Analysis of Compression

1. COMPRESSIVE STRESS

Compressive loading of a member when applied (axially) concentric with the center of gravity of the member's cross-section, results in compressive stresses distributed uniformly across the section. This compressive unit stress is —

$$\boxed{\sigma_c = \frac{P}{A}} \quad \text{.............................(1)}$$

A *short* column (slenderness ratio L/r equal to about unity or less) that is overloaded in compression may fail by crushing. From a design standpoint, short compression members present little problem. It is important to hold the compressive unit stress within the material's compressive strength.

For steel, the yield and ultimate strengths are considered to be the same in compression as in tension.

Any holes or openings in the section in the path of force translation will weaken the member, unless such openings are completely filled by another member that will carry its share of the load.

Excessive compression of *long* columns may cause failure by buckling. As compressive loading of a long column is increased, it eventually causes some eccentricity. This in turn sets up a bending moment, causing the column to deflect or buckle slightly. This deflection increases the eccentricity and thus the bending moment. This may progress to where the bending moment is increasing at a rate greater than the increase in load, and the column soon fails by buckling.

2. SLENDERNESS RATIO

As the member becomes longer or more slender, there is more of a tendency for ultimate failure to be caused by buckling. The most common way to indicate this tendency is the slenderness ratio which is equal to —

$$\frac{L}{r}$$

where L = unsupported length of member

r = the least radius of gyration of the section

and—

$$\boxed{r = \sqrt{\frac{I}{A}}} \quad \text{.............................(2)}$$

If the member is made longer, using the same cross-section and the same compressive load, the resulting compressive stress will remain the same, although the tendency for buckling will increase. The slenderness ratio increases as the radius of gyration of the section is reduced or as the length of the member is increased. The allowable compressive load which may be applied to the member decreases as the slenderness ratio increases.

The various column formulas (Tables 3 and 4) give the allowable average compressive stress ($\bar{\sigma}$) for the column. They do not give the actual unit stress developed in the column by the load. The unit stress resulting from these formulas may be multiplied by the cross-sectional area of the column to give the allowable load which may be supported.

3. RADIUS OF GYRATION

The radius of gyration (r) is the distance from the neutral axis of a section to an imaginary point at which the whole area of the section could be concentrated and still have the same amount of inertia. It is found by the expression: $r = \sqrt{I/A}$.

In the design of unsymmetrical sections to be used as columns, the least radius of gyration (r_{min}) of the section must be known in order to make use of the slenderness ratio (L/r) in the column formulas.

If the section in question is not a standard rolled section the properties of which are listed in steel handbooks, it will be necessary to compute this least radius of gyration. Since the least radius of gyration is —

$$\boxed{r_{min} = \sqrt{\frac{I_{min}}{A}}} \quad \text{.........................(3)}$$

the minimum moment of inertia of the section must be determined.

Minimum Moment of Inertia

The maximum moment of inertia (I_{max}) and the minimum moment of inertia (I_{min}) of a cross-section are

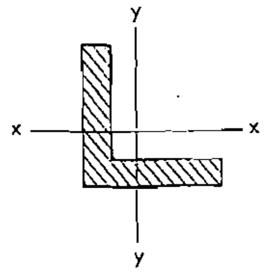


FIGURE 1

found on principal axes, 90° to each other.

$$\boxed{I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}} \quad \ldots (4)$$

Knowing I_x, I_y, and I_{xy} it will be possible to find I_{min}.

Problem 1

Locate the (neutral) x-x and y-y axes of the offset T section shown in Figure 2:

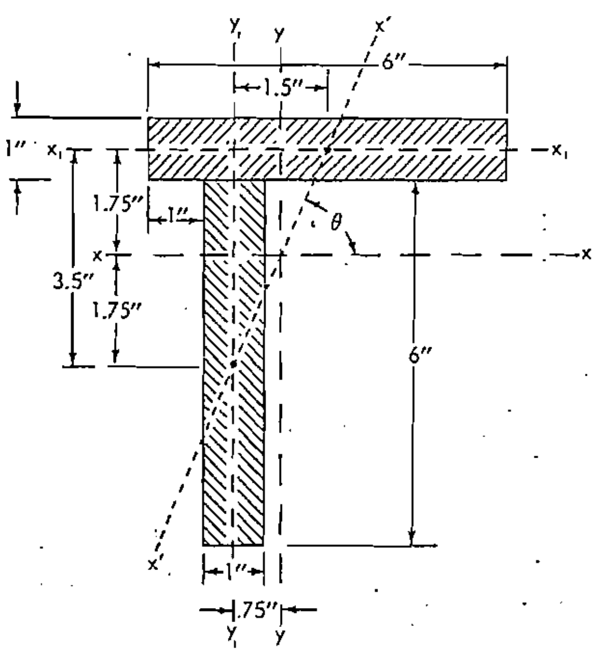


FIGURE 2

to locate neutral axis x-x:

	A	d	M
6" × 1"	6.0	0	0
1" × 6"	6.0	− 3.5	− 21.0
Total →	12.0		− 21.0

where d = distance from center of gravity of element area to parallel axis (here: x_1-x_1)

and, applying formula #1 from Section 2.3, the distance of neutral axis x-x from its parallel axis x_1-x_1 is —

$$NA_{x\text{-}x} = \frac{\Sigma M}{\Sigma A} = \frac{-21.0}{12.0} = -1.75''$$

to locate neutral axis y-y:

	A	d	M
1" × 6"	6.0	+ 1.5	+ 9.0
6" × 1"	6.0	0	0
Total →	12.0		+ 9.0

$$NA_{y\text{-}y} = \frac{\Sigma M}{\Sigma A} = \frac{+9.0}{12.0} = +.75''$$

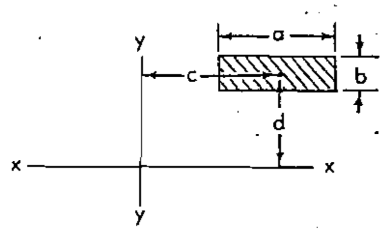
product of inertia

It will be necessary to find the product of inertia (I_{xy}) of the section. This is the area (A) times the product of distances d_x and d_y as shown in Figure 3.

(See Figure 3 on facing page).

In finding the moment of inertia of an area about a given axis (I_x or I_y), it is not necessary to consider the signs of d_x or d_y. However, in finding the product of inertia, it is necessary to know the signs of d_x and d_y because the product of these two could be either positive or negative and this will determine the sign of the resulting product of inertia. The total product of inertia of the whole section, which is the sum of the values of the individual areas, will depend upon these signs. Areas in diagonally opposite quadrants will have products of inertia having the same sign.

The product of inertia of an individual rectangular area, the sides of which are parallel to the x-x and y-y axes of the entire larger section is —



$$\boxed{I_{xy} = a\,b\,c\,d}$$

FIGURE 4

where:

a and b = dimensions of rectangle (= A)

d and c = distance of area's center of gravity to the x-x and y-y axes (= d_x and d_y)

The product of inertia of a T or angle section is — (See Figure 5).

