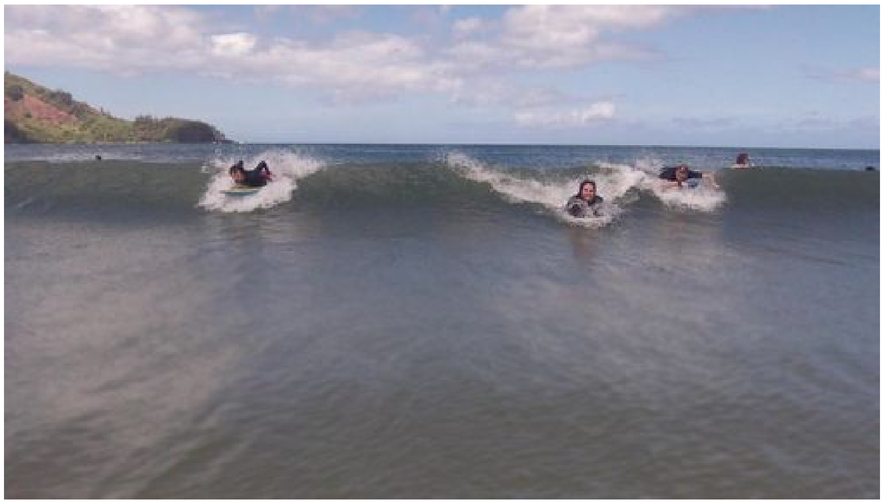
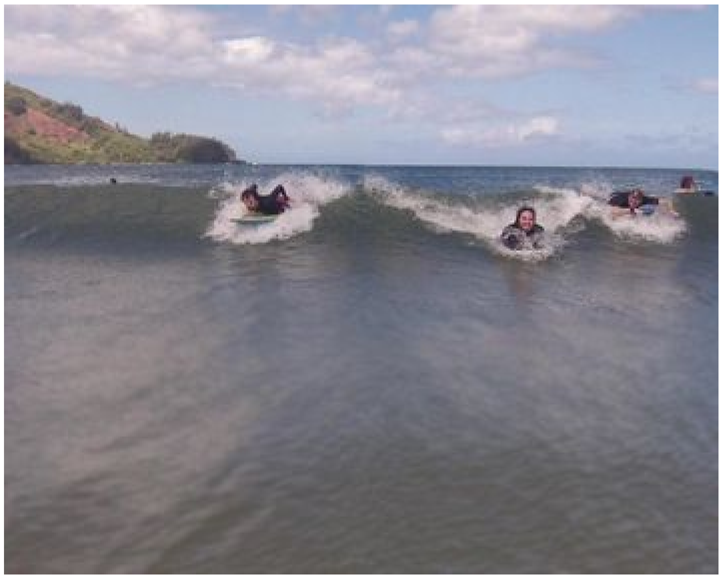
Seam Carving

Seam-carving is a content-aware image resizing technique where the image is reduced in size by one pixel of height (or width) at a time. A *vertical seam* in an image is a path of pixels connected from the top to the bottom with one pixel in each row. (A *horizontal seam* is a path of pixels connected from the left to the right with one pixel in each column.) Below left is the original 505-by-287 pixel image; below right is the result after removing 150 vertical seams, resulting in a 30% narrower image. Unlike standard content-agnostic resizing techniques (e.g. cropping and scaling), the most interesting features (aspect ratio, set of objects present, etc.) of the image are preserved.
As you'll soon see, the underlying algorithm is quite simple and elegant. Despite this fact, this technique was not discovered until 2007 by Shai Avidan and Ariel Shamir. It is now a feature in Adobe Photoshop (thanks to a Princeton graduate student), as well as other popular computer graphics applications.



