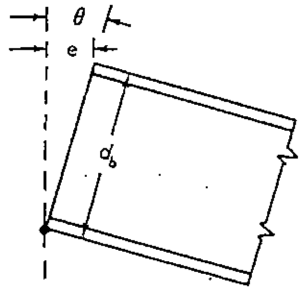
Top Connecting Plates For Semi-Rigid Connections

1. ANALYSIS OF CONNECTION

A top connecting plate designed to be stressed only below its yield point may be used as a semi-rigid connection. The reduced portion of the plate is detailed to have sufficient length (L) for elastic elongation of this section to provide the proper amount of joint rotation. See Figure 1.

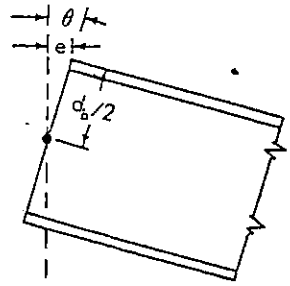
Analysis of this type of connection requires locating the center of rotation. This depends on the relative stiffness of the top and bottom portions of the connection.

For the more flexible type of semi-rigid connection, rotation will occur closer to the bottom of the beam; see Figure 2. For the more rigid connection, rotation will occur closer to the midheight of the beam; see Figure 3.



Rotation about bottom of beam

FIGURE 2



Rotation about mid-height of beam

FIGURE 3

The resisting moment of the connection is—

$$\boxed{M_c = A_p\ \sigma\ d_b} \qquad (1)$$

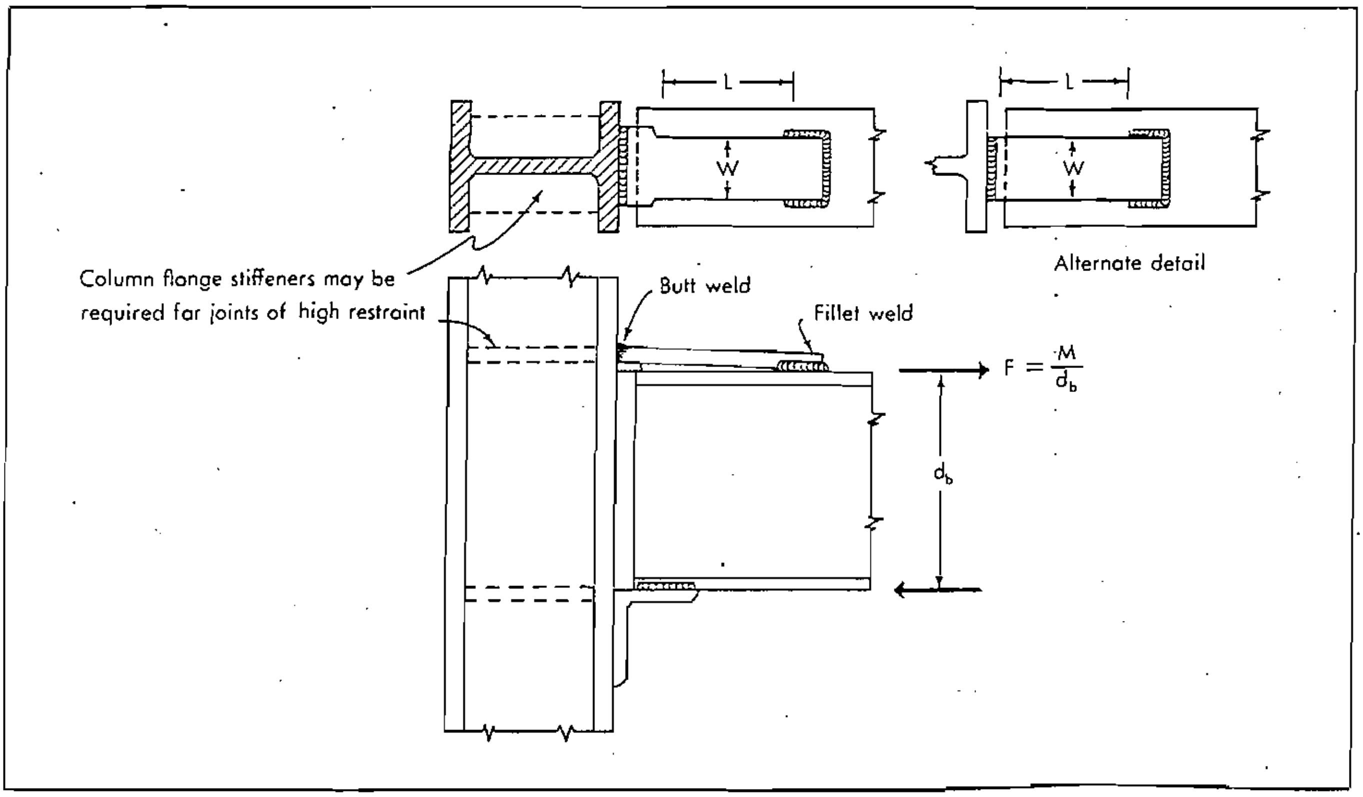


FIGURE 1

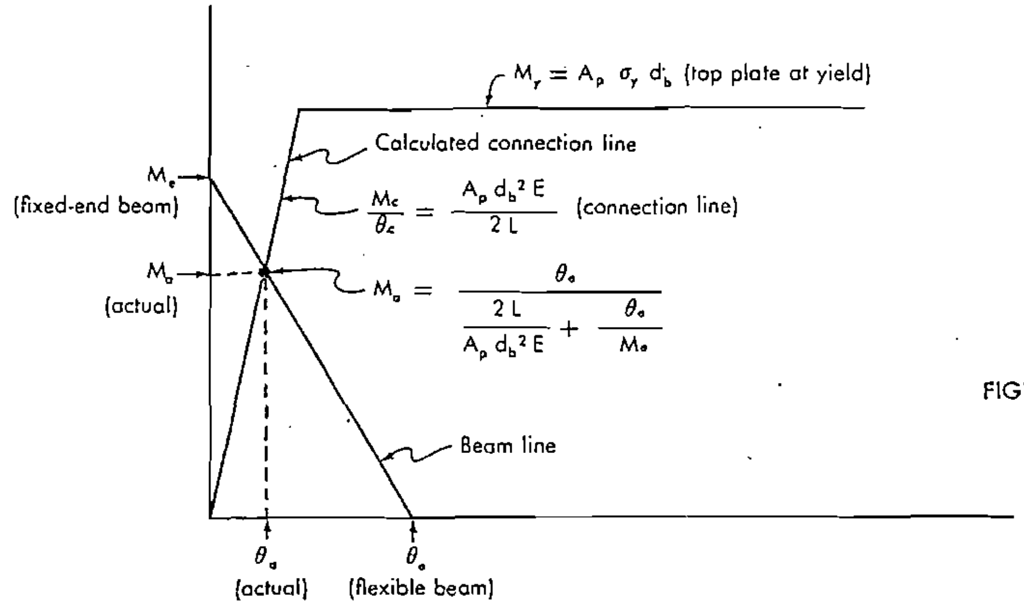


FIGURE 4

and the required cross-sectional area of the top plate is—

$$\boxed{A_p = \frac{M}{\sigma\ d_b}} \qquad (2)$$

The rotation of the connection, assuming rotation about midheight of the beam is—

