1. **Question based on Probability and statistics (Maverick Derivatives : CTC 1.5 crore - Amsterdam)**

- Consider a random permutation of the numbers 1,2,…,1000. Let A be the number of integers in the same position as before and B be the number of integers in a different position. What is the Variance of the quantity (B-A)?
- Solution - (see image below)

here, $N = 1000$.   So, $N = A + B$

$$\Rightarrow Var(B-A) = Var(N-2A)$$
$$= Var(-2A)$$
$$= 4\,Var(A)$$

Let X_i be identity random variable which becomes 1 when ith element is at ith position in the permutation.

$$A = X_1 + X_2 + \cdots + X_{1000}$$

Now, $X_i = 1$ with probability $\dfrac{1}{1000}$

and $X_i = 0$ with probability $\dfrac{999}{1000}$

By linearity of expectation, $E(A) = 1000 \times \dfrac{1}{1000}$
$$\Rightarrow E(A) = 1$$
$$\therefore E[A^2] = \sum_{n=1}^{1000} E[X_i^2] + 2\sum_{i<j} E[X_i X_j]$$

Since X_i is indicator variable, $E[X_i] = E[X_i^2] = \dfrac{1}{1000}$
If $i<j$, $E[X_i X_j] = \dfrac{(n-2)!}{n!}$

$$E[A^2] = 1 + 2\,\dfrac{n(n-1)}{2} \times \dfrac{(n-2)!}{n!} = 1+1 = 2$$

$$Var(A) = E[A^2] - (E[A])^2 = 2-1 = 1$$
$$\Rightarrow Var(B-A) = 4\,Var(A) = 4$$

Question based on DSA by DE Shaw - Quant developer role (CTC : 50 LPA)

Given an array of positive integers, each of which represent the number of litres of water in that particular bucket, we have to make the litres of water in every bucket equal.
We are allowed to do two types of operations any number of times:

1) We can altogether remove a bucket from the sequence

2) We can remove some water from a bucket

We have to tell what is the minimum number of litres removed to make all buckets have the same amount of water.

- **Solution :**

 1.) Sort the array(waters array) in descending order.

2.) Assuming every bucket as candidate find the minimum amount of water need to be removed.Result will be minimum amount of water among that.

3.) Every bucket which is in left of current bucket have more water than current bucket and every bucket which is in right of current bucket have less water than current bucket.

4.) For every bucket we remove all the bucket on its right and some amount of water from all the bucket on its left(they have more water).

5.) Amount of water removed can be calculated using (total_water - waters[i]*(i+1)).

6) res = min(res, total_water - waters[i]*(i+1)).

7.) Time complexity - O(n) and Space Complexity - O(1)

```cpp
int minLitreRemoved(vector<int>&waters){

    //sort the array

    sort(waters.begin(),waters.end(),greater<int>());//sort in descending
order

    int res = INT_MAX;

    int total_water = 0;//stores sum of litres of water from all bucket

    for(auto bucket : waters){

        total_water += bucket;

    }

    for(int i=0;i<waters.size();i++){

        res = min(res, total_water - waters[i]*(i+1));//i+1 bucket will
have atleast water[i] height.(sorted in descending order)

    }

    return res;

}
```