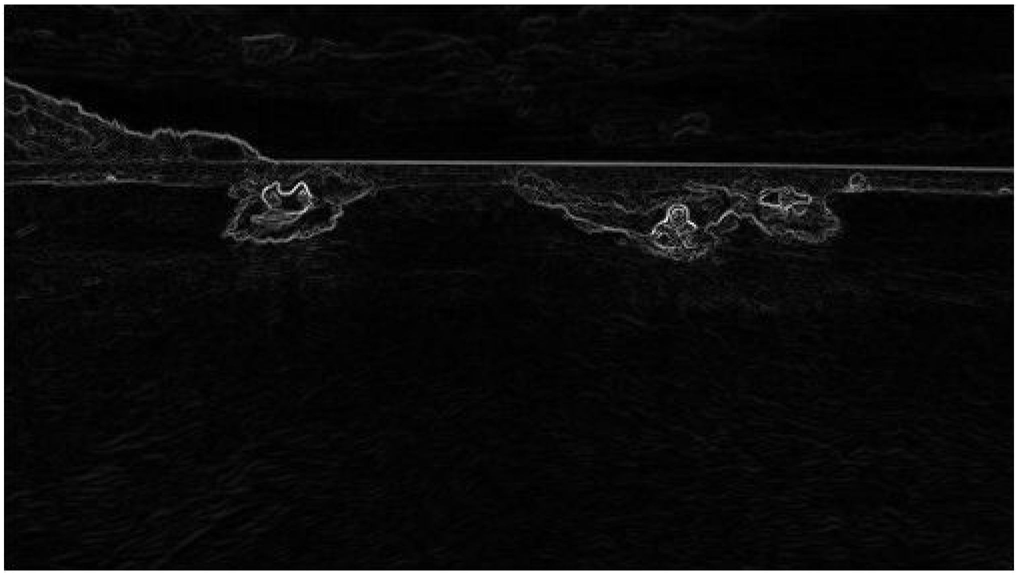


In this assignment, you will create a data type that resizes a *W*-by-*H* image using the seam-carving technique. Finding and removing a seam involves three parts and a tiny bit of notation:

0. *Notation.* In image processing, pixel (*x*, *y*) refers to the pixel in column *x* and row *y*, with pixel (0, 0) at the upper left corner and pixel (*W* − 1, *H* − 1) at the bottom right corner. This is consistent with the Picture data type in algs4.jar. *Warning*: this is the opposite of the standard mathematical notation used in linear algebra where (*i*, *j*) refers to row *i* and column *j* and with Cartesian coordinates where (0, 0) is at the lower left corner.

We also assume that the color of a pixel is represented in RGB space, using three integers between 0 and 255. This is consistent with the java.awt.Color data type.

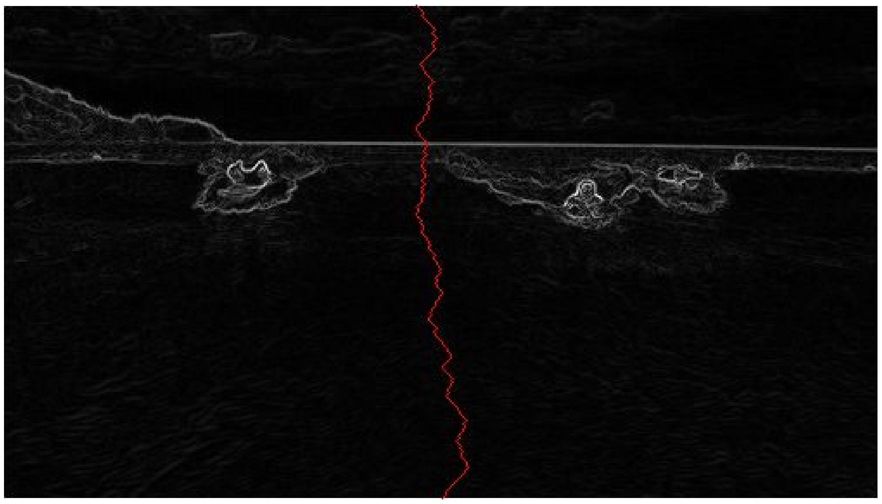
1. *Energy calculation.* The first step is to calculate the *energy* of each pixel, which is a measure of the importance of each pixel—the higher the energy, the less likely that the pixel will be included as part of a seam (as we'll see in the next step). In this assignment, you will implement the *dual-gradient energy function*, which is described below. Here is the dual-gradient energy function of the surfing image above:



The energy is high (white) for pixels in the image where there is a rapid color gradient (such as the boundary between the sea and sky and the boundary between the surfing Josh Hug on the left and the ocean behind him). The seam-carving technique avoids removing such high-energy pixels.

2. *Seam identification.* The next step is to find a vertical seam of minimum total energy. This is similar to the classic shortest path problem in an edge-weighted digraph except for the following:

- The weights are on the vertices instead of the edges.
- We want to find the shortest path from any of the *W* pixels in the top row to any of the *W* pixels in the bottom row.
- The digraph is acyclic, where there is a downward edge from pixel (x, y) to pixels ($x - 1$, $y + 1$), (x, $y + 1$), and ($x + 1$, $y + 1$), assuming that the coordinates are in the prescribed range.



3. ***Seam removal.*** The final step is to remove from the image all of the pixels along the seam.