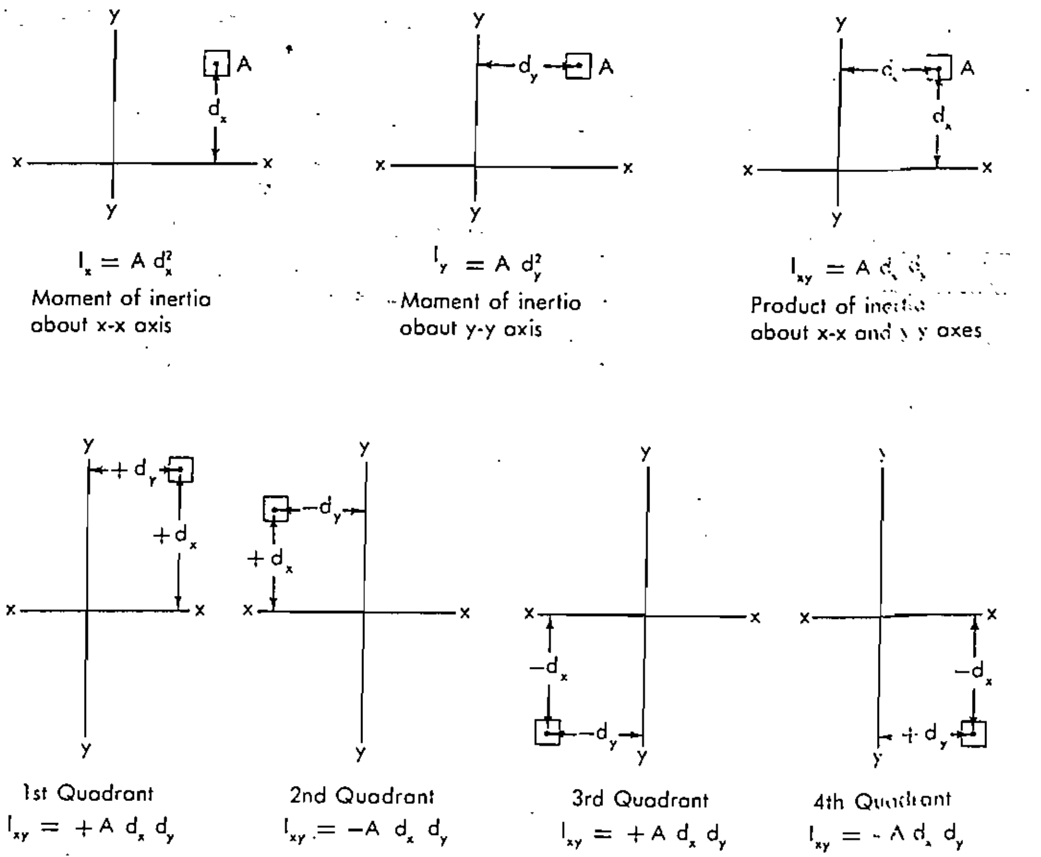
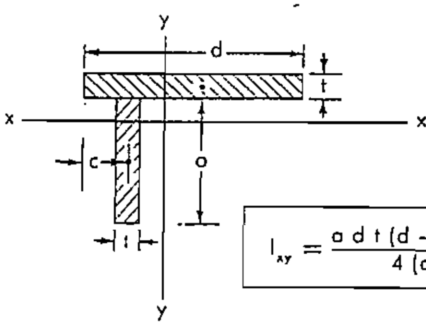


FIGURE 3



$$I_{xy} = \frac{a\,d\,t\,(d - 2c)\,(a + t)}{4\,(a + d)}$$

FIGURE 5

Here, determine sign by inspection.

Problem 2

Determine the product of inertia of this offset T section about the x-x and y-y axes:

$$\begin{aligned} I_{xy} &= \Sigma A\;(d_x)(d_y) \\ &= 2.5\;(+1)(+.555) + 2\;(-1.25)(-.695) \\ &= +1.388 + 1.737 \\ &= \underline{+\;3.125\ \text{in.}^4} \end{aligned}$$

Now use formula given previously for product of inertia of such a section:

$$\begin{aligned} I_{xy} &= \frac{a\,d\,t\,(d - 2c)(a + t)}{4\,(a + d)} \\ &= \frac{(4)(5)(\tfrac{1}{2})(5 - 2.5)(4 + \tfrac{1}{2})}{4\,(4 + 5)} \\ &= \underline{+\;3.125\ \text{in.}^4} \end{aligned}$$

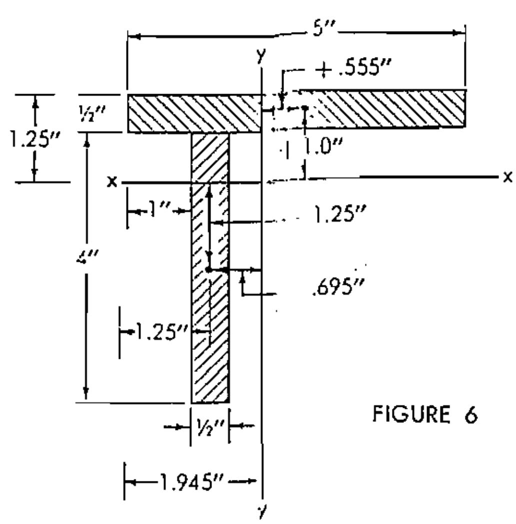


FIGURE 6

Problem 3

Determine the minimum radius of gyration of the offset T section shown previously (Fig. 2) and repeated here:

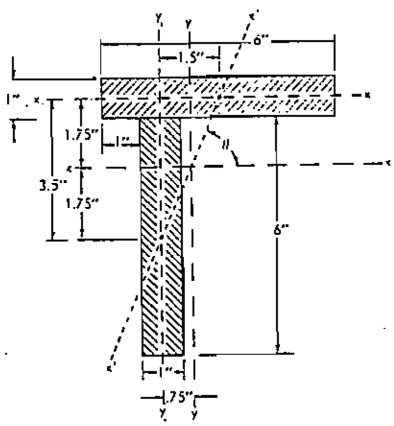


FIGURE 7

moment of inertia about axis x-x

	A	d	M	I	I_g
6″ × 1″	6.0	0	0	0	.50
1″ × 6″	6.0	− 3.5	− 21.0	+ 73.5	18.00
Total →	12.0		− 21.0	+ 92.00	

$$NA_{x\text{-}x} = \frac{\Sigma M}{\Sigma A} = \frac{-21.0}{12.0} = -1.75'' \text{ and}$$

$$I_x = I - \frac{M^2}{A} = 92.00 - 36.75 = 55.25 \text{ in.}^4$$

moment of inertia about axis y-y

	A	d	M	I	I_g
1″ × 6″	6.0	+ 1.5	+ 9.0	13.5	18.00
6″ × 1″	6.0	0	0	0	.50
Total →	12.0		+ 9.0	+ 32.00	

$$NA_{y\text{-}y} = \frac{\Sigma M}{\Sigma A} = \frac{+9.0}{12.0} = +.75'' \text{ and}$$

$$I_y = I - \frac{M^2}{A} = 32.00 - 6.75 = 25.25 \text{ in.}^4$$

product of inertia

$$I_{xy} = \Sigma A\ (d_x)(d_y)$$
$$= (1 \times 6)(+1.75)(+.75) + (1 \times 6)(-1.75)(-.75)$$
$$= +15.75 \text{ in.}^4$$

minimum moment of inertia

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x + I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{55.25 + 25.25}{2} - \sqrt{\left(\frac{55.25 - 25.25}{2}\right)^2 + (15.75)^2}$$

$$= 40.25 - 21.75$$
$$= 18.50 \text{ in.}^4$$

minimum radius of gyration

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$
$$= \sqrt{\frac{18.50}{12.0}} = \sqrt{1.542}$$
$$= \underline{1.24''}$$

As a matter of interest, this r_{min} is about axis x′-x′, the angle (θ) of which is—

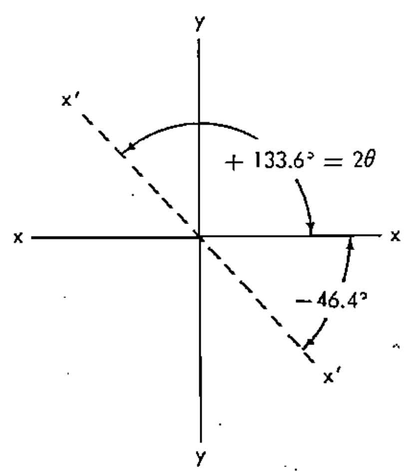
$$\tan 2\theta = -\frac{2\ I_{xy}}{I_x - I_y} \qquad \text{(See sketch below).}$$

$$= -\frac{2\ (15.75)}{55.25 - 25.25} = -1.05$$

$$2\theta = -46.4° \text{ or } +133.6°$$

and $\theta = +66.8°$

Any ultimate buckling could be expected to occur about this axis (x′-x′).