$$\boxed{\theta_c = \frac{2\ e}{d_b}} \text{ and}$$

$$e = \epsilon\ L = \frac{\sigma\ L}{E} \text{ or}$$

$$\boxed{\sigma_c = \frac{2\ \sigma\ L}{d_b\ E}} \qquad (3)$$

The slope of this connection line is—

$$\boxed{\frac{M_c}{\theta_c} = \frac{A_p\ d_b^2\ E}{2\ L}} \qquad (4)$$

This connection line breaks at the yield point, or becomes horizontal at:

$$\boxed{M_y = A_p\ \sigma_y\ d_b} \qquad (5)$$

The actual conditions of moment (M_a) and rotation (θ_a) are found at the intersection of the beam line and the connection line; see Figure 4.

Table 1 shows the moments (M) and end rotation (θ) for various load and beam conditions.

The total centerline moment ($\Sigma M_{\mathfrak{C}}$) and total end moment (ΣM_e) of a beam with any combination of the Table 1 loads equals the sum of the individual values resulting from each type of load.

When designing a beam for a given end restraint (R), the resulting maximum moment at centerline for which the beam is designed (M_b) equals the difference between the maximum centerline moment ($M_{\mathfrak{C}}$) when $R = 0$ and the actual end moment ($R\ M_e$) for the given value of R. See Figure 5.

$$M_b = \Sigma M_{\mathfrak{C}} - R\ \Sigma M_e$$

This can also be found by totaling the individual

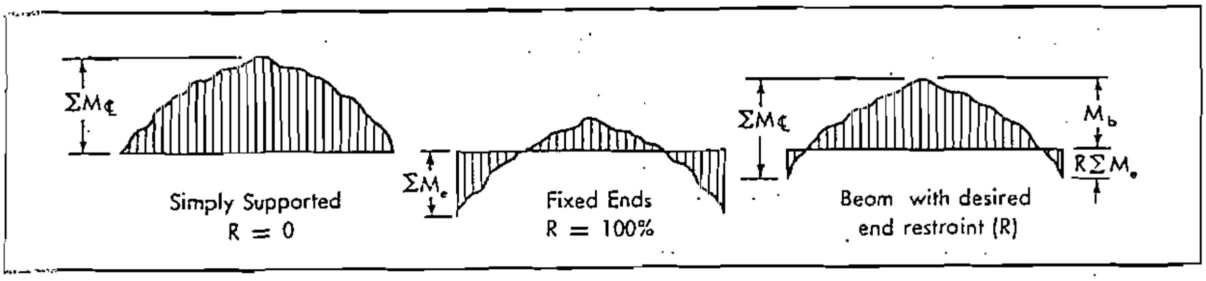


FIG. 5 Moment diagrams for different restraints (R).

