_C\right)/\left(1 + \alpha + \alpha^2\right)^{1/2} \ge \tau_C$

σ_{Cx} = critical buckling stress for uniaxial compression in the longitudinal direction, specified in 3/3.1.2, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Cy} = critical buckling stress for uniaxial compression in the transverse direction, specified in 3/3.1.2, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_C = critical buckling stress for edge shear, as specified in 3/3.1.1

β = slenderness ratio

= $\frac{s}{t}\sqrt{\frac{\sigma_0}{E}}$

E = modulus of elasticity, N/cm^2 (kgf/cm^2, lbf/in^2)

ℓ = length of long plate edge, cm (in.)

s = length of short plate edge, cm (in.)

t = thickness of plating, cm (in.)

σ_0 = yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

β, s_e and ℓ_e are as defined in 3/3.3. σ_{Cx}, σ_{Cy}, σ_0, τ_C and α are as defined in 3/3.1.

3.5 Uniform Lateral Pressure

In addition to the buckling and ultimate strength criteria in 3/3.1 through 3/3.3, the ultimate strength of a panel between stiffeners subjected to uniform lateral pressure, q_u, alone or combined with in-plane stresses is to also satisfy the following equation:

$$q_u \le 4.0\frac{\sigma_0}{\gamma_R}\left(\frac{t}{s}\right)^2\left(1+\frac{1}{\alpha^2}\right)\sqrt{1-\left(\frac{\sigma_e/\chi}{\sigma_0}\right)^2}$$

where

t = plate thickness, cm (in.)

α = aspect ratio

= ℓ/s

ℓ = length of long plate edge, cm (in.)

s = length of short plate edge, cm (in.)

σ_0 = specified minimum yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_e = equivalent stress according to von Mises, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\sqrt{\sigma_{x\max}^2 - \sigma_{x\max}\sigma_{y\max} + \sigma_{y\max}^2 + 3\tau^2}$

χ = load adjustment factor

= 1.45 for the *Static Loading Condition* (see Subsection 1/9)

= 1.00 for the *Redundancy Condition* or *Damaged Condition* (see Subsection 1/9)

= 1.25 for the other loading conditions (see Subsection 1/9)

$\sigma_{x\max}$ = maximum compressive stress in the longitudinal direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{y\max}$ = maximum compressive stress in the transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ = edge shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

5 Stiffened Panels

The failure modes of stiffened panels include beam-column buckling, torsion and flexural buckling of stiffeners, local buckling of stiffener web and faceplate, and overall buckling of the entire stiffened panel. The stiffened panel strength against these failure modes is to be checked with the criteria provided in 3/5.1 through 3/5.7. Buckling state limits for a stiffened panel are considered its ultimate state limits.

5.1 Beam-Column Buckling State Limit

A stiffened panel subjected to axial compression, or bending moment or both; with or without external pressure, is to be designed to resist beam-column buckling. Beam-column buckling is to be assessed if:

$$\lambda \geq 0.50$$

where

λ = slenderness ratio of stiffened panel

= $\sqrt{\sigma_0 / \sigma_{E(C)}}$

$\sigma_{E(C)}$ = Euler's buckling stress

= $\dfrac{\pi^2 E r_e^2}{\ell^2}$

r_e = radius of gyration of area, A_e, cm (in.)

= $\sqrt{\dfrac{I_e}{A_e}}$

I_e = moment of inertia of longitudinal or stiffener, accounting for the effective width, s_e, cm^4 (in^4)

A_e = effective sectional area, cm^2 (in^2)

= $A_s + s_e t$

A_s = sectional area of the longitudinal, excluding the associated plating, cm^2 (in^2)

t = plate thickness, cm (in.)

s_e = effective width, cm (in.)

= s when the buckling state limit of the associated plating from 3/3.1 is satisfied

= $C_x C_y C_{xy} s$ when the buckling state limit of the associated plating from 3/3.1 is not satisfied

C_x = $2/\beta - 1/\beta^2$ for $\beta > 1$

= 1.0 for $\beta \leq 1$

C_y = $0.5\varphi\left(\dfrac{\sigma_{y\max}}{\sigma_{Uy}}\right) + \sqrt{1 - \left(1 - 0.25\varphi^2\right)\left(\dfrac{\sigma_{y\max}/\chi}{\sigma_{Uy}}\right)^2}$ *

* *Note:* A limit for C_y is that the transverse loading should be less than the transverse ultimate strength of the plate panels. The buckling check for stiffeners is not to be performed until the attached plate panels satisfy the ultimate strength criteria.

$\sigma_{y\max}$ = maximum compressive stress in the transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Uy} = ultimate strength with respect to uniaxial stress in the transverse direction, as specified in 3/3.3, N/cm^2 (kgf/cm^2, lbf/in^2)

χ = load adjustment factor

= 1.45 for the *Static Loading Condition* (see Subsection 1/9)

= 1.00 for the *Redundancy Condition* or *Damaged Condition* (see Subsection 1/9)

= 1.25 for the other loading conditions (see Subsection 1/9)

C_{xy} = $\sqrt{1-\left(\dfrac{\tau/\chi}{\tau_0}\right)^2}$

φ = $1.0 - \beta/2$

β = $\dfrac{s}{t}\sqrt{\dfrac{\sigma_0}{E}}$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

τ = edge shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_0 = shear strength of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\dfrac{\sigma_0}{\sqrt{3}}$

σ_0 = specified minimum yield point of the longitudinal or stiffener under consideration. If there is a large difference between the yield points of a longitudinal or stiffener and the plating, the yield point resulting from the weighting of areas is to be used. N/cm^2 (kgf/cm^2, lbf/in^2)

The beam-column buckling state limit may be determined as follows:

$$\frac{\sigma_a}{\left(\dfrac{\sigma_{CA}}{\gamma_R}\right)\cdot\left(\dfrac{A_e}{A}\right)} + \frac{C_m \sigma_b}{\left(\dfrac{\sigma_0}{\gamma_R}\right)\left[1-\dfrac{\sigma_a}{\sigma_{E(C)}/\gamma_R}\right]} \le 1$$

where

σ_a = calculated compressive stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $-P/A$

P = total compressive load on stiffener using full width of associated plating due to factored loads, N (kgf, lbf)

σ_{CA} = critical buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\sigma_{E(C)}$ for $\sigma_{E(C)} \le P_r \sigma_0$

= $\sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{E(C)}}\right]$ for $\sigma_{E(C)} > P_r \sigma_0$

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

A = total sectional area, cm^2 (in^2)

= $A_s + st$

σ_b = bending stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= M/SM_w

M = maximum bending moment induced by lateral loads, N-cm (kgf-cm, lbf-in)

= $qs\ell^2/12$

C_m = moment adjustment coefficient, which may be taken as 0.75

q = factored lateral pressure for the region considered, N/cm^2 (kgf/cm^2, lbf/in^2)

s = spacing of the longitudinal, cm (in.)

ℓ = unsupported span of the longitudinal or stiffener, cm (in.), as defined in Section 3, Figure 7

SM_w = effective section modulus of the longitudinal at flange, accounting for the effective breadth, s_w, cm^3 (in^3)

s_w = effective breadth, as specified in Section 3, Figure 8**,** cm (in.)

γ_R = resistance factor as defined in Subsection 1/11

c = 0.578, as specified in Section 3, Figure 8**,** cm (in.)

FIGURE 7
Unsupported Span of Longitudinal

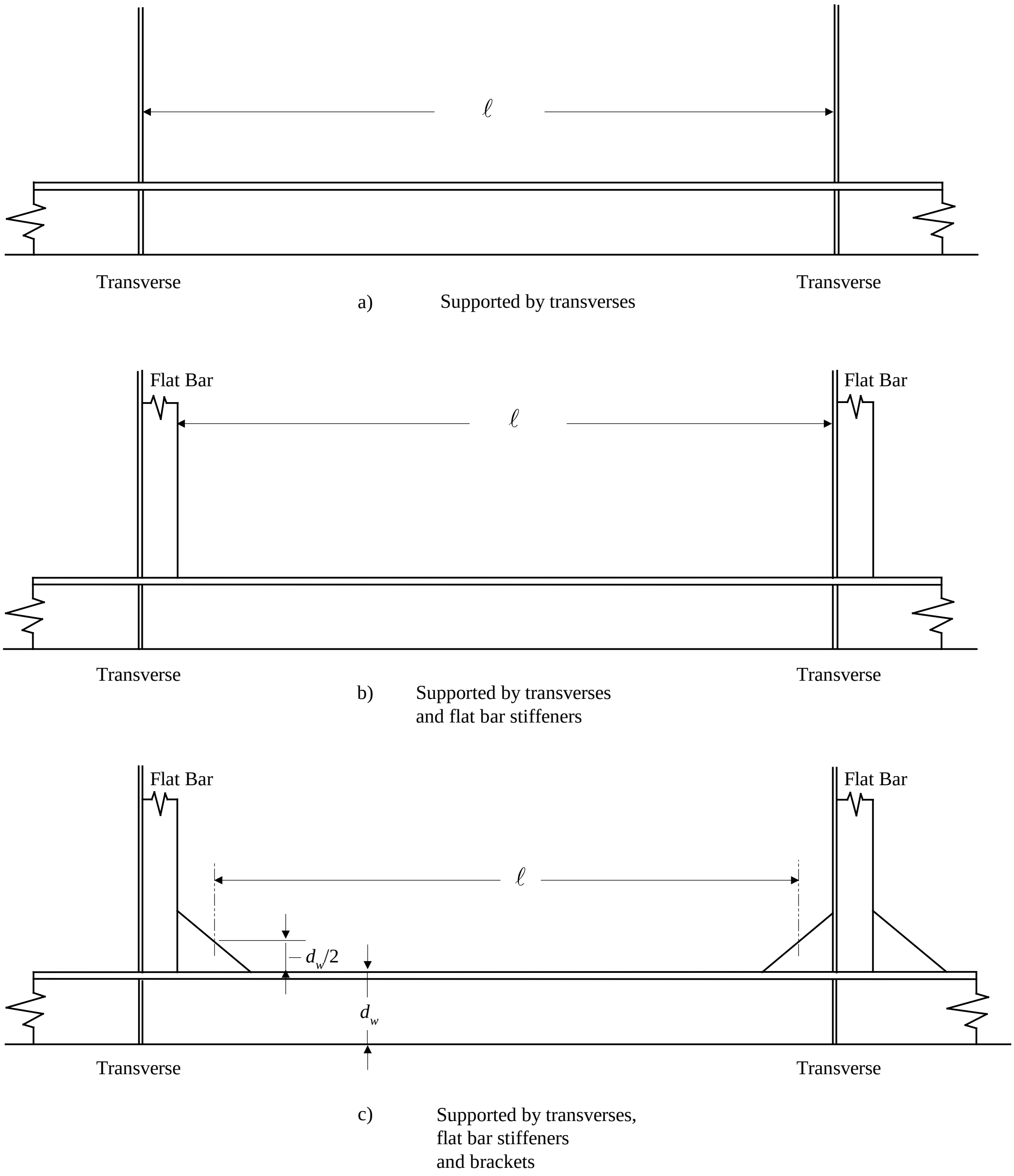
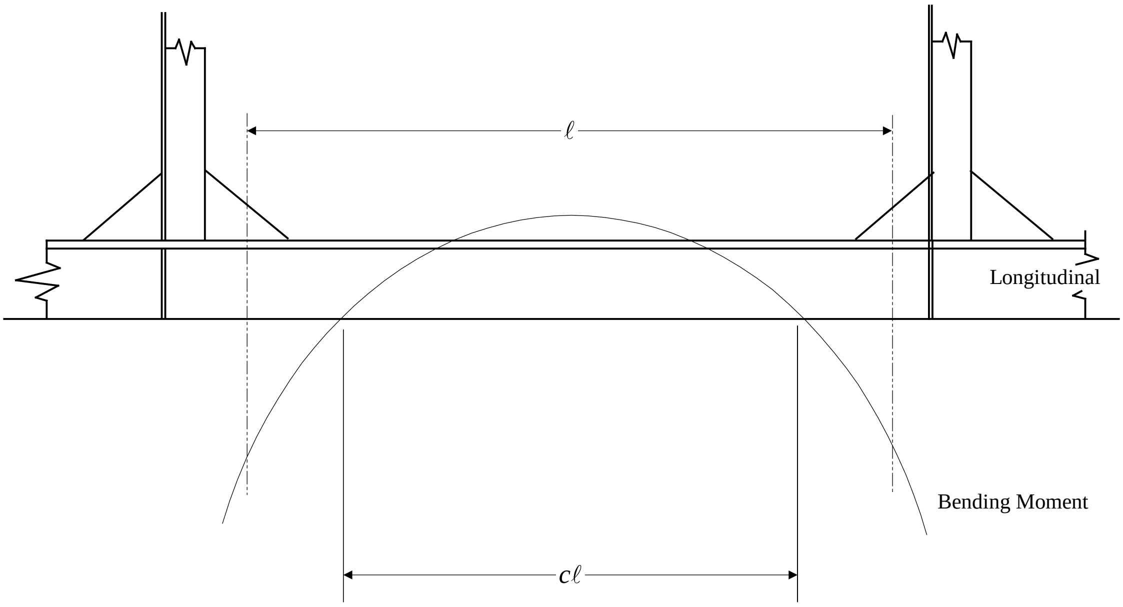


