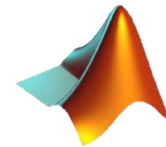


Outline

1. Introduction
2. The Bayes Filter
3. Gaussian filters
- 4. The Kalman filter

Labs:

SLAM Toolbox with Matlab



Assessment:

Labs + Exam (100%)

4.1 The Kalman Filter

Algorithm Kalman Filter ($\hat{x}_{k-1}, P_{k-1}, u_k, z_k$)

$$\begin{aligned}\hat{x}_k^- &= A_k \hat{x}_{k-1} + B_k u_k \\ P_k^- &= A_k P_{k-1} A_k^T + Q_k \\ K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \\ P_k &= (I - K_k H_k) P_k^- \\ \text{return} &(\hat{x}_k, P_k)\end{aligned}$$

Prediction:

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$$

Observation:

$$z_k$$

Update:

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1}$$

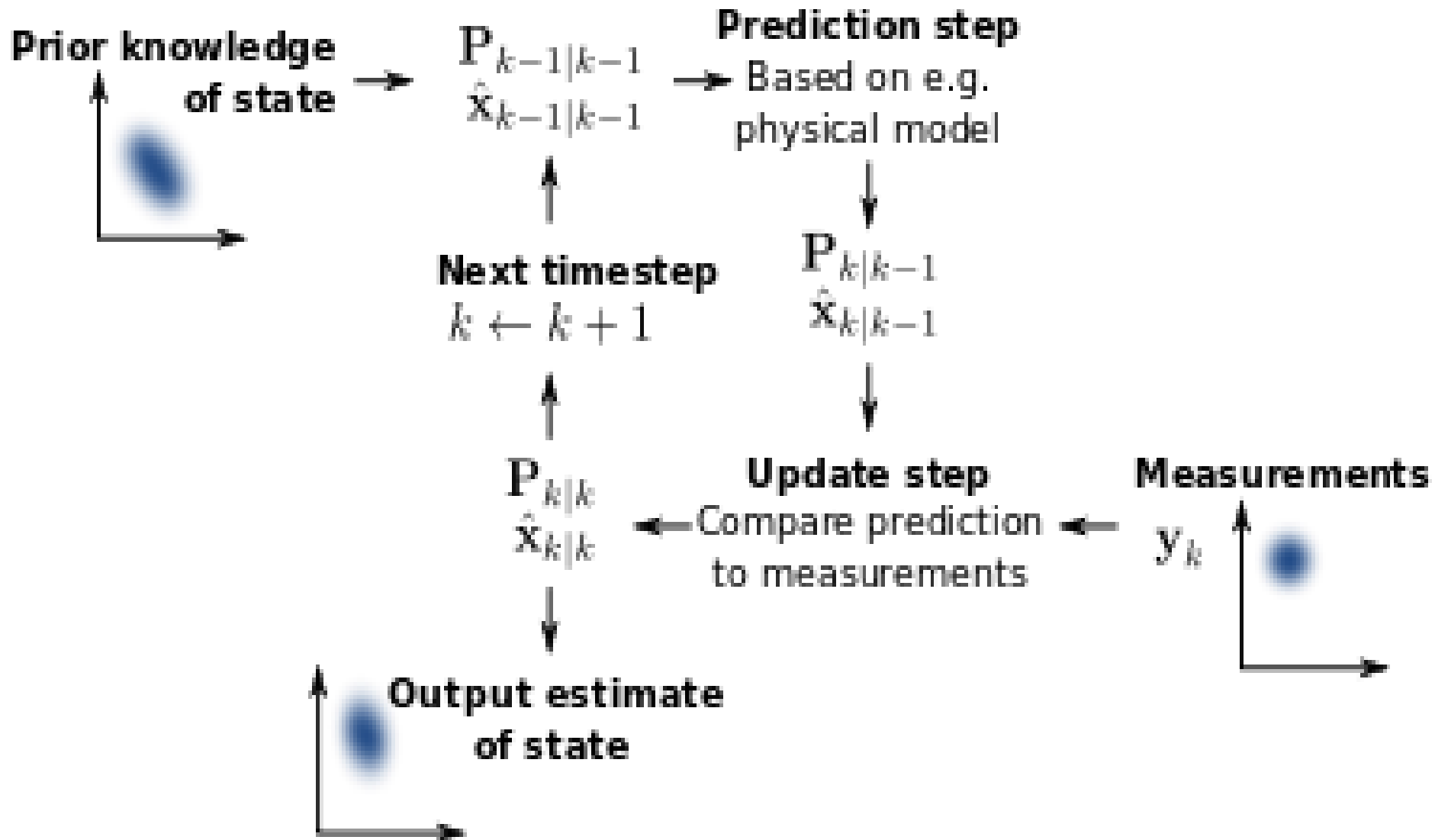
$$S_k = H_k P_{k|k-1} H_k^T + R_k$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1}$$

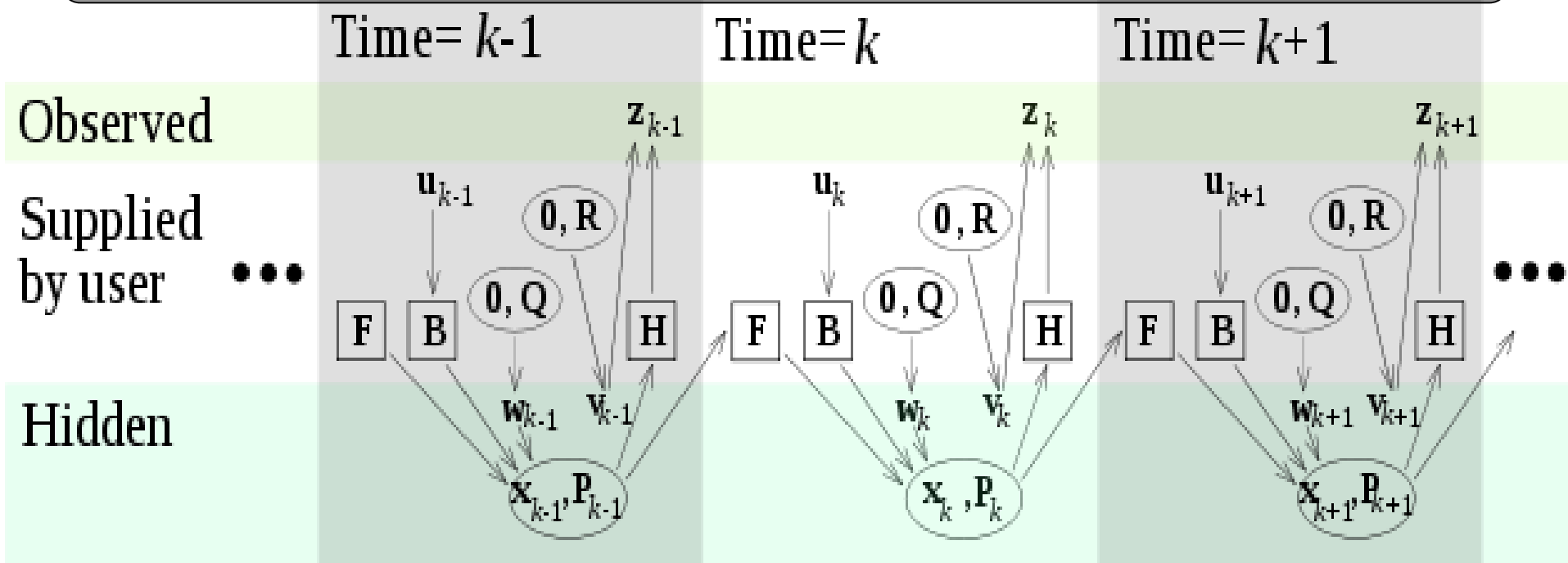
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

4.1 The Kalman Filter



4.1 The Kalman Filter



vectors

\mathbf{x}_k : state

\mathbf{u}_k : control input

\mathbf{z}_k : observation

\mathbf{w}_k : process noise = $N(0, \mathbf{Q}_k)$

\mathbf{v}_k : observation noise = $N(0, \mathbf{R}_k)$

matrices

\mathbf{P}_k : covariance of the state

\mathbf{F}_k : state-transition model

\mathbf{B}_k : control-input model

\mathbf{H}_k : observation model

\mathbf{Q}_k : covariance of the process noise

\mathbf{R}_k : covariance of the observation noise

Algorithm: SLAM

{ - Pose initialization }

$$\mathbf{x}_0^B = \hat{\mathbf{x}}_0^B; \mathbf{P}_0^B = \hat{\mathbf{P}}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[\mathbf{x}_0^B, \mathbf{P}_0^B] = \text{add_new_features}(\mathbf{x}_0^B, \mathbf{P}_0^B, z_0, R_0);$$

for k=1 to steps **do**

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B] = \text{move_vehicle}(\mathbf{x}_{k-1}^B, \mathbf{P}_{k-1}^B, u_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

$$\mathcal{H}_k = \text{data_association}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, z_k, R_k);$$