Problem 4

The channel section, Figure 8, is to be used as a column. Determine its radius of gyration about its x-x axis.

Using the conventional formulas for the properties of the section—

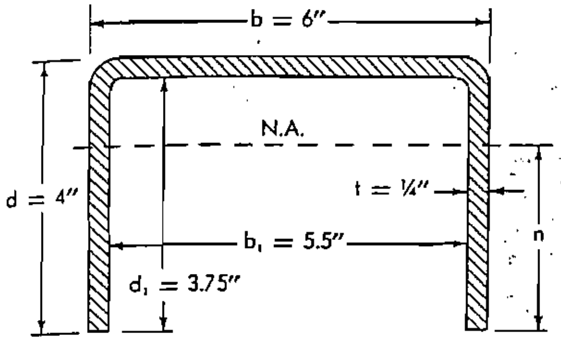


FIGURE 8

area of the section

$$A = bd - b_1d_1 = (6)(4) - (5.5)(3.75)$$
$$= 3.375 \text{ in.}^2$$

distance of neutral axis

$$n = d - \frac{2\, d^2t + b_1t^2}{2\, db - 2\, b_1d_1}$$
$$= 4 - \frac{2(4)^2(.25) + (5.5)(.25)^2}{2(4)(6) - 2(5.5)(3.75)}$$
$$= 2.764''$$

moment of inertia

$$I = \frac{2\, d^3t + b_1t^3}{3} - A(d - n)^2$$
$$= \frac{2(4)^3(.25) + (5.5)(.25)^3}{3}$$
$$- 3.375\ (4 - 2.764)^2$$
$$= 5.539''$$

radius of gyration

$$r = \sqrt{\frac{I}{A}}$$
$$= \sqrt{\frac{5.539}{3.375}}$$
$$= \underline{1.281''}$$

If a slide rule had been used, assuming a possible error of ± one part in 1000 for every operation, this answer could be as high as 1.336" and as low as 1.197". This represents an error of + 4.3% and − 6.6%. For this reason it is necessary, when using these conventional formulas, to make use of logarithms or else do the work longhand. To do this requires about 30 minutes.

The radius of gyration will now be found directly, using the properties of thin sections, treating them as a line. See Table 2. Section 2.2.

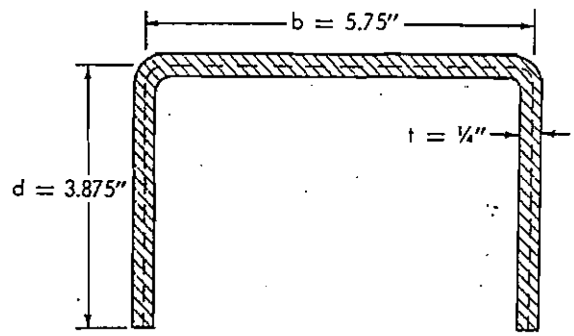


FIGURE 9

Mean dimensions b and d are used, Figure 9.

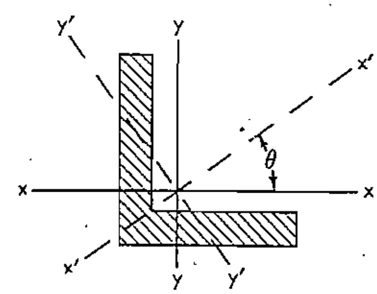
$$r_x = \frac{\sqrt{d^3/3(2b + d)}}{b + 2d}$$
$$= \frac{\sqrt{3.875^3/3(2 \times 5.75 + 3.875)}}{5.75 + 2(3.875)}$$
$$= \underline{1.279''}$$

The exact value obtained from this formula for r is 1.279". The value obtained by using the conventional formula is 1.281".

Assuming a possible error of ± one part in 1000 for every operation of the slide rule, it would be possible to get an answer as high as 1.283" and as low as 1.275". This represents an error of about ¼ of the error using the conventional formulas with slide rule. The time for this last calculation was 2 minutes.

Moment of Inertia About Any Axis

FIGURE 10



Sometimes (as in Problem 3) the moment of inertia of a section is needed about an axis lying at an angle (θ) with the conventional x-x axis. This may be found by using the product of inertia (I_{xy}) of the section about the conventional axes (x-x and y-y) with the moments of inertia (I_x) and (I_y) about these same axes in the following formula:

$$\boxed{I_x' = I_x\cos^2\theta + I_y\sin^2\theta - I_{xy}\sin^2\theta} \quad \ldots\ldots\ldots(7)$$

$$\boxed{I_y' = I_x\sin^2\theta + I_y\cos^2\theta - I_{xy}\sin^2\theta} \quad \ldots\ldots\ldots(8)$$

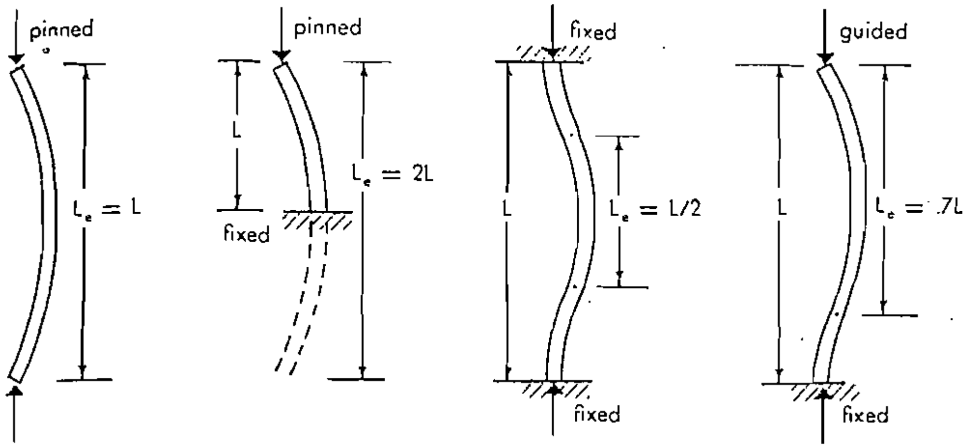


FIGURE 11

4. CRITICAL COMPRESSIVE STRESS

The critical load on a column as given by the Euler formula is —

$$\boxed{P_{cr} = \frac{\pi^2 E I}{L_e^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (9)$$

where L_e = effective length of column.

This can be changed into terms of average critical stress by dividing by the cross-sectional area of the column. Since $A = I/r^2$, this becomes —

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (10)$$

Because this formula gives excessively high values for short columns, Engesser modified it by substituting the tangent modulus (E_t) in place of the usual Young's modulus of elasticity (E).

The modified formula then becomes —

$$\boxed{\sigma_{cr} = \frac{\pi^2 E_t}{(L_e/r)^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots (11)$$

where:

E_t = tangent modulus of elasticity, corresponding to the modulus of elasticity when stressed to σ_{cr}.

r = least radius of gyration of the cross-section

L_e = effective length of the column, corresponding to the length of a pinned column that would have the same critical load. See Figure 11.

The Engesser formula is also called the Tangent Modulus formula and checks well with experimental values.