FIGURE 8
Effective Breadth of Plating s_w



$c\ell/s$	1.5	2	2.5	3	3.5	4	4.5 and greater
s_w/s	0.58	0.73	0.83	0.90	0.95	0.98	1.0

5.3 Flexural-Torsional Buckling State Limit

In general, the flexural-torsional buckling state limit of stiffeners or longitudinals is to satisfy the ultimate state limit given below:

$$\frac{\sigma_a}{\sigma_{CT}/\gamma_R} \le 1$$

where

σ_a = axial compressive stress of stiffener and its associated plating due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CT} = critical torsional/flexural buckling stress with respect to axial compression of a stiffener, including its associated plating, which may be obtained from the following equations:

$$= \begin{cases} \sigma_{ET} & \text{if} \quad \sigma_{ET} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{ET}}\right] & \text{if} \quad \sigma_{ET} > P_r\sigma_0 \end{cases}$$

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{ET} = elastic flexural-torsional-buckling stress with respect to the axial compression of a stiffener, including its associated plating, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{\dfrac{K}{2.6} + \left(\dfrac{n\pi}{\ell}\right)^2 \Gamma + \dfrac{C_0}{E}\left(\dfrac{\ell}{n\pi}\right)^2}{I_0 + \dfrac{C_0}{\sigma_{cL}}\left(\dfrac{\ell}{n\pi}\right)^2} E$$

K	=	St. Venant torsion constant for the stiffener cross section, excluding the associated plating, cm^4 (in^4)
	=	$\dfrac{b_f t_f^3 + d_w t_w^3}{3}$
I_0	=	polar moment of inertia of the stiffener, excluding the associated plating (considered at the intersection of the web and plate), cm^4 (in^4)
	=	$I_y + mI_z + A_s(y_0^2 + z_0^2)$
I_y, I_z	=	moment of inertia of the stiffener about the *y*- and *z*-axis, respectively, through the centroid of the longitudinal, excluding the plating (*x*-axis perpendicular to the *y*-*z* plane shown in Section 3, Figure 2), cm^4 (in^4)
m	=	$1.0 - u\left(0.7 - 0.1\dfrac{d_w}{b_f}\right)$
u	=	$1 - 2\dfrac{b_1}{b_f}$, unsymmetrical factor
y_0	=	horizontal distance between centroid of stiffener, A_s, and web plate centerline (see Section 3, Figure 2), cm (in.)
z_0	=	vertical distance between centroid of stiffener, A_s, and its toe (see Section 3, Figure 2), cm (in.)
d_w	=	depth of the web, cm (in.)
t_w	=	thickness of the web, cm (in.)
b_f	=	total width of the flange/face plate, cm (in.)
b_1	=	smaller outstand dimension of flange/face plate with respect to web's centerline, cm (in.)
t_f	=	thickness of the flange/face, cm (in.)
C_0	=	$\dfrac{Et^3}{3s}$
Γ	$\cong$	warping constant, cm^6 (in^6)
	$\cong$	$mI_{zf} d_w^2 + \dfrac{d_w^3 t_w^3}{36}$
I_{zf}	=	$\dfrac{t_f b_f^3}{12}\left(1.0 + 3.0\dfrac{u^2 d_w t_w}{A_s}\right)$, cm^4 (in^4)
σ_{cL}	=	critical buckling stress for associated plating corresponding to *n*-half waves, N/cm^2 (kgf/cm^2, lbf/in^2)
	=	$\dfrac{\pi^2 E\left(\dfrac{n}{\alpha} + \dfrac{\alpha}{n}\right)^2\left(\dfrac{t}{s}\right)^2}{12(1-\nu^2)}$
α	=	$\dfrac{\ell}{s}$
n	=	number of half-waves that yield the smallest σ_{ET}

E	=	modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel
ν	=	Poisson's ratio, 0.3 for steel
σ_0	=	specified minimum yield point of the material, N/cm^2 (kgf/cm^2, lbf/in^2)
s	=	spacing of longitudinal/stiffeners, cm (in.)
A_s	=	sectional area of the longitudinal or stiffener, excluding the associated plating, cm^2 (in^2)
t	=	thickness of the plating, cm (in.)
ℓ	=	unsupported span of the longitudinal or stiffener, cm (in.)
γ_R	=	resistance factor as defined in Subsection 1/11

5.5 Local Buckling of Web, Flange and Face Plate

The local buckling of stiffeners is to be assessed if the proportions of stiffeners specified in Subsection 3/9 are not satisfied.

5.5.1 Web

Critical buckling stress can be obtained from 3/3.1 by replacing s with the web depth and ℓ with the unsupported span, and taking:

$$k_s = 4C_s$$

where

C_s	=	1.0	for angle or tee bar
	=	0.33	for bulb plates
	=	0.11	for flat bar

5.5.2 Flange and Face Plate

Critical buckling stress can be obtained from 3/3.1 by replacing s with the larger outstanding dimension of flange, b_2 (see Section 3, Figure 2), and ℓ with the unsupported span, and taking:

$$k_s = 0.44$$

5.7 Overall Buckling State Limit

The overall buckling strength of the entire stiffened panels is to satisfy the following equation with respect to the biaxial compression:

$$\left(\frac{\sigma_x}{\sigma_{Gx}/\gamma_R}\right)^2 + \left(\frac{\sigma_y}{\sigma_{Gy}/\gamma_R}\right)^2 \le 1$$

where

σ_x	=	calculated average compressive stress in the longitudinal direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)
σ_y	=	calculated average compressive stress in the transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)
σ_{Gx}	=	critical buckling stress for uniaxial compression in the longitudinal direction, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \begin{cases} \sigma_{Ex} & \text{if} \quad \sigma_{Ex} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{Ex}}\right] & \text{if} \quad \sigma_{Ex} > P_r\sigma_0 \end{cases}$$

σ_{Gy} = critical buckling stress for uniaxial compression in the transverse direction, N/cm² (kgf/cm², lbf/in²)

$$= \begin{cases} \sigma_{Ey} & \text{if} \quad \sigma_{Ey} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{Ey}}\right] & \text{if} \quad \sigma_{Ey} > P_r\sigma_0 \end{cases}$$

σ_{Ex} = elastic buckling stress in the longitudinal direction, N/cm² (kgf/cm², lbf/in²)

= $k_x\pi^2(D_xD_y)^{1/2}/(t_xb^2)$

σ_{Ey} = elastic buckling stress in the transverse direction, N/cm² (kgf/cm², lbf/in²)

= $k_y\pi^2(D_xD_y)^{1/2}/(t_y\ell^2)$

k_x = 4 for $\ell/b \ge 1$

= $\dfrac{1}{\eta_x^2} + 2\rho + \eta_x^2$ for $\ell/b < 1$

k_y = 4 for $b/\ell \ge 1$

= $\dfrac{1}{\eta_y^2} + 2\rho + \eta_y^2$ for $b/\ell < 1$

η_x = $(\ell/b)(D_y/D_x)^{1/4}$

η_y = $(b/\ell)(D_x/D_y)^{1/4}$

D_x = $EI_x/s_x(1 - \nu^2)$

D_y = $EI_y/s_y(1 - \nu^2)$

= $Et^3/12(1 - \nu^2)$ if no stiffener in the transverse direction

ρ = $[(I_{px}I_{py})/(I_xI_y)]^{1/2}$

t = thickness of the plate, cm (in.)

ℓ, b = length and width of stiffened panel, respectively, cm (in.)

t_x, t_y = equivalent thickness of the plate and stiffener in the longitudinal and transverse direction, respectively, cm (in.)

= $(s_xt+A_{sx})/s_x$ or $(s_yt+A_{sy})/s_y$

s_x, s_y = spacing of stiffeners and girders, respectively, cm (in.)

A_{sx}, A_{sy} = sectional area of stiffeners and girders, excluding the associated plate, respectively, cm (in.)

I_{px}, I_{py} = moment of inertia of the effective plate alone about the neutral axis of the combined cross section, including stiffener and plate, cm⁴ (in⁴)

I_x, I_y = moment of inertia of the stiffener with effective plate in the longitudinal or transverse direction, respectively, cm⁴ (in⁴). If no stiffener, the moment of inertia is calculated for the plate only.

E = modulus of elasticity, 2.06×10^7 N/cm² (2.1×10^6 kgf/cm², 30×10^6 lbf/in²) for steel

ν = Poisson's ratio, 0.3 for steel

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_0 = specified minimum yield point of the material, N/cm² (kgf/cm², lbf/in²)

γ_R = resistance factor as defined in Subsection 1/11

7 Girders and Webs

In general, the stiffness of web stiffeners fitted to the depth of web plating is to be in compliance with 3/9.3. Web stiffeners that are oriented parallel to the face plate, and thus subject to axial compression, are to also satisfy 3/3.1, considering the combined effects of the compressive and bending stresses in the web. In this case, the unsupported span of these parallel stiffeners may be taken as the distance between tripping brackets, as applicable.