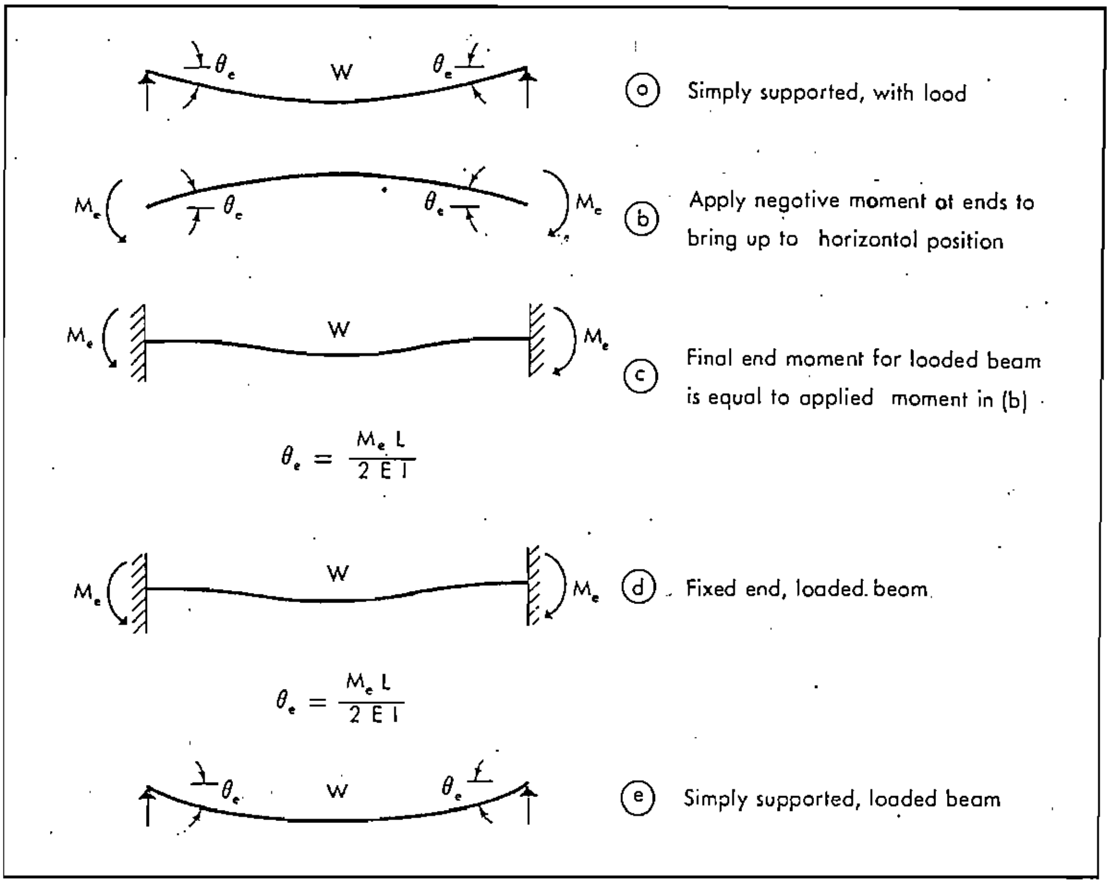


FIGURE 6

TABLE 1—Moments and End Rotation for Various Load/Beam Conditions

	1 Force W	2 Forces W	3 Forces W	4 Forces W	5 Forces W	Uniform Load W
Moment Diagram — Simply Supported						
Center Moment $M_{℄}$ Simply Supported	$+\frac{WL}{4}$	$+\frac{WL}{6}$	$+\frac{WL}{6}$	$+\frac{3WL}{20}$	$+\frac{3WL}{20}$	$+\frac{WL}{8}$
End Rotation θ_e Simply Supported (R = 0)	$\frac{WL^2}{16EI}$	$\frac{WL^2}{18EI}$	$\frac{5WL^2}{96EI}$	$\frac{WL^2}{20EI}$	$\frac{7WL^2}{144EI}$	$\frac{WL^2}{24EI}$
End Moment M_e Fixed Ends (R = 100%)	$-\frac{WL}{8}$	$-\frac{WL}{9}$	$-\frac{5WL}{48}$	$-\frac{WL}{10}$	$-\frac{7WL}{92}$	$-\frac{WL}{12}$
Beam Moment M_b at ℄ For Given Value of R	$\frac{WL}{8}(2-R)$	$\frac{WL}{18}(3-2R)$	$\frac{WL}{48}(8-5R)$	$\frac{WL}{20}(3-2R)$	$\frac{WL}{360}(54-35R)$	$\frac{WL}{24}(3-2R)$

values of M_b for a given value of R resulting from each of the types of loads; see Table 1.

We must now obtain the two points for the beam line with all of its loads (W): the total end moment (M_e) when beam ends are fixed and the angle rotation (θ_e) when beam ends are simply supported.

The fixed moments (M_e) from all the loads are totaled, and the angle rotation (θ_e) may be found from this total fixed end moment (M_e):

$$\theta_e = \frac{M_e \ L}{2 \ E \ I}$$

This relationship may be found by determining the end moment required to rotate the end of a simply supported beam back to a horizontal position; see Figure 6a, b and c.

It will be easier then, to total the individual end moments for all of the types of applied loads; this becomes the final end moment when treated as a fixed-end beam, Figure 6d. Use this formula to determine the final end rotation (θ_e) of this beam with all of its applied loads when simply supported, Figure 6e.

Problem 1

Design a beam of A373 steel and detail the connection to support a uniformly distributed load of 22 kips (Fig. 7) and four concentrated loads of 6 kips each on L/5 centers along a 15-ft span (Fig. 8). The beam's design will be based on an end restraint of R = 50%, and the connection's design for R = 75%.

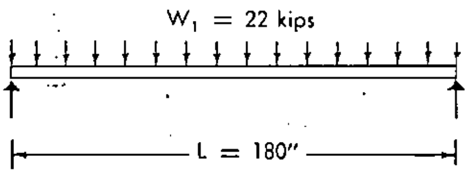


FIGURE 7

Here, the beam moment at centerline:

$$M_{b1} = \frac{W \ L}{24} \ (3 - 2 \ R)$$
$$= \frac{(22{,}000)(180)}{24} \ [3 - 2 \ (.50)]$$
$$= 330 \text{ in.-kips}$$

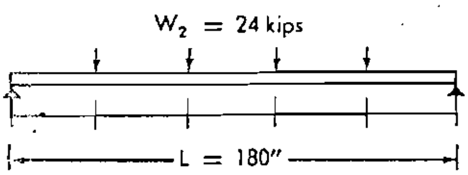


FIGURE 8

Here, the beam moment at centerline:

$$M_{b2} = \frac{W \ L}{20} \ (3 - 2 \ R)$$
$$= \frac{(24{,}000)(180)}{20} \ [3 - 2 \ (.50)]$$
$$= 432 \text{ in.-kips}$$

Thus the total moment on the beam at its centerline is—

$$M_b = 330 + 432 \qquad (R = 50\%)$$
$$= 762 \text{ in.-kips}$$

The beam's required section modulus is—

$$S = \frac{M_b}{\sigma}$$
$$= \frac{(762)}{(20{,}000)}$$
$$= 38.1 \text{ in.}^3$$

A 14" WF 30# beam could be used, since it has—

$S = 41.8$ in.^{3} OK

In order to plot this as a beam line, it is necessary to know 1) the end rotation (θ_e) of the beam under the total load when simply supported, and 2) the end moment (M_e) on the beam under the total load considering the beam as having fixed ends.

$$M_{e1} = \frac{W \ L}{12}$$
$$= \frac{(22{,}000)(180)}{12}$$
$$= 330 \text{ in.-kips}$$

$$M_{e2} = \frac{W \ L}{10}$$
$$= \frac{(24{,}000)(180)}{10}$$
$$= 432 \text{ in.-kips}$$

Total end moment (R = 100%):

$$M_e = M_{e1} + M_{e2}$$
$$= 330 + 432$$
$$= 762 \text{ in.-kips.}$$

Resulting end rotation of beam, with combined loads, simply supported (R = 0):

$$\theta_e = \frac{M_e\ L}{2\ E\ I}$$

$$= \frac{(762)\ (180)}{2(30 \times 10^6)(289.6)}$$

$$= \underline{7.9 \times 10^{-3} \text{ radians}}$$

Design top plate for an end moment of 75% M_e = .75 (762 in.-kips) = 571 in.-kips.

Cross-sectional area of top plate:

$$A_p = \frac{M}{\sigma\ d_b}$$

$$= \frac{(571{,}000)}{(20{,}000)(13.86)}$$

$$= 2.06 \text{ in.}^2$$

or use a 3/8" x 5½" plate, having A_p = 2.06 in.²

The length of the reduced portion of the top plate will be made L = 7".