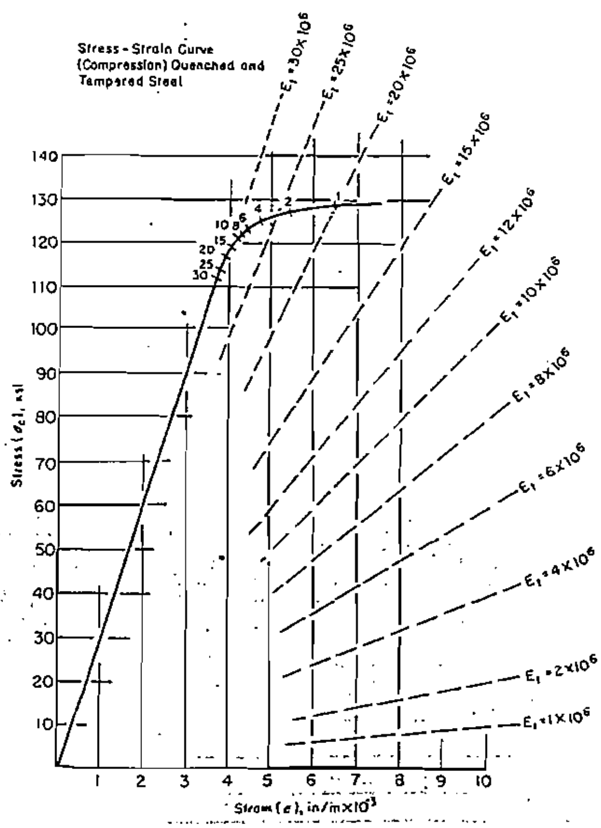


FIGURE 12

5. TANGENT MODULUS

Use of the Tangent Modulus formula necessitates a stress-strain curve (preferably in compression) of the material. See Figure 12, stress-strain curve for a quenched and tempered steel in compression. Whereas the usual Young's modulus of elasticity represents a fixed value for steel (30×10^6) according to the ratio

Slenderness Ratios: Quenched & Tempered Steel

TABLE 1

σ_c	E_t	L_e/r
110,000	30.2×10^6	52.1
112,000	30.0	51.4
114,000	26.5	47.9
116,000	22.0	43.4
118,000	17.5	38.3
120,000	13.0	32.7
122,000	9.0	27.0
124,000	5.5	20.9
126,000	3.3	16.1
128,000	1.5	10.8

Engesser portion of curve (inelastic bending)

TABLE 2

L_e/r	E_t	σ_c
50	30.2×10^6	119,500
60	30.2	82,900
70	30.2	60,900
75	30.2	53,000
80	30.2	46,600
90	30.2	36,800
100	30.2	29,850
110	30.2	27,700
125	30.2	19,100
140	30.2	15,200

Euler portion of curve (elastic bending)

of stress to strain below the proportional limit, the tangent modulus of elasticity takes into consideration the changing effect of plastic strain beyond this point corresponding to the actual stress involved.

Notice, in Figure 12, the broken lines representing the slope for various values of tangent modulus of elasticity (E_t), in this case from 1×10^6 psi up to 30×10^6. The compressive stress level (σ_c) at which a given E_t value applies is determined by moving out parallel from that reference modulus line (dotted), by means of parallel rule or other suitable device, until the stress-strain curve is intersected at one point only. The line is tangent at this point.

The compressive stress-strain curve for any material can be superimposed on this graph and the values of E_t at a given stress level (σ_c) read by the same technique.

The values of tangent modulus (E_t) for quenched and tempered steel, as read from Figure 12, are now plotted against the corresponding compressive stress (σ_c). This is shown in Figure 13.

The Engesser or tangent modulus formula for critical stress (σ_{cr}) is then put into the following form —

$$\boxed{\frac{L_e}{r} = \pi \sqrt{\frac{E_t}{\sigma_{cr}}}} \quad \ldots\ldots\ldots\ldots (12)$$

Tangent Modulus for Quenched and Tempered Steel

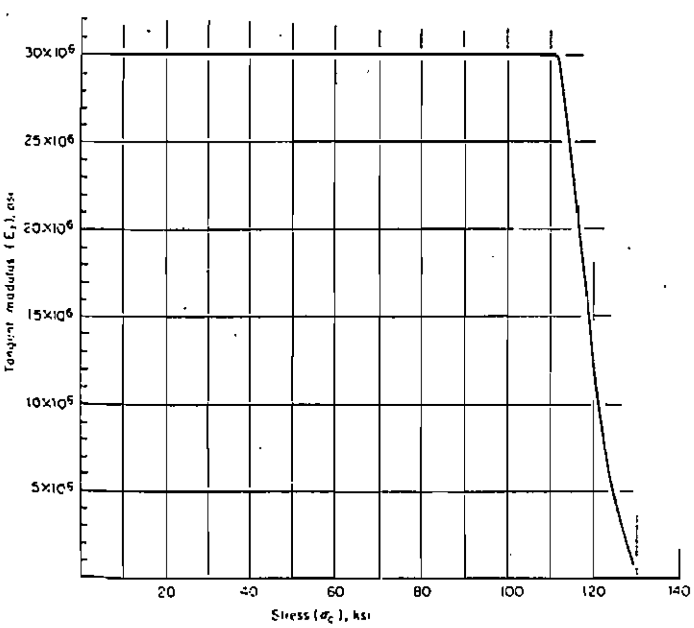


FIGURE 13

Resulting Critical Compressive Stress for Quenched and Tempered Steel (A suitable factor of safety must be applied to these values)

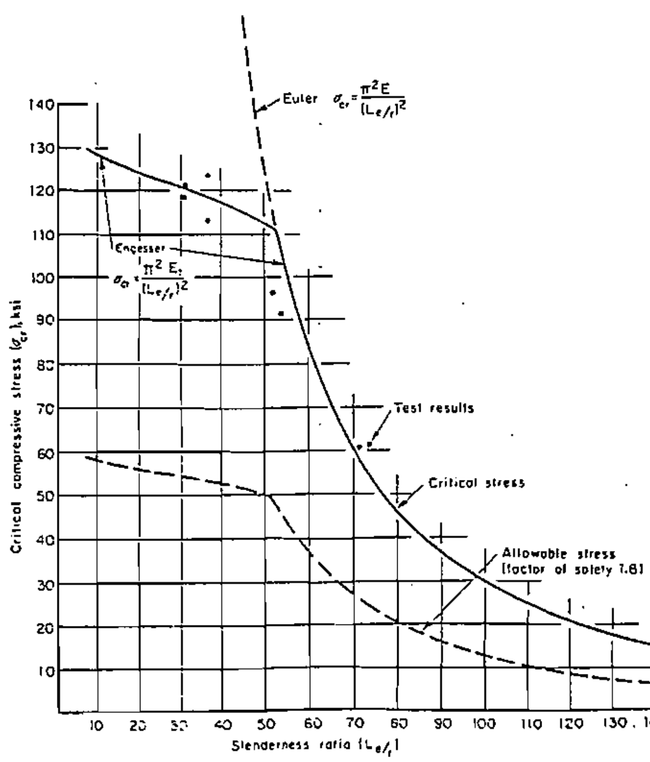


FIGURE 14

and the critical slenderness ratio (L_e/r) is determined for various values of stress (σ_e), resulting in Tables 1 and 2 for quenched and tempered steel only.

Table 1 gives corresponding values of slenderness ratio (L_e/r) for given values of stress (σ_e) above the proportional limit of a quenched and tempered steel.

Below the material's proportional limit, the use of Young's modulus (E) or tangent modulus (E_t) provide the same value. Table 2 for quenched and tempered steel gives the slenderness ratio (L_e/r) for stress levels (σ_e) within the proportional portion of the stress-strain curve. Since the original Euler formula for σ_{cr} applies here, this portion of the curve is often called the Euler curve.

6. PLOTTING ALLOWABLE STRESS CURVE

These values from Tables 1 and 2 are now plotted to form the curve in Figure 14. The Euler portion of the curve is extended upward by a broken line to indicate the variance that would be obtained by continuing to use the Euler formula beyond the proportional limit. This must be kept in mind in designing compression members having a low slenderness ratio (L/r).

A few test results are also shown to indicate the close relationship between the Tangent Modulus formula and actual values.

Note that a corresponding curve has been plotted below the main curve, representing the allowable stress ($\underline{\sigma}$) after applying a factor of safety of 1.8.