The SeamCarver API. Your task is to implement the following mutable data type:

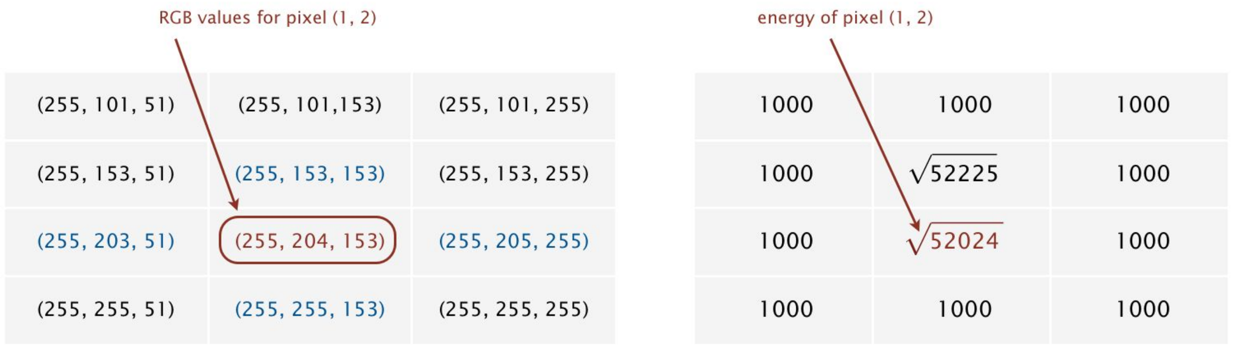
```
public class SeamCarver {
  public SeamCarver(Picture picture) // create a seam carver object based on the given picture
  public Picture picture()                    // current picture
  public     int width()                    // width of current picture
  public     int height()                   // height of current picture
  public  double energy(int x, int y)            // energy of pixel at column x and row y
  public   int[] findHorizontalSeam()            // sequence of indices for horizontal seam
  public int[] findVerticalSeam()              // sequence of indices for vertical seam
  public void removeHorizontalSeam(int[] seam)// remove horizontal seam from current picture
  public    void removeVerticalSeam(int[] seam)    // remove vertical seam from current picture
}
```

- **Corner cases.** Your code should throw a java.lang.IllegalArgumentException with a message **"picture is null"** when a constructor or method is called with an invalid argument, as documented below:
  - ✓ By convention, the indices *x* and *y* are integers between 0 and *width* − 1 and between 0 and *height* − 1 respectively, where *width* is the width of the current image and *height* is the height. Throw a java.lang.IllegalArgumentException with a message **"IllegalArgumentException"** if either *x* or *y* is outside its prescribed range.
  - ✓ Throw a java.lang.IllegalArgumentException with a message **"IllegalArgumentException"** if the constructor, removeVerticalSeam(), or removeHorizontalSeam() is called with a null argument.
  - ✓ Throw a java.lang.IllegalArgumentException with a message **"IllegalArgumentException"** if removeVerticalSeam() or removeHorizontalSeam() is called with an array of the wrong length or if the array is not a valid seam (i.e., either an entry is outside its prescribed range or two adjacent entries differ by more than 1).
  - ✓ Throw a java.lang.IllegalArgumentException with a message **"IllegalArgumentException"** if removeVerticalSeam() is called when the width of the picture is less than or equal to 1 or if removeHorizontalSeam() is called when the height of the picture is less than or equal to 1.

- **Constructor.** The data type may not mutate the Picture argument to the constructor.

- **Computing the energy of a pixel.** You will use the *dual-gradient energy function*: The energy of pixel (x,y) is square root of ($\Delta 2x(x,y)+\Delta 2y(x,y)$), where the square of the x-gradient $\Delta 2x(x,y)=Rx(x,y)2+Gx(x,y)2+Bx(x,y)2$, and where the central differences $Rx(x,y)$, $Gx(x,y)$, and $Bx(x,y)$ are the differences in the red, green, and blue components between pixel (x + 1, y) and pixel (x − 1, y), respectively. The square of the y-gradient $\Delta 2y(x,y)$ is defined in an analogous manner. We define the energy of a pixel at the border of the image to be 1000, so that it is strictly larger than the energy of any interior pixel.

  As an example, consider the 3-by-4 image (supplied as 3x4.png) with RGB values—each component is an integer between 0 and 255—as shown in the table below:


RGB values for pixel (1, 2)
energy of pixel (1, 2)

(255, 101, 51)	(255, 101,153)	(255, 101, 255)
(255, 153, 51)	(255, 153, 153)	(255, 153, 255)
(255, 203, 51)	(255, 204, 153)	(255, 205, 255)
(255, 255, 51)	(255, 255, 153)	(255, 255, 255)

a 3-by-4 image (RGB values)