The buckling strength of the web plate between stiffeners and flange/face plate is to satisfy the limits specified in 3/3.1 through 3/3.5. When cutouts are present in the web plate, the effects of the cutouts on the reduction of the critical buckling stresses should be considered (See 3/7.9).

In general, girders are to be designed as stocky so that lateral buckling may be disregarded and torsional buckling also may be disregarded if tripping brackets are provided (See 3/7.7). The lateral buckling check of girders is necessary if:

$$\lambda \geq 0.50$$

where

λ = slenderness ratio of girder

= $\sqrt{\sigma_0 / \sigma_{E(C)}}$

$\sigma_{E(C)}$ = Euler's buckling stress

= $\dfrac{\pi^2 E R_e{}^2}{L^2}$

R_e = radius of gyration of area, A_e, cm (in.)

= $\sqrt{\dfrac{I_e}{A_e}}$

I_e = moment of inertia of girder, accounting for the effective width, ℓ_e, cm^4 (in^4)

A_e = effective sectional area, cm^2 (in^2)

= $A_G + \ell_e t$

A_G = sectional area of the girder, excluding the associated plating, cm^2 (in^2)

t = plate thickness, cm (in.)

L = unsupported span of the girder, cm (in.)

ℓ_e = effective width, cm (in.)

= $\min(1/3L,\ G_x G_y G_{xy} \ell)$

G_y = $\dfrac{s}{\ell}C + 0.1(1-\dfrac{s}{\ell})(1+\dfrac{1}{\beta^2})^2$

C = $2/\beta - 1/\beta^2$ for $\beta > 1$

= 1.0 for $\beta \leq 1$

G_x = $0.5\varphi\left(\dfrac{\sigma_{x\max}}{\sigma_{Ux}}\right) + \sqrt{1-\left(1-0.25\varphi^2\right)\left(\dfrac{\sigma_{x\max}/\chi}{\sigma_{Ux}}\right)^2}$

$\sigma_{x\max}$ = maximum compressive stress in the longitudinal direction on small plating due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Ux} = $\sigma_0 C$

χ = load adjustment factor

= 1.45 for the *Static Loading Condition* (see Subsection 1/9)

= 1.00 for the *Redundancy Condition* or *Damaged Condition* (see Subsection 1/9)

= 1.25 for the other loading conditions (see Subsection 1/9)

G_{xy} = $\sqrt{1-\left(\frac{\tau/\chi}{\tau_0}\right)^2}$

φ = $1.0 - \beta/2$

β = $\frac{s}{t}\sqrt{\frac{\sigma_0}{E}}$

s = spacing of the longitudinal, cm (in.)

ℓ = unsupported span of the longitudinal or stiffener, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

τ = edge shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_0 = shear strength of plating, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_0 = specified minimum yield point. N/cm^2 (kgf/cm^2, lbf/in^2)

The lateral buckling state limit of girders may be determined as follows:

$$\frac{\sigma_a}{\left(\frac{\sigma_{CA}}{\gamma_R}\right)\cdot\left(\frac{A_e}{A}\right)} + \frac{C_m \sigma_b}{\left(\frac{\sigma_0}{\gamma_R}\right)\left[1-\frac{\sigma_a}{\sigma_{E(C)}/\gamma_R}\right]} \le 1$$

where

σ_a = calculated compressive stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $-P/A$

P = total compressive load on girder due to factored loads using full width of associated plating, N (kgf, lbf)

σ_{CA} = critical buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\sigma_{E(C)}$ for $\sigma_{E(C)} \le P_r\sigma_0$

= $\sigma_0\left[1-P_r(1-P_r)\frac{\sigma_0}{\sigma_{E(C)}}\right]$ for $\sigma_{E(C)} > P_r\sigma_0$

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

A = total sectional area of girder, cm^2 (in^2)

= $A_G + \ell t$

σ_b = bending stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= M/SM_w

M = maximum bending moment induced by lateral loads, N-cm (kgf-cm, lbf-in)

= $q\ell L^2/12$

C_m = moment adjustment coefficient, which may be taken as 0.75

q = factored lateral pressure for the region considered, N/cm^2 (kgf/cm^2, lbf/in^2)

SM_w = effective section modulus of the girder at flange, accounting for the effective breadth, ℓ_g, cm^3 (in^3)

ℓ_g = min(1/3L, ℓ_w), cm (in.)

ℓ_w = effective breadth**,** cm (in.) (the calculation of ℓ_w is the same as s_w specified in Section 3, Figure 8. Replace s with ℓ and ℓ with L)

γ_R = resistance factor as defined in Subsection 1/11

7.1 Web Plate

The buckling limit state for a web plate is considered as the ultimate state limit and is given in 3/3.1.

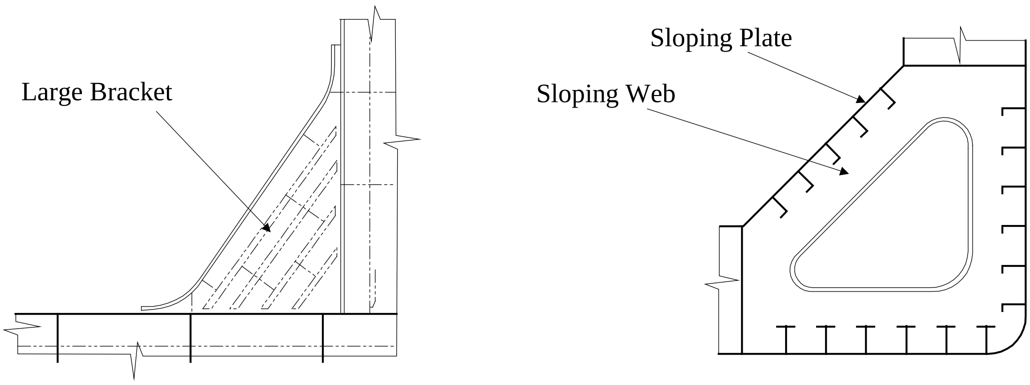
7.3 Face Plate and Flange

The breadth to thickness ratio of faceplate and flange is to satisfy the limits given in 3/9.7.

7.5 Large Brackets and Sloping Webs

The buckling strength is to satisfy the limits specified in 3/3.1 for the web plate.

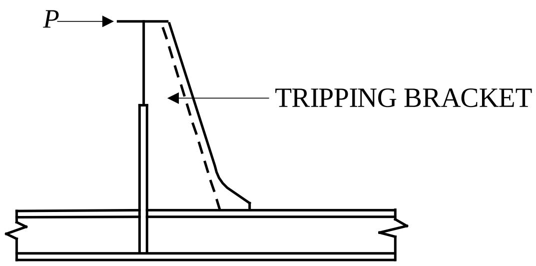
FIGURE 9
Large Brackets and Sloping Webs



7.7 Tripping Brackets

To prevent tripping of deep girders and webs with wide flanges, tripping brackets are to be installed with spacing generally not greater than 3 meters (9.84 feet).

FIGURE 10
Tripping Brackets



The design of tripping brackets may be based on the force, P, acting on the flange, as given by the following equation:

$$P = 0.02\sigma_{c\ell}(b_f t_f + \frac{1}{3} d_w t_w)$$

where

$\sigma_{c\ell}$ = critical lateral buckling stress with respect to axial compression between tripping brackets, N/cm^2 (kgf/cm^2, lbf/in^2)

= σ_{ce} for $\sigma_{ce} \le P_r\sigma_0$

= $\sigma_0 [1 - P_r(1 - P_r)\ \sigma_0/\sigma_{ce}]$ for $\sigma_{ce} > P_r\sigma_0$

σ_{ce} = $0.6E[(b_f/t_f)(t_w/d_w)^3]$, N/cm^2 (kgf/cm^2, lbf/in^2)

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_0 = specified minimum yield point of the material, N/cm^2 (kgf/cm^2, lbf/in^2)

b_f, t_f, d_w, t_w are defined in Section 3, Figure 2.

7.9 Effects of Cutouts

The depth of a cutout, in general, is to be not greater than $d_w/3$, and the calculated stresses in the area are to account for the local increase due to the cutout.

7.9.1 Reinforced by Stiffeners around Boundaries of Cut-outs

When reinforcement is made by installing straight stiffeners along boundaries of a cutout, the critical buckling stresses of the web plate between stiffeners with respect to compression, in-plane bending and shear may be obtained from 3/3.1.

7.9.2 Reinforced by Face Plates around Contour of Cut-outs

When reinforcement is made by adding face plates along the contour of a cut-out, the critical buckling stresses with respect to compression, bending and shear may be obtained from 3/3.1, without reduction, provided that the cross sectional area of the face plate is not less than $8t_w^2$, where t_w is the thickness of the web plate, and the depth of the cut-out is not greater than $d_w/3$, where d_w is the depth of the web.

7.9.3 No Reinforcement Provided

When reinforcement is not provided, the buckling strength of the web plate surrounding the cutout may be treated as a strip of plate with one edge free and the other edge simply supported.

$$k_s = 0.44$$

9 Stiffness and Proportions

To fully develop the intended buckling strength of assemblies of structural members and panels, supporting elements of plate panels and stiffeners are to satisfy the following requirements for stiffness and proportion in highly stressed regions.

9.1 Stiffness of Stiffeners

In the plane perpendicular to the plating, the moment of inertia of a stiffener, i_0, with an effective breadth of plating, is not to be less than that given by the following equation:

$$i_0 = \frac{st^3}{12(1-\nu^2)}\gamma_0$$

where

γ_0	=	$(2.6 + 4.0\delta)\alpha^2 + 12.4\alpha - 13.2\alpha^{1/2}$
δ	=	$A_s/(st)$
α	=	ℓ/s
s	=	spacing of longitudinal, cm (in.)
t	=	thickness of plating supported by the longitudinal, cm (in.)
ν	=	Poisson's ratio, 0.3 for steel
A_s	=	cross sectional area of the stiffener (excluding plating), cm^2 (in^2)
ℓ	=	unsupported span of the stiffener, cm (in.)

9.3 Stiffness of Web Stiffeners

The moment of inertia, I_e, of a web stiffener, with the effective breadth of plating not exceeding s or 0.33ℓ, whichever is less, is not to be less than the value obtained from the following equations:

$$I_e = 0.17\ell t^3(\ell/s)^3 \qquad \text{for } \ell/s \le 2.0$$

$$I_e = 0.34\ell t^3(\ell/s)^2 \qquad \text{for } \ell/s > 2.0$$

where

ℓ	=	length of stiffener between effective supports, cm (in.)
t	=	required thickness of web plating, cm (in.)
s	=	spacing of stiffeners, cm (in.)