TABLE 3—Allowable Compressive Stress (AISC)

Range of $\frac{L_e}{r}$ Values	Average Allowable Compressive Unit Stress ($\underline{\sigma}$)
0 to C_c	$\underline{\sigma} = \left[1 - \dfrac{\left(\frac{KL}{r}\right)^2}{2C_c^2}\right]\dfrac{\sigma_y}{F.S.}$
C_c to 200	$\underline{\sigma} = \dfrac{149{,}000{,}000}{\left(\frac{KL}{r}\right)^2} = \left(\dfrac{12{,}210}{\frac{KL}{r}}\right)^2$

where:

$$C_c = \sqrt{\frac{2\,\pi^2\,E}{\sigma_y}}$$

$$F.S. = \frac{5}{3} + \frac{3}{8}\left(\frac{\frac{KL}{r}}{C_c}\right) - \frac{1}{8}\left(\frac{\frac{KL}{r}}{C_c}\right)^3$$

For very short columns, this factor of safety (F.S.) is equal to that of members in tension (F.S. = 1.67). For longer columns, the safety of factor increases gradually to a maximum of F.S. = 1.92.

K = effective length factor

7. BASIC FORMULAS FOR COMPRESSION MEMBERS

In "Buckling Strength of Metal Structures," page 53, Bleich introduces a parabolic formula to express this tangent modulus curve for compression. By applying a factor of safety (F.S.), this becomes the allowable compressive stress. The basic parabolic formula thus modified is —

$$\boxed{\underline{\sigma} = \frac{\sigma_y}{F.S.} - \frac{\sigma_p(\sigma_y - \sigma_p)}{\pi^2\,E\,F.S.}\left(\frac{L_e}{r}\right)^2} \quad \ldots\ldots(13)$$

E = modulus of elasticity

σ_p = proportional limit

σ_y = yield point

F.S. = factor of safety

Any residual compressive stress (σ_{rc}) in the member tends to lower the proportional limit (σ_p), or straight-line portion of the stress-strain curve in compression, without affecting the yield point. For the purpose of the above formula, it is assumed that

$$\sigma_p = \sigma_y - \sigma_{rc}$$

Also assuming this value of residual compressive stress is about half of the yield point, or $\sigma_{rc} = ½\,\sigma_y$, Formula #13 becomes:

$$\boxed{\underline{\sigma} = \frac{\sigma_y}{F.S.} - \frac{\sigma_y^2}{4\,\pi^2\,E\,F.S.}\left(\frac{L_e}{r}\right)^2} \quad \ldots\ldots(14)$$

This formula provides a parabolic curve, starting at a slenderness ratio of ($L_e/r = 0$) with values at yield stress (σ_y), and extending down to one-half of this stress where it becomes tangent with the Euler curve at the upper limit of elastic bending.

The slenderness ratio at this point is:

$$\boxed{\frac{L_e}{r} = \sqrt{\frac{2\,\pi^2\,E}{\sigma_y}} = \frac{23{,}925}{\sqrt{\sigma_y}} \text{ for steel}} \quad \ldots(15)$$

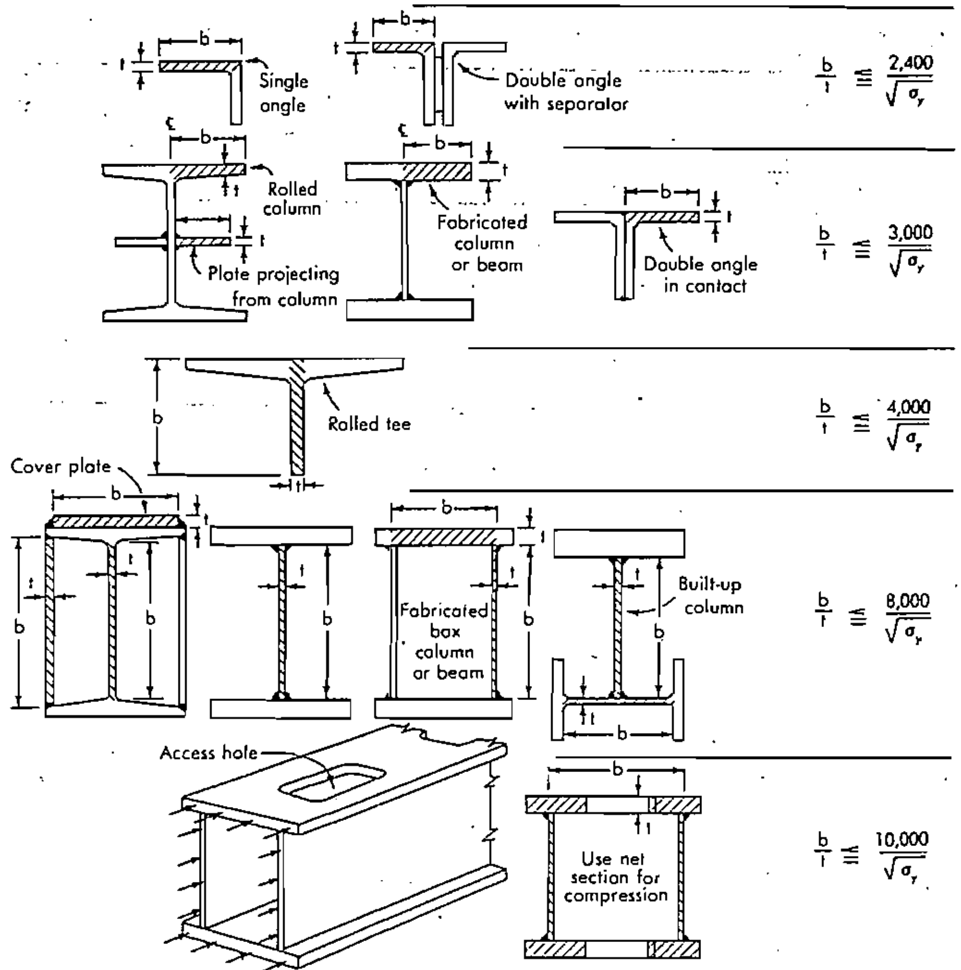
Above this slenderness ratio, the Euler formula is used:

$$\boxed{\underline{\sigma} = \frac{\pi^2\,E}{F.S.\left(\frac{L_e}{r}\right)^2} = \frac{1}{F.S.}\left[\frac{16{,}918}{\frac{L_e}{r}}\right]^2 \text{ for steel}} \quad (16)$$

MAXIMUM WIDTH-TO-THICKNESS RATIOS

For Elements of Members Under Axial Compression or Compression Due to Bending

Adapted from 1961 AISC, Sec. 1.9.1 and 1.9.2



The above ratios of b/t may be exceeded if, by using in the calculations a width equal to the maximum of these limits, the compressive stress value obtained is within the allowable stress.

FIGURE 15

8. AISC FORMULAS FOR COMPRESSION MEMBERS

The AISC has incorporated (1963) these basic column formulas endorsed by the Column Research Council Report in its specifications for structural buildings.

The slenderness ratio where the Euler and parabolic portions of the curve intersect, Formula 15, has been designated in the AISC Specification as (C_c). This is also incorporated into Formula 13.

AISC uses a value of $E = 29{,}000{,}000$ psi (instead of the usual 30,000,000 psi) for the modulus of elasticity of steel. For the Euler portion of the curve, Formula 16, AISC uses a factor of safety of 1.92.

The resulting new AISC column formulas are shown in Table 3.

Tables 6 through 14 give the AISC compression allowables for several strengths of structural steel.

For various conditions of column cross-section, Figure 15, there is a limiting ratio of element width to thickness (b/t). This ratio is expressed as being equal to or less than ($\leqq$) a certain value divided by the square root of the material's yield strength. The related Table 4 permits direct reading of a compression element's b/t ratio for various yield strengths of steel.

At times it may be desirable to exceed the limiting b/t ratio of an element. This can be done if, in the calculations, substituting the shorter maximum width allowed (by the Fig. 15 limits) would give a compressive unit stress value within the allowable stress.

To help in visualizing relative savings in metal by the use of higher-strength steels, Figure 16 indicates the allowable compressive strength (σ) obtained from the Table 3 formulas for 8 different yield strengths. Notice that the advantage of the higher strengths drops off as the column becomes more slender.

