

# **PROJECT REPORT – VEHICLE DYNAMICS**

## **MINI PROJECT**

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MECHANICAL ENGINEERING

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## Table of contents:

<b>Table of contents:</b>	<b>2</b>
<b>1. Introduction</b>	<b>4</b>
2. Literature review	5
2.1 Vehicle Dynamics and Ride Comfort:	5
2.2 Simulation Techniques:	5
2.3 Quantifiable Metrics:	5
2.4 Suspension Parameter Optimization:	5
2.5 Challenges and Future Directions:	5
<b>3. Methodology</b>	<b>6</b>
3.1 Model Inspiration:	6
3.2 Half-Car Model:	6
3.2.1 Advantages of the Half-Car Model:	6
3.3 Half Car Model Overview	7
3.3.1 Dynamic Governing Equation:	7
3.3 Ride Comfort Analysis:	9
3.3.1 Simulation Setup:	9
3.3.2 Quantification of Ride Comfort Parameters:	9
3.4 Road Profile Generation:	10
3.4.1 Profile Generation Process:	10
3.4.2 Significance of Realistic Road Profiles:	11
3.4.3 Application in Ride Comfort Analysis:	11
<b>4. Results and Analysis</b>	<b>11</b>
4.1 Small Semicircle Bump on Flat Road	11
4.1.1 Vertical Displacement Analysis	11
4.1.2 Acceleration Analysis	12
4.1.3 Power Spectral Density (PSD)	12
4.2 White Noise Road Simulation	13
4.2.1 Vertical Displacement Analysis	13
4.2.2 Acceleration Analysis	14
4.2.3 Power Spectral Density (PSD)	14
4.3 Stiffness Variation Effect Analysis	15
4.3.1 Stiffness Variation	15
4.4 Damping Coefficient Variation Effect Analysis	17
4.4.1 Damping Coefficient Variation	17
<b>5. Conclusions</b>	<b>20</b>

5.1 Half-Car Model Insights	20
5.2 Ride Comfort Analysis	20
5.3 Parametric Variation Analysis	20
5.4 Future Directions	20
<b>APPENDIX</b>	<b>21</b>

# **1. Introduction**

In the contemporary landscape of automotive engineering, this project, executed within the MATLAB environment, scrutinizes the intricate facets of ride comfort, acknowledging its conventional role as an imperative consideration within the domain of vehicle dynamics. Defined by the ease and fluidity of motion experienced by vehicle occupants, ride comfort assumes a pivotal standing at the intersection of engineering precision and user contentment. This study adopts a comprehensive approach to vehicle design, scrutinizing critical elements such as suspension systems, tire characteristics, structural design, and seating comfort—all encapsulated within the rigorous framework of MATLAB.

While not a groundbreaking innovation, the project recognizes the enduring significance of ride comfort. It stands as a determining factor influencing consumer preferences, augmenting safety measures, and contributing to market competitiveness. A vehicle that seamlessly aligns with the needs of its occupants, mitigating discomfort and fatigue, not only allures customers but substantially fortifies brand perception and customer loyalty. Addressing the commonplace challenges associated with discomfort, the project meticulously quantifies parameters such as vertical acceleration, pitching acceleration, power spectral density, natural frequencies, and the bump toe range.

Through the lens of MATLAB, the project seeks to refine, leveraging the robust capabilities of this computational tool to meticulously analyze and optimize ride comfort. The results are stated for specific cases but the model can be utilized for mass-data based optimization algorithms since it is a parametric model.

## **2. Literature review**

### **2.1 Vehicle Dynamics and Ride Comfort:**

Within the realm of vehicle dynamics, seminal contributions from researchers such as Pacejka and Sharp have meticulously unraveled the intricate dynamics governing tire-road interactions. Serving as the bedrock of understanding vehicular motion, their work lays the foundation for comprehending the fundamental forces steering a vehicle's trajectory. Complementing this theoretical framework, Wong and Khajepour provide invaluable tools, including quarter-car, half-car, and full-car models, empowering engineers to fine-tune suspension parameters for an optimal amalgamation of ride comfort and handling prowess.

### **2.2 Simulation Techniques:**

In the pursuit of elevating ride comfort, simulation techniques emerge as transformative tools. Mendes underscores the versatility of MATLAB, portraying it as a robust platform for modeling vehicle behavior and dissecting the influence of variable parameters. Gillespie and Milliken deepen this exploration by delving into multibody dynamics simulations, offering a meticulous examination of the intricate interplay between suspension systems and structural elements shaping the overall ride experience.

### **2.3 Quantifiable Metrics:**

The assessment of ride comfort necessitates objective metrics, and herein lies the significance of the work by Crolla and Jazar. Their emphasis on quantifiable parameters like Root Mean Square (RMS) Acceleration and Pitch Acceleration provides a rigorous framework for evaluating the smoothness and stability of a vehicle. Augmenting this approach, Guiggiani and Lotfi introduce additional metrics, employing Power Spectral Density (PSD) analysis to discern the frequency distribution of vibrations on the road.

### **2.4 Suspension Parameter Optimization:**

Gillich and Li assume the role of adept orchestrators, manipulating and optimizing suspension parameters to craft the ideal recipe for a smooth ride. Delving into the nuanced interplay of stiffness and damping coefficients, they contribute to the ongoing quest for achieving an optimal balance between comfort and stability. In alignment with this pursuit, our project navigates the same terrain, striving to identify the most effective configuration for a comfortable and stable ride.

### **2.5 Challenges and Future Directions:**

Envisioning a future where road trips unfold seamlessly, Rakheja and Su advocate for cutting-edge technologies such as real-time data and intelligent algorithms to elevate ride comfort further. This forward-looking perspective entails the integration of sophisticated systems capable of predicting and adapting to varying road conditions, ushering in an era where rides transcend the boundaries of smoothness.

In essence, this literature review encapsulates the fundamental principles of vehicle dynamics, the application of simulation techniques for optimizing ride comfort, the importance of quantifiable metrics in evaluating comfort levels, and the ongoing quest to refine suspension parameters for an optimal ride experience.

---

## 3. Methodology

### 3.1 Model Inspiration:

Our model draws inspiration from the work of André de Souza Mendes, specifically the "Half car model Version 1.0.1." While incorporating elements from Mendes's implementation, our focus diverges toward an in-depth analysis of ride comfort, with an emphasis on quantifiable metrics rather than simulation and animation aspects. This adaptation allows us to delve into the intricate dynamics of a half-car model, seeking to optimize ride comfort through systematic parameter variations.

### 3.2 Half-Car Model:

The Half-Car Model divides the vehicle into two main components: the front half and the rear half. Each half is treated as a separate mass-spring-damper system, connected by the vehicle's chassis. This simplification allows for a more manageable representation of the complex interactions between the vehicle and the road surface.

#### 3.2.1 Advantages of the Half-Car Model:

1. **Simplicity and Computational Efficiency:** The Half-Car Model significantly reduces the complexity of the suspension system while retaining essential features. This simplicity enhances computational efficiency, making it a preferred choice for initial design and analysis phases.
2. **Focused Ride Comfort Analysis:** The model excels in analyzing ride comfort, particularly in terms of vertical motion and pitch behavior. It provides insights into how different parameters, such as suspension stiffness and damping, impact the vehicle's response to road irregularities.
3. **Effective for Parametric Studies:** The Half-Car Model is well-suited for parametric

studies where variations in specific parameters, such as suspension stiffness or damping, need to be systematically analyzed for their effects on ride comfort. This capability is valuable for optimization processes.

4. **Insights into Pitch Dynamics:** As pitch motion (rotation about the vehicle's longitudinal axis) significantly influences ride comfort, the Half-Car Model offers a clear representation of pitch dynamics. This insight is crucial for designing suspensions that minimize pitching acceleration and, consequently, enhance passenger comfort.
5. **Integration with Road Profiles:** The model can be integrated with different road profiles to simulate real-world conditions. This integration allows for the evaluation of how the vehicle responds to variations in road surfaces and the identification of optimal suspension settings.

### 3.3 Half Car Model Overview

This model captures the essential elements influencing ride comfort—mass (M), damping (C), and stiffness (K) matrices. These matrices are crucial for formulating the dynamic equation that governs the motion of the vehicle.

#### 3.3.1 Dynamic Governing Equation:

$$Mx''(t) + Cx'(t) + Kx(t) = Fu(t)$$

**Mass Matrix (M):** The mass matrix represents the distribution of mass within the vehicle and is a key determinant of its inertia. For our half-car model, the mass matrix M is a 4x4 matrix, where the diagonal elements correspond to the total mass (m) of the car and the masses of the front (m<sub>1</sub>) and rear (m<sub>2</sub>) unsprung masses, along with the lateral moment of inertia (I<sub>y</sub>).

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_y & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}$$

**Damping Matrix (C):** The damping matrix accounts for the dissipation of energy due to the internal friction within the suspension components. It includes the damping coefficients for the front (c<sub>1</sub>) and rear (c<sub>2</sub>) suspension systems.

$$C = \begin{bmatrix} c_1 + c_2 & a_2 c_2 - a_1 c_1 & -c_1 & -c_2 \\ a_2 c_2 - a_1 c_1 & a_1^2 c_1 + a_2^2 c_2 & a_1 c_1 & -a_2 c_2 \\ -c_1 & a_1 c_1 & c_1 & 0 \\ -c_2 & -a_2 c_2 & 0 & c_2 \end{bmatrix}$$

**Stiffness Matrix (K):** The stiffness matrix characterizes the resistance of the suspension components to deformation. It includes the stiffness coefficients for the front ( $k_1$ ) and rear ( $k_2$ ) suspension systems, as well as the stiffness of the front ( $k_{t1}$ ) and rear ( $k_{t2}$ ) tires.

$$K = \begin{bmatrix} k_1 + k_2 & a_2 k_2 - a_1 k_1 & -k_1 & -k_2 \\ a_2 k_2 - a_1 k_1 & a_1^2 k_1 + a_2^2 k_2 & a_1 k_1 & -a_2 k_2 \\ -k_1 & a_1 k_1 & k_1 + k_{t1} & 0 \\ -k_2 & -a_2 k_2 & 0 & k_2 + k_{t2} \end{bmatrix}$$

**Forcing Vector ( $F$ ):** The forcing vector represents the external forces acting on the system, primarily originating from the road profile. In our model, the forcing vector is a 4x2 matrix, accounting for the forces exerted on the front and rear tires. Here,  $u_1(t)$  and  $u_2(t)$  denote the displacements of the front and rear wheels, respectively. The entries  $k_{t1}$  and  $k_{t2}$  represent the tire stiffness for the front and rear wheels.

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_{t1} u_1(t) & 0 \\ 0 & k_{t2} u_2(t) \end{bmatrix}$$

**State Vector ( $x$ ):** The state vector  $x$  is a column vector with eight components representing dynamic variables: vertical displacement, pitch angle, vertical displacement of unsprung masses.

$$X = \begin{bmatrix} z \\ \theta \\ z_{u1} \\ z_{u2} \end{bmatrix}$$

These matrices form the basis for the dynamic governing equation:  $Mx''(t) + Cx'(t) + Kx(t) = F(t)$ , where  $x(t)$  represents the displacement vector, and  $u(t)$  represents the input force from the road profile. The Half-Car Model, with its detailed representation of mass, damping, and stiffness, lays the groundwork for a nuanced analysis of ride comfort parameters.



### 3.3 Ride Comfort Analysis:

Ride comfort analysis involves assessing the response of the vehicle to external disturbances, focusing on minimizing discomfort for occupants. The Half-Car Model, with its detailed representation of vehicle dynamics, is employed to simulate and analyze the ride comfort parameters.

#### 3.3.1 Simulation Setup:

The governing equations of motion for the Half-Car Model are solved numerically to obtain displacement, velocity, and acceleration profiles of key components, including the body, front and rear unsprung masses, and pitch motion. The simulation incorporates a realistic road profile, characterized by bumps and irregularities, to emulate actual driving conditions.

Utilizing the developed equations, a simulation is conducted in MATLAB to analyze the response of the Half-Car Model to a given road profile. The simulation yields displacement, velocity, and acceleration profiles for various components, including the vehicle body, front and rear unsprung masses, and pitch angle.

#### 3.3.2 Quantification of Ride Comfort Parameters:

Several ride comfort parameters are quantified to assess the performance of the vehicle suspension system. These include:

1. **Root Mean Square (RMS) Acceleration:** A measure of the average acceleration experienced by the vehicle occupants. Lower RMS acceleration values indicate a smoother ride.

$$\text{RMS Acceleration} = \sqrt{\frac{1}{N} \sum_{i=1}^N a_i^2}$$

2. **RMS Pitch Acceleration:** Gauges the average angular acceleration around the pitch axis, providing insights into pitch motion and stability.

$$\text{RMS Pitch Acceleration} = \sqrt{\frac{1}{N} \sum_{i=1}^N \theta_i^2}$$

3. **Transmissibility (TR):** This metric gauges the ratio of body acceleration to unsprung mass acceleration, indicating the level of isolation provided by the suspension system.

$$\text{Transmissibility} = \frac{\text{Maximum Acceleration of Body}}{\text{Maximum Acceleration of Unsprung Mass}}$$

4. **Power Spectral Density (PSD):** Analyzes the frequency distribution of power in the vehicle's response. The PSD helps identify dominant frequencies contributing to vibrations.

$$\text{PSD}(f) = \frac{1}{T} \left| \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \right|^2$$

5. **Natural Frequencies:** Represents the inherent vibrational frequencies of the vehicle structure. Optimizing these frequencies is crucial for minimizing resonance effects and enhancing ride comfort.

The quantification of these parameters allows for a comprehensive evaluation of the ride comfort performance, guiding engineers in refining the suspension design for optimal passenger comfort under varying road conditions. The simulation results provide valuable insights into the dynamic behavior of the vehicle and aid in the identification of key factors influencing ride quality.

### 3.4 Road Profile Generation:

The road profile is a crucial element in simulating realistic driving conditions and evaluating a vehicle's suspension system. The generation process involves synthesizing a road profile that replicates typical road irregularities, forming the basis for a comprehensive ride comfort analysis.

#### 3.4.1 Profile Generation Process:

The road profile, denoted by  $Z_r(X_r)$ , is generated by combining white noise with sinusoidal bumps:

$$Z_r(X_r) = \text{WhiteNoise} + \sum_{i=1}^n A_i \cdot \sin(2\pi f_i X_r)$$

In this equation,  $X_r$  is the longitudinal position along the road profile.  $A_i$  is the amplitude of the  $i$ th sinusoidal bump.  $f_i$  is the frequency of the  $i$ th sinusoidal bump.

Parameters such as profile length, sampling rate, amplitude factor, number of bumps, bump amplitude, and bump frequency influence the profile's characteristics. The white noise component adds unpredictability, while sinusoidal bumps simulate specific road features like speed bumps or undulations.

### 3.4.2 Significance of Realistic Road Profiles:

Realistic road profiles are essential for accurate vehicle dynamic simulations. They enable a thorough evaluation of the vehicle's response to diverse driving conditions, allowing engineers to assess ride comfort, suspension effectiveness, and overall performance. The road profile's dynamic nature introduces variability, making the analysis more representative of actual road scenarios.