9.5 Stiffness of Supporting Girders

The moment of inertia of a supporting member is not to be less than that obtained from the following equation:

$$I_G/i_0 \ge 0.2(B/\ell)^3(B/s)$$

where

I_G	=	moment of inertia of the supporting girders, including the effective plating, cm^4 (in^4)
i_0	=	moment of inertia of the stiffeners, including the effective plating, cm^4 (in^4)
B	=	unsupported span of the supporting girders, cm (in.)
ℓ	=	unsupported span of the stiffener, cm (in.), as defined in Section 3, Figure 7

9.7 Proportions of Flanges and Faceplates

The breadth-to-thickness ratio of flanges and faceplates of stiffeners and girders is to satisfy the limits given below.

$$b_2/t_f \le 0.4(E/\sigma_0)^{1/2}$$

where

b_2	=	larger outstand dimension of flange (See Section 3, Figure 2), cm (in.)
t_f	=	thickness of flange/face plate, cm (in.)
σ_0	=	specified minimum yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)
E	=	modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

9.9 Proportions of Webs of Stiffeners

The depth to thickness ratio of webs of stiffeners is to satisfy the limits given below.

$d_w/t_w \le 1.5(E/\sigma_0)^{1/2}$ for angles and tee bars

$d_w/t_w \le 0.85(E/\sigma_0)^{1/2}$ for bulb plates

$d_w/t_w \le 0.4(E/\sigma_0)^{1/2}$ for flat bars

where

σ_0 = specified minimum yield point of plate, N/cm^2 (kgf/cm^2, lbf/in^2)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

d_w and t_w are as defined in Section 3, Figure 2.

11 Corrugated Panels

This Subsection includes criteria for the buckling and ultimate strength for corrugated panels.

11.1 Local Plate Panels

The buckling strength of the flange and web plate panels is to satisfy the following state limit:

$$\left(\frac{\sigma_{x\max}}{\sigma_{Cx}/\gamma_R}\right)^2 + \left(\frac{\sigma_{y\max}}{\sigma_{Cy}/\gamma_R}\right)^2 + \left(\frac{\tau}{\tau_C/\gamma_R}\right)^2 \le 1$$

where

$\sigma_{x\max}$ = maximum compressive stress in corrugation direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{y\max}$ = maximum compressive stress in transverse direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

τ = in-plane shear stress due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Cx} = critical buckling stress in corrugation direction from 3/3.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Cy} = critical buckling stress in transverse direction from 3/3.1, N/cm^2 (kgf/cm^2, lbf/in^2)

τ_C = critical buckling stress for edge shear from 3/3.1, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

11.3 Unit Corrugation

Any unit corrugation of the corrugated panel may be treated as a beam column and is to satisfy the following state limit:

$$\frac{\sigma_a}{\left(\dfrac{\sigma_{CA}}{\gamma_R}\right)} + \frac{C_m \sigma_b}{\left(\dfrac{\sigma_{CB}}{\gamma_R}\right)\left[1 - \dfrac{\sigma_a}{\sigma_{E(C)}/\gamma_R}\right]} \le 1$$

where

σ_a = maximum compressive stress in the corrugation direction due to factored loads, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_b = maximum bending stress along the length due to lateral pressure, N/cm^2 (kgf/cm^2, lbf/in^2)

= M_b/SM

M_b = maximum bending moment induced by the factored lateral pressure, N-cm (kgf-cm, lbf-in)

= $\left(\dfrac{q_u + q_\ell}{2}\right) sL^2/12$

SM = $I_y/(d - z_0)$ for $M_b \geq 0$

= $I_y/(z_0 + t)$ for $M_b < 0$

σ_{CA} = critical buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\sigma_{E(C)}$ for $\sigma_{E(C)} \leq P_r\sigma_0$

= $\sigma_o\left[1 - P_r(1 - P_r)\dfrac{\sigma_0}{\sigma_{E(C)}}\right]$ for $\sigma_{E(C)} > P_r\sigma_0$

$\sigma_{E(C)}$ = elastic buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\dfrac{\pi^2 E r^2}{L^2}$

r = radius of gyration of area A, cm (in.)

= $\sqrt{\dfrac{I_y}{A}}$

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_{CB} = critical bending buckling stress

= $\sigma_{E(B)}$ for $\sigma_{E(B)} \leq P_r\sigma_0$

= $\sigma_0\left[1 - P_r(1 - P_r)\dfrac{\sigma_0}{\sigma_{E(B)}}\right]$ for $\sigma_{E(B)} > P_r\sigma_0$

$\sigma_{E(B)}$ = elastic buckling stress of unit corrugation

= $k_c \dfrac{E}{12(1-\nu^2)}\left(\dfrac{t}{a}\right)^2$

k_c = coefficient

= $[7.65 - 0.26(c/a)^2]^2$

C_m = bending moment factor determined by rational analysis, which may be taken as 1.5 for a panel whose ends are simply supported

A, I_y = area and moment of inertia of unit corrugation, as specified in 3/13.3

SM = sectional modulus of unit corrugation, as specified in 3/13.3, cm^3 (in^3)

s = width of unit corrugation, as defined in Section 3, Figure 4 and specified in 3/13.3

a, c = width of the compressed flange and web plating, respectively, as defined in Section 3, Figure 4

t = thickness of the unit corrugation, cm (in.)

L = length of corrugated panel, cm (in.)

q_u, q_ℓ = factored lateral pressure at the upper and lower ends of the corrugation, N/cm² (kgf/cm², lbf/in²) ($q_u \geq 0$, $q_\ell \leq 0$)

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

E = modulus of elasticity, 2.06×10^7 N/cm² (2.1×10^6 kgf/cm², 30×10^6 lbf/in²) for steel

ν = Poisson's ratio, 0.3 for steel

σ_0 = specified minimum yield point, N/cm² (kgf/cm², lbf/in²)

γ_R = resistance factor as defined in Subsection 1/11

11.5 Overall Buckling

The overall buckling strength of the entire corrugated panels is to satisfy the following equation with respect to the biaxial compression and edge shear:

$$\left(\frac{\sigma_x}{\sigma_{Gx}/\gamma_R}\right)^2 + \left(\frac{\sigma_y}{\sigma_{Gy}/\gamma_R}\right)^2 + \left(\frac{\tau}{\tau_G/\gamma_R}\right)^2 \leq 1$$

where

σ_x = calculated average compressive stress in the corrugation direction due to factored loads, N/cm² (kgf/cm², lbf/in²)

σ_y = calculated average compressive stress in the transverse direction due to factored loads, N/cm² (kgf/cm², lbf/in²)

τ = in-plane shear stress due to factored loads, N/cm² (kgf/cm², lbf/in²)

σ_{Gx} = critical buckling stress for uniaxial compression in the corrugation direction, N/cm² (kgf/cm², lbf/in²)

$$= \begin{cases} \sigma_{Ex} & \text{if} \quad \sigma_{Ex} \leq P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1 - P_r)\dfrac{\sigma_0}{\sigma_{Ex}}\right] & \text{if} \quad \sigma_{Ex} > P_r\sigma_0 \end{cases}$$

σ_{Gy} = critical buckling stress for uniaxial compression in the transverse direction, N/cm² (kgf/cm², lbf/in²)

$$= \begin{cases} \sigma_{Ey} & \text{if} \quad \sigma_{Ey} \leq P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1 - P_r)\dfrac{\sigma_0}{\sigma_{Ey}}\right] & \text{if} \quad \sigma_{Ey} > P_r\sigma_0 \end{cases}$$

τ_G = critical buckling stress for shear stress, N/cm² (kgf/cm², lbf/in²)

$$= \begin{cases} \tau_E & \text{if} \quad \tau_E \leq P_r\tau_0 \\ \tau_0\left[1 - P_r(1 - P_r)\dfrac{\tau_0}{\tau_E}\right] & \text{if} \quad \tau_E > P_r\tau_0 \end{cases}$$

σ_{Ex} = elastic buckling stress in the corrugation direction, N/cm² (kgf/cm², lbf/in²)

= $k_x\pi^2(D_xD_y)^{1/2}/(t_xB^2)$

σ_{Ey} = elastic buckling stress in the transverse direction, N/cm² (kgf/cm², lbf/in²)

= $k_y\pi^2(D_xD_y)^{1/2}/(tL^2)$

τ_E = elastic shear buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $k_S\pi^2 D_x^{3/4} D_y^{1/4}/(tL^2)$

k_x = 4 for $L/B \geq 0.5176(D_x/D_y)^{1/4}$

= $\dfrac{1}{\eta_x^2} + \eta_x^2$ for $L/B < 0.5176(D_x/D_y)^{1/4}$

k_y = 4 for $B/L \geq 0.5176(D_y/D_x)^{1/4}$

= $\dfrac{1}{\eta_y^2} + \eta_y^2$ for $B/L < 0.5176(D_y/D_x)^{1/4}$

k_S = 3.65

L, B = length and width of corrugated panel

t_x = equivalent thickness of the corrugation in the corrugation direction, as specified in 3/13.3, cm (in.)

t = thickness of the corrugation, cm (in.)

η_x = $(L/B)(D_y/D_x)^{1/4}$

η_y = $(B/L)(D_x/D_y)^{1/4}$

D_x = EI_y/s

D_y = $\dfrac{Et^3}{12(1-\nu^2)}\dfrac{s}{a+b+2c}$

I_y = moment of inertia of a corrugation with spacing s

a, b, c = width of the flanges and web plating, respectively, as defined in Section 3, Figure 4, cm (in.)

s = width of the unit corrugation, as defined in Section 3, Figure 4, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν = Poisson's ratio, 0.3 for steel

σ_0 = specified minimum yield point of the material, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

13 Geometric Properties

This Subsection includes the formulations for the geometric properties of stiffened panels and corrugated panels. The effective width, s_e, and effective breadth, s_w, can be obtained from 3/5.1 and Section 3, Figure 8, respectively.

13.1 Stiffened Panels

13.1.1 Beam-Column Buckling

b_f = 0 for flat-bar

t_f = 0 for flat-bar

b_1 = $0.5\, t_w$ for angle bar

$$A_s = d_w t_w + b_f t_f$$

$$A_e = s_e t + A_s$$

$$z_{ep} = [0.5(t + d_w)d_w t_w + (0.5t + d_w + 0.5t_f)b_f t_f]/A_e$$

$$I_e = \frac{t_p^3 s_e}{12} + \frac{d_w^3 t_w}{12} + \frac{t_f^3 b_f}{12} + 0.25(t + d_w)^2 d_w t_w + b_f t_f(0.5t + d_w + 0.5t_f)^2 - A_e z_{ep}^2$$

$$r_e = \sqrt{I_e / A_e}$$

$$A_w = s_w t + A_s$$

$$z_{wp} = [0.5(t + d_w)d_w t_w + (0.5t + d_w + 0.5t_f)b_f t_f]/A_w$$

$$I_w = \frac{t_p^3 s_e}{12} + \frac{d_w^3 t_w}{12} + \frac{t_f^3 b_f}{12} + 0.25(t + d_w)^2 d_w t_w + b_f t_f(0.5t + d_w + t_f)^2 - A_w z_{wp}^2$$

$$SM_w = \frac{I_w}{(0.5t + d_w + t_f) - z_{wp}}$$

t, b_f, b_1, t_f, d_w, t_w are defined in Section 3, Figure 2.