TABLE 4—Limiting b/t Ratios of Section Elements Under Compression

Limits of Ratio of Width to Thickness of Compression Elements for Different Yield Strengths of Steel

Fig. 15 Ratio \ σ_Y	33,000	36,000	42,000	45,000	46,000	50,000	55,000	60,000	65,000	90,000*	95,000*	100,000*
$\frac{2{,}400}{\sqrt{\sigma_Y}}$	13.2	12.6	11.7	11.3	11.2	10.7	10.2	9.8	9.4	8.0	7.8	7.6
$\frac{3{,}000}{\sqrt{\sigma_Y}}$	16.5	15.8	14.6	14.1	14.0	13.4	12.8	12.2	11.8	10.0	9.7	9.5
$\frac{4{,}000}{\sqrt{\sigma_Y}}$	22.0	21.0	19.5	18.9	18.7	17.9	17.1	16.3	15.7	13.3	13.0	12.6
$\frac{8{,}000}{\sqrt{\sigma_Y}}$	44.0	42.1	39.0	37.7	37.3	35.8	34.1	32.6	31.4	26.6	25.9	25.3
$\frac{10{,}000}{\sqrt{\sigma_Y}}$	55.0	52.6	48.7	47.1	46.6	44.7	42.6	40.8	39.2	33.4	32.4	31.6

Round off to the nearest whole number.
* Quenched and tempered steels: yield strength at 0.2% offset.

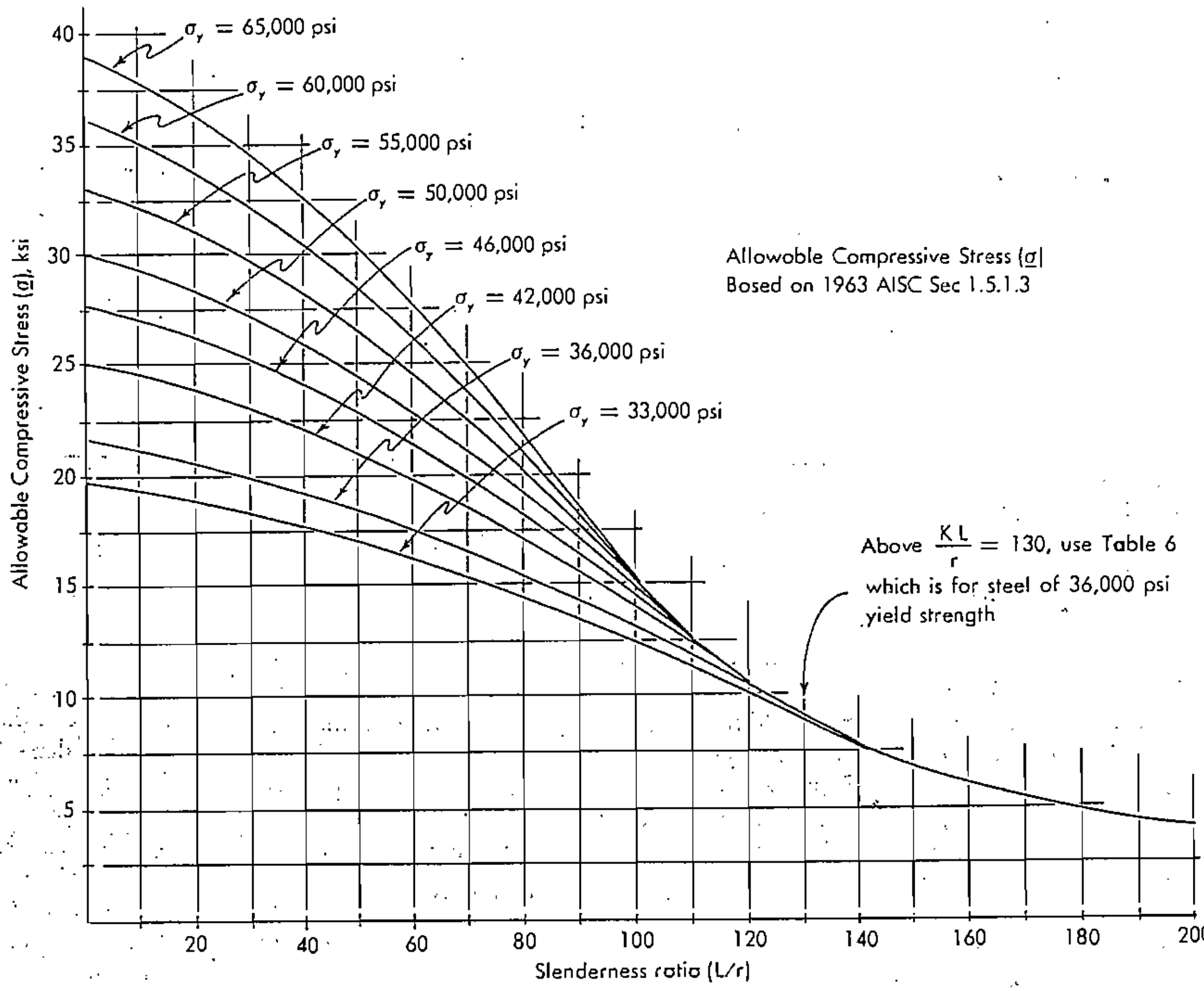


FIGURE 16

If the allowable stress curve of quenched and tempered steel (Fig. 14) were now superimposed on this graph, the even greater strength advantage of quenched and tempered steel at lower slenderness ratios would be readily apparent.

The allowable compressive unit stress ($\underline{\sigma}$) for a given slenderness ratio (KL/r), from unity through 200, is quickly read from Tables 6 through 14 for steels of various yield strengths.

Above KL/r of 130, the higher-strength steels offer no advantage as to allowable compressive stress ($\underline{\sigma}$). Above this point, use Table 7 for the more economical steel of 36,000 psi yield strength.

9. OTHER FORMULAS FOR COMPRESSION MEMBERS

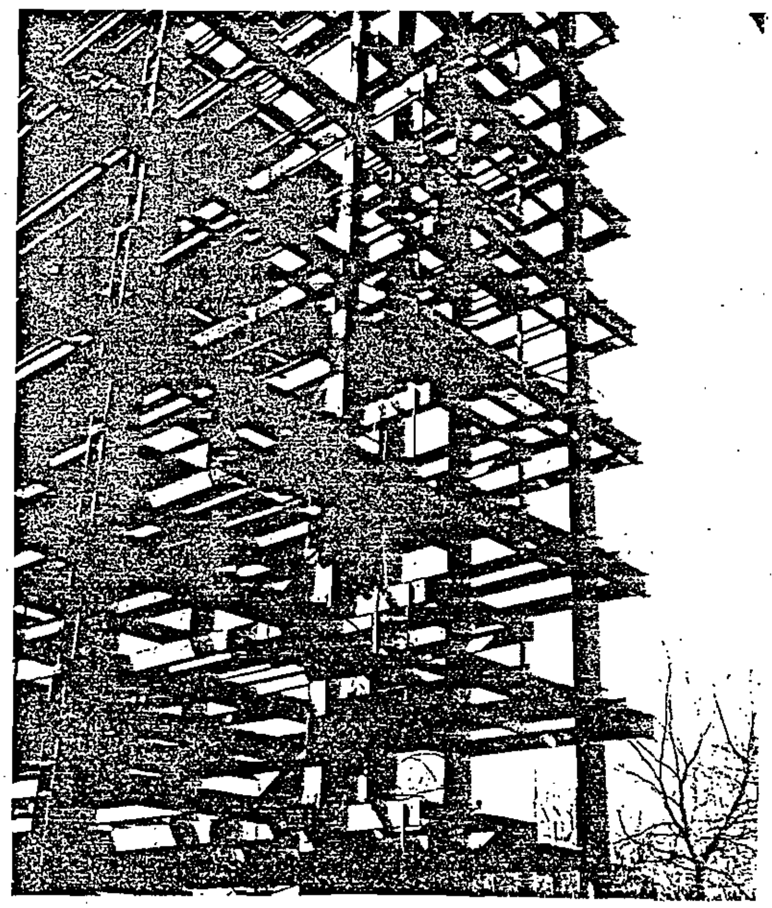
Table 5 gives the AASHO formulas, which are applicable to bridge design.