### 3.4.3 Application in Ride Comfort Analysis:

The generated road profile serves as the input to the Half-Car Model, influencing the dynamic response of the vehicle. By incorporating variations in road surfaces, the analysis becomes more nuanced, providing insights into how the vehicle interacts with different terrains and road conditions. This realistic input is crucial for a comprehensive understanding of ride comfort parameters.

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## 4. Results and Analysis

### 4.1 Small Semicircle Bump on Flat Road

#### 4.1.1 Vertical Displacement Analysis

The simulation of a small semicircle bump on a flat road demonstrated that the vehicle's body exhibited significantly less motion compared to the road. The graph depicting vertical displacement of unsprung and sprung masses, along with pitch, against longitudinal displacement illustrated the effective isolation of the vehicle body from road irregularities.

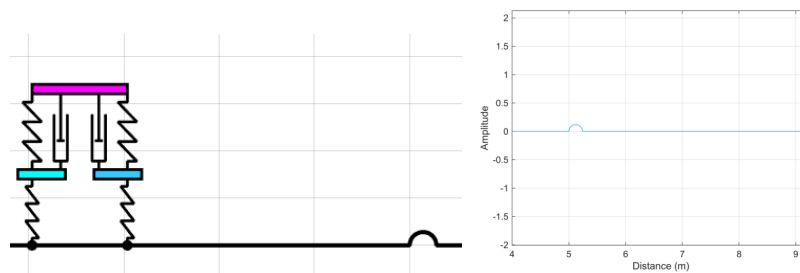


Fig: Visualization of road bump

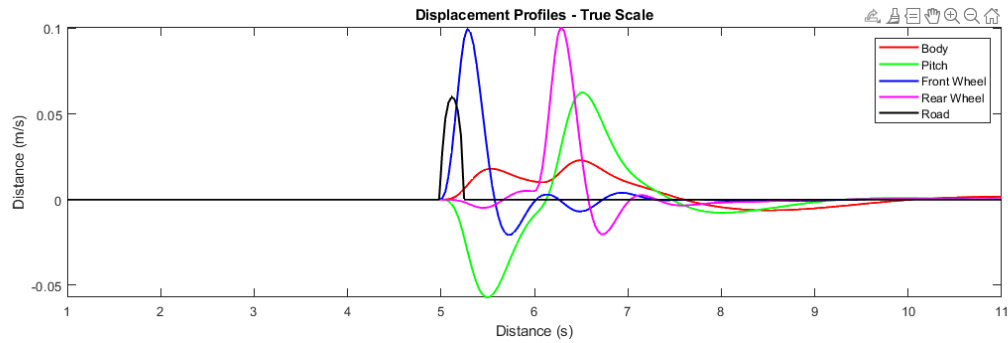


Fig: Vertical displacement v/s longitudinal position

#### 4.1.2 Acceleration Analysis

The analysis of acceleration profiles for both sprung and unsprung masses revealed a transmissibility of 0.08. This low transmissibility value indicated that the disturbances from the road had minimal impact on the body, ensuring a smoother ride experience for passengers.

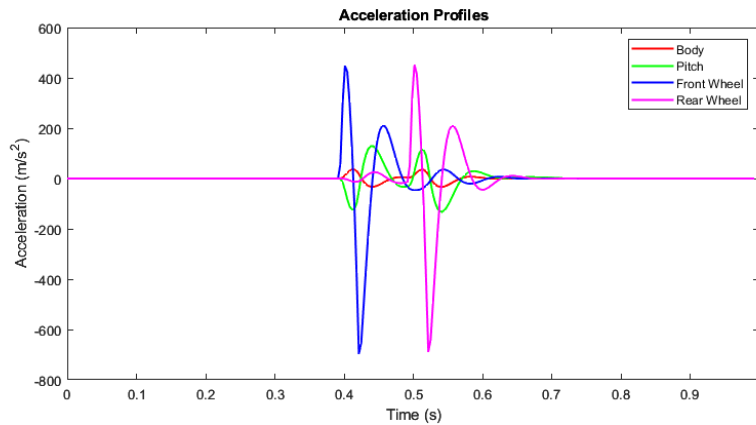


Fig: Vertical acceleration v/s time

RMS Acceleration:

1. Body: 9.0862 m/s<sup>2</sup>
2. Pitch: 34.0564 rad/s<sup>2</sup>

#### 4.1.3 Power Spectral Density (PSD)

The PSD graphs provided insights into the first 5 natural frequencies of the system, showing

peaks at [3.0101, 11.0369, 19.0638, 29.0973, 39.1309] in Hertz. The total spectral power of 62.49 indicated the distribution of power across different frequency components.

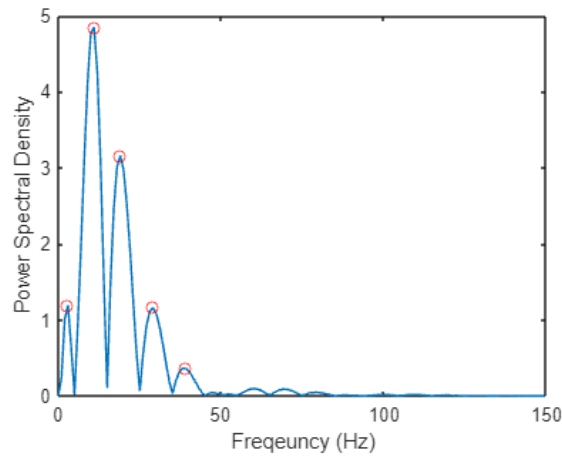


Fig: Power Spectral Density after FFT of Acceleration

## 4.2 White Noise Road Simulation

### 4.2.1 Vertical Displacement Analysis

Employing a white noise-based road input with realistic parameters, the simulation showcased reduced body motion compared to the road. The graph depicting vertical displacement of unsprung and sprung masses, along with pitch, against longitudinal displacement emphasized the effective dampening of road irregularities.

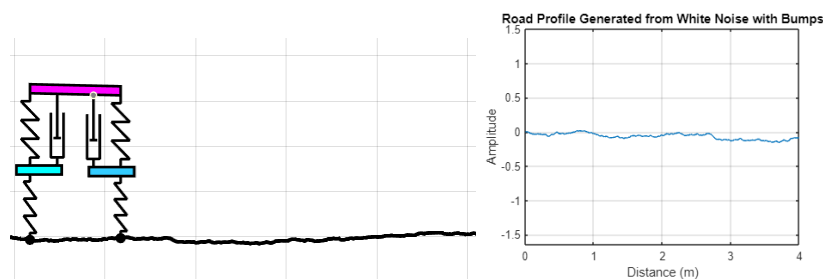


Fig: Visualization of road profile

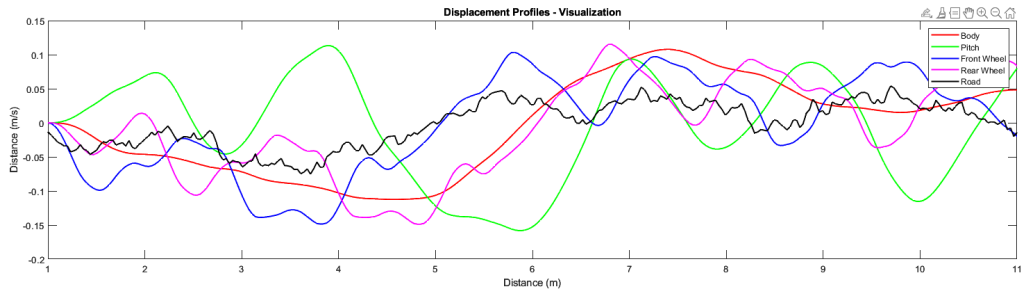


Fig: Vertical displacement v/s longitudinal position

#### 4.2.2 Acceleration Analysis

Acceleration profiles for both sprung and unsprung masses yielded a transmissibility of 0.1147. This slightly higher transmissibility value indicated a controlled transmission of road disturbances, contributing to an acceptable compromise between comfort and road feedback.

RMS Acceleration:

1. Body: 6.8532  $\text{m/s}^2$
2. Pitch: 29.7704  $\text{rad/s}^2$

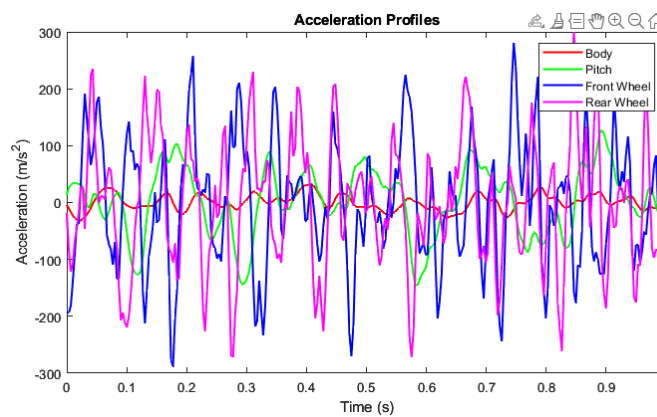


Fig: Vertical acceleration profile v/s time

#### 4.2.3 Power Spectral Density (PSD)

PSD graphs exhibited the lowest 5 natural frequencies at [2.0067, 6.0201, 8.0268, 11.0369, 13.0436] in Hertz, with a total spectral power of 52.1049. The distribution of power across various frequencies provided insights into the system's response to road inputs.

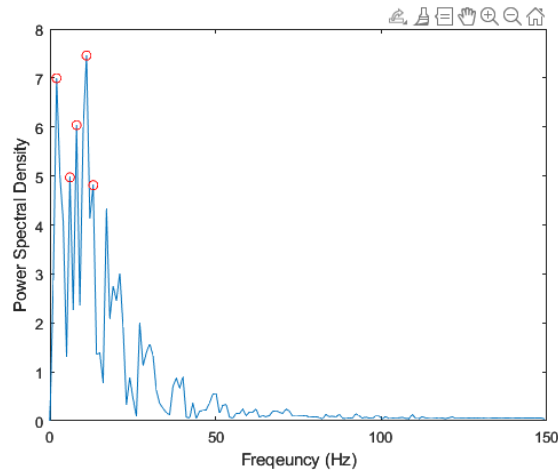


Fig: Power Spectral Density after FFT of Acceleration

## 4.3 Stiffness Variation Effect Analysis

### 4.3.1 Stiffness Variation

Systematic variation of stiffness from 200 to 30000 N/m highlighted several key observations.

1. RMS acceleration:

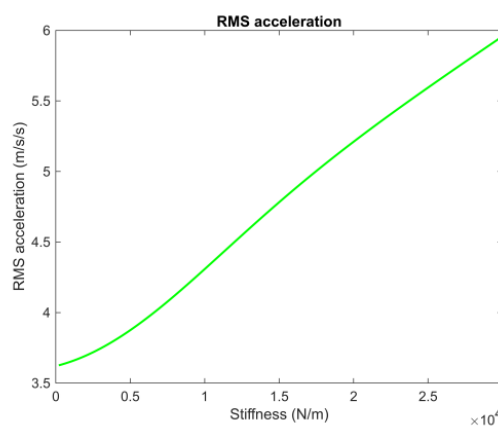


Fig: Stiffness Variation Analysis - RMS acceleration

Exhibited a monotonically increasing trend with stiffness, indicating that higher stiffness resulted in more pronounced accelerations.

## 2. RMS Pitch Acceleration:

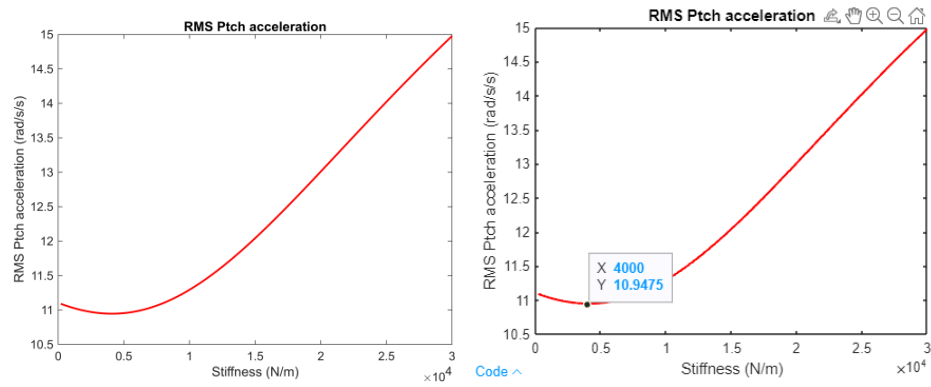


Fig: Stiffness Variation Analysis - RMS pitch acceleration

Reached a minimum at around 4000 N/m, suggesting an optimal stiffness value for minimizing pitch motion.

## 3. Transmissibility:

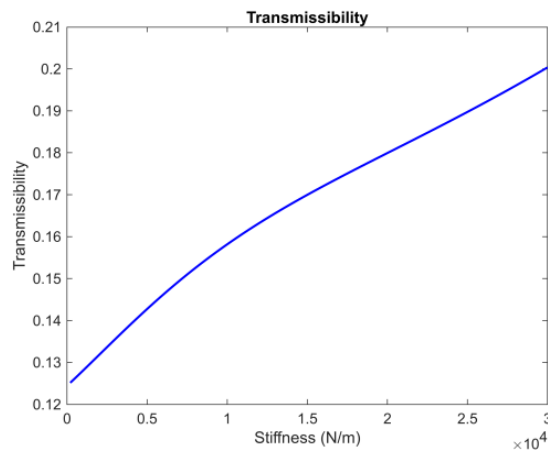


Fig: Stiffness Variation Analysis - Transmissibility

Increased with stiffness, but the slope decreased, indicating a balance between suspension stiffness and ride comfort.

## 4. Total Spectral Power:



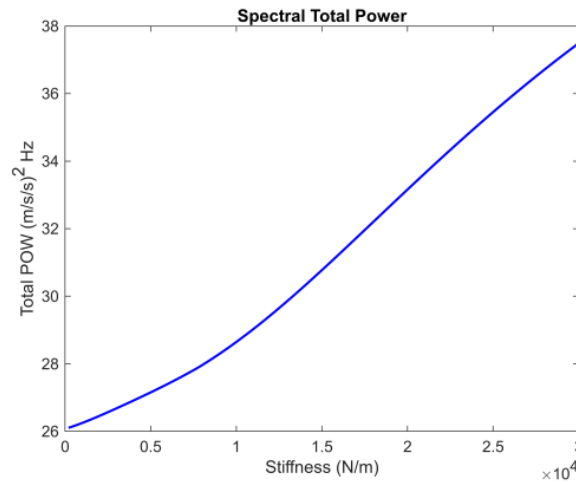


Fig: Stiffness Variation Analysis - Total Spectral Power

Monotonically increased with stiffness, indicating a higher distribution of power across various frequency components.

## 4.4 Damping Coefficient Variation Effect Analysis

### 4.4.1 Damping Coefficient Variation

Systematic variation of damping coefficients from 0 to 10000, with a constant stiffness of 15000, resulted in noteworthy observations.