13.1.2 Torsional/Flexural Buckling

$$A_s = d_w t_w + b_f t_f$$

$$y_0 = (b_1 - 0.5b_f)b_f t_f/A_s$$

$$z_0 = [0.5d_w^2 t_w + (d_w + 0.5t_f)b_f t_f]/A_s$$

$$I_y = \frac{d_w^3 t_w}{12} + \frac{t_f^3 b_f}{12} + 0.25 d_w^3 t_w + b_f t_f \left(d_w + 0.5t_f\right)^2 - A_s z_0^2$$

$$I_z = \frac{t_w^3 d_w}{12} + \frac{b_f^3 t_f}{12} + b_f t_f \left(b_1 - 0.5b_f\right)^2 - A_s z_0^2$$

b_f, b_1, t_f, d_w, t_w, y_0 and z_0 are defined in Section 3, Figure 2.

13.3 Corrugated Panels

The following formulations of geometrical properties are derived, provided that the section is thin-walled and the thickness is small.

$$s = a + b + 2c\cos\phi$$

$$t_x = (st + A_{sx})/s$$

$$A = (a + b)t + 2ct$$

$$A_{sx} = 2ct\sin\phi$$

$$z_o = dt(a + c)/A$$

$$I_y = \frac{(a + b)t^3}{12} + ad^2 t + \frac{2}{3}cd^2 t - Az_0^2$$

a, b, c, d, t, ϕ and z_0 are defined in Section 3, Figure 4.

SECTION 4 Cylindrical Shells

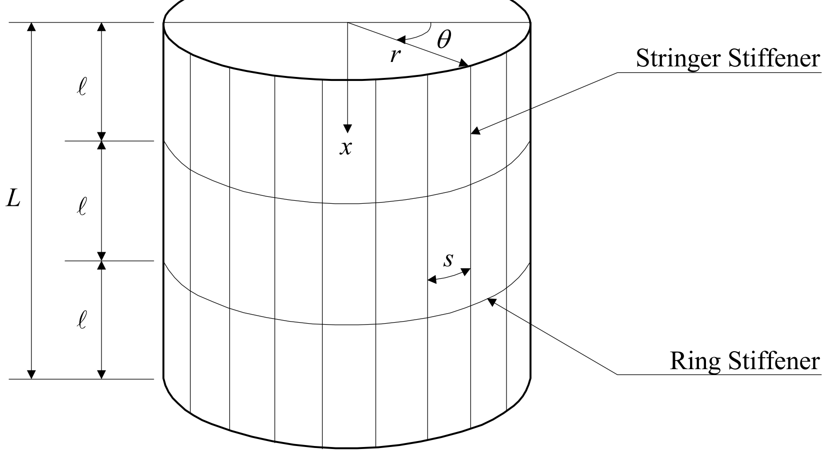
1 General

This Section presents criteria for calculating the strength of ring- and/or stringer-stiffened cylindrical shells subjected to axial loading, bending moment, radial pressure or a combination of these loads when performing the Ultimate Limit State strength checks of cylindrical shells. The buckling limit state of a stiffened cylindrical shell is to be determined based on the formulations provided below. Alternatively, either well-documented experimental data or a verified analytical approach may be employed.

1.1 Geometry of Cylindrical Shells

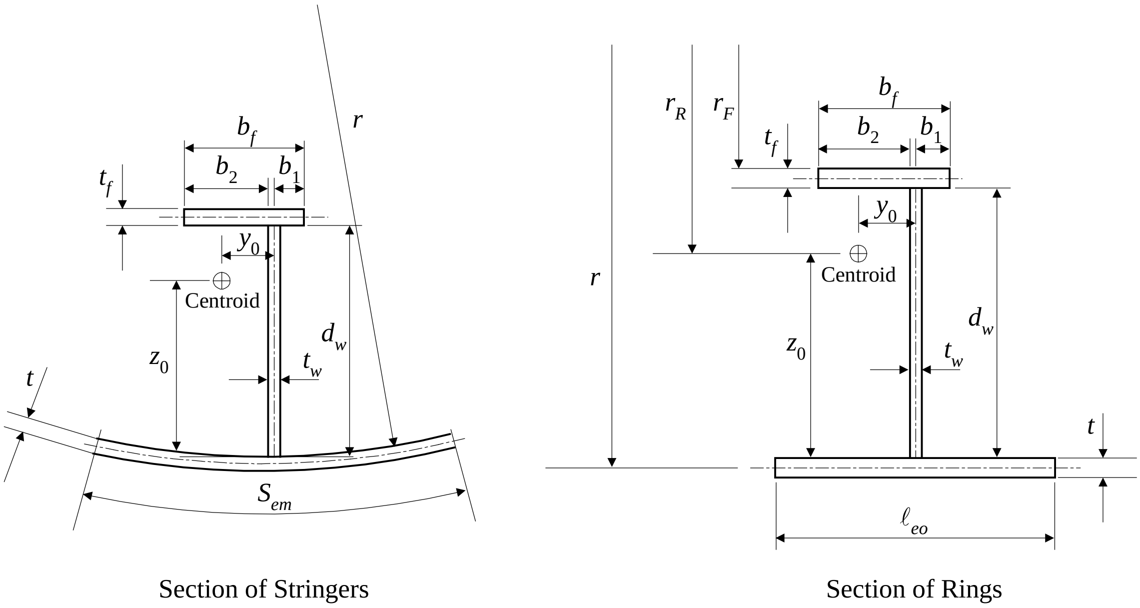
The criteria given below apply to ring- and/or stringer-stiffened cylindrical shells, as depicted in Section 4, Figure 1, where coordinates (x, r, θ) denote the longitudinal, radial and circumferential directions, respectively. Stiffeners in a given direction are to be equally spaced, parallel and perpendiculars to panel edges, and have identical material and geometric properties. General types of stiffener profiles, such as flat bar, T-bar, angle and bulb plate, may be used. The dimensions and properties of a ring or stringer stiffener are described in Section 4, Figure 2. The material properties of the stiffeners may be different from those of the shell plating.

FIGURE 1
Ring and Stringer-stiffened Cylindrical Shell



The formulations given for ring- and/or stringer-stiffened shells are applicable for offshore structures with the diameter to thickness ratio in the range of $E/(4.5\sigma_0)$ to 1000.

FIGURE 2
Dimensions of Stiffeners



1.3 Load Application

This Section includes the buckling state limit criteria for the following loads and load effects.

- Uniform compression in the longitudinal direction, σ_a *
- Bending of the overall cylinder, σ_b
- External pressure, p
- Combinations of the above

* *Note:* If uniform stress, σ_a, is tensile rather than compressive, it may be set equal to zero.

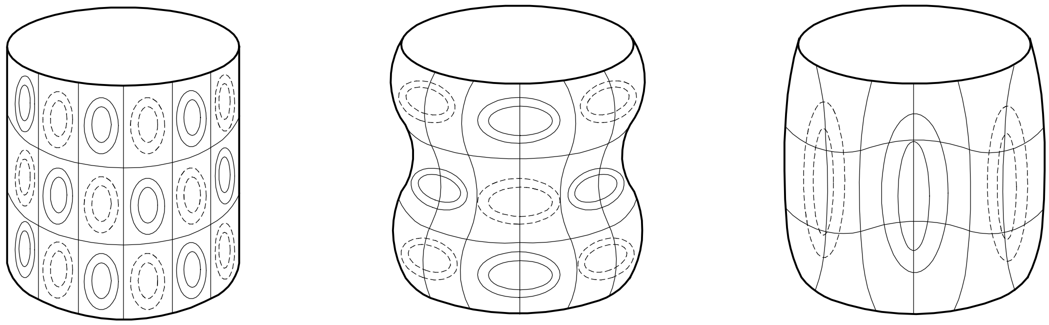
1.5 Buckling Control Concepts

The probable buckling modes of ring- and/or stringer-stiffened cylindrical shells can be sorted as follows:

- Local shell or curved panel buckling (i.e., buckling of the shell between adjacent stiffeners). The stringers remain straight and the ring stiffeners remain round.
- Bay buckling (i.e., buckling of the shell plating together with the stringers, if present, between adjacent ring stiffeners). The ring stiffeners and the ends of the cylindrical shells remain round.
- General buckling (i.e., buckling of one or more ring stiffeners together with the attached shell plus stringers, if present).
- Local stiffener buckling (i.e., torsional/flexural buckling of stiffeners, ring or stringer, or local buckling of the web and flange). The shell remains undeformed.
- Column buckling (i.e., buckling of cylindrical shell as a column).

The first three failure modes for ring and stringer-stiffened cylindrical shells are illustrated in Section 4, Figure 3.

FIGURE 3
Typical Buckling Modes of Ring and Stringer Cylindrical Shells



Local Shell Buckling — Bay Buckling — General Buckling

A stiffened cylindrical shell is to be designed such that a general buckling failure is preceded by bay instability, and local shell buckling precedes bay instability.

The buckling strength criteria presented below are based on the following assumptions and limitations:

- Ring stiffeners with their associated effective shell plating are to have moments of inertia not less than i_r, as given in 4/15.1.
- Stringer stiffeners with their associated effective shell plating are to have moments of inertia not less than i_s, as given in 4/15.3.
- Faceplates and flanges of stiffener are proportioned such that local instability is prevented, as given in 4/15.7.
- Webs of stiffeners are proportioned such that local instability is prevented, as given in 4/15.5.

For stiffened cylindrical shells that do not satisfy these assumptions, a detailed analysis of buckling strength using an acceptable method should be pursued.

1.7 Adjustment Factor

Strength formulations given in subsequent subsections below may entail adjustment factors as follows.

For shell buckling: *

$$\psi = 0.833 \qquad \text{if } \sigma_{Cij} \leq 0.55\sigma_0$$

$$= 0.629 + 0.371\sigma_{Cij}/\sigma_0 \qquad \text{if } \sigma_{Cij} > 0.55\sigma_0$$

where

σ_{Cij} = critical buckling stress of cylindrical shell, representing σ_{CxR}, $\sigma_{C\theta R}$, σ_{CxP}, $\sigma_{C\theta P}$, σ_{CxB} or $\sigma_{C\theta B}$, which are specified in Subsections 4/3, 4/5 and 4/7, respectively, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

* *Note:* The maximum allowable strength factor for shell buckling should be based on the critical buckling stress, which implies that it may be different for axial compression and external pressure in local shell or bay buckling. The smallest maximum allowable strength factor should be used in the corresponding buckling state limit.