The slope of the connection line:

$$\frac{M_c}{\theta_c} = \frac{A_p\ d_b^2\ E}{2\ L}$$

$$= \frac{(2.06)(13.86)^2(30 \times 10^6)}{2\ (7)}$$

$$= 8.47 \times 10^5 \text{ in.-lbs/radian}$$

This connection line can also be constructed by solving for end moment (M_c) and end rotation (θ_c) when stressed to yield, σ_y = 33,000 psi:

$$M_c = A_p\ \sigma_y\ d_b$$

$$= (2.06)(33{,}000)(13.86)$$

$$= 943 \text{ in.-kips}$$

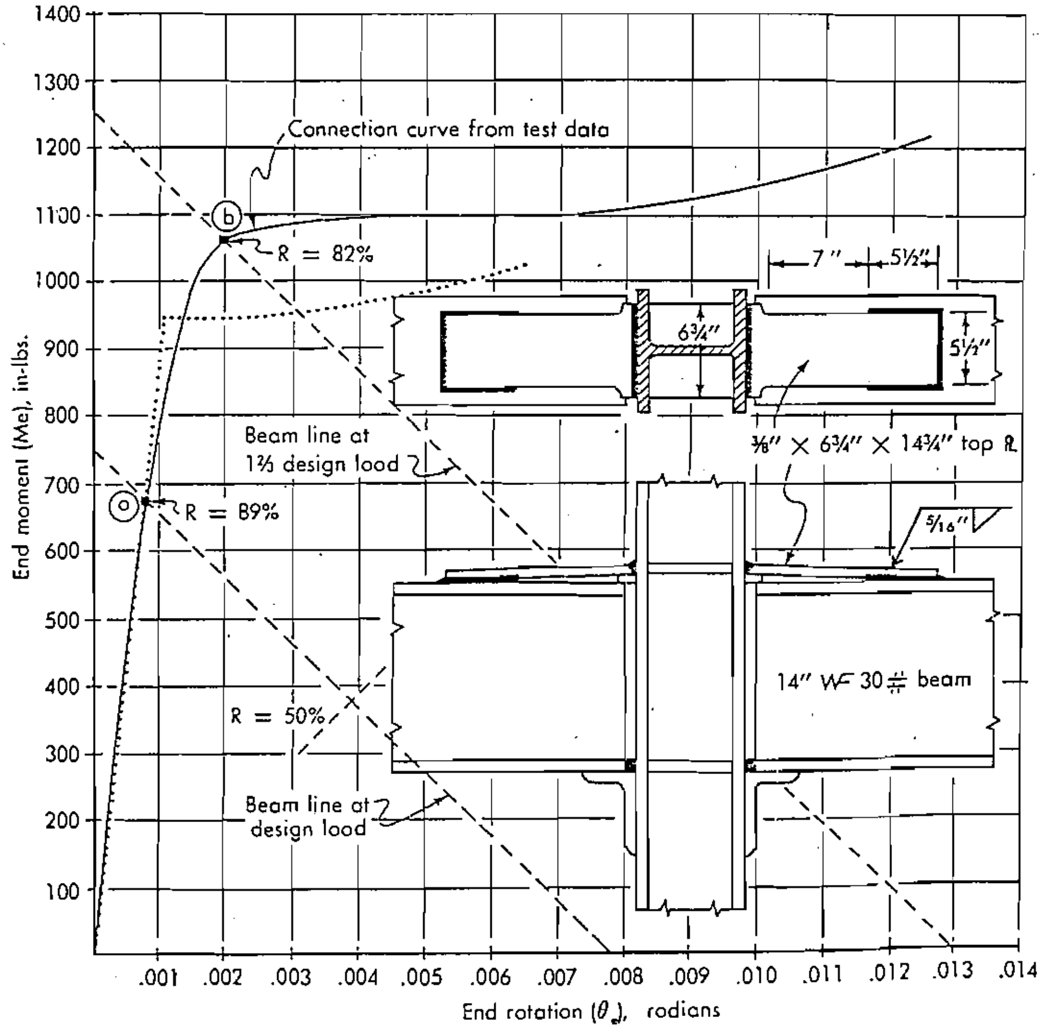
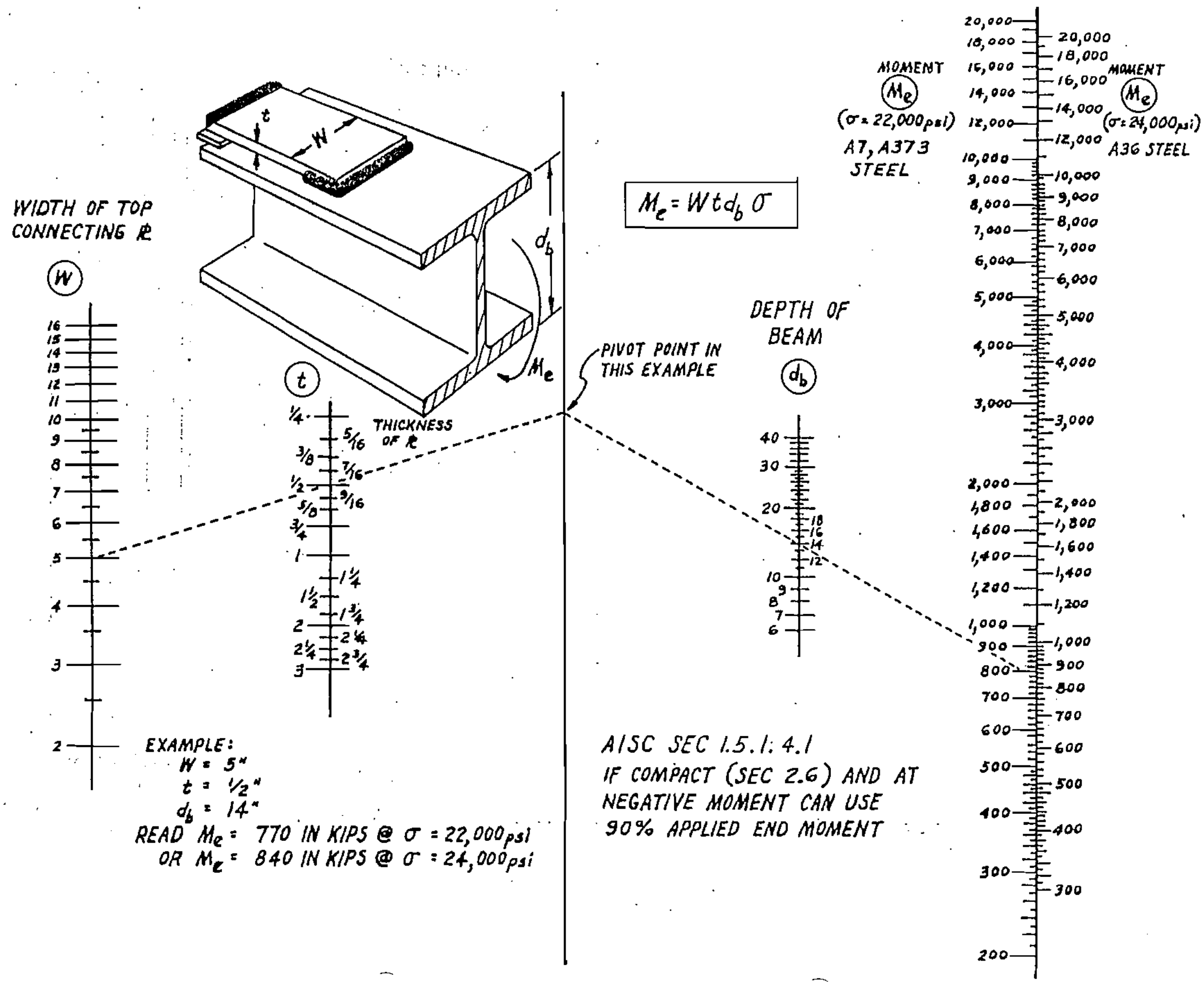