1. The RMS acceleration:

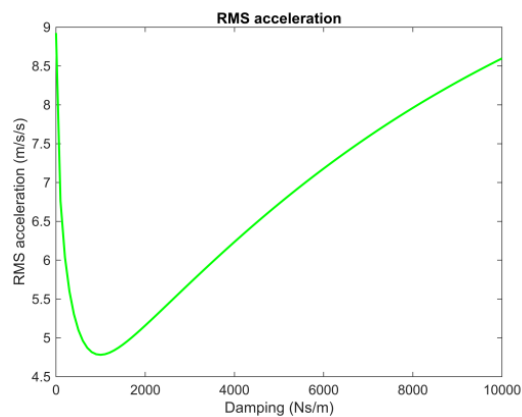


Fig: Stiffness Variation Analysis - RMS Acceleration

Demonstrated a decreasing trend to a minimum at around 900 Ns/m, suggesting an optimal damping value for minimizing accelerations.

## 2. RMS Pitch Acceleration:

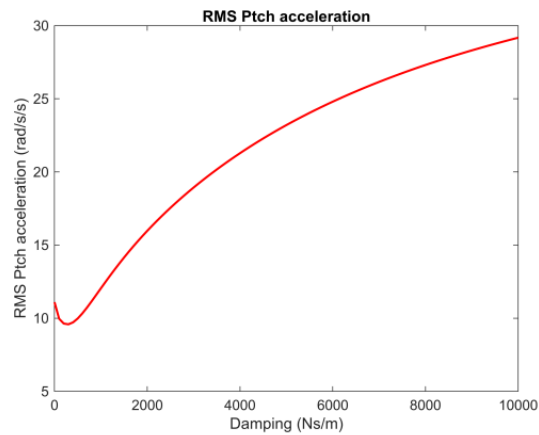


Fig: Stiffness Variation Analysis - RMS Pitch Acceleration

Exhibited a minimum at approximately 300, indicating an optimal damping coefficient for minimizing pitch motion.

## 3. Transmissibility:

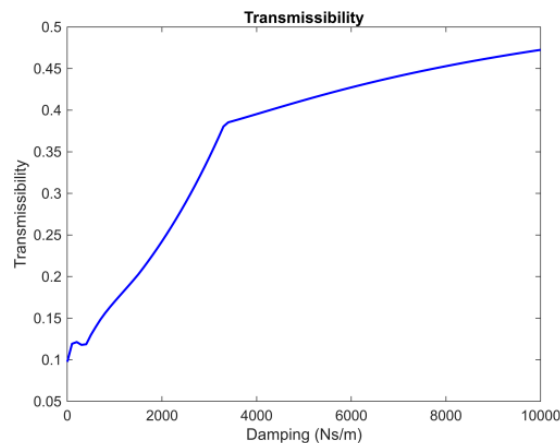


Fig: Stiffness Variation Analysis - Transmissibility

Increased with damping but showed an irregularity at low damping, likely due to superposition effects.

#### 4. Total Spectral Power:

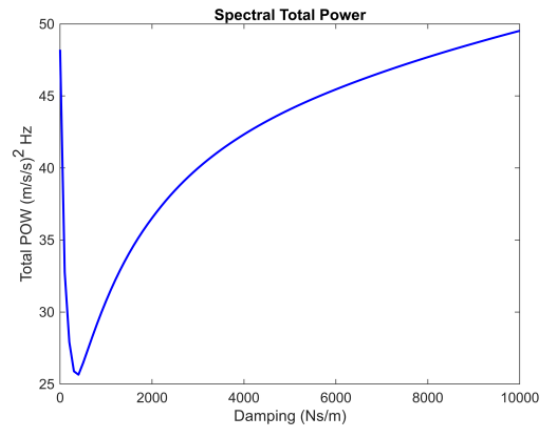


Fig: Stiffness Variation Analysis - Spectral Total Power

Decreased to a minimum at 400 Ns/m, suggesting an optimal damping value for enhanced overall ride comfort.

## 5. Conclusions

In conclusion, this research delves into the realm of ride comfort analysis with a focus on a parametrically modeled half-car system. Drawing inspiration from the Half Car Model Version 1.0.1 by André de Souza Mendes, this implementation departs from simulation aspects, emphasizing real-world dynamics. The half-car model, encapsulated by the dynamic equation  $M\ddot{x} + C\dot{x} + Kx = Fu$ , provides a comprehensive representation of the vehicle's response to external forces.

### 5.1 Half-Car Model Insights

Section 3.2 elucidates the matrices involved in the dynamic equation. The mass ( $M$ ), damping ( $C$ ), and stiffness ( $K$ ) matrices, along with the road input ( $F$ ), collectively determine the system's behavior.

### 5.2 Ride Comfort Analysis

The ride comfort analysis in Section 3.3 involves road profile generation, utilizing white noise for realistic simulations. The obtained results, displayed in Section 4, underscore the model's efficacy. A semicircle bump simulation elucidates the system's response, while a white noise road input introduces complexity.

### 5.3 Parametric Variation Analysis

Sections 4.1 and 4.2 explore the effects of stiffness and damping coefficient variations on ride comfort. Notably, damping optimization proves critical, showcasing diverse minima in RMS acceleration, RMS pitch acceleration, transmissibility, and PSD power. This variability highlights the potential for a combined optimization algorithm to enhance overall suspension system performance.

### 5.4 Future Directions

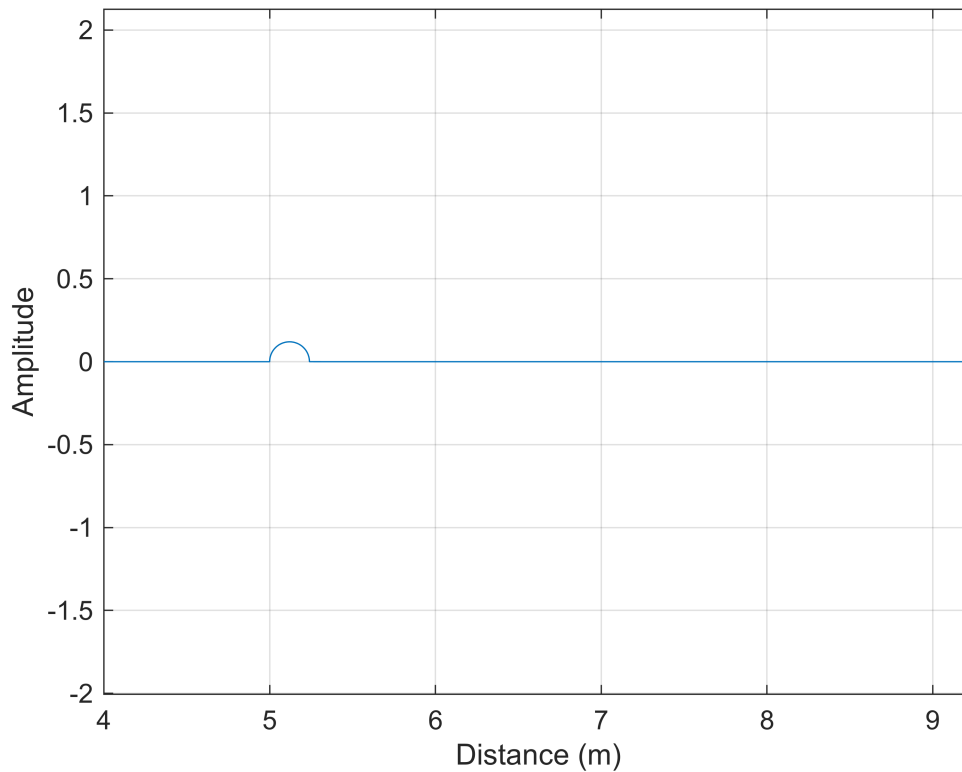
This research opens avenues for further exploration. Future work could delve into comprehensive optimization algorithms considering both stiffness and damping. Moreover, a closer integration with real-world data acquisition systems and subjective evaluations can provide a holistic understanding of ride comfort.

In summary, this study underscores the importance of a parametrically modeled half-car system in analyzing ride comfort, offering valuable insights for future advancements in vehicle suspension design and optimization.

# **APPENDIX**

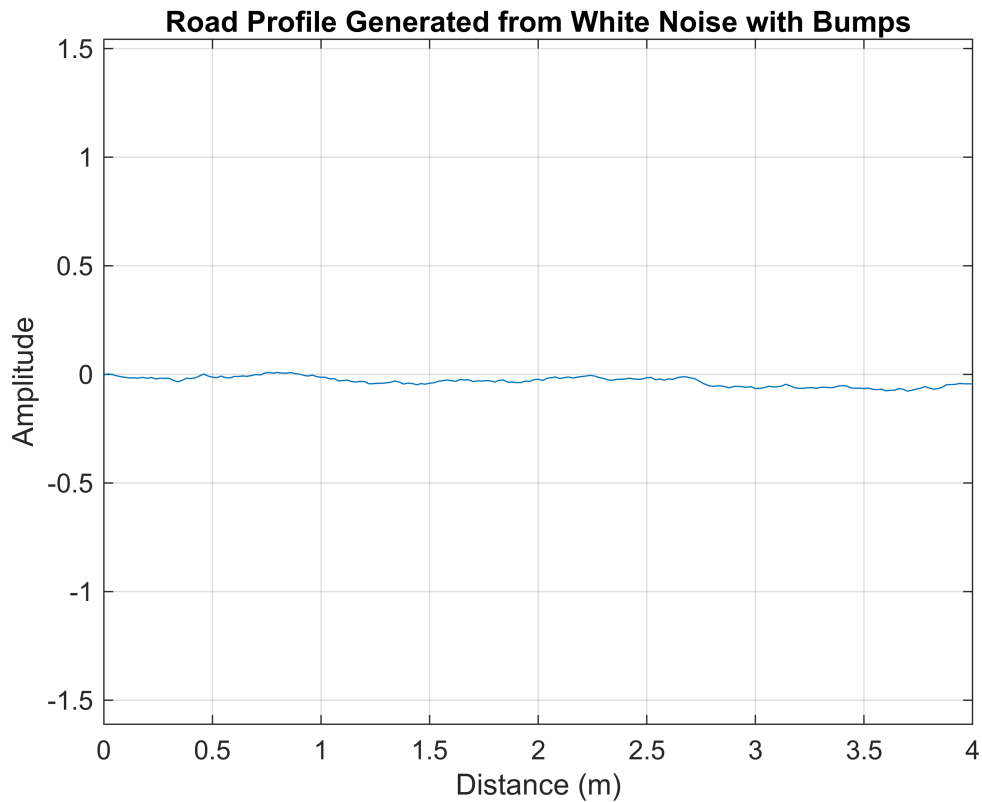
## Road Profile - Generation

```
[X_r, Z_r] = bump_road_input(5,0.12,15);
```



```
profile_length = 200;           % Length of the road profile (meters)
sampling_rate = 50;             % Numberof points per meter
amplitude_factor = 0.005;       % Roughness
num_bumps = 5;                  % Number of bumps
bump_amplitude = 0.001;         % Amplitude of bumps
bump_frequency = 0.5;           % Frequency of bumps
```

```
[X_r, Z_r] = generateRoadProfileWithBumps(profile_length, sampling_rate, amplitude_factor, num_bumps, bump_amplitude, bump_frequency);
```



```
road.X_r = X_r;
road.Z_r = Z_r;
```

## Car Model

```
%Vehicle
scooter.mass = 150;
scooter.front_unsprung_mass = 15;
scooter.rear_unsprung_mass = 15;
scooter.Lateral_MOI = 20;
scooter.CG_2_Front = 0.5;
scooter.CG_2_Rear = 0.5;

stiffness.front_strut = 15000;
stiffness.rear_strut = 15000;
stiffness.tire_front = 100000;
stiffness.tire_rear = 100000;

damping.strut_front = 1000;
damping.strut_rear = 1000;

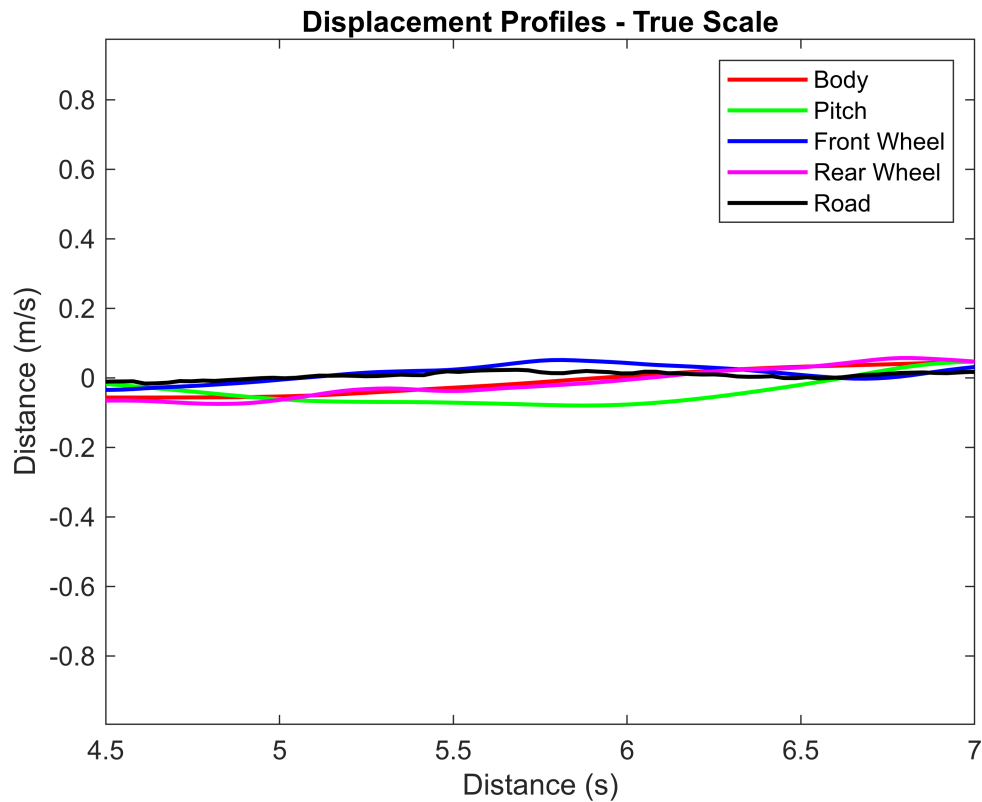
%Velocity
initial_vel = 10; %velocity in m/s
acc = 0;
```