For column buckling:

$$\psi = 0.87 \qquad \text{if } \sigma_{E(C)} \leq P_r\sigma_0$$

$$= 1 - 0.13\sqrt{P_r\sigma_0 / \sigma_{E(C)}} \qquad \text{if } \sigma_{E(C)} > P_r\sigma_0$$

where

$\sigma_{E(C)}$ = Euler's buckling stress, as specified in Subsection 4/11, N/cm^2 (kgf/cm^2, lbf/in^2)

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

For tripping of stringer stiffeners:

$\psi = 1.0$

3 Unstiffened or Ring-stiffened Cylinders

3.1 Bay Buckling Limit State

For the buckling limit state of unstiffened or ring-stiffened cylindrical shells between adjacent ring stiffeners subjected to axial compression, bending moment and external pressure, the following strength criterion is to be satisfied:

$$\left(\frac{\sigma_x}{\psi\sigma_{CxR}/\gamma_R}\right)^2 - \varphi_R\left(\frac{\sigma_x}{\psi\sigma_{CxR}/\gamma_R}\right)\left(\frac{\sigma_\theta}{\psi\sigma_{C\theta R}/\gamma_R}\right) + \left(\frac{\sigma_\theta}{\psi\sigma_{C\theta R}/\gamma_R}\right)^2 \le 1$$

where

σ_x = compressive stress in longitudinal direction due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = compressive hoop stress due to factored loads from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CxR} = critical buckling stress for axial compression or bending moment from 4/3.3, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{C\theta R}$ = critical buckling stress for external pressure from 4/3.5, N/cm^2 (kgf/cm^2, lbf/in^2)

φ_R = coefficient to reflect interaction between longitudinal and hoop stresses (negative values are acceptable)

$$= \frac{\sigma_{CxR} + \sigma_{C\theta R}}{\sigma_0} - 1.0$$

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ = maximum allowable strength adjustment factor of shell buckling, as specified in 4/1.7, for ring-stiffened cylindrical shells subjected to axial compression or external pressure, whichever is less.

3.3 Critical Buckling Stress for Axial Compression or Bending Moment

The critical buckling stress of unstiffened or ring-stiffened cylindrical shell subjected to axial compression or bending moment may be taken as:

$$\sigma_{CxR} = \begin{cases} \sigma_{ExR} & \text{for} \quad \sigma_{ExR} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{ExR}}\right] & \text{for} \quad \sigma_{ExR} > P_r\sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{ExR} = elastic compressive buckling stress for an imperfect cylindrical shell, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\rho_{xR} C \sigma_{CExR}$

σ_{CExR} = classical compressive buckling stress for a perfect cylindrical shell, N/cm^2 (kgf/cm^2, lbf/in^2)

= $0.605 \dfrac{Et}{r}$

C = length dependent coefficient

$$= \begin{cases} 1.0 & \text{for} \quad z \geq 2.85 \\ 1.425/z + 0.175z & \text{for} \quad z < 2.85 \end{cases}$$

ρ_{xR} = nominal or lower bound knock-down factor to allow for shape imperfections

$$= \begin{cases} 0.75 + 0.003z\left(1 - \dfrac{r}{300t}\right) & \text{for} \quad z < 1 \\ 0.75 - 0.142(z-1)^{0.4} + 0.003z\left(1 - \dfrac{r}{300t}\right) & \text{for} \quad 1 \leq z < 20 \\ 0.35 - 0.0002\dfrac{r}{t} & \text{for} \quad 20 \leq z \end{cases}$$

z = Batdorf parameter

= $\dfrac{\ell^2}{rt}\sqrt{1-\nu^2}$

ℓ = length between adjacent ring stiffeners (unsupported)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν = Poisson's ratio, 0.3 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

3.5 Critical Buckling Stress for External Pressure

The critical buckling stress for an unstiffened or ring-stiffened cylindrical shell subjected to external pressure may be taken as:

$$\sigma_{C\theta R} = \Phi \sigma_{E\theta R}$$

where

Φ = plasticity reduction factor

= 1 for $\Delta \leq 0.55$

= $\dfrac{0.45}{\Delta} + 0.18$ for $0.55 < \Delta \leq 1.6$

= $\dfrac{1.31}{1+1.15\Delta}$ for $1.6 < \Delta < 6.25$

= $1/\Delta$ for $\Delta \geq 6.25$

Δ = $\sigma_{E\theta R}/\sigma_0$

$\sigma_{E\theta R}$ = elastic hoop buckling stress for an imperfect cylindrical shell, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\rho_{\theta R}\dfrac{q_{CE\theta R}(r+0.5t)}{t}K_\theta$

$\rho_{\theta R}$ = nominal or lower bound knock-down factor to allow for shape imperfections

= 0.8

K_θ = coefficient to account for the effect of ring stiffener, as determined from 4/13.3

$q_{CE\theta R}$ = elastic buckling pressure, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \begin{cases} \dfrac{1.27E}{A_L{}^{1.18}+0.5}\left(\dfrac{t}{r}\right)^2 & \text{for} \quad A_L \le 2.5 \\ \dfrac{0.92E}{A_L}\left(\dfrac{t}{r}\right)^2 & \text{for} \quad 2.5 < A_L \le 0.208\dfrac{r}{t} \\ 0.836C_p{}^{-1.061}E\left(\dfrac{t}{r}\right)^3 & \text{for} \quad 0.208\dfrac{r}{t} < A_L \le 2.85\dfrac{r}{t} \\ 0.275E\left(\dfrac{t}{r}\right)^3 & \text{for} \quad 2.85\dfrac{r}{t} < A_L \end{cases}$$

A_L = $\dfrac{\sqrt{z}}{(1-\nu^2)^{1/4}} - 1.17 + 1.068k$

C_p = $A_L/(r/t)$

k = 0 for lateral pressure

= 0.5 for hydrostatic pressure

z = Batdorf parameter

= $\dfrac{\ell^2}{rt}\sqrt{1-\nu^2}$

ℓ = length between adjacent ring stiffeners (unsupported)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν = Poisson’s ratio, 0.3 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

3.7 General Buckling

The general buckling of a ring-stiffened cylindrical shell involves the collapse of one or more ring stiffeners together with the shell plating and is to be avoided due to its catastrophic consequences. The ring stiffeners are to be proportioned in accordance with Subsection 4/15 to exclude the general buckling failure mode.

5 Curved Panels

Local curved panel buckling of ring and stringer-stiffened cylindrical shells will not necessarily lead to complete failure of the shell, as stresses can be redistributed to the remaining effective section associated with the stringer. However, knowledge of local buckling behavior is necessary in order to control local deflections, in accordance with serviceability requirements, and to determine the effective width to be associated with the stringer when determining buckling strength of the stringer-stiffened shells.

5.1 Buckling State Limit

The buckling state limit of curved panels between adjacent stiffeners can be defined by the following equation:

$$\left(\frac{\sigma_x}{\psi\sigma_{CxP}/\gamma_R}\right)^2 - \varphi_P\left(\frac{\sigma_x}{\psi\sigma_{CxP}/\gamma_R}\right)\left(\frac{\sigma_\theta}{\psi\sigma_{C\theta P}/\gamma_R}\right) + \left(\frac{\sigma_\theta}{\psi\sigma_{C\theta P}/\gamma_R}\right)^2 \le 1$$

where

σ_x = compressive stress in the longitudinal direction due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = compressive hoop stress due to factored loads from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CxP} = critical buckling stress for axial compression or bending moment from 4/5.3, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{C\theta P}$ = critical buckling stress for external pressure from 4/5.5, N/cm^2 (kgf/cm^2, lbf/in^2)

φ_P = coefficient to reflect interaction between longitudinal and hoop stresses (negative values are acceptable),

$$= \frac{0.4(\sigma_{CxP} + \sigma_{C\theta P})}{\sigma_0} - 0.8$$

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ = maximum allowable strength adjustment factor of shell buckling, as specified in 4/1.7 for curved panels in axial compression or external pressure, whichever is the lesser

5.3 Critical Buckling Stress for Axial Compression or Bending Moment

The critical buckling stress for curved panels bounded by adjacent pairs of ring and stringer stiffeners subjected to axial compression or bending moment may be taken as:

$$\sigma_{CxP} = \begin{cases} \sigma_{ExP} & \text{for } \sigma_{ExP} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{ExP}}\right] & \text{for } \sigma_{ExP} > P_r\sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{ExP} = elastic buckling stress for an imperfect curved panel, N/cm^2 (kgf/cm^2, lbf/in^2)

= $B_{xP}\rho_{xP}\sigma_{CExP}$

σ_{CExP} = classical buckling stress for a perfect curved panel between adjacent stringer stiffeners, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= K_{xP}\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{s}\right)^2$$

K_{xP} = $4+\dfrac{3z_s^2}{\pi^4}$ for $z_s \le 11.4$

= $0.702z_s$ for $z_s > 11.4$

ρ_{xP} = nominal or lower bound knock-down factor to allow for shape imperfections

= $1-0.019z_s^{1.25}+0.0024z_s\left(1-\dfrac{r}{300t}\right)$ for $z_s \le 11.4$

= $0.27+\dfrac{1.5}{z_s}+\dfrac{27}{z_s^2}+0.008\sqrt{z_s}\left(1-\dfrac{r}{300t}\right)$ for $z_s > 11.4$

B_{xP} = factor compensating for the lower bound nature of ρ_{xP}

= $\begin{cases} 1.15 & \text{for } \lambda_n > 1 \\ 1+0.15\lambda_n & \text{for } \lambda_n \le 1 \end{cases}$

λ_n = $\sqrt{\dfrac{\sigma_0}{\rho_{xP}\sigma_{CExP}}}$

z_s = $\sqrt{1-\nu^2}\,\dfrac{s^2}{rt}$

s = spacing of stringer stiffeners, cm (in.)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

ν = Poisson's ratio, 0.3 for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

5.5 Critical Buckling Stress under External Pressure

The critical buckling stress for curved panels bounded by adjacent pairs of ring and stringer stiffeners subjected to external pressure may be taken as:

$$\sigma_{C\theta P} = \Phi\sigma_{E\theta P}$$

where

Φ = plasticity reduction factor

= 1 for $\Delta \le 0.55$

= $\dfrac{0.45}{\Delta}+0.18$ for $0.55 < \Delta \le 1.6$

= $\dfrac{1.31}{1+1.15\Delta}$ for $1.6 < \Delta < 6.25$

= $1/\Delta$ for $\Delta \ge 6.25$

Δ = $\sigma_{E\theta P}/\sigma_0$

$\sigma_{E\theta P}$ = elastic hoop buckling stress of imperfect curved panel, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\dfrac{q_{CE\theta P}(r+0.5t)}{t}K_\theta$

K_θ = coefficient to account for the strengthening effect of ring stiffener from 4/13.3

$q_{CE\theta P}$ = elastic buckling pressure, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{\frac{Et}{r}}{n^2 + k\alpha^2 - 1}\left[\frac{\left(n^2+\alpha^2-1\right)^2}{12\left(1-\nu^2\right)}\left(\frac{t}{r}\right)^2 + \frac{\alpha^4}{\left(n^2+\alpha^2\right)^2}\right]$$

n = circumferential wave number starting at $0.5N_s$ and increasing until a minimum value of $q_{CE\theta P}$ is attained

α = $\frac{\pi r}{\ell}$

k = 0 for lateral pressure

= 0.5 for hydrostatic pressure

ℓ = length between adjacent ring stiffeners (unsupported)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

