Submitted by:
VAMSHI KODIPAKA
MSCV-1 (2nd sem)
University of Condorcet
Le Creusot, France.

Home Work: Visual Perception

Course Evaluator: Dr. Djamila Aouada.
Submitted on: 10th June, 2019.

IMAGE FILTERING:-

PART-1:- Given, observed corrupted image is : $y = x + w$ → (1)

- x – noise free $m \times m$ with additive Gaussian noise w.

- $$\hat{x}_p = \frac{\sum_{q \in N(p)} f(p,q) \cdot y_q}{\sum_{q \in N(p)} f(p,q)}$$ ← linear filtering. — (2)

1Q:- Write the analytical expression for the Gaussian kernal f with standard deviation σ_f ?

A:- Gaussian kernal describes the normal distributions in signal processing, 2D-gaussians are used for Gaussian blurring in image processing. They are also used to solve Heat eqns. and diffusion eqns. They are used to define ~~th~~ statistical models.

Gaussian Kernel in 1D: $G_{1D}(x/\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$

Gaussian Kernal in 2D: $G_{2D}(x; y/\sigma) = \frac{1}{(\sqrt{2\pi\sigma^2})} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$

Gaussian Kernal in ND: $G_{ND}(x|\sigma) = \frac{1}{(\sqrt{2\pi}\sigma)^N} e^{-\frac{|x|^2}{2\sigma^2}}$

σ is the width of the Gaussian kernel.

The factor of 2 in the exponent is a convention, because we have cleaner formula for diffusion eqn. as; between spatial & scale parameters are conventionally put ~~to~~ order to make difference b/w parameters explicitly.

2Q:- Rewrite (2) using convolutions.

A:-
$$\boxed{(f * g) = \sum_{k=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} f(k, i)\, g(u-k, u-i)}$$

3Q:- Assuming pixels y_p are i.i.d. with std. dev. σ_y. Compute noise red. $r = \frac{\sigma_{\hat{x}}}{\sigma_y}$. $\sigma_{\hat{x}}^2$ - variance of filtered image $\hat{x}_p$.

We know,
$$\boxed{r = \frac{\sigma_x}{\sigma_y} = \frac{1}{\sqrt{\pi N(p)}}}$$

$N(p)$ are the neighbouring pixels.

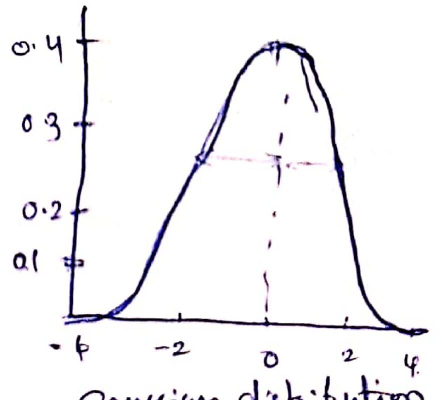
4Q:- Give a suitable integer-valued convolution mask f of size (5x5) that approximates Gaussian filter f with a standard deviation $\sigma_f = 1.4$.

Gaussian filtering blurs the images & removes noises.

1D Gaussian : $G_{1D}(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}}$

where σ is stand. dev. of the distribution. (Assumption mean=0)

This shown as in the plot:



Gaussian distribution
mean=0 & variance σ=1.