## SOLVING

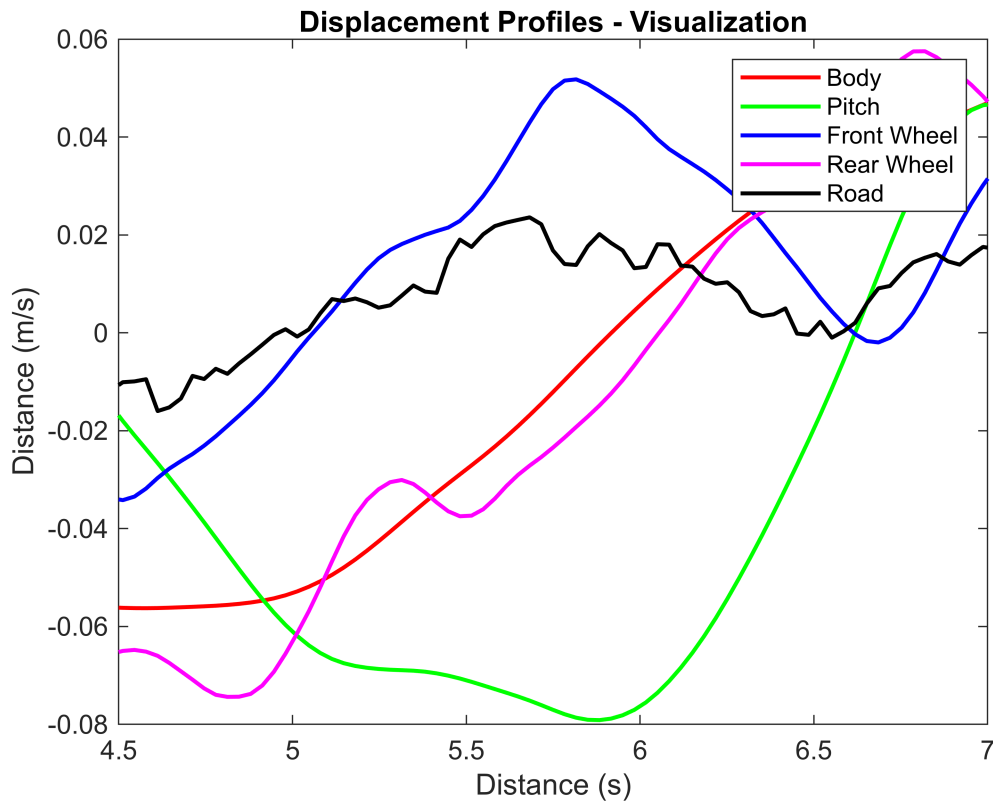
```
[displacement, velocity, acceleration] = Ride_Comfort_Analysis(scooter, stiffness, damping, road)
```

```
displacement = struct with fields:
    z_body: [300x1 double]
    z_unsprung_front: [300x1 double]
    z_unsprung_rear: [300x1 double]
    theta: [300x1 double]
    time: [300x1 double]
    tire_front: [-0.0066 -0.0091 -0.0114 -0.0142 -0.0154 -0.0170 -0.0169 -0.0214 -0.0206 -0.0193 -0.0166 -0.0142 -0.0114 -0.0091 -0.0066 -0.0041 -0.0016 5.6040e-05 -0.0041 -0.0069 -0.0080 -0.0078 -0.0089 -0.0096 -0.0088 -0.0090 -0.0092 -0.0094 -0.0096 -0.0098 -0.0100 -0.0102 -0.0104 -0.0106 -0.0108 -0.0110 -0.0112 -0.0114 -0.0116 -0.0118 -0.0120 -0.0122 -0.0124 -0.0126 -0.0128 -0.0130 -0.0132 -0.0134 -0.0136 -0.0138 -0.0140 -0.0142 -0.0144 -0.0146 -0.0148 -0.0150 -0.0152 -0.0154 -0.0156 -0.0158 -0.0160 -0.0162 -0.0164 -0.0166 -0.0168 -0.0170 -0.0172 -0.0174 -0.0176 -0.0178 -0.0180 -0.0182 -0.0184 -0.0186 -0.0188 -0.0190 -0.0192 -0.0194 -0.0196 -0.0198 -0.0200 -0.0202 -0.0204 -0.0206 -0.0208 -0.0210 -0.0212 -0.0214 -0.0216 -0.0218 -0.0220 -0.0222 -0.0224 -0.0226 -0.0228 -0.0230 -0.0232 -0.0234 -0.0236 -0.0238 -0.0240 -0.0242 -0.0244 -0.0246 -0.0248 -0.0250 -0.0252 -0.0254 -0.0256 -0.0258 -0.0260 -0.0262 -0.0264 -0.0266 -0.0268 -0.0270 -0.0272 -0.0274 -0.0276 -0.0278 -0.0280 -0.0282 -0.0284 -0.0286 -0.0288 -0.0290 -0.0292 -0.0294 -0.0296 -0.0298 -0.0300]
    tire_rear: [-0.0016 5.6040e-05 -0.0041 -0.0069 -0.0080 -0.0078 -0.0089 -0.0096 -0.0088 -0.0090 -0.0092 -0.0094 -0.0096 -0.0098 -0.0099 -0.0101 -0.0103 -0.0105 -0.0107 -0.0109 -0.0111 -0.0113 -0.0115 -0.0117 -0.0119 -0.0121 -0.0123 -0.0125 -0.0127 -0.0129 -0.0131 -0.0133 -0.0135 -0.0137 -0.0139 -0.0141 -0.0143 -0.0145 -0.0147 -0.0149 -0.0151 -0.0153 -0.0155 -0.0157 -0.0159 -0.0161 -0.0163 -0.0165 -0.0167 -0.0169 -0.0171 -0.0173 -0.0175 -0.0177 -0.0179 -0.0181 -0.0183 -0.0185 -0.0187 -0.0189 -0.0191 -0.0193 -0.0195 -0.0197 -0.0199 -0.0201 -0.0203 -0.0205 -0.0207 -0.0209 -0.0211 -0.0213 -0.0215 -0.0217 -0.0219 -0.0221 -0.0223 -0.0225 -0.0227 -0.0229 -0.0231 -0.0233 -0.0235 -0.0237 -0.0239 -0.0241 -0.0243 -0.0245 -0.0247 -0.0249 -0.0251 -0.0253 -0.0255 -0.0257 -0.0259 -0.0261 -0.0263 -0.0265 -0.0267 -0.0269 -0.0271 -0.0273 -0.0275 -0.0277 -0.0279 -0.0281 -0.0283 -0.0285 -0.0287 -0.0289 -0.0291 -0.0293 -0.0295 -0.0297 -0.0299 -0.0301]
    longitudinal_pos_front: [1 1.0334 1.0669 1.1003 1.1338 1.1672 1.2007 1.2341 1.2676 1.3010 1.3344 1.3679 1.4013 1.4348 1.4682 1.5016 1.5350 1.5684 1.6018 1.6352 1.6686 1.7020 1.7354 1.7688 1.8022 1.8356 1.8690 1.9024 1.9358 1.9692 2.0026 2.0360 2.0694 2.1028 2.1362 2.1696 2.2030 2.2364 2.2698 2.3032 2.3366 2.3700 2.4034 2.4368 2.4702 2.5036 2.5370 2.5704 2.6038 2.6372 2.6706 2.7040 2.7374 2.7708 2.8042 2.8376 2.8710 2.9044 2.9378 2.9712 3.0046 3.0380 3.0714 3.1048 3.1382 3.1716 3.2050 3.2384 3.2718 3.3052 3.3386 3.3720 3.4054 3.4388 3.4722 3.5056 3.5390 3.5724 3.6058 3.6392 3.6726 3.7060 3.7394 3.7728 3.8062 3.8396 3.8730 3.9064 3.9398 3.9732 4.0066 4.0400 4.0734 4.1068 4.1402 4.1736 4.2070 4.2404 4.2738 4.3072 4.3406 4.3740 4.4074 4.4408 4.4742 4.5076 4.5410 4.5744 4.6078 4.6412 4.6746 4.7080 4.7414 4.7748 4.8082 4.8416 4.8750 4.9084 4.9418 4.9752 5.0086 5.0420 5.0754 5.1088 5.1422 5.1756 5.2090 5.2424 5.2758 5.3092 5.3426 5.3760 5.4094 5.4428 5.4762 5.5096 5.5430 5.5764 5.6098 5.6432 5.6766 5.7100 5.7434 5.7768 5.8102 5.8436 5.8770 5.9104 5.9438 5.9772 6.0106 6.0440 6.0774 6.1108 6.1442 6.1776 6.2110 6.2444 6.2778 6.3112 6.3446 6.3780 6.4114 6.4448 6.4782 6.5116 6.5450 6.5784 6.6118 6.6452 6.6786 6.7120 6.7454 6.7788 6.8122 6.8456 6.8790 6.9124 6.9458 6.9792 7.0126 7.0460 7.0794 7.1128 7.1462 7.1796 7.2130 7.2464 7.2798 7.3132 7.3466 7.3800 7.4134 7.4468 7.4802 7.5136 7.5470 7.5804 7.6138 7.6472 7.6806 7.7140 7.7474 7.7808 7.8142 7.8476 7.8810 7.9144 7.9478 7.9812 8.0146 8.0480 8.0814 8.1148 8.1482 8.1816 8.2150 8.2484 8.2818 8.3152 8.3486 8.3820 8.4154 8.4488 8.4822 8.5156 8.5490 8.5824 8.6158 8.6492 8.6826 8.7160 8.7494 8.7828 8.8162 8.8496 8.8830 8.9164 8.9498 8.9832 9.0166 9.0500 9.0834 9.1168 9.1502 9.1836 9.2170 9.2504 9.2838 9.3172 9.3506 9.3840 9.4174 9.4508 9.4842 9.5176 9.5510 9.5844 9.6178 9.6512 9.6846 9.7180 9.7514 9.7848 9.8182 9.8516 9.8850 9.9184 9.9518 9.9852 10.0186 10.0520 10.0854 10.1188 10.1522 10.1856 10.2190 10.2524 10.2858 10.3192 10.3526 10.3860 10.4194 10.4528 10.4862 10.5196 10.5530 10.5864 10.6198 10.6532 10.6866 10.7200 10.7534 10.7868 10.8202 10.8536 10.8870 10.9204 10.9538 10.9872 11.0206 11.0540 11.0874 11.1208 11.1542 11.1876 11.2210 11.2544 11.2878 11.3212 11.3546 11.3880 11.4214 11.4548 11.4882 11.5216 11.5550 11.5884 11.6218 11.6552 11.6886 11.7220 11.7554 11.7888 11.8222 11.8556 11.8890 11.9224 11.9558 11.9892 12.0226 12.0560 12.0894 12.1228 12.1562 12.1896 12.2230 12.2564 12.2898 12.3232 12.3566 12.3900 12.4234 12.4568 12.4902 12.5236 12.5570 12.5904 12.6238 12.6572 12.6906 12.7240 12.7574 12.7908 12.8242 12.8576 12.8910 12.9244 12.9578 12.9912 13.0246 13.0580 13.0914 13.1248 13.1582 13.1916 13.2250 13.2584 13.2918 13.3252 13.3586 13.3920 13.4254 13.4588 13.4922 13.5256 13.5590 13.5924 13.6258 13.6592 13.6926 13.7260 13.7594 13.7928 13.8262 13.8596 13.8930 13.9264 13.9598 13.9932 14.0266 14.0600 14.0934 14.1268 14.1602 14.1936 14.2270 14.2604 14.2938 14.3272 14.3606 14.3940 14.4274 14.4608 14.4942 14.5276 14.5610 14.5944 14.6278 14.6612 14.6946 14.7280 14.7614 14.7948 14.8282 14.8616 14.8950 14.9284 14.9618 14.9952 15.0286 15.0620 15.0954 15.1288 15.1622 15.1956 15.2290 15.2624 15.2958 15.3292 15.3626 15.3960 15.4294 15.4628 15.4962 15.5296 15.5630 15.5964 15.6298 15.6632 15.6966 15.7300 15.7634 15.7968 15.8302 15.8636 15.8970 15.9304 15.9638 15.9972 16.0306 16.0640 16.0974 16.1308 16.1642 16.1976 16.2310 16.2644 16.2978 16.3312 16.3646 16.3980 16.4314 16.4648 16.4982 16.5316 16.5650 16.5984 16.6318 16.6652 16.6986 16.7320 16.7654 16.7988 16.8322 16.8656 16.8990 16.9324 16.9658 16.9992 17.0326 17.0660 17.0994 17.1328 17.1662 17.1996 17.2330 17.2664 17.2998 17.3332 17.3666 17.3999 17.4333 17.4667 17.5001 17.5335 17.5669 17.6003 17.6337 17.6671 17.7005 17.7339 17.7673 17.8007 17.8341 17.8675 17.9009 17.9343 17.9677 18.0011 18.0345 18.0679 18.1013 18.1347 18.1681 18.2015 18.2349 18.2683 18.3017 18.3351 18.3685 18.4019 18.4353 18.4687 18.5021 18.5355 18.5689 18.6023 18.6357 18.6691 18.7025 18.7359 18.7693 18.8027 18.8361 18.8695 18.9029 18.9363 18.9697 19.0031 19.0365 19.0699 19.1033 19.1367 19.1701 19.2035 19.2369 19.2703 19.3037 19.3371 19.3705 19.4039 19.4373 19.4707 19.5041 19.5375 19.5709 19.6043 19.6377 19.6711 19.7045 19.7379 19.7713 19.8047 19.8381 19.8715 19.9049 19.9383 19.9717 20.0051 20.0385 20.0719 20.1053 20.1387 20.1721 20.2055 20.2389 20.2723 20.3057 20.3391 20.3725 20.4059 20.4393 20.4727 20.5061 20.5395 20.5729 20.6063 20.6397 20.6731 20.7065 20.7399 20.7733 20.8067 20.8401 20.8735 20.9069 20.9403 20.9737 21.0071 21.0405 21.0739 21.1073 21.1407 21.1741 21.2075 21.2409 21.2743 21.3077 21.3411 21.3745 21.4079 21.4413 21.4747 21.5081 21.5415 21.5749 21.6083 21.6417 21.6751 21.7085 21.7419 21.7753 21.8087 21.8421 21.8755 21.9089 21.9423 21.9757 22.0091 22.0425 22.0759 22.1093 22.1427 22.1761 22.2095 22.2429 22.2763 22.3097 22.3431 22.3765 22.4099 22.4433 22.4767 22.5101 22.5435 22.5769 22.6103 22.6437 22.6771 22.7105 22.7439 22.7773 22.8107 22.8441 22.8775 22.9109 22.9443 22.9777 23.0111 23.0445 23.0779 23.1113 23.1447 23.1781 23.2115 23.2449 23.2783 23.3117 23.3451 23.3785 23.4119 23.4453 23.4787 23.5121 23.5455 23.5789 23.6123 23.6457 23.6791 23.7125 23.7459 23.7793 23.8127 23.8461 23.8795 23.9129 23.9463 23.9797 24.0131 24.0465 24.0799 24.1133 24.1467 24.1801 24.2135 24.2469 24.2803 24.3137 24.3471 24.3805 24.4139 24.4473 24.4807 24.5141 24.5475 24.5809 24.6143 24.6477 24.6811 24.7145 24.7479 24.7813 24.8147 24.8481 24.8815 24.9149 24.9483 24.9817 25.0151 25.0485 25.0819 25.1153 25.1487 25.1821 25.2155 25.2489 25.2823 25.3157 25.3491 25.3825 25.4159 25.4493 25.4827 25.5161 25.5495 25.5829 25.6163 25.6497 25.6831 25.7165 25.7499 25.7833 25.8167 25.8501 25.8835 25.9169 25.9503 25.9837 26.0171 26.0505 26.0839 26.1173 26.1507 26.1841 26.2175 26.2509 26.2843 26.3177 26.3511 26.3845 26.4179 26.4513 26.4847 26.5181 26.5515 26.5849 26.6183 26.6517 26.6851 26.7185 26.7519 26.7853 26.8187 26.8521 26.8855 26.9189 26.9523 26.9857 27.0191 27.0525 27.0859 27.1193 27.1527 27.1861 27.2195 27.2529 27.2863 27.3197 27.3531 27.3865 27.4199 27.4533 27.4867 27.5201 27.5535 27.5869 27.6203 27.6537 27.6871 27.7205 27.7539 27.7873 27.8207 27.8541 27.8875 27.9209 27.9543 27.9877 28.0211 28.0545 28.0879 28.1213 28.1547 28.1881 28.2215 28.2549 28.2883 28.3217 28.3551 28.3885 28.4219 28.4553 28.4887 28.5221 28.5555 28.5889 28.6223 28.6557 28.6891 28.7225 28.7559 28.7893 28.8227 28.8561 28.8895 28.9229 28.9563 28.9897 29.0231 29.0565 29.0899 29.1233 29.1567 29.1901 29.2235 29.2569 29.2903 29.3237 29.3571 29.3905 29.4239 29.4573 29.4907 29.5241 29.5575 29.5909 29.6243 29.6577 29.6911 29.7245 29.7579 29.7913 29.8247 29.8581 29.8915 29.9249 29.9583 29.9917 30.0251 30.0585 30.0919 30.1253 30.1587 30.1921 30.2255 30.2589 30.2923 30.3257 30.3591 30.3925 30.4259 30.4593 30.4927 30.5261 30.5595 30.5929 30.6263 30.6597 30.6931 30.7265 30.7599 30.7933 30.8267 30.8601 30.8935 30.9269 30.9603 30.9937 31.0271 31.0605 31.0939 31.1273 31.1607 31.1941 31.2275 31.2609 31.2943 31.3277 31.3611 31.3945 31.4279 31.4613 31.4947 31.5281 31.5615 31.5949 31.6283 31.6617 31.6951 31.7285 31.7619 31.7953 31.8287 31.8621 31.8955 31.9289 31.9623 31.9957 32.0291 32.0625 32.0959 32.1293 32.1627 32.1961 32.2295 32.2629 32.2963 32.3297 32.3631 32.3965 32.4299 32.4633 32.4967 32.5301 32.5635 32.5969 32.6303 32.6637 32.6971 32.7305 32.7639 32.7973 32.8307 32.8641 32.8975 32.9309 32.9643 32.9977 33.0311 33.0645 33.0979 33.1313 33.1647 33.1981 33.2315 33.2649 33.2983 33.3317 33.3651 33.3985 33.4319 33.4653 33.4987 33.5321 33.5655 33.5989 33.6323 33.6657 33.6991 33.7325 33.7659 33.7993 33.8327 33.8661 33.8995 33.9329 33.9663 33.9997 34.0331 34.0665 34.0999 34.1333 34.1667 34.1999 34.2333 34.2667 34.2999 34.3333 34.3667 34.3999 34.4333 34.4667 34.4999 34.5333 34.5667 34.5999 34.6333 34.6667 34.6999 34.7333 34.7667 34.7999 34.8333 34.8667 34.8999 34.9333 34.9667 35.0000]
    longitudinal_pos_rear: [0 0.0334 0.0669 0.1003 0.1338 0.1672 0.2007 0.2341 0.2676 0.3010 0.3344 0.3679 0.4013 0.4348 0.4682 0.5016 0.5350 0.5684 0.6018 0.6352 0.6686 0.7020 0.7354 0.7688 0.8022 0.8356 0.8690 0.9024 0.9358 0.9692 1.0026 1.0360 1.0694 1.1028 1.1362 1.1696 1.2030 1.2364 1.2698 1.3032 1.3366 1.3700 1.4034 1.4368 1.4702 1.5036 1.5370 1.5704 1.6038 1.6372 1.6706 1.7040 1.7374 1.7708 1.8042 1.8376 1.8710 1.9044 1.9378 1.9712 2.0046 2.0380 2.0714 2.1048 2.1382 2.1716 2.2050 2.2384 2.2718 2.3052 2.3386 2.3720 2.4054 2.4388 2.4722 2.5056 2.5390 2.5724 2.6058 2.6392 2.6726 2.7060 2.7394 2.7728 2.8062 2.8396 2.8730 2.9064 2.9398 2.9732 3.0066 3.0400 3.0734 3.1068 3.1402 3.1736 3.2070 3.2404 3.2738 3.3072 3.3406 3.3740 3.4074 3.4408 3.4742 3.5076 3.5410 3.5744 3.6078 3.6412 3.6746 3.7080 3.7414 3.7748 3.8082 3.8416 3.8750 3.9084 3.9418 3.9752 4.0086 4.0420 4.0754 4.1088 4.1422 4.1756 4.2090 4.2424 4.2758 4.3092 4.3426 4.3760 4.4094 4.4428 4.4762 4.5096 4.5430 4.5764 4.6098 4.6432 4.6766 4.7100 4.7434 4.7768 4.8102 4.8436 4.8770 4.9104 4.9438 4.9772 5.0106 5.0440 5.0774 5.1108 5.1442 5.1776 5.2110 5.2444 5.2778 5.3112 5.3446 5.3780 5.4114 5.4448 5.4782 5.5116 5.5450 5.5784 5.6118 5.6452 5.6786 5.7120 5.7454 5.7788 5.8122 5.8456 5.8790 5.9124 5.9458 5.9792 6.0126 6.0460 6.0794 6.1128 6.1462 6.1796 6.2130 6.2464 6.2798 6.3132 6.3466 6.3800 6.4134 6.4468 6.4802 6.5136 6.5470 6.5804 6.6138 6.6472 6.6806 6.7140 6.7474 6.7808 6.8142 6.8476 6.8810 6.9144 6.9478 6.9812 7.0146 7.0480 7.0814 7.1148 7.1482 7.1816 7.2150 7.2484 7.2818 7.3152 7.3486 7.3820 7.4154 7.4488 7.4822 7.5156 7.5490 7.5824 7.6158 7.6492 7.6826 7.7160 7.7494 7.7828 7.8162 7.8496 7.8830 7.9164 7.9498 7.9832 8.0166 8.0500 8.0834 8.1168 8.1502 8.1836 8.2170 8.2504 8.2838 8.3172 8.3506 8.3840 8.4174 8.4508 8.4842 8.5176 8.5510 8.5844 8.6178 8.6512 8.6846 8.7180 8.7514 8.7848 8.8182 8.8516 8.8850 8.9184 8.9518 8.9852 9.0186 9.0520 9.0854 9.1188 9.1522 9.1856 9.2190 9.2524 9.2858 9.3192 9.3526 9.3860 9.4194 9.4528 9.4862 9.5196 9.5530 9.5864 9.6198 9.6532 9.6866 9.7200 9.7534 9.7868 9.8202 9.8536 9.8870 9.9204 9.9538 9.9872 10.0206 10.0540 10.0874 10.1208 10.1542 10.1876 10.2210 10.2544 10.2878 10.3212 10.3546 10.3880 10.4214 10.4548 10.4882 10.5216 10.5550 10.5884 10.6218 10.6552 10.6886 10.7220 10.7554 10.7888 10.8222 10.8556 10.8890 10.9224 10.9558 10.9892 11.0226 11.0560 11.0894 11.1228 11.1562 11.1896 11.2230 11.2564 11.2898 11.3232 11.3566 11.3900 11.4234 11.4568 11.4902 11.5236 11.5570 11.5904 11.6238 11.6572 11.6906 11.7240 11.7574 11.7908 11.8242 11.8576 11.
```