N_s = number of stringer stiffeners

7 Ring and Stringer-stiffened Shells

7.1 Bay Buckling Limit State

For the buckling limit state of ring and stringer-stiffened cylindrical shells between adjacent ring stiffeners subjected to axial compression, bending moment and external pressure, the following strength criteria is to be satisfied:

$$\left[\frac{\sigma_x}{\psi(\sigma_{CxB}/\gamma_R)(A_e/A)}\right]^2 - \varphi_B\left[\frac{\sigma_x}{\psi(\sigma_{CxB}/\gamma_R)(A_e/A)}\right]\left(\frac{\sigma_\theta}{\psi\,\sigma_{C\theta B}/\gamma_R}\right) + \left(\frac{\sigma_\theta}{\psi\,\sigma_{C\theta B}/\gamma_R}\right)^2 \le 1$$

where

σ_x = compressive stress in longitudinal direction due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = compressive hoop stress due to factored loads from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CxB} = critical buckling stress for axial compression or bending moment from 4/7.3, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{C\theta B}$ = critical buckling stress for external pressure from 4/7.5, N/cm^2 (kgf/cm^2, lbf/in^2)

φ_B = coefficient to reflect interaction between longitudinal and hoop stresses (negative values are acceptable)

$$= \frac{1.5(\sigma_{CxB} + \sigma_{C\theta B})}{\sigma_0} - 2.0$$

A_e = effective cross sectional area, cm^2 (in^2)

= $A_s + s_{em}t$

A = total cross sectional area, cm^2 (in^2)

= $A_s + st$

A_s = cross sectional area of stringer stiffener, cm^2 (in^2)

t = thickness of cylindrical shell, cm (in.)

s = spacing of stringers

s_{em} = modified effective shell plate width

= $\left(\frac{1.05}{\lambda_m} - \frac{0.28}{\lambda_m^2}\right)s$ for $\lambda_m > 0.53$

= s for $\lambda_m \le 0.53$

λ_m = modified reduced slenderness ratio

= $\sqrt{\frac{\sigma_{CxB}}{\sigma_{ExP}}}$

σ_{ExP} = elastic buckling stress for imperfect curved panel between adjacent stringer stiffeners subjected to axial compression from 4/5.3, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

γ_R = resistance factor as defined in Subsection 1/11

ψ = maximum allowable strength adjustment factor of shell buckling, as specified in 4/1.7, for ring and stringer-stiffened cylindrical shells in axial compression or external pressure, whichever is the lesser

7.3 Critical Buckling Stress for Axial Compression or Bending Moment

The critical buckling stress of ring and stringer-stiffened cylindrical shells subjected to axial compression or bending may be taken as:

$$\sigma_{CxB} = \begin{cases} \sigma_{ExB} & \text{for} \quad \sigma_{ExB} \le P_r\sigma_0 \\ \sigma_0\left[1 - P_r(1-P_r)\dfrac{\sigma_0}{\sigma_{ExB}}\right] & \text{for} \quad \sigma_{ExB} > P_r\sigma_0 \end{cases}$$

where

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{ExB} = elastic compressive buckling stress of imperfect stringer-stiffened shell, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\sigma_c + \sigma_s$

σ_s = elastic compressive buckling stress of stringer-stiffened shell, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\rho_{xB}\dfrac{0.605E\left(\frac{t}{r}\right)}{1 + \frac{A_s}{st}}$

ρ_{xB} = 0.75

σ_c = elastic buckling stress of column, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\dfrac{\pi^2 EI_{se}}{\ell^2(A_s + s_e t)}$

I_{se} = moment of inertia of stringer stiffener plus associated effective shell plate width, cm^4 (in^4)

$$= I_s + A_s z_{st}^2 \frac{s_e t}{A_s + s_e t} + \frac{s_e t^3}{12}$$

I_s = moment of inertia of stringer stiffener about its own centroid axis, cm^4 (in^4)

z_{st} = distance from centerline of shell to the centroid of stringer stiffener, cm (in.)

A_s = cross sectional area of stringer stiffener, cm^2 (in^2)

s_e = reduced effective width of shell, cm (in.)

$$= \frac{0.53}{\lambda_{xP}} s \qquad \text{for } \lambda_{xP} > 0.53$$

$$= s \qquad \text{for } \lambda_{xP} \le 0.53$$

s = shell plate width between adjacent stringers, cm (in.)

λ_{xP} = reduced shell slenderness ratio

$$= \sqrt{\frac{\sigma_0}{\sigma_{ExP}}}$$

σ_{ExP} = elastic compressive buckling stress for imperfect curved panel between adjacent stringer stiffeners from 4/5.3, N/cm^2 (kgf/cm^2, lbf/in^2)

ℓ = length between adjacent ring stiffeners (unsupported), cm (in.)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

7.5 Critical Buckling Stress for External Pressure

The critical buckling stress for ring and stringer-stiffened cylindrical shells subjected to external pressure may be taken as

$$\sigma_{C\theta B} = (\sigma_{C\theta R} + \sigma_{sp})\, K_p \le \sigma_0$$

where

$\sigma_{C\theta R}$ = critical hoop buckling stress for the unstiffened shell from 4/3.5, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{sp} = collapse hoop stress for a stringer stiffener plus its associated shell plating, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{q_s (r + 0.5t)}{t} K_\theta$$

K_θ = coefficient to account for the strengthening effect of ring stiffener from 4/13.3

q_s = collapse pressure of a stringer stiffener plus its associated shell plating, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{16}{s\ell^2} A_s |z_{st}| \sigma_0$$

z_{st} = distance from centerline of shell to the centroid of stringer stiffener, cm (in.)

A_s = cross sectional area of stringer stiffener, cm^2 (in^2)

K_p = effective pressure correction factor

$= 0.25 + \dfrac{0.85}{500} g$ for $g \le 500$

$= 1.10$ for $g > 500$

g = geometrical parameter

$$= 2\pi \frac{\ell^2 A_s}{N_s I_s}$$

I_s = sectional moment area of inertia of stringer stiffener, cm^4 (in^4)

N_s = number of stringer stiffeners

ℓ = length between adjacent ring stiffeners (unsupported), cm (in.)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

7.7 General Buckling

The general buckling of a ring and stringer-stiffened cylindrical shell involves the collapse of one or more ring stiffeners together with shell plating plus stringer stiffeners and should be avoided due to its catastrophic consequences. The ring and stringer stiffeners are to be proportioned, in accordance with 4/15.1 and 4/15.3, to exclude the general buckling failure mode.

9 Local Buckling Limit State for Ring and Stringer Stiffeners

9.1 Flexural-Torsional Buckling

When the torsional stiffness of the stiffeners is low and the slenderness ratio of the curved panels is relatively high, the stiffeners can suffer torsional-flexural buckling (tripping) at a stress level lower than that resulting in local or bay buckling. When the stiffener buckles, it loses a large part of its effectiveness to maintain the initial shape of the shell. The buckled stiffener sheds load to the shell, and therefore, should be suppressed.

The flexural-torsional buckling limit state of stringer stiffeners is to satisfy the ultimate state limit given below:

$$\frac{\sigma_x}{\psi \, \sigma_{CT} / \gamma_R} \le 1$$

where

σ_x = compressive stress in the longitudinal direction due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{CT} = flexural-torsional buckling stress with respect to axial compression of a stiffener, including its associated shell plating, may be obtained from the following equations:

$$= \begin{cases} \sigma_{ET} & \text{if} \quad \sigma_{ET} \le P_r \sigma_0 \\ \sigma_0 \left[1 - P_r (1 - P_r) \dfrac{\sigma_0}{\sigma_{ET}} \right] & \text{if} \quad \sigma_{ET} > P_r \sigma_0 \end{cases}$$

σ_0 = specified minimum yield point of the stringer under consideration, N/cm^2 (kgf/cm^2, lbf/in^2)

P_r = proportional linear elastic limit of the structure, which may be taken as 0.6 for steel

σ_{ET} = ideal elastic flexural-torsional buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{\frac{K}{2.6}+\left(\frac{n\pi}{\ell}\right)^2 \Gamma + \frac{C_0}{E}\left(\frac{\ell}{n\pi}\right)^2}{I_0+\frac{C_0}{\sigma_{CL}}\left(\frac{\ell}{n\pi}\right)^2} E$$

K = St. Venant torsion constant for the stiffener cross-section, excluding the associated shell plating, cm^4 (in^4)

$$= \frac{b_f t_f^3 + d_w t_w^3}{3}$$

I_0 = polar moment of inertia of the stiffener, excluding the associated shell plating, cm^4 (in^4)

$$= I_y + mI_z + A_s(y_0^2 + z_0^2)$$

I_y, I_z = moment of inertia of the stiffener about the y- and z-axis, respectively, through the centroid of the longitudinal, excluding the shell plating (y-axis perpendicular to the web, see Section 4, Figure 2), cm^4 (in^4)

m = $1.0 - u\left(0.7 - 0.1\frac{d_w}{b_f}\right)$

u = non-symmetry factor

$$= 1 - 2\frac{b_1}{b_f}$$

y_0 = horizontal distance between centroid of stiffener and web plate centerline (see Section 4, Figure 2), cm (in.)

z_0 = vertical distance between centroid of stiffener and its toe (see Section 4, Figure 2), cm (in.)

d_w = depth of the web, cm (in.)

t_w = thickness of the web, cm (in.)

b_f = total width of the flange/face plate, cm (in.)

b_1 = smaller outstanding dimension of flange or face plate with respect to web's centerline, cm (in.)

t_f = thickness of the flange or face plate, cm (in.)

C_0 = $\frac{Et^3}{3s}$

Γ ≅ warping constant, cm^6 (in^6)

$$\cong mI_{zf} d_w^2 + \frac{d_w^3 t_w^3}{36}$$

I_{zf} = $\frac{t_f b_f^3}{12}\left(1.0 + 3.0\frac{u^2 d_w t_w}{A_s}\right)$, cm^4 (in^4)

σ_{CL} = critical buckling stress for associated shell plating corresponding to *n*-half waves, N/cm^2 (kgf/cm^2, lbf/in^2)

= $$\frac{\pi^2 E\left(\frac{n}{\alpha}+\frac{\alpha}{n}\right)^2\left(\frac{t}{s}\right)^2}{12\left(1-\upsilon^2\right)}$$

α = ℓ/s

n = number of half-waves which yields the smallest σ_E

σ_0 = specified minimum yield point of the material, N/cm^2 (kgf/cm^2, lbf/in^2)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

s = spacing of stringer stiffeners, cm (in.)

A_s = sectional area of stringer stiffener, excluding the associated shell plating, cm^2 (in^2)

t = thickness of shell plating, cm (in.)

ℓ = length between adjacent ring stiffeners (unsupported), cm (in.)

γ_R = resistance factor as defined in Subsection 1/11

ψ = maximum allowable strength adjustment factor, as specified in 4/1.7, for tripping of stringer stiffeners

9.3 Web Plate Buckling

The depth to thickness ratio of the web plate is to satisfy the limit given in 4/15.5.

9.5 Faceplate and Flange Buckling

The breadth to thickness ratio of the faceplate or flange is to satisfy the limit given in 4/15.7.