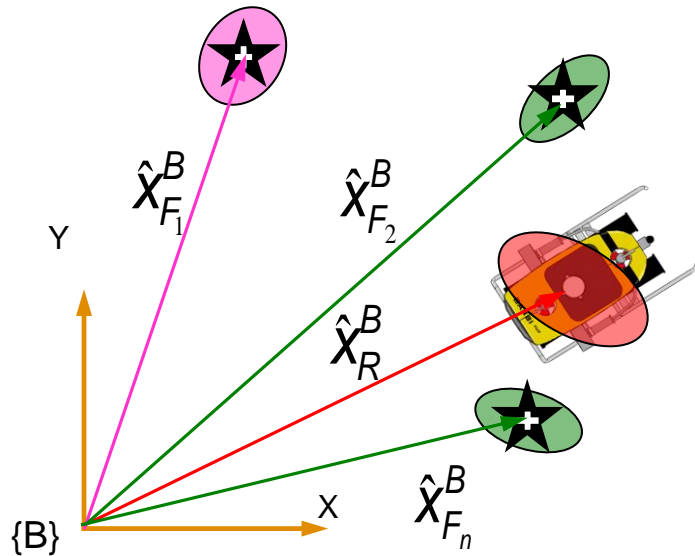
{ -EKF update }

$$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{update_position}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, z_k, R_k);$$

$$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{add_new_features}(\mathbf{x}_k^B, \mathbf{P}_k^B, z_k, R_k, \mathcal{H}_k);$$

end for

4.2 SLAM



Spatial Relationships

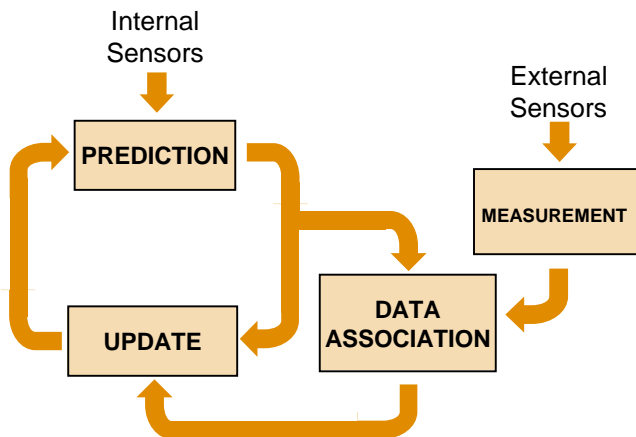
$$\hat{x}^B = E[x^B] = \begin{bmatrix} \hat{x}_R^B \\ \hat{x}_{F_1}^B \\ \hat{x}_{F_2}^B \\ \vdots \\ \hat{x}_{F_n}^B \end{bmatrix}$$

$$P^B = E[(x^B - \hat{x}^B)(x^B - \hat{x}^B)^T] = \begin{bmatrix} P_R^B & P_{F_1 R}^B & P_{F_2 R}^B & \dots & P_{F_n R}^B \\ P_{F_1 R}^B & P_{F_1}^B & P_{F_1 F_2}^B & \dots & P_{F_1 F_n}^B \\ P_{F_2 R}^B & P_{F_2 F_1}^B & P_{F_2}^B & \dots & P_{F_2 F_n}^B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{F_n R}^B & P_{F_n F_1}^B & \dots & \dots & P_{F_n}^B \end{bmatrix}$$

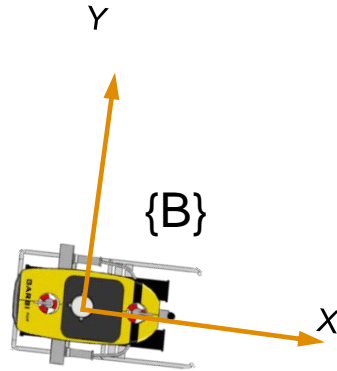
> A map is represented by:

- > The robot pose referenced to {B}
- > The location of a set of features referenced to {B}
- > The covariances between them

> The map is built incrementally



4.2 SLAM



SLAM Algorithm

Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$$

for k=1 to steps do

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, x_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

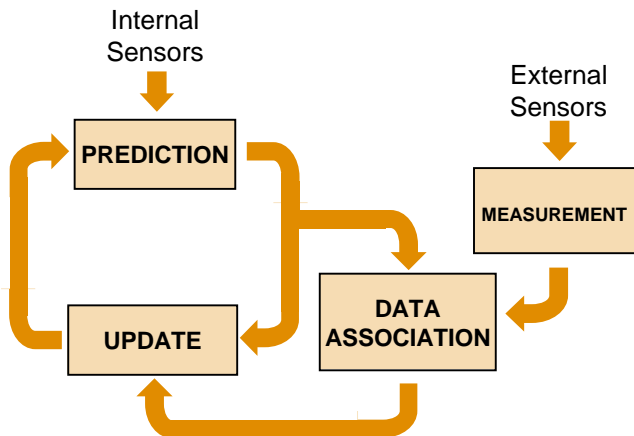
$$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

{ -EKF update }

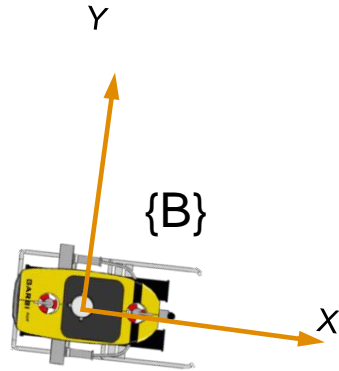
$$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

$$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k, \mathcal{H}_k);$$

end for



4.2 SLAM

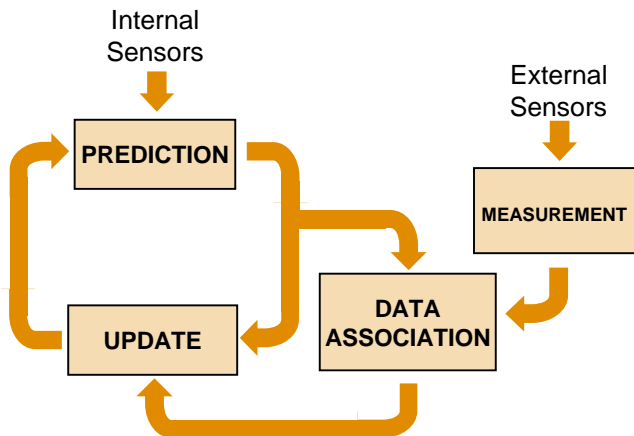


Initialization (II)

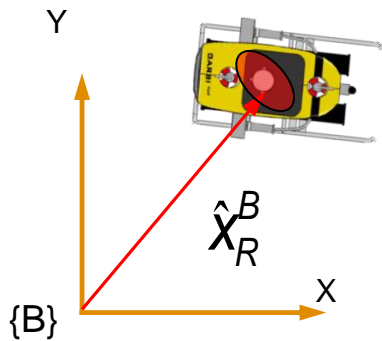
$$\hat{x}^B = \begin{bmatrix} 0 \end{bmatrix}, \quad P^B = \begin{bmatrix} 0 \end{bmatrix}$$

> The process starts Selecting the current position of the robot as the Reference frame

$$\hat{x}_R^B = 0 \quad P_R^B = 0$$



4.2 SLAM

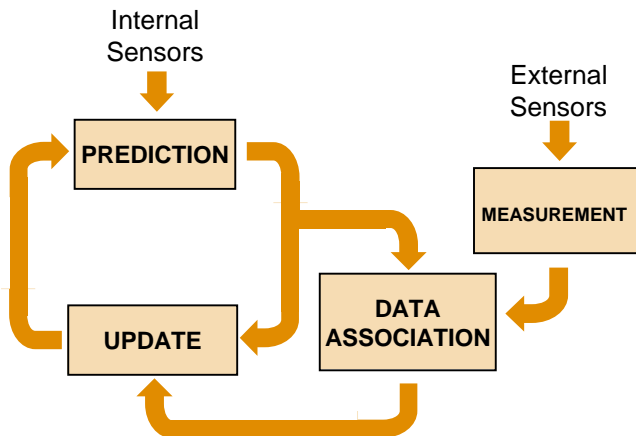


Initialization (III)

$$\hat{x}_k^B = \begin{bmatrix} \hat{x}_R^B \\ \vdots \end{bmatrix}, \quad P_k^B = \begin{bmatrix} P_R^B & \vdots \\ \vdots & \ddots \end{bmatrix}$$

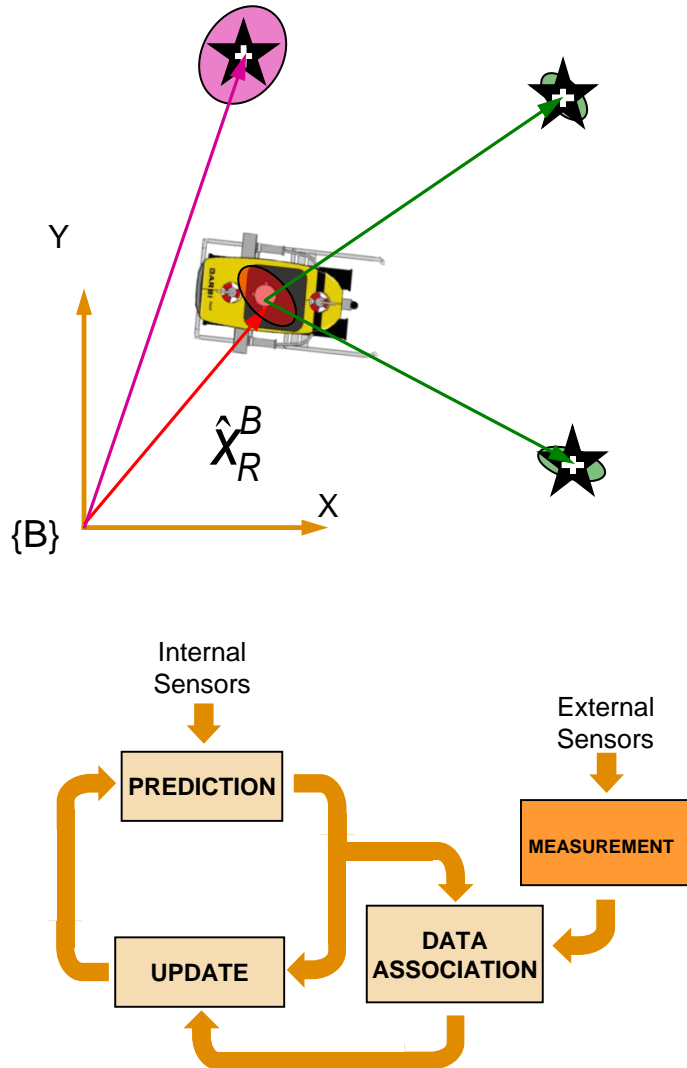
> Sometimes it is interesting to select another frame as the global reference

> In this case an estimation of the position mean and covariance is needed: \hat{x}_R^B P_R^B



