MSc Computer Vision / Vibot

Probabilistic Robotics

Practice Session

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Bayesian Probability

NOTE
The goal of this lab is to use/implement/understand Bayes theorem .

Problem
Suppose you live at a place where days are either sunny, cloudy, or rainy (a place like Le Creusot). The weather transition function is a Markov chain with the following transition table:

		tomorrow will be...		
		sunny	cloudy	rainy
today it's...	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

1. Suppose Day 1 is a sunny day. What is the probability of the following sequence of days: Day 2 = *cloudy*, Day 3 = *cloudy*, Day 4 = *rainy*?
2. Write a simulator (Matlab) that can randomly generate sequences of "weather" from this transition function.
3. Use your simulator to determine the stationary distribution of this Markov chain. The stationary distribution measures the probability that a random day will be sunny, cloudy or rainy.
4. Can you devise a closed-form solution to calculate the stationary distribution based on the state transition matrix above?
5. What is the entropy of the stationary distribution?
6. Using Bayes rule, compute the probability table of yesterday's weather given today's weather. (It is okay to provide the probabilities numerically, and it is also okay to rely on results from previous questions).
7. Suppose we added seasons to our model. The state transition function above would only apply to the Summer, whereas different ones would apply to Winter, Spring and Fall. Would this violate the Markov property of this process? Explain your answer.
8. Suppose you cannot observe the weather directly, but instead rely on a sensor. The problem is that your sensor is noisy. Its measurements are governed by the following model:

		our sensor tells us...		
		sunny	cloudy	rainy
	sunny	.6	.4	0
the actual weather is...	cloudy	.3	.7	0
	rainy	0	0	1

(a) Suppose Day 1 in sunny (this is known for a fact), and in the subsequent four days our sensor observes *cloudy*, *cloudy*, *rainy*, *sunny*. What is the probability that Day 5 is indeed sunny as predicted by the sensor?

(b) Again, suppose Day 1 is known to be sunny. At Days 2, 3 and 4, the sensor measures *sunny*, *sunny*, *rainny*. What is the most likely weather for each of the Days 2, 3 and 4?

(c) Consider the same situation (Day 1 is sunny, and the meqsurements for Days 2, 3 and 4 are *sunny*, *sunny*, *rainny*). What is the most likely sequence of weather for Days 2, 3 and 4? What is the probability of this most likely sequence?