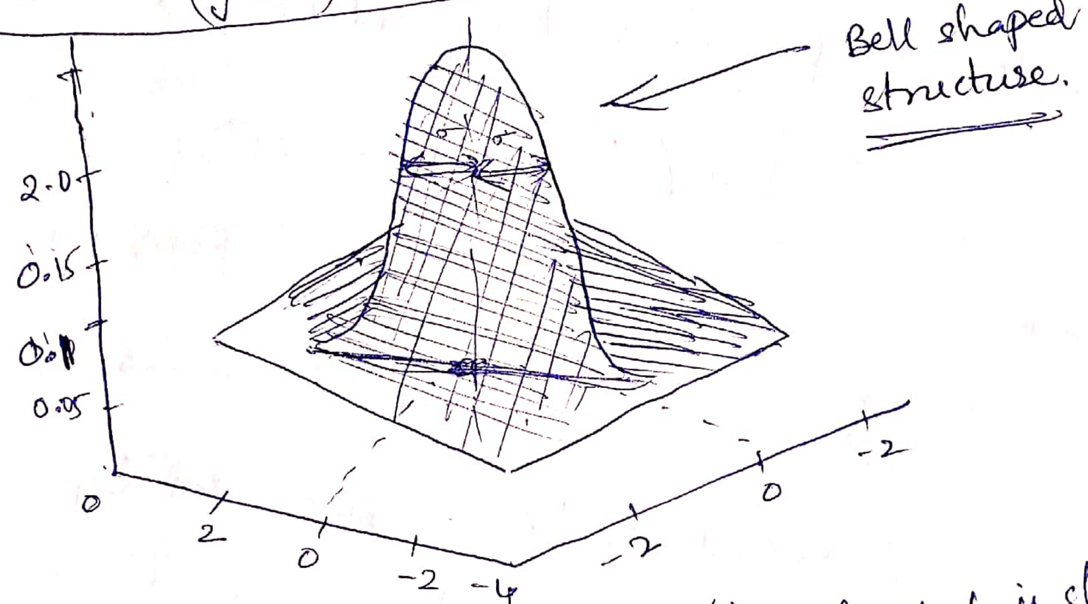
Gaussian function's standard deviation explains the behaviour of the distributed data.

The Gaussian function has parameters which are varied with respect to integral.

$$I = \int_{-\infty}^{\infty} \exp(-x^2)\, dx = \sqrt{\pi}$$

when observed for 2D image, we take 2D guassian:

$$\boxed{G_{2D}(x,y) = \frac{1}{(\sqrt{2\pi}\sigma)^2} e^{-\frac{x^2+y^2}{2\sigma^2}}}$$



A 2D-gaussian distrib. with mean (0,0) and $\sigma = 1$ is shown above.

PART-2 :- EDGE DETECTION:

1Q:- Given T is the transpose of matrix and $*$ the operation of convolution, What does expression S estimate?

$$S = (y*f)^2 + (y*f^T)^2 \qquad f = \begin{bmatrix} -0.05 & 0.08 & 0.05 \\ -0.136 & -0.225 & -0.136 \\ 0.05 & 0.08 & 0.05 \end{bmatrix}$$

Sol:- Solution is (e). It is the quadratic variation.

$$\left(\frac{\partial^2 f}{\partial x^2}\right)^2 + \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

This expression has 2 square terms with 2 different directions in x and y directions. Since we also have convolutions in these 2 terms; we have third directions x, y.

2Q: What is the advantage of the quadratic variation as compared to the Laplacian.

A:- 1. Laplacian's brightness at a point (x, y) is given by:

$$\left(\frac{\partial^2 f}{\partial x^2}\right)^2 + \left(\frac{\partial^2 f}{\partial y^2}\right) \longrightarrow ①$$

where as; quadratic variation is given by:

$$\left(\frac{\partial^2 f}{\partial x^2}\right)^2 + \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 \longrightarrow ②$$

2. Laplacian Kernal can be approximated by several kernals like

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

and quad. var. can be estimated by three partial second deriv. kernals

$$\Delta_{xx} = \frac{1}{\sigma}\begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Delta_{yy} = \frac{1}{\sigma}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Delta_{xy} = 2\begin{bmatrix} -0.25 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.25 & 0 & -0.25 \end{bmatrix} \leftarrow$$

high freq. than Δ_{xx} Δ_{yy} as it has both x and y components.

PART-III :- IMAGE FEATURES.