4.2 SLAM

Adding New Features



Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$$

for k=1 to steps do

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, x_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

$$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

{ -EKF update }

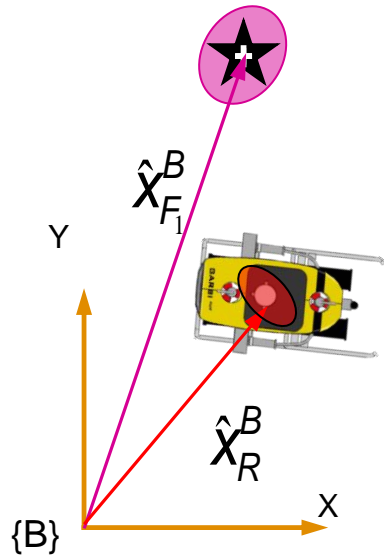
$$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

$$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k, \mathcal{H}_k);$$

end for

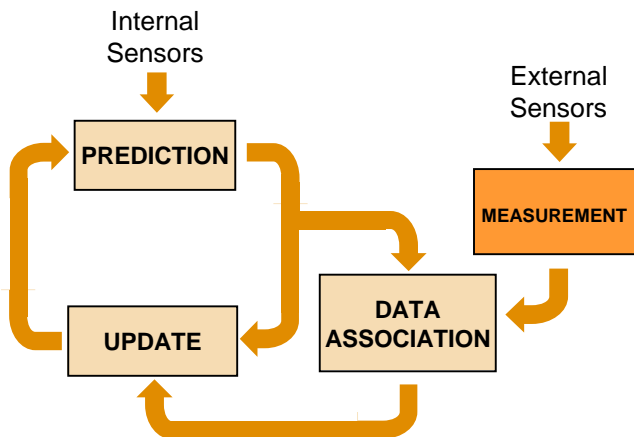
4.2 SLAM

Adding New Features: World Referenced

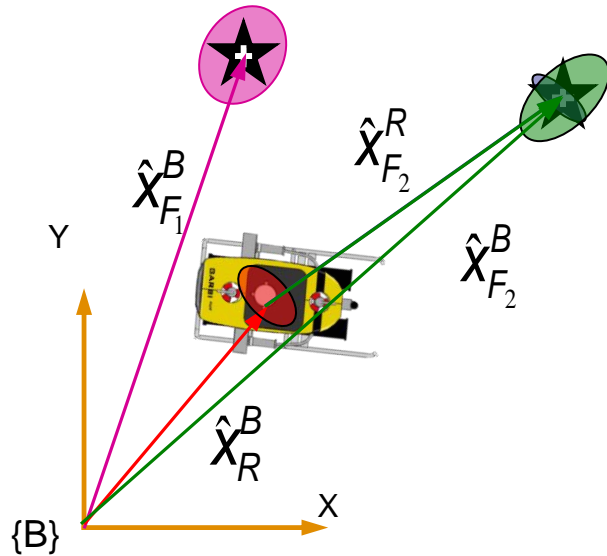


$$\hat{x}_k^B = \begin{bmatrix} \hat{x}_R^B \\ \hat{x}_{F_1}^B \\ \vdots \end{bmatrix}, \quad P_k^B = \begin{bmatrix} P_R^B & 0 \\ 0 & P_{F_1}^B \\ & & \ddots \end{bmatrix}$$

- > A New Feature is observed: $\hat{x}_{F_1}^B \quad P_{F_1}^B$
- > Feature & Robot position are uncorrelated



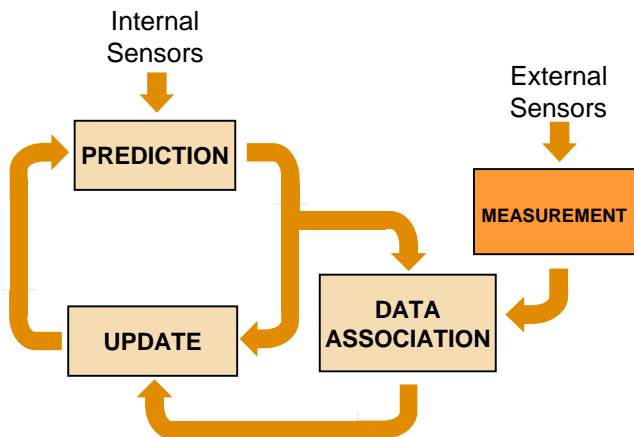
4.2 SLAM



Adding New Features: Robot Referenced

$$\hat{x}_k^B = \begin{bmatrix} \hat{x}_R^B \\ \hat{x}_{F_1}^B \\ \hat{x}_{F_2}^B \\ \vdots \end{bmatrix}, \quad P_k^B = \begin{bmatrix} P_R^B & 0 & ? \\ 0 & P_{F_1}^B & ? \\ ? & ? & P_{F_2}^B \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

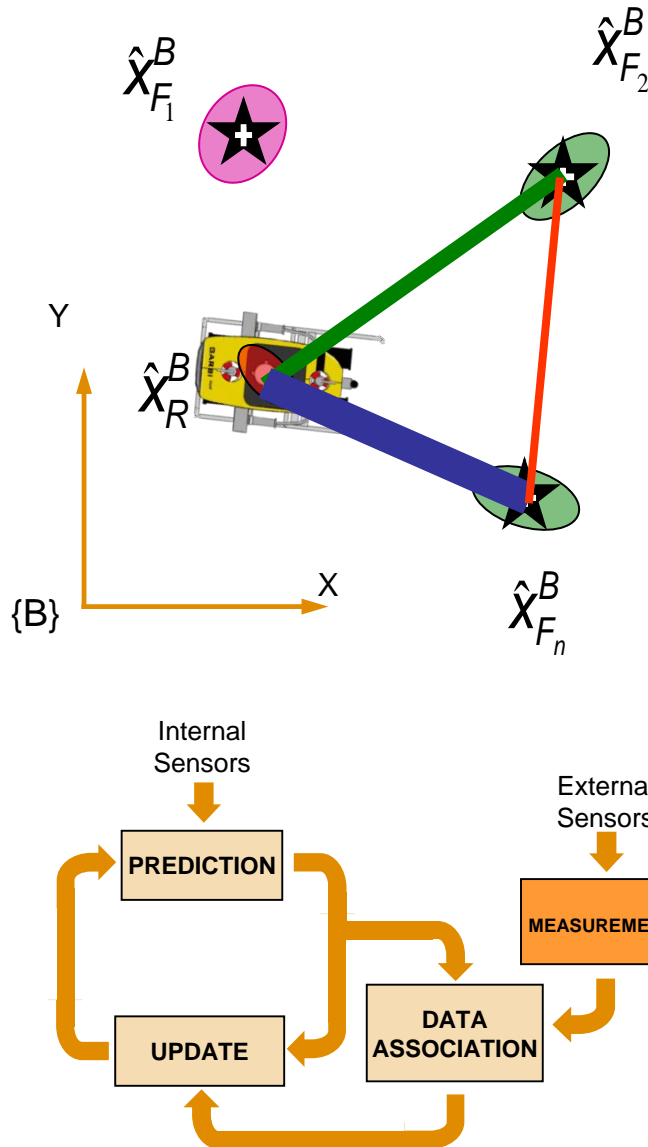
- > A New Feature is observed: $\hat{x}_{F_2}^R \quad R_k$
- > Feature uncertainty includes the noise of the robot & the noise of the measurement.



$$\hat{x}_{F_2}^B = \hat{x}_R^B \oplus \hat{x}_{F_2}^R$$

$$P_{F_2}^B = J_{1\oplus} P_R^B J_{1\oplus}^T + J_{2\oplus} R_k J_{2\oplus}^T$$

4.2 SLAM



What does correlation mean?

> Imagine correlation as a network of springs interconnecting all the features

$$P^B = \begin{bmatrix} P_R^B & P_{RF_1}^B & P_{RF_2}^B & \dots & P_{RF_n}^B \\ P_{F_1R}^B & P_{F_1}^B & P_{F_1F_2}^B & \dots & P_{F_1F_n}^B \\ P_{F_2R}^B & P_{F_2F_1}^B & P_{F_2}^B & \dots & P_{F_2F_n}^B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{F_nR}^B & P_{F_nF_1}^B & P_{F_nF_2}^B & \dots & P_{F_n}^B \end{bmatrix}$$

> Do you remember the correlation matrix?

$$r = \begin{bmatrix} 1 & r_{x_R y_R} & r_{x_R q_R} & r_{x_R x_F} & r_{x_R y_F} \\ r_{y_R x_R} & 1 & r_{y_R q_R} & r_{y_R x_F} & r_{y_R y_F} \\ r_{q_R x_R} & r_{q_R y_R} & 1 & r_{q_R x_F} & r_{q_R y_F} \\ r_{x_F x_R} & r_{x_F y_R} & r_{x_F q_R} & 1 & r_{x_F y_F} \\ r_{y_F x_R} & r_{y_F y_R} & r_{y_F q_R} & r_{y_F x_F} & 1 \end{bmatrix} \quad \rho_{12} \triangleq \frac{\sigma_{x_1 x_2}^2}{\sigma_{x_1} \sigma_{x_2}}$$

> \uparrow Covariance $\Rightarrow \uparrow$ Correlation

> \downarrow Covariance $\Rightarrow \downarrow$ Correlation

> \uparrow Correlation $\Rightarrow \uparrow$ Stiffness of the spring

4.2 SLAM

What does correlation mean?