As a matter of general interest, the column formula established for use of quenched and tempered steel on the Carquinez Strait Bridge (California) is —

$$\boxed{\sigma = 36{,}000 - 1.75 \left(\frac{L}{r}\right)^2}$$

TABLE 5—AASHO Allowable Stress for Compression Members *Having Rigid Ends and Concentric Loads*

A-7 and A-373	A-441 (or A-242)		
	¾" and under $\sigma_y = 50{,}000$ psi	over ¾" to 1½" $\sigma_y = 46{,}000$ psi	over 1½" to 4" $\sigma_y = 42{,}000$ psi
$\sigma = 15{,}000 - \frac{1}{4}\left(\frac{L}{r}\right)^2$	$\sigma = 22{,}000 - .56\left(\frac{L}{r}\right)^2$	$\sigma = 20{,}000 - .46\left(\frac{L}{r}\right)^2$	$\sigma = 18{,}000 - .39\left(\frac{L}{r}\right)^2$
$\frac{L}{r}$ to 140	$\frac{L}{r}$ to 125	$\frac{L}{r}$ to 125	$\frac{L}{r}$ to 125



Steel skeleton for 10-story Buffalo, New York apartment building features unique shop-welded construction. Principal erection element is a "bent" consisting of a 50' floor girder or "needle beam" threaded through the web of column section near each end and welded. Girder is supported mainly by an angle bracket or "saddle" previously welded to the column web. Girders cantilever out as much as 13' from column.

TABLES 6 through 14—Allowable Compressive ($\underline{\sigma}$) Values (1963 AISC), Main Members

TABLE 6—33,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		19,770	19,730	19,690	19,660	19,620	19,580	19,540	19,500	19,460
10	19,410	19,370	19,320	19,280	19,230	19,180	19,130	19,080	19,030	18,980
20	18,930	18,880	18,820	18,770	18,710	18,660	18,600	18,540	18,840	18,420
30	18,360	18,300	18,240	18,180	18,110	18,050	17,980	17,920	17,850	17,780
40	17,710	17,640	17,570	17,500	17,430	17,360	17,290	17,220	17,140	17,070
50	16,990	16,920	16,840	16,760	16,680	16,600	16,520	16,440	16,360	16,280
60	16,200	16,120	16,030	15,950	15,860	15,780	15,690	15,610	15,520	15,430
70	15,340	15,250	15,160	15,070	14,980	14,890	14,800	14,700	14,610	14,510
80	14,420	14,320	14,230	14,130	14,030	15,930	13,840	13,740	13,640	13,530
90	13,430	13,330	13,230	13,130	13,020	12,920	12,810	12,710	12,600	12,490
100	12,380	12,280	12,170	12,060	11,950	11,830	11,720	11,610	11,900	11,380
110	11,270	11,150	11,040	10,920	10,800	10,690	10,570	10,450	10,330	10,210
120	10,090	9,996	9,840	9,720	9,590	9,470	9,340	9,220	9,090	8,960
130	8,830	8,700	8,570	8,440	8,320	8,190	8,070	7,960	7,840	7,730
140	7,620	7,510	7,410	7,300	7,200	7,100	7,010	6,910	6,820	6,730
150	6,640	6,550	6,460	6,380	6,300	6,220	6,140	6,060	5,980	5,910
160	5,830	5,760	5,690	5,620	5,550	5,490	5,420	5,350	5,290	5,230
170	5,170	5,110	5,050	4,990	4,930	4,880	4,820	4,770	4,710	4,660
180	4,610	4,560	4,510	4,460	4,410	4,360	4,320	4,270	4,230	4,180
190	4,140	4,090	4,050	4,010	3,970	3,930	3,890	3,850	3,810	3,770
200	3,730									

TABLE 7—36,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		21,560	21,520	21,480	21,440	21,390	21,350	21,300	21,250	21,210
10	21,160	21,100	21,050	21,000	20,950	20,890	20,830	20,780	20,720	20,660
20	20,600	20,540	20,480	20,410	20,350	20,280	20,220	20,150	20,080	20,010
30	19,940	19,870	19,800	19,730	19,650	19,580	19,500	19,420	19,350	19,270
40	19,190	19,110	19,030	18,950	18,860	18,780	18,700	18,610	18,530	18,440
50	18,350	18,260	18,170	18,080	17,990	17,900	17,810	17,710	17,620	17,530
60	17,430	17,330	17,240	17,140	17,040	16,940	16,840	16,740	16,640	16,530
70	16,430	16,330	16,220	16,120	16,010	15,900	15,790	15,690	15,580	15,470
80	15,360	15,240	15,130	15,020	14,900	14,790	14,670	14,560	14,440	14,320
90	14,200	14,090	13,970	13,840	13,720	13,600	13,480	13,350	13,230	13,100
100	12,980	12,850	12,720	12,590	12,470	12,330	12,200	12,070	11,940	11,810
110	11,670	11,540	11,400	11,260	11,130	10,990	10,850	10,710	10,570	10,430
120	10,280	10,140	9,990	9,850	9,700	9,550	9,410	9,260	9,110	8,970
130	8,840	8,700	8,570	8,410	8,320	8,190	8,070	7,960	7,840	7,730
140	7,620	7,510	7,410	7,300	7,200	7,100	7,010	6,910	6,820	6,730
150	6,640	6,550	6,460	6,380	6,300	6,220	6,140	6,060	5,980	5,910
160	5,830	5,760	5,690	5,620	5,550	5,490	5,420	5,350	5,290	5,230
170	5,170	5,110	5,050	4,990	4,930	4,880	4,820	4,770	4,710	4,660
180	4,610	4,560	4,510	4,460	4,410	4,360	4,320	4,270	4,230	4,180
190	4,140	4,090	4,050	4,010	3,970	3,930	3,890	3,850	3,810	3,770
200	3,730									

TABLE 8—42,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		25,150	25,100	25,050	24,990	24,940	24,880	24,820	24,760	24,700
10	24,630	24,570	24,500	24,430	24,360	24,290	24,220	24,150	24,070	24,000
20	23,920	23,840	23,760	23,680	23,590	23,510	23,420	23,330	23,240	23,150
30	23,060	22,970	22,880	22,780	22,690	22,590	22,490	22,390	22,290	22,190
40	22,080	21,980	21,870	21,770	21,660	21,550	21,440	21,330	21,220	21,100
50	20,990	20,870	20,760	20,640	20,520	20,400	20,280	20,160	20,030	19,910
60	19,790	19,660	19,530	19,400	19,270	19,140	19,010	18,880	18,750	18,610
70	18,480	18,340	18,200	18,060	17,920	17,780	17,640	17,500	17,350	17,210
80	17,060	16,920	16,770	16,620	16,470	16,320	16,170	16,010	15,860	15,710
90	15,550	15,390	15,230	15,070	14,910	14,750	14,590	14,430	14,260	14,090
100	13,930	13,760	13,590	13,420	13,250	13,080	12,900	12,730	12,550	12,370
110	12,190	12,010	11,830	11,650	11,470	11,280	11,100	10,910	10,720	10,550
120	10,370	10,200	10,030	9,870	9,710	9,560	9,410	9,260	9,110	8,970

*

TABLE 9—45,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		26,950	26,890	26,830	26,770	26,710	26,640	26,580	26,510	26,440
10	26,370	26,300	26,220	26,150	26,070	25,990	25,910	25,820	25,740	25,650
20	25,570	25,480	25,390	25,290	25,200	25,110	25,010	24,910	24,810	24,710
30	24,610	24,500	24,400	24,290	24,180	24,070	23,960	23,850	23,740	23,620
40	23,510	23,390	23,270	23,150	23,030	22,900	22,780	22,660	22,530	22,400
50	22,270	22,140	22,010	21,880	21,740	21,610	21,470	21,330	21,190	21,050
60	20,910	20,770	20,630	20,480	20,340	20,190	20,040	19,890	19,740	19,590
70	19,440	19,280	19,130	18,970	18,810	18,650	18,490	18,330	18,170	18,000
80	17,840	17,670	17,510	17,340	17,170	17,000	16,830	16,650	16,480	16,300
90	16,130	15,950	15,770	15,590	15,410	15,220	15,040	14,850	14,660	14,480
100	14,290	14,100	13,900	13,710	13,510	13,320	13,120	12,920	12,720	12,520
110	12,320	12,110	11,910	11,670	11,470	11,270	11,070	10,880	10,700	10,520
120	10,350	10,180	10,010	9,850	9,690	9,540	9,390	9,240	9,090	8,950