FIGURE 9

FIG. 10. Moment Capacity of Top Plate Connection.



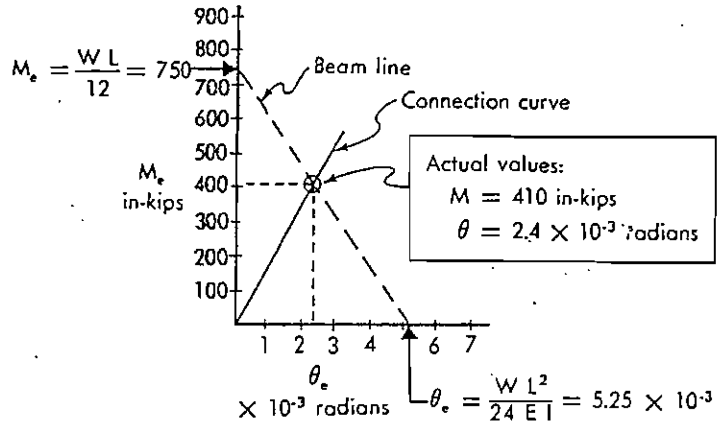


FIGURE 11

$$\theta_c = \frac{2\ \sigma_y\ L}{d_b\ E}$$

$$= \frac{2(33{,}000)(7)}{(13.86)(30 \times 10^6)}$$

$$= 1.11 \times 10^{-3} \text{ radians}$$

This calculated connection line is shown as a dotted line in Figure 9. It rises to a moment of M = 943 in.-kips at which time the top plate should reach yield stress. From there on, this plate will yield plastically and build up a higher resistance as it work hardens. It would finally reach the ultimate tensile strength of the plate unless some other portion of the connection would fail first.

Superimposed upon this graph in solid lines are the actual test results of this particular connection, from the paper "Welded Top Plate Beam-Column Connections" by Pray and Jensen, AWS Welding Journal, July 1956, p 338-s.

The beam lines of the particular example are shown as broken lines in the figure. Notice that the beam line at working load intersects the connection curve (point a) well within the capacity of the connection.

The second beam line at 1⅔ working load also is well within the ultimate capacity of the connection (point b).

Holding the length of the reduced portion of the top plate to L = 7″ has resulted in an end moment of M = 680 in.-kips instead of the 75% value or M = 571 in.-kips as originally planned. This is a restraint of R = 89.3% instead of R = 75%.

A lower restraint could be obtained by increasing the length of the reduced portion (L) of the top plate. However with the present connection the top plate has sufficient strength:

$$\sigma = \frac{.9\ M}{A_p\ d_b}$$ 90% M used at negative moment; (AISC Sec 1.5.1.4.1)

$$= \frac{.9(680{,}000)}{(2.06)(13.86)}$$

$$= 21{,}400 \text{ psi} < 22{,}000 \text{ psi} \quad \underline{\text{OK}}$$

(AISC Sec 1.5.1.4.1)

Notice also that the connection curve lies quite a distance above the R = 50% point of the beam line. Since the beam is designed on the basis of R = 50%, the connection could drop down to this value before the beam would be overstressed.

The moment capacity of a proposed top plate connection can be readily obtained from the nomograph, Figure 10.

2. CONNECTION BEHAVIOR UNDER ASYMMETRICAL CONDITIONS

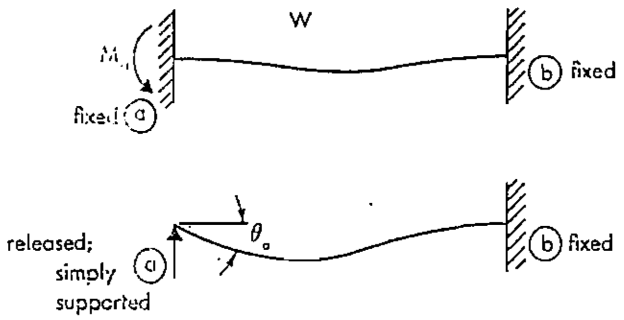
In the usual analysis of a connection made by superimposing a beam line on a connection curve, it is assumed that the beam is symmetrically loaded and has identical connections on both ends.

This is illustrated in Figure 11, where the member is a 14″ WF 43# beam, and:

W = 50 kips
L = 15 ft
I = 429 in.^{4}

When these conditions of symmetrical loading and identical connections do not exist, the following method may be used to better understand the behavior of the connection under a given load. The above beam and load value will be used.