Full Correlation

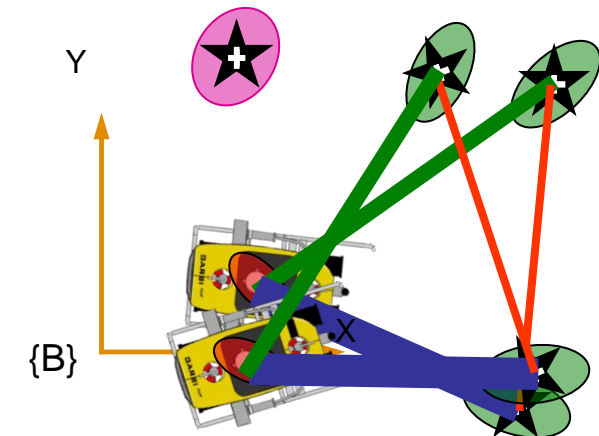
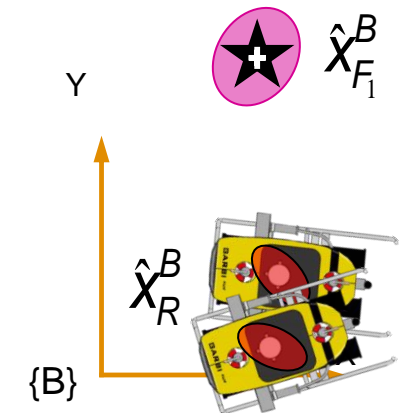
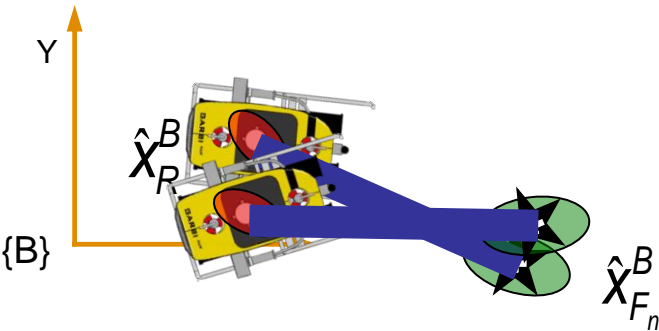
- > When fully correlated $r_{RF_n} = \pm 1$ the spring belongs rigid
- > If we update the Robot pose, the feature moves together

Uncorrelation

- > When uncorrelated $r_{RF_n} = 0$ the spring disappears
- > If we update the Robot pose, the feature remains static

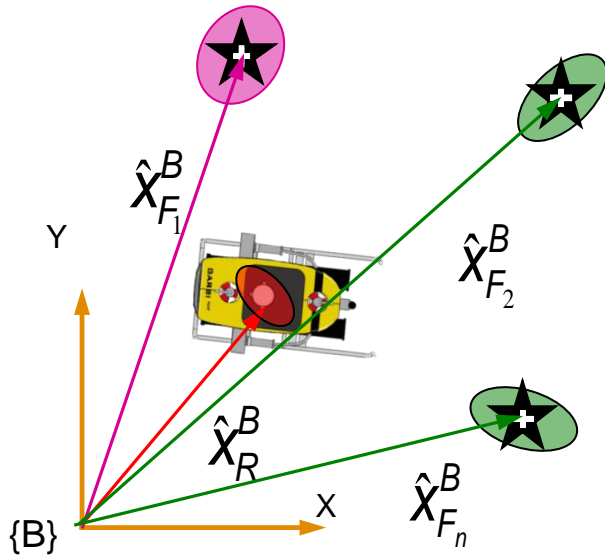
Partial Correlation

- > When partially correlated $0 < |\rho_{RF_n}| < 1$:
 $\uparrow \text{Correlation} \Rightarrow \uparrow \text{Stiffness of the spring}$
- > If we update the Robot pose, the network of springs adapt the position of the features



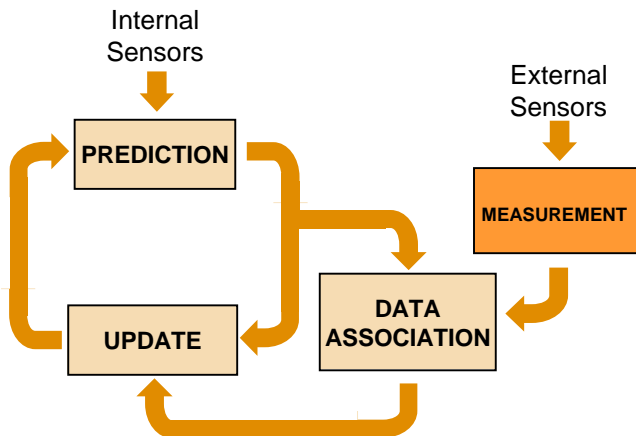
4.2 SLAM

Adding New Features

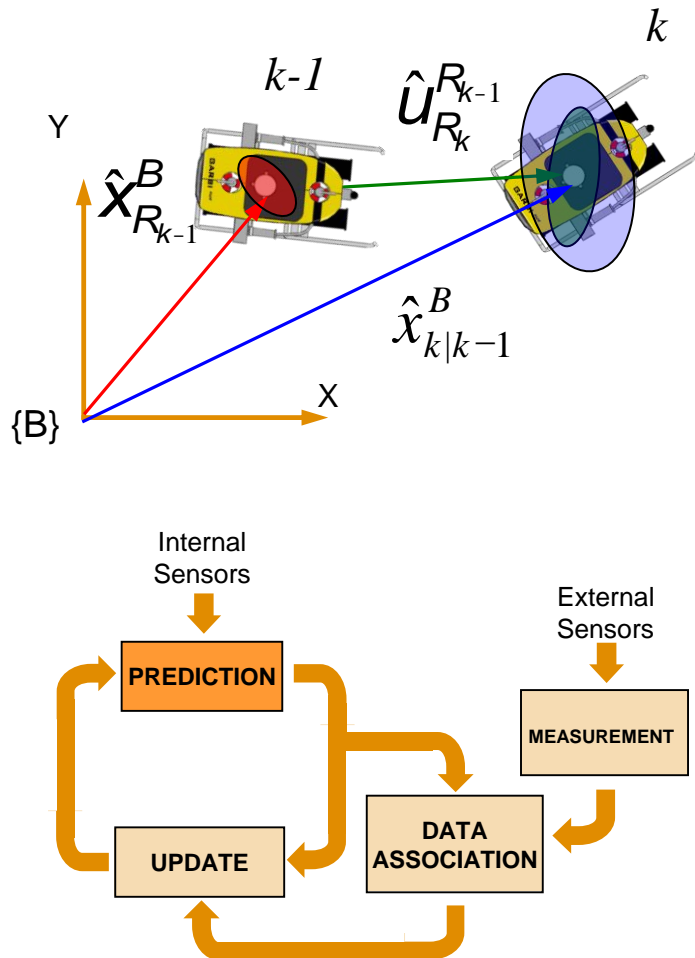


$$\hat{x}^B = \begin{bmatrix} \hat{x}_R^B \\ \hat{x}_{F_1}^B \\ \hat{x}_{F_2}^B \\ \vdots \\ \hat{x}_{F_n}^B \end{bmatrix}, \quad P^B = \begin{bmatrix} P_R^B & P_{RF_1}^B & P_{RF_2}^B & \dots & P_{RF_n}^B \\ P_{F_1R}^B & P_{F_1}^B & P_{F_1F_2}^B & \dots & P_{F_1F_n}^B \\ P_{F_2R}^B & P_{F_2F_1}^B & P_{F_2}^B & \dots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{F_nR}^B & P_{F_nF_1}^B & \dots & \dots & P_{F_n}^B \end{bmatrix}$$

> This process is repeated whenever a new feature is observed



4.2 SLAM



State Prediction

Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$[z_0, R_0] = \text{get_measurements}$

$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$

for k=1 to steps **do**

$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$

{ -EKF prediction }

$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, u_{R_k}^{R_{k-1}}, Q_k)$

$[z_k, R_k] = \text{get_measurements}$

$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$

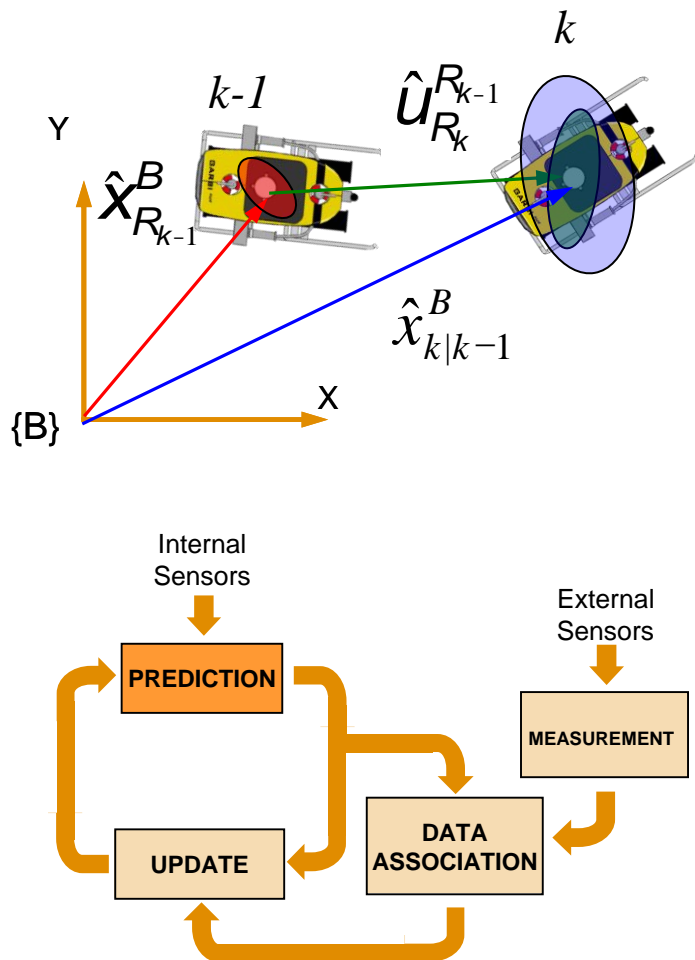
{ -EKF update }

$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$

$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k, \mathcal{H}_k);$

end for

4.2 SLAM



State Prediction

$$\hat{x}_{k|k-1}^B = \begin{bmatrix} \hat{x}_{R_{k-1}}^B \oplus \hat{u}_{R_k}^{R_{k-1}} \\ \hat{x}_{R_{k-1}}^B \\ \vdots \\ \hat{x}_{F_n}^B \end{bmatrix} \quad P_{k|k-1}^B = \begin{bmatrix} P_R^B & ? & ? & \dots & ? \\ ? & P_{F_1}^B & P_{F_1 F_2}^B & \dots & P_{F_1 F_n}^B \\ ? & P_{F_2 F_1}^B & P_{F_2}^B & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & P_{F_n F_1}^B & \dots & \dots & P_{F_n}^B \end{bmatrix}$$