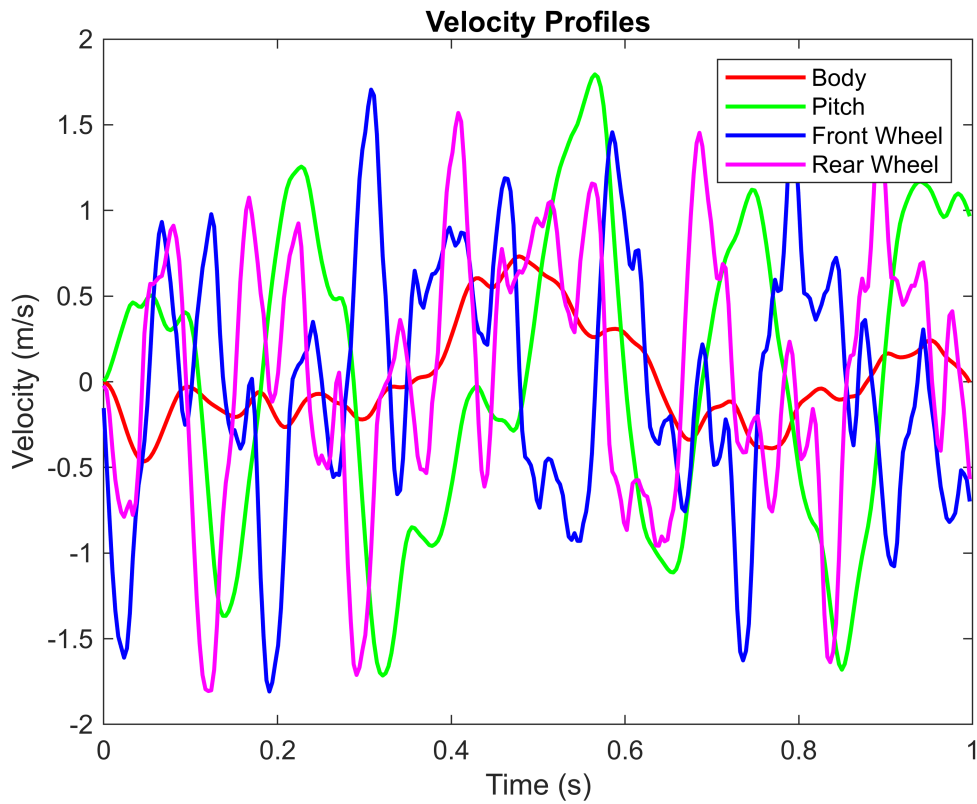


```
figure
u = displacement.longitudinal_pos_front;
plot(u , displacement.z_body, 'r', 'LineWidth', 1.5);
hold on;
plot(u, displacement.theta, 'g', 'LineWidth', 1.5);
plot(u, displacement.z_unsprung_front, 'b', 'LineWidth', 1.5);
plot(u, displacement.z_unsprung_rear, 'm', 'LineWidth', 1.5);
plot(u, displacement.tire_front, 'k', 'LineWidth', 1.5);
xlabel('Distance (s)');
ylabel('Distance (m/s)');
legend('Body', 'Pitch', 'Front Wheel', 'Rear Wheel','Road');
title('Displacement Profiles - Visualization');
xlim([4.5 7])
hold off
```

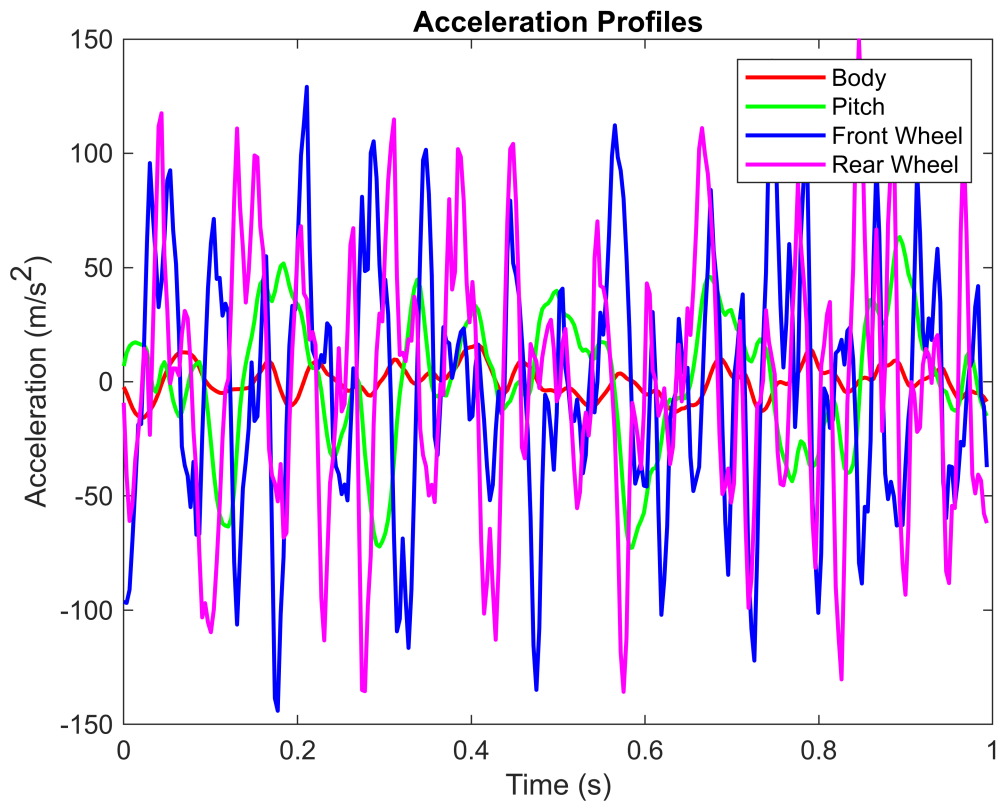


## Time Based Analysis

```
figure;
plot(velocity.time, velocity.v_body, 'r', 'LineWidth', 1.5);
hold on;
plot(velocity.time, velocity.v_theta, 'g', 'LineWidth', 1.5);
plot(velocity.time, velocity.v_unsprung_front, 'b', 'LineWidth', 1.5);
plot(velocity.time, velocity.v_unsprung_rear, 'm', 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Velocity (m/s)');
legend('Body', 'Pitch', 'Front Wheel', 'Rear Wheel');
title('Velocity Profiles');
hold off
```



```
figure;
plot(acceleration.time, acceleration.a_body, 'r', 'LineWidth', 1.5);
hold on;
plot(acceleration.time, acceleration.a_theta, 'g', 'LineWidth', 1.5);
plot(acceleration.time, acceleration.a_unsprung_front, 'b', 'LineWidth', 1.5);
plot(acceleration.time, acceleration.a_unsprung_rear, 'm', 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Acceleration (m/s^2)');
legend('Body', 'Pitch', 'Front Wheel', 'Rear Wheel');
title('Acceleration Profiles');
hold off
```



## RIDE COMFORT QUANTIZATION

Root Mean Square Acceleration

```
%Body
RMS_Acc_Body = rms(acceleration.a_body)
```

```
RMS_Acc_Body = 6.8532
```

```
%Pitch
RMS_Acc_Pitch = rms(acceleration.a_theta)
```

```
RMS_Acc_Pitch = 29.7704
```