1Q:- Give the expressions of a (3×3) filter g and the expression of Mp, each time explaining your notation; for:

Sol:-

a) The Harris detector

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

E is the summed squared difference of two image influeed by two windows original and the next window at (u,v).

u — displacement in x-direction.

v — displacement in y-direction.

$$E(u,v) = [u, v]\left(\sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}\right)\begin{bmatrix} u \\ v \end{bmatrix}$$

b). The Hessian Detector

$$b_y(p) = \sum b_y(i,j) = \sum_{1}^{3} \int (p,q)\, y_q.$$

2Q:- Let λ_p^1 and λ_p^2 denote the eigenvalues of Mp. What kind of image feature is present at p when:

Sol:- $\lambda_p^1 << \lambda_p^2$ of the Harris Detector.

(a) $0 < \lambda_p^1 << \lambda_p^2$

The feature is Edge.

(b) $\lambda_p^1 \approx \lambda_p^2 >> 0$ represents Hessian Detector.

The feature is Corner.

(C) $\lambda_p^1 = \lambda_p^2 = 0$. represents Hessian Detector &.

The feature is Flat Region.

(e) $0 < \lambda_p^1$ or $0 < \lambda_p^2$ of harris detector?

Either of them is > 0. So, it is an edge.

3Q: What are the invariance properties of:

a) Harris detector b) Hessian detector.

Sol: a) The Harris Detector

1. Rotation Invariant (corner response R, invariant to position)
2. Affine intensity change. (partial invariant to affine intensity change, depends on type of theshold).
3. Scaling (not invariant to scaling).
4. Triangulation.

b) The Hessian detector

1. Response mainly contains corners and strongly textured areas.
2. Considers only points having higher values then its 8-neighborhoods.
3. Rotation invariant.
4. Not invariant to affine intensity changes.

4Q: The shift descriptor is known for its scale invariance property where the idea is to find local extrema in the scale space parameterized by standard dev. σ_f such that: $Z = (f_{\sigma_f} - f_{k\sigma_f}) * y$;

where k is fixed scalar factor. & f_{σ_f} is gaussian kernel with (std. dev) σ_f. variance.

Sol:- a) why is Kernal f chosen to be Gaussian? (4)

Because f is independent on Image Gradients those create an array of orientations in Histograms.

e) How is the factor k typically chooseen?

Value of k is chooosen in such a way that we obtain fixed no. of convoluted images per octave.

PART-IV: IMAGE MATCHING

Given, $\{V_1^i\}$ & $\{V_2^j\}$ are two set of descriptors in two images.

18: Give the expression of $d(V_1^i, V_2^j)$ if:-

Sol:- a) d is sum of square difference (SSD):

Then SSD is defined as:-

$$SSD(m,n) = \sum_i \sum_j \left[g(i,j) \pm f(i-m, i-n)\right]^2.$$

and expended as.

$$SSD(m,n) = \sum_i \sum_j \left[\underbrace{g(i,j)^2 + f(i-m,j-n)^2}_{\text{constants}} - 2g(i,j) + (i-m)(j-n)\right]$$

∴ The cross correlation remains:

$$R(m,n) = \sum_i \sum_j \left[g(i,j) \pm f(i-m, j-n)\right].$$

b) d is the cross correlation (CC):

$$R(m,n) = \sum_i \sum_j \left[g(i,j) + f(i-m, j-n)\right];$$

8Q:- What is the relationship b/w the two distances?

SSD is given by : $SSD = \sum_{(i,j)\in R} (f-g)^2$

$$\therefore SSD = \sum_{[i,j]\in R} f^2 + \sum_{[i,j]\in R} g^2 - 2 \sum_{[i,j]\in R} f \cdot g$$

$$\boxed{C_{fg} = \sum_{[i,j]\in R} f(i,j) \cdot g(i,j)}$$ ← describes the correlation.

REFERENCES:

[1]. "Cyrill Stachniss" – Youtube lectures.

[2]. Wikipedia.

[3]. "Dr. RigDas" – class notes.

— * * * —