> Next Pose is the previous one + the displacement

$$\hat{x}_{k|k-1}^B = \hat{x}_{R_{k-1}}^B \oplus \hat{u}_{R_k}^{R_{k-1}}$$

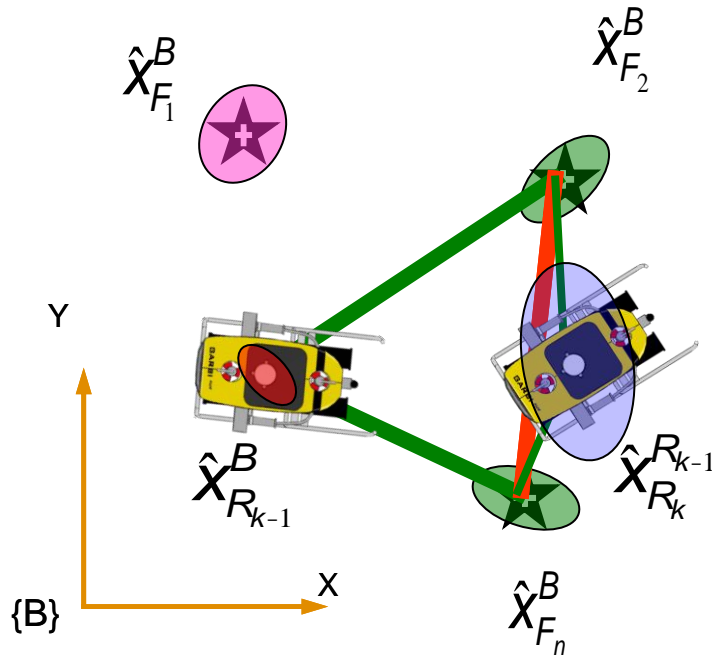
> Covariance inflation

$$P_{k|k-1}^B = F_k P_{k-1}^B F_k^T + G_k Q_k G_k^T$$

> Features are static

> F_k & G_k are the model Jacobians

4.2 SLAM



State Prediction

What happens to the correlations?

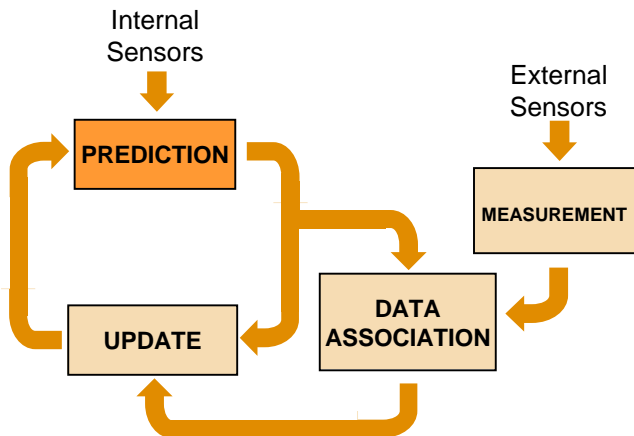
- > The correlations of the features with respect to the robot decrease

$$F_k = \begin{bmatrix} J_{1\oplus} \left\{ \hat{x}_{R_{k-1}}^B, \hat{u}_{R_k}^{R_{k-1}} \right\} & 0 & \dots & 0 \\ 0 & I & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & I \end{bmatrix} \quad G_k = \begin{bmatrix} J_{2\oplus} \left\{ \hat{x}_{R_{k-1}}^B, \hat{u}_{R_k}^{R_{k-1}} \right\} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

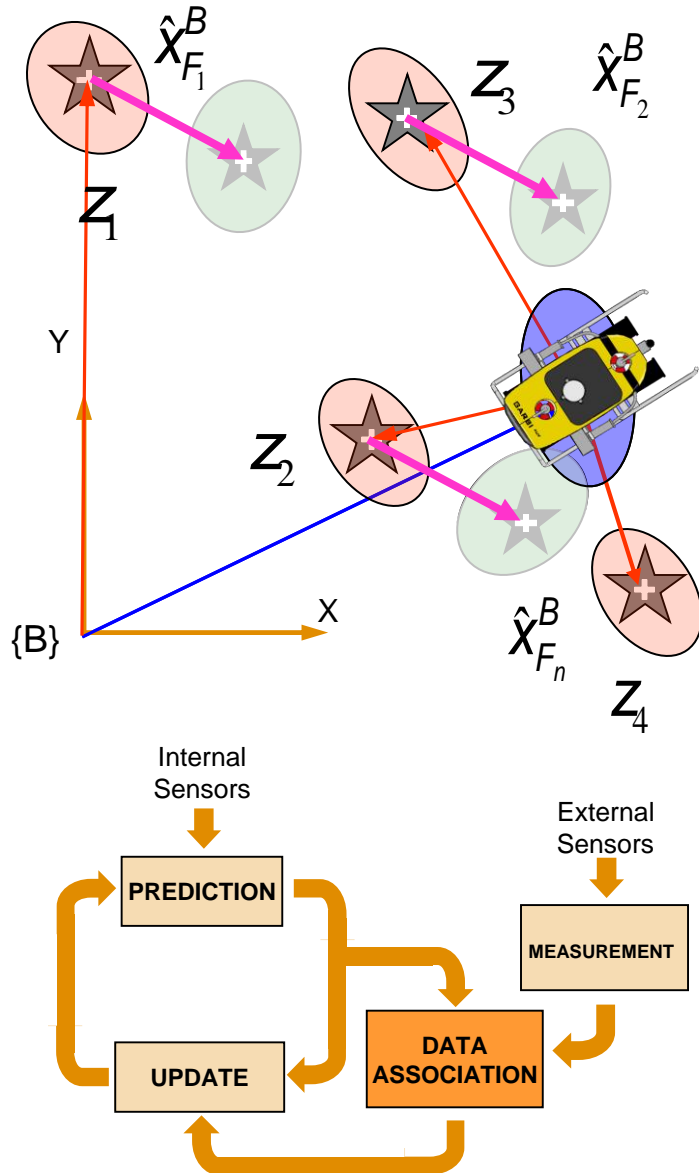
$$P_{k|k-1}^B = F_k P_{k-1}^B F_k^T + G_k Q_k G_k^T$$

$$P_{k|k-1}^B = \begin{bmatrix} J_{1\oplus} P_R^B J_{1\oplus}^T & J_{1\oplus} P_{R_{F_1}}^B & J_{1\oplus} P_{R_{F_2}}^B & \dots & J_{1\oplus} P_{R_{F_n}}^B \\ P_{F_1 R}^B J_{1\oplus}^T & P_{F_1}^B & P_{F_1 F_2}^B & \dots & P_{F_1 F_n}^B \\ P_{F_2 R}^B J_{1\oplus}^T & P_{F_2 F_1}^B & P_{F_2}^B & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{F_n R}^B J_{1\oplus}^T & P_{F_n F_1}^B & \dots & \dots & P_{F_n}^B \end{bmatrix} + \begin{bmatrix} J_{2\oplus} Q_k J_{2\oplus}^T & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

- > The correlations among the features does not change



4.2 SLAM



Data Association

Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$$

for k=1 to steps do

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, u_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

$$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

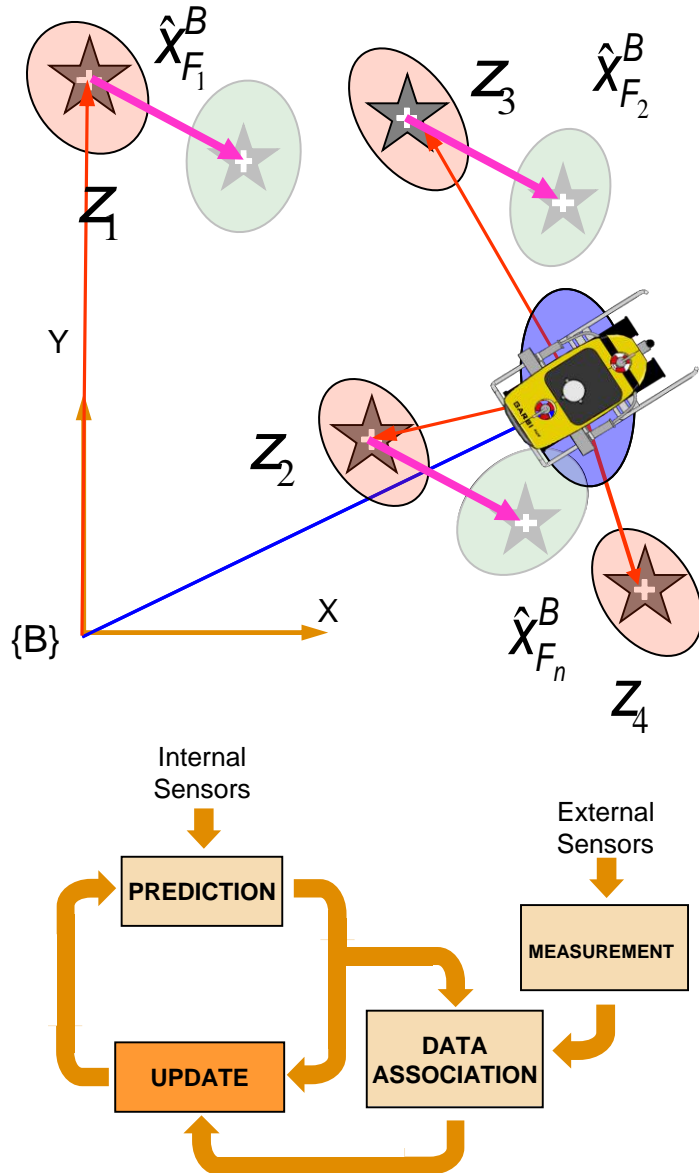
{ -EKF update }

$$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, \mathcal{H}_k)$$

$$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k);$$

end for

4.2 SLAM



State Update

Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$$

for k=1 to steps **do**

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, u_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

