EE3025-Assignment 1

Vamshika K - EE17BTECH11045

Download all python codes from

https://github.com/vamshikak/EE3025/tree/main/Assignment_1/codes

and latex-tikz codes from

https://github.com/vamshikak/EE3025/tree/main/Assignment_1

1 Problem

5.3 The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

2 Solution

For the given system

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.1}$$

$$y(n) = 0 \text{ for } n < 0 \tag{2.0.2}$$

On applying Z-transform on both sides, we get

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{2.0.3}$$

$$Y(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}}X(z) \tag{2.0.4}$$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2.0.5}$$

Relation between z-transforms of input, output and impulse response is as follows

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.6}$$

Hence

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2.0.7}$$

Now, applying inverse Z-transform on both sides

$$h(n) = \left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.0.8}$$

Checking for stability: We check if the system is stable using **BIBO stability condition** as follows

$$|x(n)| \le B_x < \infty \implies |y(n)| \le B_y < \infty \tag{2.0.9}$$

For every bounded input (x(n)) output (y(n)) of the system is bounded.
We know that in time domain,

$$y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k) \tag{2.0.10}$$

After applying mod on both sides,

$$|y(n)| = |\sum_{-\infty}^{\infty} h(k)x(n-k)| \tag{2.0.11}$$

From 2.0.9 lets suppose x(n) is bounded by Bx, then

$$|y(n)| \le |\sum_{-\infty}^{\infty} h(k)B_x| \tag{2.0.12}$$

$$|y(n)| \le B_x|\sum_{-\infty}^{\infty} h(k)| \tag{2.0.13}$$

Since $|y(n)| < \infty$ for BIBO stability condition to hold for bounded input and holds only if

$$|\sum_{-\infty}^{\infty} h(k)| < \infty \tag{2.0.14}$$

Hence, impulse response of system in time domain must be absolutely sumable for it to be BIBO stable.

Now, checking if system is BIBO stable for 2.0.8

$$\sum_{n=-\infty}^{\infty} \left| \left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \right| < \infty \quad (2.0.15)$$

$$\sum_{n=-\infty}^{\infty} \left| \left[\frac{1}{2}\right]^{n} u(n) + \left[\frac{1}{2}\right]^{n-2} u(n-2) \right| < \infty \quad (2.0.16)$$

$$2\sum_{n=-\infty}^{\infty} \left| \left[\frac{1}{2}\right]^{n} u(n) \right| < \infty \quad (2.0.17)$$

$$2\left[\frac{1}{1-\frac{1}{2}}\right] < \infty \quad (2.0.18)$$

$$4 < \infty \quad (2.0.19)$$

Our system is BIBO stable since the condition hold.

Example Input: Our system,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \quad (2.0.20)$$

$$y(n) = 0 \text{ for } n < 0 \quad (2.0.21)$$

Lets consider the input to be

$$x(n) = [1.0, 2.0, 1.0, 4.0, 3.0, 3.0] \quad (2.0.22)$$

Figure

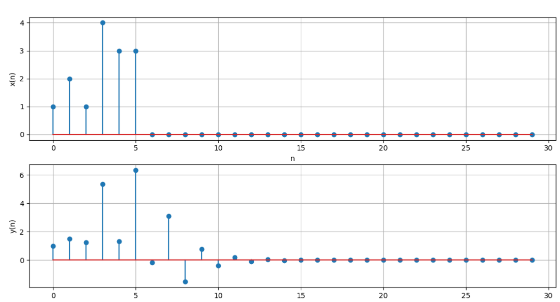

Fig. A1: Input signal x(n) and Output signal y(n)

We know that for our input signal $B_x = 4$ and after calculating for output we get $B_y = 6.34375$ as shown in Fig. A1. Since both input and output are less then infinity(bounded), BIBO stability holds.

Stability by Region Of Convergence(ROC):
Method 1: For a system to be stable, its ROC must include unit circle. By solving 2.0.7 for poles and zeros we get as follows.

$$Poles = 0, -\frac{1}{2} \quad (2.0.23)$$

$$Zeros = +1j, -1j \quad (2.0.24)$$

After plotting it can be seen that ROC includes unit circle and hence stable.
Method 2: We know that

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (2.0.25)$$

Considering h(n) is absolutely summable

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \infty \quad (2.0.26)$$

By triangle inequality, following holds

$$\implies \left| \sum_{n=-\infty}^{\infty} h(n)z^{-n} \right|_{|z|=1} < \sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} \quad (2.0.27)$$

by substituting 2.0.27 in 2.0.26

$$|H(z)|_{|z|=1} < \infty \quad (2.0.28)$$

Hence, ROC includes unit circle as shown in Fig. A2. By both the above methods it can be said that system is stability

Figure

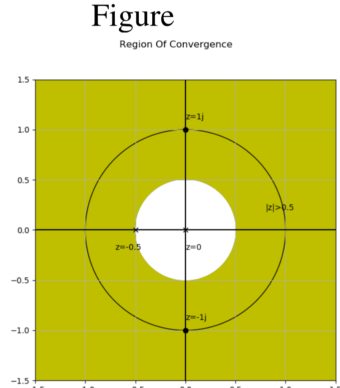

Fig. A2: Plot for Pole-Zero