1000	1000	1000
1000	$\sqrt{52225}$	1000
1000	$\sqrt{52024}$	1000
1000	1000	1000

dual-gradient energies

  The ten border pixels have energy 1000. Only the pixels (1, 1) and (1, 2) are nontrivial. We calculate the energy of pixel (1, 2) in detail:
  $R_x(1, 2) = 255 - 255 = 0$,
  $G_x(1, 2) = 205 - 203 = 2$,
  $B_x(1, 2) = 255 - 51 = 204$,
  yielding $\Delta_x^2(1, 2) = 2^2 + 204^2 = 41620$.
  $R_y(1, 2) = 255 - 255 = 0$,
  $G_y(1, 2) = 255 - 153 = 102$,
  $B_y(1, 2) = 153 - 153 = 0$,
  yielding $\Delta_y^2(1, 2) = 102^2 = 10404$.
  Thus, the energy of pixel (1, 2) is √41620+10404 = √52024
  Similarly, the energy of pixel (1, 1) is √2042+1032 = √52225

- **Finding a vertical seam.** The findVerticalSeam() method returns an array of length H such that entry y is the column number of the pixel to be removed from row y of the image. For example, the dual-gradient energies of a 6-by-5 image (supplied as 6x5.png).

energy of seam = 1000.00 + 107.89 + 133.07 + 174.01 + 1000.00 = 2414.97

1000.00	1000.00	1000.00	**1000.00**	1000.00	1000.00
1000.00	237.35	151.02	234.09	**107.89**	1000.00
1000.00	138.69	228.10	**133.07**	211.51	1000.00
1000.00	153.88	**174.01**	284.01	194.50	1000.00
1000.00	1000.00	**1000.00**	1000.00	1000.00	1000.00

a minimum energy vertical seam in a 6-by-5 image

- The minimum energy vertical seam is highlighted in blue. In this case, the method findVerticalSeam() returns the array { 3, 4, 3, 2, 2 } because the pixels in a minimum energy vertical seam are (3, 0), (4, 1), (3, 2), (2, 3), and (2, 4). When there are multiple vertical seams with minimal total energy, your method can return any such seam.

- **Finding a horizontal seam.** The behavior of findHorizontalSeam() is analogous to that of findVerticalSeam() except that it returns an array of length *width* such that entry *x* is the row number of the pixel to be removed from column *x* of the image. For the 6-by-5 image, the method findHorizontalSeam() returns the array { 2, 2, 1, 2, 1, 2 } because the pixels in a minimum energy horizontal seam are (0, 2), (1, 2), (2, 1), (3, 2), (4, 1), and (5, 2).

energy of seam = 1000.00 + 138.69 + 151.02 + 133.07 + 107.89 + 1000.00 = 2530.68

1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
1000.00	237.35	**151.02**	234.09	**107.89**	1000.00
1000.00	**138.69**	228.10	**133.07**	211.51	**1000.00**
1000.00	153.88	174.01	284.01	194.50	1000.00
1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

a minimum energy horizontal seam in a 6-by-5 image

- **Performance requirements.** The width(), height(), and energy() methods should take constant time in the worst case. All other methods should run in time at most proportional to *width* × *height* in the worst case. For faster performance, do not construct explicit DirectedEdge and EdgeWeightedDigraph objects.

Analysis of running time (optional and not graded).

- Give the worst-case running time to remove *r* rows and *c* columns from a *width*-by-*height* image as a function of *r*, *c*, *width*, and *height*.

- Estimate empirically the running time (in seconds) to remove *r* rows and *c* columns from a *width*-by-*height* image as a function of *r*, *c*, *width*, and *H*. Use tilde notation to simplify your answer.

Steps to start:

1. Your task is to complete the methods given in SeamCarver.java.
2. In Solution.java, do not modify main method. It is used for passing test cases based on the client requirements.
3. You can feel free to use other clients given in the file, which can be used for testing.
4. Start by writing the constructor as well as picture(), width() and height(). These should be very easy.

5. From there, write energy(). Calculating Δx^2 and Δy^2 are very similar. Using two private methods will keep your code simple. To test that your code works, use the client PrintEnergy described in the testing section above.

6. To write findVerticalSeam(), you will want to first make sure you understand the topologial sort algorithm for computing a shortest path in a DAG. Do not create an EdgeWeightedDigraph. Instead construct a 2d energy array using the energy() method that you have already written. Your algorithm can traverse this matrix treating some select entries as reachable from (x, y) to calculate where the seam is located. To test that your code works, use the client PrintSeams described in the testing section above.

7. To write findHorizontalSeam(), transpose the image, call findVerticalSeam(), and transpose it back.

8. Now implement removeVerticalSeam(). Typically, this method will be called with the output of findVerticalSeam(), but be sure that they work for any seam. To test that your code words, use the client ResizeDemo described in the testing section above.

9. To write removeHorizontalSeam(), transpose the image, call removeVerticalSeam(), and transpose it back.