$$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

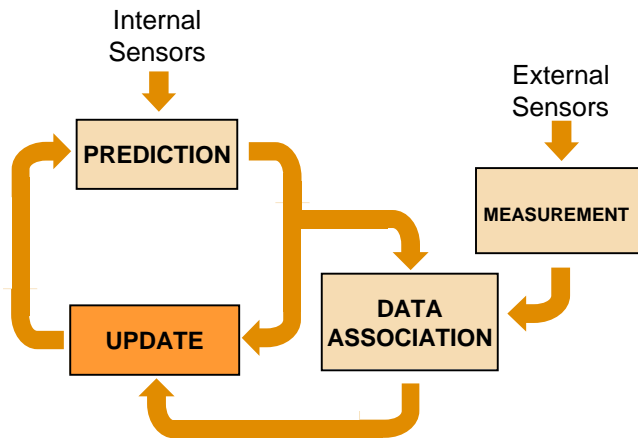
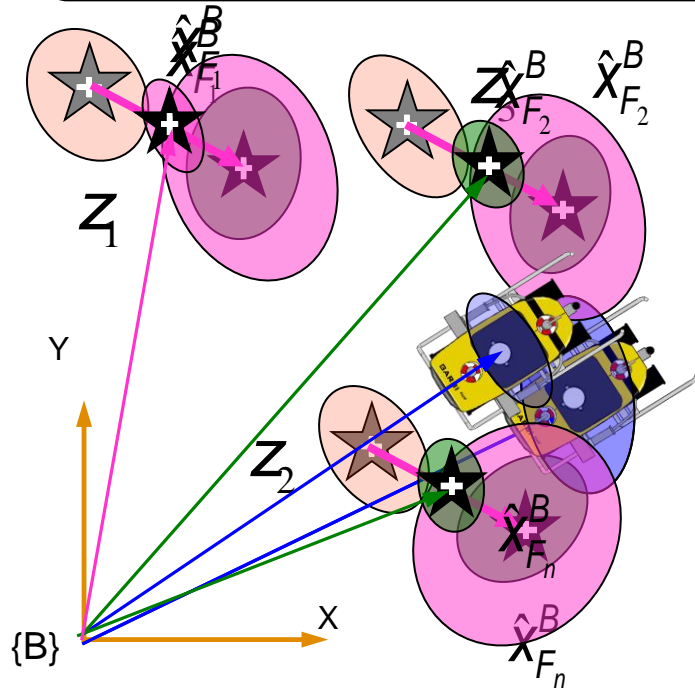
{ -EKF update }

$$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

$$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k, \mathcal{H}_k);$$

end for

4.2 SLAM



State Update

> First the discrepancies (innovation) between the features and the measurements are computed.

$$n_{H_k} = z_k - h_{H_k}(\hat{x}_{k|k-1}^B)$$

$$S_{H_k} = H_{H_k} P_{k|k-1}^B H_{H_k}^T + R_k$$

> Then the state is updated

$$K_{H_k} = P_{k|k-1}^B H_{H_k}^T S_{H_k}^{-1}$$

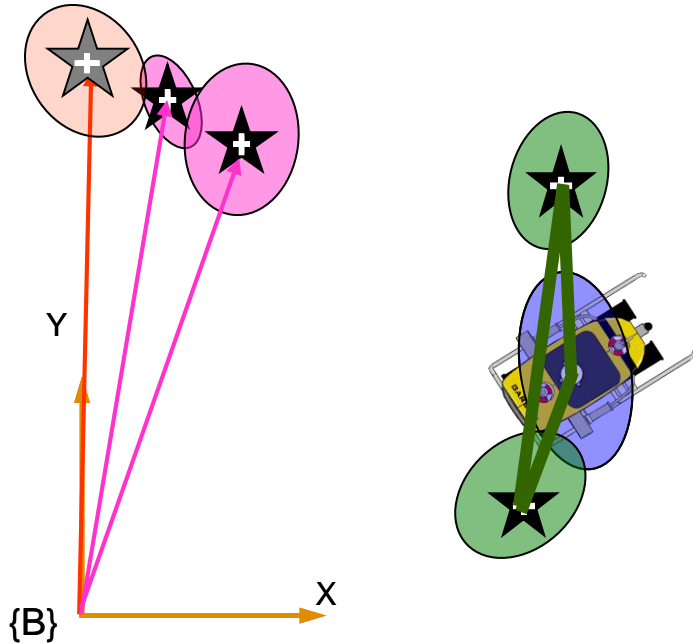
$$\hat{x}_k^B = \hat{x}_{k|k-1}^B + K_{H_k} n_{H_k}$$

$$P_k^B = (I - K_{H_k} H_{H_k}) P_{k|k-1}^B$$

$$= (I - K_{H_k} H_{H_k}) P_{k|k-1}^B (I - K_{H_k} H_{H_k})^T + K_{H_k} R_k K_{H_k}^T$$

> H_k is the jacobian of the measurement function

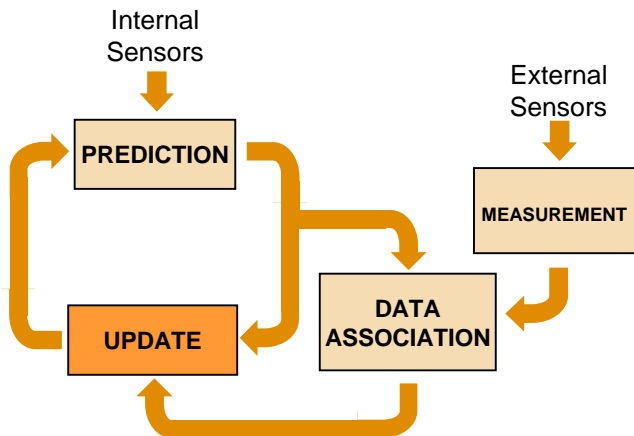
4.2 SLAM



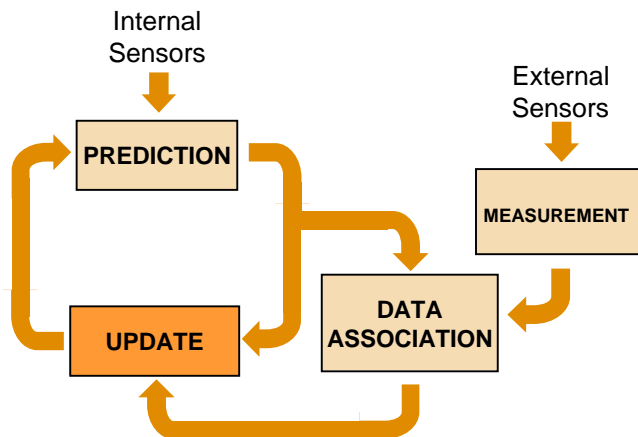
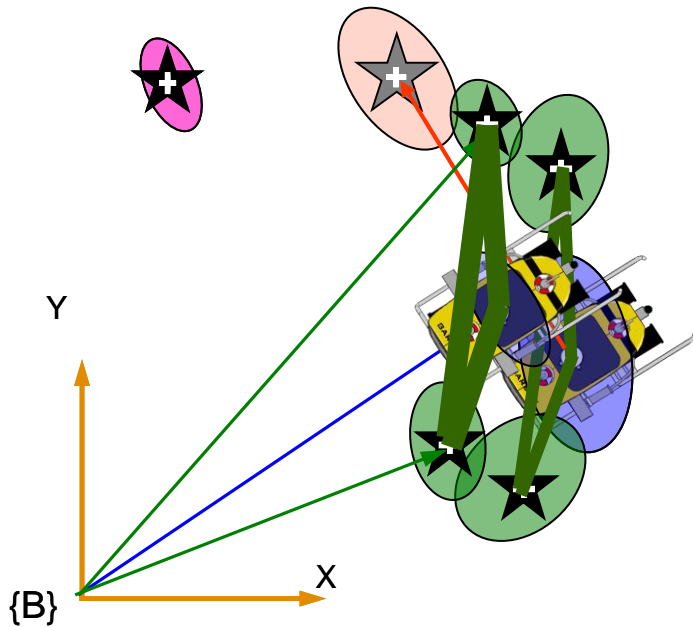
What happens to the correlations?

Observation of an uncorrelated feature

- > The uncertainty of the feature decrease
- > No other feature nor the robot decrease their uncertainty
- > There are no changes in the correlations



4.2 SLAM



What happens to the correlations?

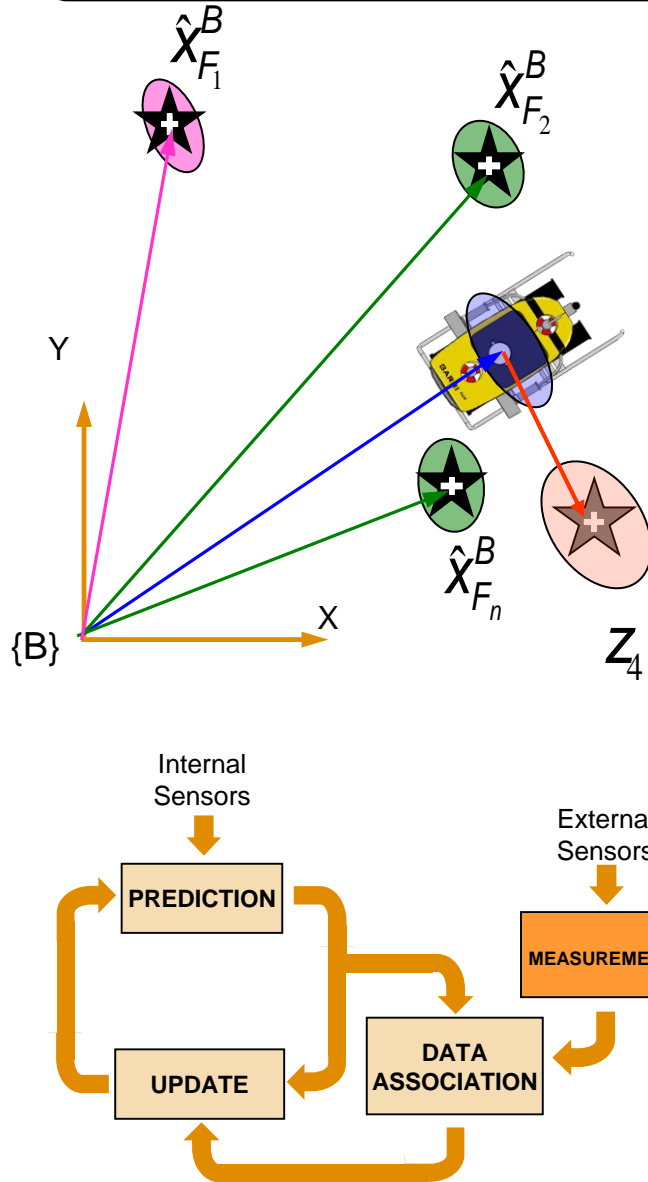
Observation of an uncorrelated feature

- > The uncertainty of the feature decrease
- > No other feature nor the robot decrease their uncertainty
- > There are no changes in the correlations