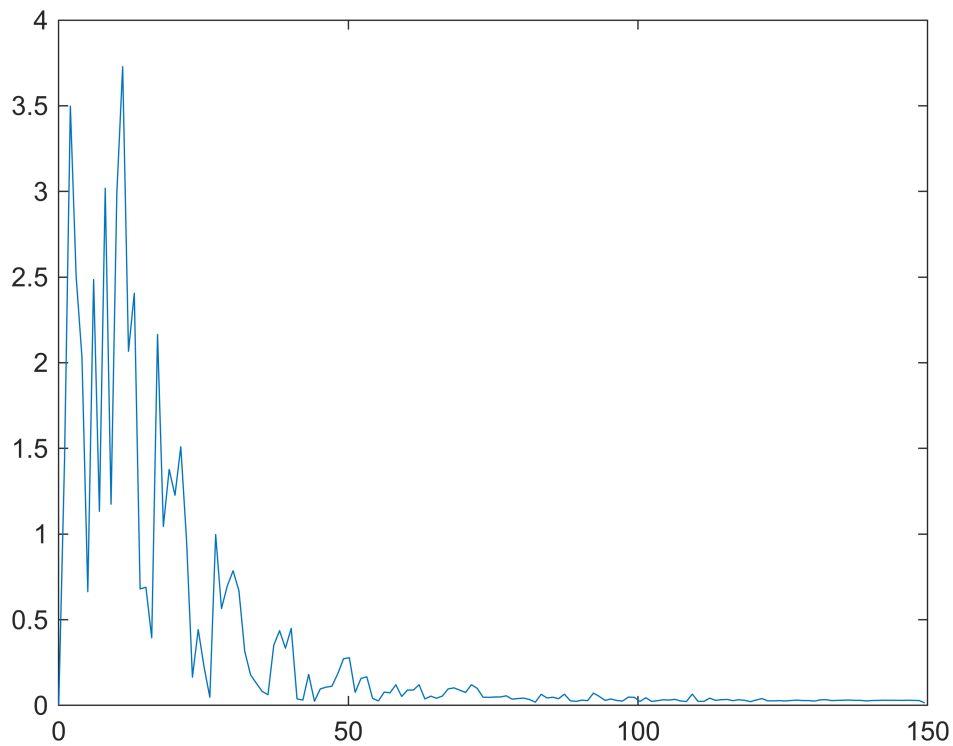
Transmissibility (acceleration based)

```
max(acceleration.a_body)/max(acceleration.a_unsprung_front)
```

```
ans = 0.1147
```

Power Spectral Density - Frequency based

```
[Weighted_PSD, frequency_arr, PSD] = Frequency_analysis(acceleration.a_body,acceleration.time);
plot(frequency_arr, PSD)
```



Total Power (PSD integral)

```
Total_Power = trapz(frequency_arr, PSD)
```

```
Total_Power = 52.1049
```

Finding Natural Frequencies

```
% Find and display the first 5 peaks in the amplitude spectrum (simple peak detection)
peaks = [];
locs = [];

for i = 2:length(frequency_arr)-1
    if PSD(i) > PSD(i-1) && PSD(i) > PSD(i+1)
        peaks = [peaks, PSD(i)];
        locs = [locs, frequency_arr(i)];
    end
end

n_modes = 5;
```

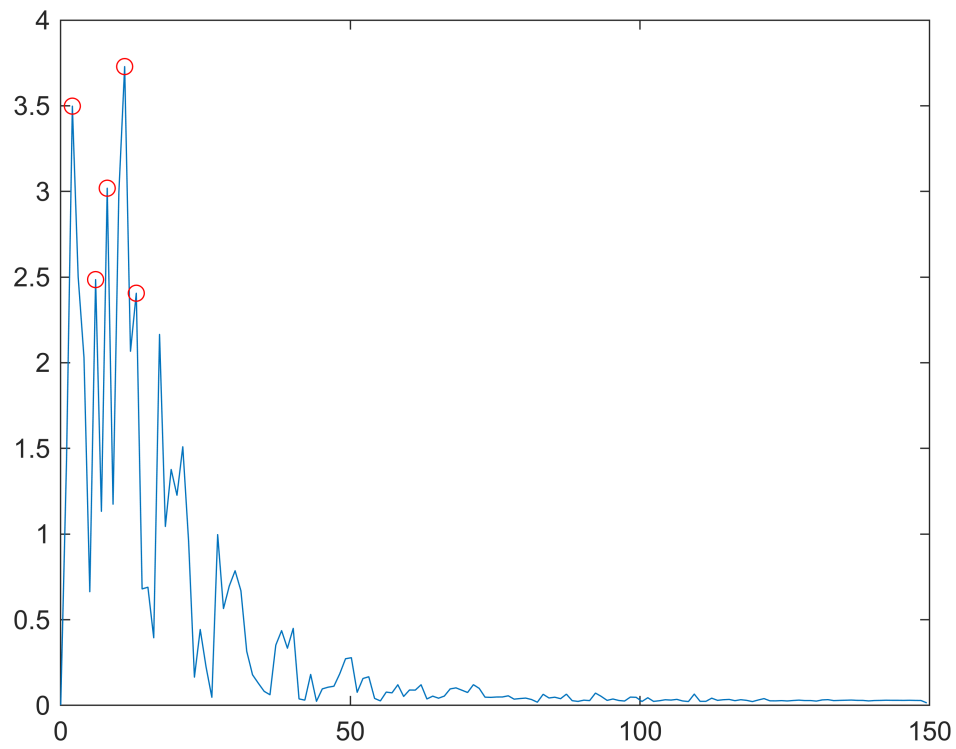
Natural Frequencies (First 5)

```
peaks = peaks(1:n_modes);
freqs = locs(1:n_modes)
```

```
freqs = 1x5
```

2.0067    6.0201    8.0268    11.0369    13.0436

```
plot(frequency_arr, PSD)
hold on;
plot(freqs, peaks, 'ro');
hold off;
```



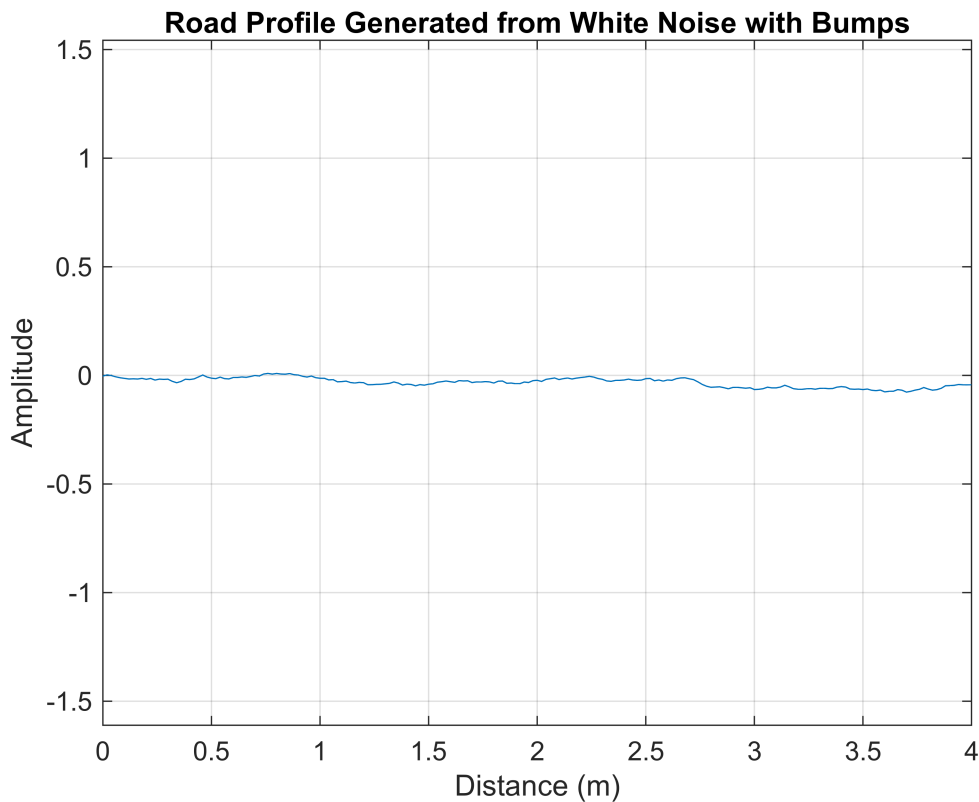
# PARAMETER EFFECT ANALYSIS

## Road Profile - Generation

```
% [X_r, Z_r] = bump_road_input(5,0.12,15);

profile_length = 200;           % Length of the road profile (meters)
sampling_rate = 50;             % Number of points per meter
amplitude_factor = 0.005;       % Roughness
num_bumps = 5;                  % Number of bumps
bump_amplitude = 0.001;         % Amplitude of bumps
bump_frequency = 0.5;           % Frequency of bumps

[X_r, Z_r] = generateRoadProfileWithBumps(profile_length, sampling_rate, amplitude_factor, num_bumps, bump_amplitude, bump_frequency);
```



```
road.X_r = X_r;
road.Z_r = Z_r;
```

## Car Model

```
%Vehicle
scooter.mass = 150;
scooter.front_unsprung_mass = 15;
scooter.rear_unsprung_mass = 15;
scooter.Lateral_MOI = 40;
scooter.CG_2_Front = 0.5;
```

```

scooter.CG_2_Rear = 0.5;

stiffness.front_strut = 15000;
stiffness.rear_strut = 15000;
stiffness.tire_front = 44000;
stiffness.tire_rear = 44000;

damping.strut_front = 1000;
damping.strut_rear = 1000;

%Velocity
initial_vel = 10; %velocity in m/s
acc = 0;

```

## Stiffness Variation

```

% Stiffness variation from 200 to 30000
% Damping kept at 1000 for analysis
k = 200:100:30000;
rms_acc = k;
natural_frequency = [k;k];
rms_pitch = k;
TRS = k;
Total_POW = k;

```

## Simulating

```

for i = 1:length(k)

    stiffness.front_strut = k(i);
    stiffness.rear_strut = k(i);
    [displacement, velocity, acceleration] = Ride_Comfort_Analysis(scooter, stiffness, damping);
    rms_acc(i) = rms(acceleration.a_body);
    rms_pitch(i) = rms(acceleration.a_theta);
    TRS(i) = max(acceleration.a_body)/max(acceleration.a_unsprung_front);

    [Weighted_PSD, frequency_arr, PSD] = Frequency_analysis(acceleration.a_body,acceleration.t);
    Total_POW(i) = trapz(frequency_arr, PSD);

    peaks = [];
    locs = [];
    for j = 2:length(frequency_arr)-1
        if PSD(j) > PSD(j-1) && PSD(j) > PSD(j+1)
            peaks = [peaks, PSD(j)];
            locs = [locs, frequency_arr(j)];
        end
    end

    natural_frequency(:,i) = (locs(1:2))';
end

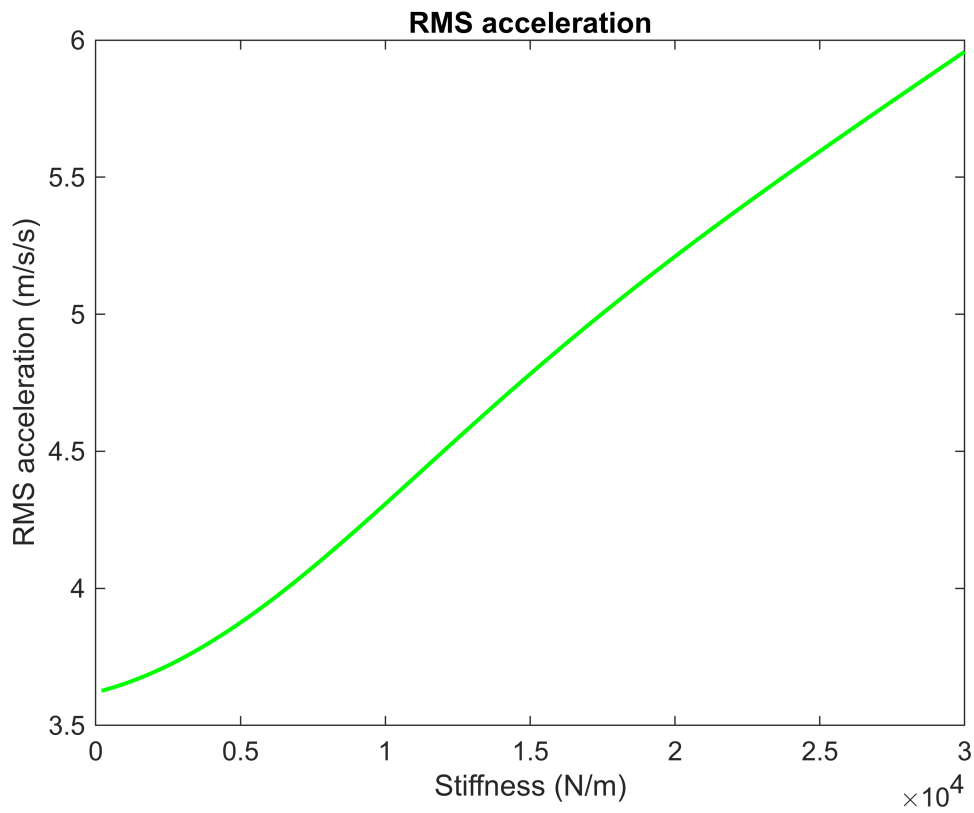
```