Step 1. Start at the left end (a) of the beam with the right end (b) held fixed. The left end (a) is first held fixed ($\theta_a = 0$) and the end moment (M_a) determined; the left end is then released and simply supported ($M_a = 0$) and the end rotation (θ_a) determined. See Figure 12.



$$\theta_a = \frac{W\ L^2}{48\ E\ I} = 2.62 \times 10^{-3}$$

FIGURE 12

From these two points (M_a = 750 in.-kips and θ_a = 2.62 x 10^{-3} radians), the beam line for the left end (a) is drawn, Figure 13. Upon this is superimposed the connection line, and the point at which it intersects the beam line represents the actual end moment and end rotation after the connection has allowed the beam end to move.

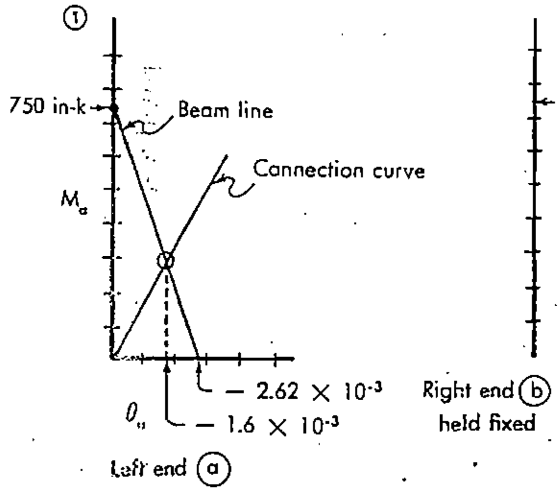


FIGURE 13

This relaxing or movement of the left end (a), from $\theta_a = 0$ to $\theta_a = 1.6 \times 10^{-3}$ radians, causes the fixed opposite end (b) to increase in end moment (M_b). This increase may be found by the following:

If a uniformly loaded beam is supported by fixed ends which have previously rotated (θ_a and θ_b), the two end moments (M_a and M_b) are—

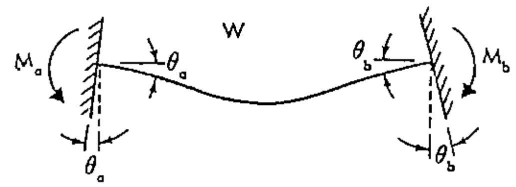


FIGURE 14

$$M_a = -\frac{4\ E\ I}{L}\ \theta_a - \frac{2\ E\ I}{L}\ \theta_b - \frac{W\ L}{12}$$

$$M_b = +\frac{2\ E\ I}{L}\ \theta_a + \frac{4\ E\ I}{L}\ \theta_b - \frac{W\ L}{12}$$

Step 2. Thus with the right end held fixed (σ_b = 0), the resulting moment at the right end (b) consisting of the initial moment and the additional moment due to movement of the left end (a), is—

where:
$\theta_a = -1.6 \times 10^{-3}$
$\theta_b = 0$

$$M_b = \frac{2\ E\ I}{L}\ \theta_a + \frac{4\ E\ I}{L}\ \theta_b - \frac{W\ L}{12}$$
$$= -979 \text{ in.-kips}$$

Now the left end (a) of the beam is held fixed at $\theta_a = -1.6 \times 10^{-3}$ while the right end (b) is released and simply supported ($M_b = 0$) and the end rotation (θ_b) determined. See Figure 15.

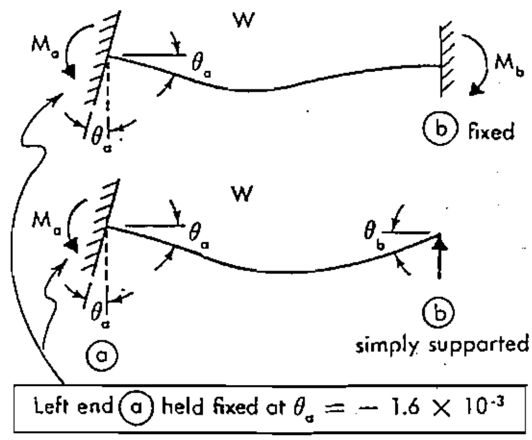


FIGURE 15

From:

$$M_b = +\frac{2\ E\ I}{L}\ \theta_a + \frac{4\ E\ I}{L}\ \theta_b - \frac{W\ L}{12}$$

when:

$M_b = 0$ and $\theta_a = -1.6 \times 10^{-3}$

the rotation of the beam at the right end (b), if simply supported and no restraint from the connection, would be:

$$\theta_b = + 3.42 \times 10^{-3}$$

These two points (M_b = —979 and $\theta_b = +3.42 \times 10^{-3}$) determine the beam line for the right end (b); Figure 16. Its intersection with the connection curve represents the actual end moment and end rotation after the connection has allowed the end to move.

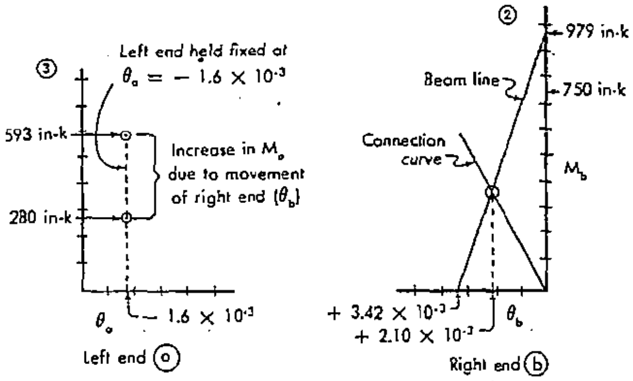


FIGURE 16

Step 3. As before, this movement of the right end (b) from $\theta_b = 0$ to $\theta_b = +2.1 \times 10^{-3}$ causes an increase in the moment on the left end (a); Figure 16, left.

From:

$$M_a = - \frac{4\,E\,I}{L}\,\theta_a - \frac{2\,E\,I}{L}\,\theta_b - \frac{W\,L}{12}$$

when:

$$\theta_a = -1.6 \times 10^{-3} \text{ and } \theta_b = +2.1 \times 10^{-3}$$

the moment on the left end (a) is found to be

$$M_a = -593 \text{ in.-kips}$$

This entire procedure is repeated until the corrections become very small, Figures 17 and 18.