Observation of a correlated feature

- > The uncertainty of the feature decrease
- > The uncertainty of the correlated features decrease
- > The uncertainty of the robot decrease
- > The correlations become stronger

4.2 SLAM



Adding New Features

Algorithm: SLAM

{ - Pose initialization }

$$x_0^B = \hat{x}_0^B; P_0^B = \hat{P}_0^B;$$

$$[z_0, R_0] = \text{get_measurements}$$

$$[x_0^B, P_0^B] = \text{add_new_features}(x_0^B, P_0^B, z_0, R_0);$$

for k=1 to steps do

$$[u_{R_k}^{R_{k-1}}, Q_k] = \text{get_odometry}$$

{ -EKF prediction }

$$[x_{k|k-1}^B, P_{k|k-1}^B] = \text{move_vehicle}(x_{k-1}^B, P_{k-1}^B, u_{R_k}^{R_{k-1}}, Q_k)$$

$$[z_k, R_k] = \text{get_measurements}$$

$$\mathcal{H}_k = \text{data_association}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

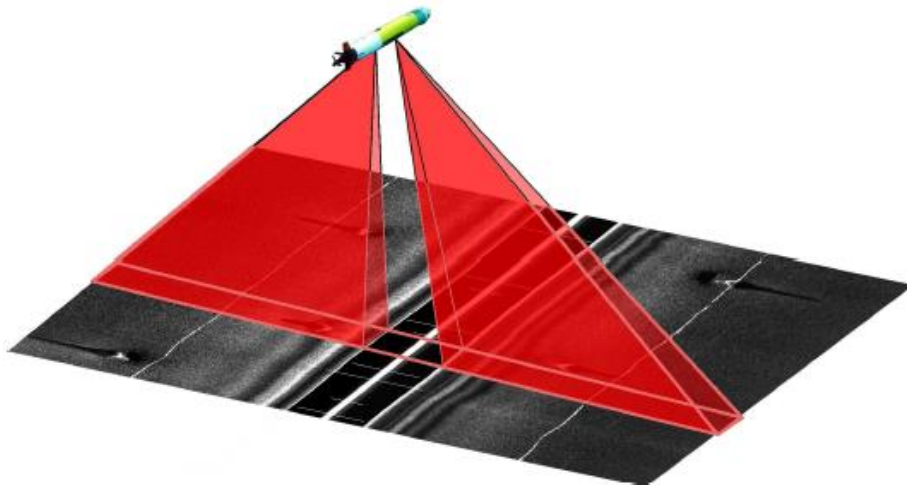
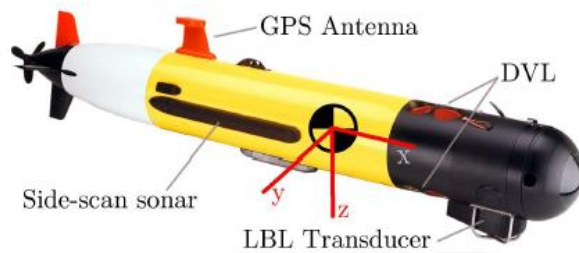
{ -EKF update }

$$[x_k^B, P_k^B] = \text{update_position}(x_{k|k-1}^B, P_{k|k-1}^B, z_k, R_k);$$

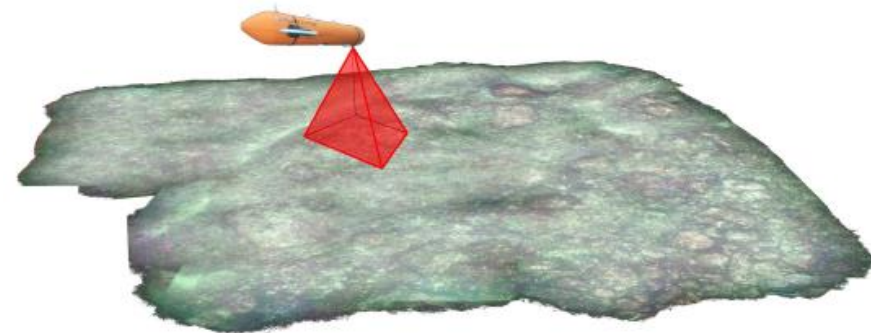
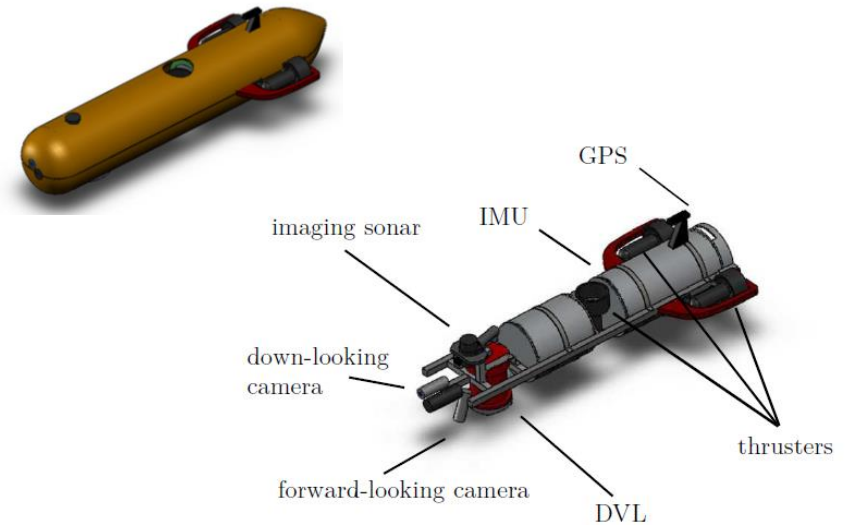
$$[x_k^B, P_k^B] = \text{add_new_features}(x_k^B, P_k^B, z_k, R_k, \mathcal{H}_k);$$

end for

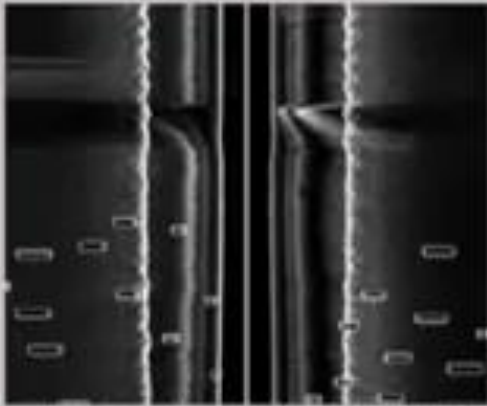
REMUS 100 AUV



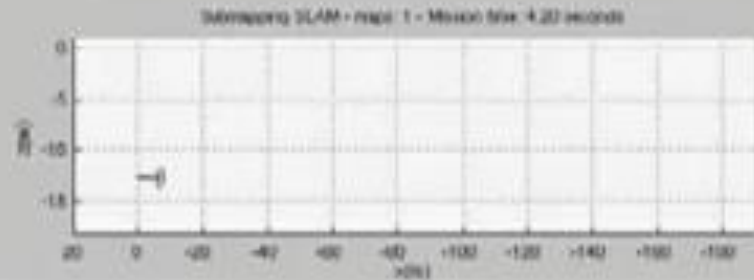
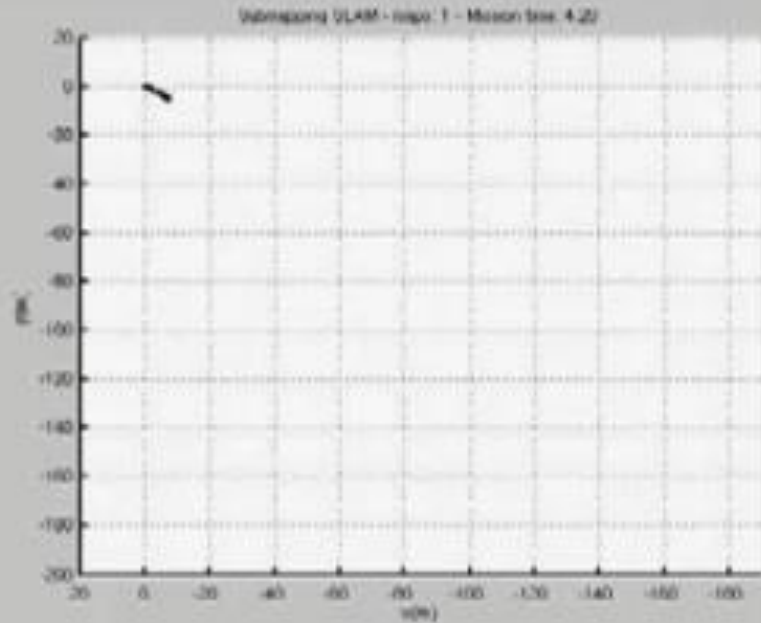
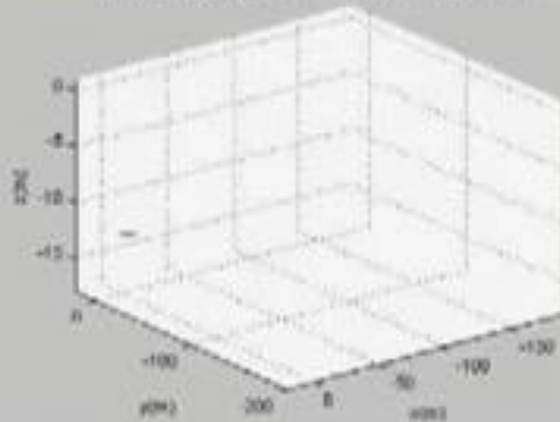
SPARUS AUV