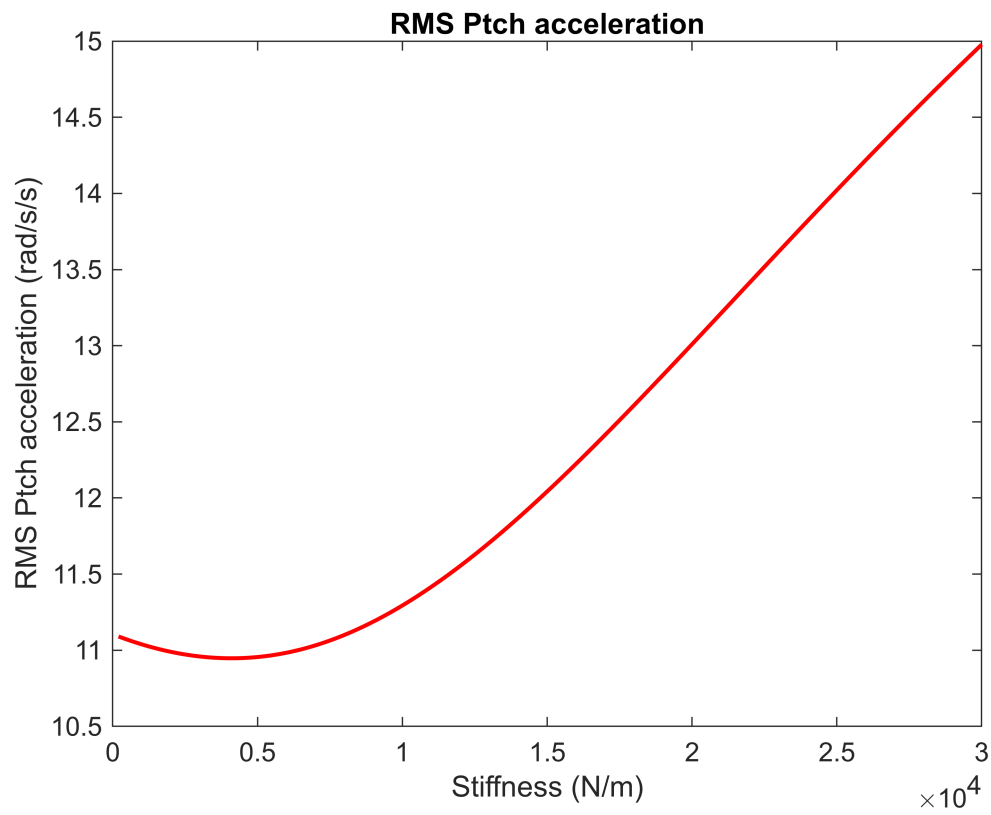
```
end
```

## Plotting

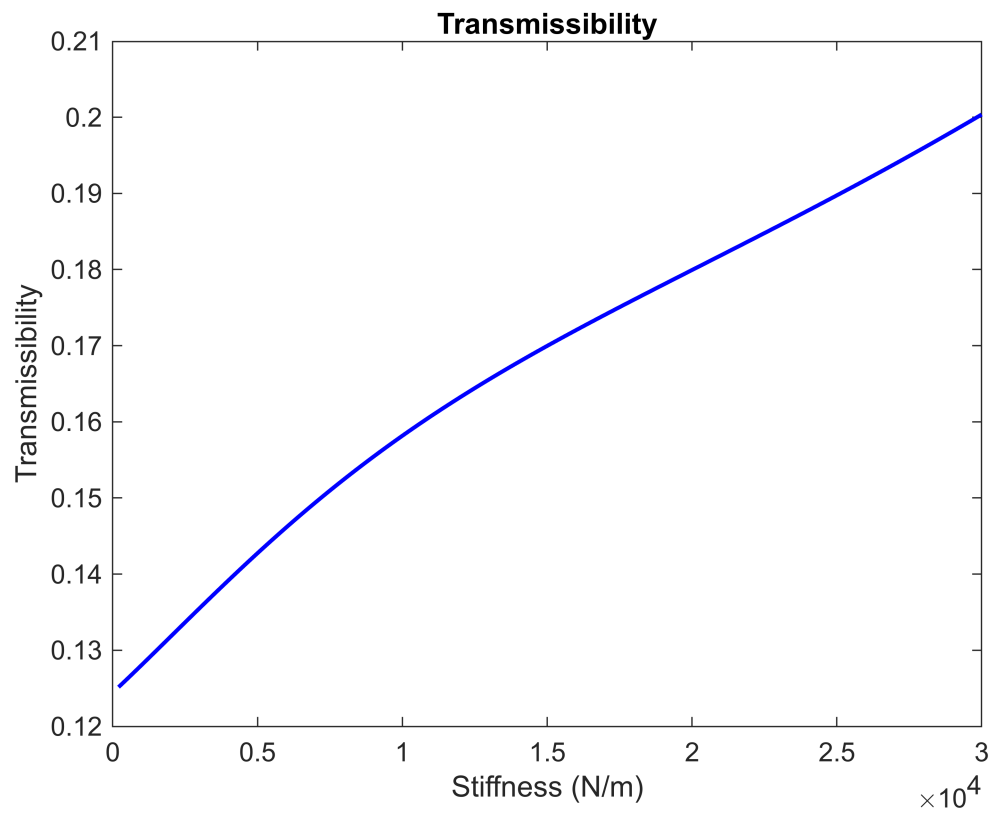
```
plot(k, rms_acc, 'g', 'LineWidth', 1.5);  
xlabel('Stiffness (N/m)');  
ylabel('RMS acceleration (m/s/s)');  
title('RMS acceleration');
```



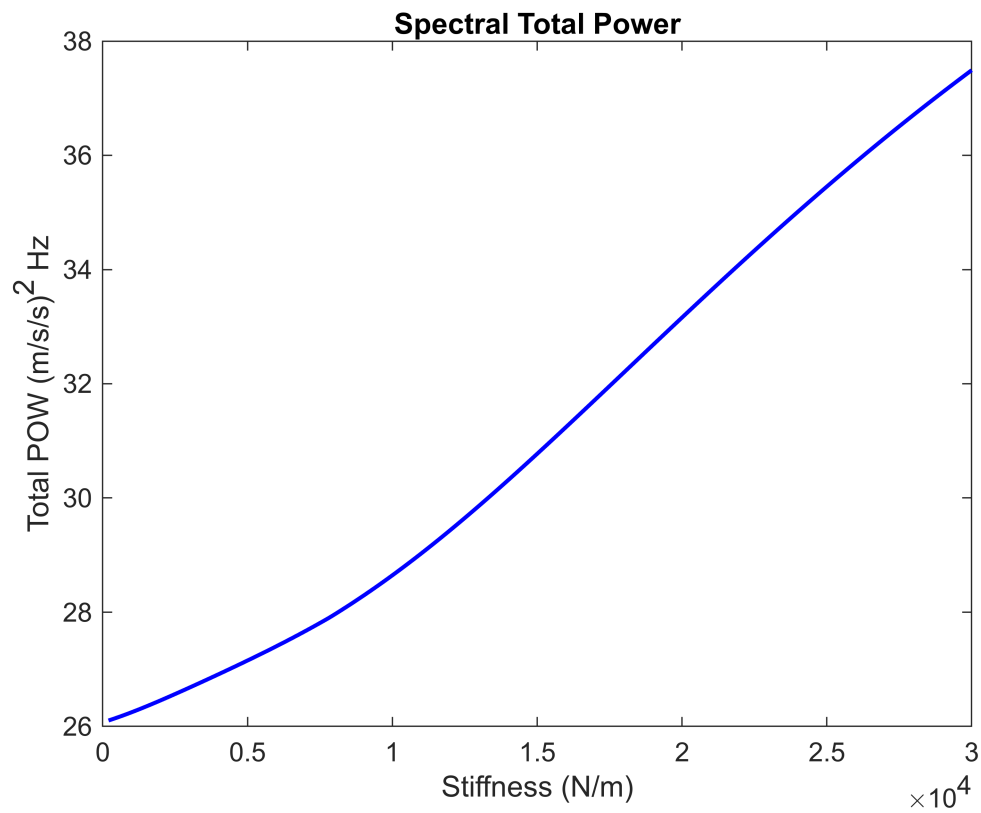
```
plot(k, rms_pitch, 'r', 'LineWidth', 1.5);  
xlabel('Stiffness (N/m)');  
ylabel('RMS Ptch acceleration (rad/s/s)');  
title('RMS Ptch acceleration');
```



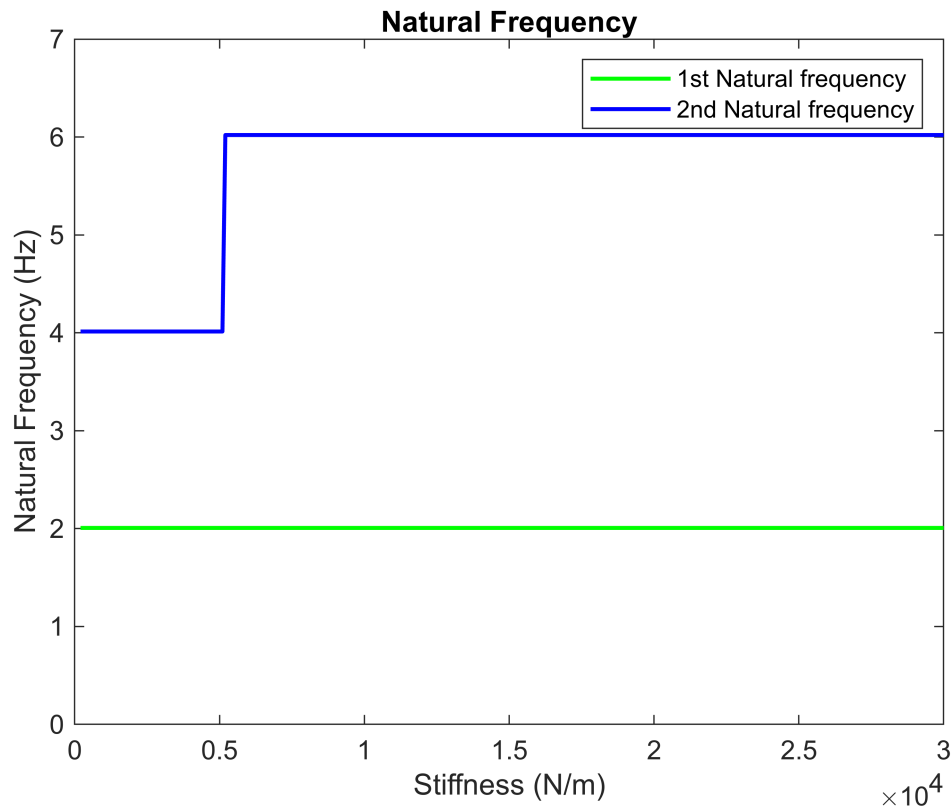
```
plot(k, TRS, 'b', 'LineWidth', 1.5);  
xlabel('Stiffness (N/m)');  
ylabel('Transmissibility');  
title('Transmissibility');
```



```
plot(k, Total_POW, 'b', 'LineWidth', 1.5);  
xlabel('Stiffness (N/m)');  
ylabel('Total POW (m/s/s)^2 Hz');  
title('Spectral Total Power');
```



```
plot(k, natural_frequency(1,:), 'g', 'LineWidth', 1.5);
hold on;
plot(k, natural_frequency(2,:), 'b', 'LineWidth', 1.5);
xlabel('Stiffness (N/m)');
ylabel('Natural Frequency (Hz)');
legend('1st Natural frequency', '2nd Natural frequency');
title('Natural Frequency');
ylim([0 7])
hold off
```



## Damping Variation

```
% Stiffness kept at 15000 for analysis
C = 0:100:10000;
rms_acc = C;
natural_frequency = [C;C];
stiffness.front_strut = 15000;
stiffness.rear_strut = 15000;

rms_pitch = C;
TRS = C;
Total_POW = C;
```

Simulating

```
for i = 1:length(C)

    damping.strut_front = C(i);
    damping.strut_rear = C(i);
    [displacement, velocity, acceleration] = Ride_Comfort_Analysis(scooter, stiffness, damping);
    rms_acc(i) = rms(acceleration.a_body);
    rms_pitch(i) = rms(acceleration.a_theta);
    TRS(i) = max(acceleration.a_body)/max(acceleration.a_unsprung_front);

end
```

```

[Weighted_PSD, frequency_arr, PSD] = Frequency_analysis(acceleration.a_body, acceleration.t);
Total_POW(i) = trapz(frequency_arr, PSD);

peaks = [];
locs = [];
for j = 2:length(frequency_arr)-1
    if PSD(j) > PSD(j-1) && PSD(j) > PSD(j+1)
        peaks = [peaks, PSD(j)];
        locs = [locs, frequency_arr(j)];
    end
end
[peaks, sortIdx] = sort(peaks, 'descend');
locs = locs(sortIdx);

natural_frequency(:,i) = (locs(1:2))';

end

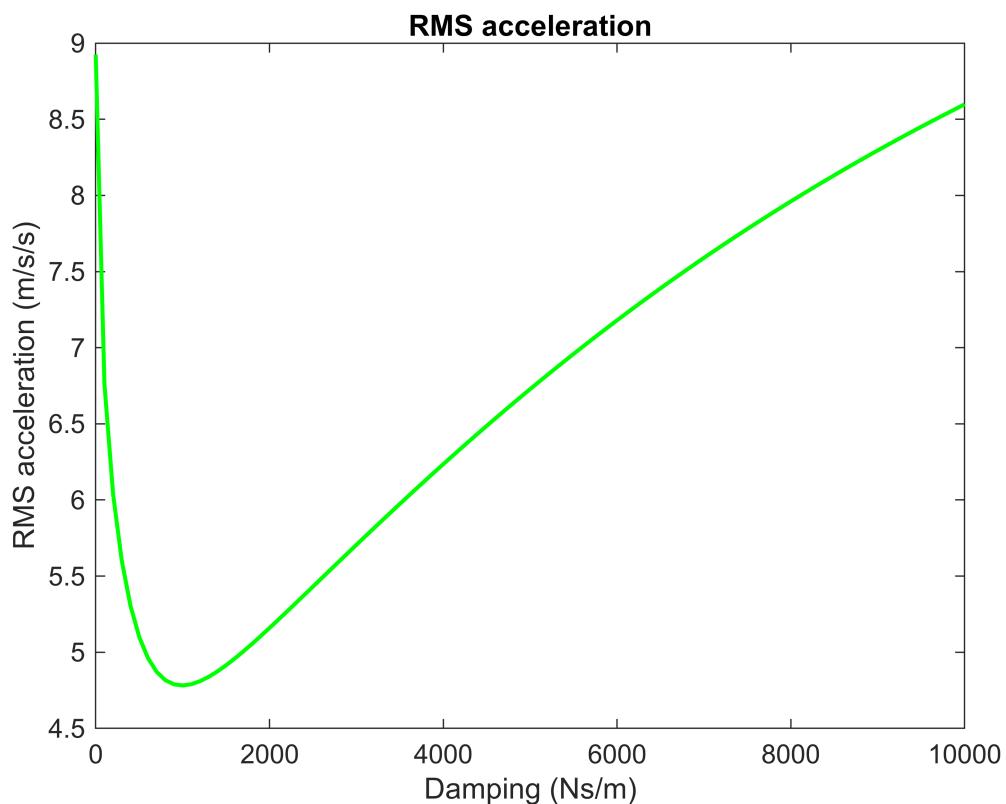
```