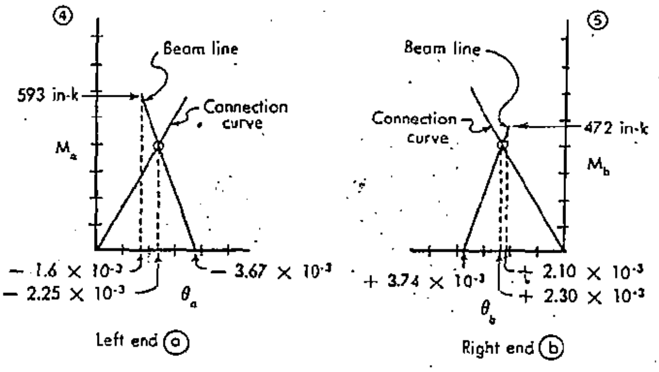


FIGURE 17

Step 4. When the left end (a) is simply supported ($M_a = 0$), the end rotation would be $\theta_a = -3.67 \times 10^{-3}$. Releasing the left end (a) allows it to rotate to $\theta_a = -2.25 \times 10^{-3}$.

Step 5. This movement θ_a from -1.6×10^{-3} to -2.25×10^{-3} on the left end causes the right moment to increase to M_b = —472 in.-kips. When the right end (b) is simply supported ($M_b = 0$), the end rotation would be $\theta_b = +3.74 \times 10^{-3}$. Releasing the right end (b) allows it to rotate to $\theta_b = +2.3 \times 10^{-3}$.

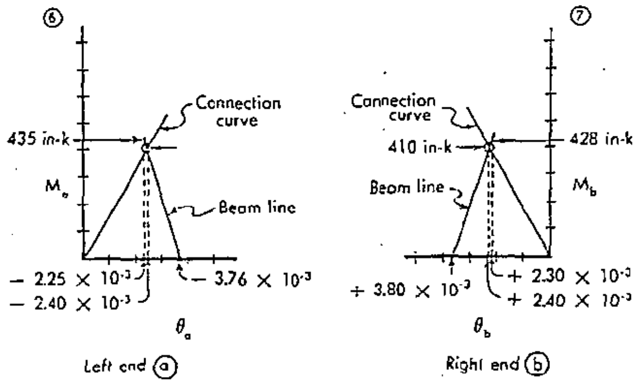


FIGURE 18

Step 6. This movement of θ_b from $+2.1 \times 10^{-3}$ to

+2.3 x 10^{-3} on the right end causes the left moment to increase to M_a = —435 in.-kips. When the left end (a) is simply supported (M_a = 0), the end rotation would be θ_a = —3.76 x 10^{-3}. Releasing the left end (a) allows it to rotate to θ_a = —2.40 x 10^{-3}.

Step 7. This movement of θ_a from —2.25 x 10^{-3} to —2.40 x 10^{-3} on the left end causes the right moment to increase to M_b = —428 in.-kips. When the right end (b) is simply supported (M_b = 0), the end rotation would be θ_b = +3.80 x 10^{-3}. Releasing the right end (b) allows it to rotate to: θ_b = +2.40 x 10^{-3}.

Conclusion: The final end conditions resulting from this sequential handling of the given connection and beam loading are—

M_e = —410 in.-kips

θ_e = 2.40 x 10^{-3} radians

Reference to Figure 11 shows that these are the same values as obtained when the beam was considered to be symmetrically loaded with identical conditions on both ends.

3. BEHAVIOR OF CONNECTIONS STRESSED ABOVE YIELD

The same method used previously may also be applied to connections that are stressed above their yield points and thus yield plastically. See Figure 19, using same beam as before.

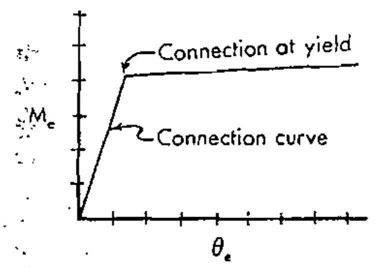


FIGURE 19

To simplify this analysis, two changes will be made.

First. In computing the two points of the beam line (M_e) for fixed ends and (θ_e) for this end simply supported, it is noticed that these same values can be obtained by considering the beam as fixed at one end and supported at the other, with no gravity load. A moment (M_e) is applied at the supported end and the resulting end rotation (θ_e) is found at this same end, Figure 20.

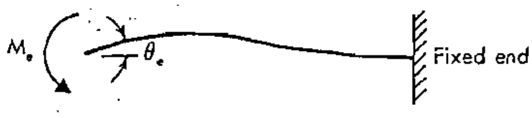


FIGURE 20

Here:

$$\theta_e = \frac{M_e\ L}{4\ E\ I} \quad \text{or} \quad \frac{M_e}{\theta_e} = \frac{4\ E\ I}{L}$$

In this particular example:

$$\frac{M_e}{\theta_e} = \frac{4\ E\ I}{L} = \frac{4(30 \times 10^6)(429)}{(180)} = 286 \times 10^6$$

With the particular scale used in the original construction of Figure 19,

1″ = 4 x 10^{-3} radians

or 1 radian = ¼ x 10^3 inch

and 1″ = 400 in-kips = 400,000 in.-lbs

or 1 in.-lb = ¼ x 10^{-5} inch

The slope of this beam line is—

$$\frac{M_e}{\theta_e} = 286 \times 10^6 \frac{\text{inch-lbs}}{\text{radians}} = \frac{286 \times 10^6(\frac{1}{4} \times 10^{-5})}{(\frac{1}{4} \times 10^3)} = 2.86$$

or an angle of 70.7°, Figure 21.

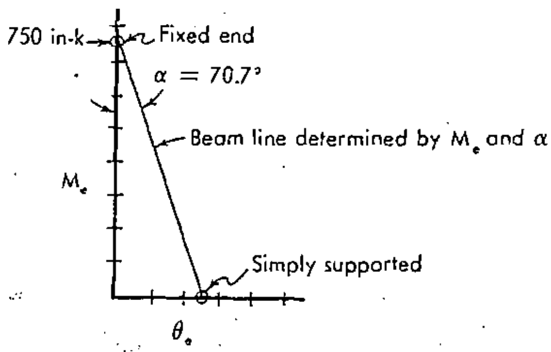


FIGURE 21

Another method of constructing this slope is to use a convenient value of θ_e; for example, θ_e = 5 x 10^{-3}. The corresponding end moment would be—

$$M_e = (286 \times 10^6)\ \theta_e = (286 \times 10^6)(5 \times 10^{-3}) = 1430 \text{ in.-kips}$$

These two values are plotted on the figure and the slope determined by protractor; Figure 22.