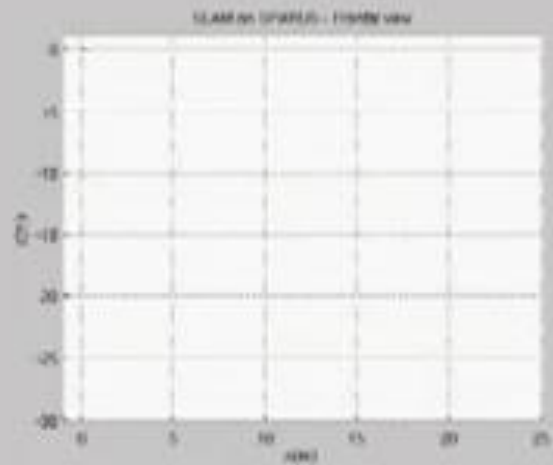
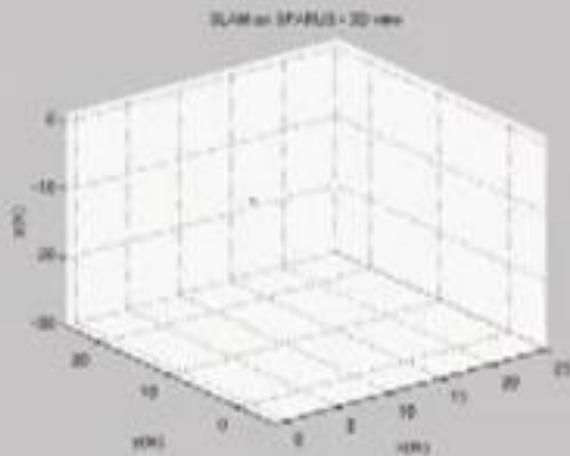
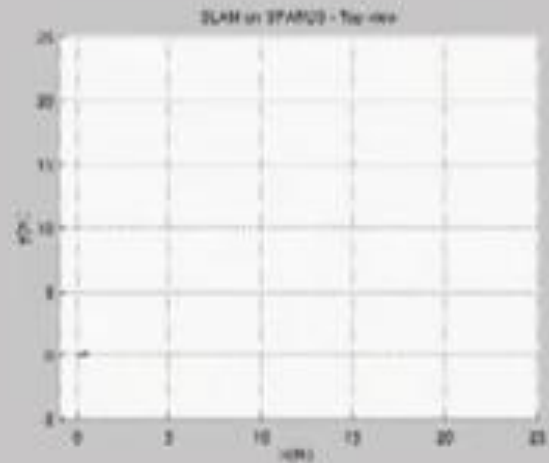
REMUS 100 AUV



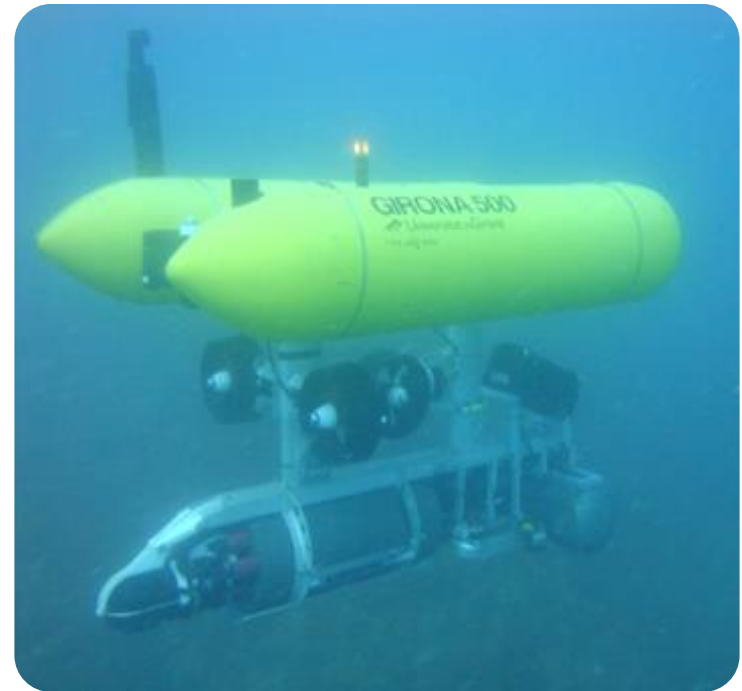
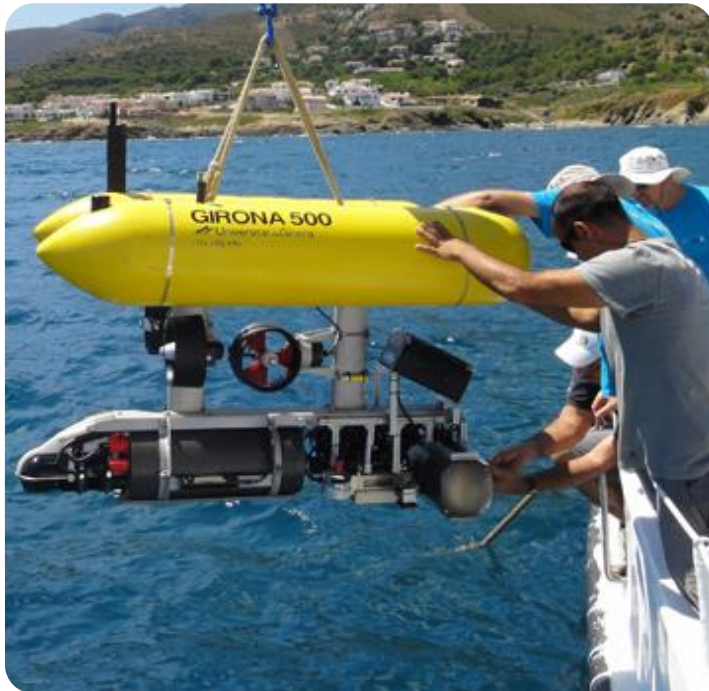
Submapping SLAM - image 1 - Mission time: 4.20 seconds



SPARUS AUV



GIRONA 500 AUV



AUTONOMOUS UNDERWATER INTERVENTION

Black-Box Recovery



UNIVERSITAT JAUME I



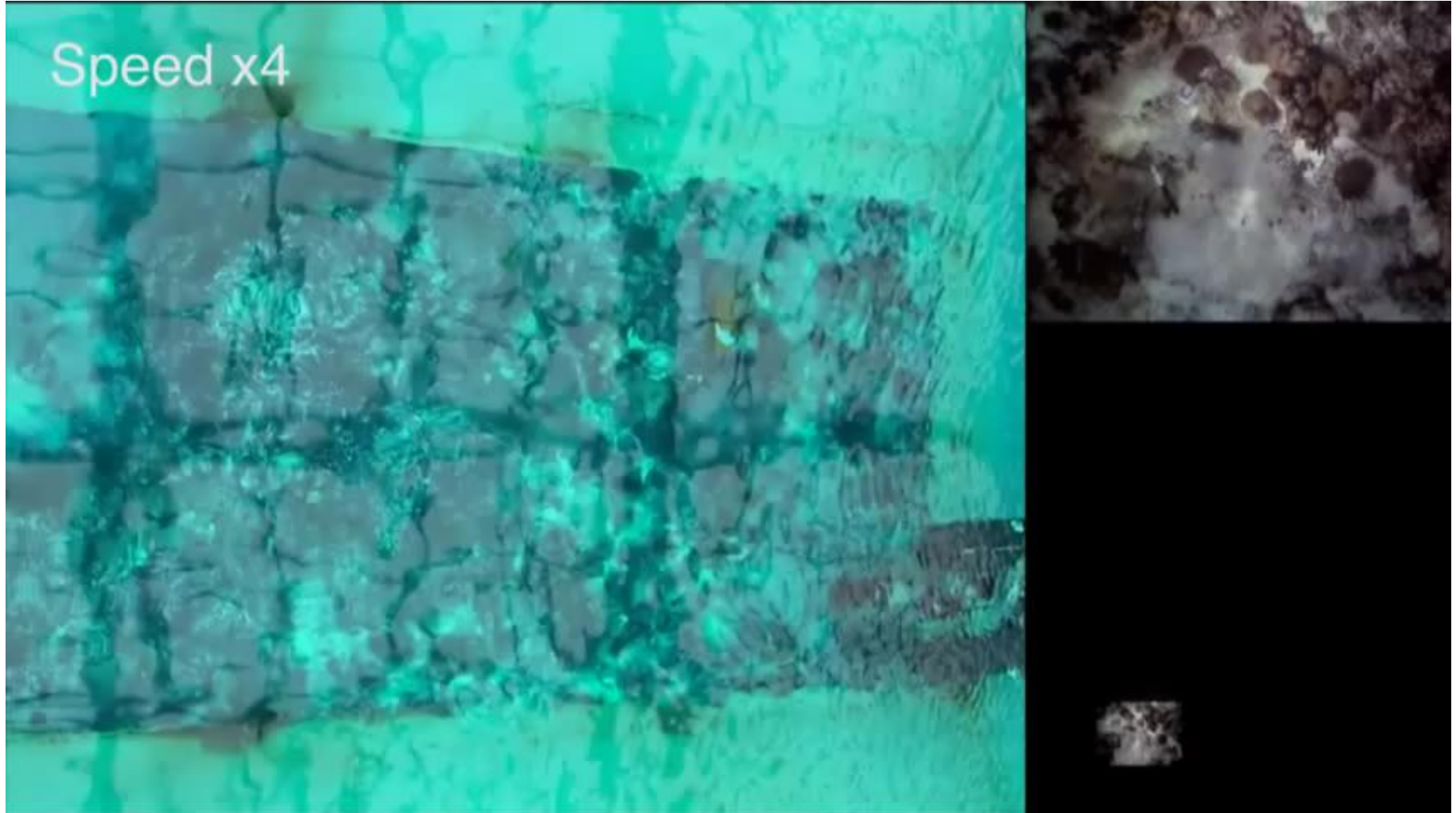
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GIRONA 500 AUV



GIRONA 500 AUV