11 Beam-Column Buckling

A cylindrical shell subjected to axial compression, or bending moment or both; with or without external pressure, is to be designed to resist beam-column buckling. Beam-column buckling is to be assessed if:

$$\lambda_{xE} \geq 0.50$$

where

λ_{xE} = slenderness ratio of cylindrical shell

= $\sqrt{\sigma_0 / \sigma_{E(C)}}$

$\sigma_{E(C)}$ = Euler buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

= $\pi^2 E r_i^2/(kL)^2$

r_i = radius of gyration of the cross section of the cylindrical shell, cm (in.)

= $\sqrt{\dfrac{I_T}{A_T}}$

I_T = moment of inertia of the cross section of the cylindrical shell; if the cross section is variable along the length, the minimum value is to be used, cm^4 (in^4)

A_T = cross sectional area of the cylindrical shell; if the cross section is variable along the length, the minimum value is to be used, cm^2 (in^2)

kL = effective length of the cylinder, as defined in 2/3.3

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

The beam-column buckling limit state of a cylindrical shell subjected to axial compresion, or bending or both; with or without external pressure, is to satisfy the following criteria at all cross-sections along its length:

$$\frac{\sigma_a}{\psi\left(\dfrac{\sigma_{Ca}}{\gamma_R}\right)}+\frac{\sigma_b}{\psi\left(\dfrac{\sigma_{Cx}}{\gamma_R}\right)\left[1-\dfrac{\sigma_a}{\psi\,\sigma_{E(C)}/\gamma_R}\right]}\le 1$$

where

σ_a = calculated axial normal compressive stress due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_b = calculated bending stress due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_{Ca} = critical compressive buckling stress, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \begin{cases} \sigma_{E(C)} & \text{if} \quad \sigma_{E(C)} \le P_r\sigma_{Cx} \\ \sigma_{Cx}\left[1-P_r(1-P_r)\dfrac{\sigma_{Cx}}{\sigma_{E(C)}}\right] & \text{if} \quad \sigma_{E(C)} > P_r\sigma_{Cx} \end{cases}$$

σ_{Cx} = critical axial or bending buckling stress of bay

for ring-stiffened cylindrical shell

$$= \sigma_{CxR}\left[0.5\varphi_R\left(\frac{\sigma_\theta/\chi}{\sigma_{C\theta R}}\right)+\sqrt{1-(1-0.25\varphi_R^2)\left(\frac{\sigma_\theta/\chi}{\sigma_{C\theta R}}\right)^2}\right]$$

for ring and stringer-stiffened cylindrical shell

$$= \frac{A_e}{A}\sigma_{CxB}\left[0.5\varphi_B\left(\frac{\sigma_\theta/\chi}{\sigma_{C\theta B}}\right)+\sqrt{1-(1-0.25\varphi_R^2)\left(\frac{\sigma_\theta/\chi}{\sigma_{C\theta B}}\right)^2}\right]$$

σ_θ = calculated hoop stress from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

A = cross sectional area as defined in 4/7.1

A_e = effective cross sectional area as defined in 4/7.1

χ = load adjustment factor

= 1.45 for the *Static Loading Condition* (see Subsection 1/9)

= 1.00 for the *Redundancy Condition* or *Damaged Condition* (see Subsection 1/9)

= 1.25 for the other loading conditions (see Subsection 1/9)

ψ = maximum allowable strength adjustment factor, as specified in 4/1.7, for column buckling

σ_{CxR}, $\sigma_{C\theta R}$, φ_R, σ_{CxB}, $\sigma_{C\theta B}$ and φ_B are as defined in Subsections 4/3 and 4/7.

13 Stress Calculations

13.1 Longitudinal Stress

The longitudinal stress in accordance with beam theory may be taken as:

$$\sigma_x = \sigma_a + b_a$$

where

σ_a = stress due to axial force, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{P}{2\pi rt(1+\delta)}$$

σ_b = stress due to bending moment, N/cm^2 (kgf/cm^2, lbf/in^2)

$$= \frac{M}{\pi r^2 t(1+\delta)}$$

P = axial force due to factored loads, N (kgf, lbf)

M = bending moment due to factored loads, N-cm (kgf-cm, lbf-in)

δ = $\dfrac{A_{st}}{st}$

A_{st} = cross sectional area of stringer stiffener, cm^2 (in^2)

s = shell plate width between adjacent stringer stiffeners, cm (in.)

r = mean radius of cylindrical shell, cm (in.)

t = thickness of cylindrical shell, cm (in.)

13.3 Hoop Stress

The hoop stress may be taken as

At midway of shell between adjacent ring stiffeners:

$$\sigma_\theta = \frac{q(r+0.5t)}{t} K_\theta$$

At inner face of ring flange, (i.e., radius r_F in Section 4, Figure 2):

$$\sigma_{\theta R} = \frac{q(r+0.5t)}{t}\frac{r}{r_F} K_{\theta R}$$

where

K_θ = $1 - \dfrac{1-k\nu}{1+t(t_w+\ell\varpi)/\overline{A}_R} G_\alpha$

$K_{\theta R}$ = $\dfrac{1-k\nu}{1+\overline{A}_R/[t(t_w+\ell\varpi)]}$

$\overline{A}_R$ = $A_R\left(\dfrac{r}{r_R}\right)^2$, cm^2 (in^2)

ϖ = $\dfrac{\cosh 2\alpha - \cos 2\alpha}{\alpha(\sinh 2\alpha + \sin 2\alpha)} \geq 0$

α = $\dfrac{\ell}{1.56\sqrt{rt}}$

G_α = $2\dfrac{\sinh\alpha\cos\alpha+\cosh\alpha\sin\alpha}{\sinh 2\alpha+\sin 2\alpha}\geq 0$

k = N_x/N_θ for radial pressure

= $N_x/N_\theta + 0.5$ for hydrostatic pressure

A_R = cross sectional area of ring stiffener, cm^2 (in^2)

q = factored external pressure, N/cm^2 (kgf/cm^2, lbf/in^2)

N_x = axial load per unit length, excluding the capped-end actions due to hydrostatic pressure, N/cm (kg/cm, lbf/in)

N_θ = circumferential load per unit length, N/cm (kg/cm, lbf/in)

r = mean radius of cylindrical shell, cm (in.)

r_R = radius to centroid of ring stiffener, as defined in Section 4, Figure 2, cm (in.)

r_F = radius to inner face of ring flange, as defined in Section 4, Figure 2, cm (in.)

t = thickness of cylindrical shell, cm (in.)

t_w = stiffener web thickness, cm (in.)

ℓ = length between adjacent ring stiffeners (unsupported), cm (in.)

ν = Poisson's ratio

r, r_R and r_F are described in Section 4, Figure 2.

15 Stiffness and Proportions

To fully develop the intended buckling strength of the assemblies of a stiffened cylindrical shell, ring and stringer stiffeners are to satisfy the following requirements for stiffness and proportions.

15.1 Stiffness of Ring Stiffeners

The moment of inertia of the ring stiffeners, i_r, together with the effective length of shell plating, ℓ_{eo}, should not be less than that given by the following equation:

$$i_r = \frac{(\sigma_x/\chi)(1+\delta)tr_e^4}{500E\ell} + \frac{(\sigma_\theta/\chi)r_e^2\ell t}{2EK_\theta}\left(1+\frac{z_e}{100r}\frac{1.36E}{\psi\,\sigma_0/\gamma_R}\frac{1}{1-(\sigma_{\theta R}/\chi)/(\psi\,\sigma_0/\gamma_R)}\right)$$

where

σ_x = compressive stress in longitudinal direction due to factored loads from 4/13.1, N/cm^2 (kgf/cm^2, lbf/in^2)

σ_θ = compressive hoop stress midway between adjacent ring stiffeners due to factored loads from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

$\sigma_{\theta R}$ = compressive hoop stress at outer edge of ring flange due to factored loads from 4/13.3, N/cm^2 (kgf/cm^2, lbf/in^2)

δ = A_s/st

i_r = moment of inertia of the ring stiffeners with associated effective shell length, ℓ_{eo}

ℓ_{eo} = $1.56\sqrt{rt}\ \leq \ell$

r_e	=	radius to the centroid of ring stiffener, accounting for the effective length of shell plating, cm (in.)
z_e	=	distance from inner face of ring flange to centroid of ring stiffener, accounting for the effective length of shell plating, cm (in.)
K_θ	=	coefficient from 4/13.3
σ_0	=	specified minimum yield point of ring stiffeners, N/cm^2 (kgf/cm^2, lbf/in^2)
E	=	modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel
s	=	spacing of stringer stiffeners, cm (in.)
A_s	=	cross sectional area of stringer, cm^2 (in^2)
t	=	thickness of shell plating, cm (in.)
ℓ	=	length between adjacent ring stiffeners (unsupported), cm (in.)
χ	=	load adjustment factor
	=	1.45 for the *Static Loading Condition* (see Subsection 1/9)
	=	1.00 for the *Redundancy Condition* or *Damaged Condition* (see Subsection 1/9)
	=	1.25 for the other loading conditions (see Subsection 1/9)
γ_R	=	resistance factor as defined in Subsection 1/11
ψ	=	maximum allowable strength adjustment factor for stiffened cylindrical shells subjected to external pressure, see 4/1.7

15.3 Stiffness of Stringer Stiffeners

The moment of inertia of the stringer stiffeners, i_s, with effective breadth of shell plating, s_{em}, is not to be less than:

$$i_o = \frac{st^3}{12(1-\nu^2)}\gamma_0$$

where

γ_0	=	$(2.6 + 4.0\delta)\alpha^2 + 12.4\alpha - 13.2\alpha^{1/2}$
δ	=	$A_s/(st)$
α	=	ℓ/s
s	=	spacing of stringer stiffeners, cm (in.)
t	=	thickness of shell plate, cm (in.)
ν	=	Poisson's ratio
A_s	=	cross sectional area of stringer stiffener, cm^2 (in^2)
ℓ	=	length between adjacent ring stiffeners (unsupported), cm (in.)

15.5 Proportions of Webs of Stiffeners

The depth to thickness ratio of webs of stiffeners is to satisfy the applicable limit given below.

$d_w/t_w \le 1.5(E/\sigma_0)^{1/2}$	for angles and tee bars
$d_w/t_w \le 0.85(E/\sigma_0)^{1/2}$	for bulb plates
$d_w/t_w \le 0.4(E/\sigma_0)^{1/2}$	for flat bars

where

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel

d_w and t_w are as defined in Section 4, Figure 2.

15.7 Proportions of Flanges and Faceplates

The breadth to thickness ratio of flanges and faceplates of stiffeners is to satisfy the limit given below.

$$b_2/t_f \leq 0.4(E/\sigma_0)^{1/2}$$

where

b_2 = larger outstanding dimension of the flange/faceplate, cm (in.)

t_f = thickness of flange/face plate, cm (in.)

σ_0 = specified minimum yield point, N/cm^2 (kgf/cm^2, lbf/in^2)

E = modulus of elasticity, 2.06×10^7 N/cm^2 (2.1×10^6 kgf/cm^2, 30×10^6 lbf/in^2) for steel