Since the slope of the beam line remains constant, it won't be necessary to compute the value of θ_e for the simply supported end for each step.

Second. Instead of computing the end moment after it has been increased by the angle movement on the other end of the beam, it is seen that the actual increase in moment is—

$$M_x = \frac{2\,E\,I}{L}\,\theta_x$$

This may be drawn on the figure from any convenient value of θ_e and M_e. Any given increase in θ_x is laid off horizontally on this line, and the increase in moment (M_x) is measured off as the vertical distance and added to the moment on the opposite end of the beam. See Figure 23.

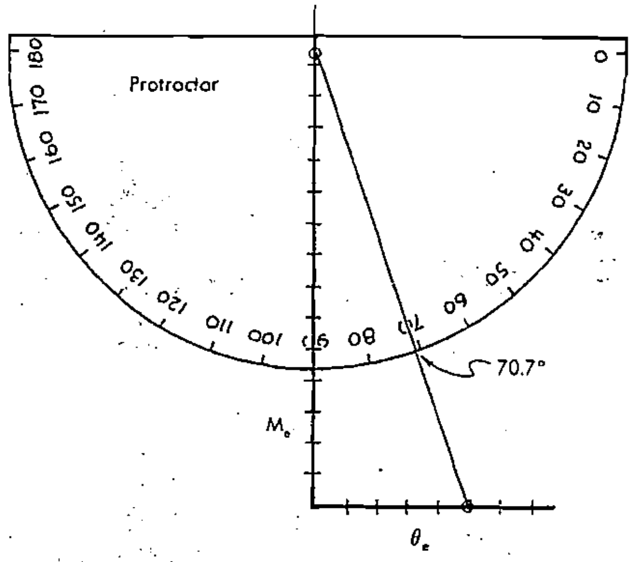


FIGURE 22

Application of Method

This method is now used on the same 14" WF 43# beam, uniformly loaded with 50 kips on a 15-ft span; Figure 24. The connection is made with a top connecting plate, 5/16" x 3", which is stressed to yield (σ = 33,000 psi) at a moment of 423 in.-kips.

With additional movement, the plate will strain harden and its resisting moment will very gradually increase. This accounts for the slight rise in the connection line above the point of initial yield.

FIGURE 23

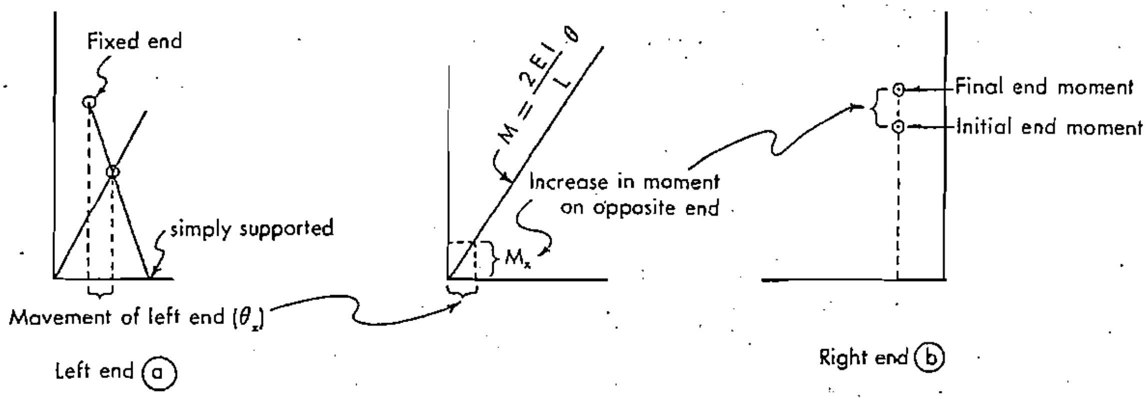
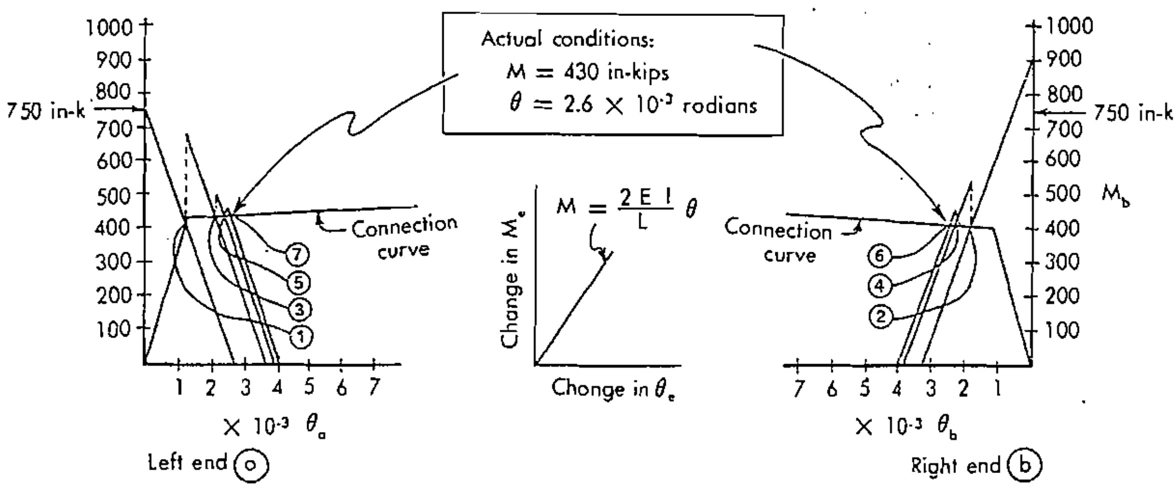


FIGURE 24



On the Ainsley Building in Miami, weldor is completing fillet weld on top connecting plate, leaving an unwelded length 1.2 times the plate width. Plate is beveled and groove welded to the column.