## Plotting

```

plot(C, rms_acc, 'g', 'LineWidth', 1.5);
xlabel('Damping (Ns/m)');
ylabel('RMS acceleration (m/s/s)');
title('RMS acceleration');

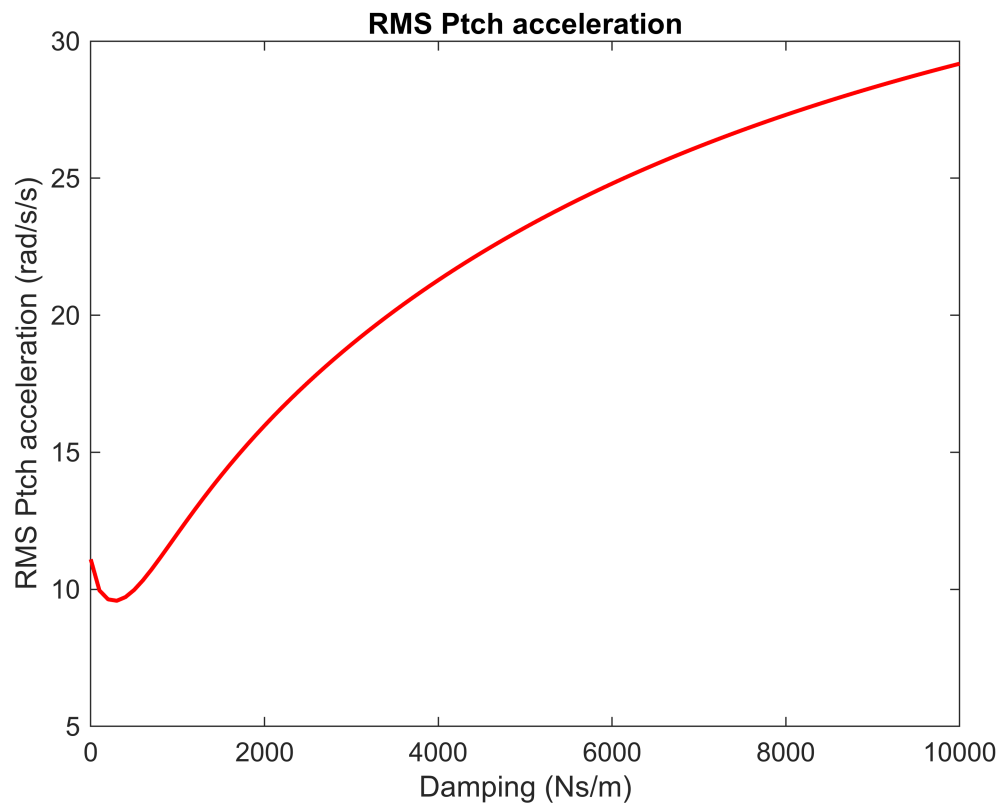
```



```

plot(C, rms_pitch, 'r', 'LineWidth', 1.5);
xlabel('Damping (Ns/m)');
ylabel('RMS Ptch acceleration (rad/s/s)');
title('RMS Ptch acceleration');

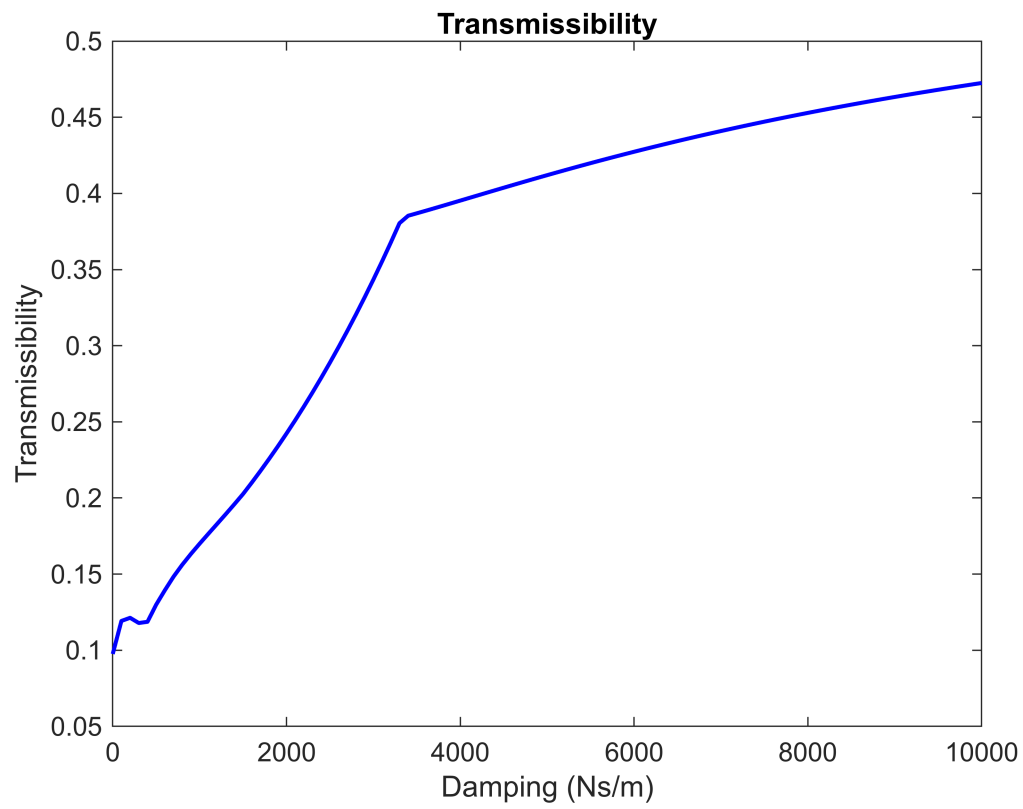
```



```

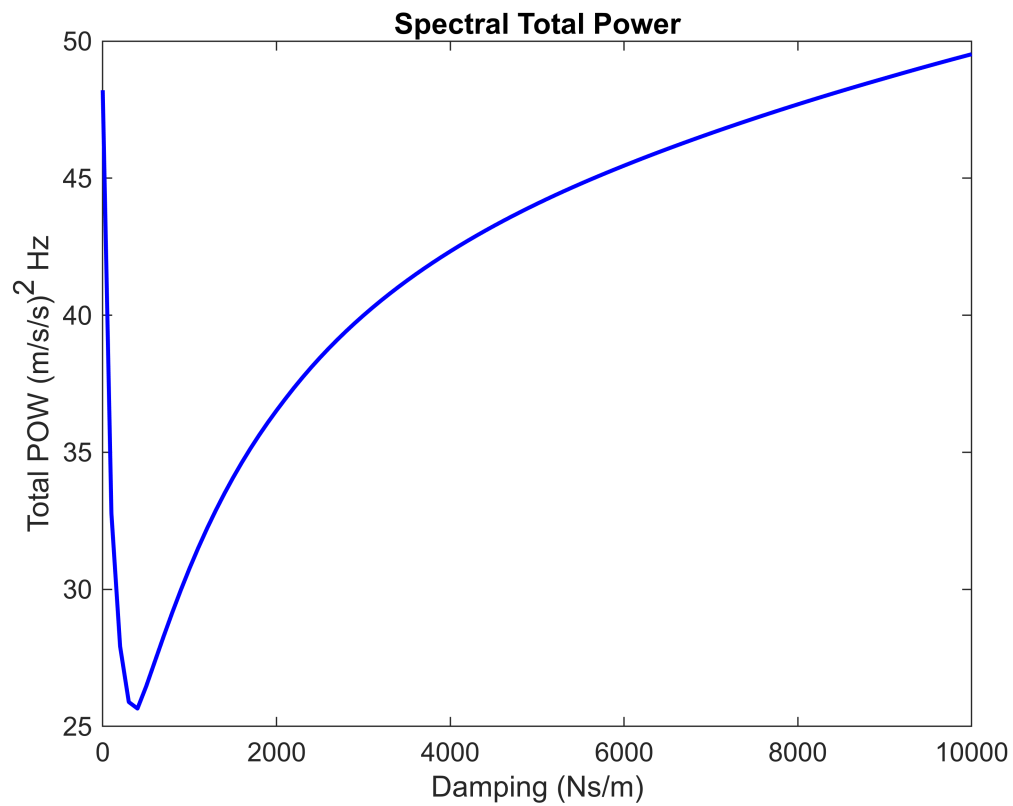
plot(C, TRS, 'b', 'LineWidth', 1.5);
xlabel('Damping (Ns/m)');
ylabel('Transmissibility');
title('Transmissibility');

```



```
plot(C, Total_POW, 'b', 'LineWidth', 1.5);  
xlabel('Damping (Ns/m)');  
ylabel('Total POW (m/s/s)^2 Hz');  
title('Spectral Total Power');
```

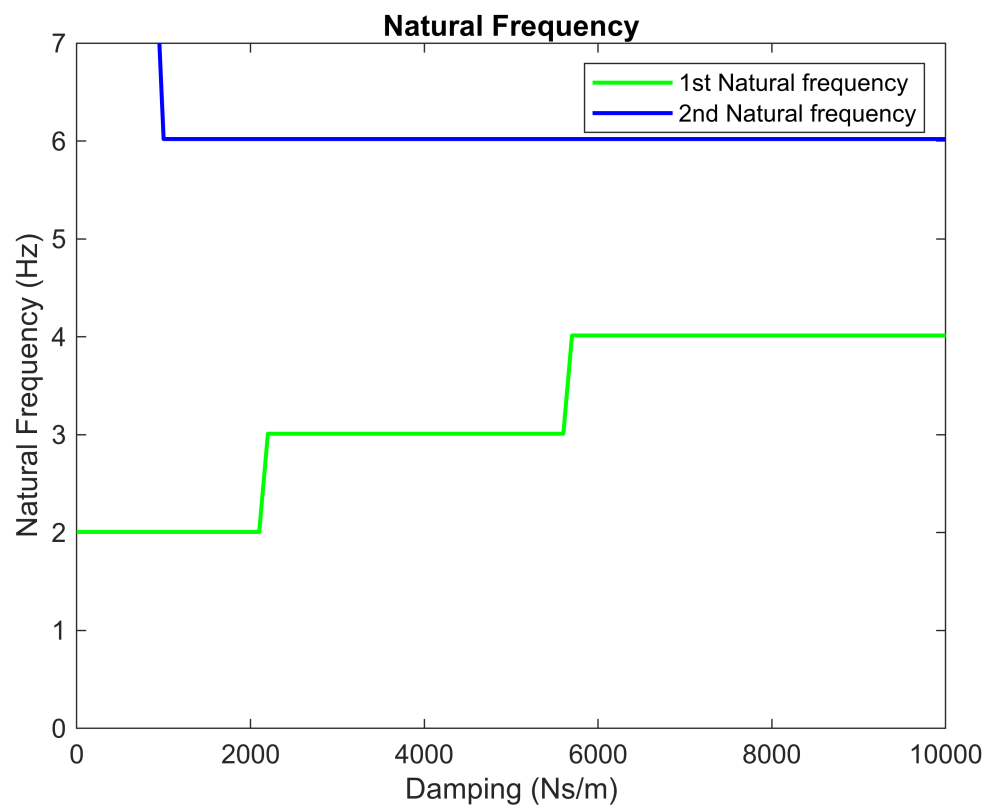




```

plot(C, natural_frequency(1,:), 'g', 'LineWidth', 1.5);
hold on;
plot(C, natural_frequency(2,:), 'b', 'LineWidth', 1.5);
xlabel('Damping (Ns/m)');
ylabel('Natural Frequency (Hz)');
legend('1st Natural frequency', '2nd Natural frequency');
title('Natural Frequency');
ylim([0 7])
hold off

```



```

function [displacement, velocity, acceleration] = Ride_Comfort_Analysis(car, stiffness, damping

m    = car.mass;
m1   = car.front_unsprung_mass;
m2   = car.rear_unsprung_mass;
Iy   = car.Lateral_MOI;
a1   = car.CG_2_Front;
a2   = car.CG_2_Rear;
k1   = stiffness.front_strut;
k2   = stiffness.rear_strut;
kt1  = stiffness.tire_front;
kt2  = stiffness.tire_rear;
c1   = damping.strut_front;
c2   = damping.strut_rear;

 playback_speed = 0.1;
tF    = 1;
fR    = 30/playback_speed;
dt    = 1/fR;
time  = linspace(0,tF,tF*fR);

X_r = road.X_r;
Z_r = road.Z_r;

M = [  m    0    0    0    ;
      0    Iy    0    0    ;
      0    0    m1    0    ;
      0    0    0    m2   ];

C = [  c1+c2          a2*c2-a1*c1      -c1      -c2      ;
      a2*c2-a1*c2    c1*a1^2+c2*a2^2    a1*c1    -a2*c2  ;
      -c1            a1*c1              c1        0       ;
      -c2            -a2*c2             0         c2      ];

K = [  k1+k2          a2*k2-a1*k1      -k1      -k2      ;
      a2*k2-a1*k1    k1*a1^2+k2*a2^2    a1*k1    -a2*k2  ;
      -k1            a1*k1              k1+kt1    0       ;
      -k2            -a2*k2             0         k2+kt2  ];

F = [  0      0    ;
      0      0    ;
      kt1     0    ;
      0      kt2  ];

% State space model
A = [  zeros(4,4)      eye(4,4)    ;
      -M\K            -M\C        ];
B = [  zeros(4,2)    ;
      M\F            ];
C = [  1 0 0 0 0 0 0 0 ;

```

```

    0 1 0 0 0 0 0 0 ;
    0 0 1 0 0 0 0 0 ;
    0 0 0 1 0 0 0 0 ;
    0 0 0 0 0 0 0 0 ;
    0 0 0 0 0 0 0 0 ;
    0 0 0 0 0 0 0 0 ;
    0 0 0 0 0 0 0 0 ];
D = zeros(8,2);

sys = ss(A,B,C,D);

% Input
lon_pos_2 = vel*time + 0.5*acc*(time.*time);           % Longitudinal position of the rear axle
lon_pos_1 = lon_pos_2 + a1+a2;   % Longitudinal position of the front axle [m]
% OBS: Added wheelbase!
%
u1 = interp1(X_r,Z_r,lon_pos_1);
u2 = interp1(X_r,Z_r,lon_pos_2);

u_vet = [u1' u2'];

[y,time,x] = lsim(sys,u_vet,time);

z      = y(:,1); % Body vertical motion coordinate      [m]
theta  = y(:,2); % Body pitch motion coordinate        [rad]
zu1    = y(:,3); % Front wheel vertical motion coordinate [m]
zu2    = y(:,4); % Rear wheel vertical motion coordinate [m]

% Time step
dt = mean(diff(time));

% Velocity calculation
v_z = diff(z) / dt; % Body vertical velocity
v_theta = diff(theta) / dt; % Body pitch velocity
v_zu1 = diff(zu1) / dt; % Front wheel velocity
v_zu2 = diff(zu2) / dt; % Rear wheel velocity

% Acceleration calculation
a_z = diff(v_z) / dt; % Body vertical acceleration
a_theta = diff(v_theta) / dt; % Body pitch acceleration
a_zu1 = diff(v_zu1) / dt; % Front wheel acceleration
a_zu2 = diff(v_zu2) / dt; % Rear wheel acceleration

% Time vector for velocity and acceleration (one element less due to differentiation)

displacement.z_body = z;
displacement.z_unsprung_front = zu1;
displacement.z_unsprung_rear = zu2;
displacement.theta = theta;
displacement.time = time;

```

```
displacement.tire_front = u1/2;  
displacement.tire_rear = u2/2;  
displacement.longitudinal_pos_front = lon_pos_1;  
displacement.longitudinal_pos_rear = lon_pos_2;  
  
velocity.v_body = v_z;  
velocity.v_unsprung_front = v_zu1;  
velocity.v_unsprung_rear = v_zu2;  
velocity.v_theta = v_theta;  
velocity.time = time(1:end-1);  
  
acceleration.a_body = a_z;  
acceleration.a_unsprung_front = a_zu1;  
acceleration.a_unsprung_rear = a_zu2;  
acceleration.a_theta = a_theta;  
acceleration.time = time(1:end-2);  
  
end
```