*

TABLE 10—46,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		27,540	27,480	27,420	27,360	27,300	27,230	27,160	27,090	27,020
10	26,950	26,870	26,790	26,720	26,630	26,550	26,470	26,380	26,290	26,210
20	26,110	26,020	25,930	25,830	25,730	25,640	25,540	25,430	25,330	25,230
30	25,120	25,010	24,900	24,790	24,680	24,560	24,450	24,330	24,210	24,100
40	23,970	23,850	23,730	23,600	23,480	23,350	23,220	23,090	22,960	22,830
50	22,690	22,560	22,420	22,280	22,140	22,000	21,860	21,720	21,570	21,430
60	21,280	21,130	20,980	20,830	20,680	20,530	20,370	20,220	20,060	19,900
70	19,740	19,580	19,420	19,260	19,100	18,930	18,760	18,600	18,430	18,260
80	18,080	17,910	17,740	17,560	17,390	17,210	17,030	16,850	16,670	16,480
90	16,300	16,120	15,930	15,740	15,550	15,360	15,170	14,970	14,780	14,580
100	14,390	14,190	13,990	13,790	13,580	13,380	13,170	12,960	12,750	12,540
110	12,330	12,120	11,900	11,690	11,490	11,290	11,100	10,910	10,720	10,550
120	10,370	10,200	10,030	9,870	9,710	9,560	9,410	9,260	9,110	8,970

*

* Above $\frac{KL}{r}$ of 130, the higher-strength steels offer no advantage as to allowable compressive stress ($\underline{\sigma}$). Above this point, use Table 7 for the more economical steel of 36,000 psi yield strength.

K multiplied by actual length (L) = effective length.

Table values computed by Research Dept., Bethlehem Steel Co.

TABLE 11—50,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		29,940	29,870	29,800	29,730	29,660	29,580	29,500	29,420	29,340
10	29,260	29,170	29,080	28,990	28,900	28,800	28,710	28,610	28,510	28,400
20	28,300	28,190	28,080	27,970	27,860	27,750	27,630	27,520	27,400	27,280
30	27,150	27,030	26,900	26,770	26,640	26,510	26,380	26,250	26,110	25,970
40	25,830	25,690	25,550	25,400	25,260	25,110	24,960	24,810	24,660	24,510
50	24,350	24,190	24,040	23,880	23,720	23,560	23,390	23,220	23,060	22,890
60	22,720	22,550	22,370	22,200	22,020	21,860	21,670	21,490	21,310	21,120
70	20,940	20,750	20,560	20,380	20,190	19,990	19,800	19,610	19,410	19,210
80	19,010	18,810	18,610	18,410	18,200	17,990	17,790	17,580	17,370	17,150
90	16,940	16,720	16,500	16,290	16,060	15,840	15,620	15,390	15,170	14,940
100	14,710	14,470	14,240	14,000	13,770	13,530	13,290	13,040	12,800	12,570
110	12,340	12,120	11,900	11,690	11,490	11,290	11,100	10,910	10,720	10,550
120	10,370	10,200	10,030	9,870	9,710	9,560	9,410	9,260	9,110	8,970

TABLE 12—55,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		32,930	32,850	32,770	32,690	32,600	32,510	32,420	32,330	32,230
10	32,130	32,030	31,930	31,820	31,720	31,600	31,490	31,380	31,260	31,140
20	31,010	30,890	30,760	30,630	30,500	30,370	30,230	30,090	29,950	29,810
30	29,670	29,520	29,370	29,220	29,070	28,910	26,760	28,600	28,440	28,270
40	28,110	27,940	27,770	27,600	27,430	27,260	27,080	26,900	26,730	26,540
50	26,360	26,180	25,990	25,800	25,610	25,420	25,220	25,030	24,830	24,630
60	24,430	24,230	24,020	23,820	23,610	23,400	23,190	22,970	22,760	22,540
70	22,320	22,100	21,880	21,660	21,430	21,200	20,970	20,740	20,510	20,280
80	20,040	19,800	19,560	19,320	19,070	18,830	18,580	18,330	18,080	17,830
90	17,570	17,310	17,050	16,790	16,530	16,260	16,000	15,730	15,460	15,180
100	14,910	14,630	14,350	14,040	13,780	13,510	13,260	13,010	12,770	12,540
110	12,310	12,090	11,880	11,670	11,470	11,270	11,070	10,880	10,700	10,520
120	10,350	10,180	10,010	9,850	9,690	9,540	9,390	9,240	9,090	8,950

TABLE 13—60,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		35,920	35,830	35,740	35,640	36,540	35,440	35,340	35,230	35,120
10	35,010	34,890	34,770	34,650	34,520	34,400	34,270	34,130	34,000	33,860
20	33,710	33,570	33,420	33,270	33,120	32,960	32,810	32,650	32,480	32,320
30	32,150	31,980	31,810	31,630	31,460	31,280	31,090	30,910	30,720	30,530
40	30,340	30,150	29,950	29,760	29,560	29,350	29,150	28,940	28,730	28,520
50	28,310	28,100	27,880	27,660	27,440	27,210	26,990	26,760	26,530	26,300
60	26,060	25,830	25,590	25,350	25,110	24,860	24,610	24,360	24,110	23,860
70	23,610	23,350	23,090	22,830	22,560	22,300	22,030	21,760	21,490	21,210
80	20,940	20,660	20,380	20,090	19,810	19,520	19,230	18,940	18,640	18,340
90	18,040	17,740	17,440	17,130	16,820	16,510	16,190	15,880	15,510	15,200
100	14,900	14,610	14,320	14,040	13,780	13,510	13,260	13,010	12,770	12,540
110	12,310	12,090	11,880	11,670	11,470	11,270	11,070	10,880	10,700	10,520
120	10,350	10,180	10,010	9,850	9,690	9,540	9,390	9,240	9,090	8,950

* See note on previous page

TABLE 14—65,000 psi yield steel

$\frac{KL}{r}$ ratio		1	2	3	4	5	6	7	8	9
		38,900	38,810	38,700	38,590	38,480	38,370	38,250	38,130	38,000
10	37,870	37,740	37,600	37,460	37,320	37,180	37,030	36,870	36,720	36,560
20	36,390	36,230	36,060	35,890	35,710	35,530	35,350	35,170	34,980	34,790
30	34,600	34,400	34,210	34,010	33,800	33,600	33,390	33,170	32,960	32,740
40	32,520	32,300	32,070	31,850	31,620	31,380	31,150	30,910	30,670	30,430
50	30,180	29,930	29,680	29,430	29,180	28,920	28,660	28,390	28,130	27,860
60	27,590	27,320	27,040	26,770	26,490	26,200	25,920	25,630	25,340	25,050
70	24,760	24,460	24,160	23,860	23,550	23,240	22,930	22,620	22,300	21,990
80	21,670	21,340	21,020	20,690	20,360	20,020	19,690	19,350	19,000	18,660
90	18,310	17,960	17,600	17,240	16,860	16,510	16,170	15,840	15,510	15,200
100	14,900	14,610	14,320	14,040	13,780	13,510	13,260	13,010	12,770	12,540
110	12,310	12,090	11,880	11,670	11,470	11,270	11,070	10,880	10,700	10,520
120	10,350	10,180	10,010	9,850	9,690	9,540	9,390	9,240	9,090	8,950